I-3 (cont) Linear Independance & Bases

Find a basis of $W < V$

1) Row space method

$W = Sp\{u_1, \dots, u_r\}$ say

Put $\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_r \end{bmatrix}$ a matrix

W = Row space of this
row ops → echelon form
no change in span
⇒ basis is non zero rows of echelon form

Exer $B = \{(1,1,1), (1,1,0), (1,0,0)\}$

$B' = \{(1,0,1), (1,1,0), (0,1,1)\}$

Find $V_{B'}$ give $V_B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}_B$ and V_S with a) scalars R
b) scalars binary.

2) Col space method.

To extract a basis for
$W = Sp\{u_1, \dots, u_n\} < R^m$ need to identify the linear dependencies in $\{u_1, \dots, u_n\}$

→

Say we have

$k_1 u_1 + \dots + k_n u_n = 0$ linear dependency

write as

$$\begin{bmatrix} u_1 & u_2 & \dots & u_n \end{bmatrix} \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$$

zero vector

The scalars required are soln of the homogenous ~~sol~~ sys

$$A\underline{k} = \underline{0}$$

Row reduction of A to echelon form does <u>not</u> change the solns $(k_1, \dots, k_n)$.

- change the dependencies b/w the colns

Identify the independant cols in each matrix ✓ use pivots

- Same cols in A are also independant
  ↳ required basis of w

Ex Find the basis for

$$W = Sp\{(1,-2,5,-3), (2,3,1,-4), (3,8,-3,-5)\} < R^4$$

= using the col sp method ~~consisting~~

$\boxed{\text{OR}}$

consisting only of original vectors given.

soln.

$$\begin{bmatrix} 1 & 2 & 3 \\ -2 & 3 & 8 \\ 5 & 1 & -3 \\ -3 & -4 & 5 \end{bmatrix} \sim \cdots \sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

independant $\Rightarrow$ basis of col sp $\{c_1', c_2'\}$

$c_3' = -c_1' + 2c_2'$

$\Rightarrow \{c_1, c_2\}$ is a basis for the col sp of A

Required basis of W is

$\{(1, -2, 5, -3), (2, 3, 1, -4)\}$

Ex Same problem with.

$W = sp\{(1,2,3), (-1,0,2), (-1,2,7)\}$

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & 2 \\ 3 & 2 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

row canonical.

$c_3' = c_1' + 2c_2'$

$\Rightarrow c_3 = c_1 + 2c_2$

Basis of W is $\{c_1, c_2\}$ or $\{(1,2,3)(-1,0,2)\}$

Imp Result

$$\boxed{\begin{aligned} &\text{rank}(A) = \text{rank}(A^T) \\ &\dim(\text{row sp}(A)) = \dim(\text{col space}(A)) \end{aligned}}$$

ISOMORHISM

= sameness

There is one to one correspondence b/w vectors in a n-dimensional vector space V over $\mathbb{R}$ & vectors in $\mathbb{R}^n$.

How? choose a basis B of V & use the coordinate map

$$v \in V \rightarrow [v]_B \in \mathbb{R}^n$$

↑ unique!!!

Can use this idea to do calcs in $\mathbb{R}^n$ instead of a difference vector space

Ex Find a basis for

$W = sp\{p_1, p_2, p_3, p_4\} < P_3$ $\qquad S = \{t^3, t^2, t, 1\}$

$p_1(t) = t^3 - 2t^2 + 4t + 1$ $\qquad$ & $p_1 \rightarrow [P_1]_S \rightarrow (1, -2, 4, 1)$

$p_2(t) = 2t^3 - 3t^2 + 9t + 1$ $\qquad$ $[P_2]_S \rightarrow (2, -3, 9, 1)$

$p_3(t) = t^3 + 6t - 5$ $\qquad$ $[P_3]_S \rightarrow (1, 0, 6, -5)$

$p_4(t) = 2t^3 - 5t^2 + 7t + 5$ $\qquad$ $[P_4]_S \rightarrow (2, -5, 7, 5)$

use row sp method

$$\begin{bmatrix} 1 & -2 & 4 & 1 \\ 2 & -3 & 9 & 1 \\ 1 & 0 & 6 & -5 \\ 2 & -5 & 7 & 5 \end{bmatrix} \sim \dots 2 \begin{bmatrix} \boxed{1} & -2 & 4 & 1 \\ 0 & \boxed{1} & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ Basis in $\{R_1, R_2\}$

$$\rightarrow \{t^3 - 2t^2 + 4t + 1,\ t^2 + t - 3\}$$