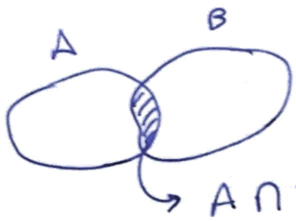
13/01/16 $\underline{\underline{312}}$ Lecture 4

I-1 (cont) subspaces

Set Notation

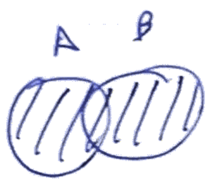
$A \subset B$ subset

$\emptyset = \{ . \}$ empty set



$A \cap B$ = intersection

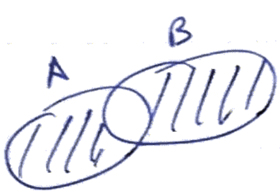
$= \{ x \mid x \in A \mid x \in B \}$



$A \cup B$ = union

$= \{ x \mid x \in A$ or $x \in B \}$

$A - B$ difference $= \{ x \mid x \in$ and $x \notin B \}$



$(A \cup B) - (A \cap B)$ symmetric difference

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Back to subspaces

V is a vector space ove F (scalars)

$W \subseteq V$ (subset of vectors)

W is a v.s. with the same operation as V provided:

W is closed to the vector space operation.

ie. $u, v \in W \Rightarrow u+v \in W$

Note : 1) can combine as
$u, v \in W \Rightarrow ku + v \in W$

2) #2 guarantees $\underline{0} \in W$

put ($k=0$)

Examples

1) Euclidean subspaces

eg R^3 $\{0\}$
$\{u \mid u = ku_0\}$ line through $(0,0,0)$
$\{u \mid \sum_{i=1}^{3} a_i u_i = 0\}$ plane through origin

In R^n : $\left\{ u \mid \sum_{i=1}^{n} a_i u_i = 0 \right\}$

hyperplane

To check $u, v \in W \Rightarrow \sum_{i=1}^{n} a_i u_i = 0$, $\sum_{i=1}^{n} a_i v_i = 0$

$u = (u_1, ---, u_n) \in R^n$
$v = (v_1, ----, v_n) \in R^n$

$\rightarrow$

Now $u+v = (u_1+v_1, \ldots, u_n+v_n)$

$\Sigma a_i(u_i+v_i)$

$= \Sigma a_i u_i + \Sigma a_i v_i$

$= 0$

Also :

$\Sigma a_i (k u_i)$

$= k \Sigma a_i u_i$

$= 0$ b/u $u \in W$

Note: $W = \{ u \mid \sum_{i=1}^{n} a_i u_i = k \}$ is a subspace of R^n

$\Leftrightarrow k = 0$ (check it)

Throug the origin only

Ex 2) $M_{n,n}$ Say $M_{n,n}$

$W = \{$ Symmetric matrices $A = A^T \}$

$A, B \in W$ check AB & kA

$A = A^T$
$B = B^T$

chek A

$(A+B)^T = A^T + B^T = A + B$

$(kA)^T = kA^T = kA$

$\boxed{W \subset M_{n,n}}$ symmetric matrices subspace

Also antisymmetric matrices.

$A = -A^T$ subspace

$\{ A \in M_{m,n} \mid a_{11} = 0 \} \in M_{m,n}$

Ex 3) P

$P_n = \{ p \mid \deg p \le n \}$

$p, q \in P_n \Rightarrow \deg(p+q) \le n$

& $\deg(kp) \le n$

So $P_n \subset P$

Chain of subspaces of P

$P_0 \subset P_1 \subset P_2 \subset P_3 \ldots\ldots \subset P$

indistinguishable from R

even degree polynomial is a $\subset P$

Ex 4 Function subspaces

Take $F(X, R)$ real-val functions from $X \to R$

X maybe an interval $[a,b]$ or $[a,\infty]$ or R etc

OR infact any set

$W_1 = \{ f \mid f(x) = f(-x) \}$ even
$W_2 = \{ f \mid f(x) = -f(-x) \}$ odd.
} function/form

Check $f, g \in W_1$

$(f+g)(-x)$

$= f(-x) + g(-x)$ defn of $f+g$

$= f(x) + g(x)$ $f, g \in W_1$

$= (f+g)(x)$ defn of $f+g$

$(kf)(-x)$

$= k(f(-x))$

$= k(f(x))$ $f \in W_1$

$= (kf)(x)$

Another ex bounded fns

$W = \{ f \mid |f(x)| \leq M \text{ ; some } M \}$

subspace?

$f, g \in W$ $\quad |(f+g)(x)| \leq M_1 + M_2$

$f \to M_1$, $g \to M_2$ $\quad |(kf)(x)| \leq kM_1$