

Indian Institute of Technology Bhilai

IC105: Probability and Statistics

Assignment 5

February 5, 2022

1. Suppose X denotes the time (in hours) needed to locate and rectify a problem in the software that governs the timing of lights in a large city. Let X follows $N(10, 9)$. Find the probability that the next problem will require at most 15 hours to locate and rectify.
2. In a town adult population have equally divided opinion on the implementation of a municipality project. A random sample of 100 adults is asked to give their opinion. Using binomial approximation to normal determine the probability that at least 60% of adults in the sample are in favour of the project.
3. In an industrial process the diameter of a ball bearing is an important component. The buyer sets specifications on the diameter to be 3.0 ± 0.01 cm. The diameter has a normal distribution with mean 3 cm and standard deviation 0.005 cm. On the average how many manufactured balls will be scrapped?
4. The height a university high jumper will clear, each time he jumps, is a normal random variable with mean 2 meters and standard deviation 10 cm. What is the greatest height that he will jump with probability 0.95? What is the height that he will clear only 10% of the time?
5. If the set of Students' total marks in IC105 follows $N(74, 62.41)$, find (a) the lowest passing marks if the lowest 10% of the students are given Grade F ; (b) the cutoff marks for Grade A if the top 5% of the students are given Grade A .
6. A box contain 5 white balls numbered 0, 1, 2, 3, 4; 4 red balls numbered 0,1,2,3; 3 black balls numbered 0, 1, 2 and 2 blue balls numbered 0,1. A ball was drawn at random from the bag. Let the random variables X and Y defined as follows: X takes values 0, 1, 2, 3 respectively for white, red, black and blue balls. Y denotes the number on the ball. Find the joint distribution of X and Y . Also find the marginal distribution of X and Y .
7. A boy and a girl plan to meet a coffee shop between 4 pm and 5 pm each agreeing not to wait more than 20 minutes for the other. They arrived at the coffee shop independently between 4 pm to 5 pm. What is the probability that they will meet?

8. The joint pdf of X and Y is given as

$$f_{X,Y}(x,y) = \begin{cases} k(1-x-y), & x > 0, y > 0, x+y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of k .
- (b) Find the marginal pdf of X and Y .
- (c) Find $E(X)$, $E(Y)$, $E(XY)$ and $\text{Cov}(X,Y)$ and $\rho_{X,Y}$.

9. A two dimensional discrete random vector (X,Y) having pmf as

$$f_{X,Y}(x,y) = P(X=x, Y=y) = \begin{cases} c(3x+4y), & x=0,1,2,3, y=1,2,3,4, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the value of c .
- (b) Find the marginal distribution of X and Y .
- (c) Find $P(X \geq 2|Y \leq 3)$ and $P(Y=2|X=3)$.

10. The joint pmf of a discrete random vector (X,Y) is given as

X/Y	-1	0	1
0	0	1/3	0
1	1/3	0	1/3

- (a) Find the marginal distribution of X and Y .
- (b) Find $E(X)$, $E(Y)$, $E(XY)$ and $\rho_{X,Y}$.
- (c) Are X and Y independent?

11. Let (X,Y) be a random vector with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 6xy(2-x-y), & 0 < x < 1, 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the marginal distribution of X and Y .
- (b) Find $E(X)$, $E(Y)$, $E(XY)$ and $\rho_{X,Y}$.
- (c) Are X and Y independent?

12. Let (X,Y) be a random vector with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} \frac{6-x-y}{8}, & 0 < x < 2, 2 < y < 4, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Find the marginal distribution of X and Y .
- (b) $P(X < 1, Y < 3)$, $P(X+Y < 3)$, $P(X < 1|Y=3)$ and $P(X < 1|Y < 3)$.