INDIAN INSTITUTE OF TECHNOLOGY BHILAI

CS203: Theory of Computation I

Tutorial Sheet 2

- *Solve the following problems before the Tutorial.*

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1. Prove or disprove the following statement: "If M_1 is an NFA recognizes a language A, then the NFA we get after swapping the accept and non accept states recognizes A^c".

2. Let $M_1 = \{\{p, q, r\}, \{0, 1\}, \delta, p, \{q, r\}\}$ be an NFA given below. Find the equivalent DFA of M_1.

δ	0	1
p	{p}	{p,q}
q	{r}	ϕ
r	ϕ	ϕ

3. Let $M_1 = \{\{p, q, r, s\}, \{0, 1\}, \delta, p, \{q, s\}\}$ be an NFA given below. Find the equivalent DFA of M_1.

δ	0	1
p	{q,s}	{q}
q	{r}	{q,r}
r	{s}	{p}
s	ϕ	{p}

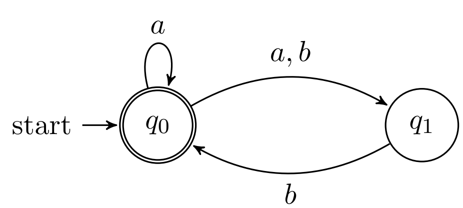

Figure 1: M_1

4. Find the DFA equivalent to the following NFA: M_1, shown in Figure 1.

5. Find DFA from the ϵ-NFA: M_2, shown in Figure 2
OR First find NFA equivalent to the following ϵ-NFA then find DFA equivalent to the calculated NFA.

6. Prove that intersection of two regular language is regular.

7. Prove that concatenation of two regular language is also regular.

8. Prove that reversal of a regular language is regular.

9. Give NFAs with the specified number of states recognizing each of the following languages. Further, convert each of the NFA into an equivalent regular expression. In all cases, the alphabet is $\sum = \{0, 1\}$.
   (a) $L_1 =\{ w \in \sum^* \mid w$ ends with 00$\}$ with three states.
   (b) $L_2 = \{ w \in \sum^* \mid 0101$ as substring of $w\}$ with five states.
   (c) $L_3 = \{ w \in \sum^* \mid w$ has at least two 0's or exactly two 1's $\}$ with six states.
   (d) $L_4 = \{\epsilon\}$ with one state.

10. For each of the following parts, give an example satisfying the given conditions. Give a brief justification for each of your examples. Note that $S^+ = SS^*$.

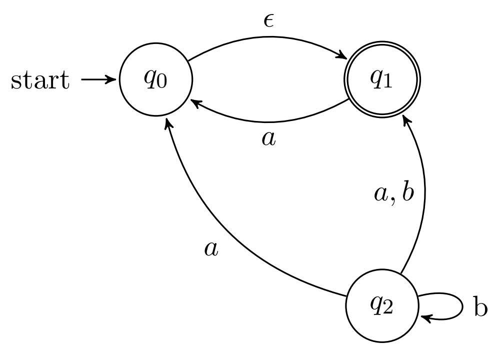

Figure 2: M_2

(a) Give an example of a set S of strings such that $S^* = S^+$

(b) Give an example of a set S of strings such that $S^* \neq S^+$

(c) Give an example of a set S of strings such that $S = S^*$

(d) Give an example of a set S of strings such that $S \neq S^*$

(e) Give an example of a set S of strings such that S^* is finite.

11. Let $L = \{w \in \{0,1\}^* | w \text{ starts or ends with } 11\}$.

    (a) Give a regular expression E such that $L(E) = L$.

    (b) Convert E to an ϵ-NFA N.

    (c) Convert N to an NFA N'.

    (d) Convert N' into one equivalent DFA.

    (e) Convert the DFA to the equivalent regular expression to verify that the expression is same as E.

12. Let w be a string of symbols and let T be a language adding w to a language S, i.e., $T = S \bigcup \{w\}$. Suppose further that $T^* = S^*$. Give the answer of the followings:

    (a) Is it necessarily true that $w \in S$. If this is necessarily true, give a proof. If this is not necessarily true, give a counterexample.

    (b) Is it necessarily true that $w \in S^*$. If this is necessarily true, give a proof. If this is not necessarily true, give a counterexample.

13. Give regular expressions that generate each of the following languages. In all cases, the alphabet is $\sum = \{a, b\}$.

    (a) The language $\{w| \ \ |w| \text{ is odd } \}$.

    (b) The language $\{w| \ w$ has an odd number of a's $\}$.

    (c) The language $\{w| \ w$ contains at least two a's, or exactly two b's $\}$.

    (d) The language $\{w| \ w$ ends in a double letter $\}$. (A string contains a double letter if it contains aa or bb as a substring.)

    (e) The language $\{w| \ w$ does not end in a double letter$\}$.

    (f) The language $\{w| \ w$ contains exactly one double letter$\}$. For example, $baaba$ has exactly one double letter, but $baaaba$ has two double letters.

14. Let A be the language over the alphabet $\sum = \{a, b\}$ defined by regular expression $((aa)^*b \cup a)^*$. Give an NFA that recognizes A.

15. Give a regular expression for the language recognized by the DFA below

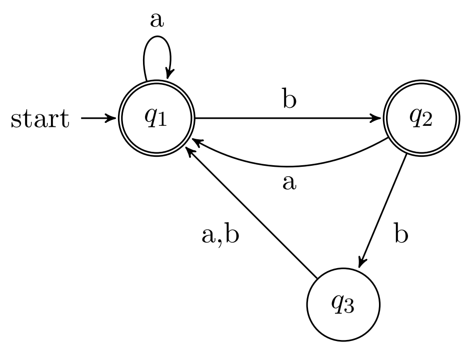

Figure 3: M_1

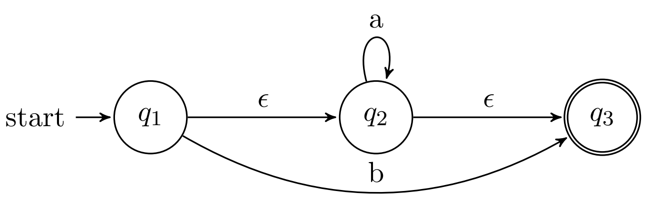

Figure 4: M_1

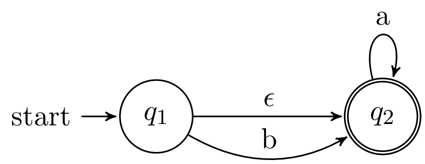

Figure 5: M_1

16. Give a regular expression for the language recognized by the NFA's below.
17. Convert the following regular expressions to ϵ-NFA. In all parts $\sum = \{a, b\}$.
    (a) $a(aaa)^* + b$
    (b) $a^+ \cup (ab)^+$
    (c) $(a + b^+)a^+b^+$
18. Prove that $(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1) = 0^*1(0 + 10^*1)^*$
19. Construct a regular grammar accepting $L = \{w \in \{a, b\}^* | w$ is a string over $\{a, b\}$ such that the number of b's is 3 mod 4$\}$