

# Tutorial - 1

## MA202 : Calculus II

1. Determine whether the limits of the following functions exist as  $(x, y) \rightarrow (0, 0)$

- (a)  $f(x, y) = \frac{xy}{x^2+y^2}, (x, y) \neq (0, 0),$
- (b)  $f(x, y) = \frac{|x|}{y^2} e^{-\frac{|x|}{y^2}}, y \neq 0,$
- (c)  $f(x, y) = \frac{x^2 y}{x^4+y^2}, (x, y) \neq (0, 0),$
- (d)  $f(x, y) = \log \left( \frac{\sqrt{x^2+y^2+x}}{\sqrt{x^2+y^2-x}} \right), y \neq 0,$

2. Using  $\epsilon - \delta$  method prove the followings

- (a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{4xy^2}{x^2+y^2} = 0$
- (b)  $\lim_{(x,y) \rightarrow (-1,-1)} (xy - 2x^2) = -1$
- (c)  $\lim_{(x,y) \rightarrow (-2,-2)} \frac{x^2-y^2}{x-y} = -4$
- (d)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y^2}{x^2+y^2} = 0.$

3. Using  $\epsilon - \delta$  method prove that the following functions are continuous at the given points.

- (a)  $f(x, y) = \begin{cases} xy & \text{if } (x, y) \neq (2, 3) \\ 6 & \text{if } (x, y) = (2, 3) \end{cases}$  Check at  $(2, 3)$
- (b)  $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$  Check at  $(0, 0)$
- (c)  $f(x, y) = \begin{cases} xy \frac{x^2-y^2}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$  Check at  $(0, 0)$
- (d)  $f(x, y) = \begin{cases} \frac{x^3}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$  Check at  $(0, 0)$

4. Show that the function is continuous at  $(0, 0)$  but partial derivatives do not exist.

$$f(x, y) = \begin{cases} \frac{x^2+y^2}{|x|+|y|} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

5. Show that the following function possesses partial derivatives at  $(0, 0)$  but not continuous.

$$f(x, y) = \begin{cases} \frac{x^2 y}{x^4+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

6. Find  $f_x(0, 0)$ ,  $f_y(0, 0)$ ,  $f_x(0, y)$  and  $f_y(x, 0)$  of the following functions.

- (a)  $f(x, y) = \begin{cases} 1 & \text{if } (x, y) = (0, 0), \text{ or } x = 0, \text{ or } y = 0 \\ 0 & \text{otherwise} \end{cases}$
- (b)  $f(x, y) = \begin{cases} \frac{xy}{x+y} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

7. Discuss the differentiability of the following functions at  $(0, 0)$

- (a)  $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$
- (b)  $f(x, y) = \begin{cases} (x^2 + y^2) \sin \frac{1}{x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$