Given (may be required in calculations):

$$\Gamma(n) = \int_0^\infty dx e^{-x} x^{n-1} \tag{1}$$

$$\zeta(n) = \frac{1}{\Gamma(n)} \int_0^\infty dx \frac{x^{n-1}}{e^x - 1} \tag{2}$$

$$L(n) = \int_0^\infty dx \frac{x^n}{e^x + 1} = (1 - 2^{-n})\Gamma(n+1)\zeta(n+1) \tag{3}$$

1. No of photons N, emitting from black body at temperature T (and chemical potential $\mu = 0$) can be expressed as

$$N = 2 \int_0^\infty \frac{d^3x d^3p}{h^3} \frac{1}{e^{\beta\epsilon} - 1} \tag{4}$$

with photon's energy $\epsilon = pc$. Using the Riemann zeta function

$$\zeta(n) = \frac{1}{\Gamma(n)} \int_0^\infty \frac{x^{n-1}}{e^x - 1} \tag{5}$$

with Gamma function $\Gamma(n) = (n-1)!$ (when n is integer), we get number density as

(a) $\frac{N}{V} = 8\pi \left(\frac{K_B T}{hc}\right)^3 \zeta(1)\Gamma(1)$

(b) $\frac{N}{V} = 8\pi \left(\frac{K_B T}{hc}\right)^3 \zeta(2)\Gamma(2)$

(c) $\frac{N}{V} = 8\pi \left(\frac{K_B T}{hc}\right)^3 \zeta(3)\Gamma(3)$

(d) none of the above

2. Total energy (internal energy) of photon gas at temperature T (and chemical potential $\mu = 0$) can be expressed as

$$U = 2 \int_0^\infty \frac{d^3x d^3p}{h^3} \frac{\epsilon}{e^{\beta\epsilon} - 1} \tag{6}$$

with photon's energy $\epsilon = pc$. Using the Riemann zeta function (given in earlier question), we get energy density as

(a) $\frac{U}{V} = 8\pi K_B T \left(\frac{K_B T}{hc}\right)^3 \zeta(2)\Gamma(2)$

(b) $\frac{U}{V} = 8\pi K_B T \left(\frac{K_B T}{hc}\right)^3 \zeta(3)\Gamma(3)$

(c) $\frac{U}{V} = 8\pi K_B T \left(\frac{K_B T}{hc}\right)^3 \zeta(4)\Gamma(4)$

(d) none of the above

3. Energy density U/V and intensity I of photon gas is connected through relation $I = \frac{c}{4}\frac{U}{V}$, which can reproduce the famous empirical law - Stefan-Boltzmann law $I = \sigma T^4$, where

(a) $\sigma = 8\pi \left(\frac{K_B^4}{h^3 c^2}\right) \zeta(2)\Gamma(2)$

(b) $\sigma = 8\pi \left(\frac{K_B^4}{h^3 c^2}\right) \zeta(3)\Gamma(3)$

(c) $\sigma = 8\pi \left(\frac{K_B^4}{h^3 c^2}\right) \zeta(4)\Gamma(4)$

(d) none of the above

4. If we see the integrand of energy density or intensity of photon gas, then it provide us the black body spectrum by using the quantum relation $\epsilon = pc = h\nu = hc/\lambda$. Which observation can not be connected with the integrand or which observation is wrong?
(a) energy density first increases the decreases along ν or λ axis
(b) Peak value spectrum depends on T
(c) Peak value spectrum does not depend on T
(d) none of the above

5. In the Eq. (6), replacing BE distribution by MB, we can get

$$U = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{\epsilon}{e^{\beta\epsilon}} \tag{7}$$

with photon's energy $\epsilon = pc$. Using the Gamma function $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$, we get energy density as
(a) $\frac{U}{V} \propto T^2$
(b) $\frac{U}{V} \propto T^3$
(c) $\frac{U}{V} \propto T^4$
(d) none of the above

6. Pressure (P) of photon gas at temperature T (and chemical potential $\mu = 0$) can be expressed as

$$\frac{PV}{K_BT} = 2\int_0^\infty \frac{d^3x d^3p}{h^3}\frac{pc/3}{e^{\beta\epsilon}-1} \tag{8}$$

with photon's energy $\epsilon = pc$. Using the Riemann zeta function (given in earlier question), we get
(a) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(2)\Gamma(2)$
(b) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(3)\Gamma(3)$
(c) $P = \frac{8\pi}{3}K_BT\left(\frac{K_BT}{hc}\right)^3\zeta(4)\Gamma(4)$
(d) none of the above

7. Photons are
(a) Fermions and follow FD distribution
(b) Bosons and follow BE distribution
(c) classical particles and follow MB distribution
(d) none of the above

8. For any particle with spin $s\hbar$ has spin-degeneracy factor $2s+1$. e.g. electron's spin is $\hbar/2$ and spin-degeneracy factor 2. In this regards, photon has an interesting properties:
(a) photon has spin $\hbar$ but its spin-degeneracy factor is 2
(b) photon has spin $\hbar/2$ but its spin-degeneracy factor is 3
(c) photon has spin $\hbar$ but its spin-degeneracy factor is 1
(d) none of the above

9. White Dwarf form a degenerate electron gas. We know that electrons are
(a) Fermions and follow FD distribution
(b) Bosons and follow BE distribution
(c) classical particles and follow MB distribution
(d) none of the above

10. Assuming white Dwarf (WD) as degenerate non-relativistic electron gas, whose temperature T is quite smaller than Fermi energy E_F or μ (i.e. $\mu/(KT) \gg 1$). Its equation of state (EoS) become different from ideal gas equation $P = nKT$, where P, n are pressure and number density. EoS of WD will be
(a) $\frac{P}{n} \propto KTn^{4/3}$
(b) $\frac{P}{n} \propto KTn^{2/3}$
(c) $\frac{P}{n} \propto KTn^{2/3}$
(d) none of the above

11. For above problem but for ultra-relativistic case ($E = pc$), EoS of WD will be
(a) $\frac{P}{n} \propto KTn^{4/3}$
(b) $\frac{P}{n} \propto KTn^{2/3}$
(c) $\frac{P}{n} \propto KTn^{2/3}$
(d) none of the above

12. Assuming white Dwarf (WD) as degenerate non-relativistic electron gas, whose temperature T is quite smaller than Fermi energy E_F or μ (i.e. $\mu/(KT) \gg 1$). Its Equi-partition law become different from ideal gas case $u = \frac{3}{2}nKT$, where u, n are internal energy density and number density. Equi-partition law of WD will be
(a) $\frac{u}{n} \propto \frac{3}{2}KTn^{4/3}$
(b) $\frac{u}{n} \propto \frac{3}{2}KTn^{2/3}$
(c) $\frac{u}{n} \propto \frac{3}{2}KTn^{1/3}$
(d) none of the above

13. For above problem but for ultra-relativistic case ($E = pc$), Equi-partition law of WD will be
(a) $\frac{u}{n} \propto \frac{3}{2}KTn^{4/3}$
(b) $\frac{u}{n} \propto \frac{3}{2}KTn^{2/3}$
(c) $\frac{u}{n} \propto \frac{3}{2}KTn^{1/3}$
(d) none of the above

14. At low temperature, n number of Fermions and m number of Bosons can occupy the lowest quantum state, where values of n and m are
(a) $n \geq 0$, $m = 1$
(b) $n = 1.4$, $m \geq 1$
(c) $n = 1$, $m \to \infty$
(d) none of the above

15. Internal energy density $u = U/V$ for massless quark-gluon plasma with quark degeneracy factors 36 and gluon degeneracy factor 16 will be (assuming h=c=K=1 in natural unit)
(a) $u = 15.6T^4$
(b) $u = 42T^4$
(c) $u = 42T^3$
(d) none of the above