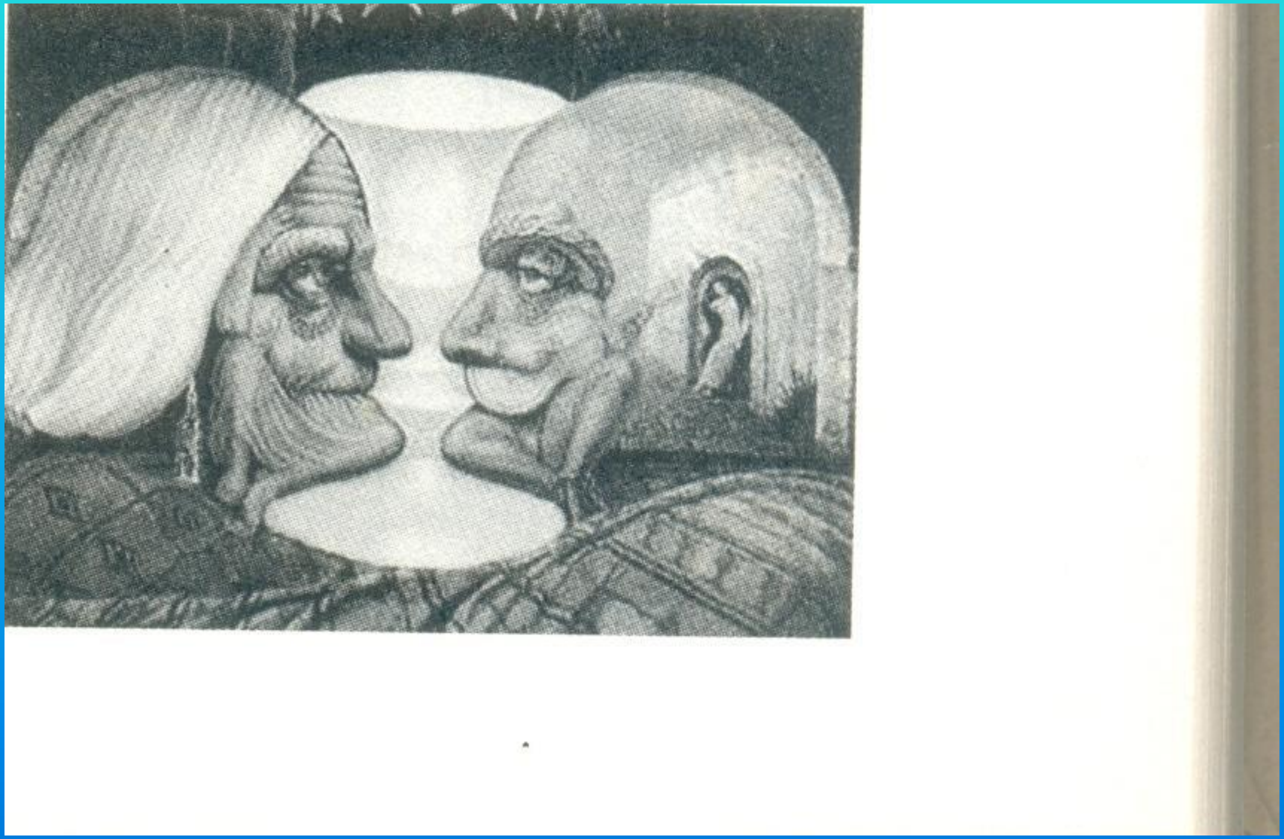
VEDIC MATHEMATICS

Prepared By:
KRISHNA PRASAD K
Associate Professor
Institute of Computer Science and Information Science
Srinivas University
City Campus, Pandeshwar
Mangalore-575001

Memory Power-(Optical Illusion) How Many people you can see in figure



Why Vedic Mathematics?

- It helps a person to solve problems 10-15 times faster.
- It reduces burden (Need to learn tables up to nine only)
- It provides one line answer.
- It is a magical tool to reduce scratch work and finger counting.
- It increases concentration.
- Time saved can be used to answer more questions.
- Logical thinking process gets enhanced.

List of 16 Sutras with their Meanings and Uses

S.No.	Sutras Name	Meaning	Where to use
Sutra 1	Ekadhikina Purvena	By one more than the previous one	Squaring of a number ending with 5
Sutra 2	Nikhilam Navatashcaramam Dashatah	All from 9 and the last from 10	Multiplication of numbers, which are near to base like 10, 100, 1000
Sutra 3	Urdhva-Tiryagbyham	Vertically and crosswise	It is the general formula, applicable to all cases of multiplication of two large number
Sutra 4	Paraavartya Yojayet	Transpose and adjust	When divisor greater than 10

Sutra 5	Shunyam Saamyasamuccaye	When the sum is the same that sum is zero	-
Sutra 6	Anurupyena- Sunyamanyat	If one is in ratio, the other is zero	To find out the product of two number when both are near the common base like 40, 40, etc. (multiples of powers of 10).
Sutra 7	Sankalana- Vyavakalanabhyam	By addition and by subtraction	It is used to solve simultaneous simple equations which have the co-efficient of the variables interchanged.
Sutra 8	Puranapuranabyham	By the completion or Non-completion	Used to simplify or solve the algebra problems.
Sutra 9	Chalana-Kalanabyham	Differences and Similarities	-
Sutra 10	Yaavadunam	Whatever the extent of its deficiency	Applicable to obtain sq. of a number close to bases of powers of 10

Sutra 11	Vyashtisamanstih	Part and Whole	Help in the factorisation of the quadratic equation of types
Sutra 12	Shesanyankena Charamena	The remainders by the last digit	It is to express a fraction as a decimal to all its decimal places
Sutra 13	Sopaantyadvayamantyam	The ultimate and twice the penultimate	-
Sutra 14	Ekanyunena Purvena	By one less than the previous one	This sutra is used in case of multiplication by 9, 99…
Sutra 15	Gunitasamuchyah	The product of the sum is equal to the sum of the product	Used to verify the correctness of obtained answers in multiplications, divisions and factorizations.
Sutra 16	Gunakasamuchyah	The factors of the sum are equal to the sum of the factors	

List of 13 Sub-Sutras with their Meanings

S.No.	Sub-Sutras Name	Meaning
Sub-Sutra 1	Anurupyena	Proportionately
Sub-Sutra 2	Sisyate Sesasamjnah	Remainder remains constant
Sub-Sutra 3	Adyamdyenantya-mantye-na	First by first and last by last
Sub-Sutra 4	Kevalaih Saptakam Gunyat	For 7 the Multiplicand is 143
Sub-Sutra 5	Vestanam	By Osculation

Sub-Sutra 6	Yavadunam Tavadunam	Lessen by the Deficiency
Sub-Sutra 7	Yavadunam Tavadunam Varganca Yojayet	Whatever the Deficiency lessen by that amount and set up the Square of the Deficiency
Sub-Sutra 8	Antyayordasake	Last Totalling 10
Sub-Sutra 9	Antyayoreva	Only the Last Terms
Sub-Sutra 10	Samuccayagunitah	The Sum of the coefficients in the product
Sub-Sutra 11	Lopanasthapanabhyam	By Alternate Elimination and Retention
Sub-Sutra 12	Vilokanam	By Mere Observation
Sub-Sutra 13	Gunitasmuccayah Samuccayagunitah	The Product of the Sum is the Sum of the Products

Baudhāyana Sulba Sūtra

- The *Baudhāyana Sulba Sūtra* states the rule referred to today in most of the world as the Pythagorean Theorem.
- The rule was known to a number of ancient civilizations, including also the Greek and the Chinese, and was recorded in Mesopotamia as far back as 1800 BCE.

Miscellaneous Simple Method

Squaring of Numbers Ending With ‘5’ (last digits add to ten)

- Conventional Method

65 X 65

```
   6 5
 X 6 5
 -----
 3 2 5
3 9 0
-------
4 2 2 5
```

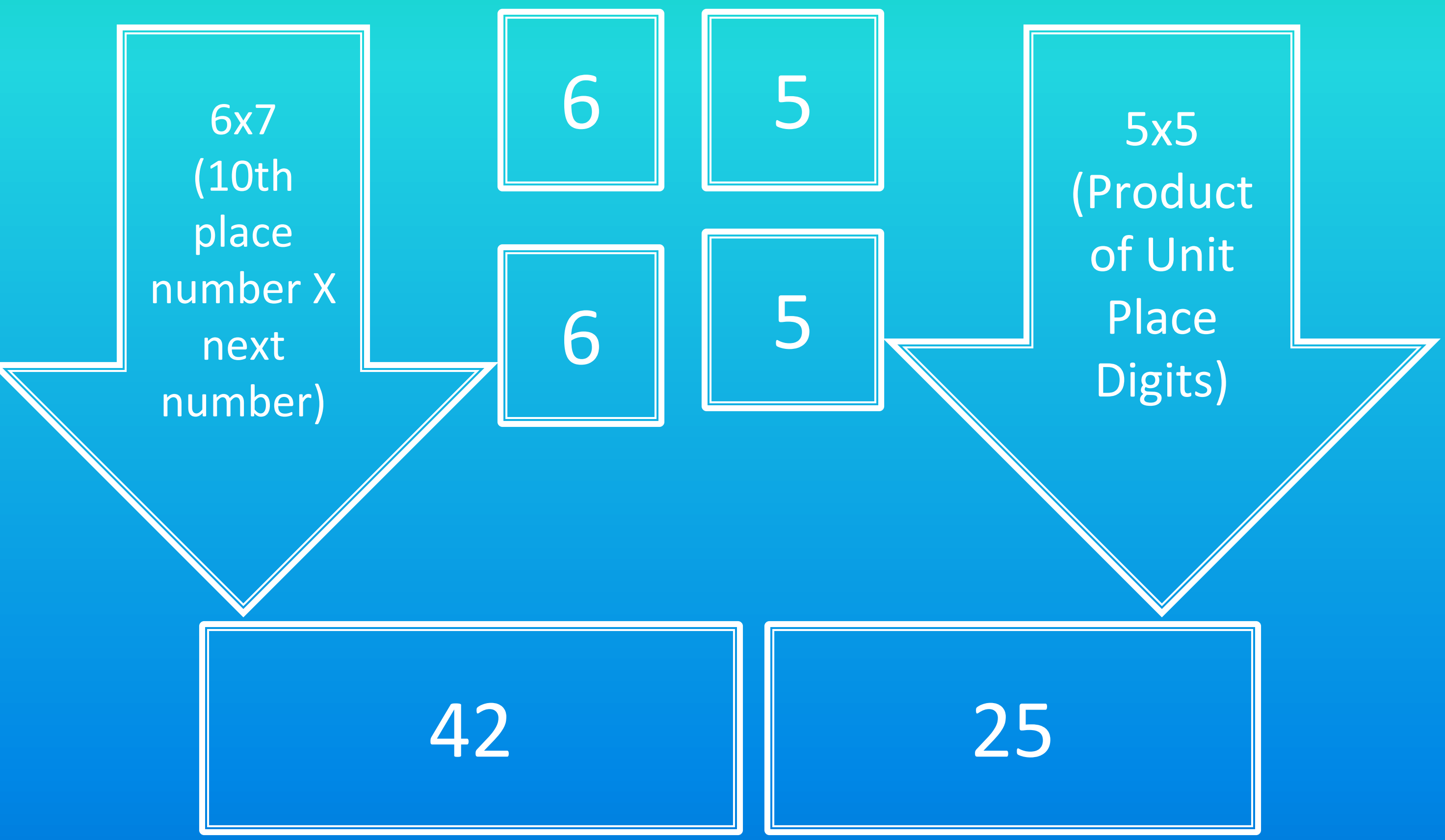
- **Vedic Mathematics**

65 X 65 = 4225

'multiply the previous digit 6 by one more than itself .

Multiply last digits viz. (5x5) and write down 25 to the right of 42 viz. (6x7)

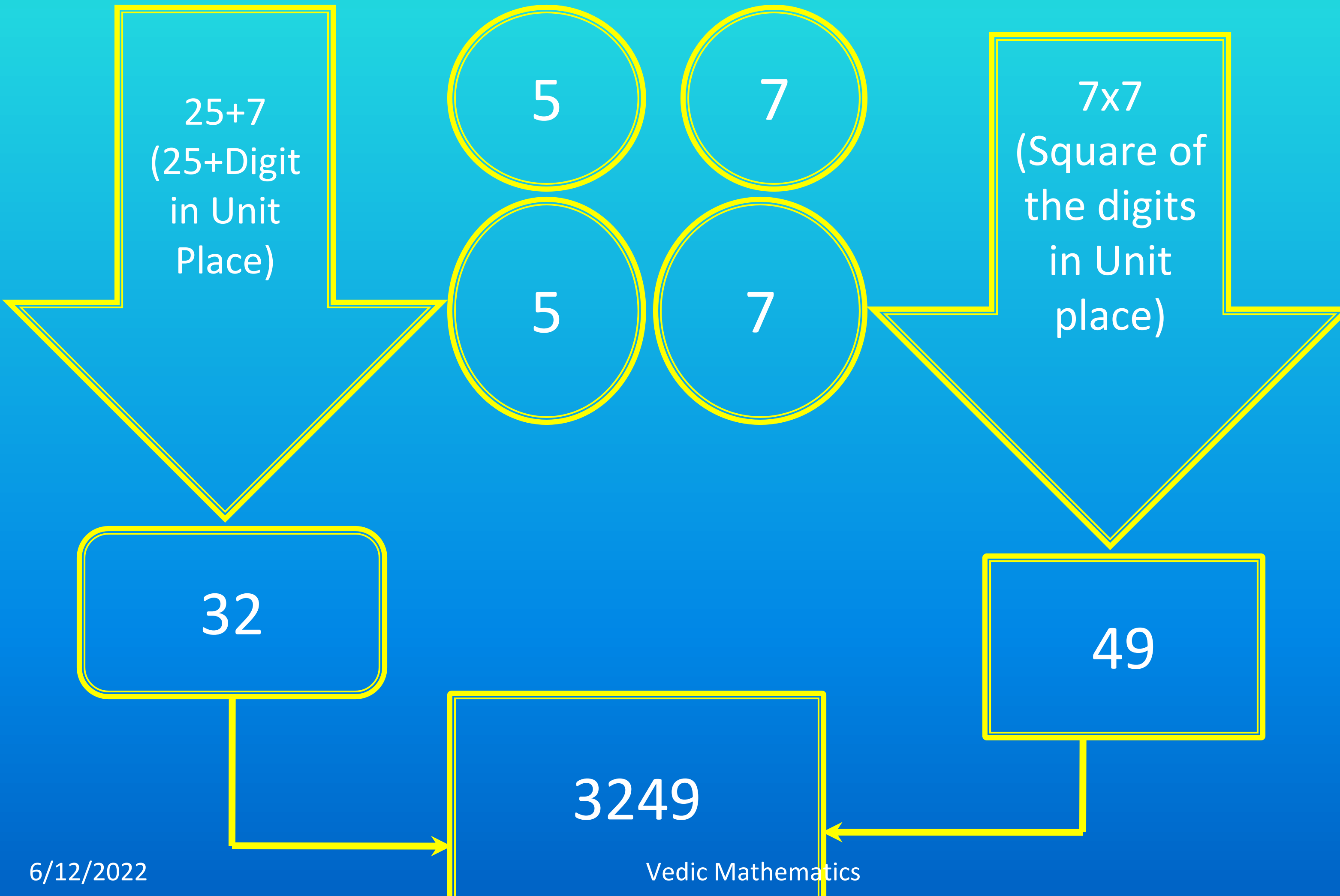
Squaring of Numbers Ending With ‘5’- Example



Squaring of numbers between 50 and 60

- Method:
- Add 25 to the digit in the unit place and put it left hand part of the answer
- Square the digits in the unit place and put it as the right hand part of the answer (if it is single digit then convert it to two digits)

Squaring of numbers between 50 and 60-Example



Multiplication of Numbers with a series of 9's

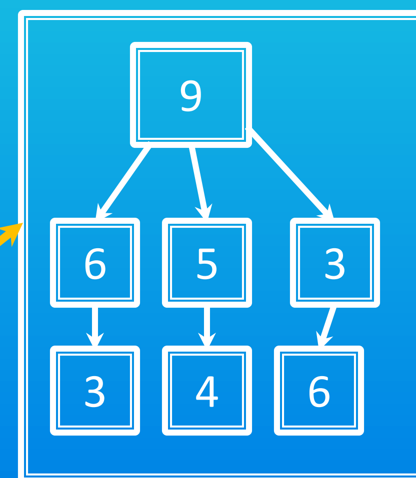
654

X 999

653 346

Subtract 1 from 654 and put it on left side of the answer

Subtract Each of the Digit (653) from nine viz. 9-6, 9-5, 9-3



Case 1: Multiplying a number with an equal number of nines

Multiplication of Numbers with a series of 9's (Cont.)

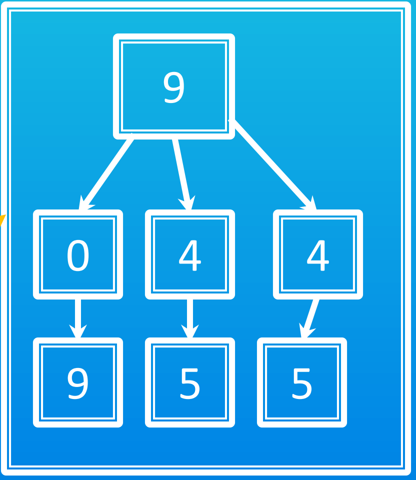
045

X 999

044 955

Subtract 1 from 45 and put it on left side of the answer (add 0 to 45 & write it as 045)

Subtract Each of the Digit (044) from nine viz. 9-0, 9-4, 9-4



Case 2: Multiplying a number with higher number of nines

Multiplication of Numbers with a series of 9's (Cont.)

654 X 99

```
6    53    99
      6    53
---------------
     64746
```

1. Subtract one from multiplicand and group Multiplicand two- two digit from right to left
2. In next line shift one group to right
3. Subtract 53 from 99
4. Subtract remaining methods using traditional method

Case 2a: Multiplying a number with higher number of nines (Digits in first number more than the number of 9's

Multiplication of Numbers with a series of 1's-Example-1

32

X 11

352

First we write the right-hand most digit 2 of first number as it is. (Answer = _______2)

Next, we add 2 to the number in left 3 and write 5. (Answer = ______52)

Last, we write the left-hand most digit 3 as it is. (Answer = 352)

- 1275
- 11
- -------------
- 1402 5
- -----------

Multiplication of Numbers with a series of 1's-Example-2

652

X 11

7172

First we write the right-hand most digit 2 of first number as it is. (Answer = _______2)

Next, we add 2 to 5 and write 7. (Answer is ______72)

Next, we add 5 to 6 and make it 11. We write down 1 and carry over 1. (Answer = 172)

Last, we take 6 and add the one carried over to make it 7. (Final answer = 7172)

Multiplication of Numbers with a series of 1’s-Example-3

3102

X 11

34122

We write down 2 as it is. (Answer =______2)

We add 2 to 0 and make it 2.(Answer is__22

We add 0 to 1 and make it 1. (Answer is __122

We add 1 to 3 and make it 4 (Answer is_____4122

We write first digit 3 as it is (Final Answer is 34122

Multiplication of Numbers with a series of 1's-Example-4

We write down 2 in the unit place as it is. (2)

```
 201432
X   111
-------
22358952
```

We move to the left and add (2+3) and write 5

We move to the left and add (2+3+4) and write 9

We move to the left and add (3+4+1) and write 8

We move to the left and add (4+1+0) and write 5

We move to the left and add (1+0+2) and write 3

We move to the left and add (0+2) and write 2

We move to the left and write the single digit 2 as it is –Final answer 22358952

Multiplication of Numbers with a series of 1's-Example-5

We write down 2 in the unit place as it is. -2

210432	Add (2+3) =5	-52
	Add (2+3+4) =9	-952
X 1111	Add (2+3+4+0) =9	-9952
223789952	Add (3+4+0+1) =8	-89952
	Add (4+0+1+2) =7	-789952
	Add (0+1+2) =3	-3789952
	Add (1+2) =3	-23789952
	Add (2) =2	233789952

- 3541234
- 1111
- ---------------
- 434210 974

Multiplication of numbers with a series of similar digits in multiplier

Multiply 333 by 222

333 X 222
=333 X 2 X 111 (Because 222 is Multiplied by 111)
=666 X 111 (because 333 multiplied by 2 is 666)

Carefully observe the Logic applied Here

$$\begin{array}{r} 666 \\ \times 111 \\ \hline 73926 \end{array}$$

Subtraction using the rule 'All from 9 and last from 10' (used for power of 10)

Subtract 54.36 from 1000

Conventional Method

```
  100.00
-   54.36
---------
   45.64
```

Generally start from right and subtract 6 from 0. We realize that it is not possible to subtract 6 from 0 so we move number to the left and then borrow and give it to 0 and so on.

Vedic Method

9-5=4

9-4=5

9-3=6

10-6=4

Start from Left & subtract all from 9 and the last from 10

Speed Addition

- 7
- 8
- 9
- 2
- 4
- 6
- 3

39

7

8

9

2

4

6

3

Mental Calculation of Numbers-Addition

4639 +1235

4639 +1000=5639
This 5639+200=5839
5839+30=5869
And 5869 +5 =5874.

Start from left to right, keep 4639 as it is in your mind . Break up second number

Mental Calculation of Numbers- Subtraction

7580 - 4142

Start from left to right, keep 7580 as it is in your mind . Break up second number

4142 (4000 + 100+40 +2)

7580-4000=3580

3580-100=3480

3480-40=3448

3480-2=3438

Mental Calculation of Numbers- Multiplication

76 X 7

Mentally break up the number 76 as 70 + 6 and then multiply each of these values by 7

70 + 6

7 + 7

70 x 7 is 490

6 X 7 is 42

490+42= (490 +10 +32)

490+10=500

500+32=532

Fractions, percentage & Decimals

Number	Fractional Value	Decimal value
With 2	1/2	0.5
With 3	1/3 2/3	0.33 0.67
With 4	1/4 2/4 3/4	0.25 0.5 0.75
With 5	1/5 2/5 3/5 4/5	0.2 0.4 0.6 0.8
With 6	1/6 2/6 3/6 4/6 5/6	0.16 0.33 0.50 0.67 0.83

Fractions, percentage & Decimals(cont.)

Number	Fractional Value	Decimal value
With 7	1/7 2/7 3/7 4/7 5/7 6/7	0.14 (approx) 0.28 0.42 0.57 0.71 0.85
With 8	1/8 2/8 3/8 4/8 5/8 6/8 7/8	0.125 0.250 0.375 0.50 0.625 0.750 0.875

Fractions, percentage & Decimals(cont.)

Number	Fractional Value	Decimal value
With 9	1/9 2/9 3/9 4/9 5/9 6/9 7/9 8/9	0.11 0.22 0.33 0.44 0.55 0.66 0.77 0.88

Exercises

1. Find the product in the following numbers whose last digits add to ten

45 X 45

95 X 95

111 X 119

107 X 103

123 X 147

88 X 92

65 X 65

2. Find the squares of the following numbers between 50 & 60

56

52

57

58

54

3. Find the sum of following numbers using mental calculations

567 X 999

23249 X 99999

66 X 9999

302 X 99999

412 X 99

4. Find the product of the following numbers which are multiplied by a series of ones

32221 X 11

64609 X 11

12021 X 111

80041 X 111

900021X 111

Exercises (Cont.)

5. Find the product in the following numbers which are multiplied by a series of same numbers

7005 X 77

1234 X 22

2222 X 222

1203 X 333

1407 X 444

1212 X 666

6. Subtract the following numbers from a given power of ten

1000-675.43

10000-7609.98

10000-666

1000-2.653

10000-2.876

7. Find the sum of following numbers using mental calculations

10980+5680

11764+6480

23452+5730

8. Find the difference of the following numbers using mental calculation

34576-4320

5734-2200

89765-3478

9. Find the product of the following numbers using mental calculation

88 X 7

99 X 8

66 X6

44 X 9

CRISS-CROSS SYSTEM OF MULTIPLICATION

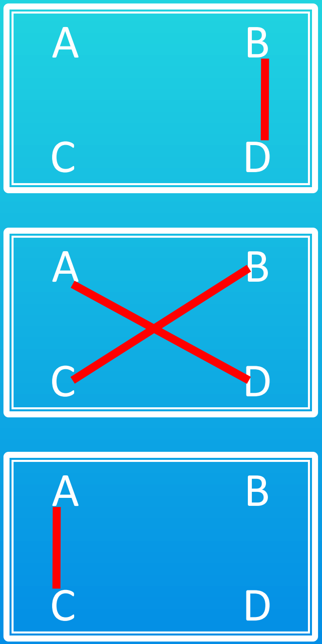
Criss-Cross System of Multiplication

- **This the general formula applicable to all cases of multiplication.**
- **It means ‘Vertically and cross-wise’**

Criss-Cross Multiplication-2 digits numbers

- Vedic Method

$$\begin{array}{r} 4\ 6 \\ \underline{\times\ 4\ 3} \\ \underline{1\ 9\ 7\ 8} \end{array}$$



Step 1: B X D

Step 2: A X D +B X C

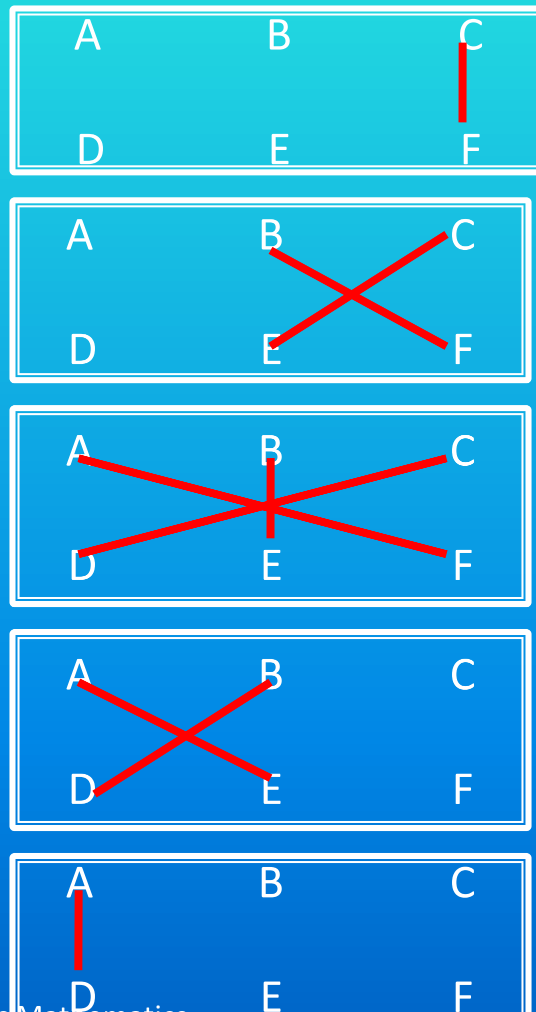
Step c: A X C

- **<u>Step 1</u>: 6×3=18, write down 8 and carry 1**
- **<u>Step 2</u>: 4×3 + 6×4 = 12+24=36, add to it previous carry over value 1, so we have 37, now write down 7 and carry 3**
- **<u>Step 3</u>: 4×4=16, add previous carry over value of 3 to get 19, write it down.**
- **So we have 1978 as the answer.**

Criss-Cross Multiplication-3 digits numbers

- **Vedic Method**

```
  103
X 105
10,815
```



C X F

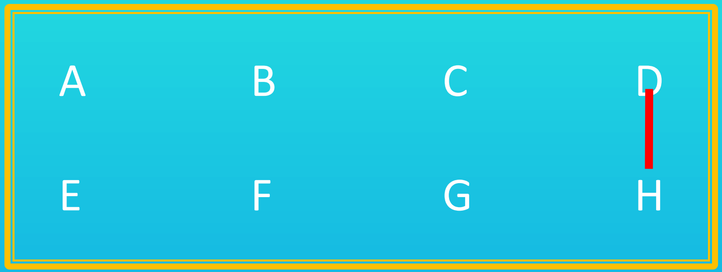
B X F + C X E

A X F + C X D + B X E

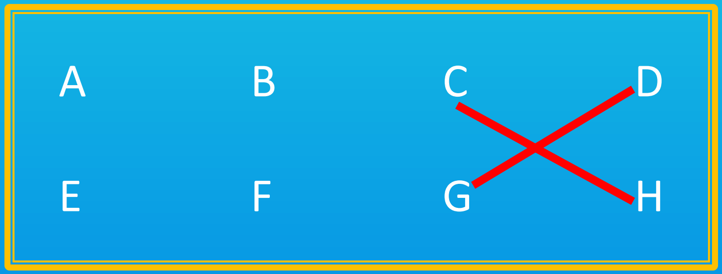
A X E + B X D

A X D

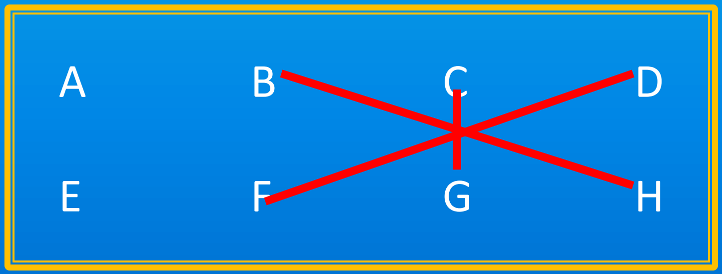
Criss-Cross Multiplication-4 digits numbers



Step 1: D X H

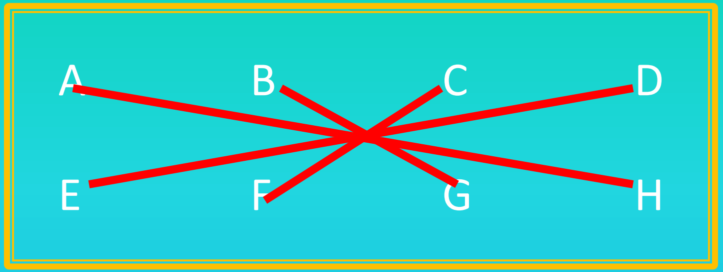


Step 2: C X H + G X D

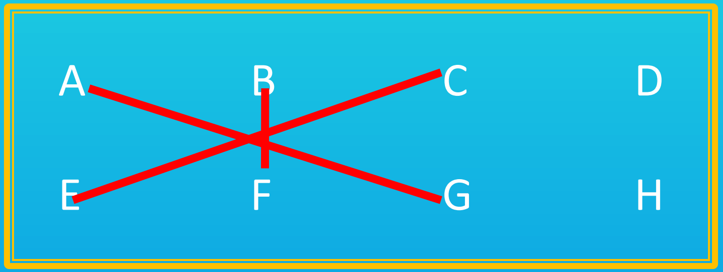


Step 3: B X H + F X D + C X G

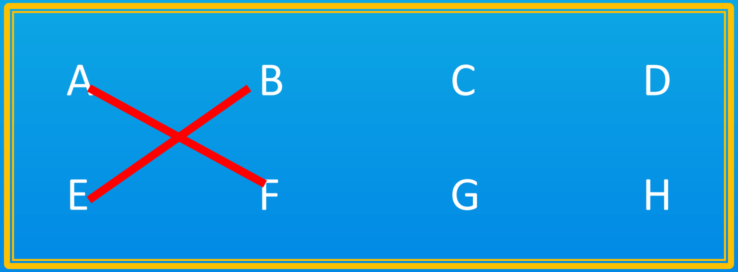
Criss-Cross Multiplication-4 digits numbers (Cont.)



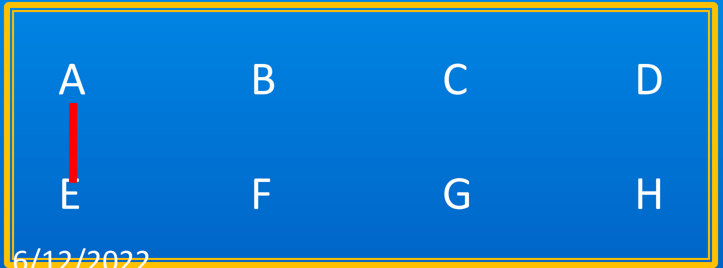
Step 4: (E X D) + (A X H) + (B X G) + (C X F)



Step 5: (A X G) + (C X E) + (B X F)

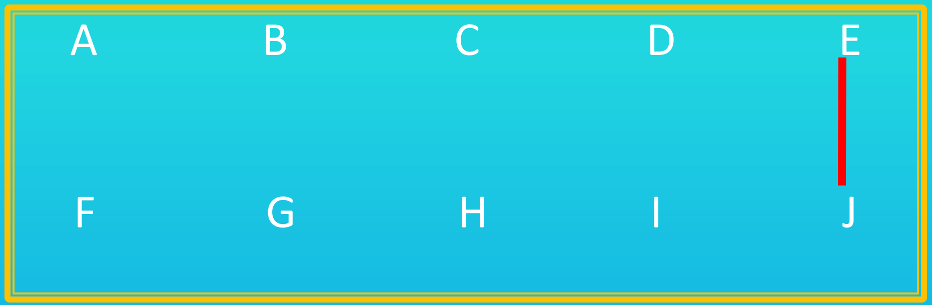


Step 6: A X F + B X E

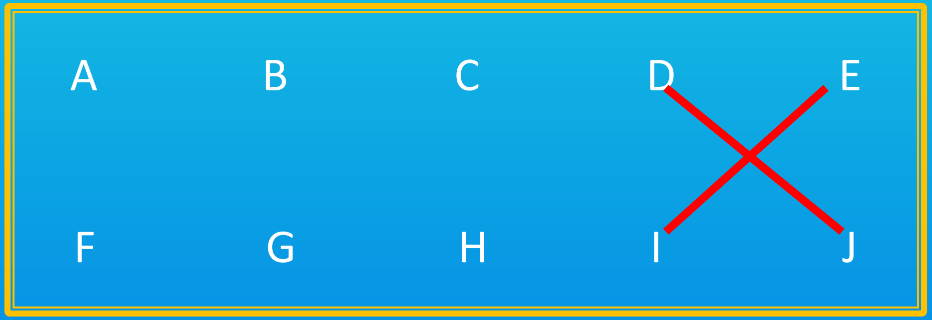


Step 7: A X E

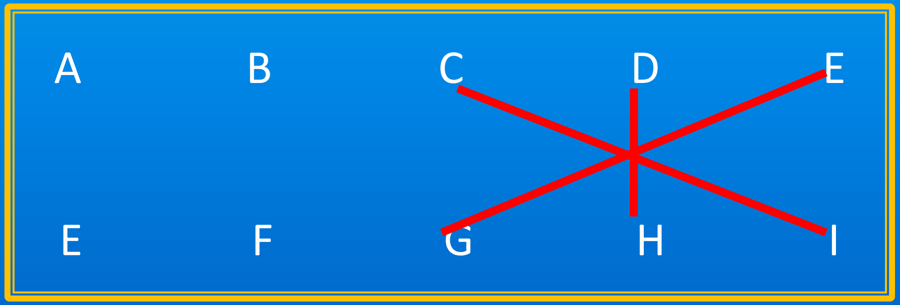
Criss-Cross Multiplication-5 digits numbers



Step 1: E X J

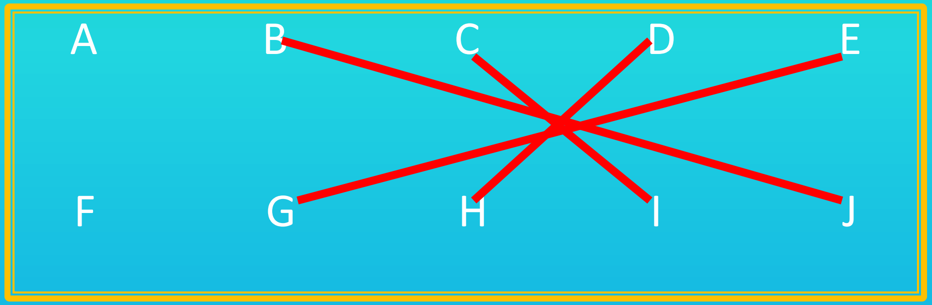


Step 2: D X J + E X I

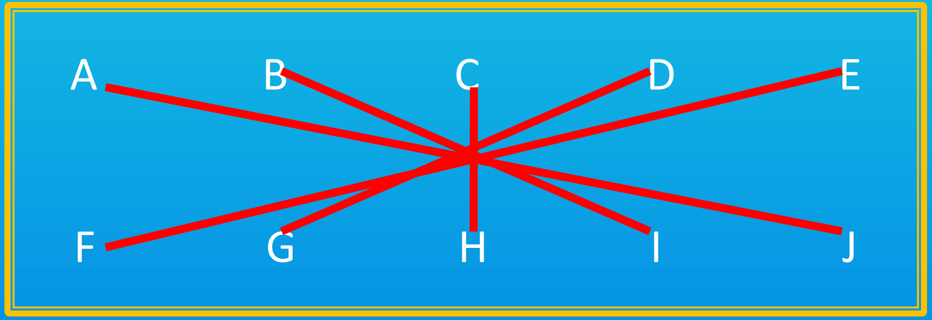


Step 3: B X H + F X D + C X G

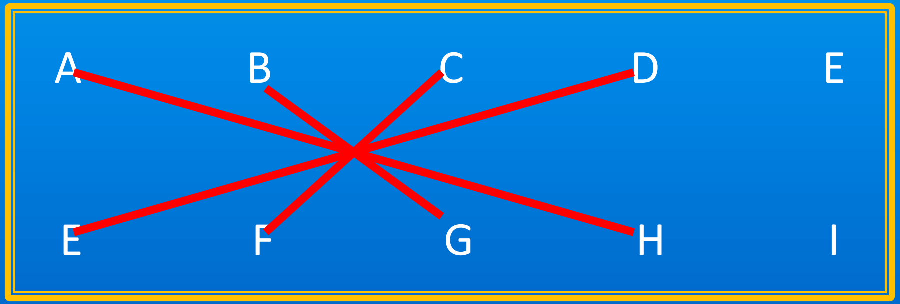
Criss-Cross Multiplication-5 digits numbers (Cont.)



Step 4: (B X J) + (G X E) + (C X I) + (H X D)

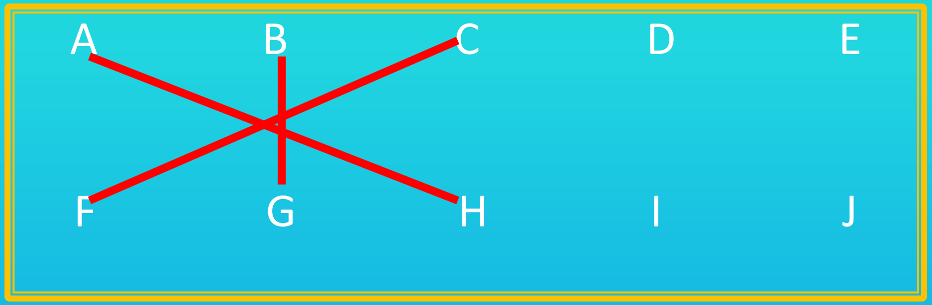


Step 5: (A X J) + (F X E) + (D X J) + (B X I) + G X D + C X H

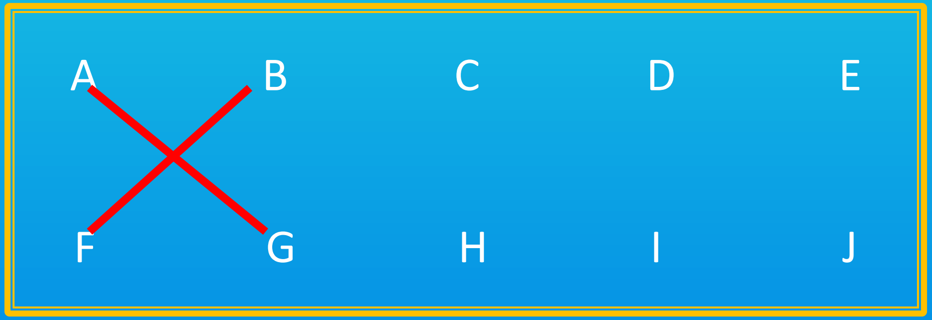


Step 6: (A X H) + (E X D) + (BX G) + (F X C)

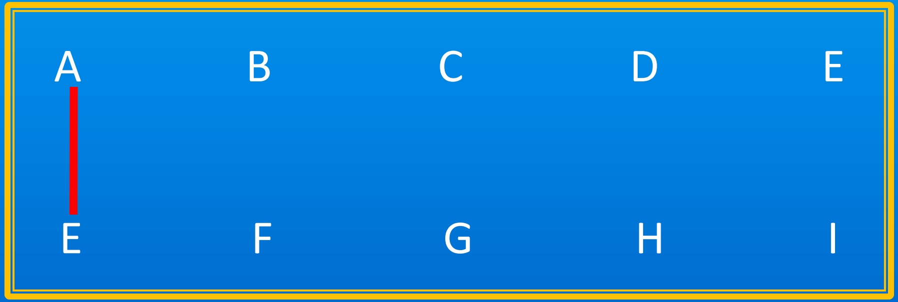
Criss-Cross Multiplication-5 digits numbers (Cont.)



Step 7: A X H + C X F + B X G



Step 8: A X G + B X F



Step 9: A X E

Characteristics of Criss-Cross Multiplication

- The Number of steps used for any multiplication can be found using the formula '(2 X number of digits)-1
- If there are unequal number of digits in multiplicand and multiplier, they should be made equal by inserting 0's at the appropriate places
- The number of steps used will be always an odd number
- In this first and last step, second and second-to-last and so on are mirror image of each other

Exercises

PART A

(a) 23 X 12 (b) 34 X 11 (c)33 X 21 (d) 41 X 13

(e) 211 X 320 (f) 222 X 111 (g) 303 X 210
(h)1111 X 1111

PART B

(a) 44 X 22 (b) 33 X41 (c) 91 X 31 (d) 24 X 51

(e) 358 X 111 (f) 423 X 202 (g) 801 X 601

(h) 2323 X 3232

Squaring Numbers

Squaring Numbers

- Vedic Method

$$\begin{array}{r} 2\ 3 \\ \underline{\times\ 2\ 3} \\ \underline{5\ 2\ 9} \end{array}$$

- **Step 1: $3 \times 3 = 9$, write down 9**
- **Step 2: $2 \times 3 + 2 \times 3 = 6 + 6 = 12$, write down 2 and carry 1**

Step 3: $2 \times 2 = 4$, add previous carry over value of 1 to get 5, write it down.

- **So we have 529 as the answer.**

Squaring Numbers (Cont.)- Formula Method

- $(a+b)^2 = a^2 + 2ab + b^2$
- $(a-b)^2 = a^2 - 2ab + b^2$
- $(a^2 - b^2) = (a+b)(a-b)$

Therefore $a^2 = (a+b)(a-b) + b^2$

Formula Method-Example1

- Find Square of 1009 & 995

$$(1000+9)^2 = (1000)^2 + 2(1000)(9) + (9)^2$$
$$=1000000 + 18000 + 81$$
$$=1018081$$

$$(1000-5)^2 = (1000)^2 - 2(1000)(5) + (5)^2$$
$$=1000000 - 10000 + 25$$
$$=990025$$

Formula Method-Example2

- Find Square of 72

$(72)^2 = (70 + 2) + (70\text{-}2) + (b)^2$

$=(72 + 2)(72\text{-}2) + (2)^2$

$=(74)(70) + 4$

$=(70 \text{ X } 70) + (4 \text{ X } 70) + 4$

$=4900 + 280 + 4$

=Thus square of 72 is 5184

Substitute the value of b with such a number that the whole equation becomes easy to solve.

Exercises

- Find the square of the following numbers using the Criss-cross System.
- 45 66 118
- Find the Square of the numbers using the formula for $(a+b)^2$
- 206 3005 5050
- Find the Square of the numbers using the formula for $(a-b)^2$
- 8991 9900 1090
- Find the Square of the numbers using the formula for $(a^2 - b^2) = (a + b)(a - b)$
- 92 82 109 97 99

Cube Root of Perfect Cubes

Cube Root of Perfect Cubes

1	1
2	8
3	27
4	64
5	125
6	216
7	343
8	512
9	729
10	1000

Note that all cube roots end with same number as their corresponding cubes except 3 & 7 and 8 & 2 which end with each other

Cube Root of Perfect Cubes -Example

Find the cube root of 287496

- Step 1: We shall represent the number as
- 287 | 496
- Step 2: Cube root ends with 6, thus answer at this stage is ___6
- Step 3: To find the left hand of answer we take number which lies left of the slash is 287
- Step 4: Find the two perfect cubes between which the number 287 lies in the number line (216 <287 <343) viz. between 6 & 7
- Step 5: Out of the above 2 numbers, take smallest one viz. 6 we write answer as 66
- Thus 66 is cube root of 287496

Cube Root of Perfect Cubes (Cont.)

Note:-

- When ever a cube is given to you calculate its cube root, you must put a slash before the last three digits
- Number of digits in cube is immaterial

Comparison

Traditional Method

2	262144
2	131072
2	65536
2	32768
2	16384
2	8192
2	4096
2	2048
2	1024
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

2 X 2 X 2 X 2 X 2 X 2 = 64

The Vedic Method

262 | 144 = 64

Exercises

- Find the cube root of the following numbers with the aid of writing material
- (a) 970299 (b) 658503 (c) 314432 (d) 110592
- (e) 466566 (f) 5832 (g) 421875 (h) 1030301
- (i) 857375 (j) 592704

- Find the cube root of the following numbers without the aid of writing material
- (a) 132651 (b) 238328 (c) 250047 (d) 941192
- (e) 474552 (f) 24389 (g) 32768 (h) 9261
- (i) 59319 (j)74088 (k) 10648

Square Roots of Perfect Squares

Square Roots of Perfect Squares

1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

Compare last digit of the square and square Root

1	1 or 9
4	2 or 8
9	3 or 7
6	4 or 6
5	5
0	0

A perfect square will never end with the digits 2,3, 7, 8

Square Root-Example

- Find the square root of 7744.
- Step 1: The number 7744 ends with 4. Therefore square root ends with __2 or __8.
- Step 2: Take complete Number 7744
- Step 3: 7744 lies between 6400 (which is square of 80) and 8100 (which is square of 90)
- Step 4: From Step 2 we know that square root ends with 2 or 8. Of all the numbers between 80 & 90 (81, 82, 83, 84, 85, 86, 87, 88, 89). Thus out of 82 & 89 one is the correct answer
- Step 5: Observe the Number (7744) is ether closer to 6400 or 8400. It is closer to 8400 . So Answer is 88.

Comparison

Traditional Method

2	576
2	288
2	144
2	72
2	36
2	18
2	9
3	3
3	1

2 X 2 X 2 X 3 = 24

The Vedic Method

20= 400 24

576

30= 900 26

Exercises

- Find the square roots of the following numbers with the aid of writing material
- (a) 9216 (b) 7569 (c) 5329 (d) 3364
- (e) 1681 (f) 1081 (g) 2304
- Find the square roots of the following numbers without the aid of writing material
- (a) 9801 (b) 5625 (c) 1936 (d) 3481 (e) 1369
- Find the square roots of the following numbers with or without the aid of writing material
- (a) 12769 (b) 15625 (c) 23104 (d) 11881
- (e) 17689

Base Method of Multiplication

Base Method of Multiplication

- ***"All from 9 and the last from 10"***

- **This formula can be very effectively applied in multiplication of numbers, which are nearer to bases like 10, 100, 1000 i.e., to the powers of 10 (eg: 96 x 98 or 102 x 104).**

Steps

1. Find the Base and Difference
2. Number of Digits on the RHS= Number of zeros in the base
3. Multiply the difference on the RHS
4. Put the cross answer on the LHS

Case I-when both the numbers are lower than the base

- Find 97 X 99

97-3
99-1

Step 1: Find base and difference. Base is 100

97-3
99-1
| _ _

Step 2: Number of Digits in RHS =Number of zeros in base

97-3
99-1
| 03

Step 3: Multiply the difference in RHS

97-3
99-1
96 | 03

Step 4: Put the cross answer in LHS (97-1 or 99-3)

Case II-when both the numbers are above the base

- Find 1007 X 1010

Step 1: Find base and difference. Base is 1000

1007+7
1010+10

Step 2: Number of Digits in RHS =Number of zeros in base

1007+7	
1010+10	
	_ _ _

Step 3: Multiply the difference in RHS

1007+7	
1010+10	
	070

Step 4: Put the cross answer in LHS (1007+10 or 1010+7)

1007+7	
1010+10	
1017	070

Case III-when the number of digits in RHS exceeds number of zeros in the base

❖ Find 950 X 950

950-50
950-50

Step 1: Find base and difference. Base is 1000

950-50
950-50

	$_2$500

Step 3: Multiply the difference in RHS and move carry to LHS

950-50
950-50

	_ _ _

Step 2: Number of Digits in RHS =Number of zeros in base

Step 4: Put the cross answer in LHS (950-50) &carry 2

950-50
950-50

902	500

Case IV-Multiplying a number above the base with number below the base

- Find 95 X 115

Step 1: Find base and difference. Base is 100

95-5
115+15

Step 2: Number of Digits in RHS =Number of zeros in base

95-5
115+15
| – –

Step 3: Multiply the difference on RHS

95-5
115+15
| (-75)

Step 4: Multiply the LHS with base and subtract RHS

=110 X 100-75
=11000-75
=10925

Case V-Multiplying numbers with different bases

- Find 85 X 995

850-150
995-5

Step 1: Multiply 85 by 10 to make bases same

850-150
995-5

	– – –

Step 2: Number of Digits in RHS =Number of zeros in base

850-150
995-5

	750

Step 3: Multiply the difference on RHS

Step 4: Put the cross answer in LHS (850-5 or 990-5)

850-50
995-50

845	750

Final Answer= 845750/10 =84575

Case VI- When the base is not power of ten

- Two bases are maintained-Actual Base and Working Base
- Actual base is power of 10
- Working base will be obtained by dividing or multiplying the actual base by a suitable number
- Eg: Actual Base: 10, 100, 1000, etc.
- Eg: Working Base: 40, 60, 500, 250, etc.

Example

Method 1

Actual Base = 100

Working Base=100/2 = 50

48 - 2

48 - 2

46/04

Final Answer 46/2

2304

Method 2

Actual Base = 10

Working Base=10 X 5 = 50

48 - 2

48 - 2

46/4

Final Answer 46 X 5

230/4

2304

Exercises

1. Multiply the following

(a) 990 X 994 (b) 999993 X 999999

(c) 102 X 10100 (d) 1050 X 1005 (e) 106 X 104

2. Multiply the following numbers when the answer in RHS exceeds the number of zeros in the base

(a) 16 X 17 (b) 1500 X 1040 (c) 9300 X 9500

(d) 860 X 997

3. Calculate the product of the Following (one number is above the base and the other number is below the base

(a) 96 X 104 (b) 890 X 1004 (c) 10080 X 9960

(d) 970 X 1010

Exercises (Cont.)

4. Multiply the following numbers using different bases.

(a) 73 X 997 (b) 94 X 990 (c) 82 X 9995

(b) (d) 102 X 1010 (e) 104 X 1020 (f) 12 X 109

5. Multiply the numbers using actual and working base

(a) 49 X 48 (b) 22 X 22 (c) 53 X 49 (d) 18 X 17

(e) 499 X 496 (f) 32 X 34

Base method for Squaring

Base method for Squaring

- ‘Whatever the extent of its deficiency, lessen it to the same extent and also set up the square of deficiency’
- The first part says that Whatever the extent of its deficiency, lessen it to the same extent
- The Second part simply says- square the deficiency

Base Method for Squaring-Example

- Find square of 97

97-3
97-3

Step 1: Find base and difference. Base is 100

97-3
97-3

_	_

Step 2: Number of Digits in RHS =Number of zeros in base

97-3
97-3

	03

Step 3: Multiply the difference in RHS

97-3
97-3

94	09

Step 4: Put the cross answer in LHS (97-3)

Exercises

1. Find the square of the following numbers using Yavadunam Rule.

(a) 7 (b) 95 (c) 986 (d) 1025 (e) 1012 (f) 999

2. (a) 85 (b) 880 (c) 910 (d) 18 (e) 1120 (f) 2102

3. (a) 22 (b) 203 (c) 303 (d) 405 (e) 498 (f) 225 (g) 247

Digit-Sum Method

Digit- Sum Method

- This method is used for quick checking of answer rather than quick calculation.
- This technique has wonderful different applications in competitive exams as they are already provided with four alternatives to every answer.
- This can be used to check answers involving multiplication, division, addition, subtraction, squares, square roots, cube roots, etc.

Example-1

- Find the digit sum of 2467539
- The number is 2467539.
- We add all the digits of that number.
- $2 + 4 + 6 + 7 + 5 + 3 + 9 = 36$
- Now take number 36 and add its digits $3 + 6 = 9$

Mathematical Operation	Procedure for checking answer
Multiplication	The digit sum of multiplicand when multiplied with the digit sum of the multiplier should equal to the digit sum of the product.
Division	Use the formula dividend =divisor multiplied by quotient + remainder . (Use digit sum instead of actual numbers)
Addition	The digit sum of the final sum should be equal to the digit sum of all the numbers used in addition process.
Subtraction	The digit sum of the smaller number as subtracted from the digit sum of the bigger number should equal the digit sum of the difference.
Squaring/Square Rooting	The digit sum of the square root as multiplied by itself should equal to the digit sum of the square. E.g.: whether 23 is square root of 529. 5x5=25 =7(digit sum of 25) . 529 digit sum. 16 =7(1+6).
cube/cube Rooting	The digit sum cube-root when multiplied by itself and once again by itself should equal to the digit sum of the cube. E.g.: Whether 1331 is cube root of 11. 2 x 2x 2 =8 . 1+3+3+1=8.

Example-2

Verify Whether 467532 multiplied by 107777 equals 50389196364

The digit sum of 467532 = 4+6+7+5+3+2 is 9

The digit sum of 107777=29=9+2 is 11 is 2

When we multiply 9 by 2 we get 18. digit sum is 9

Digit sum of 50389196364 = 54=5+4 is 9

We can assume answer is correct because The digit sum of multiplicand when multiplied with the digit sum of the multiplier = digit sum of product

Example-3

- Verify whether 2308682040 divided by 36524 equals 63210.
- Dividend=divisor x Quotient +remainder
- The digit sum of dividend is 6
- The digit sum of divisor, quotient and remainder is 2, 3, and 0 respectively.
- Since 6 = 2 x 3 + 0, we can assume our answer is coorect

Important points to be noted

- When calculating digit sum of a number, you can eliminate all the nines and all the digits that add up to nine.
- The elimination will have no effect on final answer.
- Example- digit sum of 637281995
- =6+3+7+2+8+1+9+9+5
- =50 and again 5 + 0 = 5
- Now we will eliminate the numbers that add up to 9 (6 & 3, 7 &2, 8 & 1 and also eliminate the two 9's)

Important points to be noted (Cont.)

- ***The digit sum method can only tell us whether an answer is wrong or not. It cannot tell us with complete accuracy whether an answer is correct or not.***
- ***However if the digit-sum of the answer does not match with the digit sum of the question then you can be 100% sure that the answer is wrong.***

Exercises

1. Instantly calculate the digit sum of the following

- (a) 23456789 (b)123456789 (c) 27690815
- (d)988655543 (e) 918273645

2. Verify whether the following answer are correct or incorrect without actual calculations

(a) 95123 x 66666 = 6341469918

(b) 838102050 divided by 12345 = 67890

Exercises (Cont.)

- (c) 88^2 =744
- (d) 88^3 = 681472
- (e) 475210
- (f) 900/120m gives quotient 7 and reminder 60

Magic Squares

Magic Squares

- ‘Magic Squares’ is a term given to squares which are filled with consecutive integers and the total of whose rows, columns and diagonals is always the same.
- In lower level competitive exams, questions on magic squares are often asked.

3 X 3 Magic Square

4	3	8
9	5	1
2	7	6

We can verify the various totals.

Row 1: 4 + 3+ 8 = 15

Row 2: 9 + 5 + 1 = 15

Row 3: 2 + 7 + 6 = 15

Column 1: 4 + 9 + 2 = 15

Column 2 : 3 + 5 + 7 =15

Column 3: 8 + 1+ 6 =15

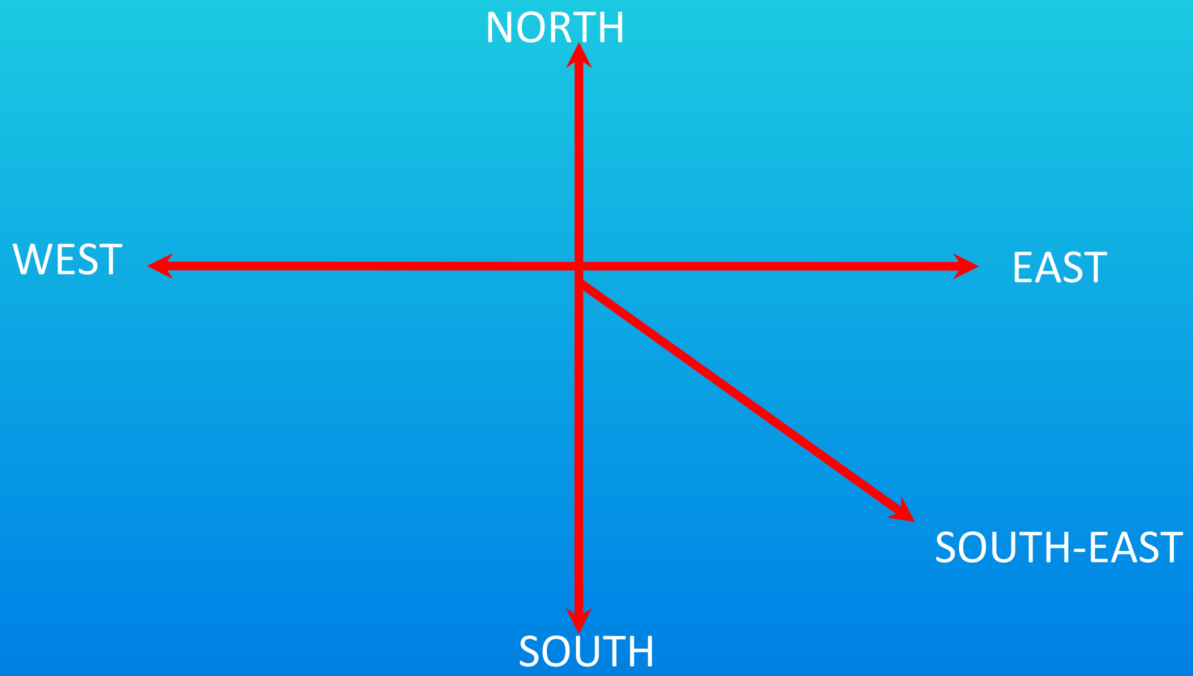
Diagonal 1: 4 + 5 + 6 =15

Diagonal 2: 2 + 5 + 8 =15

Rules to Construct Magic Square

1. Always put the number 1 in the centre most square of the last column.
2. After inserting a number in a square move to the square in the south east direction and fill it with the next number.
3. If the square in the south-east direction cannot be filled, then move to the square in the west and fill it with the next number.
4. When you have filled a number in the last square of the grid, fill the next number in the square to its west.

Direction

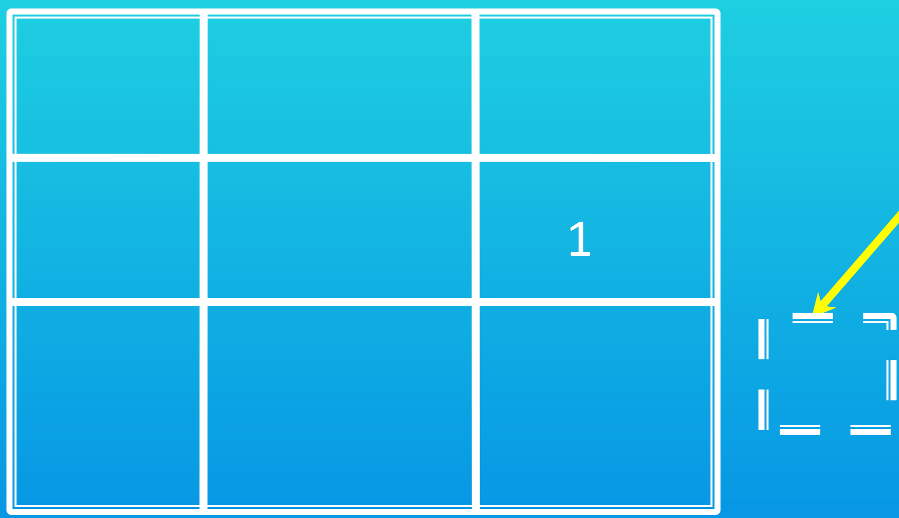


Step by Step to Construct 3 x 3 Magic Square

		1

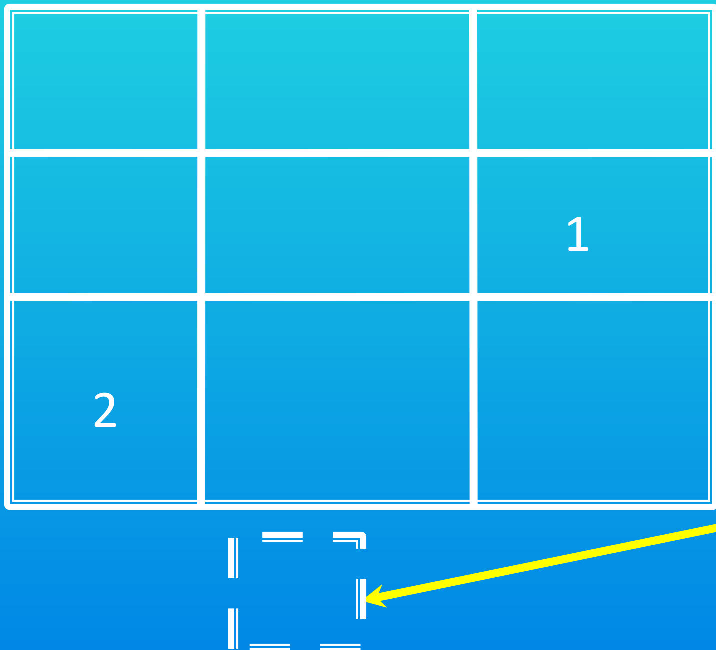
First, we follow the rule 1 and place the first number 1 in the centre most square of the last column

Step by Step to Construct 3 x 3 Magic Square (Cont.)



We move south east direction from square 1. There is nothing in the south-east direction. As per the rules the digit 2 will come in imaginary square (Dotted Square) . Number 2 will be written in a square farthest from it in same direction.

Step by Step to Construct 3 x 3 Magic Square (Cont.)



We move south east direction from square 2. There is nothing in the south-east direction. As per the rules the digit 3 will come in imaginary square (Dotted Square) . Number 3 will be written in a square farthest from it in same direction.

Step by Step to Construct 3 x 3 Magic Square (Cont.)

	3	
		1
2		

Use rule 3. If the square in the south-east direction is already filled, then move to the square in the west and fill it with the next number. viz. 4

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	
		1
2		

Use rule 2. We move south east direction from square 4 and fill it with next number 5.

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	
	5	1
2		

Use rule 2. We move south east direction from square 5 and fill it with next number 6.

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	
	5	1
2		6

Use rule 4. When you have filled a number in the last square of the grid, fill the next number, 7 in the square to its west

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	
	5	1
2	7	6

We move south east direction from square 7. There is nothing in the south-east direction. As per the rules the digit 8 will come in imaginary square (Dotted Square) . Number 8 will be written in a square farthest from it in same direction.

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	8
	5	1
2	7	6

We move south east direction from square 8. There is nothing in the south-east direction. As per the rules the digit 9 will come in imaginary square (Dotted Square) . Number 9 will be written in a square farthest from it in same direction.

Step by Step to Construct 3 x 3 Magic Square (Cont.)

4	3	8
9	5	1
2	7	6

All Squares of the grid are filled.

Properties of Magic Square

(a) The number of rows and columns will always equal. Using this rules only odd pair of magic grid will be filled. Like 3x3, 5x5, 7x7. Not 2x2 or 4x4, etc.

(b) The first and last number always lie in the same row and exactly opposite to each other.

(c) The total of any side can be found out by multiplying the number in the center most square of the grid with number of squares in any side. In 3x3 grid center most number is 5. number of squares in any side is 3. 5x3=15.

Properties of Magic Square (Cont.)

(d) You can find out which number will come in the centre most square of any grid by ***"Taking the maximum number, dividing it by 2 and rounding it off to the next higher number"***

- E.g.: In 3x3 grid maximum number is 9. 9/2=4.5. i.e. 5

(e) There are many possible ways by which a magic square can be made out of certain grid. If we take 3x3 grid, then we can form a magic square out of it in a few different manner as shown below.

Different possible ways of 3x3 Grid

4	3	8
9	5	1
2	7	6

8	1	6
3	5	7
4	9	2

6	7	2
1	5	9
8	3	4

2	9	4
7	5	3
6	1	8

Exercises

- Make a 3x3 grid using first 9 even numbers (2,4,6,8….18)
- Make a 5x5 grid
- Make a 5x5 grid using multiplies of 3 (3,6,9,etc.)
- Using the multiples of 5, make a 3x3 grid and represent in four different ways.
- Construct a 7x7 grid

Dates & Calendars

Dates & Calendars

- Single Year Calendar for year 2012

154 163 152 742

There are 12 numbers in the box
Each number represents month of the year
The number 1 represents January, the next number 5 represents February, the next number 4 represents March and so on up to last number 2 which represents December.

Here, 1 January is Sunday
2 February is a Sunday
4 March is a Sunday
1 April is s Sunday
And so on….

Technique

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	4	4	0	2	5	0	3	6	1	4	6

Remainder	Day
1	Sun
2	Mon
3	Tue
4	Wed
5	Thu
6	Fri
7	Sat

Works only for the century 1901-2000

How to Remember (Using small Verse)

- ***It's the square of twelve***
- ***And the square of five***
- ***And the square of six***
- ***And one-four-six***

Steps:

1. Take the last two digit of the year
2. Add the number of leap years from the beginning of the century
3. Add the month Key
4. Add the date
5. Divide the total by 7
6. Take the remainder and verify it with the day-key

Example

- What day is 1 Jan 1941
- First we take the last two digits of the year = **41**
- We add to it the number of leap year from 1901 to 1941 = **10**
- We add month key for January (refer to the month-key) = **1**
- We add the date(as given in the question) = **1**
- Total = **53**
- Divide 53 by 7 = quotient 7 and the reminder =4
- From the day key remainder 4 corresponds to Wednesday

Characteristics of Dates

- There are exactly 52.1 weeks in a year.
- In other words a year is made up of 52 complete week and an extra day.
- As every year progresses, a date moves a day later.
- Every 4 year’s once in February maximum date becomes 29. A year which contains 366 days is called leap year. If the last 2 digits of a number is perfectly divisible by 4, then it is a leap year. A century is a leap year if the first 2 digits are perfectly divisible by 4. (Eg. 2000, 400, 800,2400)

Characteristics of Dates (Cont.)

For January & February	For March to December
Rule A: A date moves a step ahead as every year passes by	Rule A: A date moves a step ahead as every year passes by
Rule C: A date moves two steps ahead in a year succeeding a leap year	Rule B: A date moves two steps ahead in a leap year

Example-2

- If 31st December 2000 is a Sunday, what day will it be on 2nd January 2005.
- 31st December 2000 is a Sunday. It will fall on Monday, Tuesday and Wednesday in the years 2001, 2002 and 2003 respectively. In 2004, it will become a Friday. Hence on 2nd January 2005 (2days later) it will be a Sunday.

Formula to find day on which any date falls (Zeller's Rule)

- $F = K + [(13 \times m - 1)/5] + D + [D/4] + [C/4] - 2 \times C$
- Where k-Date
- m- Month number
- D-Last two digit of the year
- C-First two digit of the year

Zeller’s Rule (Cont.)

- In Zeller’s rule year begins with March and ends with February. Hence, the month Number for march 1, April 2, and son on and January-11 & February-12.
- So when we calculating the day of any day on January (for eg. 2026, month=11, year=25 instead of month=1 & year 26)
- While calculating we drop off every number after decimal point.
- Once we found the answer we divide it by 7 and take the remainder.
- Remainder 0, 1, 2 corresponds to Sunday, Monday and Tuesday and so on.
- If the remainder is negative then add seven.

Example

- Find the day on 26th June 1983
- Here k=26, m is 4, d is 83 and c is 19.
- F= K+ [(13xm-1)/5] + D + [D/4] + [C/4] -2 x C
- =26 + [(13 x 4-1)/5 + 83 +[83/4] + [19/4]-2 x 19
- =26 + [51/5] + 83 + 20.75 + 4.75-38
- =26 +10+83+20+4-38 (We drop the digits after decimal point)
- =105
- 105/7, remainder is 0.
- Hence The day is Sunday.

Exercises

- Part A:

1. Find which of the following years are leap years and which are not:

- 2000, 2100, 2101, 2040, 2004, 1004, 2404, 1404, 4404

2. Given that the key for the current year 2005 is 266315374264. Find the days corresponding to the following dates:

7th Jan, 3rd Dec, 14th Nov, 28th Aug, 26th June, 30th Dec

Exercises (Cont.)

- Part B:

1. Harry has provided us with the details of the birthdays of his families. Find the days on which they were born

(a) Father: 1 December 1953

(b) Mother: 4 January 1957

(c) Grandpa: 9 December 1924

(d) Brother: 26 January 1984

Part C:

1. Given that 31^{st} March 2002 is a Sunday. Find the days on which the following dates will fall:

(a) 31 March 2005 (b) 2 April 1999 (c) 23 March 2004

(d) 7 April 2000 (e) 29^{th} March 2003

General Equations

General Equations

- Let us assume that we have to solve equation
- $ax + b = cx + d$.
- ***x=d-b/a-c***
- Example: Solve $5x + 3 = 4x + 7$
- (a) (b) (c) (d)
- Values of a, b, c, d are 5, 3, 4, 7 respectively
- $x= d-b/a-c$
- $=7-3/5-4$
- $= 4$

General Equations (Cont.)

- Method II
- If the equation is of the form
- $(x + a)(x + b) = (x + c)(x + d)$
- Then the value of x can be found using formula
- ***$x = (cd - ab) / (a + b - c - d)$***
- Solve the equation
- $(x + 7)(x + 12) = (x + 6)(x + 15)$
- $a=7$, $b=12$, $c=6$, $d=15$
- $x= 90-84/ 7 +12 -6-15$
- $= 6/-2 = -3$

Criss-Cross Multiplication for Algebraic identities

- Multiply $(a + b)$ by $(a + 3b)$
- $a + b$
- $a + 3b$
- $a^2 + (3ab + ab) + 3b^2$
- $a^2 + 4ab + 3b^2$

Simultaneous Linear Equations

Simultaneous Linear Equations

- Simultaneous Linear equations have two variables in them. Let us say x and y.
- When these two equation are solved we get the values of the variables x and y.
- For example:
- $2x + 4y = 10$ ________________(1)
- $3x + 2y = 11$________________ (2)

Traditional Method

- $2x + 4y = 10$ ______________(1)
- $3x + 2y = 11$______________ (2)
- The co-efficient of x are 2 and 3 respectively and co-efficient of y are 4 and 2 respectively.
- In order to solve the equation we have to equalize either the co-efficient of x or co-efficient of y.
- This can be done multiplying equation with suitable numbers
- Multiply equation (1) with 3 and equation (2) with 2. The equations are
- $6x + 12y =30$_____________(1)
- $6x + 4y = 22$______________(2)
- Subtract (2) form (1)
- $8y = 8$
- $y= 1$
- Substitute value of y in equation (1)
- $2x=6$, $x=3$

Speed Method

- In traditional method Forming a equation is time consuming process.
- Secondly, equalizing the co-efficient is not always an easy task, if the co-efficient have big numbers or decimal values in it.
- This method not forms new equation, instead it calculates value of x and y from the given equation itself.
- ***x = Numerator / Denominator***
- ***y= Numerator / Denominator***

Example-1 (Calculating value of ‘x’)

- Find the value of the variables x and y for the equations $2x + 4y = 10$ and $3x + 2y = 11$
- The value of Numerator & Denominator
- $2x + 4y = 10$ __________ (1)

  $3x + 2y = 11$ ___________ (2)

The numerator is obtained by cross-multiplying (4 x 11) and subtracting from it cross product of (2 x 10)
i.e. $(4 \times 11)-(2 \times 10) = 24$

$2x + 4y = 10$ __________ (1)

$3x + 2y = 11$ ___________ (2)

The denominator is obtained by cross-multiplying (4 x 3) and subtracting from it cross product of (2 x 2)
i.e. $(4 \times 3)-(2 \times 2) = 8$

Example-1 (Cont.)

- Thus we have obtained the value of x as 3.
- Now we will substitute the value of x in the equation $2x + 4y = 10$
- $2(3) + 4y = 10$
- $6 + 4y = 10$
- $y = 1$

Calculating value of y-Example-2

- Find the value of the variables x and y for the equations $2x + 4y = 10$ and $3x + 2y = 11$
- The value of Numerator & Denominator

$2x + 4y = 10$ ________ (1)

$3x + 2y = 11$ __________ (2)

The numerator is obtained by cross-multiplying (2 x 11) and subtracting from it cross product of (10 x 3)
i.e. (2x11)-(10x3) =-8

$2x + 4y = 10$ ________ (1)

$3x + 2y = 11$ __________ (2)

The denominator is obtained by cross-multiplying (2 x 2) and subtracting from it cross product of (4 x 3)
i.e. (2 x 2)-(4 x 3) =-8

Rule of thumb

- How to decide which variable to solve either x or y?
- If the co-efficient of x are big numbers then calculate the value of x and substitute for y and If the co-efficient of y are big numbers then calculate the value of y and substitute for x.
- This happens because when you calculate the value of x you will be dealing with the y co-efficient twice and hence avoid the big x co-efficients and vice versa.

Specific Case

- Special Rule which states that 'If one is in ration, the other is zero'. This is useful when the co-efficients of either x or y are in certain ratio.
- Example:
- $5x + 8y = 40$
- $10x + 11y = 80$
- Co-efficient are in the ratio 1: 2 (5:10) and constants are also 1:2 (40:80)
- As per the rule variable y is zero

Exercises

1. Solve the first three equations by calculating for x and the next three equations y. Write the answer in the form of (value of x, value of y)

   (a) $4x + 3y = 25$ and $2x + 6y = 26$

   (b) $9x + 10y = 65$ and $8x + 20y = 80$

   (c) $8x + 4y = 6$ and $4x + 6y = 5$

   (d) $7x + 2y = 19$ and $4x + 3y = 22$

   (e) $2x + 9y = 27$ and $4x + 4y = 26$

   (f) $40x + 20y = 400$ and $80x + 10y = 500$

Exercises (Cont.)

- 2. Solve the following equations by detecting a ratio amongst any variable:
- (a) $39x + 64y = 128$
- $63x + 128y = 256$
- (b) $507x + 922y = 1000$

  $2028x + 1634y = 4000$

Exercises (Cont.)

- Solve the following word problem:

(a) A man has one rupee and two rupee coins in his purse. The total number of coins is 52 and the total monetary value of the coin is 81 rupees. Find the number of one rupee and two rupee coins.

(b) The monthly incomes of Tom and Harry are in the ratio of 4:3. Both of them save 800 per month. Their expenditure are in the ratio 3:2. Find the monthly income of tom.

Exercises (Cont.)

- The average of two numbers is 45. Twice the first number equals thrice the second number. Find the numbers.
- There are two classrooms having certain number of students. If ten students are transferred from the first classroom to the second the ratio becomes 5: 9. If ten students are transferred from the second classroom to the first, the ratio becomes 1:1. Find the number of students in each classroom.

Square Roots of Imperfect Squares

Square Roots of Imperfect Squares

1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100

Compare last digit of the square and square Root

1	1 or 9
4	2 or 8
9	3 or 7
6	4 or 6
5	5
0	0

A perfect square will never end with the digits 2,3, 7, 8

Two Important Rule for imperfect Square root

- ***Rule 1: 'After every step, add the quotient to the divisor and get a new divisor'***
- ***Rule 2: 'A new divisor can be multiplied by only that number which is suffixed to it'***
- ***While calculating square roots, divide the number into group of two digits each starting from right to left. If a single digit is left in the extreme left it will be considered a group in itself.***

Example

- Find the square root of 529
- Form two group containing the digits 5 and 29

5 29

- Start from digit 5. Try to find perfect square just smaller to 5. (2x2=4). Put 2 in divisor column and 2 in quotient column.

2	5	29	2

Example (Cont.)

- Write the product (2 x 2 =4) below 5. When 4 is subtracted from 5 the remainder is 1. The remainder 1 cannot divisible by 2. Bring down the next group of digits 29 and make the dividend 129. We add 2 (quotient) to 2 (divisor) and make it 4.

2	5	29	2
	-4		
4	1	29	

Example (Cont.)

- As per rule 2 a new divisor can be multiplied by only that number which is suffix to it. If we take suffix ‘one’ to 4 it will become 41 and (41 x 1 = 41)
- If we suffix ‘two’ to 4 it will become 42 and (42 x 2 =84)
- If we suffix ‘three’ to 4 it will become 42 and (43x 3 =129)
- If we suffix ‘three’ to 4 it will become 43 and the product 129 so obtain will be complete the division. The remainder is zero.

Example (Cont.)

- There fore square root is 23

2	5	29	23
	-4		
4	1	29	
	-1	29	
		0	

Example for imperfect Square Root

- Find the square root of 656
- 31/50=0.62 Answer is 25.62

2	6	56	25
	4		
45	2	56	
	2	25	
50 ( )		31	

Square root of decimal numbers

- ***Rule 1: The grouping of the integral part will be done from right to left, and grouping of the decimal part will be done from left to right***

For Eg. 538.7041---- 5 38. 70 41

0.055696----- 0. 05 56 96

0. 6-------0.6

Rule 2: if there are odd number of places after decimal, make them even by putting a zero. Thus 0.6 will be converted to 0.60

Example

- Find the square root of 538. 7041

2	5 38.70 41	23.21
	4	
43	1 38	
	1 29	
462	970	
	924	
4641	4641	
	4641	
	0	

Estimation of imperfect Square roots

- Find the square root of 70.
- First find a perfect square root less than 70.
- 64 viz. 8
- Divide 70/8=8.75
- Take the average of 2 numbers 8 & 8.75
- 8.37 is the approx square root of 70

Exercises

1. Find the square roots of the following perfect squares.

   (a) 961 (b) 6889 (c) 12321 (d) 4084441

2. Find the square roots of the following imperfect squares

   (a) 700 (b) 1550 (c) 15641

3. Find the square root of the following decimals (up to 2 decimal places)

   (a) 0.4 (b) 150.3432

Cubing Numbers

Cubing Numbers

Formula Method

The cube of any number can be found using formulae:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Eg. Find the cube of 102

We know that 102 is (100 +2). Values of a and b are 100 and 2

$$(100 +2)^2 = (100)^3 + 3 \text{ x } (100)^2 \text{ x } 2 + 3 \text{ x } 100 \text{ x } 2^2 + 2^3$$

$$=1000000 + 3(10000) \text{ x } 2 + 300 \text{ x } 4 + 8$$

$$=1000000 + 60000 + 1200 + 8$$

$$= 1061208$$

Example-2

- Find the cube of 97

We know that 97 is (100 -3). Values of a and b are 100 and 3

$$(100 - 3)^2 = (100)^3 - 3 \times (100)^2 \times 3 + 3 \times 100 \times 3^2 - 3^3$$
$$= 1000000 - 3(10000) \times 3 + 300 \times 9 - 27$$
$$= 1000000 - 90000 + 2700 - 27$$
$$= 912673$$

Cube: The Anurupya Suthra

- The Anurupya Suthra is based on following formulae
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- The Expression of RHS can be broken into two parts as shown below

$a^3 + a^2b + ab^2 + b^3$ ______________(1)

$+ \quad 2a^2b + 2ab^2$ ____________(2)

Equals $a^3 + 3a^2b + 3ab^2 + b^3$

Cube: The Anurupya Suthra (Cont.)

- Note that if we take the first term a^3 and multiply it by b/a we get the second term a^2b and if we multiply the second term a^2b by b/a we get the third term ab^2
- If the third term ab^2 is multiplied by b/a we get the final term that is b^3
- $a^3 \times b/a = a^2b$ $\qquad a^2b \times b/a = ab^2$
- $ab^2 \times b/a = b^3$

Rules

- The value of first row can be obtained by moving in a geometric progression of b/a (left to right) a/b (right to left).
- The values of second row are obtained by doubling the middle terms in the first row.
- The cube is obtained by adding the two rows.
- For the final answer add three zero behind the first term, add two aero behind second term, add one zero behind third term and no zero behind the last term and add all the term.

Example-1

- Find the cube of 52
- $a=5, \; b=2$
- First row
- $5^3 = 125, \quad 2/5 \times 125 = 50, \quad 2/5 \times 50 = 20,$
- $2^3 = 8$
- The second row is obtained by doubling the middle terms of the first row.

Example-1 (Cont.)

$$
\begin{array}{r}
125000 \\
15000 \\
600 \\
+ \quad 8 \\
\hline
140608
\end{array}
$$

125	50	20	8
	100	40	
125	150	60	8

Example-2

- Find the cube of 31
- a=3, b=1 (start from right to left)
- First row
- $1^3 = 1$, $3/1 \times 1 = 3$, $3/1 \times 3 = 9$,
- $3^3 = 27$
- The second row is obtained by doubling the middle terms of the first row.

Example-2 (Cont.)

$$
\begin{array}{lr}
 & 27000 \\
 & 2700 \\
 & 90 \\
+ & 1 \\
\hline
 & 29791
\end{array}
$$

27	9	3	1
	18	6	
27	27	9	1

The rule of Zero

- ***In previous examples we put 3, 2, 1 and no zeros after each step.***
- ***However this rule is applicable only up to the number 999.***
- ***From the number 1000 onwards we double the number of zeros that you used to put in the former case. i.e. 6, 4, 2 and no zero.***

Example-3

- Find the cube of 1001
- a=10, b=01 (start from left to right)
- First row
- $10^3 = 1000$, $1/10 \times 1000 = 100$,
- $1/10 \times 100 = 10$,
- $1/10 \times 10 = 1$
- $1^3 = 1$
- The second row is obtained by doubling the middle terms of the first row.

Example-3 (Cont.)

$$
\begin{array}{r}
1000000000 \\
3000000 \\
3000 \\
+ \quad 1 \\
\hline
1003003001
\end{array}
$$

1000	100	10	1
	200	20	
1000	300	30	1

Exercise

- 1. Find the cube of the following numbers using the formula $(a + b)^3$.
- (a) 105 (b) 41 (c) 54 (d) 23 (e) 34
- 2. Find the cube of the following numbers using the formula $(a - b)^3$.
- (a) 49 (b) 90 (c) 199 (d) 96 (e) 98
- 3. Find the cube of the following using the Anurupya Rule.
- (a) 66 (b) 77 (c) 91 (d) 19
- 4. Find the cube of the following using the Anurupya Rule.
- (a) 43 (b) 72 (c) 101

Base Method of Division

Base Method of Division

- Divide the dividend into two parts. The RHS will contain as many digits as the number of zeros in the base.
- The final answer obtained on the LHS is the quotient and RHS is the remainder.
- **Base**

Divisor

Difference

Dividend

Quotient | Remainder

Example-1

- Divide 23 by 9.
- The divisor is 9, Dividend is 23, the base is 10 and the difference is 1
- Since the base 10 has one zero in it, we divide the dividend in such a way that the RHS has one digit
- We now bring down the first digit of the dividend, viz. 3, as shown in the diagram below

Example-1 (Cont.)

Step 1: Group the dividend into two parts. Number of zeros in the base = number of digits in the RHS

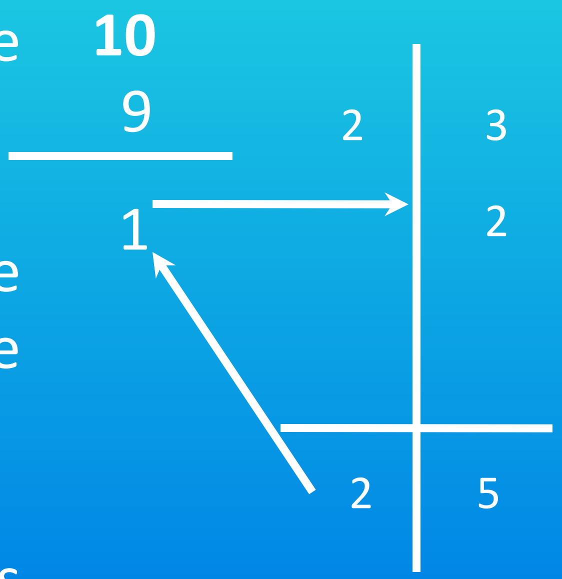
Step 2: Write the difference of base and divisor as shown in diagram.

Step 3: Bring down the first digit of the dividend as shown in diagram.

10	2	3
9		
1		
	2	

Example-1 (Cont.)

Step 4: Multiply 2 with difference 1 and add the answer to the next digit of the dividend.



Step 5: The product of 2 and 1 (the difference) is 2 which is written 3. The sum of 3 and 2 is 5.

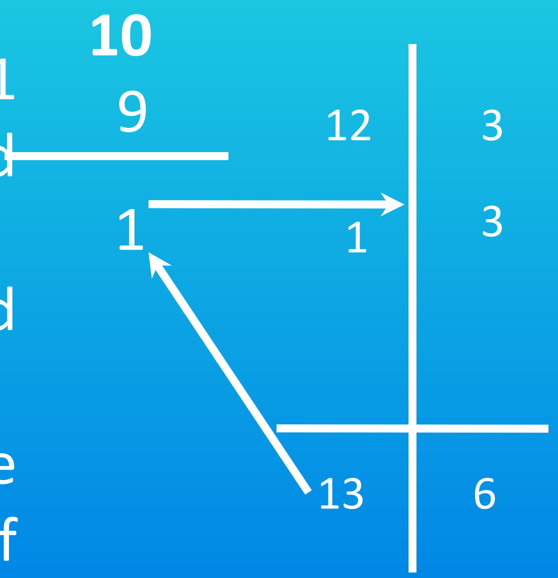
When 23 is divided by 9 the quotient is 2 and the remainder is 5.

Example-2

- Divide 123 by 9.
- The divisor is 9, Dividend is 123, the base is 10 and the difference is 1
- Since the base 10 has one zero in it, we divide the dividend in such a way that the RHS has one digit
- We now bring down the first digit of the dividend, viz. 1, as shown in the diagram below

Example-1 (Cont.)

Now we are left with two digits on the LHS.
We bring down the first digit 1 as it is.
We multiply the 1 with the difference 1 and put the answer below the second digit of the dividend
The second digit of the dividend is 2 and we add 1 to it. The total is 3.
Multiply 3 with difference 1 and write the product below the third digit of dividend
The total is 6. Quotient is 13 and remainder is 6.

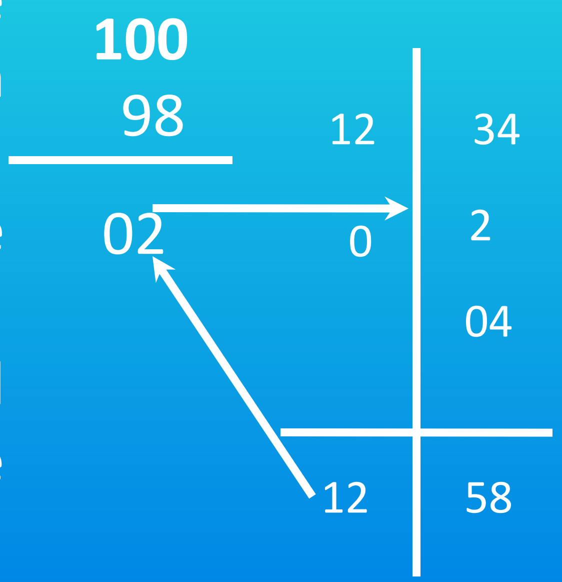


Example-3

- Divide 1234 by 98.
- The divisor is 98, Dividend is 1234, the base is 100 and the difference is 02
- Since the base 10 has two zero in it, we divide the dividend in such a way that the RHS has two digit
- We now bring down the first digit of the dividend, viz. 1, as shown in the diagram below

Example-3 (Cont.)

- ❖ We bring down the first digit 1 as it is.
- ❖ We multiply the 1 with the difference 02. The product 02 is written down from the second digit of dividend.
- ❖ Add 2 plus 0 downwards and get the 2 digit of the quotient.
- ❖ Multiply 2 with 02 and the final answer 04 is written down from the third digit of the dividend.
- ❖ Add up the number on RHS
- ❖ Hence the product is 12 and the remainder is 58.



Exercises

- 1.
- (a) Divide 102 by 74   (b) Divide 10113 by 898
- (c) Divide 102030 by 7999
- (d) Divide 1005 by 99
- 2.
- (a) Divide 431 by 98   (b) Divide 10301 by 97
- (c) Divide 12000 by 889 (d) Divide 111099 by 8987   (e) Divide 30111 by 87

Division (Part Two)

Division (Part Two)-Example-1

- ❖Divide 1296 by 113
- ❖The divisor is related to the base 100 . So split the dividend such a way that RHS has two digits.
- ❖The base 100 and difference is -13 (negative)
- ❖Write down first digit 1 of dividend as it is.
- ❖Multiply 1 with the difference -13 and write down answer as -1 & -3 below second & third digit of the dividend
- ❖Next move to second column of the dividend. Bring down 2 minus 1 is 1
- ❖Multiply 1 with -13 and write down the answer -1 & -3 below the last two digits of the dividend.
- ❖Thus quotient is 11 & remainder is 53

100

113

-(13)

12	96
-1	-3
	-1-3
11	53

Division (Part Two)-Example-2

- ❖Divide 2688 by 120
- ❖The divisor is related to the base 100 . So split the dividend such a way that RHS has two digits.
- ❖The base 100 and difference is -2-0 (negative)
- ❖Write down first digit 2 of dividend as it is.
- ❖Multiply 2 with the difference -2-0 and write down answer as -1 & -3 below second & third digit of the dividend
- ❖Next move to second column of the dividend. Bring down 6 minus -4 is 2
- ❖Multiply 2 with -2-0 and write down the answer -4 & -0 below the last two digits of the dividend.
- ❖Thus quotient is 22 & remainder is 48

100

120	26	88
-2-0	-4	-0
		-4-0
	22	48

Example -3

- Divide 110999 by 1321

1000		
1321	110	9 9 9
-3-2-1	-3-2	-1
	6	4 2
		-12 -8 -4
	1-2 4	0 3 5

Here the quotient is 100 -20 + 4 equals 84. The remainder is 035

Example-4

- Divide 1693 by 131

❖The quotient is 13 and the remainder is -10

❖Reduce the quotient by 1 and subtract the remainder from the divisor.

❖Hence quotient is 13-1=12 & remainder is 131-10=121

❖This is because (for example) 890 by 100

❖Quotient we have 9 & remainder -10 (because 100 multiplied by 9 minus 10 is 890)

❖Quotient =9; Remainder =-10

❖Or

❖Quotient =8; Remainder = 90

100

131

-3-1

	1 6	9 3
	-3	-1
		-9 -3
	1 3	-1 0

Example-5

- Divide 16379 by 1222

- Here the quotient is 14 and remainder is (-700 -30 +1) . Which is equals -720.
- Reduce quotient by 1 and subtract remainder from divisor.
- The final quotient is 13 and remainder is (1222-729) equals 493.

1000

1222

-2-2-2

1 6	3 7 9
-2	-2 -2
	-8 -8 -8
1 4	-7 -3 1

Substitution Method

- Divide 10030 by 827
- Normal Method

1000
827
173

1 0	0 3 0
1	7 3
	1 7 3
1 1	9 3 3

12 / 106

Substitution Method

1000
827
2-33

1 0	0 3 0
2	-3 3
	4 -6 6
1 2	1 0 6

1000-827=173
173= 200-30+3
=2-3+3

Example-2

- Divide 1459 by 242
- Divide 242/2=121
- Divide quotient 12/2
- =6,
- remainder remains same

100
121
-2-1

1 4	5 9
-2	-1
	-4 -2
1 2	0 7

Exercises

PART A

(a) Divide 1389 by 113

(b) Divide 145516 by 1321

(c) Divide 136789 by 12131

(d) Divide 246406 by 112

PART B

(a) Divide 13592 by 114

(b) Divide 25430 by 1230

(c) Divide 15549 by 142

(d) Divide 101156 by 808

PART C

(a) Divide 4949 by 601 (Hint: use 601 x 2 = 1202 as divisor)

(b) Divide 14799 by 492 (Hint: use 492 / 4 = 123 as divisor)

Other Topics

Pythagorean Values

- We know that square of the hypotenuse of a right angled triangle is equal to the sum of the other two sides.
- If the side of right angled triangle are 3, 4 and 5 then the square of 5 equals the square of 3 plus the square of 4.
- We can express square of a number as the sum of two squared numbers. (case 1: Odd Numbers, case 2: even numbers)
- We can express a given number as the difference of two squared numbers.

Case 1: Odd Numbers

- The square of an odd number is also odd. This square is the sum of two consecutive middle digits.
- Example: $3^2 = 9 = 4 + 5$
- $5^2 = 25 = 12 + 13$
- $9^2 = 81 = 40 + 41$

Case 2: Even Numbers

- The square of an even number is even
- We cannot have two middle digits on dividing it by 2.
- Thus divide the even numbers by 2, 4, 8, 16 etc, until we get an odd number.
- Example: One value of the Pythagorean triplet is 6. find the other two values
- We divide 6 by 2 to get odd number 3. (3-4-5)
- Since we have divided the number by 2 we multiply all the values of (3-4-5) by 2 to form (6-8-25)

Expressing given number as a difference of two squares

- We express given number ‘n’ as a product of two numbers ‘a’ and ‘b’ and then express it as
- $n = [(a + b)/2]^2 – [(a-b)/2]^2$
- Express 15 as a difference of two squared numbers.
- $15 = 5 \times 3$
- $= [(5+3)/2]^2-[(5-3)/2]^2 = (8/2)^2 –(2/2)^2$
- $= (4)^2-(1)^2$

Divisibility Test

Divisible by	Condition
2	If the last bit is multiplier of 2 or last bit is exactly divisible by 2
3	Add up the digits. If the sum is divisible by 3 then the number is divisible by 3.
4	If the number formed by last two digits is divisible by 4, then whole number is divisible by 4
5	if the last digit is either 5 or 0
6	Check for divisibility of 2 and 3. if divisible by 2 & 3 then divisible by 6
7	Double the last digit and subtract it from the remaining number. If what is left is divisible by 7, then the original number is also divisible by 7. For e.g. (9+9=18-4=14)
8	If the number formed by last three digits is divisible by 8, then whole number is divisible by 8
9	Add the digits. If sum is divisible by 9 then the whole number is divisible by 9. This holds good for any power of three)

Divisibility Test (Cont.)

Divisible by	Condition
10	If the number ends in 0
11	If the difference between the sum of 1^{st} , 3^{rd} , 5^{th} digits and sum of 2^{nd} , 4^{th} , 6^{th} digits is a multiple of 11 or 0.
12	Check for divisible by 3 and 4
13	Delete the last digit from the given number. Then subtract 9 times the deleted digit from the remaining number. If what is left is divisible by 13, then so is the original number
For divisibility by 14, check for divisibility by 2 and 7. and so on.	

Raising to fourth and Higher power

- $(a + b)^4 = a4 + 4a^3b + 6a^2b^2 + 4ab^2 + b^4$
- We can represent it as

$a^4 + a^3b + a^2b^2 + ab^3 + b^4$ ______________(1)

$+ \underline{3a^3b + 5a^2b^2 + 3ab^3}$ ____________(2)

Which on addition gives $(a + b)^2$

Example

- Find $(21)^4$

16	8	4	2	1
	24	20	2	
16	32	24	8	1

$$
\begin{array}{r}
160000 \\
32000 \\
2400 \\
80 \\
+ \qquad 1 \\
\hline
194481
\end{array}
$$

y-

Co-ordinate Geometry

- Find the equation of a straight line passing the points (7, 5) and (2, -8)
- Two approaches for solving the questions using traditional method.
- First approach is using the formula

$$y-y_1 = y_2-y_1 / x_2-x_1 \ (x-x_1)$$

Traditional Method 1

We have $7m + c = 5$; $2m + c = -8$

We solve the equation simultaneously

$7m + c$ ________________(1)

$- 2m + c$ ________________(2)

Therefore $5m = 13$; $m = 13/5$

Traditional Method 1 (Cont.)

- Substitute the value of in equation-1
- 7 x 13/5 + c = 5;   91/5 +c = 5
- C= 5-91/5; -66/5
- Substitute the value of m and c in the original equation (y = mx + c), we have
- y = 13/5x -66/5
- And therefore equation of the line as 13x-5y = 66

Traditional Method 2

- On substituting the values of (7, 5) and (2, -8), we have
- $y-5 = (-8 -5 / 2 -7) x (x-7)$
- $y-5 = -13/-5 (x-7)$
- $-5(y-5) = -13 (x -7)$
- $-5y +25 = -13 x + 91$
- $13x -5y =66$

Speed Mathematics method

- Put the difference of y co-ordinates as the x co-efficient & the put the difference of the x co-ordinates as y co-efficient
- The given co-ordinates are (7, 5) and (2, 8)
- Therefore x-coefficient is 5-(-8) =13 and
- Our y co-efficient is 7-2 = 5
- We have the answer 13 and 5 with us. Thus LHS is 13x-5y.
- RHS can be easily obtained by substituting the values of x and y of any co-ordinate in the LHS.

Speed Mathematics method (Cont.)

- For example 13 (7) -5 (5) =66
- Thus final answer is 13x- 5y = 66.
- An alternative way of RHS is using the rule:
- ‘Product of the means minus the product of the extremes’
- Therefore we have (7, 5) and (2, 8)
- (5 x 2) – (-8 x 7)
- =66