CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Circuits and Protocols

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Quantum Algorithms

Basic Framework

Quantum Interference

- Application of a Hadamard gate to an arbitrary qubit is an example of quantum interference
- Recall for $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$,

$$H |\psi\rangle = \left(\frac{\alpha + \beta}{\sqrt{2}}\right) |0\rangle + \left(\frac{\alpha - \beta}{\sqrt{2}}\right) |1\rangle$$

- Note probabilities of obtaining $|0\rangle$ and $|1\rangle$ have changed.

$$\alpha \rightarrow \frac{\alpha + \beta}{\sqrt{2}} \qquad \beta \rightarrow \frac{\alpha - \beta}{\sqrt{2}}$$

- For the state:

$$|\psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad H|\psi\rangle = H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = |0\rangle$$

- This is a manifestation of *quantum interference*
- Mathematically this means the addition of probability amplitudes.
- There are two types of interference,
    - *positive interference* in which probability amplitudes add constructively to increase or
    - *negative interference* in which probability amplitudes add destructively to decrease

$$
\begin{aligned}
H|\psi\rangle &= H\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}} H|0\rangle + \frac{1}{\sqrt{2}} H|1\rangle \\
&= \frac{1}{\sqrt{2}}\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + \frac{1}{\sqrt{2}}\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right) \\
&= \frac{1}{2}\left(\underbrace{|0\rangle + |0\rangle}_{+ve\ interference} + \underbrace{|1\rangle - |1\rangle}_{-ve\ interference}\right) = |0\rangle
\end{aligned}
$$

- Probabilities of measurements of $|0\rangle$ and $|1\rangle$ change w.r.t $|\psi\rangle \rightarrow H|\psi\rangle$:

$$
|0\rangle : \quad \underbrace{\frac{1}{2} \rightarrow 1}_{+ve\ interference} \qquad |1\rangle : \quad \underbrace{\frac{1}{2} \rightarrow 0}_{-ve\ interference}
$$

Quantum Interference in Quantum Algorithms

Quantum interference

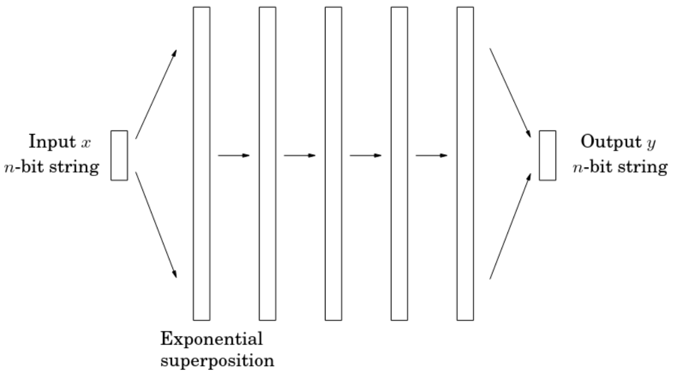
- An integral part of of the basic quantum algorithm toolkit
- Allows to gain information about a function $f(x)$ that depends on evaluating the function *at many values* of x
- Allows to deduce certain *global properties* of the function
- Plays important role in quantum parallelism

Quantum Parallelism and Function Evaluation

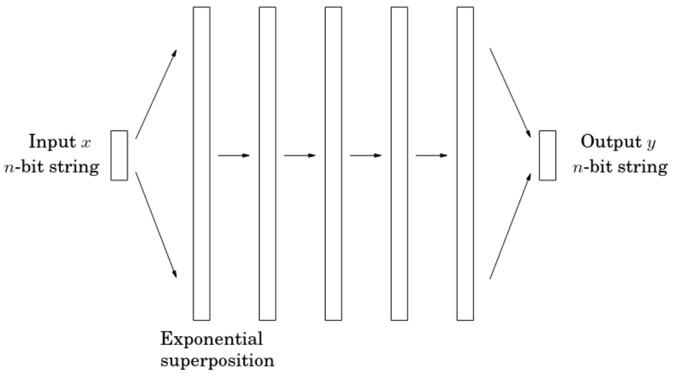
Quantum parallelism can be described as the ability to evaluate the function $f(x)$ at many values of x simultaneously

- Deutsch's algorithm helps demonstrate the power of quantum parallelism using a very simple problem

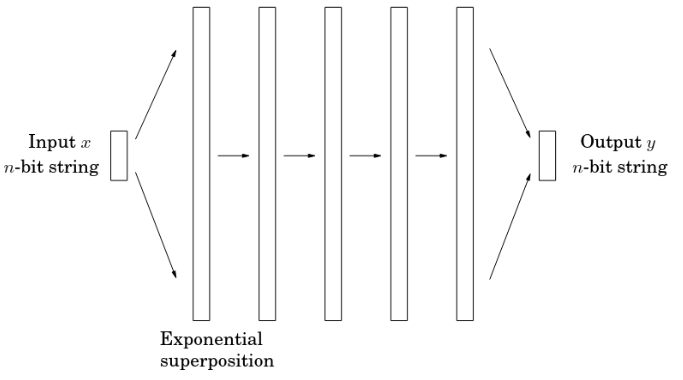
- A quantum algorithm takes n "classical" bits as its input



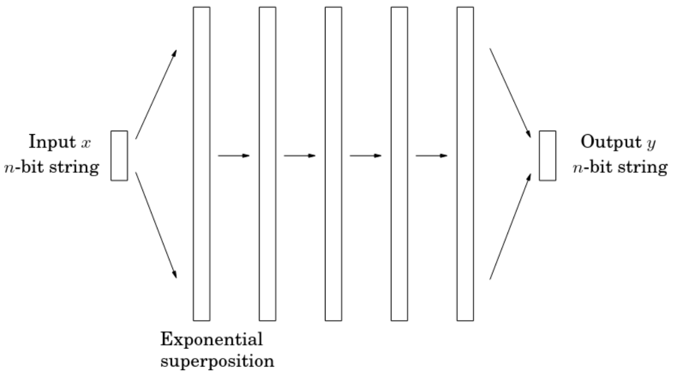
- Manipulates them so as to create a superposition of their 2^n possible states



- Manipulates this exponentially large superposition to obtain the final quantum result

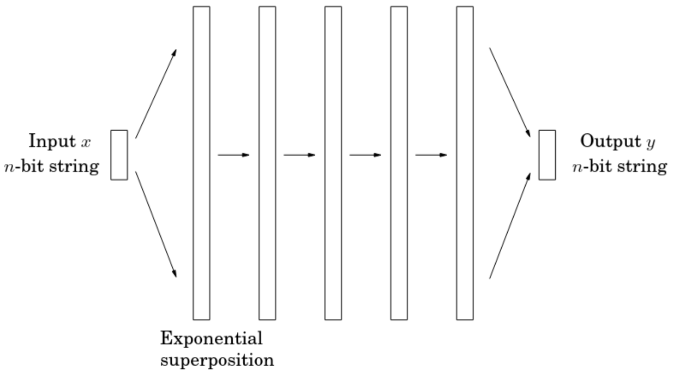


- Then measures the result to get (with the appropriate probability distribution) the n output bits.



Quantum Parallelism

For the middle phase, there are elementary operations which count as one step and yet manipulate all the exponentially many amplitudes of the superposition.



References

1. Quantum Computing for Computer Scientists, by Noson S. Yanofsky, Mirco A. Mannucci
2. Quantum Computing Explained, David Mcmahon. John Wiley & Sons
3. Lecture Notes on Quantum Computation, John Watrous, University of Calgary
   - `https://cs.uwaterloo.ca/~watrous/QC-notes/`