Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Assignment 4

January 27, 2022

1. Suppose 15% of items produced at a manufacturing facility are defective. What is the probability that a lot of randomly selected 10 items contains more than 3 defective items?

2. The average number of trains either arriving at or departing from a railway station is one every 5 minutes. What is the probability that at least 10 trains arrive/depart during a selected hour? What is the probability that fewer than 4 such train will take place in an hour?

3. A electronic system consists of n parts each of which function independently with probability p. The entire system will be able to operate effectively, if at least one-half of its components function. For what values of p, a 5-component system more likely to operate effectively than a 3-component system?

4. The DVD produced by a certain company are defective with probability 0.01, independently of each other. The company sells the DVDs in packs of size 10 and offers a money-back guarantee if more than one of the 10 DVDs in the pack is found to be defective. If you buy 3 packs, what is the probability that at most one pack will be returned.

5. The number of times that an individual contracts cold in a given year is a Poisson random variable with parameter $\lambda = 3$. Suppose that a new drug has been just marketed that reduces the parameter λ to 2 for 75% of the population. For the other 25% of the population the drug has no appreciable effect on the cold. If an individual tries the drug for a year and has no cold in that time, how likely is it that the drug is beneficial for him?

6. A point P is chosen at random on a line segment AB of length $2a$. Find the expected values of (i) $AP.PB$ (ii) $|AP - PB|$ (iii) $\max\{AP, PB\}$.

7. In a precision bombing attack there is a 50% chance that a bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give at least 99% chance of completely destroying the target?

8. Suppose 5% of chips manufactured at a plant are defective. How many should a person buy in order that there is more than 99% chance of having at most one defective chip?

9. The daily consumption of oil in a city, in excess of $30,000$ gallons, is approximately distributed as a gamma distributed with parameters $\alpha = 2$ (two consumptions in a day) and $\beta = 1/10000$ (time taken in days per consumption of one gallon of oil). The city has a daily stock of $40,000$ gallons. What is the probability that the stock is insufficient on a particular day?

10. TV manufacturer offers a one year warranty. If a TV fails during this warranty period, it is replaced free of cost. The time to failure (in years) X has a shifted exponential distribution with location parameter 0 and scale parameter 8. What percentage of TVs will fail within the warranty period? The manufacturing cost of a TV is Rs $25,000$ and the net profit per unit scale is Rs $10,000$. If in the first week of January 2010, thousand TVs are sold, what is the expected profit after a year?

11. The lead time for orders of diodes from a certain manufacturer is known to have a gamma distribution with a mean of 20 days and a standard deviation of 10 days. Determine the probability of receiving an order within 15 days of placement date.

12. A large microprocessor chip contains multiple copies of circuits. If a circuit fails, the chip knows it and knows how to select the proper logic to repair itself. The average number of defects per chip is 300. What is the probability that no more than 4 defects will be found in a randomly selected area that comprises 2% of the total surface area?