CE

$$Z_N^{CE}(N,T,V) = \sum_i e^{-\beta E_i}$$

$$= \sum_\nu \left(\frac{d^{3N}x\, d^{3N}p}{N!\, h^{3N}}\right)_{E_\nu} e^{-\beta E_\nu} \quad (E_\nu = N\epsilon)$$

$$= \frac{V^N}{N!\, h^{3N}} \left[\int d^3p\, e^{-\beta\epsilon}\right]^N$$

$$= \frac{Z_{CE}^N}{N!}$$

GCE

$$Z^{GCE} = \sum e^{-\beta(E-\mu)}$$

$$= \sum_{\nu,\alpha} \left(\frac{d^{3N}x\, d^{3N}p}{N_\alpha!\, h^{3N}}\right)_{E_\nu, N_\alpha} e^{-\beta(E_{\nu\alpha}-\mu)}$$

$$= \sum_\alpha \frac{V^N}{N!\, h^{3N}} \left[\int d^3p\, e^{-\beta(\epsilon-\mu)}\right]^N \quad (\because \mu_\alpha = N\mu)$$

$$= \sum_\alpha \frac{(e^{\beta\mu} Z_{CE})^{N_\alpha}}{N_\alpha!}$$

where $Z = \frac{V}{h^3}\int d^3p\, e^{-\beta\epsilon}$ is single/one particle partition function of CE system

$= \frac{V}{\lambda^3}$ for $\epsilon = \frac{p^2}{2m}$

with $\lambda = \frac{h}{\sqrt{2\pi m K T}}$ (thermal de Broglie wavelength)

So if we take all possible N_α from 0 to α

$$Z^{GCE}(\mu, T, V) = \sum_{N_\alpha = 0}^{\alpha} \frac{(e^{\beta\mu} Z_{CE})^{N_\alpha}}{N_\alpha!}$$

$$= \exp\{e^{\beta\mu} Z_{CE}\}$$

$$\Rightarrow -PV = \Phi = -KT \ln Z^{GCE} = -KT\{e^{\beta\mu} Z_{CE}\}$$

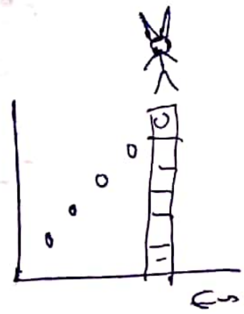
$$= -KT \underbrace{\int \frac{V d^3p}{h^3} e^{-\beta(\epsilon-\mu)}}_{N}$$

GCE (Classica Ideal gas)

One-particle

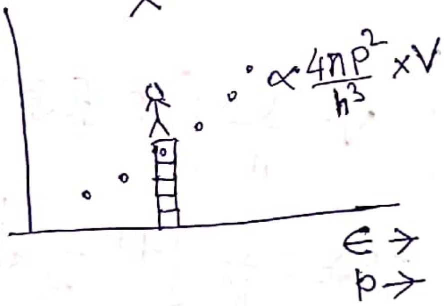
Partition function $Z_1 = \sum e^{-\beta(\epsilon-\mu)}$

$$= \int \frac{d^3x\, d^3p}{h^3} e^{-\beta(\epsilon-\mu)}$$



Remember

$$\boxed{\sum \equiv \int \frac{d^3x\, d^3p}{h^3}}$$



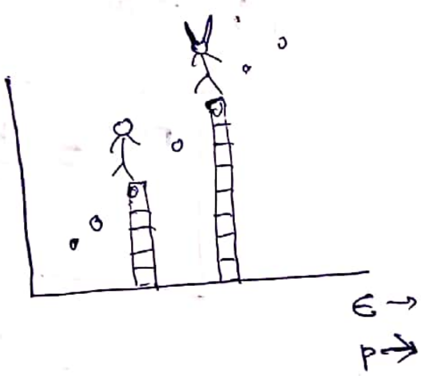
Possible Homes or states for one particle

For many or N particle,

Partition function $Z_N = Z_1 \times Z_2 \times Z_3 \times \cdots$

$$= \cancel{0} \prod_i Z_i$$

$$= \prod_i e^{-\beta(\epsilon_i - \mu_i)}$$



$$-PV = \Phi = -KT \ln Z_N = -KT \ln\left\{\prod_i Z_i\right\}$$

$$= -KT \sum_i \ln Z_i$$

$$= -KT \int \frac{d^3x\, d^3p}{h^3} \underbrace{\ln Z}$$

$$Z = \exp\left\{e^{-\beta(\epsilon-\mu)}\right\}$$

$$= \sum_{N_\alpha=0}^{\infty} \frac{\left\{e^{-\beta(\epsilon-\mu)}\right\}^{N_\alpha}}{N_\alpha!}$$

$$\boxed{P = KT \int \frac{d^3p}{h^3} e^{-\beta(\epsilon-\mu)}}$$

$$= \frac{KTN}{V}$$

$$\Phi = -PV = -KT\int \frac{Vd^3p}{h^3}\, e^{-\beta(\epsilon-\mu)} \quad \text{--- (1)}$$

$$\Rightarrow P = \frac{KT}{V} N \,, \text{ where } N = \frac{V}{h^3}\int d^3p\, e^{-\beta(\epsilon-\mu)}$$

$$= \frac{V}{h^3}\int d^3p\, f_0(\epsilon)$$

MB distribution func^n

$$N = -\left(\frac{\partial \Phi}{\partial \mu}\right)_{V,T}$$

$$= -(-KT)\beta \int \frac{Vd^3p}{h^3}\, e^{-\beta(\epsilon-\mu)}$$

No. $\boxed{N = \frac{V}{h^3}\int d^3p\, f_0(\epsilon)}$

$$= V \times \frac{1}{\lambda^3(T)} \quad \text{for } \epsilon = \frac{p^2}{2m}$$

Thermal distribution function of particle with energy ϵ.

$$S = -\left(\frac{\partial \Phi}{\partial T}\right)_{\mu,V} = K\underbrace{\int \frac{Vd^3p}{h^3}\, e^{-\beta(\epsilon-\mu)}}_{N} + KT\frac{(\mu-\epsilon)}{-KT^2}\underbrace{\int \frac{Vd^3p}{h^3}\, e^{-\beta(\epsilon-\mu)}}_{N}$$

$\times T$

$$TS = \underbrace{KTN}_{PV} - \mu N + \underbrace{\int \frac{Vd^3p}{h^3}\, \epsilon\, f_0(\epsilon)}_{U}$$

$$\Rightarrow TS = U + PV - \mu N$$

Internal Energy $\boxed{U = \int \frac{Vd^3p}{h^3}[\epsilon]\, f_0}$

~~We can~~ $= \frac{3}{2}NKT$

$$= \frac{3}{2}KT\left\{\frac{V}{\lambda^3(T)}\right\} \quad \text{for } \epsilon = \frac{p^2}{2m}$$

After knowing N, U, we get

$$S = \frac{U + PV - \mu N}{T}$$

$$= \frac{5}{2}NK - \frac{\mu N}{T}$$

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Single / One particle

Partition function $Z = e^{-\beta(\epsilon - \mu)}$ classical

Quantum Degeneracy → if 'n' no of degeneracy is available for energy ϵ, then

$$Z = \sum_n e^{-\beta n(\epsilon - \mu)}$$

According Pauli-Exclusion principle $n = 1$ (for fermion)

But for Boson, n can be any values from $1, 2, \dots, \infty$

Classical picture also miss $n=0$ possibility (which is valid for Boson and Fermion.

So $$Z = \sum_{n=0}^{1} e^{-\beta n(\epsilon - \mu)}$$ for Fermion

$$= \left[1 + e^{-\beta(\epsilon - \mu)}\right]$$

$$Z = \sum_{n=0}^{\infty} e^{-\beta n(\epsilon - \mu)}$$ for Boson

$$= \frac{1}{1 - e^{-\beta(\epsilon - \mu)}}$$

Total partition function $Z_t = \prod_i Z_i = Z_1 \cdot Z_2 \cdot Z_3 \cdots$

$$\Rightarrow \ln Z_t = \ln\{\prod_i Z_i\}$$

$$= \sum_i \ln Z_i$$

Boson → Spin in integer $\hbar$ ⇒ $S = (0, 1, 2, \cdots) \times \hbar$

Fermion → " " half-integer of $\hbar$

⇒ $S = \left(\frac{1}{2}, \frac{3}{2}, \cdots\right) \times \hbar$

$$-PV = \Phi = -KT \ln Z_t$$

$$= -KT \sum \ln Z = -KT \sum \ln\left[1 + e^{-\beta(\epsilon-\mu)}\right] \quad \text{for Fermion}$$

$$= -KT \sum \ln\left[1 - e^{-\beta(\epsilon-\mu)}\right]^{-1} \quad \text{for Boson}$$

In general, we can write

$$-PV = \Phi = \frac{-KT}{\eta} \sum \ln\left[1 + \eta e^{-\beta(\epsilon-\mu)}\right] \quad \begin{cases} \eta = +1 \ (F) \\ \eta = -1 \ (B) \end{cases}$$

$$= \frac{-KT}{\eta} \int \frac{V d^3p}{h^3} \ln\left[1 + \eta e^{-\beta(\epsilon-\mu)}\right]$$

$$N = -\left(\frac{\partial \Phi}{\partial \mu}\right)_{T,V} = \frac{1}{\eta} \sum \frac{\eta e^{-\beta(\epsilon-\mu)}}{1 + \eta e^{-\beta(\epsilon-\mu)}}$$

$$= \sum \frac{1}{e^{\beta(\epsilon-\mu)} + \eta}$$

$$= \int \frac{V d^3p}{h^3} \frac{1}{e^{\beta(\epsilon-\mu)} + \eta}$$

$$\left.\begin{matrix} N_{FD} \\ N_{BE} \\ N_{MB} \end{matrix}\right\} = \int \frac{V d^3p}{h^3} f_0 \quad \begin{cases} \frac{1}{e^{\beta(\epsilon-\mu)} + 1} \\ \frac{1}{e^{\beta(\epsilon-\mu)} - 1} \\ e^{-\beta(\epsilon-\mu)} \end{cases}$$

Classical distribution $\longrightarrow$ Quantum distribution

$$f_0 = e^{-\beta(\epsilon-\mu)} \text{ (MB)} \quad \begin{cases} f_0 = \dfrac{1}{e^{\beta(\epsilon-\mu)}+1} & \text{(FD)} \\[2ex] f_0 = \dfrac{1}{e^{\beta(\epsilon-\mu)}-1} & \text{(BE)} \end{cases}$$

$$\sum f_0 = \boxed{\begin{aligned} N &= \frac{V}{h^3}\int d^3p\, f_0(\epsilon) \\ N &= \frac{V}{h^3}\int d^3p \frac{1}{e^{\beta(\epsilon-\mu)}+\eta} \end{aligned}} \quad \begin{cases} \eta = 0 \text{ (MB)} \\ \eta = 1 \text{ (FD)} \\ \eta = -1 \text{ (BE)} \end{cases}$$

$$\sum \epsilon f_0 = \boxed{\begin{aligned} U &= \frac{V}{h^3}\int d^3p . [\epsilon]\, f_0(\epsilon) \\ &= \frac{V}{h^3}\int d^3p\, [\epsilon] \frac{1}{e^{\beta(\epsilon-\mu)}+\eta} \end{aligned}}$$

$$PV = -\Phi = \frac{KT}{\eta}\sum \ln\left\{1+\eta e^{\beta(\mu-\epsilon)}\right\}$$

$$\boxed{PV = \frac{KT}{\eta}\int \frac{Vd^3p}{h^3} \ln\left\{1+\eta e^{\beta(\mu-\epsilon)}\right\}}$$

$$= \frac{V}{h^3}\int d^3p\, [\ ?\ ] \frac{1}{e^{\beta(\epsilon-\mu)}+\eta}$$

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check!

$$\ln Z = -\frac{1}{\eta} \sum_i \ln\left[1 + \eta e^{\beta(\mu-\epsilon)}\right]$$

$$= \lim_{\eta \to 0} \frac{-\sum_i \frac{\partial}{\partial \eta} \ln\left[1+\eta e^{\beta(\mu-\epsilon)}\right]}{\frac{\partial \eta}{\partial \eta}}$$

$$= -\sum_i \lim_{\eta \to 0} \frac{e^{\beta(\mu-\epsilon)}}{1+\eta e^{\beta(\mu-\epsilon)}}$$

$$= -\sum_i e^{\beta(\mu-\epsilon)}$$

$$pV = \Phi = -KT \ln Z = KT \sum_i e^{-\beta(\epsilon-\mu)} = KT \underbrace{\int \frac{d^3x\, d^3p}{h^3} e^{-\beta(\epsilon-\mu)}}_{N}$$

$$N = -\left(\frac{\partial \Phi}{\partial \mu}\right)_{T,V} = KT \times \beta \sum e^{-\beta(\epsilon-\mu)} = \int \frac{d^3x\, d^3p}{h^3} e^{-\beta(\epsilon-\mu)}$$

$$\left.\begin{matrix} P_{FD} \\ P_{BE} \\ P_{MB} \end{matrix}\right\} = KT \int \frac{V d^3p}{h^3} \left[\ \ \right] \quad \begin{cases} -\ln\left\{1+e^{-\beta(\epsilon-\mu)}\right\} \\ \ln\left\{1-e^{-\beta(\epsilon-\mu)}\right\} \\ e^{-\beta(\epsilon-\mu)} \end{cases}$$