

① Inserting  $n$  elements using

a) Aggregate method

→ The table doubles in size when it runs out of space.

→ So, if the original size is 1, after insertion it doubles to size 2, after 2 more insertion it doubles to size 4 etc

→ In general, after  $k$  doublings the size is  $2^k$

pseudo code

→ initialize table with capacity = 1

for  $i$  1 to  $n$ :

if table is full:

new table = Create new table  
with size 2 to current size copy element  
from old to its new table.  
table = new table

insert element  $i$  into table

let  $k = \log(n+1) - 1$

Total cost =  $O(n) * k$

=  $O(n \log n)$

Amortize cost per insertion =  $O(\log n)$

Runtime per insertion is  $O(\log n)$

Total time is  $O(n) + \log(n+1)$

### (b) Accounting method

- \* Charge 2 units for each insertion
- when the table doubles in size from  $m$  to  $2m$ , Credit  $m$  units
- The Credit exactly pay for copy cost at  $O(m)$
- Total Credit is  $m + 2m + 4m + \dots + \frac{n}{2}m = O(n)$

pseudo code :

→ initialize table with capacity = 1

for  $i = 0$  to  $n$ :

if table is full:

newtable = Create new table with size  
2 in current size

copy elements from old table to new  
table

table = newtable

insert element  $i$  into table

initialize charges = 0

initialize credits = 0

for  $i = 1$  to  $n$ :

charges += 2

if table doubled in size from  $m$  to  $2m$ :  
Credits +=  $m$

Total Charges =  $2 \times n = O(n)$

Total Credits =  $m + 2m + \dots + \frac{n}{2}m = O(n)$

Amortized cost per insertion = Total /  $n = O(n)/n$   
= 1 //

Runtime per insertion =  $O(1)$

Total time =  $O(n)$  //