Department of Mathematics
Indian Institute of Technology Bhilai
IC152: Linear Algebra-II
Tutorial Sheet 1

---

1. Find the eigenvalues of zero and identity linear operators on a $n-$dimensional vector space by exhibiting the characteristic polynomials.

2. Suppose $T : V \to V$ be a linear operator on a vector space over a field $\mathbb{F}$ such that every vector in V is an eigenvector of T. Prove that T is a scalar multiple of the identity operator.

3. Let U and T are linear operators on a finite dimensional vector space V over a field $\mathbb{F}$. Prove that UT and TU have the same eigen values. What if V is not of finite dimension?

4. Find the eigenvalues and eigenvectors of the following operators

   (i) $T : \mathbb{R}^2 \to \mathbb{R}^2$, defined as $T(x, y) = (x + y, x)$.

   (ii) $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$, defined as $T(f(x)) = f(x) + (x + 1)f'(x)$.

   (iii) $T : M_{n\times n}(\mathbb{R}) \to M_{n\times n}(\mathbb{R})$, defined as $T(A) = A^t$.

   (iv) $T : \mathbb{C}^2(\mathbb{C}) \to \mathbb{C}^2(\mathbb{C})$, defined as $T(x, y) = (y, -x)$.

5. Does $T : \mathcal{C}(\mathbb{R}; \mathbb{R}) \to \mathcal{C}(\mathbb{R}; \mathbb{R})$, where $\mathcal{C}(\mathbb{R}; \mathbb{R})$ denotes the space of continuous real valued functions defined on $\mathbb{R}$, defined as

$$(Tf)(x) = \int_0^x f(t)dt$$

   has an eigenvector?

6. Let T be a linear operator on a vector space over a filed $\mathbb{F}$ with $T\alpha = c\alpha$ for some $c \in \mathbb{F}$, then for any polynomial f over the field F, $f(T)\alpha = f(c)\alpha$.

7. Define determinant and trace of a linear operator on a finite dimensional vector space. Justify that your definitions are well defined.

8. Let $A \in M_{n\times n}(\mathbb{R})$ with characteristic polynomial

$$f(t) = (-1)^n t^n + a_{n-1}t^{n-1} + \cdots a_1 t + a_0.$$

   Prove that $f(0) = a_0 = \det(A)$. Is it possible for an invertible matrix to afford 0 as an eigenvalue?