

Trying to orient our brain about entire **time axis** of our universe life time

## How old is it?



**The Universe**  
~13.8 billion years



**The Milky Way**  
~13.6 billion years



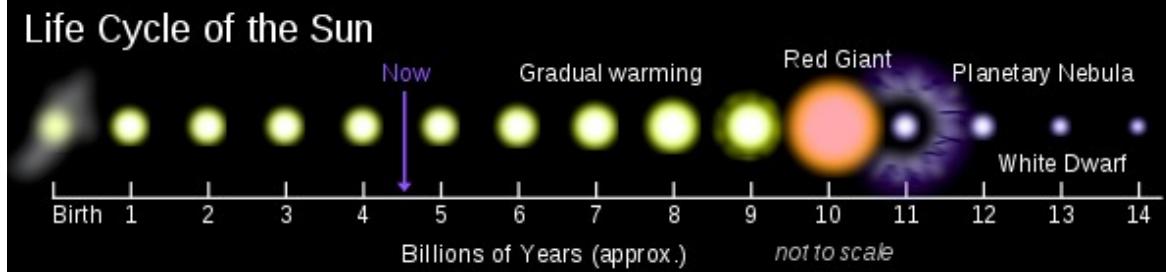
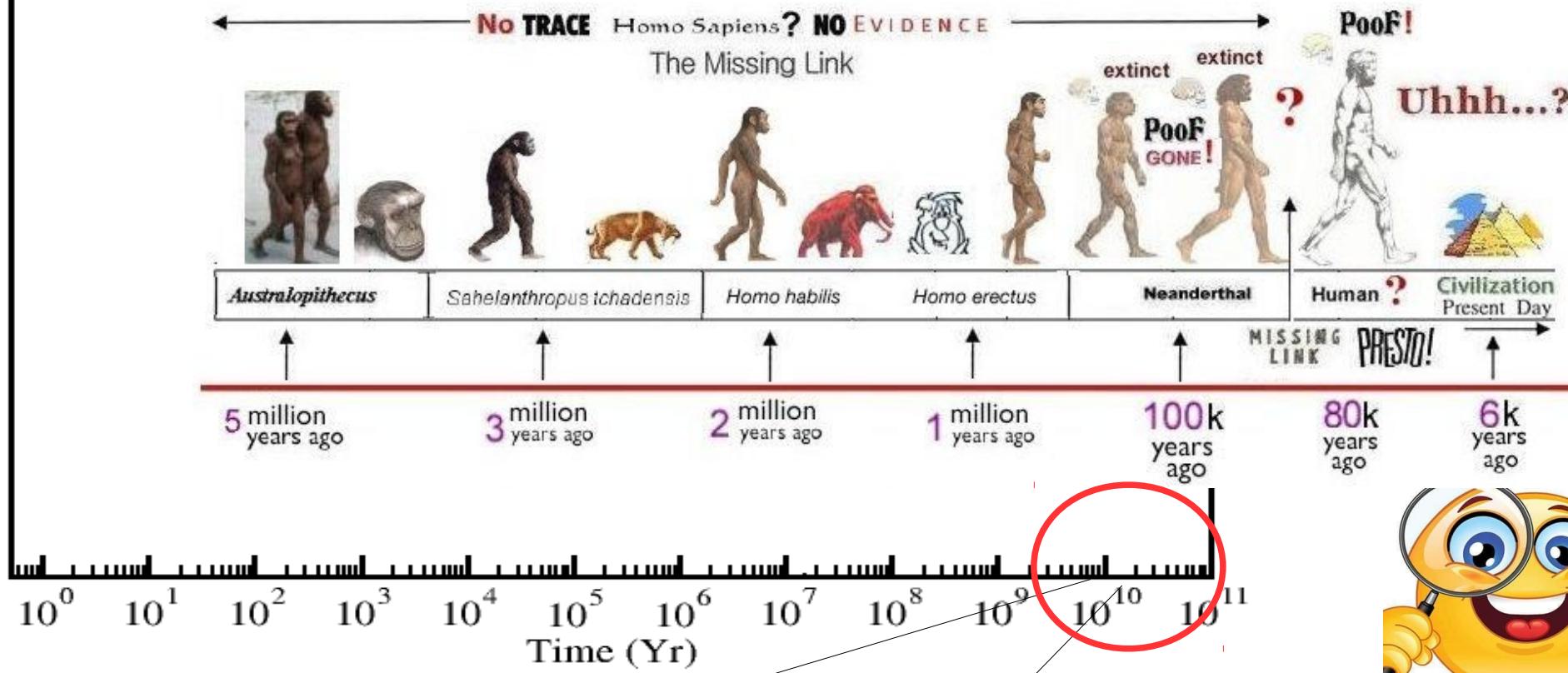
**Our Sun**  
~4.5 billion years



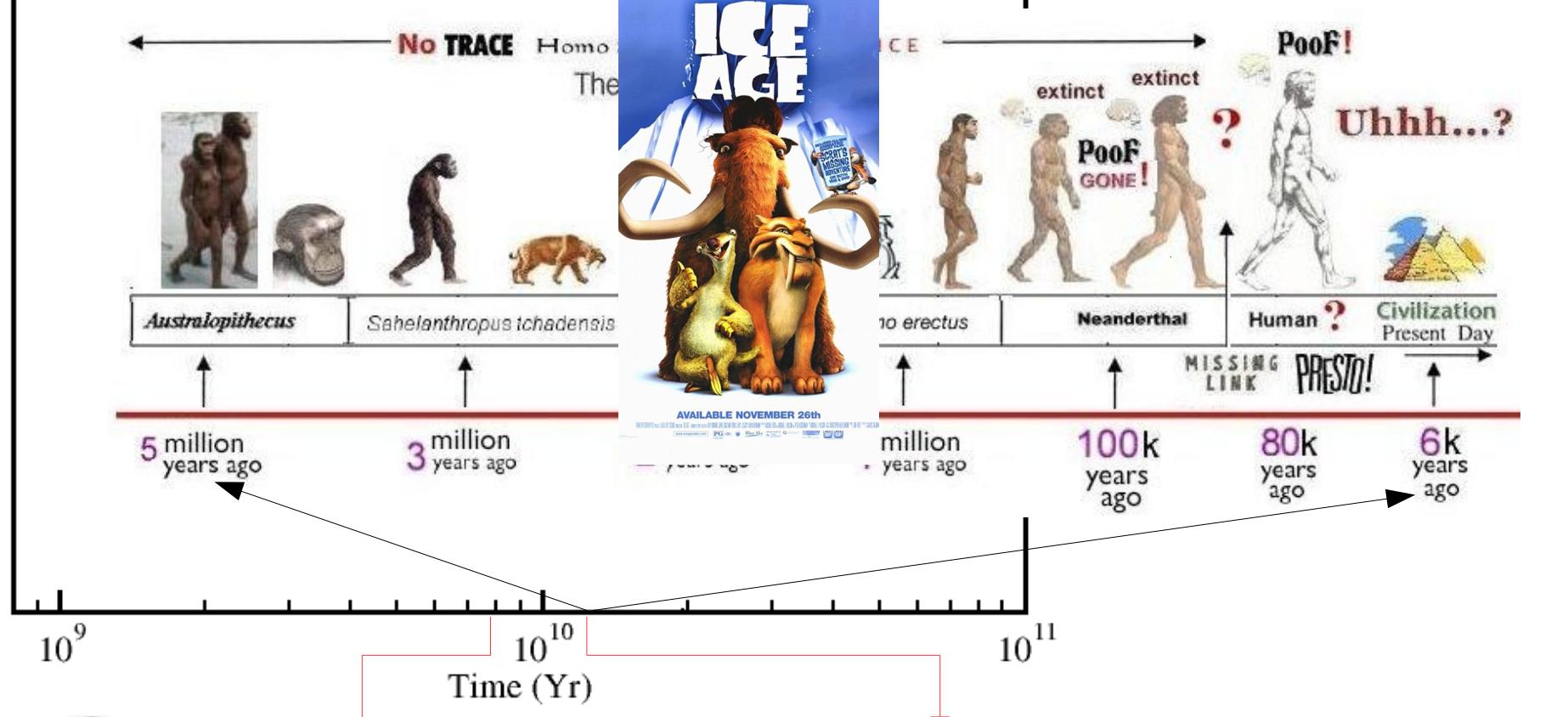
**Earth**  
~4.5 billion years



<i>Number in words</i>	<i>Number in figures</i>	<i>Number in standard form</i>	<i>Number written as a decimal</i>
One thousand	1,000	$10^3$	
Ten thousand	10,000	$10^4$	0.01 million
One hundred thousand	100,000	$10^5$	0.1 million
One million	1,000,000	$10^6$	
Ten million	10,000,000	$10^7$	0.01 billion
One hundred million	100,000,000	$10^8$	0.1 billion
One billion	1,000,000,000	$10^9$	
Ten billion	10,000,000,000	$10^{10}$	0.01 trillion
One hundred billion	100,000,000,000	$10^{11}$	0.1 trillion
One trillion	1,000,000,000,000	$10^{12}$	
One quadrillion	1,000,000,000,000,000	$10^{15}$	



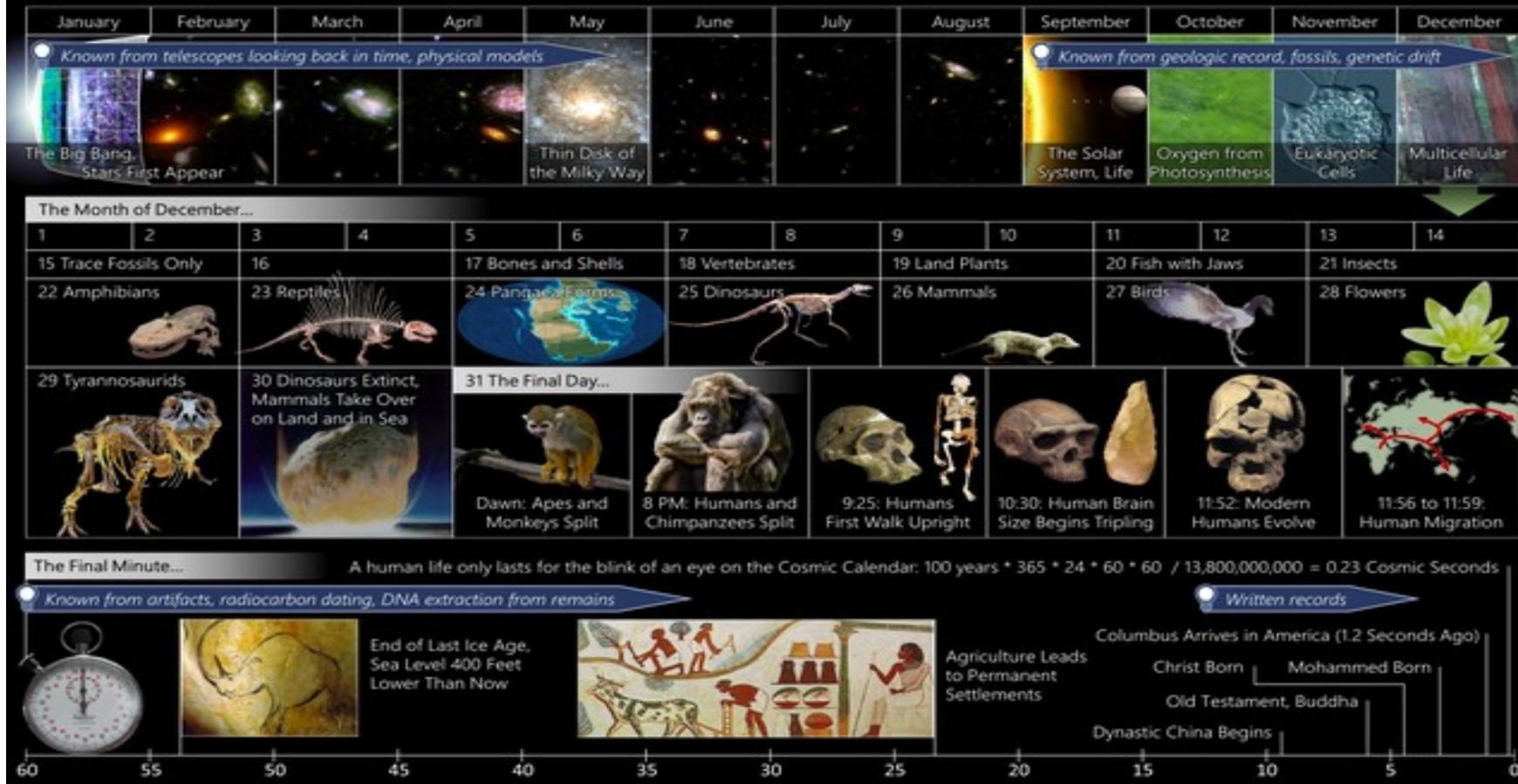
[https://www.youtube.com/watch?v=JgPwTA\\_o90s](https://www.youtube.com/watch?v=JgPwTA_o90s)



Wikipedia Link: [https://en.wikipedia.org/wiki/Cosmic\\_Calendar](https://en.wikipedia.org/wiki/Cosmic_Calendar)

# The Cosmic Calendar

The 13.8 billion year history of the universe scaled down to a single year, where the Big Bang is January 1<sup>st</sup> at midnight, and right now is midnight 1 year later

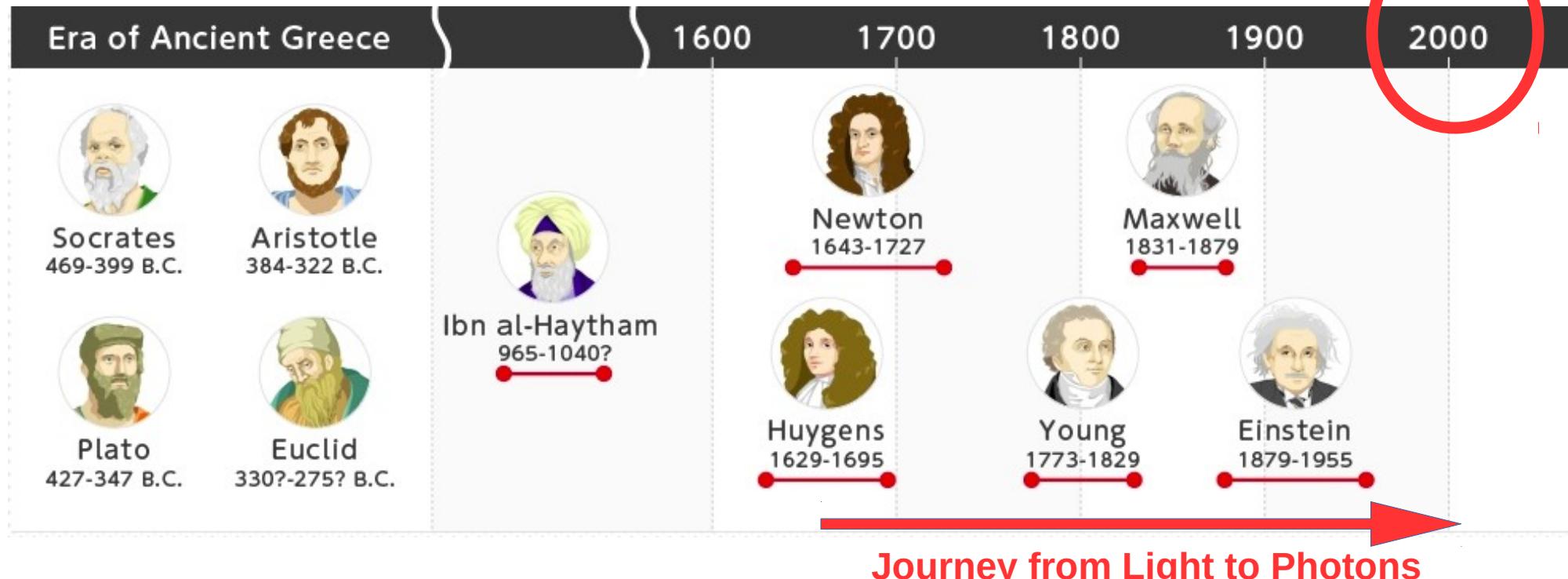


# Ancient Light for Universe and Human history



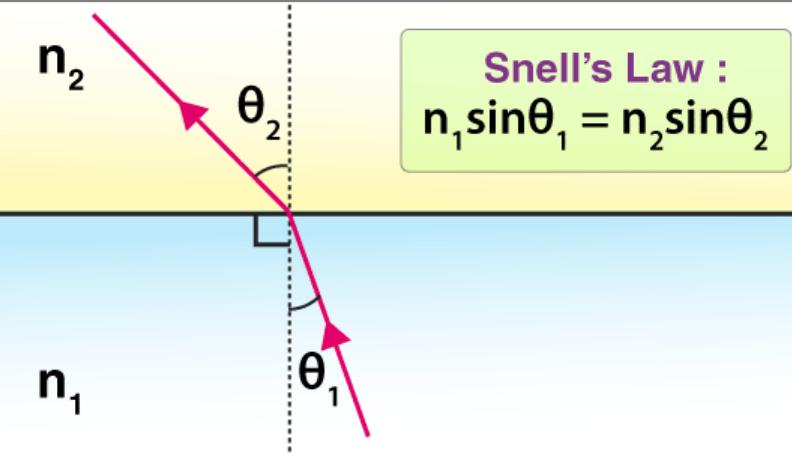
# Duration when scientific thought on Light started

## Photonics Timeline (1960-Present)

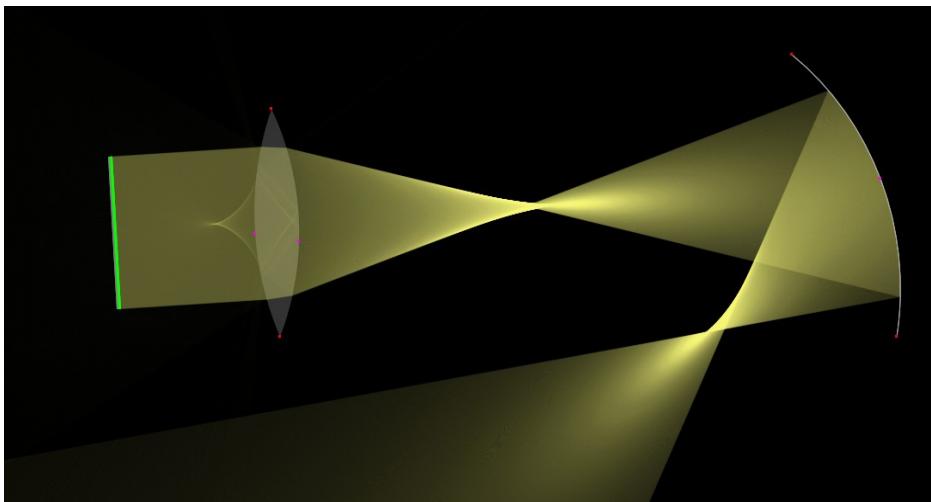
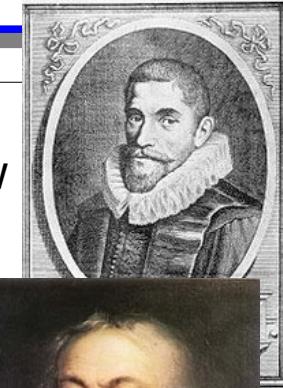


Link : <https://photonterrace.net/en/photon/history/>

**What is light ?** Ans: Just consider as Ray/something flowing. Just focus on its direction (Ray Optics/Geometrical Optics)



- Laws of Reflection
- Laws of Refraction : Snell's law (Willebrord Snellius 1580–1626)
- Fermat's principle (1607-65)



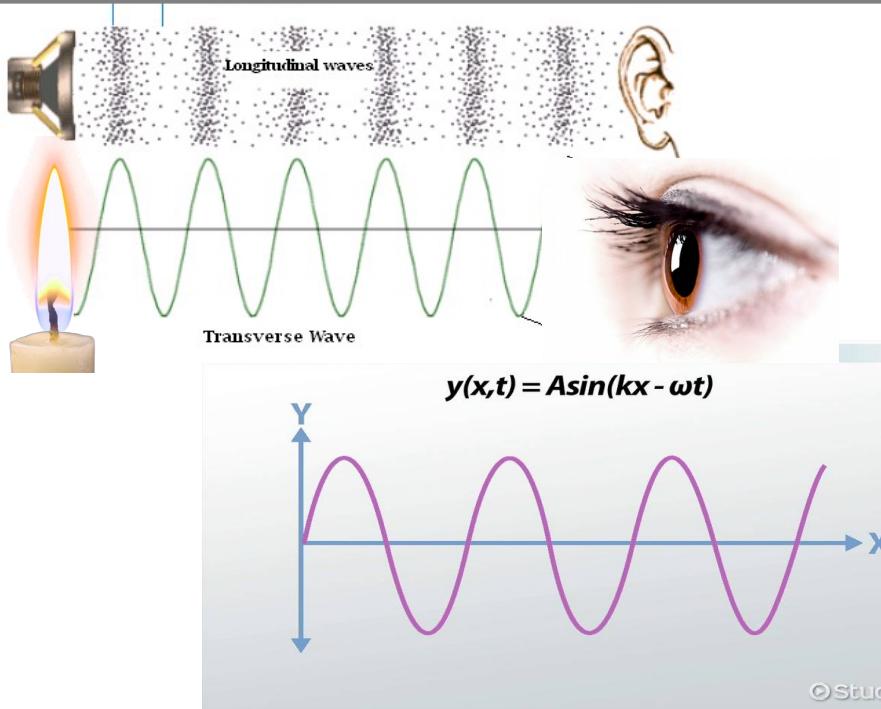
The lens formula

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$f$  = focal length (m)  
 $u$  = object distance (m)  
 $v$  = image distance (m)



**What is light ? Ans: It is a Wave. Just like Sound, **some quantity** (??) is fluctuating with time and space (Physical Optics)**



Christiaan Huygens  
(1629 - 1695)

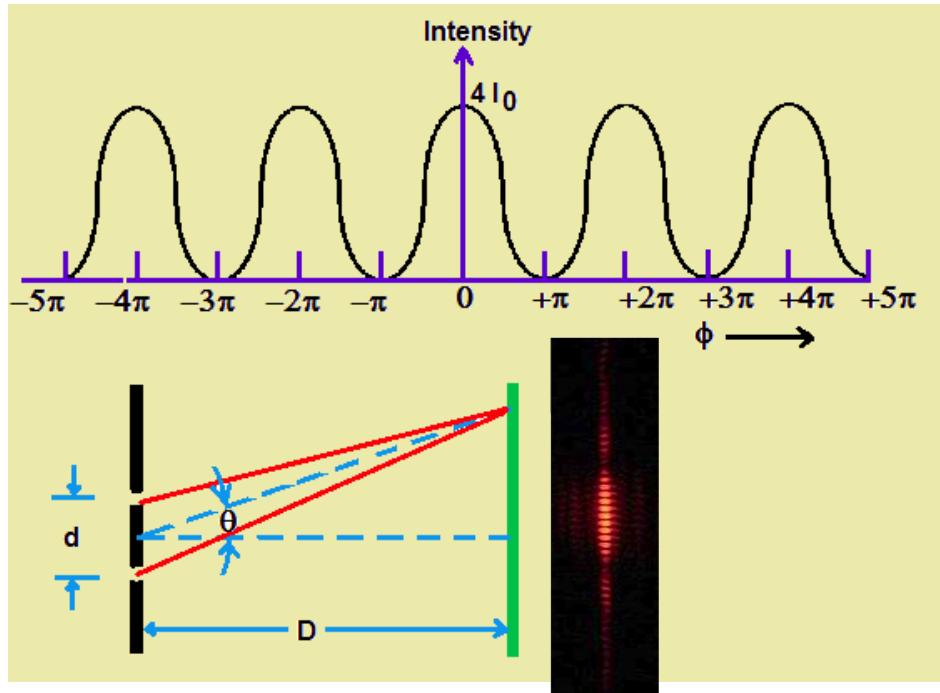


Thomas Young  
(1773 - 1829)

Amplitude A is constant with space x and time t

Wave Optics  
Diffraction & Interference

**What is light ? Ans:** It is a Wave. Just like Sound, **some quantity (??)** is fluctuating with time and space (Physical Optics)



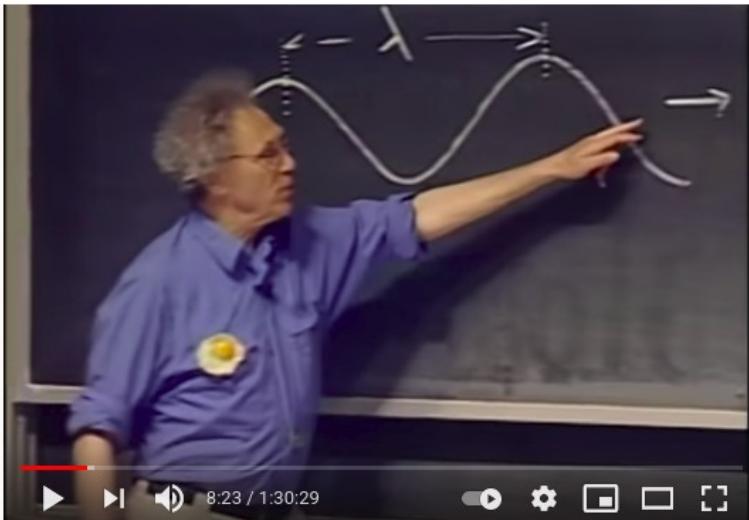
Christiaan Huygens  
(1629 - 1695)



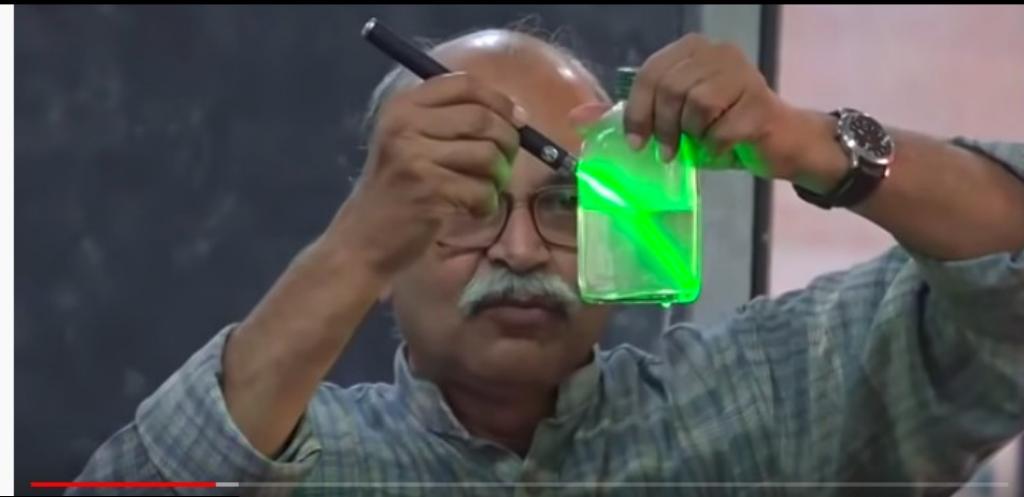
Thomas Young  
(1773 - 1829)

If we make Amplitude  $A$  as a function of space  $x$

Wave Optics  
Diffraction & Interference



- <https://www.youtube.com/watch?v=irpjwXpa4xU>
- <https://www.youtube.com/watch?v=WEMdxBKRXhE>
- [https://www.youtube.com/watch?v=LnJIZ\\_lwSBM](https://www.youtube.com/watch?v=LnJIZ_lwSBM)



The Mystery of Light - Walter Lewin - July 19, 2005

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Lectures by Walter Lewin. They will make you ♥ Physics.  
1.08M subscribers

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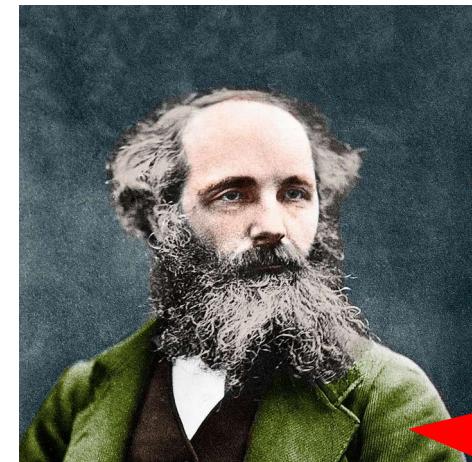
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**What is light ? Which quantity is fluctuating?** Ans: Interestingly the answer was hidden in the branch of Electricity & Magnetism.



James Clerk Maxwell  
(1831-1879)

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

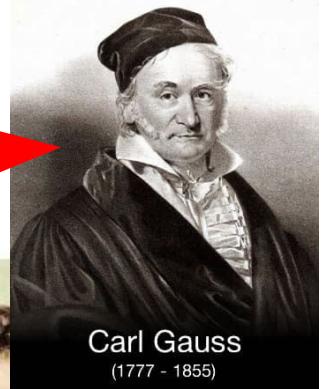
$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



Michael Faraday  
(1791-1867)

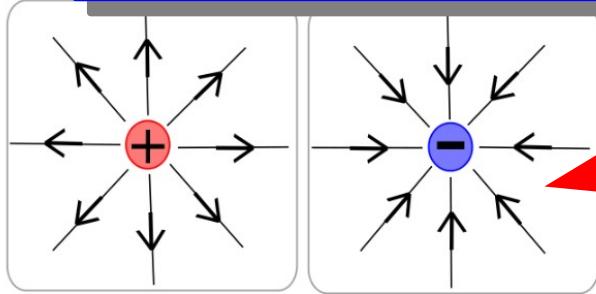


Carl Gauss  
(1777 - 1855)



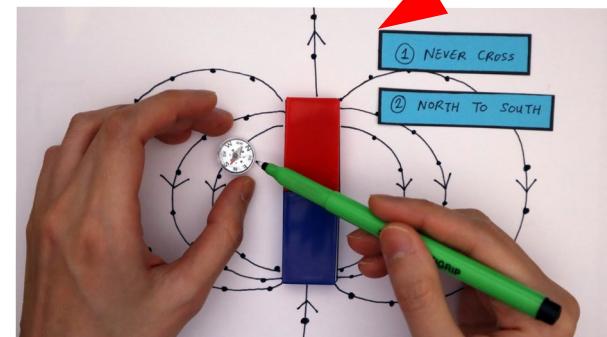
Andre Marie Ampere  
(1755-1836)

**What is light ? Which quantity is fluctuating? Ans:** Interestingly the answer was hidden in the branch of Electricity & Magnetism.



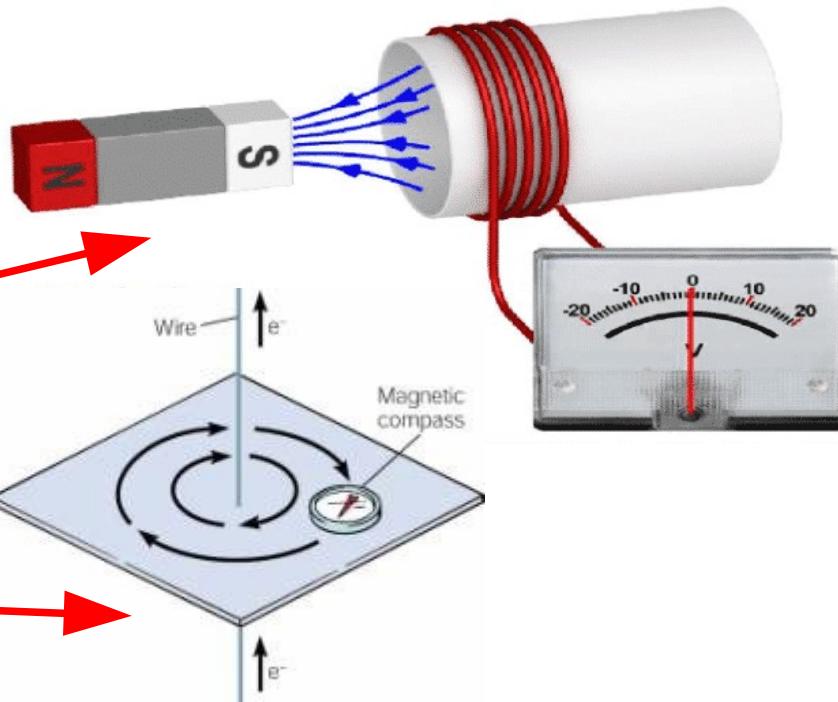
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$



$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$



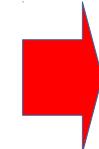
# What is light ? Ans: Light is fluctuating Electric & Magnetic field – An Electro-magnetic Wave

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \left( \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \right)$$

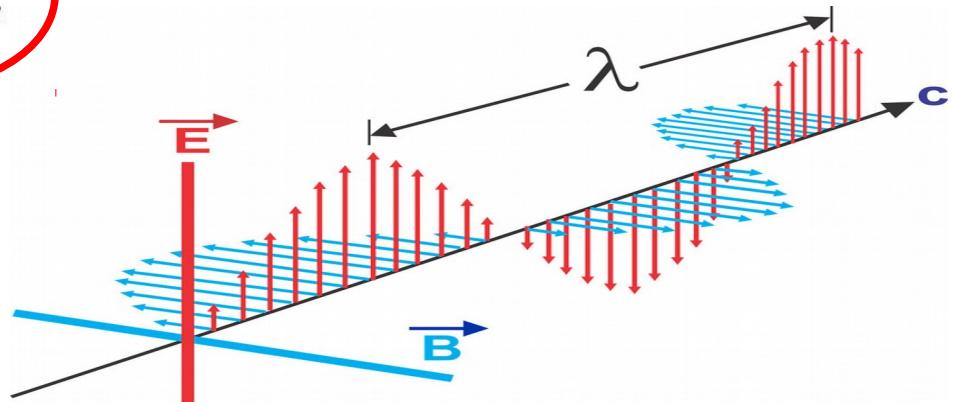
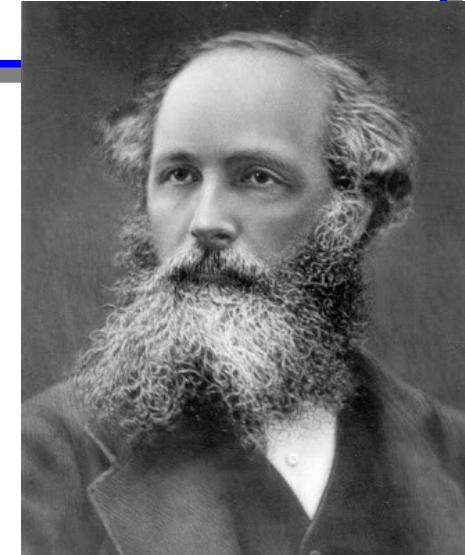


$$\left( v_{ph}^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \mathbf{E} = 0$$

$$\left( v_{ph}^2 \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \mathbf{B} = 0$$

where

$$v_{ph} = \frac{1}{\sqrt{\mu \epsilon}}$$

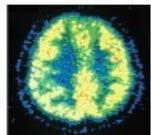


# VISIBLE light is a small part of Electro-magnetic Wave

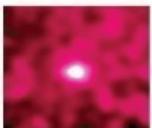
Increasing energy  $E$   
Increasing frequency  $\nu$   
Increasing wavelength  $\lambda$



PET scan



Cosmic ray



X-ray

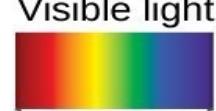
Dental curing



Night vision



Visible light



Gamma

X-ray

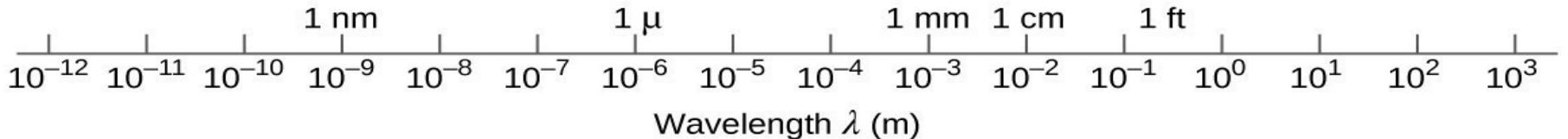
Ultraviolet

Infrared

Terahertz

Microwave

Broadcast and wireless radio



Microwave oven



Ultrasound



Wireless data



Radar

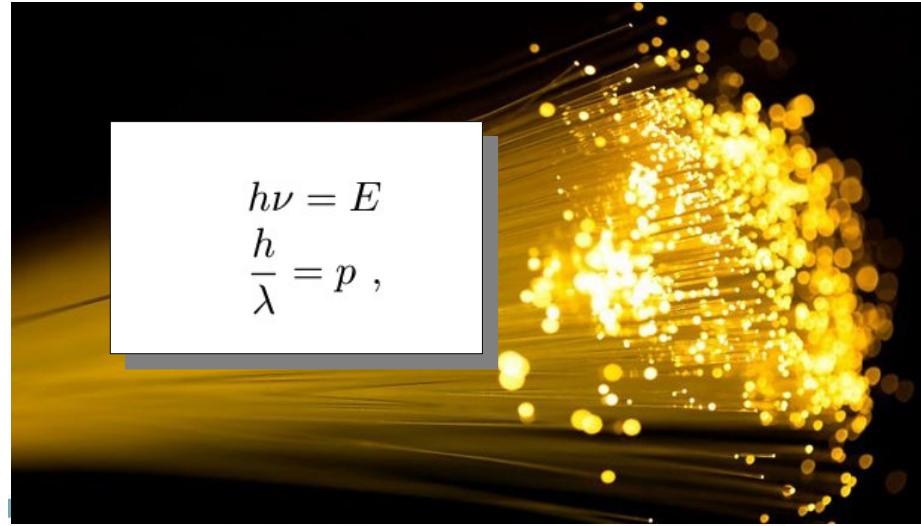
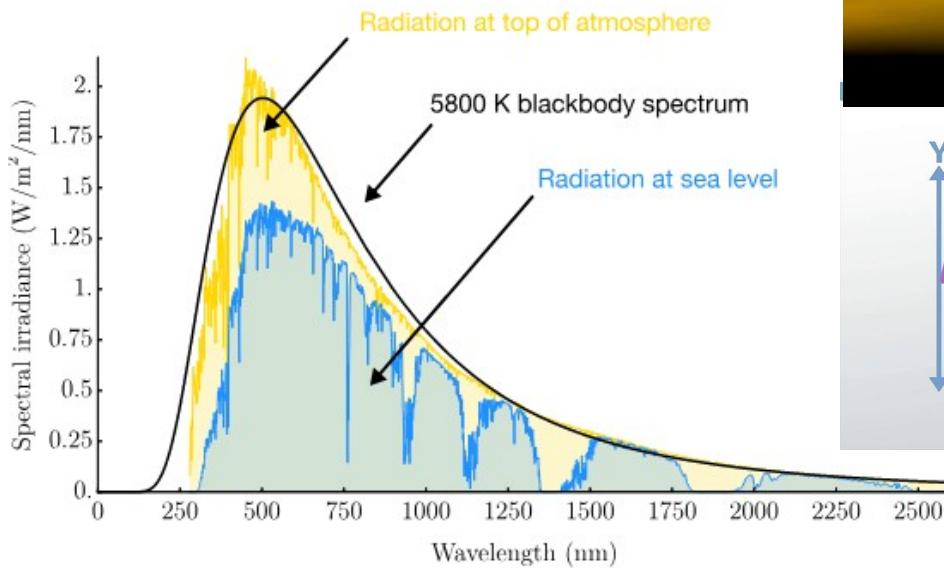
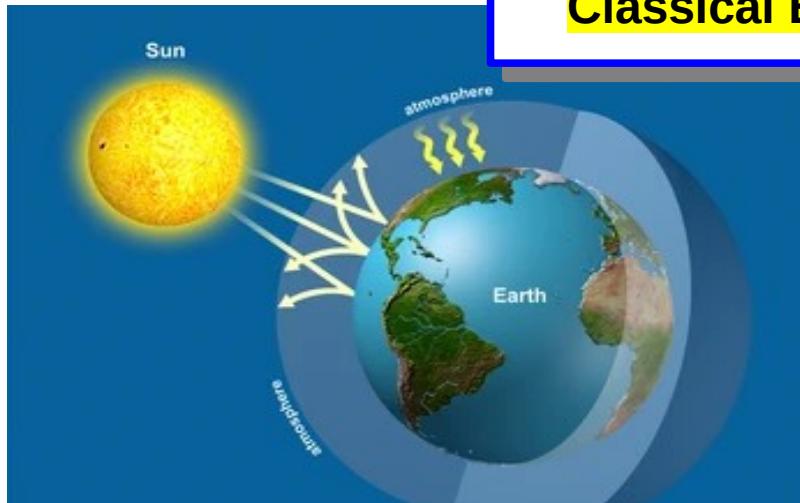


AM radio

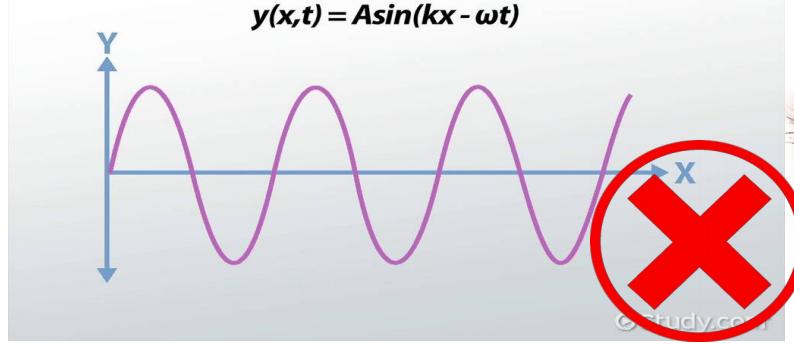


Cell phone

# Classical EM Wave failed to explain Black Body Radiation



$$h\nu = E$$
$$\frac{h}{\lambda} = p ,$$



© rudy.com

What is light ? Ans: Billions of photon particles with speed c

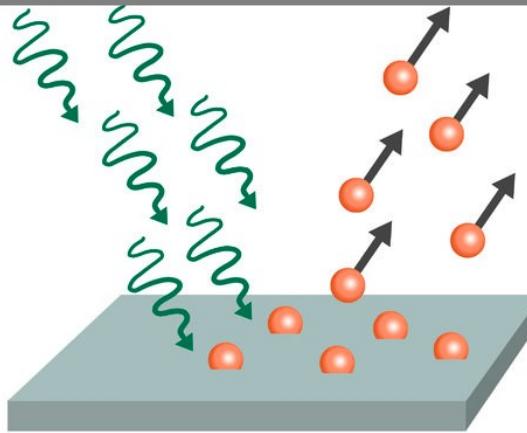
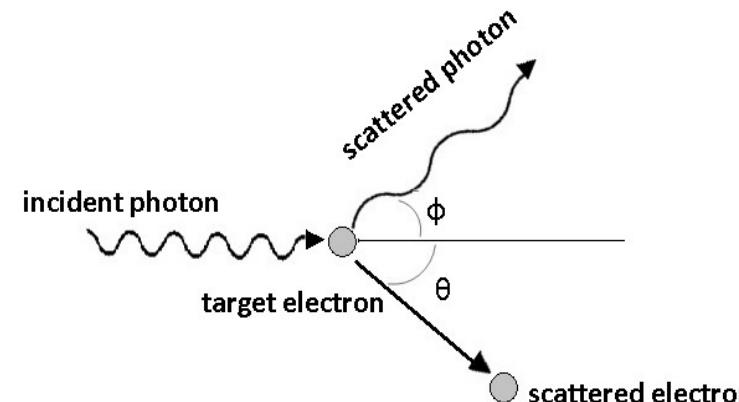


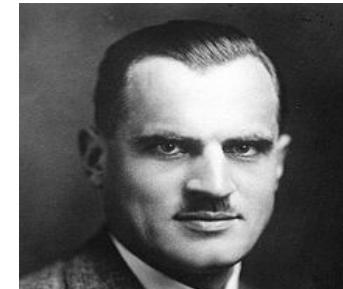
Photo electric effect



Albert Einstein  
(1879-1955)

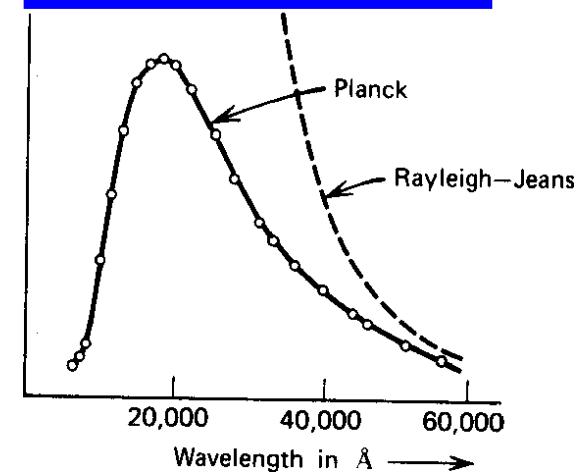


Compton Scattering



Arthur Compton  
(1892-1962)

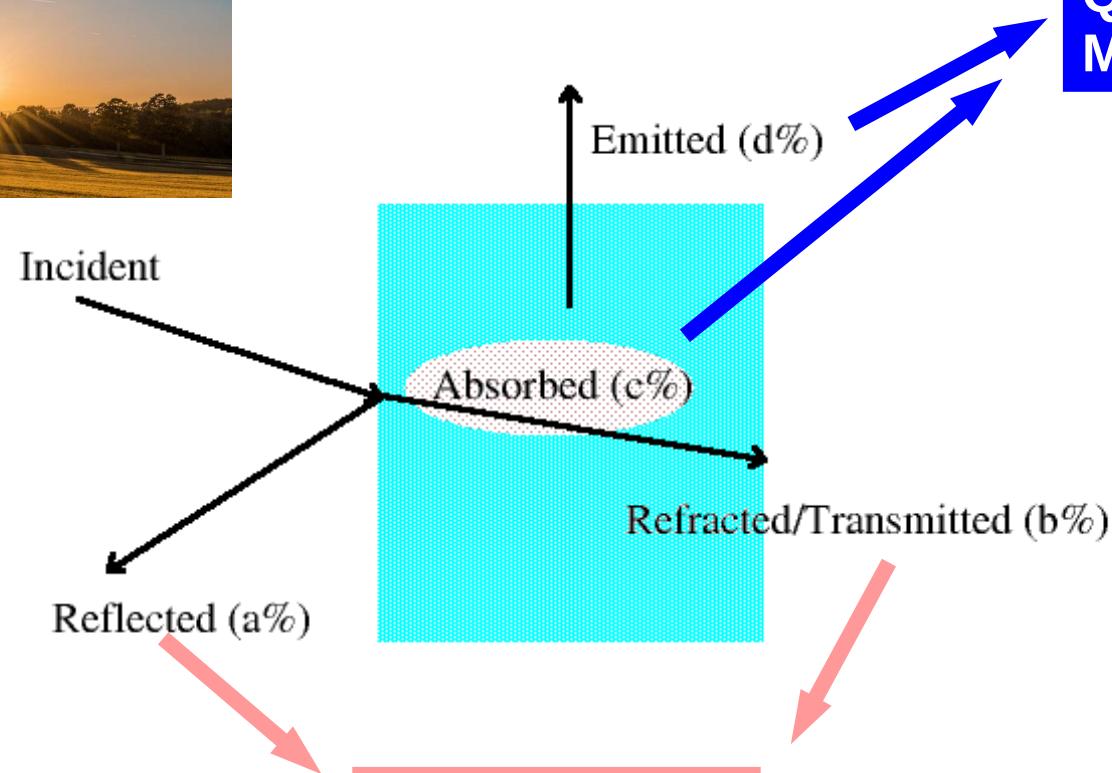
Black Body Radiation



Max Planck  
(1858-1947)

# Black Body Radiation (100% absorbtion or Emission)

## Normal Radiation



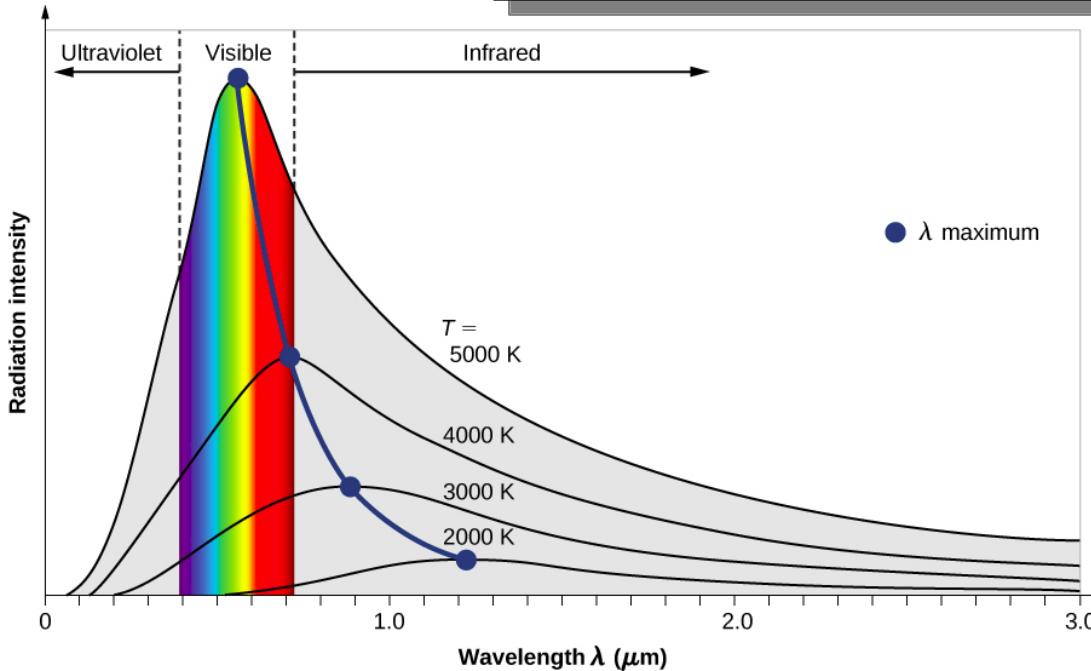
Explanation from  
**Quantum Mechanics**

Explanation from  
**Classical picture**

# Black Body Radiation

## Wien's parametrization

$$\epsilon(\lambda) = \frac{A}{\lambda^5} e^{-B/\lambda T}$$



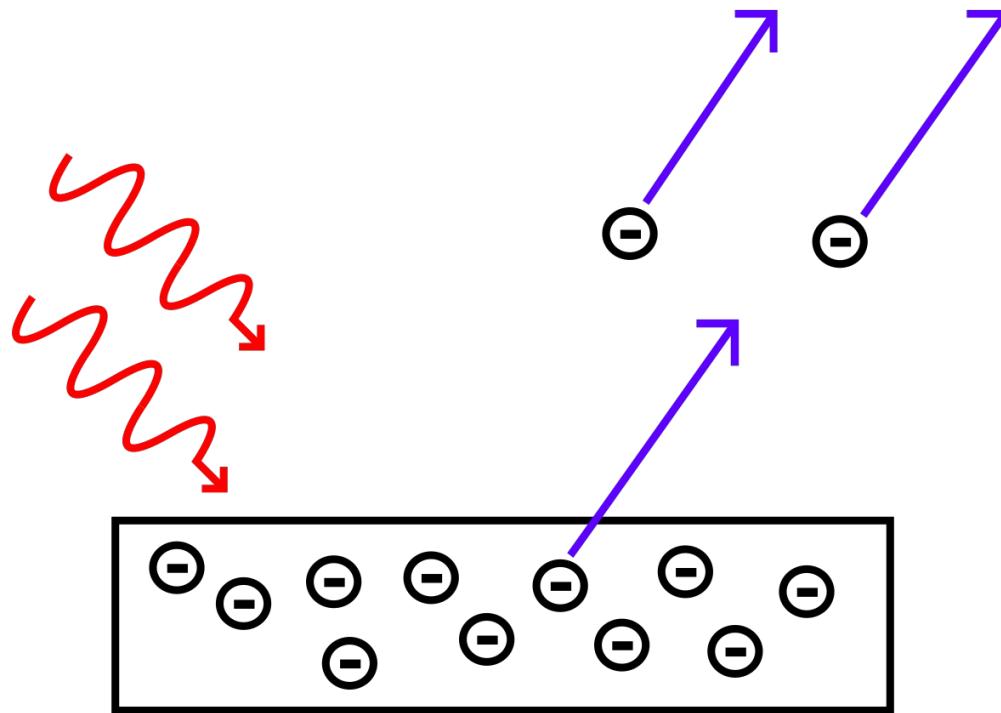
## Planck's Radiation Law

$$\begin{aligned}\epsilon(\nu)d\nu &= \text{Density of stationary light} \times \text{Average energy} \\ &= 2 \frac{4\pi\nu^2 d\nu}{c^3} \times 2 \frac{1}{2} KT \\ \Rightarrow \epsilon(\lambda) &= \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1}.\end{aligned}\tag{6}$$

## Rayleigh-Jeans Law

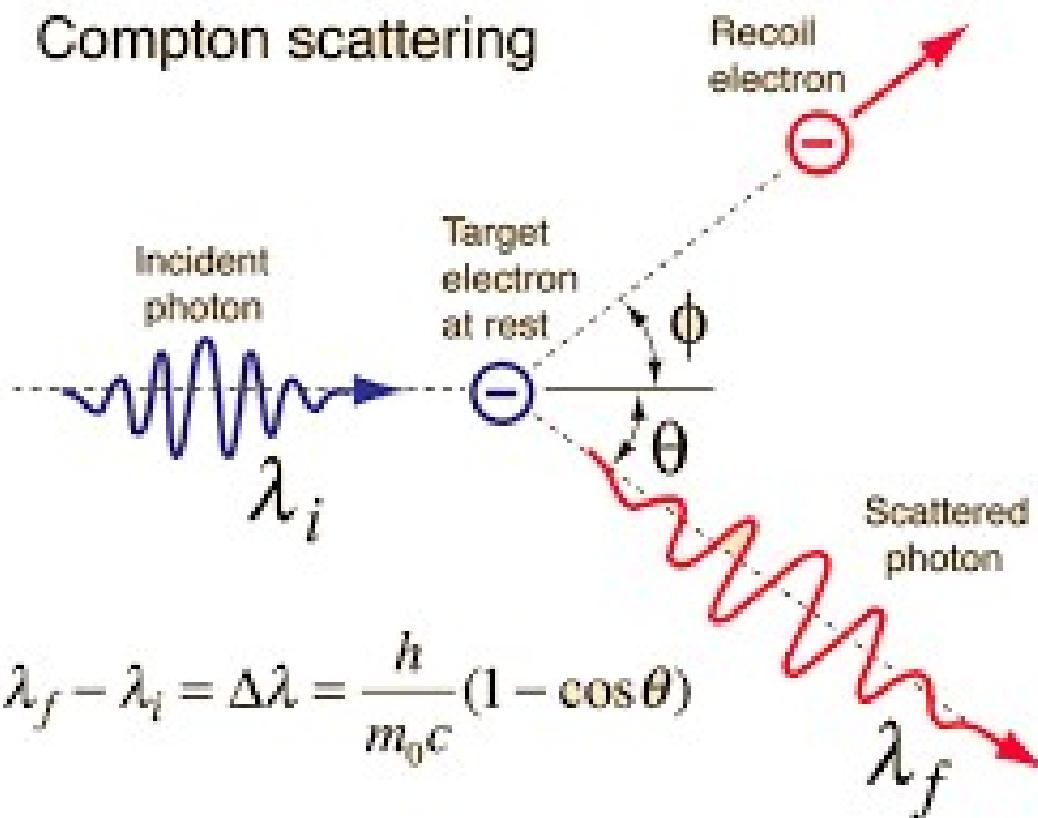
$$\begin{aligned}\epsilon(\nu)d\nu &= \text{Density of stationary light} \times \text{Average energy} \\ &= 2 \frac{4\pi\nu^2 d\nu}{c^3} \times 2 \frac{1}{2} KT \\ \Rightarrow \epsilon(\lambda) &= \frac{8\pi KT}{\lambda^4},\end{aligned}\tag{4}$$

# Photo-electric Effect



$$\begin{aligned} \text{Light Energy} - \text{WorkFunction} &= \text{Electron Energy} \\ h\nu - \Phi &= \frac{1}{2}mv^2 \\ h\nu - h\nu_0 &= eV \end{aligned} \tag{7}$$

## Compton Scattering:



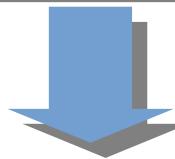
# Summary

- **Question:** What is Light?
- **Ans:** Just consider as Ray/something flowing. Just focus on its direction (**Ray Optics/Geometrical Optics**)
- **Ans:** It is a Wave. Just like Sound, **some quantity (??)** is fluctuating with time ans space (**Physical Optics**)
- **Ans:** Light is fluctuating Electric & Magnetic field – An Electro-magnetic Wave (**Classical Electrodynamics**)
- **Ans:** Light is basically billions of photon particles with cpeed c (**Quantum Mechanics, Special Theory of Relativity**)
- **Last Question:** What is Light? Rays?, Wave? Or Particles?
- **Ans:** .....No Answer....:)

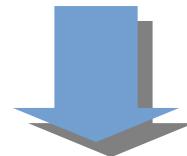


Niels Bohr  
(1885-1962)

**Previous Courses :** De Broglie hypothesis, Wave-particle duality, Group and phase velocities, Heisenberg's uncertainty etc.



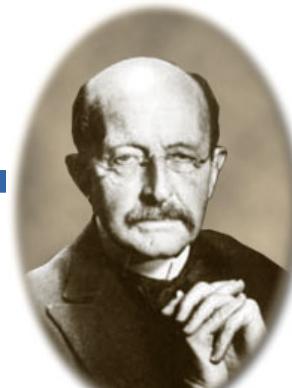
## **Particle in a Box (Quantum Mechanics)**



**Latter Courses :** Few more application of Schrodenger's Equation,..... Hydrogen Atom Problem (Origin of Atomic spectra) etc.

## Experimental Facts

- Black Body Radiation
- Photo electric Effect
- Compton Scattering



Max Planck  
(1858-1947)

Photon

Light/ E.M. Wave

Particle quantities

Energy

$$E = \hbar\omega$$

Wave quantities

Angular Frequency

Momentum

$$p = \hbar k$$

Propagation constant

Particle

Matter Wave



Louis de Broglie  
(1892-1987)

# de-Broglie wave equation

$$\lambda = \frac{h}{p}$$

Particle

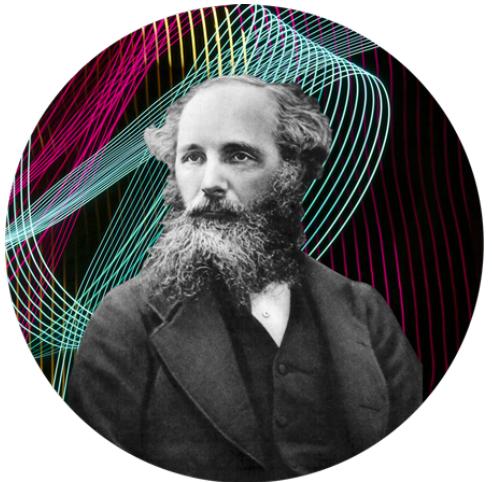


Louis de Broglie  
(1892-1987)

Matter Wave

- Newton's Equation
- Euler-Lagrangian Equation
- Hamiltonian Equation





Wave equation Light wave:

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 \psi}{\partial t^2}$$

Solution:

$$\begin{aligned}\psi(x, t) &= Ae^{i(kx - \omega t)} \\ &= Ae^{i2\pi(\frac{x}{\lambda} - \nu t)},\end{aligned}$$

where  $c = \frac{\omega}{k} = \nu\lambda$ .

## Electro Magnetic Wave



**Erwin Schrödinger**  
(1887-1961)

Wave equation of particle/Matter wave:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi = i\hbar \frac{\partial \psi}{\partial t}$$

Solution:

$$\psi(x, t) = Ae^{i(px - Et)/\hbar}$$



**Matter Wave**

Wave function

$$\psi \approx e^{j(\omega t - k_x x)}$$

$$\frac{\partial}{\partial t} \psi = j\omega \psi$$



$$E\psi = \hbar\omega\psi = -j\hbar \frac{\partial}{\partial t} \psi$$

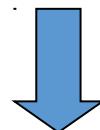
$$\frac{\partial}{\partial x} \psi = -jk_x \psi$$



$$p_x \psi = \hbar k_x \psi = j\hbar \frac{\partial}{\partial x} \psi$$

Particle

$$E = \frac{p^2}{2m} \quad (\text{free-particle})$$



$$-j\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}$$

Matter Wave associated  
with free particle

## Schrodinger equation with potential term

$$-j\hbar \frac{\partial}{\partial t} \psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x) \psi$$

- Time-harmonic solutions to Schrodinger equation are of the form:  $\Psi(x, t) = e^{-iEt/\hbar} \psi(x)$
- $\Psi(x, t)$  is a measurable quantity and represents the probability distribution of finding the particle.

## Time-Dependent Schrodinger Wave Equation

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x) \Psi(x, t)$$

↑  
**Total E  
term**

↑  
**K.E. term**

↑  
**P.E. term**

$$\Psi(x, t) = e^{-iEt/\hbar} \psi(x)$$

## Time-Independent Schrodinger Wave Equation

$$E\psi(x) = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi(x) + V(x) \psi(x)$$

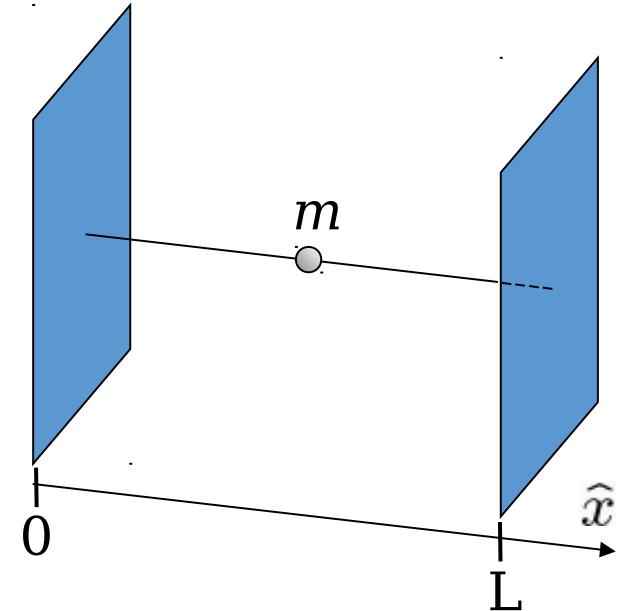
A point mass  $m$  constrained to move on an infinitely-thin, frictionless wire which is strung tightly between two impenetrable walls a distance  $L$  apart

for  $(x \leq 0, x \geq L)$

$$V(x) = \infty$$

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi + (\infty)\psi$$

$\longrightarrow \psi = 0$



for  $(0 < x < L)$

$$V(x) = 0$$

$$E\psi = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \psi$$

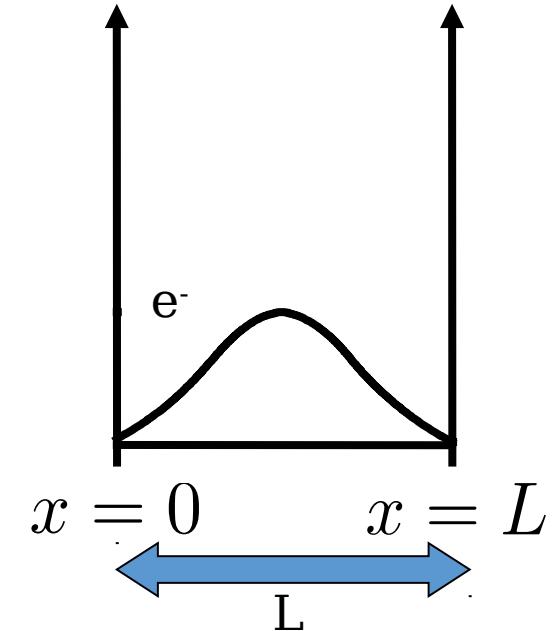
$\longrightarrow \psi(0) = \psi(L) = 0$

As wave function must be  
CONTINUOUS

for  $(0 < x < L) : V(x) = 0$

$$E_n \psi_n = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_n}{\partial x^2}$$

WE WILL HAVE  
MULTIPLE  
SOLUTIONS,  
SO WE  
INTRODUCE  
LABEL  $n$



REWRITE AS:

$$\frac{\partial^2 \psi_n}{\partial x^2} + k_n^2 \psi_n = 0 \quad \text{WHERE} \quad k_n^2 = \frac{2mE_n}{\hbar^2}$$

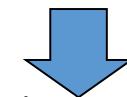
GENERAL SOLUTION:

$$\psi_n(x) = A \sin k_n x + B \cos k_n x \quad \text{OR} \quad \psi_n = C_1 e^{j k_n x} + C_2 e^{-j k_n x}$$

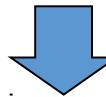
## USE BOUNDARY CONDITIONS TO DETERMINE COEFFICIENTS A and B

$$\psi(0) = 0 \quad \xrightarrow{\hspace{1cm}} \quad B = 0$$

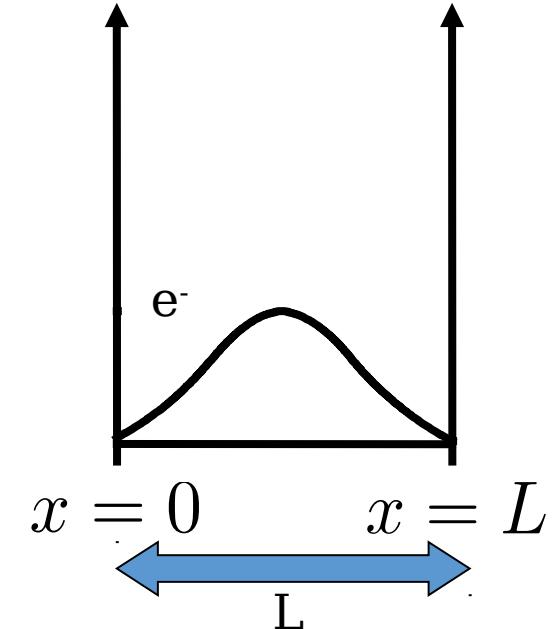
$$\psi(L) = 0 \quad \xrightarrow{\hspace{1cm}} \quad k_n L = n\pi$$



$$k_n^2 = \frac{2mE_n}{\hbar^2}$$



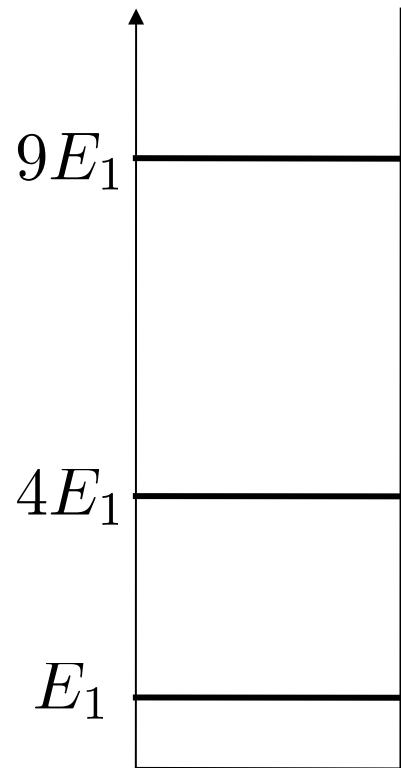
$$E_n = n^2 E_1 \quad \xleftarrow{\hspace{1cm}} \quad E_1 = \frac{\hbar^2 k_1^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2}$$



## NORMALIZE THE INTEGRAL OF PROBABILITY TO 1

$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

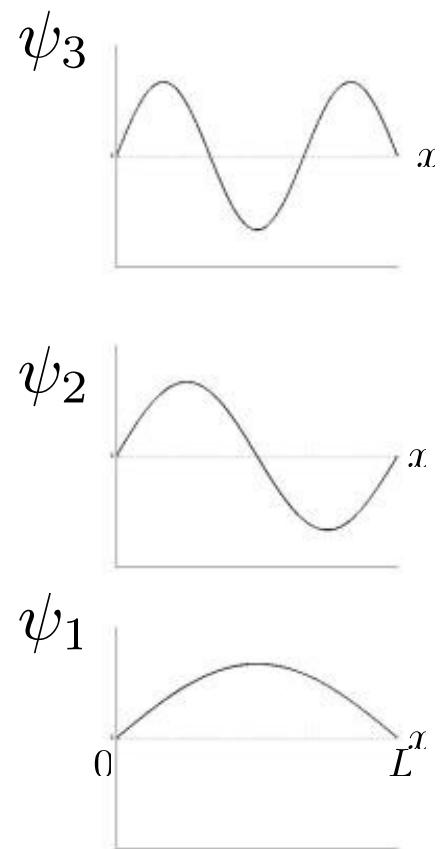
## EIGEN ENERGIES for 1-D BOX



$$E_n = n^2 E_1$$



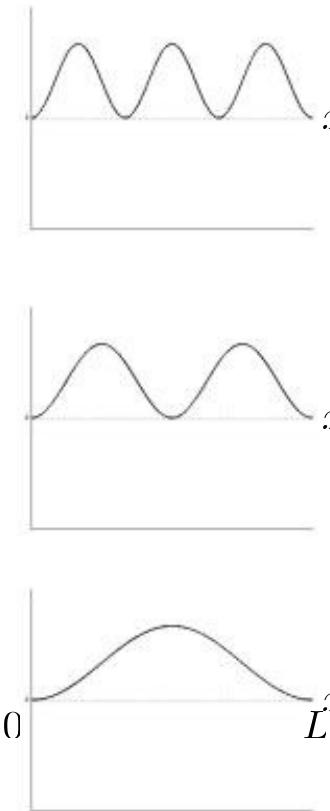
## EIGEN STATES For 1-D BOX



$$\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

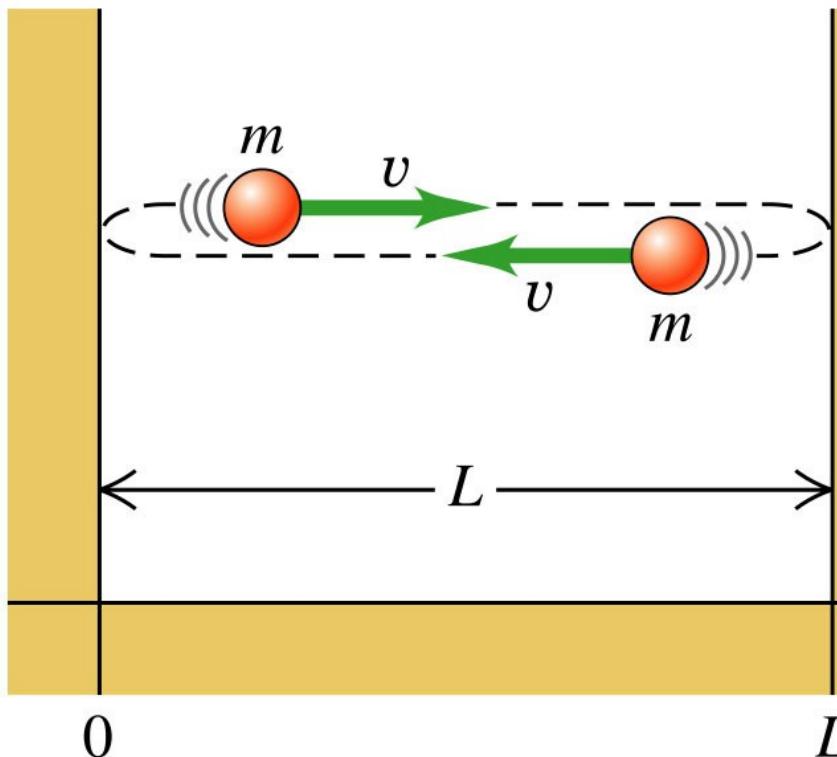
$$P(x) = |\psi(x)|^2 dx = \frac{2}{L} \sin^2 \frac{n\pi x}{L} dx$$

## PROBABILITY DENSITIES



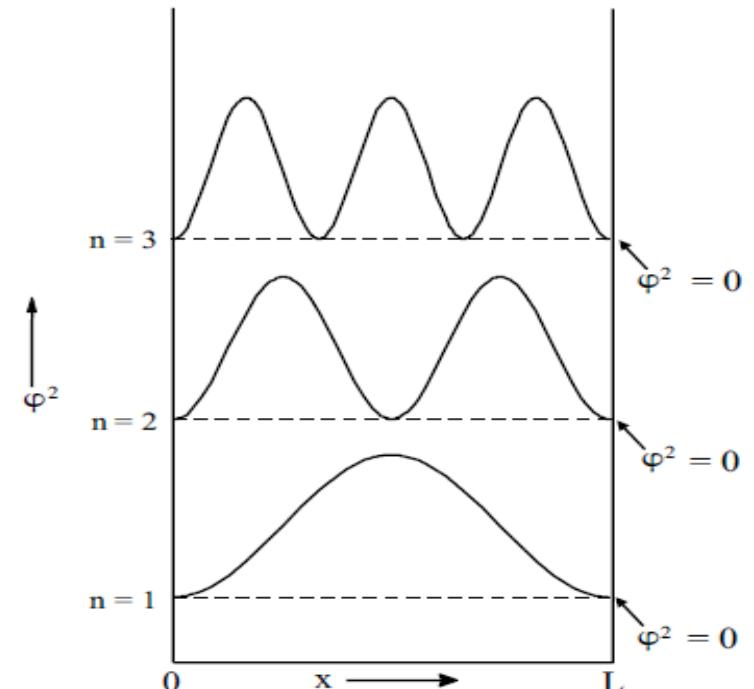
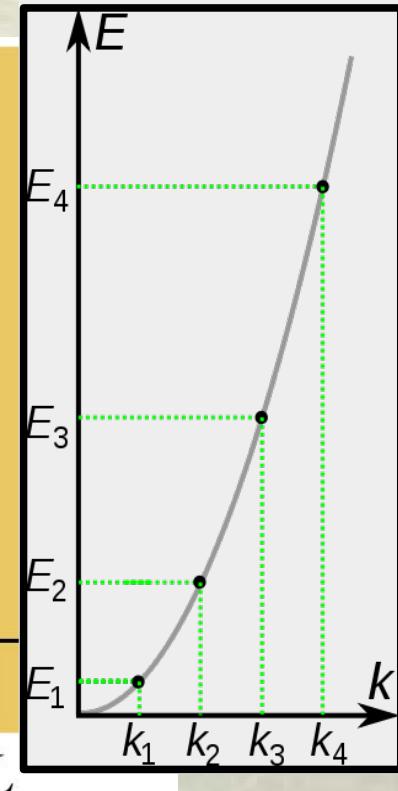
## Classical View :

- Definite Existence in space



## Quantum View :

- Probabilistic Existence in space



$$E = \frac{p^2}{2m}$$

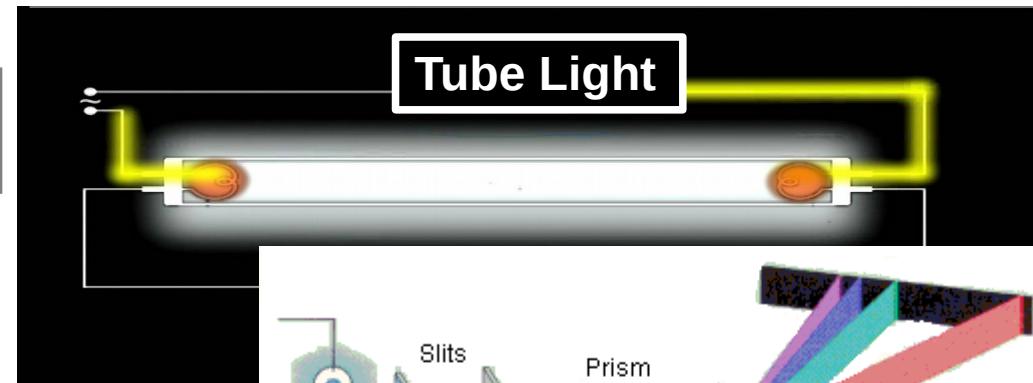
$$E_n = n^2 E_1$$

$$E_1 = \frac{\hbar^2 k_1^2}{2m} = \frac{\hbar^2 \pi^2}{2mL^2}$$

- Energy can vary continuously

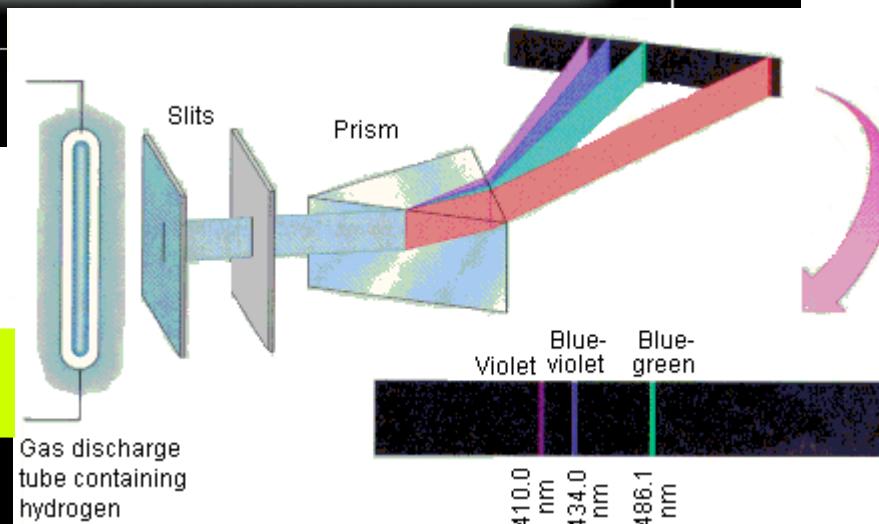
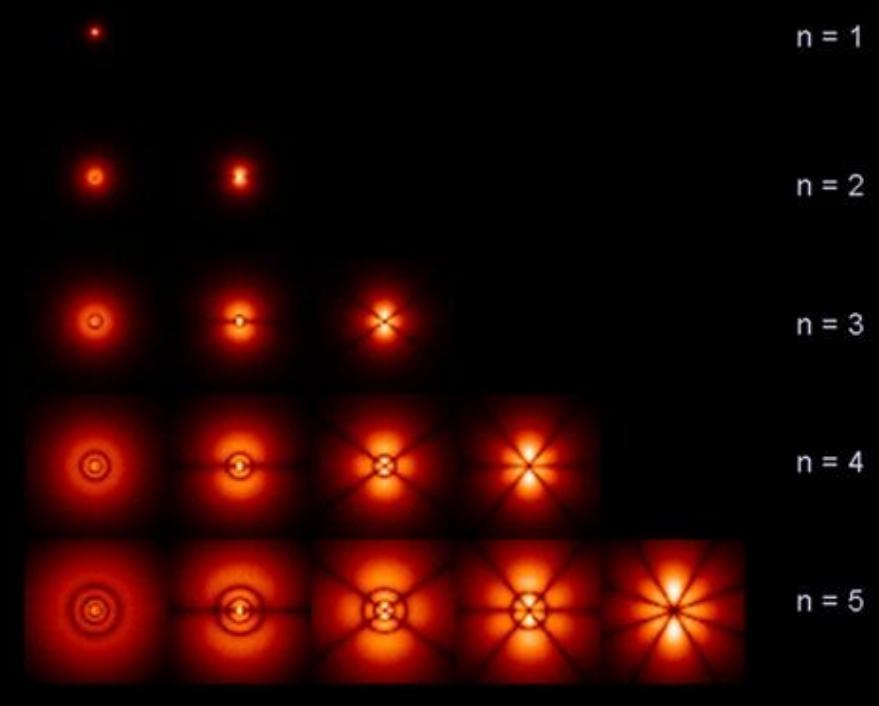
- Available Energies are quantized

Aim is to understand the origin of Atomic Spectra

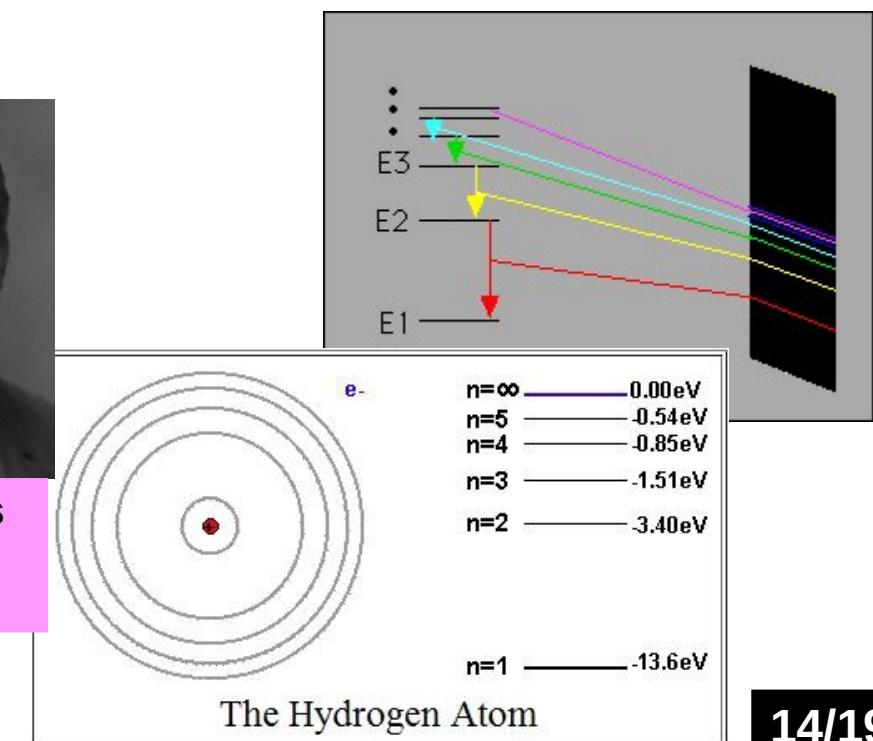


Picture from Schrodinger's Equation

Hydrogen atom orbitals  
l=0      l=1      l=2      l=3      l=4



Bohr's atomic model



## Tutorial Part

Question: An electron is in 1D box of 1nm length. What is the probability of locating the electron between x=0 and x=0.2nm in its lowest energy state?

Solution:

$$P(x_1, x_2) = \int_{x_1}^{x_2} |\psi(x)|^2 dx$$

$$\int_{x=0 \text{ nm}}^{0.2 \text{ nm}} |\psi_1(x)|^2 dx = \underbrace{\frac{2}{1.0 \text{ nm}} \int_{0 \text{ nm}}^{0.2 \text{ nm}} \sin^2\left(\frac{\pi x}{1.0 \text{ nm}}\right) dx}_{n=1, a=1 \text{ nm}}$$

$$\int \sin^2\left(\frac{\pi}{a}x\right) dx = \frac{x}{2} - \frac{\sin(2\pi x/a)}{4\pi/a}$$

$$= \frac{2}{1.0 \text{ nm}} \left( \frac{0.2 \text{ nm}}{2} - \frac{\sin(2\pi \times (0.2 \text{ nm})/(1.0 \text{ nm}))}{4\pi/(1.0 \text{ nm})} \right) \approx 0.05$$

## Example: What are the most likely locations of a particle in a box of length L in the state n=3

$$\psi_3 = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$$

$$P(x) \propto \psi_3 \psi_3^* \propto \sin^2\left(\frac{3\pi x}{L}\right)$$

The maxima and minima in  $P(x)$  corresponds to  $\frac{dP(x)}{dx} = 0$

$$\frac{dP(x)}{dx} \propto \sin\left(\frac{3\pi x}{L}\right) \cos\left(\frac{3\pi x}{L}\right) \propto \sin\left(\frac{6\pi x}{L}\right)$$

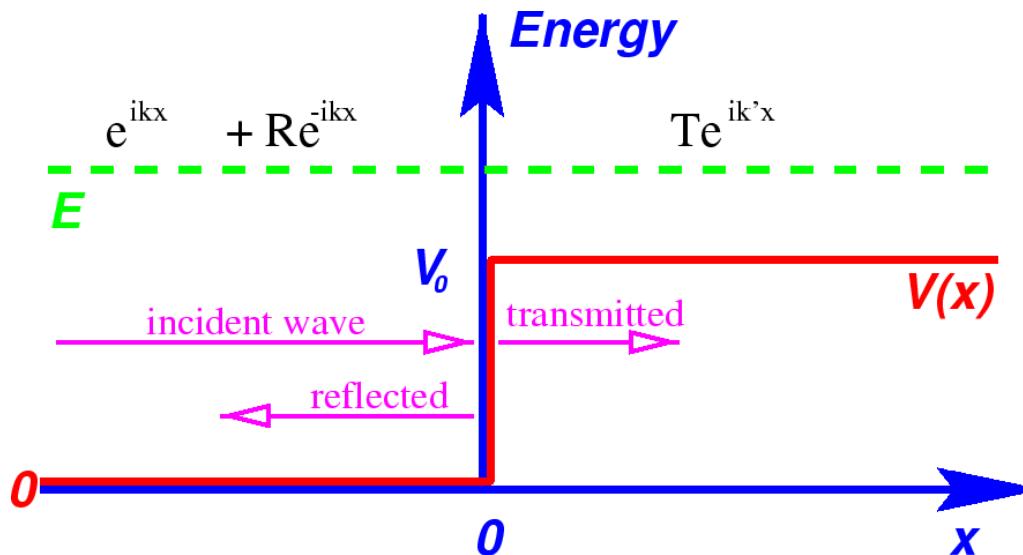
$$\sin \theta = 0 \text{ when } \theta = \left(\frac{6\pi x}{L}\right) = n' \pi, \quad n = 0, 1, 2, \dots$$

$$\text{which corresponds to } x = \frac{n' L}{6}, \quad n' < 6.$$

$n' = 0, 2, 4$ , and  $6$  corresponds to minima in

$n' = 1, 3$ , and  $5$  to maxima

## Step potential:



Potential :

$$\begin{aligned} V(x < 0) &= 0 , \\ V(x \geq 0) &= V_0 , \end{aligned} \quad (21)$$

Schrodenger's Equations :

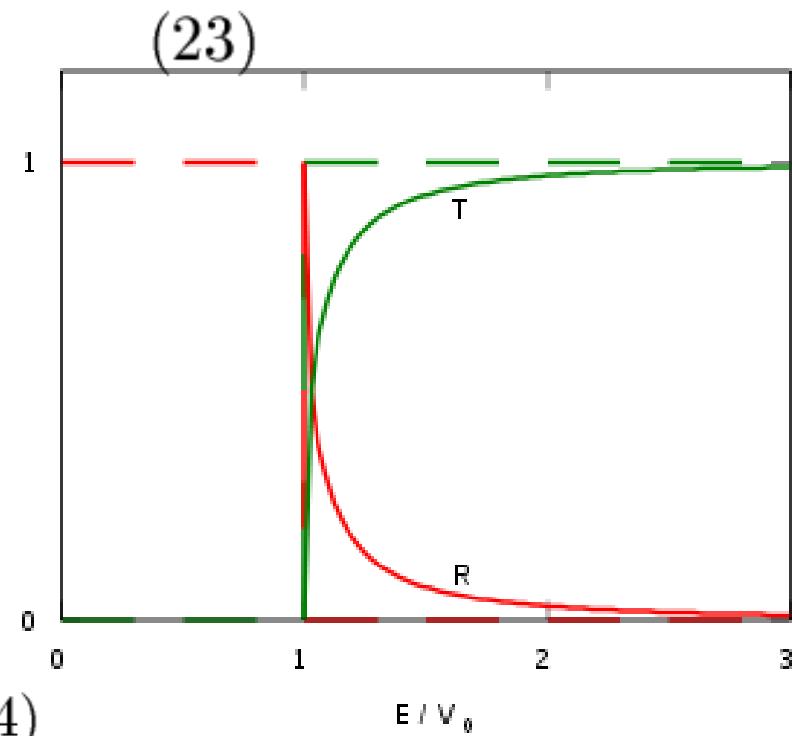
$$\begin{aligned} \frac{\partial^2 \psi_1}{\partial x^2} + k_1^2 \psi_1 &= 0 , \text{ with } k_1^2 = \frac{2mE}{\hbar^2} \\ \frac{\partial^2 \psi_1}{\partial x^2} + k_2^2 \psi_1 &= 0 , \text{ with } k_2^2 = \frac{2m(E - V_0)}{\hbar^2} \quad E < V_0 \end{aligned} \quad (22)$$

## Guess Solutions:

$$\begin{array}{lcl} \psi_1 & = & Ae^{ik_1 x} + Be^{-ik_1 x} \\ \text{E} < V_0 & \psi_2 & = Ce^{ik_2 x} + De^{-ik_2 x} \end{array}$$

## Boundary Conditions:

$$\begin{aligned} \psi_1(x=0) = \psi_2(x=0) &\Rightarrow A + B = C \\ \left(\frac{\partial\psi_1}{\partial x}\right)_{x=0} = \left(\frac{\partial\psi_2}{\partial x}\right)_{x=0} &\Rightarrow A - B = \frac{k_2}{k_1}C \end{aligned} \quad (24)$$



So the transmitted and reflection coefficients are

$$\begin{aligned} T &= \frac{\hbar k_2}{\hbar k_1} \left| \frac{C}{A} \right|^2 = \frac{4k_1 k_2}{(k_1 + k_2)^2} \\ R &= \frac{\hbar k_1}{\hbar k_1} \left| \frac{B}{A} \right|^2 = \frac{(k_1 - k_2)^2}{(k_1 + k_2)^2}. \end{aligned} \quad (25)$$

# Tunneling Effect:



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## Trailanga

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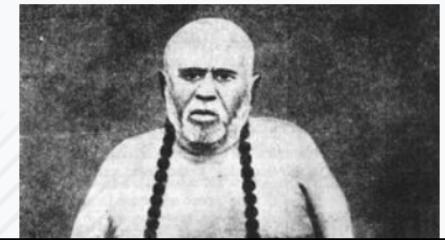


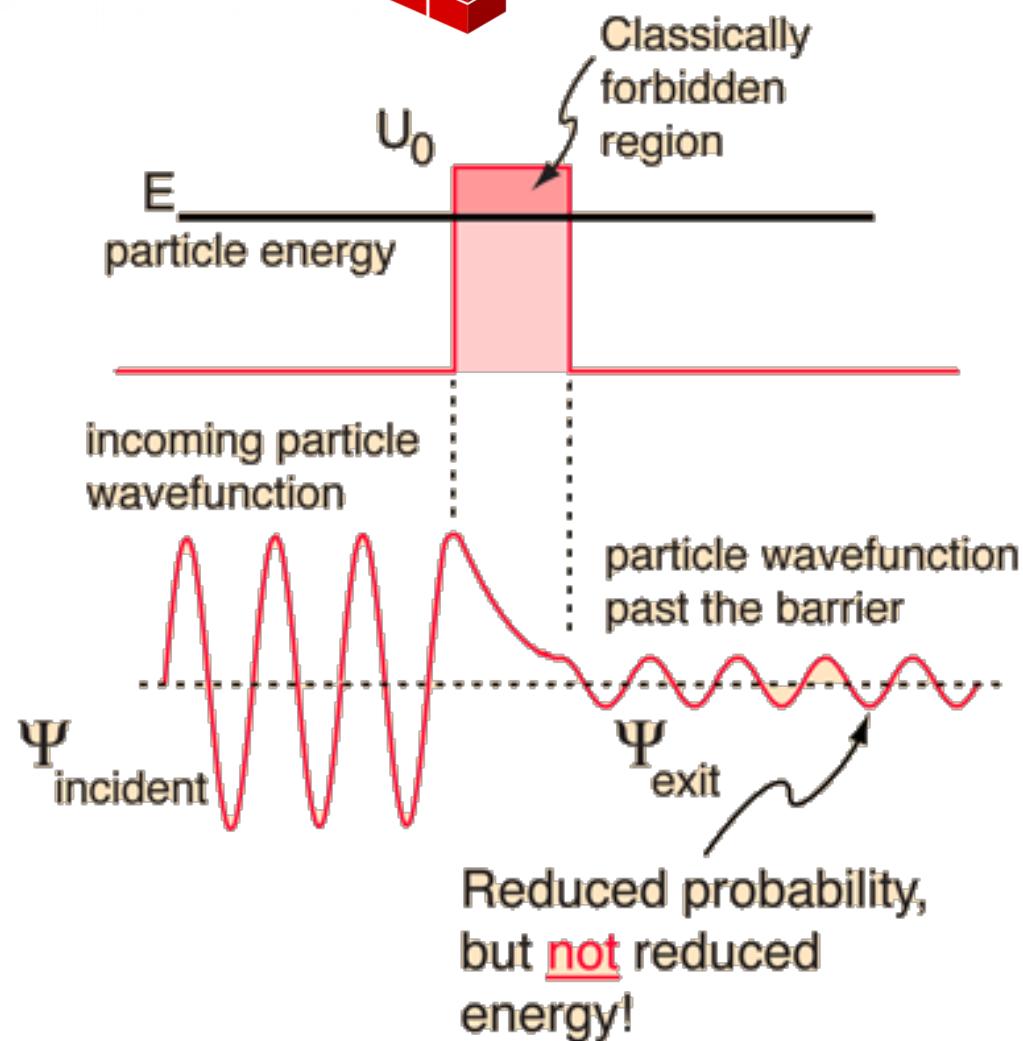
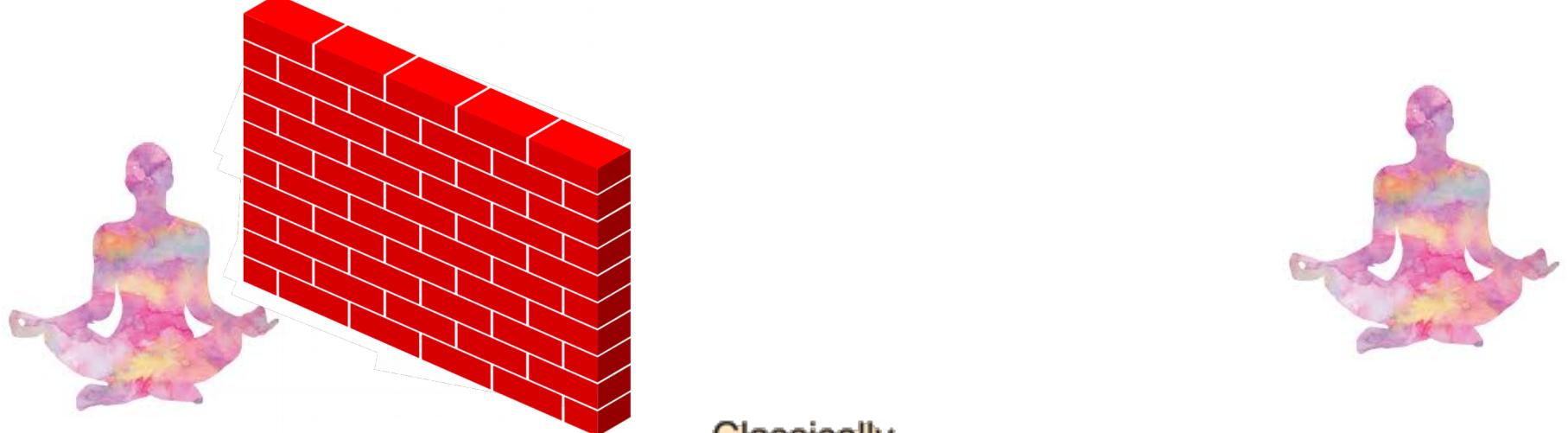
This article uncritically uses texts from within a religion or faith system **without referring to secondary sources that critically analyze them**. Please help [improve this article](#) by adding references to reliable secondary sources, with multiple points of view. (November 2013) ([Learn how and when to remove this template message](#))

**Trailanga Swami** (also **Tailang Swami, Telang Swami**) (reportedly<sup>[nb 1]</sup> 1607<sup>[2]</sup>–1887<sup>[2][3]</sup>) was a Hindu yogi and mystic famed for his spiritual powers who lived in Varanasi, India.<sup>[2]</sup> He is a legendary figure in Bengal, with stories told of his yogic powers and longevity. According to some accounts, Trailanga Swami lived to be 280 years old,<sup>[2][4]</sup> residing at Varanasi between 1737 and 1887.<sup>[3]</sup> He is regarded by devotees as an incarnation of Shiva. Sri Ramakrishna referred to him as "The walking Shiva of Varanasi".<sup>[5]</sup>

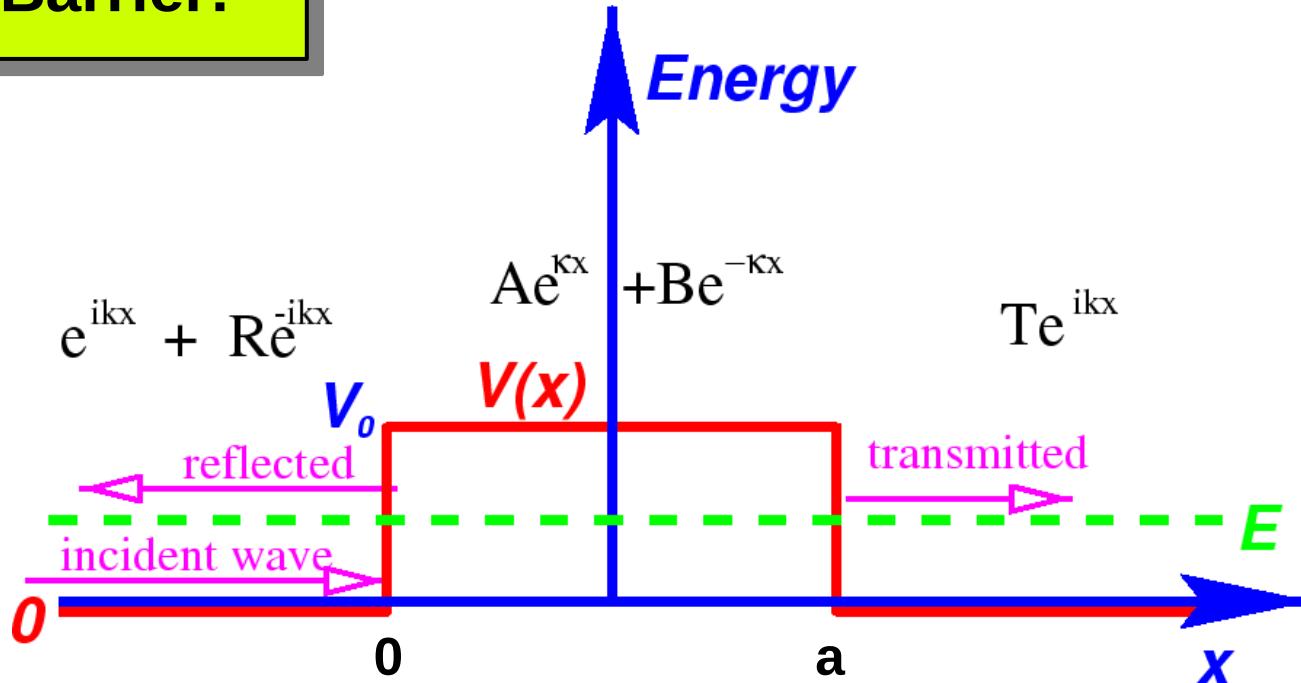
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**Swami Ganapati Saraswati**





## Potential Barrier:



$$V(x) = \begin{cases} 0 & \text{if } x < 0 \\ V_0 & \text{if } 0 < x < a \\ 0 & \text{if } a < x \end{cases}$$

Schrodenger Equation

Guess Solutions  
in 3 region

Use Boundary  
Conditions to  
find unknown  
constants

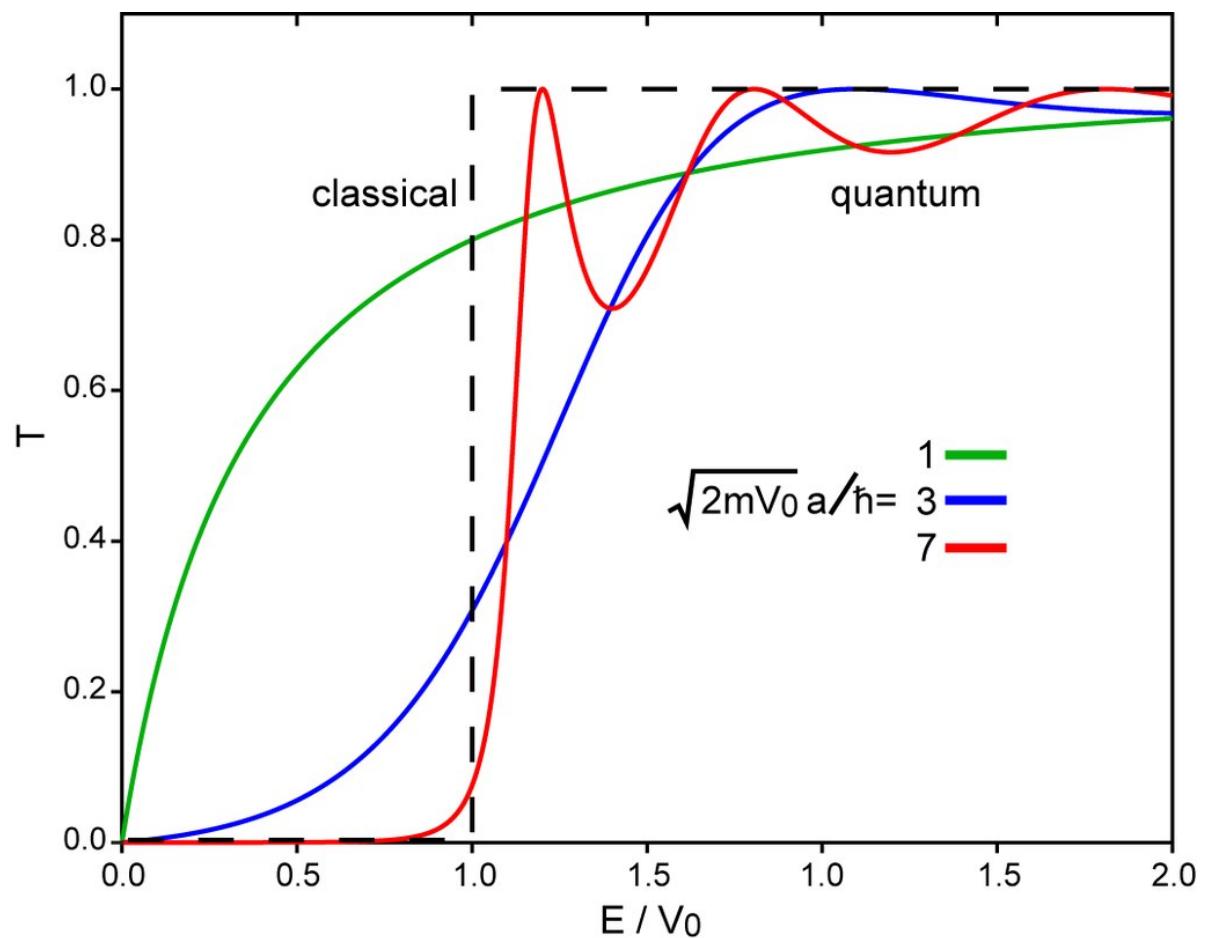
Calculate  
Reflection and  
Transmission  
Coefficients

## Tunneling Effect:

$E < V_0$  [edit]

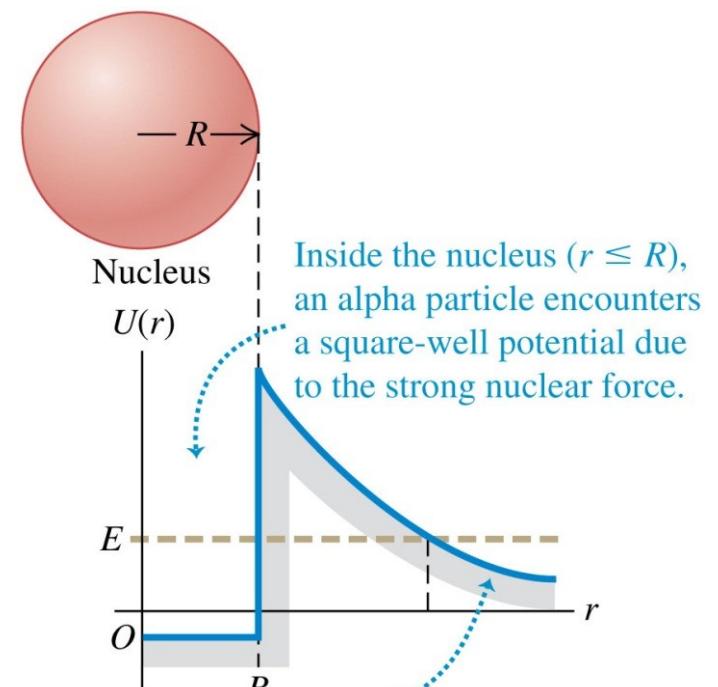
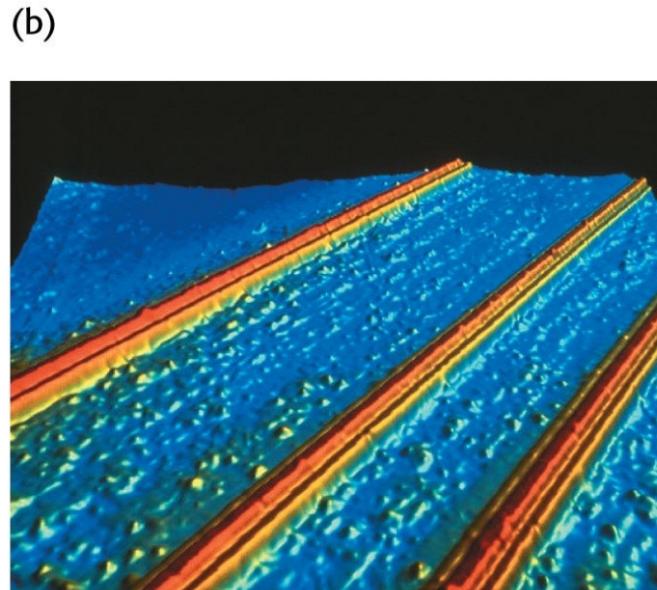
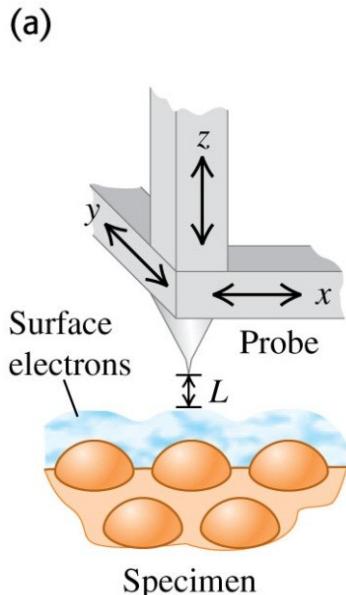
The surprising result is that for energies less than the barrier height,  $E < V_0$  there is a non-zero probability

$$T = |t|^2 = \frac{1}{1 + \frac{V_0^2 \sinh^2(k_1 a)}{4E(V_0 - E)}}$$



# Applications of tunneling

- A scanning tunneling microscope measures the atomic topography of a surface. It does this by measuring the current of electrons tunneling between the surface and a probe with a sharp tip. As the tip nears an atom, the barrier gets thinner, so the current indicates how close the atom is to the tip (the height of the surface)
- An alpha particle inside an unstable nucleus can only escape via tunneling. The reverse happens in fusion, such as what goes on in the Sun.



Inside the nucleus ( $r \leq R$ ), an alpha particle encounters a square-well potential due to the strong nuclear force.

$E$

$O$

$R$

$r$

Outside the nucleus ( $r > R$ ), an alpha particle experiences a  $1/r$  potential due to electrostatic repulsion.