

CS621/CSL611

Quantum Computing For Computer Scientists

The Leap from Classical to Quantum

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The Leap from Classical to Quantum

- Graphs **without weights** will model classical **deterministic** systems.
- Graphs **weighted with real** numbers will **model classical probabilistic** systems.
- Graphs **weighted with complex** numbers and will model **quantum systems**.

The double-slit experiment

Computer science/graph-theoretic version of the double-slit experiment, perhaps the **most important experiment** in quantum mechanics.

Classical Deterministic Systems

Graph Modeling of the “State” of a Deterministic System

Example

Let there be 6 vertices in a graph and a total of 27 marbles. We might place 6 marbles on vertex 0, 2 marbles on vertex 1, and the rest as described by this picture

0 • 6

1 • 2

2 • 1

3 • 5

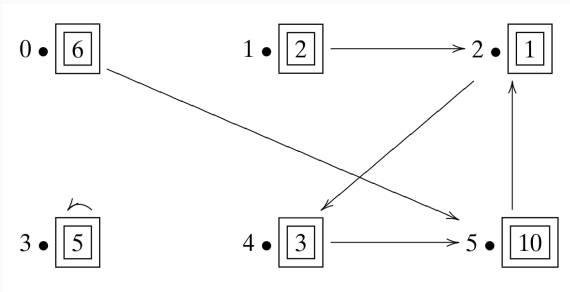
4 • 3

5 • 10

- We shall denote this state as $X = [6, 2, 1, 5, 3, 10]^T$

Capturing the “Dynamics” of the System

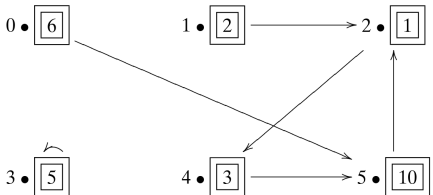
- Dynamics of the system can be represented by a graph with directed edges



- The idea is that if an arrow exists from vertex i to vertex j , then in one time click, all the marbles on vertex i will shift to vertex j .

Boolean Adjacency Matrix

$$M = \begin{matrix} & \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{matrix},$$



- Here $M[i, j] = 1$ if and only if there is an arrow from vertex j to vertex i ¹.
- The requirement that every vertex has exactly one outgoing edge corresponds to the fact that every column of the Boolean adjacency matrix contains exactly one 1.

¹Note that the direction is reversed for reasons to be described later.

- Lets say that we multiply M by a state of the system $X = [6, 2, 1, 5, 3, 10]^T$. Then we have

$$MX = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 6 \\ 2 \\ 1 \\ 5 \\ 3 \\ 10 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 12 \\ 5 \\ 1 \\ 9 \end{bmatrix} = Y.$$

Interpretation

If X describes the state of the system m at time t , then Y is the state of the system at time $t + 1$, i.e., after one time click.

- Using the dynamics given below, determine what the state of the system would be if you start with the state

$$[5, 5, 0, 2, 0, 15]^T$$

$$M = \begin{array}{c} \mathbf{0} \quad \mathbf{1} \quad \mathbf{2} \quad \mathbf{3} \quad \mathbf{4} \quad \mathbf{5} \\ \mathbf{0} \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right. \\ \mathbf{1} \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right. \\ \mathbf{2} \left[\begin{array}{cccccc} 0 & 1 & 0 & 0 & 0 & 1 \end{array} \right. \\ \mathbf{3} \left[\begin{array}{cccccc} 0 & 0 & 0 & 1 & 0 & 0 \end{array} \right. \\ \mathbf{4} \left[\begin{array}{cccccc} 0 & 0 & 1 & 0 & 0 & 0 \end{array} \right. \\ \mathbf{5} \left[\begin{array}{cccccc} 1 & 0 & 0 & 0 & 1 & 0 \end{array} \right. \end{array} \right],$$

Generalizing single iteration

- In general, any simple directed graph with n vertices can be represented by an n – by – n matrix M having entries as

$M[i, j] = 1$ if and only if there is an edge from vertex j to vertex i .

$= 1$ if and only if there is a path of length 1 from vertex j to vertex

- If $X = [x_0, x_1, \dots, x_{n-1}]^T$ is a column vector that corresponds to placing x_i marbles on vertex i , and
- If $MX = Y$ where $Y = [y_0, y_1, \dots, y_{n-1}]^T$, then there are y_j marbles on vertex j after one time click.
- M is thus a way of describing how the state of the marbles can change from time t to time $t + 1$.

Relation with Interpreting Quantum Systems

- (finite-dimensional) quantum mechanics works the same way.
- **States** of a system are represented by **column vectors**, and the way in which the **system changes** in one time click is **represented by matrices**.
- **Multiplying a matrix** with **a column vector** yields a **subsequent state of the system**.

- In general, multiplying an $n - by - n$ matrix by itself several times will produce another matrix whose i, j th entry will indicate whether there is a path after several time clicks.
- Consider $X = [x_0, x_1, \dots, x_{n-1}]^T$ to be the state where one places x_0 marbles on vertex 0, x_1 marbles on vertex 1, \dots, x_{n-1} marbles on vertex $n - 1$.
- Then, after k steps, the state of the marbles is Y , where $Y = [y_0, y_1, \dots, y_{n-1}]^T = M^k X$.
- In other words, y_j is the number of marbles on vertex j after k steps.

Relation with Interpreting Quantum Systems

- In quantum mechanics, if there are two or more matrices that manipulate states, the action of one followed by another is described by their **product**.
- We shall take different states of systems and multiply the states by various matrices (of the appropriate type) to obtain other ones.
- These new states will again be multiplied by other matrices until we attain the desired end state.

- In quantum computing, we shall start with an initial state, described by a vector of numbers.
- The initial state will essentially be the input to the system.
- Operations in a quantum computer will correspond to multiplying the vector with matrices.
- The output will be the state of the system when we are finished carrying out all the operations.

- The **states of a system** correspond to **column vectors** (state vectors).
- The **dynamics** of a system correspond to **matrices**.
- To progress from one state to another **in one time step**, one must multiply the **state vector by a matrix**.
- **Multiple step dynamics** are obtained via **matrix multiplication**.

Write a program that performs our little marble experiment.

- The program should allow the user to enter a Boolean matrix that describes the ways that marbles move.
- Make sure that the matrix follows our requirement.
- The user should also be permitted to enter a starting state of how many marbles are on each vertex.
- Then the user enters how many time clicks she wants to proceed.
- The computer should then calculate and output the state of the system after those time clicks.