Hierarchical Bayesian Inference of Cosmic Shear & Intrinsic Galaxy Properties

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Abstract

The gravitational lensing of distant galaxies by mass along the line-of-sight is a sensitive probe of both the expansion and structure growth rates of the Universe. The massive amount of data from the Large Synoptic Survey Telescope (LSST) and other surveys can be used to infer the properties of cosmic shear. We describe how Hierarchical Bayesian Models can be used to infer these properties while marginalizing nuisance parameters.

Weak lensing and cosmology

The presence of mass (dark and luminous matter) in the line-of-sight path of light distorts the observed images of light sources. The distribution of this foreground mass can be inferred by measuring slight correlations in the observed properties of light sources in a given patch of the sky. Estimating the shear is not easy as we do not know the intrinsic properties of the light sources.

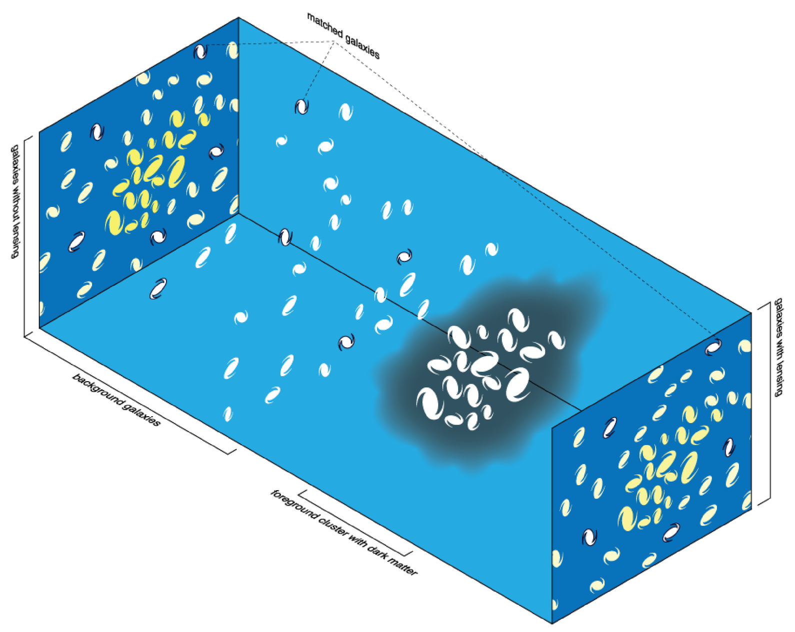


Figure: Geometry for gravitational lensing of distant galaxies.

Probabilistic Model

Inferring the cosmic shear from observed data is an intractable inverse modeling problem. By using a hierarchical bayesian model, we can take effects due to different factors like image noise, atmospheric distortions and cosmic shear into account infer mass distribution and intrinsic properties. The model also allows us to update our beliefs with incoming data and to marginalize out unknowable nuisance parameters.

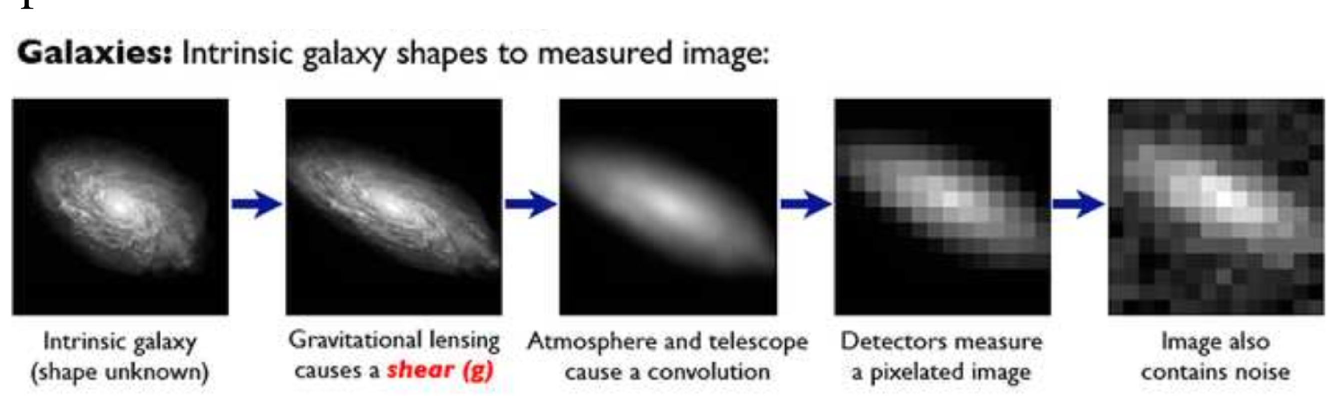


Figure: Illustration of the forward process.

$$P(\gamma|\{D_i\}_1^N) \propto P(\{D_i\}_1^N|\gamma)P(\gamma) \qquad (1)$$

Where γ is the property we are investigating, D_i is the data for the i^{th} galaxy, $P(\{D_i\}_1^N|\gamma)$ is the likelihood and $P(\gamma)$ encodes our prior belief.

Univariate Toy Model

To determine the performance and validity of different techniques, we used a toy model where the data consisted only of observed galaxy ellipticities ϵ_{sh}. In the toy universe, the intrinsic ellipticities follow a Normal distribution $N_{\epsilon_{int}}(0, \sigma_e)$, with shear g added within weak lensing limits: $\epsilon_{sh} \approx \epsilon_{int} + g$. We also added gaussian noise σ_n to observed ellipticities.

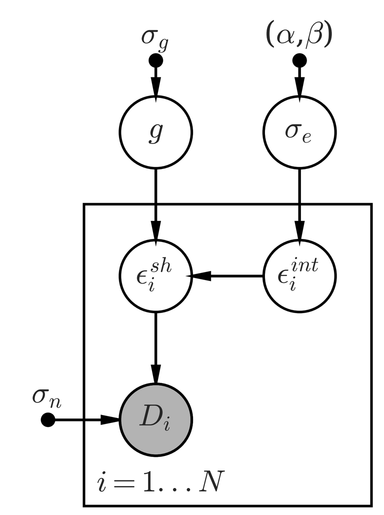


Figure: Probabilistic Graphical Model of our toy model.

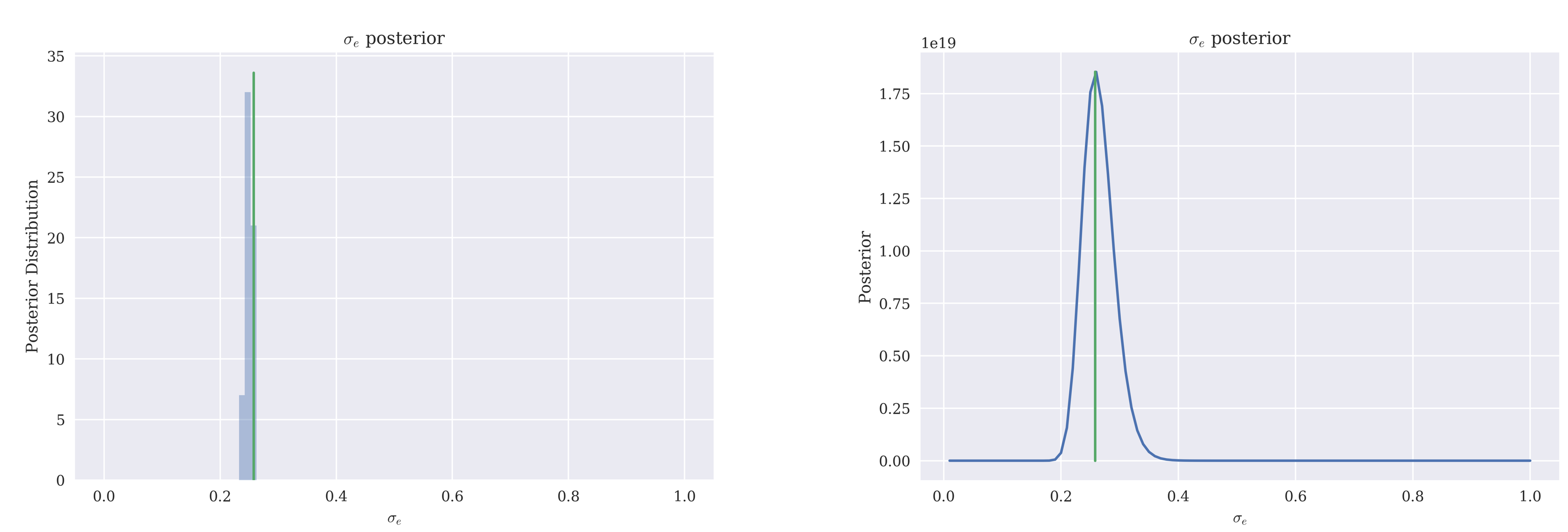
Our goal was to infer the value of g and σ_e from data generated using the above scheme. For this model, the analytical solution was a simple integral and we used it to check other techniques.

The observed properties depend on intrinsic properties which we do not know about. In this model, ϵ_{sh} depends on ϵ_{int}. We marginalize out such nuisance parameters.

$$P(D_n|\gamma) \propto \int d\epsilon_{(int,n)} P(D_n|\epsilon_{(int,n)}, \gamma) P(\epsilon_{(int,n)}|I) \qquad (2)$$

We used the following techniques to generate posterior distributions:

- Markov Chain Monte Carlo through Gibbs Sampling
- Importance Sampling with an interim prior



(a) Gibbs Sampling with Inv Gamma prior. (b) Imp. Sampling with normal interim prior.

Figure: Posterior distributions generated using Gibbs Sampling and Importance Sampling. True value denoted by green line ($\sigma_e = 0.258$).

While Gibbs sampling performed slightly faster in the toy model, it required more data as compared to importance sampling to get comparable results. Also, Gibbs sampling requires closed form conditional distributions, which can not always be derived for multivariate models. Importance sampling gave results with less variance and can be generalized for multivariate models.

Multivariate Model

To apply a hierarchical probabilistic model to real data, we can't ignore non-shape parameters like flux, size and light profile. As a proof of concept, we extended the toy model to handle multiple correlated parameters. The figure below shows how the posterior distribution generated using importance sampling varies with two correlated parameters in the extended toy model.

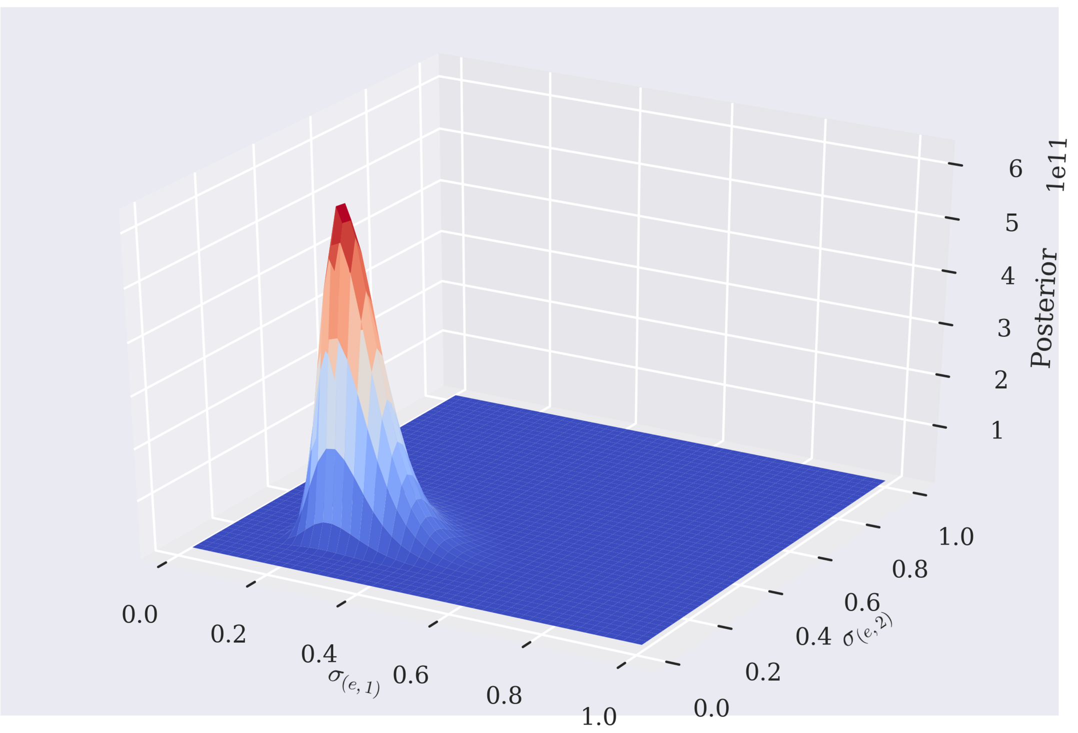


Figure: Posterior distribution for diagonal elements of a 2X2 covariance matrix Σ. The peak corresponds to the true value (0.258,0.258).

Results

- We validated monte carlo methods for marginal probability inference.
- The Jupyter notebooks created for the toy model can be used as tutorials on hierarchical bayesian models for cosmic shear.
- The extended toy model is the first demonstration of multivariate modeling for galaxies in a hierarchical framework.

Future Directions

- Generalize the multivariate model to include any number of property parameters. Also, there are many design questions like the choice of priors, number of samples etc that need to be explored.
- Add Pareto Smoothed Importance Sampling.
- Add better uncertainty quantification (eg Kullback-Liebler divergence to compare distributions).

References

Hierarchical probabilistic inference of cosmic shear M. D. Schneider, D. W. Hogg, P. J. Marshall, W. A. Dawson, J. Meyers, D. J. Bard, D. Lang, The Astrophysical Journal, 807, 87 (2015)

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