NU Lecture 3

15/02/16

Recall

Given values $x = a_0$ and $x = b_0$ satisfying $f(a_0)f(b_0) < 0$ & $a_0 < b_0$

Assumption : $f \in C^0[a_0, b_0]$

For iteration $k = 0, 1, \ldots$

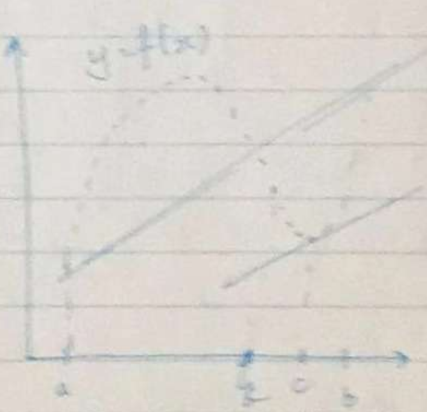
$$\text{Set } m_k \leftarrow \begin{cases} \dfrac{a_k + b_k}{2} & \text{Bolzano's Method} \\ \dfrac{a_k f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)} & \text{False Position or Regula Falsi} \end{cases}$$

$$\text{Set } [a_{k+1}, b_{k+1}] \leftarrow \begin{cases} [a_k, m_k] & \text{if } f(a_k)f(m_k) < 0 \\ [m_k, b_k] & \text{if } f(m_k)f(b_k) < 0 \end{cases}$$

Mean Value Theorem

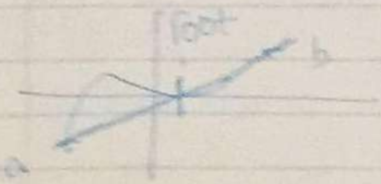
If the functions $f(x)$ and $f'(x)$ are continuous on some ~~interval~~ interval $[a, b]$ (i.e. $f \in C'[a,b]$) then $\exists c \in \mathbb{R}$ in the interval $[a,b]$ satisfying

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



Secant line ∥ tangent line

Learn → Method of false position:



Join the pts a, b & use the x-intercept as the root

Secant Method

- Another derivative free root finding method
- not an enclosure method
- models $f(x)$ locally by a linear function

Given: Values $x = x_0$ and $x = x_1$ of x with $f_0 = f(x_0)$ and $f_1 = f(x_1)$ that do not necessarily satisfy $f(x_0) f(x_1) < 0$

let $f_i = f(x_i)$

At iteration $k = 1, 2, 3, \ldots$ model $f(x)$ locally by the secant line through the two pts

$$(x_{k-1}, f_{k-1}) \ \& \ (x_k, f_k)$$

and use x_{k+1}, the x-intercept of this secant line as our next pt.

$$x_{k+1} = \frac{f_k x_{k-1} - f_{k-1} x_k}{f_k - f_{k-1}}$$

We then discard the oldest pt x_{k-1} & repeat this process using x_k & x_{k+1}.