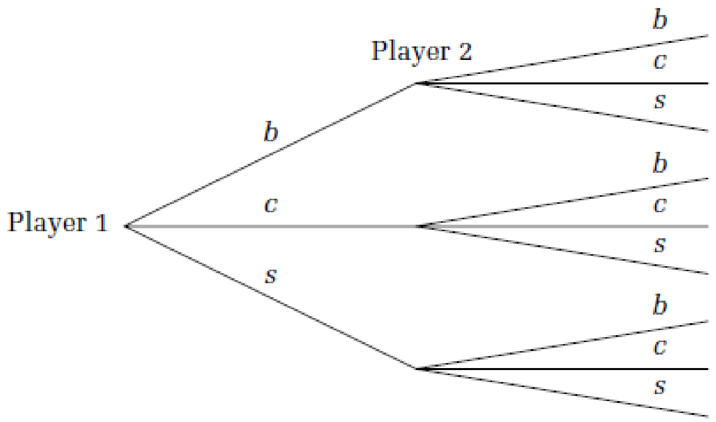
GAME THEORY

LA358

Extensive form games, Sub-game perfect equilibrium and Backward Induction

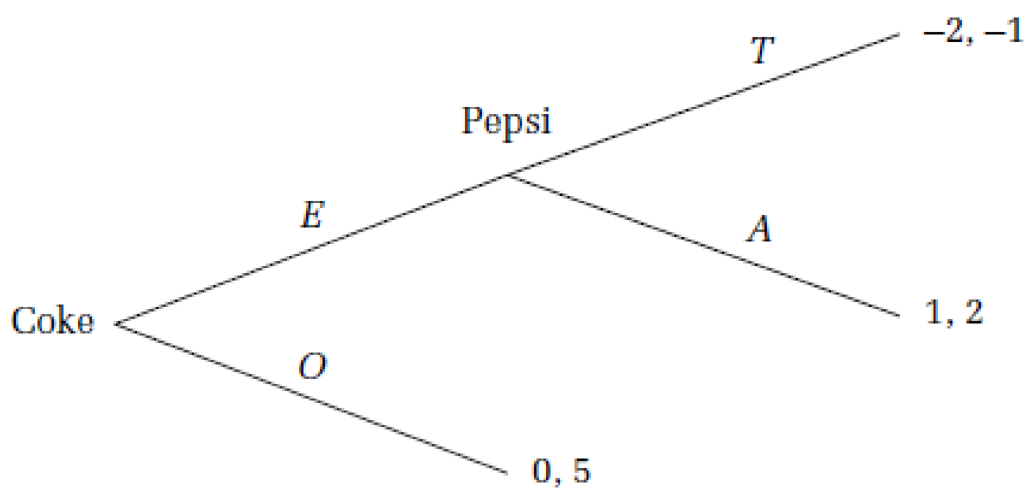
Extensive form games with perfect information

- Structure of extensive form games – A game tree
- Starting point – root
- From root there is/are branches representing Strategies/Options/ choices of first player
- Each Strategies/Options/ branches lead to next players **decision node/terminal history**
- If a decision node end without further branches it is called as **terminal node**
- **Each player has perfect/complete information about the payoffs for both players**



Extensive form games with PI

- Example 1: Market entry –players make decision about how much to invest in the market
- Coke (player 1) has two options – enter the market (E) /stay out (O)
- Pepsi Player 2 has two choices –Accommodative (A) or Tough competition (T)



Extensive form games with PI

- Extension with next level of decision point for player 1 (Coke) = If it enters it can also play T (spend a lot on ads and other promotion) or Accommodate strategy

Extensive form games with PI

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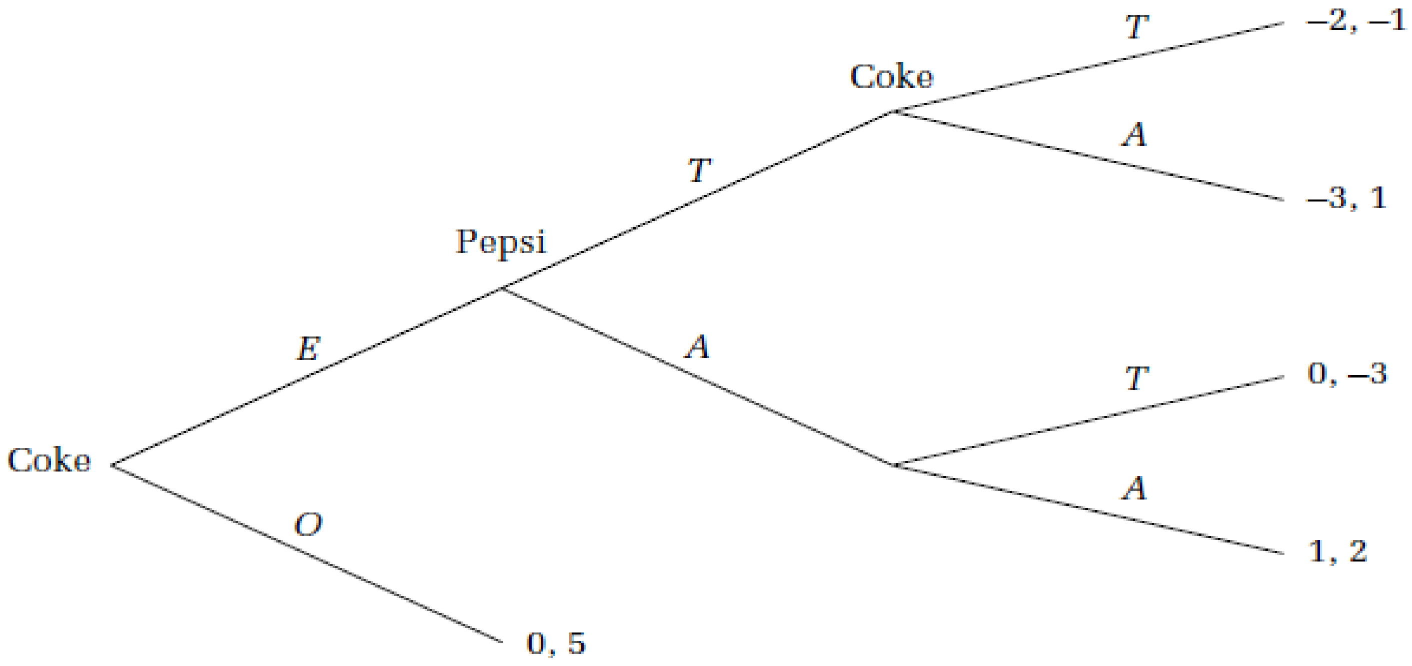
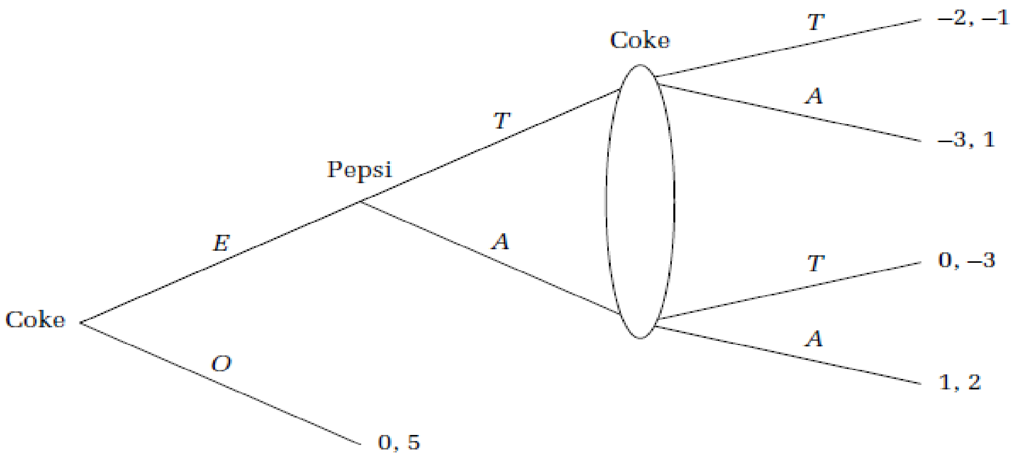


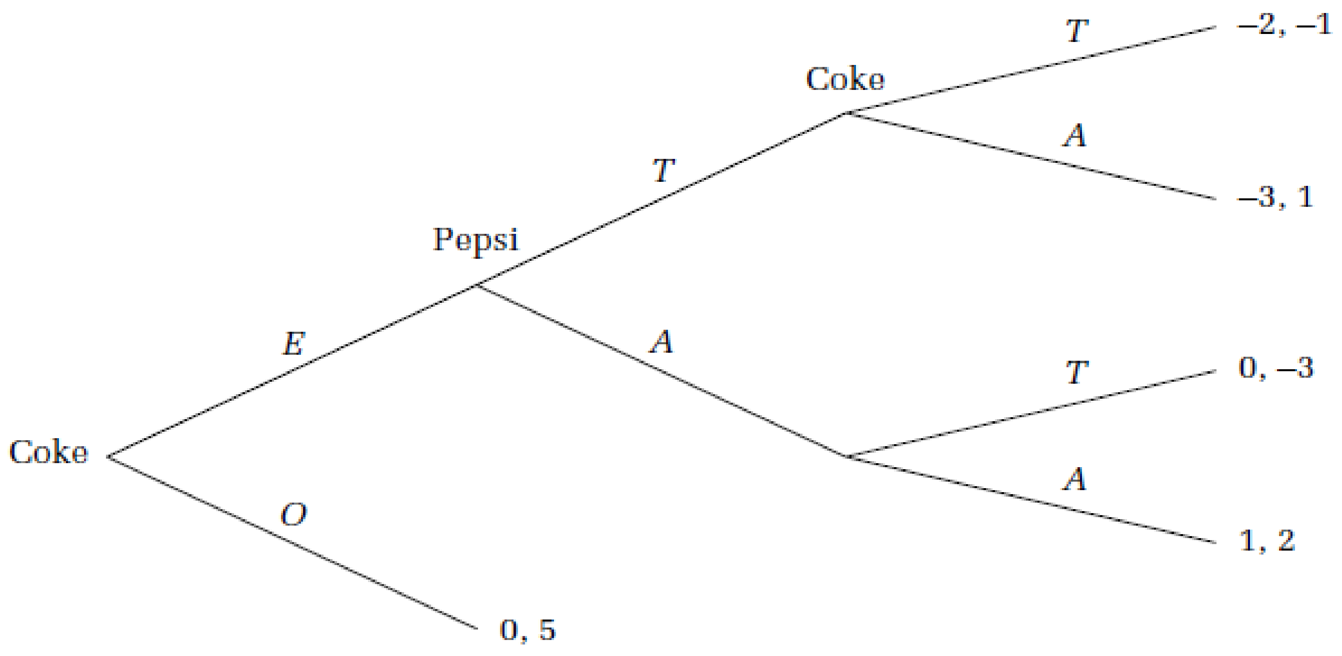
➢ Example 2: Both players make decision about how much to invest in the market .

However what if these **decisions are taken simultaneously.** Then it is **not a game of perfect information**



Sub-game perfect equilibroum

- SGPE: sub-game perfect equilibrium

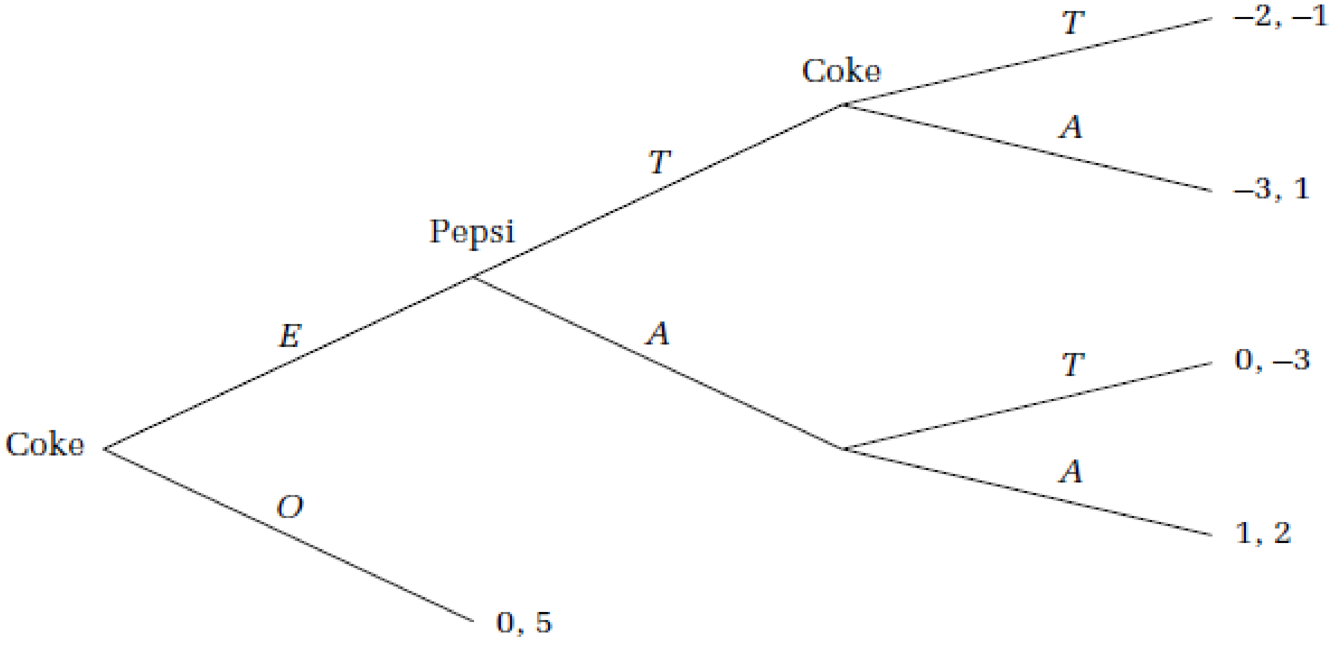


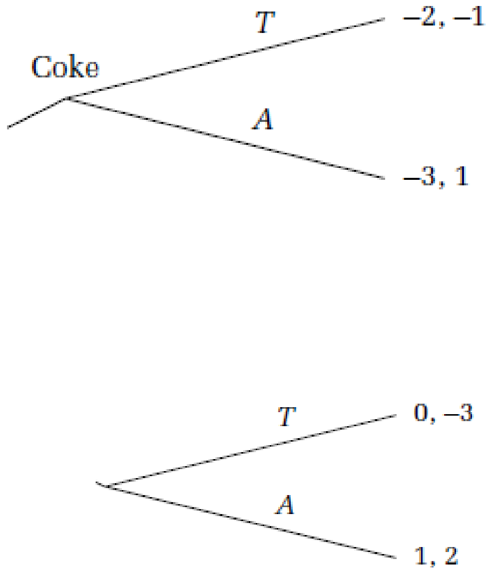
Extensive form games with PI- Backward Induction

- We fold the game tree back one step at a time till we reach the beginning . **One step at a time** is finding **sub-game perfect equilibrium** , which is know as **induction**. And since we are working it backward-the process is called **backward induction**
- Kuhn's and Zermelo's Theorem: Every game of perfect information with a finite number of nodes has a solution to backward induction.

Sub-game perfect equilibroum

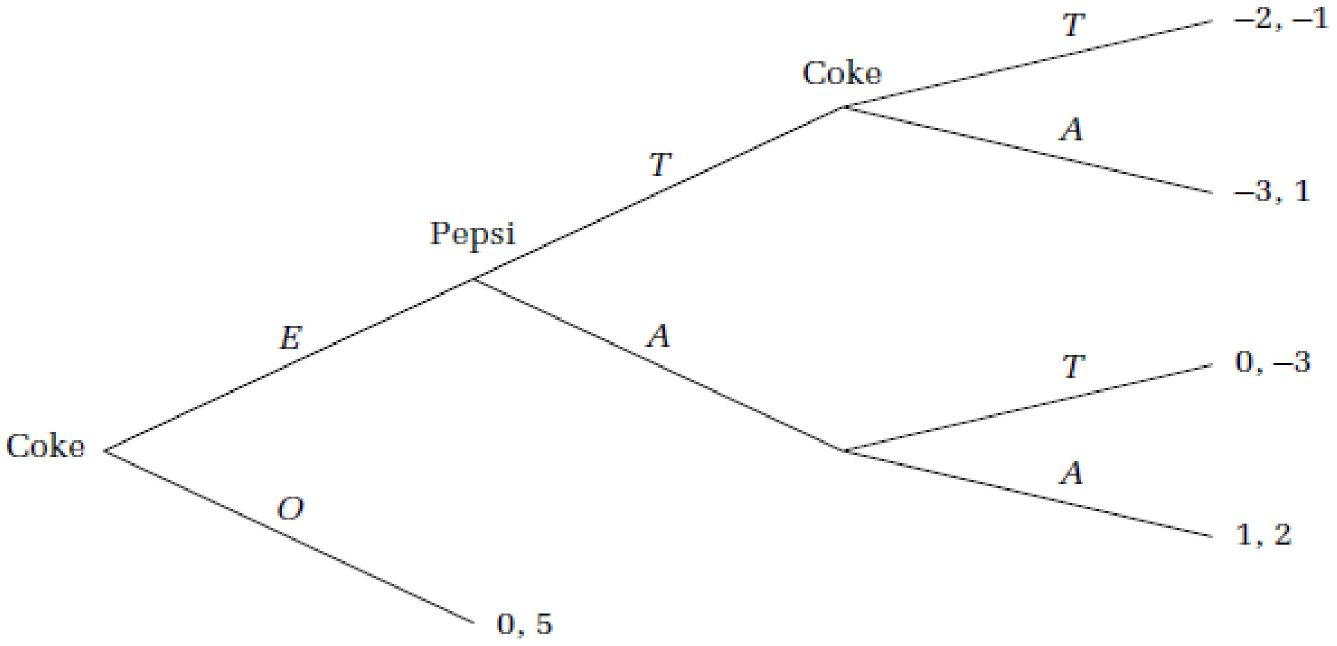
➢Solution- Backward induction through SPGE. Starts from Player 1 (coke's) decision. Based on it, then P2 (Pepsi's) make decision. Depending on these sub-game solutions Player 1 chose its strategy in the last stage

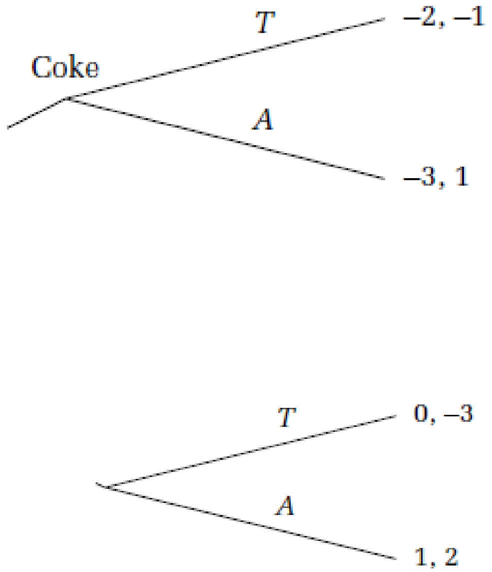




Sub-game perfect equilibrium- Phase I of Backward Induction (BI)

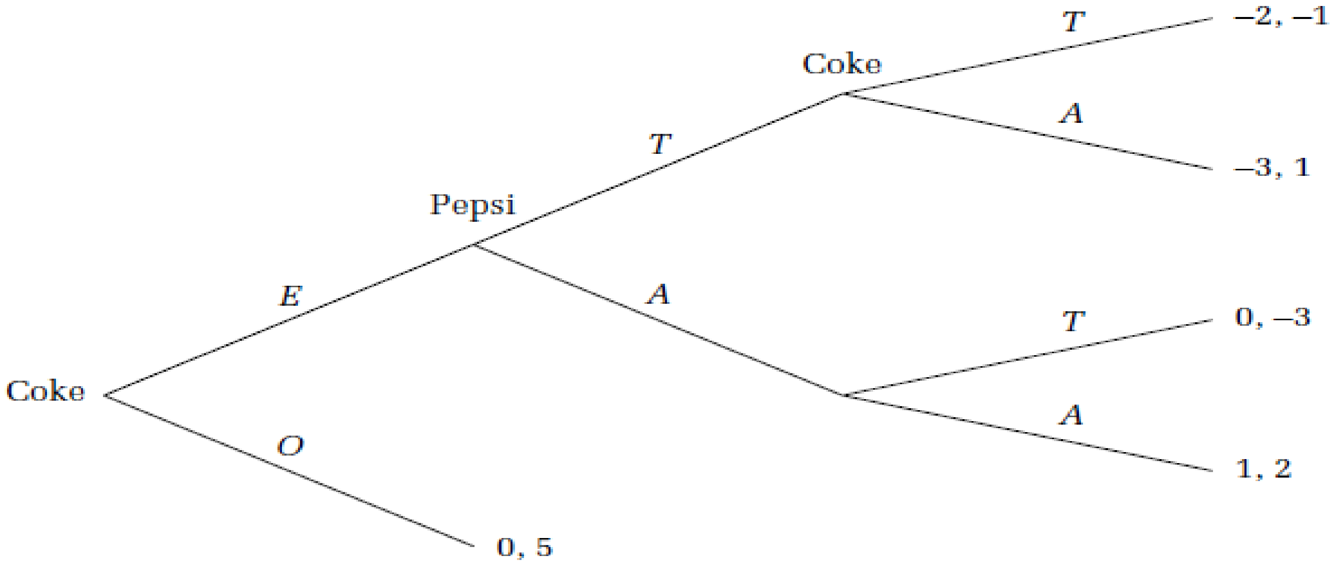
Game tree-last phase is played by player 1. Hence, we start backward Induction from Player 1 (Coke).
Coke's best response= if Pepsi adopt tough (T) then Coke choose Tough; (-2,-1) v/s (-3,1) = (-2,-1) opted (as -2>-3)
Coke's best response= If Pepsi adopt Accommodative (A), then coke choose Accommodative ; (0,-3) v/s (1,2)=(1,2)
These two decisions are depicted as sub-games in figure below

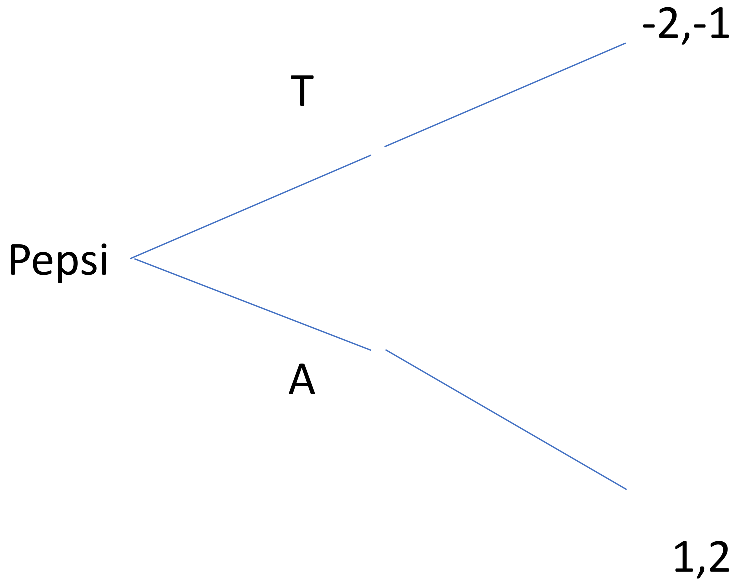




Sub-game perfect equilibrium- Phase I of Backward Induction (BI)

Game Tree: Player 2 (Pepsi) face two pay-offs in the next sub-game , which were opted by Player 1 in previous phase of the game. Choosing from -1 v/s 2 ; Pepsi will opt for Accommodative strategy (1,2)

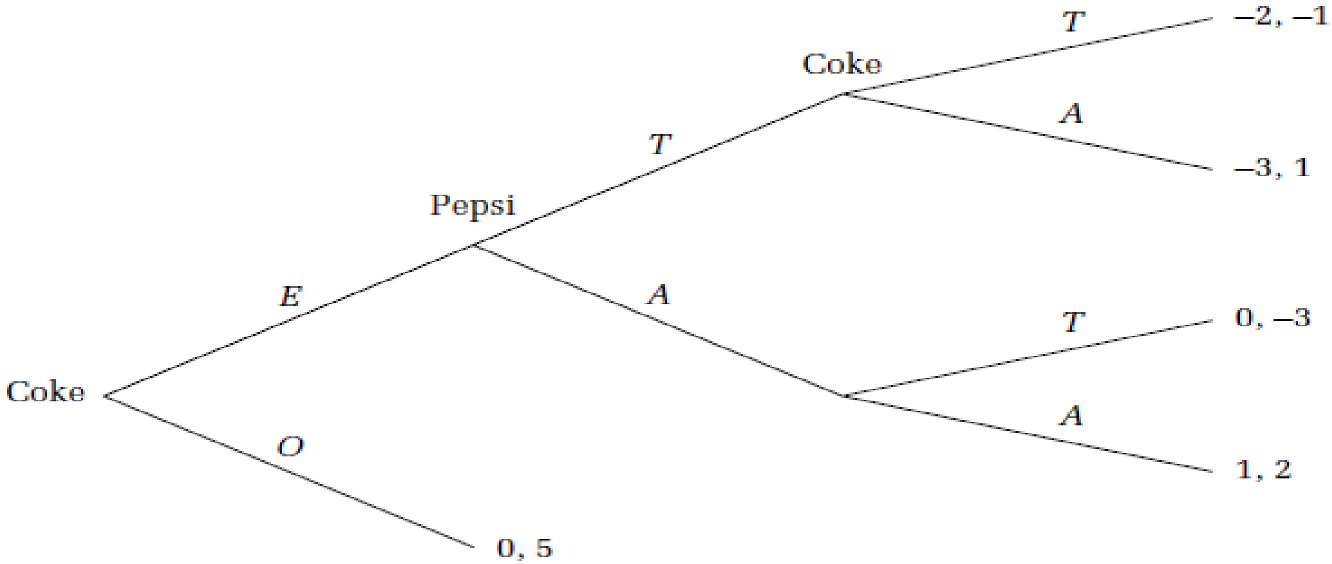


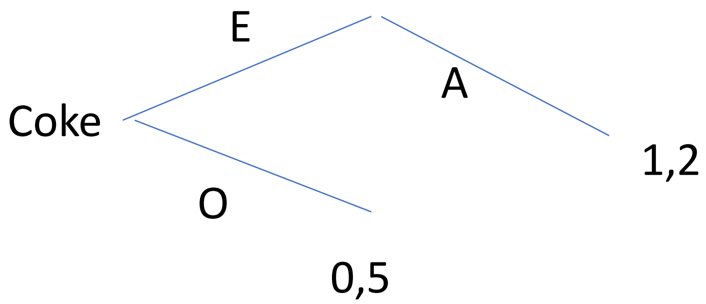


Sub-game perfect equilibrium- Phase I of Backward Induction (BI)

Game Tree: Player 1 (coke) also face two pay-offs in the next sub-game , based on previous phase outcomes of the game. Choosing from (1,2) v/s (0,5) ; Coke will opt for Enter-Accommodative strategy (1,2)

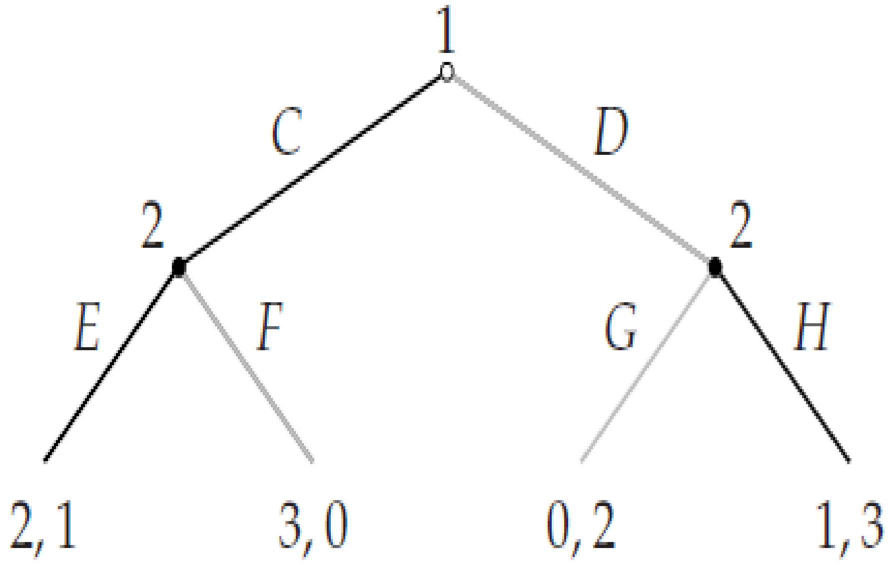
Hence the game solution through BI and SGPE is Coke (Player 1) enters the market, Pepsi (Player 2) opt for accommodative strategy ; and player 1 also respond with accommodative strategy with the pay-off (1,2).





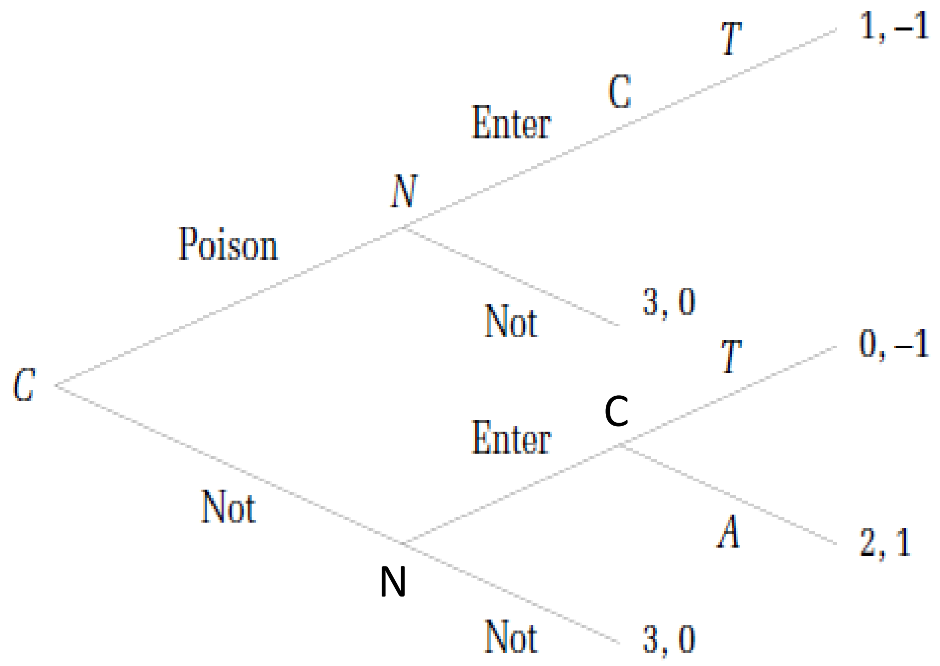
Extensive form games with PI

- Example: Player 1 (C v/s D) and Player 2 (E v/s F; C) and (G v/s H; D).
- Solution: Backward Induction
- Player 2: E >F (1 >0); G<H (2 <3)
- Player 1: CE (2,1) v/s DH (1,3)= (2>1)
- Hence Player 1 opt for CE
- And Game solution is CE(2,1)



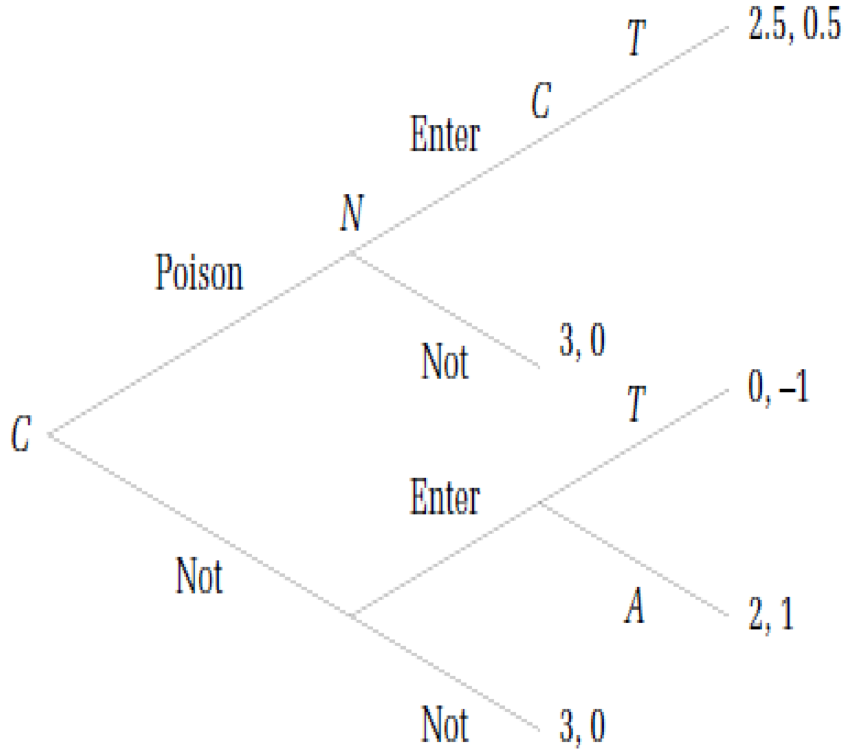
Extensive form games with PI

- Player's = Firms C & N
- Strategy : Player 1- **Poison** (self destruction strategy by the firm to avoid take over by competitor) and **Not** Poison.
- Player 2 (N): **Enter** and **Not** Enter
- Player 1 (C): Tough (**T**) and Accommodative (**A**)
- Solution: C-Poison; Then N-not enter (or not takeover) . Pay-off at (3,0)



Extensive form games with PI

- Solution: C-Poison; Even then N-will enter (attempt takeover); C (player 1) will resist takeover by playing Tough (T) . Pay-off at (2.5,0.5)



Extensive form games with PI

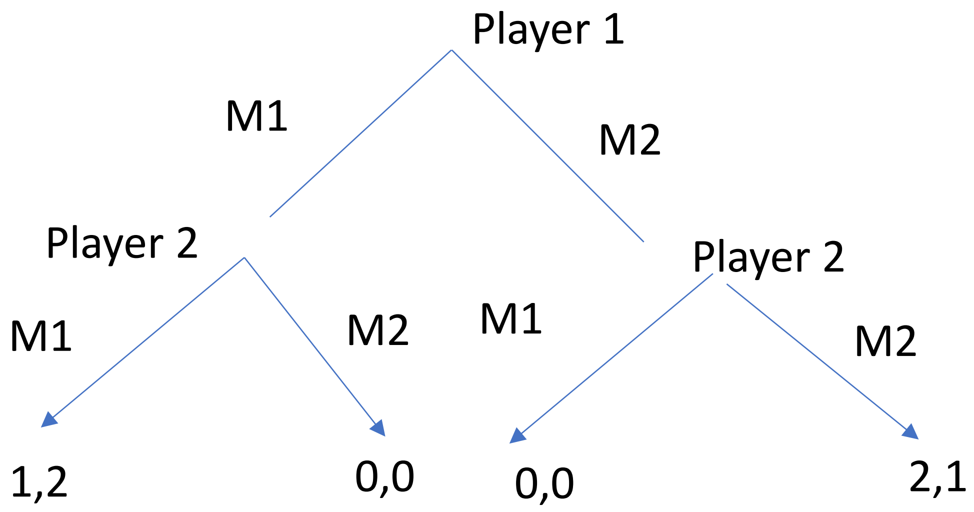
- *Extensive form game-player 1 usually have advantage of taking the first move in the game*

Extensive form games with PI

- BoS game: Wife prefer M2>M1; Husband prefer M1>M2
- Pay-off matrix in simultaneous game

		Player2 (Husband)	
		M1	M2
Player 1 (wife)	M1	1,2	0,0
	M2	0,0	2,1

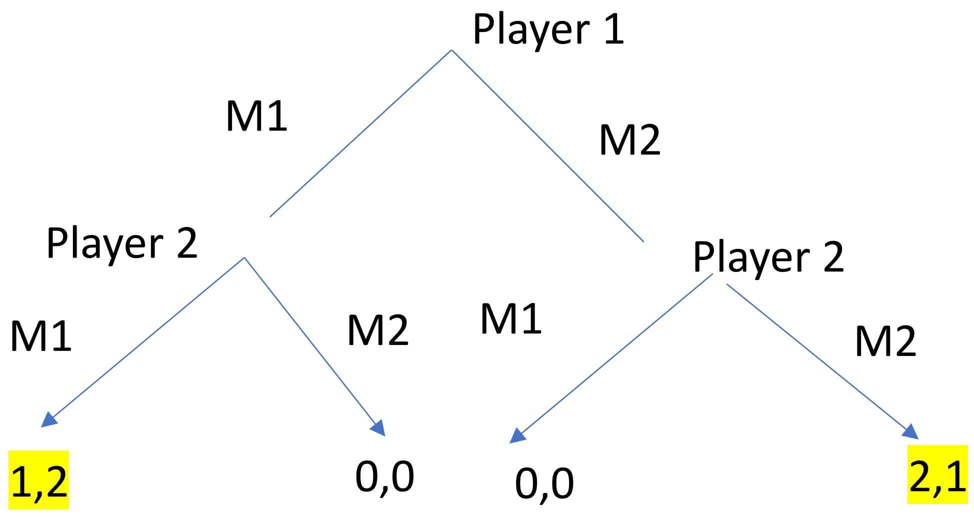
Extensive form games with PI



- Pay-off in sequential/extensive game

Extensive form games with PI

- NE for the game



Extensive form games with PI

- Sub-game perfect equilibrium