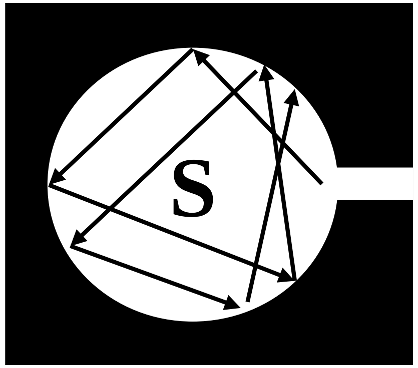
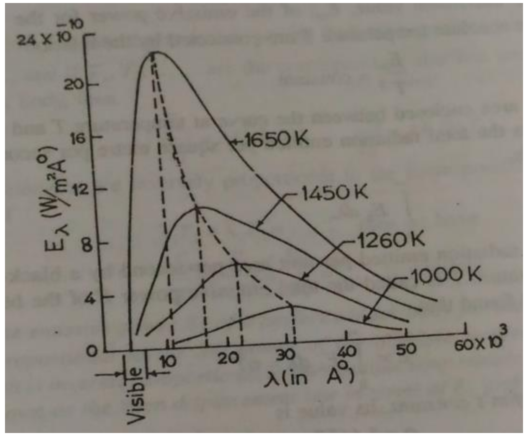
Day 2

Summary-Day 1

Day 1.Black Body Radiation (Summary:L1)





1.Wien's radiation Formula

$$E_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-\frac{C_2}{\lambda T}} d\lambda$$

2. Stefan-Boltzmann Law

$$R_B = \int_0^\infty E(\lambda) d\lambda = \sigma T^4$$

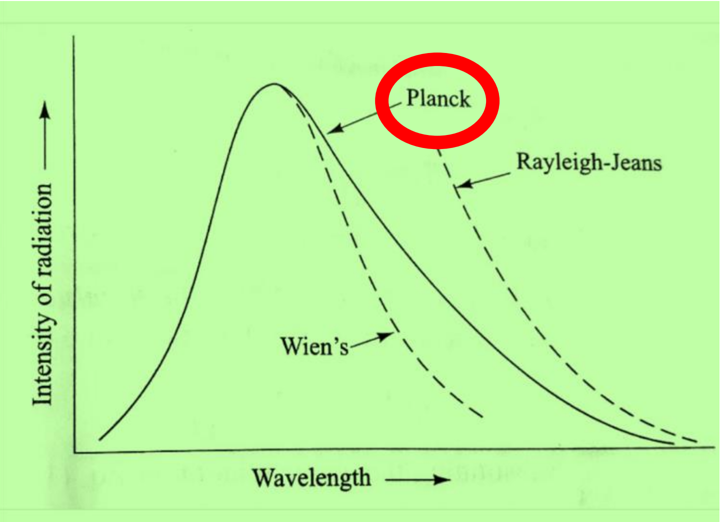
3. Rayleigh-Jeans Law

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

Failure of classical mechanics!

Planck's Law

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)} d\lambda$$



Energy of the oscillator in black body has to be integral multiple of hν;

E=nhν

The oscillator of the black-body cannot have any amount of energy but has a discrete energy equal to the integral multiple of some minimum energy ϵ=hν

$\epsilon_i = n\epsilon$, where n = 0, 1, 2, 3, …

Energy Level	0	hν	2hν	2hν	3hν	4hν
E_i	E_0	E_1	E_2	E_3	E_4	E_5
N_i	N_0	N_1	N_2	N_3	N_4	N_5

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)} d\lambda$$

Assuming 'λ' is small

1.Wien's radiation Formula

$$E_\lambda d\lambda = \frac{C_1}{\lambda^5} e^{-\frac{C_2}{\lambda T}} d\lambda$$

Integration

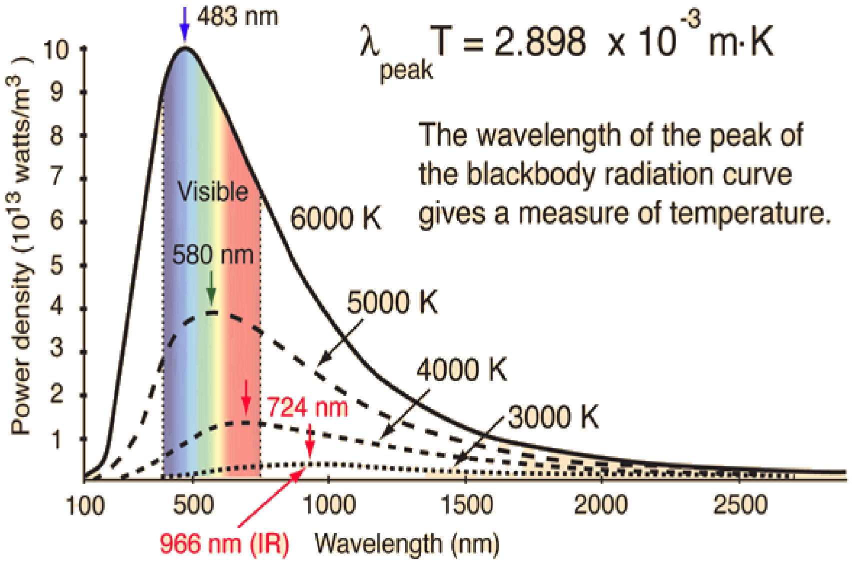
2. Stefan-Boltzmann Law

$$R_B = \int_0^\infty E(\lambda) d\lambda = \sigma T^4$$

Assuming 'λ' is large

3. Rayleigh-Jeans Law

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

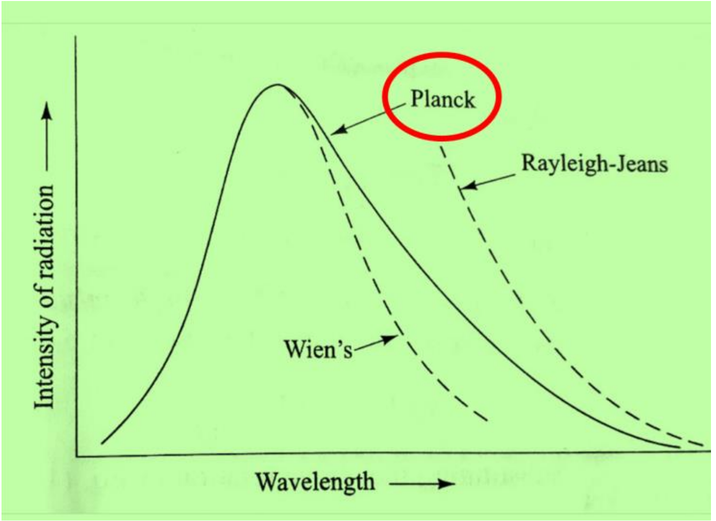


1) Derive Planck's radiation formula.
2) Show that Planck's radiation formula can be reduced to (i) Wien's radiation formula, and (ii) Rayleigh-Jeans radiation formula in suitable range of wavelengths.
3) Derive the Wien's displacement formula from Planck's radiation formula.
4) Calculate the average energy of an oscillator of frequency $10^{15} s^{-1}$ at 300 K and 5000 K using the classical and quantum nature of light. Show that at 5000 K the average energy tends to the equipartition value (classical value).
5) A 45kW broadcasting antenna emits radio waves at a frequency of 4MHz. (a) How many photons are emitted per second?
6) Starting from Plank's radiation law, obtain Stefan-Boltzmann law: $E = AT^4$; where E=total radiation energy from $\lambda=0$ to $\propto$. $A=8\pi^5 k^4/15(hc)^3$

Given:

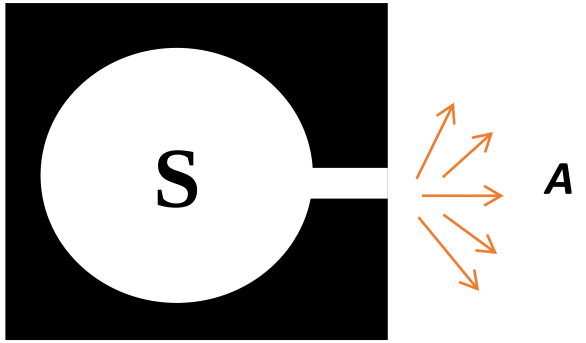
$$\int_0^{\propto} \frac{X^3}{(e^x - 1)} = \frac{\pi^4}{15}$$

Planck's Law (1900)



$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)}d\lambda$$

- It perfectly describes the black-body radiation in whole range of wavelength
- All the laws (Wien, Rayleigh-Jeans and Stephan) can be arrived from this law.
- **Energy of the oscillator has to be integral multiple of hν; E=nhν**



Atoms in the walls are like simple harmonic oscillators having a fixed frequency ν. 1901 Planks proposed the oscillator emits radiation in discrete amount.
(The electrons are pictured as oscillators like they oscillates in antenna to give radiowaves. But in black body they oscillates at a much higher frequency that why we see emission in visible/UV/IR ranges)

Assumptions:

1) Oscillators of black body can't have any amounts of energy but have a discrete energy. It can have only those values of total energy E, which satisfy the relation E=nhν. (n=0,1,2…) hν is the basic unit of energy. [h=6.625×10^{-34} J s]

2) The oscillator does not emits continuously. The emission/absorption only occurs when they jumps from one energy level to another.

The oscillator of the black-body cannot have any amount of energy but has a discrete energy equal to the integral multiple of some minimum energy ϵ=hν

$$\epsilon_i = n\epsilon, \text{where n} = 0, 1, 2, 3, \ldots$$

Energy Level	0	hν	2hν	2hν	3hν	4hν
E_i	E_0	E_1	E_2	E_3	E_4	E_5
N_i	N_0	N_1	N_2	N_3	N_4	N_5

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)}d\lambda$$

Derivation of Planks Law!

1. $\left(1+x+x^2+\cdots\right)=1/(1-x)$

2. $\left(1+2x+3x^2\ \ldots\right)$

$=d/dx(1+x+x^2+x^3+x^4+\ldots)$

$=d/dx[1/(1-x)]=1/(1-x)^2$

Thus:

$$\left(x+2x^2+3x^3\ \ldots\right)=x/(1-x)^2$$

Derivation of Planck's Law

- **The total energy emitted per unit wavelength region is given by**

$$E_\lambda d\lambda = no. of\ oscillators\ per\ unit\ volume\ \times\ average\ energy\ of\ each\ oscillator$$

$$= dn\ \times Eavg$$

- **The number of oscillators per unit volume can be derived by the same way as done Rayleigh and Jeans:**

$$dn = \frac{8\pi}{\lambda^4} d\lambda$$

KL Kapoor, 5e

- The oscillator of the black-body cannot have any amount of energy but has a discrete energy equal to the integral multiple of some minimum energy $\boldsymbol{\epsilon}$

$$\boldsymbol{\epsilon_i = n\epsilon}, \textbf{where n} = \mathbf{0, 1, 2, 3, \ldots}$$

- After this he used the Boltzmann expression to compute the average energy of the oscillator. According to which, the number of oscillators having energy $\boldsymbol{\epsilon_i}$ at temperature T is given by

$$\boldsymbol{N_i = n_0 e^{-\frac{\epsilon_i}{kT}}}$$

- Total number of oscillators, $\boldsymbol{N = \sum_i N_i}$

E_{avg} can be derived in the following manner:

$$N_i = n_0 e^{-\frac{\epsilon_i}{kT}}$$

$$N = N_0 + N_1 + N_2 + \cdots = n_0 e^{-\frac{\epsilon_0}{kT}} + n_0 e^{-\frac{\epsilon_1}{kT}} + n_0 e^{-\frac{\epsilon_2}{kT}} + \cdots$$

$\epsilon_i = \mathrm{n}\epsilon$ [n = 0, 1, 2, 3 …]

$$N = N_0 + N_1 + N_2 + \cdots = n_0\left(1 + e^{-\frac{\epsilon}{kT}} + e^{-\frac{2\epsilon}{kT}} + \cdots\right) = n_0\left(1 + x + x^2 + \cdots\right)$$

Where, $x = e^{-\frac{\epsilon}{kT}}$

Remember: $\left(1 + x + x^2 + \cdots\right) = 1/(1 - x)$

Thus we get: $N = \dfrac{n_0}{1 - x}$

Average Energy of Oscillator

$$\bar{E} = \frac{\sum_i N_i \epsilon_i}{N}$$

$$= \frac{1}{N}[[n_0 \times \varepsilon_0 + n_1 \times \varepsilon_1 + n_2 \times \varepsilon_2 + n_3 \times \varepsilon_3 + ...]$$

$$= \frac{1}{N}[(n_0 \times 0) + n_1\epsilon + 2n_2\epsilon + 3n_3\epsilon \ldots]$$

$$= \frac{1}{N}\left[(n_0 \times 0) + n_0 e^{-\frac{\epsilon_1}{kT}}\epsilon + n_0 e^{-\frac{\epsilon_2}{kT}} 2\epsilon + \cdots\right]$$

$$= \frac{n_0 \epsilon x/(1-x)^2}{n_0/(1-x)} = \frac{\epsilon x}{(1-x)} = \frac{\epsilon}{\left(e^{\frac{\epsilon}{kT}} - 1\right)}$$

$$E_\lambda d\lambda = \frac{8\pi}{\lambda^4} \frac{\epsilon}{\left(e^{\frac{\epsilon}{kT}} - 1\right)} d\lambda$$

Remember:

$(1 + 2x + 3x^2 \ldots)$

$= d/dx(1+x+x^2+x^3+x^4+\ldots)$

$= d/dx[1/(1-x)] = 1/(1-x)^2$

Thus:

$\left(x + 2x^2 + 3x^3 \ldots\right) = x/(1-x)^2$

Where, $x = e^{-\frac{\epsilon}{kT}}$

Considering the continuous nature of energy, average energy can be calculated as:

$$\bar{E} = \frac{\int_0^\infty N\epsilon \, d\epsilon}{\int_0^\infty N \, d\epsilon}$$

Where, $N = n_0 e^{-\frac{\epsilon}{kT}}$

If we Solve, the modified $average\ energy\ (Eavg) = kT$

(Same as derived by equipartition principle)

1) Derive Planck's radiation formula.
2) Show that Planck's radiation formula can be reduced to (i) Wien's radiation formula, and (ii) Rayleigh-Jeans radiation formula in suitable range of wavelengths.
3) Derive the Wien's displacement formula from Planck's radiation formula.
4) Calculate the average energy of an oscillator of frequency $10^{15}s^{-1}$ at 300 K and 5000 K using the classical and quantum nature of light. Show that at 5000 K the average energy tends to the equipartition value (classical value).
5) A 45kW broadcasting antenna emits radio waves at a frequency of 4MHz. (a) How many photons are emitted per second?
6) Starting from Plank's radiation law, obtain Stefan-Boltzmann law: $E= AT^4$; where E=total radiation energy from λ=0 to $\propto$. $A=8\pi^5k^4/15(hc)^3$

Given:

$$\int_0^{\propto} \frac{X^3}{(e^x-1)} = \frac{\pi^4}{15}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{hc}{\lambda kT}}-1\right)}d\lambda$$

Assuming 'λ' is small

1.Wien's radiation Formula

$$E_\lambda d\lambda = \frac{C_1}{\lambda^5}e^{-\frac{C_2}{\lambda T}}d\lambda$$

Wien's displacement formula

Assuming 'λ' is large

3. Rayleigh-Jeans Law

$$E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda$$

1) Derive Planck's radiation formula.
2) Show that Planck's radiation formula can be reduced to (i) Wien's radiation formula, and (ii) Rayleigh-Jeans radiation formula in suitable range of wavelengths.
3) Derive the Wien's displacement formula from Planck's radiation formula.
4) Calculate the average energy of an oscillator of frequency $10^{15}s^{-1}$ at 300 K and 5000 K using the classical and quantum nature of light. Show that at 5000 K the average energy tends to the equipartition value (classical value).
5) A 45kW broadcasting antenna emits radio waves at a frequency of 4MHz. (a) How many photons are emitted per second?
6) Starting from Plank's radiation law, obtain Stefan-Boltzmann law: $E= AT^4$; where E=total radiation energy from $\lambda=0$ to $\propto$. $A=8\pi^5k^4/15(hc)^3$

Given:

$$\int_0^{\propto} \frac{X^3}{(e^x - 1)} = \frac{\pi^4}{15}$$

The followings are the example, where classical mechanics fails

- ***Black body radiation***
- ***Photoelectric effect***
- ***Compton effect***

L2: Objective

1) Photoelectric effect!
2) Effect of intensity, frequency on photo-emission
3) Concept of stopping potential!
4) Determination of h-Millikan’s experiment
5) Basic of Compton’s effect (T2)
6) Generation of X-Rays!
7) Dual nature of light!

Photo-electric Effect

Photo electric effect is the ejection of electrons from various metal surfaces when exposed to the UV or visible light

Chamber:

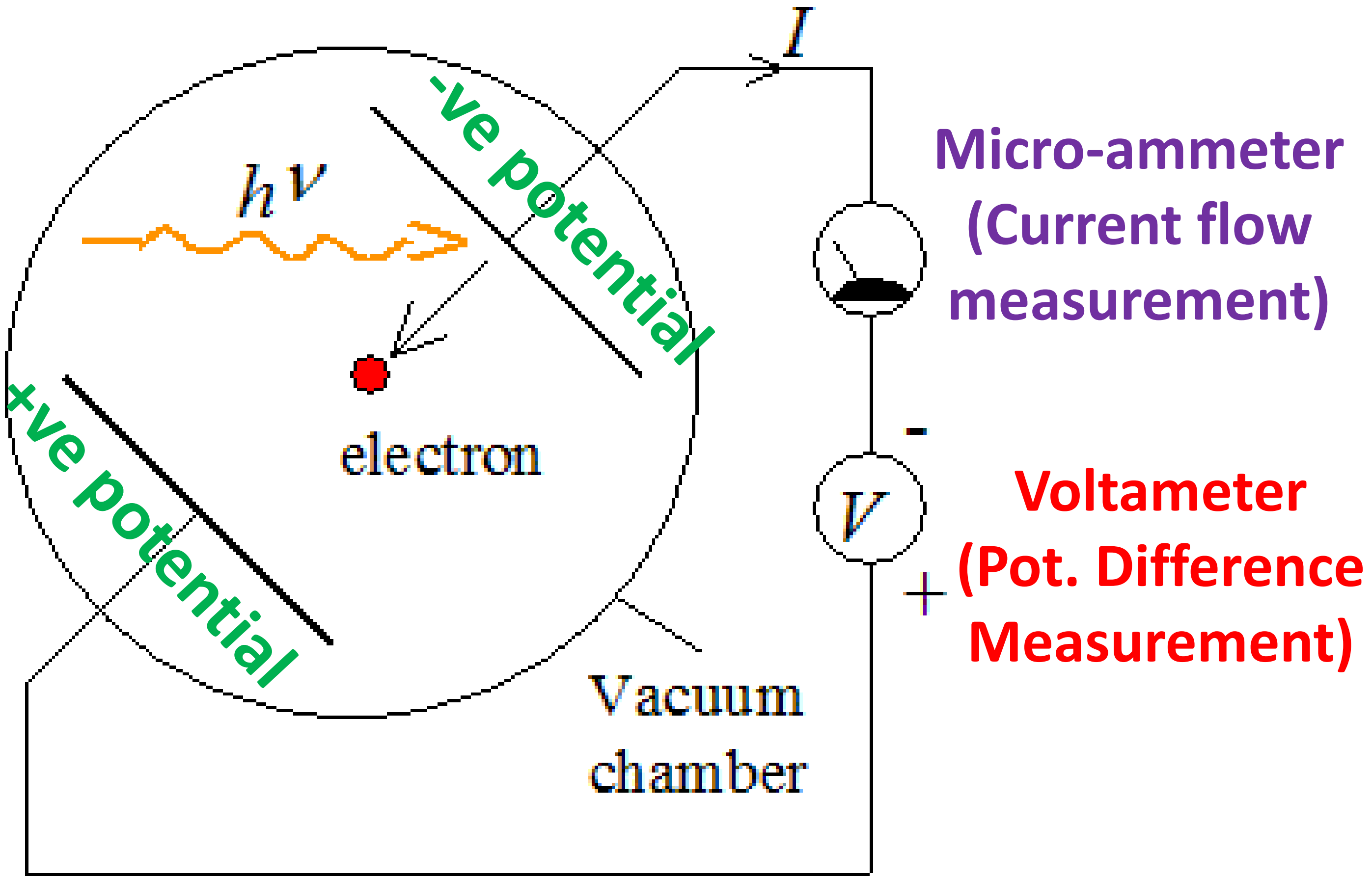
Under Vacuum

Quartz window

Light:

UV

Frequency of light can be varied



~~Cs~~

Low melting point

Photoelectric effect (Major observations)

Electrons are emitted instantaneously from a given metal plate when it is irradiated with the frequency equal or greater than (≥) some minimum frequency called ‘thresold frequency’. The value depends upon the material and nature of emitting surface.

The kinetic energy of the emitted electrons depends on frequency of the incident irradiation and not its intensity.

The number of electrons emitted is proportional to the intensity of radiation.

Photo-current=function of (frequency of incident radiation, intensity, pot difference, surface nature)

Failure of Classical Mechanics

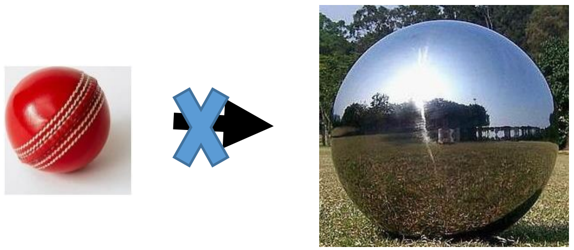
- **Existence of threshold frequency.**
- **Electrons are ejected instantaneously, could not be explained.**
- **If intensity increased, e^- should come with higher energies. But practically intensity had no effect on kinetic energy!**

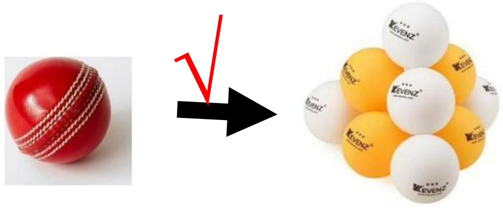
(All are due to the fact that quantization of energy was not known!)

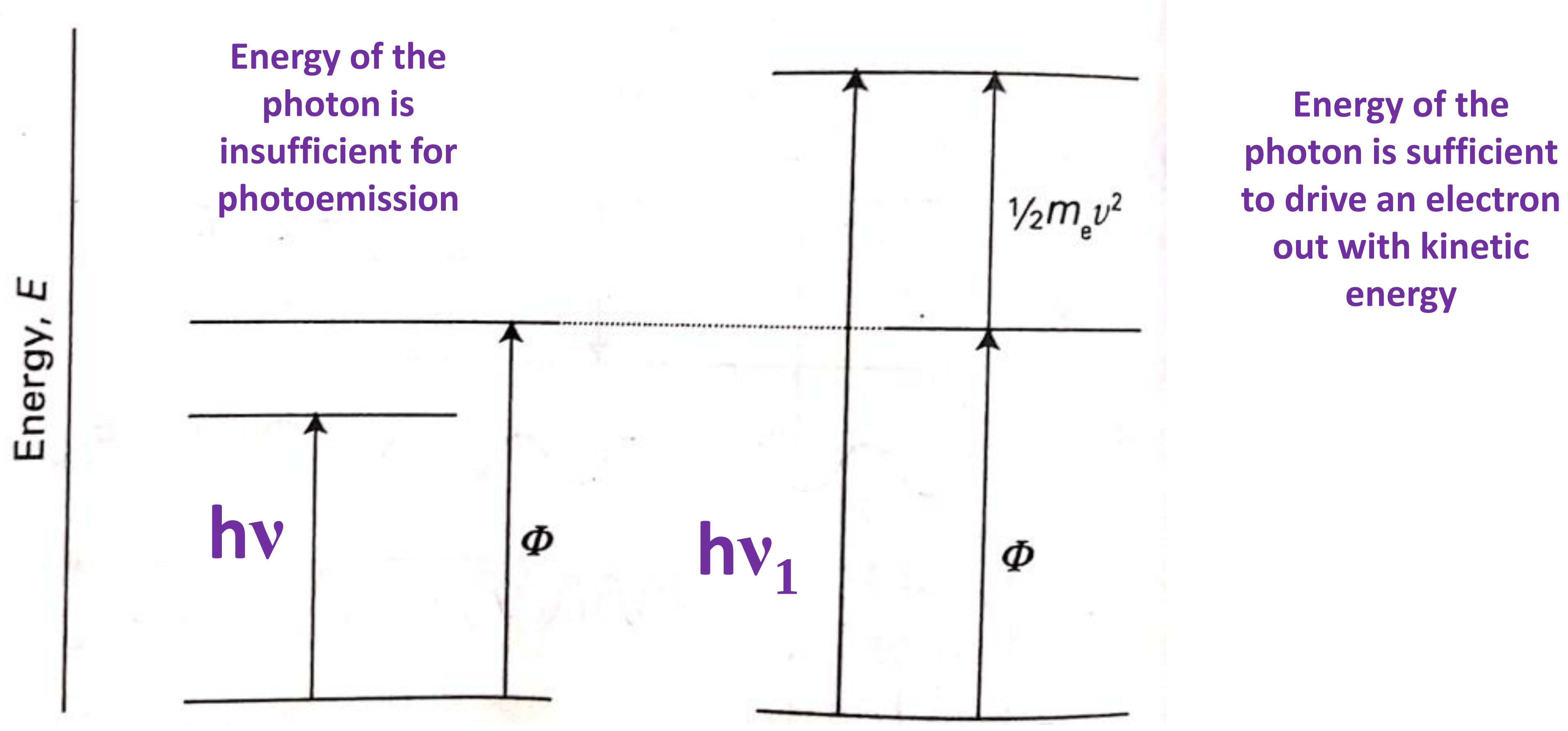
- **Einstein explained the observations with the fact that energy carried by light existed in 'packets' of an amount hν.**
- **When a photon of energy hν is incident on a metal surface, the entire energy is absorbed by a single electron without time lag. This is like having a moving particle collide with and transfer energy to a stationary particle without any time-lag**

Explains the Instanteneous process!

- **The number of electrons ($\nu > \nu_0$) ejected therefore depends upon the number of photons, i.e. the intensity of the light.**
- **Some (%) of the total energy in the packet is used to overcome the binding energy of the electron in the metal, called as the work function, Φ. The remaining energy appears as the kinetic energy , of the emitted electron.**







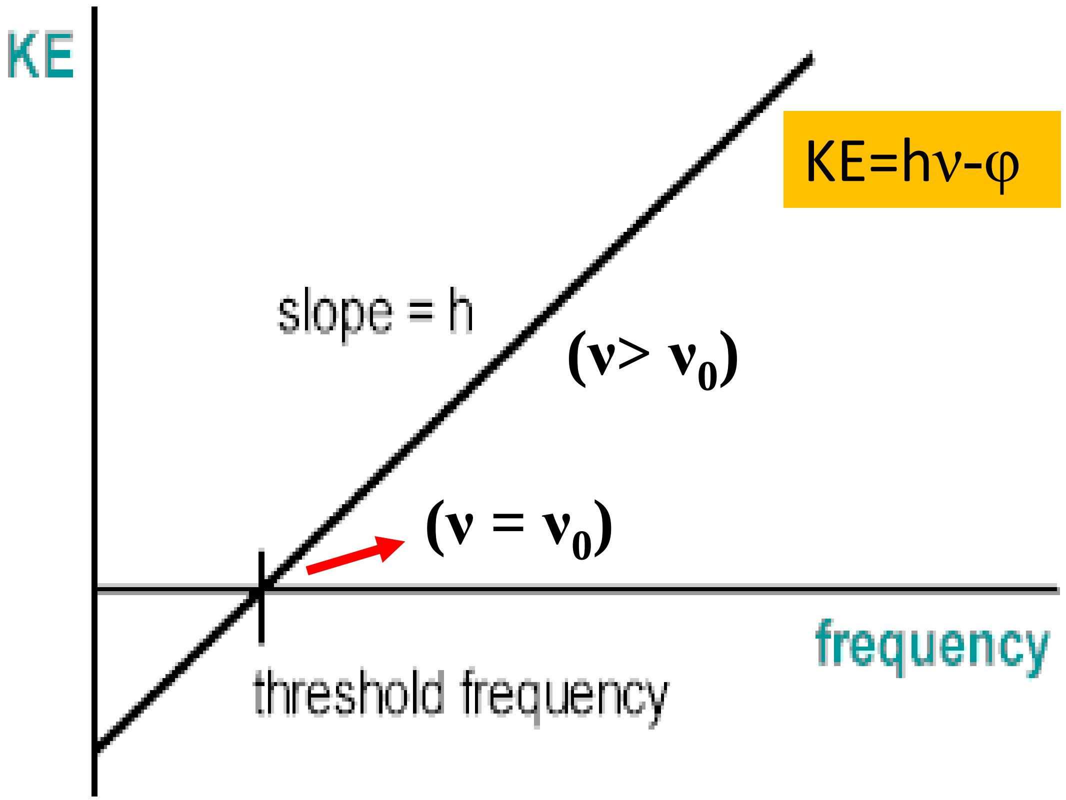
Energy, E
Energy of the photon is insufficient for photoemission
$h\nu$
Φ
$h\nu_1$
$\frac{1}{2}m_e v^2$
Φ
Energy of the photon is sufficient to drive an electron out with kinetic energy

$\nu_{1.} > \nu$

- ✓ **The electrons in a metal posses potential energy which must have to overcome before removal of the metal surface and this energy is called work function of the electrons.**

From conservation of energy:
Total energy of electrons = E_{Photon} = KE of electron + Pot. energy

$$h\nu = \frac{1}{2} mV^2 + \text{Work-function}$$
$$\frac{1}{2} mV^2 = h\nu - \text{Work-function}$$



Total energy of electrons = KE + Pot. energy

hν = ½ mV²+Work-function

The slope of the straight line obtained by plotting the kinetic energy as a function of frequency above the threshold frequency is just Planck's constant, and the x-intercept, where $(1/2)mv^2=0$, is just the work function of the metal, $\Phi=h\nu_0$.

- For a given frequency ν, ($\nu > \nu_0$) the number of photoelectrons emitted/sec from a surface under constant potential difference, is directly proportional to the intensity of the incident radiation.

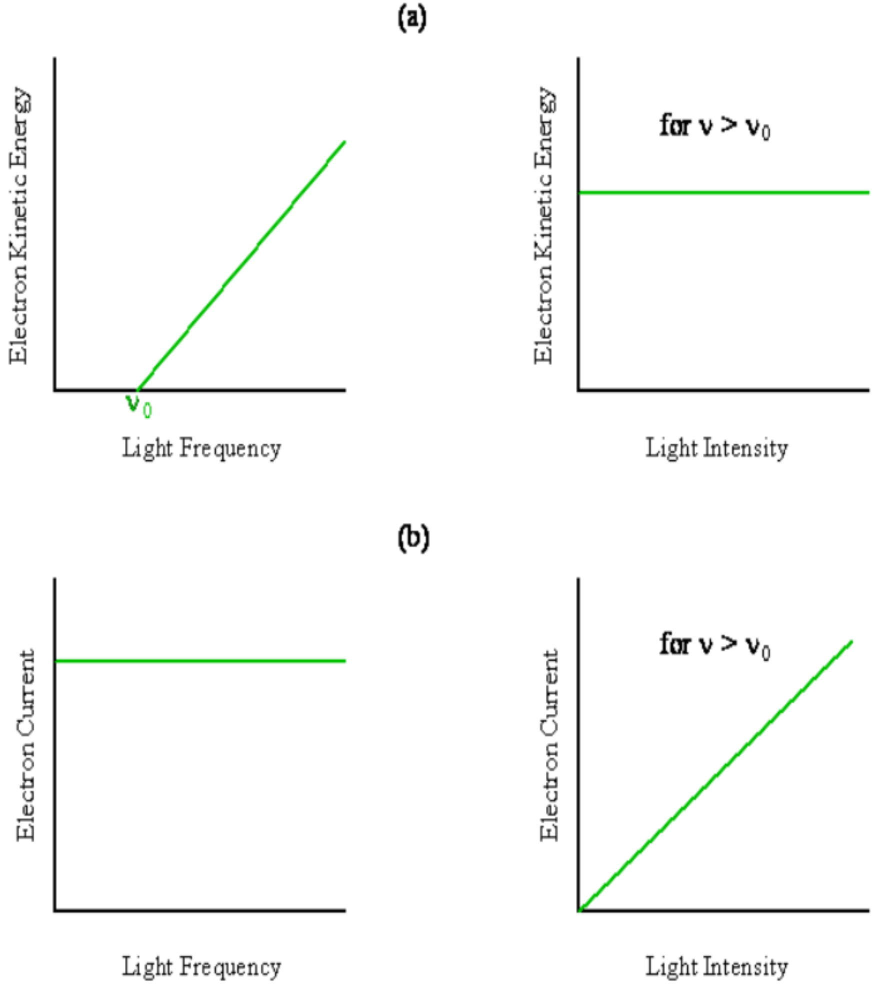


Figure 2.3.1: Schematic drawings showing the characteristics of the photoelectric effect. (a) The kinetic energy of any single emitted electron increases linearly with frequency above some threshold value and is independent of the light intensity. (b) The number of electrons emitted per second (i.e. the electric current) is independent of frequency and increases linearly with the light intensity.

https://chem.libretexts.org/Bookshelves/Physical_and_Theoretical_Chemistry_Textbook_Maps/Book%3A_Quantum_States_of_Atoms_and_Molecules_(Zielinksi_et_al)/02%3A_Foundations_of_Quantum_Mechanics/2.03%3A_Photoelectric_Effect

Concept of Stopping Potential

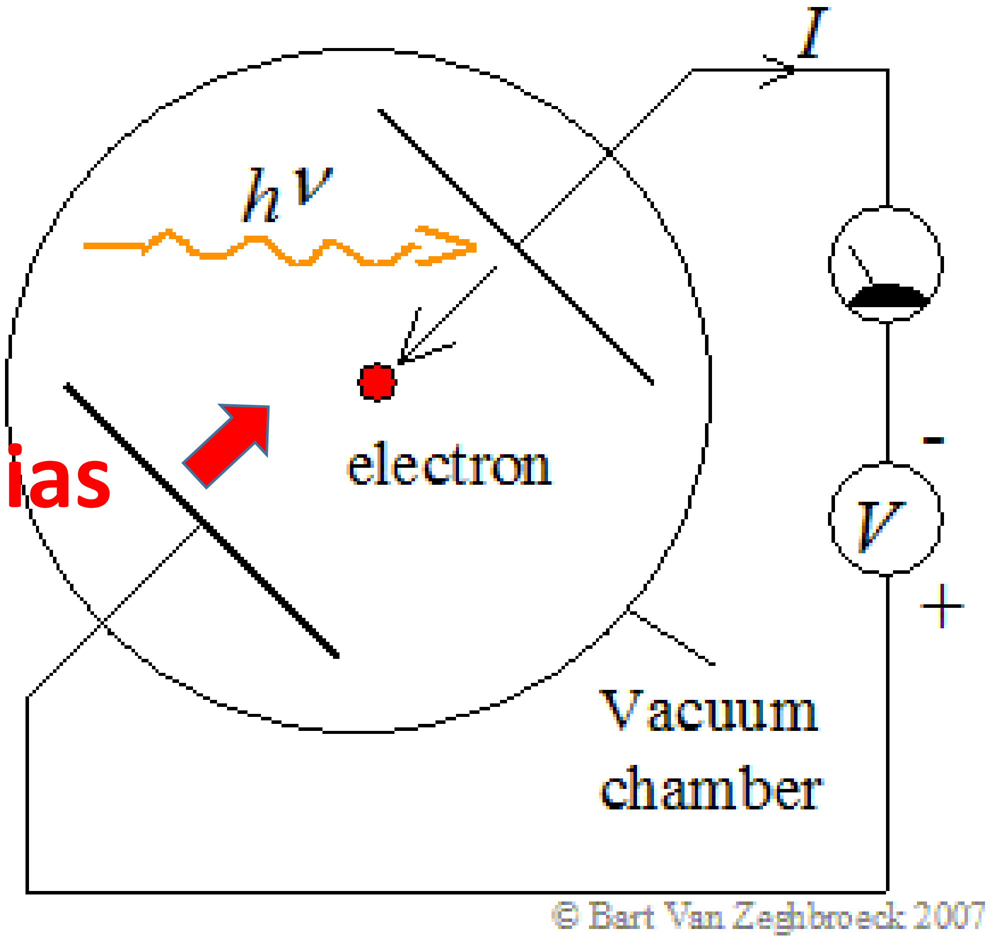
The retarding potential, that stops the photo-current is called stopping potential.

When light of a certain frequency ($\nu > \nu_0$) is incident at a surface, kinetic energies ranging from 0 to maximum kinetic energy electrons are emitted from the surface.

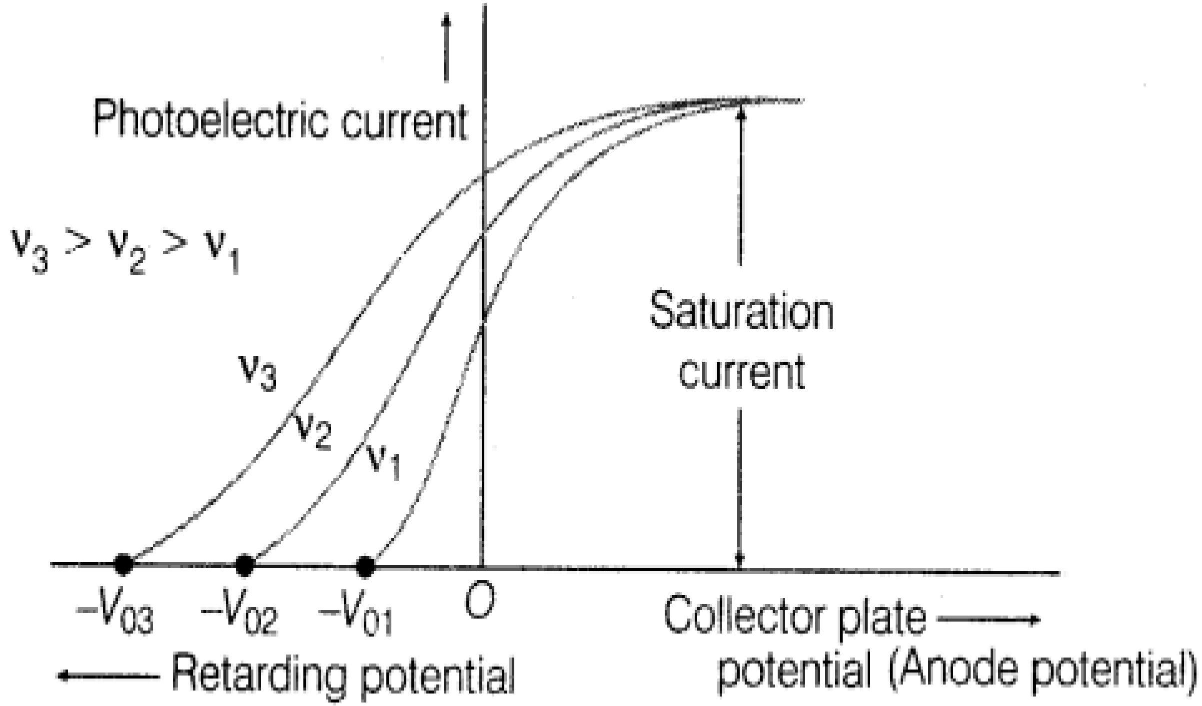
The maximum kinetic energy electrons are stopped by the stopping potential (V_0 volts).

$$eV_0 = (1/2)mV_{max}^2$$

-ve bias



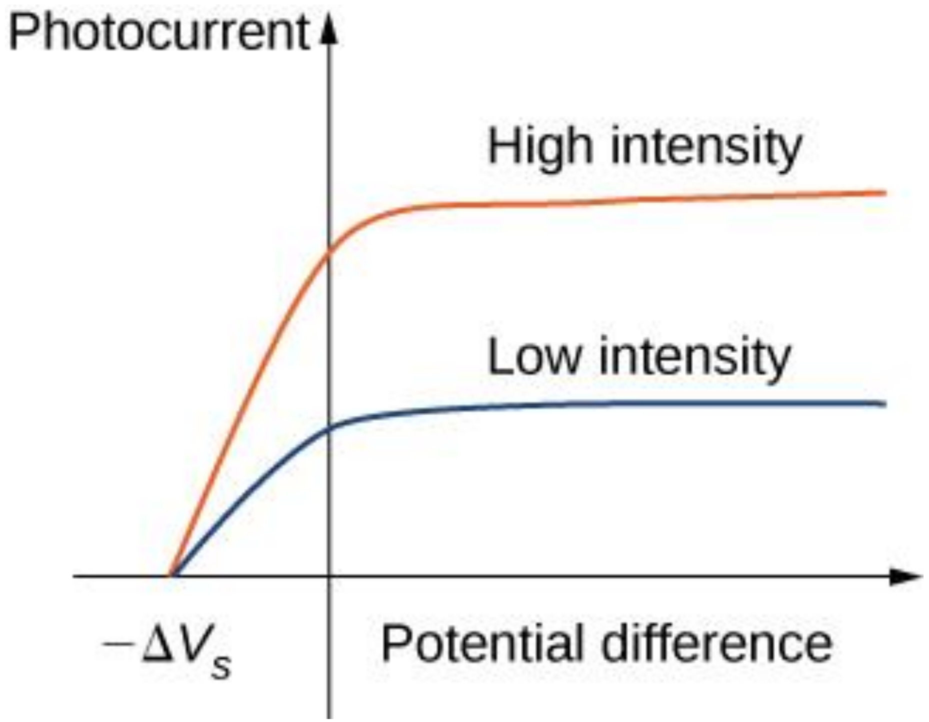
Effect of Frequency and Intensity



If frequency of incident light is increased, stopping potential will decrease

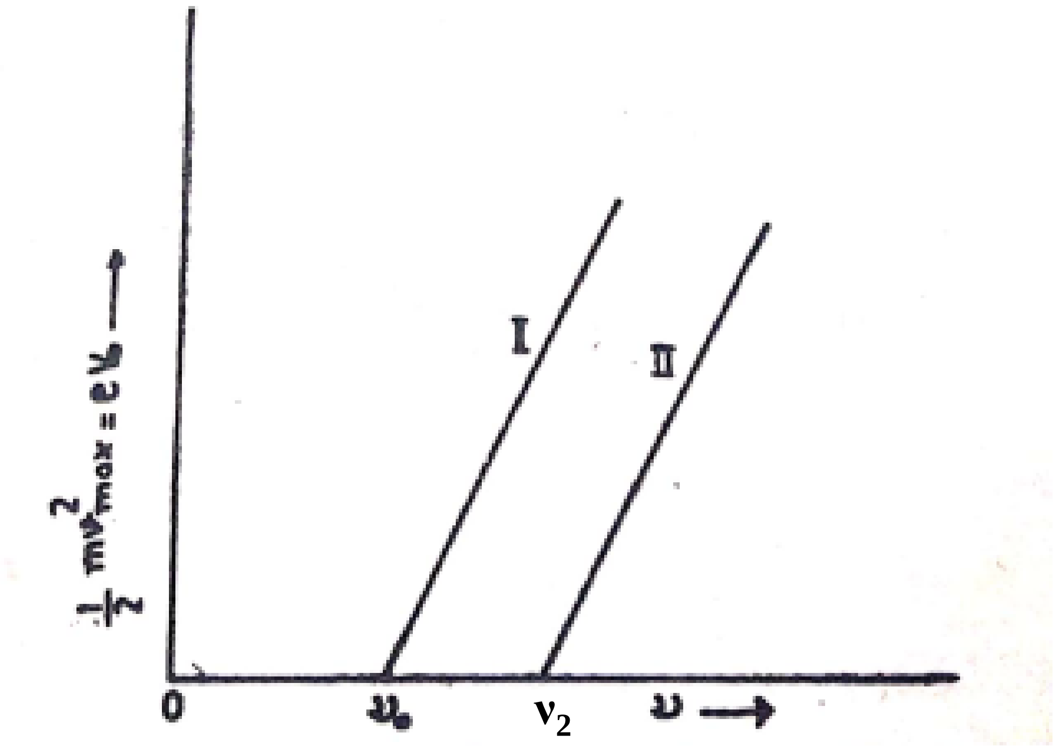
Effect of Intensity

Even light intensity is increased for a frequency ($\nu > \nu_0$), the stopping potential will be the same.



Effect of frequency on maximum kinetic energy

Metal I and Metal II



- **The energy hν gained by the electron is used up in the following two ways:**

1) **A part of energy is used to overcome the attractive forces and the minimum energy is called work function (W_0).**

2) **The remaining energy ($h\nu - W_0$) appears as kinetic energy of the electrons. If the electrons with this kinetic energy comes from the surface without any collision with another electron, then it will be emitted from the surface into vacuum with maximum kinetic energy $\frac{1}{2} m v_{max}^2$.**

Thus,

$$\frac{1}{2} m v_{max}^2 = h\nu - W_0 \qquad (1)$$

If the frequency of incident radiation is reduced to a critical value ν_0 *(threshold frequency)*, such that an electron emitted from surface with zero kinetic energy. [at $\nu = \nu_0$, $\frac{1}{2} mv_{max}^2 = 0$]

$$\frac{1}{2} mv_{max}^2 = h\nu - W_0 \qquad (1)$$

$$0 = h\nu - W_0$$

$$h\nu_0 = W_0$$

Substituting in (1): $\frac{1}{2} mv_{max}^2 = h\nu - h\nu_0 = h(\nu - \nu_0)$

This is another form of Einstein's equation.

Summary

- **When $\nu<\nu_0$ [no photoelectrons]**

- **When $\nu=\nu_0$ [photoelectrons with zero kinetic energy]**

- **When $\nu>\nu_0$ [photoelectrons emitted with kinetic energies ranging from zero to a maximum value]**

- **When $\nu>\nu_0$ and intensity is increased, the photo-electrons emitted/second will increase and thus the current will increase.**

- **Kinetic energy of electrons $\neq$ f (Intensity of light) [at $\nu>\nu_0$]**
- **Kinetic energy of electrons =f(frequency of light) [at $\nu>\nu_0$]**
- **Measured Photocurrent = f(Intensity of light) [at $\nu>\nu_0$]**

MilliKan (1916)

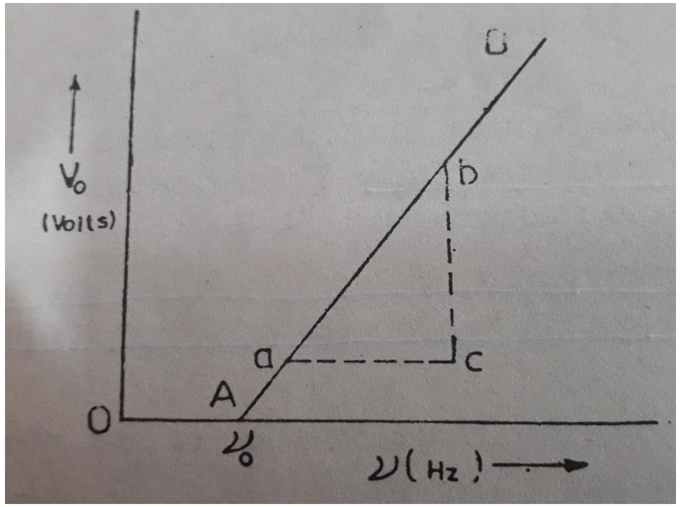
- As you have seen at stopping potential $eV_0 = \frac{1}{2} mv_{max}^2$ [at a particular ν; when $\nu > \nu_0$]
- Then the following can be written:
- **$\frac{1}{2} mv_{max}^2 = eV_0 = h(\nu - \nu_0)$**

$$V_0 = (h/e) \times (\nu - \nu_0)$$

***In the experiment, stopping potential (V_0) was measured by changing frequency -*‘ν’ over a particular metal surface and ‘V_0’*vs* ‘ν’ was plotted. The slope of the straight line can be measured as [bc/ac].**

Thus, h/e = [bc/ac]

[e=1.602×10^{-19} coulomb]



Summary

- ✓ **Planks statement that oscillator in black body radiates energy in discrete unit was extended by Einstein.**

- ✓ **Einstein extended that the radiation it self is quantized, consisting of light quanta called photons. Hence, Einstein considered that light wave to be considered in nature with each photon carrying an energy, E=hν.**

End of Lecture 2