

Problem 3

Training data :

Sample	X_1	X_2	X_3	Y
1	0	0	0	1
2	1	0	0	1
3	1	1	0	-1

First, initialize weights to zero.

w_0	w_1	w_2	w_3
0	0	0	0

In class, we had been asked to use the same activation function as the one used in the example, i.e.

$$\phi(w^T x) = \begin{cases} 1 & \text{if } \phi(w^T x) \geq 0 \\ -1 & \text{otherwise} \end{cases}$$

∴ Predicted output label for the first example:

X_1	X_2	X_3	Y	Predicted
0	0	0	1	1

Calculating weights :

$$w_j = w_j + \eta (\text{target}^{(i)} - \text{output}^{(i)}) x_j^{(i)}$$

$$\Delta w_0 = \eta (\text{target}^{(i)} - \text{output}^{(i)})$$

$$\Delta w_0 = 0.1(1-1) = 0$$

$$\Delta w_1 = \eta (\text{target}^{(i)} - \text{output}^{(i)}) x_1^{(i)}$$

$$\Delta w_1 = 0.1(1-1) \times 0 = 0$$

$$\Delta w_2 = \eta (\text{target}^{(i)} - \text{output}^{(i)}) x_2^{(i)}$$

$$\Delta w_2 = 0.1(1-1) \times 0 = 0$$

$$\Delta w_3 = \eta (\text{target}^{(i)} - \text{output}^{(i)}) x_3^{(i)}$$

$$\Delta w_3 = 0.1(1-1) \times 0 = 0$$

Updating weights :

$$w_0 = w_0 + \Delta w_0 = 0$$

$$w_1 = w_1 + \Delta w_1 = 0$$

$$w_2 = w_2 + \Delta w_2 = 0$$

$$w_3 = w_3 + \Delta w_3 = 0$$

Now we have,

X ₁	X ₂	X ₃	Y
0	0	0	1
1	0	0	1

Predicted
1

w ₀	w ₁	w ₂	w ₃
0	0	0	0
0	0	0	0

$$\Delta w_0 = 0 \cdot 1 (1-1) = 0$$

$$\Delta w_1 = 0 \cdot 1 (1-1) \times 1 = 0$$

$$\Delta w_2 = 0 \cdot 1 (1-1) \times 0 = 0$$

$$\Delta w_3 = 0 \cdot 1 (1-1) \times 0 = 0$$

$$w_0 = w_0 + \Delta w_0 = 0$$

$$w_1 = w_1 + \Delta w_1 = 0$$

$$w_2 = w_2 + \Delta w_2 = 0$$

$$w_3 = w_3 + \Delta w_3 = 0$$

∴ now we have,

w_0	w_1	w_2	w_3
0	0	0	0
0	0	0	0
0	0	0	0

Now we have,

Pred. labels

x_1	x_2	x_3	y
0	0	0	1
1	0	0	1
1	1	0	-1

y
1
1
1

Weights after last example :

$$\Delta w_0 = 0.1(-1-1) = -0.2$$

$$\Delta w_1 = 0.1(-1-1)1 = -0.2$$

$$\Delta w_2 = 0.1(-1-1)1 = -0.2$$

$$\Delta w_3 = 0.1(-1-1)0 = 0$$

$$\therefore w_0 = -0.2$$

$$w_1 = -0.2$$

$$w_2 = -0.2$$

$$w_3 = 0$$

Weights at the end of the first epoch :

	w_0	w_1	w_2	w_3
0	0	0	0	0
1	-0.2	-0.2	-0.2	0

We carry these weights forward to the next epoch :

X_1	X_2	X_3	Y	Predicted	w_0	w_1	w_2	w_3
0	0	0	1	?	-0.2	-0.2	-0.2	0

Predicted output for first example :

$$\begin{aligned} w^T x &= 1 \times (-0.2) + 0 \times (-0.2) + 0 \times (-0.2) \\ &\quad + 0 \times (-0.2) \\ &= -0.2 \end{aligned}$$

$$\therefore \phi(w^T x) = -1$$

X_1	X_2	X_3	Y	Predicted	w_0	w_1	w_2	w_3
0	0	0	1	-1	-0.2	-0.2	-0.2	0

Updating weights :

$$\Delta w_0 = 0.1 (1 - (-1)) = 0.2$$

$$\Delta w_1 = 0.1 (1 - (-1)) \times 0 = 0$$

$$\Delta w_2 = 0.1 (1 - (-1)) \times 0 = 0$$

$$\Delta w_3 = 0.1 (1 - (-1)) \times 0 = 0$$

$$\therefore w_0 = 0 \quad w_1 = -0.2 \\ w_2 = -0.2 \quad w_3 = 0$$

Predicting label for second example.

X_1	X_2	X_3	Y	Predicted	w_0	w_1	w_2	w_3
0	0	0	1	-1	-0.2	-0.2	-0.2	0
1	0	0	1	?	0	-0.2	-0.2	0

$$w^T x = 0 + (-0.2) \times 1 + (-0.2) \times 0 + (-0.2) \times 0 \\ = -0.2 \quad \therefore \phi(w^T x) = -1$$

X_1	X_2	X_3	Y	Predicted	w_0	w_1	w_2	w_3
0	0	0	1	-1	-0.2	-0.2	-0.2	0
1	0	0	1	-1	0	-0.2	-0.2	0

Updating weights

$$\Delta w_0 = 0.1(1 - (-1)) = 0.2$$

$$\Delta w_1 = 0.1(1 - (-1)) \times 1 = 0.2$$

$$\Delta w_2 = 0.1(1 - (-1)) \times 0 = 0$$

$$\Delta w_3 = 0.1(1 - (-1)) \times 0 = 0$$

$$w_0 = w_0 + \Delta w_0 = 0.2$$

$$w_1 = w_1 + \Delta w_1 = 0$$

$$w_2 = w_2 + \Delta w_2 = -0.2$$

$$w_3 = w_3 + \Delta w_3 = 0$$

∴ now we have,

w_0	w_1	w_2	w_3
-0.2	-0.2	-0.2	0
0	-0.2	-0.2	0
0.2	0	-0.2	0

Predicting label for last example :

$$\begin{aligned} w^T x &= (0.2 \times 1) + (0 \times 1) + (-0.2 \times 1) + (0 \times 0) \\ &= 0 \end{aligned}$$

$$\therefore \phi(w^T x) = 1$$

Now we have,

pred. labels

x_1	x_2	x_3	y
0	0	0	1
1	0	0	1
1	1	0	-1

1	-1	-1	1
---	----	----	---

Weights after last example :

$$\Delta w_0 = 0.1(-1 - 1) = -0.2$$

$$\Delta w_1 = 0.1(-1 - 1) 1 = -0.2$$

$$\Delta w_2 = 0.1(-1 - 1) 1 = -0.2$$

$$\Delta w_3 = 0.1(-1 - 1) 0 = 0$$

$$\therefore w_0 = 0.2 - 0.2 = 0$$

$$w_1 = 0 - 0.2 = -0.2$$

$$w_2 = -0.2 - 0.2 = -0.4$$

$$w_3 = 0 + 0 = 0$$

\therefore Weights at the end of 2 epochs:

Epoch ↓	w ₀	w ₁	w ₂	w ₃
0	0	0	0	0
1	-0.2	-0.2	-0.2	0
2	0	-0.2	-0.4	0

(b)

Model testing

	Sample	X_1	X_2	X_3	Y
Test	1	1	1	0	-1
	2	1	0	1	-1
	3	1	1	1	1
	4	0	0	0	1

The weights (at this stage, i.e., after 2 epochs) are :

$$w_0 = 0$$

$$w_1 = -0.2$$

$$w_2 = -0.4$$

$$w_3 = 0$$

Predicting example 1 label :

$$\begin{aligned} w^T x &= 0 + (-0.2 \times 1) + (-0.4 \times 1) + (0 \times 0) \\ &= -0.2 - 0.4 = -0.6 \end{aligned}$$

$$\therefore \phi(w^T x) = -1 \quad \text{pred.}$$

label for x_{test}^1

Predicting example 2 label

$$\begin{aligned} w^T x &= 0 + (-0.2 \times 1) + (-0.4 \times 0) + (0 \times 1) \\ &= -0.2 \end{aligned}$$

$$\therefore \phi(w^T x) = -1$$


pred.
label for x^2_{test} .

Predicting example 3 label

$$\begin{aligned} w^T x &= 0 + (-0.2 \times 1) + (-0.4 \times 1) + (0 \times 1) \\ &= -0.6 \end{aligned}$$

$$\therefore \phi(w^T x) = -1$$


pred.
label for x^3_{test} .

Predicting example 4 label :

$$w^T x = 0 + (-0.2 \times 0) + (-0.4 \times 0) + (0 \times 0)$$

$$= 0$$

$$\therefore \phi(w^T x) = 1$$

pred.
 label for x_{test}^4

	Sample	X_1	X_2	X_3	Y	
Test	1	1	1	0	-1	$\begin{matrix} 1 \\ -1 \\ -1 \\ -1 \end{matrix}$
1	1	0	1	-1	-1	✓
2	1	1	1	1	-1	✗
3	0	0	0	1	1	✓

(c) Model evaluation \rightarrow confusion matrix :

	Predicted: +1	Predicted: -1
Actual: +1	$TP = 1$	$FN = 1$
Actual: -1	$FP = 0$	$TN = 2$

$$(d) \text{ Precision} = \frac{TP}{TP + FP} = \frac{1}{1+0} = 1$$

$$\text{Recall} = \frac{TP}{TP + FN} = \frac{1}{1+1} = 0.5$$

$$\therefore \begin{cases} \text{Precision} = 1 \\ \text{Recall} = 0.5 \end{cases}$$