IC200: Tierce Exam 1

Dr. Mahavir Sharma

In S.I. units: $G = 6.67 \times 10^{-11}$, $M_\odot = 2 \times 10^{30}$, $R_\odot = 7 \times 10^{8}$, $k_B = 1.38 \times 10^{-23}$

1. For a potential $V(r)$, the trajectory of the particle in r, θ plane turns out to be $r = r_0\, e^{A\theta}$, where r_0 and A are constants. Find out the expression for $V(r)$ and plot it as a function of r. **[5 marks]**

2. A photon of wavelength λ_{21} is emitted when the electron jumps from shell 2 to 1 in hydrogen atom, and a photon of wavelength λ_{51} is emitted if the electron jumps from shell 5 to 1. Calculate the ratio $\lambda_{21}/\lambda_{51}$. **[5 marks]**

3. In a gas cloud that is spread across 1 km, the atoms can both absorb and emit the radiation. The coefficient of emission in the cloud is $j_\nu = A\nu^3 e^{-h\nu/(k_B T)}$ and the absorption coefficient is $\alpha_\nu = 0.1$ cm^{-1} where A, h and k_B are constants, ν is the frequency and T is the temperature. If light of intensity $I_{\nu,0}$ is incident on this cloud then derive the intensity I_ν that comes out of the cloud and plot it as a function of ν. **[5 marks]**

4. Equations of the stellar structure can be written as,

$$\frac{dM}{dr} = 4\pi r^2 \rho \quad ; \qquad \frac{dP}{dr} = -\frac{GM}{r^2}\rho$$

where ρ and P are the density and pressure at a given r, and M is the mass enclosed within r. Say, the equation of state is not given but instead it has been given that the density is (somehow) constant. Then calculate and plot the pressure P as a function of r. **[5 marks]**

5. Imagine that the nuclear fusion is switched off, and the Sun starts shrinking slowly (pressure $\neq 0$). Prove that the total energy of the Sun ($E = E_{\rm internal} + E_{\rm gravitational}$) decreases as it shrinks and the temperature increases. *(Hint: use the virial theorem)*
If the loss in the E is what the Sun radiates away as luminosity $L_\odot = 4 \times 10^{26}$ J/s, then calculate in how much time, all of the energy of the Sun would be lost. **[5 marks]**

6. Prove that the equation of state for non-relativistic degenerate white dwarf star is $P \propto \rho^{5/3}$ where ρ is the density (You can use the approximate approach we followed in the class that begins with the uncertainty principle). **[5 marks]**

7. A spherical gas cloud has uniform density $\rho = 10^{-10}$ kg/m^3 and radius $R = R_\odot$. How much time will it take to collapse freely due to gravity. Also calculate the same for another cloud that has the same density but is a million time bigger in size (i.e. $R = 10^6 R_\odot$). **[5 marks]**

8. Derive the mass as a function of radial distance, M(r), for a sphere of dark matter which has a radial density profile given by $\rho = \rho_0 e^{-r/L}$ where ρ_0 and L are constants. Also, plot M(r) as a function of r/L. **[5 marks]**

Do not forget to write your roll number on the answer sheet.