INDIAN INSTITUTE OF TECHNOLOGY BHILAI

CS203: Theory of Computation I

**Tutorial Sheet 2**

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- *Solve the following problems before the Tutorial.*

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1. Two DFAs, $M_1$ and $M_2$ are given below. Answer the following questions for each of these machines.

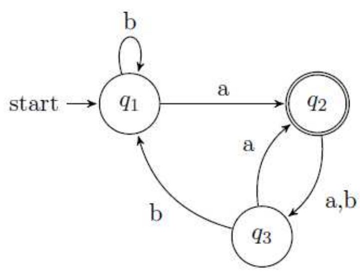



Figure 1: $M_1$

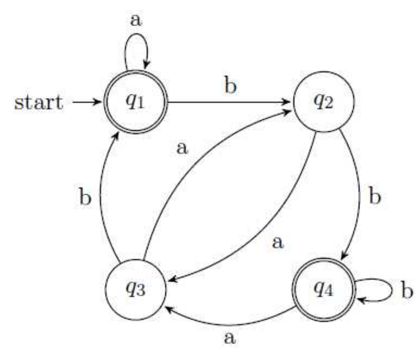



Figure 2: $M_1$

(a) What is the start state?

(b) What is the set of accept states?

(c) What sequence of states does the machine go through on input *aabb*?

(d) Does the machine accept the string *aabb*?

(e) Does the machine accept the string $\epsilon$?

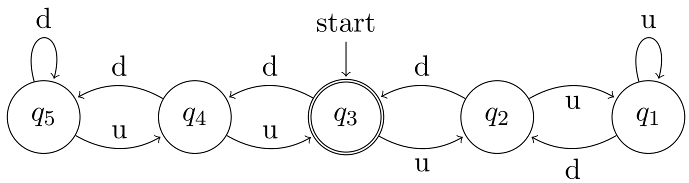
**Solution:**

| Part | Question | $M_1$ | $M_2$ |
|---|---|---|---|
| a | Start State ? | $\{q_1\}$ | $\{q_1\}$ |
| b | Accepting States | $\{q_2\}$ | $\{q_1, q_4\}$ |
| c | Sequence of states for input aabb | $q_1q_2q_3q_1q_1$ | $q_1q_1q_1q_2q_4$ |
| d | Acceptance for string aabb | No | Yes |
| e | Acceptance for string $\epsilon$ | No | Yes |

2. The formal description of a DFA, $M$ is $(\{q_1, q_2, q_3, q_4, q_5\}, \{u, d\}, \delta, q_3, \{q_3\})$, where $\delta$ is given by the following table. Give the state diagram of this machine.

| | $u$ | $d$ |
|---|---|---|
| $q_1$ | $q_1$ | $q_2$ |
| $q_2$ | $q_1$ | $q_3$ |
| $q_3$ | $q_2$ | $q_4$ |
| $q_4$ | $q_3$ | $q_5$ |
| $q_5$ | $q_4$ | $q_5$ |

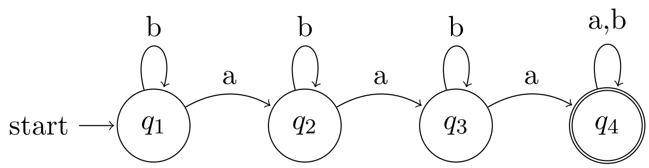
**Solution:**




3. Construct and give the state diagram of DFAs for the following given languages. In all parts $\Sigma = \{a, b\}$.

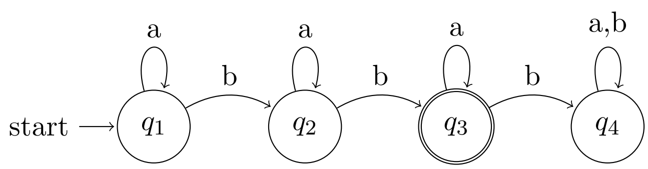
   (a) $\{\ w|w$ has at least three $a$'s$\}$

   **Solution:**

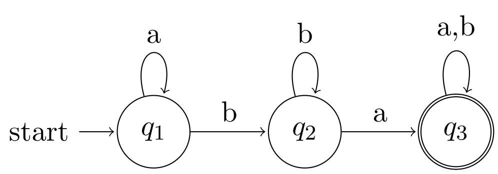
   (b) $\{\ w|w$ has exactly two $b$'s$\}$

   **Solution:**

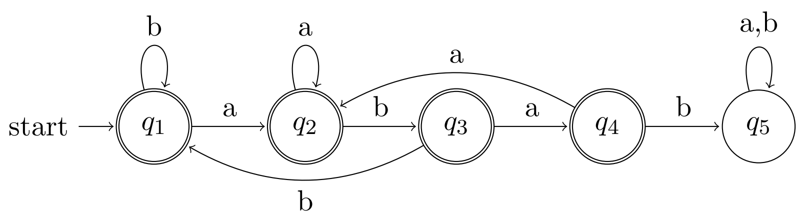
   (c) $\{\ w|w$ contains substring $ba\}$

   **Solution:**

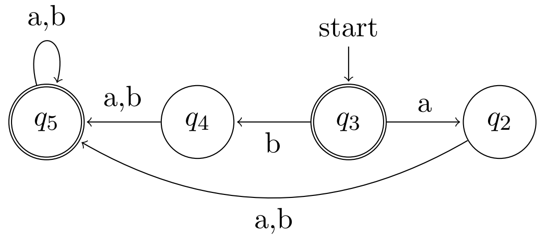
   (d) $\{\ w|w$ does not contain substring $abab\}$

   **Solution:**

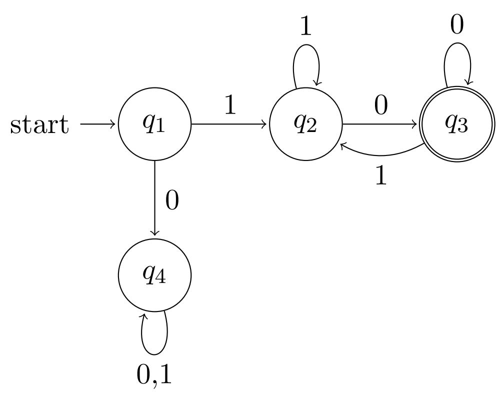
(e) $\{\ w | w$ is any string except $a$ and $b\}$

**Solution:**



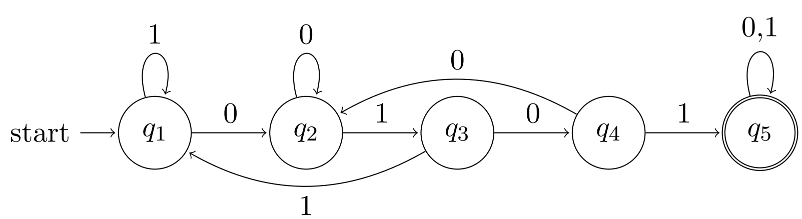

4. Give state diagrams of DFAs recognizing the following languages.
   In all parts the alphabet $\Sigma = \{0, 1\}$.

(a) $\{\ w | w$ begins with a 1 and ends with a 0$\}$



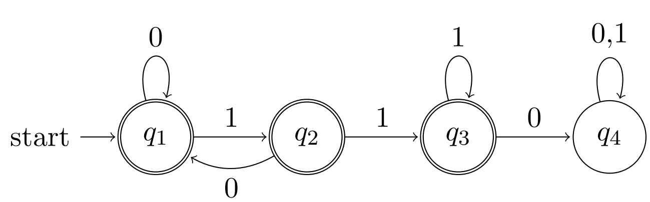

**Solution:**

(b) $\{\ w | w$ contains the substring 0101, i.e., $w = x0101y$ for some $x$ and $y\}$



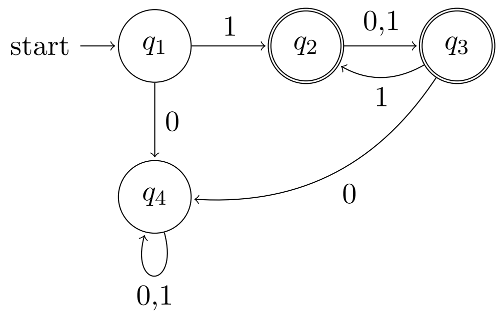

**Solution:**

(c) $\{\ w | w$ does not contain the substring 110$\}$



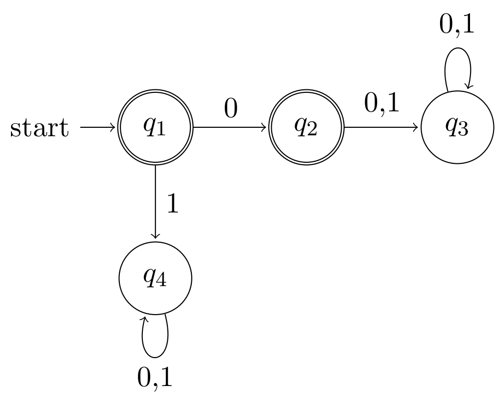

**Solution:**

(d) $\{\ w |$ every odd position of $w$ is a 1$\}$




**Solution:**

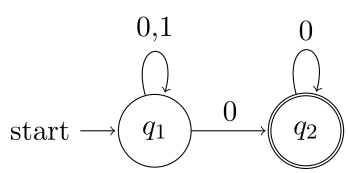
(e) $\{\epsilon, 0\}$




**Solution:**

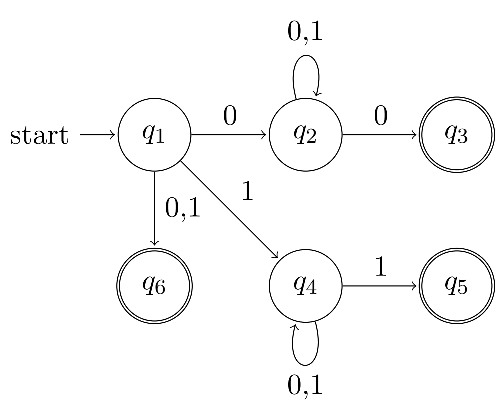
5. Construct and give the state diagram of NFAs for the following given languages.

(a) Set of all string in {0,1} which are the binary representation of integers divisible by 2.



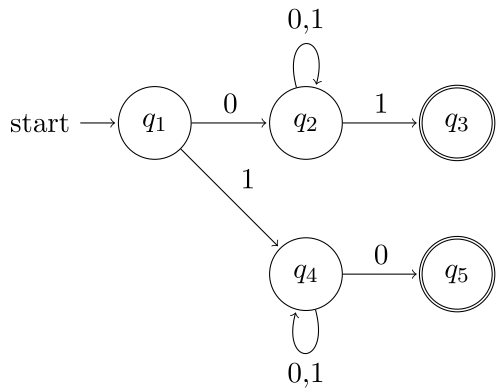

**Solution:**

(b) Set of strings consisting all strings over $\{0, 1\}$ starts and ends with same symbol.




**Solution:**

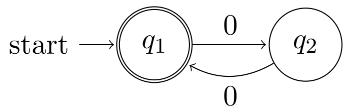
(c) Set of strings consisting all strings over $\{0, 1\}$ does not start and end with same symbol.



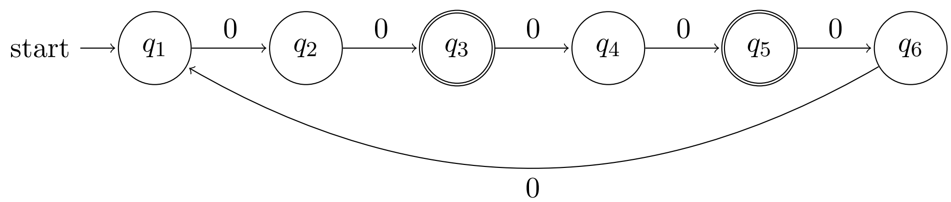

**Solution:**

(d) Set of strings consisting all strings over $\{0\}$ of the form $0^k$, for $k$ is even.

**Solution:**

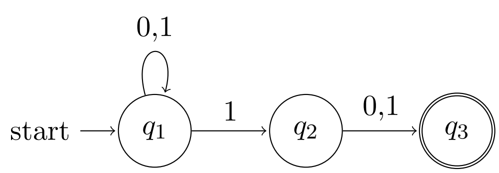



(e) Set of strings consisting all strings over $\{0\}$ of the form $0^k$, for $k$ is even and not divisible by 3.



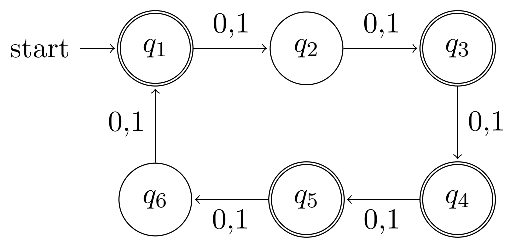

**Solution:**

(f) Set of strings consisting all strings over $\{0, 1\}$ containing an 1 in the 2nd position from end.




**Solution:**

(g) Set of strings over $\{0, 1\}$ of length either divisible by 2 or 3.




**Solution:**

6. Prove that if $M_1 = \{Q, \sum, \delta, q_0, F\}$ is a DFA recognizes a language $A$, Then $M_1 = \{Q, \sum, \delta, q_0, Q \backslash F\}$ recognizes $A^c$.

**Solution:** $M_1 = \{Q, \sum, \delta, q_0, F\}$ is a DFA recognizes a language $A$.

$M_2 = \{Q, \sum, \delta, q_0, Q \backslash F\}$ recognizes $A^c$.

Reasons why $M_2$ recognizes $A^c$. Since $M_1$ and $M_2$ have the same transition function $\delta$, therefore if $M_1$ is deterministic, $M_2$ is also deterministic. Consider any string $w \in \sum^*$. Running $M_1$ on string $w$ will result in $M_1$ ending in some state $q_r \in Q$. Since $M_1$ is deterministic, there is only one possible state that $M_1$ can end in on input $w$. If we run $M_2$ on the same input $w$, then $M_2$ will end in the same state $q_r$ since $M_1$ and $M_2$ have the same transition function. Also, since $M_2$ is deterministic, there is only one possible ending state that $M_2$ can be in on input $w$.

Now suppose that $w \in A$. Then, $M_1$ will accept $w$, which means that the ending state $q_r \in F$, i.e., $q_r$ is an accept state of $M_1$. But then $q_r \notin Q \backslash F$, so $M_2$ does not accept $w$ since $M_2$ has $Q \backslash F$ as its set of accept states. Similarly, suppose that $w \notin A$. Then, $M_1$ will not accept $w$, which means that the ending state $q_r \notin F$. But then $q_r \in Q \backslash F$, so $M_2$ accepts $w$. Therefore, $M_2$ accepts string $w$ if and only $M_1$ does not accept string $w$, so $M_2$ recognizes language $A^c$.