CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Circuits and Protocols

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Quantum Algorithms

Basic Framework

- Application of a Hadamard gate to an arbitrary qubit is an example of quantum interference
- Recall for $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$,

$$H|\psi\rangle = \left(\frac{\alpha+\beta}{\sqrt{2}}\right)|0\rangle + \left(\frac{\alpha-\beta}{\sqrt{2}}\right)|1\rangle$$

- Note probabilities of obtaining $|0\rangle$ and $|1\rangle$ have changed.

$$\alpha \to \frac{\alpha+\beta}{\sqrt{2}} \qquad \beta \to \frac{\alpha-\beta}{\sqrt{2}}$$

- For the state:

$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad H|\psi\rangle = H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = |0\rangle$$

- This is a manifestation of *quantum interference*
- Mathematically this means the addition of probability amplitudes.
- There are two types of interference,
    - *positive interference* in which probability amplitudes add constructively to increase or
    - *negative interference* in which probability amplitudes add destructively to decrease

+ve/-ve Interference

$$
\begin{aligned}
H|\psi\rangle &= H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}H|0\rangle + \frac{1}{\sqrt{2}}H|1\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\
&= \frac{1}{2}\left(\underbrace{|0\rangle + |0\rangle}_{+ve\ interference} + \underbrace{|1\rangle - |1\rangle}_{-ve\ interference}\right) = |0\rangle
\end{aligned}
$$

- Probabilities of measurements of $|0\rangle$ and $|1\rangle$ change w.r.t $|\psi\rangle \rightarrow H|\psi\rangle$:

$$
|0\rangle : \quad \underbrace{\frac{1}{2} \rightarrow 1}_{+ve\ interference} \qquad |1\rangle : \quad \underbrace{\frac{1}{2} \rightarrow 0}_{-ve\ interference}
$$

Quantum Interference in Quantum Algorithms

Quantum interference

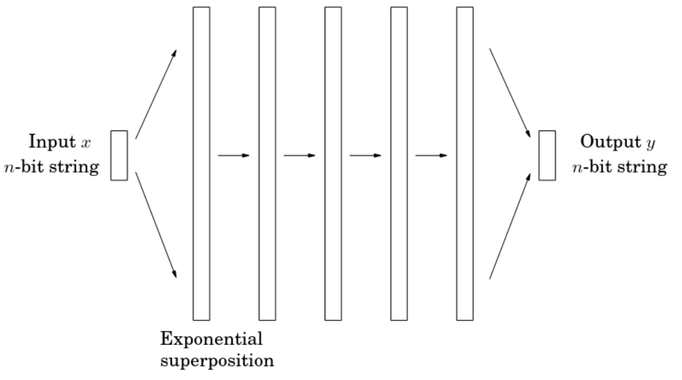
- An integral part of of the basic quantum algorithm toolkit
- Allows to gain information about a function $f(x)$ that depends on evaluating the function *at many values* of x
- Allows to deduce certain *global properties* of the function
- Plays important role in quantum parallelism

Quantum Parallelism and Function Evaluation

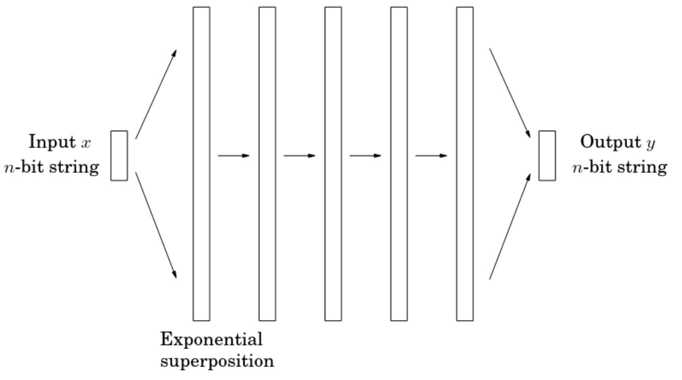
Quantum parallelism can be described as the ability to evaluate the function $f(x)$ at many values of x simultaneously

- Deutsch's algorithm helps demonstrate the power of quantum parallelism using a very simple problem

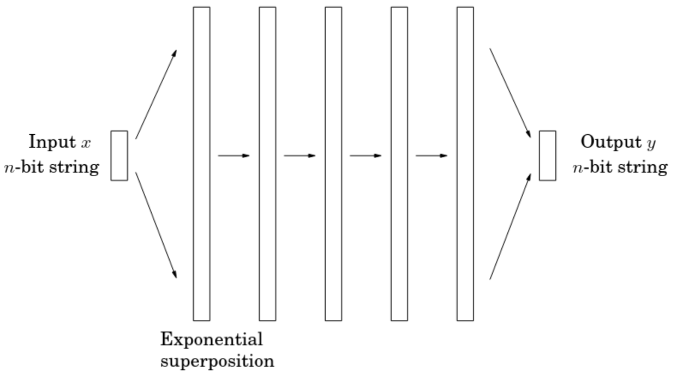
- A quantum algorithm takes n “classical” bits as its input



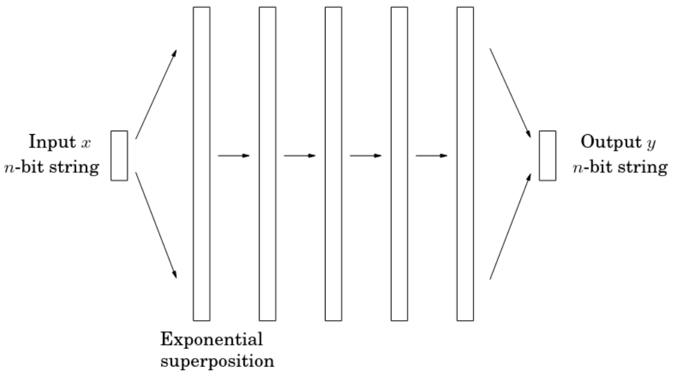
- Manipulates them so as to create a superposition of their 2^n possible states



- Manipulates this exponentially large superposition to obtain the final quantum result

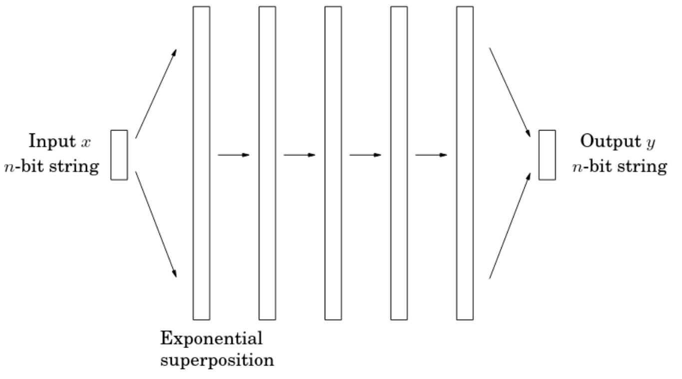


- Then measures the result to get (with the appropriate probability distribution) the *n* output bits.



Quantum Parallelism

For the middle phase, there are elementary operations which count as one step and yet manipulate all the exponentially many amplitudes of the superposition.



References

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3. Lecture Notes on Quantum Computation, John Watrous, University of Calgary
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