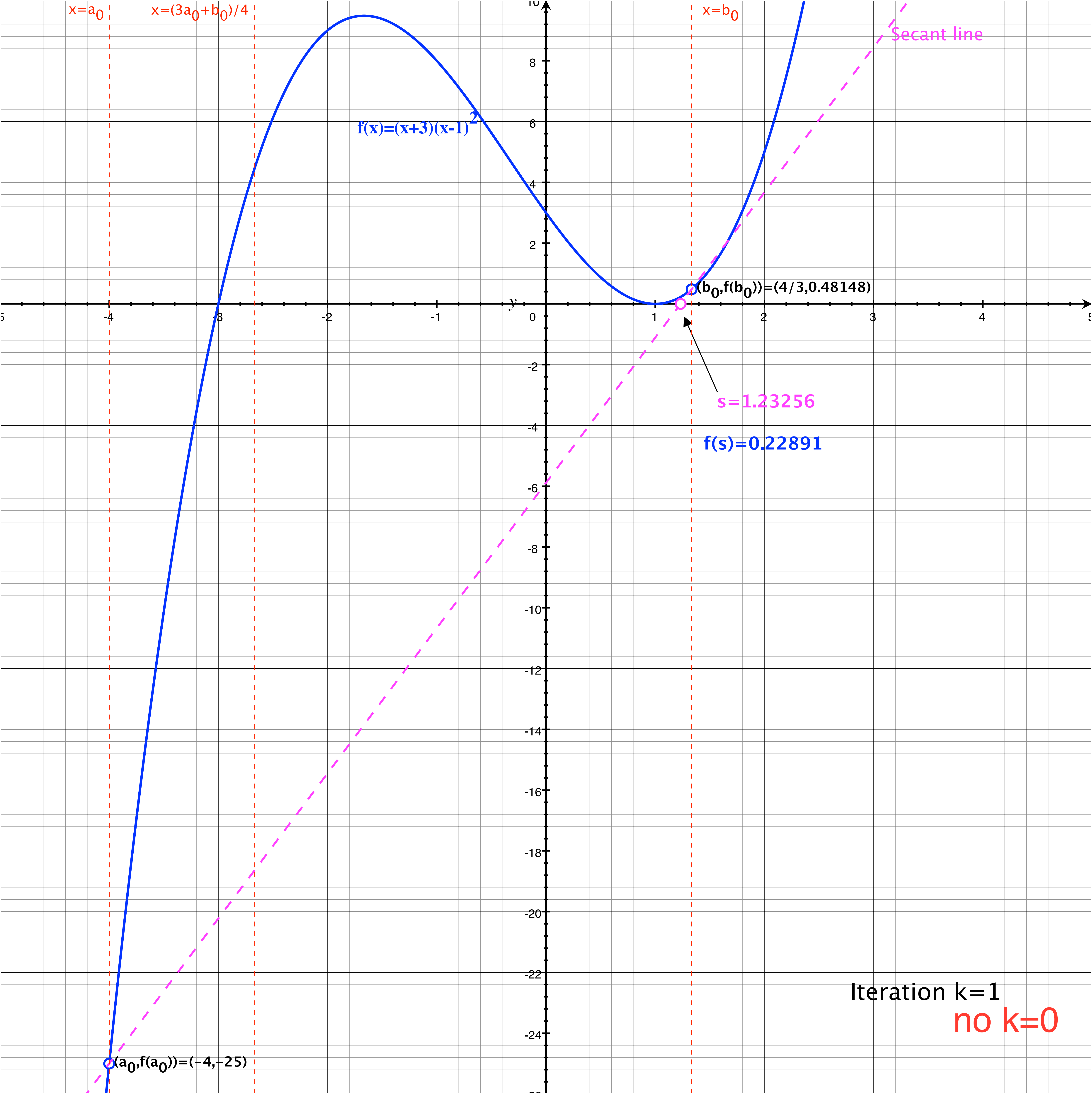


x=a0
x=(3a0+b0)/4
x=b0
Secant line
f(x)=(x+3)(x-1)2
(b0,f(b0))=(4/3,0.48148)
s=1.23256
f(s)=0.22891
(a0,f(a0))=(−4,−25)
Iteration k=1
no k=0

Detailed Example of Brent's Method

$k=0$ $(a_0, f(a_0)) = (-4, -25)$ $(b_0, f(b_0)) = (4/3, 0.48148)$ mflag $= 1$

Since $f(b_{-1}) = f(a_0)$ we use the Secant Method to compute s_0

$$s_0 = \frac{a_0 f(b_0) - b_0 f(a_0)}{f(b_0) - f(a_0)} = 1.23256$$

$$s_0 \in \left[\frac{3a_0 + b_0}{4}, b_0\right]$$

mflag $= 1$ but $|s_0 - b_0| < \frac{|b_0 - a_0|}{2}$

$\therefore$ $c_0 \leftarrow s_0 = 1.23256$ mflag $\leftarrow 0$

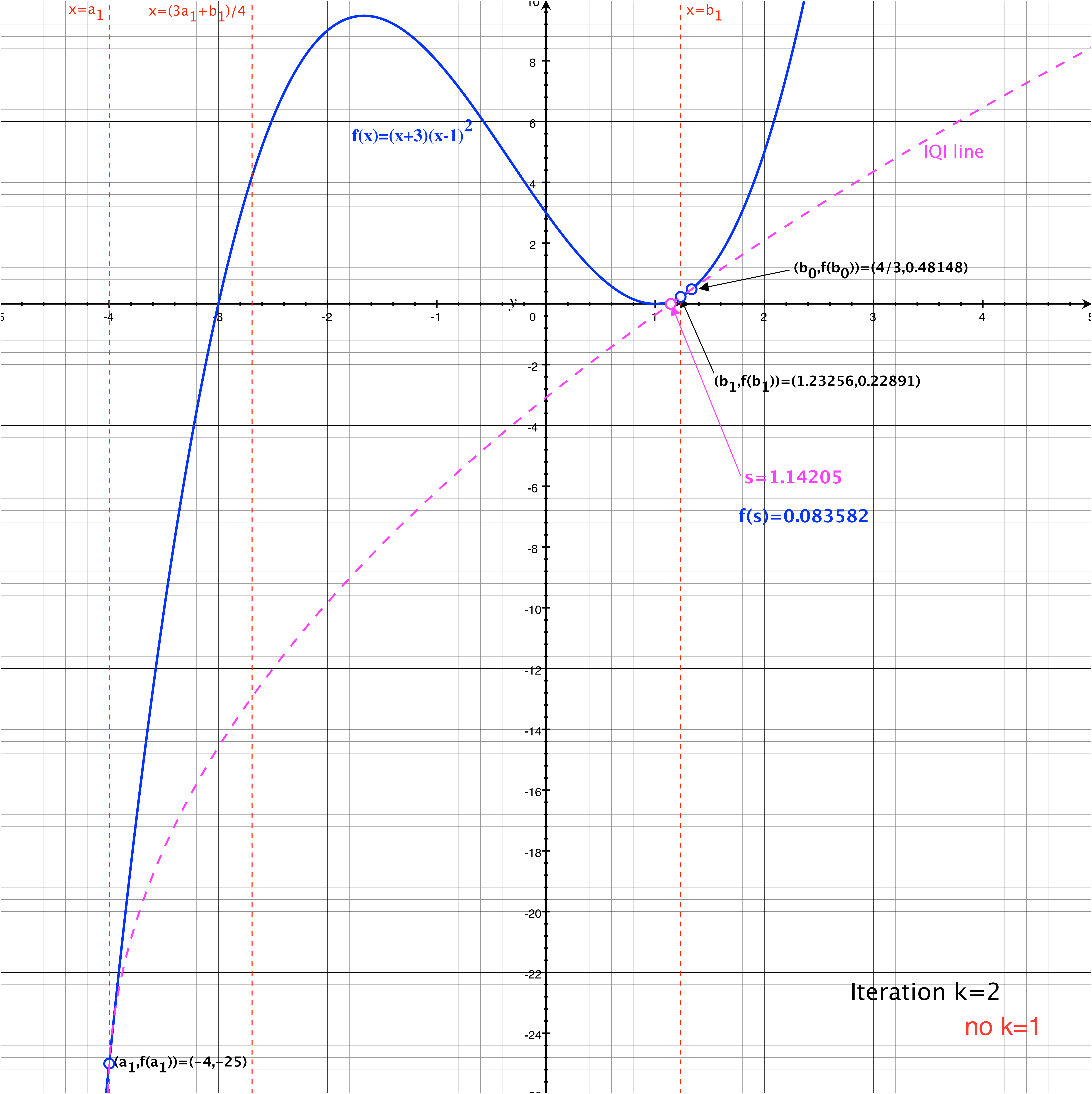
Since $f(c_0) = f(s_0) = 0.22891$ has the opposite sign

then does $f(a_0) = -25$

set $(a_1, b_1) = (a_0, c_0) = (-4, 1.23256)$

Since $|f(a_1)| = 25 > f(b_1) = 0.22891$

we don't swap the values of a_1 & b_1



x=a1
x=(3a1+b1)/4
x=b1
f(x)=(x+3)(x-1)2
IQI line
(b0,f(b0))=(4/3,0.48148)
(b1,f(b1))=(1.23256,0.22891)
s=1.14205
f(s)=0.083582
(a1,f(a1))=(-4,-25)
Iteration k=2
no k=1

$k=1$ $(a_1, f(a_1)) = (-4, -25)$ $(b_1, f(b_1)) = (1.23256, 0.22891)$

$(b_0, f(b_0)) = (4/3, 0.48148)$ $mflag = 0$

Since $f(a_1) \neq f(b_0)$ and $f(b_1) \neq f(b_0)$ we use IQI to compute s_1

$$s_1 = a_1 \frac{f(b_1) f(b_0)}{(f(a_1) - f(b_1))(f(a_1) - f(b_0))} + b_1 \frac{f(a_1) f(b_0)}{(f(b_1) - f(a_1))(f(b_1) - f(b_0))}$$

$$+ b_0 \frac{f(a_1) f(b_1)}{(f(b_0) - f(a_1))(f(b_0) - f(b_1))} = 1.14205$$

$$s \in \left[\frac{3a_1 + b_1}{4}, b_1\right]$$

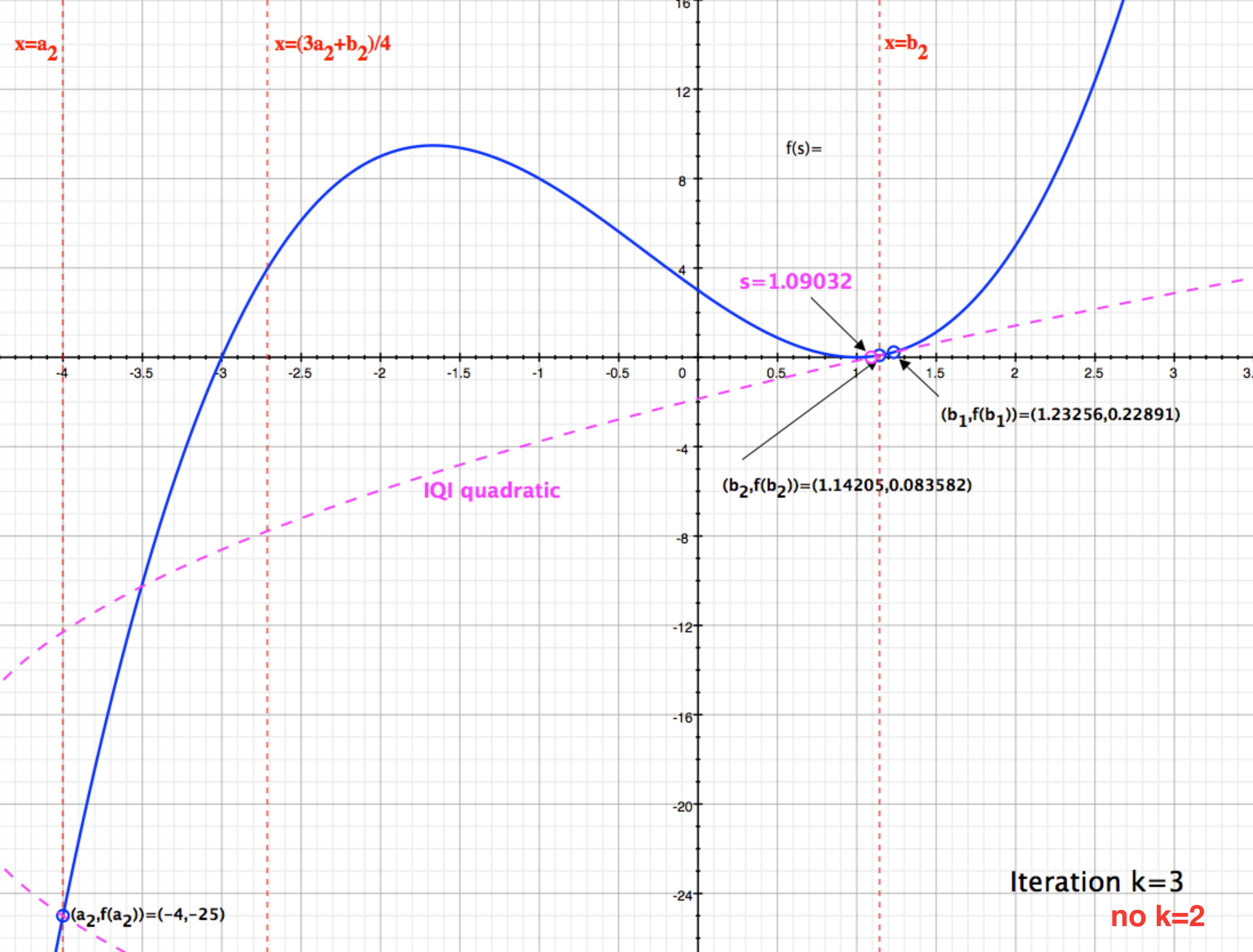
$mflag = 0$ but $|s_1 - b_1| < \frac{|b_0 - b_{-1}|}{2}$

$\therefore c_1 = s_1 = 1.14205$ $mflag \leftarrow 0$

Since $f(c_1) = 0.083582$ has the opposite sign then does $f(a_1) = -25$

set $(a_2, b_2) = (a_1, c_1) = (-4, 1.14205)$

Since $|f(a_2)| = 25 > |f(b_2)| = 0.083582$ we don't swap the values of a_2 and b_2



x=a2
x=(3a2+b2)/4
x=b2
f(s)=
s=1.09032
(b1,f(b1))=(1.23256,0.22891)
(b2,f(b2))=(1.14205,0.083582)
IQI quadratic
(a2,f(a2))=(−4,−25)
Iteration k=3
no k=2

$k=2 \quad (a_2, f(a_2)) = (-4, -25) \quad (b_2, f(b_2)) = (1.14205, 0.083582)$

$(b_1, f(b_1)) = (1.23256, 0.22891) \quad mflag = 0$

Since $f(a_2) \neq f(b_1)$ AND $f(b_2) \neq f(b_1)$ we use IQI to compute s_2

$$s_2 = a_2 \frac{f(b_2) f(b_1)}{(f(a_2) - f(b_2))(f(a_2) - f(b_1))} + b_2 \frac{f(a_2) f(b_1)}{(f(b_2) - f(a_2))(f(b_2) - f(b_1))}$$

$$+ b_1 \frac{f(a_2) f(b_2)}{(f(b_1) - f(a_2))(f(b_1) - f(b_2))} = 1.09032$$

$$s_2 \in \left[\frac{3a_2 + b_2}{4}, b_2\right]$$

$mflag = 0$ AND $|s_2 - b_2| > \frac{|b_1 - b_0|}{2}$

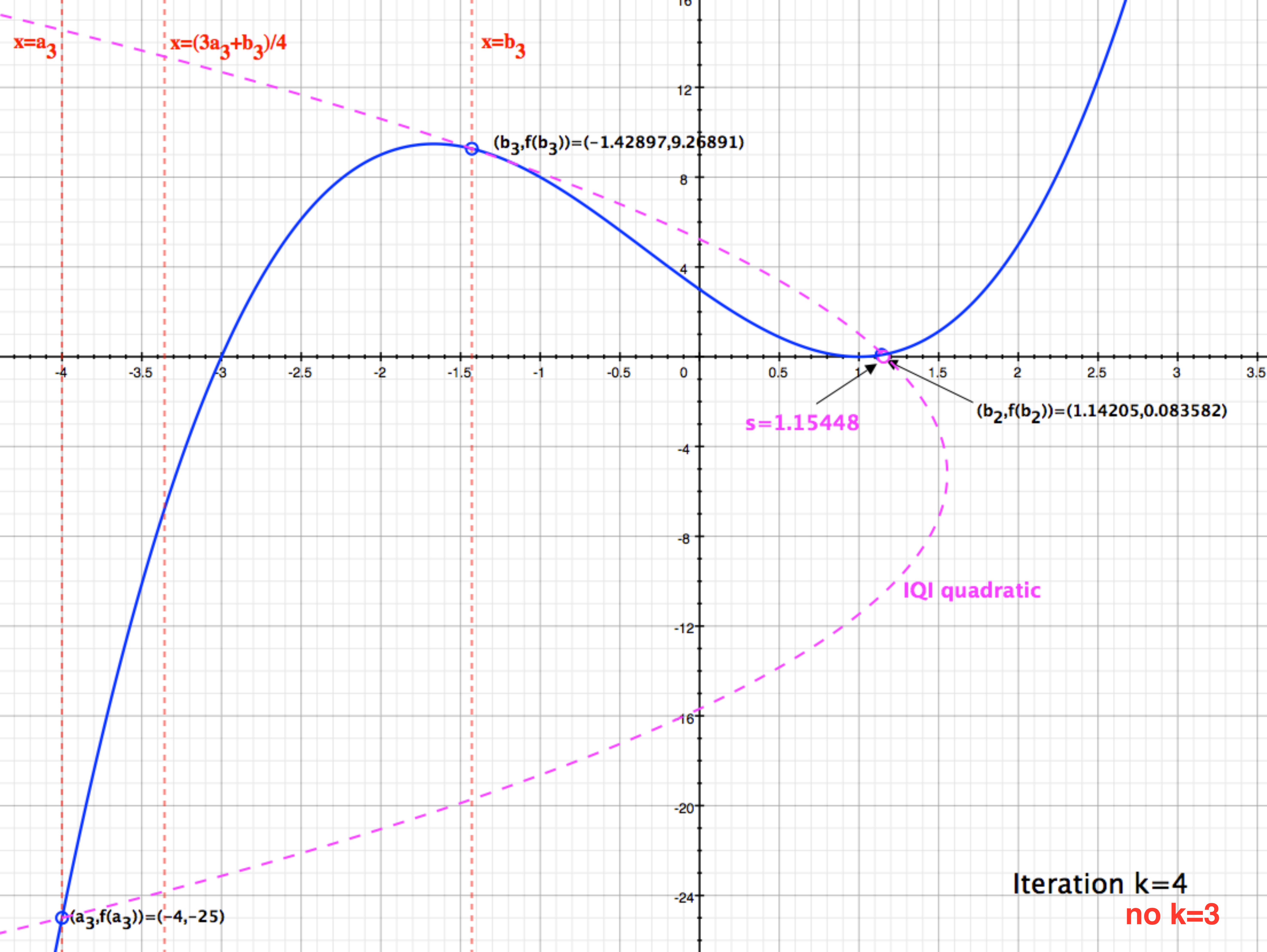
so s_2 is rejected for c_2 and instead

$c_2 = m_2 = \frac{a_2 + b_2}{2} = -1.42897 \quad mflag \leftarrow 1$

Since $f(c_2) = 9.26891$ has the opposite sign then does $f(a_2)$

set $(a_3, b_3) = (a_2, c_2) = (-4, -1.42897)$

Since $|f(a_3)| = 25 > |f(b_3)| = 9.26891$ we don't swap the values of a_3 and b_3



x=a3
x=(3a3+b3)/4
x=b3
(b3,f(b3))=(−1.42897,9.26891)
s=1.15448
(b2,f(b2))=(1.14205,0.083582)
IQI quadratic
(a3,f(a3))=(−4,−25)
Iteration k=4
no k=3

$k=3$ $(a_3, f(a_3)) = (-4, -25)$ $(b_3, f(b_3)) = (-1.42897, 9.26891)$

$(b_2, f(b_2)) = (1.14205, 0.083582)$ mflag $= 1$

Since $f(a_3) \neq f(b_2)$ AND $f(b_3) \neq f(b_2)$ we use

IQI to compute s_3

$$s_3 = a_3 \frac{f(b_3)f(b_2)}{(f(a_3)-f(b_3))(f(a_3)-f(b_2))} + b_3 \frac{f(a_3)f(b_2)}{(f(b_3)-f(a_3))(f(b_3)-f(b_2))}$$

$$+ b_2 \frac{f(a_3)f(b_3)}{(f(b_2)-f(a_3))(f(b_2)-f(b_3))} = 1.15448$$

Since $s_3 \notin \left[\frac{3a_3+b_3}{4}, b_3\right]$, s_3 is rejected for

c_3 and instead

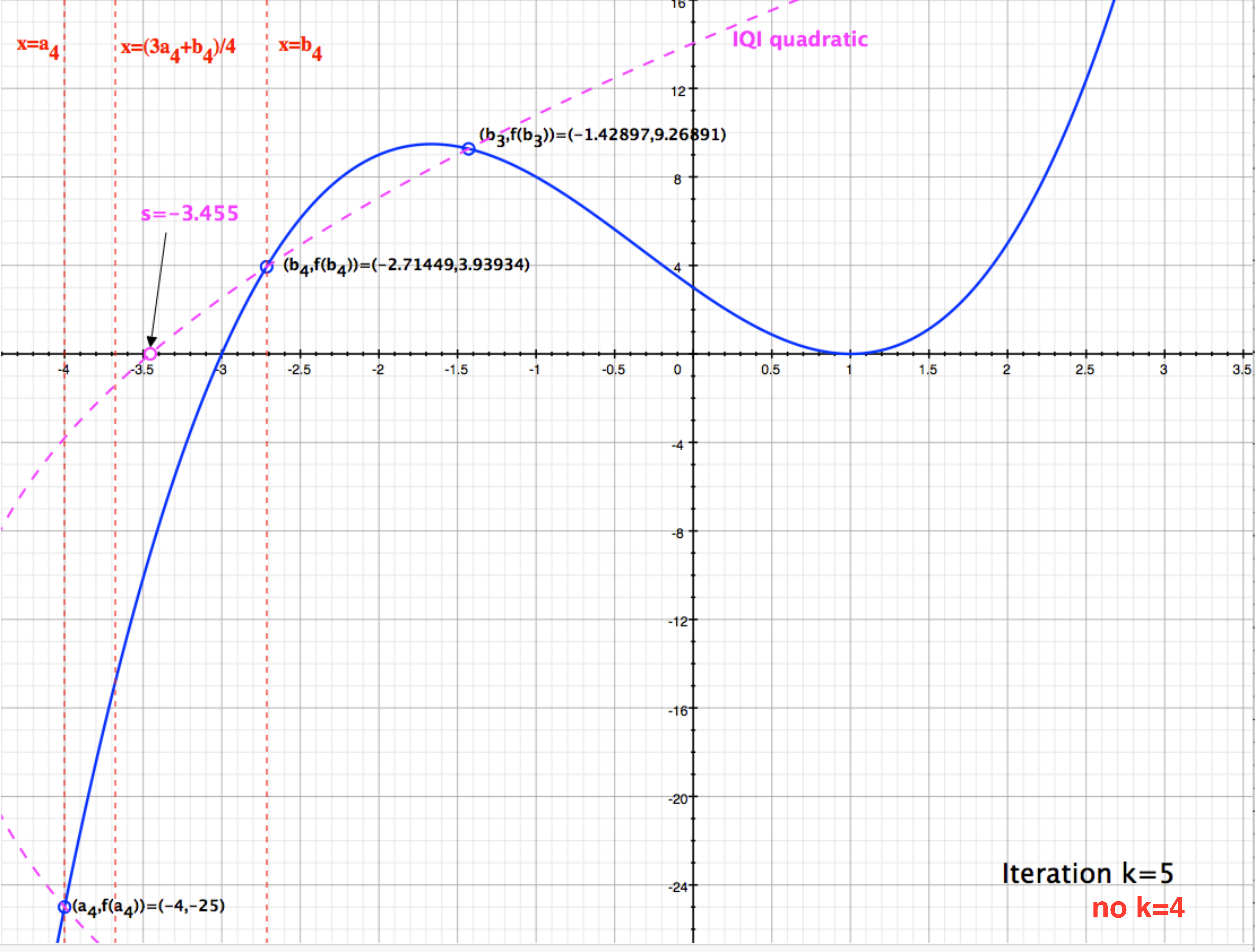
$$c_3 = m_3 = \frac{(a_3+b_3)}{2} = -2.71449 \quad \text{mflag} \leftarrow 1$$

Since $f(c_3) = 3.93934$ has the opposite sign to $f(a_3)$

set $(a_4, b_4) = (a_3, c_3) = (-4, -2.71449)$

Since $|f(a_4)| = 25 > |f(b_4)| = 3.93934$ we don't

swap the values of a_4 and b_4



x=a4
x=(3a4+b4)/4
x=b4
IQI quadratic
(b3,f(b3))=(−1.42897,9.26891)
s=−3.455
(b4,f(b4))=(−2.71449,3.93934)
(a4,f(a4))=(−4,−25)
Iteration k=5
no k=4

$k=4$ $\quad (a_4, f(a_4)) = (-4, -25)$ $\quad (b_4, f(b_4)) = (-2.71449, 3.93934)$

$(b_3, f(b_3)) = (-1.42897, 9.26891)$ $\quad$ mflag $= 1$

Since $f(a_4) \neq f(b_3)$ AND $f(b_4) \neq f(b_3)$ we use IQI to compute s_4

$$s_4 = a_4 \frac{f(b_4) f(b_3)}{(f(a_4) - f(b_4))(f(a_4) - f(b_3))} + b_4 \frac{f(a_4) f(b_3)}{(f(b_4) - f(a_4))(f(b_4) - f(b_3))}$$

$$+ b_3 \frac{f(a_4) f(b_4)}{(f(b_3) - f(a_4))(f(b_3) - f(b_4))} = -3.45500$$

$$s_4 \in \left[\frac{3a_4 + b_4}{4}, b_4 \right].$$

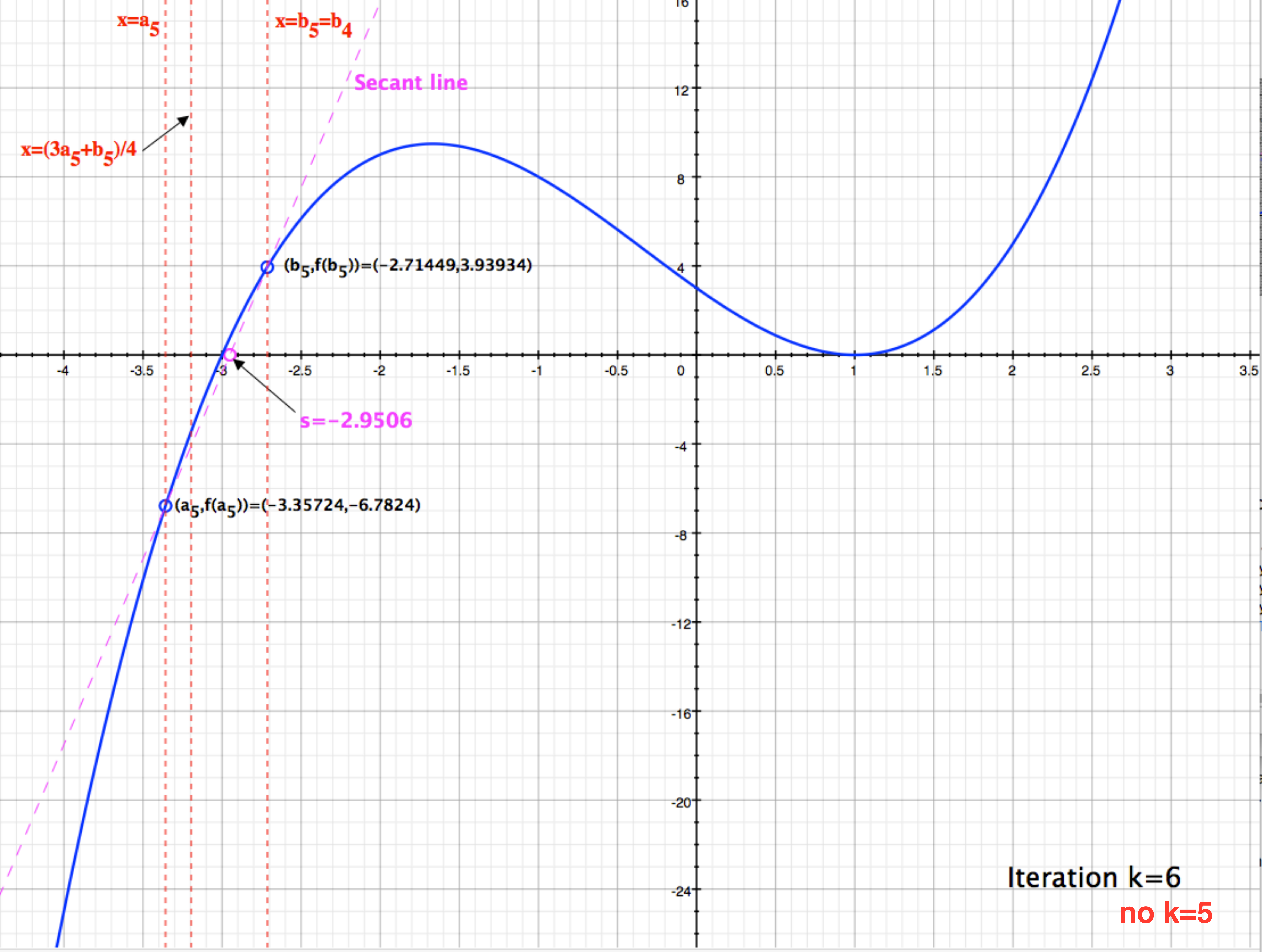
mflag $= 1$ AND $|s_4 - b_4| > \dfrac{|b_4 - b_3|}{2}$, so

s_4 is rejected for c_4 and instead

$c_4 = m_4 = \dfrac{(a_4 + b_4)}{2} = -3.35725$ $\quad$ mflag $\leftarrow 1$

Since $f(c_4) = -6.78239$ has the opposite sign than does $f(b_4)$ set $(a_5, b_5) = (c_4, b_4) = (-3.35724, -2.71449)$

Since $|f(a_5)| = 6.78239 > |f(b_5)| = 3.93934$ we don't swap the values of a_5 and b_5



x=a5
x=b5=b4
Secant line
x=(3a5+b5)/4
(b5,f(b5))=(−2.71449,3.93934)
s=−2.9506
(a5,f(a5))=(−3.35724,−6.7824)
Iteration k=6
no k=5

$k=5$ $\quad (a_5, f(a_5)) = (-3.35724, -6.7824)$

$(b_5, f(b_5)) = (-2.71449, 3.93934)$

and $b_4 = b_5$ $\quad$ mflag $=1$

Since $f(b_4) = f(b_5)$ we use the Secant Method to compute s_5

$$s_5 = \frac{a_5 f(b_5) - b_5 f(a_5)}{f(b_5) - f(a_5)} = -2.95064$$

$$s_5 \in \left[\frac{3a_5 + b_5}{4}, b_5\right]$$

$\vdots$