

GAME THEORY

LA358

L2

- *Interdependence* of players in a game
- GT is a formal way to analyse **interaction** among a **group** of **rational** agents who behave strategically
 - Group: More than 1 decision maker (or player)
 - Interaction: What any one individual player does affect at least one other player in the group
 - Strategic: An individual player accounts for this interdependence in deciding what action to take
 - Rational: Accounting for interdependence, each player chooses best action

➤ Example: Group project

- Group: Group of students
- Interaction: Certain amount of work needs to be done by each member to finish the project. If anyone slack, others have to do extra work (i.e. interdependence)
- Strategic: Free-rider problem (each player can think they can free ride while other complete the project by doing extra work)
- Rational: Accounting for interdependence (here free rider problem), each member takes a decision to do extra work depending on the pay-off (expected grades and satisfaction from it)

➤ When can a situation fail to be a game?

➤ 2 cases:

❑ the one: Your decision affects only you

- Eg: Choice of exercise, hours to spend on study etc

❑ The infinity: your decision affect others but the group is so large that one

individual player action need not be traced or need not influence the whole group

- Eg: purchase/sale of shares (stock) of a company by an individual wont affect a large body of stock holders
- A person's decision to buy 'Onions' doesn't affect the market price of onions in that city

Example 1: Card Game

- Two piles of cards: A and B
- Two players: P1 and P2
- Two cases : Balanced Game (BG) and Unbalanced Game (UBG)
- Rules:
 - R1: Any number of cards can be taken from a pile. If either piles have cards remaining, then players are required to take at least one card
 - R2: Players can only remove cards from one plie at a time
 - R3: The game starts with player 1 and then each player gets alternative chances (P1-P2-P1-P2 and so on until the game ends)
 - R4: One who picks the last card wins

Example 1: Card Game-Balanced (BG)

➤ $G1: [A, B] = [\text{Number of cards in Pile A}, \text{number of cards in Pile B}]$

➤ Case 1: $[A, B] = [1, 1]$

➤ Game: $P1[0, 1] - P2[0, 0] = P2$ wins

➤ $G2: [A, B]: [2, 2]$

➤ Case 1

➤ $P1[0, 2] - P2[0, 0] = P2$ wins

➤ Case 2

➤ $P1[1, 2] - P2[1, 1] - P1[0, 1] - P2[0, 0] = P2$ wins

Example 1: Card Game-Balanced (BG)

- P1: Best strategy given P2 action affect P1 is that to ensure both piles have cards. Otherwise P2 will grab all remaining cards in the available pile and wins the game
- P2: Best strategy given P1's strategy is that to ensure both piles have equal number of cards after his/her move ; which the P2 can achieve by exactly removing the same number of cards as P1 but from the other pile (mimicking P1 action in pile B, if P1 removed from pile A in the previous turn)
- Outcome/result: P2 has a winning strategy for BG

Example 1: Card Game-Unbalanced (UBG)

- P1 begins the game hence adopt P2 strategy in BG, which is to ensure that both piles have equal number of cards after P2's move. For that P1 just need to mimick P2's action in the other pile
- P2: Best strategy is that to ensure both piles have cards
- For UBG the strategy got reversed for both players
- Outcome/result: P1 has a winning strategy for UBG