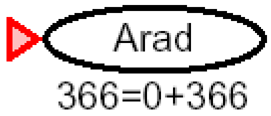
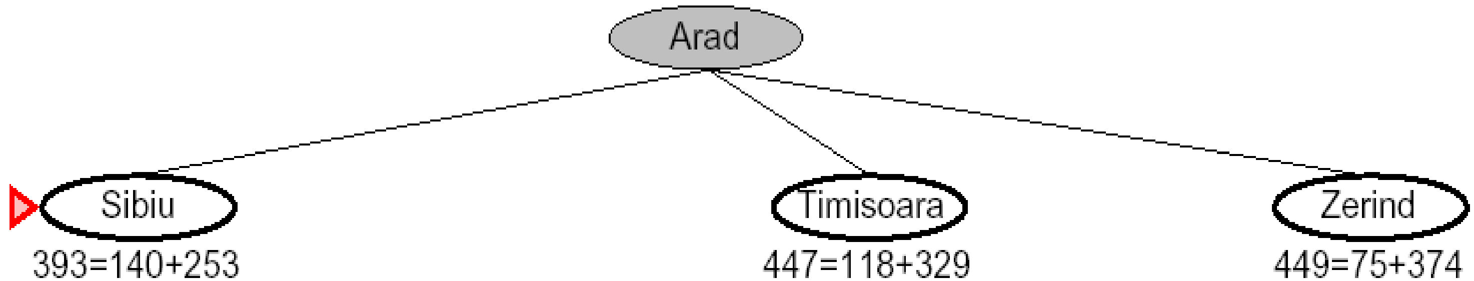
A^* search

- Idea: avoid expanding paths that are already expensive
- Evaluation function $f(n) = g(n) + h(n)$

- $g(n)$ = cost so far to reach n
- $h(n)$ = estimated cost from n to goal
- $f(n)$ = estimated total cost of path through n to goal

A* for Romanian Shortest Path

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Timisoara
447=118+329
Zerind
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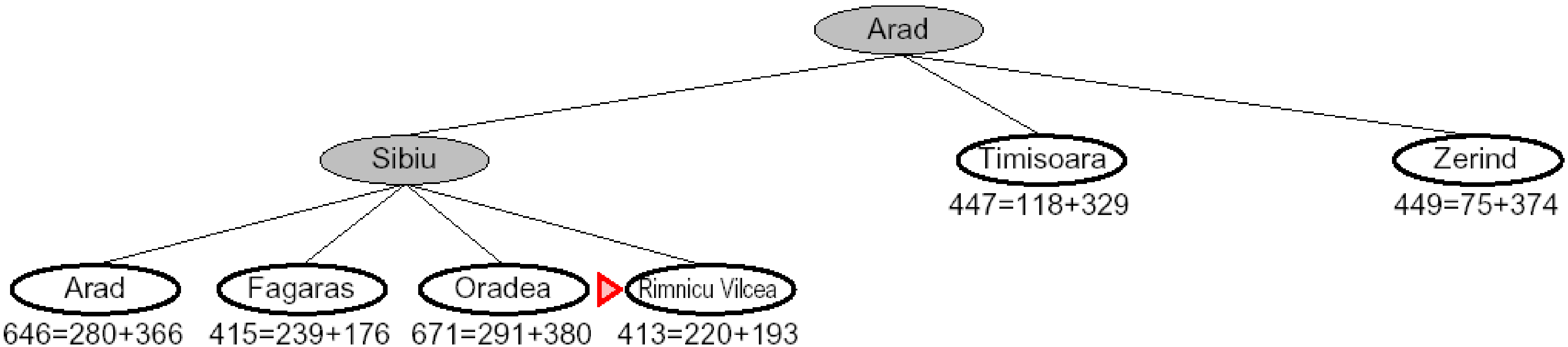
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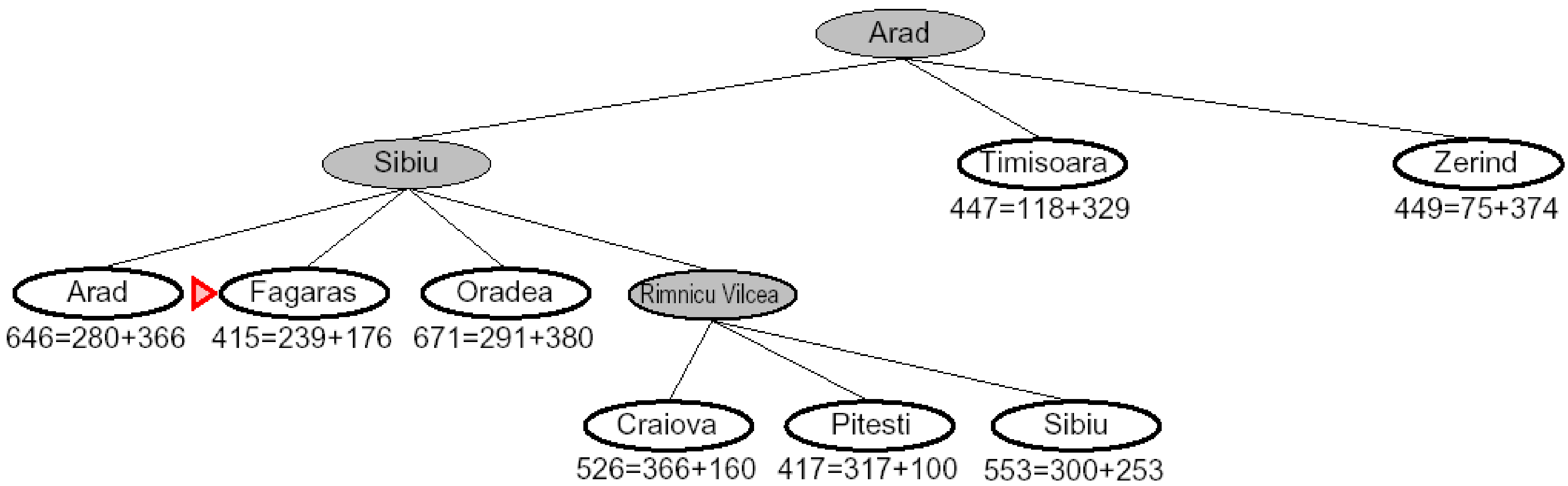
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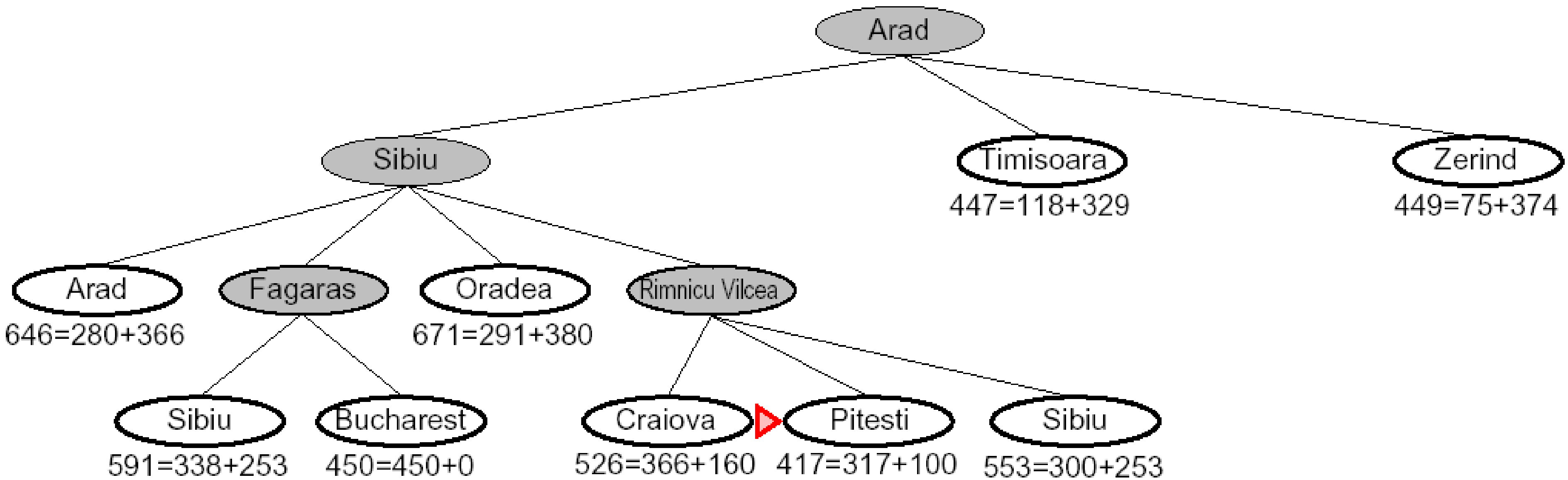
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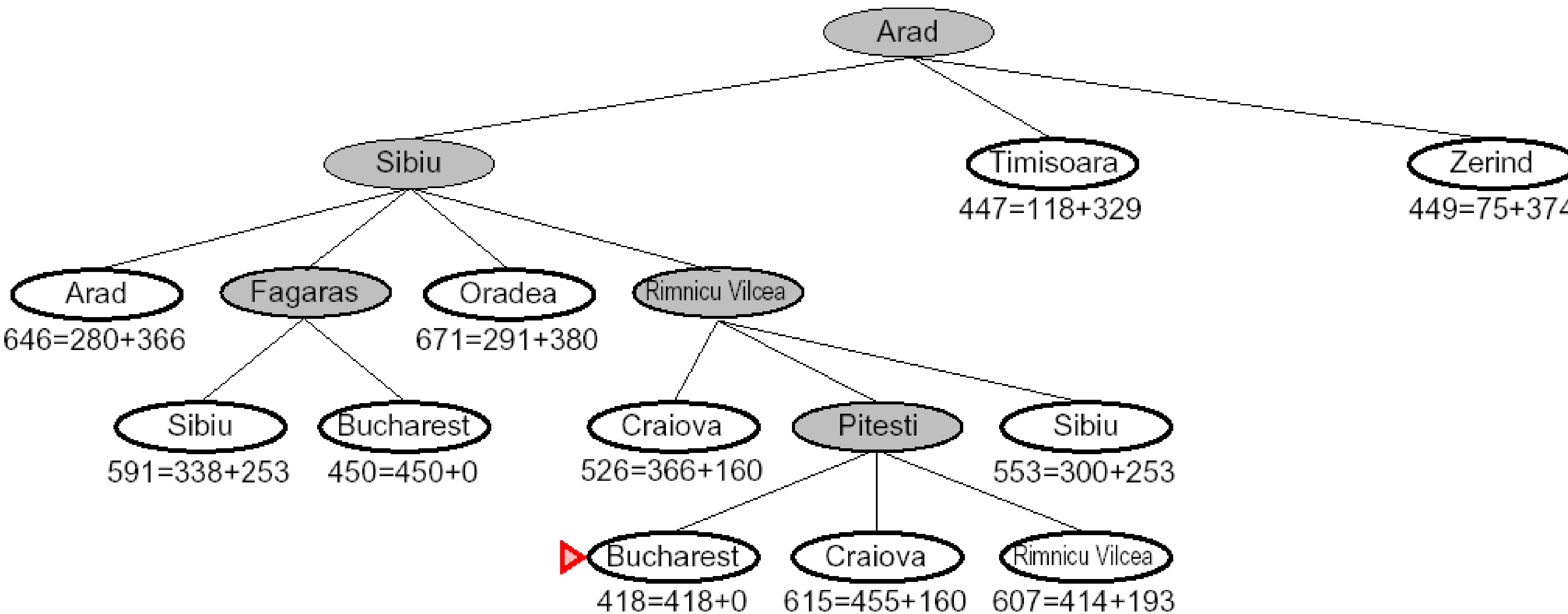
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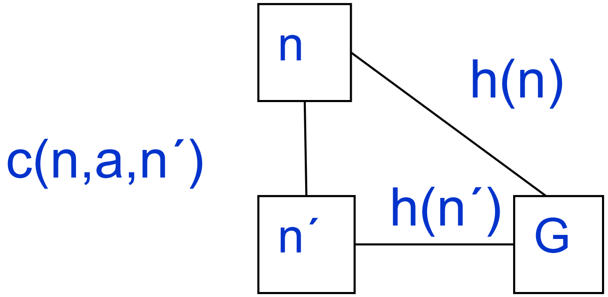
Admissible heuristics

- A heuristic function $h(n)$ is admissible if for every node n, $h(n) \le h^*(n)$, where $h^*(n)$ is the true cost to reach the goal state from n.
- An admissible heuristic never overestimates the cost to reach the goal, i.e., it is optimistic
- Example: $h_{SLD}(n)$ (never overestimates the actual road distance)
- Theorem: If $h(n)$ is admissible, A^* using TREE-SEARCH is optimal

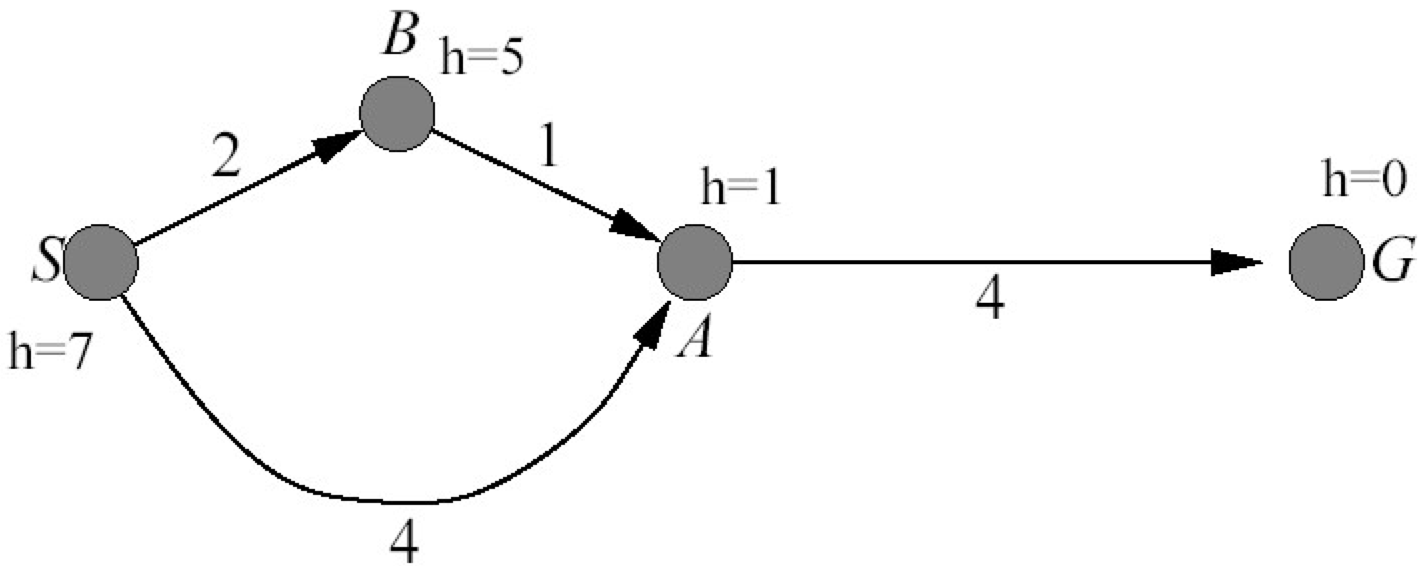
Consistent Heuristics

- h(n) is consistent if
  - for every node n
  - for every successor n´ due to legal action a
  - h(n) <= c(n,a,n´) + h(n´)



- Every consistent heuristic is also admissible.
- Theorem: If *h(n)* is consistent, A^* using GRAPH–SEARCH is optimal

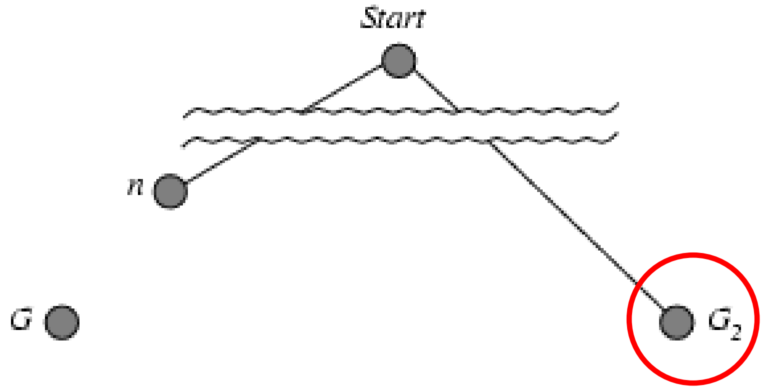
Example



Source: http://stackoverflow.com/questions/25823391/suboptimal-solution-given-by-a-search

Proof of Optimality of (Tree) A^*

- Assume $h()$ is admissible.
  Say some sub-optimal goal state G_2 has been generated and is on the frontier.
  Let n be an unexpanded state such that n is on an optimal path to the optimal goal G.



Focus on G_2:

$f(G_2) = g(G_2)$ since $h(G_2) = 0$

$g(G_2) > g(G)$ since G_2 is suboptimal

Proof of Optimality of (Tree) A*

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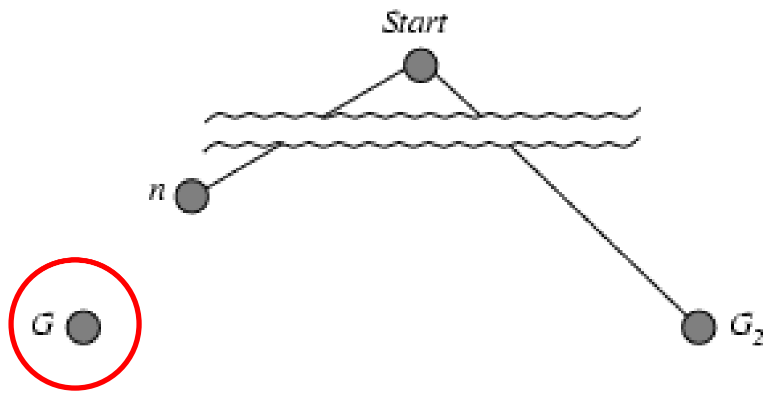


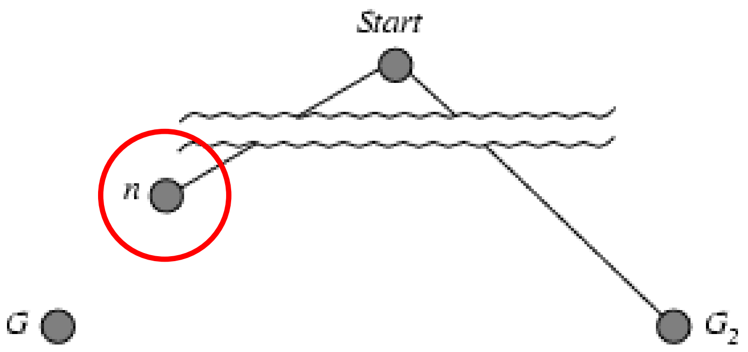
$f(G_2) = g(G_2)$	since $h(G_2) = 0$
$g(G_2) > g(G)$	since G_2 is suboptimal

Focus on G:

$f(G) = g(G)$	since $h(G) = 0$
$f(G_2) > f(G)$	substitution

Proof of Optimality of (Tree)A*

- Assume *h*() is admissible.
  Say some sub-optimal goal state G_2 has been generated and is on the frontier.
  Let *n* be an unexpanded state such that *n* is on an optimal path to the optimal goal *G*.



$f(G_2) = g(G_2)$ since $h(G_2) = 0$
$g(G_2) > g(G)$ since G_2 is suboptimal

$f(G) = g(G)$ since $h(G) = 0$
$f(G_2) > f(G)$ substitution

Now focus on n:

$h(n) \leq h^*(n)$ since h is admissible
$g(n) + h(n) \leq g(n) + h^*(n)$ algebra
$f(n) = g(n) + h(n)$ definition
$f(G) = g(n) + h^*(n)$ by assumption
$f(n) \leq f(G)$ substitution

Hence $f(G_2) > f(n)$, and A* will never select G_2 for expansion.

Properties of A*

- Complete?

  Yes (unless there are infinitely many nodes with f $\leq$ *f(G)*)

- Time? Exponential (worst case all nodes are added)

- Space? Keeps all nodes in memory

- Optimal?

  Yes (depending upon search algo and heuristic property)

A*

http://www.youtube.com/watch?v=huJEgJ82360

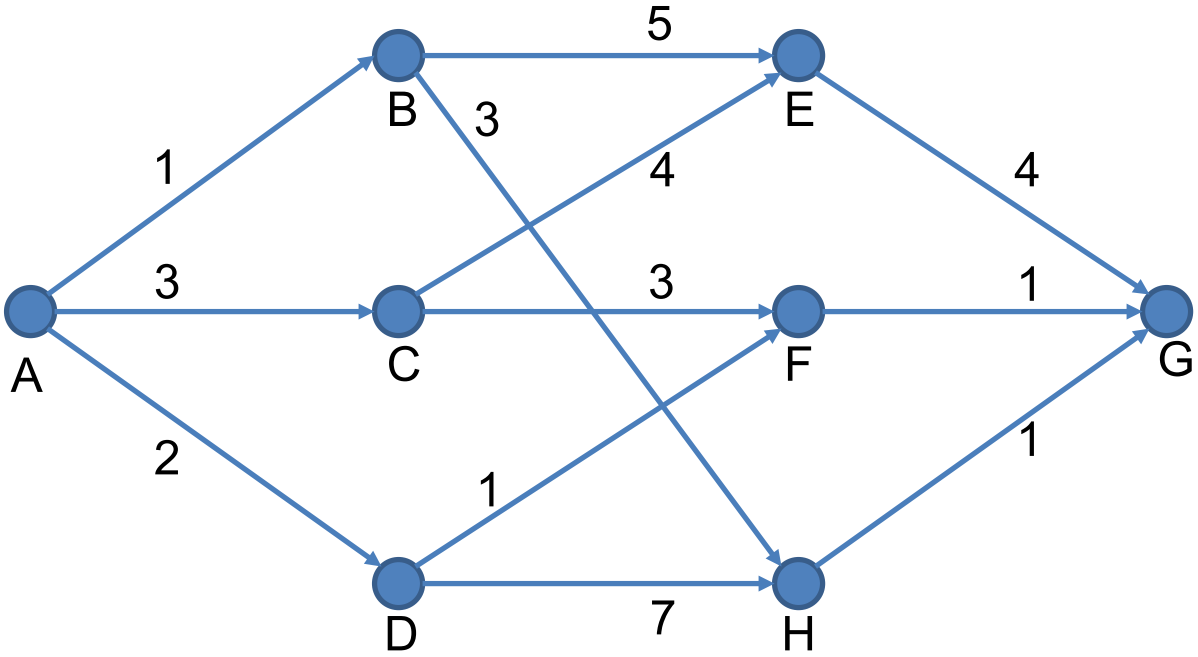
Memory Problem?

- Iterative deepening A*
  - Similar to ID search
  - While (solution not found)
    - Do DFS but prune when cost (f) > current bound
    - Increase bound

Depth First Branch and Bound

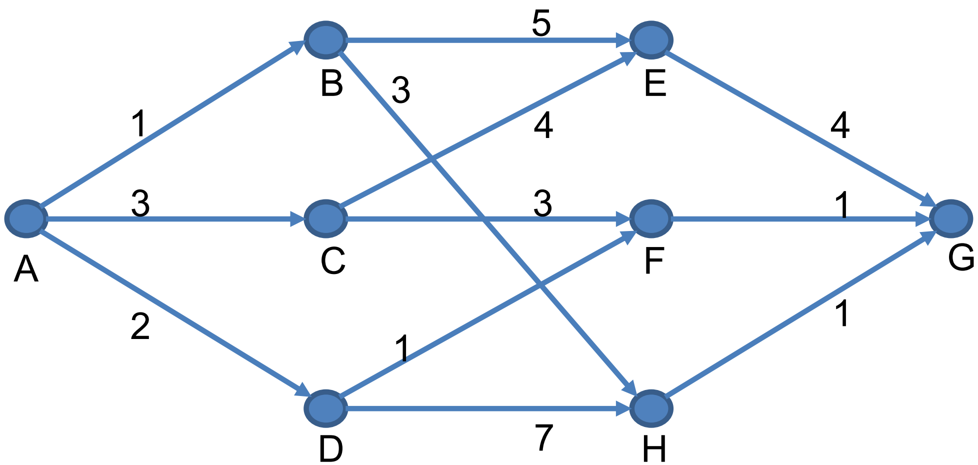
- 2 mechanisms:
  - BRANCH: A mechanism to generate branches when searching the solution space
    - Heuristic strategy for picking which one to try first.
  - BOUND: A mechanism to generate a bound so that many branches can be terminated

Example



Find optimal path from A to G

Search Tree


A
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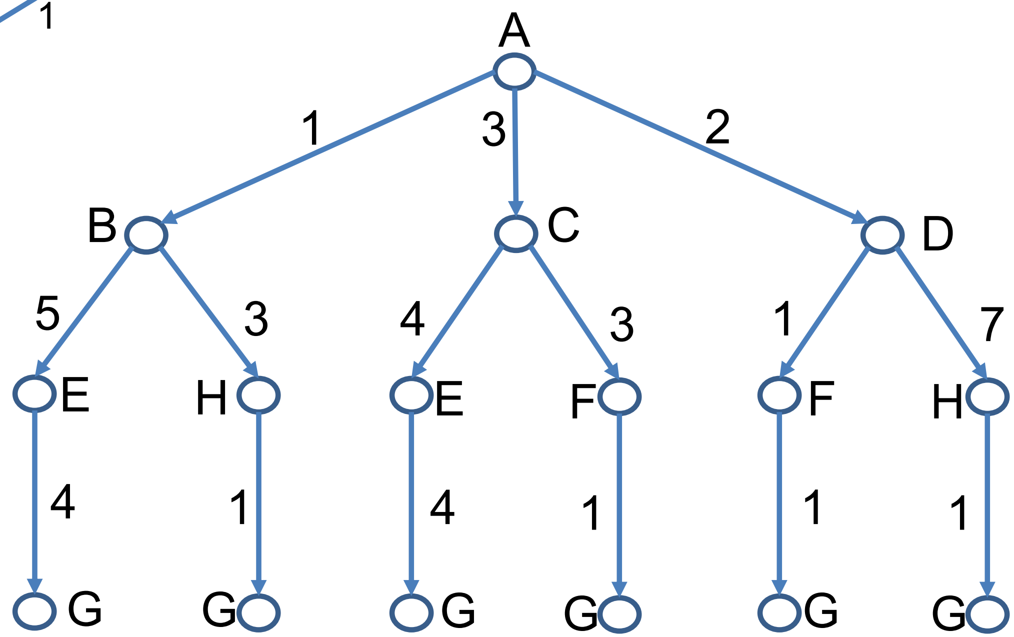
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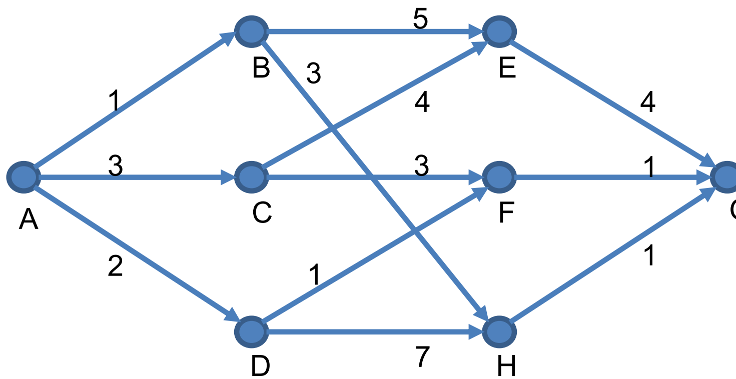
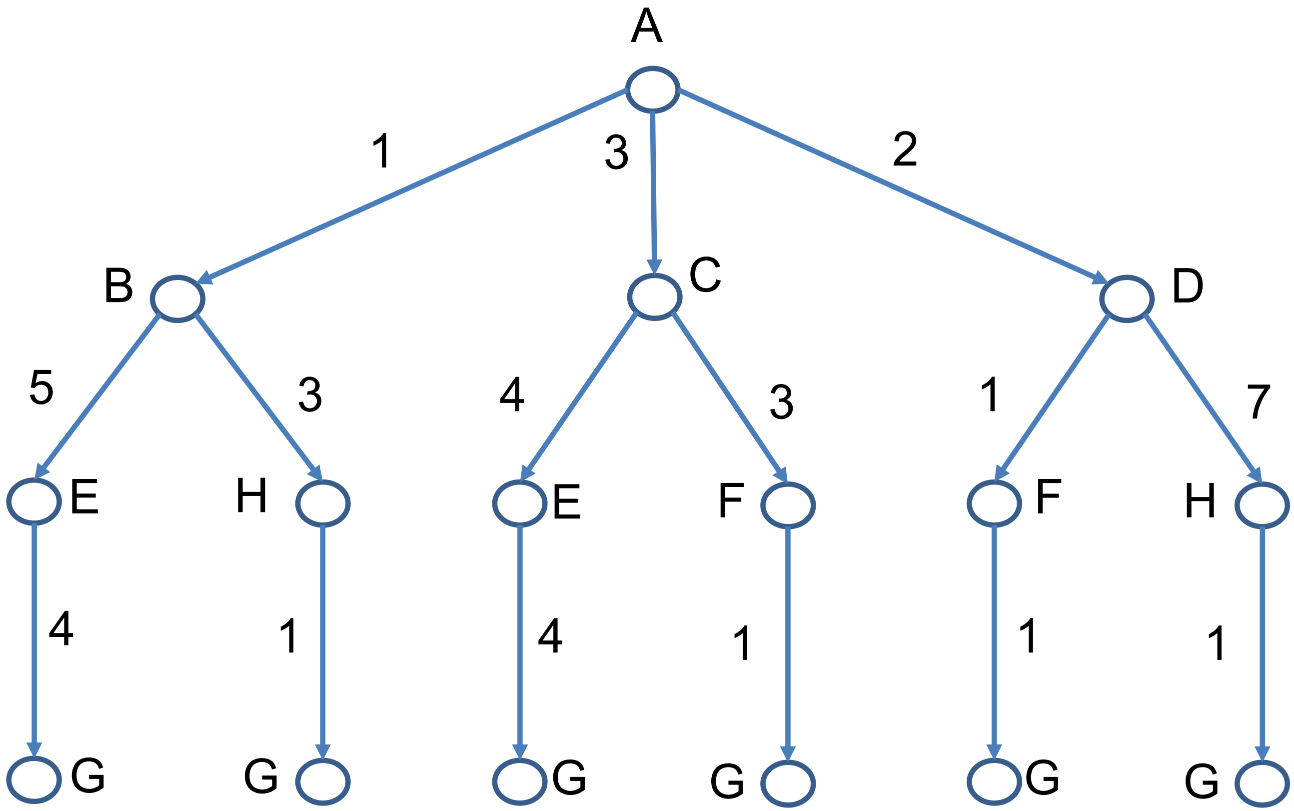
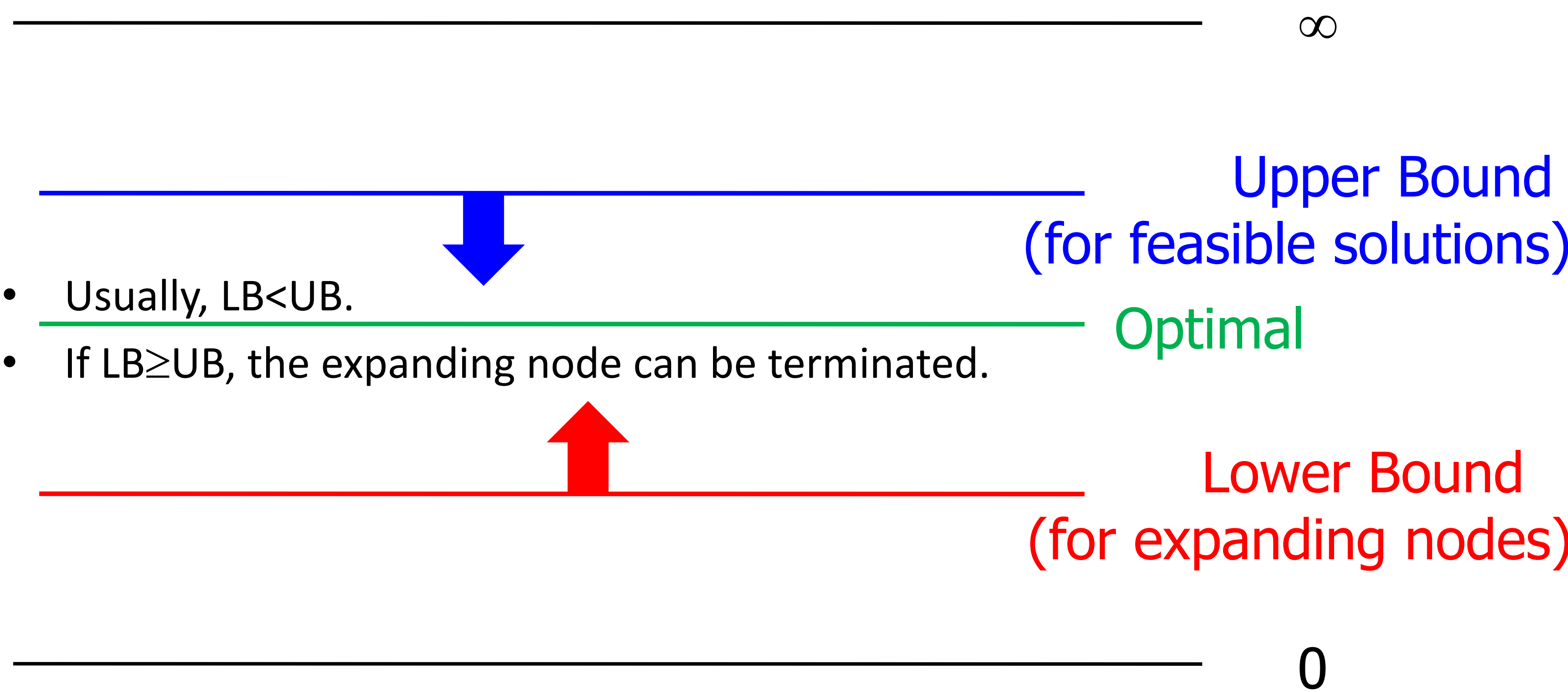
DFS B&B

E.g., Branch policy: take lowest cost edge first





For Minimization Problems



- Usually, LB<UB.
- If LB≥UB, the expanding node can be terminated.

DFS B&B vs. IDA*

- Both optimal
- IDA* never expands a node with f > optimal cost
  - But not systematic
- DFb&b systematic never expands a node twice
  - But expands suboptimal nodes also
- Search tree of bounded depth?
- Easy to find suboptimal solution?
- Infinite search trees?
- Difficult to construct a single solution?

Non-optimal variations

- Use more informative, but inadmissible heuristics

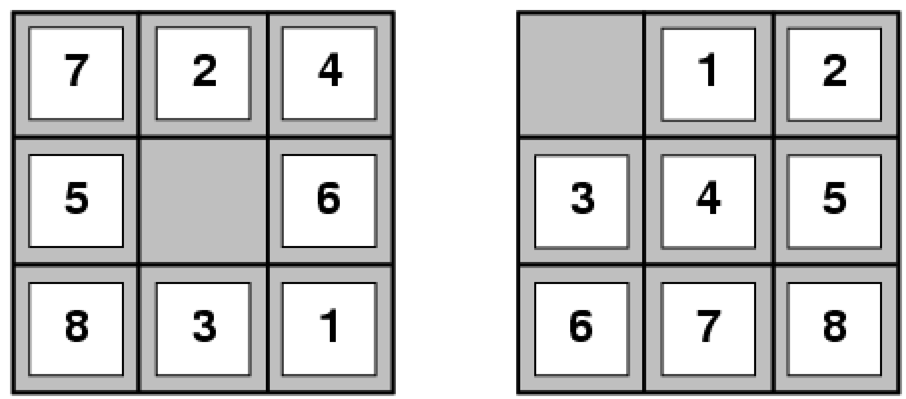
- Weighted A*
  - $f(n) = g(n) + w.h(n)$ where $w>1$
  - Typically $w=5$.
  - Solution quality bounded by w for admissible h

Admissible heuristics

E.g., for the 8-puzzle:

- $h_1(n)$ = number of misplaced tiles
- $h_2(n)$ = total Manhattan distance

(i.e., no. of squares from desired location of each tile)



Start State Goal State

- <u>$h_1(S)$ = ?</u>
- <u>$h_2(S)$ = ?</u>

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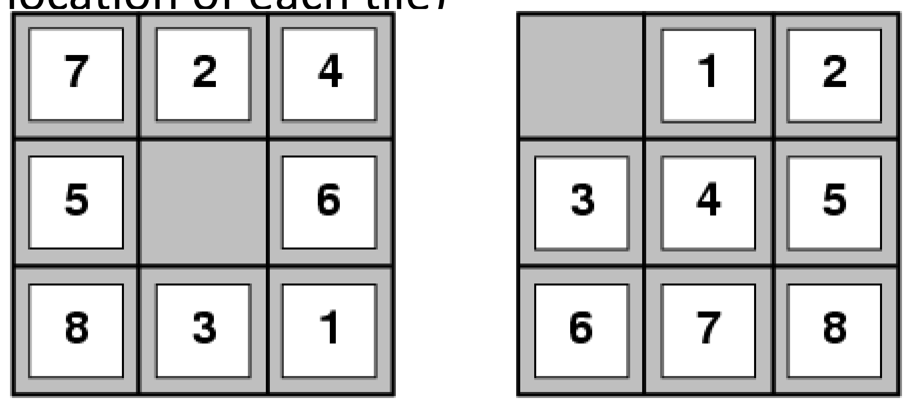

Start State Goal State

- $h_1(S)$ = ? 8
- $h_2(S)$ = ? 3+1+2+2+2+3+3+2 = 18

Dominance

- If $h_2(n) \geq h_1(n)$ for all n (both admissible)
  then h_2 dominates h_1
- h_2 is better for search

- Typical search costs (average number of node expanded):

- *d=12* IDS = 3,644,035 nodes
  $A^*(h_1)$ = 227 nodes
  $A^*(h_2)$ = 73 nodes
- *d=24* IDS = too many nodes
  $A^*(h_1)$ = 39,135 nodes
  $A^*(h_2)$ = 1,641 nodes

Relaxed problems

- A problem with fewer restrictions on the actions is called a relaxed problem
- The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem
- If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
- If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Hamiltonian Cycle Problem

What can be relaxed?

Solution =

1) Each node degree 2
2) Visit all nodes
3) Visit nodes exactly once

What is a good admissible heuristic for (a1→a2→…→ak)

- length of the cheapest edge leaving ak +
  length of cheapest edge entering a1

- length of shortest path from ak to a1

- length of minimum spanning tree of rest of the nodes