GAME THEORY

LA358

Dominant Strategy

➢Problems:

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢Solution: Regardless of the other player strategy, we need to check if any player has a dominant strategy (DS)

➢Step 1: Check DS of Player 1-i.e in both possible cases of P2's strategy (A or B), can P1 choose one dominant strategy (I or II)?

➢Step 2: Check DS of Player 2-i.e in both possible cases of P1's strategy(I or II), can P2 choose one dominant strategy (A or B)?

Dominant Strategy

➢ DS of P1 (I or II?)

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Case 1: P2 choose **A**, then P1 choose which one?

Dominant Strategy

➢ Step 1: DS of P1(I or II?)

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Case 1: if P2 choose A, then P1 choose which one?

➢ Fix A as P2's strategy and then compare P1's payoff b/w strategy I and II

➢ Hence compare (2 v/s 1; 2>1), therefore if P2 A= then P1 choose strategy I

Dominant Strategy

➢ Step 1: DS of P1(I or II?)

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Case 2: if P2 choose B, then P1 choose which one?

➢ Fix B as P2's strategy and then compare P1's payoff b/w strategy I and II

➢ Hence compare (3 v/s 0, 3>0), therefore if P2 B= then P1 choose strategy I

Dominant Strategy

➢ Step 1: DS of P1

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Hence, regardless of P2's strategy (A or B); P1 will always choose I over II.

➢ Thus, I is DS for P1 (Strategy I is strictly DS for P1)

➢ Note: When we check P1 payoff don't consider P2 payoff (here comparison with same color)

Dominant Strategy

➢ Step 2: DS of P2 (A or B?)

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Case 1: P1 choose I, then P2 choose which one?

➢ Fix I as P1's strategy and then compare P2's payoff b/w strategy A and B

➢ Hence compare (0 v/s 2; 0<2), therefore if P1choose I= then P2 choose strategy B

Dominant Strategy

➢ Step 2: DS of P2

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Case 1: P1 choose II, then P2 choose which one?

➢ Fix II as P1's strategy and then compare P2's payoff b/w strategy A and B

➢ Hence compare (1v/s 2; 1<2), therefore if P1choose II= then P2 choose strategy B

Dominant Strategy

➢ Step 2: DS of P2

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Hence, regardless of P1's strategy (I or II); P2 will always choose B over A.

➢ Thus, B is DS for P2 (B is strictly DS)

➢ Note: When we check P2 payoff don't consider P1's payoff

Dominant Strategy

➢ Dominant Strategy Equilibrium (DSE)

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,2

➢ Since both players have DS: P1(I) and P2(B)=cell IB=(3,2)= is the DSE of the game

Weak Dominant Strategy

➢ Problems:

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,1

➢ P1- DS is I (from previous example)

➢ Strategy I is the strictly dominant strategy for player 1

Weakly Dominant Strategy

➢ Problems:

		Player 2	
		A	B
Player 1	I	2,0	3,2
	II	1,1	0,1

➢ P1- DS is I

➢ P2 : **case 1-** P2 choose B(2) over A(0) if P1 chooses I

➢ P2: **Case 2-** P2 payoff is equal for A(1) and B(1) if P1 chooses II

➢ Here P2 don't have a strictly dominant strategy,

➢ However P2 has **B preferred over A in atleast one case** (here, case 1) and **A similar to B** (A=B) for the **other cases (here**, case 2) and **in no case A preferred over B**. Then, Strategy B is weakly dominant over A

Dominant Strategy

➢ Problems:

		Player 2		
		A	B	C
Player 1	I	2,0	3,2	4,1
	II	1,3	0,3	5,2

Dominant Strategy

➢ Sol:

		Player 2		
		A	B	C
Player 1	I	2,0	3,2	1,1
	II	2,3	3,3	0,2

Dominant Strategy

➢ Sol:

		Player 2		
		A	B	C
Player 1	I	2,0	3,2	1,1
	II	2,3	3,3	0,2

Dominant Strategy

- Sol: P1 WDS is strategy I; and P2 WDS is B

		Player 2		
		A	B	C
Player 1	I	2,0	3,2	1,1
	II	2,3	3,3	0,2

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Check each cell if there is motivation to change strategy for each player
- Fixing the other player strategy check if the concerned player has motivation to change (based on the pay-off)

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Start with cell IA (1,1)
- Start with player 1 : P1 thinks that from IA (where P2 strategy A is fixed), P1 has only option to choose strategy II which leads to cell (IIA)(2,3)
- No other cell is possible as P1 takes P2 choose A

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Start with cell IA (1,1)
- Start with player 1 : P1 thinks that from IA (where P2 strategy A is fixed), P1 has only option to choose strategy II and therefore change to cell (IIA)(2,3)
- Between these two cells, compare P1 pay off (1 v/s 2) = (1< 2) which results in P1 has motivation to change strategy to II (given Player 2 is at A). Thus from cell IA players will end up at cell IIA
- Since at least one player has a motivation to change strategy, the cell IA is not NE. Hence, we need not have to check if player 2 has a motivation to change from cell IA

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Consider cell IIA (2,3)
- Player 1 : P1 thinks that from IIA (where P2 strategy A is fixed), P1 has only option to choose strategy I. However, this will lead them back to cell IA which P1 doesn't prefer due to pay off (1<2). Hence P1 has no motivation to change

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Consider cell IIA (2,3)
- Player 1 : P1 thinks that from IIA (where P2 strategy A is fixed), P1 has only option to choose strategy I. However, this will lead them back to cell IA which P1 doesn't prefer due to pay off (1<2). Hence P1 has no motivation to change
- At cell IIA, P2 consider P1 fixed strategy II, and P2 has option to choose between A and B.
- This implies P2 will have to choose between cells IIA and IIB (as P1 is fixed at II). Payoff (3 v/s 1)= 3>1
- Thus player 2 also has no motivation to change from cell IIA.

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- Consider cell IIA (2,3)
- Player 1 : P1 thinks that from IIA (where P2 strategy A is fixed), P1 has only option to choose strategy I. However, this will lead them back to cell IA which P1 doesn't prefer due to pay off (1<2). Hence P1 has no motivation to change
- At cell IIA, P2 consider P1 fixed strategy II, and P2 has option to choose between A and B.
- This implies P2 will have to choose between cells IIA and IIB (as P1 is fixed at II). Payoff (3 v/s 1)= 3>1
- Thus player 2 also has no motivation to change from cell IIA.
- Since both players have no motivation to change from **cell IIA, it is a Nash Equilibrium**
- **Check for cell IB and IIB likewise and see if there is/are any other NE?**

Nash Equilibrium

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

- **Check for cell IB and IIB likewise and see if there is/are any other NE?**
- Cell IB is NE
- Cell IIB not a NE

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1

		Player 2	
		A	B
Player 1	I	1,1	3,2
	II	2,3	0,1