January 26, 2016 8:34 AM

System Stab; lity

V(s) N(s)

System poles are given by roots (zeros) of 6(5)

Note: For Stability, we only care if poles ore m LHP us RHP, not exact locations

Routh - Harwitz Criteria

Step 1

Expand  $\delta(s) = 0$ ; into the form  $\delta(s) = a_n s^n + ... + a_n s + a_n$ 

Step 2
Apply the Hurwitz Test - Examine the signs of all
the coefficients of any as

If the coefficients are not all the same sign, or if some are missing, then there will be roots of  $\triangle(s)$  in the RHP

If all the cofficients are non-zero and the same sign the system may still be unstable

 $E_{X}$   $\Delta(S) = S^{3} + \frac{1}{2} S^{2} + \frac{7}{2} S + 4$   $= \frac{(S+1)}{LHP} \left( S - \frac{1}{4} + \frac{1}{3} \frac{1}{4} \right) \left( S - \frac{1}{4} - \frac{1}{3} \frac{1}{4} \right)$   $\therefore This System is unstable$ 

Step 3 Use the Rowth Criterion

3a) Construct the Routh array

For (S) = an 5" + ... + a, S + a.

Form:				_					
s <sup>~</sup>	an.	Gn. 2. 7	٨,	~	row	C	Note	Si.	
52-1	ا ممر	Ga - 5	٠, ۵،	_	CON	1		1	
5 n-2	5, ;	$S_{2,2}$ $S_{2,2}$		4	rew	5	Row	Col	
:	82,	8 32 83							
٤,	,	ν,	. •						
۲,	'	1							
	1	Pirat C	olumn						
		· _							

Where  $S_{i,i} = -\underbrace{\left(S_{i-2,1}, S_{i-1,i+1} - S_{i-1,1} \cdot S_{i-2,i+1}\right)}_{S_{i-1,1}}$ 

36) Apply the Routh criterion:

The characteristic polynomial  $\Delta(s)=0$  has roots in the RHP or on the jw axis : F there are any zeros or sign changes in the privid column of the Routh array

 $\triangle(s) = \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_2 = 0$ ,  $\alpha_i > 0$ , i = 1, 2, 3, 4

This passes the Hurwitz test, so construct the Ruth array

S	$A_3$ , $A_1$ , $O$ $A_2$ , $A_0$ , $O$ $A_2$ , $A_0$	S2.1 - a.a2 - a3a.
S2	a2 100 0	2.1 A2
s '	S2 S2=0	•
ς °	8 = 0.	S22 = 0
	2,1 0 .	2,2

Examine the privat column as>0, a>0, a>0 So for Stability:

821>1

axa:

is required

 $E_{x}$  $\Delta(s) = a_{3}s^{2} + a_{1}s + a_{0}$  Where  $a_{0} > 0$ , i = 0, 1, 2

Ex 6(5) = 84 + 1083 + 82 + 155 +3

This passes the Hurwitz test, so form the Ruth array

$$S^{\circ}$$
 | | | 3  
 $S^{\circ}$  | 0 | 15 | 0 |  $S_{31} = (-\frac{1}{2})(15) - (\frac{1}{2})(16)$   
 $S^{\circ}$  | 75 |  $= -\frac{7}{5} - \frac{30}{2}$   
 $S^{\circ}$  | 3 |  $= -\frac{7}{5}$ 

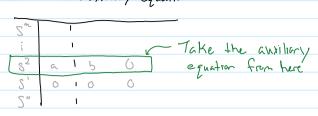
The pivot column has 2 sign changes so the system is unstable.

The actual roots of 6(s) are:
-0.1976
-10.05
+6.1218 ± 51.2234

More Information from the Kath array:

1) The # of roots of 4(5) in the RHP equals the number

ii) A row of zeros in the Routh array implies roots located synctrically about the origin. These roots can be found from the auxiliary equation:



$$\triangle_{\lambda}(s) = \alpha s^{2} + b s^{\circ} = 0$$