12/01/16 352 Lecture 3

Transfer Functions & Block Diagrams

Consider a general system described by an ODE of the form

$$\sum_{i=0}^{n} a_i \frac{d^i y(t)}{dt^i} = \sum_{k=0}^{m} b_k \frac{d^k u(t)}{dt^k} \qquad m \le n \qquad (1)$$

$y(t) \Rightarrow$ output $\qquad u(t) \Rightarrow$ input

Assume a quiescent system $\Rightarrow$ all the initial conditions are zero.

Take the Laplace Transform of (1)

$$\Rightarrow \underbrace{\sum_{i=0}^{n} a_i s^i Y(s)}_{\text{Polynomial of degree } n} = \underbrace{\sum_{k=0}^{m} b_k s^k U(s)}_{\text{Polynomial of degree } m} \qquad (2)$$

write (2) as

$$D(s)Y(s) = N(s)U(s)$$

$$Y(s) = \frac{N(s)}{D(s)} U(s)$$

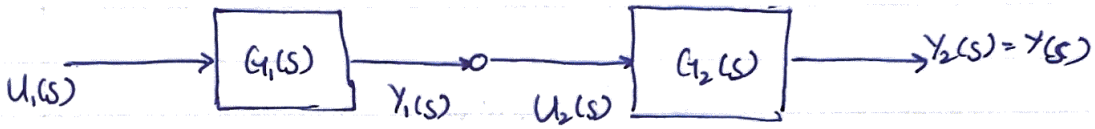
or

$$\underbrace{\frac{N(s)}{D(s)}}_{\text{system Transfer function}} = \frac{Y(s)}{U(s)} = T(s)$$

from $U(s)$ to $Y(s)$

Block diagram Algebra

① Cascade



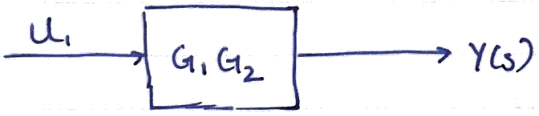
$$Y_1(s) = U_2(s)$$

$$Y_1(s) = G_1(s)\, U_1(s) = U_2(s)$$

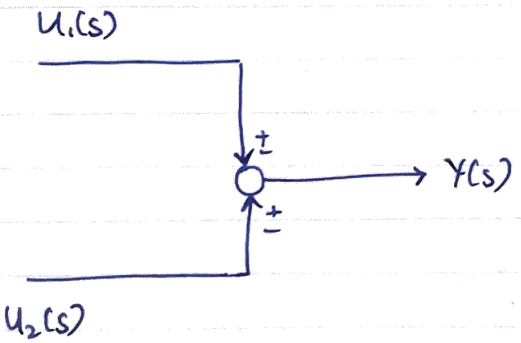
$$Y_2(s) = G_2(s)\, U_2(s)$$
$$= G_2(s)\, G_1(s)\, U_1(s)$$

$$\frac{Y}{U_1} = G_1(s)\, G_2(s)$$

$\therefore$

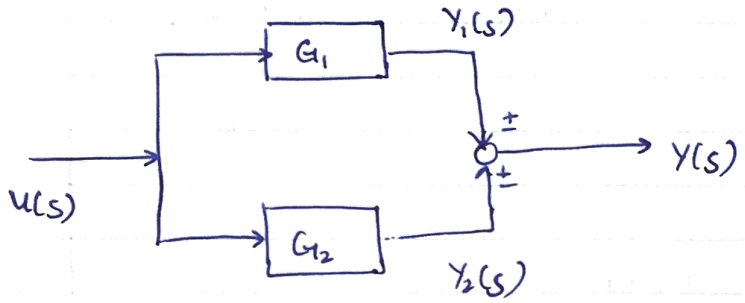


② Summing Junction

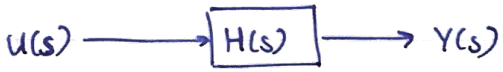


$$\pm U_1(s) \pm U_2(s) = Y(s)$$

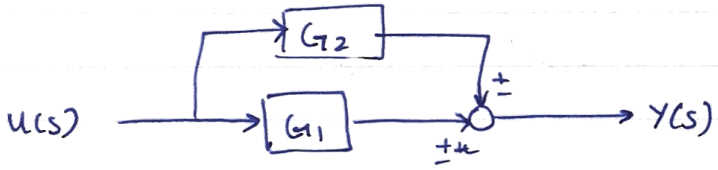
3) Blocks in Parallel (Feed forward)



find



note this is equivalent to



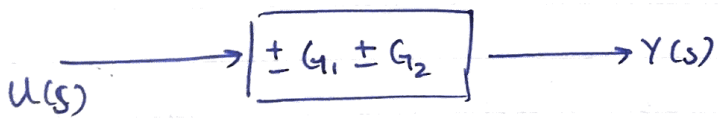
Find $\frac{Y}{U}$

$Y_1 = G_1 U \qquad Y_2 = G_2 U$

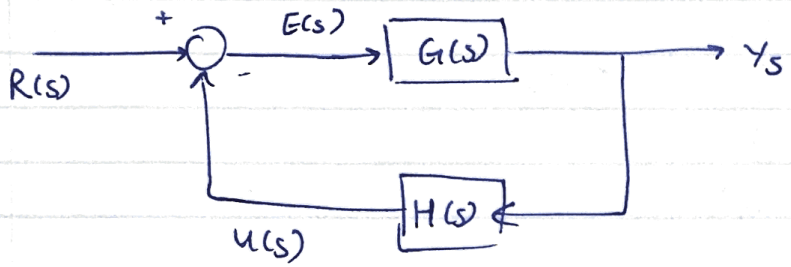
$Y(s) = \pm Y_1(s) \pm Y_2(s)$

$= \pm G_1 U \pm G_2 U$

$= (\pm G_1 \pm G_2) U$



4) Feedback loop



Find $\frac{Y}{R}$ from,

$Y(s) = G(s)\,E(s)$ — ①

$E(s) = R(s) - U(s)$ — ②

$U(s) = H(s)\,Y(s)$ — ③

$E(s) = R(s) - H(s)\,Y(s)$ — ④

$Y(s) = G(s)\,(R(s) - H(s)\,Y(s))$

$Y(s) = G\,(R - HY)$

$Y + GHY = GR$

$(GH + 1)\,Y = GR$

$$\frac{Y}{R} = \frac{G}{GH+1} \quad \text{or} \quad Y(s) = \frac{G(s)}{G(s)H(s)+1}\,R(s)$$

5) Unit Feedback Form

Given

Find $G_{eq}(s)$ such that

For equivalence

$$\frac{Y(s)}{R(s)} = T(s) = \frac{G_{eq}(s)}{1+G_{eq}(s)}$$

Solve for $G_{eq}(s)$

$$T(1+G_{eq}) = G_{eq}$$

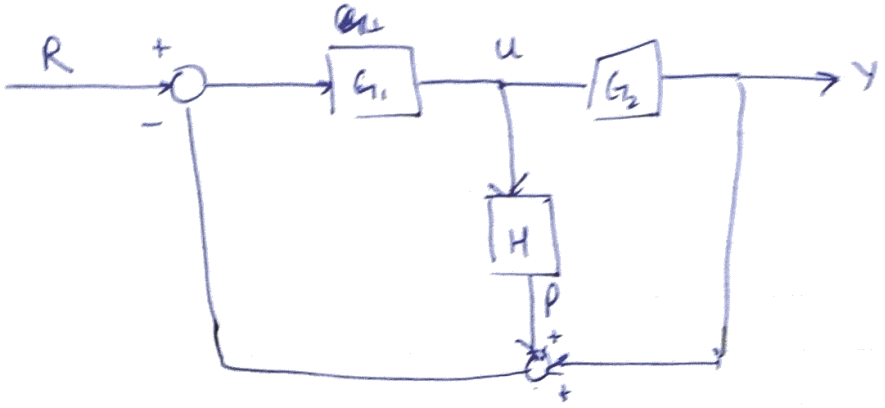
$$T = G_{eq} - G_{eq}T$$

$$T = (1-T)G_{eq}$$

$$G_{eq} = \frac{T(s)}{1-T(s)}$$

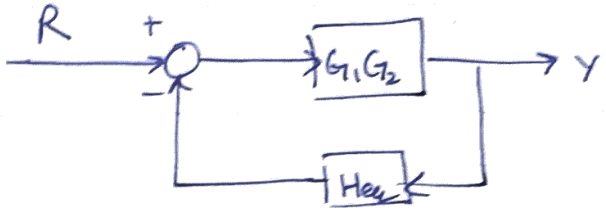
Example

Given

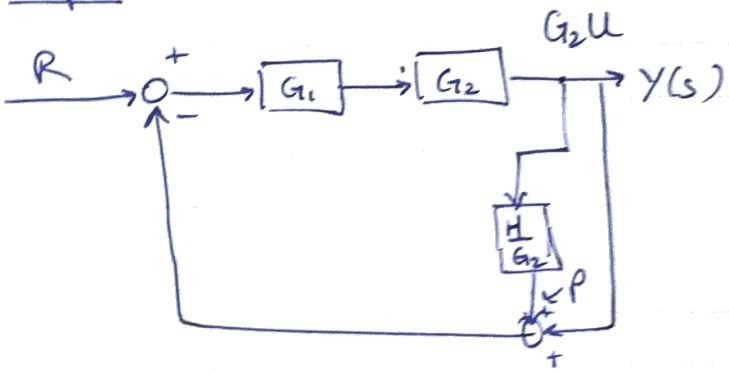


$P(s) = H(s)\,U(s)$

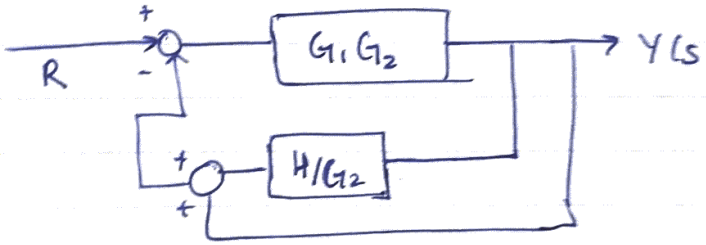
a) Find H_{eq} such that



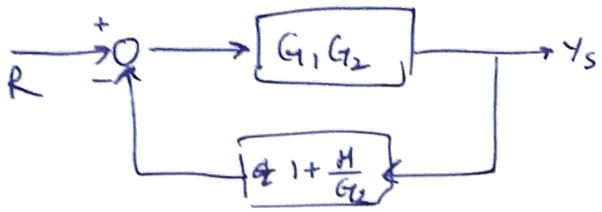
Step 1



Step 1.5

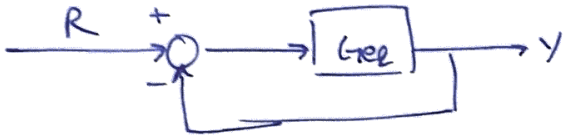


Step2 combine the blocks in parallel



use the equivalence of $\frac{Y}{R}$

b) Find the unit feedback from $G_{eq}(S)$



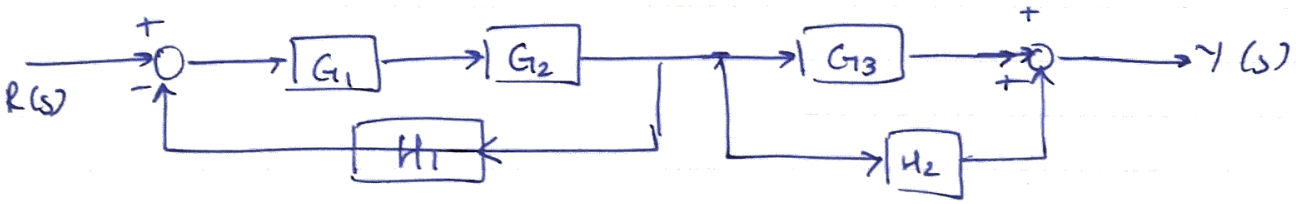
use the equivalence of Y/R

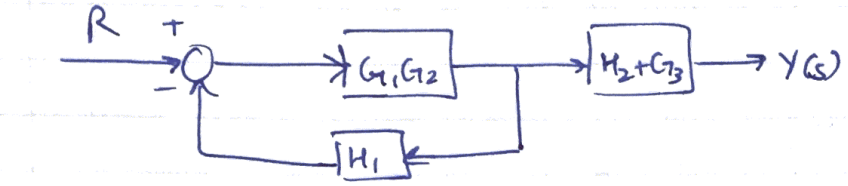
$$\frac{Y}{R} = \frac{G_{eq}}{1+G_{eq}} = \frac{G_1G_2}{1+G_1G_2\left(1+\frac{H}{G_2}\right)} = T(S)$$

solve for G_{eq}

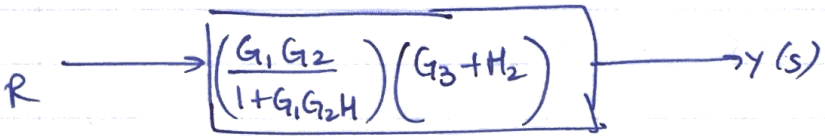
$$G_{eq} = \frac{G_1G_2}{1+G_1H} = \frac{T(S)}{1-T(S)}$$

Find Y/R for




R
+
−
G_1G_2
H_2+G_3
Y(s)
H_1


R
+
−
$\frac{G_1G_2}{1+G_1G_2H_1}$
G_3+H_2
Y(s)


R
$\left(\frac{G_1G_2}{1+G_1G_2H}\right)\left(G_3+H_2\right)$
Y(s)