Quantum Mechanics (Part-2) : PHL502/PH502
Time : 1 hour 15 min

Name:

ID:

- **No mobile, No calculator.**
- **Don't forget to write your names and ID in your question paper.**
- **Put a circle or tick on the right answer on this question paper. However, you can use rough papers (if required), which are not to be submitted. Submit your question paper to the TAs, as your answers are there.**
- **It is recommended to make a copy of your answers with you (either in a paper or in your brain if it is reliable $\ddot\smile$) so that you can cross verify your marks when I reveal the answer keys after the exam.**

Values of some universal constants (if required)
Planck constant $h = 6.62 \times 10^{-34}\ m^2 Kg/s$, speed of light $c = 3 \times 10^8\ m/s$
electron mass $m_e = 9.1 \times 10^{-31}\ Kg$, proton mass $m_p = 1.6 \times 10^{-27}\ Kg$,
Length scales $1\ fm = 10^{-5} \mathring{A} = 10^{-6} nm = 10^{-15} m$
$1eV = 1.6 \times 10^{-19}$ Jule, Boltzmann constant $K = 1.38 \times 10^{-23}\ m^2 Kg/s^2/^0K$

1. In Hydrogen atom what is the potential (V) ?

   (a) $V = -\frac{2e^2}{4\pi\epsilon_0 r}$
   (b) $V = -\frac{e^2}{4\pi\epsilon_0 r}$
   (c) $V = -\frac{e^2}{2\pi\epsilon_0 r}$
   (d) $V = -\frac{e^2}{4\pi\epsilon_0 r^2}$
   (e) none of the above.

2. Which is the Bohr radius (a_H) of electron (with reduced mass μ) in hydrogen atom ?
   (a) $a_H = \frac{4\pi\epsilon_0 \hbar^2}{\mu e^2}$
   (b) $a_H = \frac{4\pi\epsilon_0}{\mu \hbar^2 e^2}$

(c) $a_H = \frac{4\pi\epsilon_0\hbar^2}{2\mu e^2}$
(d) $a_H = \frac{\pi\epsilon_0\hbar^2}{\mu e^2}$
(e) none of the above.

3. Which polynomial help to solve the hydrogen atom angular part ?
(a) Laguerre polynomial
(b) Bessel polynomial
(c) Legendre polynomial
(d) Hermite polynomial
(e) none of the above

4. Which polynomial help to solve the hydrogen atom radial part ?
(a) Laguerre polynomial
(b) Bessel polynomial
(c) Legendre polynomial
(d) Hermite polynomial
(e) none of the above

5. Which polynomial help to solve the quantum mechanical problem for charge particle in presence of magnetic field ?
(a) Laguerre polynomial
(b) Bessel polynomial
(c) Legendre polynomial
(d) Hermite polynomial
(e) none of the above

6. Which is the total cross-section formula ?
(a) $\sigma_{total} = 2\pi \int_0^{\frac{\pi}{2}} \sigma(\theta) \sin\theta d\theta$
(b) $\sigma_{total} = 2\pi \int_0^{\pi} \sigma(\theta) \cos\theta d\theta$
(c) $\sigma_{total} = 2\pi \int_0^{\pi} \sigma(\theta) \sin\theta d\theta$
(d) $\sigma_{total} = 2\pi \int_0^{\pi} \sigma(\theta) \tan\theta d\theta$
(e) none of the above

7. In Scattering theory, 1st Born approximation $f(\theta)$ will be
(a) $f(\theta) = -\frac{2\mu}{\hbar^2} \int_0^{\infty} \frac{sin(qr')}{qr'} r' dr'$
(b) $f(\theta) = -\frac{2\mu}{\hbar^2} \int_0^{\infty} \frac{\ln(qr')}{qr'} V(r') r'^2 dr'$
(c) $f(\theta) = -\frac{2\mu}{\hbar^2} \int_0^{\infty} \frac{tan(qr')}{r'} V(r') r'^2 dr'$
(d) $f(\theta) = -\frac{2\mu}{\hbar^2} \int_0^{\infty} \frac{sin(qr')}{qr'} V(r') r'^2 dr'$
(e) none of the above

8. What is the quantum condition in WKB approximation ?
(a) $\int_{x_2}^{x_1} p\ dx = (n + \frac{1}{2})\pi\hbar$
(b) $\int_{x_2}^{x_1} p\ dx = (n + \frac{1}{2})\hbar$, (where $n = 0, 1, 2...$)
(c) $\int_{x_2}^{x_1} p\ dx = (n + \frac{1}{2})\frac{\hbar}{\pi}$, (where $n = 0, 1, 2...$)
(d) $\int_{x_2}^{x_1} p\ dx = (n + \frac{1}{2})\pi^2\hbar$, (where $n = 0, 1, 2...$)
(e) none of the above

9. In scattering theory $\frac{d\sigma}{d\Omega} =$
(a) $|f(\theta)|^3$

(b) $|f(\theta)|^2$
(c) $|f(\theta)|^4$
(d)$|f(\theta)|^5$
(e) none of the above

10. Which is the final wavefunction in spherical wave with scattering
(a) $\psi = f(\theta)\frac{e^{i\vec{k}\cdot\vec{r}}}{r}$
(b) $\psi = f(\theta)\frac{e^{i\vec{k}\cdot\vec{r}}}{r^2}$
(c) $\psi = f(\theta)e^{i\vec{k}\cdot\vec{r}}$
(d)$\psi = f(\theta)\ re^{i\vec{k}\cdot\vec{r}}$
(e) none of the above

11. In scattering theory, the current density for a spherical wave is
(a)$J_s = \frac{\hbar k}{\mu}\left|\frac{Af(\theta)}{r}\right|^2$
(b) $J_s = \frac{\hbar k}{\mu}|A\ rf(\theta)|^2$
(c) $J_s = \left(\frac{\hbar k}{\mu}\right)^2\left|\frac{Af(\theta)}{r}\right|^2$
(d) $J_s = \frac{\hbar k}{\mu}|A\ f(\theta)|^2$
(e) none of the above
here, μ is the reduced mass for the system.

12. What is the total cross-section when $l \to 0$, $\delta_0 \to \frac{\pi}{2}$
(a)$\sigma = \frac{4\pi}{k^2}$
(b)$\sigma = \frac{2\pi}{k^2}$
(c)$\sigma = \frac{2\pi}{k}$
(d)$\sigma = \frac{4\pi}{k}$
(e) none of the above

13. When a low energy ($E \to 0$) incident particle is scattered through an weak potential $V(r)$, its scattering wavefunction (with respect to incident wavefunction) face very small phase difference i.e. $\delta_0 \to 0$. In the limit of $\delta_0 \to 0$, s-wave scattering amplitude $f(E) = \frac{e^{i\delta_0(E)}}{k}Sin\{\delta_0(E)\}$ give us the magnitude of scattering length a. So the expression of a will be simplified to
(a) $a = \frac{1}{k}$
(b) $a = \delta_0 k$
(c) $a = \frac{k}{\delta_0}$
(d) $a = \frac{\delta_0}{k}$
(e) none of the above

14. Let us assume total Hamiltonian $H = H^0 + H'$ is made of perturbed Hamiltonian H' and un-perturbed Hamiltonian H^0, which follow Schrodinger Equation $H^0\psi_n^0 = E_n^0\psi_n^0$, where E_n^0 are energy eigen values of wavefunctions ψ_n^0 for different integer values of quantum no. n. If total energy and wavefunction can be written as $E = E_n^0 + E_n^{(1)}$ and $\psi_n = \psi_n^0 + \psi^{(1)_n}$, then according to time independent perturbation theory, $E_n^{(1)}$ is
(a) $\langle\psi_n|H'|\psi_n\rangle$
(b) $\langle\psi_n|H^0|\psi_n\rangle$
(c) $\langle\psi_n|H'|\psi_m\rangle$

(d) $\langle\psi_n|H^0|\psi_m\rangle$.
(e) none of the above

15. If magnetic field B is applied along x-axis and electric field along z-axis, then Hall resistivity component will be
(a) ρ_{xy}
(b) ρ_{yz}
(c) ρ_{zx}
(d) ρ_{yy}
(e) none of the above

16. For above problem, one of the possible form of vector potential $\vec{A}$ is (remembering $\vec{B} = \nabla \times \vec{A}$):
(a) $\vec{A} = xB\hat{j}$
(b) $\vec{A} = yB\hat{k}$
(c) $\vec{A} = zB\hat{k}$
(d) $\vec{A} = zB\hat{i}$
(e) none of the above

17. If you find $\vec{A}$ correctly, then you can tell about expression of field momentum ($\vec{p}_F = e\vec{A}$):
(a) $\vec{p}_F = exB\hat{j}$
(b) $\vec{p}_F = eyB\hat{k}$
(c) $\vec{p}_F = ezB\hat{k}$
(d) $\vec{p}_F = ezB\hat{i}$
(e) none of the above

18. Schroedinger Equation of above problem will be
(a) $\frac{1}{2m}\Big(-i\hbar\frac{\partial}{\partial y} + exB\Big)^2\psi(y,z) - \frac{\hbar^2}{2m}\frac{\partial^2\psi(y,z)}{\partial z^2} = E\psi(y,z)$
(b) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(y,z)}{\partial y^2} + \frac{1}{2m}\Big(-i\hbar\frac{\partial}{\partial z} + ezB\Big)^2\psi(y,z) = E\psi(y,z)$
(c) $-\frac{\hbar^2}{2m}\frac{\partial^2\psi(y,z)}{\partial y^2} + \frac{1}{2m}\Big(-i\hbar\frac{\partial}{\partial z} + eyB\Big)^2\psi(y,z) = E\psi(y,z)$
(d) $-\frac{\hbar^2}{2m}\Big(\frac{\partial^2\psi(y,z)}{\partial y^2} + \frac{\partial^2\psi(y,z)}{\partial z^2}\Big) = E\psi(y,z)$
(e) none of the above

19. Quantum Hall Effect can be observed at
(a) high temperature and high magnetic field
(b) high temperature and low magnetic field
(c) low temperature and high magnetic field
(d) low temperature and low magnetic field
(e) none of the above

20. According to Classical Hall effect, Hall resistivity $\rho_H = \frac{B}{ne}$, where B is external magnetic field and n is no. density of electron. While, Quantum Hall effect says resistivity $\rho_H = \frac{h}{e^2}\frac{1}{a}$, where a is integer no, popularly called Landau levels. From the lowest value of a (called lowest Landau level), one can guess about largest value of B/n, which is known as flux quantum:
(a) $(B/n)_{\max} = \frac{h}{e^2}$

(b) $(B/n)_{\max} = \frac{h}{e}$
(c) $(B/n)_{\max} = \infty$
(d) $(B/n)_{\max} = 0$
(e) none of the above

21. Approximate value of flux quantum is
(a) 10^{-12} Wb
(b) 10^{-15} Wb
(c) 10^{-18} Wb
(d) 10^{-21} Wb
(e) none of the above

22. Quantized energy expression $E_a = (a+\frac{1}{2})\hbar\omega$ is obtained when Schroedinger Equation is solved in presence of magnetic field B. The relation between ω and B is
(a) $\omega \propto B$
(b) $\omega \propto B^0$
(c) $\omega \propto \frac{1}{B}$
(d) $\omega \propto \frac{1}{B^2}$
(e) none of the above

23. Which one is wrong?
(a) In absence of magnetic field, Hall effect is not possible
(b) Drude's electrical conductivity never depends on electric field
(c) Drude's electrical conductivity never depends on magnetic field
(d) Quantum Hall electrical conductivity proportionally increase with magnetic field
(e) none of the above

24. With respect to Bohr's semi-classical model, Schroedinger equation of hydrogen atom problem provide extra information:
(a) energy of electron is quantization
(b) electron moves in different orbits with quantized radius
(c) electron does not move in different orbits with quantized radius
(d) angular momentum of electron is quantization
(e) none of the above

25. Variation method has steps:
(a) first guess the wave function $\psi(x)$ in terms of unknown function a, then calculate unknown a by demanding $\frac{d\psi}{dx} = 0$.
(b) first guess the wave function $\psi(x)$ in terms of unknown function a, then calculate unknown a by demanding $\frac{d\psi}{da} = 0$.
(c) first guess the wave function $\psi(x)$ in terms of unknown function a, then calculate unknown a by demanding $\frac{d|\psi|^2}{dx} = 0$.
(d) first guess the wave function $\psi(x)$ in terms of unknown function a, then calculate unknown a by demanding $\frac{d|\psi|^2}{da} = 0$.
(e) none of the above