Introduction to Astronomy and Astrophysics. (IC200)

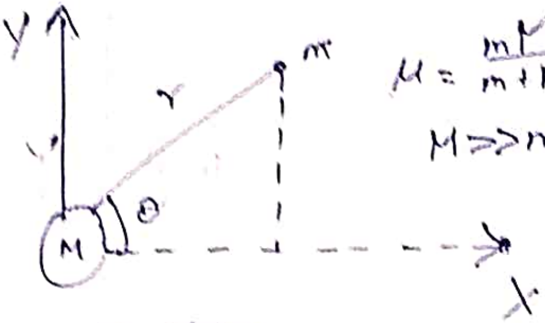
Exams :- 50%
Lab/Report :- 40%
Assessment :- 10%

Kepler's first law :-

$F = \frac{GMm}{r^2}$

① orbits are conic sections.

differential equation / Calculus



$\mu = \frac{mM}{m+M}$

$M >> m$

$x = r\cos\theta$

$y = r\sin\theta$

$$\dot{x} = \dot{r}\cos\theta - r\sin\theta\,\dot{\theta}$$

$$\dot{y} = \dot{r}\sin\theta + r\cos\theta\,\dot{\theta}$$

$$K.E = \frac{m}{2}\left[\dot{x}^2 + \dot{y}^2\right]$$

$$= \frac{m}{2}\left((\dot{r}\cos\theta)^2 + (r\sin\theta\dot{\theta})^2 \cancel{- 2r\dot{r}\cos\theta\sin\theta\dot{\theta}} + (\dot{r}\sin\theta)^2 + (r\cos\theta\dot{\theta})^2 \cancel{+ 2\dot{r}r\cos\theta\sin\theta\dot{\theta}}\right)$$

$$\text{K.E} = \frac{m}{2}\left((\dot{r})^2 + (r\dot{\theta})^2\right)$$

$$\text{Total } E = \frac{m}{2}\left(\dot{r}^2 + (r\dot{\theta})^2\right) - \frac{GMm}{r}$$

$$L = mrv = mr^2\omega = mr^2\dot{\theta}$$

L, E, m, M are Const.

$$\dot{\theta} = \frac{L}{mr^2}$$

$$E = \frac{m}{2}\left((\dot{r})^2 + \frac{r^2L^2}{m^2r^4}\right) - \frac{GMm}{r}$$

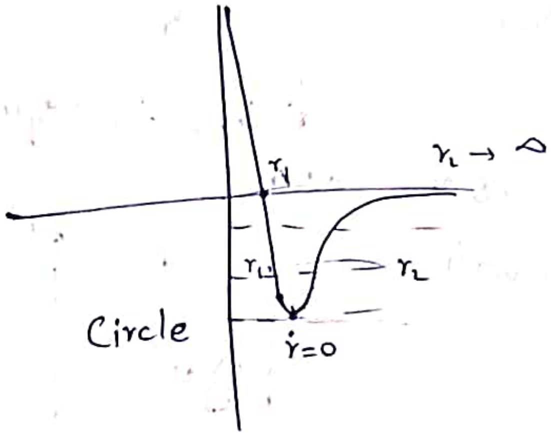
$$(\dot{r})^2 = \frac{2}{m}\left(E + \frac{GMm}{r}\right) - \frac{L^2}{m^2r^2}$$

$-\frac{2A}{r}$

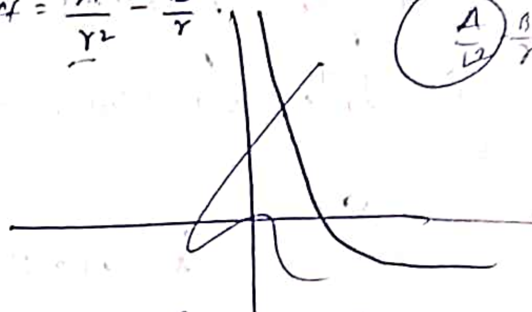
$$(\dot{r})^2 = \frac{2E}{m} + \frac{2GM}{r} - \frac{L^2}{m^2 r^2}$$

$$E = \frac{1}{2} m (\dot{r})^2 + V_{eff} \qquad \left(V_{eff} = \frac{L^2}{2mr^2} - \frac{GMm}{r} \right)$$

$$V_{eff} = \frac{A}{r^2} - \frac{B}{r} \qquad \frac{A}{L^2} \frac{B}{r}$$

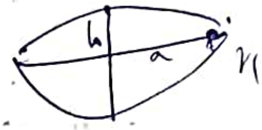


Circle

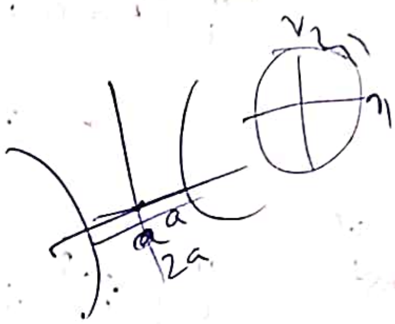


$$\frac{2A}{r^3} - \frac{B}{r^2} = 0$$

r_2



$$\dot{r} = \sqrt{\frac{2}{m}\left(E - V_{eff}\right)}$$



$$\frac{dr}{dt} = \dot{r} = \sqrt{\frac{2}{m}\left(E + \frac{GMm}{r} - \frac{L^2}{2mr^2}\right)}$$

$$\frac{d\theta}{dt} = \dot{\theta} = \frac{L}{mr^2}$$

$L = mrv$
$= mr^2\omega$

$$\frac{d\theta}{dr} = \frac{\frac{L}{mr^2}}{\sqrt{\frac{2E}{m} + \frac{2GMm}{r} - \frac{L^2}{m^2r^2}}} = \frac{\frac{1}{r^2}}{\sqrt{\frac{2E}{m}\times\frac{m^2}{L^2} + \frac{2GMm}{mr}\times\frac{m^2}{L^2} - \frac{1}{r^2}}}$$

$$\int d\theta = \int \frac{\frac{1}{r^2}\,dr}{\sqrt{\frac{2Em}{L^2} + \frac{2GMm}{L^2 r} - \frac{1}{r^2}}}$$

$$\frac{1}{r} = y \Rightarrow dy = -\frac{1}{r^2}\,dr$$

$$\int d\theta = \int \frac{-dy}{\sqrt{\frac{2Em}{L^2} + \frac{2GMm^2}{L^2}y - y^2}}$$

$GMm = \alpha$.

$$\int d\theta = \int \frac{-dy}{\sqrt{\frac{2Em}{L^2} + \frac{2\alpha m y}{L^2} - y^2}}$$

$$= \int \frac{-dy}{\sqrt{\frac{2Em}{L^2} + \frac{4\alpha^2 m^2}{L^4} - \left(y^2 - \frac{2\alpha m y}{L^2} + \frac{4\alpha^2 m^2}{L^4}\right)}}$$

$\cos^{-1}\left(\frac{x}{a}\right)$

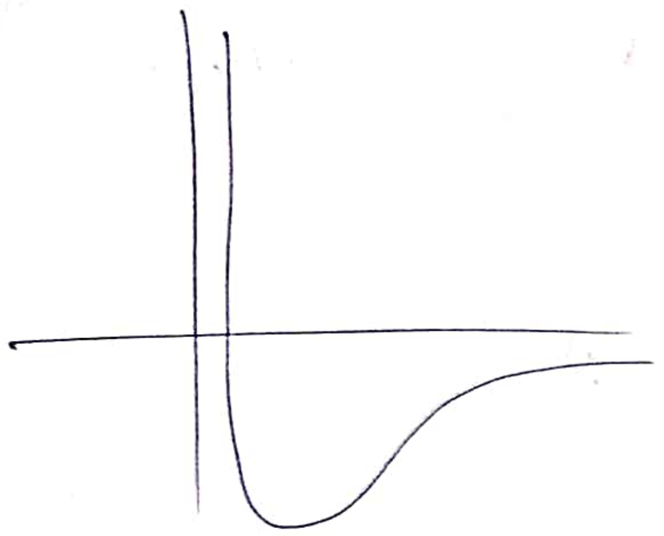
$$= \int \frac{-dy}{\sqrt{\left(\frac{2Em}{L^2} + \frac{4\alpha^2 m^2}{L^4}\right) - \left(y - \frac{\alpha m}{L^2}\right)^2}}$$

$$\theta = \cos^{-1}\left(\frac{y - \frac{\alpha m}{L^2}}{\sqrt{\frac{2Em}{L^2} + \frac{\alpha^2 m^2}{L^4}}}\right)$$

$$\cos\theta = \frac{y - \frac{\alpha m}{L^2}}{\sqrt{\frac{2Em}{L^2} + \frac{\alpha^2 m^2}{L^4}}}$$

$$y = \frac{1}{r} = \frac{\alpha m}{L^2} + \frac{\alpha m}{L^2}\cos\theta\sqrt{1 + \frac{2Em}{L^2} \times \frac{L^4}{\alpha^2 m^2}}$$

$$\frac{1}{r} = \frac{\alpha m}{L^2}\left[1 + \epsilon \cos\theta\right] \qquad \epsilon = \sqrt{1 + \frac{2EL^2}{\alpha^2 m}}$$



$$\frac{L^2}{2mr^2} - \frac{\alpha}{r} = E$$

$$L^2 - \cancel{\alpha r}\, 2m\alpha r = 2Emr^2$$

$$2Emr^2 + 2m\alpha r - L^2 = 0$$

$$2Emr^2 + 2m\alpha r + \frac{m}{2E}\alpha^2$$

$$\cancel{2Emr^2}$$

$$2Emr^2 + 2m\alpha r + \frac{m\alpha^2}{2E} - \frac{m\alpha^2}{2E} - L^2 = 0$$

$$\left(\sqrt{2Em}\, r + \sqrt{\frac{m}{2E}}\,\alpha\right)^2 = \frac{m\alpha^2}{2E} + L^2$$

$$\sqrt{2Em}\, r = \pm\sqrt{\frac{m\alpha^2}{2E} + L^2}$$

$$r = \frac{\pm\sqrt{\frac{m\alpha^2}{2E} + L^2} - \sqrt{\frac{m}{2E}}\,\alpha}{\sqrt{2Em}}$$

$$r = \frac{-\sqrt{\frac{m}{2E}}\,\alpha \pm \sqrt{\frac{m\alpha^2}{2E} + L^2}}{\sqrt{2Em}}$$

$$= \frac{-\sqrt{m}\,\alpha \pm \sqrt{m\alpha^2 + 2EL^2}}{2E\sqrt{m}}$$

$$= \frac{-m\alpha \pm \sqrt{\alpha^2 + 2mEL^2}}{2Em}$$

$$= \frac{-\alpha}{2E} \pm \frac{\alpha}{2E}\sqrt{1 + \frac{2L^2E}{\alpha^2 m}}$$

$$r = \frac{-\alpha}{2E} \pm \frac{\alpha}{2E}\sqrt{1+\frac{2L^2E}{\alpha^2 m}}$$

Let $\epsilon = \sqrt{1+\frac{2L^2E}{\alpha^2 m}}$

$$r = \frac{-\alpha}{2E} \pm \frac{\alpha}{2E}\epsilon \longrightarrow \text{(Conic sections)}$$

Bohr's model :-

Circular obits :-

$\epsilon = 0$

$\Rightarrow 1+\frac{2L^2E}{\alpha^2 m} = 0$

$L = n\hbar$

$$\boxed{\hbar = \frac{h}{2\pi}}$$

13.6

$$E = \frac{-\alpha^2 m}{2L^2} = \frac{-\alpha^2 m}{2n^2\hbar^2} = \frac{-2\alpha^2 m\pi^2}{n^2h^2} = -\frac{2\pi^2\alpha^2 m}{n^2h^2}$$

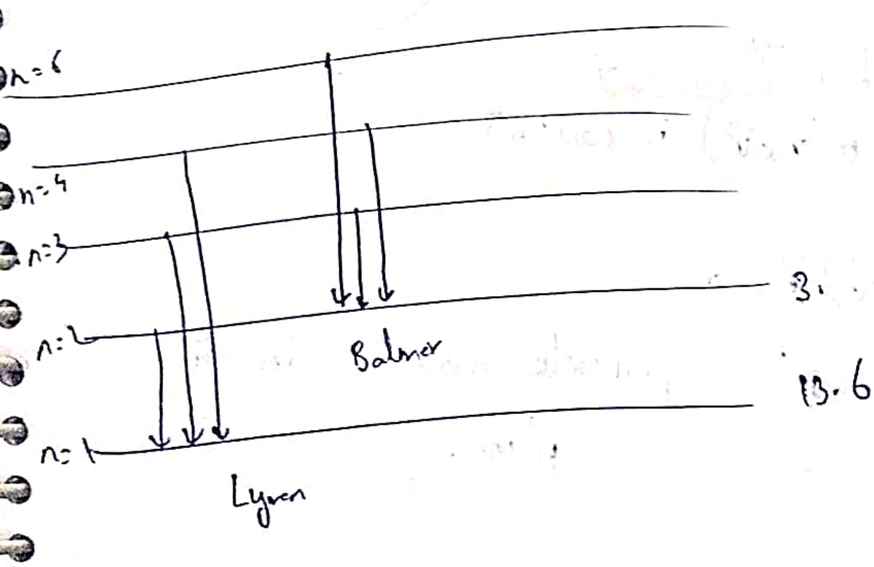
$$\cancel{= \frac{-1}{2}\left(\frac{Ze}{4\pi\epsilon_0 r}\right)^2 \times \frac{m\times 4\pi^2}{h^2}\times\frac{1}{n^2}}$$

$$= -\frac{1}{2}$$

$$= \frac{Z^2}{2n^2}\left(\frac{e^2}{4\pi\epsilon_0\hbar c}\right)^2 mc^2$$

$\frac{1}{137}$ $5\times10^5 eV$

$$= -13.6\,eV\frac{Z^2}{n^2}$$



$$r = 4\pi\epsilon_0\frac{n^2\hbar^2}{mZe^2}.$$

$r \to n=1$, $Z=1$ $= \frac{4\pi\epsilon_0\hbar^2}{me^2}$

$\frac{1}{a} = 1+\epsilon$

$r = \frac{\alpha}{2E}(1+$

$\frac{1}{a} = 1+\epsilon^2$

$\frac{1}{a} = 1-\epsilon$

$\frac{r}{\frac{\alpha}{2E}} = 1+\epsilon$

$$V = -\frac{\alpha}{r} = \frac{-Z^2e^2}{4\pi\epsilon_0} \quad \frac{me^2}{4\pi\epsilon_0 n^2\hbar^2}$$

$$\boxed{V = 2E}$$

$$\cancel{V = T}$$

$$E = T + V$$

$$\boxed{T = -E}$$

VIRIAL THeorm

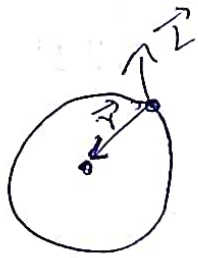
$$E = T + V \qquad E = \langle T \rangle + \langle V \rangle$$

$$E = V/2 \qquad E = \langle V/2 \rangle$$

$$T = -V/2 \qquad T = -\langle V/2 \rangle$$

Tut -1

1) $\cancel{\vec{L} = m\vec{r}}$



$$\vec{F} = F(r)\,\hat{r}$$

$$\vec{\tau} = \vec{r} \times \vec{F}_r = 0$$

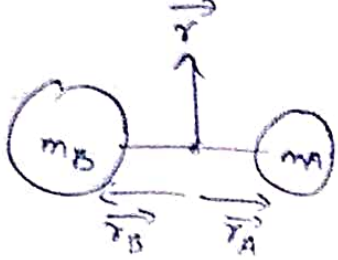
$$\frac{d\vec{L}}{dt} = \vec{\tau} = 0$$

here $\vec{L}$ constant
$m(\vec{r} \times \vec{v})$ is constant

$$\vec{J}\cdot\vec{r} = m(\vec{r}\times\vec{v})\cdot\vec{r} = 0$$

$J \perp r$ (or) particle moves in a plane.

2)



$$F_A = \frac{-G m_A m_B}{r^2}\hat{r}$$

$$= \frac{-G m_B m_A \times \vec{r}}{r^2 \times r}$$

$$\vec{r} = r_A - r_B \quad \text{— (1)}$$

$$\left(\frac{r_B}{r_A}\right) m_B = m_A \quad \text{— (2)}$$

$$m_A \vec{a_A} = \frac{-G m_B m_A}{r^2 \times r} \times r_A\left(1 + \frac{m_A}{m_B}\right)$$

from (1) and (2)
value of r_B in terms of r_A.

$$\cancel{m_A} \vec{a_A} = -\frac{G m_B \cancel{m_A}}{r^2 \times r} r_A \left(\frac{m_B + m_A}{m_B}\right)$$

$$\vec{a_A} = \frac{-G(m_A + m_B) r_A}{r^2}$$

Similarly,

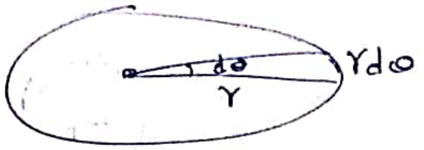
$$\vec{a_B} = \frac{G(m_A + m_B)(r_B - r_A)}{r^2}$$

$$-\left(\frac{G M \vec{r}}{r^2}\right)\mu = \frac{-G m_A m_B \vec{r}}{r^3}$$

$$M = \frac{m_A m_B}{m_A + m_B}$$

$\Rightarrow$ Orbit of a & b around each other is equivalent to orbit of M around a fixed mass m

3) Sol



$$dA = \frac{1}{2} r \times r d\theta$$

$$dA = \frac{1}{2} r^2 d\theta$$

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt} = \frac{1}{2} \cancel{r^2} \times \frac{L}{m\cancel{r^2}}$$

$$= \frac{L}{2m} = \text{Const}.$$

4) $\cancel{\frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2}$

$$\frac{1}{r} = \frac{GMm^2}{L^2}(1 + \epsilon\cos\theta)$$

At Perihelion $r = a(1+e)$

aphelian $r = a(1-e)$

$$\frac{1}{a(1-e)} = \frac{GMm^2}{L^2}(1+\epsilon)$$

$$\frac{L^2}{m^2} = GMa(1-e^2) \rightarrow \frac{L}{M} = \sqrt{GMa(1-e^2)}$$

we know that

$$\frac{dA}{dt} = \text{Const} = \frac{L}{2m}$$

$$\frac{\pi ab}{T} = \frac{1}{2}\sqrt{GMa(1-e^2)}$$

$$\frac{4\pi^2 a^2 b^2}{T^2} = GMa(1-e^2)$$

$$e^2 = 1 - \frac{b^2}{a^2}$$

$$\frac{4\pi^2 a^2 \cancel{b^2}}{T^2} = GM\frac{\cancel{a} \times \cancel{b^2}}{a^2}$$

$$\frac{4\pi a^3}{GM} = T^2$$

$$a^3 \propto T^2.$$

5)

(i) $V = \alpha r^2$

$$E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} + \alpha r^2$$

$$\dot{r} = \sqrt{\frac{2}{m}\left(E - \frac{L^2}{2mr^2} - \alpha r^2\right)}$$

$$d\dot{\theta} = \frac{L}{mr^2}$$

$$\left(\frac{d\dot{\theta}}{dr}\right) = \frac{L/mr^2}{\sqrt{\frac{2}{m}\left(E - \frac{L^2}{2mr^2} - \alpha r^2\right)}}$$

$$d\theta = \int \frac{\frac{L}{mr^2}\, dr}{\sqrt{\frac{2}{m}\left(E - \frac{L^2}{2mr^2} - \alpha r^2\right)}}$$

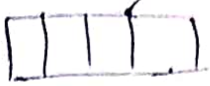
$$= \int \frac{\frac{dr}{r^2}}{\sqrt{\frac{2Em}{L^2} - \frac{2m\alpha r^2}{L^2} - \frac{1}{r^2}}} = \int \frac{\frac{dr}{r^3}}{\sqrt{\frac{2Em}{L^2 r^2} - \frac{2m\alpha}{L^2} - \frac{1}{r^4}}}$$

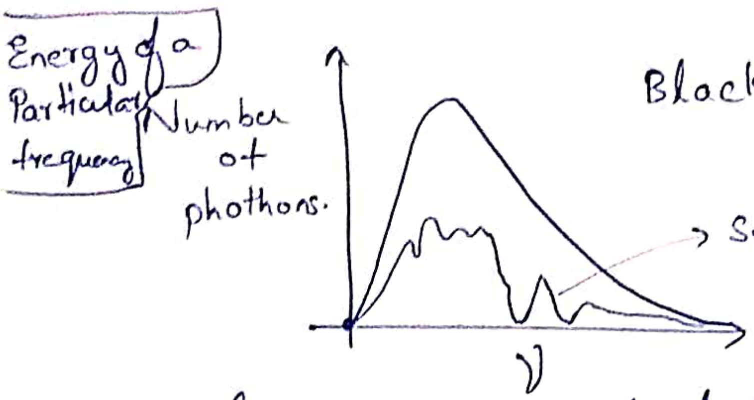
$$\frac{1}{r^2} = y$$

$$-\frac{1}{r^3}\, dr = \frac{dy}{2}$$

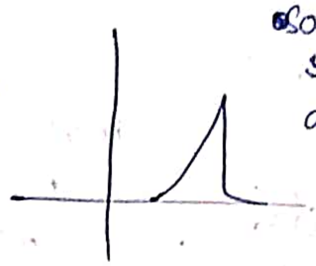
$$\theta = \int \frac{-dy}{\sqrt{ay - b - y^2}}$$

$\theta = \frac{1}{2}$

Spectrum :— 



Black body Spectrum.

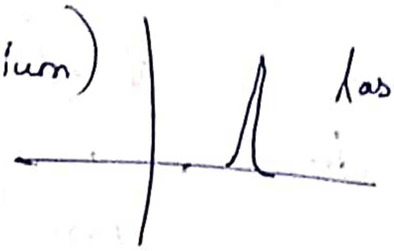


Some set of atoms Black body Spectrum

Temperature is Constant (Thermal equilibrium)

$N_{i+1} \longrightarrow i+1$

$N_i \longrightarrow i$



laser

$$\frac{N_{i+1}}{N_i} \propto e^{-\left(\frac{h\nu_i}{k_B T}\right)}$$

(Boltzman's population ratio)

$$\frac{dN_{i+1}}{dt} = -AN_{i+1}$$

Spontanious Emission

$$\frac{dN_i}{dt} \propto IN_i \Rightarrow \frac{dN_i}{dt} = -BI_{\nu_i}N_i$$

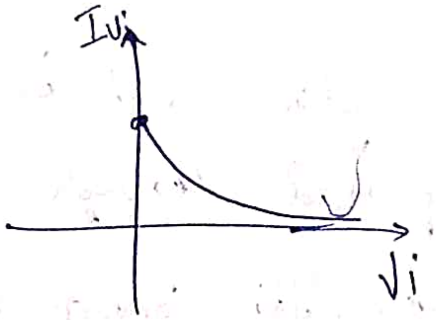
Stimulation absorption.

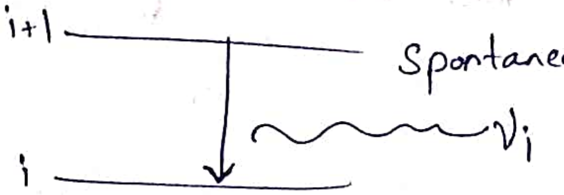
At equilibrium,

$$-AN_{i+1} = -BI_{\nu_i}N_i$$

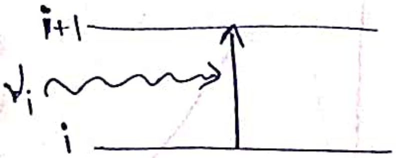
$$I_{\nu_i} = \frac{A}{B}\frac{N_{i+1}}{N_i} = \frac{A}{B}e^{-h\nu_i/k_B T}$$

$$\boxed{I_{\nu_i} = \frac{A}{B} e^{-h\nu_i/k_BT}}$$

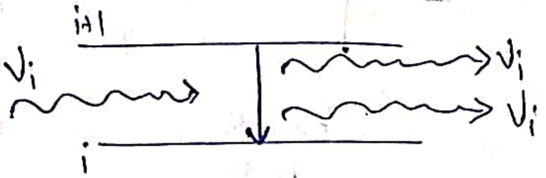




Spontaneous emission.



STIMULATED ABSORPTION.



STIMULATED EMISSION

~~So~~, Einstein Correction

$$\frac{dN_{i+1}}{dt} = -AN_{i+1} - BIN_{i+1}$$

$$\frac{dN_i}{dt} = -BIN_i$$

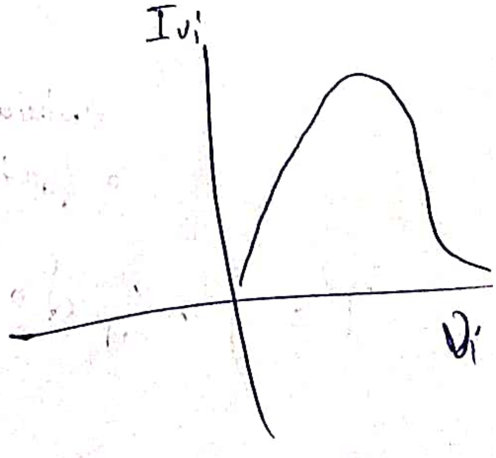
Equilibrium: $AN_{i+1} + BIN_{i+1} = BIN_i$

$$I = \frac{AN_{i+1}}{B(N_i - N_{i+1})} = \frac{A\, e^{-h\nu_i/k_BT}}{B\left(1 - e^{-h\nu_i/k_BT}\right)}$$

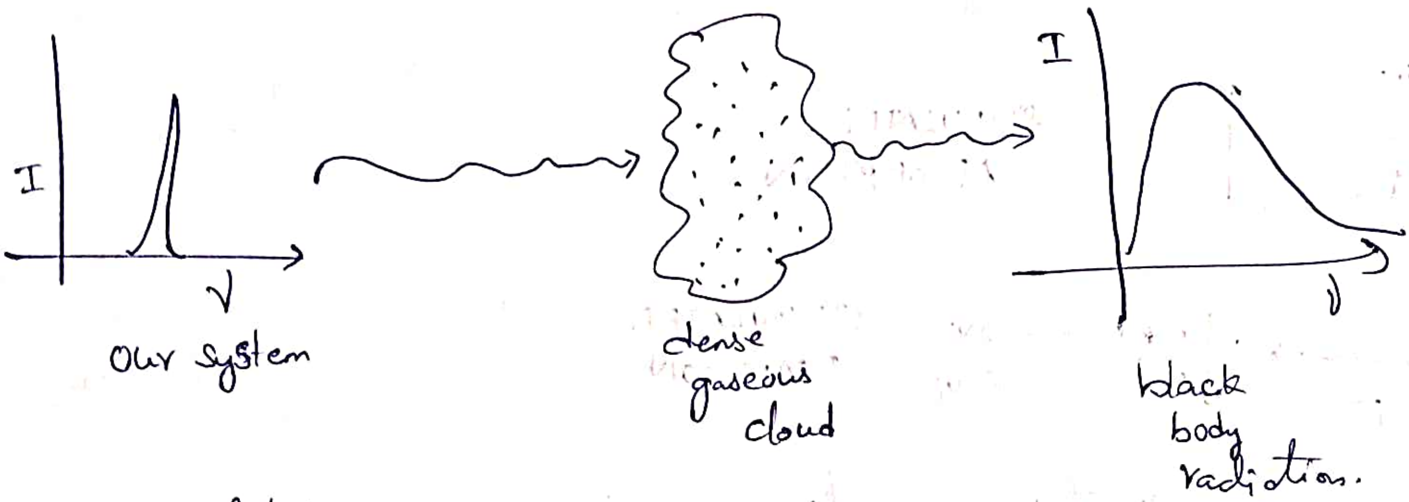
$$I = \left(\frac{A}{B}\right) \frac{e^{-h\nu_i/k_BT}}{1 - e^{-h\nu_i/k_BT}}$$

$$\left(\frac{A}{B}\right) \propto \nu^3$$

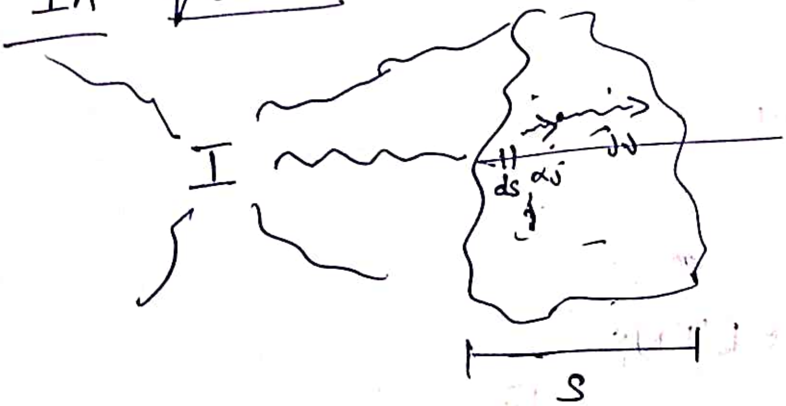
$$\Rightarrow I = \frac{\nu^3 e^{-h\nu/k_BT}}{1 - e^{-h\nu/k_BT}} = \text{Black Body function.}$$



Any system could be black body, if the emission is passed through a dense gaseous cloud of atoms. This converts our emission into black body radiation.



In Equilibrium :-



$$I = \frac{L}{d\Omega\, dA\, d\nu}$$

$$= \frac{J/s}{Hz\ Sr\ m^2}$$

$$= J\, Hz^{-1}\, Sr^{-1}\, m^{-2}\, s^{-1}$$

$$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$$

j_ν → emission ~~absorbtion~~ co-efficient ~~coefficient~~

α_ν → ~~emission coefficient~~ absorption coefficient

$$\tau = \int \alpha_\nu\, ds$$

τ → optical depth.

$$\frac{d\tau}{ds} = \alpha_\nu$$

$$\frac{dI_\nu}{d\tau} = \boxed{\frac{j_\nu}{\alpha_\nu}} - I_\nu$$

$$\frac{dI_\nu}{d\tau} = S_\nu - I_\nu$$

Photosphere - "Main Sun"

chromosphere - Moon ~~d~~ eclipse's Sun with Smaller Size.

Corona - Moon eclipse's sun with larger size.

limb dark - The central part of sun is brighter, than edges, Those edges are also called limb dark.

Temperature on Surface of Sun

$$\lambda T = 3 \times 10^{-3}$$

$$T = \frac{3 \times 10^{-3}}{500 \times 10^{-9}}$$

$$T = 6000k.$$

Virial Theorm :-

$$K.E = -\frac{1}{2} P.E$$

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$E = \frac{1}{2}mv^2 = \frac{GMm}{2r} = -\frac{1}{2}\left(-\frac{1}{2}mv^2\right)$$

$$K.E \propto T$$

$$\Rightarrow K.E = k_B T \text{ (for 1 particle).}$$

$$K.E = nk_B T \text{ (for n particles)}$$

$$P.E = -\frac{GMm}{r} \text{ (for 1 particle)}$$

$$\Sigma P.E = \Sigma -\frac{GMm_i}{r_i} = -f\frac{GM^2}{R}$$

assuming uniform density,

$$f = \frac{3}{5}$$

$$Nk_B T = \frac{f}{2}\frac{GM^2}{R}$$

$$R = R_{Sun} = 7 \times 10^8 m$$

$$M = M_{Sun} = 2 \times 10^{30} kg.$$

$$k_B = 1.38 \times 10^{-23}$$

$$N = \frac{2 \times 10^{30}}{\cancel{\frac{4}{3}\pi (7\times10^8)^3} \times 2.7 \times 10^{-27}}$$

$$\frac{M k_B T}{2.7 \times 10^{-27}} = \frac{f}{2}\frac{GM^2}{R} \quad \times 2.7\times10^{-27}$$

$$T = \frac{fGM}{2Rk_B} = \frac{f \times 6.6 \times 10^{-11} \times 2 \times 10^{30}}{2 \times 7 \times 10^8 \times 1.38 \times 10^{-23}}$$

$$= \frac{f \times 6.6 \times 10^{47}}{1.38 \times 14}$$

$$= \frac{f \times 10^7}{28} \sim 10^6 K$$

$$\frac{dI_\nu}{d\tau} = S_\nu - I_\nu$$

$$\frac{dI_\nu}{I_\nu - S_\nu} = \int -d\tau + c$$

$$I = \frac{\nu^3 e^{-h\nu/k_BT}}{1 - e^{-h\nu/k_BT}}$$

$$I_\nu - S_\nu = c' e^{-\tau}$$

$S=0,\ \tau=0,$

$I_\nu = I_{\nu_0}$

$$I_{\nu_0} - S_\nu = c'$$

$$I_\nu = I_{\nu_0} e^{-\tau} + S_\nu\left(1 - e^{-\tau}\right)$$

$$I_\nu = I_{\nu_0} e^{-\tau} + S_\nu\left(1 - e^{-\tau}\right)$$

if τ verylarge $I_\nu = \boxed{S_\nu = \frac{j_\nu}{\alpha_\nu}}$

Tiphop's law :-

$$S_\nu = \frac{j_\nu}{\alpha_\nu}$$

$$= \frac{AN_{i+1}}{BN_i - BN_{i+1}}$$

$$\frac{N_{i+1}}{N_i} = e^{-h\nu/k_BT}$$

$$\alpha_\nu = B\left[N_i - N_{i+1}\right]$$

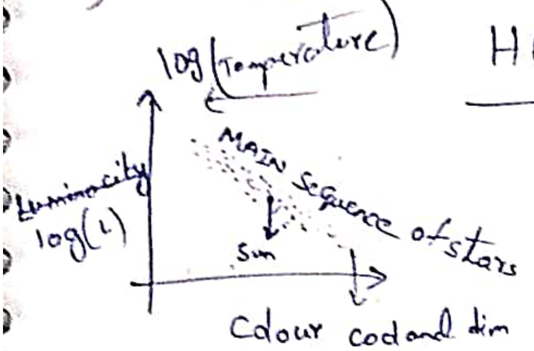
In lasers

$N_{i+1} > N_i$

So, α_ν is -ve.

$\Rightarrow$ τ is negative

1) Planetary Dynamics.
2) Fundamental of Radiative Transfer.
3) stars. → structure of stars → End state of stars.
4) Galaxies
5) Universe.



HR diagram

$$I_\nu = \frac{\nu^3 e^{-h\nu/k_B T}}{1 - e^{-h\nu/k_B T}}$$

$$L \propto 4\pi R^2 \int_0^\infty \frac{\nu^3 e^{-h\nu/k_B T}}{1 - e^{-h\nu/k_B T}} d\nu$$

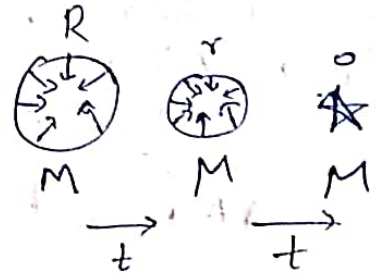
$$\frac{h\nu}{k_B T} = x$$

$$L = 4\pi R^2 \frac{k_B^3 T^3}{h^3} \int_0^\infty \frac{x^3 e^{-x}}{1-e^{-x}} \frac{k_B T}{h} dx$$

$$L = 4\pi R^2 \sigma T^4$$

Radius of star

stars — How are they
— structure
— End - stars of star



Time scale = collapse time
for a cloud of gas to convert to a star.

Time scale = Collapse time

$$\frac{d^2 r}{dt^2} = -\frac{GM}{r^2} \Rightarrow \frac{d}{dt} v = -\frac{GM}{r^2}$$

$$\frac{dr}{dt} \frac{dv}{dr} = v\frac{dv}{dr} = -\frac{GM}{r^2}$$

$$v\frac{dv}{dr} = -\frac{GM}{r^2}$$

$$\int_0^v d\left(\frac{v^2}{2}\right) = \int_R^r -\frac{GM}{r^2}\,dr$$

$$\frac{v^2}{2} = GM\left[\frac{1}{r}-\frac{1}{R}\right]$$

$$v = \pm\sqrt{2GM}\sqrt{\frac{1}{r}-\frac{1}{R}}$$

$$v = \frac{dr}{dt} = \pm\sqrt{2GM}\sqrt{\frac{1}{r}-\frac{1}{R}}$$

$$\int_R^0 \frac{dr}{\sqrt{\frac{1}{r}-\frac{1}{R}}} = \pm\sqrt{2GM}\int_0^{t_{collaspe}} dt$$

$$\frac{1}{2\sqrt{1-\frac{r}{R}}}$$

$$\frac{\sqrt{r}\,dr}{\sqrt{1-\frac{r}{R}}}$$

$$\sqrt{R(1-t^2)}\,dt$$

$\sqrt{r}\,dr$

$\sqrt{1-\frac{r}{R}} = t$

$1-t^2 = \frac{r}{R}$

$r = R(1-t^2)$

$$r = R\sin^2\theta \qquad dr = 2R\cancel{\cos\theta}\sin\theta\cos\theta\,d\theta$$

$$\int_{\pi/2}^0 \frac{dr}{\sqrt{\frac{1-\sin^2\theta}{R\sin^2\theta}}} = \pm\sqrt{2GM}\, t_{collapse}$$

$$\cancel{\sqrt{R}}\int_{\pi/2}^0 \frac{2R^{3/2}\sin\theta\,\cancel{\cos\theta\,d\theta}\times\sin\theta}{\cancel{\cos\theta}} = \pm\sqrt{2GM}\,t_{collapse}$$

$$\cancel{2R^{3/2}\left(-\cos\theta\right)\Big|_{\pi/2}^{0}}$$

$1-2\sin^2\theta/2 = \cos\theta$

$1-\cos 2\theta$

$$\int_{\pi/2}^0 2R^{3/2}\sin^2\theta\,d\theta = \pm\sqrt{2GM}\,t_{collapse}$$

$$R^{3/2}\int_{\pi/2}^0 (1-\cos 2\theta)\,d\theta = \sqrt{2GM}\,t_{collapse}$$

$$R^{3/2}\left(0-\frac{\pi}{2}-\cancel{\left(\sin 2\theta\right)}_{\pi/2}^{0}\right) = \sqrt{2GM}\, t_{collapse}$$

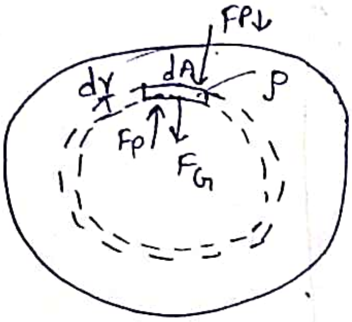
$$-\frac{R^{3/2}\pi}{2} = \sqrt{2GM}\, t_{collapse}$$

$$t_{collapse} = \sqrt{\frac{\pi^2 R^3}{8GM}} = \sqrt{\frac{3\pi}{32 G\rho}}$$

$$\frac{3\pi}{32\times 6.6\times 10^{-11}}$$

$$\frac{\cancel{3\pi}\times \cancel{\frac{4}{3}\pi\times(10^{16})^3}}{32\times 6.6\times 10^{21}\times 10^{-11}}$$

$$\frac{4\times 10^{5}\times 10}{3.2\times 6.6}$$



$$F_G + F_{P\downarrow} + F_{P\uparrow} = 0$$

$$\frac{-GM\,dm}{r^2} + P_{\downarrow} dA + P_{\uparrow} dA = 0$$

$$\cancel{\frac{-GM\,dm}{r^2} + (P+dP)A - P\,dA}$$

$$\frac{-GM\,dm}{r^2} + \left(P_{\uparrow} - P_{\downarrow}\right) dA = 0 \qquad dm = \rho\, dA\, dr$$

$$\frac{-GM\,\rho\,\cancel{dA}\,dr}{r^2} = \left(P_{\downarrow} - P_{\uparrow}\right)\cancel{dA}$$

$$\frac{-GM}{r^2}\rho = -\frac{dP}{dr}$$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \qquad \text{momentum Conservation}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \text{Mass Conservation}$$

$$M = \int_0^r 4\pi r^2 \rho\, dr$$

$$P = K\rho^{\gamma}$$ ~~Adiabatic~~ Thermodynamics.

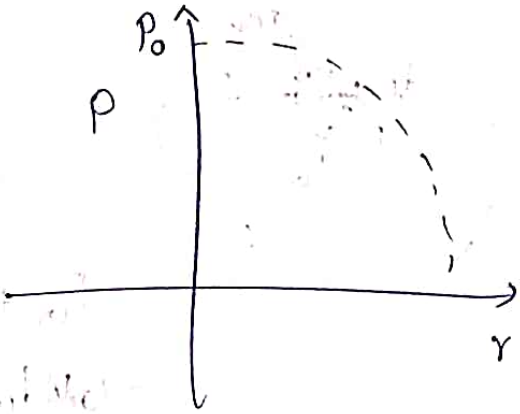
$$M = \frac{4}{3}\pi r^3 \rho$$

$$\frac{dP}{dr} = \frac{-4G\pi\rho}{3} r$$

$$\int_{P_0}^{P} dp = \frac{-4G\pi\rho}{3} \int_0^r r\, dr$$

$$P - P_0 = -\frac{2G\pi\rho}{3} r^2$$

$$P = P_0 - \frac{2\pi G\rho}{3} r^2$$



~~Tutorial - 2~~

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{d\rho}\right) = -G\frac{dM}{dr}$$

$$\frac{d}{dr}\left(\frac{r^2}{\rho}\frac{dP}{dr}\right) = -4\pi G r^2 \rho$$

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho$$

$$P = K\rho^{\gamma} = K\rho^{1+\frac{1}{n}}$$

$$\frac{dP}{dr} = K\left(1+\frac{1}{n}\right)\rho^{1/n}\frac{d\rho}{dr}$$

$$K\left(1+\frac{1}{n}\right)\frac{d}{dr}\left(\frac{r^2}{\rho}\rho^{1/n}\frac{d\rho}{dr}\right) = -4\pi G r^2 \rho$$

$$r = r_0 x$$

$$\rho = \rho_0 y^n$$

$$\frac{K\left(1+\frac{1}{n}\right) r_0^2}{r_0\ r_0}\frac{d}{dx}\left[\frac{x^2}{\rho_0 y^n}\rho_0^{1/n} y \rho_0 n y^{n-1}\frac{dy}{dx}\right]$$

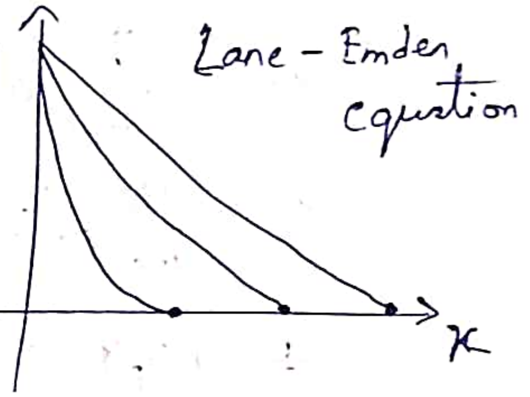
$$= -4\pi G r_0^2 \rho_0 x^2 y^n$$

$$\frac{K\left(1+\frac{1}{n}\right)\rho_0^{\frac{1}{n}}\, n}{4\pi G \rho_0 r_0^2}\frac{d}{dx}\left[x^2\frac{dy}{dx}\right] = -x^2 y^n$$

$$\Rightarrow \frac{k\left(1+\frac{1}{n}\right)\rho_0^{\frac{1}{n}-1}\cdot n}{4\pi G r_0^2} = 1$$

$$r_0 = \sqrt{\frac{k(n+1)\,\rho_0^{\frac{1}{n}-1}}{4\pi G}}$$ (appropriate constant value).

$$\frac{d}{dx}\left[x^2\frac{dy}{dx}\right] = -x^2 y^n \longrightarrow$$



$$M = \int_0^{r_{max}} 4\pi r^2 \rho\, dr = 4\pi r_0^3 \rho_0 \int_0^{x_{max}} y^n x\, dx.$$

For $n = 3$: $r_0 \propto \rho_0^{-1/3}$.

$$\frac{dM}{dr} = 4\pi r^2 \rho \qquad \frac{dP}{dM} = -\frac{GM}{4\pi r^4}$$

$$\frac{dP}{dr} = -\frac{GM}{r^2}\rho \qquad P = k\rho^{1+\frac{1}{n}}$$

$$\frac{P}{M} \propto \frac{GM}{r^4}$$

$$n=3; \quad P \propto \left(\frac{M}{R^3}\right)^{4/3}$$

$$P = \frac{GM^2}{R^4}$$

$$P = \frac{K.\, M^{4/3}}{R^4}$$

white dwarf stars

$$\Delta x \, \Delta p_e = \hbar$$

$$\Delta p_e = \frac{\hbar}{\Delta x}$$

$$\Delta v_e = \frac{\hbar}{m_e \Delta x} =$$

$$v_e = \frac{\hbar}{m_e \Delta x} = \frac{\hbar}{m_e V^{1/3}}$$

$$v_e \propto \frac{\hbar}{m_e} n_e^{1/3}$$

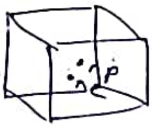
$$v_e = \frac{\hbar}{m_e} n_e^{1/3}$$

$$\cancel{P = n k_B T}$$

$$\cancel{\propto n\,(K.E)}$$

$$P_e = n_e m_e v_e^2$$

$$\boxed{P_e = \frac{\hbar^2}{m_e} n_e^{5/3}}$$



$$\rho = n_e m_e + n_p m_p$$

$$P = \frac{\hbar^2}{m_e} \frac{\rho^{5/3}}{m_p^{5/3}}$$

$$P = \frac{\hbar^2}{m_e m_p^{5/3}} \rho^{5/3}$$

Rough Calculations

$$P = \frac{GM^2}{R^4} = \frac{\hbar^2}{m_e m_p^{5/3}} \left(\frac{M}{R^3}\right)^{5/3}$$

$$\frac{GM^2}{R^4} = \frac{\hbar^2}{m_e m_p^{5/3}} \frac{M^{5/3}}{R^5}$$

$$R = \frac{\hbar^2}{G m_e m_p^{5/3}} M^{-1/3}$$

$$R = \left(\frac{M}{M_0}\right)^{-1/3}$$

$$\rho = \frac{M}{R^3} = \left(\frac{G m_e m_p^{5/3}}{\hbar^2}\right)^3 M^2$$

$$v_e = \frac{\hbar \rho^{1/3}}{m_p^{1/3}}, \quad v_e^3 = \frac{\hbar^3 \rho}{m_p}$$

$$\rho = \cancel{\frac{m_p^{5/3}}{\hbar} v_e} \ \frac{m_p}{\hbar} v_e^3$$

Let there are N particles in a volume of V, <u>Chandrasheker limit</u>

then volume occupied by one particle is $\left(\frac{V}{N}\right) = \frac{1}{n} \rightarrow$ number ~~number~~ ~~of~~ density

$$\Delta x \, \Delta p_e \leq \hbar$$

$$\frac{1}{n^{1/3}} m_e v \leq \hbar$$

In previous class,

$$\cancel{v_e = \frac{\hbar \rho^{1/3}}{m}} \qquad v_e = \frac{\hbar}{m_e}(n_e)^{1/3}$$

$$= \frac{\hbar}{m_e}\left(\frac{\rho}{m_p}\right)^{1/3} \qquad \rho = \frac{n m_e + n m_p}{m_p} = \frac{m_e}{m_p} + n$$

$$K.E \propto T \Rightarrow \text{Pressure} = n(K.E)$$

$$P = n \frac{m_e \hbar^2}{m_e^2} n^{2/3}$$

$$= \frac{\hbar^2}{m_e} n^{5/3} = \frac{\hbar^2 \rho^{5/3}}{m_e m_p^{5/3}}$$

$$\boxed{P \propto \rho^{5/3}}$$

$$\frac{dM}{dr} = 4\pi r^2 \rho$$

$$\frac{dP}{dr} = -\frac{GM}{r^2} \rho$$

$$P = f(\rho)$$

$$\frac{P}{R} = \frac{GM}{R^2}\frac{M}{R^3}$$

$$(10^{-34})^2$$

$$P = \frac{GM^2}{R^4}$$

$$\frac{\left(\frac{h}{2\pi}\right)^2}{G m_e}$$

$$\frac{10^{-64} \times 10^{11}}{(6.67 \times 9.1) \times 10^{-31} \times (1.6 \times 10^{-27})^{5/3}}$$

Star would be stable for radius R?

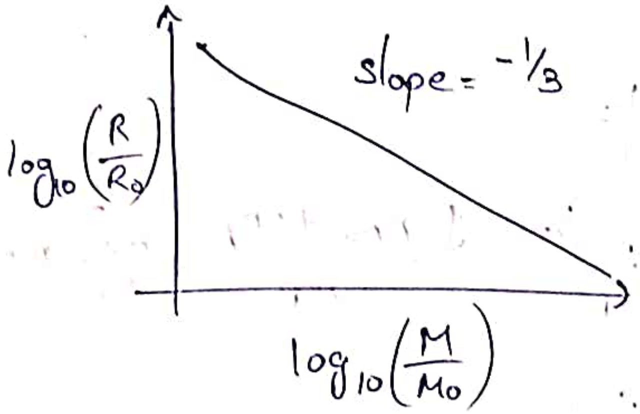
$$\frac{GM^2}{R^4} = \frac{\hbar^2}{m_e m_p^{5/3}}\left(\frac{M}{R^3}\right)^{5/3} = \frac{\hbar^2}{m_e m_p^{5/3}}\frac{M^{5/3}}{R^5}$$

$$R = \frac{\hbar^2}{G m_e m_p^{5/3}} M^{-1/3}$$

$$\left(\frac{R}{R_0}\right) \approx \frac{1}{100}\left(\frac{M}{M_0}\right)$$

where, R_0 = Radius of Sun $= 7 \times 10^8\,m$

M_0 = mass of Sun $= 2 \times 10^{30}\,kg$.



$$V = \frac{\hbar}{m_e}\left(\frac{\rho}{m_p}\right)^{1/3}$$

$$\boxed{\rho = \frac{M}{R^3} = \frac{G^3 m_e^3 m_p^5}{\hbar^6} M^2}$$

Maximum density (so that $V \leq c$)

$$\rho_{max} = \frac{c^3 m_e^3 m_p}{\hbar^3}$$

$$\rho < \rho_{max}$$

$$\frac{G^3 m_e^3 m_p^5}{\hbar^6} M^2 < \frac{c^3 m_e^3 m_p}{\hbar^3}$$

$$\Rightarrow M < \sqrt{\frac{c^3 \hbar^3}{m_p^4 \cancel{\;} G^3}}$$

~ 1.44 (chendrashaker limit)

$K.E = \frac{1}{2} m v^2$ (non-realestic relation.

$$P = pc$$

$$P = n\, p c = n \frac{\hbar}{m_e} \frac{\rho}{m_p} \quad \hbar \left(\frac{\rho}{m_p}\right)^{1/3} c$$

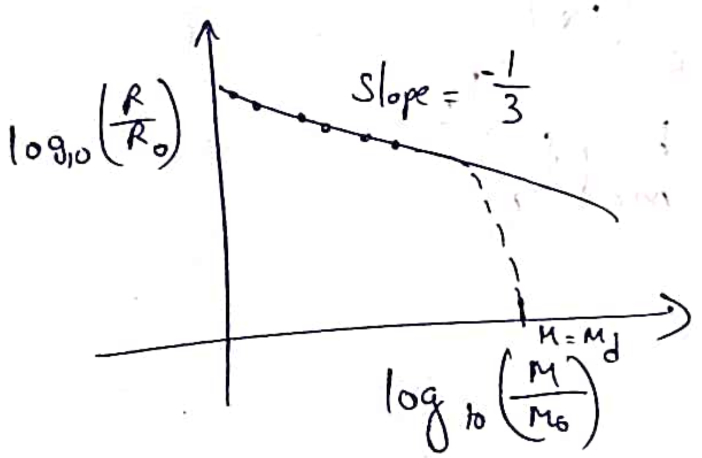
$$\boxed{P = \frac{\hbar c}{m_p^{4/3}} \rho^{4/3}} \longrightarrow \text{Pressure (Relasticvitic white drawt } \cancel{\text{celestial}} \text{ star body)}$$

Balance for relastovistic w. D star

$$\frac{GM^2}{\cancel{R^4}} = \frac{\hbar c}{m_p^{4/3}} \rho^{4/3} = \frac{\hbar c}{m_p^{4/3}} \times \frac{M^{4/3}}{\cancel{R^4}}$$

$$G^3 M^6 = \frac{\hbar^3 c^3}{m_p^4} M^4$$

$$\Rightarrow M = \sqrt{\frac{\hbar^3 c^3}{G^3 m_p^4}}$$



$7M_0 \rightarrow 18M_0$
$\downarrow$
neutron star

if $18M_0 >$
big back hole.

Tutorial - 3

1) Sol

$$t_{collapse} = \sqrt{\frac{\pi^2 R^3}{8GM}} = \sqrt{\frac{\pi^2 (7\times10^8)^3}{8 \times 6.67\times10^{-11} \times 2\times10^{30}}}$$

$$= 1770.8 \text{ sec}$$

2) $t_{uni} = \cancel{\sqrt{\frac{\pi^2 R^3}{}}} \quad \sqrt{\frac{3\pi}{32G\rho}}$ $\quad \rho_{uni} = 9\times10^{-27}$

$$= 6.9\times10^{17} \text{ sec}$$

3) VIRIAL Th: $T = \frac{Eg}{2}$

$$\cancel{\frac{Eg}{2}}\ T = Eg + \frac{Eg}{2}$$

As time passess Eg increasces, gravitational potential increasces than kinetic energy. so the star Collapse

4) $\cancel{V = \frac{\hbar}{m_e}(n_e)^{1/3}}$

$\cancel{R = \frac{\hbar^2}{}}$

$$V = \frac{\hbar}{m_e}(n_e)^{1/3}$$

$$= \frac{\hbar}{m_e}\left(\frac{\rho}{m_p}\right)^{1/3}$$

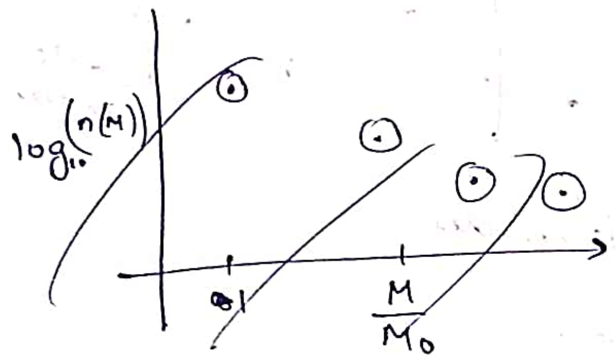
7. $L \cdot t = P.E + \Delta E$

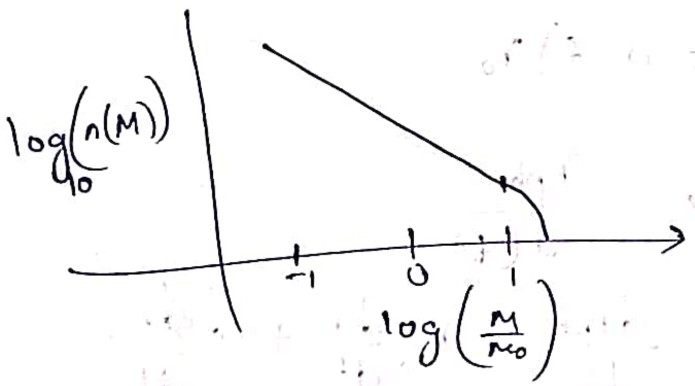
$$t = \frac{P.E + \Delta E}{L}$$

Galaxies of Universe.

Initial Mass (M)	Mass at the final stage (M_c)
$M \leq 7M_\odot$	$M_c \leq 1.44 M_\odot$
$18M_\odot \geq M > 7M_\odot$	$M_c > 1.44 M_\odot$
$M > 18M_\odot$	$M_c > 1.44 M_\odot$

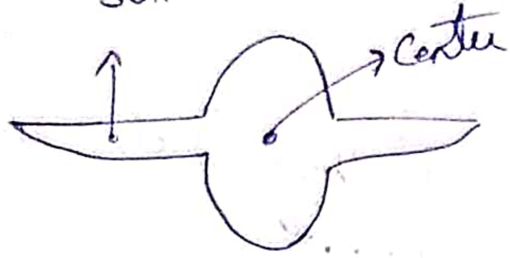
$$\frac{L_1}{4\pi r_1^2} = \frac{L_2}{4\pi r_2^2}$$ (If two stars of same mass) L_1, L_2 are their luminosities.

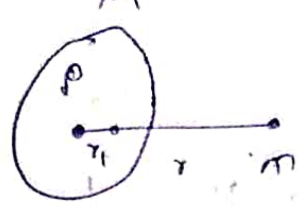




$$n \propto m^{-p} e^{\left(\frac{-m}{m_*}\right)}$$

Milkyway



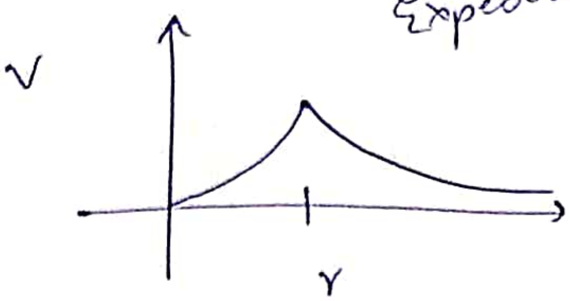


$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

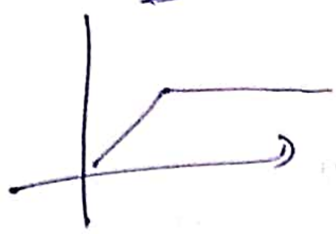
$$M = \frac{4}{3}\pi r_1^3 \rho \qquad v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{4\pi r_1^3 G \rho}{3 r_1}}$$

Expected



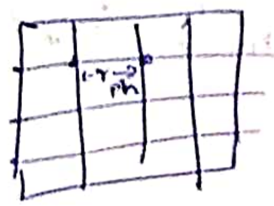
But actual



$v \propto r^0$
→ Black matter

There must be black matter

Edwin Hubble's classification scheme.



$$r_{ph} = a(t)\, r_0$$

$$r_{co} = \frac{r_{ph}}{a(t)}$$

$$v = \frac{dr_{ph}}{dt} = r_{co}\frac{da}{dt} + a\cancel{\frac{dr_{co}}{dt}}^{\,0}$$

$$v = r_{co}\dot{a}$$

$$v = a r_{co}\frac{\dot{a}}{a} = r_{ph}\frac{\dot{a}}{a}$$

$r_{ph} = a(t) r_{co}$

$H = \frac{\dot{a}}{a} \approx 70 km/s/Mpc$

$V = H r_{ph}.$

$\boxed{z = \frac{1}{a(t)} - 1} \rightarrow$ Red shift and hubble