

# Indian Institute of Technology Bhilai

## IC105: Probability and Statistics

### Assignment 6

February 11, 2022

1. The joint pdf of  $X$  and  $Y$  is given as

$$f_{X,Y}(x,y) = \begin{cases} k(1-x-y), & x > 0, y > 0, x+y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Find the  $E(X|Y = 1/2)$  and  $E(Y|X = 1/2)$ .

2. A two dimensional discrete random vector  $(X, Y)$  having p.m.f. as

$$f_{X,Y}(x,y) = P(X=x, Y=y) = \begin{cases} c(3x+4y), & x=0,1,2,3, y=1,2,3,4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the  $E(X|Y=1)$  and  $E(Y|X=0)$ .

3. The joint p.d.f. of  $X$  and  $Y$  is given as

$$f_{X,Y}(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, 2 < y < 4, \\ 0, & \text{otherwise.} \end{cases}$$

Find the  $E(X|Y=3)$  and  $E(Y|X=1)$ .

4. Let  $X$  and  $Y$  be two independent  $U(0,1)$  random variables. Then find the distributions of

(a)  $XY$ ,

(b)  $\frac{X}{Y}$ .

5. Let  $X$  be Poisson  $Po(2)$  and  $Y$  be  $Bin(10, 3/4)$  random variables. If  $X$  and  $Y$  are independent, then find the value of  $P(XY=0)$ .

6. The p.m.f. of a two dimension discrete random vector  $(X, Y)$  is given as

Y/X	-1	0	1
-2	1/6	1/12	1/6
1	1/6	1/12	1/6
2	1/12	0	1/12

Find the joint distribution of  $|X|$  and  $Y^2$ .

7. Let  $X$  and  $Y$  be independent and identically distributed uniform random variable over the interval  $(0, 1)$  and let  $S = X + Y$ . Find the probability that the quadratic equation  $9x^2 + 9Sx + 1 = 0$  has no real root.
8. Let the random variables  $X$  and  $Y$  have joint p.d.f.

$$f_{X,Y}(x,y) = \begin{cases} ce^{-(x+y)}, & y > x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of  $c$ .

(b) Find the value of  $E(Y|X = 2)$ .

9. Let  $X_1$  and  $X_2$  be iid exponential random variables with parameter  $\lambda$ . Find the conditional distribution of  $X_1$  given that  $X_1 + X_2 = 1$ .
10. Let  $X_1, X_2, X_3, X_4, X_5$  be independent random variables such that  $X_1 \sim N(200, 8)$ ,  $X_2 \sim N(104, 8)$ ,  $X_3 \sim N(108, 15)$ ,  $X_4 \sim N(120, 15)$  and  $X_5 \sim N(210, 15)$ . Let  $U = \frac{X_1+X_2}{2}$  and  $V = \frac{X_3+X_4+X_5}{3}$ . Then find the value of  $P(U > V)$ .