Matter wave :-

wave nature of particle

$$\boxed{\lambda = \frac{h}{p}} \rightarrow \text{de Broglie hypothesis}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$ for Non-Relativistic particle with K.E. 'E' $= \frac{1}{2}mv^2 = \frac{p^2}{2m}$

$$= \frac{h}{\sqrt{2m\,eV}}$$ when $\frac{1}{2}mv^2 = E = eV$

$$= \left(\frac{h}{\sqrt{2me}}\right)\frac{1}{\sqrt{V}} = \frac{12.26}{\sqrt{V}}\ \text{Å}$$

$$\frac{h}{\sqrt{2m}} = \frac{6.6\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}}} \approx \frac{6.6\times10^{-34}}{\sqrt{29.12\times10^{-50}}} \approx \frac{6.6\times10^{-34}}{5\times10^{-25}} \approx 10\times10^{-10}\ \frac{m}{\sqrt{\text{Volt}}}$$

Relativity :-

$$E = \frac{p^2}{2m}$$

$$E_{total} = E_K + m_0c^2$$

m_0c^2 → Rest mass energy

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\approx m_0 \quad \text{when } \frac{v}{c} \ll 1 \quad \because \left(1-\frac{v^2}{c^2}\right)^{\frac{1}{2}} = 1 - \frac{1}{2}\left(\frac{v^2}{c^2}\right) + \frac{\left(-\frac{1}{2}\right)\left(\frac{1}{2}-1\right)\left(\frac{v^2}{c^2}\right)}{2!} + \cdots$$

$$E_{total} = \sqrt{p^2c^2 + m_0^2c^4} \Rightarrow E_t^2 = p^2c^2 + m_0^2c^4 = (E_K + m_0c^2)^2$$

$$\Rightarrow p^2c^2 = E_K(E_K + 2m_0c^2)$$

$$\Rightarrow p = \frac{\sqrt{E_K(E_K+2m_0c^2)}}{c}$$

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de Broglie wave length $\lambda = \frac{h}{p}$ for relativistic particle with K.E. 'E_K'

$$\boxed{\lambda = \frac{hc}{\sqrt{E_K(E_K + 2m_0c^2)}}}$$

When $E_K = eV = (E_t - m_0c^2)$, then

$$\boxed{\lambda = \frac{hc}{\sqrt{eV(eV + 2m_0c^2)}}}$$

WBUT-2010

1)(viii) If ~~KE~~ $E_t \gg m_0c^2$

$\Rightarrow E_t = \sqrt{p^2c^2 + m_0^2c^4} \approx pc$

(or, $pc \gg m_0c^2$)

$\Rightarrow E_t \propto p$

5)(a) $v = 50\%$ of c then $m = ?$

$= \frac{c}{2} = 1.5 \times 10^8$ m/s $E_K = ?$

$\rightarrow m = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \frac{m_0}{\sqrt{1 - \frac{1}{4}}} = \frac{2}{\sqrt{3}} m_0$ where $m_0 = 9.1 \times 10^{-31}$ kg

$p = mv = \frac{2}{\sqrt{3}} m_0 \frac{c}{2}$

$\Rightarrow E_{tot} = \sqrt{p^2c^2 + m_0^2c^4}$

$= \sqrt{m_0^2c^4\left(\frac{1}{3} + 1\right)} = m_0c^2 \frac{2}{\sqrt{3}}$

or, $E_{tot} = mc^2$

$= \frac{2}{\sqrt{3}} m_0c^2$

So K.E., $E_K = E_{tot} - m_0c^2$

$= \left(\frac{2}{\sqrt{3}} - 1\right) m_0c^2$

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WBUT-2011

1) (x) $m_{He} > m_p > m_e$, and $v_{He} = v_p = v_e$

$\rightarrow \lambda = \frac{h}{mv} \Rightarrow \lambda_e > \lambda_p > \lambda_{He}$ for NR or R.

(xii) K.E. $E_K = m_0c^2$

$\rightarrow E_{tot} = E_K + m_0c^2 = mc^2$

$\Rightarrow m_0c^2 + m_0c^2 = mc^2 \Rightarrow \boxed{m = 2m_0}$

~~(xiii) No. of oscillation modes (for black body radiation)~~

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10) (c) $E = mc^2$ when $v \ll c$, $E_K = \frac{1}{2}m_0v^2$

$$= \frac{m_0c^2}{(1 - v^2/c^2)^{1/2}}$$

$$= m_0c^2\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

$$= m_0c^2\left[1 + \frac{1}{2}\frac{v^2}{c^2} - \underbrace{\frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2!}\left(\frac{v^2}{c^2}\right)^2}_{\to 0} + \cdots\right]$$

$$\simeq m_0c^2 + \frac{1}{2}m_0v^2$$

$$= m_0c^2 + E_K \Rightarrow E_K = \frac{1}{2}m_0v^2$$

$$E = \left[m_0^2c^4 + p^2c^2\right]^{1/2}$$

$$= m_0c^2\left[1 + \frac{p^2c^2}{m_0^2c^4}\right]^{1/2}$$

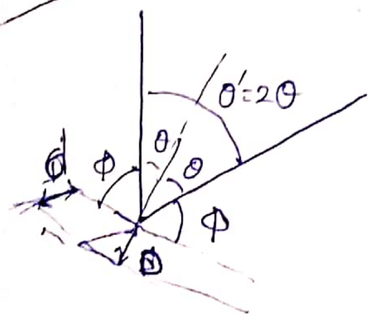
$$= m_0c^2\left[1 + \frac{1}{2}\frac{p^2c^2}{m_0^2c^4} + \underbrace{\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{p^2c^2}{m_0^2c^4}\right)^2}_{0} + \cdots\right]$$

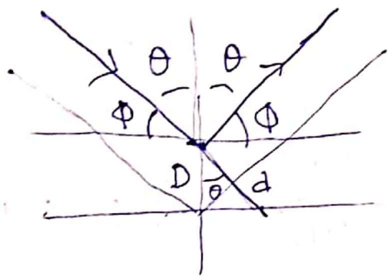
$$= m_0c^2 + \underbrace{\frac{p^2}{2m_0}}_{= E_K}$$

$2D \sin\phi = \lambda$

$\cancel{\phi = 90 - \frac{\theta}{2}}$

Devision-Germer's experiment :





$\phi = 90 - \theta$

$2D \sin\phi = \lambda$ $\qquad \boxed{D = d\sin\theta}$

$2\,d\sin\theta\cos\theta = \lambda \qquad \because \sin\phi = \sin(90-\theta) = \cos\theta$

$d\sin(2\theta) = \lambda = \frac{h}{\sqrt{2meV}}$

$d = 2.15\,Å, \quad 2\theta = 50^\circ \Rightarrow \lambda = 1.65\,Å$

$\Rightarrow V = 54\,V$

$|\psi|^2 \Rightarrow |\psi(x)|^2 =$ Probability of particle at position x.

Electron $\rightarrow$ $v = 10^6$ m/s, $m = 9\times10^{-31}$ Kg

$\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{9\times10^{-31}\times10^{6}}$

$\simeq 10^{-9}\,m \simeq 10\,Å$

Bullet $\rightarrow$ $v = 10^3$ m/s, $m = 10^{-3}$ Kg

$\lambda = \frac{h}{mv} = \frac{6\times10^{-34}}{10^{-3}\times10^{3}}$

$\sim 10^{-34}\,m$

$\sim 10^{-14}\,Å$

$\rightarrow$ too small

For wave

$$\psi = A \sin(\omega t - kx)$$

$\omega = 2\pi\nu$

$k = \frac{2\pi}{\lambda}$

$$\boxed{\text{phase velocity } v_p = \frac{\omega}{k} = \nu\lambda}$$

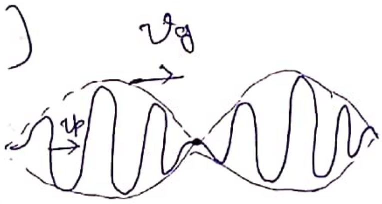
For, monochromatic ~~[illegible]~~ yellow-light.

$$\psi_1 = A \sin(\omega t - kx)$$

$$\psi_2 = A \sin[(\omega + \Delta\omega)t - (k + \Delta k)x]$$

$$\rightarrow \psi_{tot} = \psi_1 + \psi_2 = A \sin(\omega t - kx)$$

Resultant amplitude $A = 2a \cos\left(\frac{\Delta\omega}{2} t - \frac{\Delta k}{2} x\right)$



$$\boxed{\text{group velocity } v_g = \frac{d\omega}{dk}}$$

For Matter wave:

$$\boxed{v_p = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{E}{p}} \quad \text{and} \quad \boxed{v_g = \frac{d\omega}{dk} = \frac{dE}{dp}}$$

For non-relativistic, $E = \frac{p^2}{2m}$

$$\rightarrow v_p = \frac{E}{p} = \frac{p}{2m} = \frac{v}{2} < v < c$$

$$\rightarrow v_g = \frac{dE}{dp} = \frac{p}{m} = v$$

$\Rightarrow$ Group velocity of matter wave = Particle velocity,

For relativistic, $E = \sqrt{p^2c^2 + m_0^2c^4}$

$$\rightarrow v_p = \frac{E}{p} = c\left[1 + \frac{m_0^2c^4}{p^2c^2}\right]^{1/2} > c$$

$$\rightarrow v_g = \frac{dE}{dp} = \frac{1}{\cancel{2}} \frac{\cancel{2}pc^2}{[p^2c^2 + m_0^2c^4]^{1/2}} = \frac{pc^2}{E} = \frac{mv\cancel{c^2}}{m\cancel{c^2}} = v$$ same

$v_p = \frac{E}{p}$ and $v_g = \frac{pc^2}{E}$ (for relativistic)

$$\Rightarrow \boxed{v_p v_g = c^2}$$

Heisenberg's Uncertainty

$$\Delta x \, \Delta P \geq \hbar$$

x and P can't be measured simiultanously with 100% accuracy.

x and P are conjugate variables

$$[x][P] = [L][M L T^{-1}]$$
$$= [M L^2 T^{-1}]$$

similarly E and t are conjugate variable

$$[E][t] = [F L][T]$$
$$= [M L T^{-2} L][T] = [M L^2 T^{-1}]$$

$$\Delta E \, \Delta t \geq \hbar$$

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