SYDE 533 Conflict Resolution

Forms of a Game & Intro to Graph Theory

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Learning Objectives

By the end of this lesson, you will be able to:

- Identify three different forms of a conflict and represent conflicts in each of these forms
- Explain some basic graph theory concepts
- Explain basic concepts related to the Graph Model for Conflict Resolution

Normal Form of a Game

Chicken

		DM 2	
		Don't Swerve (D)	Swerve (S)
DM 1	Don't Swerve (D)	DD (1, 1)	DS (4, 2)
	Swerve (S)	SD (2, 4)	SS (3, 3)

- $N = \{1, 2, \ldots n\}$ denotes the set of DMs
- For each $i \in N$, the set S_i is a finite non-empty set called i's strategy set
- The set of states is given by $S = S_1 \times S_2 \times \ldots \times S_n$, where $\times$ denotes the Cartesian product
- The numbers in each cell represent ordinal payoffs

Normal Form of a Game

Chicken

		DM 2	
		Don't Swerve (D)	Swerve (S)
DM 1	Don't Swerve (D)	DD (1, 1)	DS (4, 2)
	Swerve (S)	SD (2, 4)	SS (3, 3)

- $N = \{DM1, DM2\}$
- $S_{DM1} =$ { Don't Swerve (D), Swerve (S) } $= S_{DM2}$
- $S = \{D, S\} \times \{D, S\} = \{DD, DS, SD, SS\}$
- Payoffs for DM 1: $P_{DM1}(DD) = 1$, $P_{DM1}(DS) = 4$, $P_{DM1}(SD) = 2$, and $P_{DM1}(SS) = 3$

Normal Form of a Game

Drawbacks

		DM 2	
		Don't Swerve (D)	Swerve (S)
DM 1	Don't Swerve (D)	DD (1, 1)	DS (4, 2)
	Swerve (S)	SD (2, 4)	SS (3, 3)

- Not practical for more conflicts with than 2 DMs

Option Form of a Game

Chicken

DM 1				
Swerve	0	1	0	1
DM 2				
Swerve	0	0	1	1
Decimal	0	1	2	3

DM 1 preferences: 2, 3, 1, 0
DM 2 preferences: 1, 3, 2, 0

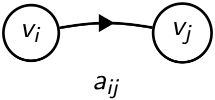
- Can represent complex conflict models (multiple DMs and options) in a clear and compact manner
- Preferences rank states from most (left) to least (right) preferred
- It is often used with the Graph Model for Conflict Resolution

Graph Theory Basics

Before discussing the graph form of a conflict, we need a few additional concepts

A **directed graph**, $D = (V, A)$, is a 2-ple where
$V = \{v_1, v_2, \ldots v_n\}$ denotes the set of vertices and
$A = \{a_1, a_2, \ldots a_k\} \in V \times V$ denotes the set of arcs

An arc $a_{ij} = (v_i, v_j) \in A$ joins vertex v_i to v_j ; v_i is the **tail** of a_{ij} and v_j is the **head** of a_{ij}



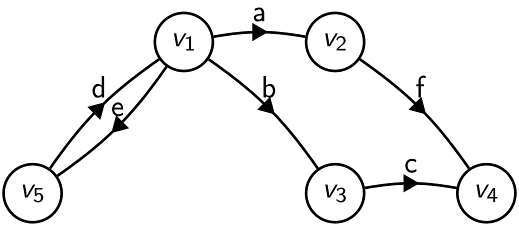
An arc with identical head and tail is called a **loop**

Graph Theory Basics

A **directed walk** in D is a finite non-null sequence $W = (v_0, a_1, v_1, \ldots a_k, v_k)$ whose terms are alternately vertices and arcs such that a_i has head v_i and tail v_{i-1}

If the arcs of a directed walk are distinct, W is called a **directed trail**

If, in addition, the vertices are distinct, W is called a **directed path**



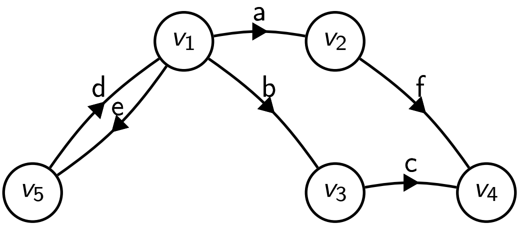
Graph Theory Basics

A vertex v_j is said to be **reachable** from vertex v_i if there is a directed (v_i, v_j)-path in D

The reachability matrix R of a directed graph D has entries

$$R_{ij} = \begin{cases} 1 & \text{if } v_j \text{ is reachable from } v_i \\ 0 & \text{otherwise} \end{cases}$$

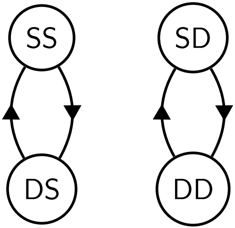
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$



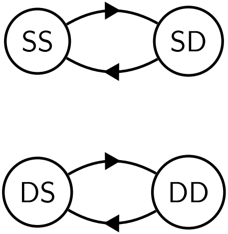
Graph Form of a Conflict
Chicken

In the graph form of a conflict:

- States → vertices
- Moves → directed arcs
- Preferences → binary relations



Graph Model for DM 1



Graph Model for DM 2

Graph Form of a Conflict

Chicken

More formally, let $N = \{1, 2, \ldots n\}$ denote the set of DMs and $S = \{s_1, s_2, \ldots s_k\}$ denote the set of states

The conflict is modelled by a set of finite directed graphs $D_i = (S, A_i)$ for $i \in N$ where

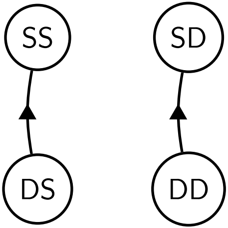
- the set of vertices S is the set of possible states of the conflict
- if DM i can (unilaterally) move (in one step) from state s_k to s_q, there is an directed arc from s_k to s_q in A_i
- there are no loops in any player's graph

We write $s_k \prec_i s_j$ if DM i prefers state s_k over state s_j and $s_k \sim_i s_j$ if DM i is indifferent between the two

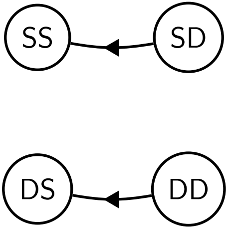
Graph Form of a Conflict

Irreversible Moves

In Chicken, if we assume that DMs cannot move back after swerving, we say that swerving constitutes an **irreversible move**



Graph Model for DM 1



Graph Model for DM 2

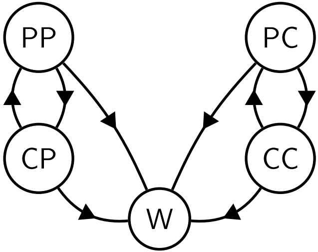
Irreversible moves are difficult to represent in normal or option form but simple to represent in graph form

Graph Form of a Conflict

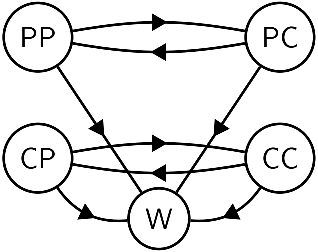
Common Moves

A **common move** occurs when two or more DMs can independently make unilateral moves that cause the game to change from one state to exactly the same other state

Assume that DM 1 and DM 2 have the same three strategies: peace (P), conventional attack (C), and full nuclear war causing nuclear winter (W)



Graph Model for DM 1



Graph Model for DM 2

Graph Form of a Conflict

Advantages

Can handle irreversible moves

Can handle common moves

Simple way to represent complex conflicts

Next Lecture

- Review
- Background Report due on October 14
- Test 1 in class on October 17