Lec-07

Tutorial session

Lec-08

$H = K + V = E$

$\downarrow$

Hamiltonian

(total energy in quantum)

$$i\hbar \frac{\partial \psi}{\partial t} = E\psi \quad \text{--- (1)}$$

$$\psi = A e^{i/\hbar (Px - Et)}$$

$$-i\hbar \frac{\partial \psi}{\partial x} = P\psi \quad \text{--- (2)}$$

$$A\psi = a\,\psi$$

operator, eigenfunction, eigen value.

for each quantity (like E, P, H, L, J etc.) there is a corresponding ~~to~~ operator.

Here $A \rightarrow$ operator corresponding to a physical quantity e.g. P, E, H.

$a \equiv$ number (value)

$$-i\hbar\frac{\partial\psi}{\partial x} = p\psi \qquad -i\hbar\frac{\partial}{\partial x} = \text{momentum operator.}$$

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \qquad -i\hbar\frac{\partial}{\partial t} = \text{Energy operator}$$

Here P, E are just value called eigen value.

since $K.E = \frac{p^2}{2m}$

so, operator for kinetic energy $= -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$

$\Rightarrow$ called p.p or $p(p)$

p operator p.

V = operator for P.E. $= V$

$H = K.E + V$

operator for Hamiltonian.

$$\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]$$

$$\boxed{H\psi = E\psi}$$

→ value (energy eigen value)

→ Other form (eigen form) of Schrodinger eqn.

→ All the operators are defined in 1-D / in X axis only.

→ for 3-D

~~1-D 3-D~~

operator	1-D	3-D
E	$i\hbar\frac{\partial}{\partial t}$	$i\hbar\frac{\partial}{\partial t}$
P	$-i\hbar\frac{\partial}{\partial x}$	$-i\hbar\nabla$
H	$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$	$-\frac{\hbar^2}{2m}\nabla^2 + V$

(sometimes we can write either V or U)

→ property of a wave function.

1. $\psi \equiv$ finite and continous and single valued, everywhere.

2. $\frac{\partial \psi}{\partial x} \equiv$ finite, single valued, continous everywhere.

3. ψ should be normalized, $\int_{-\infty}^{\infty} |\psi|^2 dv = 1$

→ the particle must be somewhere in the universe

→ If it's not a hypothetical particle then it must be somewhere in universe.

→ If a function follows these properties, it's called well behaved function.

eg. i)

$x = x_1$ It's not well behaved function (due to not single valued)

ii)

It's not well behaved function (due to not continuity of $\frac{\partial \psi}{\partial x}$)

If we find some function which follows prop. 1 & 2 of wave function but if it doesn't follow 3rd prop. then it's not well behaved function. but we can convert into well behaved function by multiplying some constant.

when.

$$\int |\psi|^2 dv \neq 1$$

$$N^2 \int |\psi|^2 dv = 1$$

$\psi' = N\psi$ (N → const.)

eg. if $\int |\psi|^2 dv = 0.5$

then $2 \times \int |\psi|^2 dv = 2 \times 0.5$

$= 1$

$$\psi' = \sqrt{2}\psi$$

$$\int |\psi'|^2 dv = 2\int |\psi|^2 dv = 1$$

N = Normalization Constant.

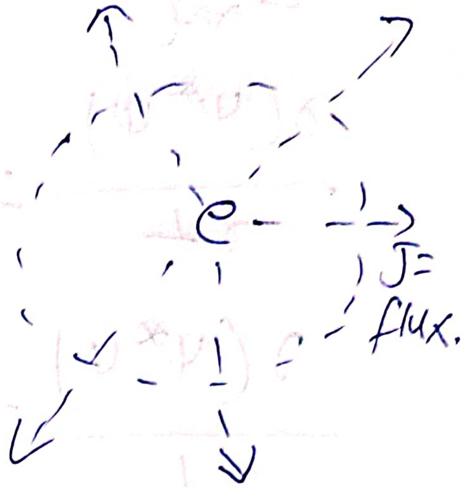
Sometimes wave function given which follows prop. 1 & 2 (but not 3) then we will not discard it as well behaved function. In that case we normalise wave function

→ probability current density:—

$$\frac{\partial \rho}{\partial t} + \nabla\cdot J = 0$$

conservation law

$$\frac{\partial \rho}{\partial t} = -\nabla\cdot J$$



$J \to$ flux.

$\nabla\cdot J$ = divergence of flux.

continuity eq^n for probability density.

probability density $= \psi\psi^*$

start with schrodinger eq^n.

$$i\hbar\frac{\partial \psi}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla^2 + V\right]\psi \quad \text{—} ①$$

and its conjugation.

$$-i\hbar\frac{\partial \psi^*}{\partial t} = \left[\frac{-\hbar^2}{2m}\nabla^2 + V\right]\psi^* \quad \text{—} ②$$

multiplying ① with ψ^* and ② with ψ and substracting ② from ①.

$$i\hbar\underbrace{\left(\psi^*\frac{\partial\psi}{\partial t} + \psi\frac{\partial\psi^*}{\partial t}\right)}_{\frac{\partial(\psi^*\psi)}{\partial t}} = \frac{-\hbar^2}{2m}\left[\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right] \quad \text{—} ③$$

$$\frac{\partial(\psi^*\psi)}{\partial t} = \frac{i\hbar}{2m}\left[\psi^*\nabla^2\psi - \psi\nabla^2\psi^*\right]$$

$$\frac{\partial(\psi^*\psi)}{\partial t} = \frac{i\hbar}{2m}\left[\nabla\cdot(\psi^*\nabla\psi - \psi\nabla\psi^*)\right]$$

or, $$\frac{\partial(\psi^*\psi)}{\partial t} + \frac{i\hbar}{2m}\left[\nabla\cdot(\psi\nabla\psi^* - \psi^*\nabla\psi)\right] = 0$$

$$\Rightarrow \frac{\partial(\psi^*\psi)}{\partial t} + \nabla\cdot\left[\frac{i\hbar}{2m}(\psi\nabla\psi^* - \psi^*\nabla\psi)\right] = 0 \quad (4)$$

$$P = \psi^*\psi = \text{probability density}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot J = 0 \quad \text{—} \quad (5)$$

comparing (4) and (5)

$$J = \frac{i\hbar}{2m}(\psi\nabla\psi^* - \psi^*\nabla\psi)$$

$\to$ probab. current density.

P = prob. density function.