CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Architecture

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Classical and Quantum Information

Definition (Bit)

A bit is a unit of information describing a **two**-dimensional classical system.

$$|0\rangle = \begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Example

- A bit is electricity traveling through a circuit or not (or high and low).
- A bit is a way of denoting "true" or "false."
- A bit is a switch turned on or off.

- In the Dirac notation, column vectors are represented by "kets", such as

$$|0\rangle \overset{def}{=} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle \overset{def}{=} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

- Dirac introduced the $|\ \rangle$ notation in the early days of the quantum theory, as a useful way to write and manipulate vectors.
- In Dirac notation you can put into the box $|\ \rangle$ anything that serves to specify what the vector is.

$$|\text{5 horizontal centimeters southwest}\rangle$$

[1] Named after its inventor Paul Dirac

Definition

A quantum bit or a qubit is a unit of information describing a **two**-dimensional quantum system.

- Representing a qubit:

$$\begin{array}{c} \mathbf{0} \\ \mathbf{1} \end{array} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix}$$

- $|c_0|^2 + |c_1|^2 = 1$
- Note: A classical bit is a *special* type of qubit

A qubit is going to look in some way superficially similar to a bit. But it is fundamentally different and that its fundamental difference allows us to do information processing in new and interesting ways.

How is a qubit any different than an ordinary bit?

- A bit in an ordinary computer can be in the state 0 or in the state 1
- A qubit can exist in the state $|0\rangle$ or the state $|1\rangle$, but it can also exist in what we call a *superposition* state.
- This is a state that is a linear combination of the states $|0\rangle$ and $|1\rangle$

Example

If we label this state $|\psi\rangle$, a superposition state is written as $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ with $|\alpha|^2 + |\beta|^2 = 1$

- If an event has N possible outcomes and we label the probability of finding result i by p_i, the condition that the probabilities sum to one is written as

$$\sum_{i=1}^{N} p_i = p_1 + p_2 + \cdots + p_N = 1$$

- When this condition is satisfied for the squares of the coefficients of a qubit, we say that the qubit is **normalized**

- Any nonzero element of $\mathbb{C}^2$ can be **converted** into a qubit

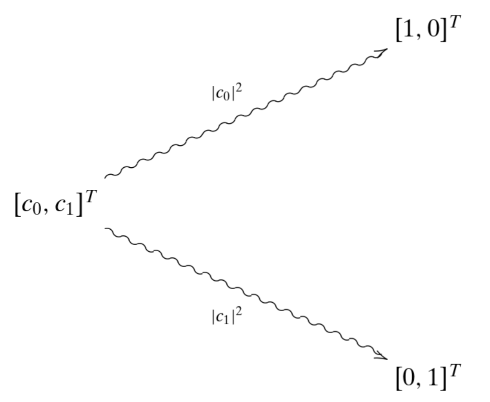
Example

$V = \begin{bmatrix} 5+3i \\ 6i \end{bmatrix}$ has norm

$$|V| = \sqrt{[5-3i, -6i]\begin{bmatrix} 5+3i \\ 6i \end{bmatrix}} = \sqrt{34+36} = \sqrt{70}$$

- V describes the **same physical state** as the qubit

$$\frac{V}{\sqrt{70}} = \begin{bmatrix} \frac{5+3i}{\sqrt{70}} \\ \frac{6i}{\sqrt{70}} \end{bmatrix} = \frac{5+3i}{\sqrt{70}}|0\rangle + \frac{6i}{\sqrt{70}}|1\rangle$$



- Will observe $|0\rangle$ with prob $|c_0|^2$
- Will observe $|1\rangle$ with prob $|c_1|^2$

- What is the probability of finding the following qubit in the state $|0\rangle$ and the state $|1\rangle$ when a measurement is made?

$$\frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$$

- $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ can be written as:

$$\begin{bmatrix}\frac{1}{\sqrt{2}}\\\frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle + \frac{1}{\sqrt{2}}\left|1\right\rangle = \frac{\left|0\right\rangle + \left|1\right\rangle}{\sqrt{2}}$$

- Similarly $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$ can be written as:

$$\begin{bmatrix}\frac{1}{\sqrt{2}}\\-\frac{1}{\sqrt{2}}\end{bmatrix} = \frac{1}{\sqrt{2}}\left|0\right\rangle - \frac{1}{\sqrt{2}}\left|1\right\rangle = \frac{\left|0\right\rangle - \left|1\right\rangle}{\sqrt{2}}$$

[2] Any vector indexed by the set $\{0, 1\}$ can be represented by a linear combination of $\left|0\right\rangle$ and $\left|1\right\rangle$, because $\{\left|0\right\rangle, \left|1\right\rangle\}$ is a basis for this space of vectors.