**Paper / Subject Code: 50921 / Engineering Mathematics-III**

Max. Marks : 80
(Time: 03 hours)

N.B. : (1) Question No.1 is **compulsory**
(2) Attempt **any three** questions from Q.2 to Q.6
(3) Figures to the right indicate full marks

Q. 1 (a) Find the Laplace Transform of $e^{2t} + 4t^3 - sin2t cos3t$  05

(b) Find the Fourier series of $f(x) = x, \quad -\pi < x < \pi$  05

(c) Calculate Spearman's coefficient of rank correlation from the following data  05

| X: | 12 | 17 | 22 | 27 | 32 |
|----|----|----|----|----|----|
| Y: | 113 | 119 | 117 | 115 | 121 |

(d) Find the constants a, b, c, d, e  05
if $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$ is analytic

Q.2 (a) Determine whether the function $f(z) = \frac{1}{2}log(x^2 + y^2) + i tan^{-1}\frac{y}{x}$ is  06
analytic and if so, find its derivative.

(b) A random variable X has the following probability distribution  06

| X | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|---|
| P(X=x) | k | 3k | 5k | 7k | 9k | 11k | 13k |

Find (i) k, (ii)$P(X < 4)$ (iii)$P(3 < X \le 6)$

(c) Evaluate $\int_0^\sigma e^{-2t} t \, cost \, dt$  08

Q.3 (a) Find the Fourier series of $f(x) = \frac{\pi^2}{12} - \frac{x^2}{4}, \quad -\pi < x < \pi$  06

(b) A continuous random variable has probability density function  06
$f(x) = k(x - x^2); \quad 0 \le x \le 1$
Find (i) k, (ii) mean, (iii) variance

(c) Find the inverse Laplace transform of $\dfrac{s^2+2s+3}{(s^2+2s+5)(s^2+2s+2)}$  08

Q.4(a) Find the Laplace Transform of $f(t)$,  06
where $f(t) = cost, \; for \; 0 < t < \pi \; and \; f(t) = sint, \; for \; t > \pi$

(b) Calculate the Karl Pearson's coefficient of correlation from the following  06
data

| X: | 65 | 66 | 67 | 67 | 68 | 69 | 70 | 72 |
|----|----|----|----|----|----|----|----|----|
| Y: | 67 | 68 | 65 | 68 | 72 | 72 | 69 | 71 |

(c)  Find the Fourier series of $f(x) = \begin{cases} x, & 0 \le x \le \pi \\ 2\pi - x, & \pi \le x \le 2\pi \end{cases}$                    08

Q.5 (a)  Find the inverse Laplace transform of $\dfrac{s}{(2s+1)^2}$                    06

(b)  Find the Laplace transform of $t\left(\dfrac{sint}{e^t}\right)^2$                    06

(c)  Find the lines of regression for the following data                    08

| X: | 78 | 36 | 98 | 25 | 75 | 82 | 90 | 62 | 65 | 39 |
|----|----|----|----|----|----|----|----|----|----|----|
| Y: | 84 | 51 | 91 | 60 | 68 | 62 | 86 | 58 | 53 | 47 |

Q.6 (a)  Find the mean and the variance of the following distribution                    06

| X | 1 | 3 | 4 | 5 |
|---|---|---|---|---|
| P(X=x) | 0.4 | 0.1 | 0.2 | 0.3 |

(b)  Find the inverse Laplace transform of $log\left(1 + \dfrac{a^2}{s^2}\right)$                    06

(c)  Find the analytic function $f(z) = u + iv$ whose imaginary part is                    08
$$v = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$$

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