Problem Set 1, Econ 220a

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1 Logit Demand with Exogenous Prices

Q1

The two most expensive products are Chobani and Dannon. Yoplait and Chobani have the largest market share.

Table 1: City 1, Period 1 Stats

Product	Price	Share	Weight	Sugar Per Gram	Protein Per Gram	Calories Per Gram
Yoplait	0.794	0.166	170	0.076	0.035	0.882
Chobani	1.144	0.164	150	0.000	0.093	0.533
Dannon	1.141	0.128	150	0.000	0.100	0.667
Stonyfield Farm	0.904	0.109	170	0.065	0.029	0.882
Activia	0.492	0.138	113	0.071	0.035	0.796

Q2

$$\text{Market Share} = s_{jct} = \frac{\exp(-\alpha p_{jct} + \beta X_j + \xi_{jct})}{1 + \sum_{k=1}^{J} \exp(-\alpha p_{kct} + \beta X_k + \xi_{kct})}$$

Let mean utility for good j in city c and time t be $\delta_{jct} = -\alpha p_{jct} + \beta X_j + \xi_{jct}$, and normalize the mean utility for the outside option to 0.

Then, using the market share formula above, $s_{0ct} = \frac{\exp(\delta_{0ct})}{\sum_{k=0}^{J} \exp(\delta_{kct})} = \frac{1}{\sum_{k=0}^{J} \exp(\delta_{kct})}$.

$$\frac{s_{jct}}{s_{0ct}} = \frac{\exp(\delta_{jct}) / \sum_{k=0}^{J} \exp(\delta_{kct})}{1 / \sum_{k=0}^{J} \exp(\delta_{kct})} = \exp(\delta_{jct})$$

$$\implies \ln(s_{jct}) - \ln(s_{0ct}) = \delta_{jct}$$

$$\ln(s_{jct}) - \ln(s_{0ct}) = -\alpha p_{jct} + \beta X_j + \xi_{jct}$$

Q3

Table 2: OLS Regression

	Market Share
Constant	-1.203***
	(0.038)
Negative Price	0.800***
	(0.020)
Weight	0.007***
	(0.000)
Sugar Per Gram	6.626***
	(0.354)
Protein Per Gram	8.761***
	(0.349)
Calories Per Gram	-0.933***
	(0.041)
N	39995

Notes: EHW robust standard errors in parentheses

Q4

Table 3: Elasticity Matrix

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	-0.5299	0.1497	0.1170	0.0785	0.0543	0.0000
Chobani	0.1053	-0.7649	0.1170	0.0785	0.0543	0.0000
Dannon	0.1053	0.1497	-0.7955	0.0785	0.0543	0.0000
Stonyfield Farm	0.1053	0.1497	0.1170	-0.6442	0.0543	0.0000
Activia	0.1053	0.1497	0.1170	0.0785	-0.3391	0.0000
Outside Option	0.1053	0.1497	0.1170	0.0785	0.0543	0.0000

Notes: Own and Cross Price Elasticities for City 1 in Year 1. Elasticity of product in row i w.r.t product in col j.

Q5

The share of sales diverted to a product j from another product i is very similar to its diversion ratio from all other products $-i$. This reflects the IIA assumption such that the share of sales diverted to a product is driven entirely by its pre-existing market share. For example, the products with the highest market shares (Yoplait, Chobani) have the highest diversion ratios across all other products. Given that we have fairly natural categories/nests of products across price (Chobani, Dannon vs Activia), sugar content (Chobani, Dannon vs others) and protein content (Chobani, Dannon vs others) it's unlikely that the diversion ratios are uniform as in Table 4.

Table 4: Diversion Matrix

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	NaN	0.1962	0.1537	0.1302	0.1653	0.3546
Chobani	0.1982	NaN	0.1533	0.1299	0.1649	0.3537
Dannon	0.1901	0.1877	NaN	0.1246	0.1582	0.3393
Stonyfield Farm	0.1860	0.1836	0.1438	NaN	0.1547	0.3319
Activia	0.1923	0.1898	0.1487	0.1260	NaN	0.3431
Outside Option	0.2354	0.2324	0.1820	0.1543	0.1959	NaN

Notes: Cell $\{i, j\}$ measures the share of sales of the product in row i that are diverted to the product in col j.

Q6

Firm's Problem:

$$\max_{p_j} \Pi_j = (p_j - mc_j)q_j = (p_j - mc_j)Ms_j$$

Where M is total market size.

FOC:

$$\frac{\partial \Pi_j}{\partial p_j} = M \cdot s_j + (p_j - mc_j) \cdot M \cdot \frac{\partial s_j}{\partial p_j} = 0$$

$$s_j + (p_j - mc_j)\frac{\partial s_j}{\partial p_j} = 0$$

$$s_j + (p_j - mc_j)[-\alpha s_j(1 - s_j)] = 0$$

$$s_j = (p_j - mc_j)[\alpha s_j(1 - s_j)]$$

$$1 = (p_j - mc_j)[\alpha(1 - s_j)]$$

$$p_j = \frac{1}{\alpha(1 - s_j)} + mc_j$$

Q7

$$mc_j = p_j - \frac{1}{\alpha(1-s_j)}$$

All the marginal costs in table 5 are negative.

Table 5: Marginal Costs

Yoplait	Chobani	Dannon	Stonyfield Farm	Activia
-0.7047	-0.3515	-0.2934	-0.4992	-0.9587

Notes: Marginal costs for city 1 in year 1 by product

Q8:

It's plausible that even conditional on observable characteristics, the price of a product is correlated with non-observed dimensions of quality. In particular, it's likely that consumers have higher WTP for products with higher prices conditional on observables because they have superior unobserved quality. I.e., $Cov(\xi_j, p_j|X_j) > 0$.

This implies that the estimated negative correlation between a product's price and it's market share will be weaker than the unbiased α (i.e., α is biased downwards), and therefore, that $\frac{1}{\alpha(1-s_j)}$ is biased upwards. This implies that $mc_j = p_j - \frac{1}{\alpha(1-s_j)}$ is biased downwards. This is likely why we observe negative MCs for all 5 products.

2 Logit Demand Model with three way fixed effects

Q9

After adding the 3 way FE, α is now identified using

1. Changes in market shares for a given product $\times$ city over time as the price changes over time.
2. Differences in market shares across different cities for a given product $\times$ period as the price varies across cities.
3. Differences in market shares across products for a given city $\times$ time period cell as the price varies across products.

This eliminates the following sources of bias:

1. $Cov(\xi_c, p_{jct}|X_{jct)} \neq 0$: Unobservables that vary across cities that affect prices. For example, cities with high average prices have higher market shares of high-price products because of a stronger preference for unobserved quality in higher income cities (which happen to be high price cities also).
2. $Cov(\xi_t, p_{jct}|X_{jct)} \neq 0$: Unobservables that vary across time that affect prices. Periods with high average prices have higher market shares of high-price products, because, for example, demand for unobservable quality and prices both increase in income, and income increases over time.
3. $Cov(\xi_j, p_{jct}|X_{jct)} \neq 0$: Unobservables that vary across products that affect prices. Products with high average prices have higher market shares, because consumers have higher WTP for higher unobserved quality.

Q10

The coefficient on negative price (α) increases slightly from 0.8 to 0.87. This likely reflects the elimination of omitted variable biases referenced in Q9. As a result the own and cross price elasticities all also rise slightly in magnitude compared to Table 3. The diversion ratio table is identical to before, since the diversion ratios only depends on the product shares.

Table 6: 3-Way FE Regression

	3 Way FE
Negative Price	0.869***
	(0.018)
Observations	39995

Notes: EHW Robust standard errors in parentheses. We include FEs for product, city and time.

Table 7: Elasticity Matrix, 3-Way FE Regression

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	-0.5758	0.1626	0.1271	0.0853	0.0590	0.0000
Chobani	0.1144	-0.8311	0.1271	0.0853	0.0590	0.0000
Dannon	0.1144	0.1626	-0.8643	0.0853	0.0590	0.0000
Stonyfield Farm	0.1144	0.1626	0.1271	-0.6999	0.0590	0.0000
Activia	0.1144	0.1626	0.1271	0.0853	-0.3684	0.0000
Outside Option	0.1144	0.1626	0.1271	0.0853	0.0590	0.0000

Notes: Own and Cross Price Elasticities for City 1 in Year 1. Elasticity of product in row i w.r.t product in col j.

Q11

All the marginal costs are still below 0. Residual omitted variable bias remains since we still fail to control for unobserved shocks at the product $\times$ city $\times$ period level. Price shocks at the product $\times$ city $\times$ time period level may still be correlated with unobserved demand for quality. For example, a successful local marketing campaign run by product j in city c and period t (e.g., at a sports event) may increase both consumer WTP and price for that product in that city, therefore confounding the price elasticity estimate.

Table 8: Marginal Costs, 3-Way FE Regression

Yoplait	Chobani	Dannon	Stonyfield Farm	Activia
-0.5852	-0.2324	-0.1791	-0.3875	-0.8432

Notes: Marginal costs for city 1 in year 1 by product

3 Logit Demand Model using cost shifters

Q12

The models aren't equivalent. The observable product traits that we see are only a subset of all the features of a product × city × period cell that affect consumer demand. Conditioning on only X_j partials out only a subset of the variation that the fixed effects do. As a result, it's possible that our distance × diesel price instrument is orthogonal to (or simply less correlated with) the error term ξ_{jct} conditional on the fixed effects, but not if we only condition on X_j. Therefore, the FE model is likely to provide less biased estimates of the price-elasticity.

If we control only for X_j, the exclusion restriction could fail if distance to distribution center is negatively correlated with city mean income, and city mean income increases both WTP for quality and price. Even if we include the FEs, the exclusion restriction could fail if changes in distance to distribution center are correlated with changes to taste for quality. For example, a growing city could have both new distribution centers popping up near them as well as an increasing taste for fancy yoghurt as upper class liberals move in.

Q13

The negative price coefficent, and the own and cross-price elasticities are all larger (in absolute value) than before. This reflects the reduced bias from using the instrument. Specifically, for the price variation we use to estimate the elasticities, price has a weaker correlation with unobserved quality. As before, the diversion ratios are unchanged.

Table 9: 3 Way FE + IV Regression

	3 Way FE + IV
Negative Price	3.434***
	(0.051)
Observations	39995

Notes: EHW Robust standard errors in parentheses. We include FEs for product, city and time. Negative price is instrumented for using distance to closest distribution center × diesel prices.

Table 10: Elasticity Matrix, 3-Way FE + IV Regression

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	-2.2757	0.6428	0.5023	0.3372	0.2330	0.0000
Chobani	0.4522	-3.2848	0.5023	0.3372	0.2330	0.0000
Dannon	0.4522	0.6428	-3.4160	0.3372	0.2330	0.0000
Stonyfield Farm	0.4522	0.6428	0.5023	-2.7663	0.2330	0.0000
Activia	0.4522	0.6428	0.5023	0.3372	-1.4562	0.0000
Outside Option	0.4522	0.6428	0.5023	0.3372	0.2330	0.0000

Notes: Own and Cross Price Elasticities for City 1 in Year 1. Elasticity of product in row i w.r.t product in col j.

Q14

All MC now > 0 for city 1 in period 1.

Table 11: Marginal Costs, 3-Way FE + IV Regression

Yoplait	Chobani	Dannon	Stonyfield Farm	Activia
0.4453	0.7956	0.8070	0.5770	0.1541

Notes: Marginal costs for city 1 in year 1 by product

Q15

Algorithm:

1. Initialize a random vector of prices: I started with p = [1, 1, 1, 1, 1].

2. Using α, FEs and taste shock estimated in Table 9, calculate Mean Utility at the product $\times$ city $\times$ period level:

$$u_{jct} = -\alpha p_{jct} + \gamma_j + \eta_c + \beta_t + \xi_{jct}$$

3. Calculate market shares given these mean utilities as

$$s_{jct} = \frac{exp(u_{jct})}{\sum_i exp(u_{jct}) + 1}$$

4. Calculate the firm's best response price given these market shares and marginal costs as

$$p_{jct} = mc_{jct} + \frac{1}{(1 - s_{jct}) * \alpha}$$

5. Repeat steps 1-4 until prices converge. I chose $\epsilon = 10^{-5}$ as the convergence condition.

$$\sum_j abs(p_{jct}^{i+1} - p_{jct}^{i}) < \epsilon$$

Table 12 calculates equilibrium prices and market shares using the algorithm above. Equilibrium prices and shares converge to be identical to observed prices and shares.

Table 12: Observed vs Equilibrium Prices, Market Shares

Product	Observed Price	Equilibrium Price	Observed Share	Equilibrium Share
Yoplait	0.7944	0.7944	0.1658	0.1658
Chobani	1.1437	1.1437	0.1637	0.1637
Dannon	1.1410	1.1410	0.1282	0.1282
Stonyfield Farm	0.9037	0.9037	0.1086	0.1086
Activia	0.4919	0.4919	0.1379	0.1379

Notes: All prices and shares are for city 1 in period 1. Equilibrium prices and shares are calculated using the algorithm above.

Q16

After the merger, the merged firm takes into account the effect of the price of Chobani and Dannon market share and vice versa when setting prices. The new optimal prices for the two merged products owned by the merged firm are:

$$p_c = mc_c + \frac{1}{\alpha(1-s_c)} + \frac{(p_d - mc_d)s_d}{1-s_c}$$

$$p_d = mc_d + \frac{1}{\alpha(1-s_d)} + \frac{(p_c - mc_c)s_c}{1-s_d}$$

The second term takes into account the effect of changing the price on one product on the profits of the other.

I re-run the pricing algorithm from **Q15** with this new pricing behavior for the merged firm while holding unmerged firms' optimal pricing function the same as before. While prices rise for all firms, the increases are largest for the merged products (3.9% and 5.1% for the Chobani and Dannon respectively), and only 0.2-0.4% for the unmerged products.

Table 13: Prices pre vs post merger

Product	Pre-Merger Price	Post-Merger Price	Percent Change
Yoplait	0.7944	0.7973	0.37%
Chobani	1.1437	1.1881	3.88%
Dannon	1.1410	1.1996	5.13%
Stonyfield Farm	0.9037	0.9056	0.20%
Activia	0.4919	0.4943	0.49%

Notes: All prices and shares are for city 1 in period 1. Equilibrium prices are calculated using the algorithm from Q15.

Q17

Given the extreme value distribution, mean welfare can be calculated as

$$E(U) = ln(\sum_{j=0}^{5} exp(u_{jct}))$$

Since α represents marginal utility per dollar, $\frac{E(U)}{\alpha}$ gives us mean welfare in $ units. Using this formula with pre and post merger prices, we get that the merge decreases average welfare by $0.014. Given initial expenditure of $0.63, this reflect a 2.4% decrease in average welfare measured in $ terms.

4 Nested Logit Demand Model

Q18

I agree with the intuition re cross-price elasticities. In addition to having the same sugar content, the two products have very similar protein and calories per gram (Table 1), which makes them likely to be closer substitutes to each other than to other goods. This could be because different types of (e.g., more health conscious) consumers purchase Chobani/Dannon vs other products, implying that a change in the price of Chobani/Dannon (holding the other constant) would disproportionately cause switching by the health-conscious consumers within the healthy product nest rather than to/from the less healthy products.

I disagree with the intuition on the welfare effects of the merger. Given that the merger is happening within a nest, the degree of substitutability between merging firms is higher than if it was happening between two arbitrary firms in a non-nested setup. A merger would give the new merged firm a much larger increase in pricing power relative to baseline compared to the world where there were no nests and the merged firms' products had relatively weak substitutability.

Q19

The market share for a good j in nest g is given by:

$$s_{jct} = s_{jct|g} \cdot s_{gct}, \text{ where} \tag{1}$$

$$s_{gct} = \frac{(\sum_{k\in g} \exp(\delta_{kct}/(1-\rho)))^{1-\rho}}{\sum_{m=0}^{G}(\sum_{k\in m} \exp(\delta_{kct}/(1-\rho)))^{1-\rho}} \tag{2}$$

$$s_{jct|g} = \frac{\exp(\delta_{jct}/(1-\rho))}{\sum_{k\in g} \exp(\delta_{kct}/(1-\rho))} \tag{3}$$

$$\text{Using (3), } \ln(s_{jct|g}) = ln(exp(\frac{\delta_{jct}}{1-\rho})) - \ln(\sum_{k\in g} \exp(\frac{\delta_{kct}}{1-\rho}))$$

$$\rightarrow \ln(s_{jct|g}) = \frac{\delta_{jct}}{1-\rho} - \ln(\sum_{k\in g} \exp(\frac{\delta_{kct}}{1-\rho}))$$

$$\delta_{jct} = (1-\rho)\ln(s_{jct|g}) + (1-\rho)\ln(\sum_{k\in g} \exp(\frac{\delta_{kct}}{1-\rho})) \tag{4}$$

$$\text{Now, using (2), } \frac{s_{gct}}{s_{0ct}} = (\sum_{k\in g} \exp(\frac{\delta_{kct}}{1-\rho}))^{1-\rho}$$

$$ln(s_{gct}) - ln(s_{0ct}) = (1-\rho)ln(\sum_{k\in g} \exp(\frac{\delta_{kct}}{1-\rho}))^{1-\rho} \tag{5}$$

Substituting (5) into (4), we get $\delta_{jct} = (1-\rho)\ln(s_{jct|g}) + ln(s_{gct}) - ln(s_{0ct})$

$$\text{Note, } \delta_{jct} = -\alpha p_{jct} + \gamma_j + \eta_c + \beta_t + \xi_{jct} :$$

$$\rightarrow -\alpha p_{jct} + \gamma_j + \eta_c + \beta_t + \xi_{jct} = (1-\rho)\ln(s_{jct|g}) + \ln(s_{gct}) - \ln(s_{0ct})$$

Substituting in (1), we get $-\alpha p_{jct}+\gamma_j+\eta_c+\beta_t+\xi_{jct}+\rho ln(s_{jct|g}) = \ln(s_{jct})-ln(s_{gct})+\ln(s_{gct})-\ln(s_{0ct})$

$$\ln(s_{jct}) - \ln(s_{0ct}) = -\alpha p_{jct} + \rho\ln(s_{jct|g}) + \gamma_j + \eta_c + \beta_t + \xi_{jct}$$

Q20

In addition to being worried about the correlation of price with unobserved measures of quality, it's also plausible that unobserved quality variables are correlated with both a product's share in a nest as well as its total share. Therefore, we instrument for Share in Nest using the number of products in the nest. The exclusion restriction implies that the number of products in a nest contains no information about the unobserved quality of the products in the nest (and therefore its aggregate share). This may be violated if, for example, nests that contain many products do so because low average quality of existing products prompted new entry.

Table 14: Nested Logit Regression, Including IV & 3 Way FEs

	Nested Logit + IV
Negative Price	2.133***
	(0.017)
Log Share in Nest	0.520***
	(0.005)
Observations	39995

Notes: EHW Robust standard errors in parentheses. We include FEs for product, city and time. Negative price and Log Share in Nest are instrumented for using distance to closest distribution center × diesel prices and # products in nest.

Q21

The own-price elasticities have roughly the same magnitude as before. However, the cross-price elasticities between goods within a nest have *increased* substantially, while the cross-price elasticities between goods in different nests has *decreased* substantially. The large changes to cross-price elasticities are a reflection of the relatively large within-nest correlation of utility ($\rho \sim= 0.52$) we estimate in Table 15. In line with the elasticities, the diversion matrix shows that diversion between goods in the same next increases substantially compared to Table 4, while diversion between goods in different nests decreases.

Table 15: Elasticity Matrix, Nested Logit + 3-Way FE + IV Regression

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	-2.511	0.399	0.312	0.759	0.525	0.000
Chobani	0.281	-3.201	1.469	0.209	0.145	0.000
Dannon	0.281	1.880	-3.600	0.209	0.145	0.000
Stonyfield Farm	1.018	0.399	0.312	-3.255	0.525	0.000
Activia	1.018	0.399	0.312	0.759	-1.661	0.000
Outside Option	0.281	0.399	0.312	0.209	0.145	0.000

Notes: Own and Cross Price Elasticities for City 1 in Year 1. Elasticity of product in row i w.r.t product in col j.

Table 16: Diversion Matrix, Nested Logit + 3-Way FE + IV Regression

	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia	Outside Option
Yoplait	NaN	0.1264	0.1121	0.3559	0.3797	0.1131
Chobani	0.1105	NaN	0.5211	0.0969	0.1034	0.1116
Dannon	0.0865	0.4601	NaN	0.0759	0.0810	0.0874
Stonyfield Farm	0.2658	0.0828	0.0735	NaN	0.2489	0.0741
Activia	0.3375	0.1052	0.0933	0.2961	NaN	0.0941
Outside Option	0.1997	0.2255	0.2001	0.1752	0.1870	NaN

Notes: Cell $\{i, j\}$ measures the share of sales of the product in row i that are diverted to the product in col j.

Q22

Yes, all the costs are > 0.

Table 17: Marginal Costs, Nested Logit + 3-Way FE + IV Regression

Nested MC	Yoplait	Chobani	Dannon	Stonyfield Farm	Activia
	0.478	0.786	0.824	0.626	0.196

Notes: Marginal costs for city 1 in year 1 by product

Q23

Table 18: Prices pre vs post merger, Including Nested Logit Model

Product	Pre-Merger	Post-Merger (Simple)	Post-Merger (Nested)	Change (Nested)
Yoplait	0.794	0.797	0.797	+0.002
Chobani	1.144	1.188	1.370	+0.226
Dannon	1.141	1.200	1.408	+0.267
Stonyfield Farm	0.904	0.906	0.920	+0.016
Activia	0.492	0.494	0.502	+0.010

Notes: All prices are for city 1 in period 1. Equilibrium prices are calculated using the algorithm from Q15, with the adjusted optimal price formulae below.

The new price setting formulae for the merged and unmerged products are now as follows:

$$\textbf{Merged} \ : p_1 = mc_1 + \frac{1-\rho+(p_2-mc_2)\alpha(\rho s_{2|g}+(1-\rho)s_2)}{\alpha\left(1-\rho s_{1|g}-(1-\rho)s_1\right)}, \text{ where 1 and 2 are the to products being me}$$

$$\textbf{Unmerged} \ : p_j = mc_j + \frac{1-\rho}{\alpha\left(1-\rho s_{j|g}-(1-\rho)s_j\right)}$$

We combine these formulae with the algorithm described in **Q15** to calculate equilibrium prices after the merger.

Compared to the simple logit, the nested logit produces substantially larger price increases (23-27%) for the merged products. For the non-merged firms, there is a small increase in price relative to pre-merger (0.2-1.6%).

Q24

There is now a mean decrease in welfare of \$0.064 compared to initial mean expenditure of \$0.67. I.e., using the nested model, mean welfare decreases by $\sim$ 10.6% compared to a decrease of just 2.4% with the non-nested model.