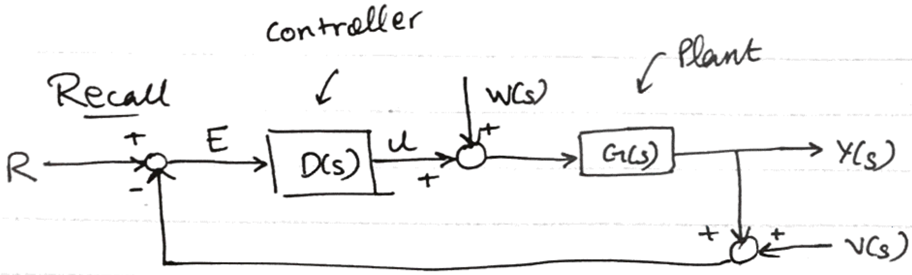
Feb 2, 2016

Recall



$$Y(s) = \frac{DG}{1+DG} R + \frac{G}{1+DG} W - \frac{DG}{1+DG} V \qquad (1)$$

$$u(s) = \frac{D}{1+DG} R - \frac{DG}{1+DG} W - \frac{D}{1+DG} V \qquad (2)$$

$$E(s) = \frac{1}{1+DG} R - \frac{G}{1+DG} W + \frac{DG}{1+DG} V \qquad (3)$$

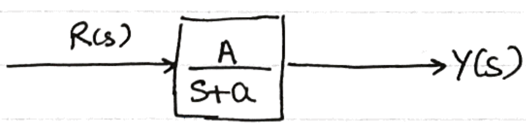
Control ~~ler~~ Problems

1) Tracking - want $Y(s)$ to follow or track the input reference $R(s)$ as clearly as possible with minimum error.

2) Regulation (set pt) - R is constant

Sensitivity

Consider

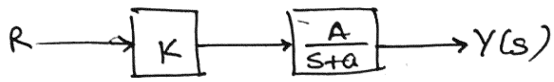


For $R(s) = \frac{1}{s}$

Then $Y(s) = \frac{A}{s(s+a)} = \frac{A}{as} - \frac{A}{a(s+a)}$

$$\Rightarrow y(t) = \frac{A}{a}(1 - e^{-at})$$

Consider adding a controller to yield $y_{ss} = 1$



$$Y(s) = \frac{KA}{s(s+a)}$$

$$y(t) = \frac{KA}{a}\left(1 - e^{-at}\right)$$

$\therefore$ Choose $K = \frac{a}{A}$ for $y_{ss} = 1$

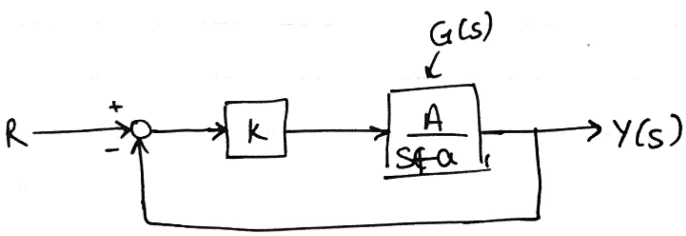
Suppose A changes to $A + \underbrace{\delta A}_{\text{change in } A}$

Then $y(t) = \frac{K(A+\delta A)}{a}\left(1 - e^{-at}\right)$

$K = \frac{a}{A}$, $\quad y(t) = \left(1 - e^{-at}\right) + \underline{\frac{\delta A}{A}\left(1 - e^{-at}\right)}$

an uncompensated change change moves our steady state response off the desired value

Try adding a feedback loop



How sensitive is the system transfer function to changes in the plant $G(s)$?

Define the relitive change in a quantity α as $\frac{d\alpha}{\alpha}$

The relative changes in β is $\dfrac{d\beta}{\beta}$

Define the sentitivity of α to changes in β as

$$\boxed{S_{\beta}^{\alpha} \triangleq \frac{d\alpha/\alpha}{d\beta/\beta} = \frac{\beta}{\alpha} \cdot \frac{d\alpha}{d\beta}}$$

The system transfer function is

$$\frac{Y}{R} = \frac{KG}{1+KG} = T(s)$$

The ~~sent~~ sensitivity of T to change in G

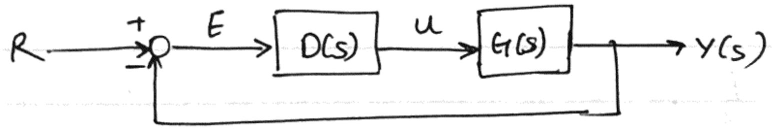
$$S_{G}^{T} = \frac{G}{T}\frac{dT}{dG}$$

$$\frac{dT}{dG} = \frac{d}{dG}\left(\frac{KG}{1+KG}\right) = \frac{K(1+KG) - K^2G}{(1+KG)^2} = \frac{K}{(1+KG)^2}$$

$$S_{G}^{T} = \frac{G}{\frac{KG}{1+KG}} \cdot \frac{K}{(1+KG)^2} = \frac{1}{1+KG}$$

$$\Rightarrow \boxed{S_{G}^{T} = \frac{1}{1+KG}}$$

Consider a more general controller



The output error

$$E(s) = R - Y = R - \frac{DG}{1+DG} R = \frac{1}{1+DG} R(s)$$

Consider the tracking problem with polynomial inputs, eg

$R(s)$ = Impulse	$1 = 1/s^0$	$\delta(t)$
position or step	$1/s$	$H(t) = 1$
velocity or ramp	$1/s^2$	t
acchuation or parabola	$1/s^3$	$\frac{1}{2} t^2$

Under what conditions on $DG(s)$ will we get no error?

$$E = \frac{1}{1+DG} R$$

$$DG = \frac{K \prod_{i=1}^{m} (1 + T_i s)}{s^{j} \prod_{k=j+1}^{n} (1 + T_k s)}$$

poles of multiplicity j at the origin

the power j is called the system type

if $j=0$ $\Rightarrow$ type "0" system

if $j=1$ $\Rightarrow$ type "1" system

etc

Recall $e_{ss} = \lim_{s \to 0} sE(s)$

Restrict our iputs to be

$$R(s) = \frac{1}{s^{p+1}} \qquad p = -1, 0, 1, 2, 3 \ldots$$

Then $$e_{ss} = \lim_{s \to 0} \frac{s}{s^{p+1} + s^{p+1} DG(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^p + s^p DG(s)}$$

$$= \lim_{s \to 0} \frac{1}{s^p DG(s)}$$

$$e_{ss} = \lim_{s \to 0} \frac{s^j}{s^p K} \qquad p \neq 0$$

$$= \lim_{s \to 0} \frac{s^{j-p}}{K}$$

$e_{ss} = 0$ we require

system type $j > p$

if $j = p$ then $e_{ss} = \frac{1}{K}$