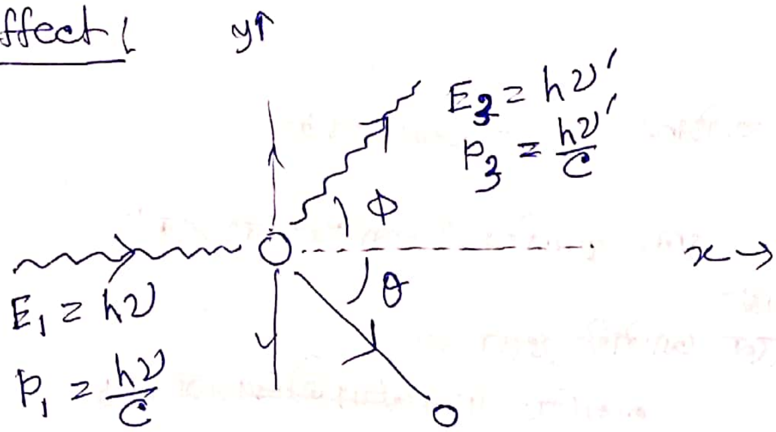
Compton Effect :



$$(E_1, P_1) \quad \begin{pmatrix} E_2 = m_0c^2 \\ P_2 = 0 \end{pmatrix} \xrightarrow[\text{Collision}]{} (E_3, P_3) \quad \begin{matrix} E_4 = mc^2 = \sqrt{p^2c^2 + m_0^2c^4} \\ P_4 = p \end{matrix}$$

Momentum conservation :-

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{(along } x\text{-axis)} \text{ — ①}$$

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad \text{(along } y\text{-axis)} \text{ — ②}$$

$$p^2c^2(\sin^2\theta + \cos^2\theta) = (h\nu - h\nu'\cos\phi)^2 + (h\nu'\sin\phi)^2$$

$$\Downarrow$$

$$p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi$$

Energy conservation :-

$$h\nu + m_0c^2 = h\nu' + \sqrt{p^2c^2 + m_0^2c^4} \text{ — ③}$$

$$\Rightarrow p^2c^2 + \cancel{m_0^2c^4} = [(h\nu - h\nu') + m_0c^2]^2$$

$$= (h\nu - h\nu')^2 + 2(h\nu - h\nu')m_0c^2 + \cancel{m_0^2c^4}$$

$$\Rightarrow \{\cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu')\} + 2(h\nu - h\nu')m_0c^2 = p^2c^2$$

$$= \cancel{(h\nu)^2} + \cancel{(h\nu')^2} - 2(h\nu)(h\nu')\cos\phi$$

$$\Rightarrow \cancel{2}(h\nu)(h\nu')\{1 - \cos\phi\} = \cancel{2}(h\nu - h\nu')m_0c^2$$

$$\Rightarrow \frac{h}{m_0c}(1 - \cos\phi) = \frac{c(\nu - \nu')}{\nu\nu'} \Rightarrow \boxed{(\lambda' - \lambda) = \frac{h}{m_0c}(1 - \cos\phi)}$$

$$\lambda' - \lambda = \Delta\lambda$$

$\Delta\lambda$ → Compton shift.

$\frac{h}{m_0c} = \lambda_0 =$ Compton wave length y of recoil electron,

We see $\nu' = \dfrac{\nu}{1 + 2\alpha \sin^2\frac{\Phi}{2}}$ $\qquad \alpha = \dfrac{h\nu}{m_0 c^2}$

electron energy (Kinetic) $= mc^2 - m_0c^2 = h\nu - h\nu'$

$$E = h\nu - \frac{h\nu}{1 + 2\alpha \sin^2\frac{\Phi}{2}}$$

$$= \frac{h\nu\, 2\alpha \sin^2\frac{\Phi}{2}}{1 + 2\alpha \sin^2\frac{\Phi}{2}} = h\nu\left[\frac{2\alpha \sin^2\frac{\Phi}{2}}{1 + 2\alpha \sin^2\frac{\Phi}{2}}\right]$$

$1 - \cos\phi$

$= 0$, $\quad E_{min} = 0 \quad$ and $\Delta\lambda = \dfrac{h}{m_0 c}(1 - \cos\phi)$

$= 0 \quad$ and $\nu' = \nu$

$\lambda' = \lambda$

, $\sin^2\frac{\pi}{4} = \frac{1}{2} \Rightarrow E = \left(\dfrac{\alpha}{1+\alpha}\right)h\nu$

$\lambda = \dfrac{h}{m_0 c} \quad \Rightarrow \lambda' = \lambda + \dfrac{h}{m_0 c} \quad$ and $\nu' = \dfrac{\nu}{1+\alpha}$

$\sin^2\frac{\pi}{2} = 1 \Rightarrow E_{max} = \left(\dfrac{2\alpha}{1+2\alpha}\right)h\nu$

$\dfrac{2h}{m_0 c} \quad$ and $\nu' = \dfrac{\nu}{1+2\alpha}$

$\dfrac{2\alpha}{+2\alpha} < 1 \Rightarrow E_{max} < h\nu$

Direction of Recoil Electron :-

From Eq ① and ②

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\phi + p\cos\theta \quad \text{--- ①}$$

$$0 = \frac{h\nu'}{c}\sin\phi - p\sin\theta \quad \text{--- ②}$$

$$\Rightarrow \tan\theta = \frac{\frac{h\nu'}{c}\sin\phi}{\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\phi} \quad \text{--- ⑤}$$

Now from compton shift relation ④,

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\phi)$$

$$\Rightarrow \frac{c}{\nu'} - \frac{c}{\nu} = \frac{h}{m_0 c}(1-\cos\phi)$$

$$\Rightarrow \frac{1}{\nu'} = \frac{1}{\nu} + \frac{h}{m_0 c^2}(1-\cos\phi)$$

$$\Rightarrow \nu' = \frac{1}{\frac{1}{\nu} + \frac{h}{m_0 c^2} 2\sin^2\frac{\phi}{2}} \Rightarrow \boxed{\nu' = \frac{\nu}{1 + \frac{h\nu}{m_0 c^2} 2\sin^2\frac{\phi}{2}}} \quad \text{--- ⑥}$$

α

Using ⑥ in ⑤, we get

$$\tan\theta = \frac{\left(\frac{\nu}{1+2\alpha\sin^2\frac{\phi}{2}}\right)\sin\phi}{\nu - \left(\frac{\nu}{1+2\alpha\sin^2\frac{\phi}{2}}\right)\cos\phi}$$

$$= \frac{\sin\phi}{1 + 2\alpha\sin^2\frac{\phi}{2} - \cos\phi} = \frac{2\sin\frac{\phi}{2}\cos\frac{\phi}{2}}{2\alpha\sin^2\frac{\phi}{2} + 2\sin^2\frac{\phi}{2}}$$

$$\boxed{\tan\theta = \frac{\cot\frac{\phi}{2}}{1+\alpha}} \quad \alpha = \frac{h\nu}{m_0 c^2}$$

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