Midori - Implementation and Analysis

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Abstract.
In this paper we discuss the algorithm to implement Midori Cipher perform Integral and Differential Cryptanalysis on it.

Keywords: Midori64 · DDT/LAT analysis · Integral Cryptanalysis · Differential Crytanalysis · MILP Model

1 Introduction

The Midori cipher, introduced by Banik et al. in 2015, is a lightweight block cipher specifically designed for resource-constrained environments such as IoT devices and embedded systems. It emphasizes energy efficiency and low power consumption while ensuring robust security against standard cryptographic attacks. Midori comes in two variants: Midori-64 (64-bit block size, 128-bit key) and Midori-128 (128-bit block size, 128-bit key), both based on a Substitution-Permutation Network (SPN) structure. By employing simple operations like XOR, substitution (S-box), and diffusion via MixColumn, Midori achieves high energy efficiency and compact hardware implementation, making it well-suited for applications in smart cards, wearables, and RFID systems. While its lightweight design balances security and performance, its reduced complexity compared to heavyweight ciphers like AES may make it vulnerable to advanced attack scenarios.

2 Integral Cryptanalysis

Similar to Differential but here we use a Set of Plaintexts known as Delta Set (Δ) instead of just two.

2.1 Basic Knowledge

2.1.1 Definition of Integral Property

- **Constant** $\mathcal{C}$: The constant byte is constant whose value is fixed and unchanged.
- **Active** $\mathcal{A}$: Bytes traverse all values of 0 - 15, which is called active byte.
- **Balance** $\mathcal{B}$: The sum of bytes is 0, which is called balanced byte.
- **Unknown** $\mathcal{U}$: If the nature of a byte is uncertain, it is called an unknown byte.

The linear combination of active bytes $\mathcal{A}$ and constant bytes $\mathcal{C}$ is active byte $\mathcal{A}$. The constant bytes $\mathcal{C}$ and balance bytes $\mathcal{B}$ are balance $\mathcal{B}$ after the linear operation. Independent active bytes $\mathcal{A}$ is active $\mathcal{A}$ after linear combination. The linear combination of two related

active $\mathcal{A}$ is balance $\mathcal{B}$. Balance $\mathcal{B}$ changes into unknown $\mathcal{U}$ through nonlinear bijection. Active $\mathcal{A}$ has the characteristics of penetration to S-box and AddRoundKey. And so on.

2.1.2 Delta Set (Δ)

- We will be taking a Set of 16 Plaintexts called Delta Set (Δ) where 4 bits of each plaintext takes all values from $\{0,1\}^4$ and the rest of the 60 bits have constant values.
- In case of Midori64 Bit Block we get the following:

$$P_0 = (0, c_1, c_2, \ldots, c_{14}, c_{15}),$$
$$P_1 = (1, c_1, c_2, \ldots, c_{14}, c_{15}),$$
$$\vdots$$
$$P_{15} = (15, c_1, c_2, \ldots, c_{14}, c_{15}),$$

  and the Delta Set (Δ) is defined as:

$$\Delta = \{P_0, P_1, \ldots, P_{15}\}.$$

- The Initial Plaintext Set (Δ) in terms of $\mathcal{A}$, $\mathcal{C}$ and $\mathcal{B}$.

$$\Delta = \{\mathcal{A}, \mathcal{C}, \mathcal{C}, \ldots, \mathcal{C}\}.$$

2.1.3 Symbol

The meaning of the symbolic representation below:

$s_j^{(i)}$: The j-th byte of the i-th round input.
$m_j^{(0)}$: The j-th byte of the plaintext.
$c_j^{(i)}$: The j-th byte in the i-th round ciphertext.
$k_j^{(i)}$: The j-th byte in the i-th round key.
R_i : The i-th round.

2.2 Description of Midori

Midori is a lightweight block ciphers which follows the Substitution-Permutation Network (SPN) design approach. The grouping of the Midori algorithm expressed by State Matrix and Each element in the matrix is a Cell. The State Matrix is a 4 × 4 matrix, as shown in Fig. 1. For the two algorithms which have different packet length, each Cell contains different vector lengths. The length of each Cell of the Midori64 is 4-bit. The length of each Cell of the Midori128 is 8-bit. And the unified definition of the left end of each Cell vector is MSB, the right end of each Cell vector is LSB.

$$\begin{bmatrix} S_0 & S_4 & S_8 & S_{12} \\ S_1 & S_5 & S_9 & S_{13} \\ S_2 & S_6 & S_{10} & S_{14} \\ S_3 & S_7 & S_{11} & S_{15} \end{bmatrix}$$

Fig.1 : State matrix of midori.

2.2.1 SubCell ($\mathcal{S}$)

The S-box type of Midori used by 4-bit S-box. The 4-bit S-boxes is shown in TABLE I. The S-boxes of Midori64 correspond to $S_0(x)$

x	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
$S_0(x)$	c	a	d	3	e	b	f	7	8	9	1	5	0	2	4	6

2.2.2 SuffleCell ($\mathcal{SC}$)

The change of the position of each Cell in the replacement process of the State Matrix is shown in Fig. 3.

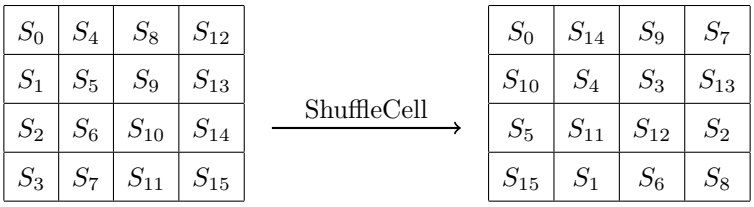


Fig.3 : ShuffleCell.

The inverse of ShuffleCell (SC) is shown in Fig. 4.

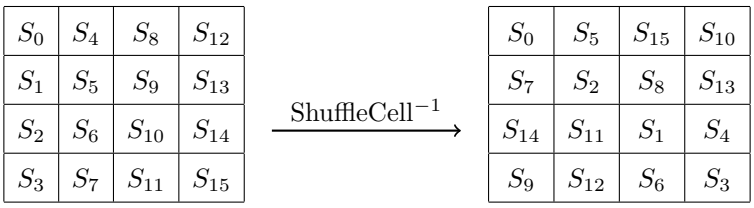


Fig.4 : ShuffleCell^{-1}

2.2.3 MixColumn ($\mathcal{MC}$)

The Cell vector in each column of State Matrix multiplies with a constant 4×4 binary matrix M, where M as given below:

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \tag{1}$$

2.2.4 KeyAdd ($\mathcal{S}$, $\mathcal{RK}_i$)

It is used bit-oriented. First, we should write round key as in (1) according to the rules. Second, the i-th m-bit round key RK_i is XORed to m-bit of State Matrix S.

2.2.5 Key Schedule

The key schedule of Midori whose design is very simple. The whiten key WK in Midori are the initial key K, as in (3)

$$WK = K \tag{3}$$

For the Midori64 algorithm, the initial key is divided into two 64-bit keys, that is K_0 and K_1 as $\mathrm{K} = \mathrm{K}_0 \,||\, \mathrm{K}_1$. The sub-key of Midori64 for round i is $\mathrm{K}_i = K_{imod2} \oplus \mathcal{B}_i$

2.3 6 - Round integral distinguiser of midori

First, constructing a 4-round integral distinguisher in the direction of encryption with an active byte as shown in Fig. 5.

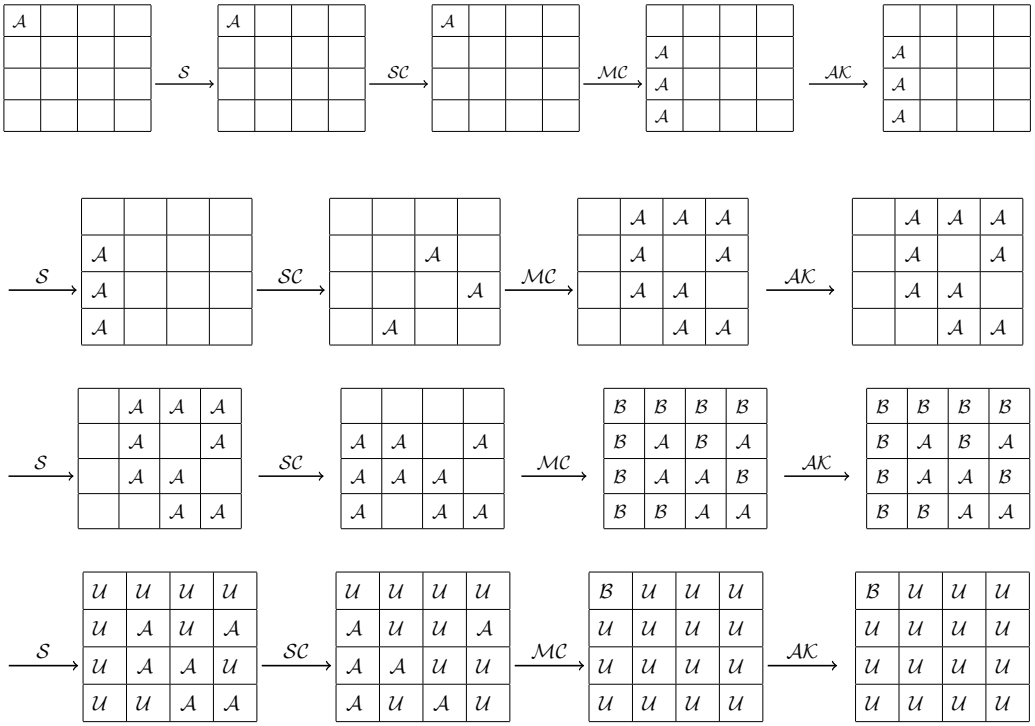


Fig.5 : 4-Round integral distinguishers on Midori.

Second, constructing a 2-round integral distinguisher in the direction of decryption as shown in Fig. 6

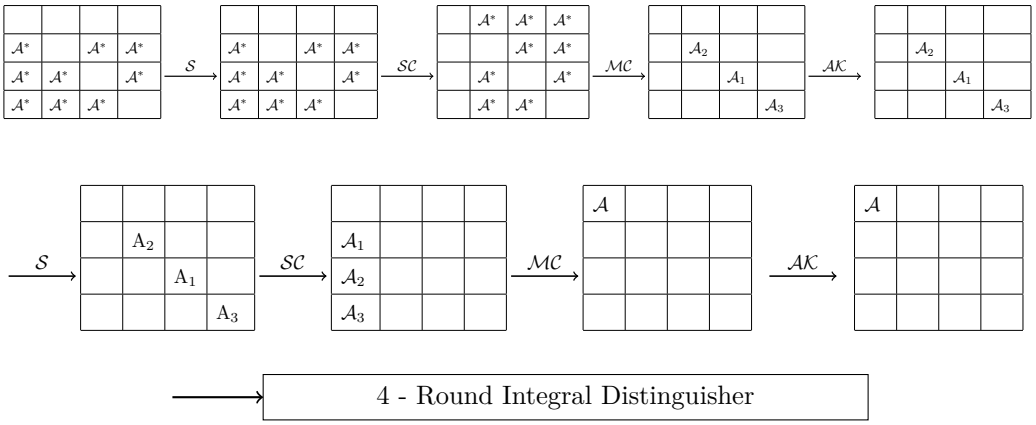


Fig.6 : 6-Round integral distinguishers on Midori.

Lemma 1: Taking nine 4 bits cells in plaintext state $S_1{}^0$, $S_2{}^0$, $S_3{}^0$, $S_6{}^0$, $S_7{}^0$, $S_9{}^0$, $S_{11}{}^0$, $S_{13}{}^0$, $S_{14}{}^0$ as active A that traverse all values, and other 4 bits cells are constants C. After two rounds of encryption, we get 2^{32} sets and in each set the byte of $S_0{}^2$ will traverse all values, and the other bytes are constants C.

Proof. It's just a case of proving when S_1, S_2, S_3 take all values in equation given below. After 2^{12} vectors $(0, S_1, S_2, S_3)^T$ are transformed by MixColumn(MC), 2^8 sets are obtained and the first component of each set takes all values of 0 to 15 and others are constants C.

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} S_1 \oplus S_2 \oplus S_3 \\ S_2 \oplus S_3 \\ S_1 \oplus S_3 \\ S_1 \oplus S_2 \end{bmatrix}$$

Let $S_1 \oplus S_2 \oplus S_3 = t_1$, $S_2 \oplus S_3 = t_2$, $S_1 \oplus S_3 = t_3$ which vectors are pairwise independence. The fourth vector is $S_1 \oplus S_2 = t_1 \oplus t_2$ that is the linear combination of the second component and the third component. Therefore, when S_2 ,S_3 are constants the first vector in the set takes all values of 0 to 15. This collection has a total of 2^8. This property is applied to the first round and second round of transformation, so we get the Lemma 1

3 S-box Analysis of Midori64

The Midori64 cipher is a lightweight block cipher designed for energy-efficient applications. It employs a 4-bit S-box in its substitution layer, which plays a critical role in ensuring the cipher's security. The S-box used in Midori64, along with its Differential Distribution Table (DDT), is analyzed to evaluate its resistance to differential cryptanalysis.

Midori64 S-box

The S-box used in the Midori64 cipher is shown below:

Table 1: Midori64 S-box

x	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
S(x)	C	A	D	3	E	B	F	7	8	9	1	5	0	2	4	6

3.1 Differential Distribution Table (DDT)

The Differential Distribution Table (DDT) of the S-box is presented below. It quantifies the probability of each output difference occurring for a given input difference.

Table 2: Differential Distribution Table (DDT)

in / out	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	16	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
1	-	2	4	-	2	2	2	-	2	-	-	-	-	-	2	-
2	-	4	-	-	4	-	-	-	-	4	-	-	4	-	-	-
3	-	-	-	-	2	-	4	2	2	2	-	-	-	2	-	2
4	-	2	4	2	2	2	-	-	2	-	-	2	-	-	-	-
5	-	2	-	-	2	-	-	4	-	2	4	-	2	-	-	-
6	-	2	-	4	-	-	-	2	2	-	-	-	2	2	-	2
7	-	-	-	2	-	4	2	-	-	-	-	2	-	4	2	-
8	-	2	-	2	2	-	2	-	-	2	-	2	2	-	2	-
9	-	-	4	2	-	2	-	-	2	2	-	2	2	-	-	-
A	-	-	-	-	-	4	-	-	-	-	4	-	-	4	-	4
B	-	-	-	-	2	-	-	2	2	2	-	4	-	2	-	2
C	-	-	4	-	-	2	2	-	2	2	-	-	2	-	2	-
D	-	-	-	2	-	-	2	4	-	-	4	2	-	-	2	-
E	-	2	-	-	-	-	-	2	2	-	-	-	2	2	4	2
F	-	-	-	2	-	-	2	-	-	-	4	2	-	-	2	4

Value Counts in the DDT:
The distribution of values in the Difference Distribution Table (DDT) is as follows:

Value	Count
0	159
2	72
4	24
16	1

3.2 Differential Distribution Table (DDT) Analysis

1. Differential Uniformity

- **Observation:** The maximum value in the DDT is 4, which is the largest count (excluding the first row). This means that the *differential uniformity* is $\Delta_{\max} = 4$.
- **Conclusion:** The differential uniformity of 4 indicates that the cipher is somewhat resistant to differential cryptanalysis, but it is not as strong as S-boxes with $\Delta_{\max} = 4$ (e.g., AES) which occurs only once in row. As in Midori cipher's DDT contains $\Delta_{\max} = 4$ more than one for the row. However, for lightweight ciphers like Midori, this trade-off is acceptable, as it balances security with efficiency.

2. Uniform Distribution of Values

- **Observation:** The values in the DDT are relatively evenly distributed, and there are no particular input-output differential pairs that dominate the table.
- **Conclusion:** This suggests that the S-box does not have significant biases or vulnerabilities that would make it easier for attackers to exploit specific differentials. This balanced distribution enhances the cipher's resistance to differential cryptanalysis.

3. Null Entries in the DDT

- **Observation:** Several entries in the DDT are 0, which means that certain input differences do not produce specific output differences. This occurs in rows like the first and others.
- **Conclusion:** Null entries limit an attacker's ability to craft specific differential trails, as they indicate that some differentials cannot happen. This property adds an extra layer of security to the S-box.

4. Symmetry

- **Observation:** The DDT is symmetric, i.e., if $D(x, y)$ is an entry, then $D(y, x)$ is also an entry.
- **Conclusion:** This symmetry is expected for well-designed S-boxes and confirms the correctness of the DDT, ensuring that the cryptographic function behaves consistently in both directions (input to output and vice versa).

5. Compliance with DDT Properties

The following propositions hold true for the DDT of Midori64:

- **Proposition 1:** Every entry in the DDT is a non-negative even integer between 0 and 2^n.
- **Proposition 2:** The top-left entry of the DDT is 2^n.
- **Proposition 3:** The first row consists of all zeros except the first entry, which is 2^n.
- **Proposition 4:** The first column consists of all zeros except the first entry, which is 2^n.
- **Proposition 5:** The sum of the entries of each row is 2^n.
- **Proposition 6:** If the S-box is bijective, then every row and column of the DDT adds up to 2^n.

The DDT of the Midori64 S-box adheres to these properties, which confirms that the S-box is well-structured and complies with the expected cryptographic behaviors.

Comparative Analysis with Other Ciphers

Table 3: Comparison of DDT Properties Across Ciphers

Cipher Name	Number of 0's	Number of 2's	Number of 4's	Number of 16's	Differential Uniformity
Pyjamask-128	-	-	-	-	-
Square	-	-	-	-	-
PRESENT	-	-	-	-	-
PRINCE	-	-	-	-	-
KLEIN	-	-	-	-	-
LED	-	-	-	-	-
BACKSHEESH	-	-	-	-	-
Midori	159	72	24	1	4
GIFT	-	-	-	-	-
PRINTcipher	-	-	-	-	-
Rectangle	-	-	-	-	-

Conclusion

The Midori64 S-box offers a reasonable trade-off between security and efficiency, making it suitable for lightweight cryptographic applications. While it does not have the optimal differential uniformity of $\Delta_{\max} = 2$ seen in AES, it remains secure against differential cryptanalysis and other attacks due to its balanced distribution and structural properties.

3.3 Linear Approximation Table (LAT) Analysis of the Midori64 S-box

The Linear Approximation Table (LAT) for the S-box is a key component for analyzing the resistance of the S-box to linear cryptanalysis. It quantifies how well the S-box behaves under linear approximations, with lower values indicating better resistance. The LAT for the Midori64 S-box is shown below:

Value Counts in the Linear Approximation Table (LAT) This table represents the counts of each value observed in the Linear Approximation Table (LAT) for the Midori64 S-box.

Table 4: Linear Approximation Table (LAT) for the Midori64 S-box

	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	8	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
1	.	2	4	2	-2	.	2	.	-2	.	2	.	4	-2	.	-2
2	.	4	.	.	4	.	.	.	-4	.	.	.	.	4	.	.
3	.	2	.	2	-2	.	2	4	2	-4	-2	.	.	2	.	2
4	.	-2	4	-2	2	.	-2	.	-2	-4	-2	.	.	-2	.	2
5	.	.	.	.	.	.	.	.	.	.	-4	-4	.	.	4	-4
6	.	2	.	2	-2	.	2	-4	-2	.	-2	.	-4	-2	.	2
7	.	.	.	4	.	.	-4	.	.	.	.	-4	.	.	-4	.
8	.	-2	-4	2	-2	.	-2	.	-4	-2	.	2	2	.	2	.
9	.	.	.	-4	-4	.	.	.	-2	2	-2	-2	2	2	-2	2
a	.	2	.	-2	-2	-4	-2	.	.	-2	4	-2	-2	.	2	.
b	.	.	.	.	.	-4	.	-4	2	-2	-2	2	2	2	-2	-2
c	.	4	.	.	.	.	-4	.	2	2	-2	2	2	-2	2	2
d	.	-2	4	2	-2	.	-2	.	.	2	.	2	-2	4	2	.
e	.	.	.	.	.	4	.	-4	2	-2	2	-2	2	2	2	2
f	.	-2	.	2	2	-4	2	.	.	2	.	-2	2	.	2	4

Value	**Count**
-4	21
-2	44
0	123
2	52
4	15
8	1

LAT Analysis

1. Linear Bias and Distribution of Values

- **Observation:** The values in the LAT table are spread between positive and negative numbers. A few entries show larger values such as ± 4 and ± 2, which represent the linear bias of the approximation. Most values are closer to 0, indicating weaker linear approximations.
- **Conclusion:** The distribution of values suggests that the S-box of Midori64 is somewhat resistant to linear cryptanalysis. Ideally, the values should be close to 0 for most of the entries, which is the case here, though some larger biases may still indicate potential vulnerabilities.

2. Symmetry in LAT

- **Observation:** The LAT is symmetric, i.e., the value at position (i, j) is equal to the value at position (j, i), which is expected from a well-designed S-box.
- **Conclusion:** The symmetry of the LAT confirms that the Midori64 S-box behaves consistently in both directions (input to output and vice versa), which is a desirable property for cryptographic functions and indicates correctness in design.

3. Even Integer Entries in LAT

- **Observation:** All the entries in the LAT are even integers, as expected from a bijective S-box. This property holds true for the Midori64 S-box, which confirms that the S-box is a bijection.
- **Conclusion:** The fact that all LAT entries are even integers, and the first row and column (except the upper-left entry) consist of zeros, is a strong indicator that the S-box is bijective. This makes the S-box more secure against linear cryptanalysis.

4. Compliance with LAT Properties

- **Observation:** The first row and the first column are zeros, except for the upper-left entry, which is 2^{n-1}, indicating that the S-box follows the LAT properties of bijective S-boxes.
- **Conclusion:** The Midori64 S-box adheres to the properties expected from a bijective S-box. This is a positive feature, enhancing its cryptographic strength by ensuring that no biases are introduced through linear approximations.

Comparison with Other Ciphers

Table 5: Comparison of LAT Properties Across Ciphers

Cipher Name	Number of -4's	Number of -2's	Number of 0's	Number of 2's	Number of 4's	Number of 8's
Pyjamask-128	-	-	-	-	-	-
Square	-	-	-	-	-	-
PRESENT	-	-	-	-	-	-
PRINCE	-	-	-	-	-	-
KLEIN	-	-	-	-	-	-
LED	-	-	-	-	-	-
BACKSHEESH	-	-	-	-	-	-
Midori	-21	44	123	52	15	1
PUFFIN	-	-	-	-	-	-
GIFT	-	-	-	-	-	-
PRINTcipher	-	-	-	-	-	-
Rectangle	-	-	-	-	-	-

Conclusion

The Midori64 S-box strikes a balance between security and efficiency, making it suitable for lightweight cryptographic applications. Although its LAT does show some biases, the values are relatively small, and the overall resistance to linear cryptanalysis is reasonable. The bijective nature of the S-box, along with its symmetry and even integer entries, further contributes to its security. While the Midori64 S-box may not be as optimal as the AES

S-box in terms of linear approximation resistance, it offers a good trade-off between security and computational efficiency, which is crucial for lightweight cryptography.

Acknowledgment

In this term paper, we have not implemented the MILP (Mixed Integer Linear Programming) model for differential trail generation. Instead, we have referred directly to the results presented in the paper titled "MILP-Based Differential Cryptanalysis on Round-Reduced Midori64" by Jiageng Cheng, Xinyu Sun, and Meiqin Wang. For further details on the MILP trails, please see the original paper available at MILP-Based Differential Cryptanalysis on Round-Reduced Midori64. [1]

4 Differential Attack on 11-Round Midori64

4.1 The Property of Probability for Round Function

Property 1: Consider four cells of the intermediate state of SC with any input difference and any output difference. However, we want one cell of these four with zero difference after the MC operation. For example, let $X\{3,6,9,12\}$ denote the positions (3, 6, 9, 12) before the SC operation, and $Y\{3,6,9,12\}$, $Z\{8,9,10,11\}$, $W\{8,9,10,11\}$ denote the corresponding positions after SC, SFC, and MC operations, respectively. Let $\Delta w_{11} = 0$, then $\Delta z_8 = \Delta z_9 \oplus \Delta z_{10}$ with the probability of $\frac{1}{16} = 2^{-4}$. Let $P((?,?,?,?) \rightarrow (?,?,?,0))$ denote $P(\mathrm{SC}(?,?,?,?) \rightarrow \mathrm{MC}(?,?,?,?) = (?,?,?,0))$. So, $P((?,?,?,?) \rightarrow (?,?,?,0)) = 2^{-4}$. Since $? \in \{0,1,\ldots,15\}$ and $* \in \{1,2,\ldots,15\}$, we can obtain $P((?,?,?,?) \rightarrow (?,?,*,0)) = \frac{15}{16} \cdot \frac{1}{16} \approx 2^{-4.09}$. Similarly, $P((?,?,?,?) \rightarrow (?,*,*,0)) \approx 2^{-4.19}$, and $P((?,?,?,?) \rightarrow (*,*,*,0)) \approx 2^{-4.28}$.

Property 2: Consider four cells of the intermediate state of SC with any input difference and any output difference. However, we want no less than one cell of these four with non-zero difference after the MC operation. We can obtain $P((?,?,?,?) \rightarrow (?,?,?,*)) = \frac{15}{16} \approx 2^{-0.09}$. Similarly, $P((?,?,?,?) \rightarrow (?,?,*,*)) \approx 2^{-0.19}$, and $P((?,?,?,?) \rightarrow (?,*,*,*)) \approx 2^{-0.28}$.

Property 3: Consider four cells of SC with two arbitrary input differences and two non-zero differences; then, we want to get two zero differences after the MC operation. We can obtain $P((?,?,*,*) \rightarrow (*,*,0,0)) = \frac{1}{16} \cdot \frac{1}{16} = 2^{-8}$ and $P((?,?,*,*) \rightarrow (?,?,0,0)) \approx 2^{-7.81}$.

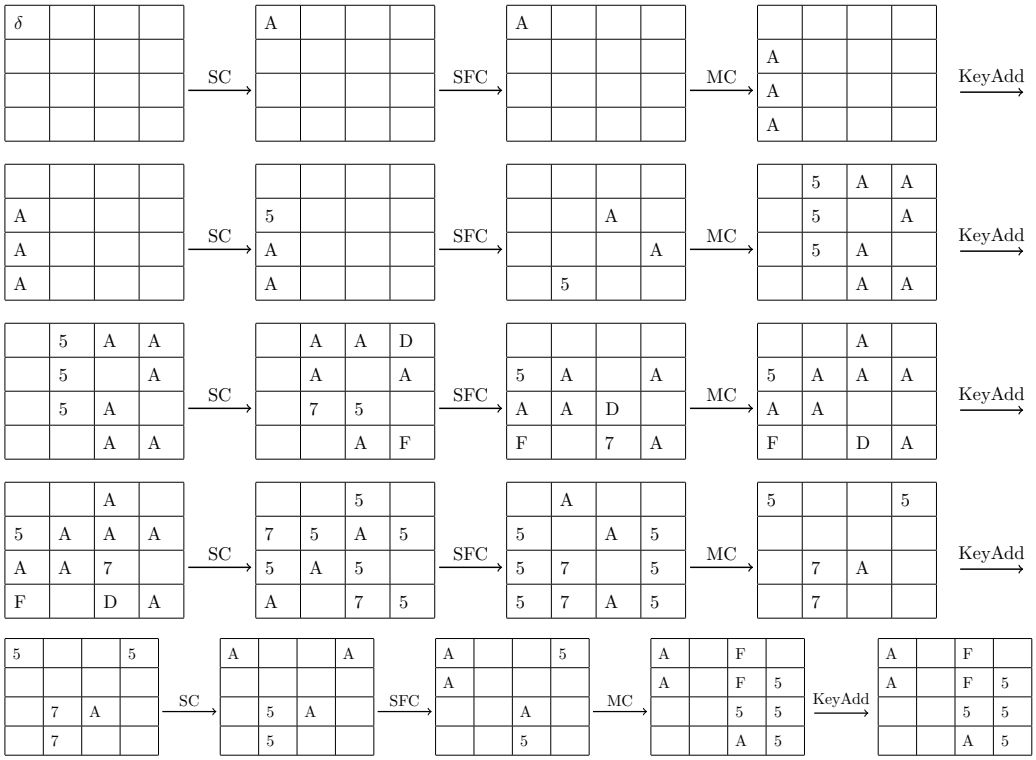
Property 4: If there are three cells with any or any non-zero input differences of SC and the same non-zero output differences of SC, we can obtain $P((?,?,?,0) \rightarrow (11,0,0,0)) = \frac{15}{16} \cdot \frac{1}{16} \cdot \frac{1}{16} \approx 2^{-8.09}$ and $P((*,*,*,0) \rightarrow (12,0,0,0)) \approx 2^{-7.81}$.

Table 6: 5-round differential path of Midori64 with probabilities 2^{-52} and 2^{-58}.

Input Round	Input Differential-1	Probability	Input Differential-2	Probability
1	α000 0000 00β0 0000	1	δ000 0000 0000 0000	1
2	2200 0000 0000 0000	2^{-4}	0AAA 0000 0000 0000	2^{-2}
3	0440 1110 0000 0000	2^{-8}	0000 5550 A0AA AA0A	2^{-8}
4	2202 0202 0202 2202	2^{-20}	05AF 0AA0 AA7D 0A0A	2^{-26}
5	0400 0011 0001 1100	2^{-40}	5000 0077 00A0 5000	2^{-48}
6	0000 0022 0032 2200	2^{-52}	AA00 0000 FF5A 0555	2^{-58}

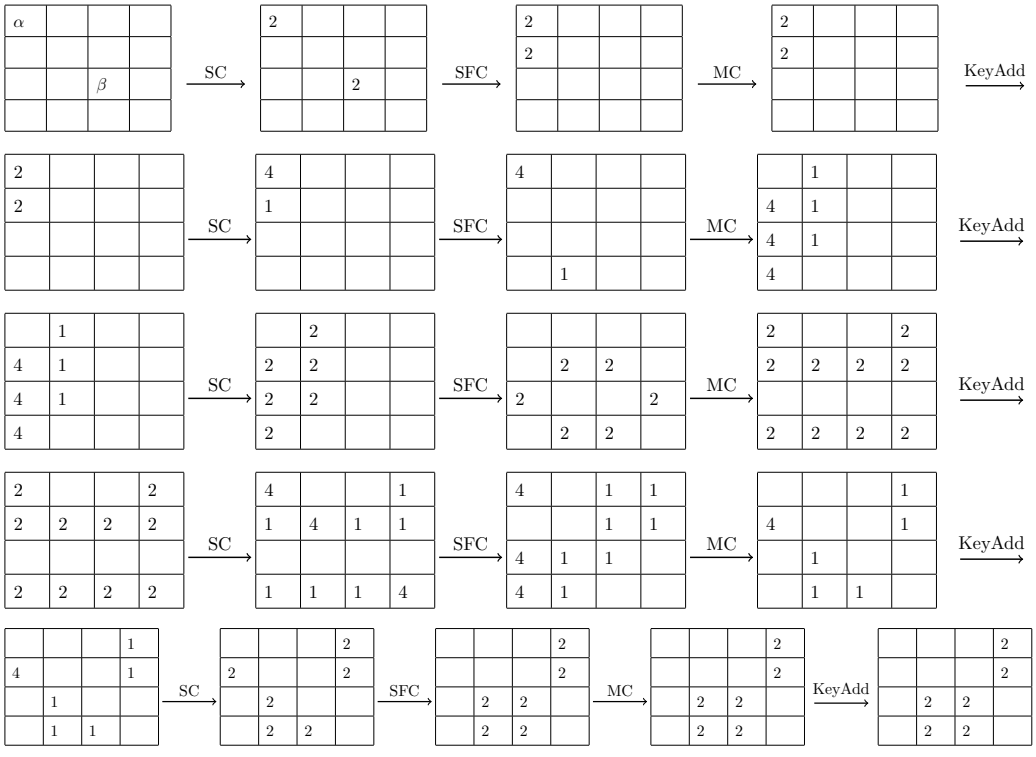
$\alpha, \beta \in \{1,4,9,C\}$ and $\delta \in \{5,A,D,F\}$.

FIGURE 1. A 5-round differential path with probability of 2^{-58}



where $\delta \in \{5, A, D, F\}$

FIGURE 2. Another 5-round differential path with probability of 2^{-52}



4.2 Attack on 11-Round Midori64

Using the 5-round differential characteristic

$$(\delta, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$
$$\downarrow$$
$$(A, A, 0, 0, 0, 0, 0, 0, F, F, 5, A, 0, 5, 5, 5)$$

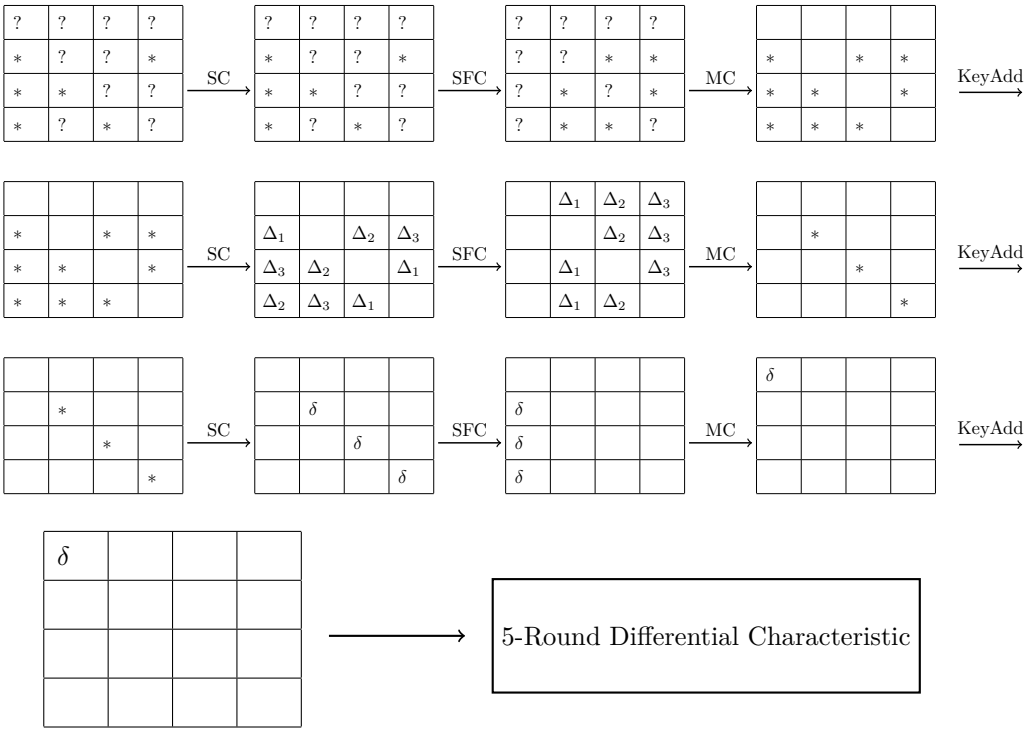
with the probability of 2^{-58} in Table 1 and Figure 1, we could launch a key-recovery attack against the 11-round Midori64. We choose the differential-2 rather than the differential-1 because the former is more effective.

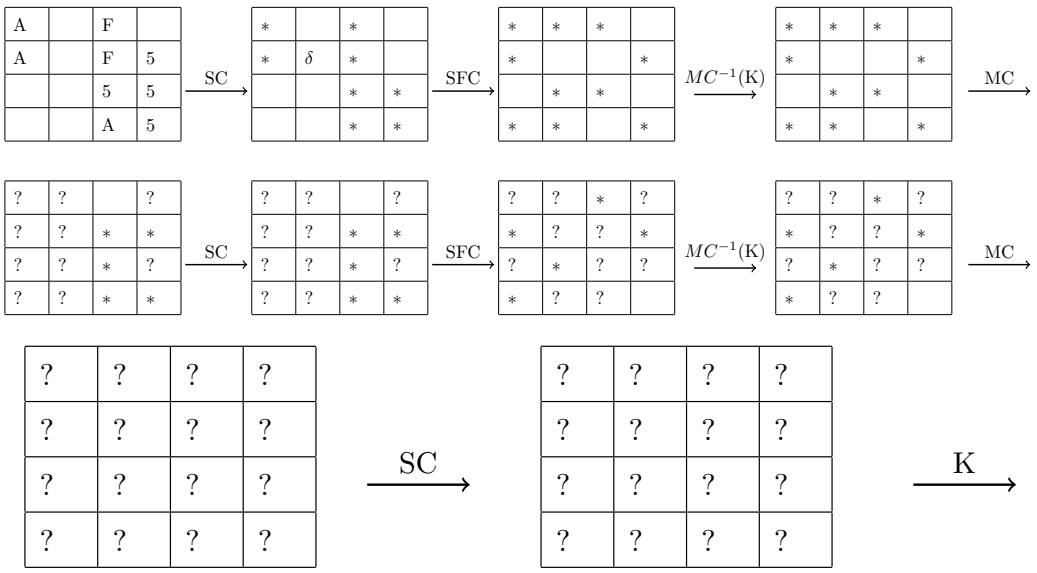
Then, add three rounds at its beginning and the end, respectively, to attack the 11-round reduced Midori64, as shown in Figure 2. The attack procedures are as follows:

4.2.1 Data Collection

Since the differences of plaintexts are all uncertain bits, plaintexts cannot be classified by inactive bits. Choose any 2^n plaintexts and form approximately 2^{2n-1} plaintext pairs. Encrypt these plaintext pairs to state W_1 and use the difference $\Delta W_1\{0,1,2,3\} = \{0,*,*,*\}$ to filter pairs. By Property 1, this provides a filtering probability of $2^{-4.28}$, and there are approximately $2^{2n-5.28}$ pairs left. Similarly, keep only the pairs such that $\Delta W_1\{4,5,6,7\} = \{0,0,*,*\}, \Delta W_1\{8,9,10,11\} = \{0,*,0,*\}$ and $\Delta W_1\{12,13,14,15\} = \{0,*,*,0\}$. By Property 3, the probability of these three cases is 2^{-8} and there are $2^{2n-29.28}$ pairs left. Therefore, in the data collection phase, the remaining number jof the plaintext/cipertext paires is approximately $2^{2n-29.28}$ only by the path choosing without guessing the key.

11 round differential





4.2.2 Key Recovery

1. **Guess 12 bits $K_0\{1, 11, 14\} \oplus \alpha_0\{1, 11, 14\}$:** Partially encrypt these plaintext pairs. The middle values of the correct pairs should satisfy

$$\Delta X_2\{1, 4, 11, 14\} = \{*, 0, *, *\}, \quad \Delta Y_2\{1, 4, 11, 14\} = \{\Delta_1, 0, \Delta_1, \Delta_1\}.$$

   the pairs can be filtered with a probability of $2^{-7.81}$ (Property 4), leaving approximately $2^{2n-37.09}$ pairs. Similarly, guess $K_0\{2, 7, 13\} \oplus \alpha_0\{2, 7, 13\}$, and the correct pairs should satisfy

$$\Delta Y_2\{2, 7, 8, 13\} = \{\Delta_2, \Delta_2, 0, \Delta_2\}.$$

   Then, guess $K_0\{3, 6, 9\} \oplus \alpha_0\{3, 6, 9\}$, and the correct pairs should satisfy

$$\Delta Y_2\{3, 6, 9, 12\} = \{\Delta_3, \Delta_3, \Delta_3, 0\}.$$

   After these steps, there are approximately $2^{2n-52.71}$ pairs left.

2. **Guess 12 bits $K_1\{5, 10, 15\} \oplus \alpha_1\{5, 10, 15\}$:** For each remaining pair, guess the 12 bits one by one and partially encrypt these pairs. The correct pairs should satisfy

$$\Delta Y_3\{5, 10, 15\} = \{\delta, \delta, \delta\}, \quad \delta \in \{5, A, D, F\}.$$

   This step has a filtering probability of $2^{-7.81} \cdot \frac{4}{15} \approx 2^{-9.72}$ (Property 4), leaving approximately $2^{2n-62.43}$ pairs.

3. **Guess $MC^{-1}(K_0 \oplus \alpha_{10})\{0, 4, 5, 8, 10, 12, 15\}$:** Decrypt the remaining pairs to state W_{10}. Use the following conditions to filter pairs:

$$\Delta W_{10}\{0, 1, 2, 3\} = \{?, *, ?, *\}, \quad \Delta W_{10}\{4, 5, 6, 7\} = \{?, ?, *, ?\},$$

$$\Delta W_{10}\{8, 9, 10, 11\} = \{*, ?, ?, ?\}, \quad \Delta W_{10}\{12, 13, 14, 15\} = \{?, *, ?, 0\}.$$

   The filtering probabilities are $2^{-0.19}$, $2^{-0.09}$, $2^{-0.09}$ (Property 2), and $2^{-4.09}$ (Property 1), respectively. After this step, there are approximately $2^{2n-66.89}$ pairs left.

4. **Guess $MC^{-1}(K_1 \oplus \alpha_9)\{0,1,2,3,4,6,7,8,9,11,12,13,14\}$:** Decrypt the remaining pairs to state W_9 and apply the following filtering conditions:

$$\Delta W_9\{0,1,2,3\} = \{*,*,0,*\}, \quad \Delta W_9\{4,5,6,7\} = \{*,0,*,*\},$$

$$\Delta W_9\{8,9,10,11\} = \{*,0,0,0\}, \quad \Delta W_9\{12,13,14,15\} = \{0,*,0,*\}.$$

   The filtering probabilities are $2^{-4.28}$, $2^{-4.28}$ (Property 1), $2^{-7.81}$ (Property 4), and 2^{-8} (Property 3), respectively. After this step, there are approximately $2^{2n-91.26}$ pairs left.

5. **Final Decryption:** Decrypt the remaining pairs to state X_9. Use the condition

$$\Delta X_9\{0,1,8,9,10,11,13,14,15\} = \{A,A,F,F,5,A,5,5,5\}$$

   to filter pairs one by one. This final step has a total filtering probability of $2^{-35.19}$. After this step, there are approximately $2^{2n-126.45}$ pairs left.

4.2.3 Complexity Analysis

Data Complexity: To distinguish the correct key from the wrong ones, choose $n = 61.2$. For a random key, there are approximately $2^{2\times 61.2-126.45} \approx 2^{-4.05}$ pairs left. However, for the correct key, there are $2^{2\times 61.2-62.43-58} \approx 4$ pairs left as the probability of the 5-round differential Path is 2^{-58}. Thus, the data complexity is $2^{61.2}$ chosen plaintexts.

Time Complexity:

1. There are $2^{2n-29.28} = 2^{93.12}$ pairs left after the phase of data collection. Guess 12 bits $K_0\{1,11,14\} \oplus \alpha_0\{1,11,14\}$, then partially encrypt these plaintext pairs for one round. The time complexity is

$$2^{93.12} \times 2 \times 2^{12} \times \frac{3}{16} \times \frac{1}{11} \approx 2^{100.25}$$

   11-round encryptions, and the number of remaining pairs is $2^{85.31}$.

   Similarly, guess $K_0\{2,7,13\} \oplus \alpha_0\{2,7,13\}$, and the time complexity is

$$2^{85.31} \times 2 \times 2^{12} \times \frac{3}{16} \times \frac{1}{11} \approx 2^{92.44}$$

   11-round encryptions, and the number of remaining pairs is $2^{77.5}$.

   Then, guess $K_0\{3,6,9\} \oplus \alpha_0\{3,6,9\}$, and the time complexity is

$$2^{84.63}$$

   11-round encryptions, and the number of remaining pairs is $2^{69.69}$.

2. For every remaining pair, guess 12 bits $K_1\{5,10,15\} \oplus \alpha_1\{5,10,15\}$, and the time complexity is

$$2^{69.69} \times 2 \times 2^{12} \times \frac{3}{16} \times \frac{1}{11} \approx 2^{76.82}$$

   11-round encryptions, and the number of remaining pairs is $2^{59.97}$.

3. Guess $MC^{-1}(K_0 \oplus \alpha_{10})\{1,2,3,6,7,9,11,13,14\}$, and for the whole round, the time complexity is

$$2^{59.97} \times 2 \times 2^{28} \times \frac{1}{11} \approx 2^{85.51}$$

   11-round encryptions, and the number of remaining pairs is $2^{55.51}$.

4. Similarly, guess $MC^{-1}(K_1 \oplus \alpha_9)\{1, 6, 8\}$, and the time complexity is

$$2^{55.51} \times 2 \times 2^{12} \times \frac{3}{16} \times \frac{1}{11} \approx 2^{62.64}$$

11-round encryptions, and the number of remaining pairs is $2^{47.7}$.

Guess $MC^{-1}(K_1 \oplus \alpha_9)\{3, 4, 13\}$, and the time complexity is

$$2^{47.7} \times 2 \times 2^{12} \times \frac{4}{16} \times \frac{1}{11} \approx 2^{55.24}$$

11-round encryptions, and the number of remaining pairs is $2^{39.7}$.

Guess $MC^{-1}(K_1 \oplus \alpha_9)\{0, 7, 9, 14\}$, and the time complexity is

$$2^{39.7} \times 2 \times 2^{16} \times \frac{4}{16} \times \frac{1}{11} \approx 2^{51.24}$$

11-round encryptions, and the number of remaining pairs is $2^{35.42}$.

Guess $MC^{-1}(K_1 \oplus \alpha_9)\{2, 11, 12\}$, and the time complexity is

$$2^{35.42} \times 2 \times 2^{12} \times \frac{3}{16} \times \frac{1}{11} \approx 2^{42.55}$$

11-round encryptions, and the number of remaining pairs is $2^{31.14}$.

5. Finally, the time complexity is

$$2^{31.14} \times 2 \times \frac{9}{16} \times \frac{1}{11} \approx 2^{27.85}$$

11-round encryptions.

Thus, the total time complexity is

$$2^{100.26}$$

11-round encryptions.

4.3 Complexity Analysis of another differential path with probability of 2^{-52}

Similarly, add 3 rounds at the beginning and at the end of the differential path with a probability of 2^{-52} to attack the 11-round reduced Midori64. It is easy to get the probability of $2^{-56.18}$ for the top 3 rounds.

Thus, we choose $n = 55.6$. For a random key, there are approximately

$$2^{2\times55.6-1-56.18-64} \approx 2^{-10}$$

pairs left. However, for the correct key, there are

$$2^{2\times55.6-1-56.18-52} \approx 4$$

pairs left, as the probability of the 5-round differential path is 2^{-52}.

Therefore, the data complexity is $2^{55.6}$ chosen plaintexts, and the time complexity is $2^{109.35}$ 11-round encryptions, correspondingly.

5 MILP Model

Introduction

The Midori64 block cipher is a lightweight substitution-permutation network (SPN) cipher, designed with a focus on simplicity and security. The MILP model for Midori64 aims to study and optimize the differential characteristics of the cipher. This model captures the linear constraints for the cipher's round operations, such as the non-linear S-box function, cell shuffling, and column mixing, to calculate the minimal number of active S-boxes. The model uses the concept of convex hulls to describe the relationship between the input and output differences, which simplifies the analysis and computation.

The MILP model can be divided into three primary operations, each described by its set of constraints.

5.1 Substitution Cell Operation

The substitution cell involves a transformation from the input variables x_i^j to the output variables y_i^j through a substitution box (S-box a_i^j). The key components are as follows:

- x_i^j: Binary input variables where i is the round number and j is the variable index (ranging from 0 to 63).
- a_i^j: Binary variable indicating whether the S-box j is active in round i.
- $p_i^j{}_k$: Probability binary variables for each S-box and each input-output pair, where k can take values 0 or 1.
- y_i^j: Binary output variables where i is the round number and j is the variable index (ranging from 0 to 63).

The transformation is done through a set of constraints for each nibble (group of 4 input/output variables) of the substitution.

Constraints

For each nibble of input $x_{i,0}, x_{i,1}, x_{i,2}, x_{i,3}$ and output $y_{i,0}, y_{i,1}, y_{i,2}, y_{i,3}$, the following constraints are applied:

$$
\begin{aligned}
x_{i,0} - a_{i,0} &\leq 0 \\
x_{i,1} - a_{i,0} &\leq 0 \\
x_{i,2} - a_{i,0} &\leq 0 \\
x_{i,3} - a_{i,0} &\leq 0 \\
x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3} - a_{i,0} &\geq 0 \\
4(x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3}) - (y_{i,0} + y_{i,1} + y_{i,2} + y_{i,3}) &\geq 0 \\
4(y_{i,0} + y_{i,1} + y_{i,2} + y_{i,3}) - (x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3}) &\geq 0
\end{aligned}
$$

- $x_{i,j} - a_{i,j} \leq 0$: This constraint ensures that the values of $x_{i,j}$ do not exceed the threshold represented by $a_{i,j}$. It limits the possible states of $x_{i,j}$, preventing values of $x_{i,j}$ from surpassing the allowable range determined by $a_{i,j}$.
- $x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3} - a_{i,0} \geq 0$: This constraint ensures that the sum of the elements in the first nibble of x_i (i.e., $x_{i,0}, x_{i,1}, x_{i,2}, x_{i,3}$) is greater than or equal to the threshold value $a_{i,0}$. It eliminates combinations where the total value of x_i is too small, ensuring it meets the minimum threshold.

- $4(x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3}) - (y_{i,0} + y_{i,1} + y_{i,2} + y_{i,3}) \geq 0$: This constraint ensures that four times the sum of the values in x_i is always greater than or equal to the sum of the corresponding values in y_i. It removes invalid combinations where the values of y_i could exceed the scaled value of x_i, maintaining a consistent relationship between the variables.
- $4(y_{i,0} + y_{i,1} + y_{i,2} + y_{i,3}) - (x_{i,0} + x_{i,1} + x_{i,2} + x_{i,3}) \geq 0$: This constraint does the reverse of the previous one, ensuring that the scaled sum of the values in y_i is greater than or equal to the sum of the corresponding values in x_i. It eliminates combinations where the x_i values could overpower the y_i values, ensuring balance and validity between the two.

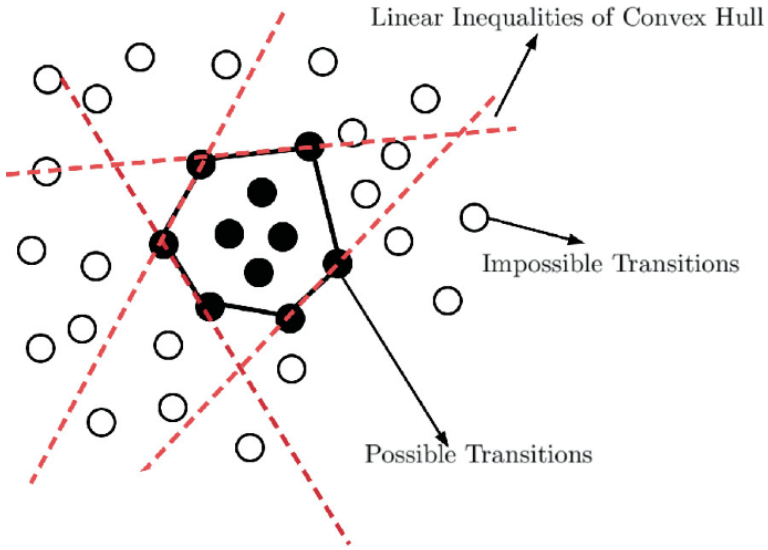
Differential Distribution Table (DDT) and Convex Hull

The Differential Distribution Table (DDT) is used to describe the probability distribution of differences in the output (delta-out) for given differences in the input (delta-in). This table helps to identify impossible transition from form input to `Sbox` and it's output from `Sbox`.

Convex Hull and its Role

The concept of a convex hull plays a significant role in optimizing the inequalities generated by the DDT. A convex hull is the smallest convex set that contains a set of points, and it helps in defining the feasible region for the inequalities.

By using the convex hull, we can construct a set of inequalities that represent the feasible region for the DDT. These inequalities will help in constraining the possible output patterns, thus making help to find differential possible pattern by eliminating impossible patterns.



Constructing Parameters and Inequalities

- We first generate the DDT, which describes how changes in the input affect the output of the S-box.
- Next, we identify the possible and impossible points based on the DDT. The possible points represent valid input-output pairs with non-zero probabilities.

- Using the convex hull, we generate inequalities that describe the feasible region of the DDT.
- Each inequality defines a region of valid patterns that satisfy the constraints imposed by the SubCell transformation.

Generating 239 Inequalities Using Convex Hull

To generate all 239 inequalities for the Sigle SBox from transformation, we first construct a polyhedron using the convex hull of the possible points. Then, we extract the inequalities from the polyhedron representation. This step provides a comprehensive set of constraints that define the valid regions of the DDT.

$$\text{Convex Hull Constraints:} \quad \sum_{i=1}^{n} \text{coeff}_i \cdot x_i + b \geq 0$$
$$x_i = (x_0, x_1, x_2, x_3, y_0, y_1, y_2, y_3, p_0, p_1)$$

and mapping for the p_i to probabilities from DDT is as follows

	(p_0, p_1)	probability
0	(0, 0)	1
1	(0, 1)	2^{-3}
2	(1, 0)	2^{-2}
3	(1, 1)	0

This inequality represents the constraints for each region of the convex hull, where the coefficients coeff_i and the constant b are derived from the convex hull construction.

Greedy Algorithm to Reduce Constraints

Once the convex hull has been constructed, the next step is to reduce the number of constraints. This is done using a greedy algorithm that selects the most effective inequalities to keep while discarding redundant ones. The greedy algorithm iteratively removes inequalities that do not significantly reduce the feasible region.

The algorithm works by comparing the number of impossible points that each inequality removes. The best inequality is selected, and the process continues until a reduced set of constraints is achieved.

The Python code for this process is provided below:

```
def reduce_inequalites(inequalities, impossible_points):
    reduced_set = set()

    while impossible_points:
        best_inequality = None
        max_removed_patterns = 0

        for inequality in inequalities:
            removed_patterns = removed_impossible_patterns(
                inequality, impossible_points)

            if len(removed_patterns) > max_removed_patterns:
                max_removed_patterns = len(removed_patterns)
                best_inequality = inequality

        if best_inequality is not None:
```

```
            removed_patterns = removed_impossible_patterns(
                best_inequality, impossible_points)

            impossible_points = [point for point in impossible_points
                                 if point not in removed_patterns]
            reduced_set.add(best_inequality)
            inequalities.remove(best_inequality)

    return reduced_set
```

This code effectively reduces the number of inequalities from 239 to 26 by selecting the most impactful constraints for Differential Distribution Table (DDT). These 26 Constraints are as follows:

$$
\begin{aligned}
-p0 - p1 &\geqslant -1 \\
-x1 - x3 - y1 - y3 + 4p0 + 3p1 &\geqslant 0 \\
-2x0 - x1 - x3 + 2y1 + 6y2 + 2y3 - 2p0 + 3p1 &\geqslant 0 \\
3x0 + 3x1 - x2 + 3x3 + y1 - 2y2 + y3 - 2p1 &\geqslant 0 \\
+2x1 + 6x2 + 2x3 - 2y0 - y1 - y3 - 2p0 + 3p1 &\geqslant 0 \\
+x1 - 2x2 + x3 + 3y0 + 3y1 - y2 + 3y3 - 2p1 &\geqslant 0 \\
-5x0 - 4x1 - 7x2 - x3 + 8y0 - y1 - 2y2 + 2y3 + 11p0 + 20p1 &\geqslant 0 \\
8x0 + 2x1 - 2x2 - x3 - 5y0 - y1 - 7y2 - 4y3 + 11p0 + 20p1 &\geqslant 0 \\
+x1 + 2x2 + x3 - 3y1 - 2y2 - 3y3 + 6p0 + 5p1 &\geqslant 0 \\
-3x1 - 2x2 - 3x3 + y1 + 2y2 + y3 + 6p0 + 5p1 &\geqslant 0 \\
2x0 + x1 + 4x2 + x3 - 3y0 - 2y1 + y2 - 2y3 + p0 + 3p1 &\geqslant 0 \\
-2x0 - x1 - 2x2 + 3x3 - 2y0 + 3y1 - y2 - 3y3 + 7p0 + 8p1 &\geqslant 0 \\
-4x0 - 2x1 + x2 - 2x3 + 2y0 + y1 + 5y2 + y3 + p0 + 4p1 &\geqslant 0 \\
-x0 + 3x1 - 2x2 - x3 - 2y0 - 2y1 - y2 + 3y3 + 6p0 + 6p1 &\geqslant 0 \\
2x0 + 2x2 - 2x3 + y1 - 2y2 - y3 + 4p0 + 3p1 &\geqslant 0 \\
-x1 - 2x2 + x3 + 2y0 - 2y1 + 2y2 + 4p0 + 3p1 &\geqslant 0 \\
+x1 - 3x2 - 2x3 + 3y0 + y1 + 2y2 - 2y3 + 5p0 + 4p1 &\geqslant 0 \\
2x0 - 2x1 + 2x2 - y1 - 2y2 + y3 + 4p0 + 3p1 &\geqslant 0 \\
+y1 - y2 + y3 + p0 &\geqslant 0 \\
+x1 - 2x2 + x3 + y1 - 2y2 + y3 + 4p0 + p1 &\geqslant 0 \\
x0 + 2x1 - x2 + 2x3 + y0 - y1 + y2 - y3 + 2p0 &\geqslant 0 \\
-2x0 + x1 - x3 - 2y1 - y2 + y3 + 4p0 + 5p1 &\geqslant 0 \\
-2x1 - x2 + x3 - 2y0 + y1 - y3 + 4p0 + 5p1 &\geqslant 0 \\
-2x0 - 2x1 + x2 + 2x3 - y0 + y1 - 2y2 - 3y3 + 6p0 + 8p1 &\geqslant 0 \\
-3x1 - x2 - x3 + 2y0 - y1 + y2 + 4y3 + 2p0 + 5p1 &\geqslant 0 \\
+x1 + 2x2 + x3 - y0 + y1 + y2 + y3 - 2p0 &\geqslant 0
\end{aligned}
$$

This pattern repeats for all 16 S-boxes (from a_0 to a_{15}) in a single round i.

5.2 Suffle Cell Operation

The **ShuffleCell** operation ensures diffusion by shuffling the input values ($y_{i,j}$) into new positions to produce the output values ($z_{i,j}$).

Transformation Rules

$$(z_0, z_1, z_2, \ldots, z_{15}) \leftarrow (y_0, y_{10}, y_5, y_{15}, y_{14}, y_4, y_{11}, y_1, y_9, y_3, y_{12}, y_6, y_7, y_{13}, y_2, y_8)$$

This rule specifies that z_k is derived from y_r, where the positions k and r are determined by the shuffle operation.

To mathematically express this transformation, we define constraints at the **4-bit nibble layer**. Each nibble consists of four binary variables, and the constraints enforce equality between corresponding input (y) and output (z) values.

For each $z_k \leftarrow y_r$,:

$$\begin{aligned} y_{i,(4r+0)} - z_{i,(4k+0)} &= 0 \\ y_{i,(4r+1)} - z_{i,(4k+1)} &= 0 \\ y_{i,(4r+2)} - z_{i,(4k+2)} &= 0 \\ y_{i,(4r+3)} - z_{i,(4k+3)} &= 0 \end{aligned}$$

This pattern repeat for all 16 nibbles.

5.3 MixColumn Operation

The MixColumn operation in Midori64 is designed to mix the state values in a manner that diffuses the bits across the cipher. For each round, the output state w_i is derived from the input state z_i using a matrix multiplication with a predefined mixing matrix. This is performed column-wise, which leads to the construction of intermediate variables and the subsequent inequalities.

The mixing matrix for Midori64 is:

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

The MixColumn operation can be viewed as a bit-level operation in which each column of the matrix produces an output w_i from the inputs z_i. Specifically, for each column, intermediate variables t_i are used to simplify the addition of the input bits. The input z_i and output w_i bits are constrained as follows:

$$w_0 = t_0 + z_{12}, \quad w_1 = t_1 + z_{13}, \quad w_2 = t_2 + z_{14}, \quad w_3 = t_3 + z_{15}$$

Where the intermediate variables are defined as:

$$t_0 = z_4 + z_8, \quad t_1 = z_5 + z_9, \quad t_2 = z_6 + z_{10}, \quad t_3 = z_7 + z_{11}$$

This allows us to break down the constraints into smaller equations.

Variables Constraints

For the each equality mentioned above we will get 4 inequlity.

For xor operation $c = a \oplus b$:

$$\begin{aligned} a + b - c &\geq 0 \\ a - b + c &\geq 0 \\ -a + b + c &\geq 0 \\ a + b + c &\leq 2 \end{aligned}$$

These constraints hold for all w_i, ensuring that each output bit is the result of the proper sum of the intermediate and input bits. Here we are indroducing 32 intermediate variables and getting 384 inequlities in total.

5.4 Optimization Function

The optimization function is maximizing summation of differential probabilities across all S-boxes for the current round i. Which result in the minimizing the summation of p_0, p_1 for all SBoxes with weights $2, 3$

$$\text{Optimization Function} = \sum_{i,j} (2p_{i,j,0} + 3p_{i,j,1})$$

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