An important point:

We know that $d\vec{s} = n\, ds$ where $n = \dfrac{\Phi_u \times \Phi_v}{\|\Phi_u \times \Phi_v\|}$ $\left(\text{or, } \dfrac{\nabla F}{\|\nabla F\|}\right)$

Also from the definition of surface integral of scalar field we have

$$|d\vec{s}| = ds = \|\Phi_u \times \Phi_v\|\, d(u,v)$$

Using these two equations we can get a relation between $d\vec{s}$ and $d(u,v)$ which is

$$d\vec{s} = n\, ds = \frac{\Phi_u \times \Phi_v}{\|\Phi_u \times \Phi_v\|}\, ds = (\Phi_u \times \Phi_v)\, d(u,v).$$

This relation $d\vec{s} = (\Phi_u \times \Phi_v)\, d(u,v)$ we use to calculate surface integral of a vector field.

* Note that, in this case it is not required to evaluate $\|\Phi_u \times \Phi_v\|$ because it will get cancelled eventually.

* Remember that we dont use the sign $d\vec{s}$. We use only ds in both cases. Here I used $d\vec{s}$ just for better understanding.