Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Assignment 2

January 6, 2022

1. Which of the following functions are distribution functions?

(i) $F(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{2}, & 0 \le x \le \frac{3}{2}, \\ 1 & x > \frac{3}{2}. \end{cases}$ (ii) $F(x) = \begin{cases} 0, & x \le 1, \\ 1 - \frac{1}{x^2}, & x > 1. \end{cases}$

(iii) $F(x) = \begin{cases} 0, & x \le 0, \\ \frac{1}{2} + \frac{e^{-x}}{2}, & x > 0. \end{cases}$ (iv) $F(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{8}, & 0 \le x < 1, \\ \frac{x+1}{8}, & 1 \le x < 2, \\ \frac{2x+1}{8}, & 2 \le x < 3, \\ 1, & x \ge 3. \end{cases}$

2. Consider a function F as

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-x}, & x \ge 0. \end{cases}$$

(a) Show that F is a distribution function.

(b) Find the value of $P(2 < X \le 3)$, $P(-2 < X \le 3)$, $P(1 \le X < 4)$, $P(5 \le X < 8)$.

3. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 2, \\ \frac{2}{3}, & 2 \le x < 5, \\ \frac{7-6k}{6}, & 5 \le x < 9, \\ \frac{3k^2-6k+7}{6}, & 9 \le x < 14, \\ \frac{16k^2-16k+19}{16}, & 14 \le x \le 20, \\ 1, & x > 20. \end{cases}.$$

(a) Find the value of constant k.

(b) Show that the r.v. X is of discrete type and find its support.

(c) Find the p.m.f. of X.

4. Let X be a random variable with distribution function given as

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x}{4}, & 0 \le x < 1, \\ \frac{x}{3}, & 1 \le x < 2, \\ \frac{3x}{8}, & 2 \le x < 5/2, \\ 1, & x \ge 5/2. \end{cases}$$

(a) Prove that X is neither continuous nor discrete random variable.

(b) Find the value of $P(1 < X \le \frac{5}{2})$, $P(1 < X < \frac{5}{2})$, $P(1 \le X < \frac{5}{2})$, $P(-2 \le X < 1)$, $P(X \ge 2)$, $P(X > 2)$.

5. Let X be a continuous type random variable with p.d.f.

$$f(x) = \begin{cases} k - |x|, & |x| < \frac{1}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

where $k \in \mathbb{R}$.

(a) Find the value of constant k.

(b) Evaluate $P(X < 0)$, $P(X \le 0)$, $P(0 < X \le 1/4)$, $P(-1/8 \le X \le 1/4)$.

(c) Find the distribution function of X.

6. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1, \\ 0, & \text{otherwise}. \end{cases}$$

(a) What is the value of c?

(b) Find the cumulative distribution function of X.

(c) Find $E(X)$ and $\text{Var}(X)$.

7. Consider a function $f(x)$ defined as

$$f(x) = \begin{cases} \frac{1}{\beta}\left(1 - \frac{|x - \alpha|}{\beta}\right), & \alpha - \beta < x < \alpha + \beta, \\ 0, & \text{otherwise}. \end{cases}$$

where $\alpha \in \mathbb{R}$, $\beta > 0$.

(a) Show that $f(x)$ is a probability density function.

(b) Find the distribution function.

(c) Find $E(X)$ and $\text{Var}(X)$.