Matrix operations

1. Basic things

I hope you are familiar with matrix, its basic properties, inverse and transpose. Here we will revise the concept of Eigen value and Eigen vector. These are used in PCA technique for dimensionality reduction. We will see this technique in future.

2. Eigen value and eigen vector

EV is defined for square matrix only. The matrix of n x n dimension has n Eigen values and n Eigen vectors. Each Eigen vector will be n x 1 vector. The Eigen value and Eigen vector will satisfy the following property :

$$Ax = \lambda x$$

Here λ is eigen value and x is eigen vector. Below are the step by step explanation to find EVs.

1.

Consider the matrix A:

$$A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$$

To find the eigenvalues, we solve the characteristic equation:

$$\det(A - \lambda I) = 0$$

$$\det\left(\begin{bmatrix} 4-\lambda & 1 \\ 2 & 3-\lambda \end{bmatrix}\right) = 0$$

$$(4-\lambda)(3-\lambda) - (1 \cdot 2) = 0$$

$$(\lambda^2 - 7\lambda + 10) = 0$$

Solving this quadratic equation gives two eigenvalues: $\lambda_1 = 5$ and $\lambda_2 = 2$.

Now, for each eigenvalue, we find the corresponding eigenvector by substituting it into $(A - \lambda I)X = 0$:

For $\lambda = 5$:

$$A - 5I = \begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix}$$

Setting up the system $(A - 5I)X = 0$:

$$\begin{bmatrix} -1 & 1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving this system, we get $x_1 = x_2$. So, one eigenvector corresponding to $\lambda_1 = 5$ is $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

Find for the remaining Eigen value also.