Tutorial-5

① Let $r = (x, y, z)$. Then prove the following

i) $\nabla \cdot \nabla \left(\frac{1}{\|r\|} \right) = 0$ $\left[\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \right]$.

ii) $\nabla \cdot \left(\frac{r}{\|r\|^2} \right) = 0$.

② Evaluate the line integral $\int_\gamma f\,ds$ where f and γ are given below

i) $f(x, y) = 3x^2 - 2y$ & γ is the line segment from $(3,6)$ to $(1,-1)$.

ii) $f(x,y) = 2yx^2 - 4x$ & γ is the lower half of the circle centred at origin of radius 3 with clockwise direction.

③ Evaluate the following line integral $\int_\gamma F \cdot d\gamma$ of the given vector field F.

i) $F(x,y) = (y^2, 3x - 6y)$ and γ is the line segment joining $(3,7)$ and $(1,2)$.

ii) $F(x,y) = (3y, x^2 - y)$ and γ be the upper half of the circle of radius 1 & centred at $(0,0)$ and the portion of $y = x^2 - 1$ from $x = -1$ to $x = 1$ with counter clockwise rotation.

④ Determine whether the following vector fields are conservative or not.

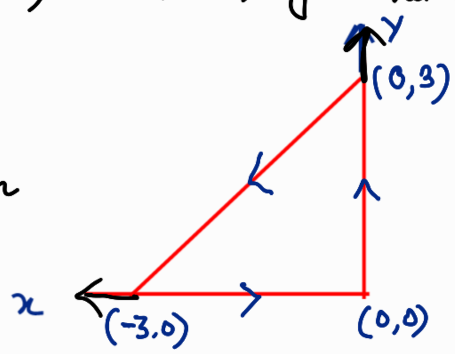
i) $F(x,y) = \left(x^3 - 4xy^2 + 2,\ 6x - 7y + x^3y^3\right)$

ii) $F(x,y) = \left(2x\sin(2y) - 3y^2,\ 2 - 6xy + 2x^2\cos(2y)\right)$

Both the vector field defined on whole $\mathbb{R}^2$.

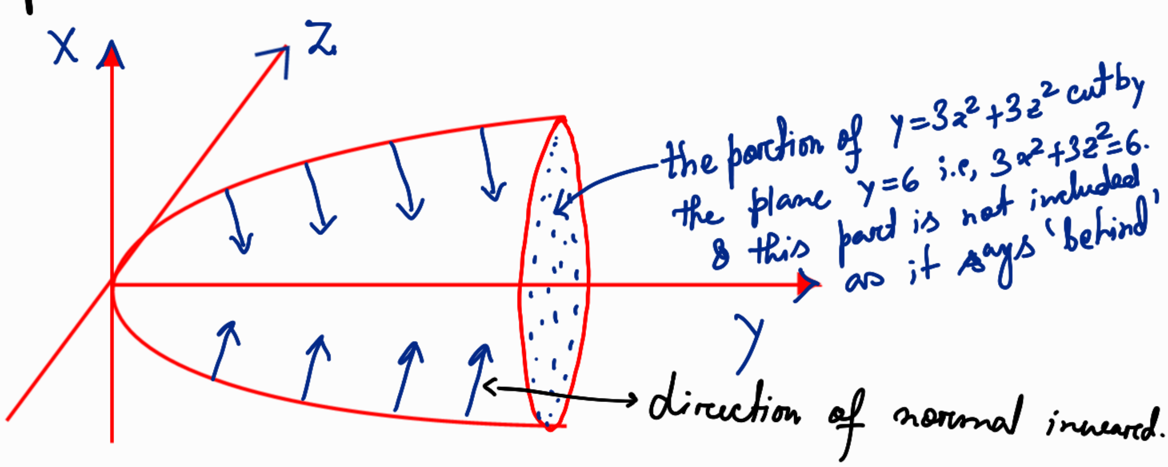
⑤ i) Verify Green's Theorem for $\int_\gamma (xy^2 + x^2)\,dx + (4x-1)\,dy$ where γ is given by the diagram

ii) Find the area of the region bounded by γ using line integral (Use Green's Theorem).



⑥ Evaluate the following surface integral $\iint_S f\,ds$ where f is a scalar function, $f(x,y,z) = 2y$ and S is surface $y^2+z^2=4$ between $x=0$ and $x+z=3$.

⑦ Evaluate $\iint_S F \cdot ds$ where $F = (-x, 2y, -z)$ and S is the portion of $y = 3x^2 + 3z^2$ that lies behind $y=6$ oriented in the positive y-axis direction (i.e, inward direction).



⑧ Use divergence Theorem to evaluate $\iint_S F \cdot ds$ where $F = \left(xy, -\frac{1}{2}y^2, z\right)$ and S is the surface consists of three surfaces, $z = 4 - 3x^2 - 3y^2$, $1 \le z \le 4$ on the top, $x^2+y^2=1$, $0 \le z \le 1$ on the sides and $z=0$ on the bottom.