CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Search

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Quantum Search

Bernstein-Vazirani algorithm

- Due to Ethan Bernstein and Umesh Vazirani in 1997
- A restricted version of the Deutsch-Jozsa problem
- Tries to learn a string encoded in a function

Problem Statement:

- Input: Quantum oracle U_f implementing $f : \{0,1\}^n \to \{0,1\}$.
- $f(x) = x \cdot s$ for all $x \in \{0,1\}^n$, where $\cdot$ is the bitwise dot product modulo 2.
- Objective: Determine the secret string s.

Quantum Oracle:

$$U_f |x\rangle |y\rangle = |x\rangle |y \oplus f(x)\rangle$$

Classical Solution: Bernstein-Vazirani Problem

Most efficient classical method to find the secret string

- Evaluate the function n times on unit vectors
- Input values:

$$x = 2^i, \forall i \in \{0, 1, \cdots, n-1\}$$

$$\begin{aligned} f(10000\ldots0_n) &= s_1 \\ f(01000\ldots0_n) &= s_2 \\ f(00100\ldots0_n) &= s_3 \\ &\vdots \\ f(00000\ldots1_n) &= s_n \end{aligned}$$

Query Complexity: $O(n)$

Quantum Solution: Bernstein-Vazirani Problem

- Classical solution $\rightarrow n$ queries
- How many queries in quantum solution? An guesses?

- Work out the quantum circuit.

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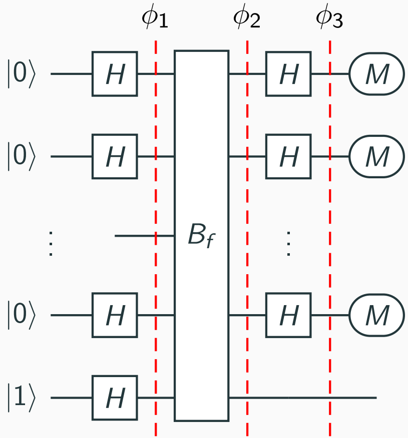
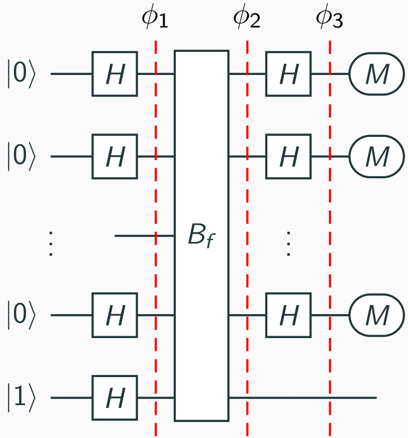


$$\phi_3 : \sum_{y \in \{0,1\}^n} \left(\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{f(x)+x\cdot y} \right) |y\rangle$$



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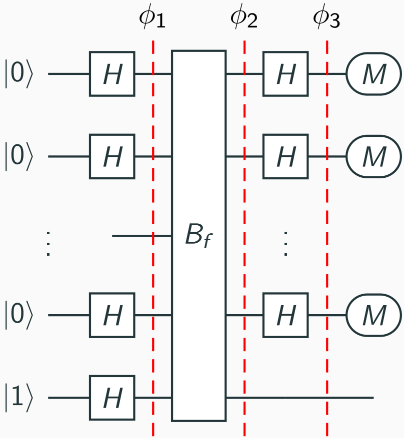

$$\phi_3 : \sum_{y \in \{0,1\}^n} \left(\frac{1}{2^n} \sum_{x \in \{0,1\}^n} (-1)^{x(s \oplus y)} \right) |y\rangle = |s\rangle$$