CS251: Introduction to Language Processing

Bottom-Up Parsing

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Acknowledgement

- Today's slides are modified from that of *Stanford University:*
    - *https://web.stanford.edu/class/archive/cs/cs143/cs143.1128/*

$S \rightarrow E$

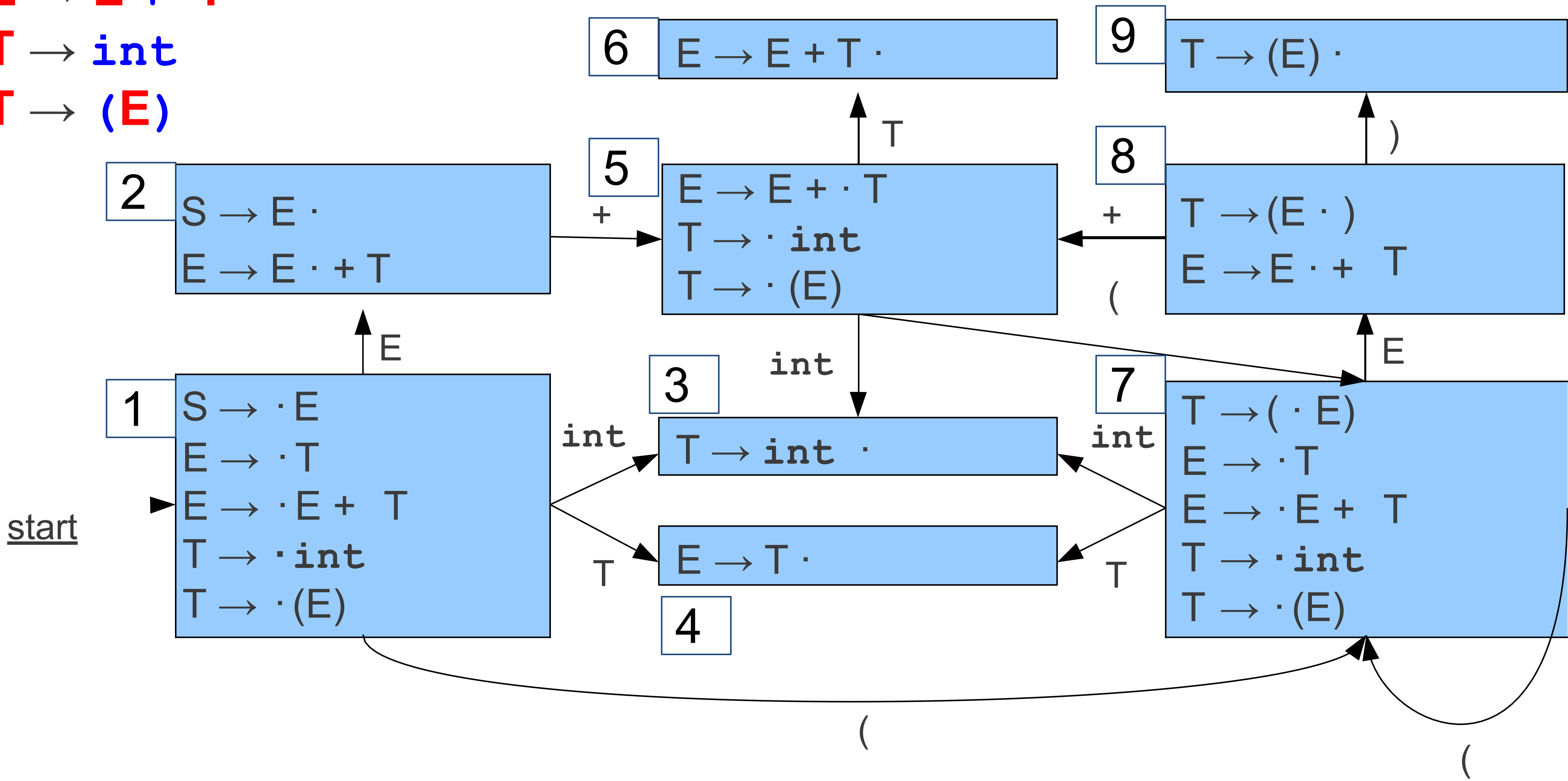
$E \rightarrow T$

$E \rightarrow E + T$

$T \rightarrow$ `int`

$T \rightarrow (E)$

LR (0) Parsing



LR(0) Table

(1) $S \rightarrow E$
(2) $E \rightarrow T$
(3) $E \rightarrow E + T$
(4) $T \rightarrow$ `int`
(5) $T \rightarrow$ `(E)`

	Action					Goto	
	`int`	`+`	`(`	`)`	`$`	E	T
1	S3					2	4
2	r1	S5/r1	r1	r1	r1		
3	r4	r4	r4	r4	r4		
4	r2	r2	r2	r2	r2		
5	S3		S7				6
6	r3	r3	r3	r3	r3		
7	S3		S7			8	4
8		S5		S9			
9	r5	r5	r5	r5	r5		

LR Conflicts

- A **shift/reduce conflict** is an error where a shift/reduce parser cannot tell whether to shift a token or perform a reduction.
  - Often happens when two productions overlap.
- A **reduce/reduce conflict** is an error where a shift/reduce parser cannot tell which of many reductions to perform.
  - Often the result of ambiguous grammars.
- A grammar whose handle-finding automaton contains a shift/reduce conflict or a reduce/reduce conflict is not LR(0).
- Can you have a shift/shift conflict?

Example

(1) $S \longrightarrow E$

(2) $E \longrightarrow T$

(3) $E \longrightarrow E + T$

(4) $T \longrightarrow$ `int`

(5) $T \longrightarrow$ `(E)`

Try to parse int+int using LR(0)

SLR(1)

- **Simple LR(1)**
- Minor modification to LR(0) automaton that uses lookahead to avoid shift/reduce conflicts.

$S \rightarrow E$

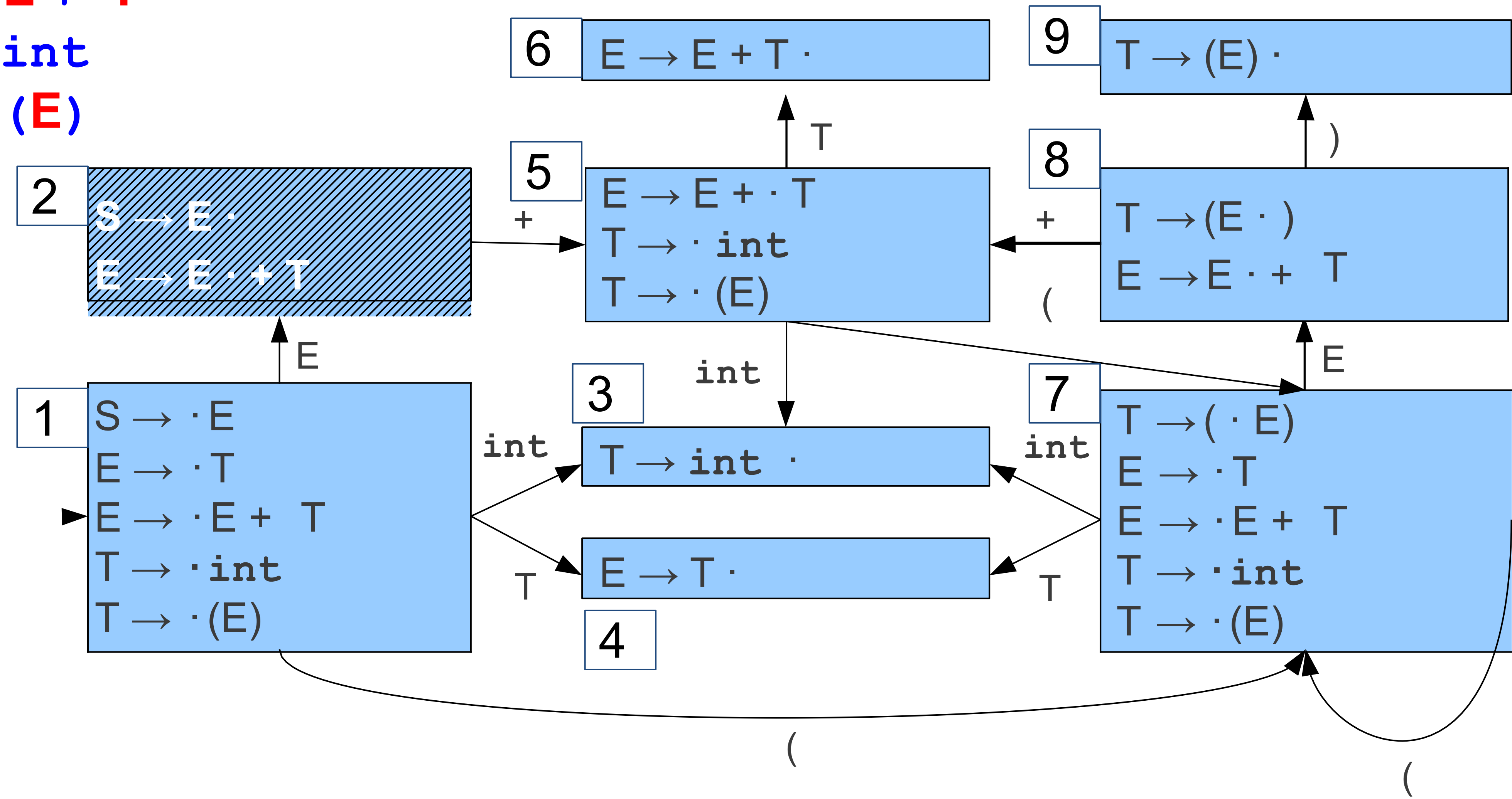
$E \rightarrow T$

$E \rightarrow E + T$

$T \rightarrow$ `int`

$T \rightarrow (E)$

SLR(1) Parsing



SLR(1) Table

(1) $S \to E$
(2) $E \to T$
(3) $E \to E + T$
(4) $T \to$ `int`
(5) $T \to$ `(E)`

	Action					Goto	
	int	+	(	)	$	E	T
1	S3					2	4
2		S5					
3							
4							
5	S3		S7				6
6							
7	S3		S7			8	4
8		S5		S9			
9							

SLR(1) Table

(1) $S \rightarrow E$

(2) $E \rightarrow T$

(3) $E \rightarrow E + T$

(4) $T \rightarrow$ `int`

(5) $T \rightarrow$ `(`E`)`

	Action					Goto	
	`int`	`+`	`(`	`)`	`$`	E	T
1	S3					2	4
2		S5			**r1**		
3		**r4**		**r4**	**r4**		
4		**r2**		**r2**	**r2**		
5	S3		S7				6
6		**r3**		**r3**	**r3**		
7	S3		S7			8	4
8		S5		S9			
9		**r5**		**r5**	**r5**		

SLR(1)

- **Simple LR(1)**
- Idea: Only reduce **A** $\rightarrow$ ***ω*** if the next token **`t`** is in FOLLOW(**A**).
- Automaton identical to LR(0) automaton; only change is when we choose to reduce.
-

Example

(1) S → E
(2) E → T
(3) E → E + T
(4) T → `int`
(5) T → `(E)`

Try to parse int+int using SLR(1)

Analysis of SLR(1)

- Exploits lookahead in a small space.
  - Small automaton – same number of states as in as LR(0).
  - Works on many more grammars than LR(0)
- Too weak for most grammars: lose context from not having extra states.

The Limits of SLR(1)

$\mathbf{S} \to \mathbf{E}$

$\mathbf{E} \to \mathbf{L}$ `=` $\mathbf{R}$

$\mathbf{E} \to \mathbf{R}$

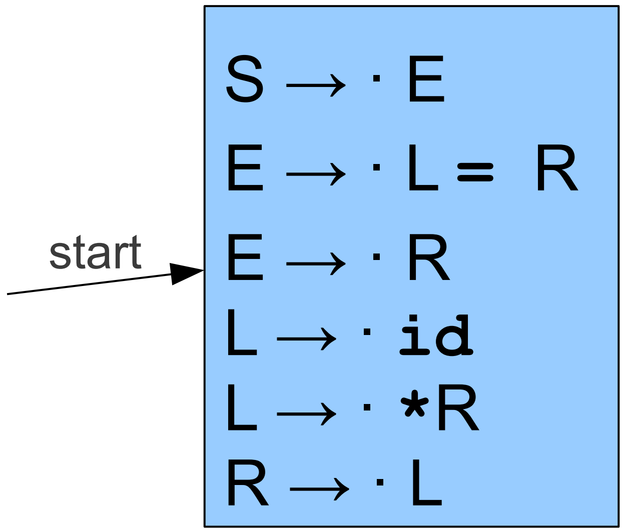
$\mathbf{L} \to$ `id`

$\mathbf{L} \to$ `*`$\mathbf{R}$

$\mathbf{R} \to \mathbf{L}$

The Limits of SLR(1)

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
$L \rightarrow \texttt{id}$
$L \rightarrow \texttt{*}R$
$R \rightarrow L$



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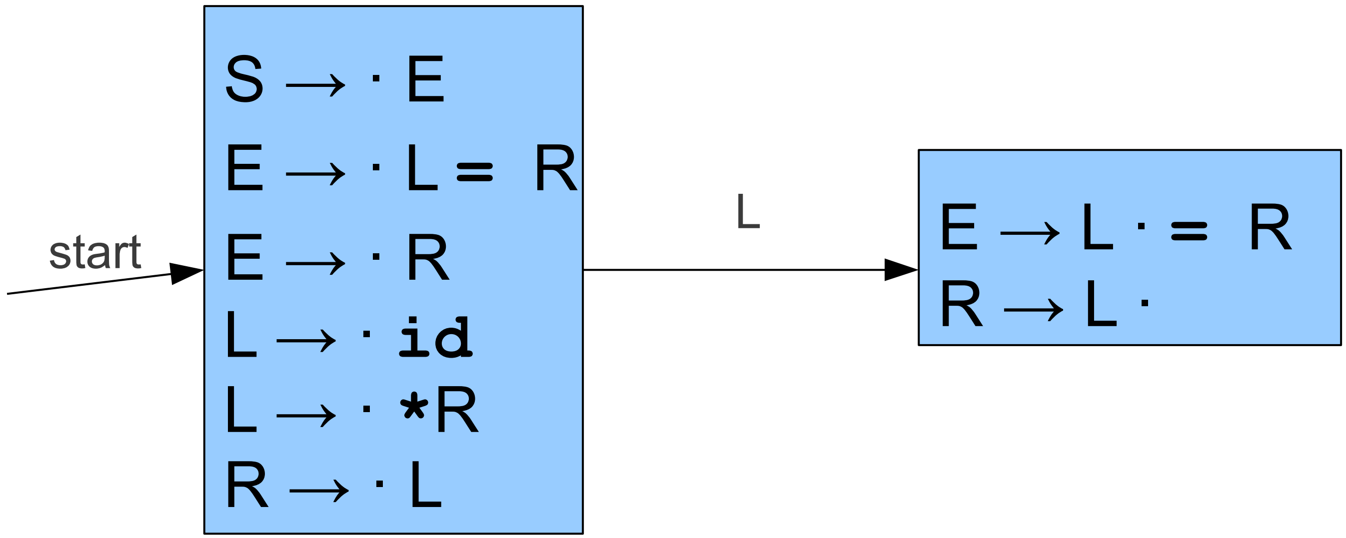

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E → L = R

E → R

L → id

L → *R

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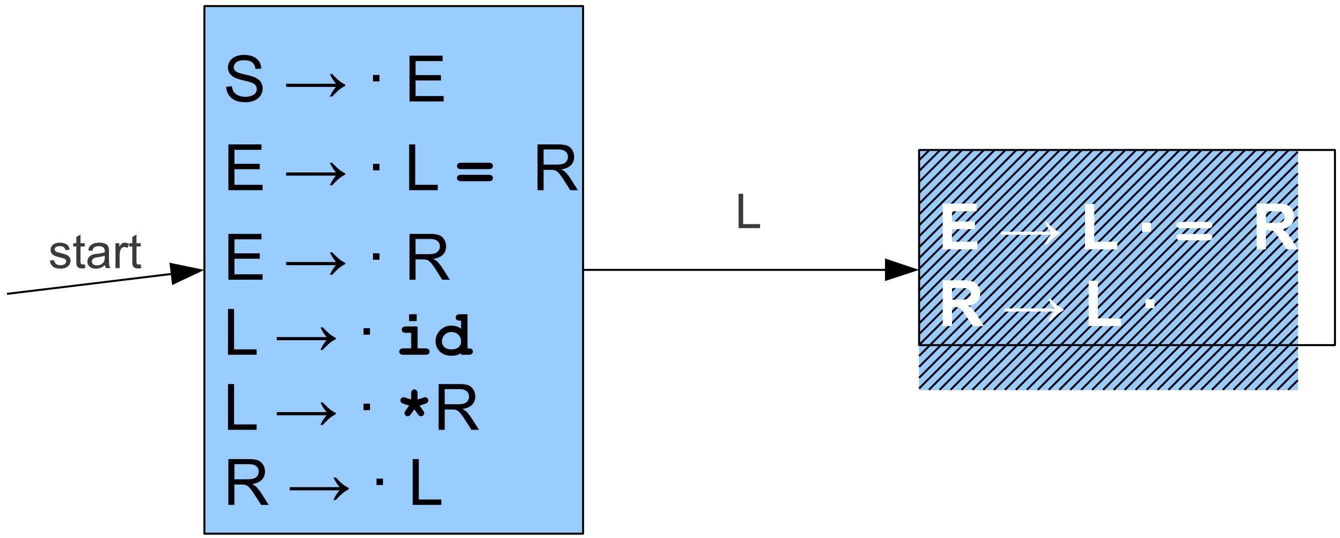

The Limits of SLR(1)

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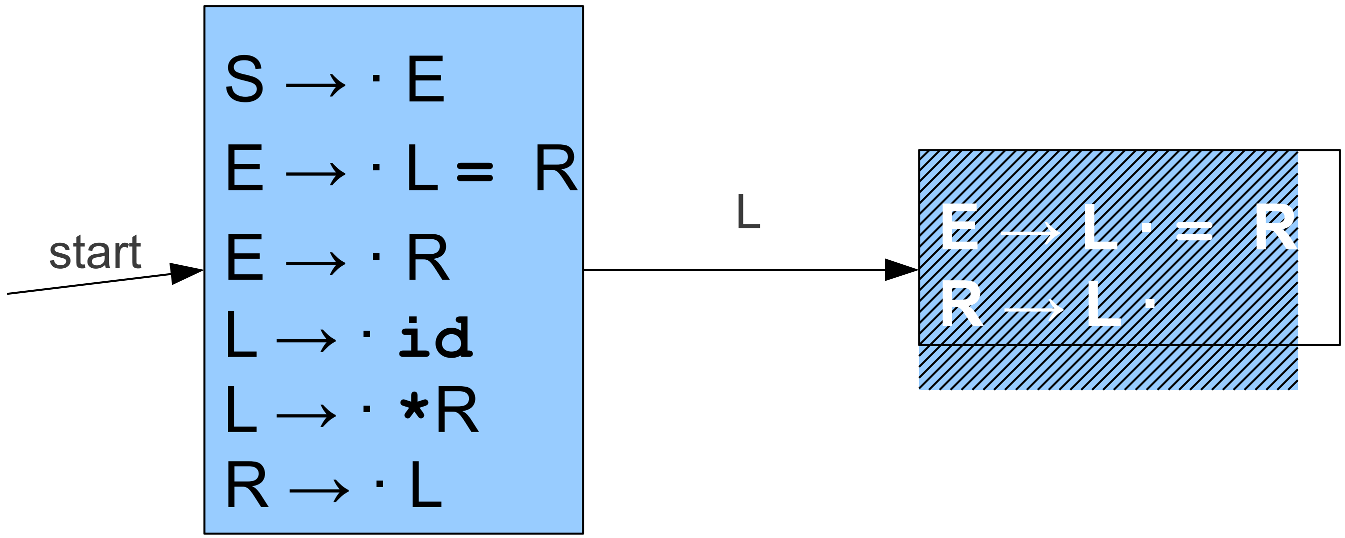

$E \rightarrow L \cdot = R$ tells us to shift on seeing $=$

$R \rightarrow L \cdot$ tells us to reduce on FOLLOW(R).

The Limits of SLR(1)

S → E
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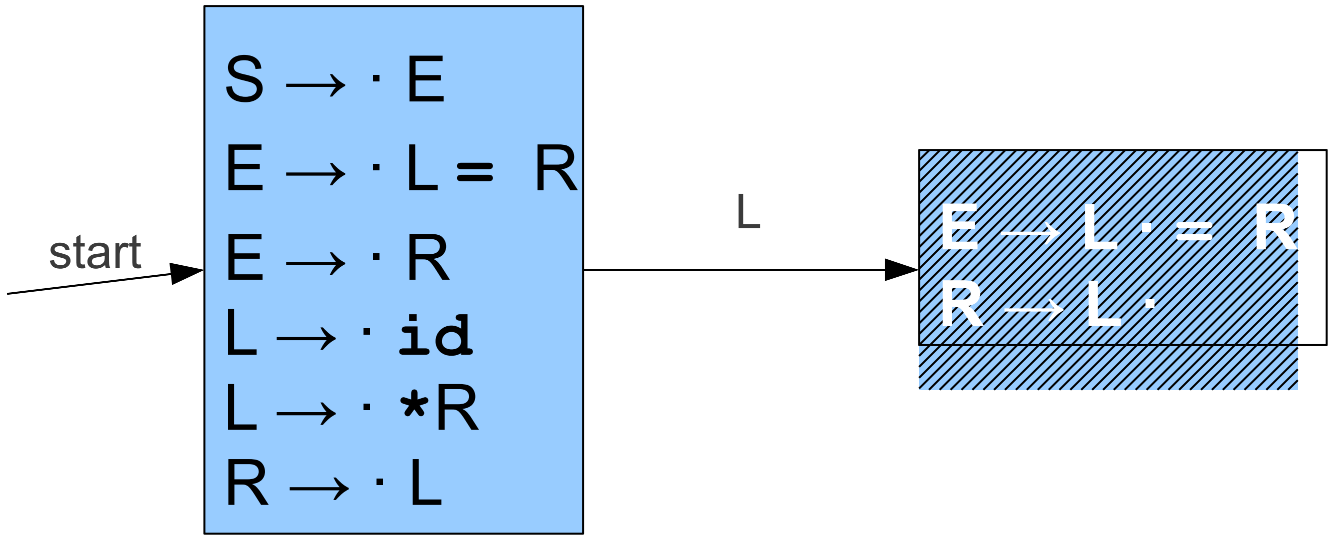
E → R
L → id
L → *R
R → L



E → L · = R tells us to shift on seeing **=**
R → L · tells us to reduce on FOLLOW(**R**).

The Limits of SLR(1)

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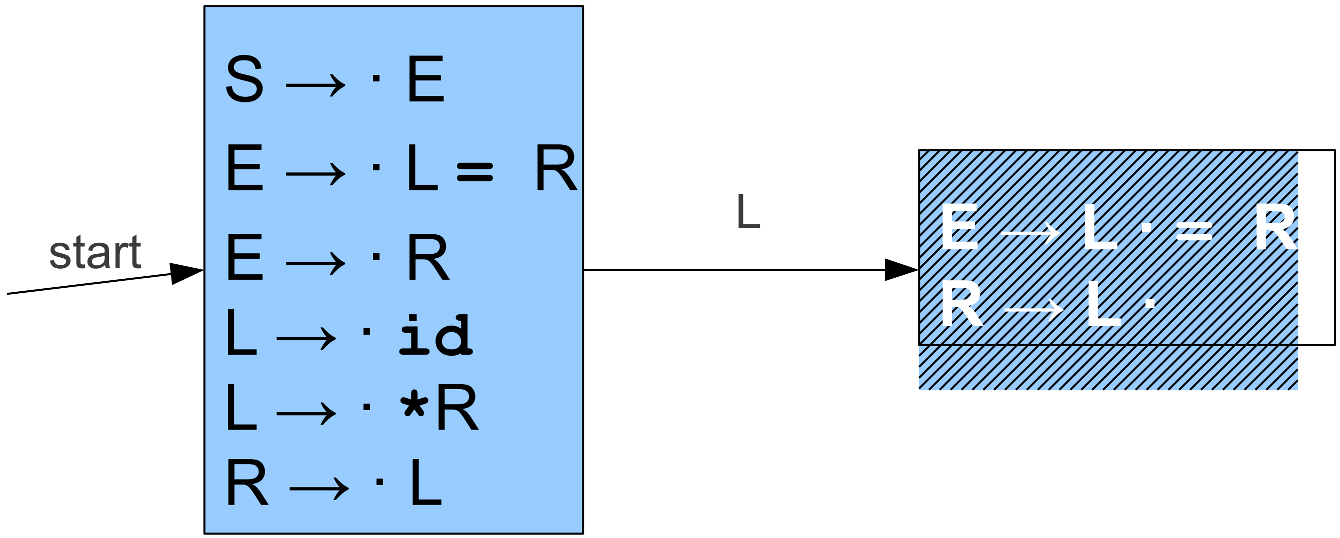

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$= \in$ FOLLOW(R).

The Limits of SLR(1)

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$L \rightarrow *R$
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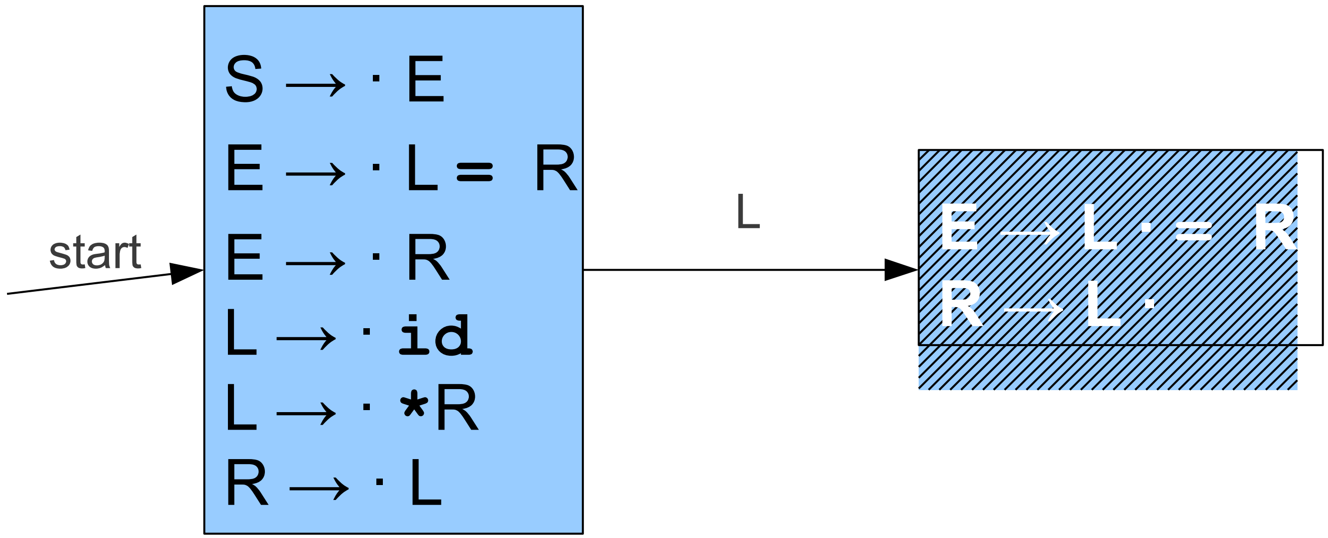


$E \rightarrow L \cdot = R$ tells us to shift on seeing $=$
$R \rightarrow L \cdot$ tells us to reduce on FOLLOW(R).

$= \in$ FOLLOW(R).

We have a conflict!

A Lack of Context

$S \rightarrow E$

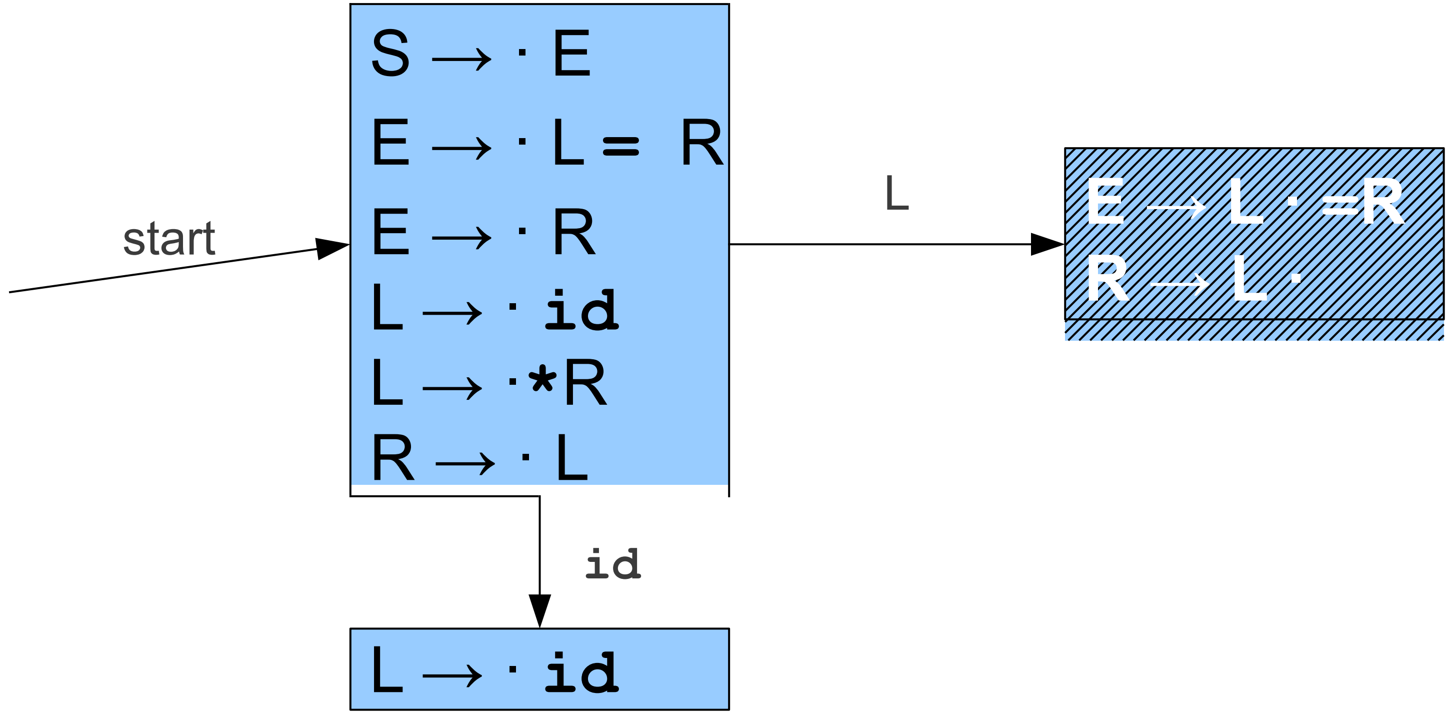
$E \rightarrow L$ `=` R

$E \rightarrow R$

$L \rightarrow$ `id`

$L \rightarrow$ `*`R

$R \rightarrow L$



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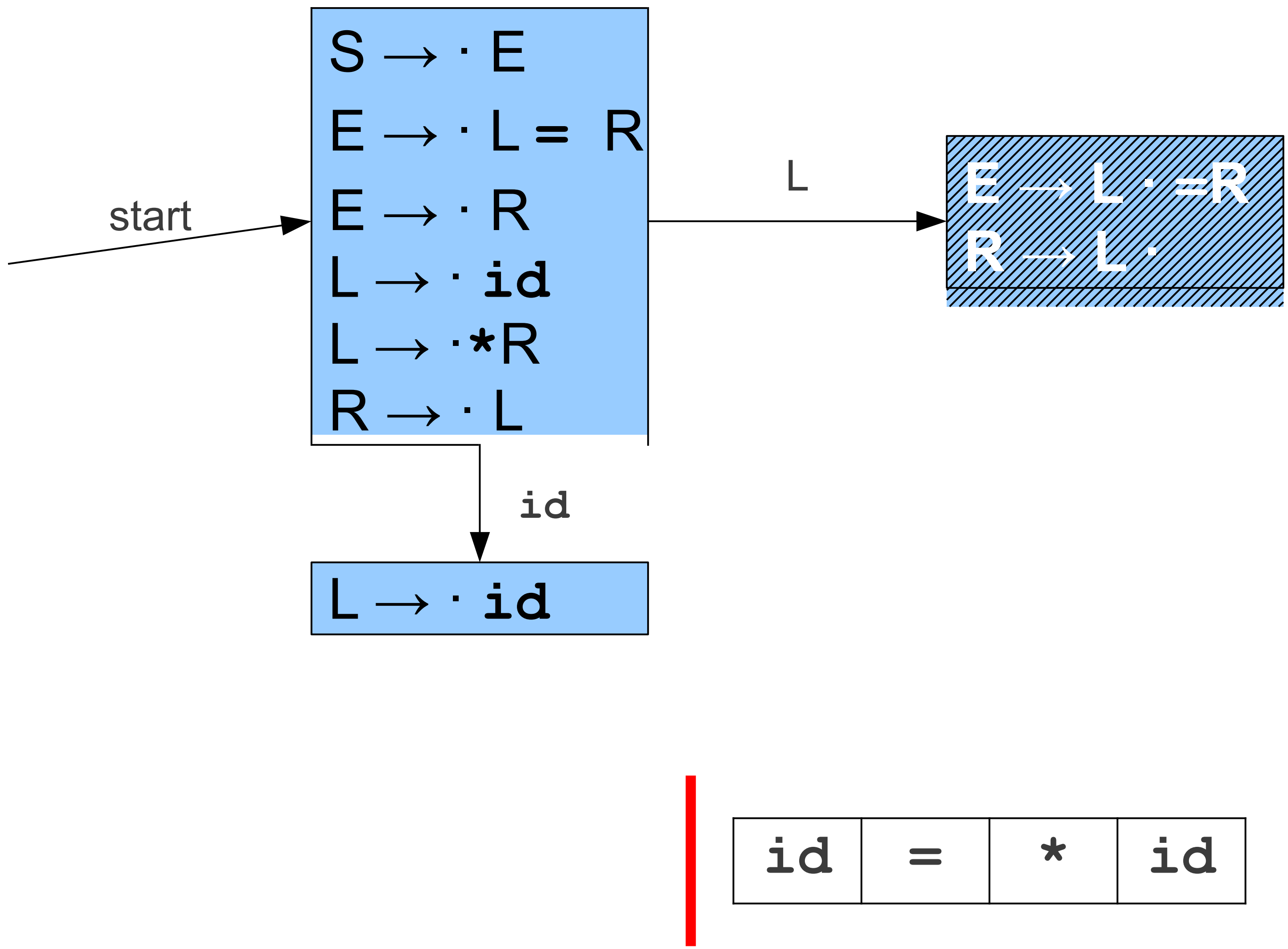

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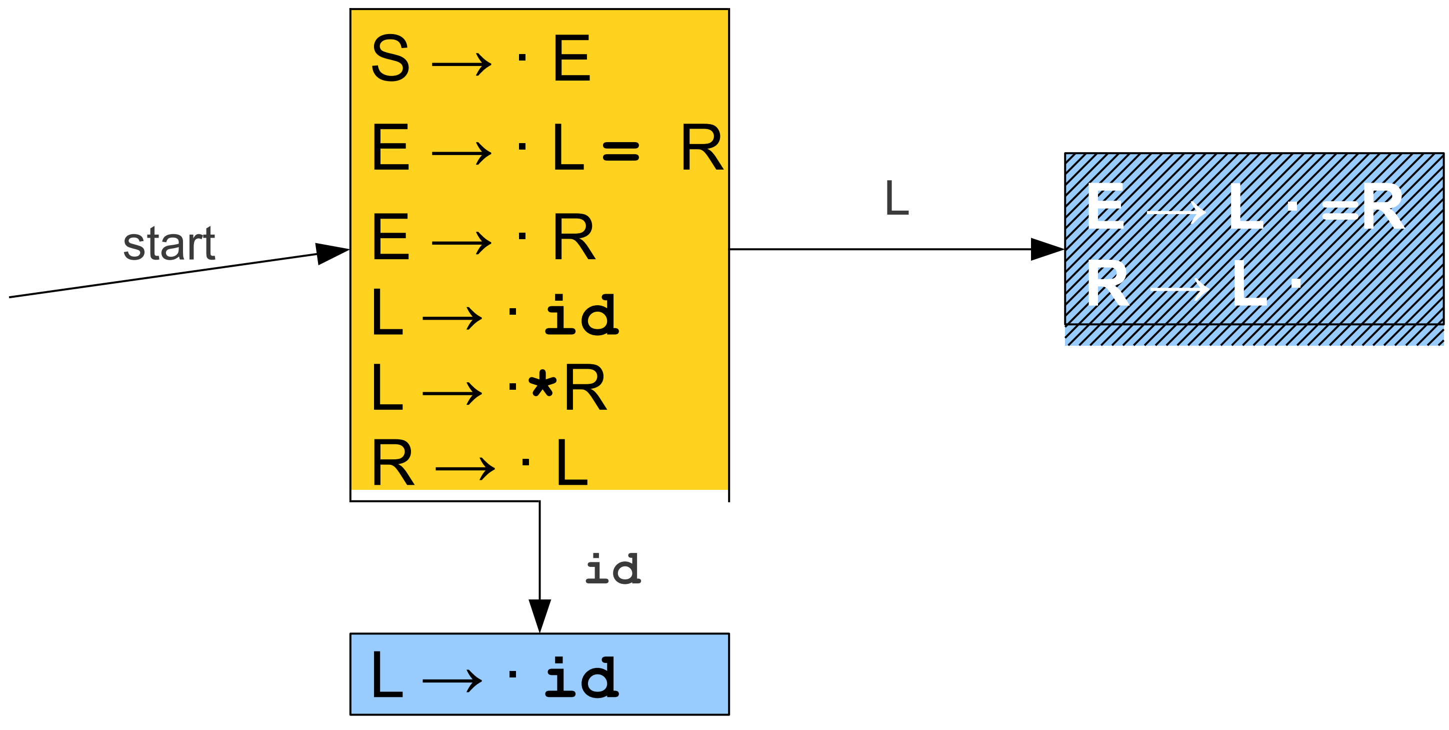

id	=	*	id

A Lack of Context

$S \rightarrow E$
$E \rightarrow L = R$
$E \rightarrow R$
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$L \rightarrow$ *R
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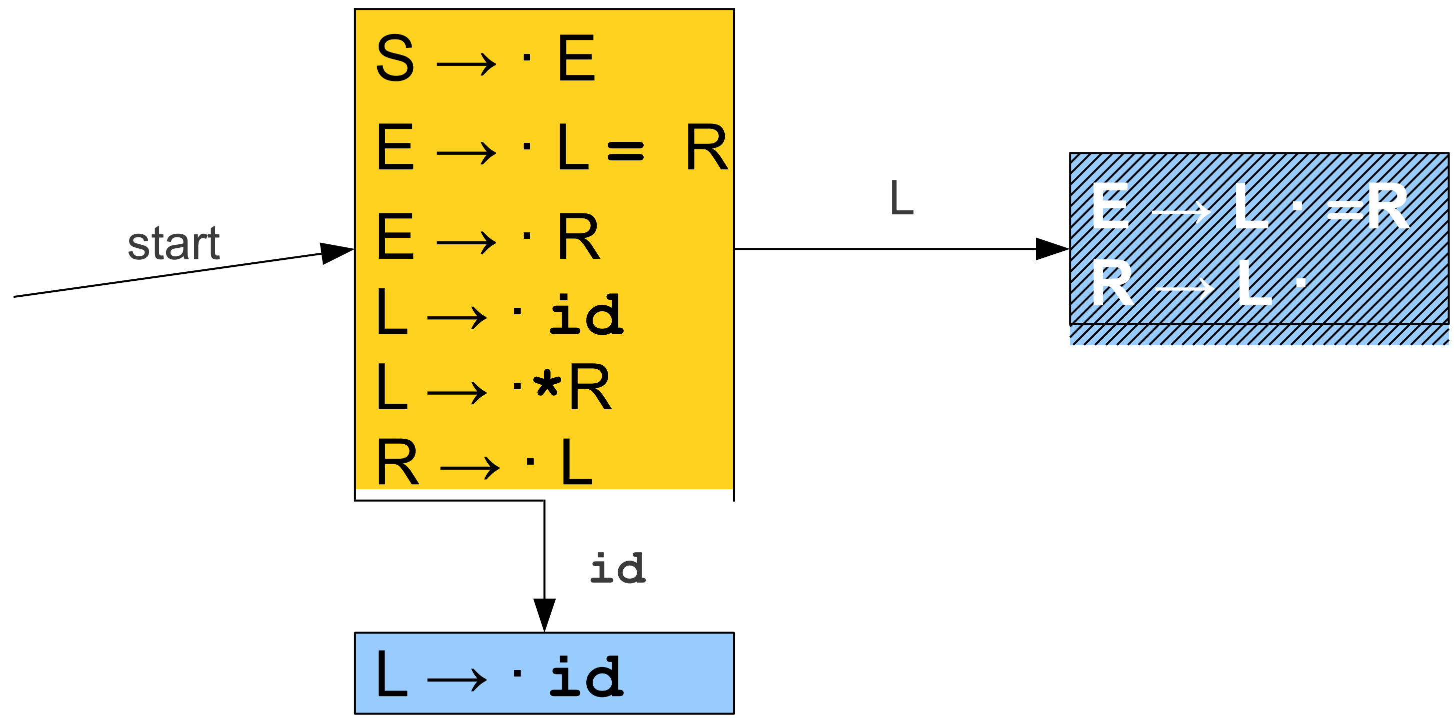

id	ǀ	=	*	id

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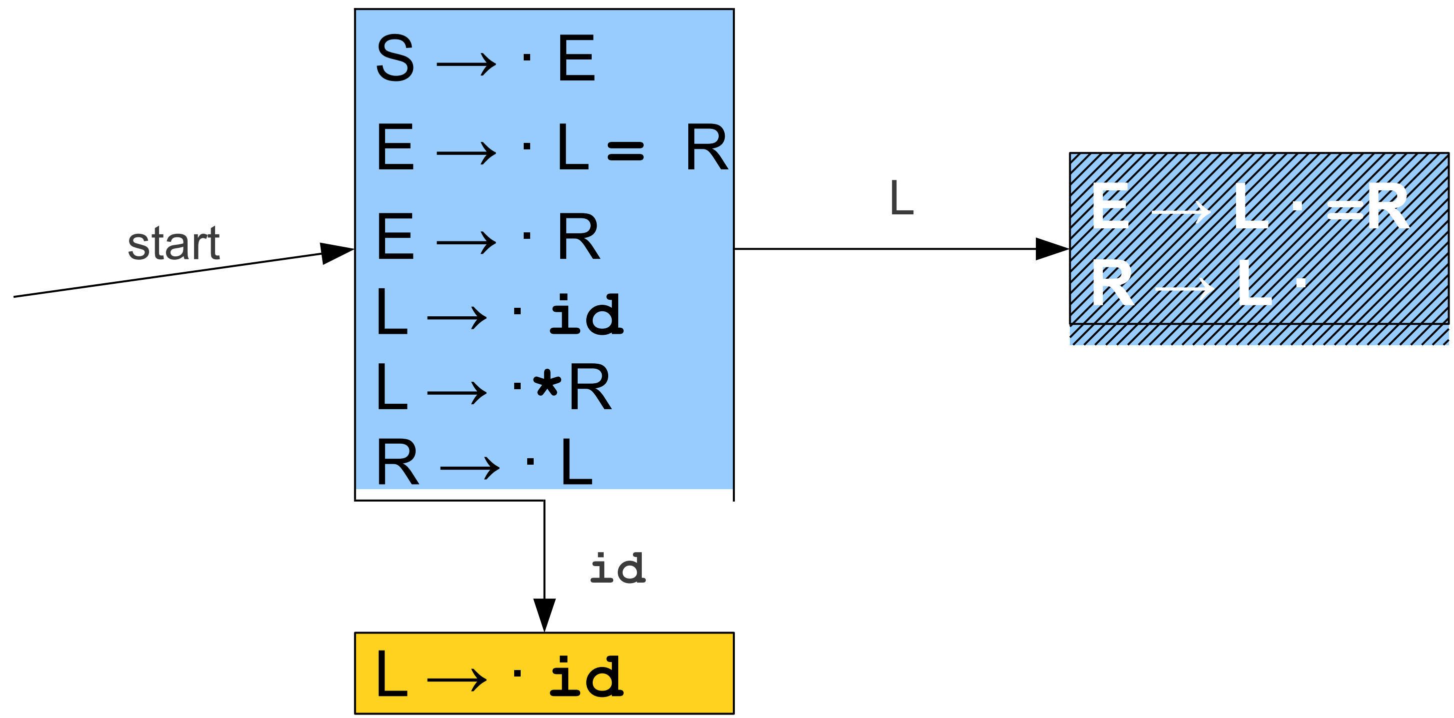
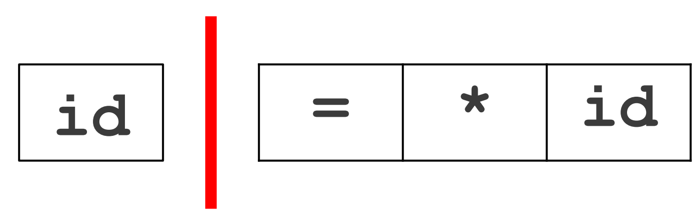




A Lack of Context

S → E

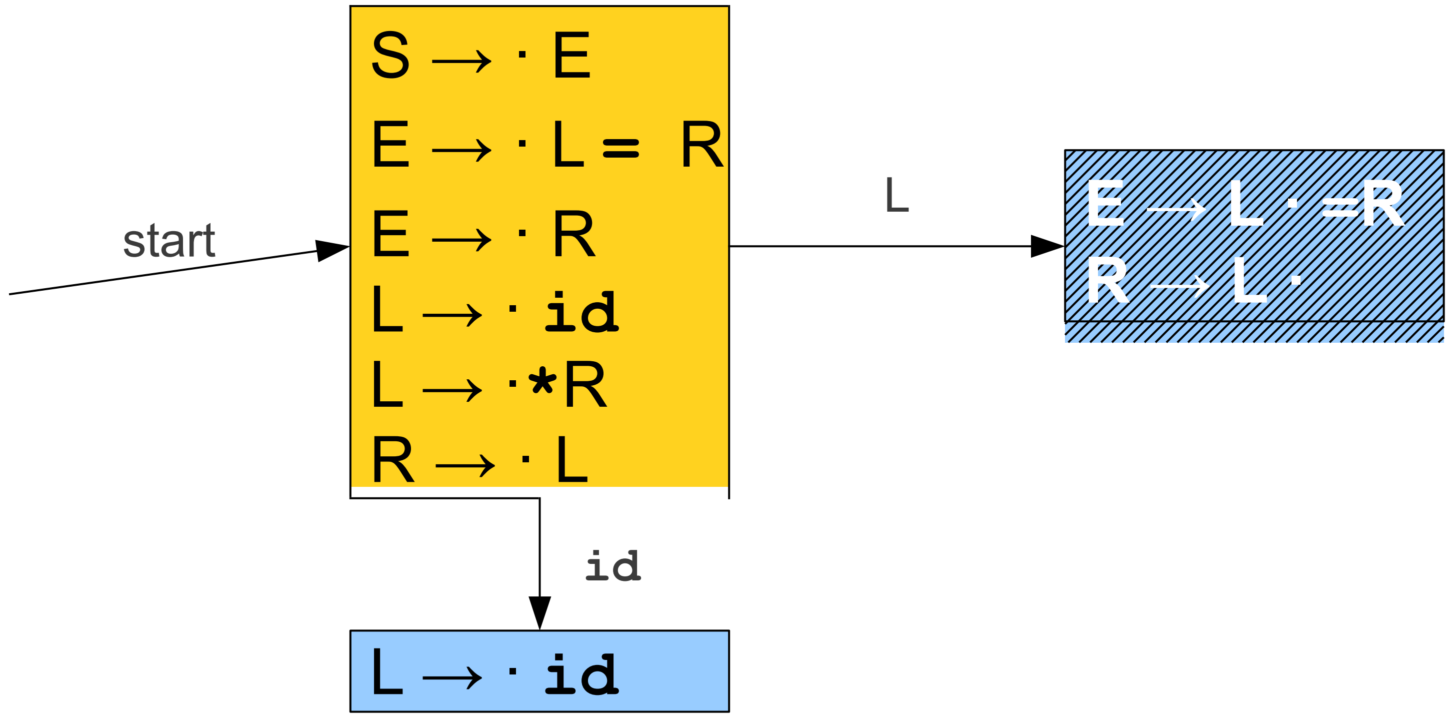
E → L = R

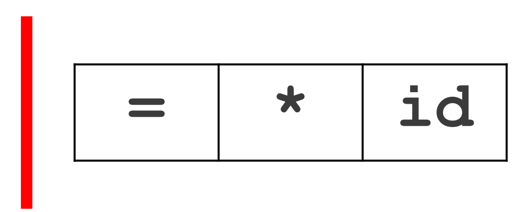
E → R

L → id

L → *R

R → L





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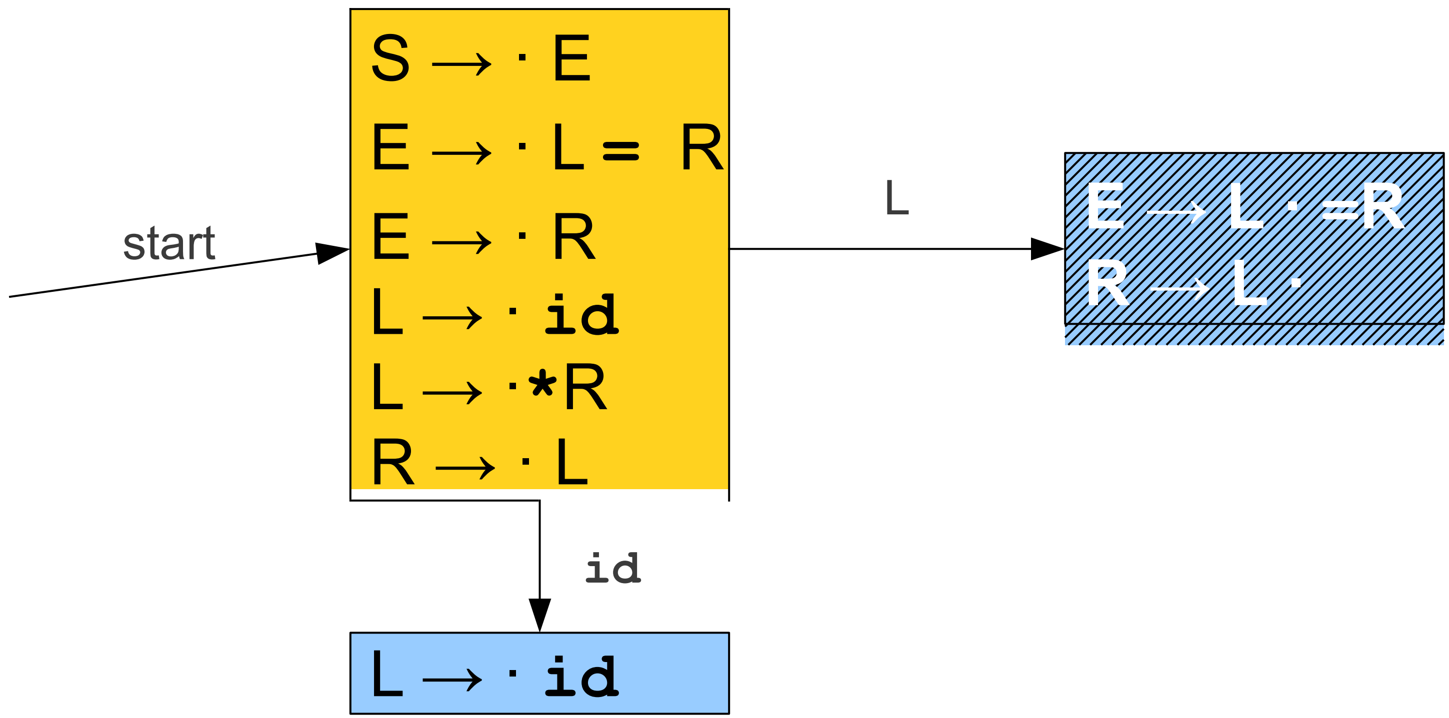

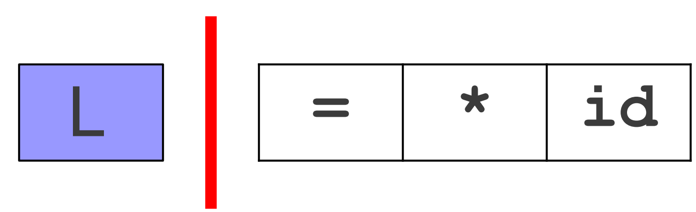

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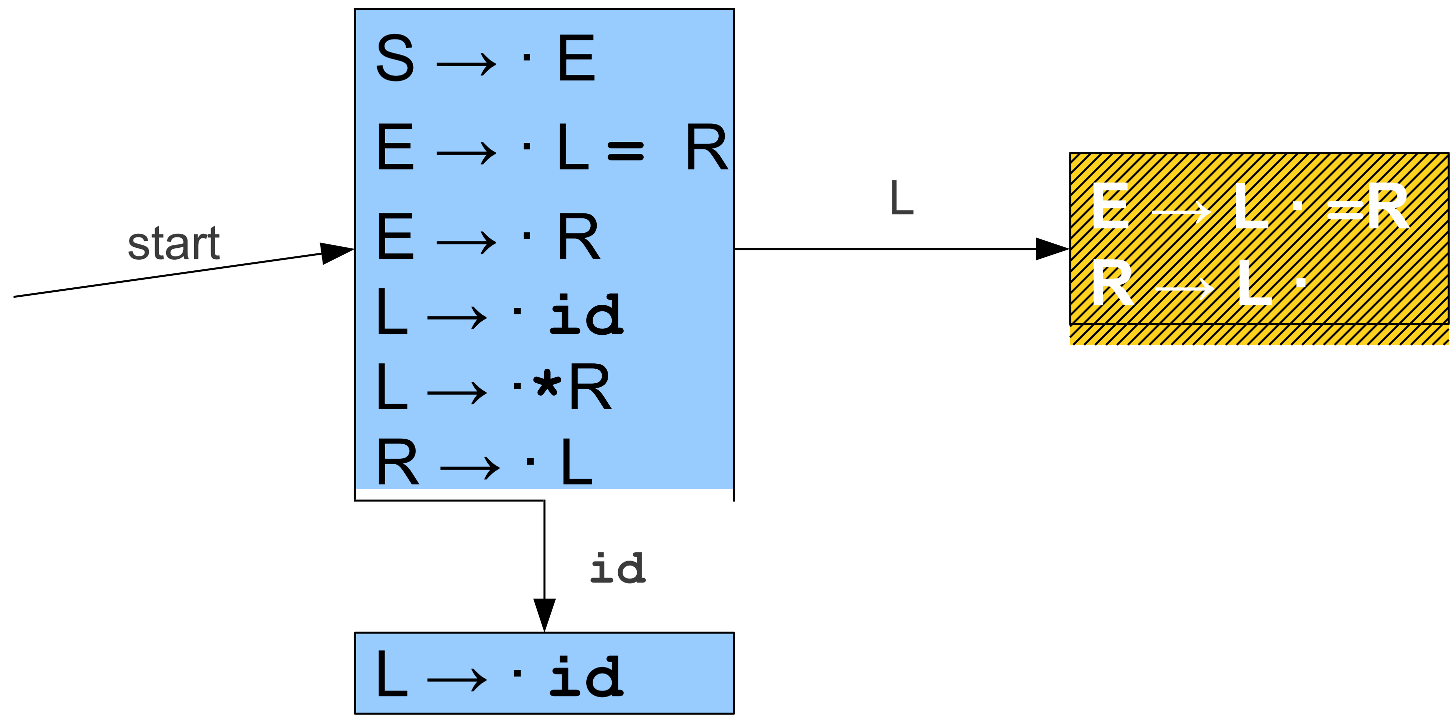

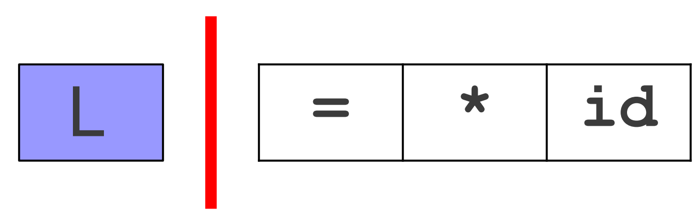

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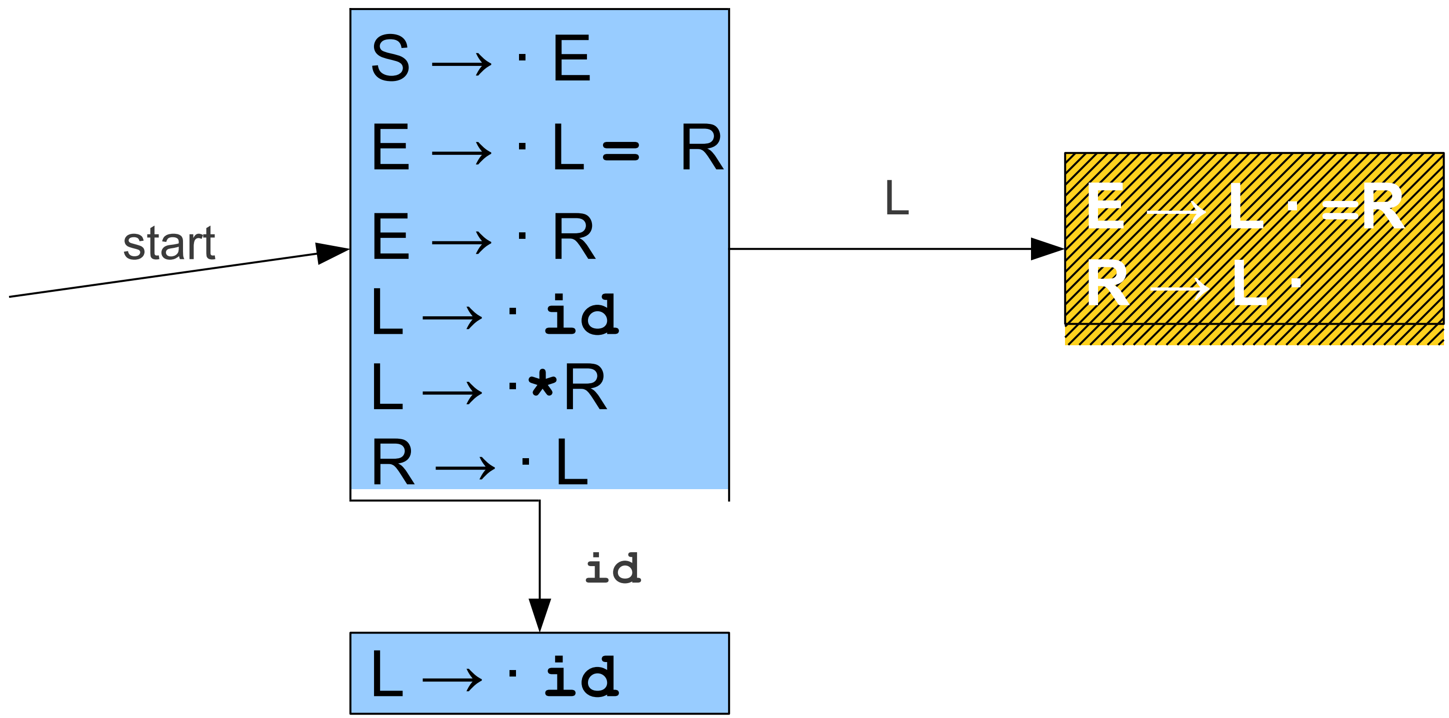

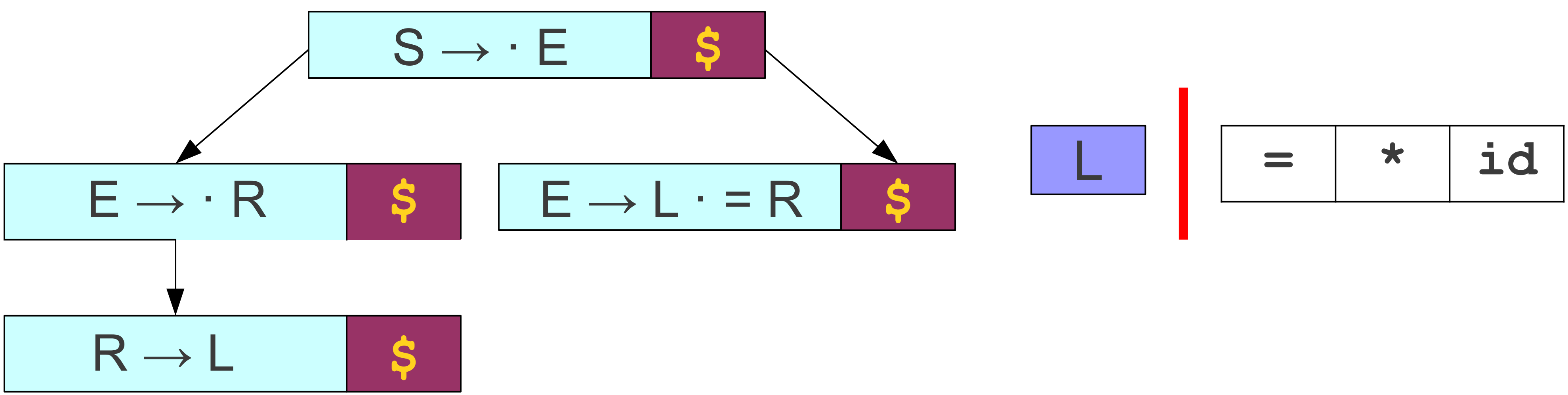

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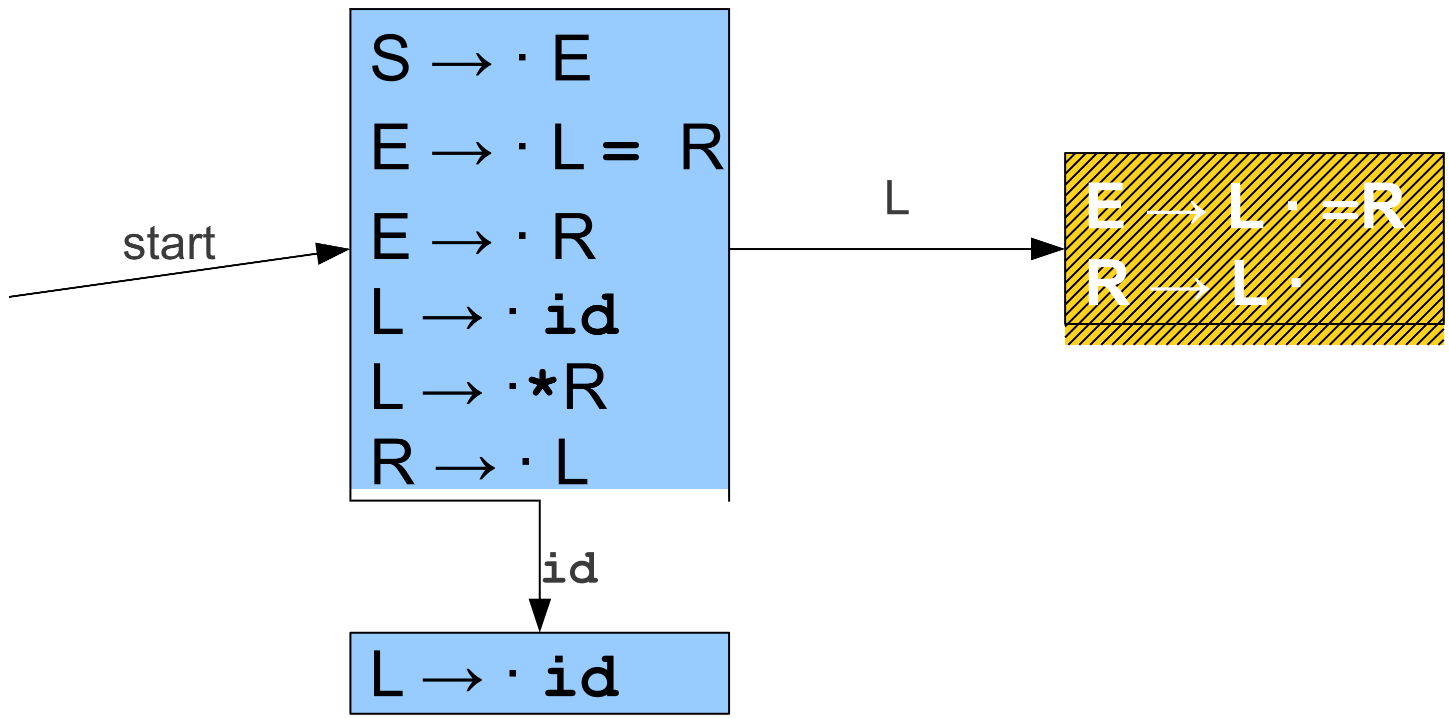
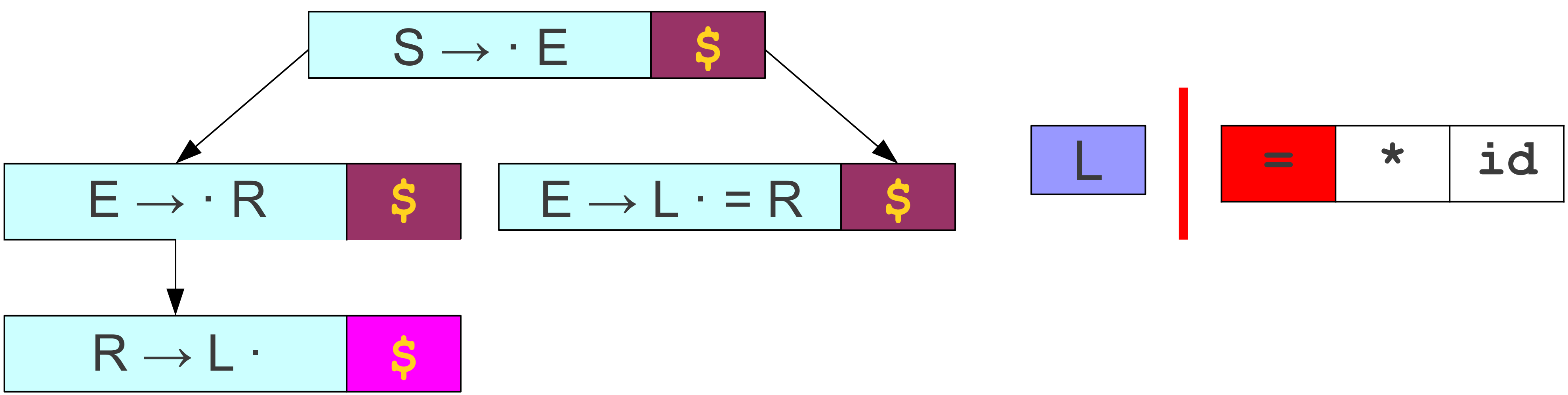




Why is SLR(1) Weak?

- With SLR(1), *minimal* context.
  - FOLLOW(**A**) means “what could follow **A** *somewhere* in the grammar?,” even if in a particular state **A** couldn't possibly have that symbol after it.
- With LR(1), we have contextual information.

Constructing LR(1) Items

- Begin in a state containing **S** $\rightarrow \cdot$ **E** , **\$** where **S** is the start symbol and **\$** `is lookahead`
- Compute the **closure** of the state:
  - If **A** $\rightarrow a \cdot$ **B**ω, l is in the state, add **B** $\rightarrow \cdot\, \gamma$, t to the state for each production **B** $\rightarrow \gamma$ and for each terminal `t` $\in$ FIRST*(ω`l`)

Deterministic LR(1) Automata

start

$S \to .E\ \$$

(1) $S \to E$

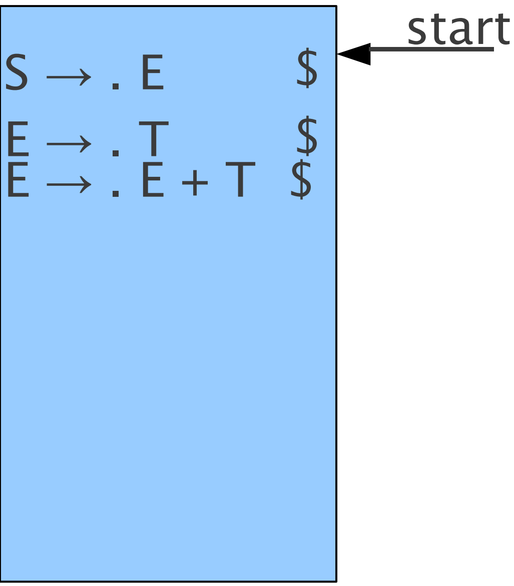
(2) $E \to T$

(3) $E \to E + T$

(4) $T \to \texttt{int}$

(5) $T \to (E)$

Deterministic LR(1) Automata



(1) S → E

(2) E → T

(3) E → E + T

(4) T → int

(5) T → (E)

Deterministic LR(1) Automata

start

$S \rightarrow .\ E$	$\$$
$E \rightarrow .\ T$	$\$$
$E \rightarrow .\ E + T$	$\$$
$E \rightarrow .\ T$	$+$
$E \rightarrow .\ E + T$	$+$

Deterministic LR(1) Automata

start

$S \rightarrow . E$	\$
$E \rightarrow . T$	\$
$E \rightarrow . E + T$	\$
$E \rightarrow . T$	+
$E \rightarrow . E + T$	+
$T \rightarrow . int$	\$
$T \rightarrow . (E)$	\$

Deterministic LR(1) Automata

start

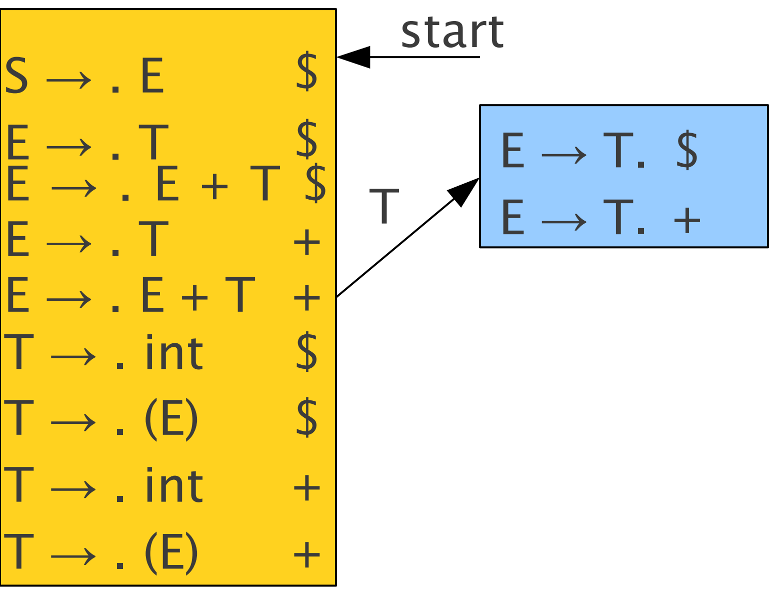
$S \rightarrow .\ E$	$
$E \rightarrow .\ T$	$
$E \rightarrow .\ E + T$	$
$E \rightarrow .\ T$	+
$E \rightarrow .\ E + T$	+
$T \rightarrow .\ int$	$
$T \rightarrow .\ (E)$	$
$T \rightarrow .\ int$	+
$T \rightarrow .\ (E)$	+

Deterministic LR(1) Automata

start

$S \rightarrow . E$	$\$$
$E \rightarrow . T$	$\$$
$E \rightarrow . E + T$	$\$$
$E \rightarrow . T$	$+$
$E \rightarrow . E + T$	$+$
$T \rightarrow . int$	$\$$
$T \rightarrow . (E)$	$\$$
$T \rightarrow . int$	$+$
$T \rightarrow . (E)$	$+$

Deterministic LR(1) Automata



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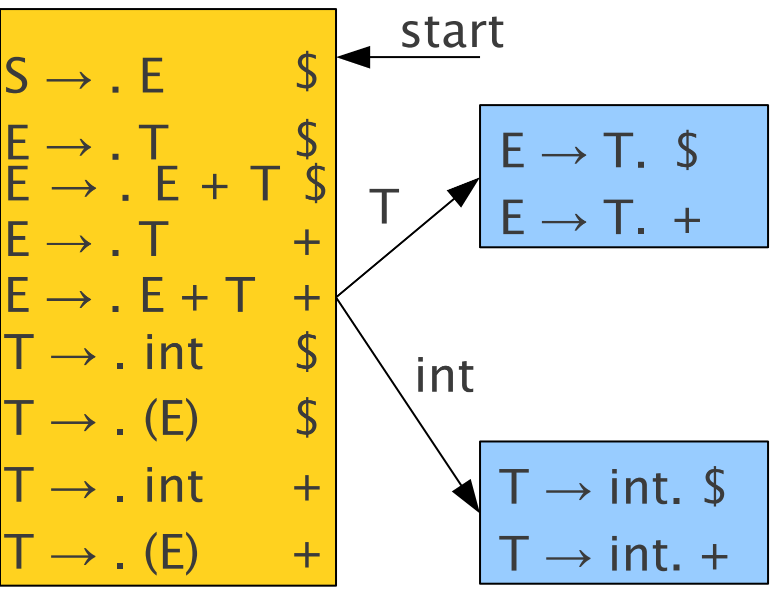

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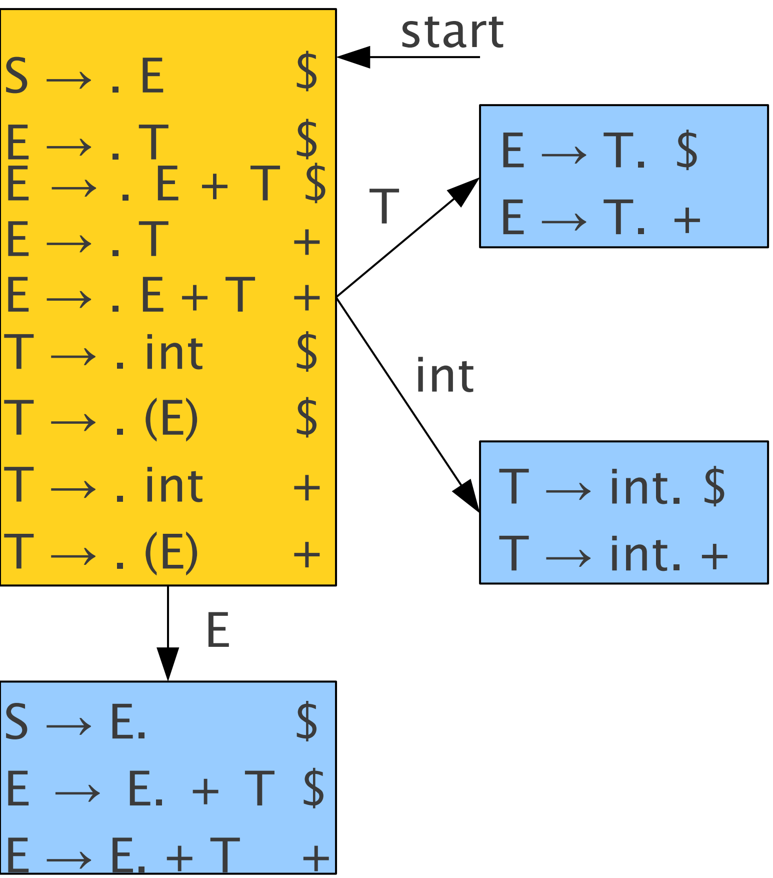

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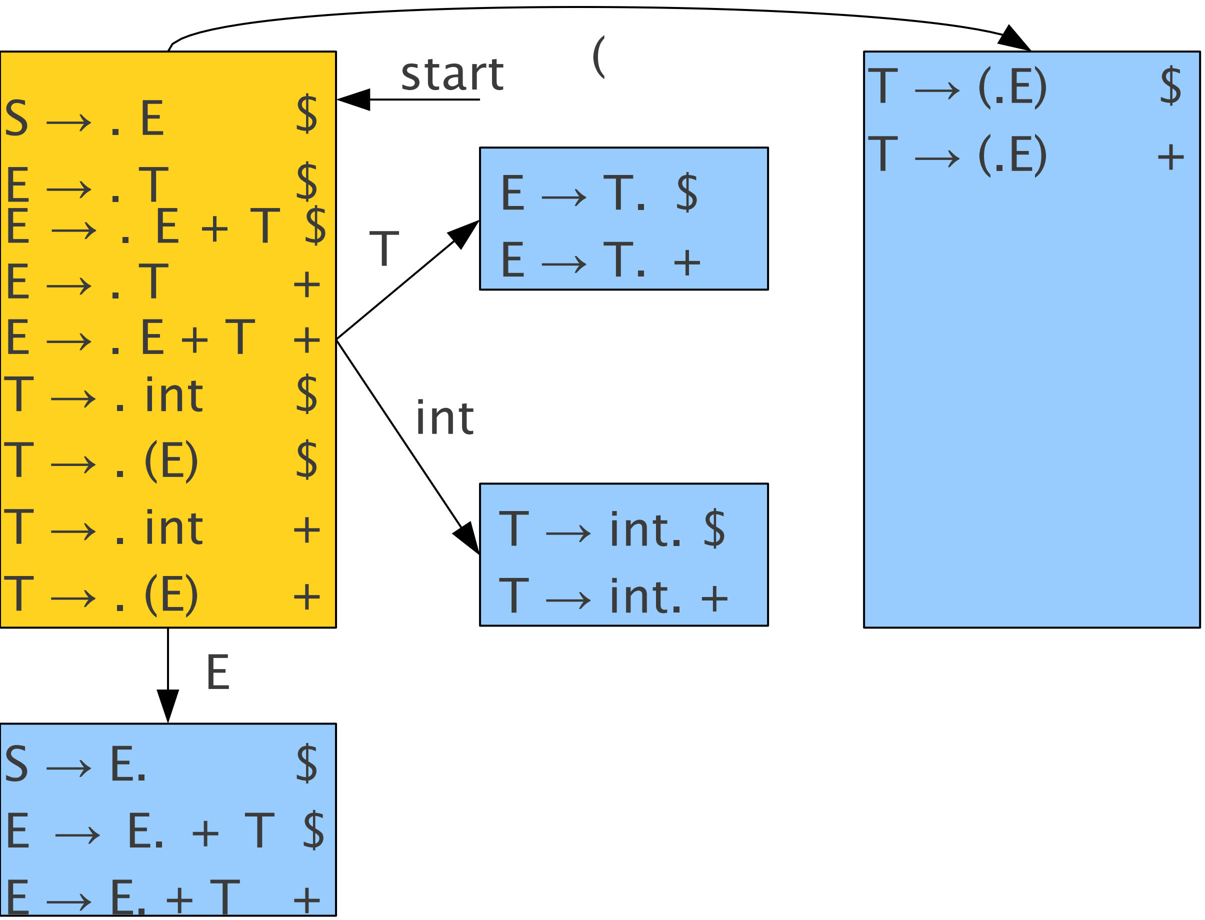

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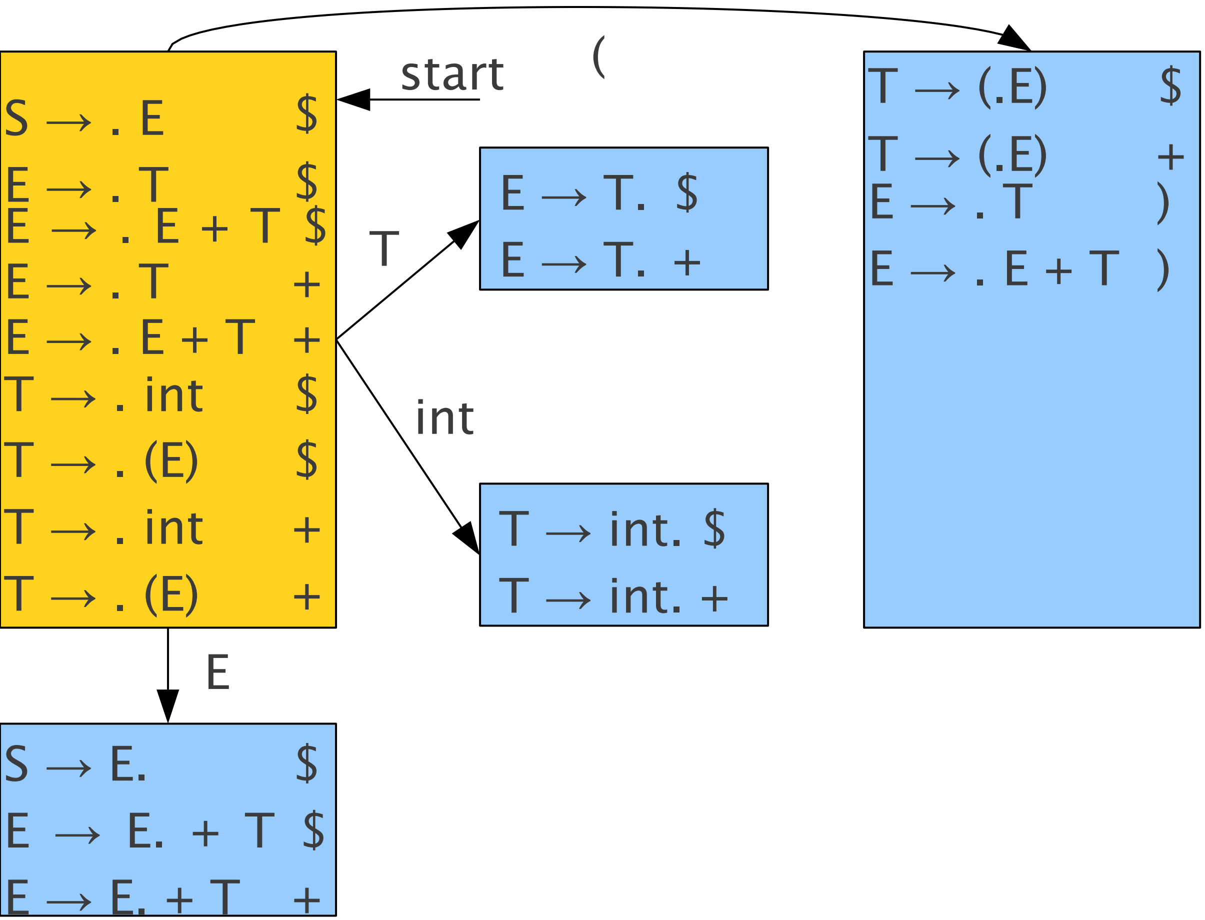

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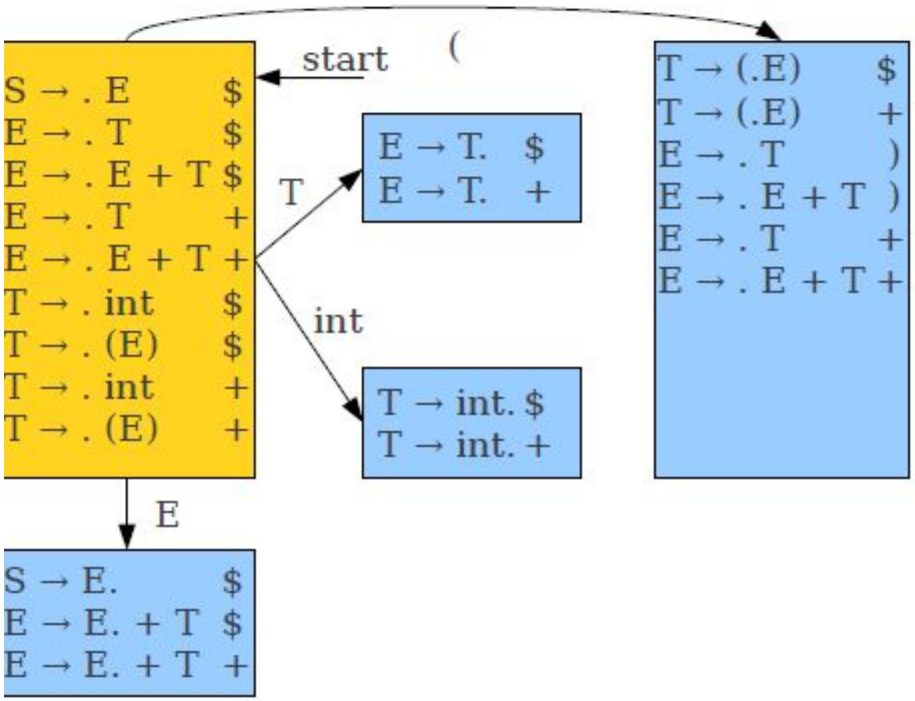

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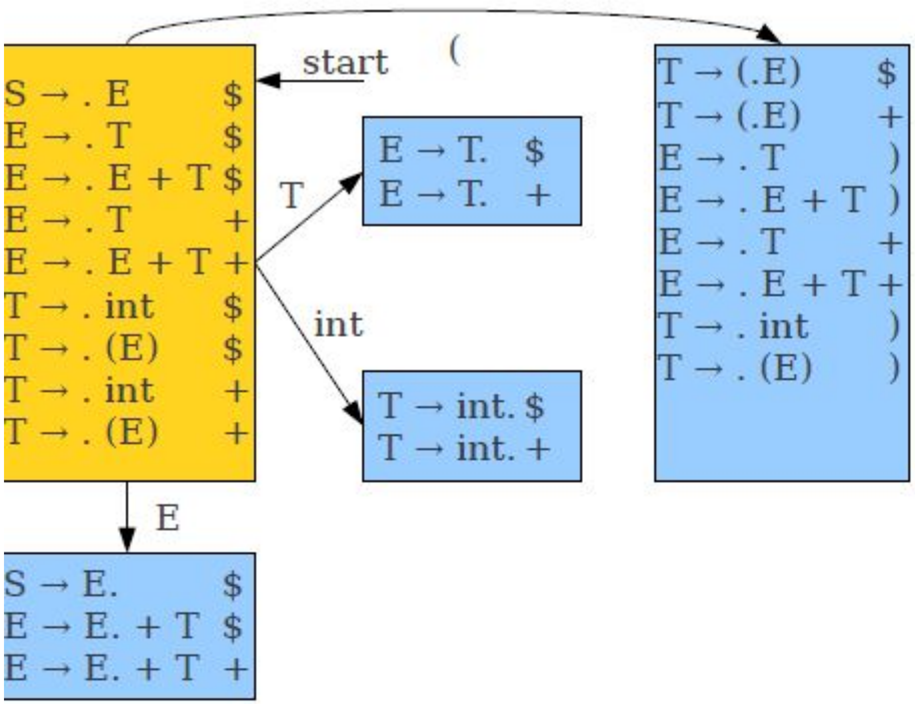

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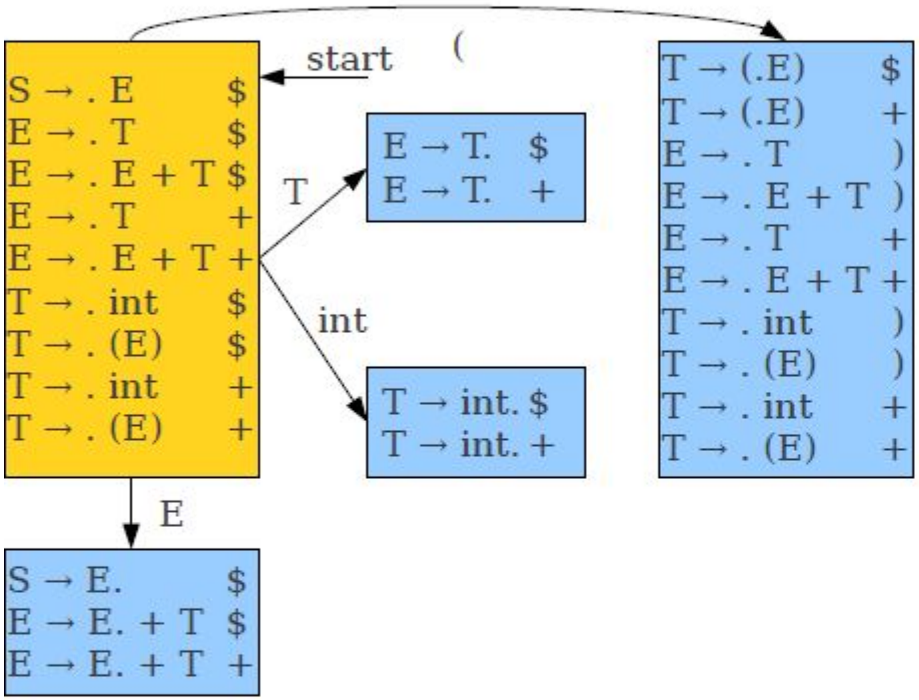

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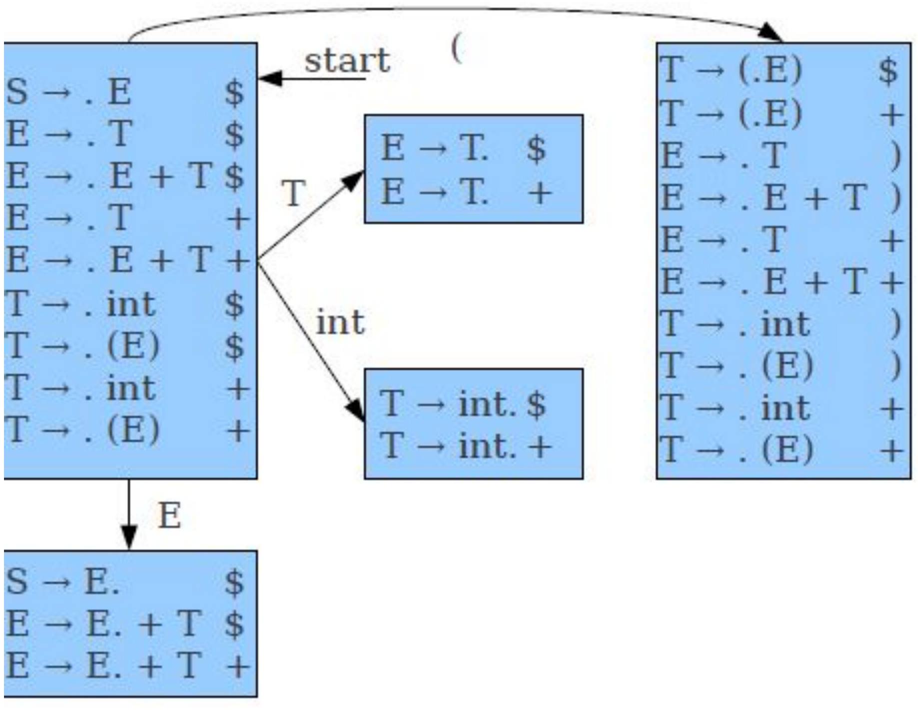

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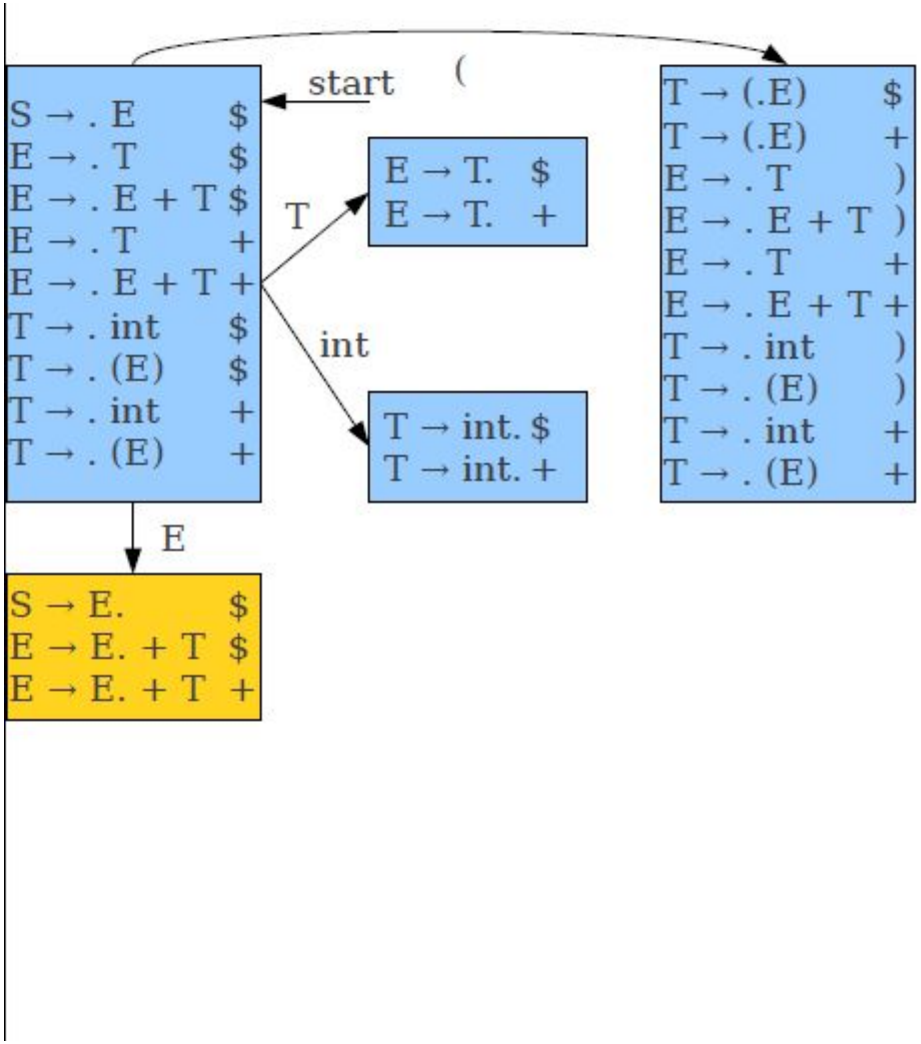

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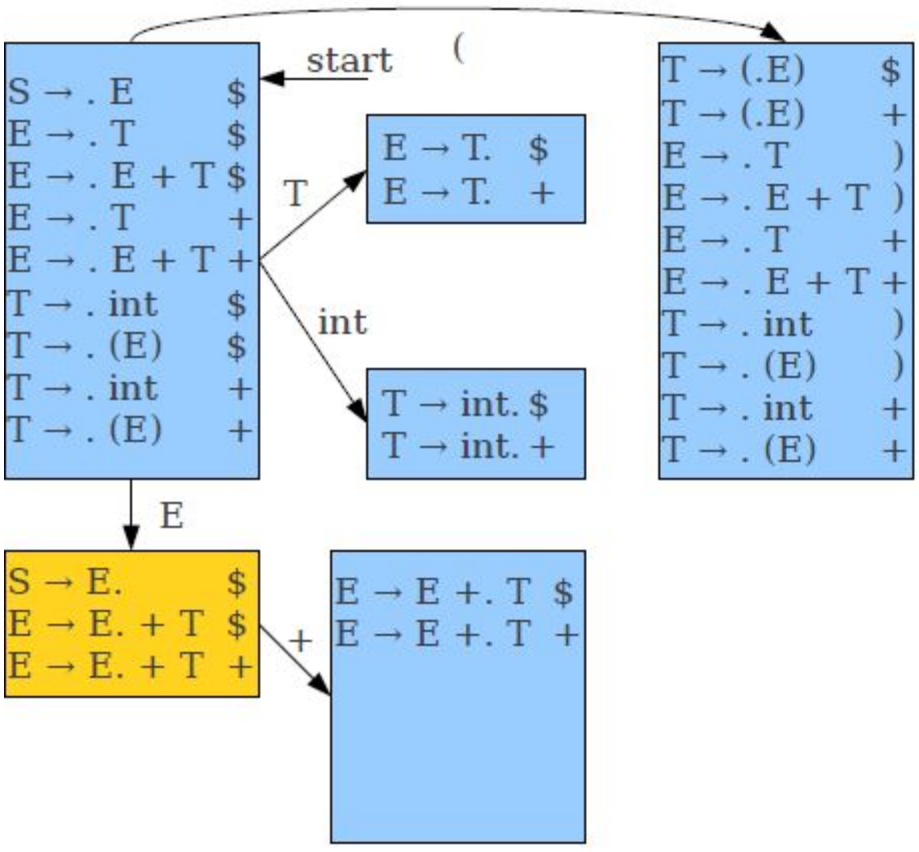

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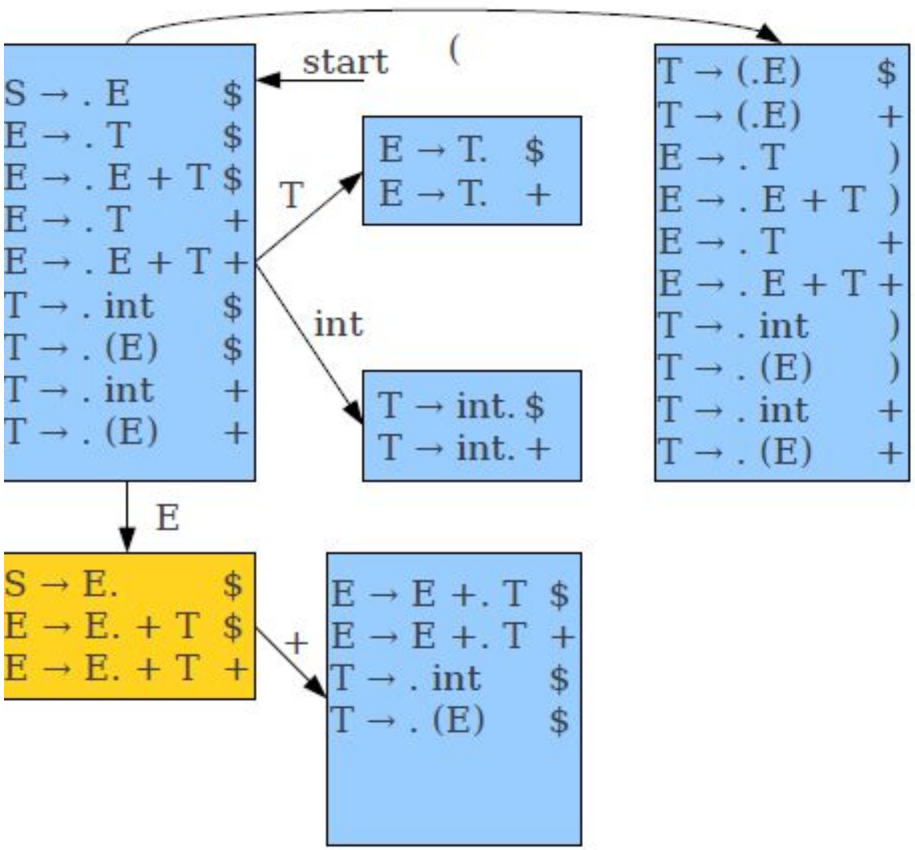

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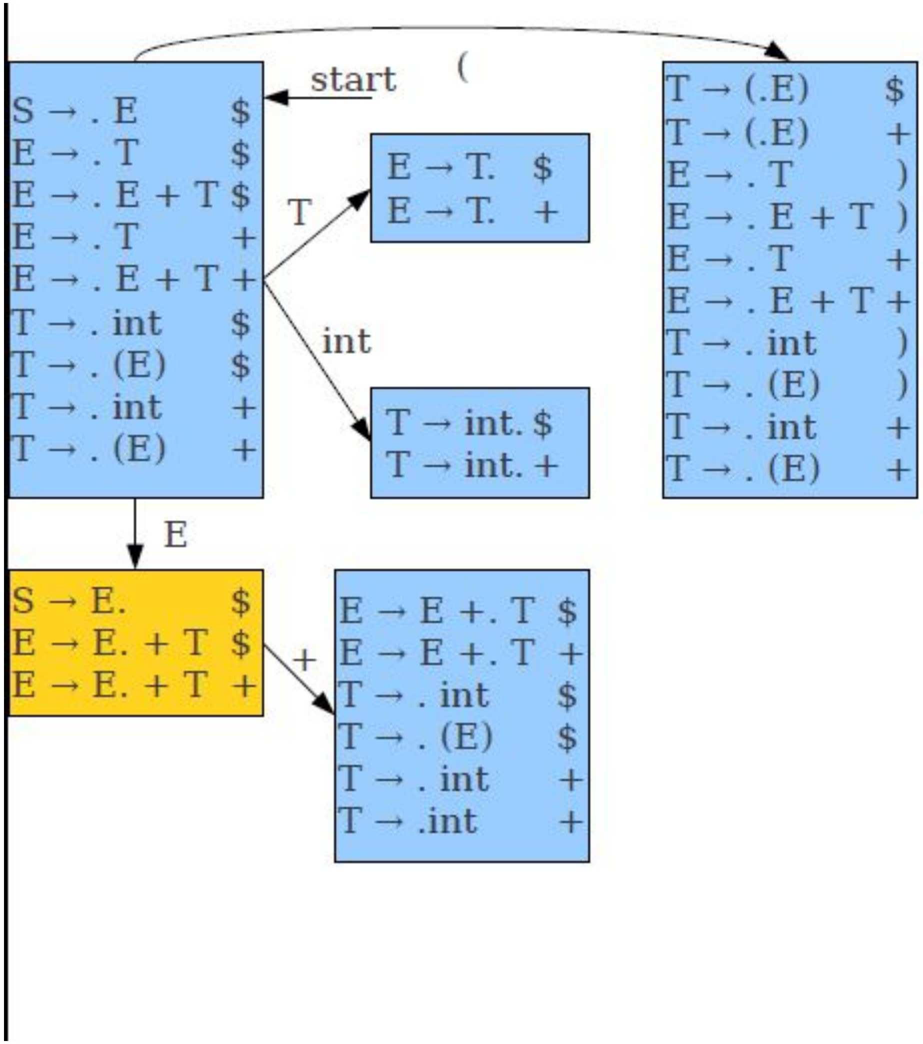

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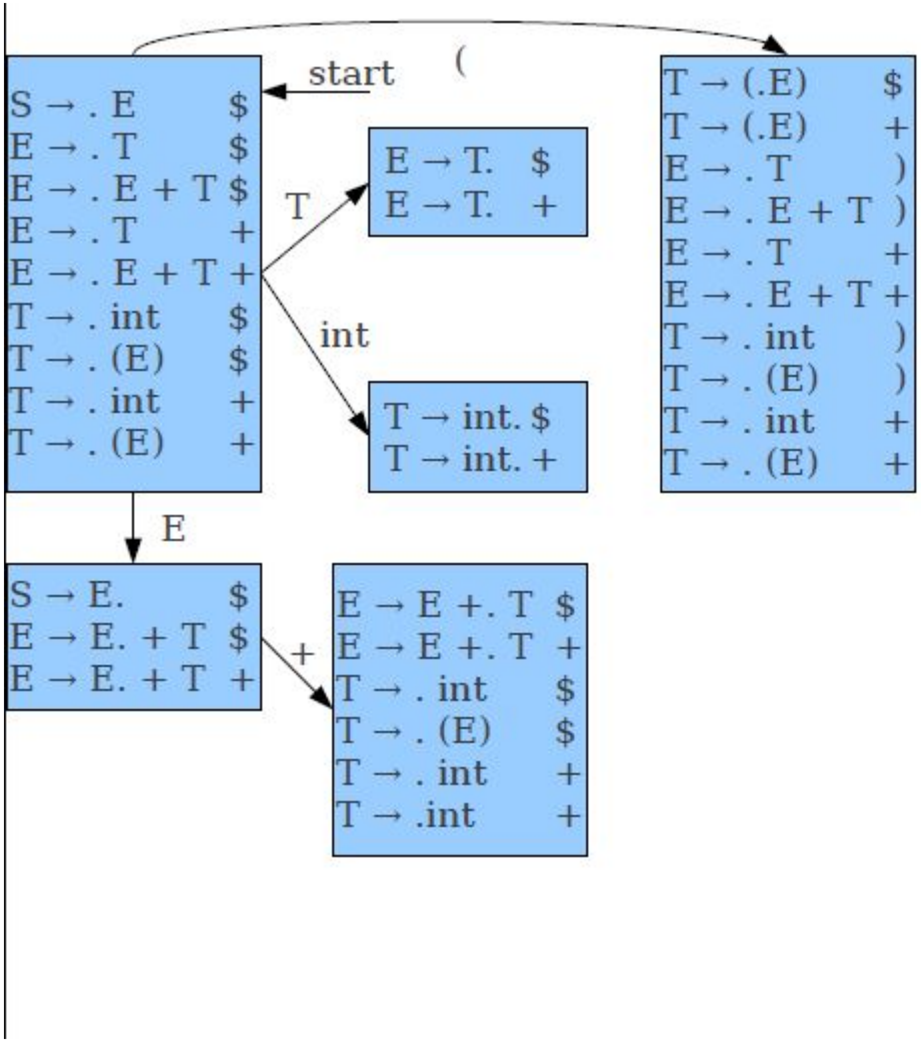

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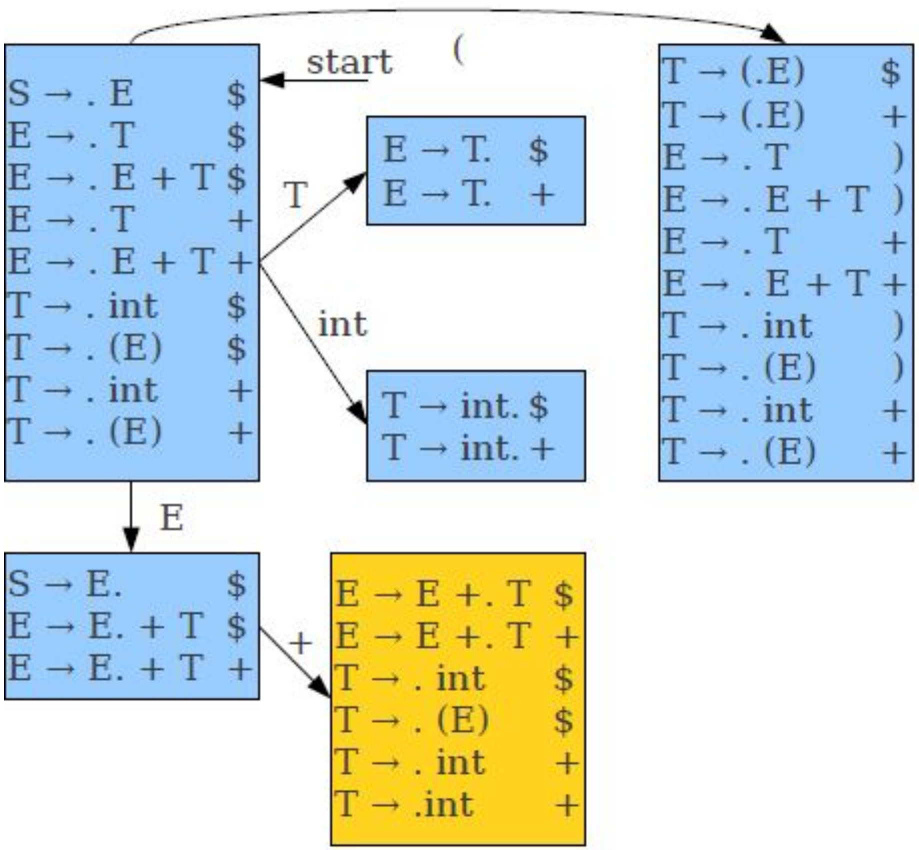

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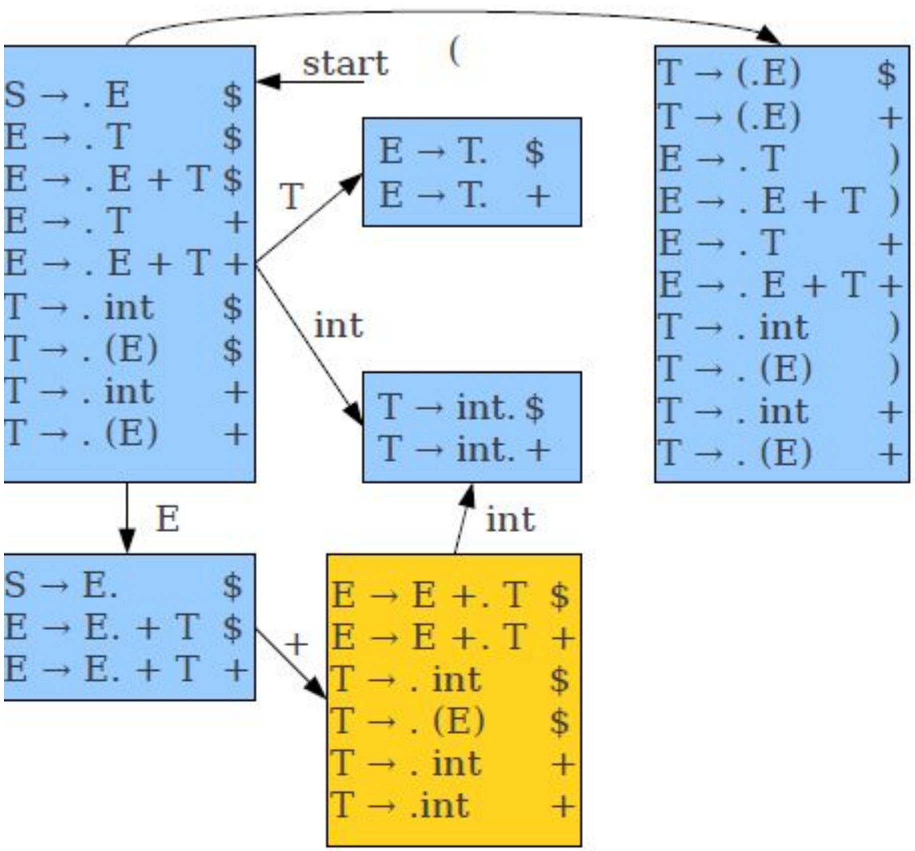

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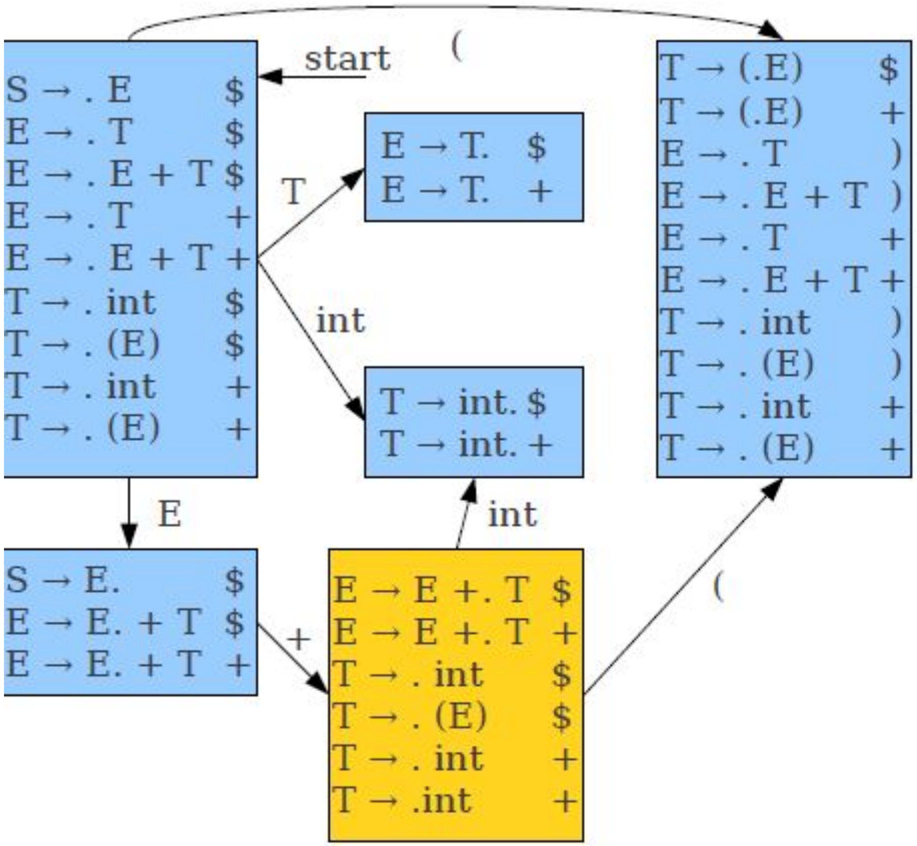
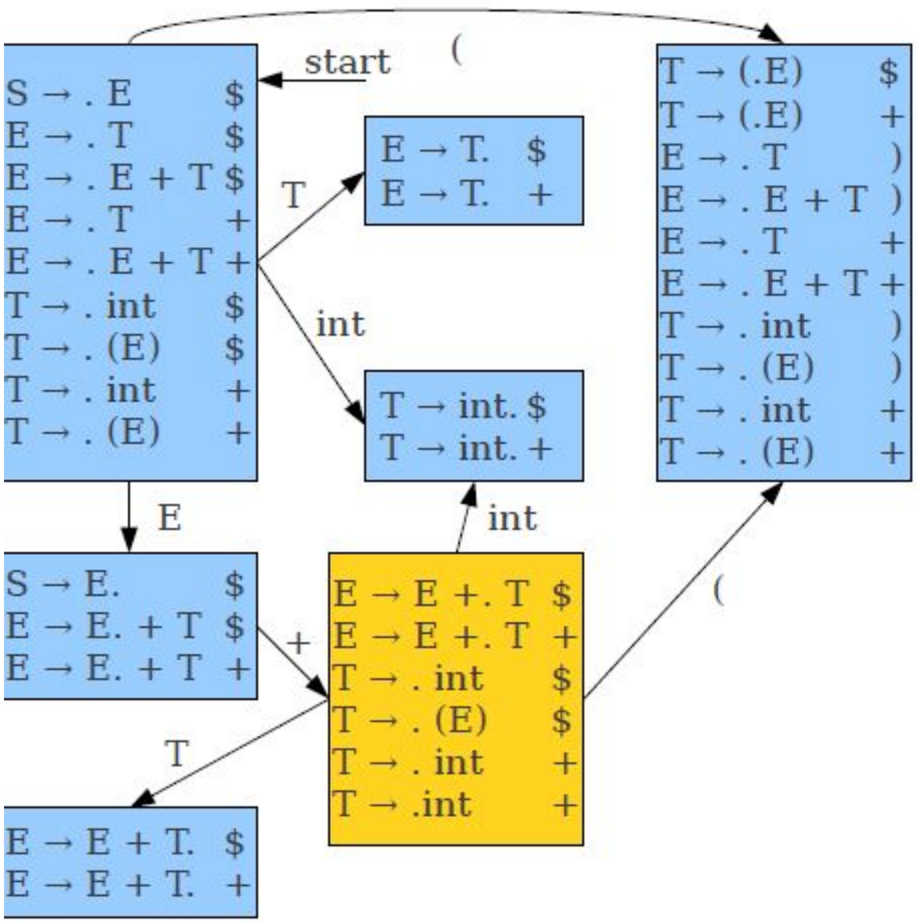
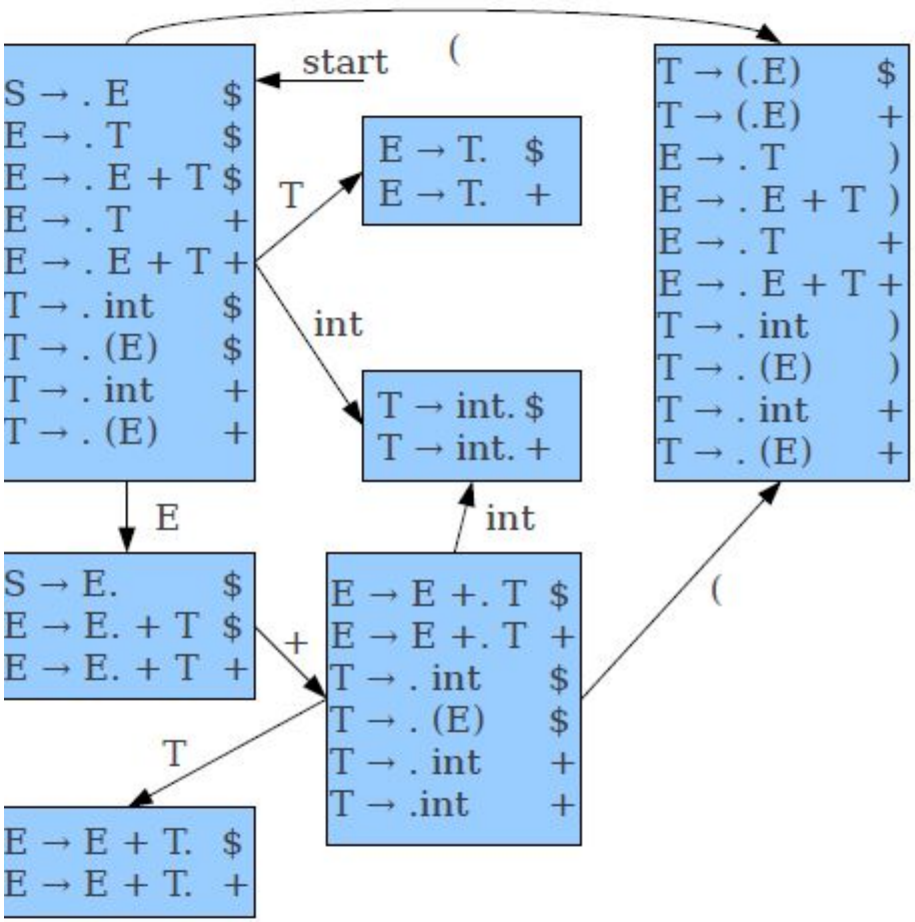


Deterministic LR(1) Automata



Deterministic LR(1) Automata



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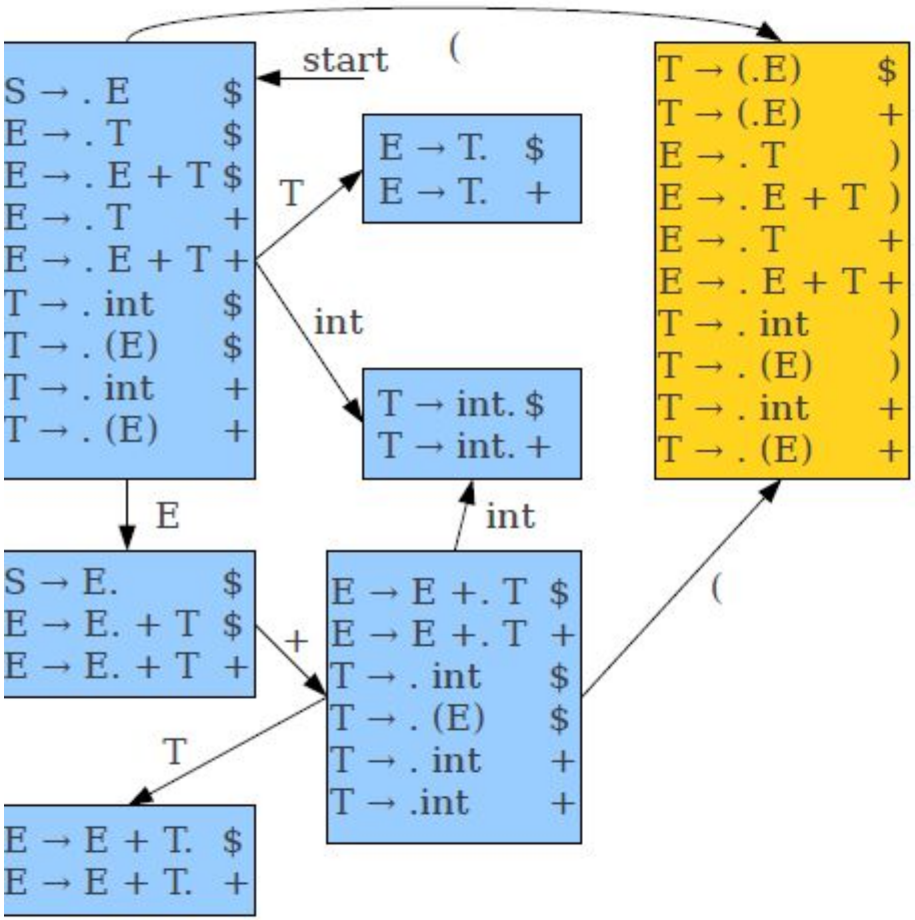

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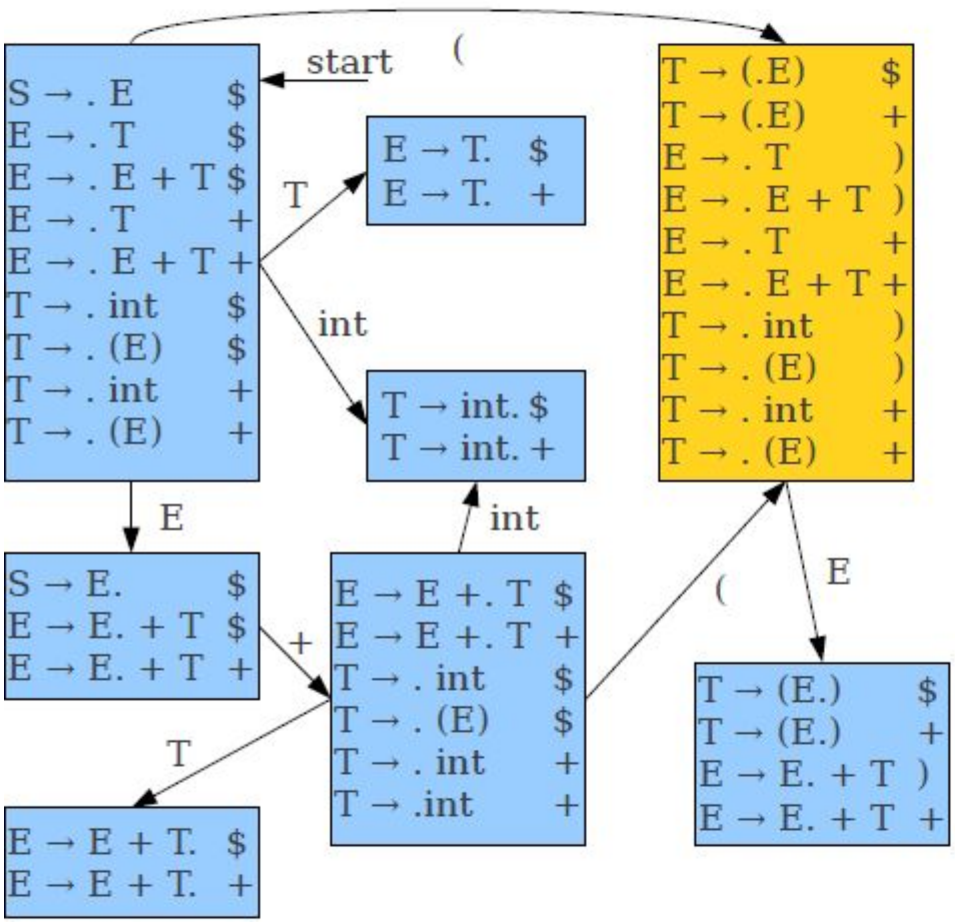

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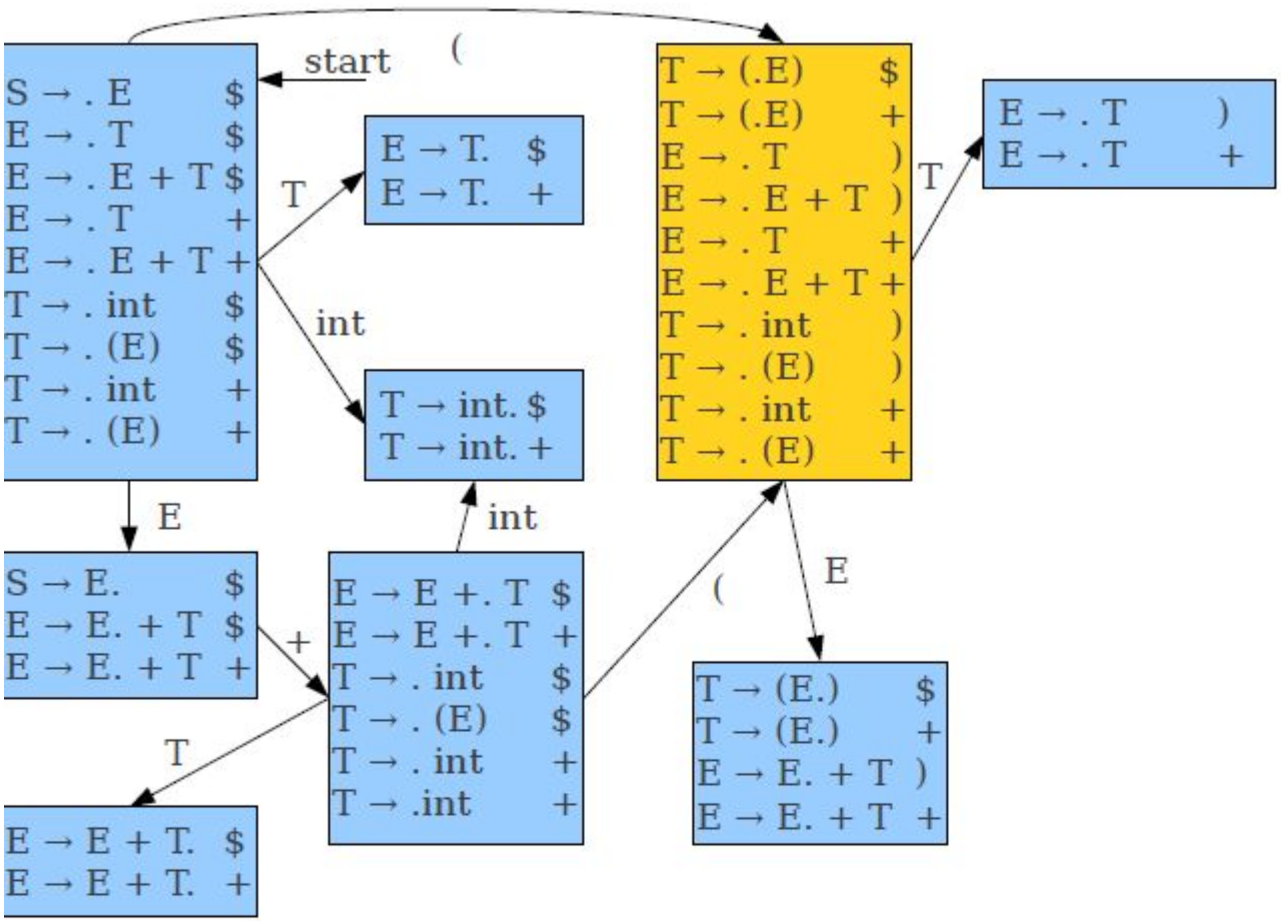

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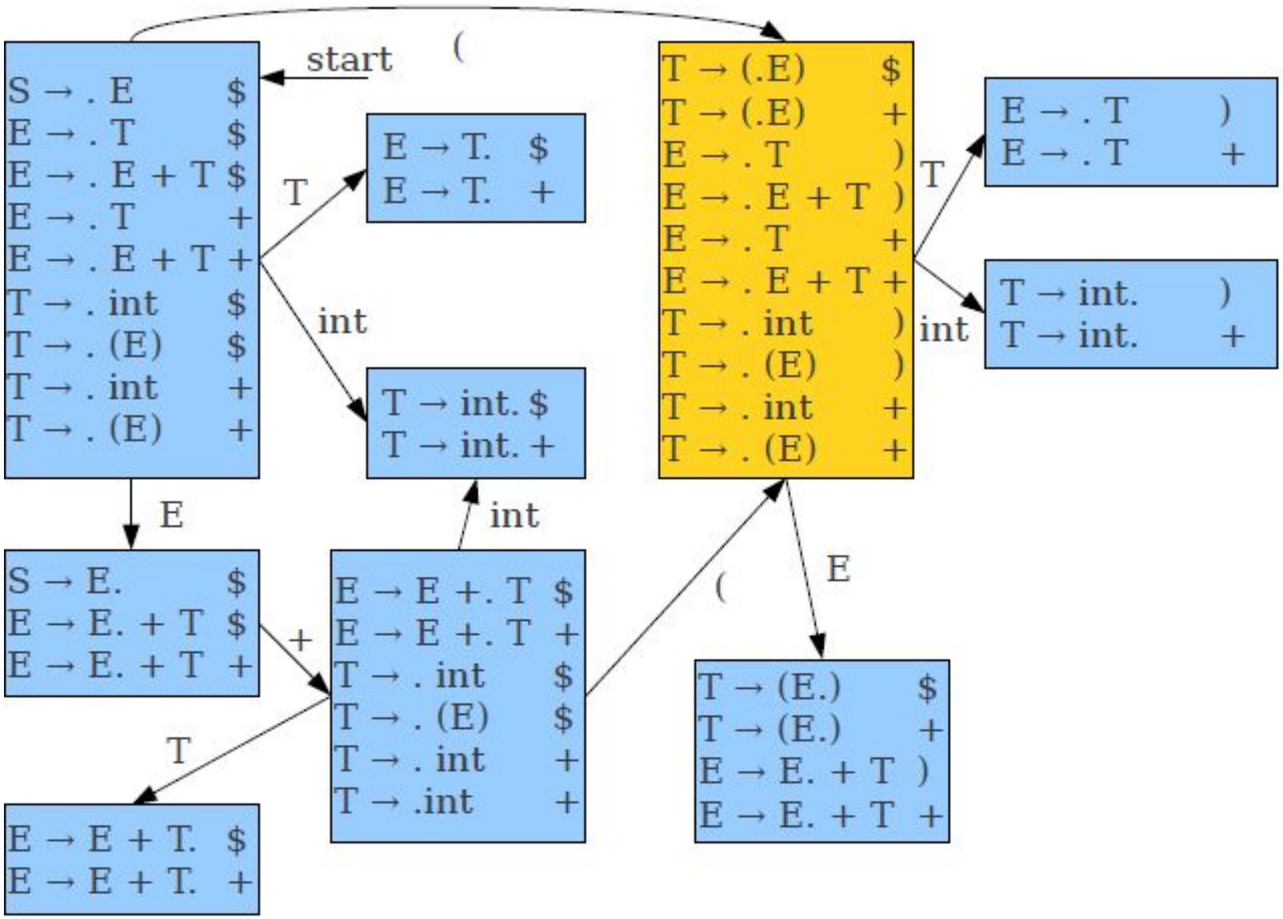

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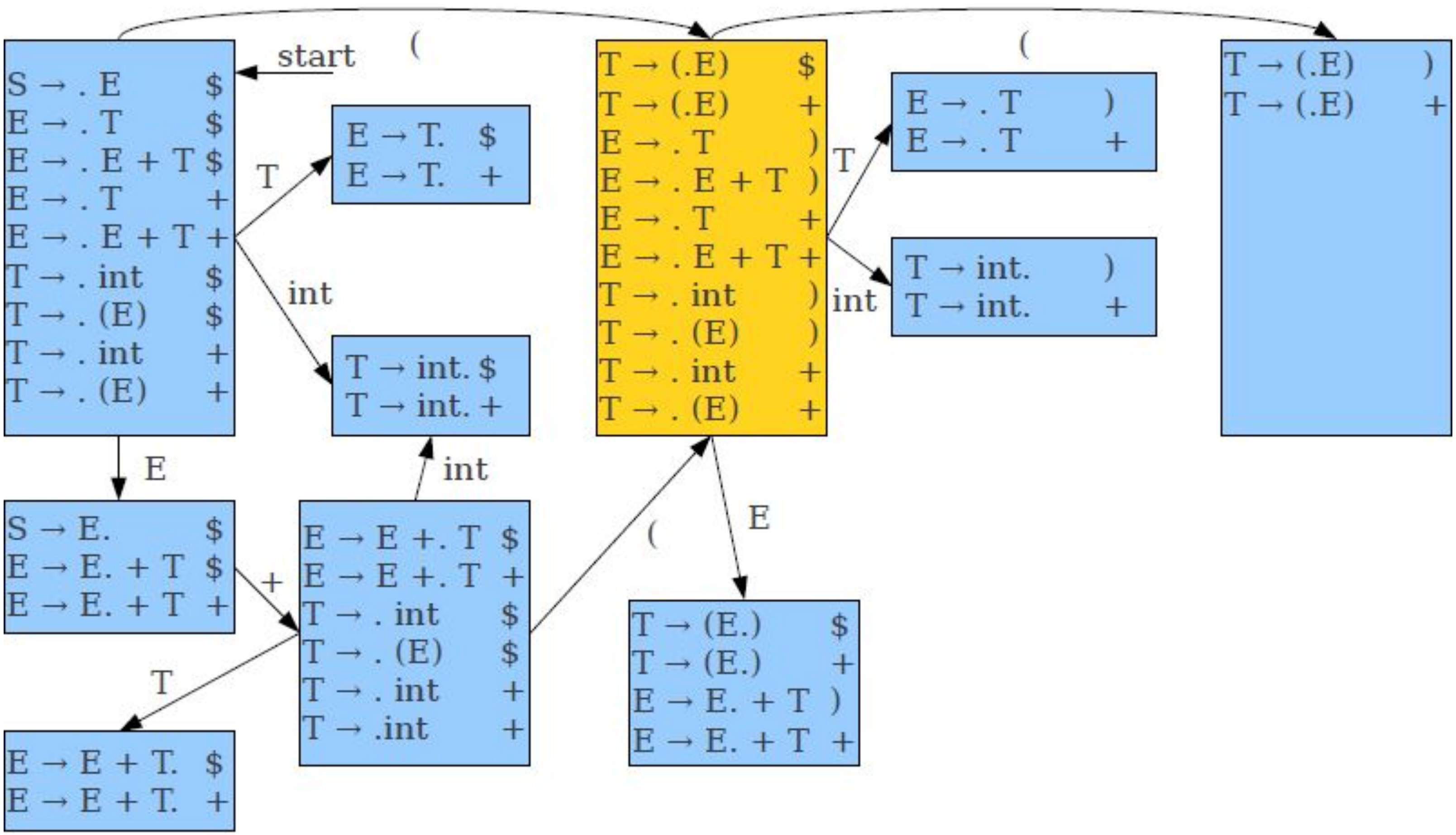

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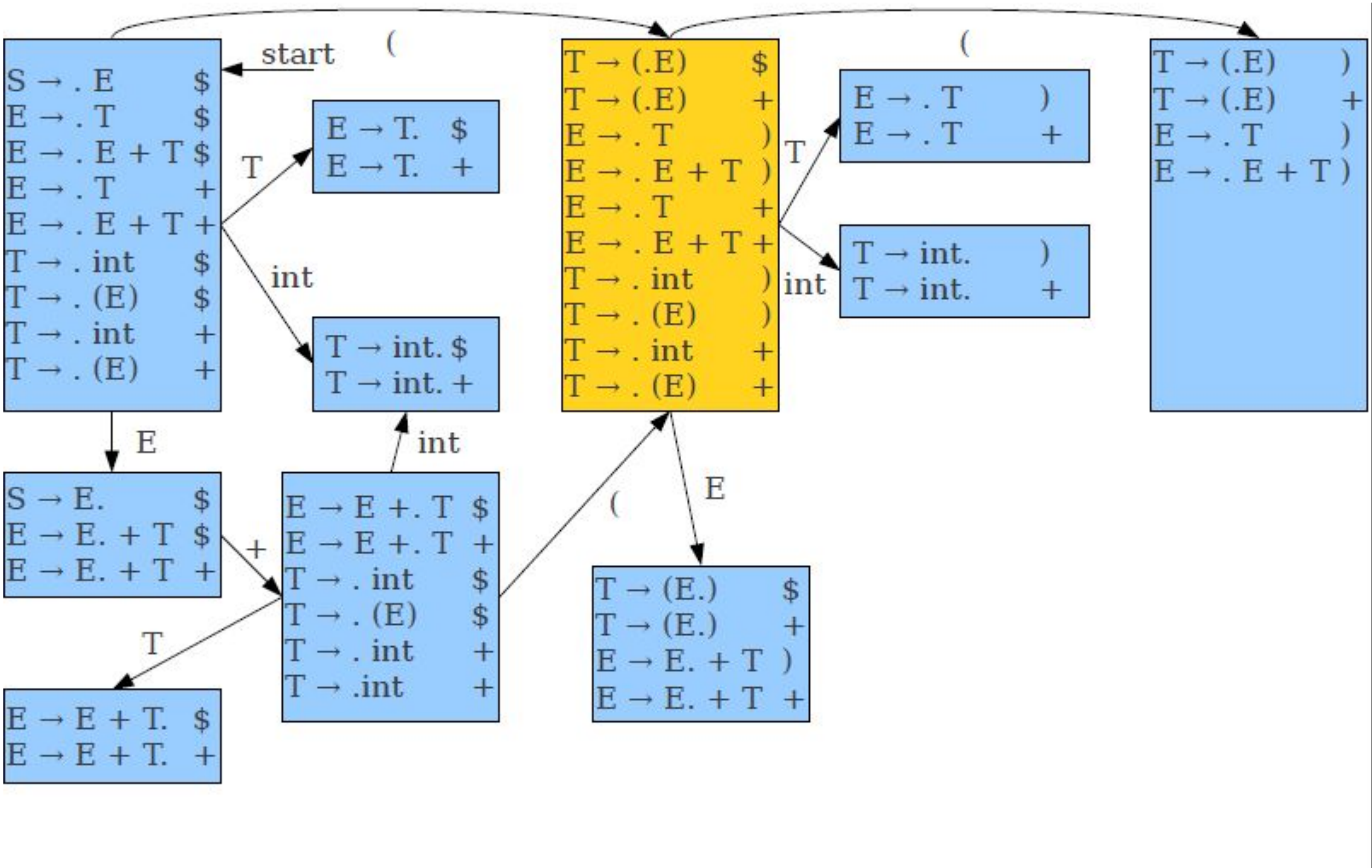

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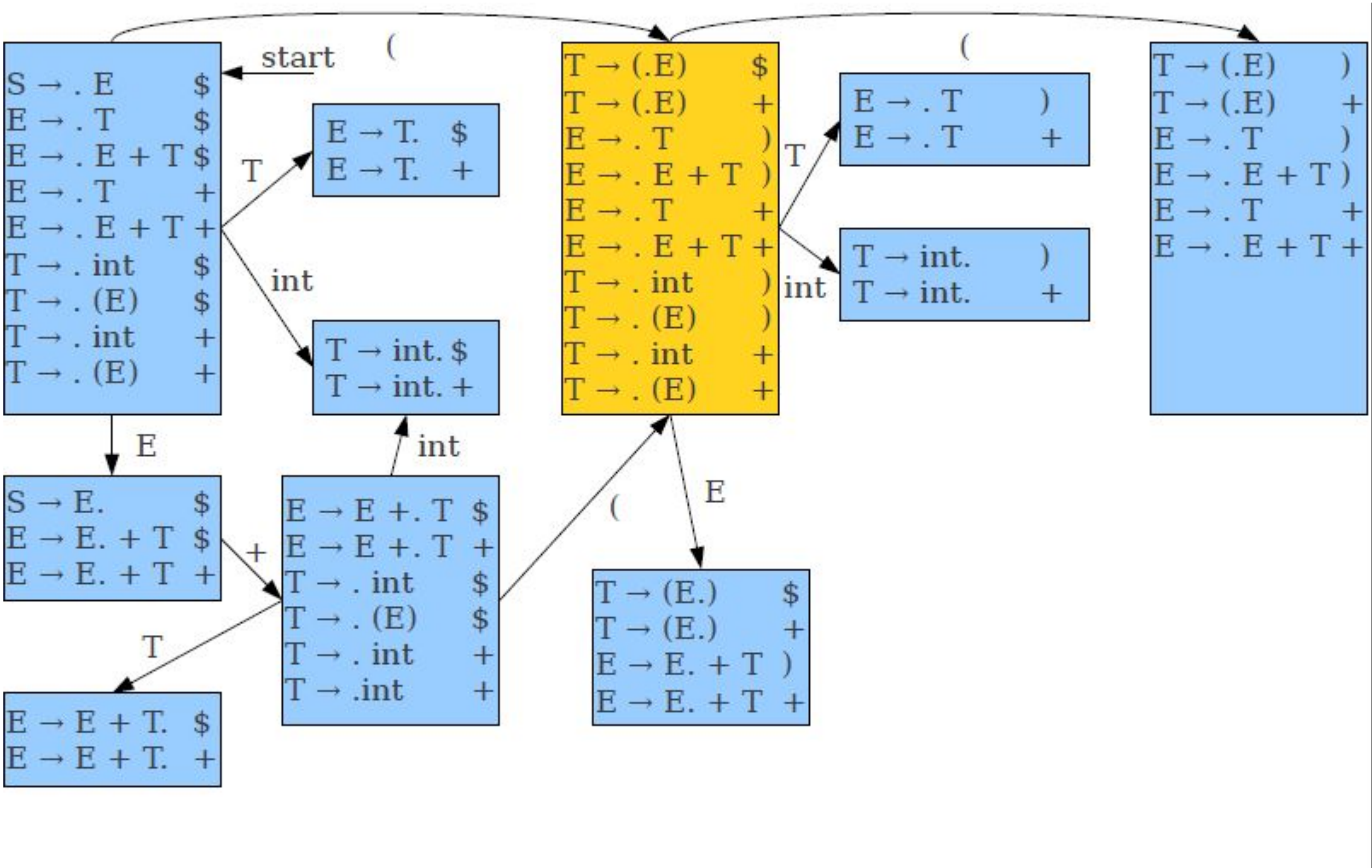

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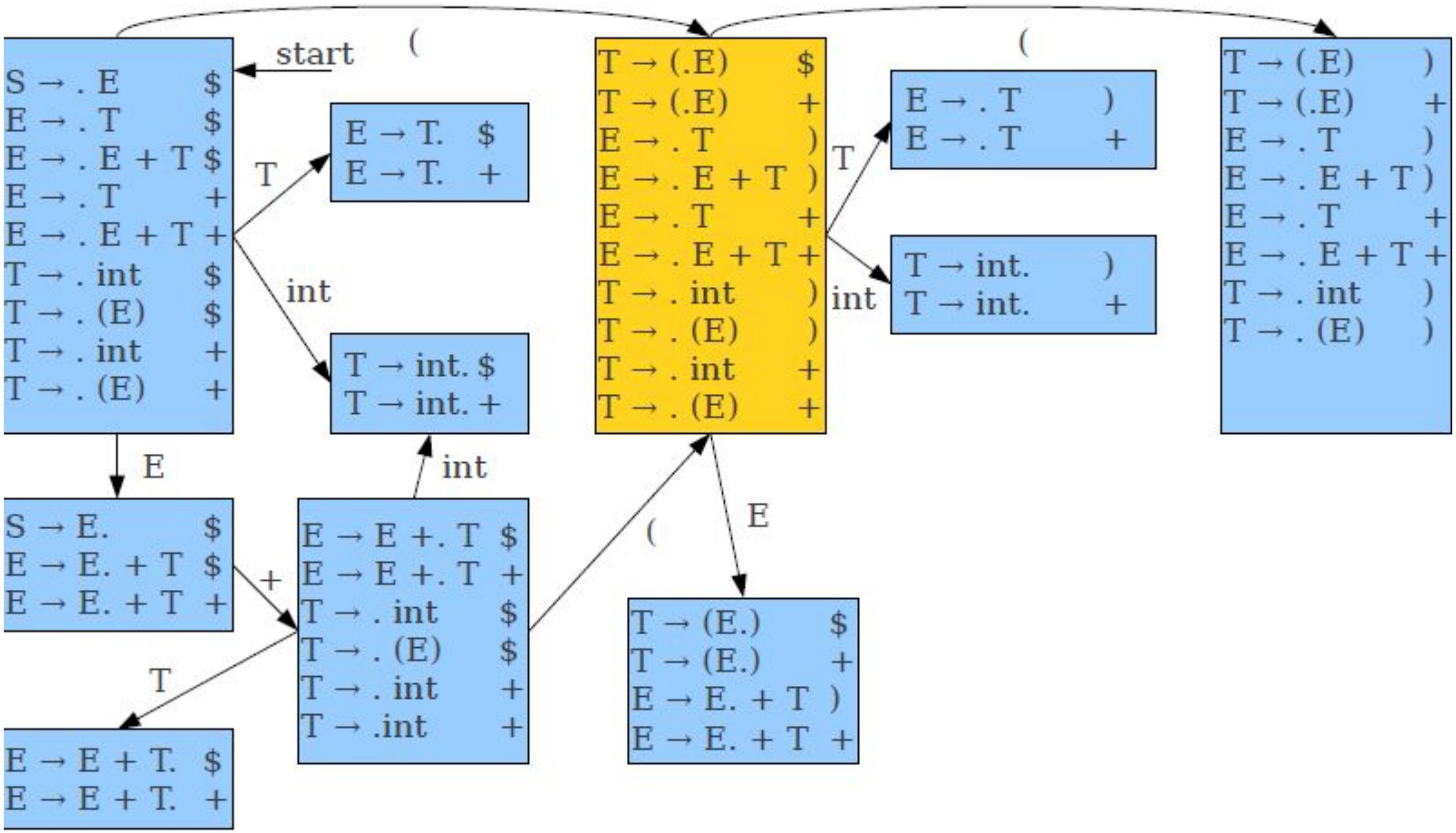

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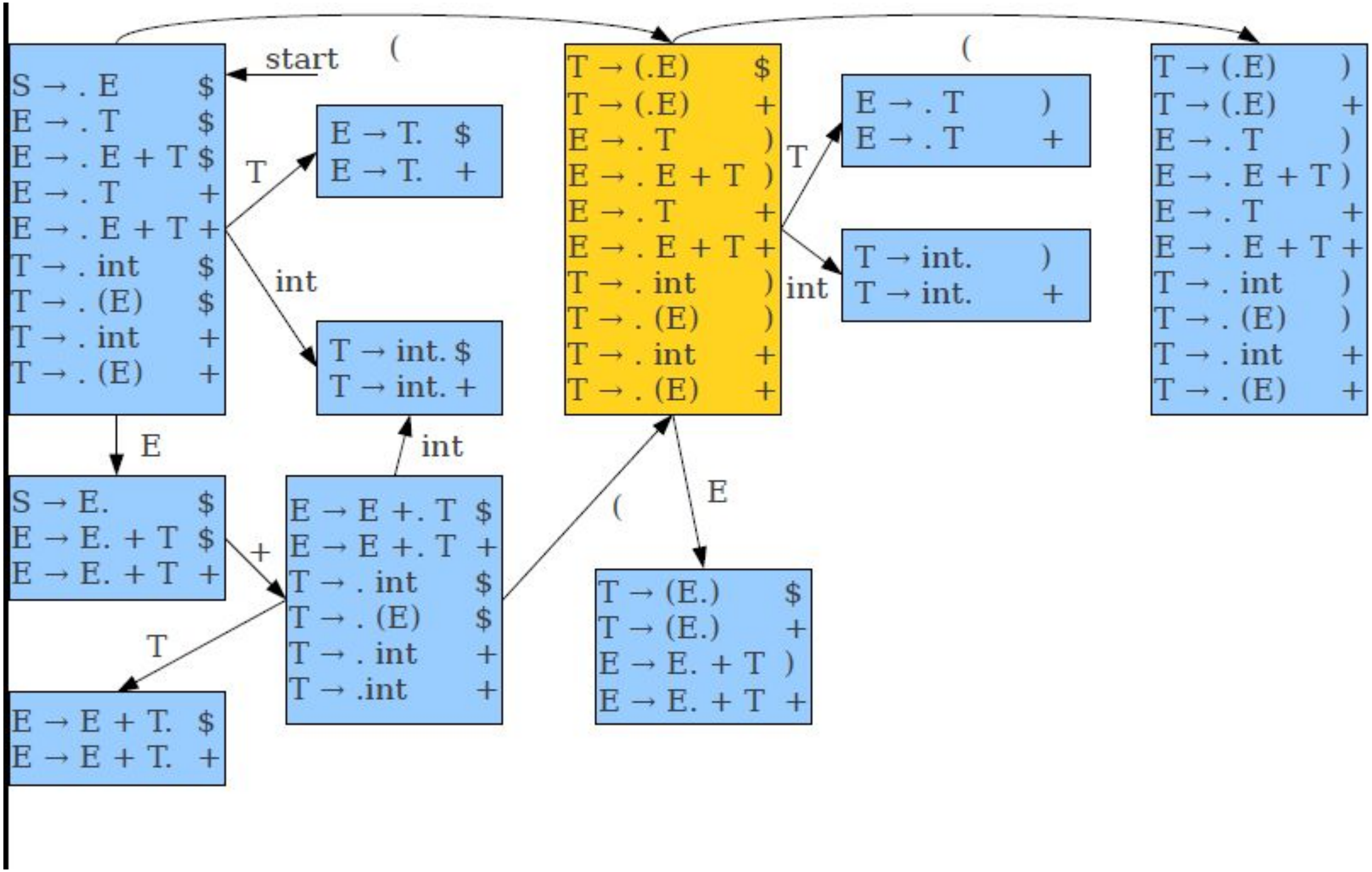

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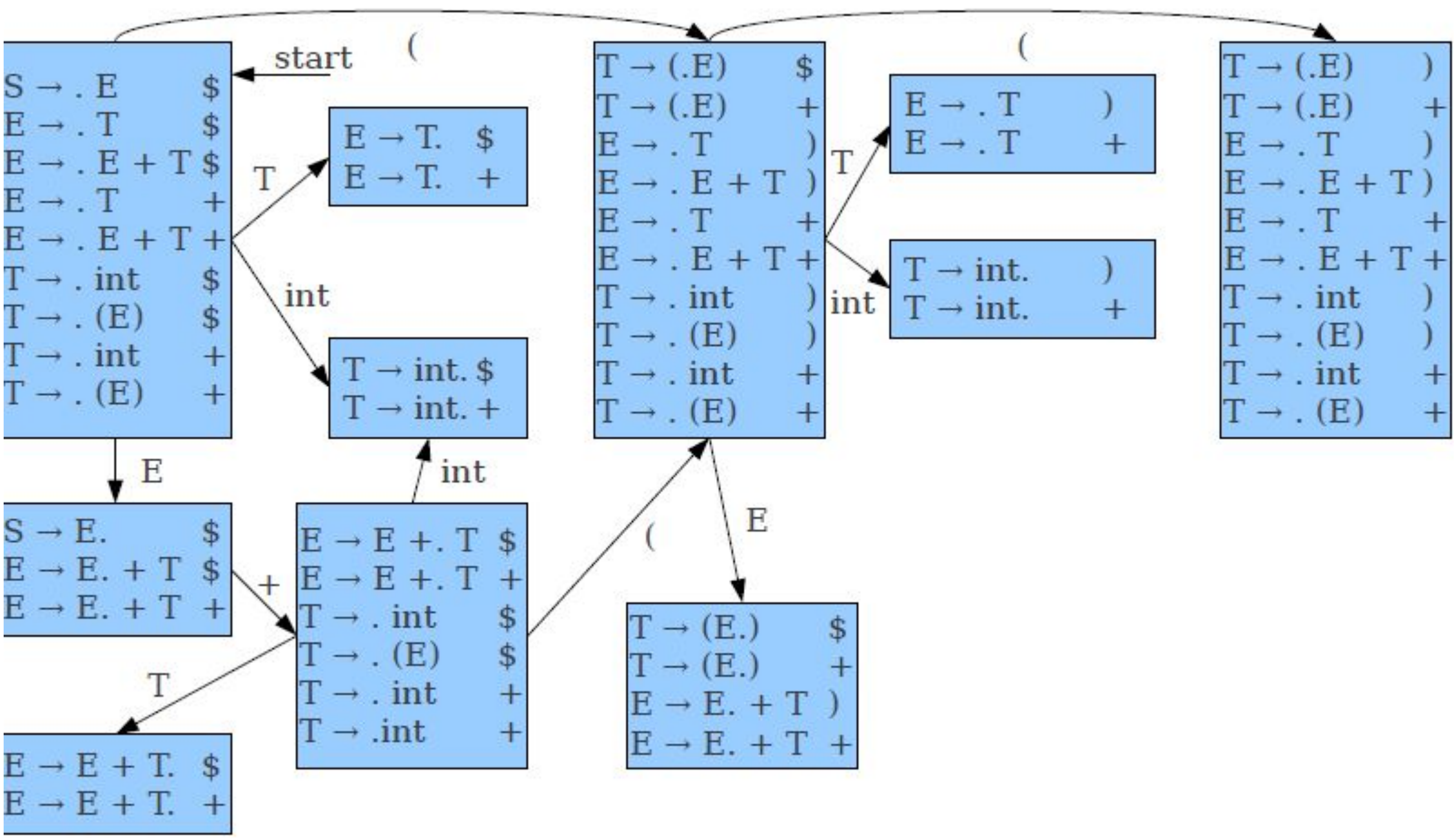

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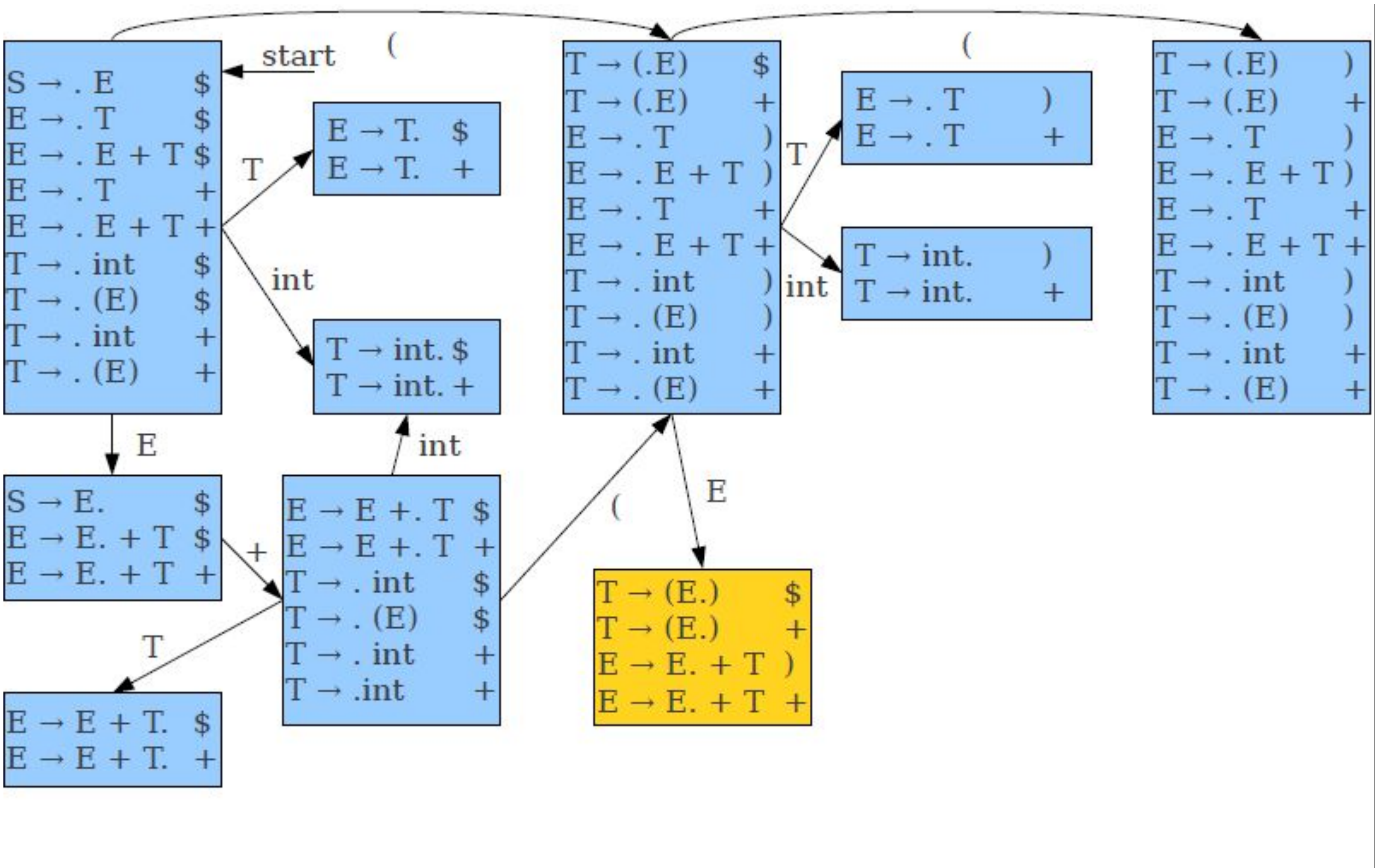

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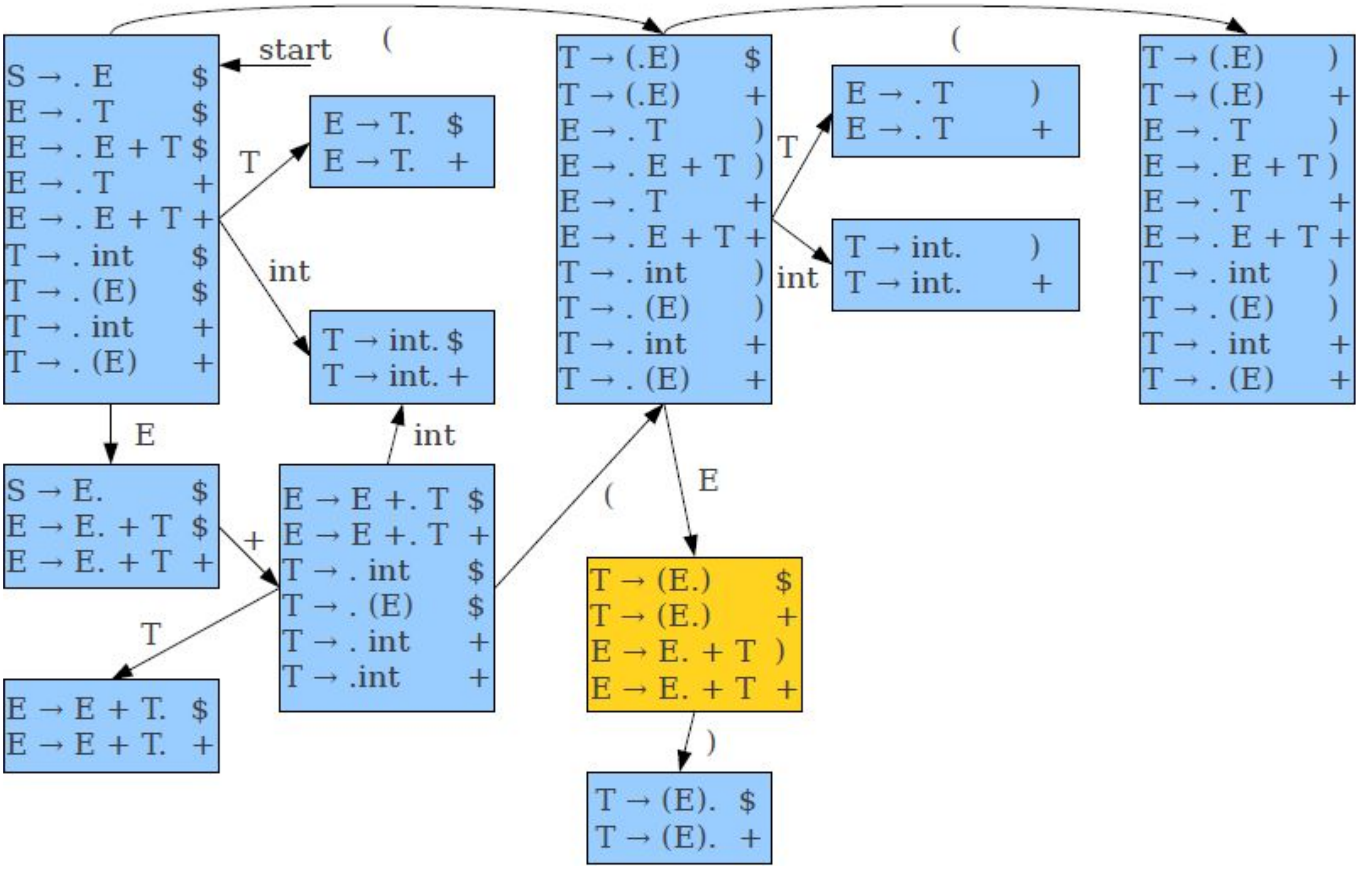

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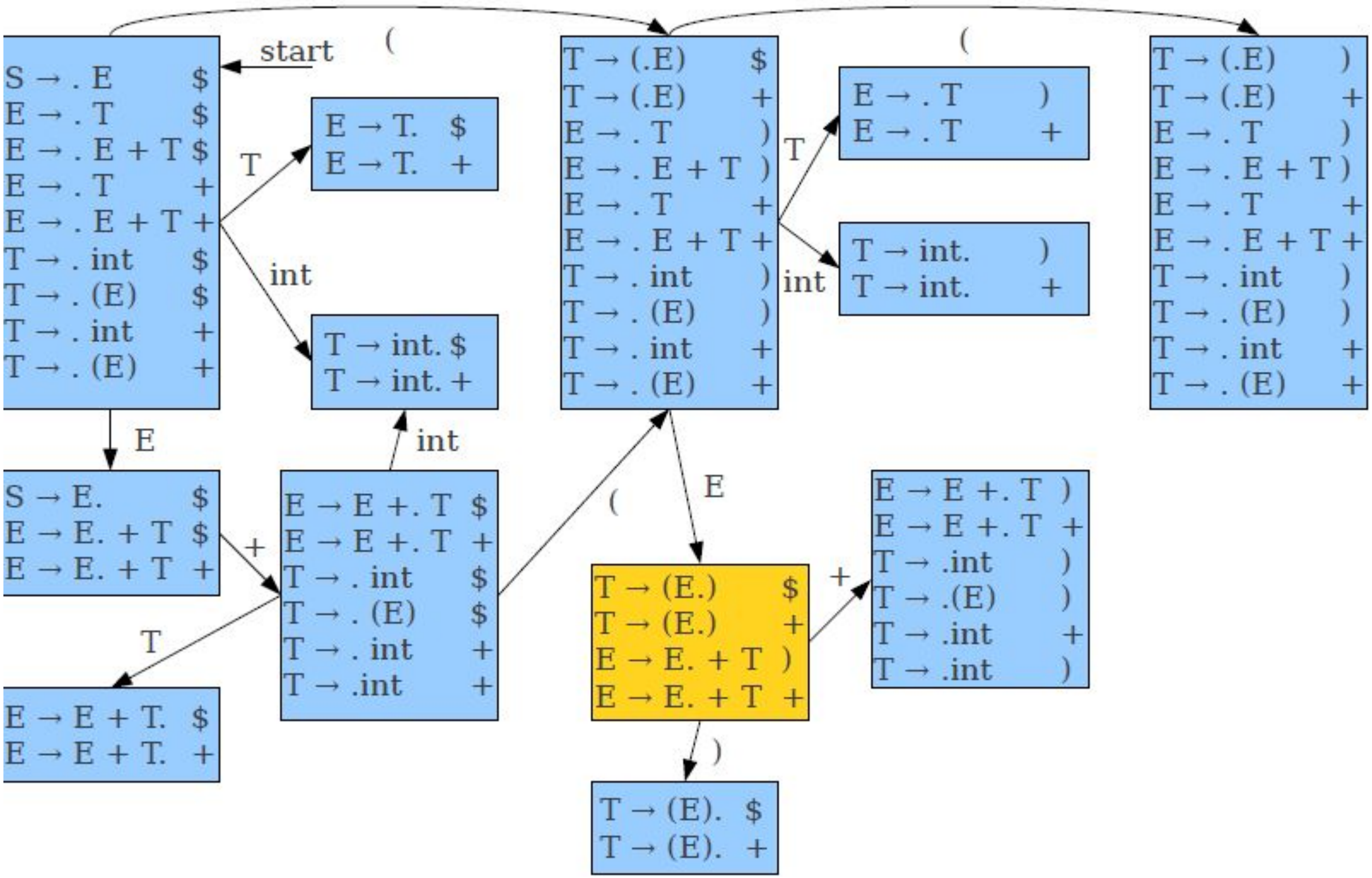

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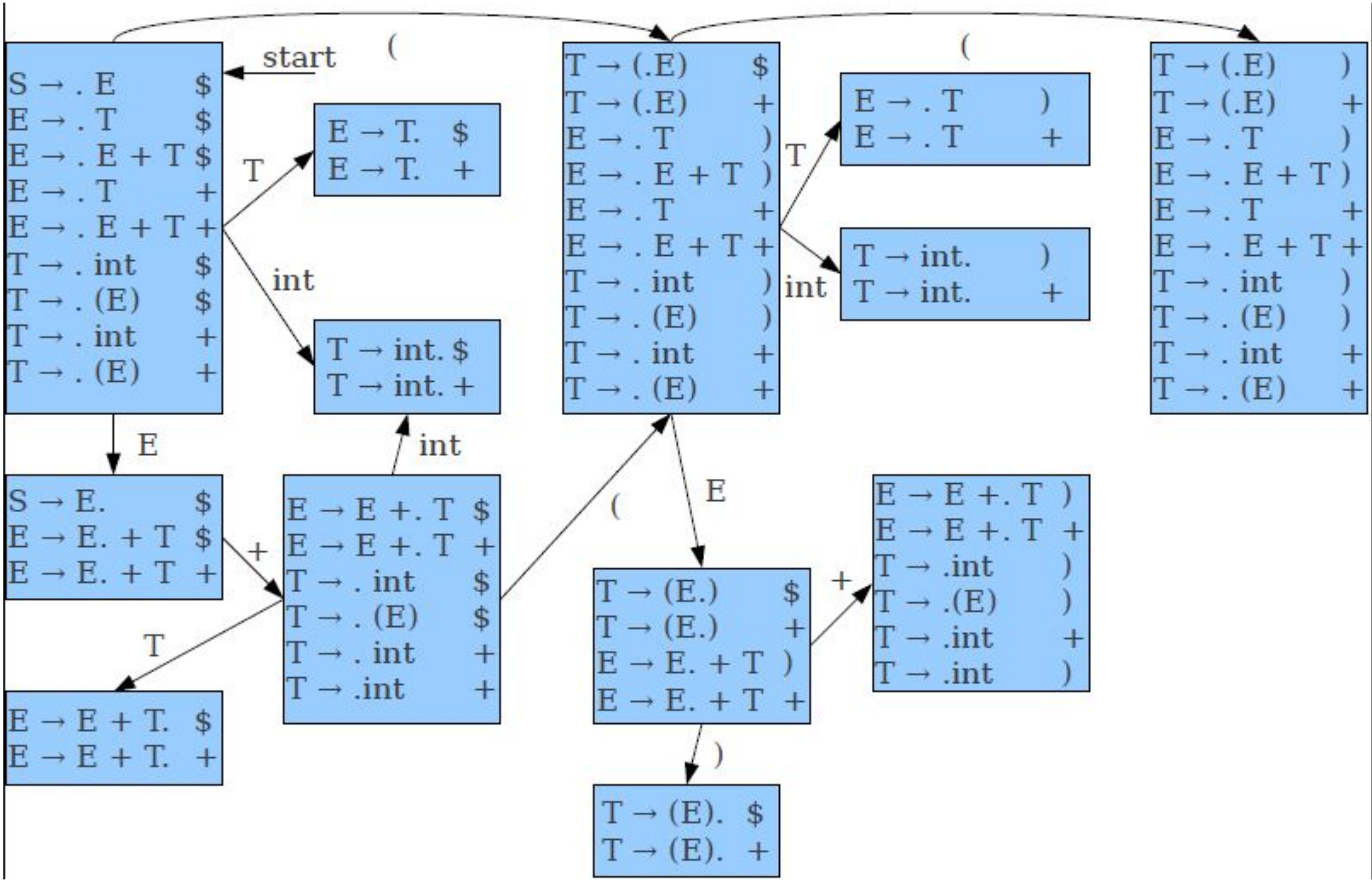

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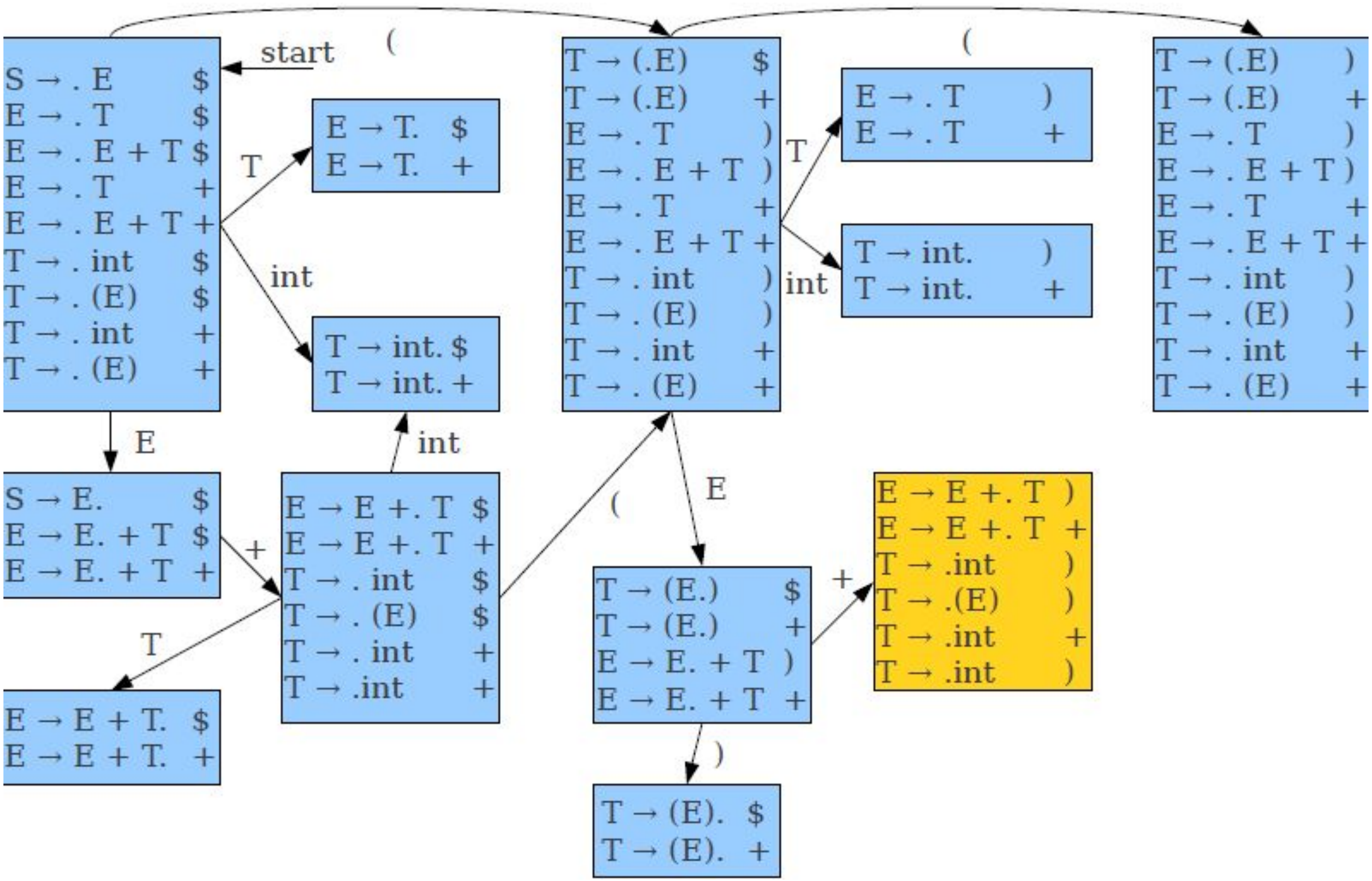

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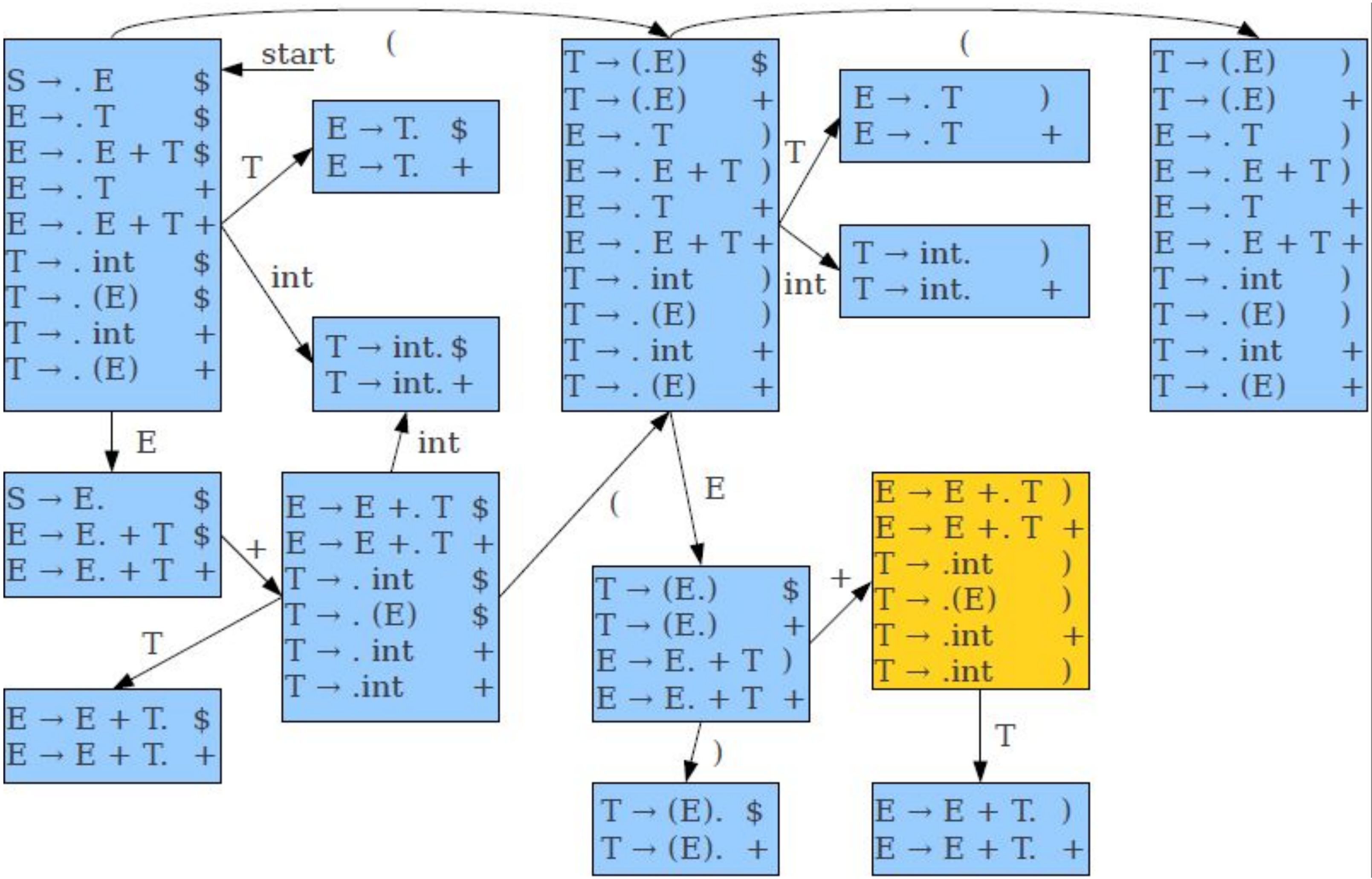

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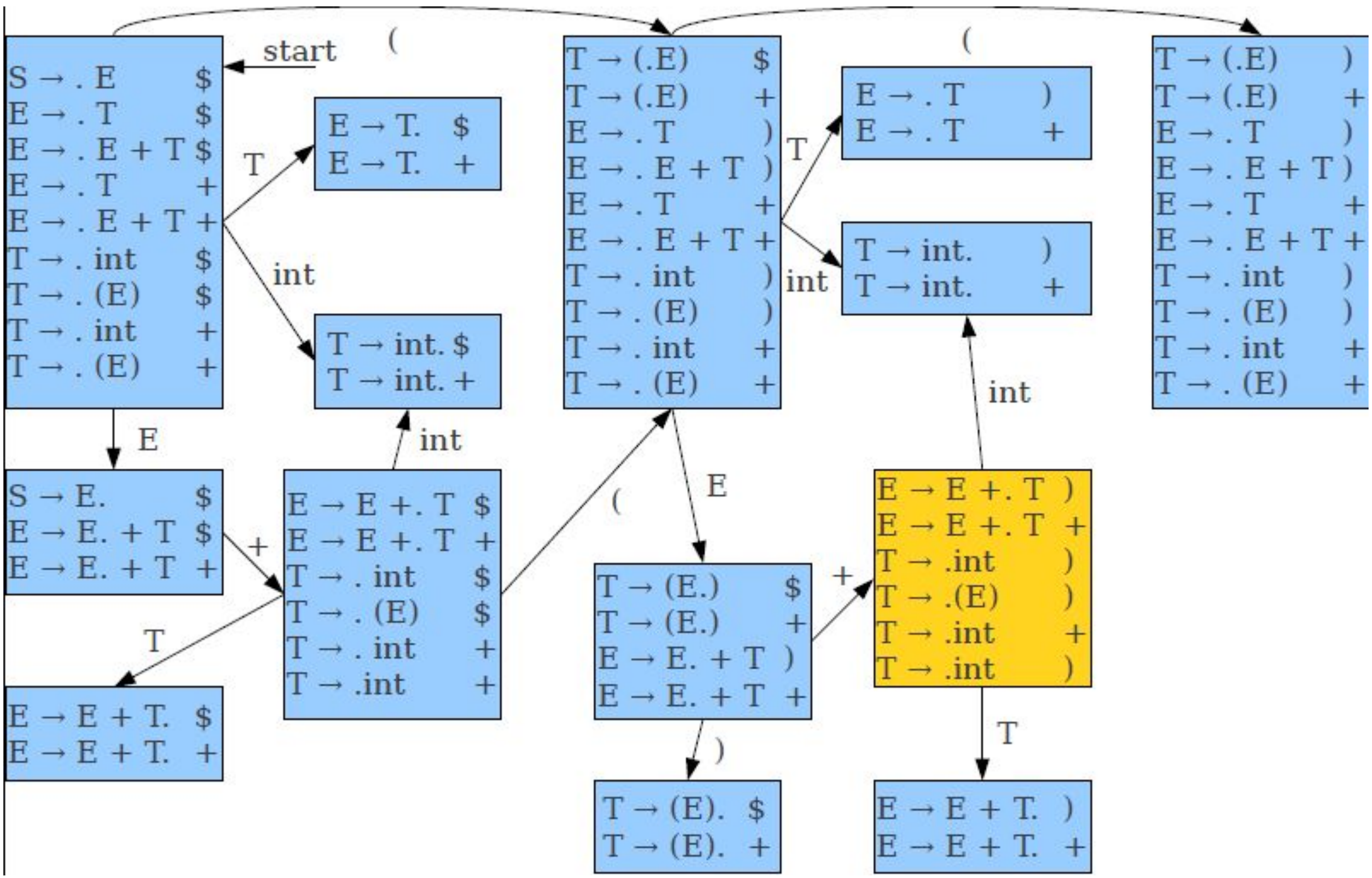

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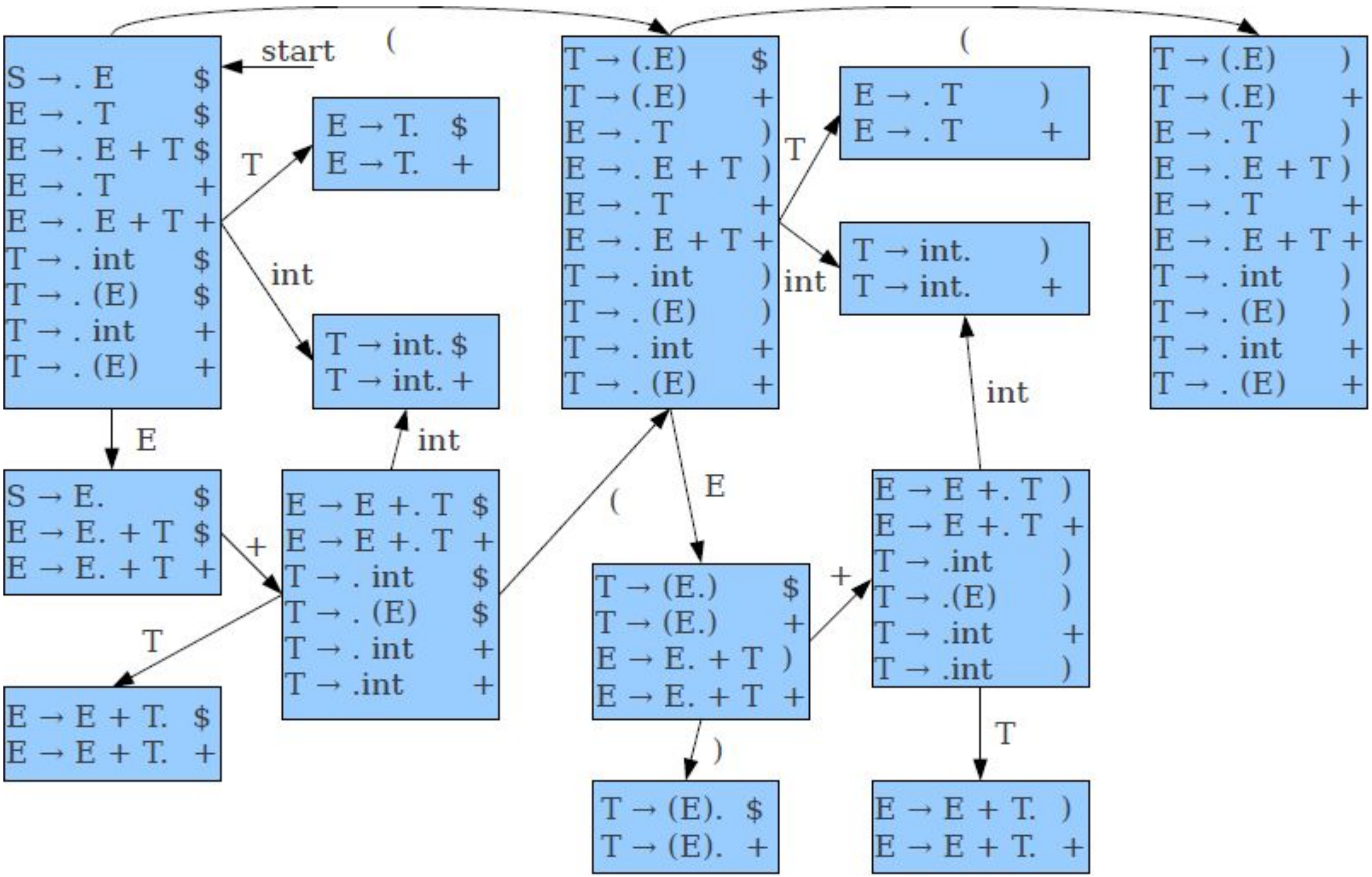

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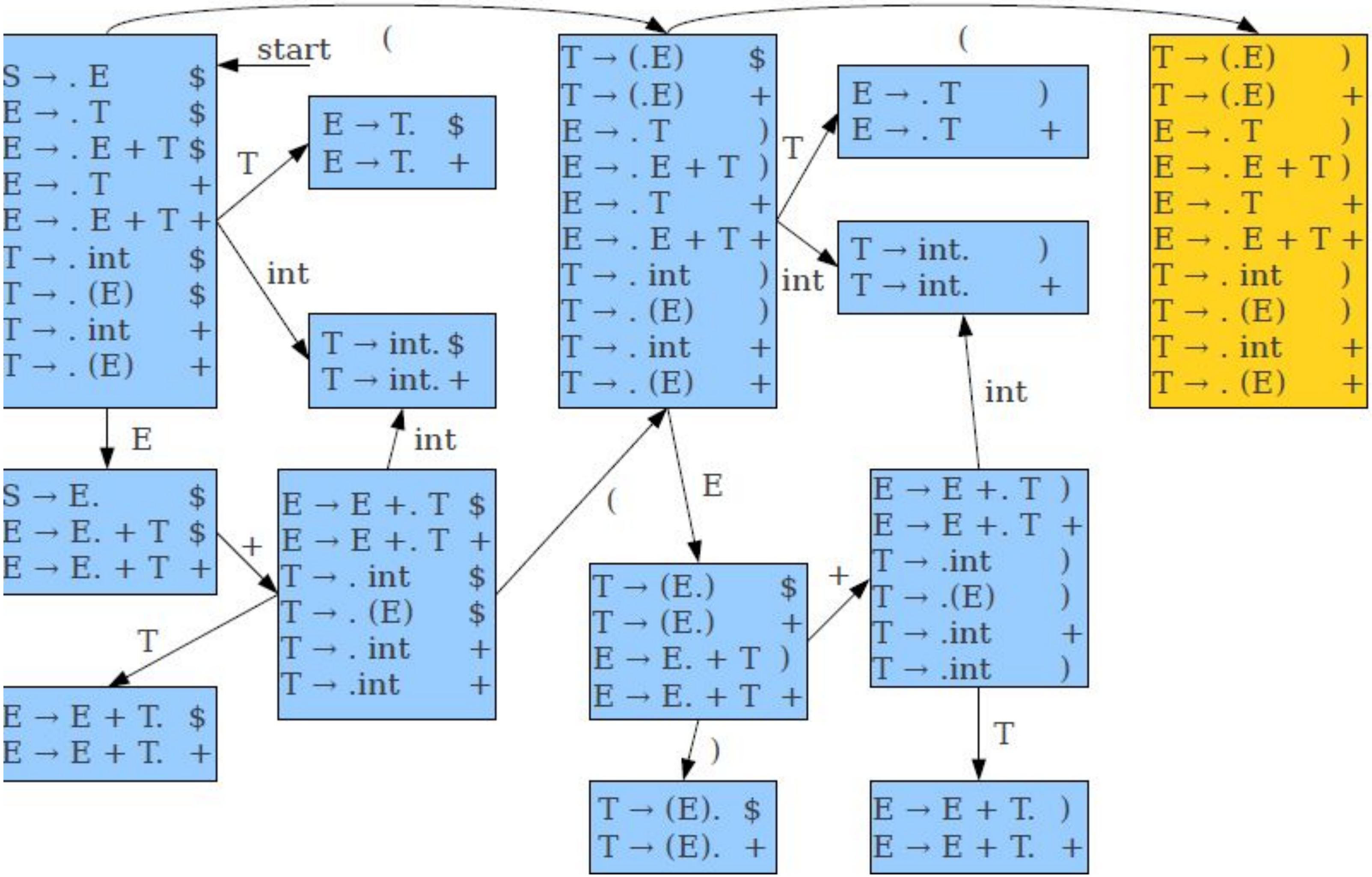

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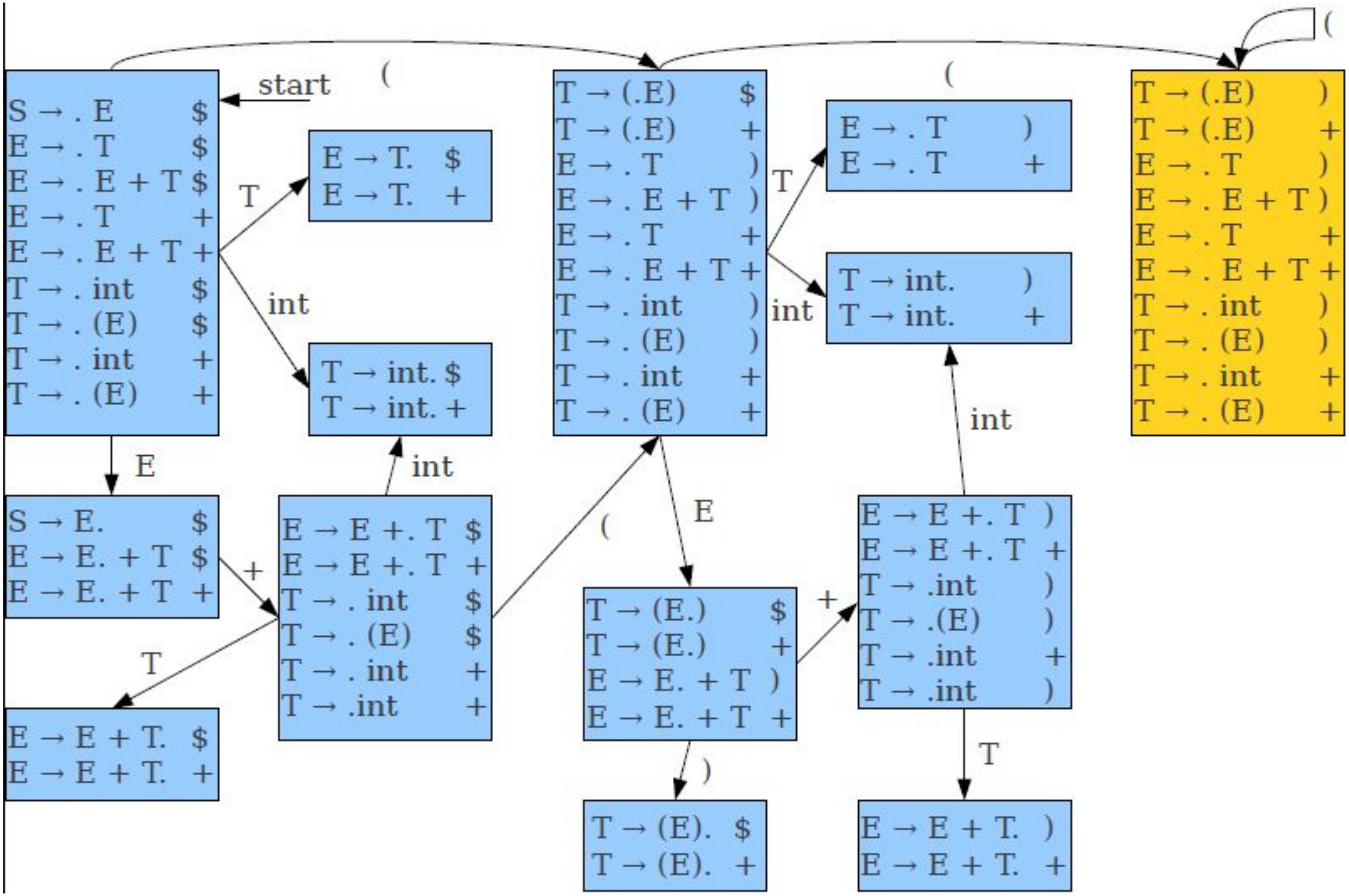

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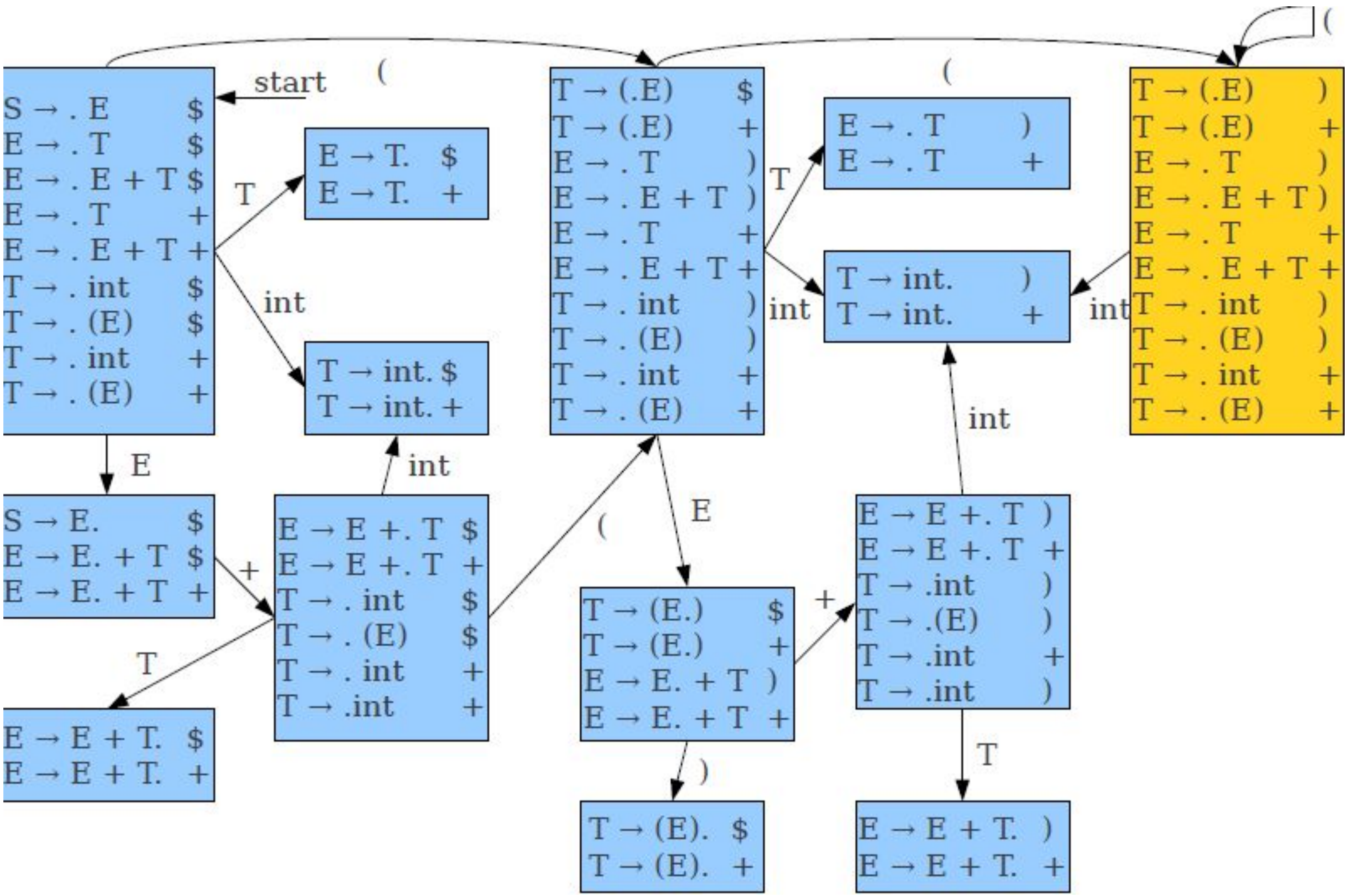

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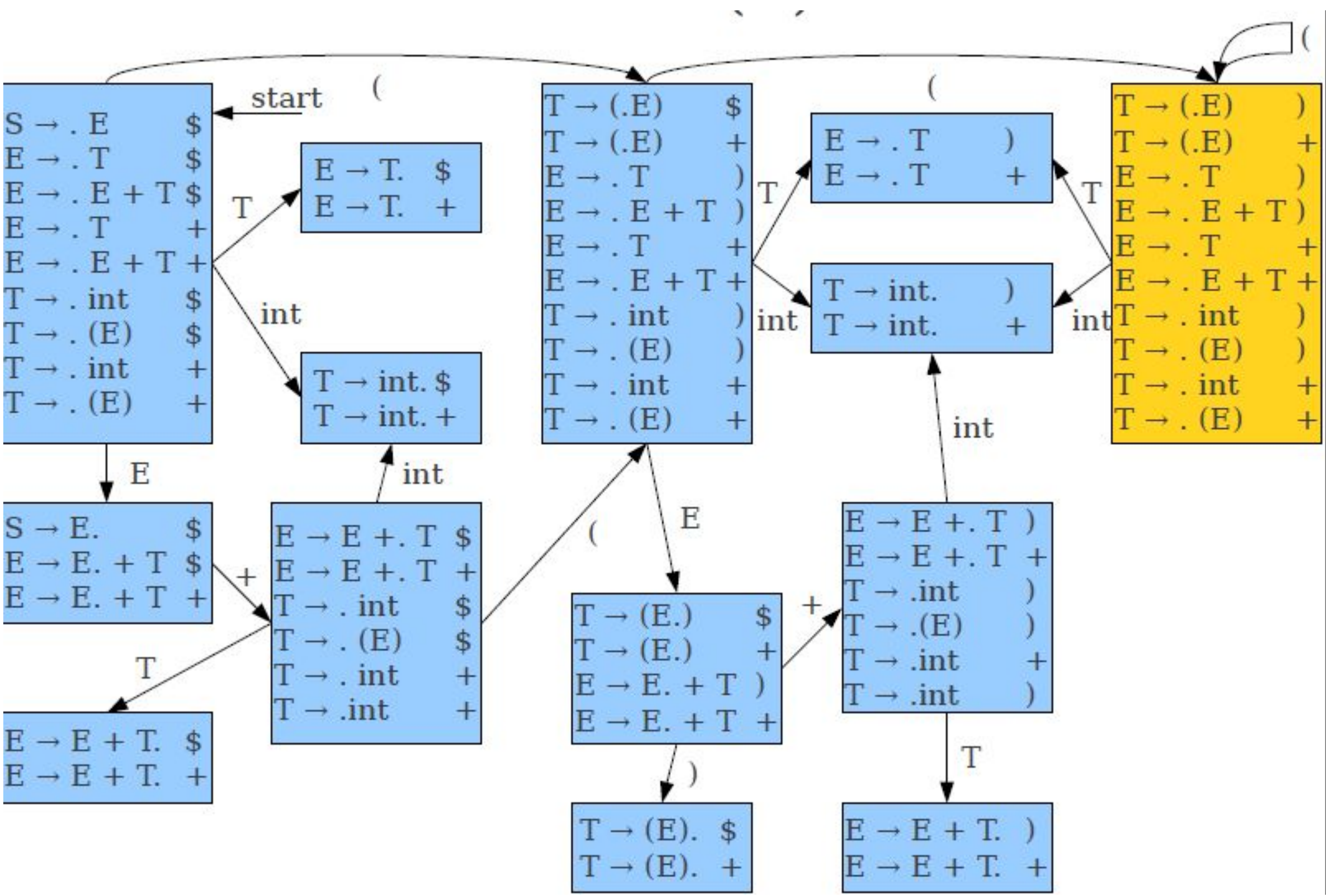

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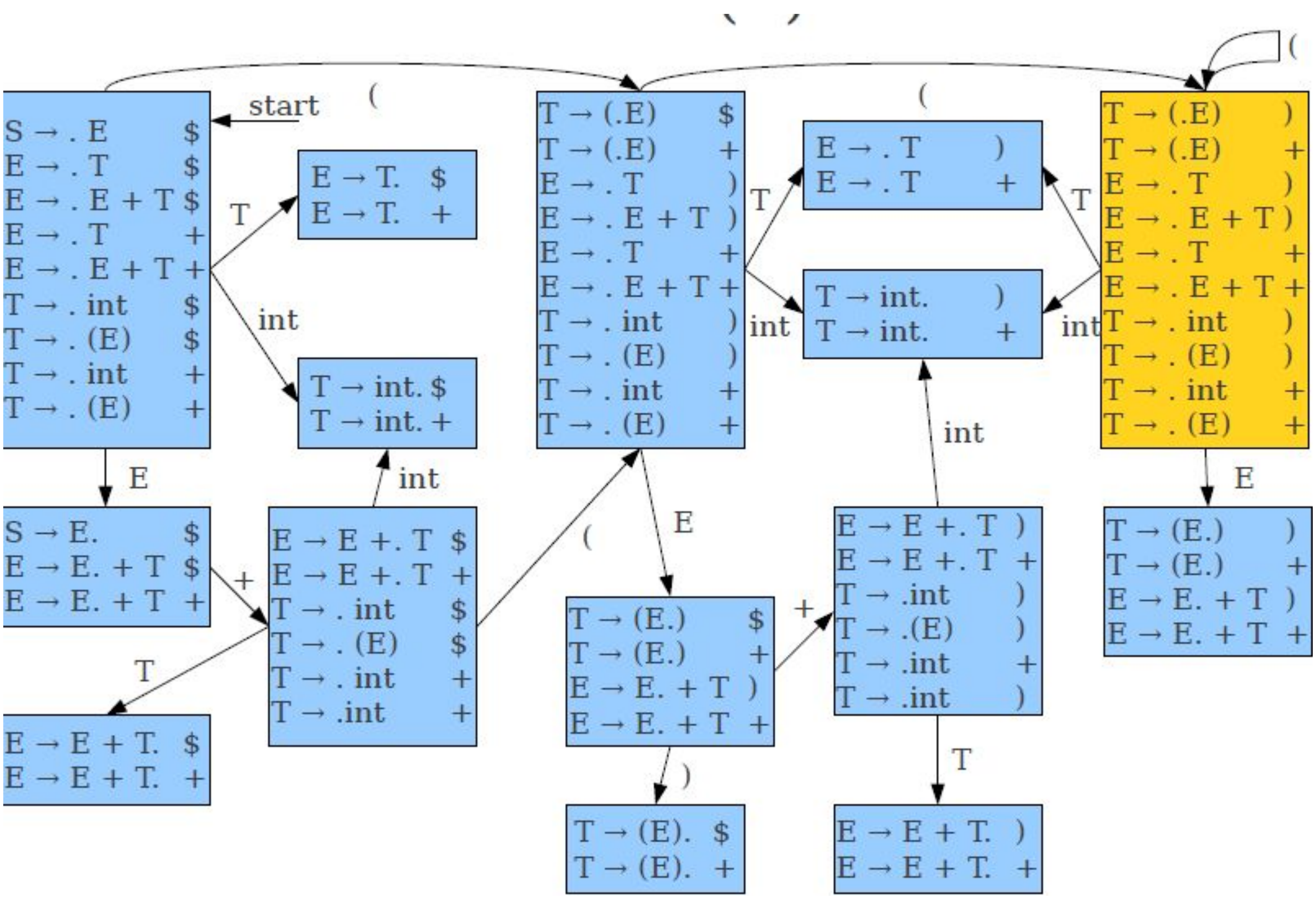

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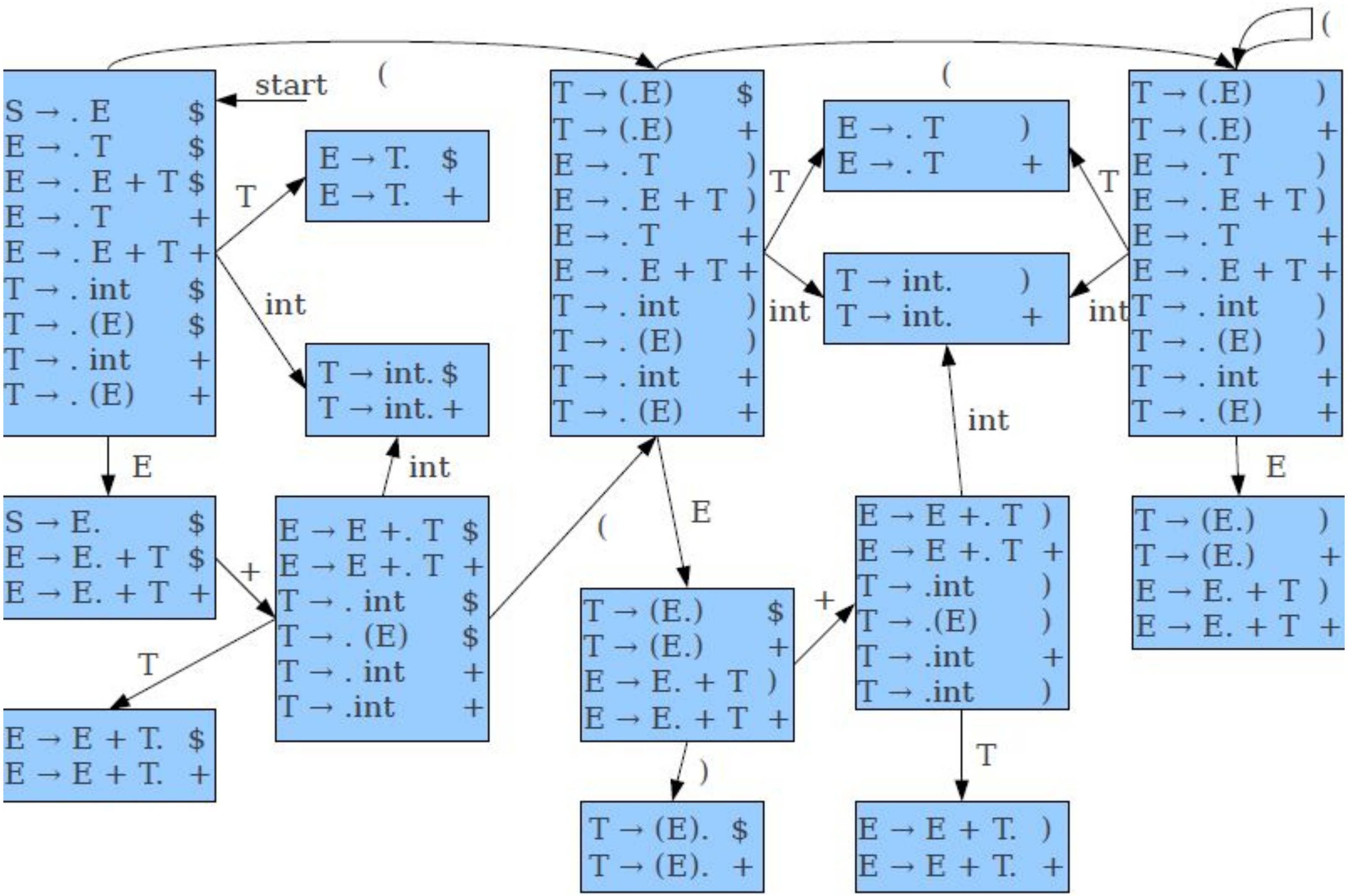

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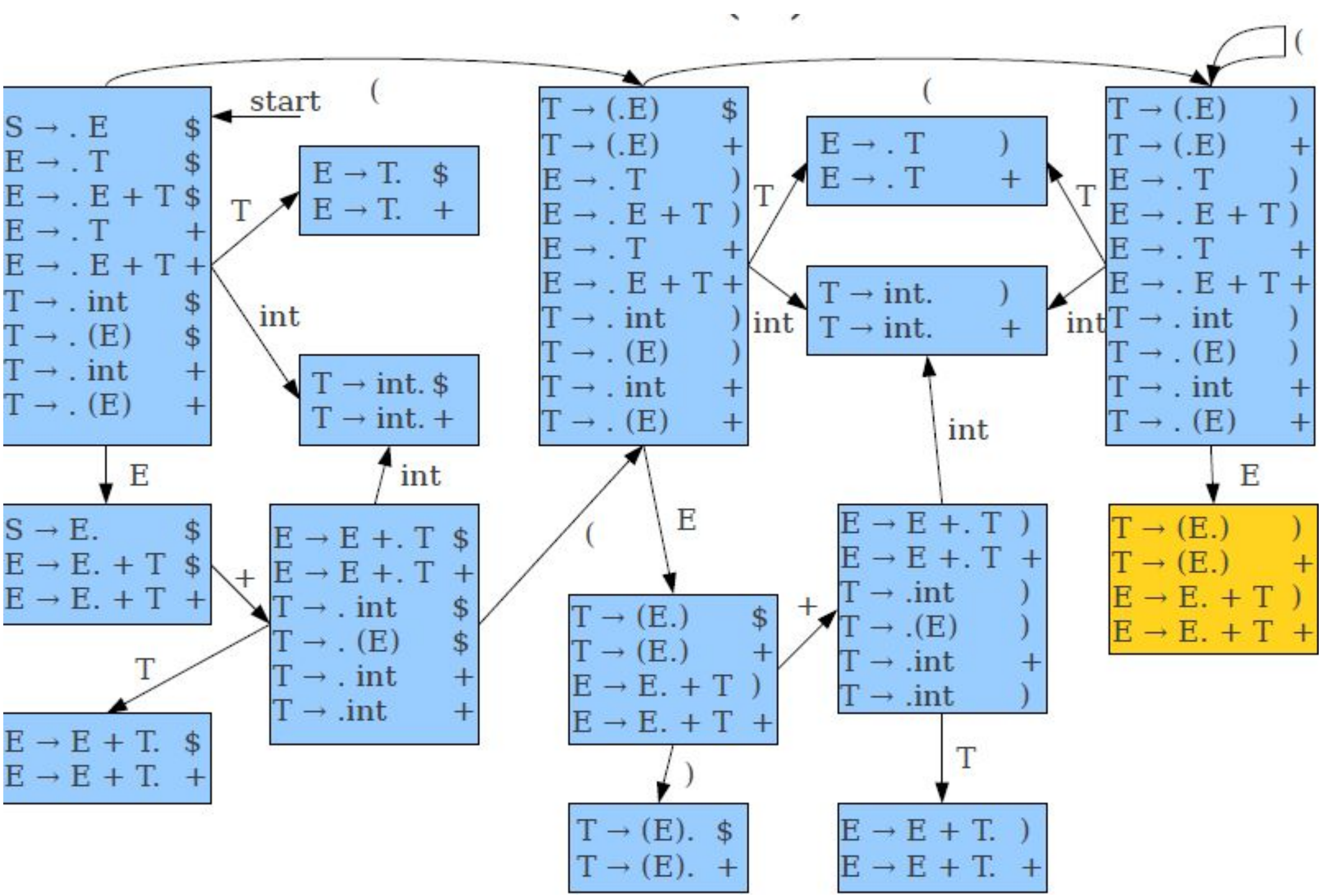

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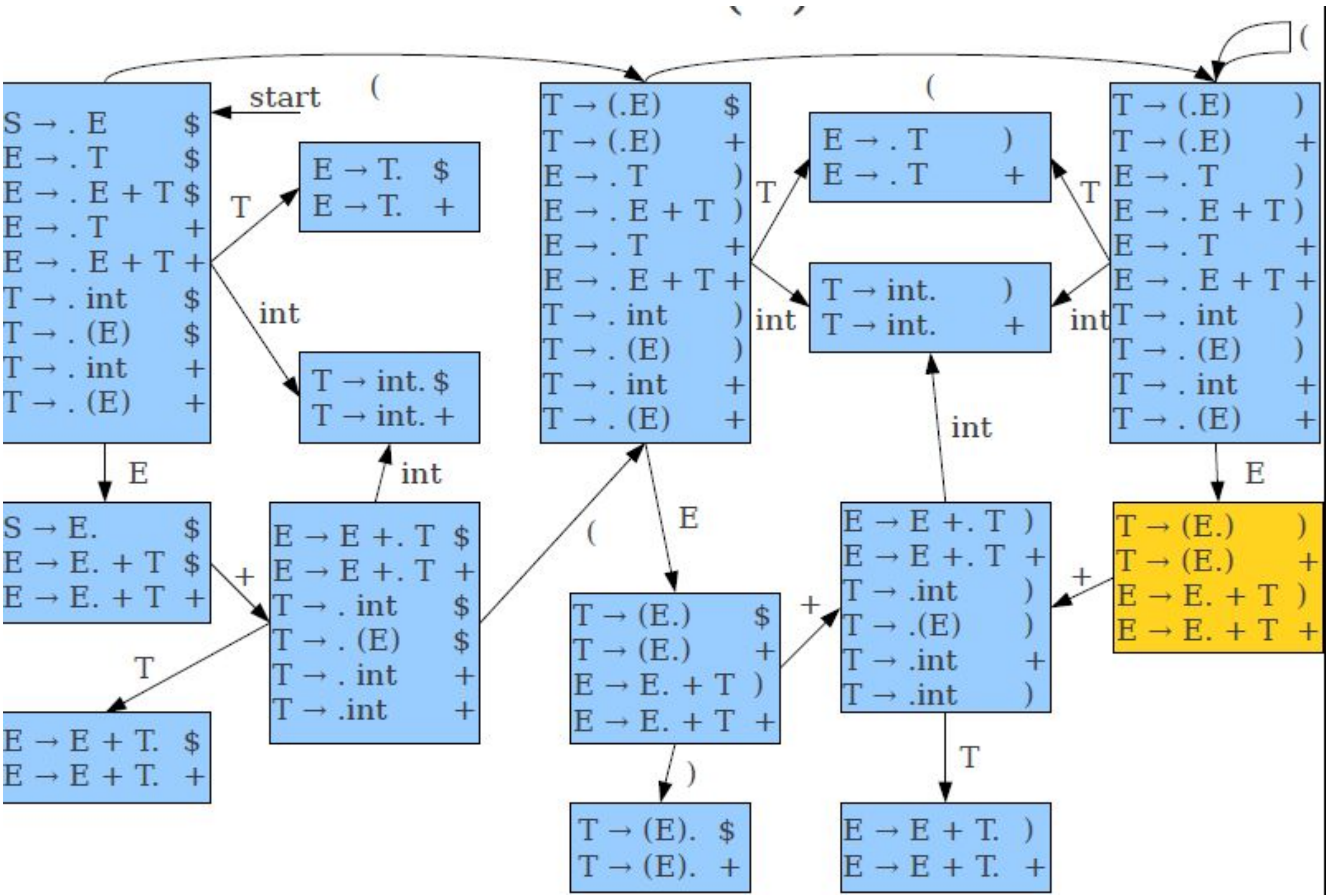

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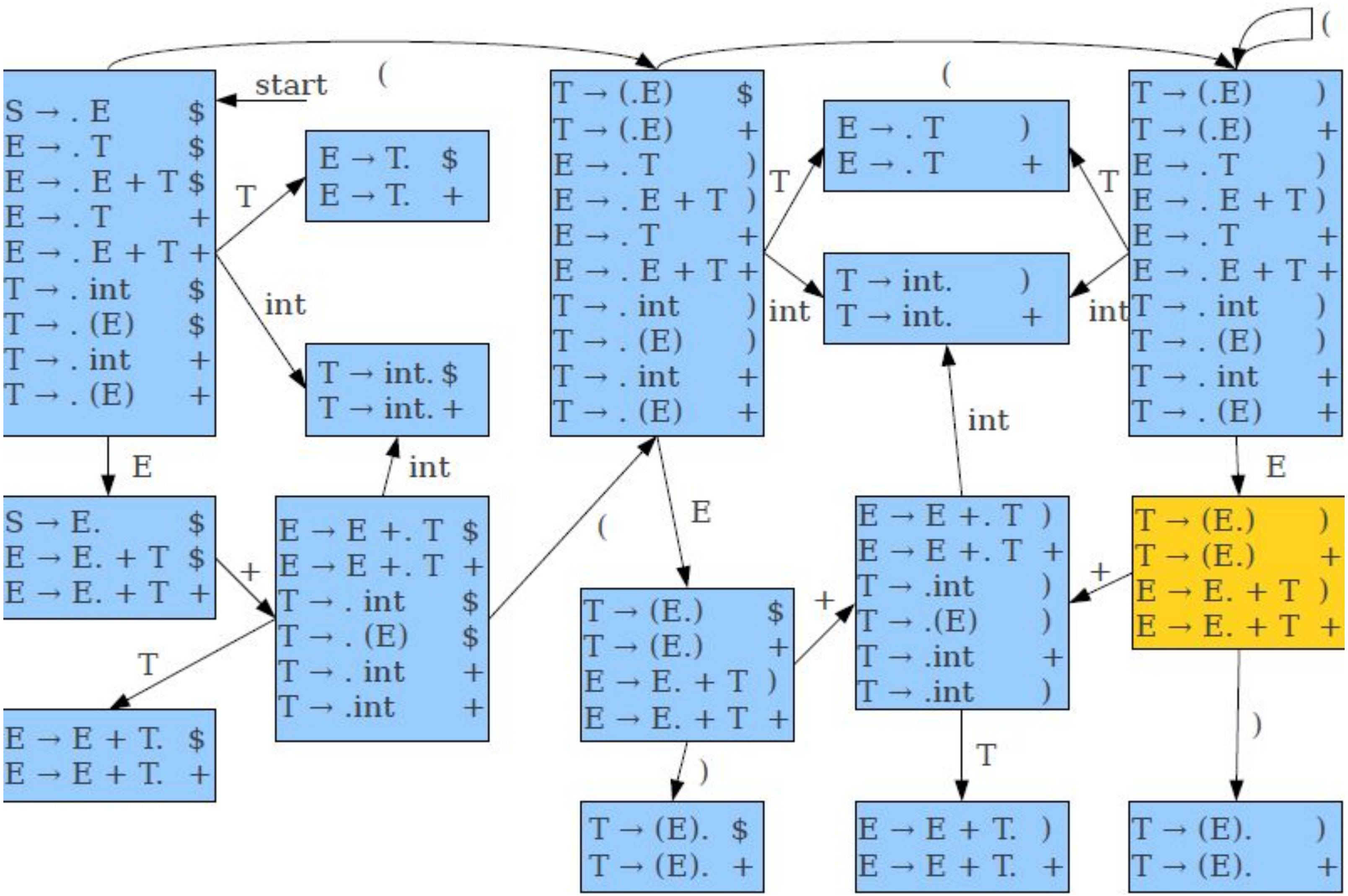

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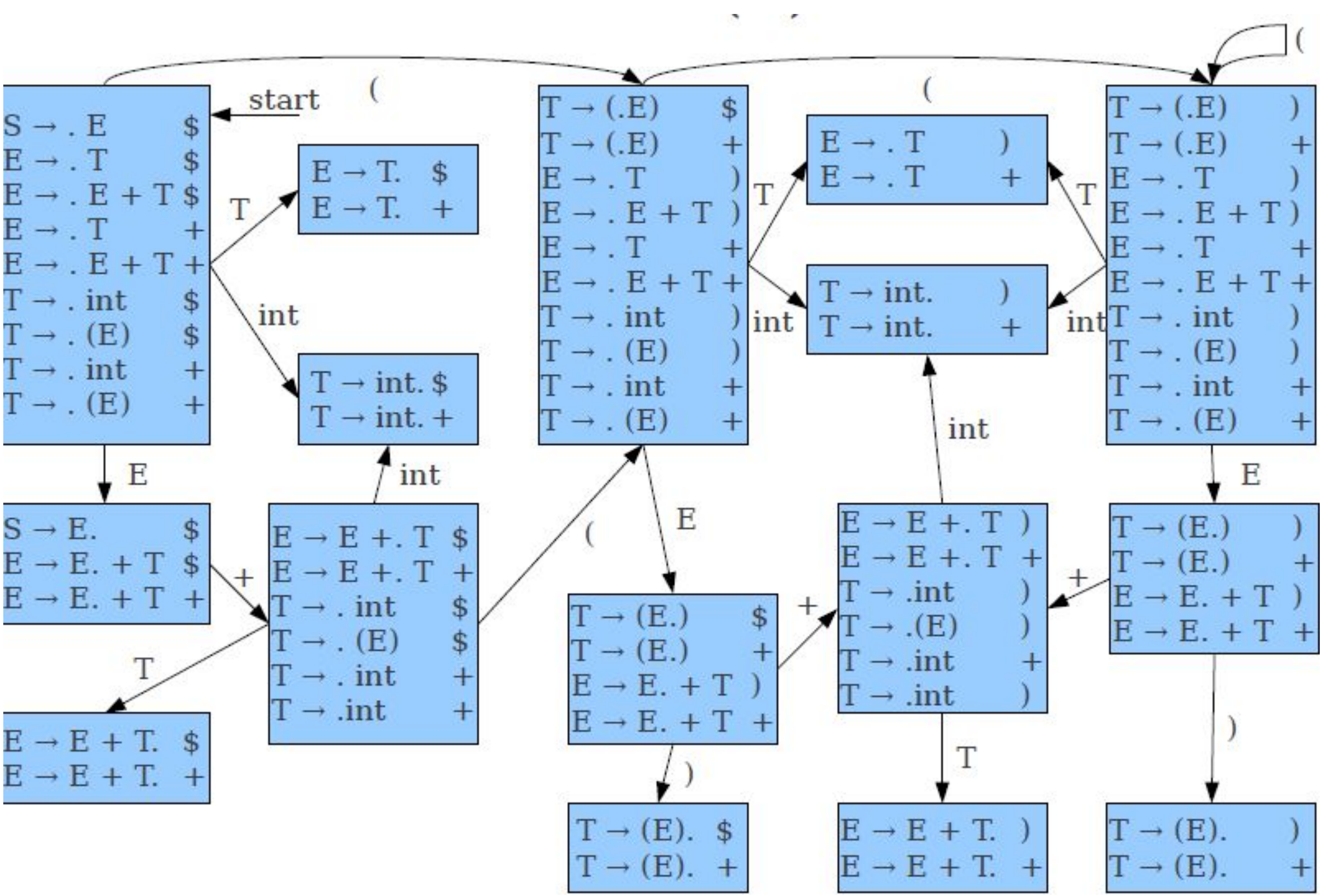

$S \longrightarrow E$ (1)
$E \longrightarrow T$ (2)
$E \longrightarrow E + T$ (3)
$T \longrightarrow$ int (4)
$T \longrightarrow (E)$ (5)

	int	(	)	+	$	T	E
1	s5					s4	s2
2				s6	ACCEPT		
3				r3	r3		
4				r2	r2		
5				r5	r5		
6	s5	s7				s3	
7	s10	s14				s10	s8
8			s9	s12			
9				r5	r5		
10			r2	r2			
11			r4	r4			
12	s11					s13	
13			r3	r3			
14	s11		s14			s10	s15
15			s16	s12			
16			r5	r5			

The LR(1) Parsing Algorithm

- Begin with an empty stack and the input set to ω**$**, where ω is the string to parse. Set **state** to the initial state.
- Repeat the following:
  - Let the next symbol of input be `t`.
  - If **action[state, `t`]** is **shift**, then shift the input and set **state** = **goto[state, `t`]**.
  - If **action[state, `t`]** is **reduce A → ω**:
    - Pop $|\omega|$ symbols off the stack; replace them with **A**.
    - Let the state atop the stack be **top-state**.
    - Set **state = goto[top-state, A]**
  - If **action[state, `t`] is accept**, then the parse is done. If
  - **action[state, `t`] is error**, report an error.

Constructing LR(1) Parse Tables

- For each state X:
  - If there is a production $\mathbf{A} \to \omega \cdot$ [**t**], set **action**[X, **t**] = reduce $\mathbf{A} \to \omega$.
  - If there is the special production $\mathbf{S} \to \mathbf{E} \cdot$ [**$**], where **S** is the start symbol, set **action**[X, **t**] = **accept**.
  - If there is a transition out of s on symbol **t**, set **action**[X, **t**] = **shift**.
- Set all other actions to **error**.
- If any table entry contains two or more actions, the grammar is not LR(1).