Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Tutorial 2

January 9, 2022

1. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, \; x < 2, \\ \frac{2}{3}, \; 2 \leq x < 5, \\ \frac{7-6k}{6}, \; 5 \leq x < 9, \\ \frac{3k^2-6k+7}{6}, \; 9 \leq x < 14, \\ \frac{16k^2-16k+19}{16}, \; 14 \leq x \leq 20, \\ 1, \; x > 20. \end{cases}.$$

   (a) Find the value of constant k.

   (b) Show that the r.v. X is of discrete type and find its support.

   (c) Find the p.m.f. of X.

2. Consider a random variable X such that

$$f_X(x) = \frac{x^2}{a}, \text{ for } x \in \{-3,-2,-1,1,2,3\} \text{ and } P(X = x) = 0 \text{ for } x \notin \{-3,-2,-1,1,2,3\},$$

   where $a > 0$ is a real parameter.
   (a) Find a.
   (b) What is the PMF of the random variable $Z = X^2$?

3. Let X be a random variable with p.d.f f_X. Find the p.d.f of the random variable $Y = |X|$

   (a) when $f(x) = \begin{cases} \frac{1}{3}, \; -2 < x \leq 1, \\ 0, \; \text{otherwise}; \end{cases}$

   (b) when $f(x) = \begin{cases} 2e^{-2x}, \; x > 0, \\ 0, \; \text{otherwise}; \end{cases}$

   .

   (c) for general $f_X(x)$.

4. A particular professor is known for his arbitrary grading policies. Each test receives a grade from the set $\{A, A-, B, B-, C, C-, D, F\}$, with equal probability, independently of other tests. How many tests do you expect to appear in before you receive each possible grade at least once?

5. Alice and Bob alternate playing at the casino table. (Alice starts and plays at odd times $i = 1, 3, \ldots$; Bob plays at even times $i = 2, 4, \ldots$.) At each time i, the net gain of whoever is playing is a random variable X_i with the following p.m.f:

$$f(x) = \begin{cases} \frac{1}{3}, & x = -2, \\ \frac{1}{2}, & x = 1, \\ \frac{1}{6}, & x = 3, \\ 0, & \text{otherwise.} \end{cases}$$

Assume that the net gains at different times are independent. We refer to an outcome of -2 as a "loss."

(a) They keep gambling until the first time where a loss by Bob immediately follows a loss by Alice. Write down the p.m.f of the total number of rounds played. (A round consists of two plays, one by Alice and then one by Bob.)

(b) Write down the p.m.f for Z, defined as the time at which Bob has his third loss.

(c) How many rounds are expected until each one of them has won at least once.