21 Jan 2016

352 — Lecture 6

2nd Order Systems

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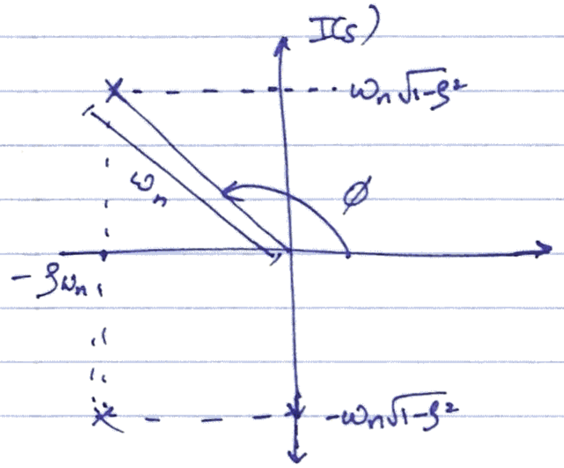
$$\frac{Y}{R} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

assume $0 \le \zeta \le 1$

system poles:

$$s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2} = -\zeta\omega_n \pm j\omega_d$$



in polar form

$$s_{1,2} = R\cos\phi + jR\sin\phi$$

$$R = |s_{1,2}| = \sqrt{(\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)} = \omega_n$$

$$\phi = \tan^{-1}\left(\frac{\omega_n\sqrt{1-\zeta^2}}{-\omega_n\zeta}\right)$$

$$\Rightarrow \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)$$

<u>Ex</u> Assume a <u>canonical</u>? 2nd order system with a unit step input

$$\frac{Y}{R} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \qquad R = \frac{1}{s}$$

with the perf characteristics/requirements.

P.O. $\le 16\%$ $\qquad$ 10% − 90% $T_{R_1} \le 1.8s$ (rise time)

Find the region of the s-plane where the poles may be placed to satisfy these conditions.

→

Step 1 Convert the perf requirements to conditions on ζ & ω_n

$$P.O. = 100 \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \le 16\%$$

solve for ζ:

$$\frac{\zeta}{\sqrt{1-\zeta^2}} \ge \frac{-\ln(0.16)}{\pi} = 0.5833$$

$$\frac{\zeta^2}{1-\zeta^2} \ge 0.3403$$

$$\Rightarrow \zeta^2 = \frac{0.3403}{1.3403} \qquad \zeta \ge 0.5039 \quad (1)$$

Rise Time requirement

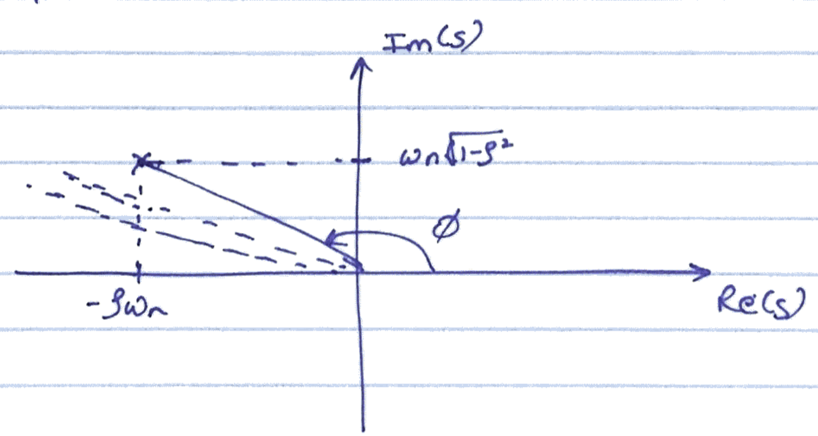
$$T_{R,\,6} = \frac{1.8}{\omega_n} \le 1.8$$

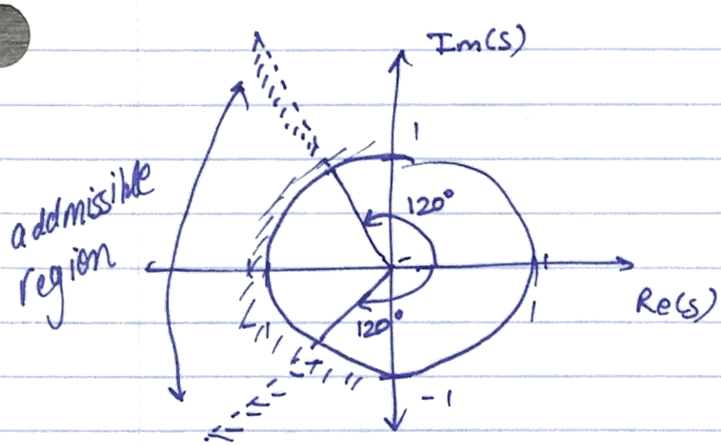
$$1 \le \omega_n \quad \Rightarrow \omega_n \ge 1 \quad (2)$$

From (1) the phase angles of the poles must satisfy

$$\phi \ge \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)\Bigg|_{0.5039} = \tan^{-1}(-1.714)$$

$$\Rightarrow \phi \ge 120^\circ$$





Question A: what happens to the overshoot for poles on the real axis.

→ as $\zeta \to 1$ P.O. → 0

b/c

$$P.O. = 100 \exp\left(\frac{-\zeta \pi}{\sqrt{1-\zeta^2}}\right)$$

~~From ① the phase ∠ of the poles must satisfy~~

~~$\phi \geq \tan^{-1}\left(\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right)\right)$~~

~~Interesting~~ Increasing ζ reduces overshoot

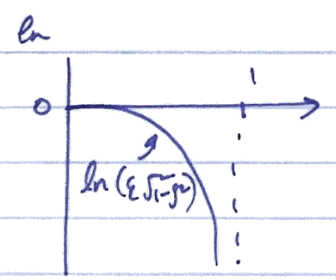
Question B How do you reduce rise time.

$$T_{R_1} = \frac{1.8}{\omega_n} \qquad T_{R_3} = \frac{1}{\omega_d} \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right)$$

Ans: increase ω_n

Question c How do you reduce settling time

$$T_s = -\left(\frac{1}{\zeta \omega_n}\right) \ln\left(\varepsilon \sqrt{1-\zeta^2}\right)$$



Ans: Increase ζ to increase T_s

Increase ω_n to decrease T_s

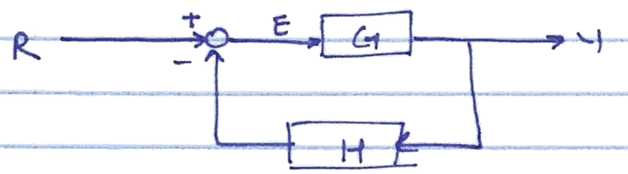
→

~~Ha~~ QuestionD : How do we reduce peak time?

$$T_p = \frac{\pi}{\omega_d}$$

Ans: inc ω_d by moving the poles away from the real axis (this will also lead to a higher frequency of oscillation)

<u>Stability</u>



$$G = \frac{N_G(s)}{D_G(s)} \quad ; \quad H = \frac{N_H(s)}{D_H(s)}$$

$$\frac{Y}{R} = \frac{G}{1+GH} = \frac{N_G D_H}{D_G D_H + N_G N_H}$$

closed loop poles are given

$\Delta(s) = D_G D_H + N_G N_H = 0$

Characteristic equation for the slosed loop system, The zeroes of $\Delta(s)$ are the ~~sol~~ poles of the closed loop system.