### A Remark about Pathological Examples

Although many powerful techniques for locating the roots of functions will be developed in this chapter, it must be kept in mind that there are problems for which it will be difficult, if not downright impossible, for even the best of techniques to find the desired solution. Consider, for example, locating the roots of

$$f(x) = 3x^2 + \frac{1}{\pi^4}\ln\left[(\pi - x)^2\right] + 1.$$

It is fairly easy to establish that $f$ has two simple real roots. First, note that since $3x^2 + 1$ is always positive, the term involving $\ln\left[(\pi - x)^2\right]$ must be negative for $f$ to evaluate to zero. This implies that any roots must lie on the interval $x \in (\pi - 1, \pi + 1)$. Combining the facts that $\lim_{x\to\pi} \ln\left[(\pi - x)^2\right] \to -\infty$ and $f$ is continuous everywhere except at $x = \pi$ with the knowledge that $f(\pi - 1) > 0$ and $f(\pi+1) > 0$ guarantees the existence of a root on each of the intervals $(\pi - 1, \pi)$ and $(\pi, \pi + 1)$. Monotonicity of $f$ on $(-\infty, \pi)$ and on $(\pi, \infty)$ guarantees the uniqueness of the root in each interval.

Since the natural logarithm term must balance $3x^2 + 1$ and the coefficient on the logarithm term is roughly 0.01, it is reasonable to assume that both roots are close to $\pi$. It follows that $\ln\left[(\pi - x)^2\right]$ must be roughly $-\left(3x^2 + 1\right)\pi^4\big|_{x=\pi}$ or $-2981.6$. Therefore, $(\pi - x)$ must be on the order of $\pm e^{-1490.788} \approx \pm 10^{-647}$, so that $x \approx \pi \pm 10^{-647}$. The floating point number system on most machines will never be able to resolve these values. The moral of the story is a simple one: Pathological problems do exist, so always perform some basic analysis before rushing to the computer.