Statistical Physics (2nd tierce exam)

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1. Non-interacting gas follow $PV = NK_BT$ and $U = \frac{3}{2}NK_BT$, when particle in the gas follow energy (ϵ) and momentum (p) relation:
(a) $\epsilon = pc$
(b) $\epsilon = p^2/(2m)$
(c) $\epsilon = pv_F$ (v_F = constant, called Fermi velocity)
(d) none of the above

2. A (one dimensional) classical harmonic oscillator (CHO) with mass m and spring constant k can oscillate with angular frequency $\omega = \sqrt{\frac{k}{m}}$. Its total energy $\epsilon = \frac{p^2}{2m} + \frac{kx^2}{2}$ is constant and so, its phase-space diagram shows an elliptical trajectory, which can be a circle for
(a) $k = b \times m^2 \quad (b = 1$ with dimension $M^2T^{-2})$
(b) $k = b \times m \quad (b = 1$ with dimension $M^2T^{-2})$
(c) $k = b/m \quad (b = 1$ with dimension $M^2T^{-2})$
(d) none of the above

3. For above question, radius R of circle in phase-space diagram will be
(a) $R = 2m\epsilon$
(b) $R = \sqrt{2m\epsilon}$
(c) $R = \sqrt{4m\epsilon/k}$
(d) none of the above

4. Assuming a canonical ensemble (CE) of N no of one dimensional CHO (as demonstrated in earlier questions), partition function $Z = \left[\int \frac{dxdp}{h} e^{(\epsilon/K_BT)}\right]^N$ (K_B is Boltzmann constant) can be obtained as
(a) $Z = \left[\frac{mKT}{\hbar\sqrt{b}}\right]^N$
(b) $Z = \left[\frac{mKT}{\hbar b}\right]^N$
(c) $Z = \left[\frac{KT}{\hbar\sqrt{bm}}\right]^N$
(d) none of the above

5. Helmholtz free energy A can be expressed in terms of partition function Z as $A(T, N, V) = -K_BTlnZ(T, N, V)$. If partition function of a gas, having three dimensional CHO with angular frequency ω is $Z = \left(\frac{KT}{\hbar\omega}\right)^{3N}$, then $A(T, N, V)$ can be expressed as
(a) $A = NK_BT\ ln\left(\frac{\hbar\omega}{K_BT}\right)$
(b) $A = 2NK_BT\ ln\left(\frac{\hbar\omega}{K_BT}\right)$
(c) $A = 3NK_BT\ ln\left(\frac{\hbar\omega}{K_BT}\right)$
(d) none of the above

6. Thermal distribution function of any particle with energy ϵ in a gas with temperature T and chemical potential μ can be written in a general form $f(\epsilon, T, \mu) = 1/[exp\{(\epsilon - \mu)/K_BT\} + \eta]$, which will be Fermi-Dirac (FD), Bose-Einstein (BE), and Maxwell-Boltzmann (MB) distribution for
(a) $\eta = +1, \ -1, \ 0$
(b) $\eta = -1, \ +1, \ 0$
(c) $\eta = 0, \ -1, \ +1$
(d) none of the above.

7. In Large Hadron Collider (LHC) experiments, apart from neutron n and proton p with spin $\hbar/2$, many other particles like pion π, Kaon K with spin 0; ρ, K^* mesons with spin $\hbar$; Δ with spin $\frac{3\hbar}{2}$ are produced. In the context of statistical mechanics, we can classify them as
(a) Bosons : π, K, n, p and Fermions : ρ, K^* Δ
(b) Bosons : π, K, ρ, K^* and Fermions : n, p, Δ
(c) Bosons : π, K, ρ, K^*, Δ and Fermions : n, p,
(d) none of the above.

8. No of photons N, emitting from black body at temperature T (and chemical potential $\mu = 0$) can be expressed as
$$N = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{1}{e^{\beta\epsilon}-1} \tag{1}$$
with photon's energy $\epsilon = pc$. Using the Riemann zeta function
$$\zeta(n) = \frac{1}{\Gamma(n)}\int_0^\infty \frac{x^{n-1}}{e^x - 1} \tag{2}$$
with Gamma function $\Gamma(n) = (n-1)!$ (when n is integer), we get number density as
(a) $\frac{N}{V} = 8\pi\left(\frac{K_BT}{hc}\right)^3\zeta(1)\Gamma(1)$
(b) $\frac{N}{V} = 8\pi\left(\frac{K_BT}{hc}\right)^3\zeta(2)\Gamma(2)$
(c) $\frac{N}{V} = 8\pi\left(\frac{K_BT}{hc}\right)^3\zeta(3)\Gamma(3)$
(d) none of the above

9. Total energy (internal energy) of photon gas at temperature T (and chemical potential $\mu = 0$) can be expressed as
$$U = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{\epsilon}{e^{\beta\epsilon}-1} \tag{3}$$
with photon's energy $\epsilon = pc$. Using the Riemann zeta function (given in earlier question), we get energy density as
(a) $\frac{U}{V} = 8\pi K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(2)\Gamma(2)$
(b) $\frac{U}{V} = 8\pi K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(3)\Gamma(3)$
(c) $\frac{U}{V} = 8\pi K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(4)\Gamma(4)$
(d) none of the above

10. Energy density U/V and intensity I of photon gas is connected through relation $I = \frac{c}{4}\frac{U}{V}$, which can reproduce the famous empirical law - Stefan-Boltzmann law $I = \sigma T^4$, where
(a) $\sigma = 8\pi\left(\frac{K_B^4}{h^3c^2}\right)\zeta(2)\Gamma(2)$
(b) $\sigma = 8\pi\left(\frac{K_B^4}{h^3c^2}\right)\zeta(3)\Gamma(3)$
(c) $\sigma = 8\pi\left(\frac{K_B^4}{h^3c^2}\right)\zeta(4)\Gamma(4)$
(d) none of the above

11. If we see the integrand of energy density or intensity of photon gas, then it provide us the black body spectrum by using the quantum relation $\epsilon = pc = h\nu = hc/\lambda$. Which observation can not be connected with the integrand or which observation is wrong?

(a) energy density first increases the decreases along ν or λ axis
(b) Peak value spectrum depends on T
(c) Peak value spectrum does not depend on T
(d) none of the above

12. In the Eq. (3), replacing BE distribution by MB, we can get

$$U = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{\epsilon}{e^{\beta\epsilon}} \tag{4}$$

with photon's energy $\epsilon = pc$. Using the Gamma function $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$, we get energy density as
(a) $\frac{U}{V} \propto T^2$
(b) $\frac{U}{V} \propto T^3$
(c) $\frac{U}{V} \propto T^4$
(d) none of the above

13. Pressure (P) of photon gas at temperature T (and chemical potential $\mu = 0$) can be expressed as

$$\frac{PV}{K_BT} = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{pc/3}{e^{\beta\epsilon}-1} \tag{5}$$

with photon's energy $\epsilon = pc$. Using the Riemann zeta function (given in earlier question), we get
(a) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(2)\Gamma(2)$
(b) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(3)\Gamma(3)$
(c) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(4)\Gamma(4)$
(d) none of the above

14. Photons are
(a) Fermions and follow FD distribution
(b) Bosons and follow BE distribution
(c) classical particles and follow MB distribution
(d) none of the above

15. For any particle with spin $s\hbar$ has spin-degeneracy factor $2s+1$. e.g. electron's spin is $\hbar/2$ and spin-degeneracy factor 2. In this regards, photon has an interesting properties:
(a) photon has spin $\hbar$ but its spin-degeneracy factor is 2
(b) photon has spin $\hbar/2$ but its spin-degeneracy factor is 3
(c) photon has spin $\hbar$ but its spin-degeneracy factor is 1
(d) none of the above