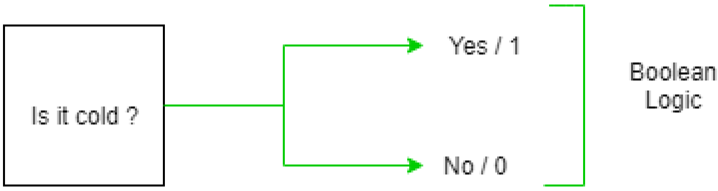
CL3 Assignment No. 3

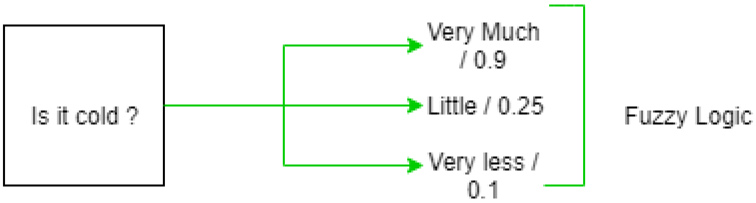
Title: Implement Union, Intersection, Complement and Difference operations on fuzzy sets. Also create fuzzy relations by Cartesian product of any two fuzzy sets and perform max-min composition on any two fuzzy relations.

Theory:

What is a Fuzzy Set?

Fuzzy refers to something that is unclear or vague. Hence, Fuzzy Set is a Set where every key is associated with value, which is between 0 to 1 based on the certainty. This value is often called as degree of membership. Fuzzy Set is denoted with a Tilde Sign on top of the normal Set notation.





Operations on Fuzzy Set:

1. Union:

Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Union of them, then for every member of A and B, Y will be:

degree_of_membership(Y)= max(degree_of_membership(A), degree_of_membership(B))

Example:
The First Fuzzy Set is : {'a': 0.2, 'b': 0.3, 'c': 0.6, 'd': 0.6}
The Second Fuzzy Set is : {'a': 0.9, 'b': 0.9, 'c': 0.4, 'd': 0.5}
Fuzzy Set Union is : {'a': 0.9, 'b': 0.9, 'c': 0.6, 'd': 0.6}

2. Intersection:

Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Intersection of them, then for every member of A and B, Y will be:

degree_of_membership(Y)= min(degree_of_membership(A), degree_of_membership(B))

Example:

The First Fuzzy Set is : {'a': 0.2, 'b': 0.3, 'c': 0.6, 'd': 0.6}
The Second Fuzzy Set is : {'a': 0.9, 'b': 0.9, 'c': 0.4, 'd': 0.5}
Fuzzy Set Intersection is : {'a': 0.2, 'b': 0.3, 'c': 0.4, 'd': 0.5}

3. Complement :

Consider a Fuzzy Sets denoted by A , then let's consider Y be the Complement of it, then for every member of A , Y will be:

degree_of_membership(Y)= 1 - degree_of_membership(A)

Example:
The Fuzzy Set is : {'a': 0.2, 'b': 0.3, 'c': 0.6, 'd': 0.6}
Fuzzy Set Complement is : {'a': 0.8, 'b': 0.7, 'c': 0.4, 'd': 0.4}

4. Difference :
Consider 2 Fuzzy Sets denoted by A and B, then let's consider Y be the Intersection of them, then for every member of A and B, Y will be:
degree_of_membership(Y)= min(degree_of_membership(A), 1- degree_of_membership(B))

Example:
The First Fuzzy Set is : {"a": 0.2, "b": 0.3, "c": 0.6, "d": 0.6}
The Second Fuzzy Set is : {"a": 0.9, "b": 0.9, "c": 0.4, "d": 0.5}
Fuzzy Set Difference is : {"a": 0.1, "b": 0.1, "c": 0.6, "d": 0.5}

5. **Cartesian Product:**
Let us consider two fuzzy sets A(x) and B(y) defined on the Universal sets X and Y, respectively.The Cartesian product of fuzzy sets A(x) and B(y), is denoted by A(x) X B(x), such that $x \in X$, $y \in Y$. It is determined, so that the following conditions satisfy
μ (AXB)(x,y)= min{μA(x),μB (y)}
Example:
The First Fuzzy Set is A: {"x1": 0.3, "x2": 0.7, "x3": 1} The
Second Fuzzy Set is B: {"y1": 0.4, "y2": 0.9}
R= A * B
R=

0.3	0.3
0.4	0.7
0.4	0.9

6. Max Min Composition:

$$\underline{T} = \underline{R} \circ \underline{S} = \mu_{\bar{T}}(x,z) = \bigvee_{y \in Y}(\mu_{\bar{R}}(x,y) \wedge \mu_{\bar{S}}(y,z))$$

$$= max_{y \in Y}\{min(\mu_{\bar{R}}(x,y), \mu_{\bar{S}}(y,z))\}$$

For example:

R=

0.6	0.5
0.1	1
0	0.7

S=

0.7	0.3	0.4
0.9	0.1	0.6

T = R * S =

0.6	0.3	0.5
0.9	0.1	0.6
0.7	0.1	0.6

Conclusion:

This assignment showcases essential operations on such as fuzzy sets union, intersection, complement, difference, and fuzzy relations such as Cartesian product, max-min composition. These operations are fundamental in fuzzy logic and set theory, enabling flexible handling of uncertainty and imprecise information in various fields like decision-making and pattern recognition.