Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Assignment 3

January 13, 2022

1. Let X be a random variable with distribution function F. Then find the distribution function for $|X|$, $aX+b$, where $a \neq 0$, $b \in \mathbb{R}$, $\max\{X, 0\}$ and $\min\{X, 0\}$.
2. Let X be a discrete random variable with p.m.f. $P(X=-2)=\frac{1}{5}$, $P(X=-1)=\frac{1}{6}$, $P(X=0)=\frac{1}{5}$, $P(X=1)=\frac{1}{15}$ and $P(X=2)=\frac{11}{30}$. Find the p.m.f. and d.f. of $Y=X^2$.
3. Let X be a random variable with p.d.f.
$$f_X(x)=\begin{cases}\frac{1}{2}, & -1<x<1,\\ 0, & \text{otherwise.}\end{cases}$$
Find the distribution function of $Y=\max\{X,0\}$.
4. The random variable X has p.d.f. $f_X(x)=\frac{1}{2}e^{-|x|}$, $-\infty<x<\infty$. Find the distribution of $Y=X^2$.
5. Suppose X have the density function
$$f_X(x)=\begin{cases}c(x+1), & -1<x<2,\\ 0, & \text{otherwise.}\end{cases}$$
Find the value of c. Hence calculate the p.d.f. and c.d.f. of $Y=X^2$.
6. Suppose X have the density function
$$f_X(x)=\begin{cases}3x^2, & 0<x<1,\\ 0, & \text{otherwise.}\end{cases}$$
Find the density function of $Y=40(1-X)$.
7. Among the 15 applicants for a job, 9 are women and 6 are men. 5 applicants are randomly selected from the applicant pool for final interviews. Let X be the number of female applicants among the final 5. (i) Give the probability mass function for X. (ii) Define Y, the number of male applicants among the final 5, as a function of X. Find the probability mass function for Y.
8. If X is a random variable such that $E(X)=3$ and $E(X^2)=13$, then determine a lower bound for $P(-2<X<8)$.
9. Let the random variable X has the m.g.f. $M(t)=\frac{e^{-2t}}{8}+\frac{e^{-t}}{4}+\frac{e^{2t}}{8}+\frac{e^{3t}}{2}$. Find the distribution function of X and find $P(X^2=4)$.
10. Let X be a random variable with m.g.f. $M(t)$, $-h<t<h$
   (a) Prove that $P(X \geq a) \leq e^{-at}M(t)$, $0<t<h$;
   (b) Prove that $P(X \leq a) \leq e^{-at}M(t)$, $-h<t<0$.