Homework 5

Q1. **(15 marks)** Suppose images are 224x224x3 and we use 64 convolutional filters that are 3x3.

a. How many responses will be computed for this layer of a CNN (stride=1, no padding)?
b. How much zero padding is necessary to produce an output of size equal to the input?
c. Repeat a for the case when the stride is 3.

Q2. (15 **marks**) Assume that the inputs are single bits 0 (white) and 1 (black). Consider a 3x3 filter, whose weights are w_{ij}, for $0 \leq i \leq 2$ and $0 \leq j \leq 2$ and whose bias is b. Suggest weights and bias so that the output of this filter will detect the following simple features.

a. Vertical boundary, where the left column is 0, and the other two columns are 1.
b. A diagonal boundary, where only the triangle of three pixels in the upper right corner are 1
c. A corner, in which the 2x2 square in the lower right is 0 and the other pixels are 1.

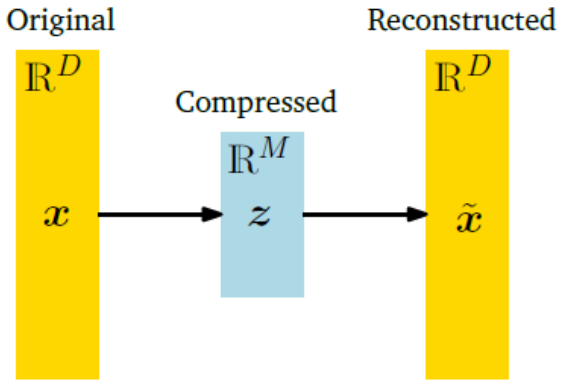
Q3. (15 marks) In this exercise, you are asked to design the input weights for one or more nodes of the hidden state of an RNN. The input is a sequence of bits, 0 or 1 only. Note that you can use other nodes to help with the node requested. Also note that you can apply a transformation to the output of the node so a "yes" answer has one value and a "no" answer has another.

a. A node to signal when the input is 1 and the previous input is 0.
b. A node to signal when the last three inputs have all been 1.
c. A node to signal when the input is the same as the previous input.

Q4. (20 marks) Let A be a matrix of 4-dimensional data points

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 \\ 2 & 4 \\ 3 & 9 \\ 4 & 16 \end{bmatrix}$$

a. Compute Eigenpairs for $\boldsymbol{A^T A}$?
b. What do you expect the eigenvalues of $\boldsymbol{AA^T}$ to be?
c. Find the eigenvectors of $\boldsymbol{AA^T}$, using the eigenvalues from part (c).
d. Write the 1-dimensional and 2-dimensional encodings (**z**) of columns of **A** using **PCA**.



Q5. (35 marks) Design a CNN that can detect the face in the given input image.

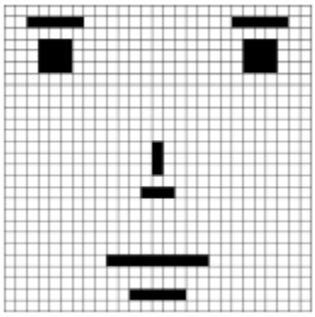


Figure 2 Face Image

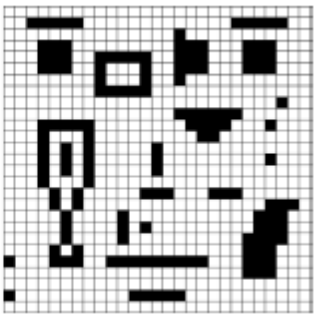


Figure 1 Input Image

a. What strategy will you use? How many layers? How many filters?
b. Describe the filters for each layer in detail.
c. Show how the convolutions will work and demonstrate that your CNN will be able to detect this pattern.