Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Tutorial 5

February 7, 2022

1. Let X and Y be Gaussian random variables, with $X \sim N(0,1)$ and $Y \sim N(1,4)$. (a) Find $P(X \leq 1.5)$ and $P(X \leq -1)$. (b) What is the distribution of $\frac{Y-1}{2}$? (c) Find $P(-1 \leq Y \leq 1)$.

2. Ben throws a dart at a circular target of radius r. We assume that he always hits the target, and that all points of impact (x, y) are equally likely. Compute the joint PDF $f_{X,Y}(x, y)$ of the random variables X and Y and compute the conditional PDF $f_{X|Y}(x|y)$.

3. Let X be a discrete random variable with p.m.f. f_X and let Y be a continuous random variable, independent from X, with p.d.f. f_Y. Derive a formula for the p.d.f. of the random variable $X + Y$.

4. The random variables X and Y are described by a joint p.d.f. which is constant within the unit area quadrilateral with vertices $(0, 0)$, $(0, 1)$, $(1, 2)$, and $(1, 1)$. Use the law of total variance to find the variance of $X + Y$.

5. (a) You roll a fair six-sided die, and then you flip a fair coin the number of times shown by the die. Find the expected value and the variance of the number of heads obtained.
   (b) Repeat part (a) for the case where you roll two dice, instead of one.

6. An ambulance travels back and forth, at a constant specific speed v, along a road of length ℓ. We may model the location of the ambulance at any moment in time to be uniformly distributed over the interval $(0, \ell)$. Also at any moment in time, an accident (not involving the ambulance itself) occurs at a point uniformly distributed on the road; that is, the accident's distance from one of the fixed ends of the road is also uniformly distributed over the interval $(0, \ell)$. Assume the location of the accident and the location of the ambulance are independent.

   Supposing the ambulance is capable of immediate U-turns, compute the c.d.f. and p.d.f. of the ambulance's travel time T to the location of the accident.

7. Random variables X and Y are distributed according to the joint p.d.f.
   $$f_{X,Y}(x, y) = \begin{cases} ax, & \text{if } 1 \leq x \leq y \leq 2, \\ 0, & \text{otherwise.} \end{cases}$$
   (a) Evaluate the constant a.
   (b) Determine the marginal p.d.f. $f_Y(y)$.
   (c) Determine the expected value of $\frac{1}{X}$, given that $Y = \frac{3}{2}$.