GAME THEORY

LA358

Dominant Strategy

➢ Problems:

Player 2

Player 1		Left	Center	Right
	up	13,3	1,4	7,3
	Middle	4,1	3,3	6,2
	Down	-1,9	2,8	8,-1

Iterated elimination of Dominant Strategy

➢ <u>Solution:</u>

Player 2

		Left	Center	Right
Player 1	up	13*,3	1,4	7,3
	Middle	4,1	3*,3	6,2
	Down	-1,9	2,8	8*,-1

➢ Player 1 doesn't have a DS, which implies player 1 always wants to change its strategy based on Player 2 strategy

➢ If P2-Left: Player 1 response is Up

➢ If P2-Center: Player 1 plays Middle

➢ If P2-Right: Player 1 plays Down

Iterated elimination of Dominant Strategy

➢ Solution:

Player 2

Player 1		Left	Center	Right
	up	13,3	1,4*	7,3
	Middle	4,1	3,3*	6,2
	Down	-1,9*	2,8	8,-1

➢ Player 2 doesn't have a DS, which implies player 2 always wants to change its strategy based on Player 1 strategy

➢ If P1- Up : Player 2 response is Center

➢ If P1- Middle : Player 2 plays Center

➢ If P1- Down : Player 2 plays Left

➢ Solution:

Player 2

Player 1		Left	Center	Right
	up	13,3	1,4*	7,3
	Middle	4,1	3,3*	6,2
	Down	-1,9*	2,8	8,-1

➢ Player 2 doesn't have a DS, however player 2 will always choose left/center and don't choose Right in any case of P1 strategy .

➢ In another way Center dominates Right in all cases of P1's actions

➢ Hence, we can eliminate Right from the game

Iterated elimination of Dominant Strategy

➢ <u>Solution:</u>

		Player 2	
		Left	Center
Player 1	up	13*,3	1,4
	Middle	4,1	3*,3
	Down	-1,9	2,8

➢ In this iterated game of 3*2 –Player 1 – no DS

➢ P2 plays Left: P1 plays Up

➢ P2 plays Center: P1 plays Middle

Iterated elimination of Dominant Strategy

- <u>Solution:</u>

Player 2

		Left	Center
Player 1	up	13*,3	1,4
	Middle	4,1	3*,3
	Down	-1,9	2,8

- In this iterated game of 3*2 –Player 1 – no DS
- P2 plays Left: P1 plays Up
- P2 plays Center: P1 plays Middle
- Here, Down is not a preferred strategy in either case of P2 playing left / center
- Hence, we can eliminate Down

Iterated elimination of Dominant Strategy

➢ Solution:

		Player 2	
		Left	Center
Player 1	up	13,3	1,4
	Middle	4,1	3,3

➢ This iterated elimination made the game into 2*2 strategy game

➢ In this game P1 no DS

➢ P2?

Iterated elimination of Dominant Strategy

➢ <u>Solution:</u>

Player 2

		Left	Center
Player 1	up	13,3	1,4*
	Middle	4,1	3,3*

➢ This iterated elimination made the game into 2*2 strategy game

➢ In this game P1 no DS,

➢ P2 has DS: Center

➢ Hence eliminate Left

Iterated elimination of Dominant Strategy

- <u>Solution:</u>

		Player 2
		Center
Player 1	up	1,4
	Middle	3*,3

- This iterated elimination made the game into 2*1 strategy game
- In this game P1 choose Middle
- Hence game equilibrium reaches at (Middle, Center): (3,3)

➢Problems:

		Player 2		
		Left	Center	Right
Player 1	up	0,2	3,1	2,3
	Middle	1,4	2,1	4,1
	Down	2,1	4,4	3,2

- <u>Solution:</u>

Player 2

		Left	Center	Right
Player 1	up	0,2	3,1	2,3
	Middle	1,4	2,1	4,1
	Down	2,1	4,4	3,2

- D,C : (4,4)
- Eliminate Up>Right>Middle>Left

Best Response function

- Solution to game (NE/DSE) : easy to find if the set of actions are few
- However, if large number of players and actions: then it is not feasible to check NE from large set of possible actions
- Then it is better to use Best Response Function (BRF/BR)
- BRF for player i:
  - Best pay-off for player-i as a response to other player’s action

Best Response function

- BRF for player i:
  - Best pay-off for player-i as a response to other player's action
  - Best pay-off =better or atleast as good as pay-off (>=)
  - Other player = -i
  - Other player action= a_{-i}
- Set of best response actions for player i=B_i (a_{-i})
- All actions of player i = A_i
- Actions of Best response set for player i = a_i
- Actions of not best response set for player i=a_i'

Best Response function

Precisely, we define the function B_i by

$$B_i(a_{-i}) = \{a_i \text{ in } A_i : u_i(a_i, a_{-i}) \geq u_i(a'_i, a_{-i}) \text{ for all } a'_i \text{ in } A_i\} :$$