A simplified Brent's Method

Preliminaries

Three points (i.e. values of x) are involved in each iteration $k = 0, 1, \ldots$ of Brent's Method

b_k - the most recent approximation to a zero

a_k - called the "contrapoint"

Satisfies $f(a_k)\, f(b_k) < 0$ AND

$|f(b_k)| \leq |f(a_k)|$

b_{k-1} - the just previous approximation to a zero

For $k = 0$ we set $b_{-1} = a_0$

We also define a flag, mflag, that will be set in each iteration. For $k = 0$ we set $mflag = 1$

In each iteration $k=0,1,\ldots$ two candidate points, m_k and s_k, for the next approximation to a zero are computed

(1) The midpoint of the bracket bounded by a_k and b_k

$$m_k = \frac{a_k + b_k}{2}$$

(2) Either a Secant or IQI approximation

$$s_k = \begin{cases} \dfrac{a_k f(b_k) - b_k f(a_k)}{f(b_k) - f(a_k)} & \text{if } f(a_k) = f(b_{k-1}) \\ & \text{or } f(b_k) = f(b_{k-1}) \\[2ex] a_k \dfrac{f(b_k)\, f(b_{k-1})}{(f(a_k)-f(b_k))(f(a_k)-f(b_{k-1}))} & \\ \quad + \; b_k \dfrac{f(a_k)\, f(b_{k-1})}{(f(b_k)-f(a_k))(f(b_k)-f(b_{k-1}))} & \text{otherwise} \\ \quad + \; b_{k-1} \dfrac{f(a_k)\, f(b_k)}{(f(b_{k-1})-f(a_k))(f(b_{k-1})-f(b_k))} & \end{cases}$$

The Simplified Algorithm

Given a_0 and b_0 satisfying $f(a_0)f(b_0)<0$, $|f(b_0)|<|f(a_0)|$

For iteration $k=0,1,\ldots$

$$c_k = \begin{cases} m_k & \begin{cases} \text{if } s_k \notin \left[\dfrac{3a_k+b_k}{4}, b_k\right] \\ \text{OR } \left(\text{mflag}=1 \text{ AND } |s_k-b_k| > \dfrac{|b_k-b_{k-1}|}{2}\right) \\ \text{OR } \left(\text{mflag}=0 \text{ AND } |s_k-b_k| > \dfrac{|b_{k-1}-b_{k-2}|}{2}\right) \end{cases} \\ s_k & \text{otherwise} \end{cases}$$

If $c_k \leftarrow m_k$ then set mflag$=1$; otherwise set mflag $=0$

Compute $f(c_k)$

Set $(a_{k+1}, b_{k+1}) = \begin{cases} (a_k, c_k) & \text{if } f(a_k)f(c_k) < 0 \\ (c_k, b_k) & \text{if } f(c_k)f(b_k) < 0 \end{cases}$

If $|f(a_{k+1})| < |f(b_{k+1})|$ then swap the values assigned to a_{k+1} and b_{k+1}

Notes

1) for IQI, s_k is set to the root $g(0)$ of the IQI approximation, $g(y)$, that interpolates the three points

$$(a_k, f(a_k)) \quad (b_k, f(b_k)) \quad (b_{k-1}, f(b_{k-1}))$$

We compute $s_k = g(0)$ when $f(a_k)$, $f(b_k)$ and $f(b_{k-1})$ have unique values

2) for Secant, s_k is set to the root of $y(x)$, the linear function that interpolates the two points

$$(a_k, f(a_k)) \quad (b_k, f(b_k))$$

We compute this root when $f(a_k)$, $f(b_k)$ and $f(b_{k-1})$ DO NOT have unique values

3) mflag is set to 1 whenever a bisection is used. Otherwise it is set to zero