Tutorial - 4
MA202 : Calculus II

1. Calculate the double integral $\iint_D f(x,y)d(x,y)$ for the given f and D after applying the given transformations.

   (a) $f(x,y) = e^{x^2+y^2}$ and D be the closed unit disk in $\mathbb{R}^2$ (transform in to polar co-ordinate)

   (b) $f = y^3(2x-y)e^{(2x-y)^2}$ and $D = \{(x,y) \in \mathbb{R}^2 : 0 \leq y \leq 2,\ \frac{y}{2} \leq x \leq \frac{y+4}{2}\}$. Apply the transformation $u = 2x - y$ and $v = y$.

   (c) $f(x,y) = y^2$ and $D = \{(x,y) \in \mathbb{R}^2 : \frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1\}$. Apply the transformation $x = ar\cos\theta$ and $y = br\sin\theta$.

   (d) $f(x,y) = 2x^2 - xy - y^2$, $D =$ the first quadrant bounded by the line $y + 2x = 4$, $y + 2x = 7$, $y = x - 2$ and $y = x + 1$. Apply the transformation $u = x - y$ and $v = 2x + y$.

2. Calculate the area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ using double integral (take the integrating function $f(x,y) = 1$) with the transformation $x = au$, $y = bv$.

3. Calculate the triple integral $\iiint_D f(x,y)d(x,y)$ for the given f and D after applying the given transformations.

   (a) $f(x,y,z) = \frac{z}{y+z}$, $D = \{(x,y,z) \in \mathbb{R}^3 : x,y,z \geq 0,\ x \leq y+z,\ 1 \leq 2(x+y+z) \leq 2\}$. Apply the transformation $u = x + y + z$, $v = \frac{y+z}{x+y+z}$, and $w = \frac{z}{y+z}$.

   (b) $f(x,y,z) = z\sqrt{1-x^2-y^2}$, $D = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2 \leq 1,\ 0 \leq z \leq 1\}$. Transform to cylindrical co-ordinates.

   (c) $f(x,y,z) = z^2$, $D = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 \leq a^2\}$. Transform to spherical co-ordinates.

   (d) $f(x,y,z) = \frac{2x-y}{2} + \frac{z}{3}$, $D = \{(x,y,z) \in \mathbb{R}^3 : 0 \leq z \leq 3,\ 0 \leq y \leq 4,\ 2x = y,\ y = 2x - 2\}$. Apply the transformation $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$, and $w = \frac{z}{3}$.

4. Let D be an open subset of $\mathbb{R}^3$. Also let $f : D \to \mathbb{R}$ be a scalar field and $F = (F_1, F_2, F_3) : D \to \mathbb{R}^3$ be a vector field where the partial derivatives of $f, F_1, F_2, F_3 : D \to \mathbb{R}$ exist and continuous. Then prove the following

   (a) $\nabla \times (\nabla f) = 0$

   (b) $\nabla.(\nabla \times F) = 0$