Examination
IC202: Calculus II

Full Marks-40 Time: 1 hour 30 minutes

Question from 1-2 carries 2 marks each and there may be multiple correct answers for a question. Write only the options which are correct. No need to copy the whole sentence. Full marks will be awarded if you identify all the correct answers and only the correct answers. In all other cases no marks will be awarded. There will be no part marks and no negative marks. Questions 3-8 carries 6 marks each and part mark may be awarded for these questions.

1. Let $E = [0,1] \times [0,1]$. Also consider
$$D_1 = \{(x,y,z) \in \mathbb{R}^3 : (x,y,z) \in [0,1]\times[0,1]\times[0,1],\ \&\ x,y,z \in \mathbb{Q}\}$$
$$D_2 = \{(x,y) \in \mathbb{R}^2 : (x,y) \in E,\ \&\ x,y \in \mathbb{Q}\}.$$
Which of the following is/are correct?
   (a) ∂D_1 is of content zero in $\mathbb{R}^3$ but ∂D_2 is not of content zero in $\mathbb{R}^2$
   (b) E is of content zero in $\mathbb{R}^3$ and ∂D_2 is not of content zero in $\mathbb{R}^2$
   (c) E is of content zero in $\mathbb{R}^2$ and ∂D_1 is not of content zero in $\mathbb{R}^3$
   (d) ∂D_1 is not of content zero in $\mathbb{R}^3$ and ∂D_2 is not of content zero in $\mathbb{R}^2$.

2. $f(x,y) = x^2+y^2$ is defined on the closed triangular plate bounded by $x=0$, $y=0$ and $y+2x=2$.
   (a) f does not have absolute maximum and absolute minimum in the given region,
   (b) absolute maximum exists at a interior point of the region and absolute minimum exists at a boundary point of the region,
   (c) both absolute maximum and absolute minimum exist at the boundary points of the region,
   (d) absolute maximum is 6 and absolute minimum is 0.

3. (a) Let $f : [0,1]\times[0,1] \to \mathbb{R}$ be defined as [3+3]
$$f(x,y) = \begin{cases} 1, & x \in \mathbb{Q} \\ 2y, & \text{otherwise} \end{cases}$$
Evaluate $\int_0^1 (\int_0^1 f(x,y)dy)dx$.
(b) Define saddle point of a real valued function of two variables.

4. Find the maximum and minimum values of the function $f(x,y,z) = x^2y^2z^2$ subject to the constraint $x^2+y^2+z^2 = 1$. Then using the maximum value of the function prove that for any $p,q,r \in \mathbb{R}$ where $(p,q,r) \neq (0,0,0)$, we have the following inequality [3+3]
$$(p^2q^2r^2)^{\frac{1}{3}} \leq \frac{p^2+q^2+r^2}{3}.$$

5. Let Let $f : [a,b]\times[c,d] \to \mathbb{R}$ be a continuous function and $P = \{(x_i,y_j) : i = 0,1,\ j = 0,1\}$ be a partition of rectangle $[a,b]\times[c,d]$. Let $x^* \in (x_0,x_1)$ and consider another partition P^* define by the following grid points $P^* = \{(x_0,y_0),(x_0,y_1),(x^*,y_0),(x^*,y_1),(x_1,y_0),(x_1,y_1)\}$. Then prove that $L(P,f) \leq L(P^*,f)$ and $U(P,f) \geq U(P^*,f)$. [6]

6. Let D be the region bounded by $x = -2y^2$ and $x = y^3$. [2+4]
   (a) Express D in the form of elementary region,
   (b) Evaluate $\iint_D (10x^2y^3 - 6)d(x,y)$.

7. Consider the region D in $\mathbb{R}^3$ bounded by $x^2 + z^2 = 4$, $x^2 + z^2 = 9$, $1 \le y \le 5$ and $z \le 0$. Also $f : D \to \mathbb{R}$ defined as $f(x, y, z) = e^{-(x^2+z^2)}$. Evaluate $\iiint_D f(x, y, z)d(x, y, z)$. [6]

8. Evaluate $\displaystyle\iiint\limits_E (10xz + 3)\, d(x, y, z)$ where E is the region bounded by the sphere $x^2+y^2+z^2 = 16$ and $z \ge 0$. [6]