Assignment #1 CS4/531

Due Date: Mon, Sep. 21, 2015

UNSUPPORTED SOLUTIONS RECEIVE NO CREDIT.

Total points: 42

- You MUST turn in your HW by 9:10am on Sep 21. After that, I will NOT accept your HW. This rule will be **STRICTLY ENFORCED.**
- Please PRINT YOUR LAST NAME, FIRST NAME and UB number on the first page.
- Write solution of each problem on a separate sheet. If a problem has several parts, please write the solution in order. Staple them in the order of problem numbers.
- If your homework solution deviates significantly from these guidelines, TA may deduct up to 20% of the points.

1. (3 pts) We want to prove the function $f(n) = 3n^2 - 4n + 20 = O(n^2)$ by using the definition of O. Namely we need to find $c > 0$ and $n_0 \geq 0$ such that:

$$3n^2 - 4n + 20 \leq cn^2 \text{ for all } n \geq n_0$$

There are many combinations of c and n_0 that will satisfy the definition. Try the following values for c.

1. Pick $c = 4$, determine the smallest n_0 that satisfies the definition.
2. Pick $c = 3$, determine the smallest n_0 that satisfies the definition.
3. Pick $c = 2$, what happens?

2. (10 pts) Using limit test, determine the relationship between the growth rates of the following function pairs. Namely, determine if $f(n) = \Theta(g(n))$, or $f(n) = o(g(n))$, or $f(n) = \omega(g(n))$, or $f(n) = g(n)$. (Recall that lg denotes the log function with base 2).

1. $f(n) = 3n^2 + 4n \lg n + 8$ and $g(n) = n^{5/2} - 4n - 16$.
2. $f(n) = 2^{n/2}$ and $g(n) = (\lg n)^{\lg n}$.
3. $f(n) = (\lg n)^{\lg n}$ and $g(n) = n^{\lg \lg n}$.
4. $f(n) = n^{\lg \lg n}$ and $g(n) = n^3$.
5. $f(n) = 8^{\lg n}$ and $g(n) = n^3$.

3. (7 pts) Let $f(n)$, $g(n)$ and $h(n)$ be asymptotically positive functions. State if each of the following statements is true or false. If the statement is true, prove it. If the statement is false, give a counterexample to disprove it.

1. $f(n) = \Omega((g(n))$ implies $g(n) = \Omega(f(n))$.
2. $f(n) = O(g(n))$ implies $2^{f(n)} = O(2^{g(n)})$.
3. $f(n) = O(h(n))$ and $g(n) = O(h(n))$ implies $f(n) \times g(n) = O(h(n)^2)$.
4. $f(n) = O(h(n))$ and $g(n) = O(h(n))$ implies $\frac{f(n)}{g(n)} = O(1)$.
5. $f(n) + g(n) = \Theta(\max\{f(n), g(n)\})$.
6. $f(n) = \Theta(2f(n))$.
7. $f(n) = \Theta(f(2n))$.

4. (4 pts) Let C_n denote the number of distinct binary trees of n leaves. We have $C_1 = 1,\ C_2 = 1,\ C_3 = 2,\ C_4 = 5 \ldots$. It is known that:

$$C_n = \frac{1}{n}\left(\begin{array}{c} 2n-2 \\ n-1 \end{array}\right) = \frac{(2n-2)!}{n!\ (n-1)!}$$

Using Stirling's formula, prove $C_n = \Theta\left(\frac{4^n}{n^{3/2}}\right)$.

5. (2+3=5 pts) Give asymptotically tight bounds on the following summations (by using the integral method.)

(a) $\sum_{k=1}^{n} k^2 - 8k^{3/2} + 4$

(b) $\sum_{k=1}^{n} 2k \ln k$

6. (2+3 = 5 pts)

(a) The sequence $< a_n >$ is defined by: $a_0 = 1$, $a_1 = 2$ and, for $n \geq 2$, $a_n = 4a_{n-1} - 3a_{n-2}$. The first few terms of the sequence are: 1, 2, 5, 14 ... Find the closed form for a_n.

(b) The sequence $< b_n >$ is defined by: $b_0 = 1$, $b_1 = 2$, $b_2 = 3$ and, for $n \geq 3$, $b_n = 3b_{n-1} - 4b_{n-3}$. The first few terms of the sequence are: 1, 2, 3, 5, 7, 9 ... Find the closed form for b_n.

7. (1 $\times$ 6 +2 =8 pts) Textbook, p107: Problem 4-1

Note: The first six recurrence relations can be solved by using the Master Theorem. The last one cannot and you have to use other method to solve it.