Spring

Syde 312

I - Vector Spaces

I.1 Vector Spaces & Subspaces

Motivation : Familiar with vectors as geometric & algebraic
(u_1, u_2, u_3)

Know how to { add two vectors
take scalar multiple of say a real # times a vector

→ // new vectors of the same type

useful properties are satisfied

Now make this more general

V = set of vectors → anything !

F = set of scalars → usually $\mathbb{R}$ or $\mathbb{C}$ sometimes {0,1} ← binary arithmetic

Define vector sum

$u, v \in V \rightarrow u + v \in V$ ← vector

& scalar multiple

$k \in F, v \in V \longrightarrow kv \in V$ (k ← scalar)

Need rules to make these useful. ~~Motd~~ Modelled on vectors in $\mathbb{R}^3$)

Reduced to a list of axioms

$u, v, w \in V$
$k, l, m \in F$

S1 $u+v = v+u$ → commutative

S2 $(u+v)+w = u+(v+w)$ → associative

S3 There is a vector $\underline{0} \in V$ so that $u+\underline{0} = u$
↳ additive identity / zero vector

S4 For all $u \in V$ there is a vector $-u \in V$ so that

$$u + (-u) = \underline{0}$$

M1 $k(u+v) = ku + kv$
vector distributive

M2 $(k+l)u = ku + lu$ scalar sum distributive

M3 $k(lu) = (kl)u$ scalar mult. associative

M4 $1u = u$
↳ scalar 1