Department of Mathematics
Indian Institute of Technology Bhilai
IC152: Linear Algebra-II
Tutorial Sheet 4

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1. Let V be a vector space over $\mathbb{F}$ and $< \cdot, \cdot >$ be an inner product on V then show that

   (i) $< 0, \alpha >= 0$ for all $\alpha \in V$.

   (ii) if $< \alpha, \beta >= 0$ for all $\beta \in V$ then $\alpha = 0$.

   (ii) if $< \alpha, \beta >=< \gamma, \beta >$ for all $\beta \in V$ then $\alpha = \gamma$.

   (iv) $< \alpha, \beta >= 0$ if and only if $\|\alpha\| \leq \|\alpha + c\beta\|$ for all $c \in \mathbb{F}$.

2. Let $< \cdot, \cdot >$ be the standard inner product on $\mathbb{R}^2$. Find $\alpha \in \mathbb{R}^2$ if $< (1,2), \alpha >= -1$ and $< (-1,1), \alpha >= 3$.

3. Which of the following are inner product?

   (i) For any $\alpha = (\alpha_2, \alpha_2), \beta = (\beta_1, \beta_2) \in \mathbb{R}^2$, define $< \alpha, \beta >= \alpha_2(\alpha_1 + 2\beta_1) + \beta_2(2\alpha_1 + 5\beta_2)$.

   (ii) For any $A, B \in M_{n\times n}(\mathbb{C})$ define $< A, B >= trace(A\bar{B})$.

   (iii) For any $A, B \in M_{n\times n}(\mathbb{R})$ define $< A, B >= trace(A + B)$.

   (iv) For any $f, g \in \mathrm{P}(\mathbb{R})$ define $< f, g >= \int_0^1 f'(x)g(x)dx$.

4. Compute $< \alpha, \beta >, \|\alpha\|, \|\beta\|, \|\alpha + \beta\|$ for the following vectors in the specified inner product spaces and verify the triangle and Cauchy Schwartz inequlaity.

   (i) $V = \mathbb{C}^3$ with standard inner product and $\alpha = (2, 1+i, i), \beta = (2-i, 2, 1+2i)$

   (ii) $V = C([0,1]; \mathbb{R})$ with $< f, g >= \int_0^1 f(x)g(x)dx$ and $\alpha = x, \beta = e^x$

   (iii) $V = M_{2\times 2}(\mathbb{C})$ with standard inner product and $\alpha = \begin{bmatrix} 1 & 2+i \\ 3 & i \end{bmatrix}, \beta = \begin{bmatrix} 1+i & 0 \\ i & -i \end{bmatrix}$

5. Suppose $u, v \in V$ are such that $\|u\| = 3, \|u+v\| = 4, \|u-v\| = 6$. Then what will be $\|v\|$?

6. Find the matrix of standard inner product $< \cdot, \cdot >$ on $\mathbb{R}^3$ relative to an ordered basis $\mathcal{B} = \{(1,-1,2), (2,0,1), (-1,2,-1)\}$.

7. Let $(V, < \cdot, \cdot >)$ be an inner product space. Then prove that, for any orthogonal vectors $\alpha, \beta \in V$

$$\|\alpha + \beta\|^2 = \|\alpha\|^2 + \|\beta\|^2.$$

8. Use standard inner product on $\mathbb{R}^2$ over $\mathbb{R}$ to prove the following statement: "A parallelogram is a rhombus if and only if its diagonals are perpendicular to each other."