Indian Institute of Technology Bhilai
CS203: Theory of Computation I
**Tutorial Sheet 4**

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- *Solve the following problems before the Tutorial.*

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1. Construct a transition system (i.e., NFA or DFA) corresponding to the regular expressions
   (a) $(ab + c^*)^*b$
   (b) $a + bb + bab * a$

2. Represent the following sets by regular expression:
   (a) $\{0, 1, 2\}$ where $\Sigma = \{0, 1, 2\}$
   (b) $\{1^{2n+1}|n > 0\}$ where $\Sigma = \{1\}$
   (c) $\{w \in \{a, b\}^*|w$ has only one $a\}$
   (d) The set of all strings over {0,1} which has at most two zeros.
   (e) $\{a^2, a^5, a^8, \cdots\}$ where $\Sigma = \{a\}$
   (f) $\{a^n|n$ is divisible by 2 or 3 or $n = 5\}$
   (g) The set of all strings over $\{a, b, c\}$ beginning with $a$ and ending not with $a$ or $c$.

3. Find a DFA for each of the following regular languages.
   Then convert the DFA into an equivalent regular expression.
   (a) $\{\ w \mid w$ is any string not in $a^*b^*\}$
   (b) $\{\ w \mid w$ does not end with $ab\ \}$
   (c) $\{\ w \mid w$ is starts with $ab$ or has a substring $bba\}$
   (d) $\{\ w \mid w$ is starts with $ab$ and has a substring $bba\}$
   (e) $\{\ w \mid w$ is starts with $ab$ or not has a substring $bba\}$
   (f) $\{\ w \mid w$ is starts with $ab$ and not a substring $bba\}$
   (g) $\{\ w \mid w$ starts with $a$ and has odd length or starts with $b$ and of even length$\}$
   (h) $\{\ w \mid w$ contains neither substring $ba$ nor $ab\}$
   (i) $\{\ w \mid$ in $w$, every a is immediately followed by $bb\}$

4. Prove by Pumping Lemma that the following languages are not regular.
   (a) $\{a^nb^n|\ n \geq 0\}$
   (b) $\{a^p|\ p$ is a prime$\}$
   (c) $\{a^q|\ q$ is a perfect square$\}$
   (d) $\{a^nba^mba^pba^q|\ n, m, p \geq 1$ and $q \equiv nm(\text{mod p})\}$
   (e) $\{a^nb^ma^pb^q|\ n + m = p + q$ and $p, q, m, n \geq 0\}$
   (f) $\{a^{pq}|\ p, q$ are primes$\}$
   (g) $\{w \in \{a + b\}^*|\ |w|_0 = |w|_1\ \}$
   (h) $\{ww|\ w \in \{a + b\}^*\ \}$
   (i) $\{w|\ w \in \{a + b\}^*$ and $w = w^R\ \}$

5. Prove by Pumping Lemma that the following languages are not regular.
   (a) $\{a^nb^n|\ n \geq 0\}$
   (b) $\{a^p|\ p$ is a prime$\}$
   (c) $\{a^q|\ q$ is a perfect square$\}$
   (d) $\{a^nba^mba^pba^q|\ n, m, p \geq 1$ and $q \equiv nm(\text{mod p})\}$
   (e) $\{a^nb^ma^pb^q|\ n + m = p + q$ and $p, q, m, n \geq 0\}$

(f) $\{a^{pq} |\ p, q \text{ are primes}\}$

(g) $\{w \in \{a+b\}^* |\ n_a(w) = n_b(w)\ \}$

(h) $\{ww |\ w \in \{a+b\}^*\ \}$

(i) $\{w |\ w \in \{a+b\}^* \text{ and } w = w^R\ \}$

(j) $\{a^n b^m a^n | m, n \geq 0\}$

(k) $\{a^n b^m | m \neq n\}$

(l) $\{w | w \in \{a, b\}^* \text{ is not palindrome }\}$

(m) $\{wtw | w, t \in \{a, b\}^+\}$

6. Give context-free grammars that generate the following languages.

   (a) $L = \{w \in \{0,1\} | w = w^R \text{ and } |w| \text{ is even }\}$.

   (b) $L = \{w \in \{0,1\}^* |$ the length of $w$ is odd and the middle symbol is 0 $\}$.

   (c) $L = \{a^i b^j c^k | i, j, k \geq 0, \text{ and } i = j \text{ or } i = k\ \}$.

   (d) $\phi$.

   (e) $L = \{a^i b^j c^k d^l | i, j, k, l \geq 0 \text{ and } i + j = k + l\ \}$.

   (f) $L = \{a^m b^n | 3m \leq 5n \leq 4m\}$.

   (g) $L = \{w \in \{a+b\}^* |$ each prefix of $w$ has at least as many $a$'s as $b$'s $\}$.

   (h) $L = \{a^m b^n | m \geq n\}$.

   (i) $L = \{a^m b^n | m \leq n\}$.