SYDE 312 Jan 26

I-3 Linear independance & bases

V v.s. over F

Take $\{u_1, \ldots, u_r\}$ vectors. Any vector $v \in S_p\{u_1, \ldots, u_r\}$ can be written as a linear combination of these r vectors

Take $v = \underline{0}$ we get : $\underline{0} = K_1 u_1 + \ldots + K_r u_r$

All $K_i = 0$ is always possible.

The vectors $\{u_1, \ldots, u_r\}$ are called linearly independant if only the zero l.c. works

If $\underline{0} = K_1 u_1 + \ldots + K_r u_r$ and at least one $K_i \neq 0$ then $\{u_1, \ldots, u_r\}$ are called linearly dependant vectors.

Ex. $\{\underline{0}\}$ is dependant ($K\underline{0} = \underline{0}$)

A set with $\underline{0}$ in it is dependant // $\{v\}$, $v \neq \underline{0}$ always independant set // $\underline{0} = Kv \Rightarrow K = 0$ // $\{u, v\}$, $u, v \neq 0$ (Put $\underline{0} = K_1 u + K_2 v$) is dependant $\Leftrightarrow u = cv$.

$\{e_1, \ldots, e_n\}$ in $\mathbb{R}^n$ independant. $e_i = (0, \ldots, 0, \underset{\uparrow_{i\text{th}}}{1}, 0, \ldots, 0)$

$\{1, t, t^2, \ldots, t^n\}$ in $\mathbb{P}_n$ monomials

$\underline{0}(t) = 0 = K_0 + K_1 t + K_2 t^2 + \ldots + K_n t^n$ all t (identity)

$\Rightarrow$ all $K_i = 0$, independant vectors in P_n

in $C(R,R)$: $\{1, \cos x, \sin x\}$

$\underline{0}(t) = 0 = K_1 + K_2 \cos x + K_3 \sin x$ All $x \in \mathbb{R}$ identity

$\Rightarrow K_i = 0$ all i

linear independant set of functions

in $C(R,R)$: $\{1, \cos^2 x, \sin^2 x\}$

$0 = K_1 + K_2 \cos^2 x + K_3 \sin^2 x$

$= 1 - \cos^2 x - \sin^2 x = 1 - (\cos^2 x + \sin^2 x) = 1 - 1 = 0$ all x

This set is dependant.

In the bin vector space $\{123, 124, 34\}$

$= \underbrace{123 + 124}_{} + 34$

$= \underbrace{34 + 34}_{}$

$= 0 = \{\}$ $\therefore$ dependant set

$\boxed{\text{A set } \{u_1, \dots, u_r\} \text{ is linearly independant } \Leftrightarrow \text{ at least one vector is a linear combination of the others}}$

Proof:

$\Rightarrow \{u_1, \dots, u_r\}$ dep. set

$\Rightarrow \underline{0} = k_1 u_1 + \dots + k_r u_r$ and some $k_i \neq 0$

say $k_1 \neq 0$ (or renumber the scalars)

$\Rightarrow -k_1 u_1 = k_2 u_2 + \dots + k_r u_r$

$u_1 = \left(\frac{-k_2}{k_1}\right) u_2 + \dots + \left(\frac{-k_r}{k_1}\right) u_r$

So, u_1 is a linear combination of the others

$u_i = k_1 u_1 + \dots k_{i-1} u_{i-1} + k_{i+1} u_{i+1} + \dots + k_r u_r$

and not all $k_i = 0$

So $u_i - k_1 u_1 - \dots - k_{i-1} u_{i-1} - k_{i+1} u_{i+1} - \dots - k_r u_r = \underline{0}$

a ~~[illegible]~~ non-zero l.c. $= 0$

i.e. $\{u_1, \dots, u_r\}$ is a dep set //

Checking independance

Ex. In $\mathbb{R}^3$ $\left.\begin{array}{l} u = (1,-2,1) \\ v = (2,1,-1) \\ w = (7,-4,1) \end{array}\right\}$ dep or indep in $\mathbb{R}^3$?

Solution $(0,0,0) = k_1(1,-2,1) + k_2(2,1,-1) + k_3(7,-4,1)$

$\rightarrow$

$$\begin{bmatrix} 1 & 2 & 7 \\ -2 & 1 & -4 \\ 1 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} \boxed{1} & 2 & 7 \\ 0 & \boxed{1} & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

rank 2
inf. # of non-0
solutions

$\{u, v, w\}$ dependant //