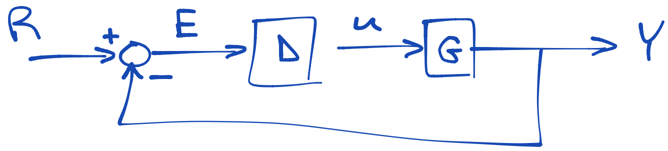


For a PID controller,

$$D(s) = K_p + K_D s + \frac{K_I}{s}$$

$$= \frac{K_D s^2 + K_p s + K_I}{s}$$

$$= \frac{K_D (s+z_1)(s+z_2)}{s}$$

$$= K_D \frac{N(s)}{D(s)}$$

$$\frac{Y}{R} = \frac{DG}{1+DG} = \frac{K_D N_D N_G}{D_D D_G + K_D N_D N_G}$$

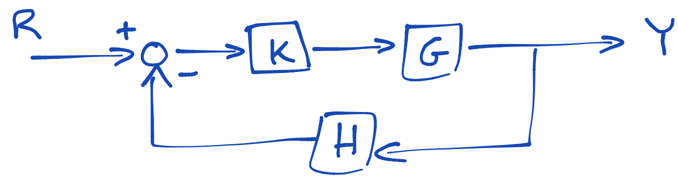
The system characteristic equation is:

$$\Delta(s) = D_D D_G + K_D N_D N_G = 0$$

How do the poles of $\frac{Y}{R}$ change as K_D changes?
Use the Root Locus Method to illustrate how the roots of $\Delta(s)$ change as K_D varies

Root Locus Method

Represent the system in the form:



$$\frac{Y}{R} = \frac{KG}{1 + KGH}$$

$$\Delta(s) = 1 + KG(s)H(s) = 0$$

$$KG(s)H(s) = -1$$

Magnitude Condition:

$$|KG(s)H(s)| = 1$$

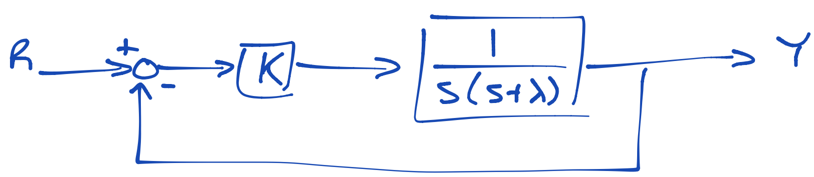
Phase Angle Condition:

$$\angle KG(s)H(s) = \pi(2\ell + 1)$$

$$\ell = 0, \pm 1, \pm 2, \ldots$$

Find the values of s that satisfy both conditions as K varies from 0 to ∞

ex.



$$\frac{Y}{R} = \frac{\frac{K}{s(s+\lambda)}}{1 + \frac{K}{s(s+\lambda)}} = \frac{K}{s^2 + s\lambda + K}$$

closed-loop transfer function

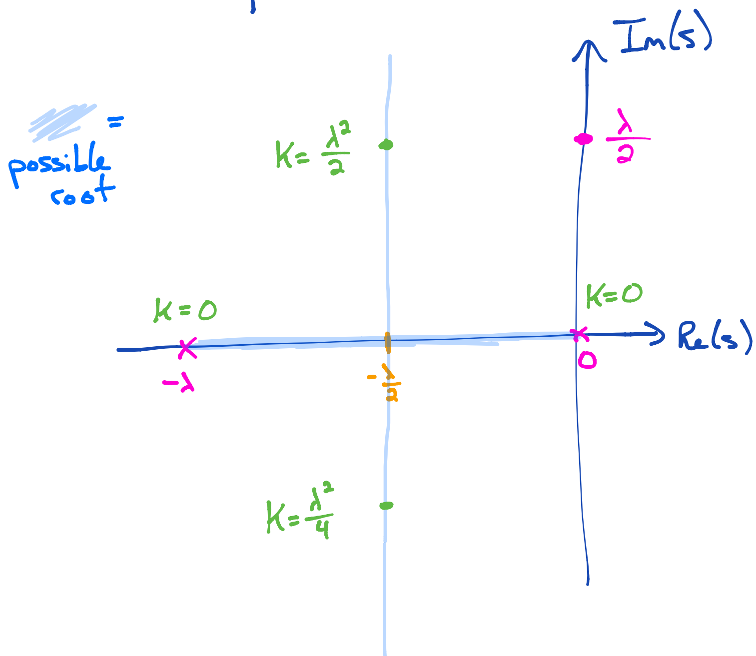
$\rightarrow$ poles @ $s_{1,2} = \frac{-\lambda}{2} \pm \sqrt{\frac{\lambda^2}{4} - K}$

also $KGH = \frac{K}{s(s+\lambda)}$ $\rightarrow$ poles $\{0, -\lambda\}$

open-loop transfer function

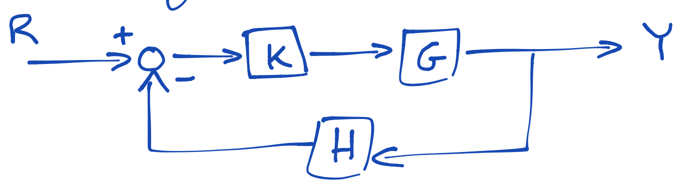
Plot the closed-loop poles in the s-plane as K varies from $0 \to \infty$

Note that where $K=0$, the C.L. poles equal the O.L. poles



Root Locus Construction Procedure

1) Put the system in the form



2) Find the O.L. transfer function

$$KGH(s)$$

3) Recall the Magnitude & phase conditions

$$|KGH(s)| = 1$$

$$\angle KGH(s) = \pi(2\ell+1) \quad , \ell = 0, \pm 1, \pm 2, \ldots$$

Satisfy the phase condition to draw the Root Locus

4) Draw the Root Locus

4a) Plot all open loop poles and zeros

4b) Sketch the Root Locus following the construction Rules

Rule 1

The number of branches of the R.L. is equal to the number of O.L. poles

Rule 2

The R.L. starts $(k=0)$ at each of the O.L. poles

Rule 3

The root locus stops at the O.L. zeros $(k=\infty)$

Rule 4

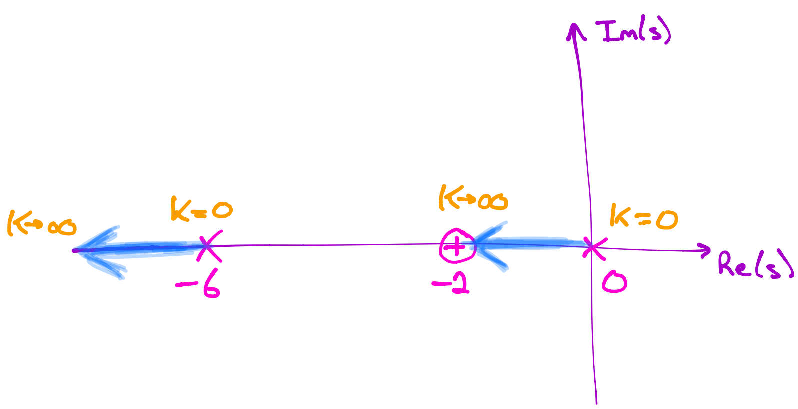
The R.L. exists on the real axis everywhere there is an odd number of poles and/or zeros to the right

Rule 5

The R.L. is symmetric through the real axis

ex. $$KG(s)H(s) = \frac{K(\frac{1}{2}s+1)}{s(s+6)}$$

O.L. zero $\{-2\}$ O.L. poles $\{0, -6\}$



Rule 6

Breakaway occurs at a local maximum of K.
Reentry occurs at a local minimum of K.
At breakaway and reentry, the branches meet at an angle of $\frac{\pi}{\alpha}$, where α is the number of branches

From $1 + KG(s)H(s) = 0 \longrightarrow K = \frac{-1}{G(s)H(s)}$

max/min @ $\frac{dK}{ds} = 0$

$$\frac{d}{ds}\left(\frac{-1}{GH(s)}\right) = 0 \qquad = \frac{\left(\frac{dGH}{ds}\right)}{[GH(s)]^2} = 0$$

equivalent

$$\frac{dGH(s)}{ds} = 0$$

The values that satisfy $\frac{dGH(s)}{ds} = 0$ will be the breakaway and reentry points, but only if they are on the R.L.

ex. $KGH(s) = \dfrac{K}{(s+1)(s+2)}$

$$\frac{d}{ds}\left(\frac{-1}{GH}\right) = \frac{d}{ds}\left(\frac{(s+1)(s+2)}{-1}\right) = 0$$

$$\frac{d}{ds}\left(s^2+3s+2\right) = 0$$

$$2s+3=0$$

$$s = -3/2$$

$$K = \frac{-1}{GH(-3/2)}$$

maximum - <u>breakaway</u>