I1 (cont) v.s & s.s   s.s → subspace

$\boxed{\begin{matrix} V, F \\ + \; k\cdot \end{matrix}}$ ku

$u \in V$ $k \in F$ all $k \in F$
$u \neq \underline{0}$ single $\underline{u} \neq 0$
closed to $+, \cdot$
⇒ v.s. same rules (i.e. s.s)

$u_1 \quad u_2$ ⟶ $k_1u_1 + k_2u_2$

$(u \neq u_2)$ neither one is $\underline{0}$ all $k_1, k_2 \in F$
$u_1 \neq u_2 \in V$
$u_1, u_2 \neq 0$

closed ⇒ s.s

$u_1, \ldots, u_r \in V$ v.s. over F

$$Sp\{u_1, \ldots, u_r\} = [k_1u_1 + \ldots + k_ru_r\} \mid k_i \in F]$$

is a subspace of V (the smallest subspace containing $u_1, \ldots, u_r$
$k_1u_1 + \ldots + k_ru_r$ is called a <u>linear combination</u> (l.c.) of $u_1, \ldots, u_r$

Ex Show that $Sp\{(1,1,1),(1,2,3),(1,5,8)\} = \mathbb{R}^3$

Soln write:

$(x,y,z) = a(1,1,1) + b(1,2,3) + c(1,5,8)$

& see any restrictions (if any) apply to (x,y,z)

As Prev (l.c.s) get a non homogenous lin system

$$\begin{array}{c} \begin{matrix} a & b & c & & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 1 & 2 & 5 & y \\ 1 & 3 & 8 & z \end{array}\right] \end{array} \sim \cdots$$

augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & x \\ 0 & 1 & 4 & y-x \\ 0 & 0 & 1 & -z+2y-x \end{array}\right]$$

a unique soln for any (x,y,z)

concluded $Sp\{\ ,\ ,\ \} = \mathbb{R}^3$

Ex Find $W = Sp\{(1,1,1),(1,2,3),(3,5,7)\}$

As prev put

$(x,y,z) = a(1,1,1) + b(1,2,3) + c(3,5,7)$

$$\begin{array}{c} \begin{matrix} a & b & c & & \end{matrix} \\ \left[\begin{array}{ccc|c} 1 & 1 & 3 & x \\ 1 & 2 & 5 & y \\ 1 & 3 & 7 & z \end{array}\right] \end{array} \sim \cdots \left[\begin{array}{ccc|c} 1 & 1 & 3 & x \\ 0 & 1 & 2 & y-x \\ 0 & 0 & 0 & x-2y+z \end{array}\right]$$

There is no sol unless $x-2y+z=0$

$\Rightarrow x = 2y - z$

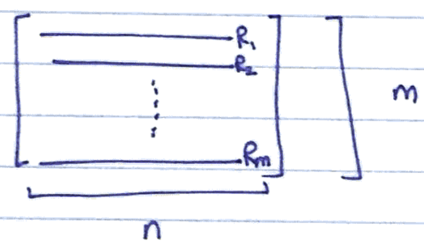
$(x,y,z) = (2y - z, y, z)$

$= y(2,1,0) + z(-1,0,1)$

so $W = Sp\{(2,1,0), (-1,0,1)\}$

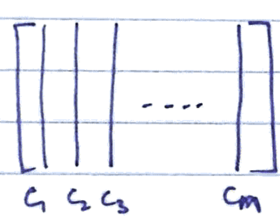
A is an $m \times n$ matrix

A =



Rows of A $R_1, \ldots, R_m$ are vectors in $\mathbb{R}^n$

A =



Cols of A $C_1, \ldots, C_n$ are vectors in $\mathbb{R}^m$

The row space of A is

$Sp\{R_1, \ldots, R_m\} < \mathbb{R}^n$

(Sp

The col space of A is

$Sp\{C_1, \ldots, C_n\} < \mathbb{R}^m$

$\rightarrow$

Elementary Row Operations

$R_i \leftrightarrow R_j$ interchange 2 rows

$R_i \leftarrow kR_i$ multiply a row by a constant

$R_i \leftarrow R_i + kR_j$ add two rows / linear combinations of rows

Two matricies A, B and B is obtained from A by a seq of elementary row operations are called <u>row equivalent</u>

$$A \sim B$$

None of the row ops change the span of rows (think about them)

Conclusion : $\boxed{\text{Row equivalent matrices have the same row space}}$

Q Do row equivalent matricies have the same col space?