Indian Institute of Technology Bhilai

IC105: Probability and Statistics
Tutorial 3

January 16, 2022

1. (a) Let X be a random variable that takes nonnegative integer values. Show that

$$E(X) = \sum_{k=1}^{\infty} P(X \geq k).$$

(b) Use the formula in the previous part to find the expectation of a random variable Y whose p.m.f is defined as follows:

$$f_Y(y) = \begin{cases} \dfrac{1}{b-a+1}, & y = a, a+1, \ldots, b, \\ 0, & \text{otherwise}; \end{cases}$$

where a and b are nonnegative integers with $b > a$. Note that for $y = a, a+1, \ldots, b$, $f_Y(y)$ does not depend explicitly on y since it is a uniform p.m.f.

2. Two fair three-sided dice are rolled simultaneously. Let X be the difference of the two rolls.
   (a) Calculate the p.m.f, the expected value, and the variance of X.
   (b) Calculate and plot the p.m.f of X^2.

3. Consider a sequence of independent tosses of a biased coin at times $t = 0, 1, 2, \ldots$. On each toss, the probability of a 'head' is p, and the probability of a 'tail' is $1 - p$. A reward of one unit is given each time that a 'tail' follows immediately after a 'head.' Let R be the total reward paid in times $1, 2, \ldots, n$. Find $E[R]$ and $var(R)$.