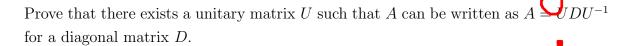
Department of Mathematics

Indian Institute of Technology Bhilai

IC152: Linear Algebra-II Tutorial Sheet 3

1. Show that the following matrix A is Hermitian

$$A = \begin{bmatrix} 3 & 2-i & -3i \\ 2+i & 0 & 1-i \\ 3i & 1+i & 0 \end{bmatrix}.$$



- 2. Let A be an $n \times n$ complex matrix. Prove that A is Hermitian if and only if X^*AX is real for all vectors X in \mathbb{C}^n .
- 3. Find out a real symmetric matrix B and a real skew-symmetric matrix C such that the following matrix A can be written as A = B + iC

$$A = \left[\begin{array}{cc} 2 & 1+i \\ 1-i & 3 \end{array} \right]$$

Can every Hermitian matrix A can be written in a similar fashion?

4. Find out Hermitian matrices B and C such that the following matrix A can be written as A = B + iC

$$A = \begin{bmatrix} i & 2 \\ 2+i & 1-2i \end{bmatrix}.$$

Generalize it for any complex $n \times n$ matrix.

5. Find the minimal polynomial for the following linear operators

$$T: P_2(\mathbb{R}) \to P_2(\mathbb{R})$$
 defined as $Tf = f'$.

(ii)
$$T: M_{n \times n}(\mathbb{R}) \to M_{n \times n}(\mathbb{R})$$
, defined as $T(A) = A^t$

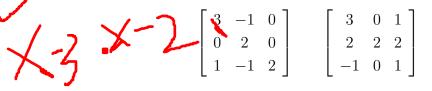
- 6 Let V be an n-dimensional vector space and let T be a linear operator on V. Suppose that there exists some positive integer k so that $T^k = 0$. Prove that $T^n = 0$.
- 7. Find a minimal polynomial of the following matrix without finding characteristic polynomial

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

8. Let $a, b, c \in \mathbb{R}$, then show that for the following matrix characteristic and minimal polynomials are same

$$A = \left[\begin{array}{ccc} 0 & 0 & c \\ 1 & 0 & b \\ 0 & 1 & a \end{array} \right] \quad \bullet$$

- 9. Prove that if $T \in L(V, V)$ is annihilated by \bullet polynomial over \mathbb{C} having distinct roots, then T is diagonalizable. As a direct application of this result, show the following
 - (a) Let T be a linear operator on a complex vector space such that $T^k = I$ for some positive integer k. Then T is diagonalizable.
 - (b) Prove that every matrix A satisfying $A^2 = A$ is diagonalizable.
- 10. Compute the minimal polynomial for the following matrices



- 11. Verify Cayley-Hamilton theorem for the following
 - (a) $T: \mathbb{R}^2 \to \mathbb{R}^2$ defined as T(x,y) = (2x + 5y, 6x + y)
 - (b) $A = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$
- 12. Let haracteristic polynomial of a matrix A be $x^2 x + 1$. Compute A^3 and A^5 .