SYDE 352 Jan 14

Pole Location, transient Response, & Stability

Consider $\dfrac{Y(s)}{R(s)} = T(s)$

with $R(s)$ any bounded input then $Y(s) = T(s)R(s)$ which can be expanded in partial fractions (use $R(s) = 1/s$)

$$Y(s) = \boxed{\frac{A_0}{s}} + \underbrace{\sum_{i=1}^{m} \frac{A_i}{s+\sigma_i}}_{\text{Real poles of } T(s)} + \underbrace{\sum_{k=1}^{n} \frac{B_k + C_k s}{s^2 + 2\alpha_k s + \alpha_k^2 + w_k^2}}_{\text{complex conjugate poles}}$$

$s = -\sigma_i$ $\qquad$ $s = -\alpha_k \pm j w_k$

$$y(t) = \boxed{A_0 H(t)} + \sum_{i=1}^{m} A_i e^{-\sigma_i t} + \sum_{k=1}^{n} \frac{D_k}{w_k} e^{-\alpha_k t} \sin(w_k t + \phi_k)$$

Observations

1. for real poles in the RHP the time domain response grows without bound
2. Real poles in the LHP cause the response to decay to zero
3. Complex poles in the RHP result in unbounded responses and complex poles in the LHP lead to decaying responses or transiets.

For stability (bounded input bounded output: BIBO), all poles must be in the LHP

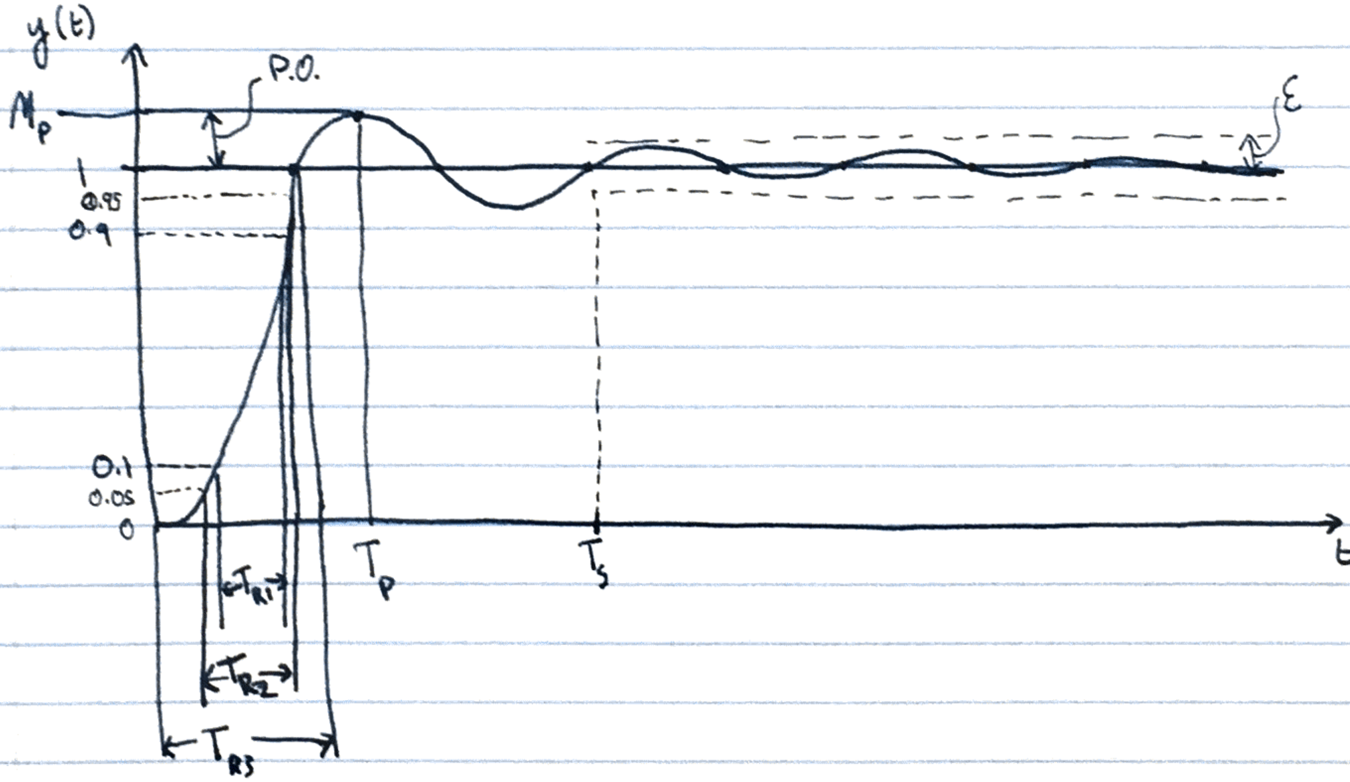
4. poles with large real parts decay faster than the responses of poles with smaller real parts

5. the responses of poles with large imaginary parts have higher frequency responses

6. If a zero is close to a pole the effect of the pole is diminished

Time domain Response Specifications

- Quiescent System
- step input



I) Rise time:

	Rise from	to
T_{R1}	10%	90%
T_{R2}	5%	95%
T_{R3}	0%	100%

II) Peak Time: T_p

time required to reach the first peak of the response

III) Maximum overshoot value: M_p

alternatively use percentage overshoot: P.O.

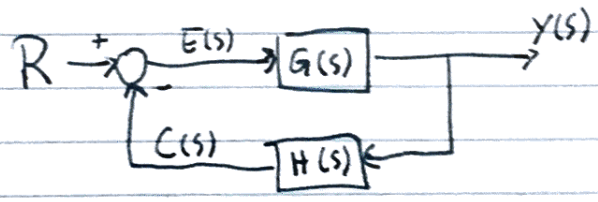
$$P.O. = \frac{y(T_p) - y(\infty)}{y(\infty)} \times 100\%$$

IV) Settling time: T_s

The time required for the response to reach and stay within a region of $\pm\varepsilon$ of the steady state value

V) Steady State Error

Consider



$E(s)$ = actuation error

alternatively there is the output error

$$\underbrace{e(t) = y(t) - r(t)}$$

If $H(s) = 1$ then the two errors are the same

Find $E(s)$ in terms of R

$$E = R(s) - C(s) = R(s) - H(s)\,Y(s)$$

let $Y(s) = \frac{G(s)}{1+GH(s)} R(s)$

$\therefore E(s) = R(s) - \frac{GH(s)}{1+GH(s)} R(s)$

$E(s) = \frac{1}{1+GH(s)} R(s)$

also valid if $H(s)=1$

The steady state error is

$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$