CS621/CSL611
Quantum Computing For Computer Scientists

Quantum Search

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Quantum Search

General Treatment $(H^{\otimes n})\,|x\rangle$

- Recall $H-$transform of basis states:

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- Further generalization

$$\boxed{H|a\rangle = \frac{1}{\sqrt{2}}\sum_{b\in\{0,1\}}(-1)^{ab}|b\rangle}$$

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$$= \frac{1}{\sqrt{2^2}} \sum_{y \in \{0,1\}^2} (-1)^{x_1 y_1 + x_2 y_2} |y\rangle$$

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- Rewriting $H^{\otimes n}|x\rangle$

$$\boxed{H^{\otimes n}|x\rangle = \frac{1}{\sqrt{2^n}} \sum_{y \in \{0,1\}^n} (-1)^{x \cdot y} |y\rangle}$$

Classwork

- State the expression for $H^{\otimes n} |0\rangle$

References

1. Quantum Computing for Computer Scientists, by Noson S. Yanofsky, Mirco A. Mannucci
2. Quantum Computing Explained, David Mcmahon. John Wiley & Sons
3. Lecture Notes on Quantum Computation, John Watrous, University of Calgary
    - `https://cs.uwaterloo.ca/~watrous/QC-notes/`