DIP EDI /302 /412

* MID - 30 %
* END - 40 %
* Ass & Project - 25%
Attendance - 5%

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~~Digi~~ DIP
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IEE
Science direct.
* object tracking
* Image denoising

$$A = \begin{bmatrix} 10 & 20 & 30 & 40 & 50 \\ 1 & 2 & 5 & 10 & 6 \\ 2 & 3 & 4 & 5 & 6 \\ 7 & 9 & 10 & 11 & 12 \\ 30 & 80 & 100 & 200 & 235 \end{bmatrix}$$

A (2:3, 2:3)

* convolution
* CVPR conference
* Paper with code

1 image representation
image Sampling & quantization
image interpolation 2X2 to → 6X6

2. Intensity transformation
3. Spatial filtering

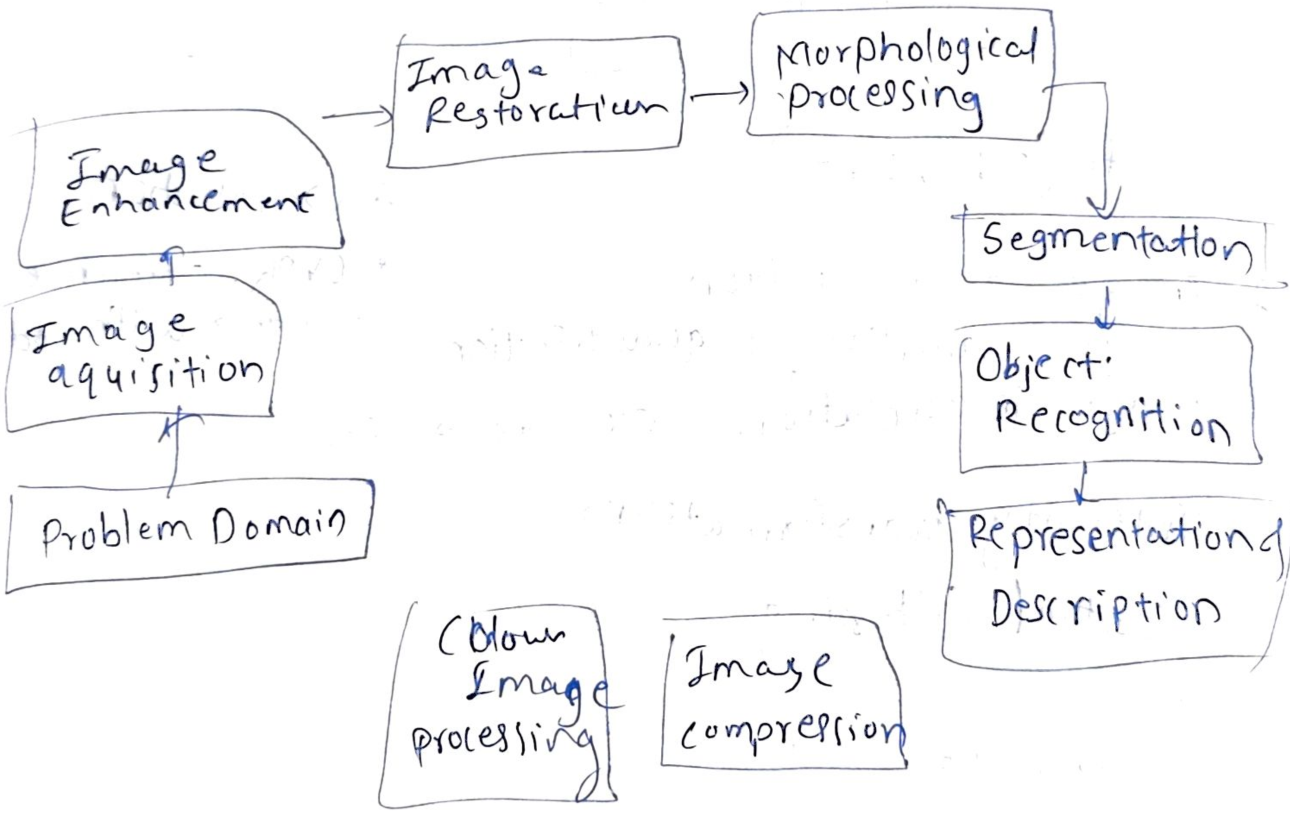
Monday
① binary image 0-1
② Gray Image 0-255 (Pixel value)

① Object tracking (single object tracking / Multiple " ")
② Image & Video dehazing
③ 2D to 3D Image Conversion

19-08-24

4. Image Inpainting
5. " Captioning
6. Underwater Image Enhancement
7. Virtual Dressing
8. Brain Tumor Detection, Breast cancer detection, Ultrasonography, COVID-19 Detection: CHEST X-RAYs, Traffic Monitoring, Face detection, Facial Expression ~~Recogtion~~ Recognition, Face Morphing, BIOMETRICS, Image Super-Resolution

* Fundamental steps in Digital Image Processing



Component of AN Image Processing System.

* Image Processing Software

Processing of digital image digital computer

$$Gray = 0.2989 \times R + 0.5870 \times G + 0.1140 \times B$$

Bartlane
5 distinct gray level

Height of Object = 100 m
distance = 500 m
What is height of Image on retina

f_{eye} 17 mm to 14 mm

$$\frac{100}{500} = \frac{h}{17} \qquad h = \frac{17}{5} \text{ mm}$$

* Brightness adaption & Discrimination of Human eye
  photopic : Bright light vision
  scotopic : dim-light vision

* A simple Image formation Model

Image → 2D function $f(x,y)$
x, y → spatial Co-ordinates

$$f(x,y) = i(x,y)\, r(x,y)$$

i - illumination r - reflectance

$[L_{min}, L_{max}]$ is called gray scale interval

$l = L - 1$

* a. Continuous Image b. Scale line from A to B
  c. sampling & Quantization d. Digital scan line

$L = 2^k$

$b = M \times N \times K$

K → bits required to represent 1 pixel
$M \times n$ → Total pixels

$M = N = 284 \quad K = 4$
$b = 284 \times 284 \times 4$
$b =$

Image Sampling & Quantization

* Spatial & Intensity Resolution

Spatial resolution ex 128 X 128 , 256 X 256

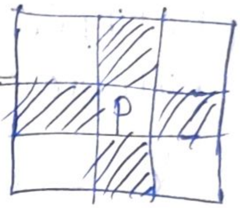
dpi dots per ~~distance of each pixel~~ distance

Intensity Resolution (0-255) for gray image

* Relationship b/w pixels.

① Neighbourhood.

Neighbours of a pixel. $N_4(P)$



$N_4(P) = (x, y-1), (x, y+1), (x-1, y)(x+1, y)$

four diagonal neighbours

$N_D(P)$

$N_D(P)$ and $N_4(P)$ together known as $N_8(P)$

② Adjacency

4- adjacency

8 - adjacency (Prefer $N_D(P)$)

m-adjacency (Prefer $N_4(P)$)

(mixed adjacency)

~~→ q is in N_4~~ (i) q is in $N_4(P)$

(ii) q is in $N_D(P)$ & $N_4(P) \cap N_4(q)$ is not from V

1	1	
	1	1
1		1

③ Connectivity

Mixed connectivity is a modification of 8-connectivity

(4) PATHS

Ex: Consider the 2 image subsets S_1 & S_2 shown in the following figure for $V = \{1\}$ determine whether this 2 subsets are
(1) 4 - adjacent
(2) 8 - adjacent
(3) m - adjacent

$V = \{1\}$

	S_1				S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1 q	1	0	0	0
0	0	1	1	1 p	0	0	0	0	0
	0	1	1	1	0	0	1	1	1

Ans
(1) 4-adjacency
q is not in the set $N_4(P)$
S_1 & S_2 are not 4-adjacent

(2) 8 - adjacency:
q is inthe set of $N_8(P)$ S_1 & S_2 are 8-adjacen

(3) m- adjacency:
q is in $N_4(P)$ ✗ OR
q is in $N_D(P)$ & $N_4(P) \cap N_4(q)$ is not from V ✓
∴ m-adjacency.

Ex 2. (a) Let $V = \{0, 1\}$ & compute the lengths of shortest 4, 8 & m-Path b/w P & Q
if a particular path doesn't exist b/w this two points. explain why.
(b) Repeat for $V = \{1, 2\}$

Ans

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

4-path V(0,1)

It is impossible to set a path b/w p, q using 4-adjacency having the values of set V

8-path V(0,1)

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

∴ 8-path length is 4

m-path V(0,1)

3	1	2	1
2	2	0	2
1	2	1	1
1	0	1	2

∴ m-path length is 5

(b) Repeat for V(1,2)

3	1	2	1 (q)
2	2	0	2
1	2	1	1
1	0	1	2

(p)

4-path length 6
8-path length 4
m-path length 6

Distance measure

(a) $D(P,q) \geq 0$

(b) $D(P,q) = D(q,P)$

(c) $D(P,Z) \leq D(P,Q) + D(Q,Z)$

Euclidean distance

D_4 Distance

D_8 Distance

Ex $P(10,12)$ $Q(15,20)$ $\rightarrow \left[(x-s)^2 + (y-t)^2\right]^{1/2}$

Euclidean $\sqrt{5^2 + 8^2} = \sqrt{25+64} \approx 9$

D_4 Distance $= |x_2 - x_1| + |y_2 - y_1|$

$= 5 + 8 = 13$

D_8 distance $= \max(|x-s|, |y-t|)$

$= \max(5, 8)$

$= 8$

* Image Enhancement technique.

1) Spatial domain

2) freq domain

1. Spatial domain

   1. Point transformation / Intensity transformation
   2. Histogram based
   3. Mask processing (spatial filtering)

$S = T(r)$

Use of Transformation for preprocessing

1) point processing Techniques

a) Image Negative

$s = T(r) = L-1-r$

$= 255 - r$ for gray scale

ex

$$\begin{bmatrix} 220 & 245 & 160 \\ 245 & 200 & 180 \\ 160 & 120 & 100 \\ 20 & 80 & 60 \end{bmatrix} \xrightarrow{\text{Image Negative}} \begin{bmatrix} 35 & 10 & 95 \\ 10 & 55 & 75 \\ 95 & 135 & 155 \\ 235 & 175 & 195 \end{bmatrix}$$

[m, n, c] = img.shape

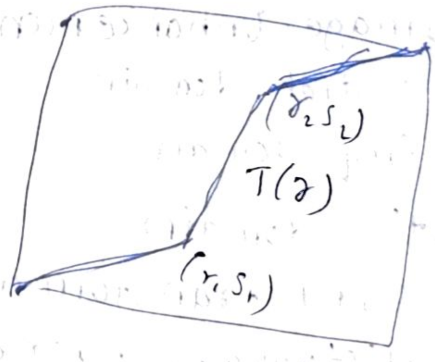
b) Log transformations

$s = c \log(1+r)$

c) Power law (Gamma) Transformation

$s = c r^{\gamma}$ → Gamma

d) contrast strecting

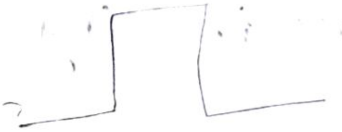


e) Intensity level slicing

Without background

$s = L-1 \quad ; \ t_1 \leq r \leq t_2$

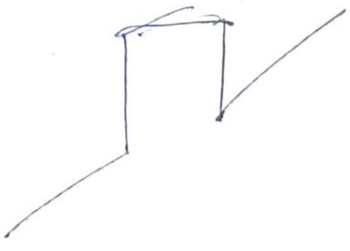
$\quad 0 \quad ; \ \text{otherwise}$



With Background

$s = L-1 \quad ; \ t_1 \leq r \leq t_2$

$\quad r \quad ; \ \text{otherwise}$



(f) bit plane slicing
changing MSB or LSB

Image Transformation

Application
- Preprocessing
  - Filtering
  - Enhancement etc.
- Data compression
- Feature extraction
  - Edge detection
  - Corner detection

What does image transformation do?
→ It represents a given image as a series summation of a set of Unitary Matrices

A is Unitary matrix if
$A^{-1} = A^{*T}$

* Histogram $h(r_k) = n_k$

r_k is the k^{th} intensity value
n_k is the no. of pixels in the image with intensity r_k

intensity value

~~20 10 15 12 13~~

$$\begin{bmatrix} 20 & 10 & 15 & 12 & 13 \\ 2 & 3 & 4 & 6 & 3 \\ 10 & 12 & 2 & 3 & 1 \\ 4 & 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 & 13 \end{bmatrix}$$

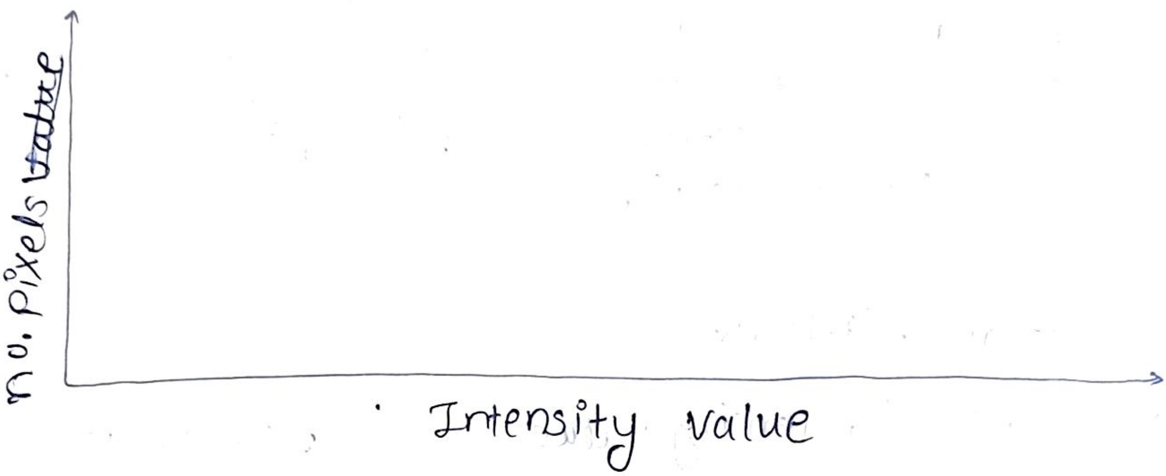
Intensity	No. of pixel
1	1
2	2
3	3
4	2
5	1

Intensity	no. of pixel
6	2
7	1
8	1
9	1
10	2
11	1

Intensity value

Intensity	Pixel value
12	3
13	2
14	0
15	1
16	0
17	
18	
19	
20	

* Good quality image has a flattered Histogram



* **Histogram Equalization** (~~To conter~~ convert to flattered Histogram)

$$\boxed{P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|}$$

$P_r(r)$ Probability density function.

Ex 1

$$P_r(r) = \begin{cases} \dfrac{2r}{(L-1)^2} & ; \ 0 \le r \le L-1 \\ 0 & ; \ \text{Otherwise} \end{cases}$$

$$\cancel{dP_r}$$

$$(L-1)\frac{d}{dr}\underbrace{\int_0^r P_r(w)\,dw}_{\text{CDF (cummulative distribution fun}^c\text{)}} = (L-1)\frac{d}{dr}\int_0^r \frac{2w}{(L-1)^2}\,dw$$

$$\frac{(L-1)}{(L-1)^2}\frac{d}{dr}r^2 = \frac{2r}{(L-1)}$$

$$\boxed{S = T(r) = (L-1)\int_0^r P_r(w)\,dw}$$

$$\cancel{P_s(s) = P_r(r)}$$

* Histogram matching

$$\frac{ds}{dr} = \frac{1}{(L-1)} \times 2r$$

$$P_s(s) = \underbrace{\frac{2r}{(L-1)^2}}_{P_r(r)} \times \underbrace{\frac{(L-1)}{2r}}_{\left|\frac{dr}{ds}\right|} = \frac{1}{L-1} = \frac{1}{255}$$

* $\boxed{P_r(r_k) = \frac{n_k}{MN} \quad \text{Normalized Histogram}}$

$n_k \rightarrow$ the no. of pixels in the image of size $M \times N$ with intensity r_k

$$\boxed{S_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_r(r_j)}$$

Perform the histogram equalization on 3-bit image $(L=8)$ of size 64×64 pixels. The intensity distribution of the image is given below

Gray level r_k	0	1	2	3	4	5	6	7
no. of pixels	790	1023	850	656	329	245	122	81

Ans Given $L = 8$

Total no. of pixels $= M \times N = 64 \times 64 = 4096$ pixels.

r_k	n_k	$p_r(r_k) = \frac{n_k}{MN}$ (N.H)	$s_k = T(r_k)$	Round off s_k	$p_s(s_k)$ (H.E)
0	790	$\frac{790}{4096} = 0.19$	$(8-1)\times 0.19 = 1.33$	1	
1	1023	0.25	$7\times 0.44 = 3.08$	3	
2	850	0.21	$7\times 0.65 = 4.55$	5	
3	656	0.16	$7\times 0.81 = 5.67$	6	$\frac{656+329}{4096} = 0.2$
4	329	0.08	$7\times 0.89 = 6.23$	6	
5	245	0.06	$7\times 0.95 = 6.65$	7	
6	122	0.03	$7\times 0.98 = 6.86$	7	
7	81	0.02	$7\times 1 = 7$	7	

Assignment without using built in func do histogram equalization.

SPATIAL FILTERING

a) a neighborhood b) a predefined operation

$$g(x,y) = \sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\, f(x+s, y+t)$$

a	b	c
d	e	f
g	h	i

*

r	s	t
u	v	w
x	y	z

2k+1

$e_{processed} = v*e +$
$r*a + s*b + t*c$
$+ u*d + w*f +$
$x*g + y*h + z*i$

$\cancel{5}+7-2-9-\cancel{5} = -4$

$$\boxed{\frac{M-K+2P}{S}+1}$$

SPATIAL CONVOLUTION

3	0	1	2	7	4
1	5	8	9	3	1
2	7	2	5	1	3
0	1	3	1	7	8
4	2	1	6	2	8
2	4	5	2	3	9

6×6

$*$

1	0	-1
1	0	-1
1	0	-1

3×3

M → Image size
K → kernel size
P → padding
S → stride

4×4

$$\frac{M-K+2P}{S}+1 = \frac{6-3+2\times 0}{1}+1$$
$$= 4$$

(Blurring & Noise reduction)

SMOOTHING SPATIAL FILTER

$\frac{1}{9}\times$

1	1	1
1	1	1
1	1	1

weighted average

$\frac{1}{16}\times$

ORDER - STATISTICS (NON · Linear) FILTERS

Examples: Median filter, Max filter, Min filter

use → salt paper noise

SHARPENING SPATIAL FILTERS: FOUNDATION

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

First order derivative

- Must be zero in constant (r)

* LAPLACIAN FILTER

LAPLACE OPERATOR

$$\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

$$\nabla^2 f = f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1) - 4f(x, y)$$

Image sharpening

$$g(x,y) = f(x,y) + c[\nabla^2 f(x,y)]$$

Unsharp Masking & High Boost Filtering & Crispening

$$g_{mask}(x,y) = f(x,y) - \bar{f}(x,y)$$

$$g(x,y) = f(x,y) + k * g_{mask}(x,y) \quad k \geq 0$$

if $k=1$ Unsharp Masking

Robert Cross-gradient operator

Sobel Operator

↳ $M(x,y) = |z_9 - z_5| + |z_8 - z_6|$

$\underline{D \geq D_0}$ | HP

$D \leq D_0$ | LP

Homomorphing

* Image enhancement in frequency domain : filters↓

① Ideal ② Butterworth ③ Gaussian

* Image Transform

$\frac{1}{1+[\frac{D}{D_0}]^{2n}}$ LP

$n=\infty \Rightarrow$ Ideal

To avoid complexity. (Transform)

We may get more features in another domain.

~~Discrete~~ fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(t).e^{-j\omega t} dt$$

eg: $x(t) = e^{-2t}u(t) \rightleftharpoons X(\omega) = \frac{1}{2+j\omega}$

Discrete fourier Transform

$$X(k) = \sum_{n=0}^{N-1} x[n].e^{-j\frac{2\pi}{N}kn}$$ where $k = 0,1,2,\ldots N-1$.

* Unitary Transform

$$A \times A^H = I$$

↳ Identity matrix

$A \rightarrow$ Transformation matrix

$A^H \rightarrow$ Hermitian matrix

$$A^H = A^{*T}$$

$$A \xrightarrow{\text{conjugate}} A^* \xrightarrow{\text{Transpose}} A^{*T}$$

IDFT

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad ; \; 0 \leq n \leq N-1$$

* Check whether the ~~Uni~~ DFT matrix is Unitary or Not.

Step 1: Determination of the matrix A:

DFT: Finding 4-point DFT (wher $N=4$)

$$X(k) = \sum_{n=0}^{3} x[n] . e^{-j\frac{2\pi}{4}kn} \quad ; \text{where } k = 0,1,2,3$$

1. Finding $X(0)$

$$X(0) = \sum_{n=0}^{3} x[n] = x(0) + x(1) + x(2) + x(3)$$

2. Finding $X(1)$

$$X(1) = \sum_{n=0}^{3} x[n] . e^{-j\pi/2 \, n}$$

$$= x[0) + -jx(1) - x(2) + jx(3)$$

$$X(1) = x(0) - jx(1) - x(2) + jx(3)$$

3. Finding $X(2) = x(0) - x(1) + x(2) - x(3)$

4. Finding ~~X(3) = j x(0)~~

$$X(3) = \sum_{n=0}^{3} x[n] . e^{-j\pi/2 \times 3n}$$

$$X(3) = x(0) + jx(1) - x(2) - jx(3)$$

coeff matrix

$$X(k) = A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$A^H = A^{*T}$$

$$A^{*T} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$A \times A^H = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}$$

$$A \times A^H = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = 4I$$

Note: Sequency → no. of sign changes

$$X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{array}{l} \rightarrow \text{zero sign change} \\ \rightarrow \text{Two sign change} \\ \rightarrow \text{Three sign change} \\ \rightarrow \text{One sign change} \end{array}$$

* 2D - Discrete fourier Transform:-

$$f(m,n) \xrightarrow{2D-DFT} F(k,l)$$

$$F(k,l) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \cdot e^{-j\frac{2\pi}{M}mk} \cdot e^{-j\frac{2\pi}{N} \cdot nl}$$

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* The inverse 2D Discrete Fourier Tranform

$$f(m,n) \xrightarrow{\text{2D-DFT}} F(k,l)$$

$$F(k,l) = R(k,l) + jI(k,l)$$

$$|F(k,l)| = \sqrt{R^2 + I^2} \qquad \cancel{\text{Deg}}\{\phi(k,l)\} = \tan^{-1}\left(\frac{I}{R}\right)$$

→ $F(k,l)$ in polar coordinates ~~$\phi(k,l)$~~ =

$$|F(k,l)| . \phi\, e^{j\phi} \rightarrow \text{phase angle or phase spectrum}$$

Magnitude spectrum

$|F(k,l)|^2$ → Power spectrum

* Note: Two dimensional DFT of image

$$F(u,v) = \text{Kernal} \times F(x,y) \times \text{Kernal}^T$$

* Properties of 2D-DFT

1. separable Property:
compute in two steps by successive 1D operations on rows & columns of an image

~~$F(k,l)$~~ =

$$\boxed{f(m,n)} \xrightarrow[\text{Transform}]{\text{Row}} \boxed{F(m,l)} \xrightarrow[\text{tranfform}]{\text{column}} \boxed{F(k,l)}$$

(n, m)

Ex 1

$$F(x,y) = \begin{matrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{matrix}$$

kernal = 1 1 1 1

kernal $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

calculate 1-D DFT along rows.

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \\ -2 \end{bmatrix}$$ → DFT of 1st Row

$\begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$ → DFT of 2nd Row

$\begin{bmatrix} 12 \\ -2 \\ 0 \\ -2 \end{bmatrix}$ → DFT of 3rd Row

$\begin{bmatrix} 8 \\ -2 \\ 0 \\ -2 \end{bmatrix}$ DFT of 4th Row

$$\Longrightarrow \begin{bmatrix} 4 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \\ 12 & -2 & 0 & -2 \\ 8 & -2 & 0 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 4 \\ 8 \\ 12 \\ 8 \end{bmatrix} = \begin{bmatrix} 32 \\ -8 \\ 16 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} -2 \\ 2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -8 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Resultant matrix:

$$\begin{bmatrix} 32 & -8 & 0 & -8 \\ -8 & 0 & 0 & 0 \\ 16 & 0 & 0 & 0 \\ -8 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{K\ F(x,y)\ K^T}$ = Resultant Verify

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 4 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$\begin{matrix} 4 & 8 & 12 & 10 \\ -2 & -2 & -2 & \end{matrix}$$

Q2 $F(x,y) = \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 5 & 5 & 5 \\ 0 & 1 & -1 & j \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & -j \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

$$= \begin{bmatrix} 19 & -1 & -1 & -1 \\ j & -j\cancel{+1-1} & -2-j & 2+j \\ 1 & -1-2j & 1 & -1+2j \\ -j & \cancel{-j+1+1} \; 2-j & -2+j & j \end{bmatrix} = \begin{bmatrix} 19 & -1 & -1 & -1 \\ j & -j & -2-j & 2+j \\ 1 & -1-2j & 1 & -1+2j \\ -j & 2-j & -2+j & j \end{bmatrix}$$

Ans

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -j \\ -1 \\ j \end{bmatrix}$$ 1st row DFT

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$ DFT of 2nd Row

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$ DFT of 3rd Row

Spatial shift property

$$DFT[f(m-m_0, n)] = e^{-j\frac{2\pi}{N}m_0 k} \quad F(k,l)$$

Periodicity property:

$$\cancel{F(K+PN, l+QN)} = \sum_{m=0}^{N-1}\sum_{n=0}^{N-1} f(m,n).e^{-j\frac{2\pi}{N}m(K+PN)}$$

$$F(K,l) \longrightarrow F(K+PN, l+QN) \qquad e^{-j\frac{2\pi}{N}n(l+QN)}$$

4. Convolution property:

$$f(m,n) * g(m,n) = \sum_{a=0}^{N-1}\sum_{b=0}^{N-1} f(a,b).g(m-a, n-b)$$

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5. Scaling Property

$$DFT\{f(am, bn)\} = \frac{1}{|ab|} F(k/a, l/b)$$

6) Conjugate symmetry.

$$DFT(f^*(m,n)) = F^*(-k,-l)$$

* Discrete cosine Transform (DCT)

$$x[k] = \alpha(k) \sum_{n=0}^{N-1} x[n] \cdot \cos\left[\frac{(2n+1)\pi k}{2N}\right]$$

where $0 \leq k \leq N-1$

$$\alpha(k) = \sqrt{\frac{1}{N}} \; ; \text{ if } k=0 \quad \alpha(k) = \sqrt{\frac{2}{N}} ; \text{ if } k \neq 0$$

1D

→ Inverse DCT:-

$$x[n] = \alpha(k) \sum^{1}$$

$$F(k,l) = \alpha(k) \cdot \alpha(l) \cdot \sum_{m=0}^{N-1} \sum_{n=0}^{N-1} f(m,n) \cdot \cos\left[\frac{(2m+1)\pi k}{2N}\right] \cdot \cos\left[\frac{(2n+1)\pi l}{2N}\right]$$

The 2D- ~~Inn~~ Inverse DCT

compute the DCT matrix for N=4

Proof: 1-D · DCT

→ In our case N=4

$$x[k] = \alpha[k] . \sum_{n=0}^{3} x[n] . \cos\left[\frac{(2n+1)\pi k}{8}\right] \quad \text{— (1)}$$

$$x[0] = \sqrt{\frac{1}{4}} . \sum_{n=0}^{3} x[n] . \cos\left[\frac{(2n+1)\pi \times 0}{8}\right]$$

$$= \frac{1}{2} . \sum_{n=0}^{3} x[n]$$

$$= \frac{1}{2}\left\{x[0] + \frac{1}{2}x(1) + \frac{1}{2}x(2) + \frac{1}{2}x(3)\right\}$$

$$x[1] = \sqrt{\frac{2}{4}} \sum_{n=0}^{3} x[n] . \cos\left[\frac{(2n+1)\pi}{8}\right]$$

$$= \frac{1}{\sqrt{2}}\left[x(0) . \cos\frac{\pi}{8} + x(1) . \cos\frac{3\pi}{8} + x(2) . \cos\frac{5\pi}{8} + x(3) . \cos\frac{7\pi}{8}\right]$$

$$x[1] = 0.6532\, x(0) + 0.2706\, x(1) - 0.2706\, x(2) - 0.653\, x(3)$$

$$x[2] = \sqrt{\frac{2}{4}} \sum_{n=0}^{3} x[n] . \cos\left[\frac{(2n+1)\pi}{4}\right]$$

$$= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}} . x(0) - \frac{1}{\sqrt{2}} x(1) - \frac{1}{\sqrt{2}} x(2) + \frac{1}{\sqrt{2}} x(3)\right)$$

$$= 0.5\, x(0) - 0.5\, x(1) - 0.5\, x(2) + 0.5\, x(3)$$

$x[3] = 0.27x(0) - 0.65\, x(1) + 0.65\, x(2) - 0.27\, x(3)$

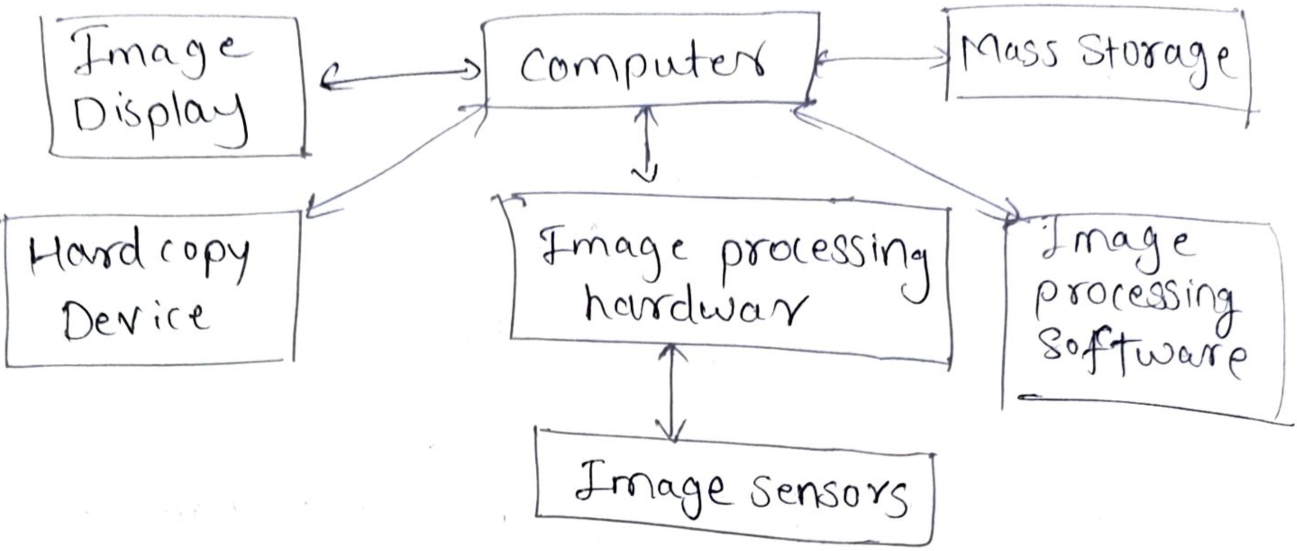
$$\begin{bmatrix} 0.5 & 0.5 & 0.5 & 0.5 \\ 0.6532 & 0.2706 & -0.2706 & -0.6532 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.27 & -0.65 & 0.65 & -0.27 \end{bmatrix}$$

* compute 2D-DCT for given gray scale image

$$f(x,y) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$K\, f(x,y)\, K^T$

Components of DIP System



* Hadamard Transform

order $N=2$ $\quad H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$\cancel{H_2 N =}$

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix}$$

$N=2$

$$H_4 = \begin{bmatrix} H_2 & H_2 \\ +H_2 & -H_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$