

$x = g(x) = 1 - x^3$. The starting point $x_0 = 0.5$ is chosen somewhat arbitrarily. Applying FPI gives the following result:

i	x_i
0	0.50000000
1	0.87500000
2	0.33007813
3	0.96403747
4	0.10405419
5	0.99887338
6	0.00337606
7	0.99999996
8	0.00000012
9	1.00000000
10	0.00000000
11	1.00000000
12	0.00000000

Instead of converging, the iteration tends to alternate between the numbers 0 and 1. Neither is a fixed point, since $g(0) = 1$ and $g(1) = 0$. The Fixed-Point Iteration fails. With the Bisection Method, we know that if f is continuous and $f(a)f(b) < 0$ on the original interval, we must see convergence to the root. This is not so for FPI.

The second choice is $g(x) = \sqrt[3]{1 - x}$. We will keep the same initial guess, $x_0 = 0.5$.

i	x_i
0	0.50000000
1	0.79370053
2	0.59088011
3	0.74236393
4	0.63631020
5	0.71380081
6	0.65900615
7	0.69863261
8	0.67044850
9	0.69072912
10	0.67625892
11	0.68664554
12	0.67922234

i	x_i
13	0.68454401
14	0.68073737
15	0.68346460
16	0.68151292
17	0.68291073
18	0.68191019
19	0.68262667
20	0.68211376
21	0.68248102
22	0.68221809
23	0.68240635
24	0.68227157
25	0.68236807

This time FPI is successful. The iterates are apparently converging to a number near 0.6823.

Finally, let's use the rearrangement $x = g(x) = (1 + 2x^3)/(1 + 3x^2)$. As in the previous case, there is convergence, but in a much more striking way.

i	x_i
0	0.50000000
1	0.71428571
2	0.68317972
3	0.68232842
4	0.68232780
5	0.68232780
6	0.68232780
7	0.68232780

Here we have four correct digits after four iterations of Fixed-Point Iteration, and many more correct digits soon after. Compared with the previous attempts, this is an astonishing result. Our next goal is to try to explain the differences between the three outcomes.