Alternative optima

Solve the following L.P.P

$$\text{Max } Z = 4x_1 + 3x_2$$

$$\text{s.t.} \quad 8x_1 + 6x_2 \le 25$$

$$3x_1 + 4x_2 \le 15, \quad x_1, x_2 \ge 0$$

Solⁿ The standard form of the L.P.P is

$$\text{Max } Z = 4x_1 + 3x_2$$

$$\text{s.t.} \quad 8x_1 + 6x_2 + x_3 = 25$$

$$3x_1 + 4x_2 + x_4 = 15$$

	C_j	4	3	0	0		
C_B	x_B	x_1	x_2	x_3	x_4	R.H.S	θ
0	x_3	8	6	1	0	25	$\frac{25}{8}$ →
0	x_4	3	4	0	1	15	$\frac{15}{3}$
	$C_j - Z_j$	4 ↑	3	0	0		
4	x_1	1	$\frac{3}{4}$	$\frac{1}{8}$	0	$\frac{25}{8}$	
0	x_4	0	$\frac{7}{4}$	$-\frac{3}{8}$	1	$\frac{45}{8}$	
	$C_j - Z_j$	0	0	$-\frac{1}{2}$	0		
4	x_1	1	0	$\frac{2}{7}$	$-\frac{3}{7}$	$\frac{5}{7}$	
3	x_2	0	1	$-\frac{3}{14}$	$\frac{4}{7}$	$\frac{45}{14}$	
	$C_j - Z_j$	0	0	$-\frac{1}{2}$	0		

$15 - \frac{75}{8}$

$= \frac{120 - 75}{}$

$\frac{9}{14} - \frac{8}{7}$

$\frac{9-16}{14}$

At the end of the first iteration or complety the second table, we observed that the non-basic variables $\{x_2, x_3\}$ have non-positive values of $C_J - Z_J$ indicating that the optimal sol^n has been reached.

However, one of the non-basic variable x_2 has a $C_J - Z_J$ value of zero.

If we enter this in the basic variables, we get another optimal solution with the same value of the objective function.

$\therefore$ The first optimal sol^n is $x_1 = \frac{25}{8}$ and $x_2 = 0$. Hence the optimal value is $Z = 4 \cdot \frac{25}{8} = \frac{25}{2}$

$= 12.5$

$\therefore$ The alternative optimum sol^n is $x_1 = \frac{5}{7}$, $x_2 = \frac{45}{14}$. Hence the optimal value is

$$Z = 4x_1 + 3x_2 = 4 \cdot \frac{5}{7} + 3 \cdot \frac{45}{14}$$

$$= \frac{40 + 135}{14} = \frac{175}{14} = \frac{25}{2} = 12.5$$

Hence there are infinite number of optimal solⁿ and it is given by

$$S = \left\{ (x_1, x_2) : \lambda \left(\frac{25}{8}, 0 \right) + (1-\lambda) \left(\frac{5}{7}, \frac{45}{14} \right) \right\}$$

where $0 \le \lambda \le 1$

Infeasible Solution

Solve the problem

$$\max Z = 4x_1 + 3x_2$$

s.t. $\quad x_1 + 4x_2 \le 3$

$\quad 3x_1 + x_2 \ge 12$

$\quad x_1, x_2 \ge 0$

The standard form of L.P.P is

$$\boxed{\begin{aligned} &\max Z = 4x_1 + 3x_2 \\ &\text{s.t.} \quad x_1 + 4x_2 + x_3 = 3 \\ &\qquad 3x_1 + x_2 - x_4 = 12 \end{aligned}}$$

we add an artificial variable a_1 to the second constraints and the transformed

L.P.P is

$$\max\ z = 4x_1 + 3x_2 - Ma_1 \qquad M > 0$$

$$\text{s.t.} \begin{cases} x_1 + 4x_2 + x_3 = 3 \\ 3x_1 + x_2 - x_4 + a_1 = 12 \end{cases}$$

C_J		4	3	0	0	$-M$		
C_B	x_B	x_1	x_2	x_3	x_4	a_1	RHS	θ
0	x_3	1	4	1	0	0	3	3 →
$-M$	a_1	3	1	0	-1	1	12	4
	$C_J - Z_J$	$3M+4$ ↑	$M+3$	0	$-M$	0		
4	x_1	1	4	1	0	0	3	
$-M$	a_1	0	-11	-3	-1	1	3	
	$C_J - Z_J$	0	$-13-11M$	$-4-3M$	$-M$	0		

Here the algorithm terminates as all the non-basic variables x_2, x_3, x_4 have negative $C_J - Z_J$. The optimal condition is satisfied but an artificial variable is present in the basis. This means that the problem is infeasible and does not have an optimal solⁿ.

Termination Conditions (Max obj)

1. All non-basic variables have negative values of $C_J - Z_J$. Basic variables are either decision or slack variables.

2. Basic variables are either decision variables or slack variables. All non-basic variables have $C_J - Z_J \leq 0$. At least one non-basic variables have $C_J - Z_J = 0$. It indicates alternative optima. Proceed to find the other optimal solⁿ and terminate.

3. Basic variables are either decision variables or slack variables. The algorithm identifies an entering

variable but is unable to decide the leaving variable because all values in the entering column are ≤ 0. It indicates unboundedness and algorithm terminates.

4. All non-basic variable have $C_j - Z_j \leq 0$ Artificial variables still exists in the basis. It indicates that the problem is infeasible and algorithm terminates.

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Surprise test

1. Solve the L.P.P by Simplex method & prove that alternative optimal sol^n exists

$$\max Z = 2x_1 - x_2 + 3x_3 + x_4$$

s.t

$$2x_1 + x_2 + 3x_3 + 5x_4 \le 12$$

$$3x_1 + 2x_2 + x_3 + 4x_4 \le 15$$