30/7/24

Game Theory (LA358)

Agents = Players

No independent actions

In Game, one player actions depends on other player's actions.

eg of payoff matrix

$$
\begin{array}{cc}
 & \text{Player B} \\
\text{Player A} & \begin{bmatrix} (10,0) & (1,10) \\ (5,7) & (7,6) \end{bmatrix}
\end{array}
$$

Evaluation Criteria

20% Quiz

30% Mid sem

50% End sem

When a situation cannot be game?

→ your decision affects only you

→ your

... decision affect other but does not influence other actions.

eg) Two piles of card : A and B
Two players : P1 and P2
Two cases : Balanced Game / Unbalanced game

R1 : Pick $n \geq 1$ cards from either pile

R2 : Can pick cards from one pile at a time

R3 : $P1 \rightarrow P2 \rightarrow P1 \rightarrow P2 - - -$

R4 : Winner is the one who picks the last card

$[1,1] \xrightarrow{P1} [0,1] \xrightarrow{P2} [0,0]$ P_2 win

$[2,2] \xrightarrow{P1} [0,2] \xrightarrow{P2} [0,0]$ P_2 win ✗

$\quad \xrightarrow{P1} [1,2] \xrightarrow{P2} [0,2] \xrightarrow{P1} [0,0]$ P_1 win ✗

$\quad\quad \xrightarrow{P2} [1,1] \xrightarrow{P1} [0,1] \xrightarrow{P2} [0,0]$ P_2 win

P_2 stragety : Pick exact # of cards P_1 has picked in previous move from other pile fr...

other pile from which P_1 has picked in previous move.

Balanced: # of cards in A = # of cards in B

$[3,3] \xrightarrow{P1} [0,3] \xrightarrow{P2} [0,0]$ X

$\quad \xrightarrow{P1} [1,3] \xrightarrow{P2} [1,1] \rightarrow \cdots$

$\quad \xrightarrow{P1} [2,3] \xrightarrow{P2} [2,2] \rightarrow$

Sequential game

Unbalanced Game

P1 winning strategy: Pick abs(|B| − |A|) from the pile having larger number of cards.

P2 strategy: Ensure both piles have cards.

<u>Voting</u>

(A) → Football court

(N)

(B) → Tennis court

	Stage 1 : A v/s B		Stage 2 : Winner (S1) v/s N		
R2	A	A	A	A	A v/s N
R3	B		A		
R4	A		N		

R2 F > N > T

R3 T > F > N

R4 N > F > T

Modified R4 : N > T > F

	Stage 1 : A v/s B		Stage 2 : Winner (S1) v/s N		
R2	A	B	N	N	B v/s N
R3	B		B		
R4	B		N		

Prisoner's Dilemma

	P_B C	P_B NC
P_A C	[5, 5]	[0, 10]
P_A NC	[10, 0]	[1, 1]

Payoff matrix

		P_2	
		C	NC
P_1	C	[5, 5]	[0, 10]
	NC	[10, 0]	[1, 1]

Country Defense Budget

		Pakistan	
		High	Low
India	High	P @ HC	War (India wins)
	Low	War (Pak wins)	P @ LC

Rational Behaviour

Action
- Not based on preferences
- Based on circumstances

Preferences
- know ordering of strategies
- Consistency

$a > b \quad b > c \quad a > c$ ✓

$a > b \quad b > c \quad a < c$ ✗

Transitivity preference

→ Rule out altruistic preferences

↳ qualitative characteristics

Pay off

Quantifying the preferences

(Giving value to each preference based on original pref. choice.)

Values does not mean how much

It just gives ordering.

Ordered information + Consistent preferences

⇓

Best option avaliable

$\boxed{\text{Games}}$

Strategic | Extensive | Collusion

Strategic

① Players shows rational behaviour

② Simultaneous Game (Time X)

③ One time game (Single Round) (Sequential X)

④ Payoff : Matrix type

Extensive

① Payoff : Decision Tree type

② Multiple Round Game

Prisoner's Dilemma as Strategic

$$
P_1 \quad \begin{array}{c} \\ C \\ NC \end{array}
\begin{array}{c} \qquad P_2 \\ \begin{array}{cc} \text{Conf} & NC \end{array} \\
\begin{bmatrix} 5,5 & 0,10 \\ 10,0 & 1,1 \end{bmatrix}
\end{array}
$$

$$P_1 \qquad \underset{3}{U(C,NC)} > \underset{2}{U(NC,NC)} > \underset{1}{U(C,C)} > \underset{0}{U(NC,C)}$$

$$P_2 \qquad \underset{3}{U(NC,C)} > \underset{2}{U(NC,NC)} > \underset{1}{U(C,C)} > \underset{0}{U(C,NC)}$$

Payoff Matrix

$$
P_1 \quad \begin{array}{c} P_2 \\ \begin{bmatrix} 1,1 & 3,0 \\ 0,3 & 2,2 \end{bmatrix} \end{array}
$$

charge

		F_2	
		H	L
F_1	H	1000, 1000	-200, 1200
	L	1200, -200	600, 600

Dominant Strategy

Low is Dominant Strategy for F_1

		F_2	
		H	L
F_1	H	1000	-200
	L	1200	600

Low is Dominant strategy for F_2

		F_2	
		H	L
F_1	H	1000	1200
	L	-200	600

Dominant

Strictly

If strategy $a'' > a'$

$u(a'', a_{-i}) > u(a', -a_i)$

Weak

If strategy $a'' > a'$

$u(a'', a_{-i}) \geq u(a', -a_i)$

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 4\ 1 & 2\ 3 \\ 2\ 0 & 3\ 2 \end{bmatrix} \end{array}$$

$$\begin{array}{ccc} & & P_2 \\ P_1 & \begin{array}{c} i \\ ii \end{array} & \begin{bmatrix} 4 & 2 \\ 2 & 3 \end{bmatrix} \end{array}$$

No dominant strategy

$$\begin{array}{cc} & \begin{array}{cc} i & ii \end{array} \\ P_1 & \begin{bmatrix} 1 & \boxed{\begin{matrix} 3 \\ 2 \end{matrix}} \\ 0 & \end{bmatrix} \end{array}$$

ii

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{bmatrix} 3\ 3 & 1\ 2 & -1\ 1 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 10\ 5 & 0\ 4 & 1\ 3 \\ 15\ 3 & 2\ 5 & 1\ 5 \end{bmatrix}$$

P_2

$$P_1 \quad \begin{array}{c} a \\ b \\ c \end{array} \begin{bmatrix} 3 & 1 & -1 \\ 10 & 0 & 1 \\ \boxed{15 \quad 2 \quad 1} \end{bmatrix}$$

Weakly

c

Highest

Col me sabse bade wali row

P_2

$$P_1 \quad \begin{array}{ccc} a & b & c \end{array}$$

$$\begin{bmatrix} 3 & 2 & 1 \\ 5 & 4 & 3 \\ 3 & 5 & 5 \end{bmatrix}$$

No dominant strategy

Prisoner's Dilemma → 1 Better to conflict

Another type : 2 Conflict is only option ⇕ Strictly conflict

eg

P_2

	H	T
H	10, -10	-10, 10

P

No dominant str...

T	-10, 10	10, -10

strategy

③ Games without conflict (Collusion)

	C_2 R LB	A HB
C_1 R LB	3 3	0 2
A HB	2 0	1 1

$$U(R,R) > U(A,R) > U(A,A) > U(R,A)$$

P_2

$$P_1 \quad \begin{array}{c|ccc} & a & b & c \\ \hline i & -2,2 & 1,3 & 1,1 \\ ii & 0,1 & 0,2 & 2,0 \\ iii & 1,0 & 2,1 & 3,-2 \end{array}$$

P_2

$$P_1 \quad \begin{array}{c|ccc} & a & b & c \\ \hline i & -2, & 1, & 1, \\ ii & 0 & 0, & 2 \end{array}$$

$P_1 = iii$

$$\begin{array}{c|ccc} \text{iii} & \boxed{1 & 2, & 3,} \end{array}$$

$$\begin{array}{cc} & P_2 \\ P_1 & \begin{array}{cccc} & a & b & c \\ i & ,2 & 3 & 1 \\ ii & ,1 & ,2 & 4 \\ iii & ,0 & ,1 & -2 \end{array} \end{array}$$

$P_2 = b$

$$P_1 \quad \begin{array}{ccc} & a & b \\ i & 20,10 & 5,10 \\ ii & 10,7 & 4,10 \\ iii & 10,5 & 7,7 \end{array}$$

$$P_1 \quad \begin{array}{ccc} & a & b \\ i & 20, & 5, \\ ii & 10, & 4, \\ iii & 10, & 7, \end{array}$$

P_1 X

$$P_1 \quad \begin{array}{ccc} & a & b \\ i & 10 & 10 \\ ii & 7 & 10 \\ iii & 5 & 7 \end{array}$$

P_2 b

$$
\begin{array}{cc|c}
 & & P_2 \\
 & & \begin{array}{ccc} a & b & c \end{array} \\
P_1 & \begin{array}{c} i \\ ii \end{array} & \begin{bmatrix} 10,5 & 1,1 & 5,5 \\ 5,10 & 0,10 & 1,20 \end{bmatrix}
\end{array}
$$

$P_i = i$

$P_{ii} = c$

$$
\begin{array}{cc|c}
 & & \begin{array}{cc} a & b \end{array} \\
P_1 & \begin{array}{c} i \\ ii \end{array} & \begin{bmatrix} 5,5 & 10,0 \\ 5,1 & 0,10 \end{bmatrix}
\end{array}
$$

$P_1 = i$

$P_2 = X$

Game Dominant Strategy Equilibrim

Battle of Couples (Better to cooperate)

$$
\begin{array}{cc|c}
 & & \text{South} \\
 & & \begin{array}{cc} \text{Shahrukh} & \text{Pawan} \end{array} \\
\text{North} & \begin{array}{c} \text{Shahrukh} \\ \text{Pawan} \end{array} & \begin{bmatrix} (2,1) & (0,0) \\ (0,0) & (1,2) \end{bmatrix}
\end{array}
$$

} No dominant strategy.

Better to cooperate even if outcome is less preferred

Strictly cooperate

Sri Lanka small country (cannot protect)
India big country

$$SL = U(\overset{3}{R,R}) > (\overset{2}{A,R}) \geq (\overset{1}{A,A}) > (\overset{0}{R,A})$$

$$India = U(\underset{3}{R,R}) > (\underset{2}{R,A}) > (\underset{1}{A,A}) > (\underset{0}{A,R})$$

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{\text{India}} \\ & & R & A \\ \hline \text{SL} & R & (3,3) & (0,2) \\ & A & (2,0) & (1,1) \end{array}$$

No dominant strategy

1. Better to conflict
2. Strictly conflict
3. Better to cooperate
4. Strictly cooperate

How to find equilibrim?

Nash Equilibrim

stable equilibrium (No motivation to change as their strategy)

Dominant Strategy Equilibrim → Nash equilibrim

Arms / Refrain in Prisoner Dilemma — Country Big v/s Big

$$U(A,R) > U(R,R)$$

Arms / Refrain in Strictly cooperate case — Big v/s Small

$$U(R,R) > U(A,R)$$

Battle of Couples

$$\begin{array}{cc|cc} & & \text{South} & \\ & & S & P \\ \text{North} & S & \boxed{2,1} & 0,0 \\ & P & 0,0 & \boxed{1,2} \end{array}$$

Is there any Nash equilibrim ?

Multiple Nash equilibrim

$\boxed{}$ → No motivation to change

⇓

Stable equilibrim

⇓

Nash equilibrim

P_2

H T

Strictly conflict

$$
\begin{array}{cc|c|c|}
 & & & \\
\hline
\multirow{2}{*}{P_1} & H & 10, -10 \; \scriptstyle P2 & -10, 10 \; \scriptstyle P1 \\
\hline
 & T & -10, 10 \; \scriptstyle P1 & 10, -10 \; \scriptstyle P2 \\
\hline
\end{array}
$$

No dominant strategy

Issue caused by red

No stable equilibrim

$\Downarrow$

No Nash equilibrim

Strictly cooperate

$$
\begin{array}{ccc}
 & & \text{India} \\
 & R & A
\end{array}
$$

$$
\begin{array}{ccc}
 & & \begin{array}{cc} R & A \end{array} \\
SL & \begin{array}{c} R \\ A \end{array} & \begin{bmatrix} \boxed{(3,3)} & (0,2)\;{\scriptstyle P1} \\ (2,0)\;{\scriptstyle P2} & \boxed{(1,1)} \end{bmatrix}
\end{array}
$$

(A, A) & (R, R) 2 Stable equilibrims

$\Downarrow$

2 Nash equilibrims

3rd Sept 2024

⑥ Duopoly market

2 firms

A \ B	H	L
H	1000, 1000	-200, 1200
L	1200, -200	600, 600

→ NE

A (L, L) > (H, H) > (L, H) > (H, L)

B (L, L) > (H, H) > (H, L) > (L, H)

// ई

लागयाई एउटा श्रम भाग

C_1 \ C_2	AD	NAD
AD	4, 6	1, 8
NAD	3, 4	7, 7

C_1 X

C_2 NAD

??

// ई

F_A \ F_B	HP	LP
HP	8, 8	3, 10
LP	10, 3	5, 5

LP

LP

LL

Eg

P_1 \ P_2	A	B
I	1,1	3,2
II	2,3	0,1

??

No dominant strategy for anyone.

Eg

P_1 \ P_2	A	B
I	2,0	3,2
II	1,1	0,2

$P_1 = I$

$P_2 = B$

Implications of dominant strategy

Eg

P_1 \ P_2	A	B	C
I	1,0	3,2	1,1
II	2,3	3,3	0,2

$P_2 = B$

$P_1 = X$

P2

P1 UP Center

P2 left P1

Middle Center

P 2

Down left

Iterative Elimation of DS

		P2		
		L	C	R
	~~U~~	~~0 2~~	~~3 1~~	~~2 3~~
P1	M	1 4	2 1	4 1
	D	2 1	4 4	3 2

P1 U — P2 R

M — P1 L

D — P2 C

P2 L — P1 D

P2 C — D

P2 R — M

P1 M P2 L

D P2 C

P2 L D

P2 C D

P2 R M

P2

		L	C	R
P1	M	1 4	2 1	4 1
	D	2 1	4 4	3 2

P1 M P2 L

D P2 C

P2

		L	C
P1	~~M~~	~~1 4~~	~~2 1~~
	D	2 1	4 4

P2 L D

C D

P2

P2

$$\begin{array}{c|cc} & L & C \\ \hline P1 \quad D & 21 & 44 \end{array}$$

DC

Best Response Function

$$\begin{array}{c|ccc} & L & C & R \\ \hline T & (1,1) & (1,0) & (0,1) \\ B & (1,0) & (0,1) & (1,0) \end{array}$$

Player i $\qquad -i$

a_i $\qquad a_i'$

BRF $\qquad$ Non BRF

$B_i(a_{-i})$

$$u_j \quad \left[U_i(a_i, a_{-i}) \geq U(a_i', a_{-i}) \right]$$

$$\begin{array}{c|ccc} & L & C & R \\ \hline T & (1,1) & (1,0) & (0,1) \end{array}$$

$$B \quad [\,(1,0)\ \ (0,1)\ \ (1,0)\,]$$

BRF	a_i	a_{-i}
p_i	T/B	L
	T	C
	B	R

BRF p_2	a_i	a_{-i}
	L/R	T
	C	B

NE

$$P_1 \quad BR(a_1^*) = P_2(a_2^*)$$

$$P_2 \quad BR(a_2^*) = P_1(a_1^*)$$

P2

P1		L	C	R
	T	1 2^*	2^* 1	1^* 0
	M	2^* 1^*	0 1^*	0 0
	B	0 1	0 0	1^* 2^*

BRF

P1

a_i	a_{-i}
M	L
T	C
T/B	R

BRF

P2

a_i	a_{-i}
L	T
L/C	M
R	B

Nash equilibrim for symmetrical games

		P2	
		L	R
P1	L	(1, 1)	(0 0)
	R	(0, 0)	(1, 1)

Oligopoly market (few firms ≤ 10)

Duopoly

Profit → Qty / Price

Price → Qty

$C_i = f(q_i)$

$$C_i = f(q_i)$$

$$P_i = P(Q)^{+ve} = P(q_1 + q_2 + \cdots + q_n)$$

$$TR_i = q_i P$$

$$= q_i P(Q)$$

$$= q_i P(q_i + q_2 \cdots q_n)$$

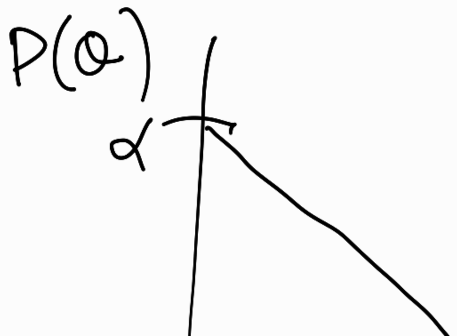
$$\text{Profit} = TR - C$$

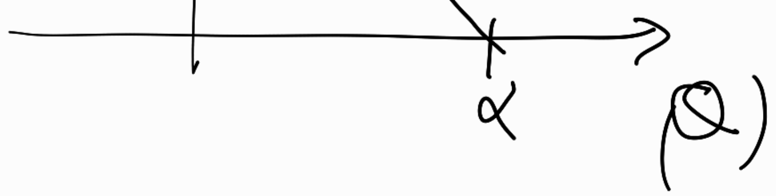
$$= q_i P(q_i + q_2 + \cdots + q_n) - C_i(q_i)$$

Duopoly   Cournot Model

$$q_1 + q_2 = Q$$

$$P(Q) = P(q_1 + q_2) = \begin{cases} \alpha - Q & \text{if } Q \leq \alpha \\ 0 & \text{o/w} \end{cases}$$





$$c_i(q_i) = cq_i$$

BRF

$$\pi_1 = \pi_1(q_1, q_2)$$

$$\Rightarrow q_1 P(Q) - cq_1$$

$$\Rightarrow q_1\alpha - q_1 Q - cq_1$$

$$\pi_1 \Rightarrow q_1\alpha - q_1 q_1 - q_1 q_2 - c q_2$$

$$\pi_2 \Rightarrow q_2(\alpha - c - q_1 - q_2)$$

$$b_1(q_2) = q_1^*$$

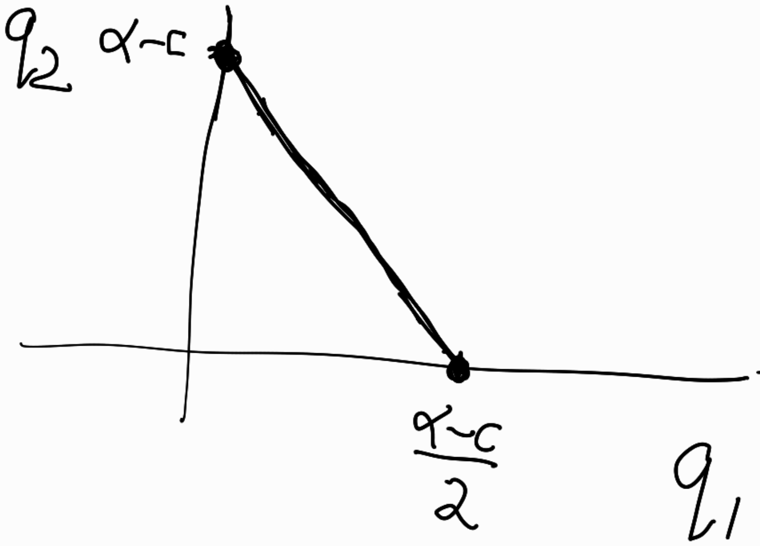
$$q_1(\alpha - c - q_1 - q_2)$$

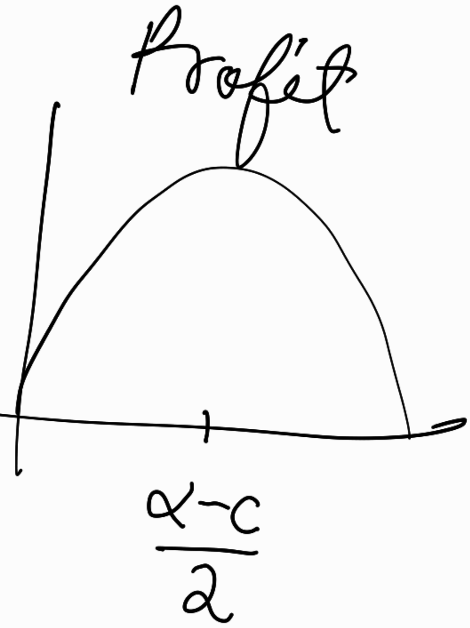
$$\alpha - c - 2q_1 - q_2 = 0$$

$$q_1 = \frac{\alpha - c - q_2}{2}$$

$q_2 = 0 \quad q_1 = \frac{\alpha - c}{2}$

$q_2 = \alpha - c \quad q_1 = 0$

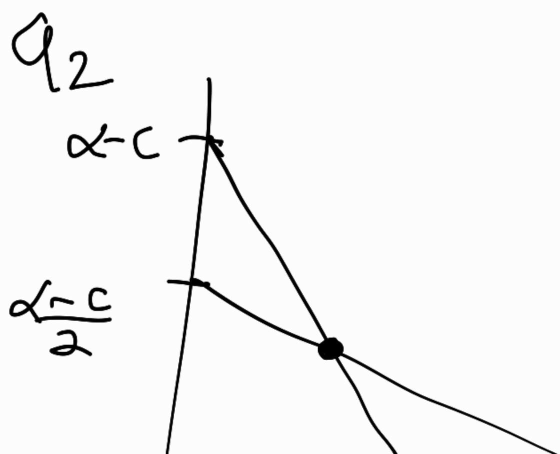




$$q_2 = \frac{\alpha - c - q_1}{2}$$

$q_1 = 0 \quad q_2 = \frac{\alpha - c}{2}$

$q_1 = \alpha - c \quad q_2 = 0$



$$y = \frac{\alpha - c - x}{2}$$

$$x = \frac{\alpha - c - y}{2}$$

$$y = \frac{\alpha - c - \frac{\alpha - c - y}{2}}{2}$$

$$4y = 2\alpha - 2c - \alpha + c + y$$

$$y = \frac{\alpha - c}{3}$$

$$x = \frac{\alpha - c - \frac{(\alpha - c)}{3}}{2}$$

$$\frac{3\alpha - 3c - \alpha + c}{}$$

$$\frac{2\alpha - 2c}{6} = \frac{\alpha - c}{3}$$

$$P = \alpha - 2\frac{(\alpha - c)}{3}$$

$$P = \frac{3\alpha - 2\alpha + 2c}{3}$$

$$P = \frac{\alpha + 2c}{3}$$

Payoff $b_1(q_2) = \begin{cases} \frac{1}{2}(\alpha - c - q_2) & q_2 \le \alpha - c \\ 0 & q_2 \ge \alpha - c \end{cases}$

Post Mid Sem Exam

01|10|24

Bertrand Duopoly Model

Cournot Model

What qty of F1 will maximize the profit of F1 given the qty of F2?

$$C = bq_i$$

$$Q = \alpha - P_i$$

Two firms i & j $\qquad \pi_i \qquad \pi_j$

	π_i	π_j
$P_i > P_j$	$\pi = 0$	$(P_j - c)(\alpha - P_j)$
$P_i = P_j$	$\pi = \underbrace{(P_i - c)(\alpha - P_i)}_{2}$	$\frac{(P_j - c)}{2}(\alpha - P_j)$
$P_i < P_j$	$\pi = (P_i - c)(\alpha - P_i)$	0

$$C = cq_i$$

$$\pi_i = P_i q_i - cq_i$$

$$\pi_i = (P_i - c)\,Q$$

$$\frac{\partial \Pi_1}{\partial P_1} = \frac{(P_1 - c)(\alpha - P_1)}{2}$$

$$= \frac{\alpha P_1 - P_1^2 - \alpha c + P_1 c}{2}$$

$$\boxed{0 = \frac{\alpha - 2P_1 + c}{2} \Rightarrow P_1 = \frac{c + \alpha}{2}}$$

$$(P_1 - c)(\alpha - P_1)$$

$$\alpha P_1 - P_1^2 - \alpha c + P_1 c$$

$$\boxed{\alpha - 2P_1 + c \Rightarrow P_1 = \frac{\alpha + c}{2}}$$

$$Q = \alpha - P_1$$

$$= \alpha - \left(\frac{\alpha + c}{2}\right)$$

$$= \alpha - c$$

$\frac{\quad}{2}$

$P_i < P_j$ then F_j will try to $P_j = P_i$

What $P_i < c$

then P_j won't follow P_i to prevent negative profit. and stays ~~get~~ 0 profit

Is $P_i = P_j = c$ NE?

Yes

		F2		
		H	M	L
	H	6,6	0,10	0,8
F1	M	10,0	5,5	0,8
	L	8,0	8,0	4,4

F1 H F2 M
M F2 L
L F2 L

Go for IE

M L

M	55	08
L	80	44

F2

M F1 L

L F1 L

15/10/24

Electoral Competition

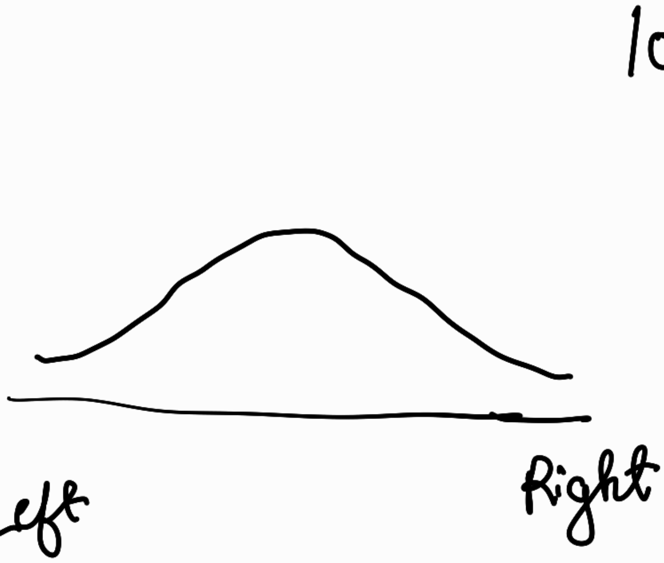
Win > Tie > Loss

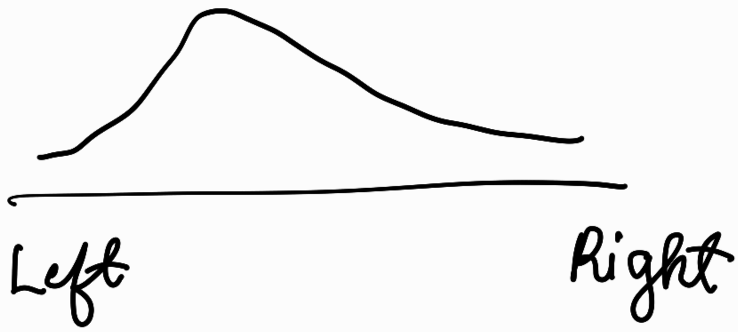
Political party → No ideological

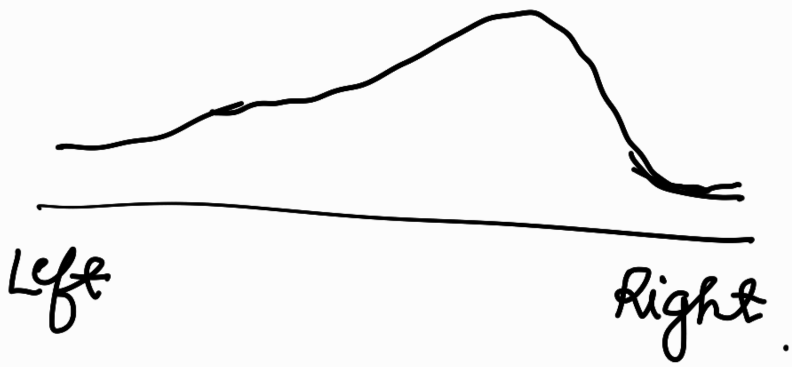
Voter → attachment

Players → Candidates

Actions → Find preferred ideological location



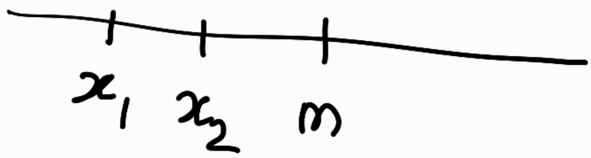




BRF

Case I $x_2 < m$

Case 1.1 $x_1 < x_2$



x_2 wins.

$x_2 < m$

1.2 if $(x_1 + x_2)$ ... x_1 wins

$\frac{\quad}{2} < m \quad x_1$ wins

1.3 else $\quad x_2$ wins.

$$BR_1(x_2) = \begin{cases} x_2 < x_1 < 2m - x_2 & \text{if } x_2 < m \\ 2m - x_2 < x_1 < x_2 & \text{if } x_2 > m \\ m & \text{if } x_2 = m \end{cases}$$

$$BR_2(x_1) = \begin{cases} x_1 < x_2 < 2m - x_1 & \text{if } x_1 < m \\ 2m - x_1 < x_2 < x_1 & \text{if } x_1 > m \\ m & \text{if } x_2 = m \end{cases}$$

Median Voter Theorem

Hotelling rule

Auction

Case of Perfect information.

n bidders

(Each bidder knows the evaluation value of other bidder)

$b_1 > b_2 > - - b_n$ $\qquad$ $v_1 > v_2 > - - - > v_n$

$b_i \neq b_j$ $\qquad$ $v_i \neq v_j$

$b_i \leq v_i$

Player i pays $v_i - b_j$

$\downarrow$

i highest bidder

j second highest bidder.

NE

$BRF_1(a_{-1})$

$b_1 > v_1$

$b_1 = v_1$

$b_1 < v_1$

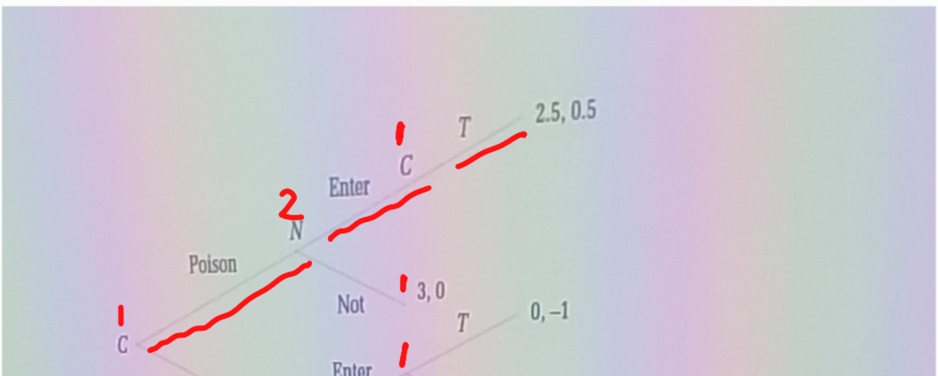
$(b_1 > b_2 > - b_n) < (v_1, v_2 - - v_n)$

$=$

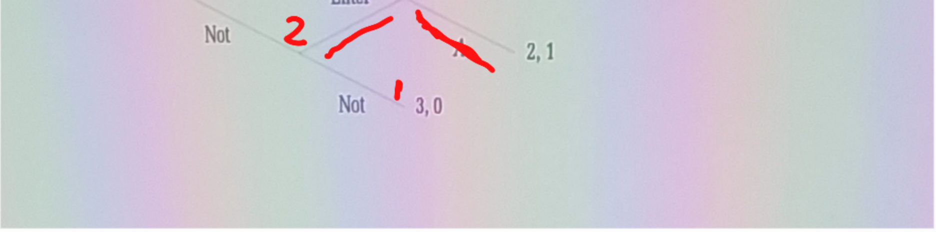
$b_i \; b_j$

Sealed

BLACK AHEAD



2.5, 0.5
T
1
C
Enter
2
N
Poison
3, 0
Not
1
C
T
0, –1
Enter


Not
2
2, 1
1
Not
3, 0