

Practice Problem: PCA

Recall that Principal Component Analysis (PCA) solves $\min_{\mathbf{U} \in \mathbb{R}^{d \times k}} \|\mathbf{X} - \mathbf{X}\mathbf{U}\mathbf{U}^\top\|_F^2$, where $\mathbf{X} \in \mathbb{R}^{n \times d}$. The value achieved for this problem is called the reconstruction error.¹ For $d = 3, n = 3$, consider the matrix

$$\mathbf{X} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & \sqrt{2} \\ -\frac{3}{2} & \frac{3}{2} & \frac{3}{\sqrt{2}} \end{bmatrix}$$

What is the reconstruction error when $k = 2$ and when $k = 3$? Note: do not reduce the mean from \mathbf{X} before applying PCA.

¹i.e. if \mathbf{U}^* is a solution retrieved by PCA, then the reconstruction error is $\|\mathbf{X} - \mathbf{X}\mathbf{U}^*(\mathbf{U}^*)^\top\|_F^2$