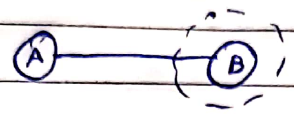
HW3

Q1 1.1 1st step -

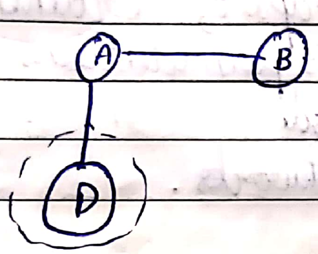
no prediction o/ps ∵ no pre-existing nodes.

2nd step -



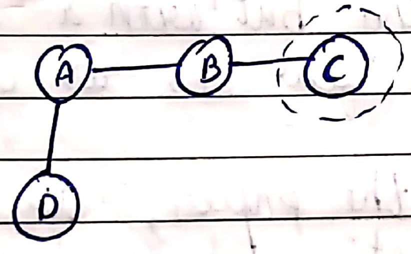
prediction o/p - $S^{\pi}_{B,A} = 1$

3rd step -



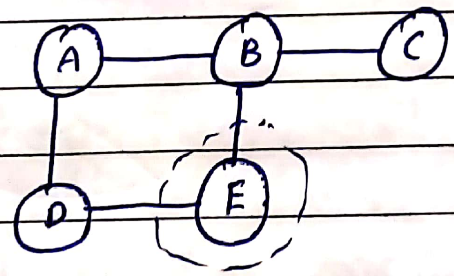
prediction o/p - $S^{\pi}_{D,A} = 1$
& $S^{\pi}_{D,B} = 0$

4th step -



prediction o/p - $S^{\pi}_{C,B} = 1$
$S^{\pi}_{C,A} = 0$
$S^{\pi}_{C,D} = 0$

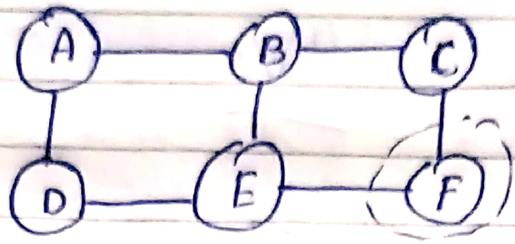
5th step -



prediction o/ps -

$S^{\pi}_{E,B} = 1$, $S^{\pi}_{E,D} = 1$
$S^{\pi}_{E,A} = 0$, $S^{\pi}_{E,C} = 0$

6^{th} step -



prediction o/p - $S^{\pi}_{F,C}=1$, $S^{\pi}_{F,E}=1$

$S^{\pi}_{F,B}=0$, $S^{\pi}_{F,D}=0$

$S^{\pi}_{F,A}=0$

1.2 (i) from the paper, training is only on possible BFS permutations not on all ~~nodes~~. possible node " , huge red^n in time for real networks.

(ii) for very sparse graphs, the algorithm can learn to predict 0 for all edges if we do random (usually) sampling but with a BFS ordering we def. have a few 1s as o/p.

(iii) from the paper, we need to make less no. of predictions for each node added in a BFS manner instead of a random order.

Q2 Q.1 given, graph A is a subgraph of graph B → ① A
" B is a " " " C. → ②

from ①,

~~∃ bijection f s.t. su~~

∃ bijection $f: V_A \to V_B$ s.t. the subgraph of B induced by this set is graph isomorphic to A.
↳ subset of nodes(B) (V_B)

from ②,

we get a similar bijection $g: V_B \to V_C$.

now suppose any subset V_S ($\subseteq V_B$) of V_B, suppose that the subgraph of B induced by $\{v \mid v \in V_S\}$ is not graph isomorphic to the subgraph of C induced by $\{g(v) \mid v \in V_S\}$.

$\Rightarrow \exists\ (u_s, v_s) \mid u_s, v_s \in V_s$ s.t. $(u_s, v_s) \notin E_C$ but $(u_s, v_s) \in E_B$
edge
OR
$(u_s, v_s) \notin E_B$ but $(u_s, v_s) \in E_C$,

but this is a contradiction ∵ the subgraph of C induced by $\{g(v) \mid v \in V_B\}$ is graph isomorphic to B. ∴ such an edge cannot exist AND so V_s does not exist ⇒ for all subgraphs of B are also subgraphs of C of corresponding nodes given by $g(\cdot)$.

DATE / /

~~∴ the biject~~

~~⊂ the subset of V_B $\{f(v) \mid v \in A\}$~~ $\exists A!$

∴ the subgraph of B induced by $\{f(v) \mid v \in A\}$ is graph isomorphic to the subgraph of C induced by $\{g(f(v)) \mid v \in V_A\}$.

⇒ A is a subgraph of C.

(∵ we have found a bijection $g(f(\cdot)) : V_A \to V_C$ s.t. the subgraph of C induced by $\{g(f(v)) \mid v \in V_A\}$ is graph isomorphic to A.)

2.2 given, A is a subgraph of B, &
B " " " " A.

∴ from the def^n given, $\exists$ $f : V_A \to V_B$ & $g : V_B \to V_A$

s.t. f & g are bijective.

clearly $|V_A| = |V_B|$ ∵ if this was not so, we would not have either f or g depending on if $|V_A| > |V_B|$ or $|V_B| > |V_A|$ correspondingly.

and now, again by the def^n, the set $\{f(v) \mid v \in V_A\}$ ~~comprises of all nodes~~ of nodes & the graph induced by these nodes in B is graph isomorphic to A BUT the give set $\{f(v) \mid v \in V_A\} = V_B$ ∵ $|V_A| = |V_B|$ & f is bijective ⇒ A & B are graph isomorphic.

23 first, "$\longrightarrow$" dir^n,

graph X is a common subgraph of A & B if
$Z_X \preccurlyeq \min\{Z_A, Z_B\}$

note that $\forall i$ s.t. $Z_A[i] \leq Z_B[i]$
$Z_X[i] \leq Z_A[i] \leq Z_B[i]$
& $\forall i$ s.t. $Z_A[i] \geq Z_B[i]$
$Z_X[i] \leq Z_B[i] \leq Z_A[i]$

$\Rightarrow Z_X[i] \leq Z_A[i] \quad \forall i$
& $Z_X[i] \leq Z_B[i] \quad \forall i$

$\Rightarrow Z_X \preccurlyeq Z_A$ & $Z_X \preccurlyeq Z_B$ (by def^n of the space)

$\Rightarrow$ graph X is a common subgraph of A & B.

second, "$\longleftarrow$" dir^n,

$Z_X \preccurlyeq \min\{Z_A, Z_B\}$ if X is a common subgraph of A & B.

it is the exact opposite sequence of reasoning from above, that will prove this claim.

2.4 with the given info.,

$$Z_A[1] < Z_B[1] < Z_C[1]$$

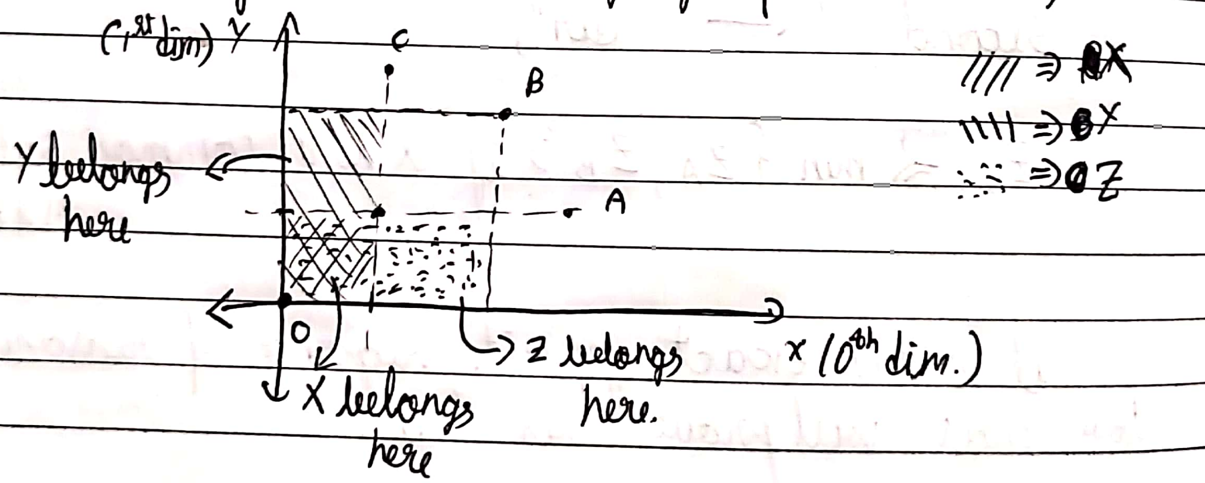
~~∵ if we~~ ∵ if $Z_B[1] \leq Z_A[1]$ then B ~~will be~~ must be a subgraph of A ∵ $Z_B \lesssim Z_A$ & by def" of the space, but it is given that B is not a subgraph of A.

similar reasoning for other inequalities.

2.5 from given info. + clarifications,

$$Z_A[1] < Z_B[1] < Z_C[1]$$
$$Z_A[0] > Z_B[0] > Z_C[0],$$

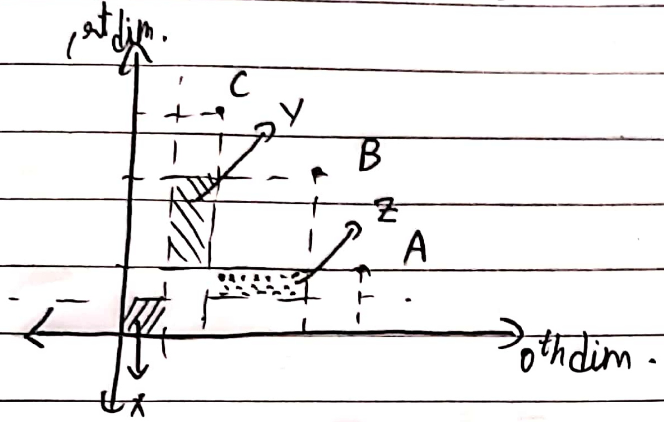
say X is a subgraph of all 3 A, B & C



say Y is a common subgraph of B & C
& Z " " " " of A & B

there are not enough to guarantee ⇒ we impose more restrictions -

X is a subgraph of A, B & C
Y " " " " B & C but <u>not</u> A
Z " " " " A & B " " C.

⇒ 

clearly, $Z_X \precsim Z_Z$ & $Z_X \precsim Z_Y$.

explicitly, set,

$$\left.\begin{aligned} Z_X[0] &\le Z_C[0]/2 \\ Z_X[1] &\le Z_A[1]/2 \end{aligned}\right\} \text{graph } X.$$

$$\left.\begin{aligned} & Z_C[0]/2 < Z_Y[0] \le Z_C[0] \\ \& \quad & Z_Y[1] \le Z_B[1] \\ \& \quad & Z_Y[1] > Z_A[1] \end{aligned}\right\} \text{graph } Y$$

$$\left.\begin{aligned} & Z_Z[0] \le Z_B[0] \\ \& \quad & Z_Z[0] > Z_C[0] \\ \& \quad & Z_A[1]/2 < Z_Z[1] \le Z_A[1] \end{aligned}\right\} \text{graph } Z.$$

∴ X is a common subgraph of Y & Z under this setting.