ECE 6143

Practice Final Exam

Solutions

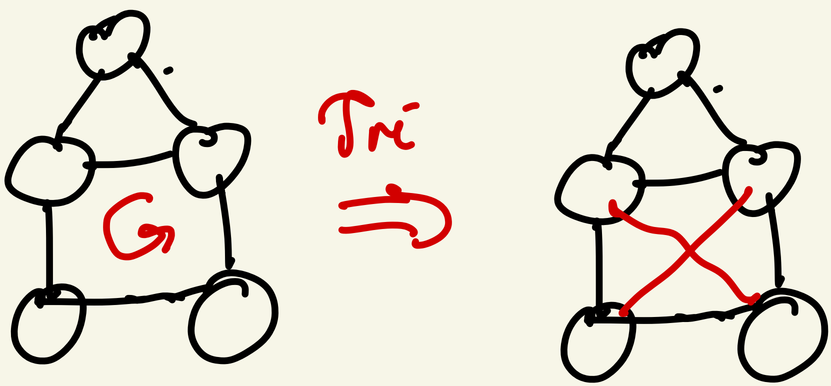
Q1) JTA:

1) Moralisation: Marry parents that share common children.

2) Drop arrow heads

3) Introduce evidence

4) Triangulation.



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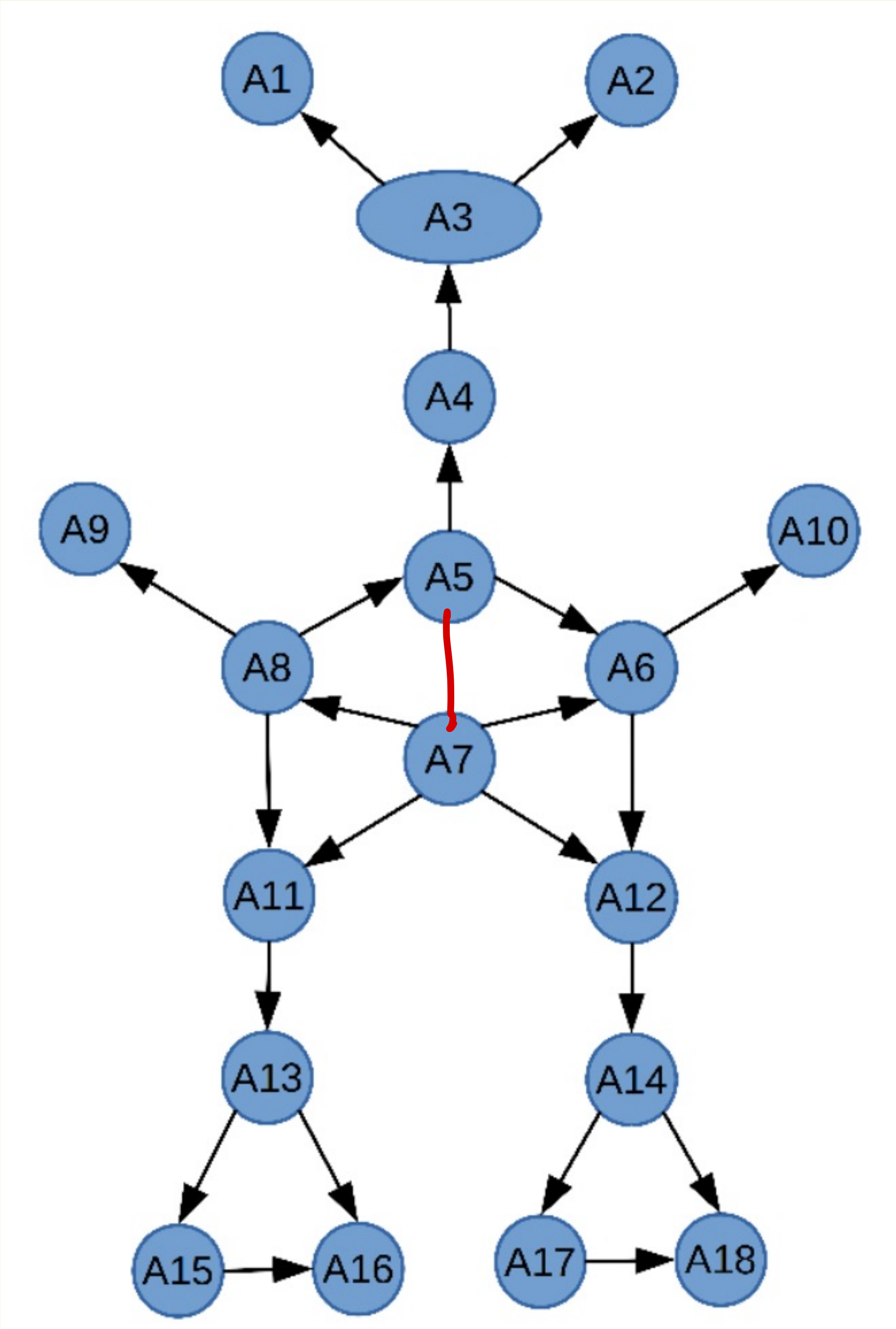
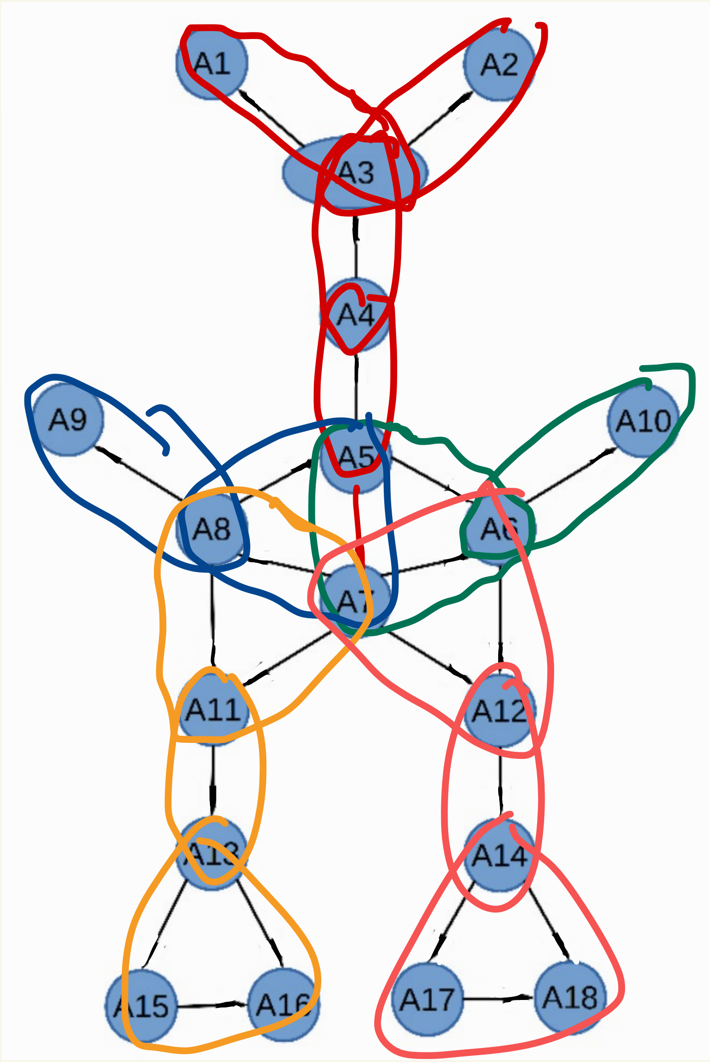


Q1)



A1A3

A2A3

A3A4

A4A5

A5A6A7 •

A6A10

A5A7A8 •

A8A9

A7A8A11 •

A11A13

A13A15A16 •

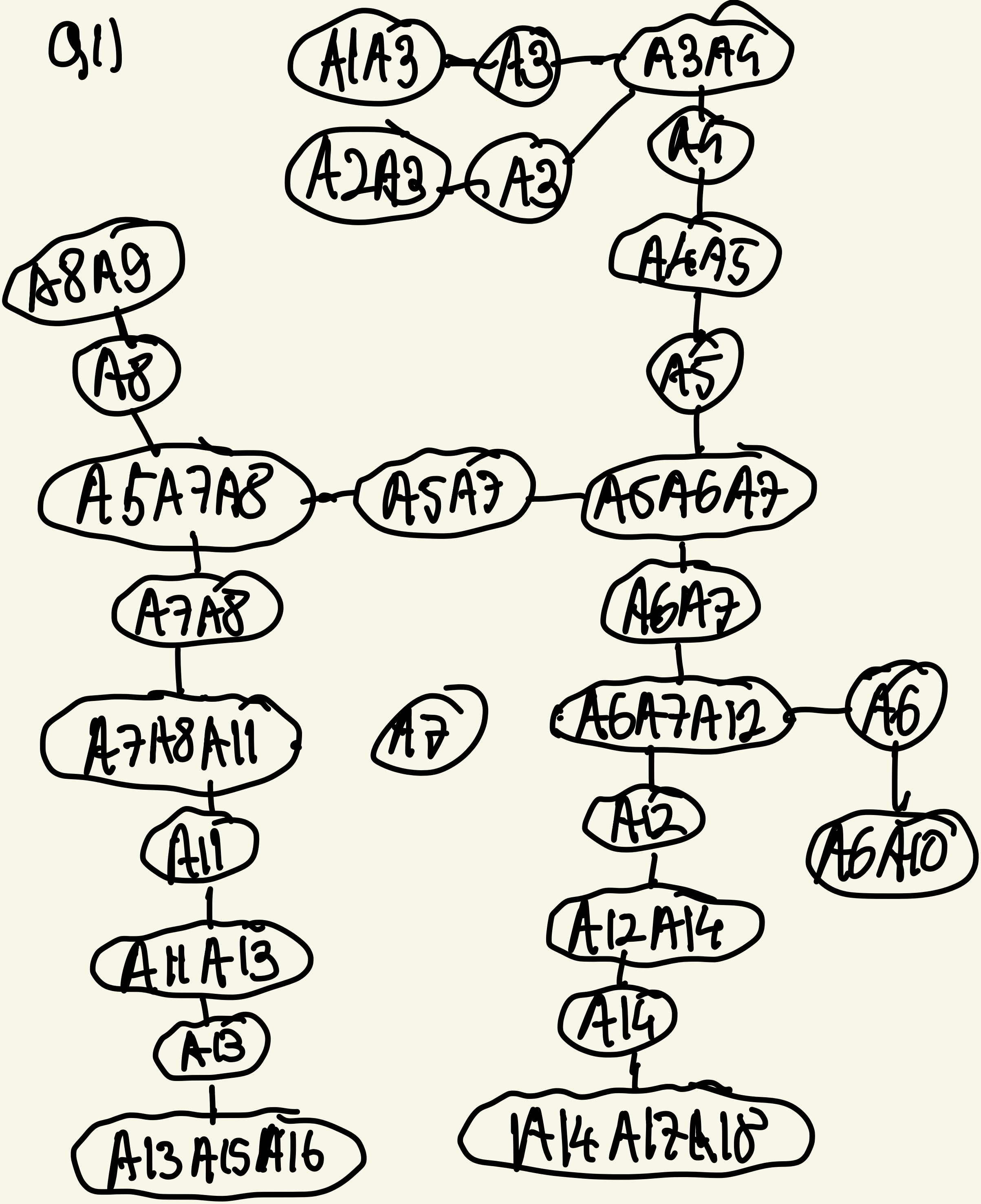
A6A7A12 •

A12A14

A14A17A18 •

Use Kruskal Alg.!

Q1)


A1A3
A3
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A8
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A5A7A8
A5A7
A5A6A7
A7A8
A6A7
A7A8A11
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A6A7A12
A6
A11
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A6A10
A11A13
A12A14
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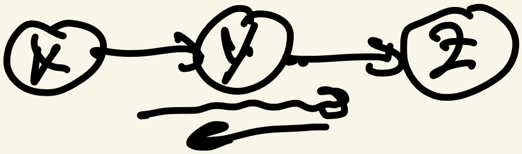
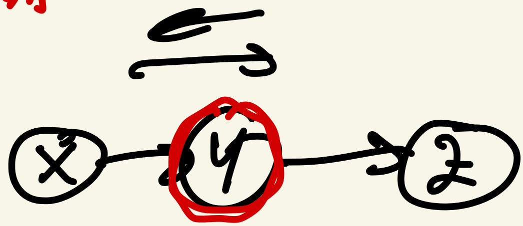
A14
A13A15A16
A14A17A18

Bayes Ball Algorithm

1) Markov Chain

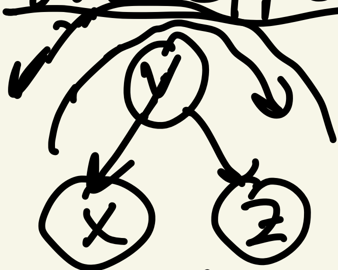


$X \not\perp\!\!\!\perp Z$

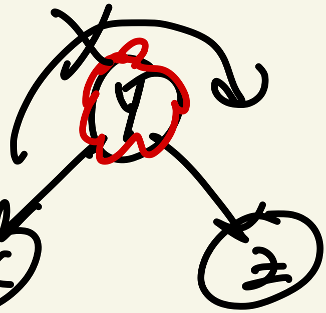


$X \perp\!\!\!\perp Z \mid Y$

2) Two Effects :-

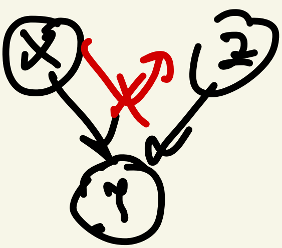


$X \not\perp\!\!\!\perp Z$

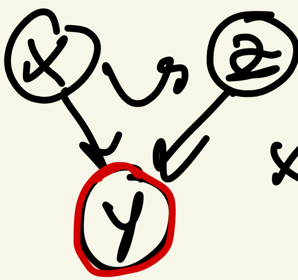


$X \perp\!\!\!\perp Z \mid Y$

3) Two Causes

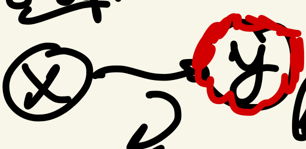


$X \perp\!\!\!\perp Z$

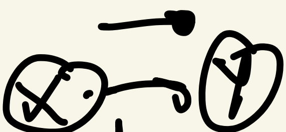


$X \perp\!\!\!\perp Z \mid Y$

4) Leaf

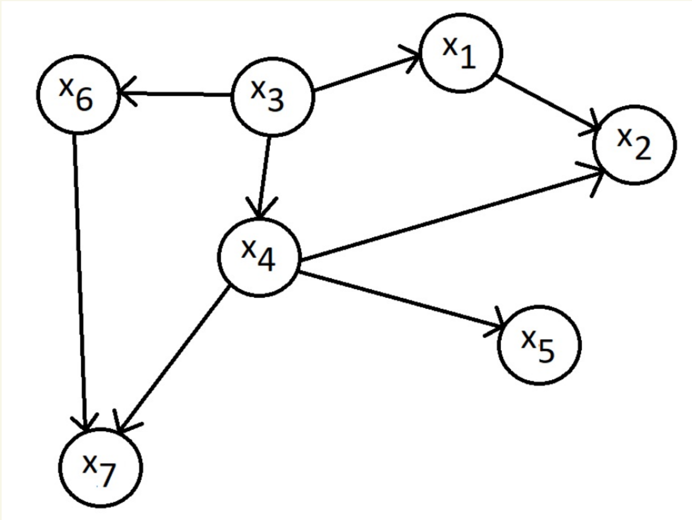


bounce



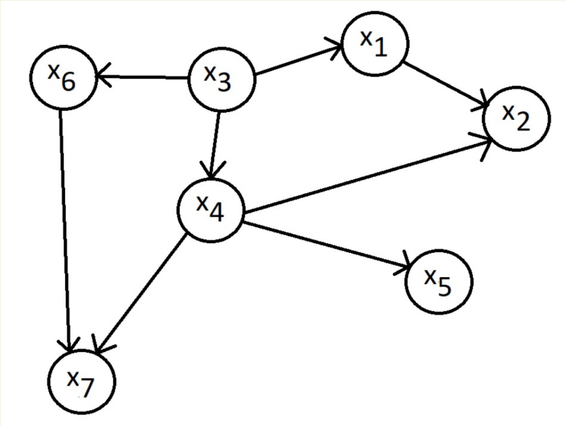
stops

Q.2)



$$p(x_1, x_2 \ldots x_6) = p(x_3) \cdot p(x_6 \mid x_3) \cdot p(x_4 \mid x_3) \cdot$$
$$p(x_7 \mid x_6, x_4)\, p(x_1 \mid x_3)$$
$$p(x_2 \mid x_1, x_4)\, p(x_5 \mid x_4)$$

Q.2)



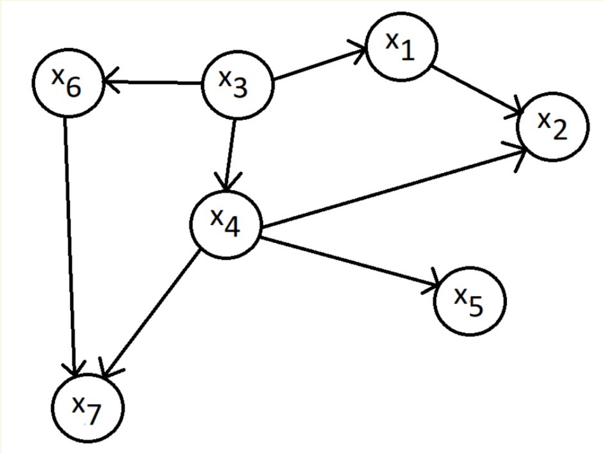
i) $x_2 \perp\!\!\!\perp x_6$?

$x_2 - x_1 - x_3$ MC ✓

$x_1 - x_3 - x_6$ 2 effects ✓

False

Q2)



True

(i) $x_2 \perp\!\!\!\perp x_6 \mid x_1, x_3, x_5$ ✓

* $x_2 - x_1 - x_3$ stops ✗
* $x_2 - x_4 - x_7$ (2 eff.) pass
* $x_4 - x_7 - x_6$ (2 causes) ✗
* $x_2 - x_4 - x_3$ (MC) ✓

$x_4 - x_3 - x_6$ ✗

* $x_2 - x_4 - x_5$ 2 effects ✓

$x_4 - x_5$ bounces

$x_5 - x_4 - x_7$ 2 effects ✓

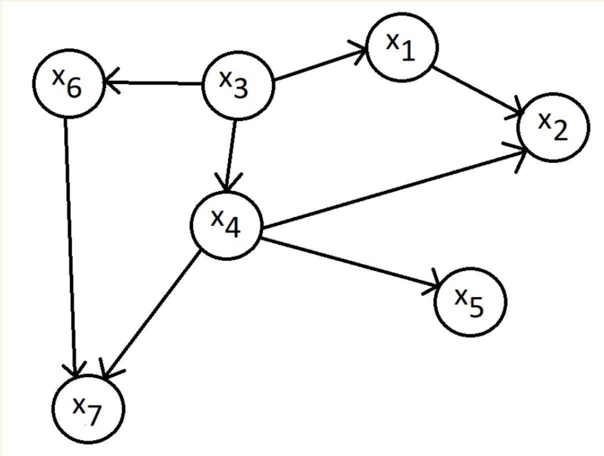
$x_4 - x_7 - x_6$ (2 causes) ✗

$x_4 - x_5$ ✓

$x_5 - x_4 - x_3$

$x_4 - x_3 - x_6$ ✗

Q2)



(ii) $x_1 \cancel{\perp\!\!\!\perp} x_7 \mid x_5$

$\propto$ $x_1 - x_2 - x_4$ (2 causes) ✗

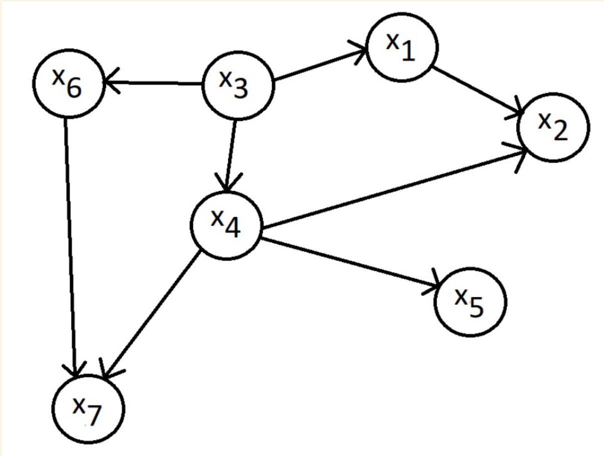
$\propto$ $x_1 - x_3 - x_4$ (2 effects) ✓

$x_3 - x_4 - x_7$ (MC) ✓

False

$x_1 \not\perp\!\!\!\perp x_7$

Q2)



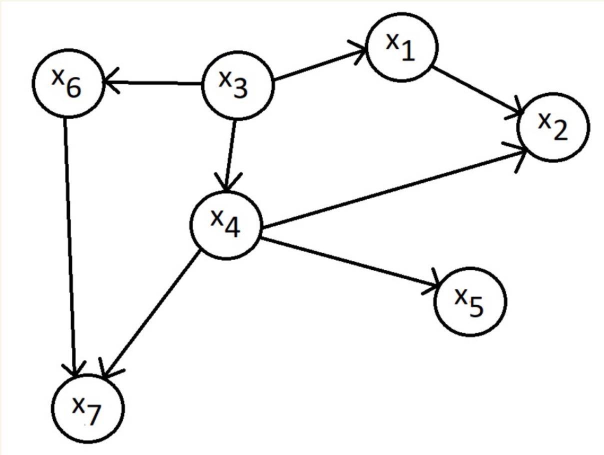
(iv) $x_5 \cancel{\perp\!\!\!\perp} x_3 \mid x_1, x_2$ ✓

$x_3 - x_4 - x_5$ (MC) ✓

False

$x_5 \not\perp\!\!\!\perp x_3$

Q2)



v) $x_5 \perp\!\!\!\perp x_6 \mid x_1 x_2 x_4$ ✓

$x_5 - x_4 - x_7$ (2 eff.) X

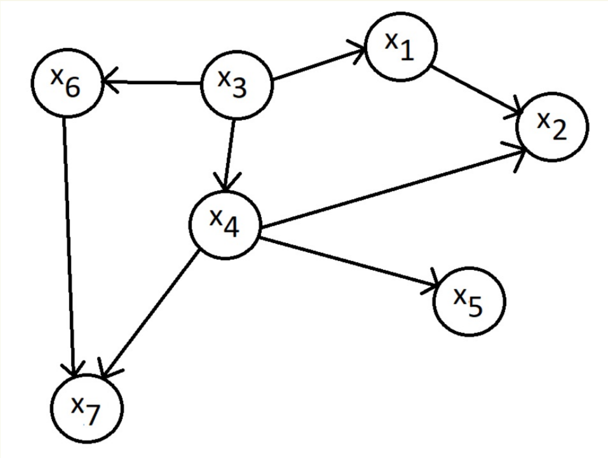
$x_5 - x_4 - x_3$ (MC) X

$x_5 - x_4 - x_2$ (2 eff.) X

True

$x_5 \perp\!\!\!\perp x_6 \mid x_4$

Q2)

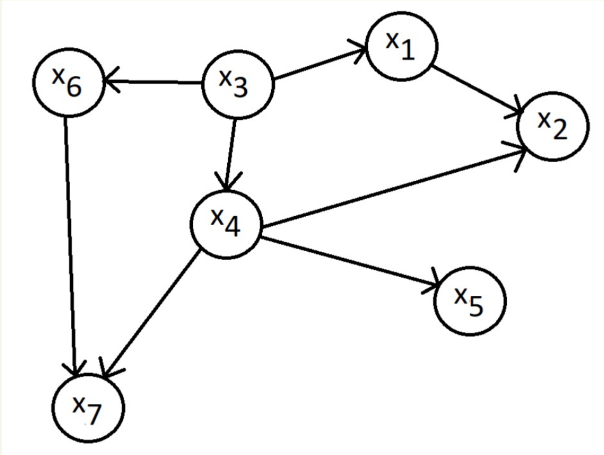


vi) $x_5 \perp\!\!\!\perp x_6 \mid x_4$

part v implies that this is

True

Q2)



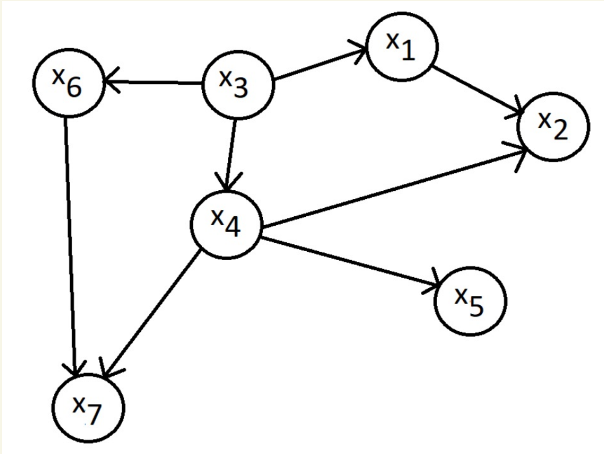
vii) $x_5 \cancel{\perp} x_6 \mid x_1$

d $x_5 - x_4 - x_3$ (MC) ✓

$x_4 - x_3 - x_6$ (2 eff.) ✓

False

Q2)



True

viii) $x_2 \perp\!\!\!\perp x_6 \mid x_3, x_5$

* $x_2 - x_1 - x_3$ MC ✓

  $x_1 - x_3 - x_6$ 2 eff. ✗

* $x_2 - x_4 - x_3$ MC ✓

  $x_4 - x_3 - x_6$ 2 eff. ✗

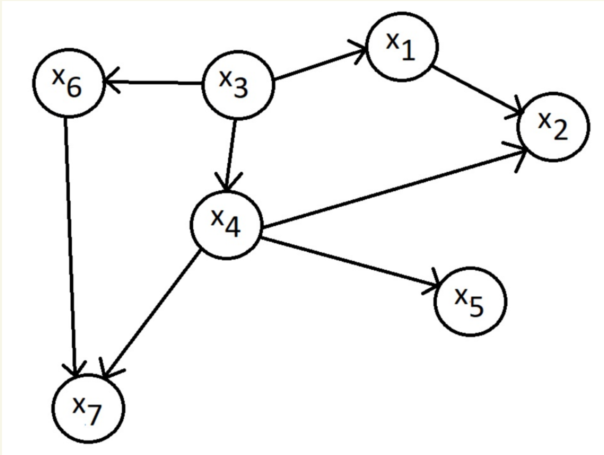
* $x_2 - x_4 - x_7$ 2 eff. ✓

  $x_4 - x_7 - x_6$ 2 caus. ✗

* leaf: $x_2 - x_4 - x_5$ 2 eff ✓

  | $x_5 - x_4 - x_7$ (2 eff.) ✓

Q2)



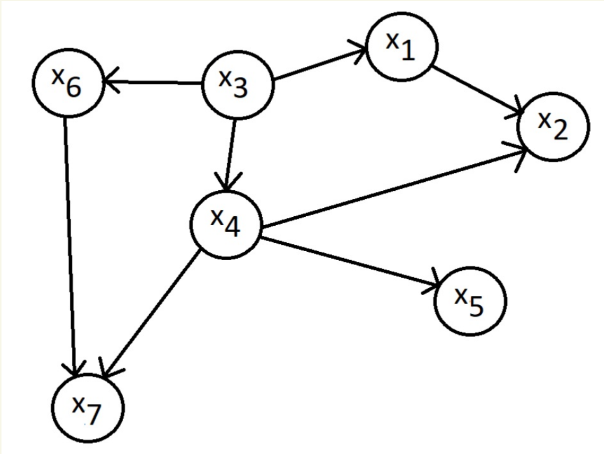
ix) $x_1 \cancel{\perp} x_7$

& $x_1 - x_3 - x_4$ (2 eff.) ~~At~~ ✓

$x_3 - x_4 - x_7$ (MC) ✓

False

Q2)



x) $x_1 \cancel{\perp\!\!\!\perp} x_7 \mid x_4$

$x_1 - x_3 - x_4$ (2 eff.) ✓

$x_3 - x_4 - x_7$ (MC) ✗

$x_1 - x_3 - x_6$ (2 eff.) ✓

$x_3 - x_6 - x_7$ (MC) ✓

False

Q3) Conv filter:

x

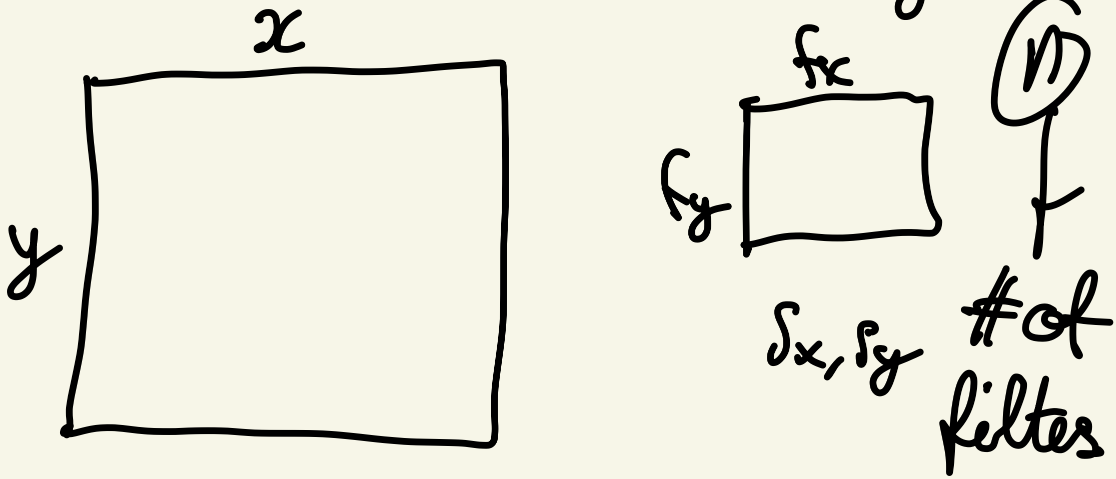
Filter size of f_x

Stride of S_x

y

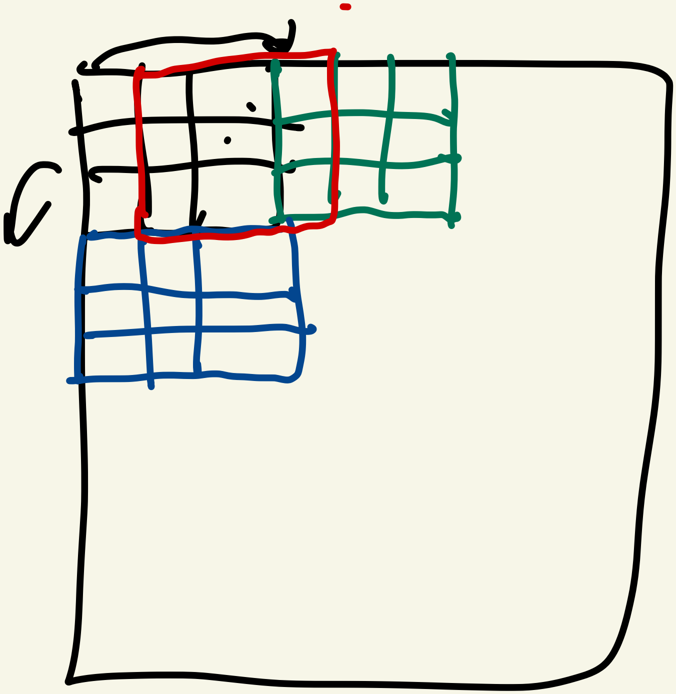
Filter size: f_y

Stride: S_y



$$\left(\underbrace{\frac{x-f_x}{S_x}+1, \quad \frac{y-f_y}{S_y}+1} \right)$$

$$(n, \qquad)$$



$(x, y) \rightarrow \left(\frac{x}{3}, \frac{y}{3}\right)$

When kernel size 3
stride is 3

$$\left(\frac{x-f_x}{S_x}+1, \frac{y-f_y}{S_y}+1\right)$$

$$\left(\frac{x-3}{3}+1, \frac{y-3}{3}+1\right) = \left(\frac{x}{3}-1+1, \frac{y}{3}-1+1\right)$$

$$= \left(\frac{x}{3}, \frac{y}{3}\right)$$

Input : $1 \times x \times y$

Conv 1 : $4 \times \left(\frac{x-5}{2}+1, \frac{y-5}{3}+1\right)$

ReLU : no effect X

Max pool : $4 \times \left(\frac{x-5}{6}+\frac{1}{3}, \frac{y-5}{9}+\frac{1}{3}\right)$

$= 4 \times \left(\frac{x-3}{6}, \frac{y-2}{9}\right)$

Conv 2 : $6 \times \left(\frac{\frac{x-3}{6}-4}{2}+1, \frac{\frac{y-2}{9}-5}{2}+1\right)$

$= 6 \times \left(\frac{x-15}{12}\right) \times \left(\frac{y-20}{18}\right)$

ReLU $\rightarrow$ no effect.

$$= 6 \times \left(\frac{x-15}{12}\right) \times \left(\frac{y-20}{18}\right)$$

Max Pool : (2x2), (2x2)

$$\rightarrow 6 \times \left(\frac{x-15}{24}\right) \times \left(\frac{y-20}{36}\right)$$

$$\rightarrow 6 \times 12 \times 8$$

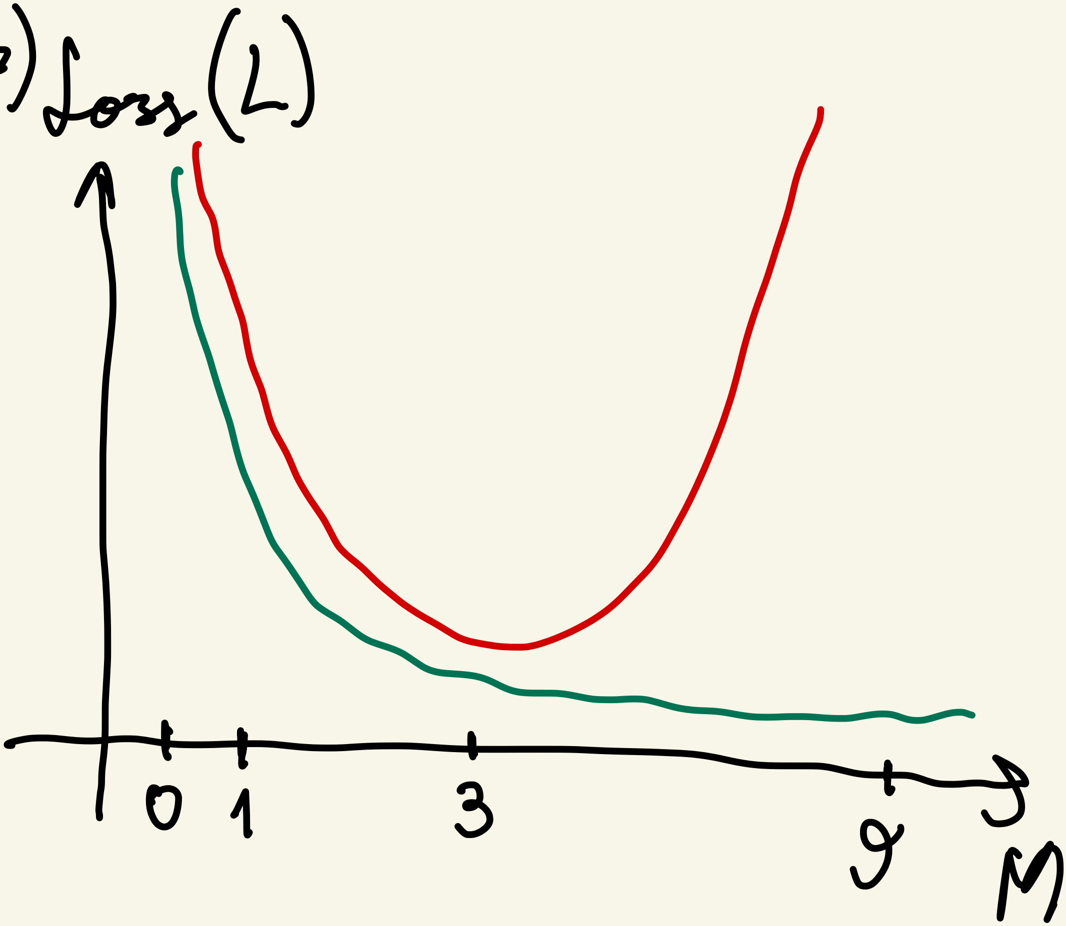
$$x - 15 = \underbrace{24 \cdot 12}_{288} \rightarrow x = \underline{303}$$

$$y - 20 = \underbrace{8 \cdot 36}_{288} \rightarrow y = \underline{308}$$

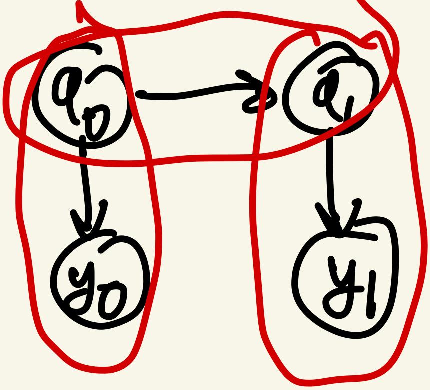
Initial Input Image Size

$$1 \times 303 \times 308$$

Q5)

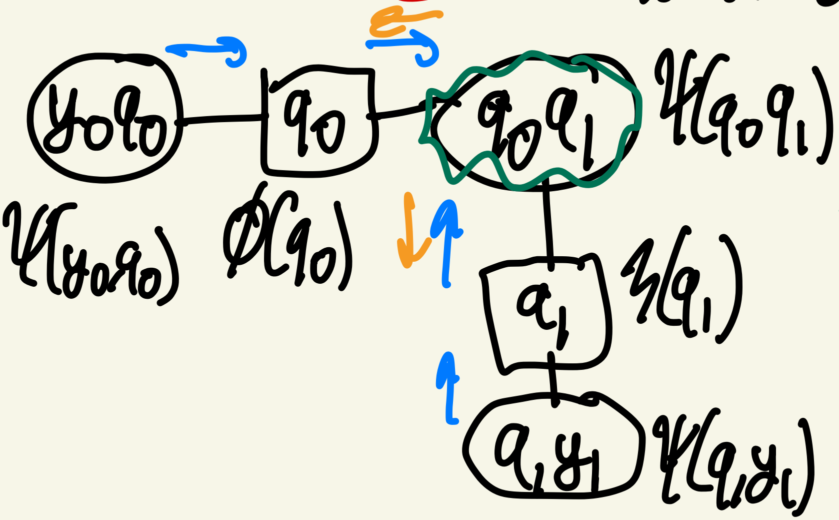


Q5)



JTA

$q_0y_0,\ q_0q_1,\ q_1y_1$



— root

1) Initialize: $\phi(q_0) = [1\ \ 1]$ $\quad \phi(q_1) = [1\ \ 1]$

Lecture 11, 12, 13

$$\eta = \begin{array}{c|cc} q_0 \backslash y_0 & A & B \\ \hline 1 & 1/4 & 3/4 \\ 2 & 1/8 & 7/8 \end{array}$$

$$\psi(q_0 y_0) = p(q_0)\,p(y_0|q_0) = \begin{array}{c} q_0 \\ 1 \\ 2 \end{array}\begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix} \begin{array}{c|cc} q_0 \backslash y_0 & & \\ \hline & 1/4 & 3/4 \\ & 1/8 & 7/8 \end{array} = \begin{array}{c|cc} q_0 \backslash y_0 & A & B \\ \hline & 1/16 & \cancel{3/16} \\ & 3/32 & \cancel{21/32} \end{array}$$

$$\psi(q_0 q_1) = p(q_1|q_0) = \begin{array}{c|cc} q_0 \backslash q_1 & & \\ \hline & 1/2 & 1/2 \\ & 1/3 & 2/3 \end{array}$$

$$\psi(q_1 y_1) = p(y_1|q_1) = \begin{array}{c|cc} q_1 \backslash y_1 & A & B \\ \hline & \cancel{1/4} & 3/4 \\ & \cancel{1/8} & 7/8 \end{array}$$

2) Collect up:

$$\eta^*(q_1) = \sum_{y_1} \psi(q_1 y_1) = \begin{bmatrix} 3/4 & 7/8 \end{bmatrix}$$

$$\psi^*(q_0 q_1) = \frac{\eta^*(q_1)}{\eta(q_1)}\,\psi(q_0 q_1)$$

$$= \frac{\begin{array}{c} q_1 \\ \begin{bmatrix} 3/4 & 7/8 \end{bmatrix} \end{array}}{\begin{array}{c} q_1 \\ \begin{bmatrix} 1 & 1 \end{bmatrix} \end{array}} \begin{array}{c|cc} q_1 \backslash q_0 & & \\ \hline & 1/2 & 1/2 \\ & 1/3 & 2/3 \end{array} = \begin{bmatrix} 3/8 & 7/16 \\ 1/4 & 7/12 \end{bmatrix}$$

3) Collect left-2-right:

$$\phi^{a}(q_0) = \sum_{y_0} \psi(q_0 y_0) = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ q_0 & \begin{bmatrix} 1/16 & 3/32 \end{bmatrix} \end{array}$$

$$\psi^{a}(q_0 q_1) = \frac{\phi^{a}(q_0)}{\phi(q_0)} \psi^{a}(q_0 q_1)$$

$$= \frac{q_0 \begin{bmatrix} 1/16 & 3/32 \end{bmatrix}}{\begin{bmatrix} 1 & 1 \end{bmatrix}} \begin{array}{c} q_1 / q_0 \\ \begin{bmatrix} 3/8 & 7/16 \\ 1/4 & 7/12 \end{bmatrix} \end{array} = \begin{array}{c} q_1 / q_0 \\ \begin{bmatrix} 3/128 & 7/256 \\ 3/128 & 7/128 \end{bmatrix} \end{array}$$

4) Distribute:

$$\phi^{a}(q_0) = \sum_{q_1} \psi^{a}(q_0 q_1) = \begin{array}{c} q_0 \\ \begin{bmatrix} 13/256 & 5/64 \end{bmatrix} \end{array}$$

$$\zeta^{a}(q_1) = \sum_{q_0} \psi^{a}(q_0 q_1) = \begin{array}{c} q_1 \\ \begin{bmatrix} 3/64 & 21/256 \end{bmatrix} \end{array}$$

$$p(y) = \frac{13}{256} + \frac{5}{64} = \frac{3}{64} + \frac{21}{256} = \underline{\underline{\frac{33}{256}}}$$

4) Distributes:

$$\phi(q_0) = \sum_{q_1} \psi(q_0 q_1) = \begin{array}{c|cc} q_0 & 1 & 2 \\ & \left[\tfrac{13}{256}\right. & \left.\tfrac{5}{64}\right] \end{array}$$

$$\xi(q_1) = \sum_{q_0} \psi(q_0 q_1) = \begin{array}{c|cc} q_1 & 1 & 2 \\ & \left[\tfrac{3}{64}\right. & \left.\tfrac{21}{256}\right] \end{array}$$

$$p(y) = \frac{13}{256} + \frac{5}{64} = \frac{3}{64} + \frac{21}{256} = \underline{\frac{33}{256}}$$

q_0	q_1
1	1
2	2

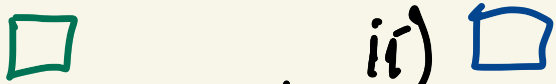
$$p(q_0 = 1 \mid y) = \frac{13/256}{13/256 + 5/64} = \frac{13}{33}$$

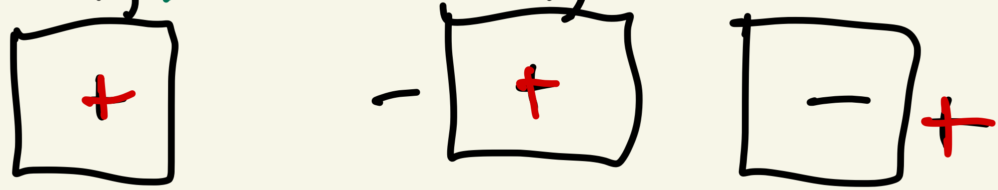
$$p(q_0 = 2 \mid y) = \frac{5/64}{p(y)} = \frac{20}{33}$$

$$p(q_1 = 1 \mid y) = \frac{3/64}{p(y)} = \frac{12}{33}$$

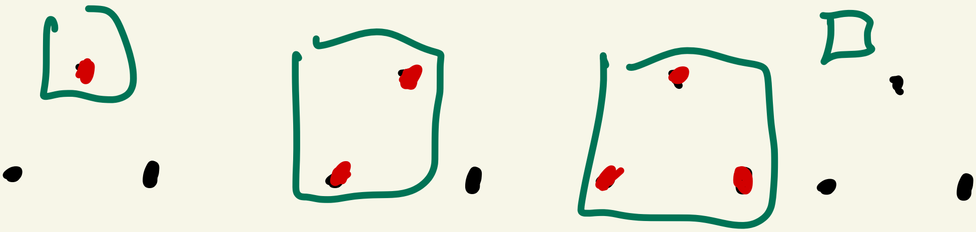
$$p(q_1 = 2 \mid y) = \frac{21/256}{p(y)} = \frac{7}{12}$$

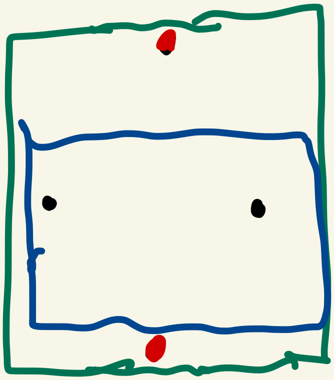
Q6) i) ii)





i)





→ failed shattering

i) VC dim = 3

ii) 5 points, it cannot shatter rectangles → general form of a square

VC dim = 4