14/Jan/2016 312 Lecture 5

I-1 (cont) Subspaces

Ex 5 (cont) subspaces of $F(X,R)$

$C^0 = \{ f \in F(X,R) \mid f \text{ is continuous} \}$

C' = space of continuously differentiable functions

C^2 : cont. 2nd deriv

C^∞ : infinitely differentiable

All subspaces of $F(X,R)$

~~$R_n < P < C^\infty < \ldots < C^3 < C^2 < C' < F(X,R)$~~

$P_0 < P_1 < P_2 < P_3 < \ldots < P < C^\infty < \ldots < C^3 < C^2 < C' < F(X,R)$

↑

"R"

Non-Examples

Ex $W = \{ v \in R^3 \mid v_1 \geq 0 \}$

$(1,1,1) \in W$ but $-1(1,1,1) \notin W$ No

Ex $W = \{ u \in R^n \mid \sum_{i=1}^{n} a_i u_i = k ; \ k \neq 0 \}$

$u, v \in W$

$\sum_{i=1}^{n} (u+v)_i = \sum_{i=1}^{n} u_i + v_i$ defn of vector sum in R^n

$= \sum_{i=1}^{n} u_i + \sum_{i=1}^{n} v_i = k + k = 2k \ (k \neq 0)$

$\Rightarrow u + v \notin W$

Shopping List

1) "Sameness" - all exs seem like R^n

isomorphic

2) dimension - max # of independant vars

3) independence of vectors

4) basis: Like $\{\hat{i}, \hat{j}, \hat{k}\}$ in R^3

The binary vector space

Vectors: all subsets of a master set (say with n elements)

Scalars: $\{0, 1\}$ with binary arithmetic

Vector Sum

$E_1, E_2 \in V$

$E_1 + E_2 = E_1 \cup E_2 - (E_1 \cap E_2)$

symmetric difference

$E + \phi = E \cup \phi - E \cap \phi$

$= E$

ϕ is the $\underline{0}$ is a vector

E is the inverse of E

$E + E = E \cup E - E \cap E$

$= E - E = \phi$

<u>Scalar Multiples</u>

$\left.\begin{array}{l} 1E = E \\ 0E = \phi \end{array}\right\}$ definitions

All axioms are satisfied V is a vector space over $\{0, 1\}$

<u>Ex</u> Binary vector space on $\{1,2,3,4,5,6\}$ V - master set

<u>eg</u> $E_1 = \{1,2,3\} \in V$
$E_2 = \{2,3,4,5\} \in V$

$E_1 + E_2 = (E_1 \cup E_2) - (E_1 \cap E_2)$

$= \{1,2,3,4,5\} - \{2,3\}$

$= \{1,4,5\}$

Notation Simplified

1) use 123 for $\{1,2,3\}$
1345 for $\{1,3,4,5\}$
etc

So $\{\{1,2,3\},\{1,3,4,5\}\}$

Can be written so $\{123, 1345\}$

2) Use coordinates $\{0,1\}^n$

eg $\{1,2,3\}$ would be $(1,1,1,0,0,0)$

$\{1,3,4,5\}$ would be $(1,0,1,1,1,0)$

Add these
$123 + 1345 = 245$

$$\begin{array}{r} (1,1,1,0,0,0) \\ + \ (1,0,1,1,1,0) \\ \hline (0,1,0,1,1,0) \end{array} \leftarrow 245$$

$1+1=0$

Subspaces of binary vector spaces

Ex V is the binary vector space on a 4 element master $\{1,2,3,4\}$

2^4 vectors

$W = \{123, 124, 34, \emptyset\}$

check Is $W < V$?

$123 + 124 = 34 \in W$

$123 + 34 = 124 \in W$

$124 + 34 = 123 \in W$

closed to vector sum

[$123 + \emptyset = 123$
$124 + \emptyset = 124$
$34 + \emptyset = 34$] unnecessary

closed to scalar prod (trivial); just check

$0 \cdot E = \emptyset \in W$

Yes $W < V$ or in coords

$(1,1,1,0) > (0,0,1,1)$
$(1,1,0,1)$
$(0,0,1,1) > (1,1,1,0)$

After lin indep is done use a matrix.

<u>Linear Combinations</u>

V is a vector space over F
$\{u_1, \ldots, u_r\}$ choose r vectors $\in V$

The expression.

$k u_1 + k u_2 + \ldots + k_r u_r$ wher $k_1, \ldots, k_r \in F$

is a vector in V (linear combination (lc))

called a linear combination of $\{u_1, \dots, u_r\}$

<u>EX</u> Show that $(-1, 1, 6, 11)$ is a lc of $(1,2,0,4)$ & $(1,1,-2,-1)$ in $\mathbb{R}^4$

<u>Soln</u> Solve for k_1, k_2 s.t.

$$(-1,1,6,11) = k(1,2,0,4) + k_2(1,1,-2,-1)$$

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 0 & -2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 6 \\ 11 \end{bmatrix}$$

4 x 2 lin system.

$$\left[\begin{array}{cc|c} 1 & 1 & -1 \\ 2 & 1 & 1 \\ 0 & -2 & 6 \\ 4 & -1 & 11 \end{array}\right] \sim \dots \sim \left[\begin{array}{cc|c} 1 & 1 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

unique sol.

$k_2 = -3$

$k_1 = 1 - k_2$

$= 2$

$$(-1, 1, 6, 11) = 2(1,2,0,4) + (-3)(1,1,-2,-1)$$

check

$\rightarrow$

Ex Express the vector $p \in P_2$ where $p(t) = t^2 + 4t - 3$

as a lin comb of the vecs $\{ t^2 - 2t + 5,\ 2t^2 - 3t,\ t+3 \}$

Soln Required:

$$t^2 + 4t - 3 = a(t^2 - 2t + 5) + b(2t^2 - 3t) + c(t+3) \quad (*)$$

To find a, b, c that work $(*)$ must be true for all t

i.e. an identity

$$\Rightarrow t^2 + 4t - 3 = (a+2b)t^2 + (-2a-3b+c)t + (5a+3c)$$

& equate coeff

$a + 2b = 1 \qquad -2a - 3b + c = 4 \qquad 5a + 3c = -3$

$$\begin{array}{cccc} a & b & c & \\ \multicolumn{4}{c}{\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ -2 & -3 & 1 & 4 \\ 5 & 0 & 3 & -3 \end{array}\right]} \end{array}$$

3+3 non homogeneous linear system.

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 6 \\ 0 & 0 & 1 & 4 \end{array}\right]$$

3 pivots in each
rank = 3
$\Rightarrow$ unique solns.

$c = 4 \quad b = ~~8~~\ 2 \quad a = -3$

V is a vector space over F
$S_1 = \{ u, ----, u_r \}$ is a set of vectors in V

$$W = \{ w \mid w = k_1 u_1 + \cdots + k_r u_r,\ k_1, \ldots, k_r \in F \}$$

is called the (linear) span of S

| Its the smallest vector space containg all vectors in S

W is a <u>subspace</u> of V

the smallest subspace contaning vectors of S

$W = Sp\{S\}$

$= Sp\{u_1, \ldots, u_r\}$

S is a spanning set of W

W is spanned by S