Steady State Error & System Type

Recall $D(s)G(s) = \dfrac{K \prod_{i=1}^{\wedge} (1+T_i s)}{s^j \prod_{k=j+1}^{\wedge} (1+T_k s)}$

where j is the system type

Example : Step input of Amplitude A

$$R(s) = \frac{A}{s}$$

$$E(s) = \frac{1}{1+DG(s)} R(s)$$

$$e_{ss} = \lim_{s \to 0} s E(s)$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} \frac{1}{1+DG} \cdot \frac{A}{\cancel{s}}$$

$$= \lim_{s \to 0} \frac{A}{1+DG}$$

$$= \frac{A}{1+\lim_{s \to 0} DG(s)} \quad \& \quad \lim_{s \to 0} D(s)G(s) = \begin{cases} K_p & j=0 \\ \infty & j>0 \end{cases}$$

$$\therefore e_{ss} = \begin{cases} A/1+K_p & j=0 \\ 0 & j \geq 1 \end{cases}$$

Ex velocity / ramp input

$$R(s) = \frac{A}{s^2}$$

Find $e_{ss} = \lim_{s \to 0} \cancel{s} \dfrac{1}{1+DG(s)} \cdot \dfrac{A}{s^2}$ →

$$e_{ss} = \lim_{s\to 0} \frac{A}{s + sDG(s)}$$

$$= \frac{A}{\lim_{s\to 0} s + \lim_{s\to 0} sDG(s)} \quad , \quad \lim_{s\to 0} sDG(s) = \begin{cases} 0 & j=0 \\ K_v & j=1 \\ \infty & j\geq 2 \end{cases}$$

$$\Rightarrow e_{ss} = \begin{cases} \infty & j=0 \\ 1/K_v & j=1 \\ 0 & j\geq 2 \end{cases}$$

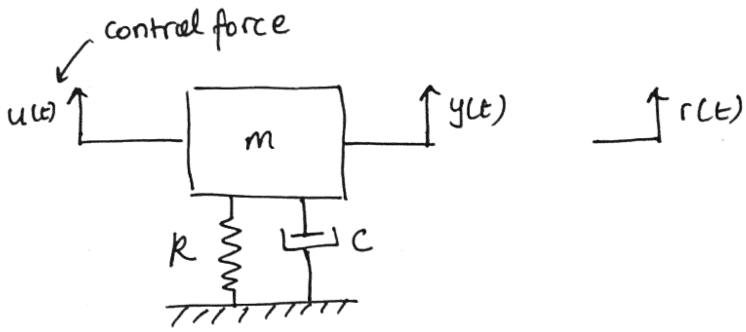
> Note: The power of the ~~demoni~~ denominator of the iput gives the bounds for the system type that will give zero steady state error. for step it was $e_{ss}=0$ if $j\geq 1$, for velocity $e_{ss}=0$ if $j\geq 2$

Proceeding similarly we find

Input \ Type	0	1	2	3
impulse	0	0	0	0
step	~~∞~~ $\frac{1}{1+K_p}$	0	0	0
ramp ~~Velocity~~	∞	$1/K_v$	0	0
parabola	∞	∞	$1/K_a$	0

→

PID Controllers



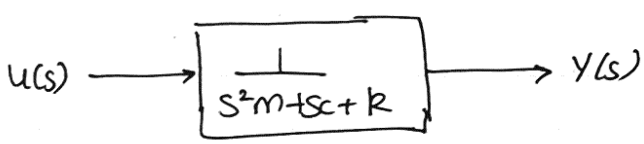
$y(t) \rightarrow$ actual output position
$r(t) \rightarrow$ reference or required position

equation of motion

$$m\ddot{y} + c\dot{y} + ky = u(t)$$

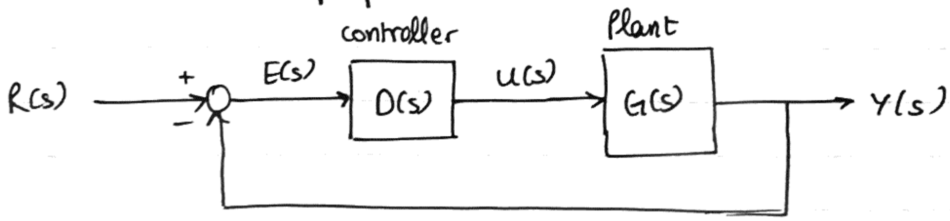
assume a quiescent system & take the Laplace transform

$$(s^2 m + sc + k)\,Y(s) = U(s)$$



output error $E(s) = R(s) - Y(s)$

make $U(s)$ proportional to $E(s)$



Case 1 Choose $D(s) = K_P$ a constant
↳ Proportional

$$Y(s) = \frac{DG}{1 + D(s)G(s)} R(s)$$

$$= \frac{K_P}{ms^2 + cs + k + K_P} R(s)$$

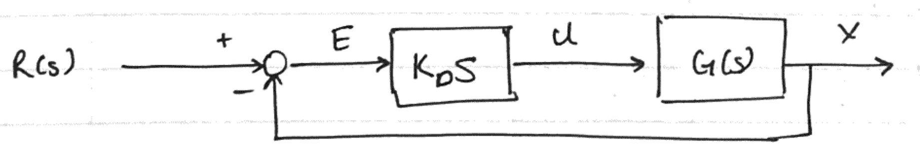
- K_P acts like adding another string
- increases ω_n
- has no effect on damping
- effects rise time
  peak time
  settling time
- no effect on overshoot

Case 2 Derivative Control

Assume the control force is proportional to the rate of change of the error.

$$u(t) = K_D \frac{de}{dt}$$

$$u(s) = K_D S E(s)$$



$$Y = \frac{K_D s}{ms^2 + (c+K_D)s + k}$$

- K_D increases the system damping
  $\Rightarrow$ the effective ζ increases

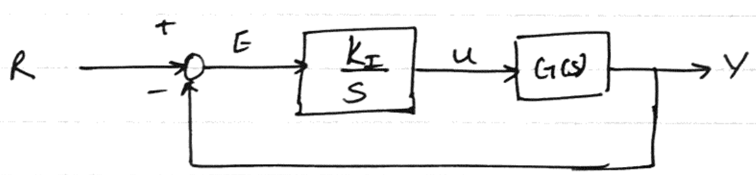
- D-control removes energy from the system
- leaves ω_n unchanged
- decreases ω_d

K_P & K_D cannot destabilize the system.

<u>Case 3</u> Integral Control

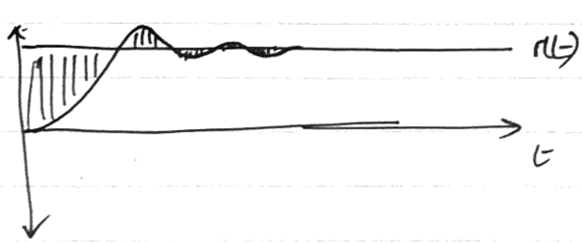
$$u(t) = K_I \int_0^t e(t)\,dt$$

$$U(s) = K_I \frac{E(s)}{s}$$



$$\frac{Y}{R} = \frac{DG}{1+DG} = \frac{K_I}{s(ms^2+cs+k)+K_I} = \frac{K_I}{ms^3+cs^2+ks+K_I}$$

- Adds another pole $\Rightarrow$ can potentially destabilize the system.



- change the system to 3rd degree
- cause $u(t) \neq 0$ even when $e(t) = 0$ due to the integral effect.

In general all are used together as