nmf

October 4, 2023

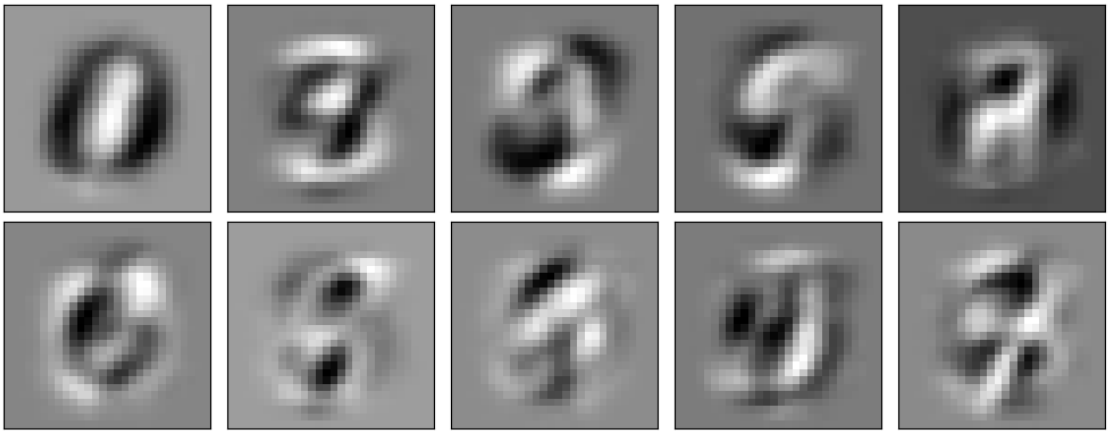
[5]:

```
"""
Inference & Representation - 2022 Fall HW1
Question 2 PCA and Non-negative matrix factorization.

"""

"""
Tools for loading the MNIST Data.
From Optimization Based Data Analysis HW1
@author: Brett
"""
import numpy as np
from mnist_tools import *
from plot_tools import *
import matplotlib.pyplot as plt

"""
Given train (in the format returned by load_train_data in mnist_tools),
and a 1d numpy array testImage you should return a tuple (digit,imageIdx). ␣
 ↪digit is
an integer giving the numerical digit value of the training image closest
to the testImage in Euclidean distance.  imageIdx is the row number of the␣
 ↪closest
training image in the 2d array train[digit].
"""

"""
Assumes the data file is in 'mnist_all.mat'.
"""

datafile = "mnist_all.mat" #Change if you put the file in a different path
train = load_train_data(datafile)

trainarr=np.asarray(train)
trainarr = np.reshape(trainarr, (trainarr.shape[0]*trainarr.shape[1],-1))
trainarr = trainarr.astype(float)
trainarr=trainarr-trainarr.mean(axis=0)
```

[6]:

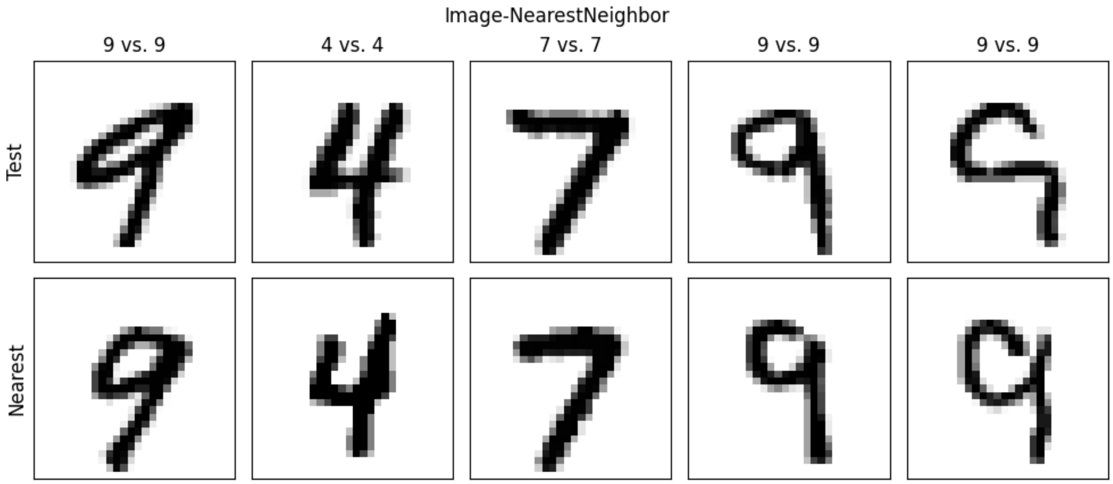
```
"""
Plot of the singular vectors corresponding
to top 10 singular values of the data.
@author: Vlad
"""
U, s, V = np.linalg.svd(trainarr, full_matrices=True)
n=10
imgs = [V[i,:] for i in range(n)]
plot_image_grid(imgs,
           "Singular vectors corresponding to top 10 singular values of the␣
 ↪data")
```

Singular vectors corresponding to top 10 singular values of the data



[7]:

```
"""
Plot of the results of the nearest neighbour test applied
to a principal component projection.
@author: Vlad
"""

def project(V, Images) :
     return np.dot(V.T, np.dot(V, Images))

def compute_nearest_neighbors(train, testImage, V) :
    train=[np.array(i, dtype=float) for i in train]
    testImage= np.array(testImage, dtype=float)
    digit=0
    imageIdx=0
    dist=np.linalg.norm (project (V, train[digit][imageIdx])-project (V,␣
 ↪testImage))
    for i in range(len(train)):
        for j in range (train[i].shape[0]):
            tempDist=np.linalg.norm (project(V,train[i][j])-project(V,␣
 ↪testImage))
            if tempDist<dist:
                digit=i
                imageIdx =j
                dist= tempDist
    return digit, imageIdx

n=8
U, s, V = np.linalg.svd(trainarr, full_matrices=False)
V=V[0:n,:]


test,testLabels = load_test_data(datafile)


imgs = []
TestLabels = []
for i in range(len(testLabels)) :
    trueDigit = testLabels[i]
    testImage = test[i]
    (nnDig,nnIdx) = compute_nearest_neighbors(train,testImage,V)
    imgs.extend( [testImage,train[nnDig][nnIdx,:]] )
    TestLabels.append(nnDig)

row_titles = ['Test','Nearest']
col_titles = ['%d vs. %d'%(i,j) for i,j in zip(testLabels,testLabels)]
plot_image_grid(imgs,
                    "Image-NearestNeighbor",
                   ␣
 ↪(28,28),len(testLabels),2,True,row_titles=row_titles,col_titles=col_titles)
```



[8]:

```python
import numpy as np
import matplotlib.pyplot as plt
import scipy.io

# Loading the MNIST data
datafile = "mnist_all.mat"
mnist_data = scipy.io.loadmat(datafile)

# Checking the keys in the loaded data
mnist_data.keys()
```

[8]:

```
dict_keys(['__header__', '__version__', '__globals__', 'train0', 'test0',
'train1', 'test1', 'train2', 'test2', 'train3', 'test3', 'train4', 'test4',
'train5', 'test5', 'train6', 'test6', 'train7', 'test7', 'train8', 'test8',
'train9', 'test9'])
```

[9]:

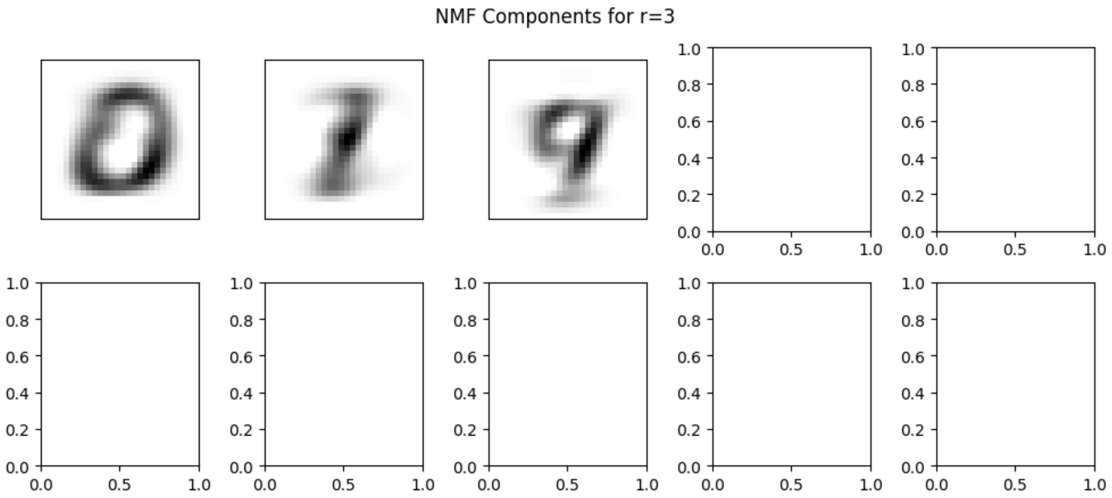
```python
# Extracting training and testing data and labels
def extract_data_and_labels(mnist_data):
    data = []
    labels = []
    for i in range(10):
        train_data_key = f'train{i}'
        test_data_key = f'test{i}'
        train_data = mnist_data[train_data_key]
        test_data = mnist_data[test_data_key]
        combined_data = np.vstack((train_data, test_data))
        data.append(combined_data)
        labels.append(np.full((combined_data.shape[0],), i))
    return data, labels

# Getting the data and labels
data, labels = extract_data_and_labels(mnist_data)

# Checking the shape of extracted data for a specific digit
data[0].shape, labels[0].shape
```

[9]:

```
((6903, 784), (6903,))
```

[12]:

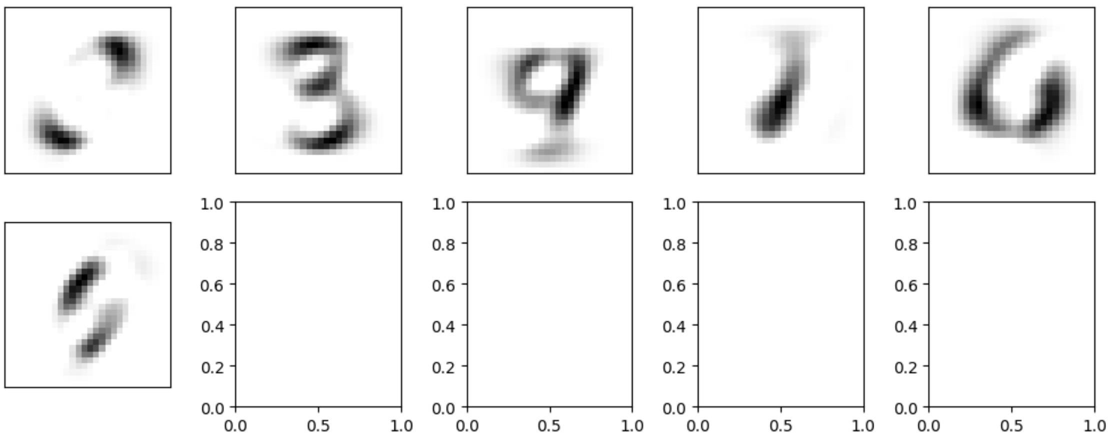
```python
from sklearn.decomposition import NMF
import plot_tools

# Function to apply NMF and plot the components
def apply_nmf_and_plot(data, n_components):
    # Combining all digit data
    all_data = np.vstack(data)
    # Applying NMF
    nmf = NMF(n_components=n_components, init='random', random_state=0)
    W = nmf.fit_transform(all_data)
    H = nmf.components_

    # Plotting the components
    plot_tools.plot_image_grid(H, f"NMF Components for r={n_components}",␣
 ↪image_shape=(28, 28))

# Plotting function (from the provided plot_tools.py with minor modifications)
# def plot_image_grid(images, title, image_shape=(28, 28), n_col=5, n_row=2):
#     fig, axes = plt.subplots(nrows=n_row, ncols=n_col, figsize=(2. * n_col, 2.
 ↪26 * n_row))
#     axes = axes.flatten()  # Flattening the axes array to simplify the␣
 ↪indexing
#     for i, comp in enumerate(images):
#         ax = axes[i]
#         ax.imshow(comp.reshape(image_shape), cmap=plt.cm.gray_r,␣
 ↪interpolation='nearest')
#         ax.set_xticks(())
#         ax.set_yticks(())

#     fig.suptitle(title)
#     plt.show()

# Re-running the NMF and plotting for r in {3, 6, 10}
for r in [3, 6, 10]:
    apply_nmf_and_plot(data, r)
```



```
/home/karanvora/miniconda3/lib/python3.8/site-
```

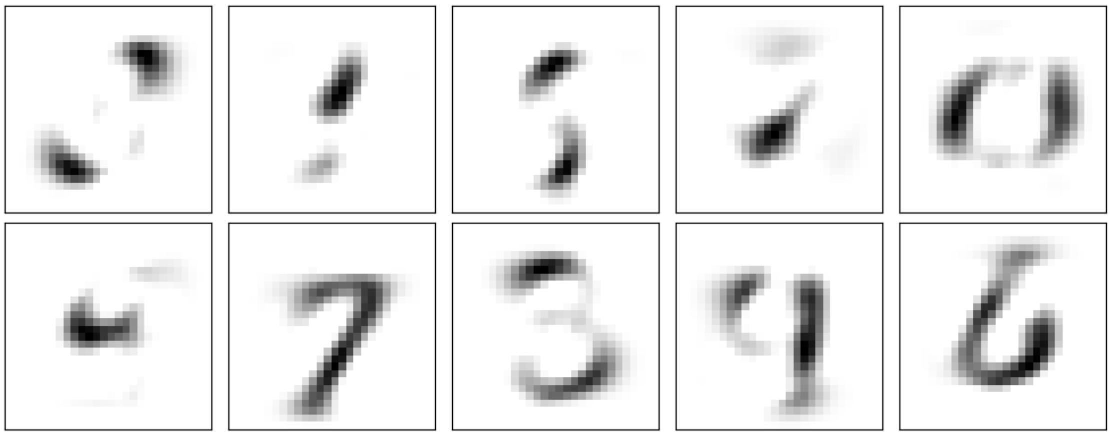
```
packages/sklearn/decomposition/_nmf.py:1710: ConvergenceWarning: Maximum number
of iterations 200 reached. Increase it to improve convergence.
  warnings.warn(
```

NMF Components for r=6

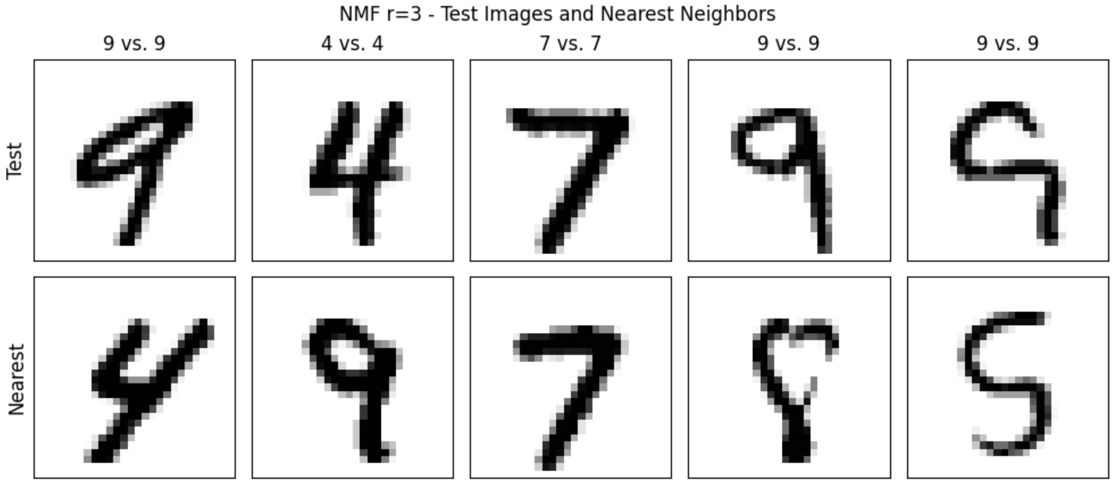


```
/home/karanvora/miniconda3/lib/python3.8/site-
packages/sklearn/decomposition/_nmf.py:1710: ConvergenceWarning: Maximum number
of iterations 200 reached. Increase it to improve convergence.
  warnings.warn(
```

NMF Components for r=10

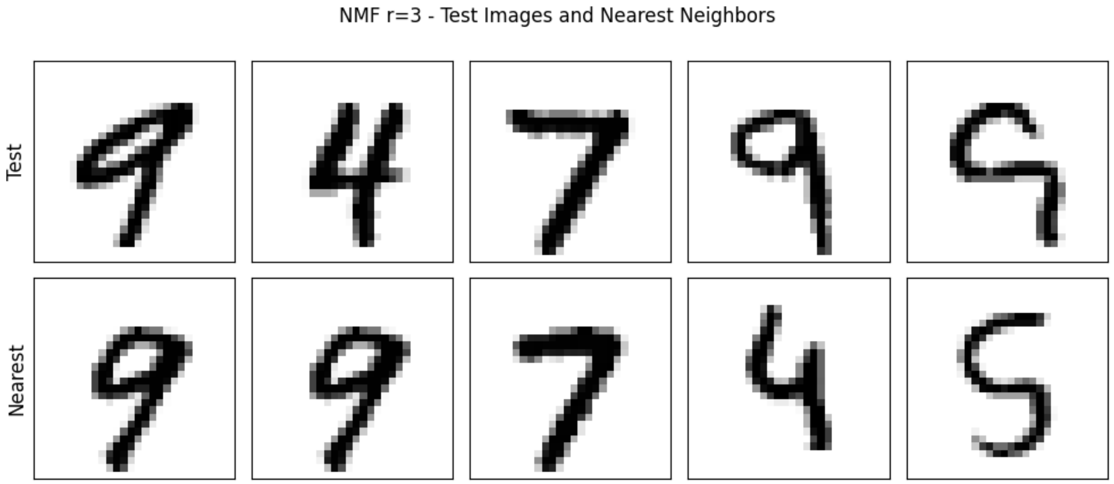


[24]:
```
# Correcting the dimension mismatch error in the compute_nearest_neighbors_NMF␣
 ↪function
def compute_nearest_neighbors_NMF(train, testImage, H):
    projected_train = [np.dot(np.array(i, dtype=float), H.T) for i in train]  #␣
 ↪Corrected this line
    projected_test = np.dot(np.array(testImage, dtype=float), H.T)  # Corrected␣
 ↪this line
    digit = 0
    imageIdx = 0
    dist = np.linalg.norm(projected_train[digit][imageIdx] - projected_test)

    for i in range(len(train)):
        for j in range(train[i].shape[0]):
            tempDist = np.linalg.norm(projected_train[i][j] - projected_test)
            if tempDist < dist:
                digit = i
                imageIdx = j
                dist = tempDist
    return digit, imageIdx

# Apply NMF with r=3
r = 3
nmf = NMF(n_components=r, init='random', random_state=0)
W = nmf.fit_transform(trainarr)
H = nmf.components_

# Find the nearest neighbors for all test images again
nmf_imgs = []
for i in range(len(testLabels)):
    nnDig, nnIdx = compute_nearest_neighbors_NMF(train, test[i], H)
    nmf_imgs.extend([test[i], train[nnDig][nnIdx,:]])

# Visualize the test images and their nearest neighbors
col_titles = ['%d vs. %d'%(i,j) for i,j in zip(testLabels,testLabels)]
plot_image_grid(nmf_imgs, "NMF r=3 - Test Images and Nearest Neighbors",
                (28,28), len(testLabels), 2, True, row_titles=['Test',␣
 ↪'Nearest'], col_titles=col_titles)
```



[22]:

```
# Correcting the dimension mismatch error in the compute_nearest_neighbors_NMF function
def compute_nearest_neighbors_NMF(train, testImage, H):
    projected_train = [np.dot(np.array(i, dtype=float), H.T) for i in train]  # Corrected this line
    projected_test = np.dot(np.array(testImage, dtype=float), H.T)  # Corrected this line
    digit = 0
    imageIdx = 0
    dist = np.linalg.norm(projected_train[digit][imageIdx] - projected_test)

    for i in range(len(train)):
        for j in range(train[i].shape[0]):
            tempDist = np.linalg.norm(projected_train[i][j] - projected_test)
            if tempDist < dist:
                digit = i
                imageIdx = j
                dist = tempDist
    return digit, imageIdx

# Apply NMF with r=3
r = 6
nmf = NMF(n_components=r, init='random', random_state=0)
W = nmf.fit_transform(trainarr)
H = nmf.components_

# Find the nearest neighbors for all test images again
nmf_imgs = []
for i in range(len(testLabels)):
    nnDig, nnIdx = compute_nearest_neighbors_NMF(train, test[i], H)
    nmf_imgs.extend([test[i], train[nnDig][nnIdx,:]])

# Visualize the test images and their nearest neighbors
col_titles = ['%d vs. %d'%(i,j) for i,j in zip(testLabels,testLabels)]
plot_image_grid(nmf_imgs, "NMF r=6 - Test Images and Nearest Neighbors",
                (28,28), len(testLabels), 2, True, row_titles=['Test',␣
 ↪'Nearest'], col_titles=col_titles)
```

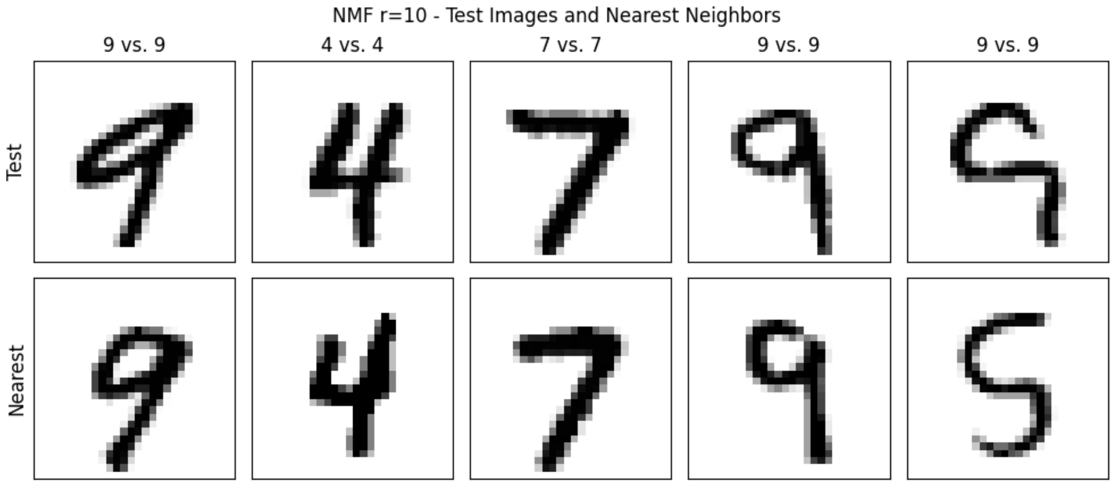


[25]:

```
# Correcting the dimension mismatch error in the compute_nearest_neighbors_NMF␣
 ↪function
def compute_nearest_neighbors_NMF(train, testImage, H):
    projected_train = [np.dot(np.array(i, dtype=float), H.T) for i in train]  #␣
 ↪Corrected this line
    projected_test = np.dot(np.array(testImage, dtype=float), H.T)  # Corrected␣
 ↪this line
    digit = 0
    imageIdx = 0
    dist = np.linalg.norm(projected_train[digit][imageIdx] - projected_test)

    for i in range(len(train)):
        for j in range(train[i].shape[0]):
            tempDist = np.linalg.norm(projected_train[i][j] - projected_test)
            if tempDist < dist:
                digit = i
                imageIdx = j
                dist = tempDist
    return digit, imageIdx

# Apply NMF with r=3
r = 10
```

```
nmf = NMF(n_components=r, init='random', random_state=0)
W = nmf.fit_transform(trainarr)
H = nmf.components_

# Find the nearest neighbors for all test images again
nmf_imgs = []
for i in range(len(testLabels)):
    nnDig, nnIdx = compute_nearest_neighbors_NMF(train, test[i], H)
    nmf_imgs.extend([test[i], train[nnDig][nnIdx,:]])

# Visualize the test images and their nearest neighbors
plot_image_grid(nmf_imgs, "NMF r=10 - Test Images and Nearest Neighbors",
                (28,28), len(testLabels), 2, True, row_titles=['Test',␣
 ↪'Nearest'], col_titles=col_titles)
```

```
/home/karanvora/miniconda3/lib/python3.8/site-
packages/sklearn/decomposition/_nmf.py:1710: ConvergenceWarning: Maximum number
of iterations 200 reached. Increase it to improve convergence.
  warnings.warn(
```



For C, Ir is clear with PCA the matches are far more closer and accurate then NMF. NMF requires higher r number to get accurate enough Information.