CS 553
CRYPTOGRAPHY

Lecture 18
Hash Functions

Instructor
Dr. Dhiman Saha

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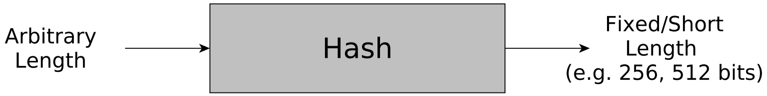
Cryptographic Hash Functions

HASH, x. There is no definition for this word – nobody knows what hash is.

Ambrose Bierce, The Devils Dictionary

A many-to-one mapping



The Cryptographers Swiss Army Knife

Cryptographic Hash Functions used in

- Digital signatures
- Public-key encryption
- Integrity verification
- Message authentication
- Password protection
- Key agreement protocols
- And many other cryptographic protocols.

Hash functions are by far the most versatile and ubiquitous of all crypto algorithms.

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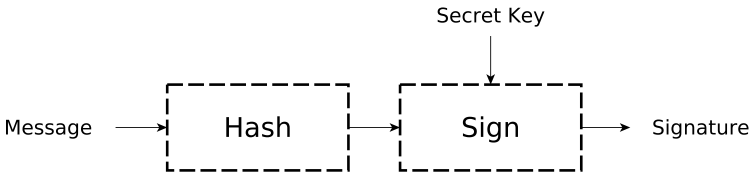
Have I had used one?

Undoubtedly Yes!

- Connecting to a HTTPS website
- Remote connection using SSH/IPSec
- Intrusion Detection Systems
- Forensic Analysis
- Bitcoin in proof-of-work
- Revision control systems like GIT
- And so on.

Use in Digital Signatures



The hash acts as an identifier for the message.

A Word of Caution!!!

Do not confuse cryptographic hash functions with **non-cryptographic** ones.

Example (Noncryptographic Hash)

Hash functions are used in

- Data structures such as hash tables
- Cyclic redundancy checks (CRCs) to detect accidental modifications of a file

No notion of security for non-cryptographic hashes

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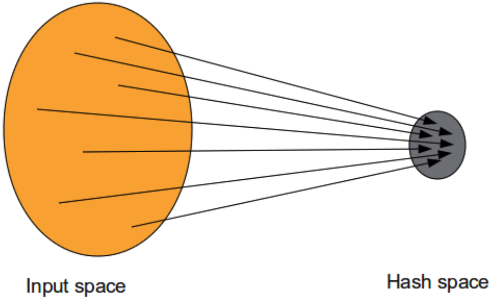
Hash Functions

- A symmetric-key primitive
- **Variable length** input
- Fixed length output called **hash** or **digest**



Recently

Notion of **variable length** output

Hash Functions

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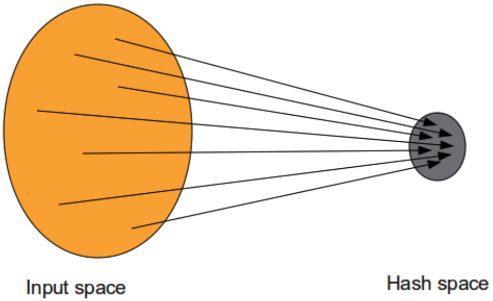

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Notion of **variable length** output

Unpredictability of Hash output

$$\text{SHA-256}\,(\text{“a”}) = \begin{cases} \texttt{87428fc522803d31065e7bce3cf03fe4} \\ \texttt{75096631e5e07bbd7a0fde60c4cf25c7} \end{cases}$$

$$\text{SHA-256}\,(\text{“b”}) = \begin{cases} \texttt{a63d8014dba891345b30174df2b2a57e} \\ \texttt{fbb65b4f9f09b98f245d1b3192277ece} \end{cases}$$

The avalanche effect

`https://passwordsgenerator.net/sha256-hash-generator/`

Theoretically Speaking

A secure hash function should behave like a **truly** random function

- Sometimes called a **random oracle**

More precisely

A secure hash function should **not** have any property or pattern that a random function would **not** have.

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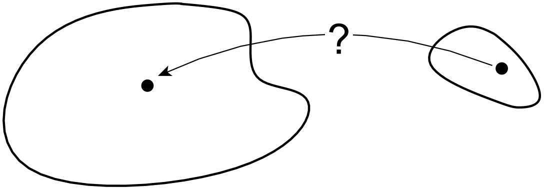
Analyzing Hash Functions

More Specific Notions

Preimage · 2nd-Preimage · Collision · Distinguishers

Pre-image Resistance

- Finding preimage
  - Get x from $h(x)$

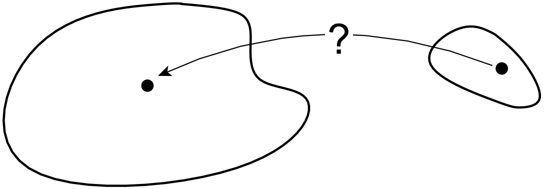


Points to Ponder

- How many pre-images on average?
- How can one be sure?

Pre-image Resistance

- Finding preimage
  - Get x from $h(x)$



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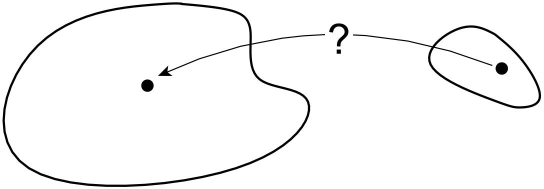

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- How many pre-images on average?
- How can one be sure?

The Cost of Preimages

The optimal preimage search algorithm

```
find-preimage(H) {
   repeat {
     M = random_message
     if Hash(M) == H then return M
   }
}
```

Complexity

2^n

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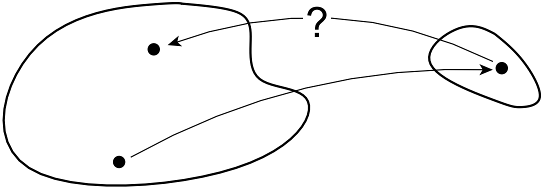
Second Pre-image Resistance

- Finding second-preimage
  - Given x, find $x' \neq x$ such that $h(x) = h(x')$



Points to Ponder

- Why Second-Preimage Resistance Is Weaker Than Preimage Resistance?

Finding Second-Preimage

For the same hash function, if you can find first preimages, you can find second preimages as well

```
solve-second-preimage(M) {
    H = Hash(M)
    return solve-preimage(H)
}
```

Why?

Any second-preimage resistant hash function is also preimage resistant.

- Complexity?

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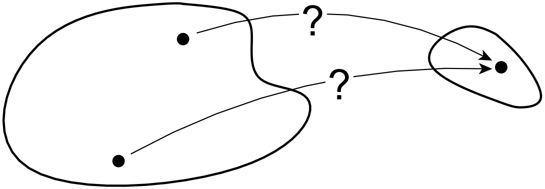
Collision Resistance

- Finding collisions
  - Get $x \neq x'$ such that $h(x) = h(x')$



Points to Ponder

- Will collisions always exist?
- Recall the pigeonhole principle

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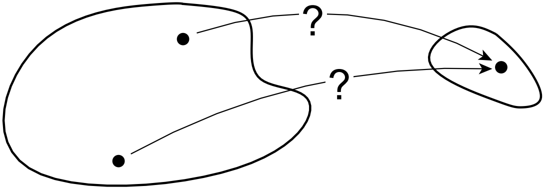

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Relation with Second Preimage Resistance

If you can find second preimages for a hash function, you can also find collisions

```
solve-collision() {
   M = random_message()
   return (M, solve-second-preimage(M))
}
```

Why?

Any collision-resistant hash is also second preimage resistant.

- Complexity of naive collision search algorithm?

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- Complexity of naive collision search algorithm?

Finding Collisions

Will be discussed in next class

The Birthday Paradox

Analyzing Hash Functions

- Finding preimage
  - Get x from $h(x)$
- Finding second-preimage
  - Given x, find $x' \neq x$ such that $h(x) = h(x')$
- Finding collisions
  - Get $x \neq x'$ such that $h(x) = h(x')$
- Devising distinguishers
  - Exhibit non-random behavior of $h(\cdot)$ or internal transformation

Above tasks should be **hard** for a good hash function

In-Class Activity

Lets design a simple hash function and then try to generate a pair of colliding messages