Review of State Removal

Players	**Options**
U.S. Support	1. Complete full GDU 2. Complete GDU modified to reduce Canadian impacts 3. Complete GDU modified to appease US environmentalists
U.S. Opposition	1. Legal action based on environmental legislation
Canadian Opposition	1. Legal action based on Boundary Treaty of 1909
IJC	1. Support completion of GDU 2. Support completion of GDU modified to reduce Canadian impacts 3. Support suspension of GDU except for the Lonetree Reservoir 4. Support the complete suspension of the GDU

Review of State Removal

Reasons	Removed
Type 1: Logically infeasible for a DM	
Mutually exclusive options for US Support	(1 1 - - - - - - -) (1 - 1 - - - - - -) (- 1 1 - - - - - -) (- - - - - 1 1 - -)
Mutually exclusive options for IJC	(- - - - - 1 - 1 -) (- - - - - 1 - - 1) (- - - - - - 1 1 -) (- - - - - - 1 - 1) (- - - - - - - 1 1)
Type 2: Preferentially infeasible for a DM	
Some sort of project will be built	(0 0 0 - - - - - - -)
IJC will make a recommendation	(- - - - - 0 0 0 0)
Type 3: Logically infeasible for a set of DMs	
None	
Type 4: Preferentially infeasible for a set of DMs	
US Opposition will pursue legal action against full project	(1 - - 0 - - - - -)
US Opposition will not pursue legal action if appeased	(- - 1 1 - - - - -)
Canadian Opposition will pursue legal action if any project larger than that approved by the IJC is built	(1 - - - 0 0 - - -) (- 1 - - 0 - - 1 -) (- 1 - - 0 - - - 1) (- - 1 - 0 - - 1 -) (- - 1 - 0 - - - 1)

Review of Sequential Stability

A state s is **sequentially sanctioned** (s) for a DM if for each UI from s, a credible action can be taken by the opponent(s) to stop the DM from taking advantage of the UI

Credible action: one that results in a more preferred outcome for the player taking the action (UI)

In n $\geq$ 2 DM games, check for sequences of UIs from opponents - each opponent can move at most one time

If there is more than one possible sequence, consider them all

Review of Sequential Stability

DM 1 Preference ranking

17	15	11	14	16	9	10	12	13	5	2	4	8	6	3	7	1
	17			17 15	11	11 9	14	14 12			5		5 4	2	8	2 3

DM 2 Preference ranking

11	17	15	1	16	5	13	9	2	10	7	8	12	6	3	4	14
	11					16	15	5	16 13	1	5 2	15 9		6	1 7	11 17

DM 3 Preference ranking

1	4	12	9	7	5	2	13	10	8	6	3	14	11	16	15	17
		4	1				5	2				6	3	8	7	

State 15 for DM 1: 15 $\rightarrow_{DM1}$ 17 $\rightarrow_{DM2}$ 11 $\rightarrow_{DM3}$ 3
Since 3 is less preferred to 15, state 15 is sequentially sanctioned for DM1
The only UI from 15 is sanctioned

Review of Sequential Stability

DM 1 Preference ranking

17	15	11	14	16	9	10	12	13	5	2	4	8	6	3	7	1
	17			17 15	11	11 9	14	14 12			5		5 4	2	8	2 3

DM 2 Preference ranking

11	17	15	1	16	5	13	9	2	10	7	8	12	6	3	4	14
	11					16	15	5	16 13	1	5 2	15 9		6	1 7	11 17

DM 3 Preference ranking

1	4	12	9	7	5	2	13	10	8	6	3	14	11	16	15	17
		4	1				5	2				6	3	8	7	

State 16 for DM 1: two UIs to check: 17 and 15

$16 \rightarrow_{DM1} 17 \rightarrow_{DM2} 11 \rightarrow_{DM3} 3 < 16$ so this UI is sanctioned

$16 \rightarrow_{DM1} 15 \rightarrow_{DM3} 7 \rightarrow_{DM2} 1 < 16$ so this UI is sanctioned

Both UIs are sanctioned, so state 16 is sequentially sanctioned for DM1

Review of Sequential Stability

DM 1 Preference ranking

r	s	r	r	s	s	s	s	s	r	r	u	r	u	u	u	u
17	15	11	14	16	9	10	12	13	5	2	4	8	6	3	7	1
	17			17	11	11	14	14			5		5	2	8	2
				15		9		12					4			3

DM 2 Preference ranking

r	s	r	r	r	r	s	s	u	s	s	u	u	r	s	u	u
11	17	15	1	16	5	13	9	2	10	7	8	12	6	3	4	14
	11					16	15	5	16	1	5	15		6	1	11
									13		2	9			7	17

DM 3 Preference ranking

r	r	s	s	r	r	r	u	u	r	r	r	u	u	u	u	r
1	4	12	9	7	5	2	13	10	8	6	3	14	11	16	15	17
		4	1				5	2				6	3	8	7	

Review of Simultaneous Sanctioning

A state s is **simultaneously sanctioned** ($\cancel{u}$) for a DM if simultaneous action by more than one DM could cause a less preferred outcome to occur

Need to check all possible combinations of simultaneous actions

For every $\Gamma \subseteq M$, $|\Gamma| \geq 2$ and $a_i \in A_i$, calculate the outcome to compare to q:

$$outcome = \sum_{i \in \Gamma} a_i - (|\Gamma| - 1) \cdot q$$

If at any point, *outcome* is less preferred to q, the state is simultaneously sanctioned for the DM in question

Review of Simultaneous Sanctioning

A state s is **simultaneously sanctioned** ($\cancel{u}$) for a DM if simultaneous action by more than one DM could cause a less preferred outcome to occur

Need to check all possible combinations of simultaneous actions

For every $\Gamma \subseteq M$, $|\Gamma| \geq 2$ and $a_i \in A_i$, calculate the outcome to compare to q:

$$outcome = \sum_{i \in \Gamma} a_i - (|\Gamma| - 1) \cdot q$$

If at any point, *outcome* is less preferred to q, the state is simultaneously sanctioned for the DM in question

Review of Simultaneous Sanctioning

DM 1 Preference Ranking:

a	e	f	g	h
b				

DM 2 Preference Ranking:

e	f	g	h	a
				c

DM 3 Preference Ranking:

f	a	e	g	h
	d			

$b + c - a = e$; $b + d - a = f$; $c + d - a = g$; $b + c + d - 2 \cdot a = h$
All of e, f, g, h are less preferred to a for DM 1 so a is ~~u~~ for DM 1
All of e, f, g, h are more preferred to a for DM 2 so a is unstable for DM 2
Some of e, f, g, h are less preferred to a for DM 3 so a is stable for DM 3

SYDE 533 Conflict Resolution

Hypergames

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Learning Objectives

By the end of this lesson, you will be able to:

- Define and explain an n-level hypergame
- Conduct stability analysis and calculations for hypergames

Hypergames

A **hypergame** is a conflict in which one or more DMs have misperceptions about the conflict

Misperceptions may include:

- preferences
- options
- players
- any combination of the above

The level of hypergame can be extended up to any levels of perception

Game of Complete Perception

In a **game of complete perception**, we define the overall game $G = \{V_1, V_2, \ldots V_n\}$ where V_i is the preference ranking for DM i

All DMs perceive each other correctly and completely ; all DMs are playing the same game

First Level Hypergame

In a **first level hypergame**, one or more players perceive different games

DM i's game is $G_i = \{V_{1i}, V_{2i}, \ldots V_{ni}\}$ where V_{ji} is the preference ranking for DM j as perceived by DM i

The collection of all DMs' subjective games mathematically defines a first-level hypergame: $H^1 = \{G_1, G_2, \ldots G_n\}$

First Level Hypergame

Hypergame Matrix

	DM perceiving			
DM perceived	1	2	$\cdots$	n
1	V_{11}	V_{12}	$\cdots$	V_{1n}
2	V_{21}	V_{22}	$\cdots$	V_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	V_{n1}	V_{n2}	$\cdots$	V_{nn}
Game	G_1	G_2	$\cdots$	G_n

V_{ij} reads as "preferences of DM i as perceived by DM j"

G_i is the game perceived by DM i

First Level Hypergame

Stability Analysis

In a **first level hypergame**, perform a stability analysis of G_i for each DM to see how they perceive individual stability results and equilibria

A state is an equilibrium if it is stable in all the preference rankings that DMs perceive for themselves

Check preferences along the main diagonal of the hypergame matrix

First Level Hypergame

Cuban Missile Crisis

It can be argued that the Russians did not expect the Americans to react as strongly as they did (i.e. blockade)

The Americans were unaware of this misunderstanding on the part of the USSR, so we model this as a first level hypergame

First Level Hypergame

Cuban Missile Crisis

The hypergame matrix is:

DM perceived	**DM perceiving** US	**DM perceiving** USSR
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8
Game	G_{US}	G_{USSR}

USSR has a misperception of the US's preferences (red)

US correctly perceives the USSR's preferences

First Level Hypergame

Cuban Missile Crisis

The hypergame matrix is:

	DM perceiving	
DM perceived	US	USSR
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8
Game	G_{US}	G_{USSR}

Separately analyse G_{US} and G_{USSR}

To determine equilibria of H^1, compare stability results for the US preferences in G_{US} and those for the USSR preferences in G_{USSR}.

First Level Hypergame

Cuban Missile Crisis

Take a few minutes to analyse G_{US}:

$V_{US,US}$:

4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,US}$:

0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Rational states for G_{US}:

$V_{US,US}$:

r				r				r			
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,US}$:

r		r		r		r					
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Sequentially sanctioned states for G_{US}:

$V_{US,US}$:

r	s			r				r			
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,US}$:

r	s	r		r		r					
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Unstable and simultaneously sanctioned states for G_{US}:

$V_{US,US}$:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,US}$:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Equilibria for G_{US}:

$V_{US,US}$:

E	E	X	X	X	X	X	X	X	X	X	X
r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,US}$:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Take a few minutes to analyse G_{USSR}:

$V_{US,USSR}$:

4	0	6	2	5	1	7	3	11	9	10	8
		4	0	4	0	4	0		11	11	11
				6	2	6	2			9	9
						5	1				10

$V_{USSR,USSR}$:

0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

Equilibria and stabilities for G_{USSR}:

$V_{US,USSR}$:

r	r	u	u	u	u	u	u	r	u	u	u
4	0	6	2	5	1	7	3	11	9	10	8
		4	0	4	0	4	0		11	11	11
				6	2	6	2			9	9
						5	1				10

$V_{USSR,USSR}$:

E	X	X	X	X	X	X	X	X	X	X	X
r	u	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

First Level Hypergame

Cuban Missile Crisis

From the US point of view, there are two equilibria: state 4 (USSR withdraws) and state 6 (US blockades and USSR withdraws)

From the USSR point of view, there is one equilibrium: state 0 (no action taken by either DM)

What game is actually being played?

First Level Hypergame

Cuban Missile Crisis

Compare stability results for $V_{US,US}$ in G_{US} to $V_{USSR,USSR}$ in G_{USSR}:

X	E	X	X	X	X	X	X	X	X	X	X

$V_{US,US}$:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

$V_{USSR,USSR}$:

r	u	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

Single equilibrium at state 6 (US blockades, USSR withdraws)

First Level Hypergame

Cuban Missile Crisis

From the US point of view, there are two equilibria: state 4 (USSR withdraws) and state 6 (US blockades and USSR withdraws)

From the USSR point of view, there is one equilibrium: state 0 (no action taken by either DM)

Hypergame analysis reveals a single equilibrium: state 6

Analysing each level of a hypergame helps determine each player's expectations and how they will behave

First Level Hypergame

Recap

- Draw hypergame matrix
- Analyse each DM's game, G_i
- Compare stability results across games (matrix diagonal) to find the hypergame equilibria

	DM perceiving	
DM perceived	US	USSR
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8
Game	G_{US}	G_{USSR}

Second Level Hypergame

In a **second level hypergame**, at least one player realizes that a first level hypergame is being played

DM i's hypergame is $H_i = \{G_{1i}, G_{2i}, \ldots G_{ni}\}$ where G_{ji} is the game for DM j as perceived by DM i

The second level hypergame is defined as $H^2 = \{H_1, H_2, \ldots H_n\}$

Second Level Hypergame

Hypergame Matrix

	DM perceiving			
DM perceived	1	2	$\cdots$	n
1	G_{11}	G_{12}	$\cdots$	G_{1n}
2	G_{21}	G_{22}	$\cdots$	G_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	G_{n1}	G_{n2}	$\cdots$	G_{nn}
Hypergame	H_1	H_2	$\cdots$	H_n

G_{ij} reads as "game for DM i as perceived by DM j"

H_i is the hypergame perceived by DM i

Second Level Hypergame

Stability Analysis

In a **second level hypergame**, perform a stability analysis of G_{ij}

A state is an equilibrium if it is stable in all the games that DMs perceive for themselves

Check along the main diagonal of the hypergame matrix

Second Level Hypergame

Cuban Missile Crisis

	US Hypergame	
	$G_{US,US}$	$G_{USSR,US}$
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

	USSR Hypergame	
	$G_{US,USSR}$	$G_{USSR,USSR}$
US	4 0 6 2 5 1 7 3 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

US knows that the USSR has a misperception of US preferences (red)

USSR is not aware of its misperception

Second Level Hypergame

Stability Analysis

- Separately analyse $G_{US,US}$, $G_{USSR,US}$, $G_{US,USSR}$, and $G_{USSR,USSR}$ to get individual stabilities and equilibria

Second Level Hypergame

Stability Analysis

- Determine equilibria in H_{US} and H_{USSR} by comparing the results along each hypergame diagonal
  - For H_{US} compare stability results from US preference ranking in $G_{US,US}$ and USSR preference ranking from $G_{US,USSR}$

	US Hypergame	
	$G_{US,US}$	$G_{USSR,US}$
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

	USSR Hypergame	
	$G_{US,USSR}$	$G_{USSR,USSR}$
US	4 0 6 2 5 1 7 3 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

Second Level Hypergame

Stability Analysis

- Determine equilibria in H_{US} and H_{USSR} by comparing the results along each hypergame diagonal
  - For H_{USSR} compare stability results from US preference ranking in $G_{US,USSR}$ and USSR preference ranking from $G_{USSR,USSR}$

	US Hypergame	
	$G_{US,US}$	$G_{USSR,US}$
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

	USSR Hypergame	
	$G_{US,USSR}$	$G_{USSR,USSR}$
US	4 0 6 2 5 1 7 3 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

Second Level Hypergame

Stability Analysis

- Determine H^2 equilibria by examining the results for the US preferences in $G_{US,US}$ and for the USSR preferences in $G_{USSR,USSR}$

	US Hypergame	
	$G_{US,US}$	$G_{USSR,US}$
US	4 6 5 7 2 1 3 0 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8
	USSR Hypergame	
	$G_{US,USSR}$	$G_{USSR,USSR}$
US	4 0 6 2 5 1 7 3 11 9 10 8	4 0 6 2 5 1 7 3 11 9 10 8
USSR	0 4 6 2 5 1 7 3 11 9 10 8	0 4 6 2 5 1 7 3 11 9 10 8

Second Level Hypergame

Cuban Missile Crisis

Compare stability results for $V_{US,US}$ in $G_{US,US}$ to $V_{USSR,USSR}$ in $G_{USSR,USSR}$:

X	E	X	X	X	X	X	X	X	X	X	X

$V_{US,US}$:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4 6	4 6 5		2	2 1	2 1 3		11	11 9	11 9 10

$V_{USSR,USSR}$:

r	u	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7 3	5 1	6 2	0 4

Single equilibrium at state 6 (US blockades, USSR withdraws)

Hypergame Analysis

Cuban Missile Crisis

H^0 (complete perception / no misperceptions):

- Two equilibria at states 4 and 6

H^1 (USSR misunderstands US preferences):

- Analyst perceives unique equilibrium at state 6
- US perceives equilibria at states 4 and 6
- USSR perceives equilibrium at state 0

H^2 (USSR misunderstands US preferences, US is aware of this):

- US perceives unique equilibrium at state 6
- USSR perceives equilibrium at state 0

Hypergame Analysis

Summary

H^0 (complete perception / no misperceptions):

- DMs are playing the same game

H^1 (at least one DM has misunderstanding):

- DMs are playing different games

H^2 (at least one DM is aware of misunderstanding):

- DMs are playing different hypergames

Can be extended to any level

Coming Up

Next Lecture:

- Hypergames (contiuned)
- Tutorial?

Next Lesson:

- Assignment 1 due on October 5th
- Forms of a game / intro to graph theory