Quantum Mechanics : PHL502/PH502

Name:

ID:

- **No mobile, No calculator.**
- **Don't forget to write your names and ID in your question paper.**
- **Put a circle or tick on the right answer on this question paper. However, you can use rough papers (if required), which are not to be submitted. Submit your question paper to the TAs, as your answers are there.**
- **It is recommended to make a copy of your answers with you (either in a paper or in your brain if it is reliable $\smile$) so that you can cross verify your marks when I reveal the answer keys after the exam.**

Values of some universal constants (if required)
Planck constant $h = 6.62 \times 10^{-34}\ m^2Kg/s$, speed of light $c = 3 \times 10^8\ m/s$
electron mass $m_e = 9.1 \times 10^{-31}\ Kg$, proton mass $m_p = 1.6 \times 10^{-27}\ Kg$,
Length scales $1\ fm = 10^{-5}\mathring{A} = 10^{-6}nm = 10^{-15}m$
$1eV = 1.6 \times 10^{-19}$ Jule, Boltzmann constant $K = 1.38 \times 10^{-23}\ m^2Kg/s^2/^0K$

1. If Aluminium (Al) is used as the material for the photoelectric effect with incident light of wavelength 415nm, what is the stopping potential needed to stop the current in the phototube? Take the work function of Al to be 4.28eV.
(a) 1.3 V
(b) 1.78 V
(c) 0 V
(d) -1.3 V
(e) none of the above

2. In a Compton Scattering experiment, one incident X-ray photon of wavelength $0.13\mathring{A}$ is scattered by an electron with a scattering angle of $120°$. What is the energy of the scattered photon?
(a) 95.4 KeV

(b) 186.3 KeV
(c) 72.8 KeV
(d) 83.7 KeV
(e) none of the above

3. In a Compton Scattering experiment, one incident X-ray photon of wavelength $\lambda = 0.15\mathring{A}$ is scattered by an electron with a scattering angle of $\phi = 60°$. The tan of the angle the recoiled electron makes with the incident X-ray direction is $tan\theta = \frac{cot\phi/2}{1+\alpha}, \alpha = \frac{h}{m_e c\lambda}$. Which of the following is true?
(a) $tan\theta = 0.5$
(b) $tan\theta = 10.7$
(c) $tan\theta = 1.5$
(d)$tan\theta = 3.5$
(e) none of the above

4. Let us assume that there are two protons of the same energy, one of which is in the ground state of a harmonic oscillator potential with frequency $\omega = 49 \times 10^{13} Hz$, and another on the ground state of an infinite 1D potential box with length L. Then which of the following is true?
(a) $L = 0.45\mathring{A}$
(b) $L = 3.6\mathring{A}$
(c) $L = 1.8\mathring{A}$
(d) $L = 0.91\mathring{A}$
(e) none of the above

5. In the photo-electric effect, stopping potentials are the same for incident light with
(a) different frequencies but the same intensities
(b) different intensities and different frequencies
(c) different intensities but the same frequencies
(d) same intensities but different wavelengths
(e) none of the above

6. In photo-electric effect, stopping potentials vs frequency curve, its slope will be Planck's constant if
(a) one divides the slope by the electric charge
(b) one divides the slope by electron mass
(c) one multiply the slope by electric charge by threshold frequency
(d) one multiplies the slope by the electric charge
(e) none of the above

7. Based on the calculation Compton wavelength of electron $\lambda_c = \frac{h}{mc}$ in meter, which rays will be the best to show the Compton effect?
(a) Radio wave ($\lambda > 10^{-3}$ m)
(b) Microwave ($\lambda = 10^{-3} - 10^{-6}$ m)
(c) Ultra-violet ray ($\lambda = 10^{-7} - 10^{-9}$ m)
(d) X-ray ($\lambda = 10^{-9} - 10^{-12}$ m).
(e) none of the above

8. In terms of Compton length $\lambda_c = \frac{h}{mc} = 0.024\mathring{A}$, Compton shift $\Delta\lambda$ follow the inequality

(a)$0 \leq \Delta\lambda \leq 0.024\mathring{A}$
(b) $0.024\mathring{A} \leq \Delta\lambda \leq 0.048\mathring{A}$
(c) $0 \leq \Delta\lambda \leq 0.048\mathring{A}$
(d) $0 < \Delta\lambda < 0.048\mathring{A}$
(e) none of the above

9. Group velocity of matter wave is v_{gr}, particle velocity is v_{part} and c is speed of light, then which relations, given below is not true
(a)$v_{gr} < c$
(b) $v_{part} < c$
(c) $v_{part} < v_{gr}$
(d) $v_{part} = v_{gr}$
(e) none of the above

10. Group velocity $v_{gr} = \frac{dE}{dp}$ and phase velocity $v_{ph} = \frac{E}{p}$ of relativistic particle with $E = \sqrt{p^2c^2 + m_0^2c^4}$ follow the relation
(a)$v_{gr} = \frac{v_{ph}^2}{c}$
(b) $v_{gr} = c$
(c) $v_{gr} = \frac{v_{ph}}{c^2}$
(d) $v_{gr} = \frac{c^2}{v_{ph}}$
(e) none of the above

11. Matter-wave of the electron, accelerated by 10^2 order voltage, is basically
(a) Radio Wave
(b) visible Ray
(c) X-Ray
(d) sound wave
(e) none of the above

12. Compton wavelength and de-Broglie wavelength of an electron can be the same when
(a) electron is at rest
(b) electron move with non-relativistic speed
(c) electron's velocity reaches the speed of light i.e., $v = c$
(d) electron move with relativistic speed $v = c/\sqrt{2}$
(e) none of the above

13. The recoil momentum of an atom is p_A when it emits an infrared photon of wavelength 1500 nm, and it is p_B when it emits a photon of visible wavelength 500 nm. What is the ratio p_A/p_B?
(a) 1 : 3
(b) 1 : 5
(c) 3 : 1
(d) 5 : 1
(e) none of the above

14. Which statement is wrong for a red ($\lambda = 7000\mathring{A}$) laser source?
(a) That source can not be considered a black body.
(b) all red photons carry the same energy ($E \approx 1.8$ eV).
(c) photoelectric effect for the sample with a work function of less than 1 eV is possible by using that laser source.

(d) one red photon energy is sufficient to be converted into an electron by following Einstein's $E = mc^2$ relation.
(e) none of the above

15. Which statement is wrong if we compare the infinite potential box problem with the picture - a student is confined in a classroom?
(a) the student and particle both can have matter waves.
(b) the student and particle both will follow Schrodinger's equation.
(c) quantum aspect can be noticed more in the particle than the student.
(d) quantum aspect can be noticed more in the student than the particle.
(e) none of the above.

16. Using Schrodinger equation $\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$, solve 1D potential box problem, whose $V(x) = 0$, (for $0 < x < a$) and ∞ elsewhere. When we put boundary conditions $\psi(x = 0) = \psi(x = a) = 0$ on the guess wave-function $\psi = A\ Sin(kx) + B\ Cos(kx)$ with $k = \sqrt{2mE}/\hbar$, then we will get
(a) $\psi_n = B\ Cos(k_n x)$ with $k_n = n\pi/a$
(b) $\psi_n = A\ Cos(k_n x)$ with $k_n = n\pi/a$
(c) $\psi_n = B\ Sin(k_n x)$ with $k_n = n\pi/a$
(d) $\psi_n = A\ Sin(k_n x)$ with $k_n = n\pi/a$
(e) none of the above.

17. After putting normalization condition $\int |\psi_n|^2 dx = 1$ on the wave function of the earlier question, we can find the unknown constant as
(a) $\sqrt{\frac{2}{a}}$
(b) $\sqrt{\frac{1}{a}}$
(c) $\frac{2}{a}$
(d) $\frac{2}{a}$
(e) none of the above.

18. It was Max Born who discover the physical meaning of wave function $\psi(x, t)$ of a particle as
(a) $\psi(x, t)$ = probability of finding the particle at the position x and at time t.
(b) $|\psi(x, t)|^2$ = probability of finding the particle at the position x and at time t.
(c) $\psi(x, t)$ = probability density of finding the particle at the position x and at time t.
(d) $|\psi(x, t)|^2$ = probability density of finding the particle at the position x and at time t.
(e) none of the above.

19. Stationary state system in quantum mechanics is understood as a system having time independent probability i.e., $|\psi(x, t)|^2$ does not have any time t dependency. So example of stationary 1D box will be $\psi(x, t) = A\ Sin(kx)e^{-iEt/\hbar}$, where
(a) E will be a real number
(b) E will be an imaginary number
(c) E will be a complex number
(d) E will be zero
(e) none of the above.

20. The energy and momentum operators are
(a) $\hat{E} = i\hbar\frac{\partial}{\partial x}$, $\hat{p} = -i\hbar\frac{\partial}{\partial x}$
(b) $\hat{E} = i\hbar\frac{\partial}{\partial t}$, $\hat{p} = -i\hbar\frac{\partial}{\partial t}$
(c) $\hat{E} = -\frac{\hbar}{i}\frac{\partial}{\partial t}$, $\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x}$
(d) $\hat{E} = \frac{\hbar}{i}\frac{\partial}{\partial t}$, $\hat{p} = -\frac{\hbar}{i}\frac{\partial}{\partial x}$
(e) none of the above.

21. The energy and momentum eigenvalues for wave function $\psi_n(x,t) = A\, sin\left(\frac{n\pi x}{L}\right)e^{-iE_n t/\hbar}$ for $0 < x < L$ are p_n and E_n, where $E_n = \frac{p_n^2}{2m}$
(a) $E_n = \frac{\hbar^2 n^2 \pi^2}{L^2 8m}$, $p_n = \frac{\hbar n\pi}{2L}$
(b) $E_n = \frac{h^2 n^2}{L^2 8m}$, $p_n = \frac{hn}{2L}$
(c) $E_n = \frac{h^2 n^2}{L^2 2m}$, $p_n = \frac{hn}{L}$
(d) $E_n = \frac{h^2 L^2}{n^2 8m}$, $p_n = \frac{hL}{2n}$
(e) none of the above.

22. For 1D potential problem (above mentioned) energy and momentum of $n = 4$ state is a and b times larger than those of $n = 3$ state. The values of a, b are
(a) $a = \frac{4}{3}$, $b = \frac{16}{9}$
(b) $a = \frac{16}{9}$, $b = \frac{4}{3}$
(c) $a = 16$, $b = 4$
(d) $a = \frac{16\hbar^2}{9L^2}$, $b = \frac{4\hbar}{3L}$
(e) none of the above.

23. For square potential barrier problem, $V(x) = V_0$ for $0 < x < a$ and else $V(x) = 0$, the expression of transmission coefficients/tunneling probability is

$$T = \frac{16E(V_0 - E)e^{-2\alpha a}}{V_0^2}\ ,\ \alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}\ . \qquad (1)$$

Using Eq. (1), the values (order of magnitude) of 'a' for electron with energy $E = 1$ eV, which has a probability $T = 10^{-3}$ for tunneling a potential barrier $V_0 = 2$ eV, will be
(a) $a = 10^{-5}$-10^{-6}m
(b) $a = 10^{-9}$-10^{-10}m
(c) $a = 10^{-11}$-10^{-12}m
(d) $a = 10^{-14}$-10^{-15}m
(e) none of the above.

24. Which statement is true for classical but not true for quantum harmonic oscillator?
(a) Total energy will be discrete,
(b) Total energy will be continuous,
(c) Total energy changes with the frequency of the oscillator,
(d) Total energy is independent of x (position).
(e) none of the above.

25. The amplitude a of quantum harmonic oscillator with mass m for ground state energy $E = \frac{1}{2}\hbar\omega$ will be
(a) $a = \frac{\hbar}{mc}$

(b) $a = \frac{c}{\omega}$
(c) $a = \sqrt{\frac{\hbar}{m\omega}}$
(d) $a = 0$
(e) none of the above.

26. For fixed values of ω, m of harmonic oscillator, quantum energy expression $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$ roughly describe the classical picture of harmonic oscillator, when
(a) $n = 0$
(b) $n > 0$ but small
(c) n is quite large
(d) $n < 0$
(e) none of the above.

27. Similarity between wave equation for Light/Sound and matter-wave is both have
(a) single time derivative
(b) double time derivative
(c) single space derivative
(d) double space derivative
(e) none of the above.

28. Commutation of position operator $\hat{x} \equiv x$ and momentum operator $\hat{p} \equiv -i\hbar\frac{\partial}{\partial x}$ is defined as $[\hat{x}, \hat{p}] = \hat{x}\hat{p} - \hat{p}\hat{x}$. Then $[\hat{p}, \hat{x}] =$?
(a) $+i\hbar$
(b) $-i\hbar$
(c) $+ih$
(d) $-ih$
(e) none of the above.

29. Using the correct answer of earlier question, derive the commutation relation between lowering operator $\hat{a} = A\hat{x}^2 + iB\hat{p}^2$ raising operator $\hat{b} = A\hat{x}^2 - iB\hat{p}^2$ [with $A = \left(\frac{m\omega}{2\hbar}\right)^{1/2}$ and $B = \left(\frac{1}{2\hbar m\omega}\right)^{1/2}$] as
(a) $[\hat{a}, \hat{b}] = +1$
(b) $[\hat{a}, \hat{b}] = -1$
(c) $[\hat{a}, \hat{b}] = +i$
(d) $[\hat{a}, \hat{b}] = -i$
(e) none of the above.

30. The degeneracy of the state having energy $\frac{14h^2}{8mL^2}$ for a particle in a 3D-cubic box of length L is
(a) 5
(b) 6
(c) 7
(d) 8
(e) none of the above.