Context free grammar

Sentence

Non leaf node → non terminals
Leaf node → Terminals

A context free grammar is a 4-tuple
< N, T, P, S > where
N is a set of non terminals
T is a set of terminals (Σ)
P set of rules
$S \in N$ is called start symbol.

Ex $G = < S, \{0, 1\}, P, S >$

P is

$S \rightarrow 0S1$
$S \rightarrow \epsilon$

$\{\epsilon, 01, 0011, 000111$

$L = \{ w \in 0^i 1^i \quad i \geq 0 \}$

Ex $L = \{ w \mid w \text{ is a palindrome} \}$

$G = \langle S, \{0,1\}, P, S \}$

P

$S \to \epsilon$
$S \to 0S0$
$S \to 1S1$
$S \to 0$
$S \to 1$

Ex $L = \{ ww^R \mid w \in \Sigma^* \}$

$G = <$

P

$S \to \epsilon$
$S \to 0S0$ Even length palindrome
0110 $S \to 1S1$

Ex $L = \{ w \mid w \text{ has equal no of } 0 \,\&\, 1\text{s} \}$

0S

$S \to 0S1S$	$S \to \epsilon$
$S \to 1S0S$	$S \to 0S1$
$S \to \epsilon$	$S \to 01S$
	$S \to S01$
	$S \to S10$
	$S \to 1S0$

1111 000

6

$S \to 10S$

Ex $L = \{ x \neq ww \text{ for } w \in (0+1)^* \}$

All odd len $\in L$

$S \to \epsilon$,
$S \to 0$

Ex $L = \{$

$S \to 0$
$S \to 1$
$S \to S01$
$S \to S10$
$S \to S00$
$S \to S11$

0S0
0S1
1S0
1S1

$S \to 0, 1$

$x = w_1 w_2$

$\boxed{a_1 a_2 - a_k} b_1 b_2 - b_k$

T

$M \rightarrow T_1 T_2$

$M \rightarrow T_3 T_4$

$T_1 \rightarrow 0T_10$
$\rightarrow 0T_11$
$\rightarrow 1T_10$
$\rightarrow 1T_11$

$T_1 \rightarrow 0$

$T_2 \rightarrow 0T_20$
$\rightarrow 0T_21$
$\rightarrow 1T_20$
$\rightarrow 1T_11$

$T_2 = 1$

$T_3 \rightarrow 0T_30$
$\rightarrow 0T_31$
$\rightarrow 1T_30$
$\rightarrow 1T_31$

$T_3 \rightarrow 1$

$T_4 \rightarrow 0$

$S' \rightarrow S$

$S' \rightarrow M$

A language is called context free if every string of the language can be generated using a context free grammer

$$P : N \rightarrow (N \cup T)^*$$

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→ To prove every language is context free
→ Closure properities of context free languages

Thm Every regular language is context free

Let L be regular language
$\Rightarrow$ L has a regular expression R

Base case if $R = a$ for some $a \in \Sigma \cup \{\epsilon\}$
Then $S \to a$

Inductive Step

Suppose every regular expression of length at most k is context free.

Let R be a regular expression of length $k+1$

(1) $R_1 + R_2$ for some $R_1 R_2 < K$
(2) $R_1 R_2$ $S \to S_1$ $S \to S_2$ ~~for some~~ R_1 $S \to S_1 S_2$
(3) R_1^*

$S \to S S_1$
$S \to \epsilon$

E $L \to 0^*1(0+1)^*$

$A \to 0A \mid \epsilon$

$B \to 0B \mid 1B \mid \epsilon$

$S \to \cancel{AB}\ A1B$

Regular Grammer

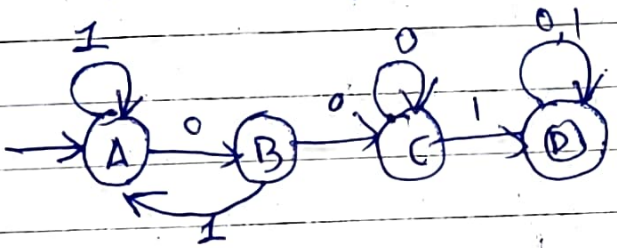
A grammer is called right linear if every tranisition is of type

$A \to \alpha B$ $\qquad A, B \in N$, $\alpha \in \Sigma^*$

left linear

$A \to B\alpha$

$(0+1)^* 001 (0+1)^*$ Regular to right lie



Prove induction

$N = \{A, B, C, D\}$

$A \to 1A$
$A \to 0B$
$B \to 0C$
$B \to 1A$
$C \to 0C$
$C \to 1D$
$D \to 0D \mid 1D \mid \epsilon$

Closure

~~1 Union of two Cu~~ C

~~2 Con~~

~~3~~

4 $\cap$ X

5 Complement X

$L = \{x \mid x \neq ww\}$
L^C is not context free.

$L_1 = \{a^m b^m c^n \mid m, n \geq 0\}$

$L_1 \cap L_2$

$L_2 = \{a^m b^n c^m \mid m, n \geq 0\}$

$\{a^n b^n c^n, n \geq 0\}$ $L_1 \cap L_2$ is not context free

$L_1 = \{a^m b^m c^n \mid m, n \geq 0\}$

N, T, P, S

$T = \{a, b, c\}$

$S \to \epsilon$

~~$S \to aSbS$~~

~~$S \to bSaS$~~

$S \to Sc$

$S \to cS$

$$\boxed{\begin{array}{l} S \to aSb \\ S \to \epsilon \\ M \to Sc \end{array}}$$

~~$S \to SM\epsilon$~~

$$\boxed{\begin{array}{l} S \to S_1 S_2 \\ S_1 \to aS_1b \mid \epsilon \\ S_2 \to S_2c \mid \epsilon \end{array}}$$

$S \to S_1 S_2 S_3$

$S_1 \to aS_2c \mid \epsilon$

$S_2 \to S_2b \mid \epsilon$

$S_1 \to S_2$

$S_1 \to S_1 b \mid \epsilon$

$S_2 \to S_1$

$S_2 \to \cancel{a d S_2 \epsilon}$

$a S_2 c \mid \epsilon$

$S \to aSc \mid \epsilon$

$S \to aSc \mid S_1 \mid \epsilon$

$S_1 \to b S_1 \mid \epsilon$

$L = \{ a^m c^p d^q \mid m = p+q \}$

$\cancel{S_1 \to a S_1 b \mid \epsilon}$

$\boxed{S \to aSc \mid aSd \mid \epsilon}$

$S_1 \to a S_1 c$

$a S_1 c$ $\quad S_2 \to a S_2 d$

$a a S_3 d c$

$$\boxed{\begin{array}{l} S_1 \to a S_1 c \mid \epsilon \\ S_3 \to S_1 \mid \epsilon \\ S_3 \to a S_3 d \mid \epsilon \end{array}}$$

$S_1 \to S a c$

S aacd

a aacdd

$S_1 \to a S_1 d \mid \epsilon$

$S_3 \to S_1 \mid \epsilon$

$S_3 \to a S_3 c \mid \epsilon$

$S \to aSd \mid S_1 \mid \epsilon$

$S_1 \to aS_1c \mid \epsilon$

$a^m b^n c^p d^q \quad m+n = p+q$

abd $\quad S \to aSd \mid S_1 \mid \epsilon$

acd $\quad \cancel{S_1 \to aS_1d \mid S_2 \mid \epsilon}$

$\cancel{S_2 \to}$

$S_1 \to bS_1d \mid S_2 \mid \epsilon$

bcd $\quad S_2 \to bS_3c \mid S$

$\cancel{A_1 \to aBd \mid A_2 \mid \epsilon}$ | $B_2 \to aSc \mid B_2$

$A_1 \to aA_1d \mid A_2 \mid \epsilon$

$S \to aSd \mid S_1 \mid S_2 \mid \epsilon$

$S_1 \to bS_1d \mid S_3 \mid \epsilon$

$S_3 \to bS_3c \mid \epsilon$

$S_2 \to aS_2c \mid S_3 \mid \epsilon$

$\cancel{S_4 \to}$

Q $L = \{x^R y \mid y \in (0+1)^* \; x \text{ is a substring of } y\}$

$S \rightarrow 0 \mid 1$
$S_1 \rightarrow SS_1 \mid \epsilon$

$A \rightarrow BS$
$B \rightarrow \epsilon$

$w = x^R u x z$

$S \rightarrow \epsilon$
$S \rightarrow 0AS0BA$
$S \rightarrow 1AS1BA$

$A \rightarrow 0 \mid 1$
$B \rightarrow AB \mid \epsilon$

$S \rightarrow SS_2 \mid \epsilon$
$S_1 \rightarrow 0S_10 \mid 1S_11 \mid S_2$

$S_2 \rightarrow 0S_2 \mid 1S_2 \mid \epsilon$

$S \rightarrow \epsilon$
$S \rightarrow 0S0$
$S \rightarrow 1S1$
$S \rightarrow S_1$
$S_1 \rightarrow AS_1A$
$A \rightarrow 0 \mid 1$
$B \rightarrow AB \mid \epsilon$

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E $L = \{ x c^n \mid x \in (a+b)^* \text{ and } |x|_a + |x|_b > n \}$

Context free grammer for L

~~$S \to aS_1S$~~

$\cancel{S \to SaSc \mid S_1}$

$\cancel{S_1 \to S_1 b S_1 c}$

~~$S \to SS_2$~~

$S \to aSc \mid bSc \mid S_1$

$S_1 \to aS_1 \mid bS_1 \mid a \mid b$

~~$S_2 \to S_2$~~

E Let ~~Prove that~~ $L = \{ a^n b^n \mid n \geq 0 \}$

Prove that L^c is context free

~~$L^c = \{ a^m b^n \mid m \neq n \}$~~

$(a+b)^* = a^* b^* + (a^* b^*)^c$

$= a^n b^n + a^m b^n + (a^* b^*)^c$

$L^c = a^m b^n + (a^* b^*)^c$

$S \to aSb \mid S_1 \mid S_2$

$S_1 \to aS_1 \mid a$

$S_2 \to bS_2 \mid b$

Regular

$\downarrow$

Complement

$\downarrow$

Regular

$\downarrow$

Context free

$15 \le 5n \le 20$

$\le 4m$ $\overset{5}{m}$ $+ n$

$\le m$

(E) $L = \{ a^m b^n \mid 3m \le 5n \le 4m \}$

Prove L is context free

$S \to aaaSbb \mid aaaaSbbb \mid aaaaaSbbb$
$\mid aaaaaSbbbb \mid \epsilon$

S_1

$a|b$

~~m is multiple~~

L_0 $m \equiv 0 \pmod 5$

L_1 $m \equiv 1 \pmod 5$

L_2 $m \equiv 2 \pmod 5$

L_3 $m \equiv 3 \bmod 5$

L_4 $m \equiv 4 \bmod 5$

$S \to A \mid B \mid C \mid D \mid E$

$\frac{3}{5p}$

L_0 $m = 5p$ $15p \le 5n \le 20p$

$a^m b^n$ $3p \le n \le 4p$

L_0

$a^{5p} b^{3p+k}$ $0 \le k \le p$

$A \to a^5 A b^3 \mid a^5 A b^4 \mid \epsilon$

$L4$ $L_1 = m = 5p + 4$

$15p + 12 \le 5n \le 20p + 16$

$3p + 2.2 \le n \le 4p + 3.2$

$3p + 3 \le n \le 4p + 3$

$a^{5p} a^4 b^{3p+k} b^3$ $0 \le k \le p$

$B \to a^4 B_1 b^3$

$B_1 \to a^5 B_1 a^3$
$a^5 B_1 a^4 \mid \epsilon$

$3p + 2.2$ 5×0.8 $3p+3$ 5×0.8 21 5.8 $3p+3$ $3p+3$

L_3

$m = 5p+3$

$15p+3 \leq 5n \leq 20p+12$

$\frac{9}{5}$ $3p+2 \leq n \leq 4p+2$

$C \rightarrow a^3 C_1 b^2$

$\frac{9}{5}$ $C_1 \rightarrow B_1$

L_2 $m = 5p+2$

$15p+6 \leq 5n \leq 20p+8$ $\frac{12}{5}$

$3p+2 \leq n \leq 4p+1$

$3p < n - k$

$5 < 4p$

$4p$ $3p+k$

$p < k < p+1$

$0 \leq k-1 \leq p$

$a^2 a^{5p} b^{3p+k} b^3$ $0 \leq k \leq p+1$

$a^2 a^{5p} b^{3p}$

$3(p-1)+5 \leq n \leq 4(p-1)+5$

$3p+2 = 3(p-1)+5$

$3p' + 5 \leq n \leq 4p'+5$

$D \rightarrow a^7 D_1 b^5$

$D_1 \rightarrow a^5 D_1 b^3 \mid \epsilon$

$a^5 D_1 b^4 \mid$

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$m = 5p+1 \qquad 5(p-1)+6$

4 $\quad 15p+3 < 5n \le 20p+4$

$3p+1 \le n \le 4p$

$3(p-1)+4 \le n \le 4(p-1)+4$

$3p-3+ \quad 3p'+4 \le n \le 4p'+4$

$D \rightarrow a^6 D_1 b^4$

$D_1 \rightarrow a^5 D_1 b^3 \mid$

$a^5 D_1 b^4 \mid$

ϵ

X ——— X ———

Parsing & Ambiguity

$S \rightarrow SS \mid (S) \mid \epsilon$

$()()$

~~$S \rightarrow SS \rightarrow (S)(S) \rightarrow ()()$~~

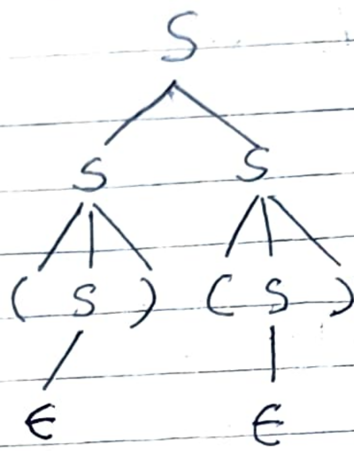
$S \rightarrow SS \rightarrow (S)S \rightarrow ()S \rightarrow ()(S) \rightarrow ()()$

$S \rightarrow SS \rightarrow S(S) \rightarrow S() \rightarrow (S)() \rightarrow ()()$

Left most derivation

A derivation of a string w from a grammer is called LMD if $\forall$ every step the leftmost variable or non terminal is substituted by a rule of the grammer

Derivation represented by tree called parse tree



$E \rightarrow E+E \mid E*E \mid (E) \mid a$ Ambi.

Not Amb.

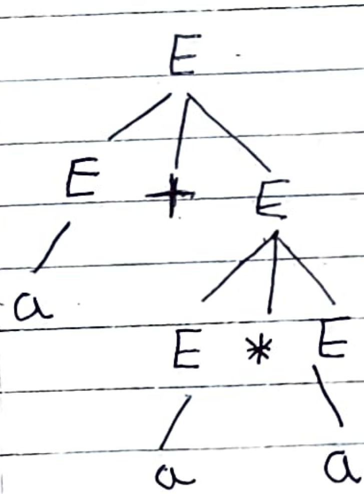
$S_1 \rightarrow S_1+T \mid T$

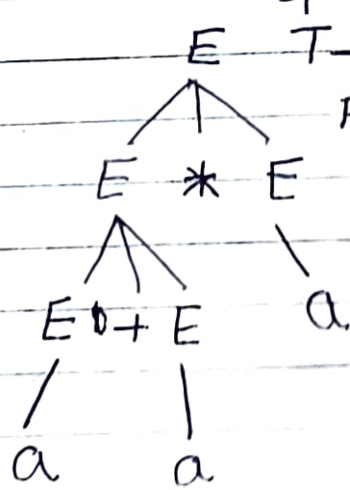
$T \rightarrow T*T \mid F$

$F \rightarrow a \mid (S_1)$

Induction

$a+a*a$





2a

a^2+a

$2a^2$

Ambiguous grammer

A grammer is called ambiguous ∃ a string w that has two different leftmost derivation or equivalently two different parse tree

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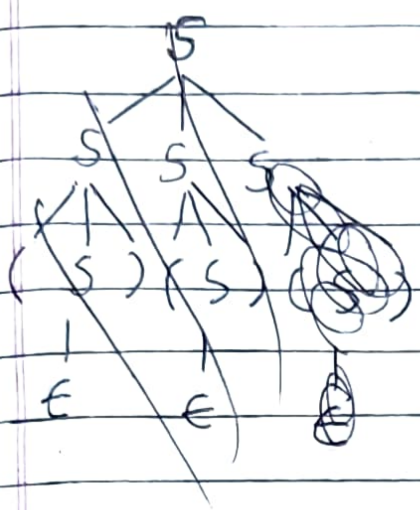

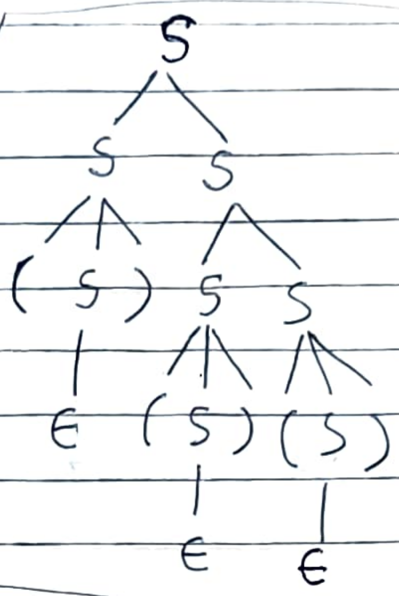

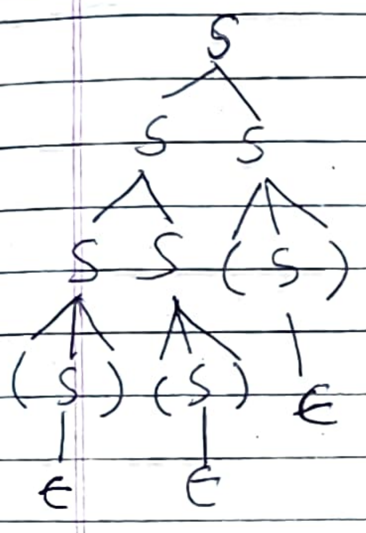

Ambiguity : property of grammer

Proof idea

+ outside bracket

$S_1 \to S_1 + T$

* outside bracket

$S_1 \to T$

$T \to T * F$

+ * inside bracket

Grammer ambiguous or not

↳ Undecidable problem TOC2

~~# whether a grammar is ambigious or not~~;

Some languages are inherently ambiguous.

eg $L = \{a^i b^j c^k \mid i = j \text{ or } j = k\}$

$S \rightarrow S_1 \mid S_2$

$S_1 \rightarrow S_3 S_4$

$S_3 \rightarrow aS_3b \mid \epsilon$

$S_4 \rightarrow cS_4 \mid \epsilon$

$S_2 \rightarrow S_5 S_6$

$S_5 \rightarrow aS_5 \mid \epsilon$

$S_6 \rightarrow bS_6c \mid \epsilon$

Ambiguity

Union

~~Intersection~~ $a^n b^n c^n$

Intersection → Ambiguity

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Pushdown Automata

Equal no of a's & b's

A PDA is a 6-tuple $\langle Q, \Sigma, \Gamma, \delta, q_0, F)$

$\delta : Q \times \Sigma_{\cup\{\epsilon\}} \times \Gamma_{\cup\{\epsilon\}} \rightarrow P(Q \times \Gamma_{\cup\{\epsilon\}})$

$0^n 1^n$

$Q = \{q_1, q_2, q_3, q_4\}$

$\Sigma = \{0, 1\}$

$\Gamma = \{\$, 0\}$

$\delta(q_1, \epsilon, \epsilon) = (q_2, \$)$

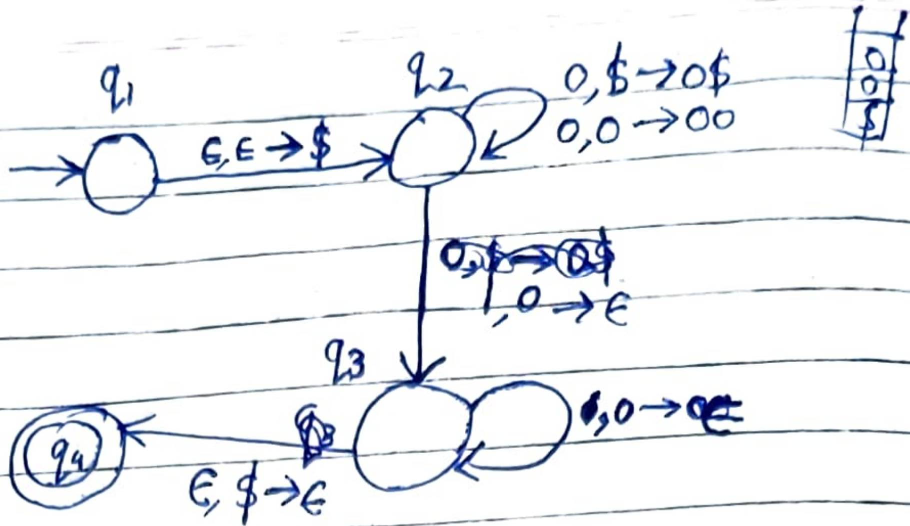
$\delta(q_2, 0, \$) = (q_2, 0\$)$

push $\delta(q_2, 0, 0) = (q_2, 00)$

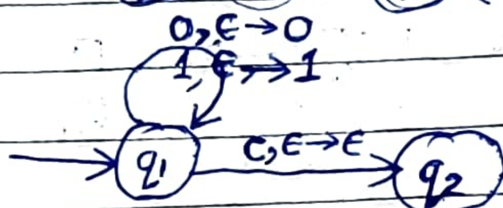
pop $\delta(q_2, 1, 0) = (q_3, \epsilon)$

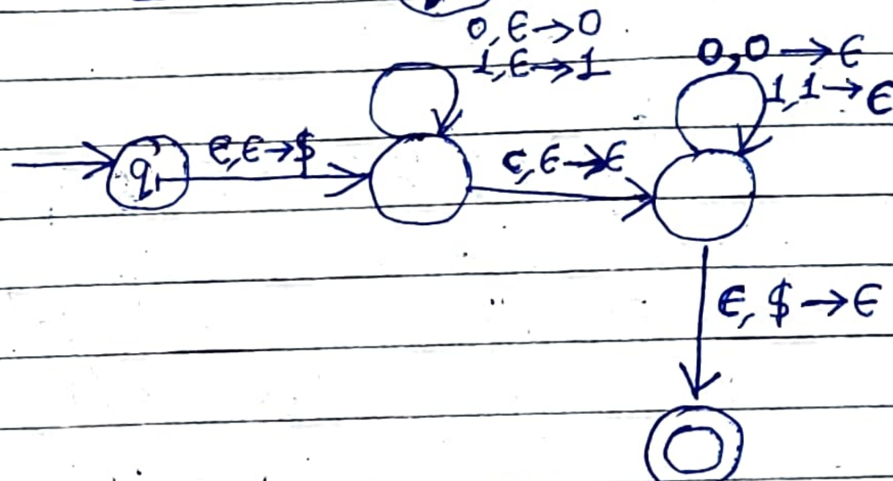
$\delta(q_3, 1, 0) = (q_3, \epsilon)$

$\delta(q_3, \epsilon, \$) = (q_4, \epsilon)$



$L = \{ w c w^R \quad w \in (0,1)^* \}$

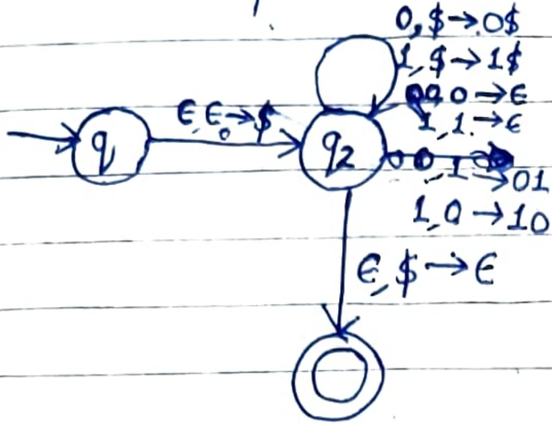




→ abcba

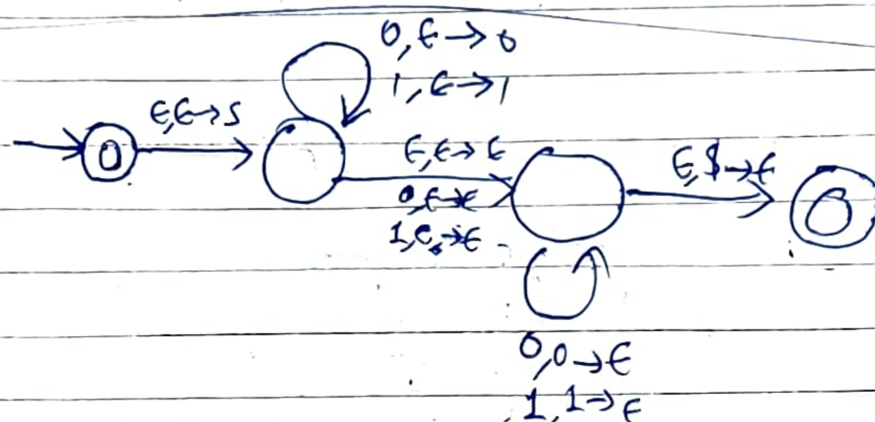
abccba

w is palindrome

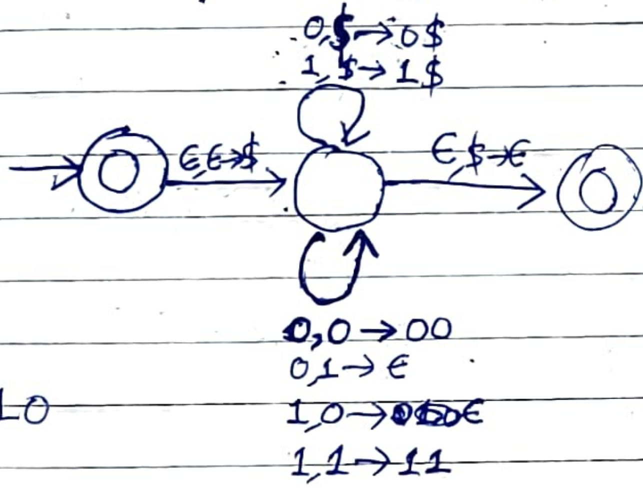


1001

10101



$|w|_0 = |w_1|$



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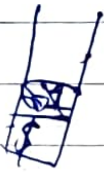
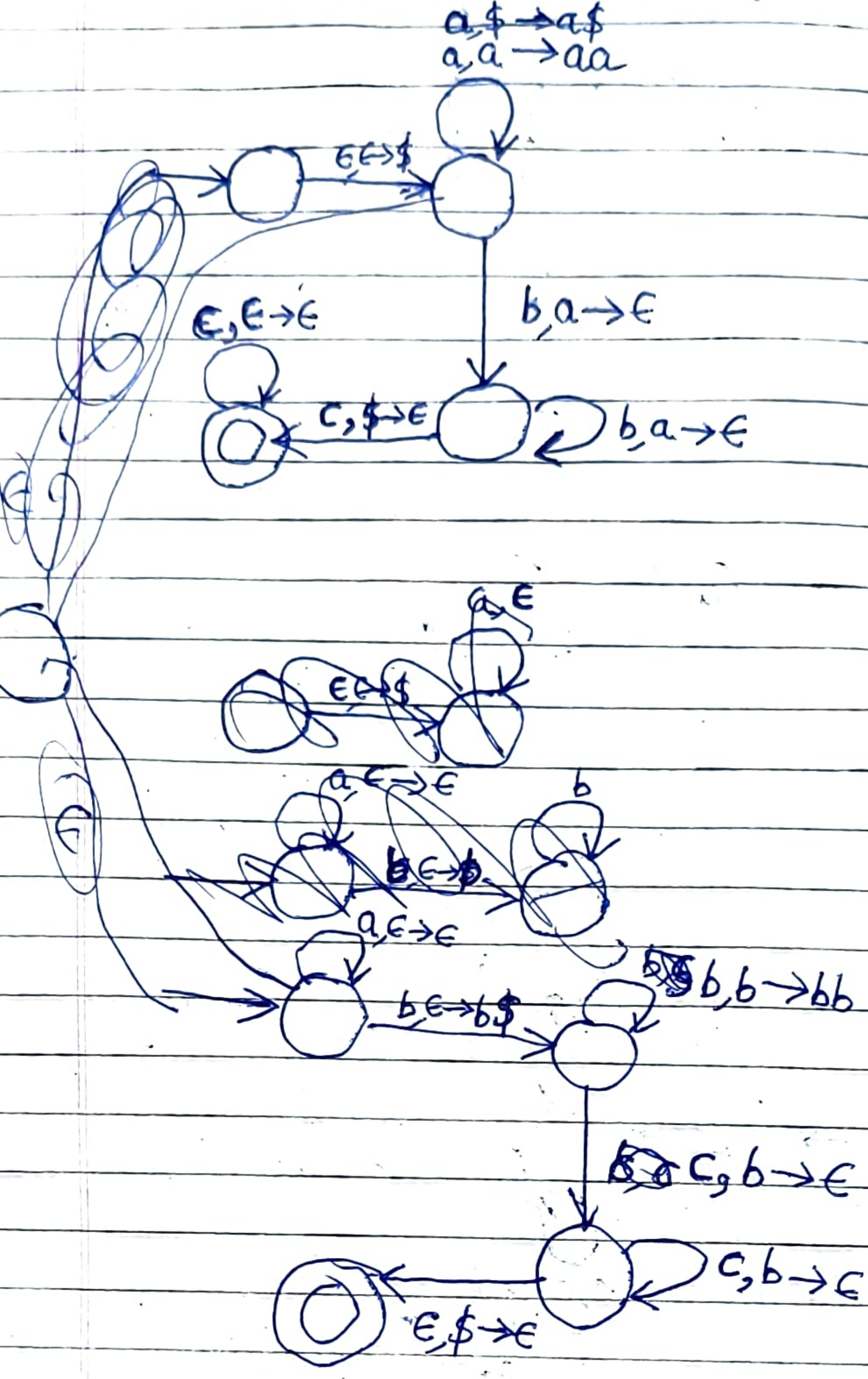
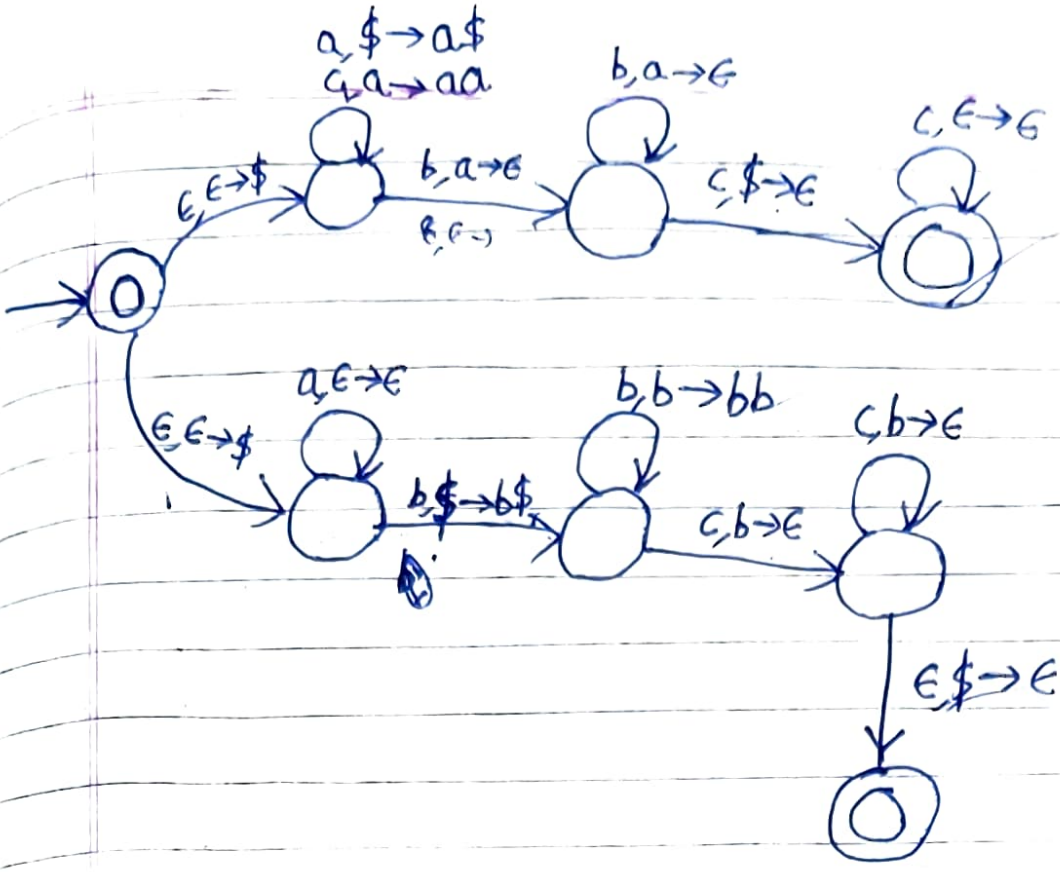
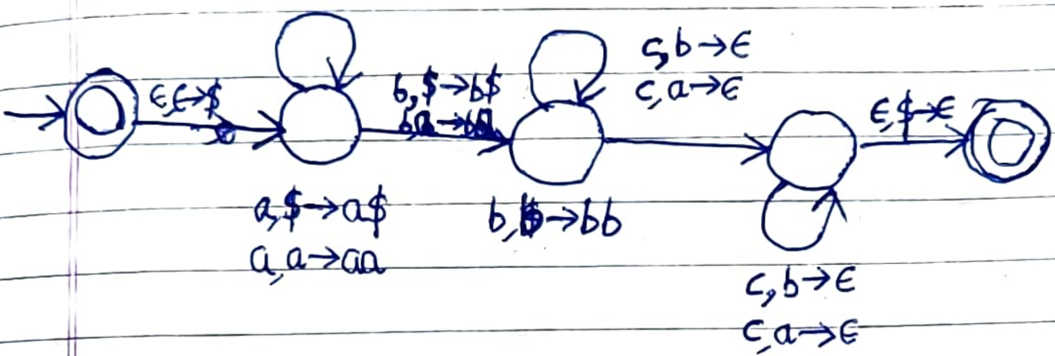


$L = \{ a^i b^j c^k \quad i=j \text{ or } j=k \}$

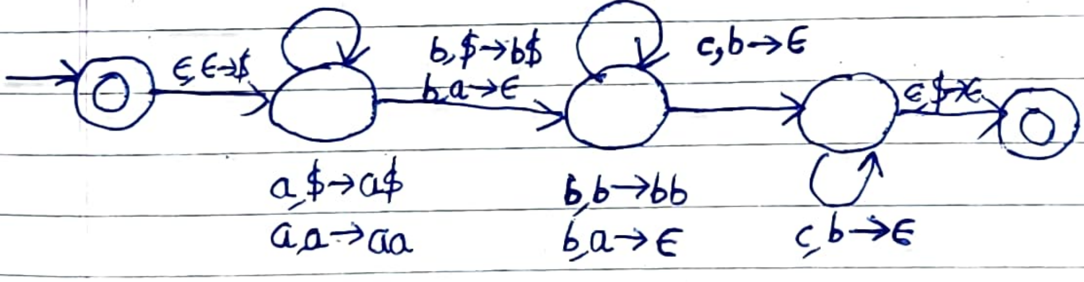




$L = \{a^i b^j c^k \mid i+j=k\}$



$L = \{a^i b^j c^k \mid j=i+k\}$



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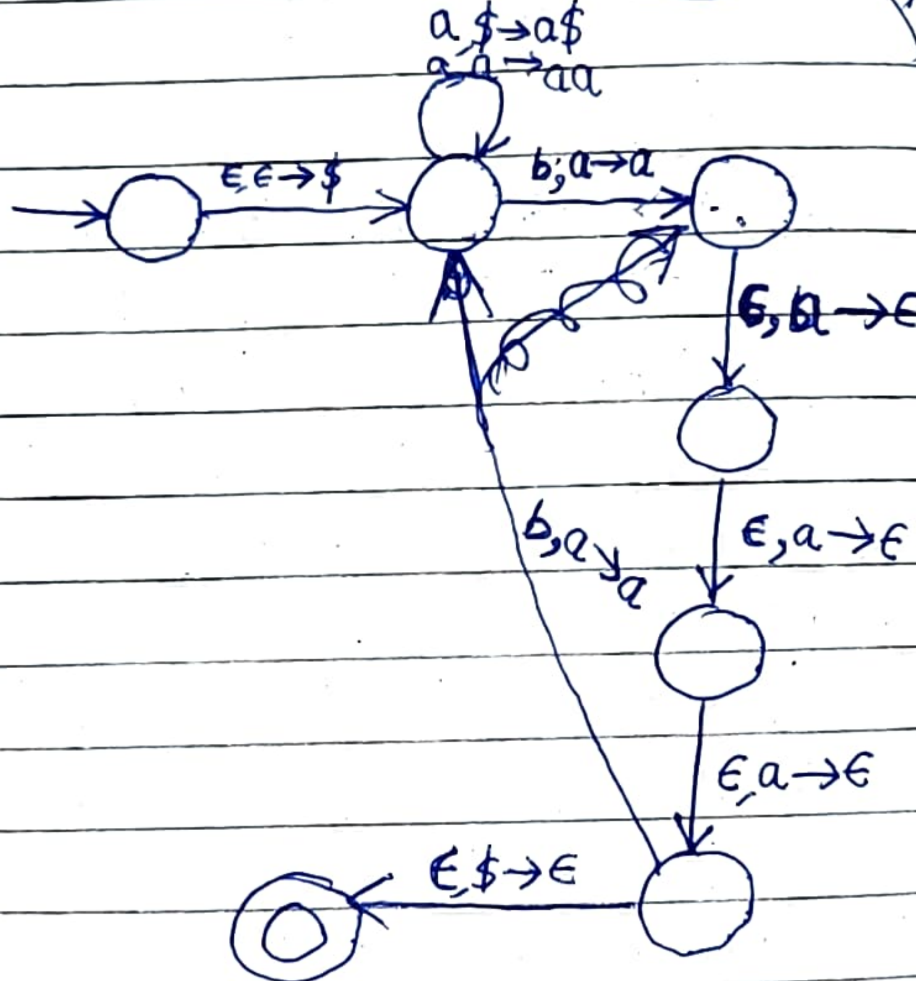
$$L = \{ a^i b^j \mid 2i = 3j \}$$

2×6

i	j
0	0
3	2
6	4
8	

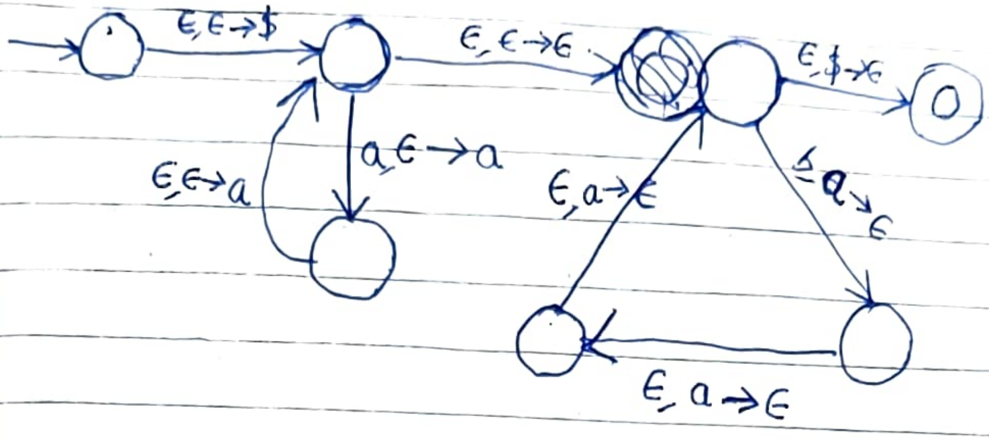
$$\boxed{S \rightarrow aaaSbb \mid \epsilon}$$



aaabb aab

~~aaaaaaabbbbbbb~~

abbbb

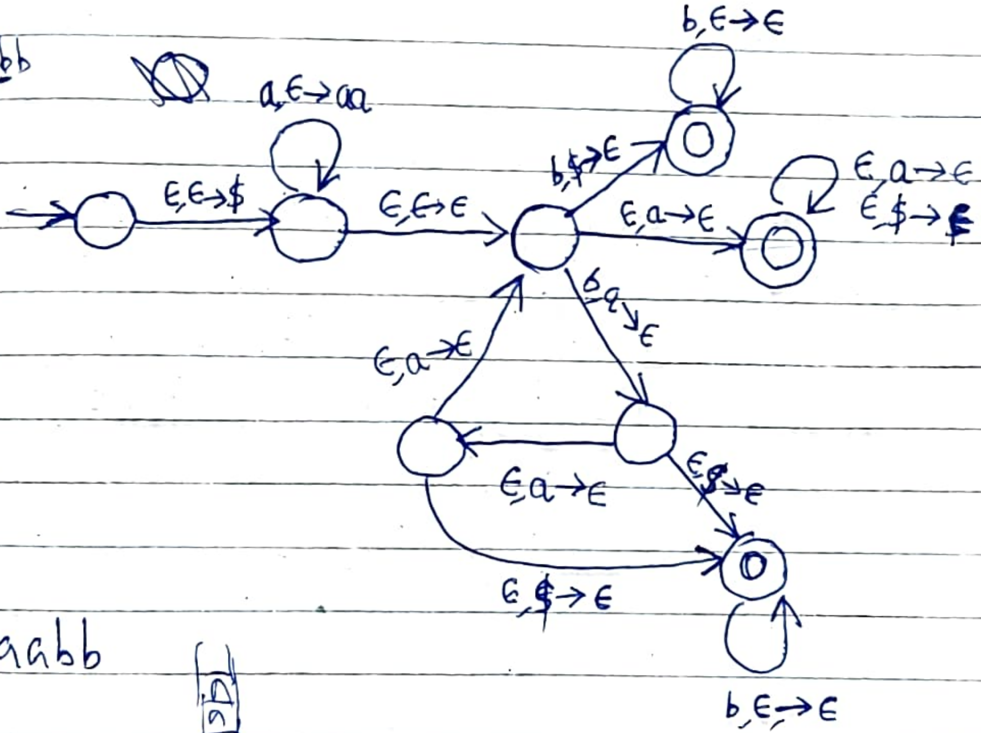


$L = \{ a^i b^j \mid 2i \neq 3j \}$

$L = (a^i b^j \mid 2i = 3j) + (a^* b^*)^c$

aab

aaaaabbb



aaabb