PH506 Statistical Mechanics (2nd tierce exam)

Name:

ID:

2023W 2nd Tierce

1. Thermal distribution function of any particle with energy E in a gas with temperature T and chemical potential μ can be written in a general form $f(E,T,\mu) = 1/[exp\{(E-\mu)/K_BT\} + \eta]$, which will be Fermi-Dirac (FD), Bose-Einstein (BE), and Maxwell-Boltzmann (MB) distribution for
(a) $\eta = +1, \ -1, \ 0$
(b) $\eta = -1, \ +1, \ 0$
(c) $\eta = 0, \ -1, \ +1$
(d) none of the above.

2. For grand canonical ensemble (GCE), pressure can be written in the general form
$$P = K_BT \int \frac{d^3p}{h^3} ln[1+\eta e^{-\beta(E-\mu)}]^{1/\eta}$$
For the MB distribution case, we have to take a limiting case
$$\lim_{\eta\to 0} = ln[1+\eta e^{-\beta(E-\mu)}]^{1/\eta} \ ,$$
which will be
(a) $ln1 = 0$
(b) $exp(e^{-\beta(E-\mu)})$
(c) $e^{-\beta(E-\mu)}$
(d) none of the above.

3. In Large Hadron Collider (LHC) experiments, apart from neutron n and proton p with spin $\hbar/2$, many other particles like pion π, kaon K with spin 0; ρ, K^* mesons with spin $\hbar$; Δ with spin $\frac{3\hbar}{2}$ are produced. In the context of statistical mechanics, we can classify them as
(a) Bosons : π, K, n, p and Fermions : ρ, K^* Δ
(b) Bosons : π, K, ρ, K^* and Fermions : n, p, Δ
(c) Bosons : π, K, ρ, K^*, Δ and Fermions : n, p,
(d) none of the above.

4. For general dispersion relation (or energy (E)-momentum (p) relation) $E = ap^n$ with constant values of a and n, Gibb's free energy $G = -\mu N$ of 3-dimensional ideal gas will be

(a) $G = NKTln\left[\frac{V}{N}\frac{(KT)^{(n+1)/2}}{h^3}x\right]$ with $x = \frac{3\pi^{3/2}\Gamma((n+1)/2)}{na^{(n+1)/2}\Gamma(5/2)}$
(b) $G = NKTln\left[\frac{V}{N}\frac{(KT)^{3/n}}{h^3}x\right]$ with $x = \frac{3\pi^{3/2}\Gamma(3/n)}{na^{3/n}\Gamma(5/2)}$
(c) $G = NKTln\left[\frac{V}{N}\frac{(KT)^{3/n}}{h^3}x\right]$ with $x = 1$
(d) none of the above.

5. For earlier 3 dimensional ideal gas problem with $E = ap^n$, Helmholtz free energy $A = U - TS$ will be
(a) $A = -NKT\left[ln\left\{\frac{V}{N}\frac{(KT)^{(n+1)/2}}{h^3}x\right\} + 1\right]$ with $x = \frac{3\pi^{3/2}\Gamma((n+1)/2)}{na^{(n+1)/2}\Gamma(5/2)}$
(b) $A = -NKT\left[ln\left\{\frac{V}{N}\frac{(KT)^{3/n}}{h^3}x\right\} + 1\right]$ with $x = \frac{3\pi^{3/2}\Gamma(3/n)}{na^{3/n}\Gamma(5/2)}$
(c) $A = -NKT\left[ln\left\{\frac{V}{N}\frac{(KT)^{3/n}}{h^3}x\right\} + 1\right]$ with $x = 1$
(d) none of the above.

6. For general dispersion relation (or energy (E)-momentum (p) relation) $E = ap^n$ with constant values of a and n, equation of state (relation between pressure and number) D-dimensional ideal gas will be
(a) $PV = \frac{n+1}{D}NKT$
(b) $PV = \frac{n}{D}NKT$
(c) $PV = \frac{n-1}{D}NKT$
(d) none of the above.

7. For earlier D-dimensional ideal gas problem with $E = ap^n$, internal energy will be
(a) $U = \frac{n+1}{d-1}NKT$
(b) $U = \frac{n}{d}NKT$
(c) $U = \frac{n-1}{d+1}NKT$
(d) none of the above.

8. For earlier D-dimensional ideal gas problem with $E = ap^n$, entropy will be
(a) $S = NK\left[(D/n+1) + ln\left\{\frac{V}{N}\frac{(KT)^{D/n}}{h^D}x\right\}\right]$ with $x = \frac{D\pi^{D/2}\Gamma(D/n)}{na^{D/n}\Gamma(D/2+1)}$
(b) $S = NK\left[(D+2)/n) + ln\left\{\frac{V}{N}\frac{(KT)^{D/n}}{h^D}x\right\}\right]$ with $x = \frac{D\pi^{D/2}\Gamma(D/n)}{na^{D/n}\Gamma(D/2+1)}$
(c) $S = NK\left[(n+3)/(D-1) + ln\left\{\frac{V}{N}\frac{(KT)^{D/n}}{h^D}x\right\}\right]$ with $x = \frac{D\pi^{D/2}\Gamma(D/n)}{na^{D/n}\Gamma(D/2+1)}$
(d) none of the above.

9. Consider a system of classically distiniguishable particles in 1D with dispersion relation(K.E - momentum relation) $K.E = ap^n$, under the influence of external potential field $V(x) = bx^m, -\infty < x < \infty$, with constant values of a, b, m and n. The partition function of the system will be
(a) $Z = \left[\frac{1}{h\beta^{mn/(m+n)}}\frac{4\pi}{mn}\left(\frac{1}{a}\right)^{1/n}\left(\frac{1}{b}\right)^{1/m}\right]^N$
(b) $Z = \left[\frac{1}{h\beta^{(m+n)/mn}}\frac{4}{mn}\left(\frac{1}{a}\right)^{1/n}\left(\frac{1}{b}\right)^{1/m}\Gamma(1/m)\Gamma(1/n)\right]^N$
(c) $Z = \left[\left(\frac{KT}{\hbar\omega}\right)^{(m+n)/mn}\right]^N$
(d) none of the above.

10. For the earlier 1D system, given in the question(9), internal energy will be
(a) $U = \left(\frac{mn}{m+n}\right)NKT$
(b) $U = \left(\frac{m+n}{mn}\right)NKT$

(c) $U = \frac{4}{mn} NKT$
(d) none of the above.

11. In the limit of $\beta \to \infty$, Fermi-Dirac distribution function

$$f_{FD} = \frac{1}{e^{\beta(E-\mu)} + 1}$$

can be converted to
(a) Sign function

$$\begin{aligned} f_{FD} = signx &= -1 \text{ (when } x < 0) \\ &= 0 \text{ (when } x = 0) \\ &= +1 \text{ (when } x > 0) \end{aligned}$$

where $x = \mu - E$.
(b) Step function

$$\begin{aligned} f_{FD} = \theta(x) &= +1 \text{ (when } x > 0) \\ &= 0 \text{ (when } x < 0) \end{aligned}$$

where $x = \mu - E$.
(c) Dirac delta function

$$\begin{aligned} f_{FD} = \delta(x) &= \infty \text{ (when } x = 0) \\ &= 0 \text{ (when } x \neq 0) \end{aligned}$$

where $x = \mu - E$.
(d) none of the above.

12. In the limit of $\beta \to \infty$, derivative of Fermi-Dirac distribution function

$$f'_{FD} = \frac{\partial f_{FD}}{\partial \mu} = \frac{\partial}{\partial \mu}\Big[\frac{1}{e^{\beta(E-\mu)} + 1}\Big]$$

can be converted to
(a) Sign function

$$\begin{aligned} f'_{FD} = signx &= -1 \text{ (when } x < 0) \\ &= 0 \text{ (when } x = 0) \\ &= +1 \text{ (when } x > 0) \end{aligned}$$

where $x = \mu - E$.
(b) Step function

$$\begin{aligned} f'_{FD} = \theta(x) &= +1 \text{ (when } x > 0) \\ &= 0 \text{ (when } x < 0) \end{aligned}$$

where $x = \mu - E$.
(c) Dirac delta function

$$\begin{aligned} f'_{FD} = \delta(x) &= \infty \text{ (when } x = 0) \\ &= 0 \text{ (when } x \neq 0) \end{aligned}$$

where $x = \mu - E$.
(d) none of the above.