SYDE 533 Conflict Resolution

Basics of 2-DM Conflicts

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Learning Objectives

By the end of this lesson, you will be able to:

- List and explain the steps in modelling
- Conduct stability analysis and calculations using the tableau form of a conflict
- Identify several types of stable outcomes and explain their relation to human behaviour

Steps in Modelling

- Select a point in time
- Choose players and options
- Remove infeasible outcomes
- Develop preference rankings for each player
- Carry out stability analysis

Select a Point in Time

Cuban Missile Crisis

- 1959: Castro overthrows Batista regime in Cuba
- American property in Cuba is nationalized
- 1961: American-sponsored Bay of Pigs invasion fails
- 1962: USSR installs nuclear missiles in Cuba
- Americans want USSR missiles out of Cuba

We will analyse the conflict situation in October 1962

Choose Players and Options

Cuban Missile Crisis

U.S.		
Air strike	1	American
Blockade	0	Strategy
U.S.S.R.		
Withdraw	1	Russian
Escalate	0	Strategy

How many possible states are there?

Remove Infeasible States

Cuban Missile Crisis

The Russians will not withdraw their missiles and escalate at the same time.

U.S.						
Air strike	0	1	0	1	$\rightarrow$	-
Blockade	0	0	1	1	$\rightarrow$	-
U.S.S.R.						
Withdraw	1	1	1	1	$\rightarrow$	1
Escalate	1	1	1	1	$\rightarrow$	1

In the compact notation, a dash " - " means a 1 or a 0

Remove Infeasible States

Cuban Missile Crisis

After removing the infeasible options, we are left with 12 feasible states:

U.S.												
Air strike	0	1	0	1	0	1	0	1	0	1	0	1
Blockade	0	0	1	1	0	0	1	1	0	0	1	1
U.S.S.R.												
Withdraw	0	0	0	0	1	1	1	1	0	0	0	0
Escalate	0	0	0	0	0	0	0	0	1	1	1	1
Decimal	0	1	2	3	4	5	6	7	8	9	10	11

Decimal Conversion for states

Example: State 11

U.S.	
Air strike	1
Blockade	1
U.S.S.R.	
Withdraw	0
Escalate	1

Read from top to bottom:
$(1 \cdot 2^0) + (1 \cdot 2^1) + (0 \cdot 2^2) + (1 \cdot 2^3) = 1 + 2 + 8 = 11$

What is the decimal expression of state (0 1 1 1)?

Preference Rankings for U.S.

Cuban Missile Crisis

Preferences for US from most to least preferred (left to right):

U.S.												
Air strike	0	0	1	1	0	1	1	0	1	1	0	0
Blockade	0	1	0	1	1	0	1	0	1	0	1	0
U.S.S.R.												
Withdraw	1	1	1	1	0	0	0	0	0	0	0	0
Escalate	0	0	0	0	0	0	0	0	1	1	1	1
Decimal	4	6	5	7	2	1	3	0	11	9	10	8

Preference Rankings for U.S.S.R.

Cuban Missile Crisis

Preferences for USSR from most to least preferred (left to right):

U.S.												
Air strike	0	0	0	0	1	1	1	1	1	1	0	0
Blockade	0	0	1	1	0	0	1	1	1	0	1	0
U.S.S.R.												
Withdraw	0	1	1	0	1	0	1	0	0	0	0	0
Escalate	0	0	0	0	0	0	0	0	1	1	1	1
Decimal	0	4	6	2	5	1	7	3	11	9	10	8

Stability Analysis

Cuban Missile Crisis

Stability analysis checks for individual stability and equilibria

- A state is stable for a DM if the DM has no incentive to move away
- A state which is stable for all DMs (under a particular definition of stability) is an equilibrium
- Solution concepts define how stability is caused

Stability Analysis

Cuban Missile Crisis

In this type of analysis, we will determine 4 possible outcomes:

- Rational (r)
- Sequentially sanctioned (s)
- Unstable (u)
- Simultaneously sanctioned (u̶)

Stability Analysis

Rational outcome

A state s is **rational** (r) for a DM if the DM has no unilateral improvements (UIs) from s.

- A UI is a move made only by the DM - fix opponent strategies - which results in a more preferred state
- From state 5, the US has UIs to states 4 and 6:

U.S.			
Air strike	0	0	1
Blockade	0	1	0
U.S.S.R.			
Withdraw	1	1	1
Escalate	0	0	0
Decimal	4	6	5

Stability Analysis

Rational outcome

A state s is **rational** (r) for a DM if the DM has no unilateral improvements (UIs) from s.

- Write r above these states
- For states with UIs, write the UIs below from most to least preferred

Stability Analysis

Rational outcome - US

Check for rational states:

US preferences:

U.S.												
Air strike	0	0	1	1	0	1	1	0	1	1	0	0
Blockade	0	1	0	1	1	0	1	0	1	0	1	0
U.S.S.R.												
Withdraw	1	1	1	1	0	0	0	0	0	0	0	0
Escalate	0	0	0	0	0	0	0	0	1	1	1	1
Decimal	4	6	5	7	2	1	3	0	11	9	10	8
		4	4	4		2	2	2		11	11	11
			6	6			1	1			9	9
				5				3				10

Stability Analysis

Rational outcome - US

In tableau form:

r				r				r			
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

Stability Analysis

Rational outcome - USSR

Check for rational states:
USSR preferences:

U.S.												
Air strike	0	0	0	0	1	1	1	1	1	1	0	0
Blockade	0	0	1	1	0	0	1	1	1	0	1	0
U.S.S.R.												
Withdraw	0	1	1	0	1	0	1	0	0	0	0	0
Escalate	0	0	0	0	0	0	0	0	1	1	1	1
Decimal	0	4	6	2	5	1	7	3	11	9	10	8

Stability Analysis

Rational outcome - USSR

In tableau form:

r		r		r		r					
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

Stability Analysis

Sequentially Sanctioned Outcome

A state s is **sequentially sanctioned** (s) for a DM if for each UI from s, a credible action can be taken by the opponent to stop the DM from taking advantage of the UI.

- Check **all** UIs for the DM from a given state
- UI for the DM leaves open a UI for the opponent which is less preferred to DM
- Write s above these states

Stability Analysis

Sequentially Sanctioned Outcome

- The US has a UI from state 6 to state 4

r				r				r			
4	**6**	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

- The USSR has a UI from state 4 to state 0

r		r		r		r					
0	**4**	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

- State 0 is less preferred to state 6 by the US

r				r				r			
4	**6**	5	7	2	1	3	**0**	11	9	10	8

- Therefore state 6 is sequentially sanctioned for the US

Stability Analysis

Sequentially Sanctioned Outcome - US

In tableau form:

r	s			r				r			
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

Stability Analysis

Unstable Outcome

A state s is **unstable** (u) for a DM if the DM has at least one UI from s for which the opponent has no credible deterrent.

- If a state is unstable, the DM will take advantage of any undeterred UI
- Write u above these states

Stability Analysis

Unstable Outcome

- The US has a two UIs from state 5: to state 4 and to state 6

r				r				r			
4	6	**5**	7	2	1	3	0	11	9	10	8
	4	**4**	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

- If the US moves to state 4, the USSR has a UI to state 0 which is less preferred to state 5 by the US so state 4 is sanctioned
- If the US moves to state 6, then the USSR will stay since state 6 is rational for the USSR

r		r		r		r					
0	**4**	**6**	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

- There is at least one UI for the US from state 5 which is not sanctioned, therefore state 5 is unstable for the US

Stability Analysis

Unstable Outcome - US

In tableau form:

r	s	u		r				r			
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

Let's fill out the rest of the table

Stability Analysis

r, s, and u - US

In tableau form:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

Stability Analysis

r, s, and u - USSR

Take a few minutes to perform stability calculations for the USSR.

USSR preferences:

r		r		r		r					
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

US preferences:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

Stability Analysis

r, s, and u - USSR

In tableau form:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

Stability Analysis

Simultaneously Sanctioned Outcome

A state s which is unstable for both DMs is **simultaneously sanctioned** ($\cancel{u}$) for a DM if both players moving together could result in an outcome which is less preferred to s by the DM.

- Only check states which are unstable for both players
- For 2-DM games, calculate as follows:
  outcome = UI for DM 1 + UI for DM 2 - unstable outcome
- If there are several UIs, check all combinations. For the state to be simultaneously sanctioned, **all** of the movements must be sanctioned.

Stability Analysis

Simultaneously Sanctioned Outcome

US:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4 6	4 6 5		2	2 1	2 1 3		11	11 9	11 9 10

USSR:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7 3	5 1	6 2	0 4

- State 1 is unstable for both the US and the USSR
- US has a UI to state 2 ; USSR has a UI to state 5
- outcome = UI for US + UI for USSR - unstable outcome
- outcome = 2 + 5 - 1 = 6
- State 6 is more preferred than state 1 for both players, so state 1 remains unstable

Stability Analysis

Simultaneous Sanctioning

Take a few minutes to check for simultaneous sanctioning.

USSR preferences:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7 3	5 1	6 2	0 4

US preferences:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4 6	4 6 5		2	2 1	2 1 3		11	11 9	11 9 10

Stability Analysis

Simultaneously Sanctioned Outcomes

Simultaneous stability calculations:

- 2 + 5 - 1 = 6, preferred by both DMs over state 1
- 2 + 7 - 3 = 6, preferred by both DMs over state 3
- 11 + 0 - 8 = 3, preferred by both DMs over state 8
- 11 + 5 - 9 = 7, preferred by both DMs over state 9
- 11 + 6 - 10 = 7, preferred by both DMs over state 10

None of the states are simultaneously sanctioned for either DM

Stability Analysis

Equilibria

A state which is stable for all DMs (under a particular definition of stability) is an **equilibrium**.

- Mark each equilibrium with an E
- Any outcome which is unstable for at least one player is indicated by an X
- Three possible types of equilibria:
  - Rational: state is r for all DMs
  - Sequentially sanctioned: state is s for at least one DM
  - Simultaneously sanctioned: state is $\cancel{u}$ for all DMs

Stability Analysis

Equilibria

Overall equilibria:

E	E	X	X	X	X	X	X	X	X	X	X

US:

r	s	u	u	r	u	u	u	r	u	u	u
4	6	5	7	2	1	3	0	11	9	10	8
	4	4	4		2	2	2		11	11	11
		6	6			1	1			9	9
			5				3				10

USSR:

r	s	r	u	r	u	r	u	u	u	u	u
0	4	6	2	5	1	7	3	11	9	10	8
	0		6		5		7	7	5	6	0
								3	1	2	4

Two sequentially stable equilibria: state 4 and state 6

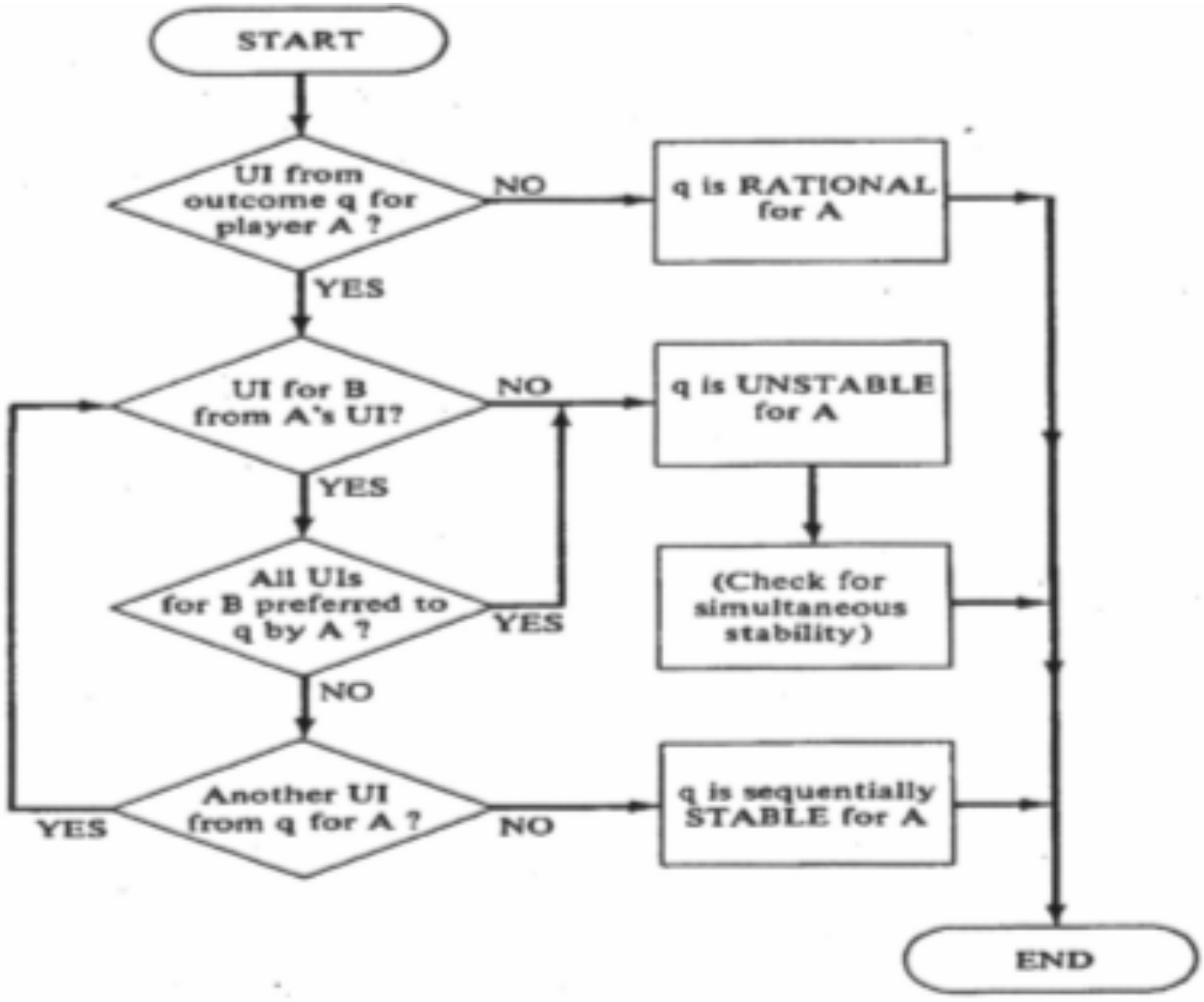
Stability Analysis

Historical Outcome

- From state 0 (status quo), the US has a UI to state 2 (blockade)
- Russia has a UI from state 2 to state 6 (withdraw) which occurred historically
- State 4 could have occurred is the USSR had been certain that the US would impose a blockade if the missiles remained in Cuba (hypergame - we will see this later)

U.S.					
Air strike	0		0		0
Blockade	0	$\longrightarrow$	1		1
U.S.S.R.					
Withdraw	0		0	$\longrightarrow$	1
Escalate	0		0		0
Decimal	0		2		6

Analysis of 2-DM Games in Chart Form



Solution Concepts and Human Behaviour

Solution Concept	**Stability Description**	**Foresight**	**Knowledge of Preferences**	**Disimprovement**	**Strategic Risk**
Rational	Focal DM cannot move unilaterally to a more preferred state	Low	Own	Never	Ignores risk
Sequential	All focal DM's UIs are sanctioned by opponent UIs	Medium	All	Never	Takes some risks; satisfies

Next Lecture

- Basics of n-DM models
- Project groups and topics due!