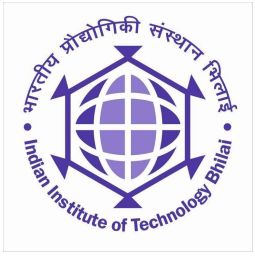


Department of Computer Science and Engineering
Indian Institute of Technology Bhilai

CS621/CSL611 - QuaCCS
Quantum Computing For Computer Scientists

Quiz 1
February 12, 2024

1. A qubit is measured with the outcome $|0\rangle$. [2+3]

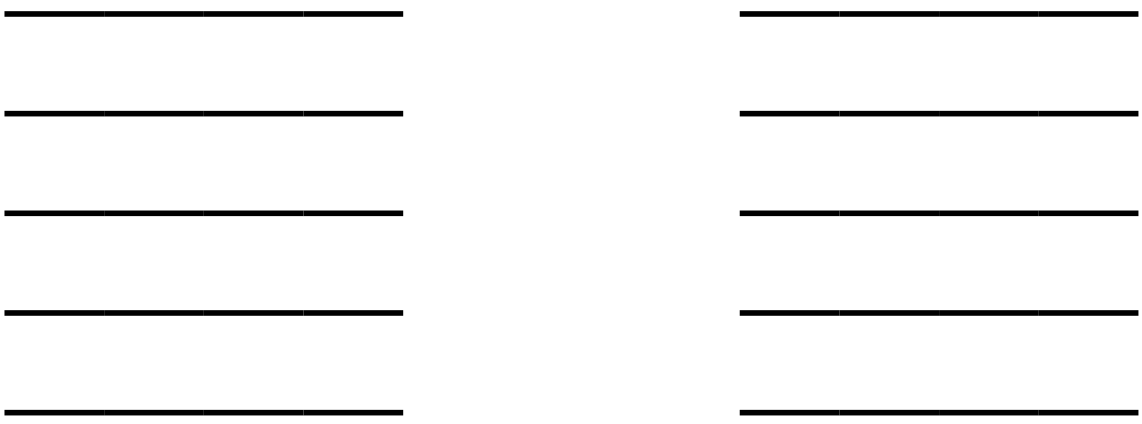
   (a) Which of the following could be its initial state before the measurement:
   $|0\rangle$ $\quad \frac{1}{\sqrt{10}}|0\rangle + \frac{3}{\sqrt{10}}|1\rangle$ $\quad \frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$ $\quad \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$

   (b) If you tried to measure the same qubit a second time, can you narrow down what the initial state was?

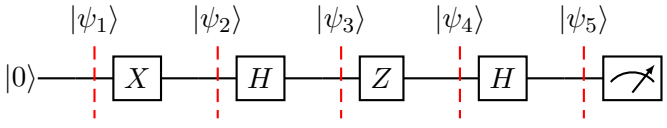
2. Show the solution to the "Know Thy Qubit" problem. The initial superpositions are: [5]

$$v_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} \quad v_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

3. Complete the cccNOT gate discussed in class. Write the intermediate qubits. What happens if we remove the last Toffoli gate. Use the following diagrams. [5]



4. Perform a circuit slicing of the the following circuit. What is the expected measurement histogram produced by it. [3 + 2]



5. Show that the following vectors form a spanning set for $\mathbb{C}^2$. Do they form a basis of $\mathbb{C}^2$? Why? [Hint: Try to represent any $|\psi\rangle = \begin{pmatrix}\alpha\\ \beta\end{pmatrix} \in \mathbb{C}^2$ in terms of $|u_1\rangle$ and $|u_2\rangle$. [7]

$|u_1\rangle = \begin{pmatrix}1\\ 1\end{pmatrix}$ $\quad$ $|u_2\rangle = \begin{pmatrix}1\\ -1\end{pmatrix}$

6. Convert the following element of $\mathbb{C}^2$ to a qubit. [3]

$V = \begin{bmatrix}5-8i\\ -2i\end{bmatrix}$

7. (a) Briefly explain the idea of entanglement. [3+2+2+3]

(b) What is a maximally entangled state?

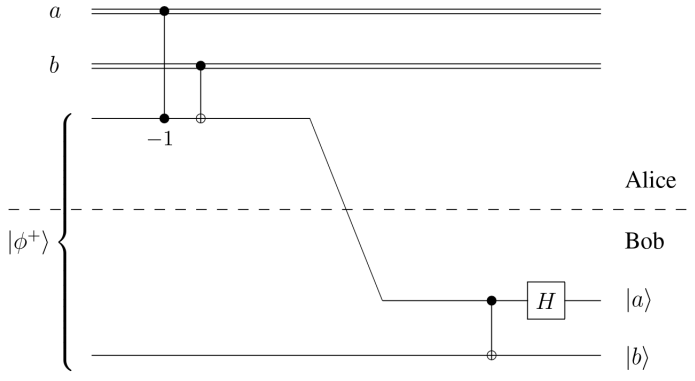
(c) Draw the quantum circuit that would allow to create one such maximally entangled pair of qubits.

(d) Suppose Alice has one half of an entangled pair and Bob has the other half. When Alice makes a measurement on her qubit, Bobs qubit instantaneously changes its state. Can Alice and Bob use entanglement to transmit information faster than the speed of light? Why or why not?

8. Construct the OR gate with two Toffoli gates. Justify the logic used **without** directly using a truth-table. [5]

9. Given below is circuit diagram of the Superdense Coding Protocol as discussed in class. [5+10]

Modify the protocol *without any additional gates* so that the order of a and b is *reversed* at Bob's side. You are allowed to change the protocol either at Alice's side or Bob's side. Give the new circuit diagram along with the steps of the modified algorithm.



 Question 9 continues on the next page...