(1) X be a r.v with d.f $F_X(x)$.

$Y_1 = |X|$ $\quad F_{Y_1}(y_1) = P(Y_1 \le y_1) = 0$ if $y_1 < 0$

$= P(-y_1 \le X \le y_1)$, if $y_1 \ge 0$

$= F_X(y_1) - P(X < -y_1)$

$= F_X(y_1) - F_X(-y_1) + P(X = -y_1)$

So the c.d.f. of Y_1 is

$$F_{Y_1}(y_1) = \begin{cases} 0, & y_1 < 0 \\ F_X(y_1) - F_X(-y_1) + P(X = -y_1), & y_1 \ge 0 \end{cases}$$

consider $Y_2 = aX + b$, $a \ne 0$, $b \in \mathbb{R}$

$$F_{Y_2}(y_2) = P(Y_2 \le y_2) = P(aX + b \le y_2)$$

$$= \begin{cases} P\left(X \le \frac{y_2 - b}{a}\right) & \text{if } a > 0 \\ P\left(X \ge \frac{y_2 - b}{a}\right) & \text{if } a < 0 \end{cases}$$

$$= \begin{cases} F_X\left(\frac{y_2 - b}{a}\right) & \text{if } a > 0 \\ 1 - F_X\left(\frac{y_2 - b}{a}\right) + P\left(X = \frac{y_2 - b}{a}\right), & \text{if } a < 0 \end{cases}$$

max (x, 0) is alway grater than equl to zero.

$$Y_3 = \max(x,0) = \begin{cases} x & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

$$F_{Y_3}(y_3) = P(Y_3 \le y_3) = \begin{cases} 0, & \text{if } y_3 < 0 \\ P(Y_3 \le 0) = P(Y_3 < 0) + P(Y_3 = 0) & \text{if } y_3 = 0 \\ P(Y_3 \le y_3) = P(Y_3 < 0) + P(Y_3 = 0) + P(0 < Y_3 \le y_3) & \text{if } y_3 > 0 \end{cases}$$

$\max(X, 0) = 0$ equivalent to $X \le 0$.

$$= \begin{cases} 0, & \text{if } y_3 < 0 \\ P(X \le 0) & \text{if } y_3 = 0 \\ P(X \le 0) + P(0 \le X \le y_3) & \text{if } y_3 > 0 \end{cases}$$

$$= \begin{cases} 0, & y_3 < 0 \\ F_X(0), & y_3 = 0 \\ F_X(y_3), & y_3 > 0 \end{cases} = \begin{cases} 0, & y_3 < 0 \\ F_X(y_3), & y_3 \geq 0. \end{cases}$$

$$Y_4 = \min(X, 0) = \begin{cases} X, & X < 0 \\ 0, & X \geq 0. \end{cases}$$

[min (x,0) is either 0 or lessthan 0.] So min(x,0) is always less than a (+ive no. or zero.]

$$P(Y_4 \le y_4) = \begin{cases} P(Y_4 \le y_4) = 1 & \text{if } y_4 \geq 0 \\ P(X \le y_4) & \text{if } y_4 < 0. \end{cases}$$

$$P(Y_4 \le y_4) = \begin{cases} 1 & \text{if } y_4 \geq 0 \\ P(X \le y_4) & \text{if } y < 0 \end{cases}$$

(2) The p.m.f. of X is given as $P(X=-2)=\frac{1}{5}$, $P(X=-1)=\frac{1}{6}$

$P(X=0)=\frac{1}{5}$, $P(X=1)=\frac{1}{15}$, $P(X=2)=\frac{11}{30}$

Let $Y=X^2$. Then the ~~set~~ $Y \in \{0,1,4\}$

$$P(Y=y)=\begin{cases}\frac{1}{5}, & y=0\\ \frac{1}{6}+\frac{1}{15}, & y=1\\ \frac{1}{5}+\frac{11}{30}, & y=4\end{cases} = \begin{cases}\frac{1}{5}, & y=0\\ \frac{7}{30}, & y=1\\ \frac{17}{30}, & y=2\end{cases}$$

C.d.f. of Y is given as

$$F_Y(y)=\begin{cases}0, & y<0\\ \frac{1}{5}, & 0\le y<1\\ \frac{1}{5}+\frac{7}{30}, & 1\le y<2\\ 1 & y\geq 2\end{cases} = \begin{cases}0, & y<0\\ \frac{1}{5}, & 0\le y<1\\ \frac{13}{30}, & 1\le y<2\\ 1, & y\geq 2.\end{cases}$$

(3) The p.d.f. of X is given as

$$f_X(x)=\begin{cases}\frac{1}{2}, & -1<x<1\\ 0, & o/w.\end{cases}$$

Let $Y=\max(X,0)$. Apply question 1 we have

$$P(Y\le y)=\begin{cases}0, & y<0\\ F_X(y), & y\geq 0\end{cases} = \begin{cases}0, & y<0\\ \frac{1}{2} & y=0\\ \frac{1}{2}+\frac{y}{2} & 0<y\le 1\\ 1, & y>1\end{cases}$$

(4) $f_X(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty$

$Y = X^2$, So we have $h(x) = x^2$ &

Now $h(x)$ is strictly decreasing in $(-\infty, 0)$ with inverse $h^{-1}(y) = -\sqrt{y}$

Again $h(x)$ is strictly increasing in $(0, \infty)$ with inverse $h^{-1}(y) = \sqrt{y}$.

Also we have $h(-\infty, 0) = h(0, \infty) = (0, \infty)$

Then the density of Y is given as

$$f_Y(y) = f_X(-\sqrt{y}) \left| -\frac{1}{2\sqrt{y}} \right| + f_X(\sqrt{y}) \left| \frac{1}{2\sqrt{y}} \right|, \quad y \in (0, \infty)$$

$$= \frac{1}{2} e^{-\sqrt{y}} \frac{1}{2\sqrt{y}} + \frac{1}{2} e^{\sqrt{y}} \frac{1}{2\sqrt{y}}, \quad y \in (0, \infty)$$

$$= \frac{1}{2\sqrt{y}} e^{-\sqrt{y}}, \quad 0 < y < \infty.$$

(5) $f_X(x) = \begin{cases} c(x+1), & -1 \le x \le 2 \\ 0, & o/w \end{cases}$

$$\int_{-\infty}^{\infty} f_X(x)\, dx = 1 \Rightarrow c = \frac{2}{9}.$$

$$f_X(x) = \begin{cases} \frac{2}{9}(x+1), & -1 \le x \le 2 \\ 0, & o/w \end{cases}$$

$Y = X^2 = h(X)$

$h(x)$ is strictly decreasing in $(-1, 0)$ with inverse

$h^{-1}(y) = -\sqrt{y}$, & $h(-1, 0) = (0, 1)$

$h(x)$ is strictly increasing in $(0, 2)$ with inverse

$h^{-1}(y) = \sqrt{y}$, & $h(0, 2) = (0, 4)$.

$$f_Y(y) = f_X(-\sqrt{y}) \left|\frac{d}{dy}(-\sqrt{y})\right| I_{(0,1)}(y) + f_X(\sqrt{y}) \left|\frac{d}{dy}(\sqrt{y})\right| I_{(0,4)}(y)$$

$$= \begin{cases} \frac{2}{9}(-\sqrt{y}+1)\frac{1}{2\sqrt{y}} + \frac{2}{9}(\sqrt{y}+1)\frac{1}{2\sqrt{y}}, & 0 < y < 1 \\ \frac{2}{9}(\sqrt{y}+1)\frac{1}{2\sqrt{y}}, & 2 < y < 4 \\ 0, & o/w. \end{cases}$$

$$= \begin{cases} \frac{2}{9\sqrt{y}}, & 0 < y < 1 \\ \frac{\sqrt{y}+1}{9\sqrt{y}}, & 1 < y < 4, \\ 0, & o/w. \end{cases}$$

[Finding c.d.f do yourself.]

(6) $f_X(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0, & o/w \end{cases}$

$Y = 40(1-X) = h(X)$. $h(x)$ is strictly increasing in $(0,1)$.

$h^{-1}(y) = x = \left(1 - \frac{y}{40}\right)$, $\frac{d h^{-1}(y)}{dy} = -\frac{1}{40}$.

$x \in (0,1)$ then $y \in (0, 40)$.

$f_Y(y) = \begin{cases} \frac{3}{40}\left(1 - \frac{y}{40}\right)^2, & 0 < y < 40 \\ 0, & o/w \end{cases}$

(7) X = number of female applicants among the final 5.

$X = 0, 1, 2, 3, 4, 5$

$P(X = i) = \dfrac{\binom{9}{i}\binom{6}{5-i}}{\binom{15}{5}}$, $i = 0, 1, 2, 3, 4, 5$

Let Y = number of male applicants $= (5 - X)$

$Y = 0, 1, 2, 3, 4, 5$

$$P(Y=y) = P(\cancel{5}-\cancel{X}=y) = P(X=5-y)$$

$$= \cancel{g} \frac{\binom{9}{5-y}\binom{6}{y}}{\binom{15}{5}}, \quad y = 0, 1, 2, 3, 4, 5.$$

(8) X be a r.v. with $E(X) = 3 = \mu$.

$E(X^2) = 13 \Rightarrow \sigma^2 = E(X^2) - (E(X))^2 = 13 - 9 = 4$

$Var(X) = 4$.

$$P(-2 < X < 8) = P(-2-3 < X-3 < 8-3)$$
$$= P(|X-3| < 5) = 1 - P(|X-3| \geq 5)$$

By chebyshev inequality

$$P(|X-3| \geq 5) \leq \frac{4}{25}$$

$$\Rightarrow 1 - P(|X-3| \geq 5) \geq 1 - \frac{4}{25} = \frac{21}{25}.$$

(9) X be a r.v. with m.g.f.

$$M(t) = \frac{e^{-2t}}{8} + \frac{e^{-t}}{4} + \frac{e^{2t}}{8} + \frac{e^{3t}}{2}$$

$$= \cancel{g}\ P(X=-2)e^{-2t} + P(X=-1)e^{-1\cdot t} + P(X=2)e^{2\cdot t} + P(X=3)e^{3\cdot t}$$

$P(X=-2)=\frac{1}{8}$, $P(X=1)=\frac{1}{4}$, $P(X=2)=\frac{1}{8}$, $P(X=3)=\frac{1}{2}$

$$P(X^2=4) = P(X=-2)+P(X=2)$$
$$= \frac{1}{8}+\frac{1}{8} = \frac{1}{4}.$$

(10) If $t>0$, $g(x)=e^{tx}$ is positive, increasing in x.

Hence $$P(X \geq a) = P(e^{tX} \geq e^{ta})$$
$$\leq \frac{E(e^{tX})}{e^{ta}} \quad \text{[By Markov's inequality]}$$
$$= e^{-at}M(t).$$

If $t<0$, then $h(x)=e^{tx}$ is positive, decreasing

and hence

$$P(X \leq a) = P(e^{tX} \geq e^{at}) = e^{-at}M(t)$$

(11) 15% of items produced at a manufacturing facility are defective

Take $p = \frac{15}{100}$

Let X denote the no. of defective items in a lot of 10 items. So $X \sim \text{Bin}\left(10, \frac{15}{100}\right)$

Required probability is

$$P(X>3) = 1-\sum P(X\leq 3)$$

$$= 1-\sum_{i=0}^{3}\binom{10}{i}(p)^{i}(1-p)^{10-i}$$

(12) Let X no of train arriving or departing from a railway Station. Then $\lambda = 1/5$

Then $X \sim \mathcal{P}\left(\frac{1}{5}\right)$

(a) ~~Poss~~ ~~Pef~~ $\lambda t = \frac{1}{5}\times 60 = 12$

$$P(X\geq 0) = 1-P(X<10) = 1-\sum_{k=0}^{9}\frac{12^{k}e^{-12}}{k!}$$

$$P(X\leq 4) = \sum_{k=0}^{3}\frac{12^{k}e^{-12}}{k!}.$$

(13) Let P_n denote the probability of an n-component system operate effectively

~~B~~ X be the number of components functioning in a n component system.

$P_3 = P(X > 1.5) = \binom{3}{2} p^2 (1-p) + \binom{3}{3} p^3$

$P_5 = P(X > 2.5) = \binom{5}{3} p^3 (1-p)^2 + \binom{5}{4} p^4 (1-p) + p^5$

5 component system is better if

$P(X > 2.5) > P(X > 2.5)$

$10 p^3 (1-p)^2 + 5 p^4 (1-p) + p^5 > 3 p^2 (1-p) + p^3$

$\Rightarrow 3(p-1)^2 (2p-1) > 0 \Rightarrow p > 1/2$

(14) The DVD produced by a company are defective with prob $p = 0.01$, independently each other

Let X = no of defective DVD in a pack of 10 DVD

$X \sim Bin(10, p)$. The prob that a pack will be ~~rege~~ returned is

$p_1 = P(X > 1) = 1 - P(X \le 1) = 1 - P(X=0) - P(X=1)$

$= 1 - (0.01)^0 (0.99)^{10} - 10 (0.01)^1 (0.99)^9$

Y = no of pack will be reduced form 3 packs.

$Y \sim Bin(3, p_1)$.

Required prob. is ~~$P(2X \leq 1)$~~

$$P(Y \leq 1) = P(Y=0) + P(Y=1)$$

(15) Let A be the event that person gets a cold

B denote the event drug is beneficial to him.

X be the number of times an individual contracts the clod in a yeare

$X|B \sim \mathcal{P}(2)$, $X|B^c \sim \mathcal{P}(3)$

P(A person does not get cold | drug is benificial) $= P(A^c|B^c)$

$$= P(X=0|B) = e^{-2}$$

P(A person not get cold | drug is not benificial) $= P(A^c|B^c)$

$$P(X=0|B^c) = e^{-3}$$

$P(B) = 0.75$, $P(B^c) = 0.25$

P(Drug is benificial to him | the person does not get cold)

$$= P(B|A^c) = \frac{P(A^c|B)\,P(B)}{P(A^c|B)\,P(B) + P(A^c|B^c)\,P(B^c)}$$