Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Tutorial 4

January 27, 2022

1. A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.
   (a) If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.
   (b) Find the probability that the next train to arrive after the first train on day 0, takes more than 3 days to arrive.
   (c) Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4th day.
   (d) Find the probability that it takes more than 2 days for the 5th train to arrive at the bridge.

2. Consider 10 independent tosses of a biased coin with a probability of heads of p.
   (a) Let A be the event that there are 6 heads in the first 8 tosses. Let B be the event that the 9th toss results in heads. Show that events A and B are independent.
   (b) Find the probability that there are 3 heads in the first 4 tosses and 2 heads in the last 3 tosses.
   (c) Given that there were 4 heads in the first 7 tosses, find the probability that the 2nd head occurred during the 4th trial.
   (d) Find the probability that there are 5 heads in the first 8 tosses and 3 heads in the last 5 tosses.

3. Buses arrive at a specified stop at 15-minute intervals starting at 7 am. That is, they arrive at 7, 7 : 15, 7 : 30, 7 : 45, and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 7 and 7 : 30, find the probability that he waits (a) less than 5 minutes for a bus; (b) more than 12 minutes for a bus.

4. If a string of 2 meter is cut into two pieces at a random point along its length, what is the probability that the length of the longer piece is at least twice the length of the shorter piece?

5. A small industrial unit has 10 bulbs whose lifetimes are independent exponentially distributed with mean 50 hours. If all the bulbs are used at a time, find the probability that even after 100 hours there are at least two bulbs working.