

type-1 Context sensitive.

$$G = (N, T, P, S)$$

$$\alpha \rightarrow \beta \quad \text{where } \alpha, \beta \in (NUT)^*$$

$$|\alpha| \leq |\beta| \quad \text{Linear ~~sensitive~~ bounded automaton.}$$

→ DCF's are not closed under union.

## Turing machine

- 1) The general model of modern computer.
- 2) Turing machine introduced by Alan Turing 1936.

DFA / PDA :

turing machine

- 1) Finite state and alphabet
- 2) Transition function

✓  
✓

## Differences

- (1) It may not stop.

A Turing machine is a 5-tuple  $(K, \Sigma, \delta, S, H)$   
where,

- (1)  $K$  is a ~~set~~ finite set of states
- 2)  $\Sigma$  is a finite alphabet combining blank symbol " $\Delta$ " and left-end symbol  $\triangleright$
- 3)  $H \subset K$  called the set of halting states.
- 4)  $\delta: (K-H) \times \Sigma \rightarrow K \times (\Sigma \cup \{L, R\})$  such that
  - (1) for  $q \in K-H$ , is  $\delta(q, \triangleright) = (p, b)$  then  $b = R$ .
  - (2)  $q \in K-H$ ,  $a \in \Sigma$  is  $\delta(q, a) = (p, b)$  then  $b \neq \Delta$

$$M = (\cancel{K}, \Sigma, \delta, S, \{h\})$$

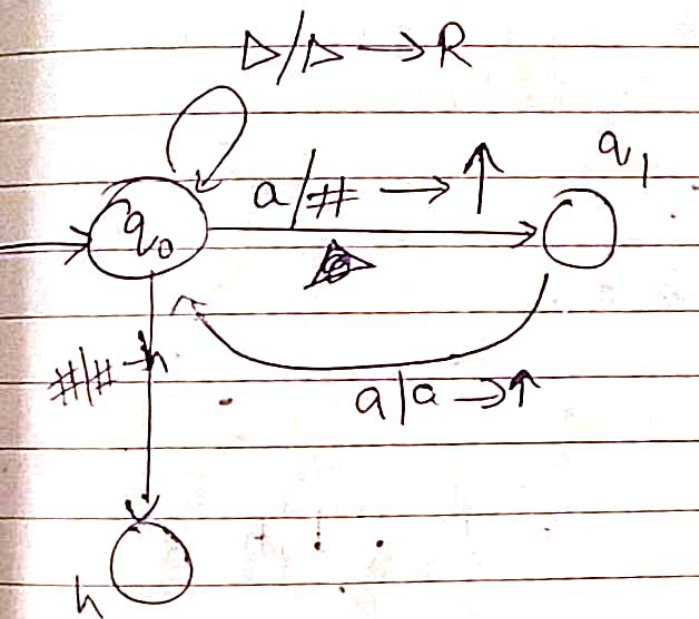
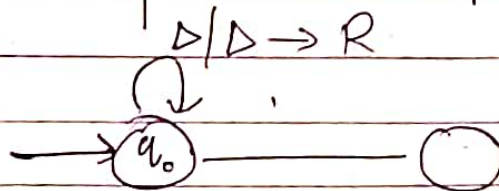
$$K = \{q_0, q_1, h\}$$

$$\Sigma = \{a, \#, \Delta\}$$

$$S = q_0.$$

$\delta$	$q$	$\gamma$	$\delta(q, \gamma)$
	$q_0$	$a$	$(q_1, \#)$
	$q_0$	$\#$	$(h, \#)$
	$q_0$	$\Delta$	$(q_0, R)$
	$q_1$	$a$	$(q_0, a)$
	$q_1$	$\#$	$(q_0, R)$
	$q_1$	$\Delta$	$(q_1, R)$

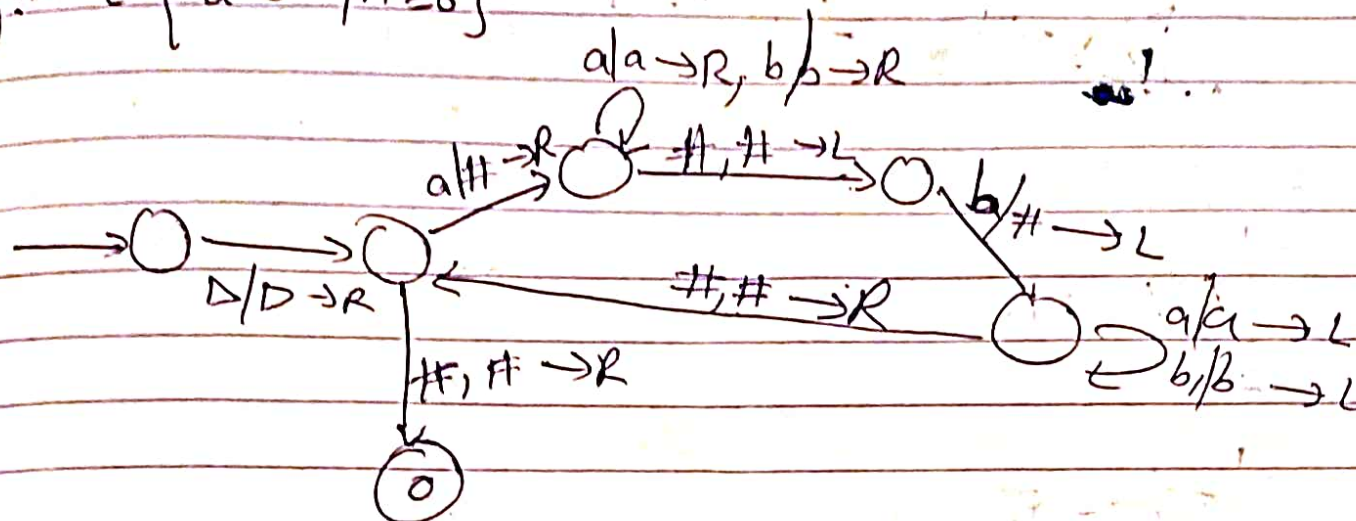
$\delta$	$q$	$\gamma$	$\delta(q, \gamma)$
	$q_0$	$a$	$(q_0, L)$
	$q_0$	$\#$	$(h, \#)$
	$q_0$	$\Delta$	$(q_0, R)$







eg:-  $L = \{ a^n b^n \mid n \geq 0 \}$



Configuration of a Turing machine:-

Configuration of a Turing machine consists of the following components:-

- (1) The current state
- (2) The current tape content
- (3) current head position

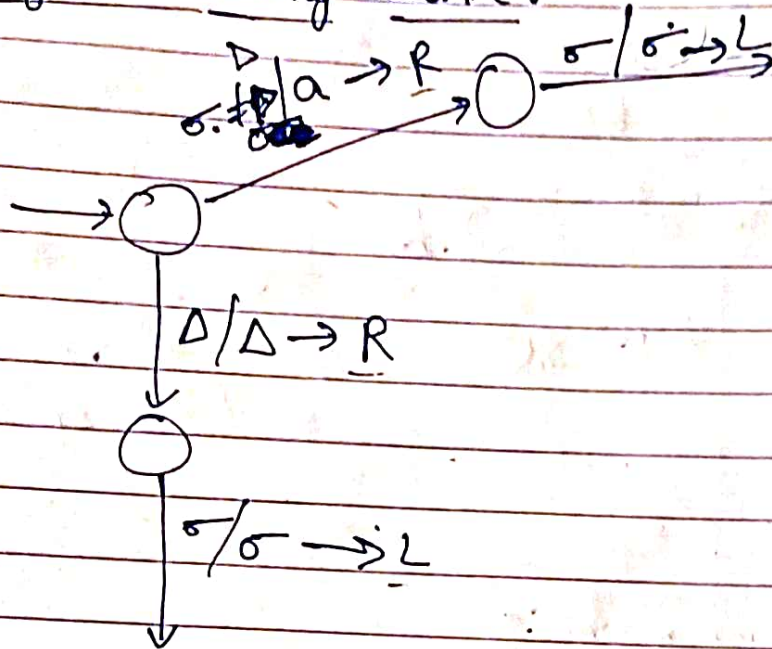
Combination of Turing Machine:-

Step 1:- Define some simple mechanism.

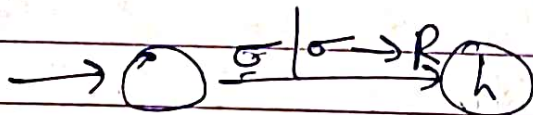
Step 2:- Combine them to make more complex machines.



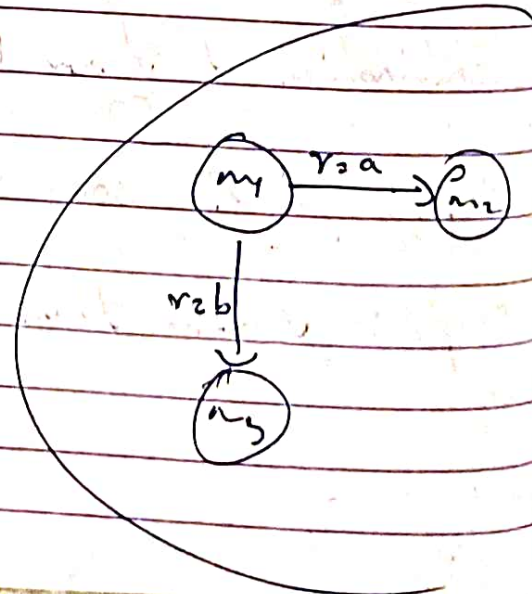
Symbol writing machine :-  $M_a$  (or)  $a$



Head moving machine :-  $M_R$  (or)  $R$



Similarly  $M_L$  :-

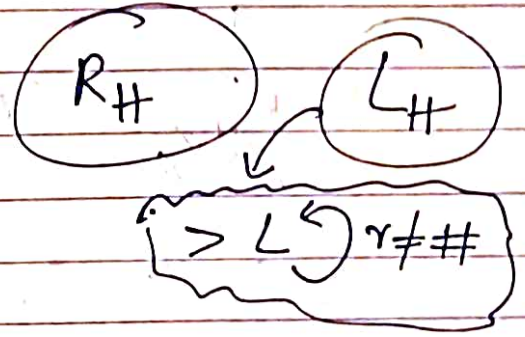


Q. ① M where the head moves two position two right and halt

Ans)  $\triangleright RR$ .

Q. ② IM that moves the head Position to the first blank symbol in the right.

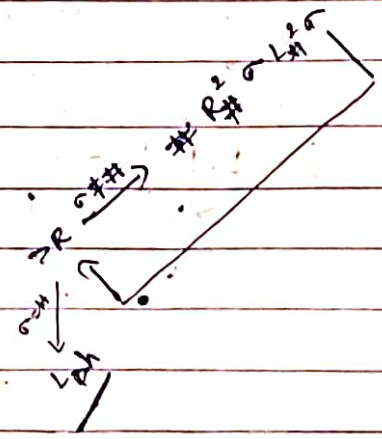
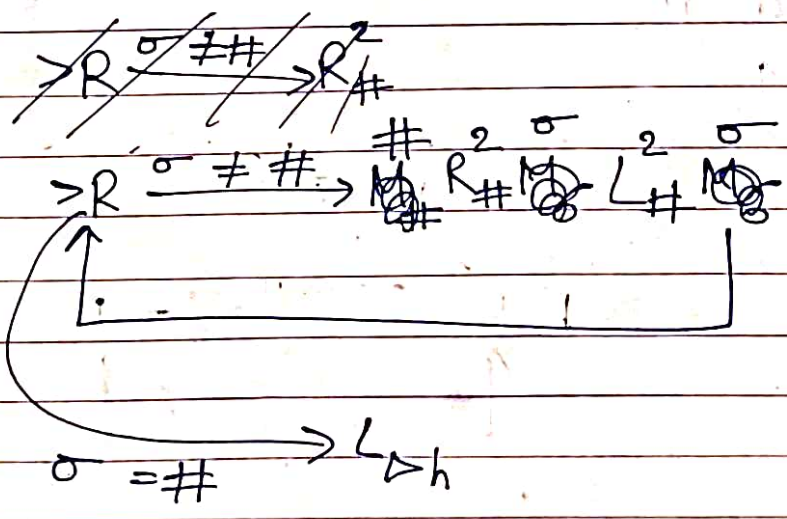
Ans)  $\triangleright R \curvearrowright r \neq \#$



Q. ③ Copy machine

$\triangleright w \# \# \dots$

$\triangleright w \# \# w \# \# \dots$

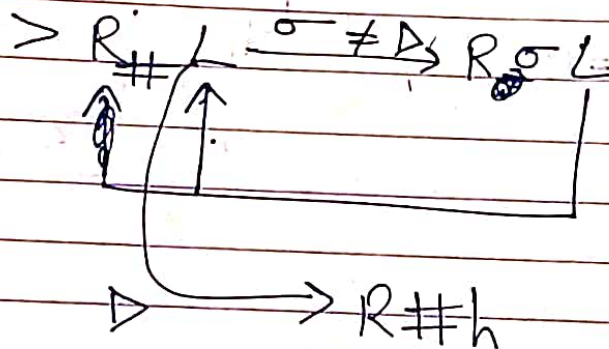




② Right shift machine :-

Δ	a	b	a	a	#	#
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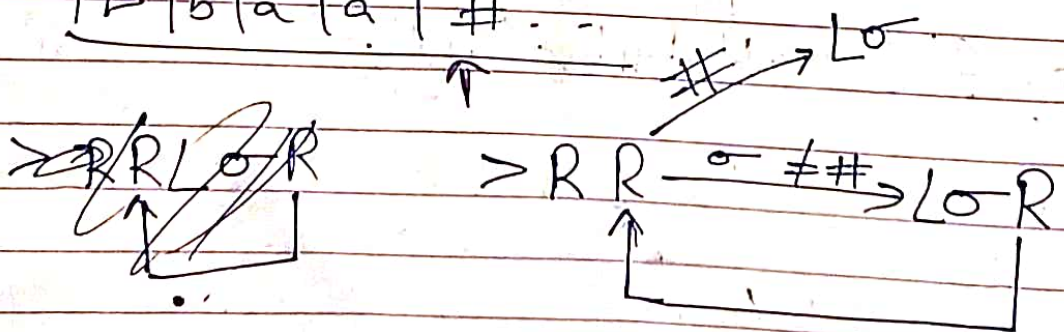
Δ	#	a	b	a	a	#	#
---	---	---	---	---	---	---	---



③ Left shift machine :-

Δ	a	b	a	a	#
---	---	---	---	---	---

Δ	b	a	a	#
---	---	---	---	---





## Turning machine as a language acceptor :-

Let  $M = (K, \Sigma, \delta, S, H)$  be a TM such that

$H = \{h_a, h_r\}$  consists of two halting states, such that

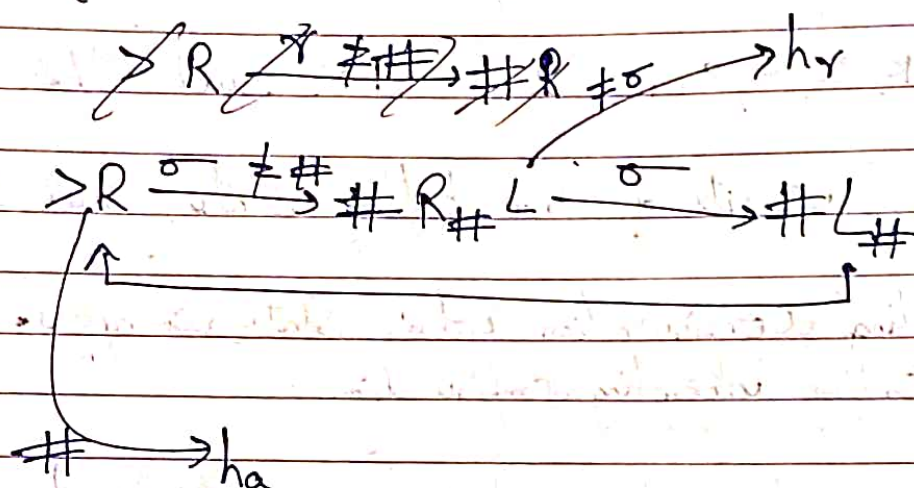
(1) any halting configuration whose state component is  $h_a$  is called accepting configuration.

(2) Any halting Config. whose state component is  $h_r$  is called rejecting configuration.

We say  $M$  accepts  $w \in \Sigma = \{\#, \Delta\}^*$  if  $(S, \Delta\#w)$  yields an accepting configuration.

- (1) If  $w \in L$ ,  $M$  accepts it
- (2) if  $w \notin L$ ,  $M$  rejects it.

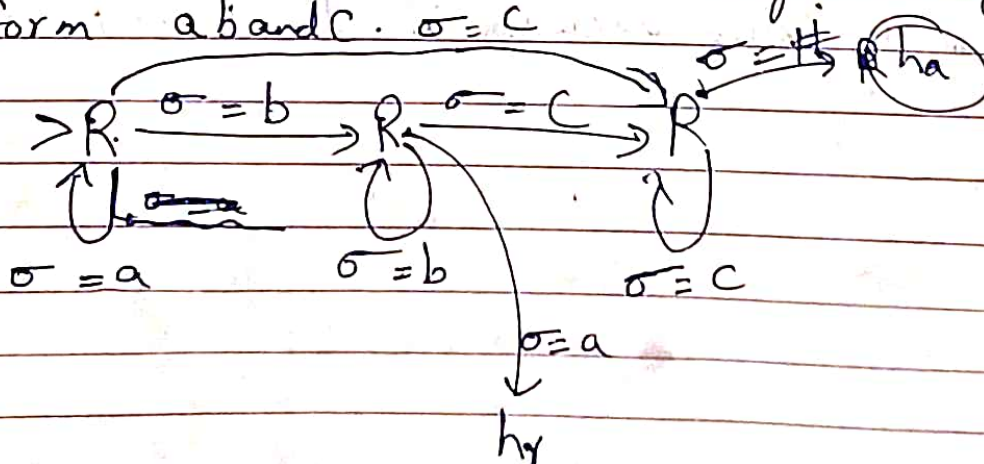
A language is called recursive if  $\exists$  a TM  $M$  that decides that.



1)  $L = \{a^n b^n c^n \mid n \geq 0\}$ .

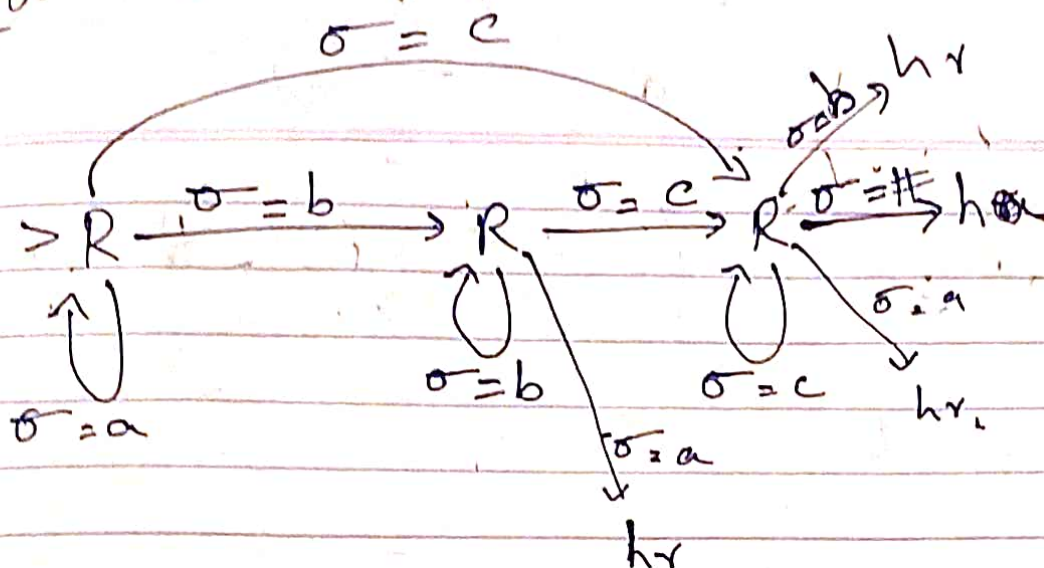
$$L = \{ ww \mid w \in (a+b)^* \}$$

1) machine to check whether the given string is of the form  $ab^nc$ .  $\sigma = c$

Sol



abc b.

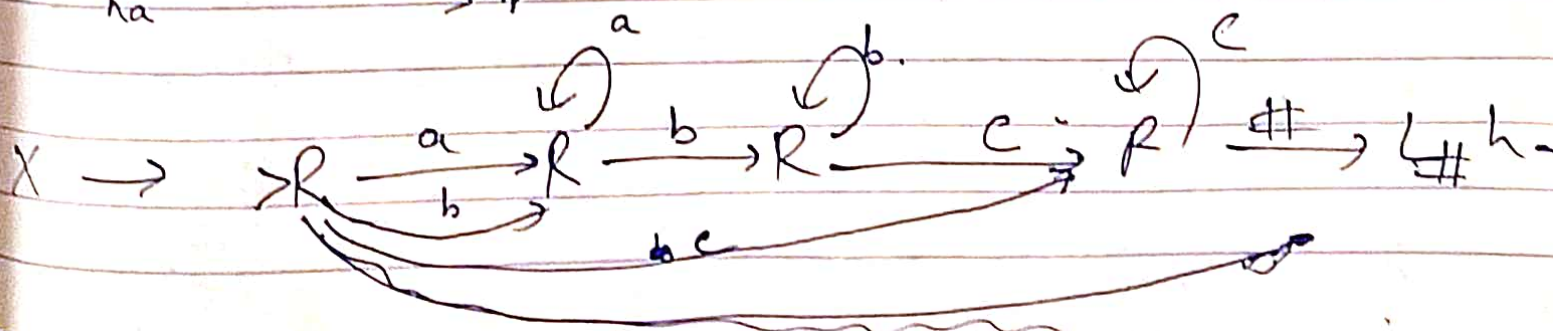
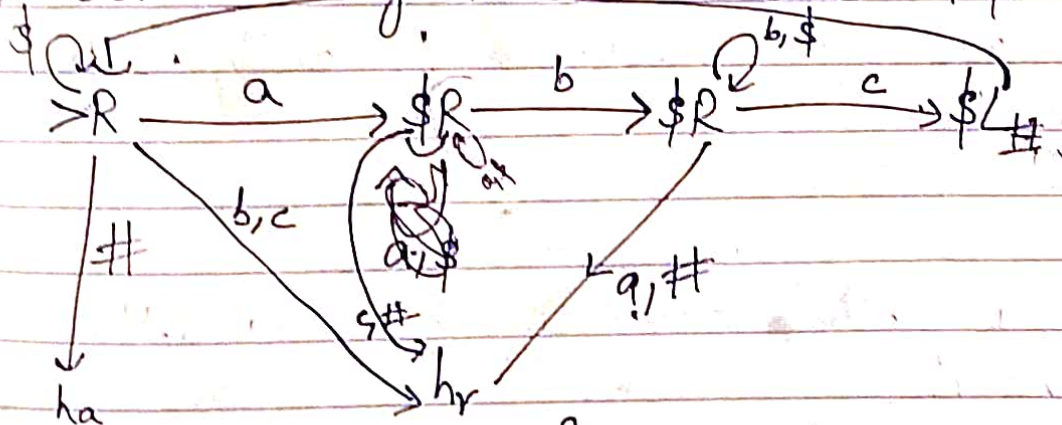


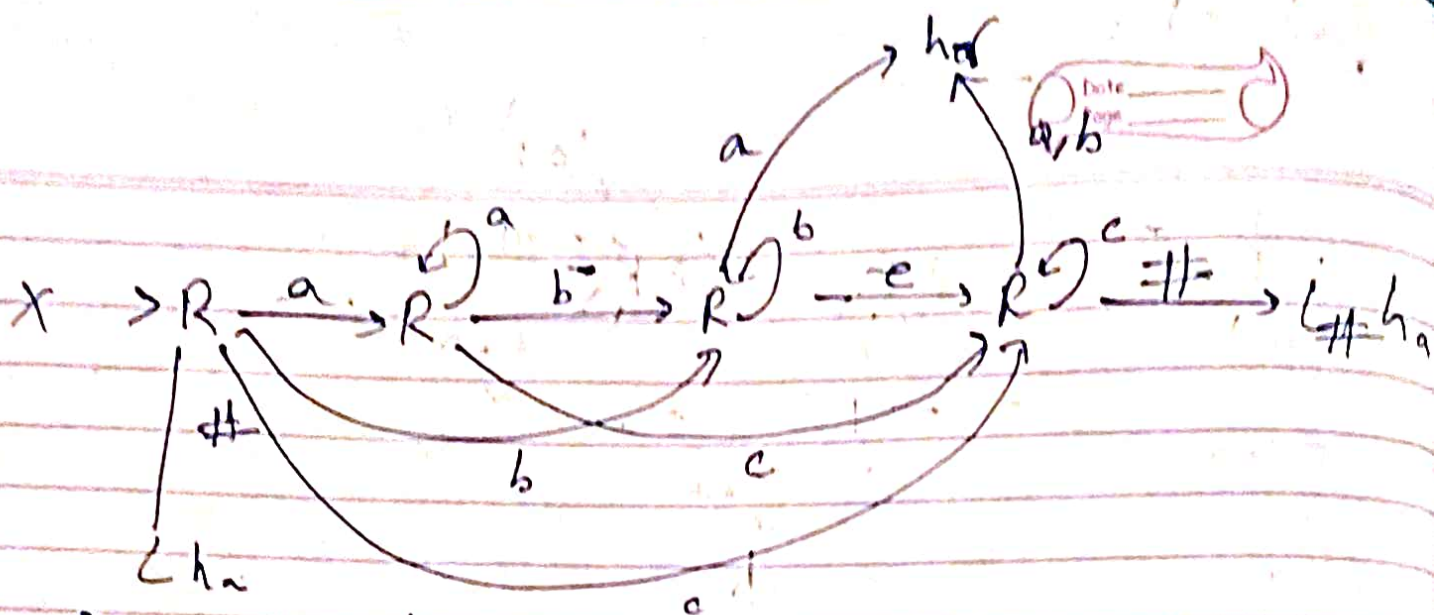
Turing Machine was a language acceptor

$$L = \{a^n \mid L = \{a^n b^n c^n \mid n \geq 0\}\}$$

The idea is:

- (1) First build a Turing Machine that checks whether the string is in the correct form (X).
- (2) build a Turing machine that checks  $|w|_a = |w|_b = |w|_c$ .





Ans  $\rightarrow XY$   
Turing Machine as a Computing device

Let  $M = (K, \Sigma, \delta, S, \{h\})$  be a Turing machine.  
 Let  $\Sigma_0 = \Sigma - \{\#, \Delta\}$  be an alphabet and  
 $w \in \Sigma_0^*$ .

$(S, \Delta \# w \#) \xrightarrow[n]{\alpha} (h, \Delta \# y \#)$  for some  $y \in \Sigma_0^*$

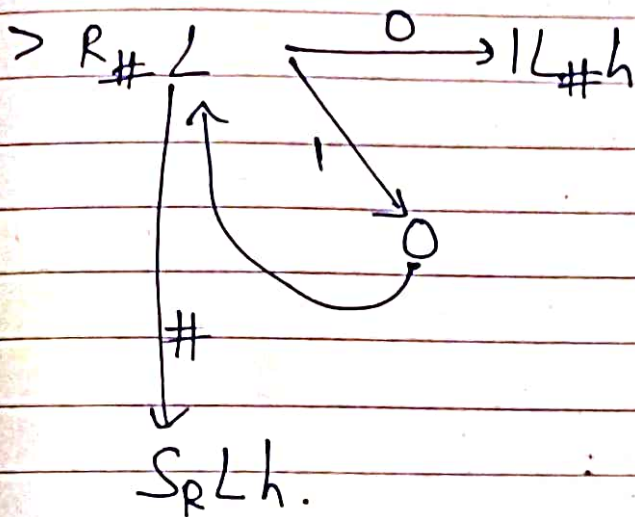
Then  $y$  is called the output of  $M$  on input  $w$  and denoted by  $M(w)$   
 $f(w) = y$ .

A function is called recursive if  $\exists$  a T.M that accepts  $f$ .



$$w \in (0+1)^*$$

$$f(n) = n+1$$



$\Delta \# 101101\#$

$\Delta \# 101110\#$

$$f(w) = ww.$$

Eg:  $\triangleright \# w \#$

$\downarrow$   
 $\triangleright \# ww \#$

~~$\triangleright \#$~~   $\triangleright CS_L \Delta R$

Recursively enumerable language:-

A language  $L$  is called r.e if for every  $w \in L$

$\exists$  a Turing machine  $M$  that halts in  $w$ .

Fact Every recursive language is <sup>r.e</sup> recursive.

Fact 2 :- Recursive languages are closed under

- (1) Intersection
- (2) Union
- (3) Complement
- (4) reversal
- (5) Concatination
- (6) \*

Variations of Turing machines:-

- (1) Non-deterministic Turing machine.
- (2) Multi tape Turing machine.
- (3) Multi head Turing machine.
- (4) Two way infinite tape Turing machine.

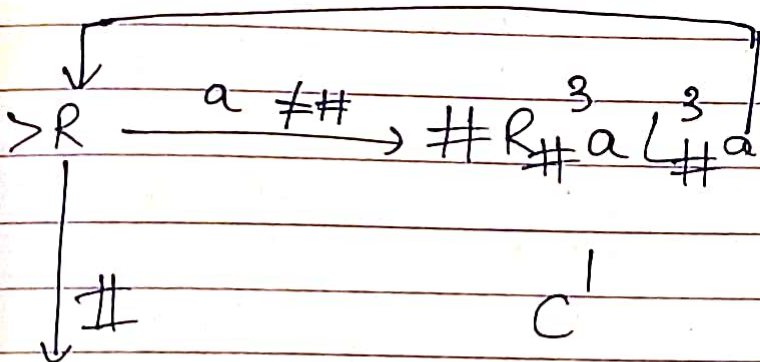


$$f(m, n) = m + n$$

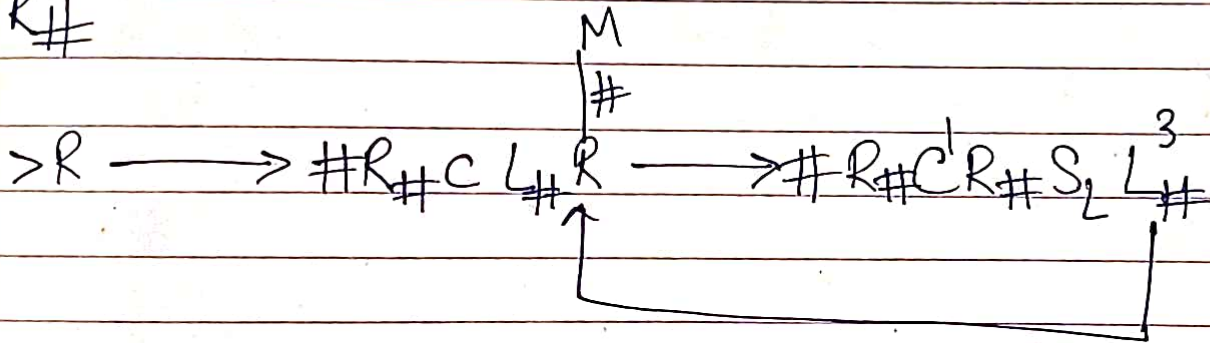
$\triangleright \# \text{||||} \# \text{||} \#$

$\triangleright \# \text{|||||} \#$

$$f(m \times n) = m \times n.$$



$R\#$



$$M >R \# L \circ \neq \Delta R S_L$$