Quiz-2
IC202: Calculus II

Full Marks-25 | Time: 45 Minutes

1. Let $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $f(x, y) = 1 - x^2y^2$. Then which of the following is true?
   (a) Determinant of the Hessian matrix at $(0, 0)$ is positive
   (b) at $(0, 0)$ the function has a local maximum
   (c) $(0, 0)$ is a saddle point
   (d) at $(1, 1)$ the function has a local maximum

2. Consider the function $f(x, y) = x^2+3y^2$ defined on the disk $D = \{(x, y) \in \mathbb{R}^2 : (x-1)^2+y^2 \leq 4\}$. Which of the following is true?
   (a) $(0, 0) \in D$, is a saddle point of f,
   (b) both maximum and minimum of f occurs on the boundary of D,
   (c) the minimum of f occurs at $\left(\frac{3}{2}, -\frac{\sqrt{15}}{2}\right)$,
   (d) $f(x, y)$ cannot be more than 14 on D.

3. Consider the function $f(x, y) = y \sin x$ defined over whole $\mathbb{R}^2$. Then which of the following is true?
   (a) there are no critical point of f,
   (b) $(0, 0)$ is a local maximum of f,
   (c) there are infinitely many saddle points of f,
   (d) not all critical points are saddle point of f.

4. Consider the function $f(x, y) = 192x^3 + y^2 - 4xy^2$ defined on the triangle D with vertices $(0, 0)$, $(4, 2)$, and $(-2, 2)$.
   (a) there are three critical points of f in the interior of D,
   (b) f does not have absolute maximum on D,
   (c) f is unbounded on D
   (d) f does not have absolute maximum and absolute minimum in the interior of D.

5. Consider the function $z = f(x, y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$ defined over the plane $5x + y = 150$. Which of the following is true?
   (a) $f(x, y)$ cannot be less than 45,
   (b) at $(27, 64)$ the function f has maximum,
   (c) at $(10, 100)$ the function f has maximum,
   (d) none of the above.

6. Consider the function $(x, y) = 8x^3 + y^3 + 6xy$. Which of the following is true?
   (a) every neighbourhood of $\left(-\frac{1}{2}, -1\right)$ contains few points for which $f(x, y) > f(-\frac{1}{2}, -1)$,
   (b) there exists a neighbourhood of $\left(-\frac{1}{2}, -1\right)$ in which $f(x, y) > f(-\frac{1}{2}, -1)$, for all (x, y) in that neighbourhood
   (c) every neighbourhood of $(0, 0)$ contains few points for which $f(x, y) < f(0, 0)$,

(d) there exists a neighbourhood of $(0,0)$ in which $f(x,y) < f(0,0)$, for all (x,y) in that neighbourhood

7. Let the curve $xy^2 = 54$ is denoted by C. Which of the following is true?

   (a) the point $(3, -3\sqrt{2})$ on C is farthest from the origin,

   (b) the point $(3, 3\sqrt{2})$ on C is farthest from the origin,

   (c) there is no point on C which is furthest from the origin

   (d) there is no point on C which is nearest to the origin

8. Let the function $w = f(x,y,z) = xyz$ be defined on the intersection of two planes $x+y+z = 40$ and $x+y = z$. Which of the following is true?

   (a) the maximum of w exists at more than one point,

   (b) the minimum of w exists at one point only,

   (c) the value of f on the intersection of the two planes cannot be less than 0,

   (d) the value of f on the intersection of the two planes cannot be more than 2000,

9. Let $f(x,y)$ be any arbitrary function having continuous first and second order partial derivatives in a neighbourhood of a point (x_0, y_0) such that $f_{xx}(x_0,y_0)f_{yy}(x_0,y_0) - (f_{xy}(x_0,y_0))^2 = 0$. Then which of the following is true?

   (a) f has a local maximum at (x_0, y_0),

   (b) f has a local minimum at (x_0, y_0),

   (c) f has a saddle at (x_0, y_0),

   (d) anyone of the above may be true.

10. Consider the problem to find the maximum/minimum of $f(x,y,z) = xyz$ subject to, $g(x,y,z) = x + 9y^2 + z^2 = 4$. Let S denotes the solution set of the Lagrange multiplier equation $\bigtriangledown f = \lambda \bigtriangledown g$ and the constraint (where λ be the Lagrange multiplier) then which of the following is true for any $(x,y,z) \in S$?

    (a) $\lambda = 0$ implies $z^2 = 18y^2$,

    (b) $x^2 = 18y^2$,

    (c) $\lambda \neq 0$ implies $z^2 = 9y^2$,

    (d) none of the above.