Game Theory

- Game Theory is a study of strategic situation which consider one players action based on other player's action/decision
- General terms:
  - → Players
  - → Strategy/actions
  - → Pay-off
  - → Pay-off matrix
- GT is a formal way of analysing interactions among a group of rational agents who behave strategically.
- Group: More than 1 decision maker (player)
- Interaction: What any one individual player does affect atleast one other player in the group.
- Strategic: An individual player accounts for this interdependence in deciding what action to take.
- Rational: Accounting for interdependence, each player chooses best action.
- When can a situation fail to be a game?

  2 cases:

  i) the one: your decision affects only you

  Eg: exercise, study

  ii) The infinity: your decision affect others but the group is so large that one individual player action need not be traced or need not influence the whole group.

Eg: Purchase/sale of shares (stock), person's decision by 'onions' doesn't affect market price

Ex: Card Game

- Two piles of cards: A and B
- Two players: P1 and P2
- Two cases: Balanced Game (BG) and Unbalanced Game (UBG)

- Rules:

R1: Any no. of cards can be taken from a pile. If either piles have cards remaining, then players are required to take atleast 1 card.

R2: Players can only remove cards from one pile at a time

R3: The game starts with player 1 & then each player gets alternative chances (P1-P2-P1-P2 and so on until the game ends)

R4: One who picks last card wins

Ex: $[A,B] = [1,1]$

$P1\,[0,1]$, $P2\,[0,0]$ = P2 wins

Balanced:

P1: Best strategy given P2 action affect P1 is that to ensure both piles have cards. Otherwise P2 will grab all remaining cards in the available pile & wins the game

P2: Best strategy given P1s strategy is that to ensure both piles have equal no. of cards after his/her move; which the P2 can achieve by exactly removing same no. of cards as P1 but from other pile (mimickin

P1 action in B if P1 removed from A)
- Outcome / result : P2 has a winning strategy for BG.

Unbalanced (UBG)
- P1 begins game hence adopt P2 strategy in BG which is to ensure that both piles have equal no. of cards after P2s move. For that P1 just need to mimick P2's action in other pile.
- P2 : Best strategy is to ensure both piles have cards.
- For UBG strategy got reversed
- Outcome / result : P1 has a winning strategy for UBG.

Actions
- The theory is based on a model with 2 components:
  i) set A consisting of all actions that under some circumstances are available to decision maker
  ii) specification of decision maker's preferences.

Preferences
- Decision maker when presented with any pair of actions, knows which of the pair she prefers or knows that she regards both actions as equally desirable.
- Preferences are consistent ⇒ if decision maker prefers action a to action b & action b to action c then she prefers a to c.
- Preferences are represented by payoff

function which associates a no. with each action in such a way that actions with higher numbers are preferred.

- Payoff function u represents a decision maker's preferences if for any actions a in A and b in A

$$u(a) > u(b) \text{ iff decision maker prefers}$$

a to b.

- Payoff function → Preference indicator function

Ex: P1 cares both about her income & about P2's income. Precisely, the value she attaches to each unit of her own income is the same as value she attaches to any 2 units of person 2's income. How do her preferences order the outcomes (1,4), (2,1) & (3,0) where the first component in each case is person 1's income & second is person 2's income? Give a payoff consistent with these preferences.

Sol

$$U(x_1, x_2) = x_1 + \frac{x_2}{2}$$

$x_1 \rightarrow$ her income

$x_2 \rightarrow$ P2's income

$U(1,4) = 1 + \frac{4}{2} = 3$

$U(2,1) = 2 + \frac{1}{2} = 2.5$

$U(3,0) = 3 + \frac{0}{2} = 3$

$U(1,4) \sim U(3,0) > U(2,1)$

- A decision-maker's preferences convey only ordinal info. They tell us that decision maker prefers a to b to c but do not tell how much she prefers a to b or whether she prefers a to b more than b to c.
- Payoff function → ordinal info
- Payoff function can only tell ordering but not how much it prefers one over other.

Ex: $u(a) = 0$, $u(b) = 1$, $u(c) = 100$ — c to b to a

$v(a) = 0$ $v(b) = 100$ $v(c) = 101$ — c to b to a

= ~~If~~ a ~~repr.~~

<u>Theory of rational choice</u>:

- is that in any given situation the decision-maker chooses the member of available subset of A that is best according to her preferences.
- All choices must be consistent.

<u>Strategic Games</u>

- Strategic game is a model of interacting decision-makers (players)
- Model captures interaction b/w players by allowing each player to be affected by actions of all players not only her own action.
- A strategic game consists of:
  - → a set of players
  - → for each player, a set of actions
  - → for each player, preferences over the set of action profiles (list of all player's actions)

- Time is absent from the model means that when analyzing a situation as a strategic game we abstract from the complications that may arise if a player is allowed to change her plan as events unfold.

Prisoner's Dilemma

- strategic game
- Its name come from a story involving suspects in a crime; its importance comes from the huge variety of situations in which participants face incentives similar to those faced by suspects in the story.

Prisoner's dilemma: Two suspects in a major crime are held in separate cells. There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against other (finks). If both stay quiet, each will be convicted of minor offense & spend 1yr in prison. If one & only one of them finks, she will be freed & used as witness against other who will spend 4 years in prison. If both fink, each will spend 3 years in prison.

Strategic game:

Players → 2 suspects

Actions → {Quiet, Fink}

Preferences → suspect 1's →

(Fink, Quiet) > (Quiet, Quiet) > (Fink, Fink) > (Quiet, Fink)

Suspect 2's →

(Quiet, Fink) > (Quiet, Quiet) > (Fink, Fink) > (Fink, Quiet)

We need a payoff function u_1 for suspect 1:

$u_1(\text{Fink}, \text{Quiet}) > u_1(\text{Quiet}, \text{Quiet}) > u_1(\text{Fink}, \text{Fink}) > u_1(\text{Quiet}, \text{Fink})$

Same for u_2, consider some values that satisfy this inequality

Suspect 2

		Quiet	Fink
Suspect 1	Quiet	2, 2	0, 3
	Fink	3, 0	1, 1

Payoff of player 1 is listed first.

- C → Quiet D → Fink

P1 prefers : (D, C) > (C, C) > (D, D) > (C, D)

P2 prefers : (C, D) > (C, C) > (D, D) > (D, C).

Ex: Joint project : You & your friend can work hard or goof off

	Workhard	Goof off
Work hard	2, 2	0, 3
Goof off	3, 0	1, 1

1st: (G, W) > (W, W) > (G, G) > (W, G)

2nd: (W, G) > (W, W) > (G, G) > (G, W)

Ex: Same as above except that each person prefers to work hard than to goof off when other person works hard.

Player 1 prefers : $(W,W) > (G,W) > (G,G) > (W,G)$

Player 2 prefers : ~~(G,W) > (W,G) > (W,G) > (G,G)~~

$(W,W) > (W,G) > (G,G) > (G,W)$

~~Duopoly:~~

~~Two firms produce same good for which each firm charges either a low~~

Duopoly: 2 firms produce same good for which each firm charges a low or high price.

	H	L
H	1000, 1000	-200, 1200
L	1200, -200	600, 600

- Issue in Prisoner's Dilemma → whether or not players will cooperate

Bach or Stravinsky (BoS) [Battle of Sexes]

2 people want to go to concert < Bach / Stravinsky

1 prefer Bach & other Stravinsky.

Both go to different ⇒ unhappy

	B	S
B	(2,1)	(0,0)
S	(0,0)	(1,2)

Matching Pennies

2 people choose Head or tail of a coin.

~~Both~~ show Same side ⇒ P2 pays P1

Different ⇒ P1 pays P2

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

Stag Hunt

Group of hunters

2 options < stag / hare

If all pursue stag, they catch it & share it equally. If any hunter devotes her energy to catching hare, stag escapes & hare belongs to defecting hunter alone.

Players → hunters

Actions → {stag, Hare}

Preferences →

$\{\cancel{Stag}, S\} > \{H, S\} = \{H, H\} > \{S, H\}$

	S	H
S	2,2	0,1
H	1,0	1,1

Ex: Security dilemma: A country prefers both countries refrain from arming to one in which it alone arms: the cost of arming outweighs the benefit if other country doesn't arm itself.

	Refrain	Arm
Refrain	3,3	0,2
Arm	2,0	1,1

Nash Equilibrium

- Experience leads to beliefs about the actions of typical opponents.
- Solution theory has 2 components:
  i) Player chooses her action according to model of rational choice, given her belief about other player's actions
  ii) Every player's belief about other player's actions is correct
- A Nash equilibrium is an action profile a^* with the property that no player i can do better by choosing an action different from a_i^* given that every other player j adheres to a_j^*.
- Nash equilibrium corresponds to a steady state.
- Expectations are coordinated.

$a \rightarrow$ action profile in which action of each player i is a_i.

$a_i' \rightarrow$ action of player i (either a_i or different)

$(a_i', a_{-i}) \rightarrow$ action profile in which every player j except i chooses her action a_j as specified by a whereas i chooses a_i' [only i deviates]

- For an action profile a^* to be a Nash equilibrium: no player i has any action a_i for which she prefers (a_i, a^*_{-1}) to a^*.

(or)

for every player i & action a_i of i, action profile a^* is atleast as good for player i as action profile (a_i, a^*_{-i})

$u_i(a^*) \geq u_i(a_i, a^*_{-i})$ for every action a_i of player i

[action profile (a_i, a^*_{-i}) → i chooses a_i while every other j chooses a^*_j]

Examples of Nash Equilibrium:

1) Prisoner's Dilemma

	Quiet	Fink
Quiet	2, 2	0, 3
Fink	3, 0	1, 1

a) (Fink, Fink) → Nash equilibrium
→ Fix P1 to Fink then P2 is better off choosing Fink than quiet ($1 > 0$)
→ Fix P2 to Fink then P1 is better off choosing Fink than Quiet ($1 > 0$)
∴ Nash equilibrium

b) (Fink, Quiet) → not nash equilibrium
→ When P1 chooses Fink, P2's payoff to Fink > payoff to Quiet

c) (Quiet, Fink) → not nash equilibrium
→ When P2 chooses Fink, P1's payoff to Fink > payoff to Quiet

d) (Quiet, Quiet) → not in nash equilibrium
→ When P_2 chooses Quiet, P_1's payoff to Fink > payoff to Quiet

Ex: Each action pair in Prisoner's Dilemma results in players' receiving amounts of money equal to no.'s corresponding actions pairs.

Payoff of $i = m_i(a) + \alpha m_j(a)$

a) $\alpha = 1$. Is this Prisoner's Dilemma?

A)

	Q	F
Q	$(2+2, 2+2) = (4,4)$	$(0+3, 3+0) = (3,3)$
F	$(3+0, 0+3) = (3,3)$	$(1+1, 1+1) = (2,2)$

(2, 2)

Payoff $= m_i(a) + m_j(a)$

We know in Prisoners dilemma the preferences of

P1 : $(F,Q) > (Q,Q) > (F,F) > (Q,F)$

P2 : $(Q,F) > (Q,Q) > (F,F) > (F,Q)$

But it is not satisfied in this case

∴ Not Prisoner's Dilemma

b) Find the range of α for which it is Prisoner's dilemma?

	Q	F
Q	$(2+2\alpha, 2+2\alpha)$	$(0+3\alpha, 3+0\alpha)$
F	$(3+0\alpha, 0+3\alpha)$	$(1+1\alpha, 1+1\alpha)$

we know

for: P_1: $(F,Q) > (Q,Q)$

$3 > 2 + 2\alpha$

$\boxed{\cancel{\alpha < 1}}$ $2\alpha < 1$

$\alpha < 0.5$

for P_2: $(Q,F) \nless (Q,Q)$

$3 > 2 + 2\alpha$:

$1 > 2\alpha$

$\alpha < 0.5$

$(F,F) > (Q,F)$

$1 + 1\alpha > 3\alpha$

$\alpha < 0.5$

So if $\alpha < 0.5 \rightarrow$ Prisoner's dilemma

<u>BoS</u>

	B	S
B	(2,1)	(0,0)
S	(0,0)	(1,2)

Fix P_1 & check if P_2 deviates & vice versa.

Nash equilibrium check:

i) $(B,B) \rightarrow$ Nash equilibrium

ii) $(B,S) \rightarrow$ not Nash

iii) $(S,B) \rightarrow$ not Nash

iv) $(S,S) \rightarrow$ Nash equilibrium

- Matching Pennies $\rightarrow$ no Nash
- Stag Hunt $\rightarrow$ (Stag, Stag) & (Hare, Hare) are in Nash equilibrium.

<u>Coordination game</u>:

Same as BoS but both prefer Bach or together.

	B	S
B	(2,2)	(0,0)
S	(0,0)	(1,1)

(B,B) & $(S,S) \rightarrow$ Nash equilibrium.

- Highest preferred equilibrium of all equilibria is focal equilibrium

Theory of Nash equilibrium is neutral about equilibrium that will occur in a game with many equilibria.

<u>Strict & Nonstrict Equilibria</u>

<u>Ex</u>:

	L	M	R
T	1,1	1,0	0,1
B	1,0	0,1	1,0

Unique Nash equilibrium → (T, L)

When P_2 chooses L, P1 is equally happy choosing T or B; if she deviates to B then she is no worse off than she is in equilibrium.

∴ (T,L) is not strict equilibrium.

- An action profile a^* is a strict Nash equilibrium if for every player i we have $u_i(a^*) > u_i(a_i, a^*_{-i})$ for every $a_i \neq a^*_i$ of player i.

<u>Best Response functions</u>

$B_i(a_{-i})$ → set of player i's best actions when list of other player's actions is a_{-i} ~~key~~

In BoS,

$B_1(\text{Bach}) = \{\text{Bach}\}$

$B_1(\text{Stravinsky}) = \{\text{Stravinsky}\}$

In above example

$B_1(L) = \{T, B\}$

$B_i(a_{-i}) = \{a_i \text{ in } A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i})$ for all a_i' in $A_i\}$

$\downarrow$

Best response function of player i.

- B_i is set-valued.
- Every member of $B_i(a_{-i})$ is a best response of player i to a_{-i} if each of other players adheres to a_{-i}.
- Action profile a^* is a Nash equilibrium of a strategic game with ordinal preferences iff for every player's action is a best response to other player's actions:

a_i^* is in $B_i(a_{-i}^*)$ for every player i

If B_i has single element,

$a_i^* = b_i(a_{-i})$ for every i.

- To find Nash equilibrium:
  i) Find best response function of each player
  ii) Find action profiles that satisfy.

Ex:

	L	C	R
T	1,2	2,1	1,0
M	2,1	0,1	0,0
B	0,1	0,0	1,2

Find Nash equilibrium

A) Find best response function of each player

i) P1:

~~$B_1(T)$~~ $B_1(L) = \{M\}$ $B_1(R) = \{T, B\}$

$B_1(C) = \{T\}$

Star player1's payoff if it is a best

response

P2:

$B_2(T) = \{L\}$

$B_2(M) = \{L, C\}$

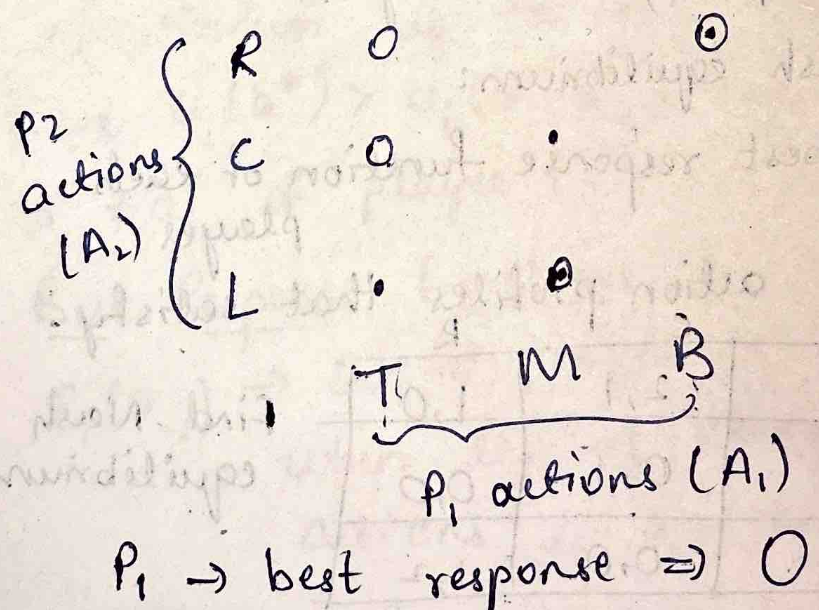
$B_2(B) = \{R\}$

Star P2 payoff if it is a best response

	L	C	R
T	1, 2*	2*, 1	1*, 0
M	2*, 1*	0, 1*	0, 0
B	0, 1	0, 0	1*, 2*

Find boxes with both player's payoffs starred.

Nash equilibrium: (M, L) & (B, R)



$P_1 \rightarrow$ best response $\Rightarrow$ O

$P_2 \rightarrow$ best response $\Rightarrow$ •

Nash equilibrium $\rightarrow$ Pair with O & •

Ex 2 2 individuals are involved in a synergistic Relationship.

$$\text{Payoff}_i = a_i(c + a_j - a_i) \qquad c > 0$$

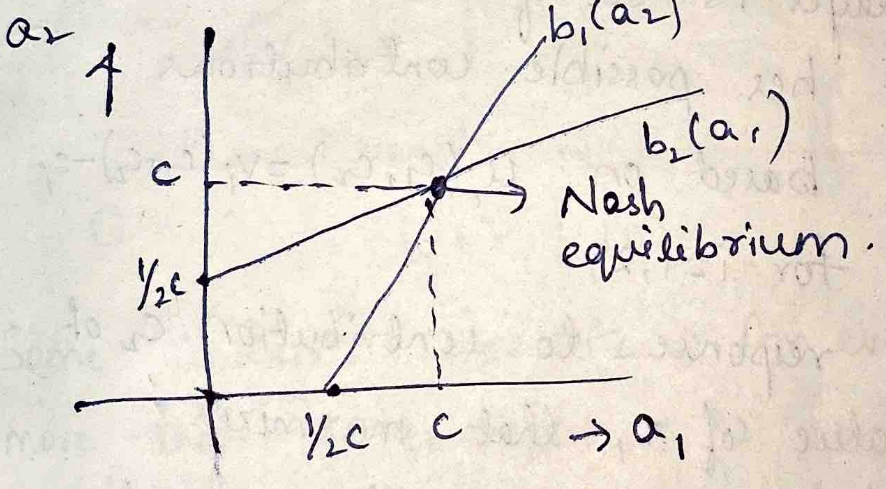
A) **Players**: 2 individuals

Actions: Set of effort levels

Preferences: based on $a_i(c + a_j - a_i)$ for $i = 1, 2$

· Infinite actions so no table.

$$b_i(a_j) = \frac{1}{2}(c + a_j) \qquad \left[\frac{d\,\text{payoff}}{da_i} = 0\right]$$



Nash equilibrium $\Rightarrow$ $b_1(a_2) = b_2(a_1)$

$$\cancel{\frac{1}{2}(c + a_2) = \frac{1}{2}(c + a_1)}$$

$$a_1 = \frac{1}{2}(c + a_2) \qquad a_2 = \frac{1}{2}(c + a_1)$$

$$a_1 = \frac{1}{2}\left(c + \frac{1}{2}(c + a_1)\right)$$

$$a_1 = \frac{3}{4}c + \frac{1}{4}a_1 \Rightarrow a_1 = c$$

$$\Rightarrow a_2 = c$$

$$(c, c)$$

Contributing to a public Good

- i^{th} Person's wealth $\rightarrow w_i$

Amt contributed to public good $\rightarrow c_i$ $(0 \leq c_i \leq w_i)$

Amt. on private good $\rightarrow w_i - c_i$

Amount of public good is equal to sum of contributions.

Payoff function $\Rightarrow u_i(c_1, c_2) = v_i(c_1 + c_2) - c_i$

Players: 2 people

Actions: Player i's set of actions is set of her possible contributions

Preferences: based on $u_i(c_1, c_2) = v_i(c_1+c_2) - c_i$ for $i = 1, 2$.

P1's best response to contribution c_2 of P2 is value of c_1 that maximizes $v_1(c_1+c_2) - c_1$. Without specifying function v_1, we cannot calculate optimal value.

We can get how it varies with c_2

[Suppose v_1 is s.t $u_1(c_1, 0)$ increases upto max & then decreases]

$c_2 = 0$

$\Rightarrow u_1(c_1, 0) = v_1(c_1) - c_1 \qquad 0 \leq c_1 \leq w_1$

$0 < b_1(0) < w_1$

$c_2 = k$

$\Rightarrow u_1(c_1, k) = v_1(c_1 + k) - c_1$

$u_1(c_1, k) = u_1(c_1 + k, 0) + c_1 + k - c_1$

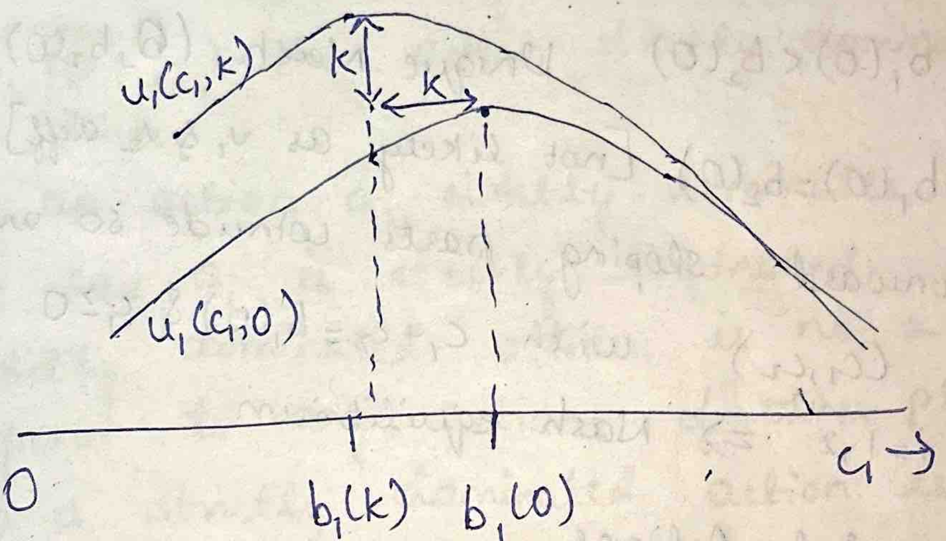
$u_1(c_1, k) = u_1(c_1 + k, 0) + k$

$\Rightarrow u_1(c_1; k) \rightarrow$ translation to left k units & up k units of $u_1(c_1, 0)$

Thus if $k \leq b_1(0) \Rightarrow b_1(k) = b_1(0) - k$

If P2's contribution $\uparrow^{ses}$ from 0 to k then P1's best response $\downarrow^{ses}$ by k.

If $k > b_1(0) \Rightarrow b_1(k) = 0$

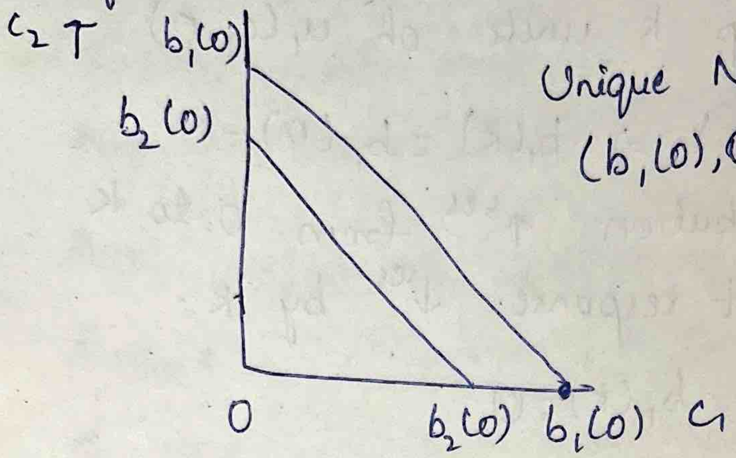


Same analysis for P2 $\Rightarrow$ for every unit more that P1 contributes, P2 contributes a unit less ~~at~~ until ~~contd~~ contribution is non-negative.

V_2 may be different from V_1 so P1's best contribution $b_1(0)$ when $c_2 = 0$ may be different from P.2's best contribution $b_2(0)$ when $c_1 = 0$

But each function has slope -1.

If $b_1(0) > b_2(0)$



Unique Nash
$(b_1(0), 0)$

If $b_1(0) < b_2(0)$ Unique Nash $(0, b_2(0))$

If $b_1(0) = b_2(0)$ [not likely as v_1 & v_2 diff]

Downward sloping parts coincide so any pair (c_1, c_2) with $c_1 + c_2 = b_1(0)$ & $c_i \geq 0$ for $i = 1,2$ => Nash equilibrium

Dominated Actions

Strict domination: In a strategic game with ordinal preferences, player i's action a_i'' strictly dominates her action a_i' if

$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ for every list a_{-i} of other players' actions where

$u_i \rightarrow$ payoff function.

- If one action is strictly better than other, no matter what other player do, it is said to strictly dominate other action.
- In iterated elimination of dominant strategy
  i) Check the dominated action for each player & remove
  ii) Continue it until you get a Nash

Ex: Prisoner's Dilemma

Fink dominates Quiet no matter what other player does

	Q	F
Q	(2,2)	(0,3)
F	(3,0)	(1,1)

→ If opponent chooses Q, you get better outcome by F.

→ If opponent chooses F, you get better outcome by ~~Q~~ F.

- In BoS, neither action strictly dominates the other.
- If an action a_i^* strictly dominates a_i we say a_i is strictly dominated.

→ Strictly dominated action is not a best response to any actions of other players so a strictly dominated action is not used in any Nash equilibrium.

→ So, eliminate all strictly dominated actions before looking for Nash equilibrium

Ex:

A)

	L	R
T	1	0
M	2	1
B	1	3

↓

M strictly dominates T but B is better than M if P2 chooses R

⇒ T is not in Nash

B)

	L	R
T	1	0
M	2	1
B	3	2

↓

M strictly dominates T & B strictly dominates M

⇒ T, M not in Nash

Weak Domination

- A player's action "weakly dominates" another action, if the first action is atleast as good as second action, no matter what other players do & is better than second action for some actions of other players

a_i'' weakly dominate a_i' if

$u_i(a_i'', a_{-i}) \geq u_i(a_i', a_{-i})$ for every list a_{-i} of other player's actions &

$u_i(a_i'', a_{-i}) > u_i(a_i', a_{-i})$ " "

Ex:

	L	R
T	1	0
M	2	0
B	2	1

M weakly dominates T, B weakly dominates M & B strongly dominates T.

- In a strict Nash equilibrium, no player's equilibrium action is weakly dominated.

Ex:

	B	C
B	1,1	0,0
C	0,0	0,0

B weakly dominates C for both players.

(B,B) is a Nash equilibrium

(C,C) " " " "

(B,B) is better than (C,C) as it doesn't have a player with weakly dominated action

	B	C
B	1,1	2,0
C	0,2	2,2

B weakly dominates C

$\underbrace{(C,C) \text{ \& } (B,B)}$
Nash equilibrium

(B,B) is not better than (C,C)

Voting

Odd no. of citizens. Everyone votes - 2 candidates
⇓
no tie

Players: Citizens

Actions: Voting for A & voting for B

Preferences: Majority prefer A. Citizens are happy until their preferred candidate wins.

~~Let citiz~~

Claim: Voting for a citizen's less preferred candidate is weakly dominated by voting for their preferred candidate.

Let citizen i prefers A. Votes of all others is fixed.

If i switches their vote from B to A:

a) If A is winning then switching from B to A doesn't change result.

b) If B is winning ⇒ might change

- Switching from B to A cannot make outcome worst. Outcome remains same or makes better. Hence, proved

- Nash equilibrium → some or all citizens are weakly dominated.

[All citizens vote for B]

Symmetric 2-player strategic game:

A 2-player strategic game with ordinal preferences is symmetric if the players' set of actions are same & players' preferences are represented by payoff functions u_1 & u_2 for which $u_1(a_1, a_2) = u_2(a_2, a_1)$ for every action pair (a_1, a_2).

[2 players having same set of actions & evaluation]

	A	B
A	w, w	x, y
B	y, x	z, z

$u_1(A, B) = u_2(B, A)$

$x = x$

$u_2(A, B) = u_1(B, A)$

$y = y$

Symmetric

- Prisoner's Dilemma & Stag Hunt — symmetric

- Matching Pennies & BoS — not symmetric

- symmetric ⇒ nothing distinguishes the players.

Symmetric Nash Equilibrium: An action profile a^* in a strategic game with ordinal preferences in which each player has same set of actions is a symmetric Nash equilibrium: (a^*, a^*)

	L	R
L	(1, 1)	(0, 0)
R	(0, 0)	(1, 1)

(L, L) & (R, R)

⇓

Symmetric Nash

- A symmetric game may have no symmetric Nash.

	X	Y
X	0,0	1,1
Y	1,1	0,0

Nash equilibrium → (X,Y), (Y,X)
but not symmetric Nash.

Cournot's model of Oligopoly.
Single good is produced by n firms.
Cost to firm i of producing q_i units of good is $C_i(q_i)$ where C_i is an ↑ing function. Price of good, determined by market, depends on total output of all firms & is
$Q = q_1 + \cdots + q_n$
given by inverse demand function $P(Q)$.
$P(Q)$ ↑ses as Q ↓ses

Each firm's revenue $= q_i P(Q)$
i^{th} firm's profit $= q_i P(q_1 + \cdots + q_n) - C_i(q_i)$

Cournot's oligopoly game

Players: Firms
Actions: Set of possible outputs (non negative numbers)
Preferences: Represented by profit.

Ex: Duopoly
2 firms. $C_i(q_i) = cq_i$ for all q_i

$$P(Q) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$

$\alpha > 0$ & $c \geq 0$

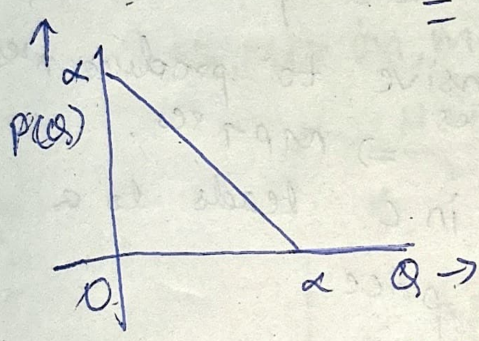
Assume $c<\alpha$, so that there is some value of total output Q for which $P(Q) > c$
[else, there would be no output for firm as they would be no profit]

Let firm's output be q_1 & q_2 then

$$P(q_1+q_2) = \begin{cases} \alpha - q_1 - q_2 & \text{if } q_1+q_2 \le \alpha \\ 0 & \text{if } q_1+q_2 > \alpha \end{cases}$$

$$\pi_1(q_1,q_2) = q_1(P(q_1+q_2) - c)$$

$$= \begin{cases} q_1(\alpha - c - q_1 - q_2) & \text{if } q_1+q_2 \le \alpha \\ -cq_1 & \text{if } q_1+q_2 \ge \alpha \end{cases}$$



1) $q_2 = 0$

$$\pi_1(q_1, 0) = q_1(\alpha - c - q_1)$$

this is quad eq. with max at $q_1 = \frac{\alpha - c}{2}$

$$b_1(0) = \frac{\alpha - c}{2}.$$

2) $q_2 > 0$

$$\pi_1(q_1,q_2) = q_1(\alpha - c - q_1 - q_2)$$

max profit at $b_1(q_2) = \frac{1}{2}(\alpha - c - q_2)$

3) $q_2 > \alpha - c$

Then firm 2 produces too much so the market price is 0 or -ve so firm 1 can't make profit so best respone is to produce nothing. $b_1(q_2) = 0$

Best response:

$$q_1 = \frac{1}{2}(\alpha - c - q_2) \quad - ①$$

$$q_2 = \frac{1}{2}(\alpha - c - q_1) \quad - ②$$

Solving $q_1 = q_2 = \frac{1}{3}(\alpha - c)$

Nash eq. : $\left(\frac{1}{3}(\alpha-c), \frac{1}{3}(\alpha-c)\right)$

Total output at equilibrium $= q_1 + q_2$
$= \frac{2}{3}(\alpha - c)$

Market price at equilibrium $= \alpha - \frac{2}{3}(\alpha-c) = \frac{\alpha+2c}{3}$
$[\alpha - Q]$

When $\alpha \uparrow^{ses}$ ⇒ consumers willing to pay more $\uparrow^{se}$
⇒ Both output & market price $\uparrow^{ses}$

→ $c \uparrow^{ses}$ ⇒ more expensive to produce $\uparrow^{ses}$
⇒ output $\downarrow^{ses}$ ⇒ MP $\uparrow^{ses}$.

- For each unit $\uparrow^{se}$ in c leads to a
$2/3^{rd}$ of unit $\uparrow^{se}$ in price.

Collusive Vs Nash Equilibrium

If 2 firms decrease their output than Nash, then MP $\uparrow^{ses}$ ⇒ both profits $\uparrow^{ses}$ ⇒ Collusive behavior

But each firm can cheat the other by deviating slightly & $\uparrow$sing its output to $\uparrow^{se}$ its profit. So the collusive state has a max profit but it is not an equilibrium state. So, Nash equilibrium is the stable state.

- Payoff function in Cournot's game

$f_i(q_i, q_1+q_2+\dots+q_n) = q_i\, P(q_1+q_2+\dots+q_n) - C_i(q_i)$

$\downarrow^{ses}$ if $q_1+q_2+\dots+q_n \uparrow^{ses}$