

SYDE 533 Conflict Resolution

Basics of n-DM Conflicts

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Review of Sequential Stability

A state s is **sequentially sanctioned** (s) for a DM if for each UI from s , a credible action can be taken by the opponent to stop the DM from taking advantage of the UI

For every UI a from s , a is sanctioned by at least one opponent UI

Check each UI from s ; the UI is sanctioned if the opponent has at least one UI which results in a less preferred state than s



Quiz!

Review of Sequential Stability

DM 1 Preference Ranking:

a	d	e
b		

DM 2 Preference Ranking:

b
d
e

$a \rightarrow b$ is blocked by $b \rightarrow d$

$a \rightarrow b$ is blocked by $b \rightarrow e$

$\Rightarrow a$ is stable for DM 1 – the UI is always sanctioned

Review of Sequential Stability

DM 1 Preference Ranking:

d	a	e
b		

DM 2 Preference Ranking:

b
d
e

$a \rightarrow b$ is NOT blocked by $b \rightarrow d$

$a \rightarrow b$ is blocked by $b \rightarrow e$

$\Rightarrow a$ is stable for DM 1 – the UI is sanctioned by at least one opponent UI

Review of Sequential Stability

DM 1 Preference Ranking:

a	d	e
b		
c		

DM 2 Preference Ranking:

b	c
d	e

$a \rightarrow b$ is blocked by $b \rightarrow d$

$a \rightarrow c$ is blocked by $c \rightarrow e$

$\Rightarrow a$ is stable for DM 1 – both ULs are sanctioned

Review of Sequential Stability

DM 1 Preference Ranking:

d	a	e
	b	
	c	

DM 2 Preference Ranking:

b	c
d	e

$a \rightarrow b$ is NOT blocked by $b \rightarrow d$

$a \rightarrow c$ is blocked by $c \rightarrow e$

$\Rightarrow a$ is unstable for DM 1 – there is an unsanctioned UI for DM 1
($a \rightarrow b$)

Review of Simultaneous Sanctioning

A state s which is unstable for both DMs is **simultaneously sanctioned** (\ddagger) for a DM if both players moving together could result in an outcome which is less preferred to s by the DM

For every combination of UIs, calculate the resulting outcome and compare to the starting state

A UI is sanctioned if at least one of the resulting states is less preferred to the starting state



Quiz!

Review of Simultaneous Sanctioning

DM 1 Preference Ranking:

a	d	f
b		
e		

DM 2 Preference Ranking:

a
c

$$b + c - a = d ; e + c - a = f$$

$a \rightarrow b$ is blocked by $a \rightarrow c$

$a \rightarrow e$ is blocked by $a \rightarrow c$

$\Rightarrow a$ is \nexists for DM 1 – both ULs are sanctioned

Review of Simultaneous Sanctioning

DM 1 Preference Ranking:

d	a	f
	b	
	e	

DM 2 Preference Ranking:

a
c

$$b + c - a = d ; e + c - a = f$$

$a \rightarrow b$ is NOT blocked by $a \rightarrow c$

$a \rightarrow e$ is blocked by $a \rightarrow c$

$\Rightarrow a$ is unstable for DM 1 – at least one unsanctioned UI

Review of Simultaneous Sanctioning

DM 1 Preference Ranking:

f	a	d	h	l
<hr/>				
b				
e				

DM 2 Preference Ranking:

a
<hr/>
c
g

$$b + c - a = d ; b + g - a = h ; e + c - a = f ; e + g - a = l$$

$a \rightarrow b$ is blocked by $a \rightarrow c$ and $a \rightarrow g$

$a \rightarrow e$ is blocked by $a \rightarrow g$

$\Rightarrow a$ is ∇ for DM 1 – both ULs are sanctioned by at least one opponent move

Review of Simultaneous Sanctioning

DM 1 Preference Ranking:

f	d	a	h	l
b				
e				

DM 2 Preference Ranking:

a
c
g

$$b + c - a = d ; b + g - a = h ; e + c - a = f ; e + g - a = l$$

$a \rightarrow b$ is blocked by $a \rightarrow g$

$a \rightarrow e$ is blocked by $a \rightarrow g$

$\Rightarrow a$ is ∇ for DM 1 – both ULs are sanctioned by at least one opponent move

Review of Equilibria

A state which is stable for all DMs (under a particular definition of stability) is an **equilibrium**

- ▶ $r + r = E$
- ▶ $s + s = E$
- ▶ $r + s = E \rightarrow$ only exception to "matching rule" so far
- ▶ $r + u = X$
- ▶ $s + u = X$
- ▶ $u + u = E$
- ▶ $u + \text{anything} = X$

Learning Objectives

Basics of n -DM Conflicts

By the end of this lesson, you will be able to:

- ▶ Identify the types of infeasible states
- ▶ Carry out the removal of states using several methods
- ▶ Conduct stability analysis and calculations for n -DM conflicts using the tableau form
- ▶ Identify several types of stable outcomes and explain their relation to human behaviour

Steps in Modelling

- ▶ Select a point in time
- ▶ Choose players and options
- ▶ Remove infeasible outcomes
- ▶ Develop preference rankings for each player
- ▶ Carry out stability analysis

Select a Point in Time

Garrison Diversion Unit

- ▶ Americans take water from the Missouri River basin to irrigate fields in the Hudson Bay drainage basin of central and eastern North Dakota
- ▶ Polluted irrigation water drains via the Red and Souris Rivers into Manitoba
- ▶ High risk of catastrophic environmental damage due to the introduction of foreign biota into the Hudson Bay drainage system

We will analyse the conflict situation in April 1967

Choose Players and Options

Garrison Diversion Unit

- ▶ US Support
- ▶ US Opposition
- ▶ Canadian Opposition
- ▶ International Joint Commission (IJC)
 - ▶ International mediator for boundary water disputes between Canada and the US, makes impartial (sometimes binding) recommendations on water disputes put forward by either country

Choose Players and Options

Garrison Diversion Unit

Players	Options
U.S. Support	<ol style="list-style-type: none">1. Complete full GDU2. Complete GDU modified to reduce Canadian impacts3. Complete GDU modified to appease US environmentalists
U.S. Opposition	<ol style="list-style-type: none">1. Legal action based on environmental legislation
Canadian Opposition	<ol style="list-style-type: none">1. Legal action based on Boundary Treaty of 1909
IJC	<ol style="list-style-type: none">1. Support completion of GDU2. Support completion of GDU modified to reduce Canadian impacts3. Support suspension of GDU except for the Lone-tree Reservoir4. Support the complete suspension of the GDU

Remove Infeasible States

Garrison Diversion Unit

There are a total of $2^9 = 512$ possible states, not all of which are feasible

Before proceeding to the analysis step, the infeasible outcomes must be removed - stability analysis is only performed on feasible states

There are 4 types of infeasible outcomes

Remove Infeasible States

Type 1

Type 1 infeasible states: logically infeasible outcomes for a given player

US Support cannot build more than one type of project

- ▶ Sets of states (1 1 - - - - - -), (1 - 1 - - - - -), and (- 1 1 - - - - -) are infeasible

Remove Infeasible States

Type 2

Type 2 infeasible states: preferentially infeasible outcomes for a given player

- ▶ States which involve a strategy selection by a player which they would not be expected to take

US Support will build some sort of project

- ▶ Set of states (0 0 0 - - - - -) is infeasible



Remove Infeasible States

Type 3

Type 3 infeasible states: logically infeasible outcomes among players

- ▶ DMs vying for indivisible resource
- ▶ States in which both DMs get resource are infeasible

Remove Infeasible States

Type 4

Type 4 infeasible states: preferentially infeasible outcomes for at least one player which involve strategy choices by at least two players

US Opposition will probably file legal action if the US Support builds the full project

- ▶ Set of states (1 - - 0 - - - - -) is preferentially infeasible for the US Opposition



Remove Infeasible States

Garrison Diversion Unit

Reasons	Removed
Type 1	
Mutually exclusive options for US Support	(1 1 - - - - -) (1 - 1 - - - -) (- 1 1 - - - -) (- - - - 1 1 - -) (- - - - 1 - 1 -) (- - - - 1 - - 1)
Mutually exclusive options for IJC	(- - - - - 1 1 -) (- - - - - 1 - 1) (- - - - - 1 1)
Type 2	
Some sort of project will be built IJC will make a recommendation	(0 0 0 - - - - -) (- - - - - 0 0 0 0)
Type 3	
None	
Type 4	
US Opposition will pursue legal action against full project US Opposition will not pursue legal action if appeased	(1 - - 0 - - - -) (- - 1 1 - - - -) (1 - - - 0 0 - -) (- 1 - - 0 - - 1 -) (- 1 - - 0 - - - 1) (- - 1 - 0 - - 1 -) (- - 1 - 0 - - - 1)

After removing all infeasible states, we are left with 23 feasible ones

Remove Infeasible States

Inspection Method

Inspection Method: for small conflicts, write down all outcomes in the game and manually remove the infeasible outcomes

This is what we did for the Cuban Missile Crisis example

Remove Infeasible States

Counting Method

Counting Method: sequentially count through all of the outcomes and write down which are not in the infeasible list

Remove Infeasible States

Subtraction Method

Subtraction Method: subtract the sets of infeasible outcomes from the total set of possible outcomes

- ▶ Keep in mind what you are removing
- ▶ Check for overlaps
- ▶ Reduce whenever possible to simplify calculations

Remove Infeasible States

Subtraction Method

Subtraction Method: subtract the sets of infeasible outcomes from the total set of possible outcomes

- ▶ In a 7 option game, the whole game is given by (- - - - -)
- ▶ Subtracting (1 0 - - - -) gives the sets (1 1 - - - -), (0 1 - - - -), and (0 0 - - - -)
 - ▶ There are 4 possible configurations for the first two bits: 00, 01, 10, and 11
 - ▶ We are removing of of these (10) and leaving the rest
- ▶ These sets can be reduced to (0 - - - -) and (1 1 - - - -)
 - ▶ The sets (0 0 - - - -) and (0 1 - - - -) have the first bit in common so can be simplified
 - ▶ There may be more than one way to simplify
- ▶ From the original $2^7 = 128$ states, we have removed $2^5 = 32$, leaving 96

Remove Infeasible States

Subtraction Method

Subtract the sets (1 1 - -) and (- - 0 -) from the game (- - - -)

- ▶ Keep in mind what you are removing
- ▶ Check for overlaps
- ▶ Reduce whenever possible to simplify calculations

Remove Infeasible States

Subtraction Method

Subtract the sets $(1\ 1\ -\ -)$ and $(-\ -\ 0\ -)$ from the game $(-\ -\ -\ -)$.

First, subtract $(1\ 1\ -\ -)$ from $(-\ -\ -\ -)$:

- ▶ We are left with $(0\ 0\ -\ -)$, $(1\ 0\ -\ -)$, and $(0\ 1\ -\ -)$
- ▶ We can simplify to $(0\ -\ -\ -)$ and $(1\ 0\ -\ -)$
- ▶ There are $2^3 + 2^2 = 12$ states remaining

Remove Infeasible States

Subtraction Method

Subtract the sets $(1\ 1\ -\ -)$ and $(-\ -\ 0\ -)$ from the game $(-\ -\ -\ -)$.

Next, subtract $(-\ -\ 0\ -)$ from $(0\ -\ -\ -)$ and from $(1\ 0\ -\ -)$:

- ▶ $(0\ -\ -\ -) - (-\ -\ 0\ -)$:
 - ▶ Keep the 0 in the first position
 - ▶ We are removing states with a "0" in the 3rd bit
 - ▶ The remaining set is $(0\ -\ 1\ -)$ which has $2^2 = 4$ states
- ▶ $(1\ 0\ -\ -) - (-\ -\ 0\ -)$
 - ▶ Keep 1 and 0 in their spots
 - ▶ We are removing states with a "0" in the 3rd bit
 - ▶ The remaining set is $(1\ 0\ 1\ -)$ which has $2^1 = 2$ states
- ▶ There are $4 + 2 = 6$ states left in $(0\ -\ 1\ -)$ and $(1\ 0\ 1\ -)$

Remove Infeasible States

Subtraction Method

Subtract the sets (1 1 - -) and (- - 0 -) from the game (- - - -)

Subtraction method: 6 states left in (0 - 1 -) and (1 0 1 -)

0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1
0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1
0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
x	x			x	x			x	x			x	x	x	x

Feasible States

Garrison Diversion Unit

USS																								
Full	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	
Reduced	1	0	0	1	1	0	0	1	1	0	1	1	0	0	1	1	0	0	1	1	0	0	1	
Appease	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0	
USO																								
Legal	0	0	1	1	0	0	1	1	0	0	1	0	0	1	1	0	0	1	1	0	0	1	1	
CO																								
Treaty	0	0	0	0	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	
IJC																								
Full	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
Reduced	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	
Lonetree	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	
Suspend	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	
Decimal	34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282	

Preferences

Garrison Diversion Unit

US Support: prefers to build the full GDU or at least some modified version

US Opposition: will press for legal action as long as the project has any adverse effects on the environment within the US

Canadian opposition: prefers no project to be built; prefers not to use legal action which goes against an IJC recommendation

IJC: prefers to make a recommendation; otherwise it is impartial

Preferences

Garrison Diversion Unit

USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57
IJC	[34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]

States between brackets [] and with a line on top are equally preferred amongst each other

Stability Analysis

Check for Rational States

A state s is **rational** (r) for a DM if the DM has no unilateral improvements (UIs) from s .

USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282		
					41			57					66	82	90		148		153		276		281		
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34		
																				282	154	90	74	58	42
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57		
										74	66	68				42	34	36					41		
IJC	[34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]		

Stability Analysis

Rational States

	r	r	r	r		r	r		r	r	r	r			r		r		r				
USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282
					41			57					66	82	90		148		153		276		281
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r						
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r	r				r	r	r			r	r	r	r		
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57
										74	66	68				42	34	36					41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]

Sequentially Sanctioned States

A state s is **sequentially sanctioned** (s) for a DM if for each UI from s , a credible action can be taken by the opponent(s) to stop the DM from taking advantage of the UI.

	r	r	r	r		r	r		r	r	r	r		r		r		r					
USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282
					41			57						66	82	90		148		153		276	281
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r						
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r	r				r	r	r				r	r	r	r	
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57
										74	66	68				42	34	36					41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]

Stability Analysis

Sequentially Sanctioned States

In 2-DM games: only need to check UIs for one opponent

In $n \geq 2$ DM games: check for **sequences of UIs** from opponents -
each opponent can move at most one time

Stability Analysis

Sequentially Sanctioned States

	r	r	r	r		r	r		r	r	r	r			r		r		r				
USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282
					41			57					66	82	90		148		153		276		281
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r						
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r					r	r	r	s	s		r	r	r	r	
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57
										74	66	68				42	34	36					41

- ▶ From 58, CO has a UI to 42. USS then has a UI from 42 to 41. 41 is less preferred by CO to 58 so 58 is stable.
- ▶ From 50, CO has a UI to 34. USO then has a UI from 34 to 42. USS then has a UI from 42 to 41. 41 is less preferred by CO to 50 so 50 is stable.

Stability Analysis

Unstable States

Any remaining states which are neither rational nor sequentially sanctioned are unstable (u).

	r	r	r	r	u	r	u	r	r	r	r	u	u	u	r	u	r	u	r	u	r	u	
USS	41	57	[34	36]	42	[50	52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282
					41			57						66	82	90		148		153		276	281
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u
USO	[276	148	84	68	52	36]	[282	154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r	r	u	u	u	r	r	r	s	s	u	r	r	r	r	u
CO	282	[274	276]	154	[146	148]	74	[66	68]	90	[82	84]	36	42	34	58	[50	52]	281	153	89	41	57
										74	66	68				42	34	36					41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34	36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]

Stability Analysis

Simultaneous Sanctioned States

If state s is unstable for at least two players, simultaneous sanctioning (\sharp) can occur if the new outcome is less preferred to s

When a state is unstable for m out of n players, simultaneous stability calculations are done for all possible combinations of two or more players for which the state is unstable and for all combinations of UIs which are not sequentially blocked

Stability Analysis

Simultaneous Sanctioned States

Formula of simultaneous sanctioning:

- ▶ q : state which is unstable for a set M of DMs, $|M| \geq 2$
- ▶ $\Gamma \subseteq M$: subset of players for which q is unstable
- ▶ A_i : set of UIs from q for DM i which are not sequentially blocked; $a_i \in A_i$ is a particular UI for DM i

For every $\Gamma \subseteq M$, $|\Gamma| \geq 2$ and $a_i \in A_i$, calculate the outcome to compare to q :

$$\text{outcome} = \sum_{i \in \Gamma} a_i - (|\Gamma| - 1) \cdot q$$

The outcome is determined for every $a_i \in A_i$ for each player in the set Γ . If all of the UIs are blocked for a DM, q is simultaneously sanctioned for that DM.

Stability Analysis

Simultaneous Sanctioned States

Example calculation: state 84

- ▶ q : 84 for a set $M = \{USS, CO\}$ of DMs, $|M| \geq 2$
- ▶ $A_{USS} = \{82\}$ and $A_{CO} = \{68\}$

For every $\Gamma \subseteq M$, $|\Gamma| \geq 2$, calculate the outcome to compare to q .
Only one possible $\Gamma \subseteq M$: $\Gamma = \{USS, CO\}$

$$\begin{aligned} outcome &= \sum_{i \in \Gamma} a_i - (|\Gamma| - 1) \cdot q \\ outcome &= (82 + 68) - (2 - 1)84 = 66 \end{aligned}$$

State 66 is more preferred than state 84 by both players, so 84 is not simultaneously sanctioned.

In fact, there are no simultaneously sanctioned states in this conflict.

Stability Analysis

Simultaneous Sanctioned States

Hypothetical calculation: state q

- ▶ q : unstable for a set $M = \{B, C, D\}$ of DMs, $|M| \geq 2$
- ▶ $A_B = \{b\}$, $A_C = \{c\}$, and $A_D = \{d\}$

For every $\Gamma \subseteq M$, $|\Gamma| \geq 2$, calculate the outcome to compare to q .

Four possible $\Gamma \subseteq M$: $\Gamma_1 = \{B, C\}$, $\Gamma_2 = \{B, D\}$, $\Gamma_3 = \{C, D\}$, and $\Gamma_4 = \{B, C, D\}$

- ▶ $outcome = \sum_{i \in \Gamma_1} a_i - (|\Gamma| - 1) \cdot q$
 $outcome = (b + c) - (2 - 1)q$
- ▶ $outcome = \sum_{i \in \Gamma_2} a_i - (|\Gamma| - 1) \cdot q$
 $outcome = (b + d) - (2 - 1)q$
- ▶ $outcome = \sum_{i \in \Gamma_3} a_i - (|\Gamma| - 1) \cdot q$
 $outcome = (c + d) - (2 - 1)q$
- ▶ $outcome = \sum_{i \in \Gamma_4} a_i - (|\Gamma| - 1) \cdot q$
 $outcome = (b + c + d) - (3 - 1)q$

Stability Analysis

Equilibria

Check for equilibria in this conflict:

	r	r	r	r	u	r	r	u	r	r	r	r	u	u	u	r	u	r	u	r	u	r	u
USS	41	57	[34 36]	42	[50 52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282		
				41			57					66	82	90		148		153		276		281	
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u	u
USO	[276 148 84 68 52 36]	[282 154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34						
																	282	154	90	74	58	42	
	r	r	r	r	r	r	r	r	u	u	u	r	r	r	s	s	u	r	r	r	r	u	
CO	282	[274 276]	154	[146 148]	74	[66 68]	90	[82 84]	36	42	34	58	[50 52]	281	153	89	41	57					
									74	66	68				42	34	36						41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34 36 41 42 50 52 57 58 66 68 74 82 84 89 90 146 148 153 154 274 276 281 282]																						

Stability Analysis

Equilibria


There are seven possible equilibria...

	E	X	X	E	X	X	X	X	X	X	E	X	X	X	X	E	X	E	X	E	X	E	X
	r	r	r	r	u	r	r	u	r	r	r	r	u	u	u	r	u	r	u	r	u	r	u
USS	41	57	[34 36]	42	[50 52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282		
				41			57					66	82	90		148		153		276		281	
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u
USO	[276 148 84	68	52	36]	[282 154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34			
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r	u	u	u	r	r	r	s	s	u	r	r	r	r	u	
CO	282	[274 276]	154	[146 148]	74	[66 68]	90	[82 84]	36	42	34	58	[50 52]	281	153	89	41	57					
									74	66	68				42	34	36						41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34 36	41	42	50	52	57	58	66	68	74	82	84	89	90	146	148	153	154	274	276	281	282]	

...but some of them are false equilibria!

Stability Analysis

False Equilibria

 False equilibria occur when Type 2 and Type 4 (preferentially infeasible) states are removed – remove Type 2 and Type 4 infeasible states with caution, or not at all

A state is a **false equilibrium** when an unsanctioned UI from it has been removed

Stability Analysis

False Equilibria

Players	Options	153	156	148
U.S. Support	1. Full GDU	1	0	0
	2. Modified GDU for Canada	0	0	0
	3. Modified GDU for environmentalists	0	1	1
U.S. Opposition	1. Legal action	1	1	0
Canadian Opposition	1. Legal action	1	1	1
IJC	1. Support full GDU	0	0	0
	2. Support mod. Canadian GDU	0	0	0
	3. Support suspension except Lonetree	1	1	1
	4. Support full suspension of GDU	0	0	0

From state 153, USS has a UI to state 156 (appease environmentalists)

From 156, USO has UI to drop legal action resulting in state 148
State 148 is more preferred to state 153 in USS's preference ranking and rational for everyone else

Stability Analysis

False Equilibria

Players	Options	36	33	44
U.S. Support	1. Full GDU	0	1	1
	2. Modified GDU for Canada	0	0	0
	3. Modified GDU for environmentalists	1	0	0
U.S. Opposition	1. Legal action	0	0	1
Canadian Opposition	1. Legal action	0	0	0
IJC	1. Support full GDU	1	1	1
	2. Support mod. Canadian GDU	0	0	0
	3. Support suspension except Lonetree	0	0	0
	4. Support full suspension of GDU	0	0	0

From state 36, USS has a UI to state 33 (build full project) which was removed since we assumed that USO would pursue legal action against the full project

From state 33, USO could take legal action and move the conflict to state 41 which is more preferred by USS to state 36 and rational for everyone else

Stability Analysis

False Equilibria

Players	Options	281	284	276
U.S. Support	1. Full GDU	1	0	0
	2. Modified GDU for Canada	0	0	0
	3. Modified GDU for environmentalists	0	1	1
U.S. Opposition	1. Legal action	1	1	0
Canadian Opposition	1. Legal action	1	1	1
IJC	1. Support full GDU	0	0	0
	2. Support mod. Canadian GDU	0	0	0
	3. Support suspension except Lonetree	0	0	0
	4. Support full suspension of GDU	1	1	1

From state 281, USS has UI to appease (284)

From state 284, USO has a UI to stop legal action, moving the conflict to state 276 which is more preferred by USS and rational for all others

Stability Analysis

False Equilibria

How to find false equilibria:

- ▶ Look only at equilibrium states
- ▶ For each DM, check if there are UIs which were removed as Type 2 or Type 4 infeasible states
- ▶ If for at least one DM, at least one of these UIs can lead to a better (feasible) outcome, the state is a false equilibrium

Stability Analysis

Equilibria

	E	X	X	E^a	X	X	X	X	X	X	E	X	X	X	X	E	X	E^a	X	E	X	E^a	X
	r	r	r	r	u	r	r	u	r	r	r	r	u	u	u	r	u	r	u	r	u	r	u
USS	41	57	[34 36]	42	[50 52]	58	66	82	74	90	68	84	89	148	146	153	154	276	274	281	282		
				41			57					66	82	90		148		153		276		281	
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	u	u	u	u	u	u
USO	[276 148 84 68 52 36]	[282 154]	90	74	58	42	281	153	89	57	41	274	146	82	66	50	34						
																		282	154	90	74	58	42
	r	r	r	r	r	r	r	r	u	u	u	r	r	r	s	s	u	r	r	r	r	u	
CO	282	[274 276]	154	[146 148]	74	[66 68]	90	[82 84]	36	42	34	58	[50 52]	281	153	89	41	57					
									74	66	68				42	34	36						41
	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r	r
IJC	[34 36 41 42 50 52 57 58 66 68 74 82 84 89 90 146 148 153 154 274 276 281 282]																						

- ▶ 41: USS builds full, USO legal, IJC recommends full
- ▶ 74: USS builds reduced, USO legal, IJC recommends reduced
- ▶ 148: USS reduced, USO legal, CO treaty, IJC recs Lonetree
- ▶ 276: USS appeases, CO treaty, IJC recommends suspension

Stability Analysis

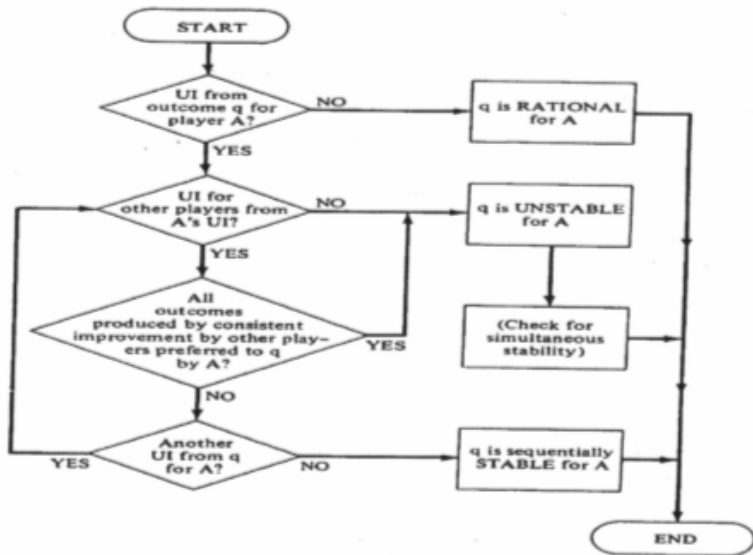
Historical Outcome

State 148 occurred historically

Given an IJC recommendation, a conflict then takes place among the remaining DMs

Solution Concept	Foresight	Knowledge of Preferences	Disimprovement	Strategic Risk
Rational	Low	Own	Never	Ignores risk
Sequential	Medium	All	Never	Takes some risks; satisfies

Analysis of n-DM Games in Chart Form



Next Week

- ▶ Basics of n-DM models (continued)

- ▶ Tutorial?

