In the last class we discussed about Pumping Lemma.

Pumping Lemma talked about a particular Property holds if a language is not regular.

But it does not say any thing about non regular language.

In fact, there exists non regular languages that satisfies pumping Lemma. Here is one such language.

$$L = \{ a^i b^i c^i \mid i \geq 0 \} \cup \{ a^m b^n c^k \mid m \neq 1 \}$$

This language is not regular but pumping lemma holds for $p = 2$.

An alternative way to prove non-regularity

Prove that $L = \{a^n b^n \mid n \geq 0\}$ is not regular.

Proof: Suppose that L is regular.

Therefore, there exists a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that accepts L.

Let $|Q| = p$.

Consider the strings $a^0, a^1, \ldots a^p$.

Let $\delta^*(q_0, a^i) = r_i$, for $i = 0 \ldots p$

Since $|Q| = p$, there exist two integers i, j such that $r_i = r_j$

i.e $\delta^*(q_0, a^i) = \delta^*(q_0, a^j)$ for $i \neq j$

$\Rightarrow \delta^*(q_0, a^i b^i) = \delta^*(q_0, a^j b^i)$ for $i \neq j$

But this is not possible as $\delta^*(q_0, a^i b^i) \in F \Rightarrow \delta^*(q_0, a^j b^i) \notin F$.

Therefore $L = \{a^n b^n \mid n \geq 0\}$ is not regular.