hw5

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Problem 1

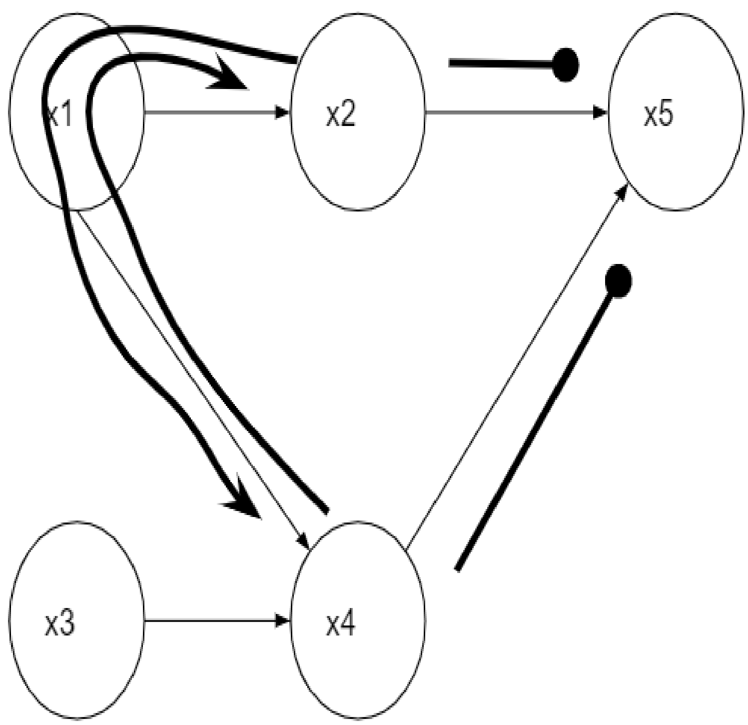
Let choosing a door be A(not revealed).
Let the door selected by host be B.
Let the probability of car being behind a door be C.
The probability of car being behind any door is
$P(C == 1) = P(C == 2) = P(C == 3) = 1/3$ Now we need the probability of the correct door being chosen given the initial door we selected and the door selected by host
$P(C/A, B)$
By applying Bayes rule we get
$P(C/A, B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}$
$P(C/A, B) = \frac{P(B/A,C) * P(A \cap C)}{P(A \cap B)}$
$P(C/A, B) = \frac{P(B/A,C) * P(A \cap C)}{P(A \cap B)}$
$P(C/A, B) = \frac{P(B/A,C) * P(C/A) * P(A)}{P(A,B)}$
$P(C/A, B) = \frac{P(B/A,C) * P(C/A) * P(A)}{\sum_{C=n} P(A,B,C)}$
$P(C/A, B) = \frac{P(B/A,C) * P(C/A) * P(A)}{\sum_{C=n} P(B/A,C) * P(A \cap C)}$
$P(C/A, B) = \frac{P(B/A,C) * P(C/A) * P(A)}{\sum_{C=n} P(B/A,C) * P(C/A) * P(A)}$
The probability of true door does not change with our choice so $P(C/A) = P(C)$
$P(C/A, B) = \frac{P(B/A,C) * P(C)}{\sum_{C=n} P(B/A,C) * P(C)}$
$P(C/A, B) = \frac{P(B/A,C) * P(C)}{\sum_{C=n} P(B/A,C) * P(C)}$
$P(C/A, B) = \frac{P(B/A,C)}{\sum_{C=n} P(B/A,C)}$
Now lets calculate these probabilities lets say we chose door 1 ,and the host chooses door 2 then we get the following probabilities
$P(B = 2/A = 1, C = 1) = 1/2$
$P(B = 2/A = 1, C = 2) = 0$
$P(B = 2/A = 1, C = 3) = 1$
Putting these values we get
$P(C = 1/A = 1, B = 2) = \frac{P(B=2/A=1,C=1)}{\sum_{C=n} P(B/A,C)}$

$P(C=1/A=1,B=2)=\frac{1/2}{\sum(0.5+0+1)}$
$P(C=1/A=1,B=2)=\frac{1}{3}$
Now
$P(C=3/A=1,B=2)=1-P(C=1/A=1,B=2)=\frac{2}{3}$
So by changing our decision we have more chance of winning

Problem 2

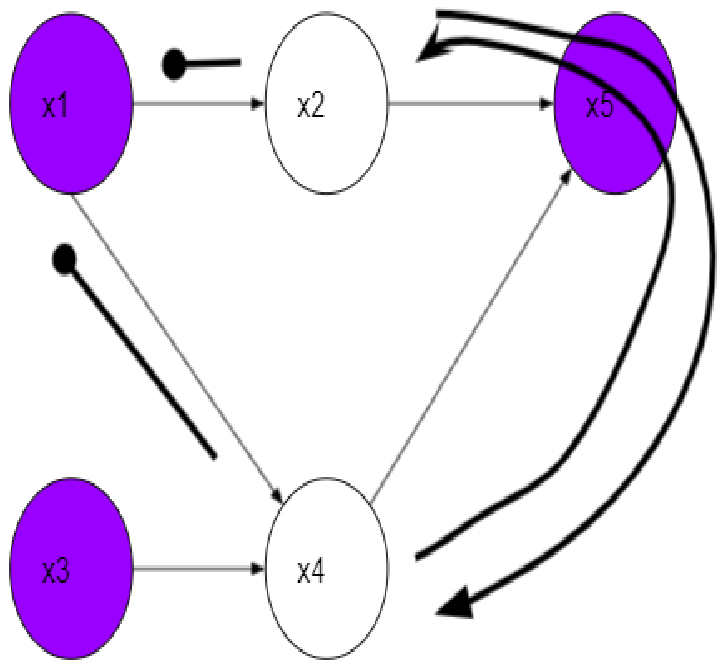
1

False



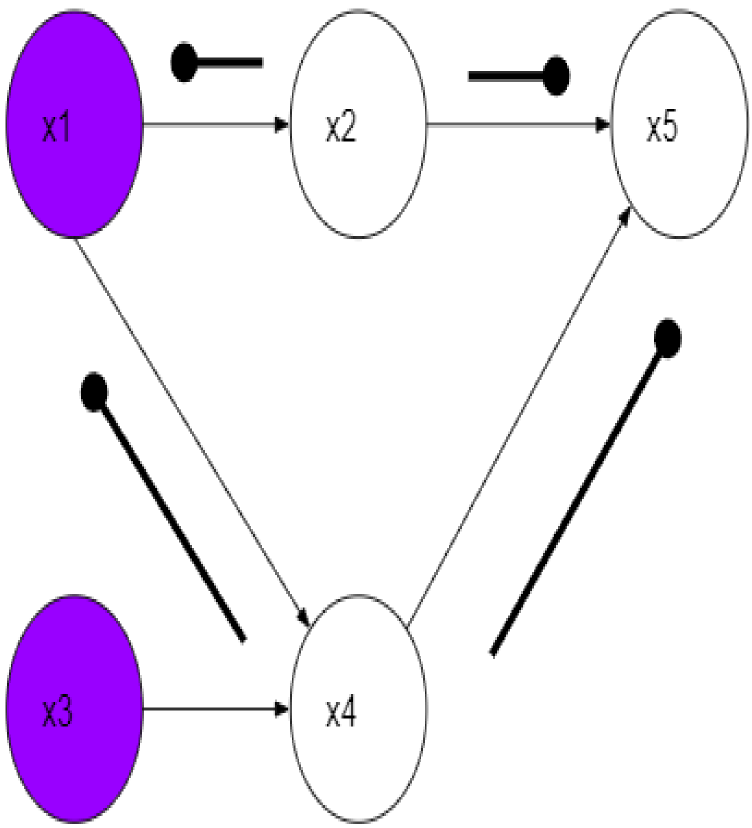
2

False



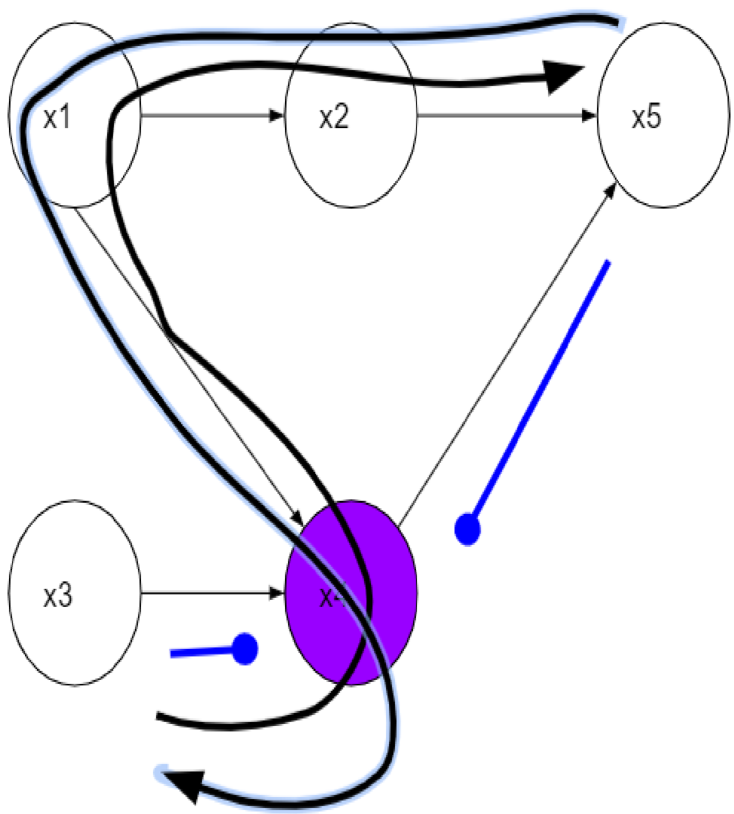
3

True



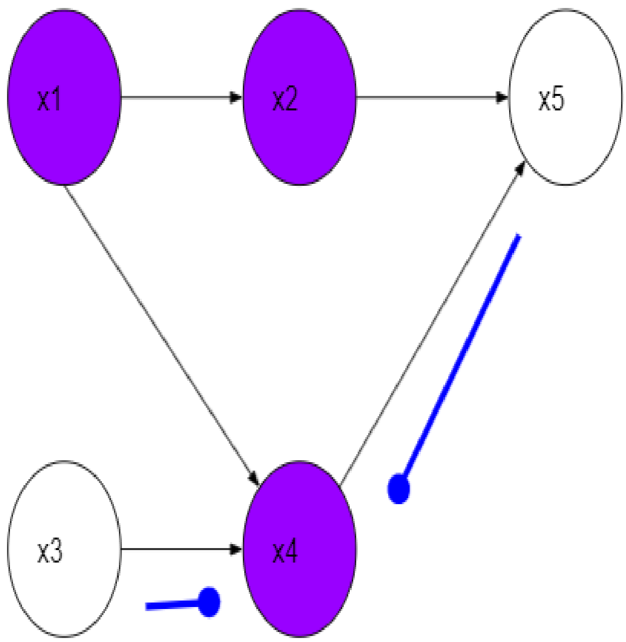
4

False



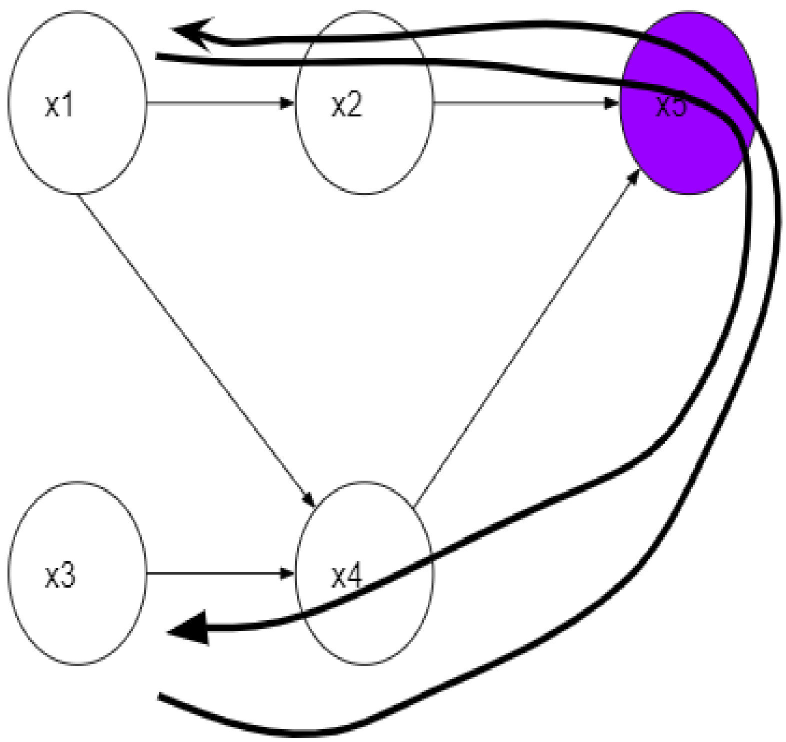
5

True



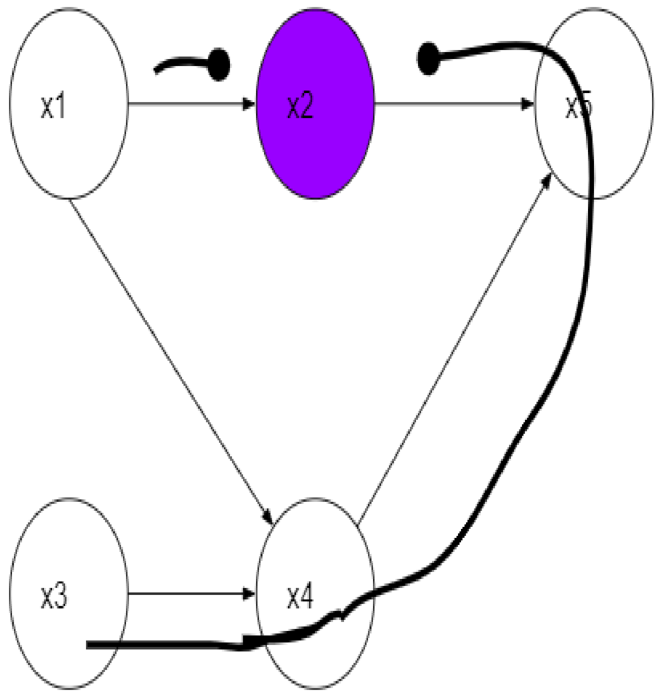
6

False



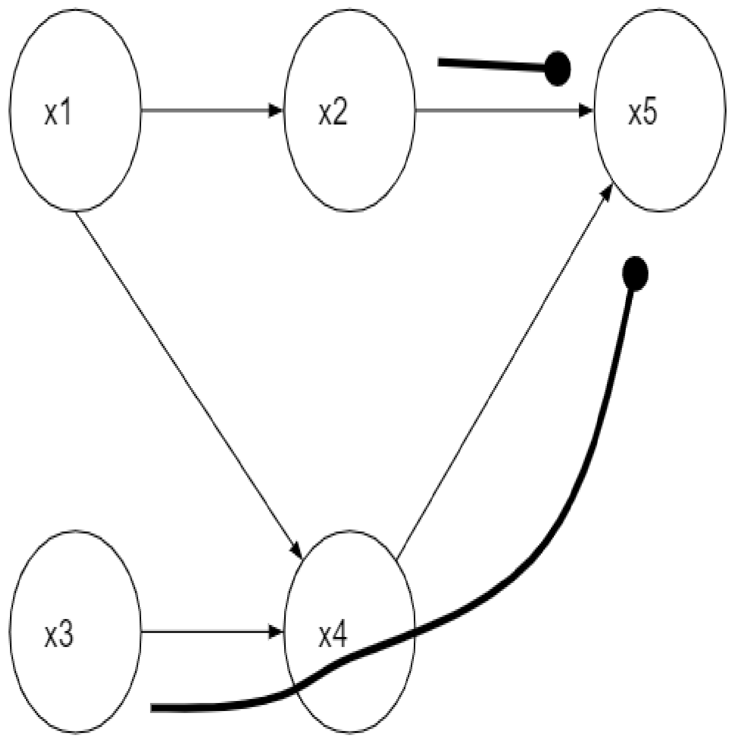
7

True



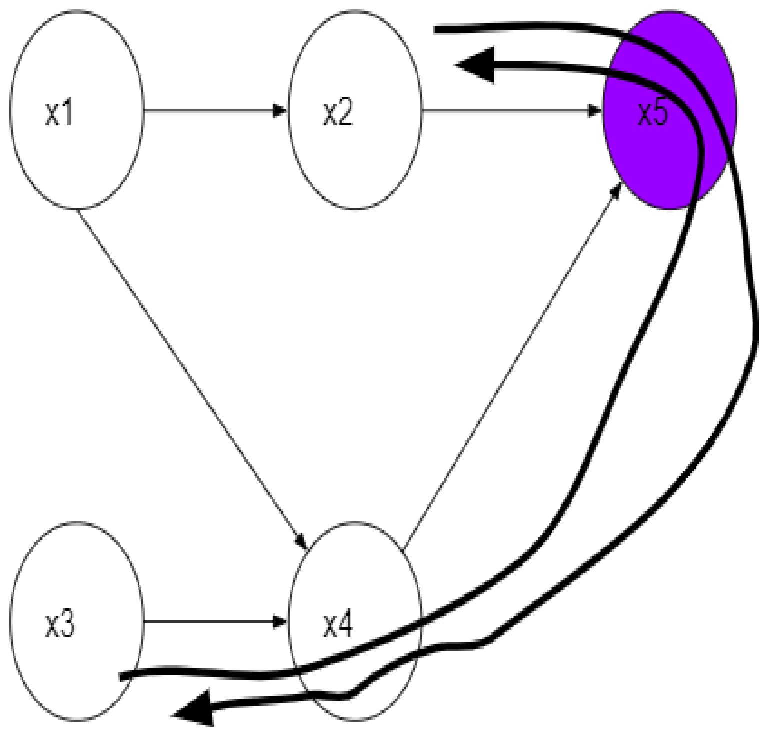
8

True



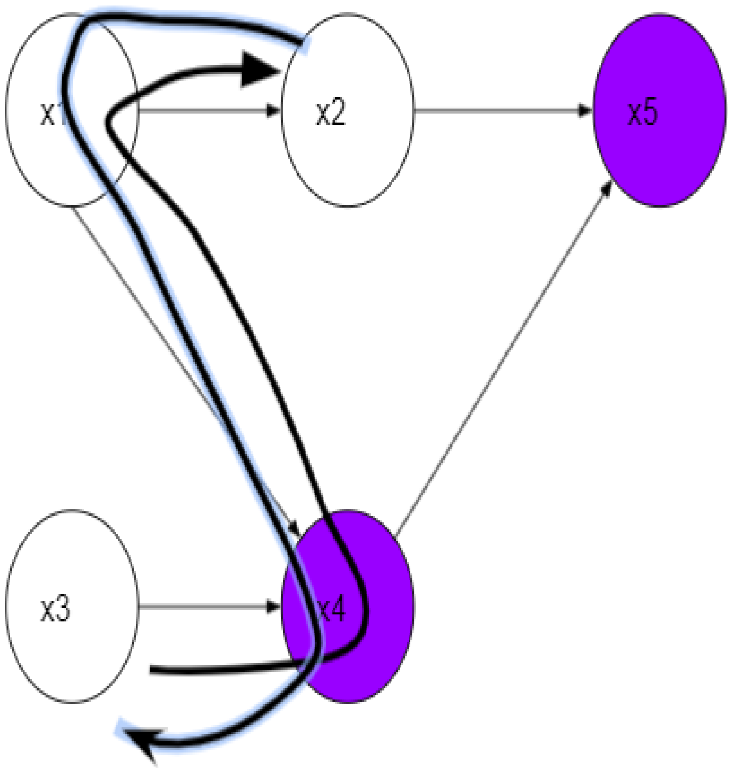
9

False



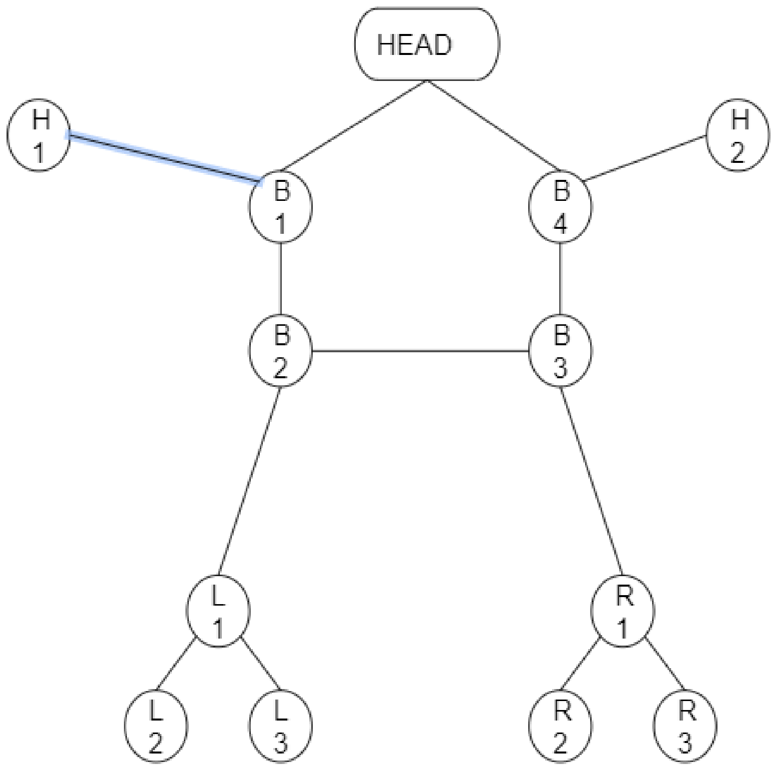
10

False

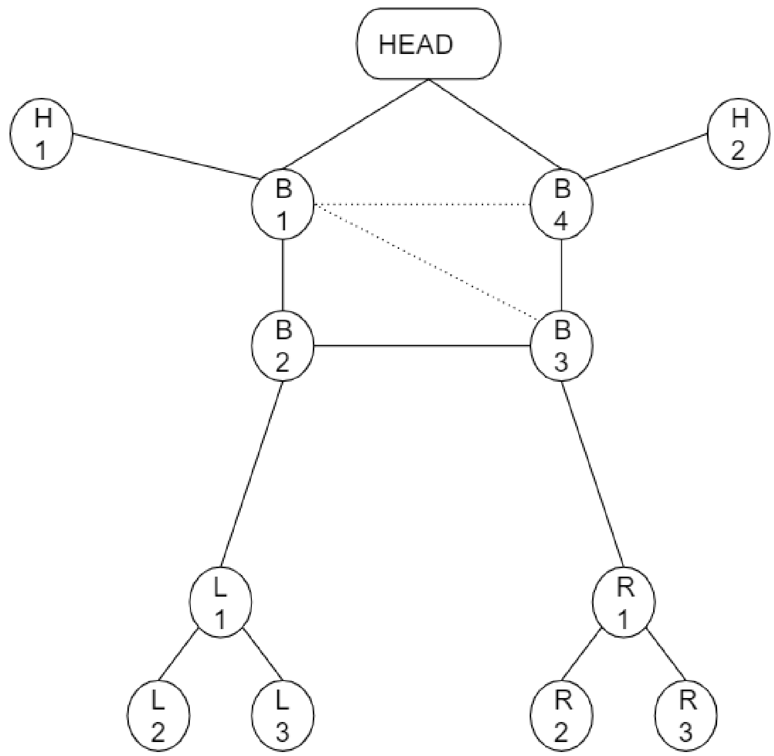


Problem 3

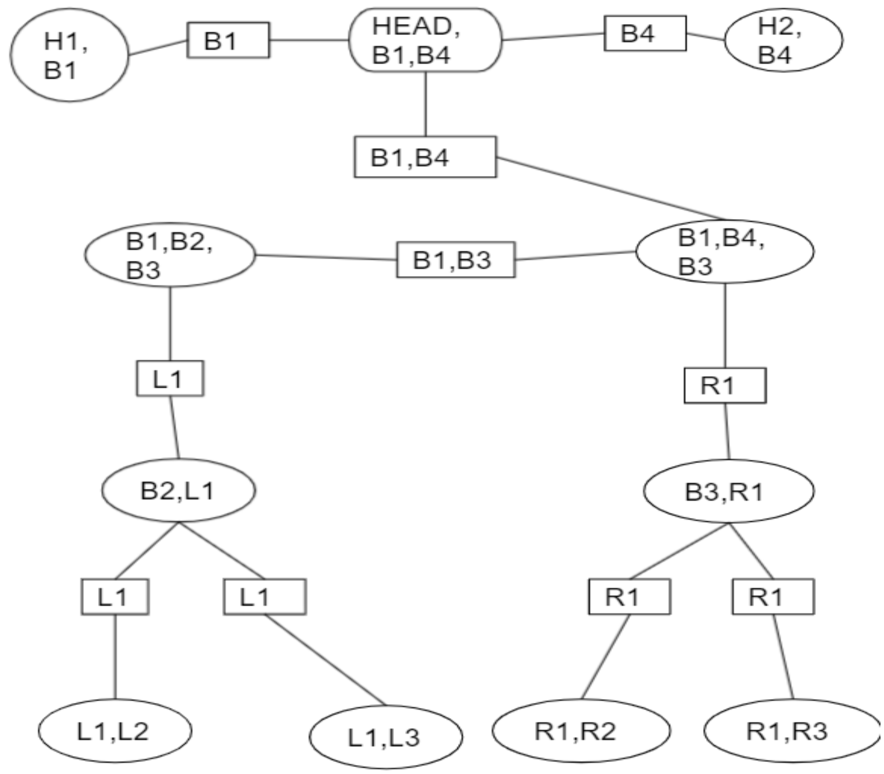
Moralization



Moralization



triangulation ,junction tree



Problem 4

The junction tree algorithm code is gievn below

```
   %Initialization of clique potential functions
 potential_functions = cell(4,1);
 potential_functions{1} = [0.1, 0.7; 0.8, 0.3];
 potential_functions{2} = [0.5, 0.1; 0.1, 0.5];
 potential_functions{3} = [0.1, 0.5; 0.5, 0.1];
 potential_functions{4} = [0.9, 0.3; 0.1, 0.3];

 marg = potential_functions;
 n = size(marg,1);
 sep = ones(n-1,2);
 %left to right message passing
 for i=1:n-1
 sep(i,:) = sum(marg{i});
 marg{i+1} = marg{i+1}.*(sep(i,:)'*[1,1]);
 end
 %right to left message passing
 for i=1:n-1
 sep1 = sep(n-i,:);
sep(n-i,:) = sum(marg{n-i+1},2)';
 marg{n-i} = marg{n-i}.*([1;1]*(sep(n-i,:)./sep1));
 end
 %Normalize
 for i=1:n
 marg{i} = marg{i}/sum(sum(marg{i}));
 end

 disp("(x1,x2)")
 disp(marg{1})
 disp("(x2,x3)")
 disp(marg{2})
 disp("(x3,x4)")
 disp(marg{3})
 disp("(x4,x5)")
 disp(marg{4})
```

The results of the final clique probabilities are

```
(x1,x2)
     0.0405     0.4451
     0.3237     0.1908

(x2,x3)
     0.2601     0.1040
     0.0578     0.5780

(x3,x4)
     0.1192     0.1987
     0.6395     0.0426

(x4,x5)
     0.5690     0.1897
     0.0603     0.1810
```

Problem 5

As given in slides we will use maxJTA algo so replace in above JTA code "sum" with "max"

```
%Use max JTA algorithm
Tran=[0.8,0.2;0.2,0.8];
Emis=[0.4,0.1,0.3,0.2;0.1,0.4,0.2,0.3];
Obs=[1,4,2,2,3];
Init=[1,0];%He is initially happy
a = size(Tran,1);

num = size(Obs,2);
p1 = zeros(a, a, num);
p2 = zeros(a, num);
%disp(size(p2(:, 1)))
p2(:, 1) = Init;

% left to right
for i = 2 : num
   pres = Obs(1, i);
p1(:, :, i) = diag(p2(:, i - 1)) *Tran * diag(Emis(:,pres));
p2(:, i) = max(p1(:, :, i));
end
% Right to left
```

```
for i = num - 1 : -1 : 1
p3 = max(p1(:, :, i + 1), [], 2);
p1(:, :, i) = p1(:, :, i) * diag(p3 ./ p2(:, i));
p2(:, i) = p3;
end
[A,H] = max(p2);
disp("final emission probabilities")
disp(p2)
disp("state")
disp(H)
```

We get the emotional state of Mario as "Happy","Angry","Angry","Angry","Angry"