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Univariate Root finding with Derivatives

- NOT Derivative free
- One such method is Newton's Method (Newton - Raphson)
- Newton-Raphson (NR) is not an enclosure method

- Given some $x_0 \in \mathbb{R}$
  For $k = 0, 1, \ldots\ldots$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)} = x_k - \frac{f_k}{f'_k}$$

where did this come from?

Consider the linear function $t(x)$ that goes through the point (x_k, f_k) and whose slope equals $f'(x_k)$

$$t_k(x) = f_k + (x - x_k)\, f'(x_k)$$

$\boxed{\text{Review taylor expansion}}$

Set $t_k(x) = 0 \Rightarrow x = x_k - f_k / f'_k$

N-R is "locally convergent"
if $f \in C^2(\mathbb{R})$ AND r is an isolated root of $f(x) = 0$ (i.e. $m = 1$) then $\exists\, \delta > 0$

such that if $|x_0 - r| \leq \delta$ the $\{x_k\}_{k=0}^{\infty} \to r$