So, $q_1^* = q_2^* = \frac{\alpha - c}{3}$ $\quad Q = \frac{2(\alpha - c)}{3}$

$$P(q_1 + q_2) = \alpha - Q = \frac{3\alpha - 2\alpha + 2c}{3} = \frac{\alpha + 2c}{3}$$

So, $P = \frac{\alpha + 2c}{3}$

~~profit~~ $NR = q_1 \cdot P = \left(\frac{\alpha - c}{3}\right) \cdot \frac{(\alpha + 2c)}{3} = \frac{\alpha^2 - c\alpha + 2c\alpha - 4c^2}{9}$

$$= \frac{\alpha^2 + c\alpha - 4c^2}{9}$$

Profit $= NR - q_1 c = \frac{\alpha^2 + c\alpha - 4c^2}{9} - \frac{c\alpha - c^2}{3} = \frac{\alpha^2 + c\alpha - 4c^2 - 3c\alpha + 3c^2}{9}$

$$= \frac{\alpha^2 - 2c\alpha - c^2}{9}$$

$c_i(q_i) = cq_i$ where c is the unit cost

BRF

π_i is $\pi_i(q_1, q_2)$ Profit is function of quantity produced by the firms

$\pi_i(q_1, q_2) = q_1 P(Q) - cq_1$

$= q_1(\alpha - Q) - cq_1$

$= q_1(\alpha - q_1 - q_2) - cq_1$

$= q_1(\alpha - c - q_1 - q_2)$

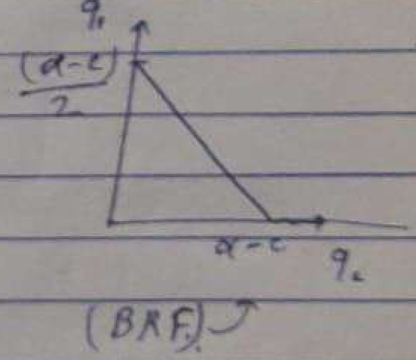
for a given q_2, maximize π_1,

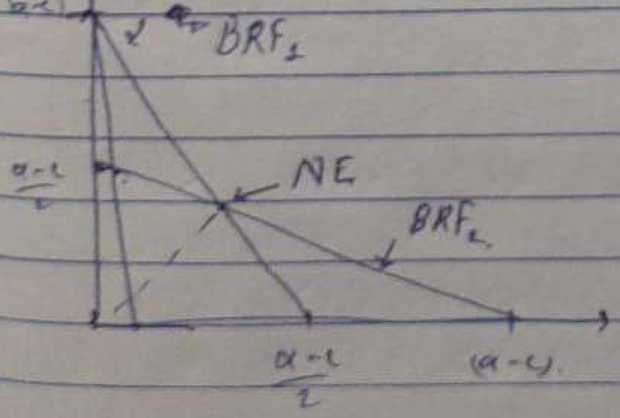
$= \cancel{q_1(\alpha - c - q_1 - q_2)}$

$\frac{d\pi_1}{dq_1} = (\alpha - c - q_1 - q_2) - q_1 = 0$ (local extrima)

$= \alpha - c - q_2 = 2q_1$

$$\boxed{q_1 = \frac{\alpha - c - q_2}{2}}$$





$2q_1^* = \alpha - c - \frac{\alpha - c - q_1}{2}$

$4q_1^* = 2\alpha - 2c - \alpha + c + q_1$

$3q_1 = \alpha - c$

$\boxed{q_1^* = \frac{\alpha - c}{3}}$ $\boxed{q_1^* = q_2^* = \frac{\alpha - c}{3}}$

* Best Response function

- When number of players or action is high, we cannot simply find NE/DSE easily, it wont be feasiable.

- For player 'i' we have set of actions as A_i.
- The best response action is defined as

$$B_i(a_{-i}) \Rightarrow \left[u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \right.$$

- The other player is denoted as '$-i$'.

$$B_i(a_{-i}) = \{ a_i \text{ in } A_i : u_i(a_i, a_{-i}) \geq u_i(a_i', a_{-i}) \; \forall a_i' \text{ in } A_i \}$$

Q.
$$\begin{array}{c} \\ T \\ B \end{array} \begin{array}{c} \begin{array}{ccc} L & C & R \end{array} \\ \begin{bmatrix} (1,1) & (1,0) & (0,1) \\ (1,0) & (0,1) & (1,0) \end{bmatrix} \end{array}$$

$B_i(L) = T/B \quad B_i(C) = T \quad B_i(R) = B.$
$$\begin{bmatrix} T/B \\ T \\ B \end{bmatrix}$$

BRF for P_2 $\Rightarrow$ for $a_{-i} = T$, is $\{L, R\}$
for $a_{-i} = B$. is $\{C\}$.
$$\begin{bmatrix} L/R \\ C \end{bmatrix}$$

- We define Nash Equilibrium using BRF as.

P_1 $BR(a_1^*) = P_2(a_2^*)$
P_2 $BR(a_2^*) = P_1(a_1^*)$.

$$\begin{array}{cc} & \begin{array}{ccc} L & C & R \end{array} \\ \begin{array}{c} T \\ M \\ B \end{array} & \begin{bmatrix} 1,2^* & 2^*,1 & 1^*,0 \\ \boxed{2^*,1^*} & 0,1^* & 0,0 \\ 0,1 & 0,0 & \boxed{1^*,2^*} \end{bmatrix} \end{array}$$

$$\text{BRF of } P_1 \Rightarrow \begin{bmatrix} M \\ T \\ T/B \end{bmatrix} \qquad P_2 \Rightarrow \begin{bmatrix} L \\ L/C \\ R \end{bmatrix}$$

* Duopoly & Oligopoly

$C_i = C_i(q_i)$ (cost is a function of quantity produced)

$P = P(Q)$ Price is a " total quantity in m

$= P(\Sigma q_i)$.

$TR_i = q_i P = q_i P(Q) = q_i P(\Sigma q_i)$

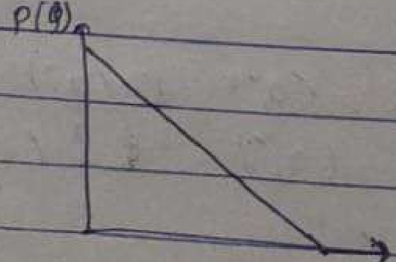
Profit $= TR - C = q_i P(Q) - C_i(q_i)$.

* Cournot Duopoly

$Q = q_1 + q_2$

$$P(Q) = P(q_1 + q_2) = \begin{cases} \alpha - Q & \text{if } Q \le \alpha \\ 0 & \text{if } Q > \alpha \end{cases}$$

• Where α is some max const price.



	WH	FR
Q. WH	3, 3	0, 4.
FR	4, 0	(2, 2).

↑ dominant strategy equilibrium
Nash equilibrium

* For Arms rate,

Q.	S	P
S	$\boxed{2, 1}$	0, 0
P	0, 0	$\boxed{1, 2}$

← Nash equilibrium.

Multiple Nash equilibrium.

Q		
P_1	$\boxed{3, 3}$	0, 2
	$\boxed{2, 0}$	$\boxed{1, 1}$

P_1 switches $1 \to 0$,

P_2 switches ✗

We have two Nash equilibrium.
(A, A) & (R, R).

Q. C_1 \ C_2	AD	NAD
AD	4, 6	1, 8
NAD	3, 4	7, 7

$C_1 \to$ NAD is dominant.
$C_2 \to$ "
Nash eq. is at (7, 7).

• Q Oligopoly

	H	L.
H.	8, 8	3, 10.
L	10, 3	5, 5.

dominant strategy are 'L'
So 5, 5 will be dominant strategy eq.
and thus Nash eq.

P_1 \ P_2	A	B
I	1,1	3,2
II	2,3	0,1.

Nash eq

Nash eq

$P_1 \rightarrow$ No dominant strategy.

$P_2 \rightarrow$. No dominant.

	A	B
I	2,0.	3,2
II	1,1.	0,2

$P_1 \rightarrow$ I is dominant

$P_2 \rightarrow$ B is dominant

(I, B). is dominant strategy eq.

* Iterated elimination of dominant strategis.

- Iteratively eleminate the strategies which are not played by ~~eith~~ the player. i.e. the ~~player~~ strategy is domindted by other strategies.

Q.

	left	center	right
up	0,2	3,1	2,3
middle	1,4	2,1	4,1
down	2,1	4,4	3,2.

up, right middle left.

		P_2	
P_1	a	b	c
i)	10,5	1,1	5,5
ii.	5,10	0,10	1,20

$P_1 \rightarrow$ (i) is strongly dominant
$P_2 \rightarrow$ (c) is weakly "
↳ c is strictly dominant over b.
c is weakly dominant over a

Q* Battle of Couples. → Better to co-operate.

$$\begin{bmatrix} 2,1 & 0,0 \\ 0,0 & 1,2 \end{bmatrix}.$$

→ Four types of games:
- Better to co-operate
- Better to conflict.
- Strictly conflict
- strictly cooperate.

→ Cooperate :- Choose same strategy.

* Nash Equilibrium.
(John Nash).
- A set of strategy is called Nash equilibrium if neither player benifits by switching. It is a stable equilibrium.

- Dominant strategy equilibrium $\Rightarrow$ Nash equilibrium
- Nash equilibrium $\not\Rightarrow$ dominant strategy

- Firm v/s Firm

		F_2	
		High	Low.
F_1	High	1000, 1000	-200, 1200
	Low	1200, -200	600,600.

- Dominant Strategy
  - For F_1, ~~the~~ "Low" is a better choice ~~in~~ regardless of strategy used by F_2.
    - $1200 > 1000$ & $600 > -200$.
  - ~~A Do~~ This is a dominant strategy.
  - A dominant strategy can be
    1) Strictly dominant $a'' > a'$
    2) weakly dominant. $u(a''_i, a_{-i}) \geq u(a'_i, a_{-i})$

Q.

		P_2		
		a	b	c
	i	3,3	1,2	-1,1
P_1	ii	10,5	0,4	1,3.
	iii	15,3	2,5	1,5.

For P_1, iii is weakly dominant
for P_2 there is no dominant strategy

Q.

		P_2	
		a	b
P_1	a	4,1	2,3
	b	2,0	3,2

P_1 - no dominant
P_2 - B is strongly dominant.

* Game of conflict.

		P_2	
		H	T
P_1	H	10,-10	-10,10
	T	-10,10	10,-10

* Conflicts is the only solution available.

* The war defense game for peaceful contries

		C_2	
		R	A
C_1	R	3,2	0,2
	A	2,0	1,1

$U(R,R) >$ others.

$C_1 \to U(R,R) > U(A,R) > U(A,A) > U(R,A)$

* Here is no conflict but no dominant solution.

Q

		P_2		
		a	b	c
	i	-2,1	1,3	1,1
P_1	ii	0,1	0,2	2,0
	iii	1,0	2,1	3,-2

$P_1 \to$ (i)

$P_2 \to$ (b)

Q2

		P_2	
		a	b
	i	20,10	0,10.
P_1	ii	10,7	4,10.
	iii	10,5	7,7

$P_1 \to$ None

$P_2 \to$ (b) is dominant.

* Dominant strategy canbe
1) Weakly dominant :-
For some choices of other player, reward with other strategy might be equal.

* Types of games :-
    1) strategic
    2) Extensive
    3) Collaborative.

* Strategic Game
    1) Rational Behaviour.
    2) Simultaneous Game (i.e. time information is missing).
    3) One time game. (not sequential).

* Extensive Game.
    - Games which are not part of strategic game
    - We use decision tree instead of payoff matrix.

* Prisoner's dilema contd
  If preferences are

$P_1 \rightarrow U[C, NC] > U[NC, NC] > U[C, C] > U[NC, C]$

3 2 1 0

$P_2 \rightarrow U[NC, NC] > U[NC, NC] > U[C, C] > U[C, NC]$

3 2 1 0

		P_2	
		NC	C
P_1	NC	(2, 2)	(0, 3)
	C	(3, 0).	(1, 1).

* History

~~Started by~~ Traces back to economic analysis

- Game theory is a formal way of analysing interaction among a group of rational agents taking decisions strategically

  - Group: More than 1 decision maker.
  - Interaction:- Action of any player at least affects one other player
  - Strategic :- Individual player accounts for interdependece in deciding action to take
  - Rational :- Accounting for interdependance every player takes the best decision.

* In a game,
  1) Your action will affect only you
  2) Interdependance.

Examples:- The card Game.
- Two players, P_1 and P_2.
- Two ~~decks~~/e piles of card A and B.
- Balanced game & Unbalanced game.

* Rules
  1). Any ~~plan~~ number of cards can be picked by player from either deck, but only one deck.
  - Player to pick last card win.

* ~~Game~~ Example Game 3 :- Priseoner's Dielem

P2 ← second element.

	NC	C
NC	(1,1)	(10,0)
C	(0,10)	(5,5)

P1

↑

First element

pay off matrix.

- Application :- Defence budget.

B

	High	low
High	Peace/ high	(win, lose)
low	(lose, win)	Peace (low cost)

A

(0.8, 0.8)	(1, 0)
(0, 1)	(1, 1)

- The outcome of prisoner's dialema is second best out come rather than the best outcome.

* Rational Decision
  - Action is not based on preference
  - Preference
    1) Player knows the order of preferences.
    2). Consistency of preferences i.e. transitivity.
- Pay Off

We ~~off~~ have a value associated with the preferences and is consistent with the preferences.

Date 30/07/24

UA358. # GAME THEORY.

– Dr. Rekha Ravindra.

Quiz – 20 % – cancelled

~~* Introduction :-~~ Midsem – 30 %

Endsem – 50 %.

* Contents
  1) Introduction to Game Theory
  2). Strategic games, Prisoner's Dilemma, Nash Equilibrium, Best response function, dominant strategy, symmetric games and its equilibria
  3). Cournout's model of oligopoly ; Bertrand's model of oligopoly, Electoral competition, Auctions.
  4). Mixed strategy equilibrium
  5). Extensive games, Collision games

• It is a study of stategy which considers one player's actions based on other player's actions.

* Glossary.
  • Player :- Participant of the game.
  • Pay-off :- The reward a player will get if he takes a given action
  • Pay-off-matrix :- Matrix of rewards coorsponding to the given strategy of each players

* Balanced Game
- Best action for P_2 will be to ensure Balanced piles.
- P_2 ~~∞~~ has a winning strategy

eg. $[1,1] \xrightarrow{P_1, A, 1} [0,1] \xrightarrow{P_2, B, 1} [0,0]$ P_2 wins.

$[2,2] \xrightarrow{P_1, A, 2} [0,2] \longrightarrow P_2$ wins.
$\quad \xrightarrow{P_1, A, 1} [1,2] \xrightarrow{P_2, B, 1} [1,1] \rightarrow P_2$ wins.

- What strategy for P_1 to delay the game?

$[3,3] \xrightarrow{P_1, A, 1} [2,3] \xrightarrow{P_2, B, 1} [2,2] \rightarrow$ continue.

* Unbalanced game
  - Here player 1 will make both the piles equal by picking card for bigger pile. This is winning strategy

* Example Game 2 :- The voting game.

Ist :- c_1 v/s c_3 $\qquad$ w v/s. c_2.

P_1 : $c_1 > c_2 > c_3$ $\qquad c_1$
P_2 : $c_3 > c_1 > c_2$ $\qquad c_3$
P_3 : $c_2 > c_1 > c_3$ $\qquad c_1$

$w = c_1$