SYDE 352 Jan 28

Routh-Hurwitz Stability criteria

Example $\Delta(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63 = 0$

s^5	1	4	3
s^4	1	24	63
s^3	-20	-60	0
s^2	21	63	0
s^1	0	0	
s^0	0/0		

$\leftarrow$ auxiliary equation $\rightarrow 21s^2 + 63 = 0$

$s_{1,2} = \pm j\sqrt{3}$

$s^2 + 3 = 0$

$s^2 = -3$

The auxiliary equation will be a factor of the characteristic equation so we can find:

$$\Delta(s) = (s^2+3)(s^3+s^2+s+21) = 0$$

s^3	1	1
s^2	1	21
s^1	-20	0
s^0	21	

- 2 sign changes
  $\Rightarrow$ 2 roots in RHP
  $\Rightarrow$ system is unstable

roots of (s^3+s^2+s+21): $s_{3,4} = 1 \pm j\sqrt{6}$ $\quad s_5 = -3$

Example $\Delta(s) = s^5 + s^4 + 2s^3 + 2s^2 + 4s + 1 = 0$

s^5	1	2	4
s^4	1	2	1
s^3	ε	3	0
s^2	$\frac{2\varepsilon-3}{\varepsilon}$ *	1	0
s^1	$3+O(\varepsilon^2)$ **	0	
s^0	1		

* $\frac{2\varepsilon-3}{\varepsilon}$ as $\varepsilon \to 0$, $\frac{2\varepsilon-3}{\varepsilon} \to \frac{-3}{\varepsilon}$

** $\dfrac{3\left(\frac{2\varepsilon-3}{\varepsilon}\right)-\varepsilon}{\frac{2\varepsilon-3}{\varepsilon}} = 3 - \dfrac{\varepsilon^2}{2\varepsilon-3}$

2 sign changes

$\Rightarrow$ 2 roots in the RHP

$\Rightarrow$ system is unstable

$$s_{1,2} = 0.5474 \pm j1.2804$$

$$s_{3,4} = -0.9278 \pm j1.0116$$

$$s_5 = -0.2792$$

Example

$$\Delta(s) = s^3 + 3s^2 + 3s + (1+K) = 0$$

For a stable system the hurwitz test requires:

$$1+K > 0$$

or $K > -1$

Use the Routh Criterion to find:

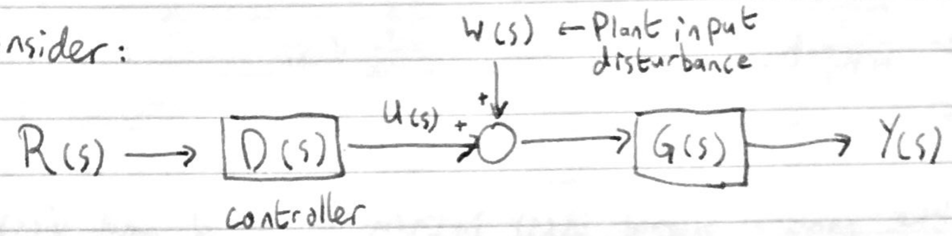
$$\frac{8-K}{3} > 0 \Rightarrow K < 8$$

s^3	1	3
s^2	3	$1+K$
s^1	$\frac{8-K}{3}$	0
s^0	$1+K$	

For a stable system $-1 < K < 8$ is required

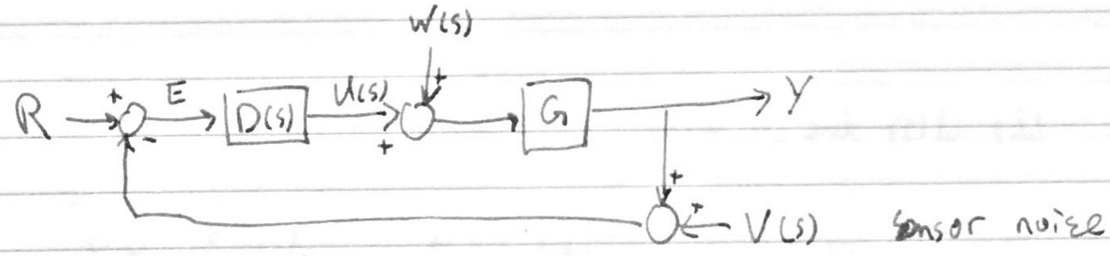
Control Concepts

Consider:



$$Y(s) = U\,G(s) + W(s)\,G(s) = R\,D(s)\,G(s) + G(s)\,W(s)$$

Examine what happens when we put a feedback loop around the system

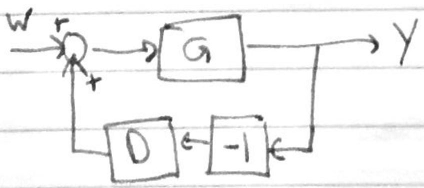


Find $Y(s)$:

a) due to $R(s)$ ($W = V = 0$)

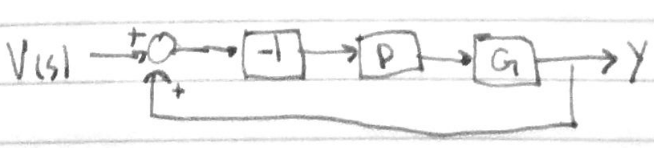
$$Y(s) = \frac{DG}{1+DG} R(s) \qquad (1)$$

b) due to W ($R = V = 0$



$$Y = \frac{G}{1-(-D)G} W = \frac{G}{1+DG} W(s) \qquad (2)$$

c) Y due to V ($R = W = 0$)



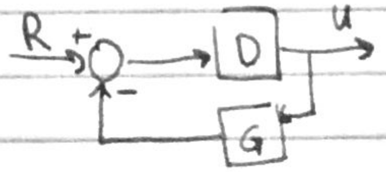
$$Y = \frac{-DG}{1-(-DG)} V = \frac{-DG}{1+DG} V(s) \qquad (3)$$

Then by superposition:

$$\boxed{Y(s) = \frac{DG}{1+DG} R(s) + \frac{G}{1+DG} W(s) - \frac{DG}{1+DG} V(s)} \quad (4)$$

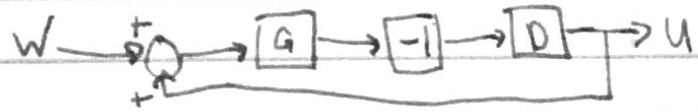
Find the control signal $U(s)$ in terms of R, W, and $V(s)$

i) $U(s)$ due to $R(s)$



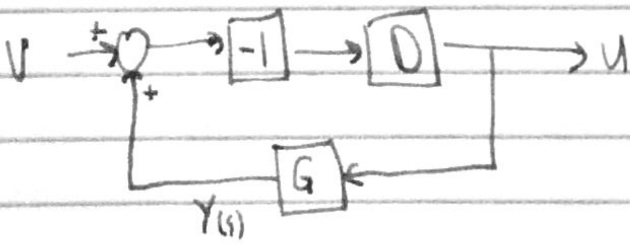
$$U(s) = \frac{D}{1+DG} R(s) \quad (5)$$

ii) $U(s)$ due to $W(s)$



$$U(s) = \frac{-DG}{1+DG} W(s) \quad (6)$$

iii) $U(s)$ due to $V(s)$



$$U(s) = \frac{-D}{1+DG} V(s) \quad (7)$$

The overall control signal will be:

$$\boxed{U(s) = \frac{D}{1+DG} R(s) - \frac{DG}{1+DG} W(s) - \frac{D}{1+DG} V(s)} \quad (8)$$

Find the error due to R, W, & V

$$\boxed{E(s) = R - Y = \frac{1}{1+DG} R(s) - \frac{G}{1+DG} W(s) + \frac{DG}{1+DG} V(s) \qquad (9)}$$