Quiz-1

1st quiz for the course IC202- Calculus II

1. Question group text

1.1. Consider the following limit

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

$$\lim_{(x,y)\to(0,0)} \tan^{-1} \frac{|x|+|y|}{x^2+y^2}.$$

Options:

0) the limit exists and equals to $\frac{\pi}{2}$,
1) the limit exists and equals to $-\frac{\pi}{2}$,
2) the limit exists but neither equals to $\frac{\pi}{2}$ nor $-\frac{\pi}{2}$,
3) the limit does not exist

Answer: [0]

1.2. Consider the function

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

$$f(x,y) = \begin{cases} y\sin\frac{1}{x} + \frac{xy}{x^2+y^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Options:

0) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists and equal to $f(0,0)$
1) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ exists but not equal to $f(0,0)$,
2) $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist,
3) All other options are incorrect.

Answer: [2]

1.3. Consider the function

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

$$f(x,y) = \begin{cases} \frac{xy^n}{x^2+y^{2n}} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

where n is any natural number.

Options:

0)

The function is continuous at $(0,0)$ for all natural number n,

1) The function is continuous at $(0,0)$ only for $n=3$,

2) The function is not continuous at $(0,0)$ for all natural number n,

3) For some natural number n the function is continuous at $(0,0)$

Answer: [2]

1.4. Consider the function

Marks: 2.5

Type: SINGLE_CORRECT_ANSWER

$$f(x,y)=\begin{cases}\frac{2xy}{(x^2+y^2)^p} & \text{if } x^2+y^2\neq 0\\ 0 & \text{if } x^2+y^2=0\end{cases}$$

where p is a positive real number.

Options:

0) the function is continuous at $(0,0)$ for $p=1$

1) the function is continuous at $(0,0)$ for $p=0.5$,

2) one of the partial derivative does not exists at $(0,0)$ for all p

3) the function is not continuous at $(0,0)$ for all p.

Answer: [1]

1.5. Consider the function

Marks: 2.5

Type: SINGLE_CORRECT_ANSWER

$$f(x,y)=\begin{cases} xy & \text{if } |x|\geq |y|\\ -xy & \text{if } |x|<|y|\end{cases}$$

The values of $\frac{\partial f}{\partial x}(0,5)$ and $\frac{\partial f}{\partial y}(5,0)$ are respectively

Options:

0) 5, -5

1) -5, 5

2) 5, 5

3) -5, -5

Answer: [1]

1.6. Consider the surface $\cos(\pi x)-x^2y+e^{xz}+yz=4$ The tangent plane of the surface at $(0,1,2)$

Marks: 2.5

Type: SINGLE_CORRECT_ANSWER

Options:

0) is parallel to xy plane

1) is parallel to xz plane,

2) intersects z axis at (0, 0, 4),

3) intersects z axis at (2, 0, 0),

Answer: [2]

1.7. Consider the followings
$z(x,y) = 4e^x \log_e y,\ x(u,v) = \log_e(u\cos v)$
$y(u,v) = u\sin v.$
The value of $\frac{\partial z}{\partial u}$ at $u = 2,\ v = \frac{\pi}{4}$ is

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

Options:

0) $\sqrt{2}(\log_e 2 + 2),$

1) $\sqrt{2}(\log_e 2 - 2),$

2) $2(\log_e \sqrt{2} - 2),$

3) $2(\log_e 2 + \sqrt{2})$

Answer: [0]

1.8. Consider the function
$f(x,y) = |x| + |y|.$
Which of the following is true?

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

Options:

0) at $(0,0)$ $f(x,y)$ is continuous but not differentiable,

1) at $(0,0)$ $f(x,y)$ is not continuous,

2) at $(0,0)$ $f(x,y)$ is continuous and differentiable,

3) both the partial derivatives exist at $(0,0)$

Answer: [0]

1.9. Consider the surface
$z = x^2 + y^2 - 2xy - x + 3y + 4$
Consider the normal line at $(2,-3,18)$ of the surface.
Which of the followings is true?

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

Options:

0) normal line is $\frac{x-2}{9} = \frac{y+3}{-7} = \frac{z-18}{1},$

1) normal line is $\frac{x-2}{9} = \frac{y+3}{7} = \frac{z-18}{-1},$

2) $(92,-73,8)$ is a point on the normal line,

3) $(92,-73,8)$ is not a point on the normal line.

Answer: [2]

1.10.

Which of the following is true for every function $f : D \to \mathbb{R}$ where $D \subseteq \mathbb{R}^2$.

Marks: 2.5
Type: SINGLE_CORRECT_ANSWER

Options:

0) If f has directional derivative along all direction then f is differentiable
1) If both the partial derivative exist then the directional derivative also exists along any direction
2) If at least one partial derivative exists then directional derivative also exists along at least one direction.
3) If f has directional derivative along all direction then f is continuous

Answer: [2]