20/09/93 411 Lecture 4

MVT : if $f \in C^1[a,b] \Rightarrow \exists c \in [a,b]$ s.t.

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Secant

$$\ell(x) = f_{k-1}\frac{(x - x_k)}{(x_{k-1} - x_k)} + f_k \frac{(x - x_{k-1})}{(x_k - x_{k-1})}$$

$$\Rightarrow x_* = \frac{x_k f_{k-1} - x_{k-1} f_k}{f_{k-1} - f_k}$$

IQI = inverse quadratic interpolation

$$g(y) = x_{k-2}\frac{(y - f_k)(y - f_{k-1})}{(f_{k-2} - f_k)(f_{k-2} - f_{k-1})}$$

$$+ x_{k-1}\frac{(y - f_k)(y - f_{k-2})}{(f_{k-1} - f_k)(f_{k-1} - f_{k-2})}$$

$$+ x_k \frac{(y - f_{k-1})(y - f_{k-2})}{(f_k - f_{k-1})(f_k - f_{k-2})}$$

$$x_{k+1} = x_{k-2}\frac{f_k f_{k-1}}{(f_{k-2} - f_k)(f_{k-2} - f_{k-1})} + x_{k-1}\frac{f_k f_{k-2}}{(f_{k-1} - f_k)(f_{k-1} - f_{k-2})}$$

$$+ x_k \frac{f_{k-1} f_{k-2}}{(f_k - f_{k-1})(f_k - f_{k-2})}$$

$\longrightarrow$

:)

IQI → after first iteration, throw away the oldest pt & replace with * $x = x_{k+1}$ (root) & repeat

IQI & secant is not an enclosure method.

A simplified Brent's Method

Preliminaries

- enclosure method
- ~~3 pts~~ hybrid (Balzano + Secant + IQI)
- derivative free
- 3 pts are involved in each iteration $k = 0, 1, \ldots$
- b_k - most recent approx to a zero
- a_k - called the "contrapoint"
  Satisfies $f(a_k)f(b_k) < 0$ AND $|f(b_k)| \leq |f(a_k)|$
- b_{k-1} - the just previous approx to a zero.

  For $k=0$ we set $b_{-1} = a_0$

- we also define a flag, mflag, that will be set in each iteration. For $k=0$ we set mflag $=1$.

Notes are online