Lec-09

In quantum physics similar to probability everything is related.

Expectation value: mathematically it's nothing but ~~weighted~~ average.
weighted

In classical physics ~~sigl~~ single particle gives single value of momentum.
But in Quntum physics we calculate weighted average of momentum (expected value)

$\langle P \rangle$ (expected value)

Expectation value: this is the probabilistic expected value of the result/measurement of an experiment.
It can be taken as an average (weighted) of all the possible values, and it's not the most probable value of the measurement. It can also be zero.
It always depends upon where we are measuring expectation value.

$$\langle A \rangle = \int \psi^* A \psi \, dx$$

Can be $-\infty \to \infty$

$$\langle p \rangle = \int_{x=x_1}^{x=x_2} \psi^* \left(-i\hbar \frac{\partial \psi}{\partial x}\right) dx$$

$\langle H \rangle = ?$

Operator of H $= \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right)$

$$\langle H \rangle = \int \psi^* H \psi \, \partial x$$

$$= \int \psi^* \left(\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \cdot \psi\right) dx + \int \psi^* V \cdot \psi \, dx$$

$$\langle H \rangle = \langle K.E \rangle + \langle V \rangle$$

Lec-08 missing part

$A \psi = a \psi$ → eigen function

operator; a → eigen value.

~~unique~~.

we can't have diff. a (eigen value) associated with single ψ.

But if operator is changed then a will change.

- One wave function can't give two eigen values for the same operation. but two or more wave function can give same eigen value for the same operation. this is called degeneracy.

If two wave function then two fold degeneracy
If three ——— then three ———.

The wave function that give the same eigen value are called degenerate wave function

Lec-09 continues :—

operator $\overset{\text{const.}}{}$

$$A\psi = a\psi$$

$$\int_{-\infty}^{\infty} \psi^* A \psi dx = \int_{-\infty}^{\infty} \psi^* a \psi dx$$

$$\left(\langle A \rangle = \int_{-\infty}^{\infty} \psi^* A \psi dx \right)$$

$$\langle A \rangle = a \underbrace{\int_{-\infty}^{\infty} \psi^* \psi dx}_{\text{equal to } 1}$$

$$\langle A \rangle = a$$

↳ eigen value.

we can say that eigen value is nothing but expectation value of an operator.

for Hydrogen atom.

$$H\psi_1 = E_1 \psi_1$$

↳ 13.6 eV

in classical phsics we can say precisly 13.6 eV but in quantum we say it's expectation value.

→ time dependent schrodinger eqⁿ.

$$\left[\frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]\psi(x) = i\hbar\frac{d\psi}{dt}$$

~~when~~ (either $\psi(x)$ or $\psi(r)$)
In basic we use only time independent
so let's find time ~~time~~ independent eqⁿ.

$$\psi'(r,t) = \psi(r)\,\phi(t)$$

using above eqⁿ.

$$\underbrace{\frac{i\hbar}{\phi(t)}\frac{\partial(\phi t)}{\partial t}}_{\text{fun. of time}} = \underbrace{\frac{1}{\psi(r)}\left[\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(r)}_{\text{funct of } r.}$$

t/r are independent quantities
so ech side (LHS/RHS) must be
const and eqal to E (energy)

$$i\hbar\frac{d\psi}{dt} = E\psi \quad \Big| \quad H\psi = (K+V)\psi$$

$$\frac{1}{\psi(r)}\left[\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(r) = E \text{ (must be)}$$

$$\Rightarrow \left[\frac{-\hbar^2}{2m}\nabla^2 + V(r)\right]\psi(r) = E\,\psi(r)$$

no involvement of time.

$$H\psi = E\psi$$

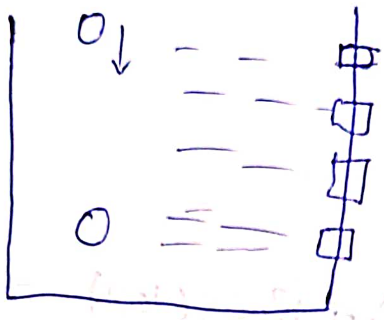
H → operator

E → eigen value

$$H = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(r)\right]$$

This was time independent schrodinger eqn.

→ eg. particle a box problem.

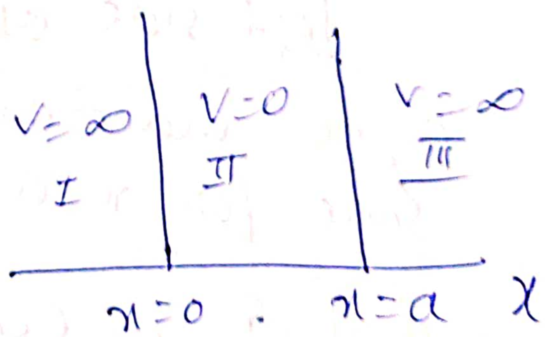


on classical physic
given energy $0 \to 100 keV$
accepted by ball $0 \to 100 keV$
energy continous

But in quantum physics heights are fixed. (Marked on boundary) but in classical it can have any height.

infinite box potential

$$V(x) = 0; \quad 0 < x \leq a$$
$$= \infty; \quad a < x < \infty$$
$$-\infty < x < 0$$



$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]\psi = E\psi$$

we solve schrodinger eqn in all three region.

In region I, III, $V = \infty$

so $\psi = 0$

i.e particle won't exist.

In region II $V = 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} = E\psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2mE}{\hbar^2}\psi \quad \left(\text{let } k^2 = \frac{2mE}{-\hbar^2}\right)$$

$$\frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0$$

we know its solution

$$\psi(x) = A\sin(kx) + B\cos(kx) \quad \text{— (3)}$$

$$\psi(x=0) \Rightarrow \psi(0) = 0 \quad \text{— (I)}$$

$$\psi(x=a) \Rightarrow \psi(a) = 0 \quad \text{— (II)}$$

left side of $x=0$, $V=\infty$ right side $V=0$

So, to maintain continuity at $x=0$, $\psi(0)=0$

Same for $x=a$ too

Putting (I) in eqn (3).

$$\psi(0) = 0 + B = 0$$

So, $B = 0$.

rewriting eq^n (3)

$$\psi = A \sin kx \quad - (4)$$

now put (II) in eq^n (4)

$$\psi(a) = A \sin ka = 0$$

for this, $ka = n\pi$

$\hookrightarrow n = 1, 2, 3, \ldots$

$$\sqrt{\frac{2mE}{\hbar^2}}\, a = n\pi$$

$$\boxed{E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}} \quad n = 1, 2, 3, \ldots$$

a = size of well/box.

m = mass of particle

$$E_1 \text{ (ground state energy)} = \frac{\pi^2 \hbar^2}{2ma^2}$$

$$E_2 = \frac{4\pi^2 \hbar^2}{2ma^2}$$

from this we can say all energy are not defined only certain energies are defined.

In quantum physics nothing will have zero energy.