CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Search

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Winter 2024

IIT Bhilai

Revisiting Simon's Algorithm

From The Index View Perspective

Problem Definition

Assumptions:

- Given any $u \in \{0,1\}^n$,
  can efficiently compute $f(u)$.
- Nonzero $s \in \{0,1\}^n$.
- $f(u) = f(u \oplus s)$ for all u.
- f has no other collisions.

Goal: Figure out s.

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Non-quantum algorithm to find s:
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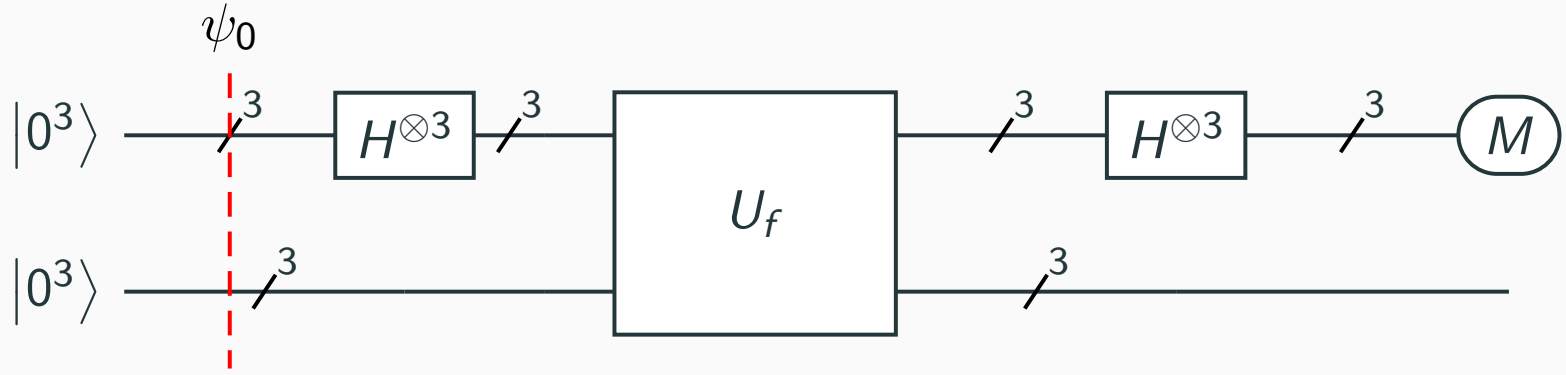
Simon's algorithm finds s with
$\approx n$ quantum evaluations of f.

Step 1. Set up pure zero state

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

- This example is for a function f with 3-bit input and 3-bit output
- Each column is a parallel universe.
- Step 3 will apply the function f (a specific function in this example), computing $f(u)$ in universe u.



Adapted from Bernstein's Invited Talk at Indocrypt 2021

Step 2.0. Hadamard$_0$

1, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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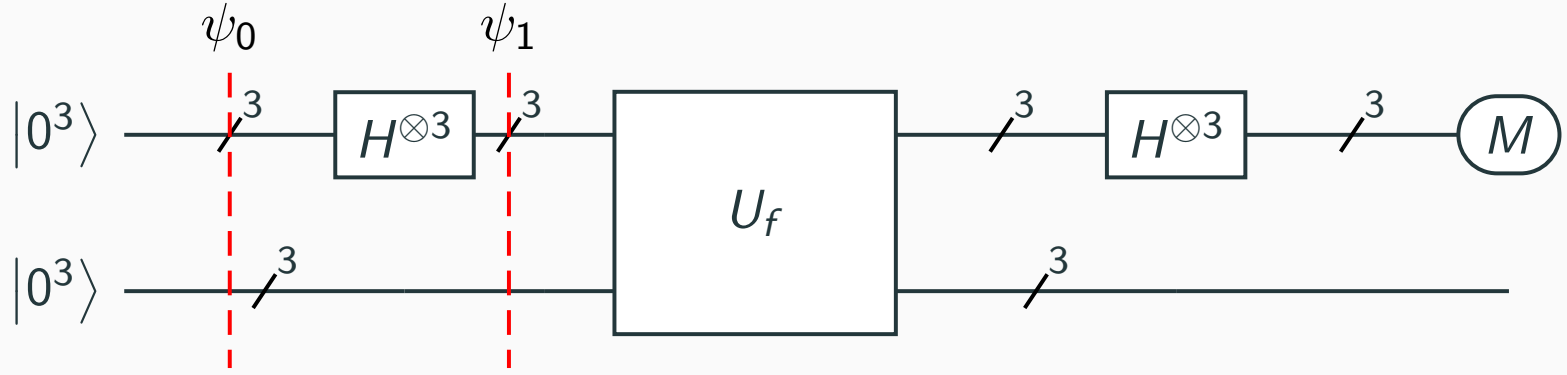

Adapted from Bernstein's Invited Talk at Indocrypt 2021

Step 2.1. Hadamard$_1$

1, 1, 1, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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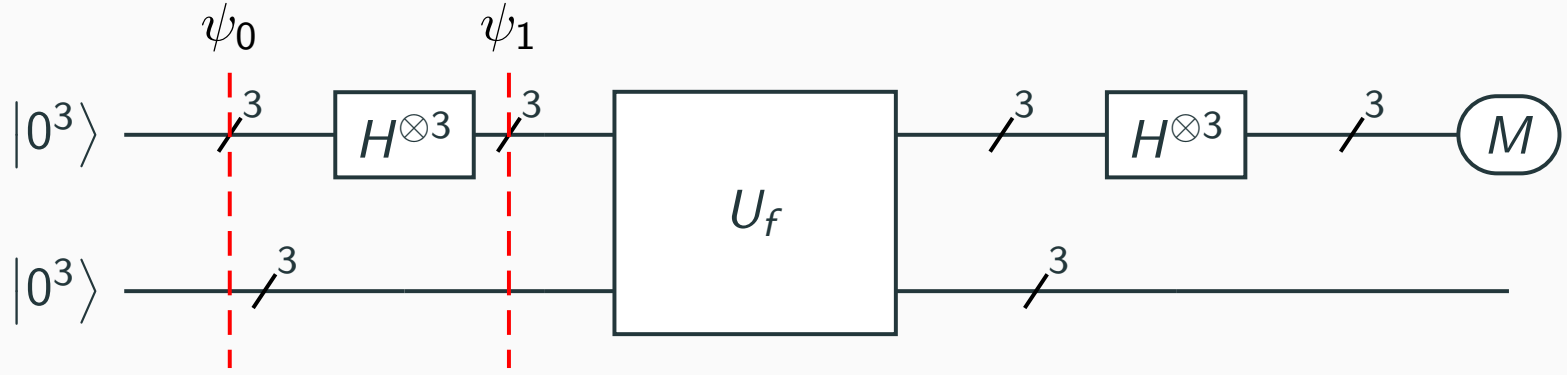

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Step 2.2. Hadamard$_2$

1, 1, 1, 1, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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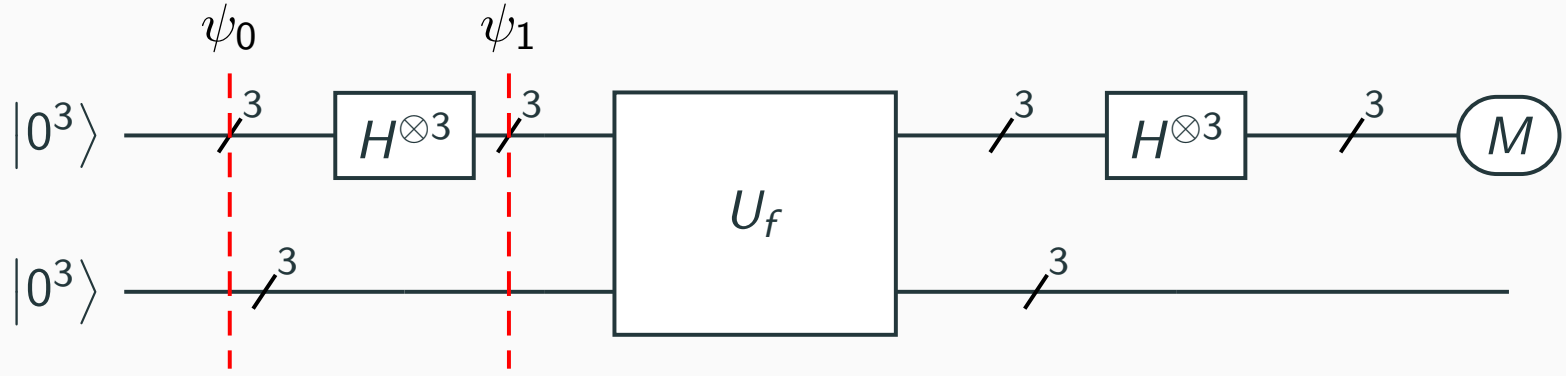
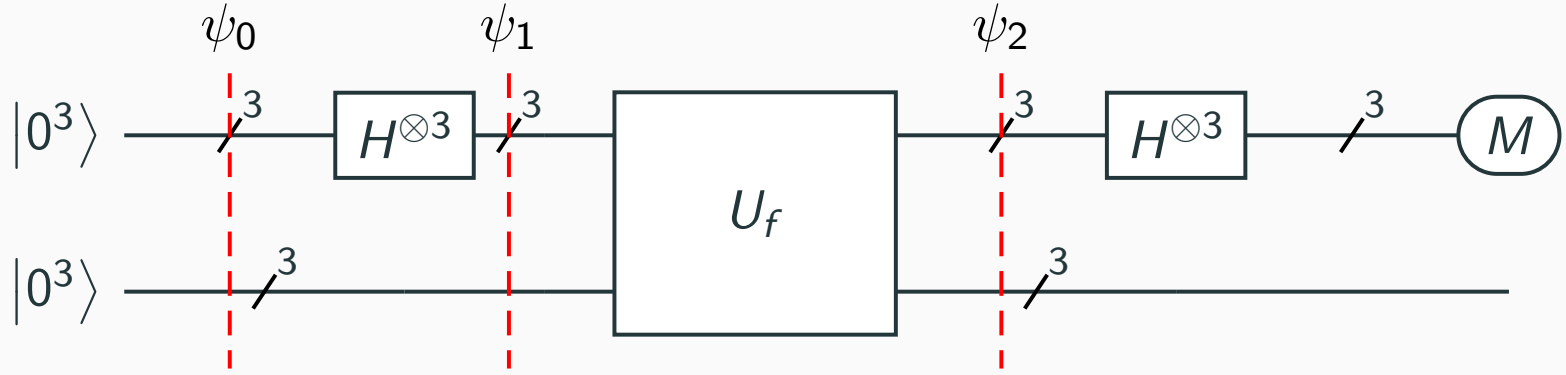


Adapted from Bernstein's Invited Talk at Indocrypt 2021

1, 0, 1, 0, 1, 0, 1, 0,
0, 1, 0, 1, 0, 1, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

- Each column is a parallel universe performing its own computations



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1, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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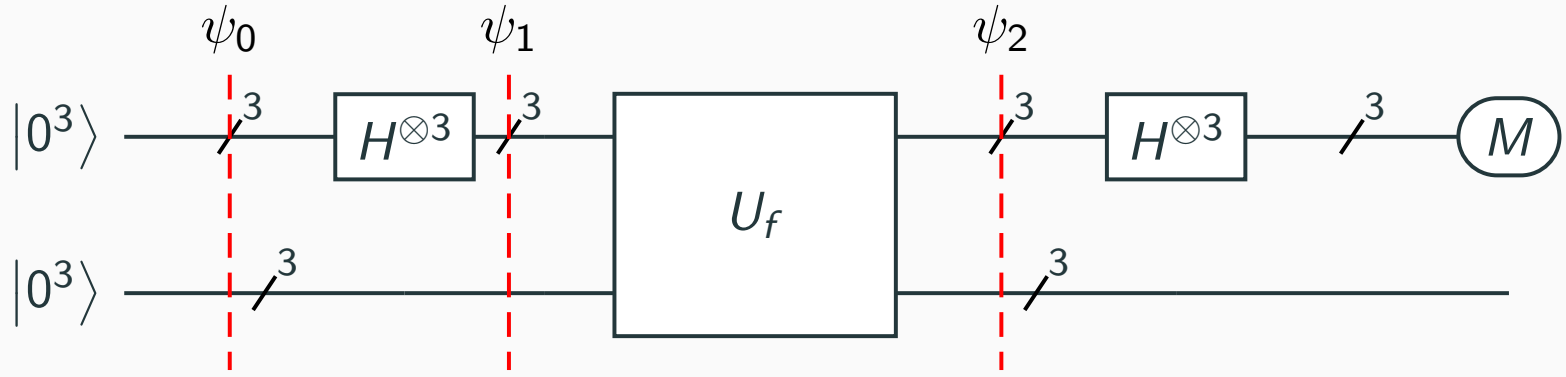

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1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 1.

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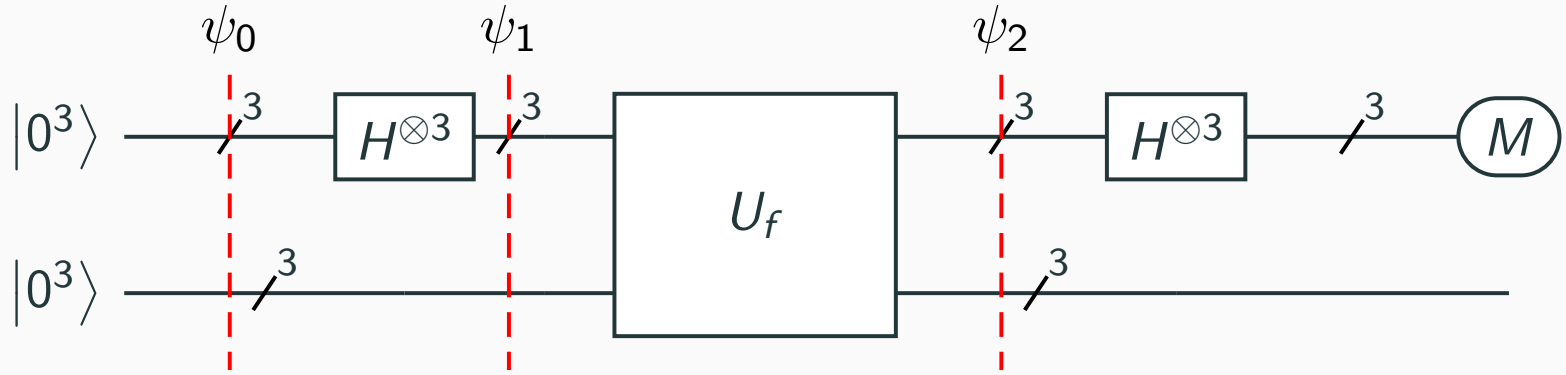

Adapted from Bernstein's Invited Talk at Indocrypt 2021

1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0.

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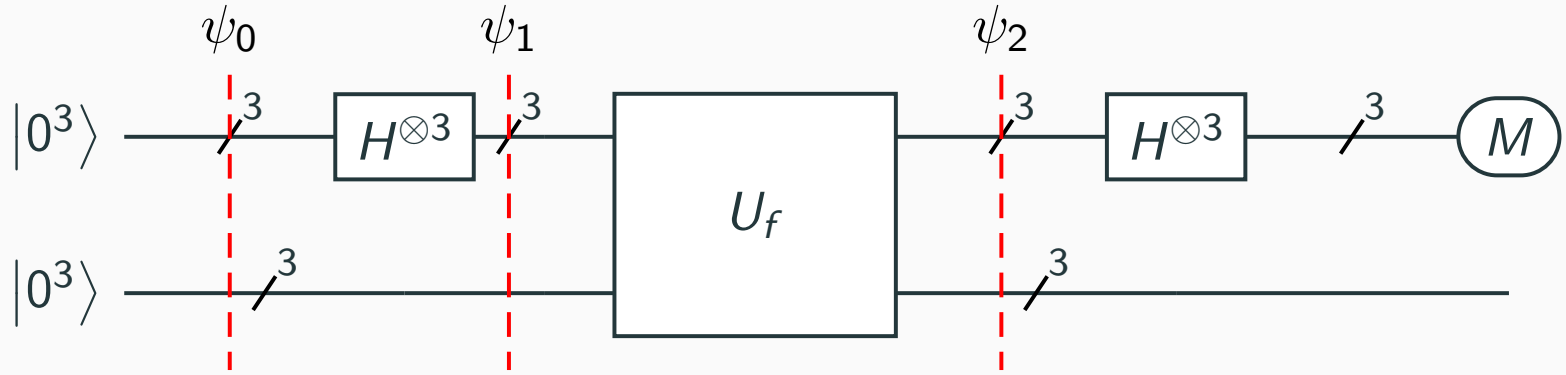

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1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0.

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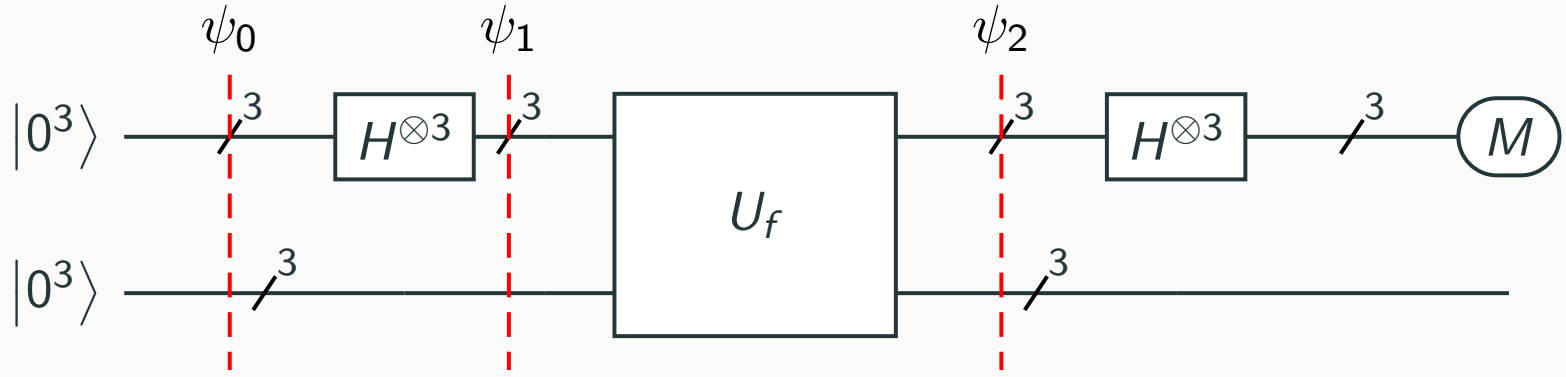

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0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0.

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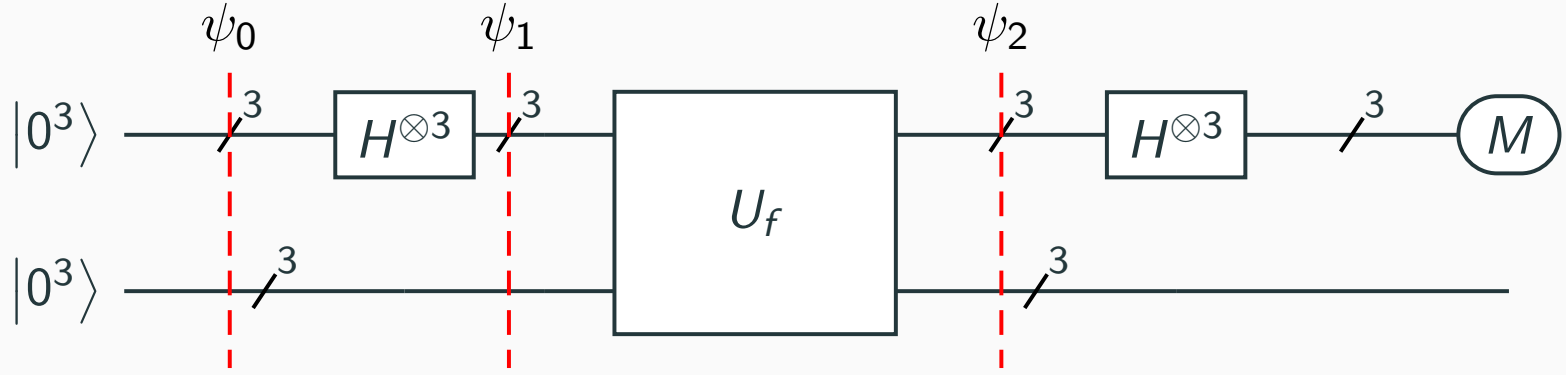

Adapted from Bernstein's Invited Talk at Indocrypt 2021

0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1.

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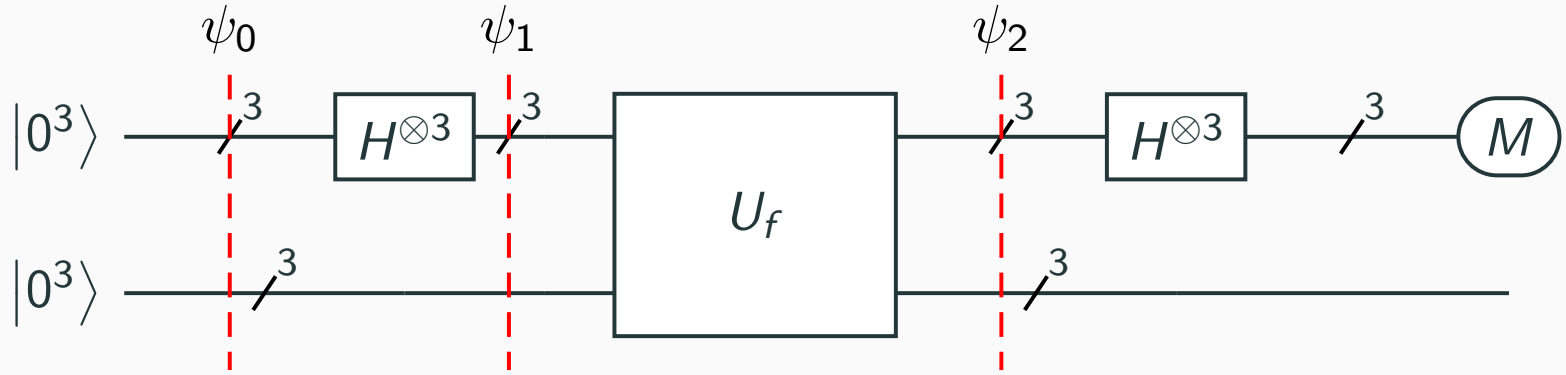

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0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

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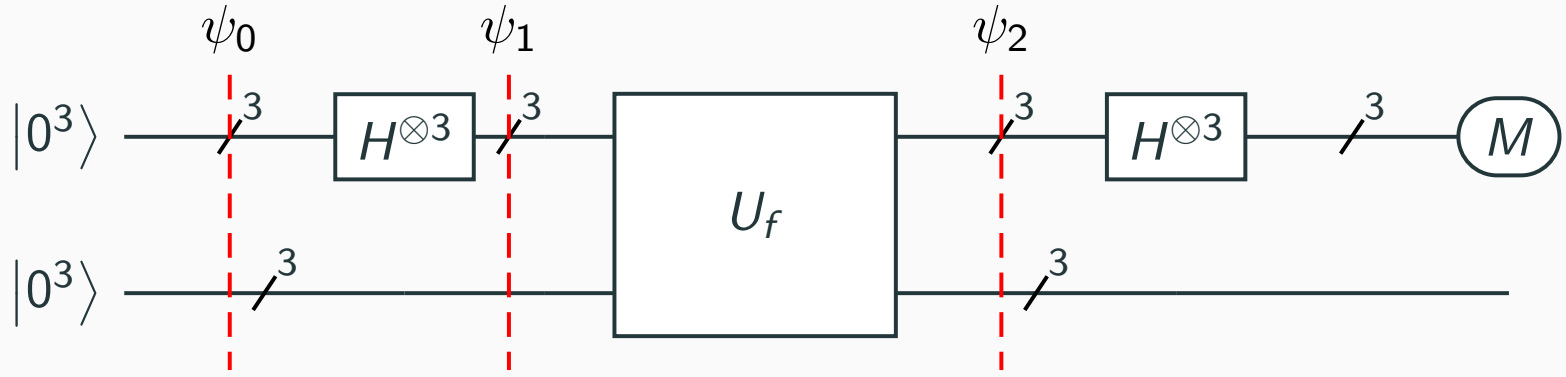

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0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 0, 0,
0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0.

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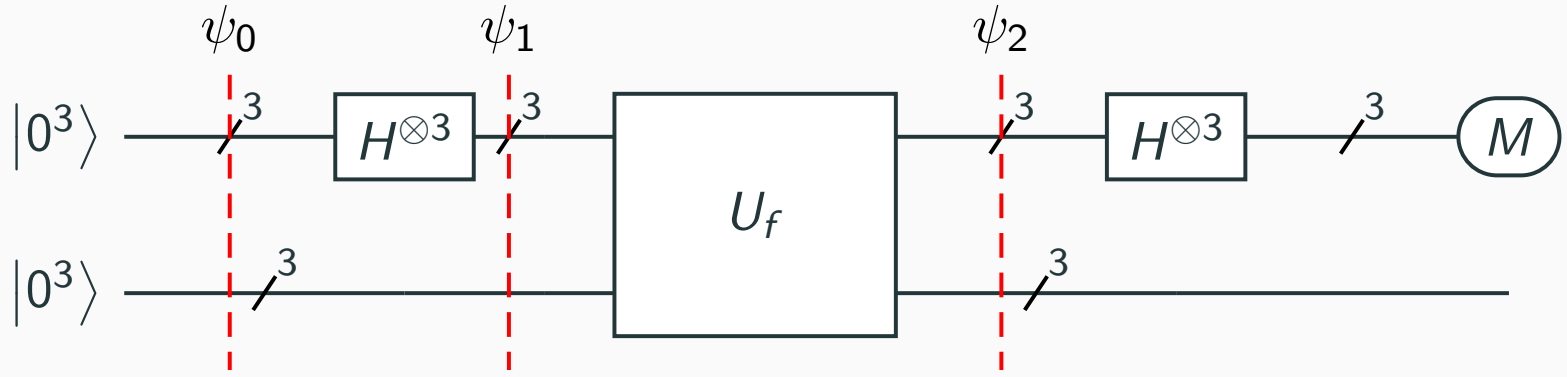

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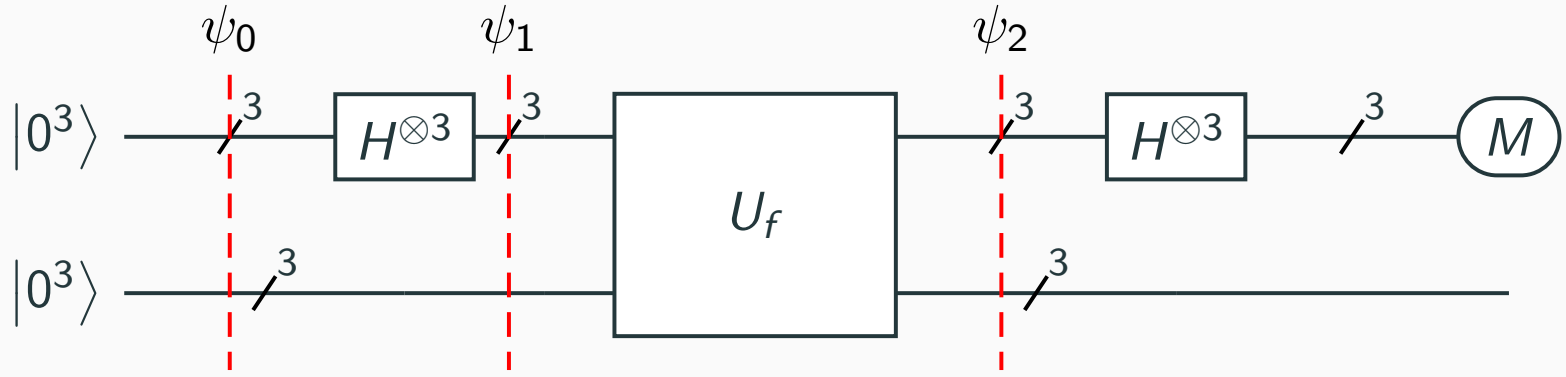
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

- Each column is a parallel universe performing its own computations



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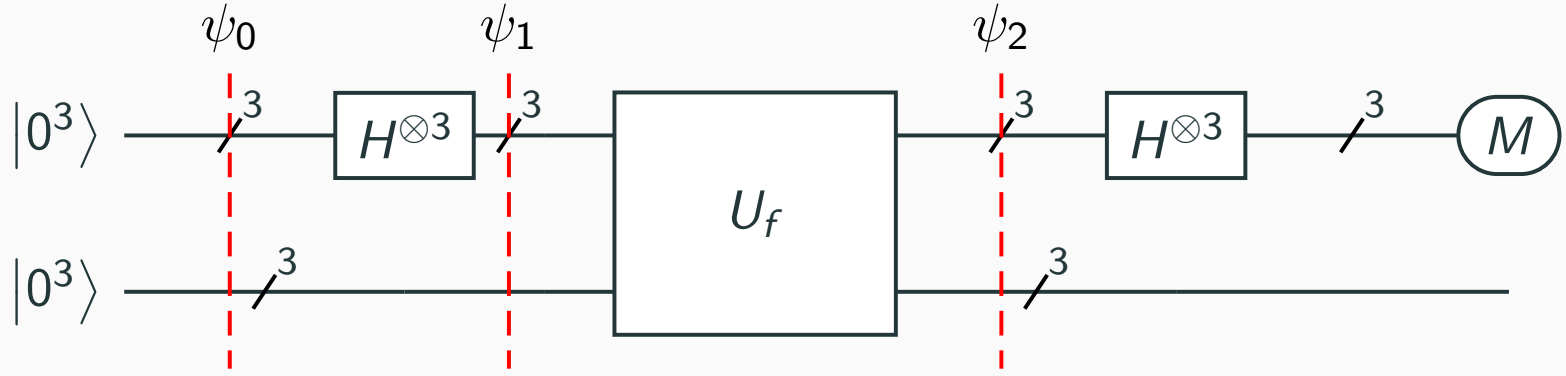
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

- Each column is a parallel universe performing its own computations
- Note: u and $u \oplus 101$ match



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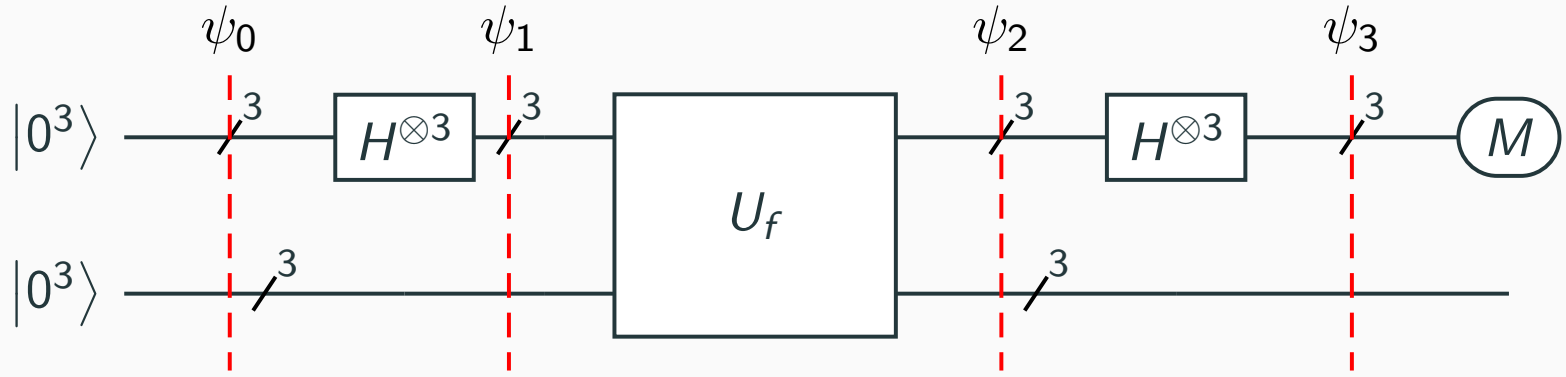
Step 4.0. Hadamard$_0$

0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 1, 0, 0.

- Notation: $\overline{1}$ means -1

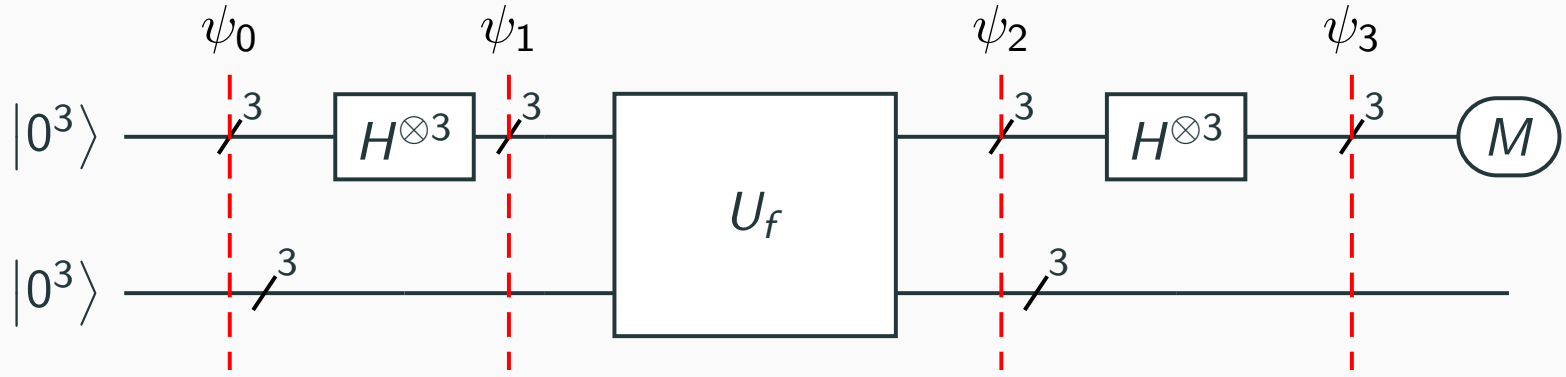


Adapted from Bernstein's Invited Talk at Indocrypt 2021

Step 4.1. Hadamard$_1$

$$
\begin{array}{cccccccc}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
0, & 0, & 1, & \overline{1}, & 0, & 0, & 1, & 1, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
0, & 0, & 1, & 1, & 0, & 0, & 1, & \overline{1}, \\
1, & \overline{1}, & 0, & 0, & 1, & 1, & 0, & 0, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
1, & 1, & 0, & 0, & 1, & \overline{1}, & 0, & 0.
\end{array}
$$

- Notation: $\overline{1}$ means -1



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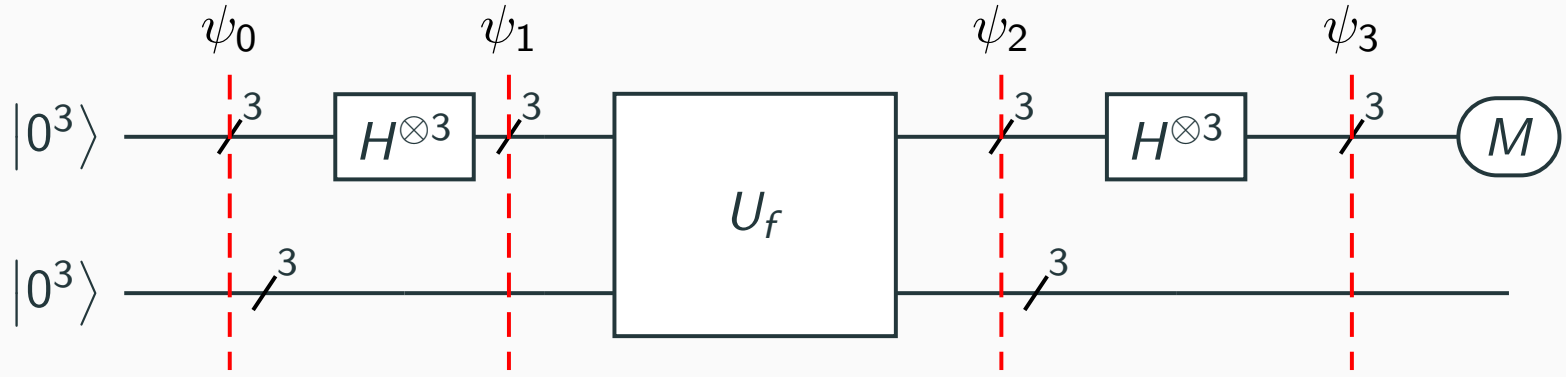
Step 4.2. Hadamard$_2$

$$
\begin{array}{cccccccc}
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
1, & \overline{1}, & \overline{1}, & 1, & 1, & 1, & \overline{1}, & \overline{1}, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
1, & 1, & \overline{1}, & \overline{1}, & 1, & \overline{1}, & \overline{1}, & 1, \\
1, & \overline{1}, & 1, & \overline{1}, & 1, & 1, & 1, & 1, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, \\
1, & 1, & 1, & 1, & 1, & \overline{1}, & 1, & \overline{1}.
\end{array}
$$

- Notation: $\overline{1}$ means -1



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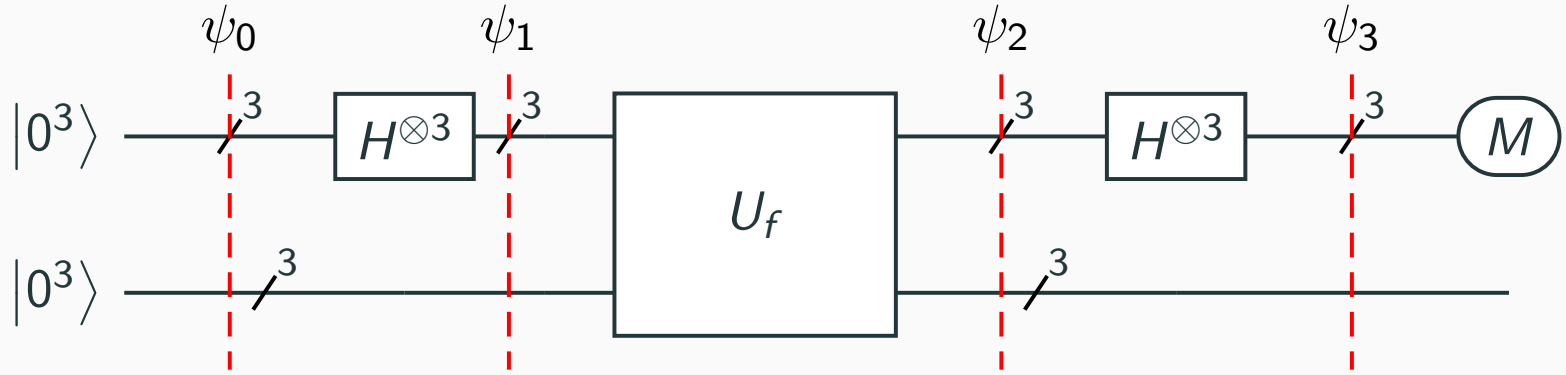
Step 5. Measure

$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ \overline{2},\ 0,\ 2,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ 2,\ 0,\ \overline{2},$
$2,\ 0,\ 2,\ 0,\ 0,\ \overline{2},\ 0,\ \overline{2},$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ 2,\ 0,\ 0,\ 2,\ 0,\ 2.$

- Notation: $\overline{1}$ means -1
- Obtain some information about the *period* of f: a random vector **orthogonal** to 101



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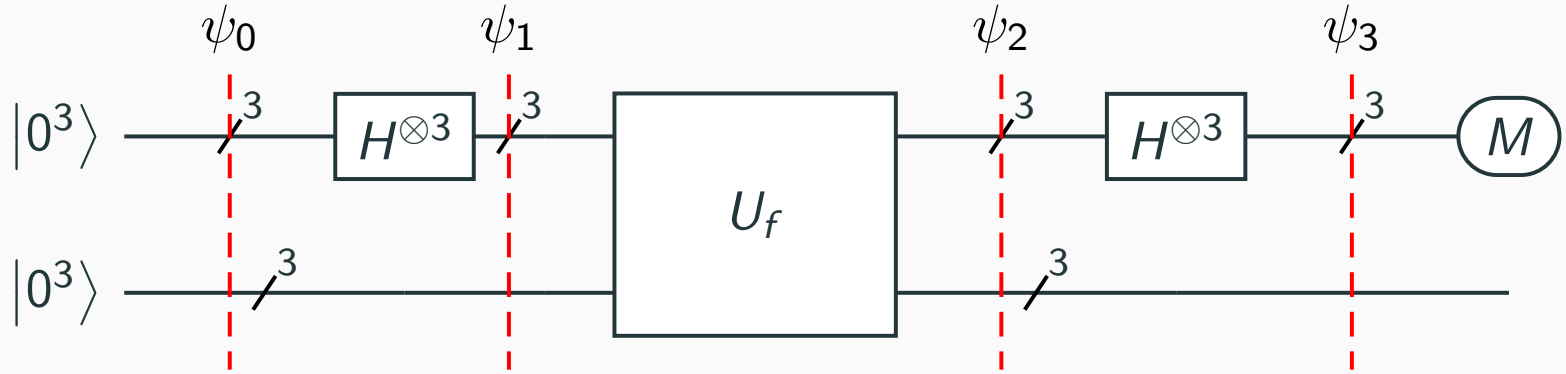
Step 5. Measure

$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ \overline{2},\ 0,\ 2,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ \overline{2},\ 0,\ 0,\ 2,\ 0,\ \overline{2},$
$2,\ 0,\ 2,\ 0,\ 0,\ \overline{2},\ 0,\ \overline{2},$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,$
$2,\ 0,\ 2,\ 0,\ 0,\ 2,\ 0,\ 2.$

- Notation: $\overline{1}$ means -1
- Obtain some information about the *period* of f: a random vector **orthogonal** to 101
- Repeat to figure out 101



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