Department of Mathematics
Indian Institute of Technology Bhilai
IC152: Linear Algebra-II
Tutorial Sheet 2

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1. Test the diagonalizability of the following linear operators

   (i) $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined as $Tf = f'$.

   (ii) $T : P_2(\mathbb{R}) \to P_2(\mathbb{R})$ defined as $(Tf)(x) = f'(x) + f''(x)$.

   (iii) $T : \mathbb{R}^3 \to \mathbb{R}^3$, defined as $T(x, y, z) = (4x + y, 2x + 3y + 2z, x + 4z)$

   (iv) $T : \mathbb{C}^2(\mathbb{C}) \to \mathbb{C}^2(\mathbb{C})$ defined as $T(w, z) = (w + iz, z + iw)$.

2. Let λ be an eigenvalue of a linear operator T on V, then show that λ^k is an eigenvalue of T^k. Can we generalize the above result, i.e., if λ is an eigenvalue of T and μ is an eigenvalue for S, then $\lambda\mu$ is an eigenvalue for TS?

3. Find out the eigenvalues of the matrix $A = \begin{bmatrix} -2 & 10 & -6 \\ 5 & -18 & 15 \\ 3 & -10 & 9 \end{bmatrix}$ without finding roots of characteristic polynomial.

4. Show that a diagonalizable linear transformation on a finite dimensional vector space having only one eigenvalue is a scalar multiple of identity operator.

5. Let trace of a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ be α and λ be one of the eignevalues of T. If the eigenspace corresponding to the eigenvalue $\lambda \in \mathbb{R}$ of T is 2- dimensional. Then find all the choices of eigenvalues of T. Is T diagonalizable for your choices of eigenvalues?

6. Let n be a positive integer. Find A^n for the following matrix $A = \begin{bmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{bmatrix}$

7. Check if the matrices $A \in M_{n\times n}(\mathbb{R})$ given below are diagonalizable. Also find an invertible matrix Q and diagonal matrix D such that $A = QDQ^{-1}$.

   (i) $\begin{bmatrix} 2 & -2 & 2 \\ 0 & 1 & 1 \\ -4 & 8 & 3 \end{bmatrix}$ (ii) $\begin{bmatrix} -3 & 1 & -3 \\ 20 & 3 & 10 \\ 2 & -2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ (iv) $\begin{bmatrix} -1 & -1 & -2 \\ 8 & -11 & -8 \\ -10 & 11 & 7 \end{bmatrix}$

8. As an application of diagonalizability: Find a general solution of the following system of differential equations $x' = x + y, \;\; y' = 4x + y$, where $x = x(r)$ and $y = y(r)$ are real valued functions of $r \in \mathbb{R}$.