519509

~~0 + 1 + 4 + 8 = 13~~

0 + 2 + 4 + 8 = 14

SYDE 411 Tutorial 1 15/09/16

Rootfinding - Background

Given f, find x such that $f(x)=0$

Given (x_1, f_1) & (x_2, f_2), we want to find the root of the line interpolating these points.

Method 1

Solve $f_1 = ax_1 + b$
$f_2 = ax_2 + b$ for a & b

→ Root would be $x^* = \frac{-b}{a}$

Given (x_1, f_1), (x_2, f_2) & (x_3, f_3). We want to find the root of the quadratic interpolating these points.

Method 1 Solve

$$f_1 = ax_1^2 + bx_1 + c$$
$$f_2 = ax_2^2 + bx_2 + c \quad \text{for } a, b \text{ \& } c$$
$$f_3 = ax_3^3 + bx_2 + c$$

$$\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix} = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

→ Roots are $x^* = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Back to the original problem of two pts

Alternate to Method 1

Use Lagrange interpolating polynomials.
Given: (x_1, f_1), (x_2, f_2)

Lagrange polynomial

$$\cancel{L(x) = \frac{f_1}{x_1 - x_2}(x - x_2) + f_2 (x - x_1)}$$

$$l(x) = f_1 \underbrace{\frac{(x - x_2)}{(x_1 - x_2)}} + f_2 \frac{(x - x_1)}{(x_2 - x_1)}$$

- Linear x
- 1 @ x_1
- 0 @ x_2

To find the root, $l(x) = 0$

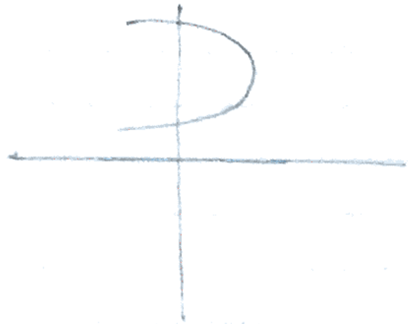
$$\Rightarrow f_1 \frac{(x - x_2)}{(x_1 - x_2)} + f_2 \frac{(x - x_1)}{(x_2 - x_1)} = 0$$

$$\Rightarrow \frac{f_1 (x - x_2) - f_2 (x - x_1)}{(x_1 - x_2)} = 0$$

$$\Rightarrow (f_1 - f_2) x = f_1 x_2 - f_2 x_1$$

$$\Rightarrow x^* = \frac{f_1 x_2 - f_2 x_1}{f_1 - f_2}$$

Inverse Quadratic



inverse quadratic is a quadratic on its side, opens left or right

quadratic: $y = ax^2 + bx + c$
inverse quadratic: $x = a'y^2 + b'y + c'$

inverse quadratic

$$\left.\begin{aligned} x_1 &= a'f_1^2 + b'f_1 + c' \\ x_2 &= a'f_2^2 + b'f_2 + c' \\ x_3 &= a'f_3^2 + b'f_3 + c' \end{aligned}\right. \Rightarrow a', b', c'$$

Alt Method
Lagrange for quadratic

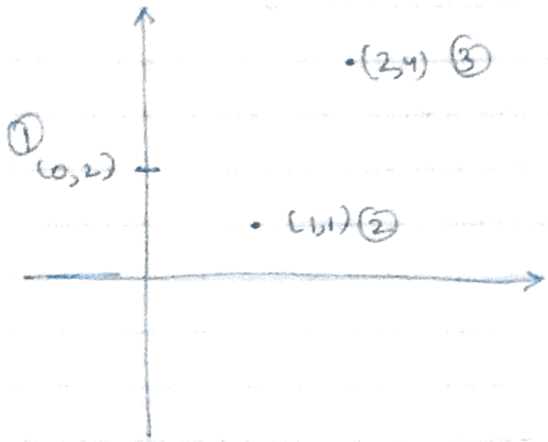
$$q(x) = f_1 \frac{(x-x_2)(x-x_3)}{(x_1-x_2)(x_1-x_3)} + f_2 \frac{(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_3)} + f_3 \frac{(x-x_1)(x-x_2)}{(x_3-x_1)(x_3-x_2)}$$

inverse quadratic for Lagrange

$$q(y) = x_1 \frac{(y-f_2)(y-f_3)}{(f_1-f_2)(f_1-f_3)} + x_2 \frac{(y-f_1)(y-f_3)}{(f_2-f_1)(f_2-f_3)} + x_3 \frac{(y-f_1)(y-f_2)}{(f_3-f_1)(f_3-f_2)}$$

$\longrightarrow$

Example



$$q(x) = 2\,\frac{(x-1)(x-2)}{(0-1)(0-2)} + 1\,\frac{(x-0)(x-2)}{(1-0)(1-2)} + 4\,\frac{(x-0)(x-1)}{(2-0)(2-1)}$$

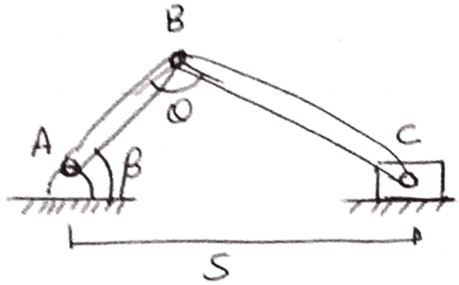
$$= (x^2-3x+2) + (-x^2+2x) + (2x^2-2x)$$

$$= 2x^2-3x+2$$

similarly $q(y) = \frac{2}{3}y^2 - 3y + \frac{10}{3}$

411 Tutorial 2

Ex: Slider Crank



Coords : $\underset{\sim}{q} = \begin{bmatrix} \beta \\ \theta \\ S \end{bmatrix}$

Constraints

Sine Law: $\dfrac{BC}{\sin\beta} = \dfrac{S}{\sin\theta}$

Cosine Law: $S^2 = AB^2 + BC^2 - 2\,(AB)(BC)\cos\theta$

$$\Phi = \begin{Bmatrix} BC\sin\theta - S\sin\beta \\ S^2 - AB^2 - BC^2 + 2(AB)BC\cos\theta \end{Bmatrix} = 0$$

suppose $\beta(t)$ is given for $t>0$

Then using $\beta_0 = \beta(0)$, we can solve

$$\Phi(\beta_0, s, \theta) = 0 \quad \text{for } s, \theta$$

To solve for $s(t)$, $\theta(t)$ discretize time

For each time t_i,
solve $\Phi(\beta(t_i), s, \theta) = 0$

for s, θ to get $s(t_i)$ & $\theta(t_i)$

* we can use our previous timestep as the initial guess

ODEs

In general,

$$F(t, x(t), \dot{x}(t)) = 0 \quad \underline{\omega} \quad x(0) = x_0$$

eg. $\dot{x} + \ln \dot{x} = 0$

To solve (approximately)

Discretize time, $t_0, t_1, \dots$

For each t solve $F\left(t_{i+1}, x(t_{i+1}), \frac{x(t_{i+1}) - x(t_i)}{\Delta t}\right) = 0$

Supose the DE is of the form

$$\dot{x} = f(t, x)$$

Thevy Euler $x(t_{i+1}) = x(t_i) + \Delta t \cdot f(t_i, x(t_i))$

Implicit Euler $x(t_{i+1}) = x(t_i) + \Delta t \cdot f(t_{i+1}, x(t_{i+1}))$

to solve we need root finding