

Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Tutorial 7

February 15, 2022

1. Let N_i denote the number of car arrivals in an interval $I_i = (t_i, t_i + c_i]$. Suppose we have n such intervals, $i = 1, 2, \dots, n$, mutually disjoint and assume that N_i 's are independent. Denote the union of these intervals by I , and their total length by $c = c_1 + c_2 + \dots + c_n$. Given $k_i \geq 0$ and with $k = k_1 + k_2 + \dots + k_n$, determine $P(N_1 = k_1, N_2 = k_2, \dots, N_n = k_n | N_I = k)$.
2. Let X and Y be two independent $U(0, 1)$ random variables. Then find the distributions of
 - (a) XY ,
 - (b) $\frac{X}{Y}$.
3. The p.m.f. of a two dimension discrete random vector (X, Y) is given as

Y/X	-1	0	1
-2	1/6	1/12	1/6
1	1/6	1/12	1/6
2	1/12	0	1/12

Find the joint distribution of $|X|$ and Y^2 .

4. Let X_1 and X_2 be iid exponential random variables with parameter λ . Find the conditional distribution of X_1 given that $X_1 + X_2 = 1$.
5. An investment bank is managing \$1 billion, which it invests in various financial instruments ("assets") related to the housing market (e.g., the infamous "mortgage backed securities"). Because the bank is investing with borrowed money, its actual assets are only \$50 million (5%). Accordingly, if the bank loses more than 5%, it becomes insolvent. (Which means that it will have to be bailed out, and the bankers may need to forgo any huge bonuses for a few months.)
 - (a) The bank considers investing in a single asset, whose gain (over a 1-year period, and measured in percentage points) is modeled as a normal random variable R , with mean 7 and standard deviation 10. (That is, the asset is expected to yield a 7% profit.) What is the probability that the bank will become insolvent? Would you accept this level of risk?

(b) In order to safeguard its position, the bank decides to diversify its investments. It considers investing \$50 million in each of twenty different assets, with the i th one having a gain R_i , which is again normal with mean 7 and standard deviation 10; the bank's gain will be $(R_1 + R_2 + \cdots + R_{20})/20$. These twenty assets are chosen to reflect the housing sectors at different states or even countries, and the bank's rocket scientists choose to model the R_i as independent random variables. According to this model, what is the probability that the bank becomes insolvent?

(c) Based on the calculations in part (b), the bank goes ahead with the diversified investment strategy. It turns out that a global economic phenomenon can affect the housing sectors in different states and countries simultaneously, and therefore the gains R_i are in fact positively correlated. Suppose that for every i and j where $i \neq j$, the correlation coefficient $\rho(R_i, R_j)$ is equal to $1/2$. What is the probability that the bank becomes insolvent? You can assume that $(R_1 + R_2 + \cdots + R_{20})/20$ is normal.