Indian Institute of Technology Bhilai

IC105: Probability and Statistics
Tutorial 6

February 7, 2022

1. A two dimensional discrete random vector (X, Y) having pmf as

$$f_{X,Y}(x, y) = P(X = x, Y = y) = \begin{cases} c(3x + 4y), & x = 0, 1, 2, 3, \ y = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the value of c.

   (b) Find the marginal distribution of X and Y.

   (c) Find $P(X \geq 2|Y \leq 3)$ and $P(Y = 2|X = 3)$.

2. Let (X, Y) be a random vector with the joint pdf

$$f_{X,Y}(x, y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, \ 2 < y < 4, \\ 0, & \text{otherwise.} \end{cases}$$

   (a) Find the marginal distribution of X and Y.

   (b) $P(X < 1, Y < 3)$, $P(X + Y < 3)$, $P(X < 1|Y = 3)$ and $P(X < 1|Y < 3)$.

3. The conditional p.d.f. of X given $Y = y \ (> 0)$ is $f_{X|Y}(x|y) = \begin{cases} ye^{-yx}, & x > 0, \\ 0, & \text{otherwise} \end{cases}$ and the

   marginal p.d.f. of Y is $f_Y(y) = \begin{cases} \alpha e^{-\alpha y}, & y > 0, \\ 0, & \text{otherwise.} \end{cases}$ Find the conditional p.d.f. of Y given $X = x$.

4. (a) Six fair dice are thrown independently. Let S denote the number of dice showing even numbers on their upper faces. Find the mean and the variance of S.

   (b) Let X_1 and X_2 be independent random variables with moment generating function $M_1(t) = \left(\frac{3}{4} + \frac{1}{4}e^t\right)^3$ and $M_2(t) = e^{2(e^t - 1)}$, respectively. Find the value of $P(X_1 + X_2 = 1)$.

5. An absent-minded professor schedules two student appointments for the same time. The appointment durations are independent and exponentially distributed with mean thirty minutes. The first student arrives on time, but the second student arrives five minutes late. What is the expected value of the time between the arrival of the first student and the departure of the second student?