Tutorial - 2
MA202 : Calculus II

1. Find the critical points for the following functions and determine their nature.
   (a) $f(x, y) = x^3 - y^3 - 2xy + 6$,
   (b) $f(x, y) = 9x^3 + \frac{y^3}{3} - 4xy$,
   (c) $f(x, y) = e^{2x} \cos y$.

2. Find the absolute maxima and minima of the functions on the given domains. (Note that the functions are continuous and defined in a closed bounded domain).
   (a) $f(x, y) = 2x^2 - 4x + y^2 - 4y + 1$ defined on the closed triangular plate bounded by the lines $x = 0,\ y = 2,\ y = 2x$ in the first quadrant.
   (b) $f(x, y) = x^2 + xy + y^2 - 6x$ on the rectangular plate $0 \leq x \leq 5,\ -3 \leq y \leq 3$.
   (c) $f(x, y) = (4x - x^2) \cos y$ on the rectangular plate $1 \leq x \leq 3$ and $-\pi/4 \leq y \leq \pi/4$.

3. Find the points on the curve $x^2 + xy + y^2 = 1$ in the xy-plane that are nearest to and furthest from the origin.

4. Find the length and width of the rectangle of greatest area that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with the sides parallel to the coordinate axes.

5. Find the point on the plane $x + 2y + 3z = 13$ closest to the point $(1, 1, 1)$.

6. Find three real numbers whose sum is 9 and sum of squares is as small as possible.

7. Find the extreme values of $f(x, y, z) = xy + z^2$ on the circle o which the plane $y - x = 0$ intersects the sphere $x^2 + y^2 + z^2 = 4$.

8. Maximize the function $f(x, y, z) = x^2 + 2y - z^2$ subject to the constraints $2x - y = 0$ and $y + z = 0$.