

THE ECB PENGUIN

CS 553

CRYPTOGRAPHY

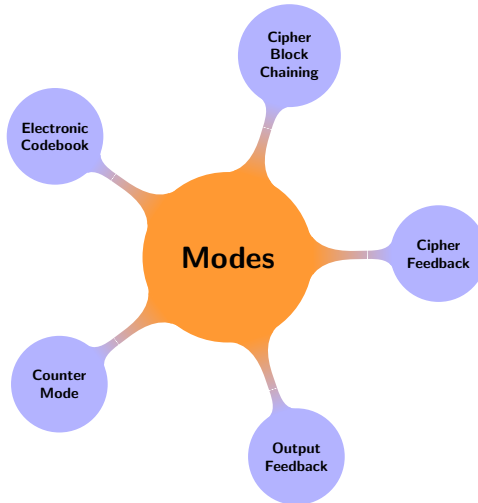
Lecture 15

Modes of Operation

Instructor
Dr. Dhiman Saha

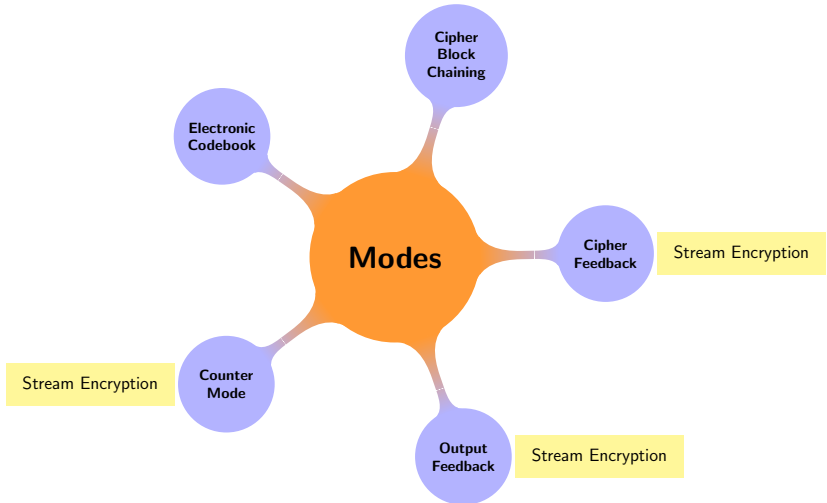
The Domain Extension Algorithm

Handling arbitrary amount of data using a fixed length function



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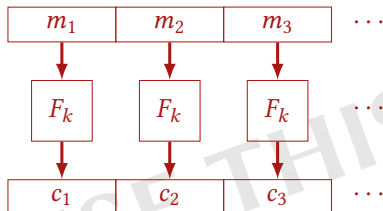


Enc($k, m_1 \cdots m_\ell$):

for $i = 1$ to ℓ :

$c_i := F(k, m_i)$

return $c_1 \cdots c_\ell$

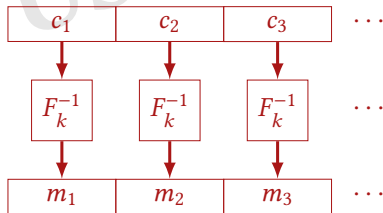


Dec($k, c_1 \cdots c_\ell$):

for $i = 1$ to ℓ :

$m_i := F^{-1}(k, c_i)$


return $m_1 \cdots m_\ell$



- ▶ Same plaintext always encrypts to same ciphertext

Recall

ECB not IND-CPA Secure

Indistinguishability under Chosen Plaintext Attack 

Possible use-cases for ECB

- ▶ Encrypting only single-block messages
- ▶ Using random characters with input blocks
- ▶ Reduces capacity of each block
- ▶ Must be undone during decryption

Advantages

- ▶ Parallelizable Encryption

Random Access Decryption



Transmission errors \implies Incorrect Ciphertext

Bit Flip ($0 \rightarrow 1 \mid 1 \rightarrow 0$)

Bit Drop ($0101 \rightarrow 001$)

Bit Flip \implies Bad plaintext block after decryption

- ▶ Limited to **one** block only
- ▶ Other blocks unaffected

Bit Drop \implies alignment lost

- ▶ All plaintext will be bad after decryption
- ▶ Fixed only if alignment recovered

CBC ($\$ \rightarrow IV$)

Cipher Block Chaining

$\text{Enc}(k, m_1 \cdots m_\ell)$:

$c_0 \leftarrow \{\mathbf{0}, \mathbf{1}\}^{\text{blen}}$:

for $i = 1$ to ℓ :

$c_i := F(k, m_i \oplus c_{i-1})$

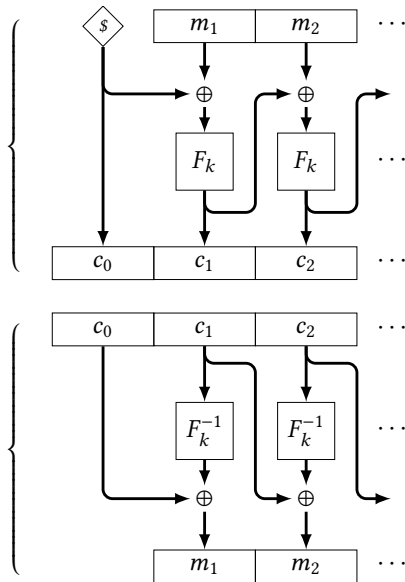
return $c_0 c_1 \cdots c_\ell$

$\text{Dec}(k, c_0 \cdots c_\ell)$:

for $i = 1$ to ℓ :

$m_i := F^{-1}(k, c_i) \oplus c_{i-1}$

return $m_1 \cdots m_\ell$



- ▶ Encryption cannot be parallelized
- ▶ No random access
- ▶ Each ciphertext block depends on all the previous blocks

IV

The Initialization Vector

Random IV \implies **Randomized** encryption \implies IND-CPA

- ▶ Identical plaintexts will encrypt to **distinct** ciphertexts
- ▶ When calling the cipher with **distinct** IVs

Note: IV must be communicated for decryption

- ▶ Encrypting l blocks under CBC mode results in $l + 1$ blocks 

What if constant IV is used?

Common prefix leakage

Assume

 $c_i = c_j$ for some $1 \leq i, j \leq n$ with $i \neq j$

$$c_i = c_j \implies F(k, m_i \oplus c_{i-1}) = F(k, m_j \oplus c_{j-1})$$

$$\implies (m_i \oplus c_{i-1}) = (m_j \oplus c_{j-1})$$

$$\implies (m_i \oplus m_j) = (c_{i-1} \oplus c_{j-1}) \quad \leftarrow \text{Info. leakage} \triangle$$

For b -bit block cipherThe Birthday Paradox¹

$$\Pr[c_i = c_j] \approx 2^{-\frac{b}{2}}$$

► Justifies need for **larger** block size

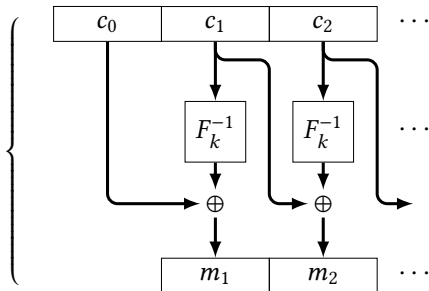
¹Will be detailed in hash function lecture


$\text{Dec}(k, c_0 \cdots c_\ell)$:

for $i = 1$ to ℓ :

$m_i := F^{-1}(k, c_i) \oplus c_{i-1}$

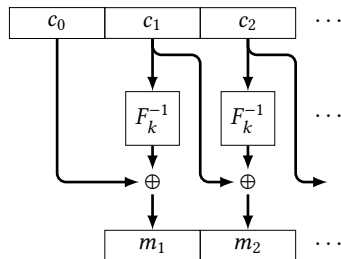
return $m_1 \cdots m_\ell$



- ▶ c_i computation depends on c_{i-1} (Encryption)
- ▶ m_i computation **does not** depends on m_{i-1} (Decryption)
- ▶ It depends on c_i, c_{i-1}
- ▶ Parallel computation **possible** if all previous c_i available (generally true) 

A single-bit transmission error in ciphertext block $c_i \Rightarrow$

- ▶ Whole plaintext block m_i corrupted
- ▶ The same bit in plaintext block m_{i+1} being corrupted



Self-Recovering/Self-Synchronizing Property



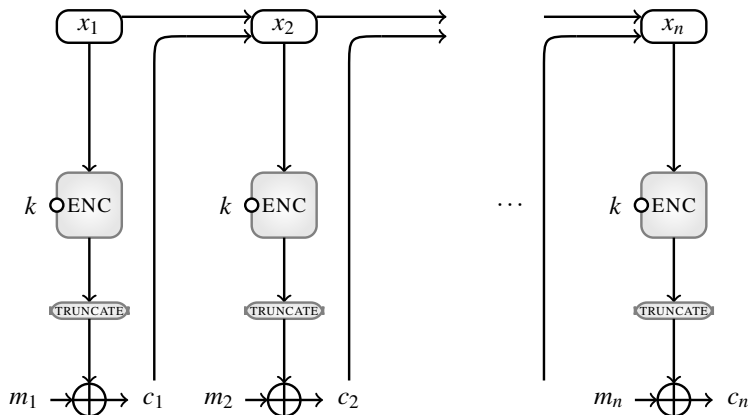
Blocks following m_{i+1} will not be affected

If a bit is added or lost from the cipher-text stream, then all subsequent blocks are garbled.

Stream Encryption

CFB · OFB · CTR

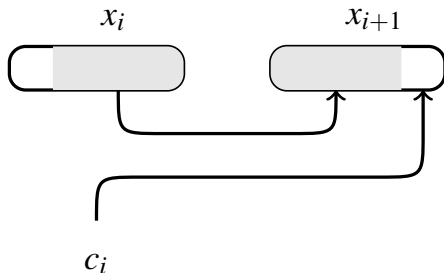
A stream cipher out of a block cipher



- t bits encrypted with each call to the block cipher²

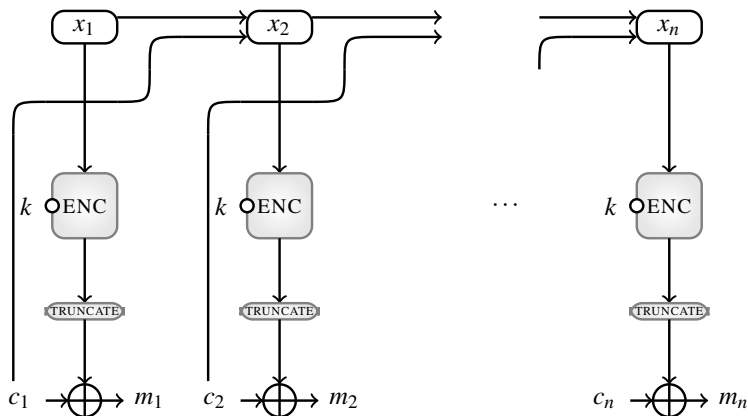
$$c_i = m_i \oplus \text{MSB}_t(\text{ENC}_k(x_i)), \quad \begin{cases} x_1 = \text{chosen IV} \\ x_{i+1} = \text{LSB}_{b-t}(x_i) || c_i \end{cases}$$

²($1 \leq t \leq b$) and ($1 \leq i \leq n$)



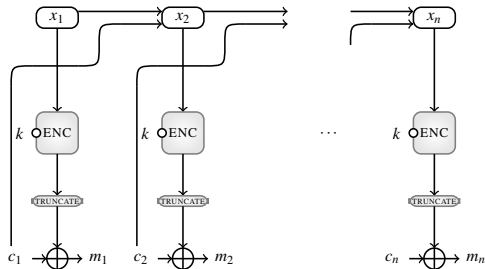
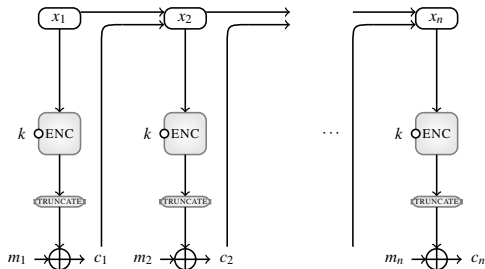
Update of state register from x_i to x_{i+1}

- ▶ First $b - t$ bits of x_i are shifted to the left
- ▶ Then c_i is used to replace the missing bits on the right



- Decryption has a similar form to encryption

$$m_i = c_i \oplus \text{MSB}_t(\text{ENC}_k(x_i)), \quad \begin{cases} x_1 = IV \text{ used in encryption} \\ x_{i+1} = \text{LSB}_{b-t}(x_i) || c_i \end{cases}$$



- Only forward encryption required
- Not parallelizable
- One call per t -bits of ciphertext

CFB Mode

Block Cipher \implies Self-synchronizing stream cipher

- ▶ An error in some CFB-encrypted ciphertext block c_i will be inherited by the corresponding plaintext block m_i that is recovered
- ▶ Since x_{i+1} will contain the incorrect c_i , the recovery of subsequent message blocks will be garbled until the source register x_j for some $j > i$ is free from the influence of c_i
- ▶ This will happen when c_i has been shifted out of the register
- ▶ #plaintext-blocks corrupted by single ciphertext error

$$\leq \left\lceil \frac{b}{t} \right\rceil + 1$$

- ▶ So, provided sufficiently many ciphertext bits are received without error, correct decryption can be recovered.

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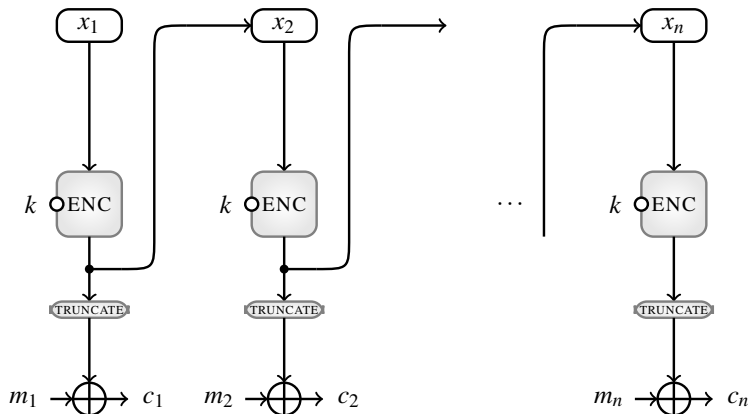
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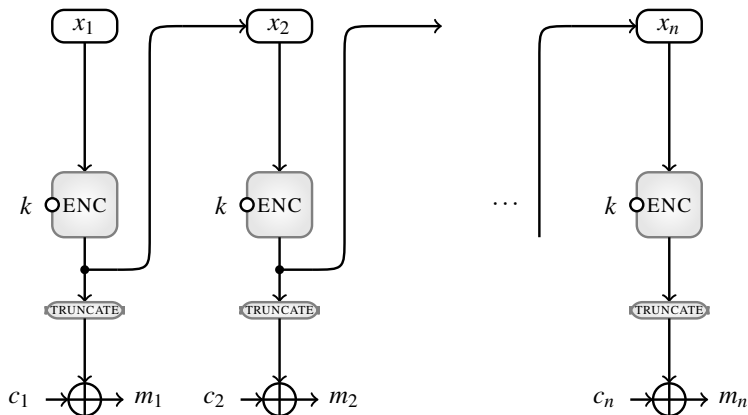
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- t bits encrypted with each call to the block cipher³

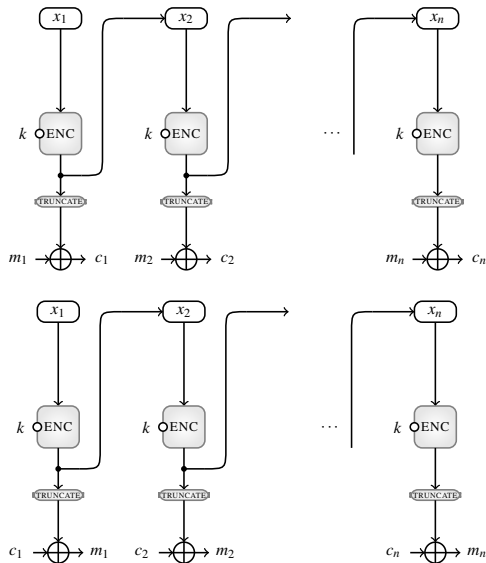
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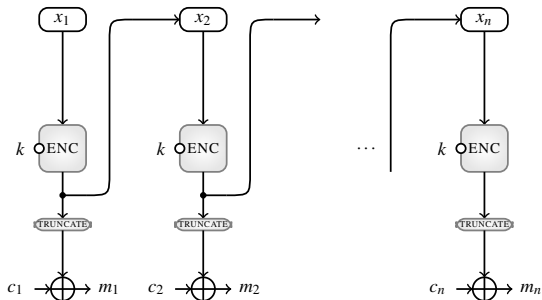
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- Only forward encryption required
- Not parallelizable
- Stream-cipher mode
- But here encryption does not depend on previous ciphertexts.

Note: (key, IV) pair should not be reused



Bit Errors

- ▶ Affect corresponding plaintext
- ▶ But no error propagation

Bit Loss

- ▶ Leads to alignment loss
- ▶ Require external resynchronization

CFB

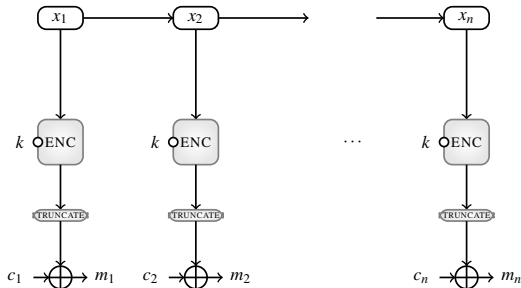
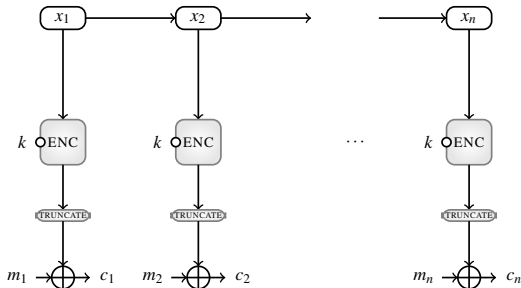
- ▶ Stream Cipher
- ▶ **Self-synchronizing**
- ▶ Key-stream is dependent on message or ciphertext

OFB

- ▶ Stream Cipher
- ▶ **Synchronous**
- ▶ Key-stream is independent of message or ciphertext

CTR (Parallelizable)

Counter Mode




- ▶ Almost similar to OFB w.r.t fault tolerance
- ▶ Also a Synchronous Stream Cipher Mode

Main Difference with OFB

- ▶ CTR supports random access for decryption
- ▶ CTR supports parallel encryption


OFB and CTR

- ▶ Same (Key, IV) \implies Same key-stream!!!
- ▶ IV can be known to some adversary, but used only **once**
- ▶ Also referred to as nonce 
- ▶ Can be a counter

CBC and CFB


- ▶ Require the IV to be unpredictable. Why? (HomeWork)
- ▶ Notion of Pre-IV
- ▶ $IV = ENC_k(\text{Pre-IV})$
- ▶ $IV = ENC_{k'}(\text{Pre-IV})$

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- ▶ Not a concern for OFB and CTR modes
 - ▶ Generate sufficient keystream
 - ▶ Encrypt the message and
 - ▶ Throw away keystream that is not needed
- ▶ For CFB, CBC, and ECB modes we might need to pad some input block to a multiple of s bits in the case of CFB mode and a multiple of b bits in the cases of CBC and ECB.
- ▶ Variety of padding methods have been proposed
- ▶ Most popular 10*
- ▶ Many attacks reported due to inapt padding 
- ▶ Padding Oracle Attack (Home Work)