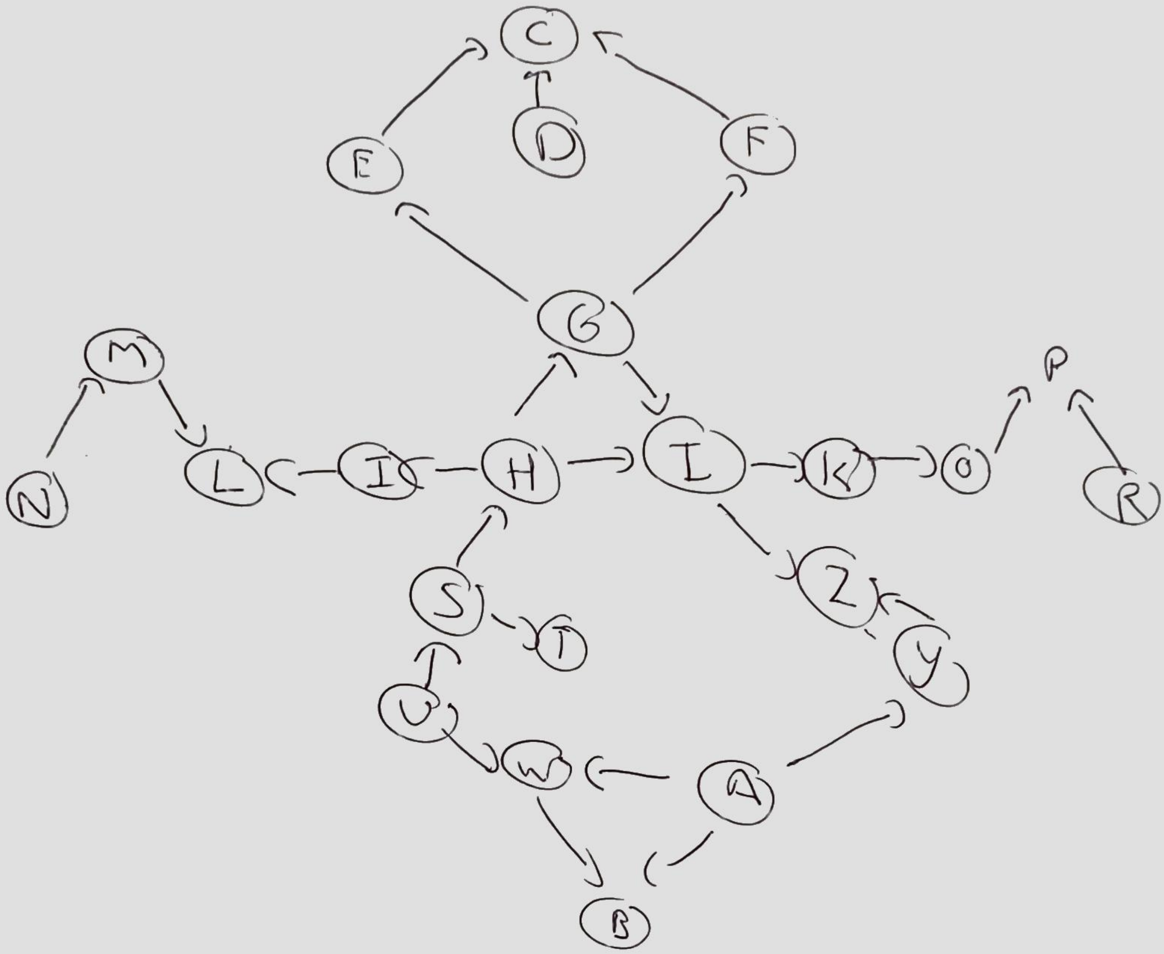
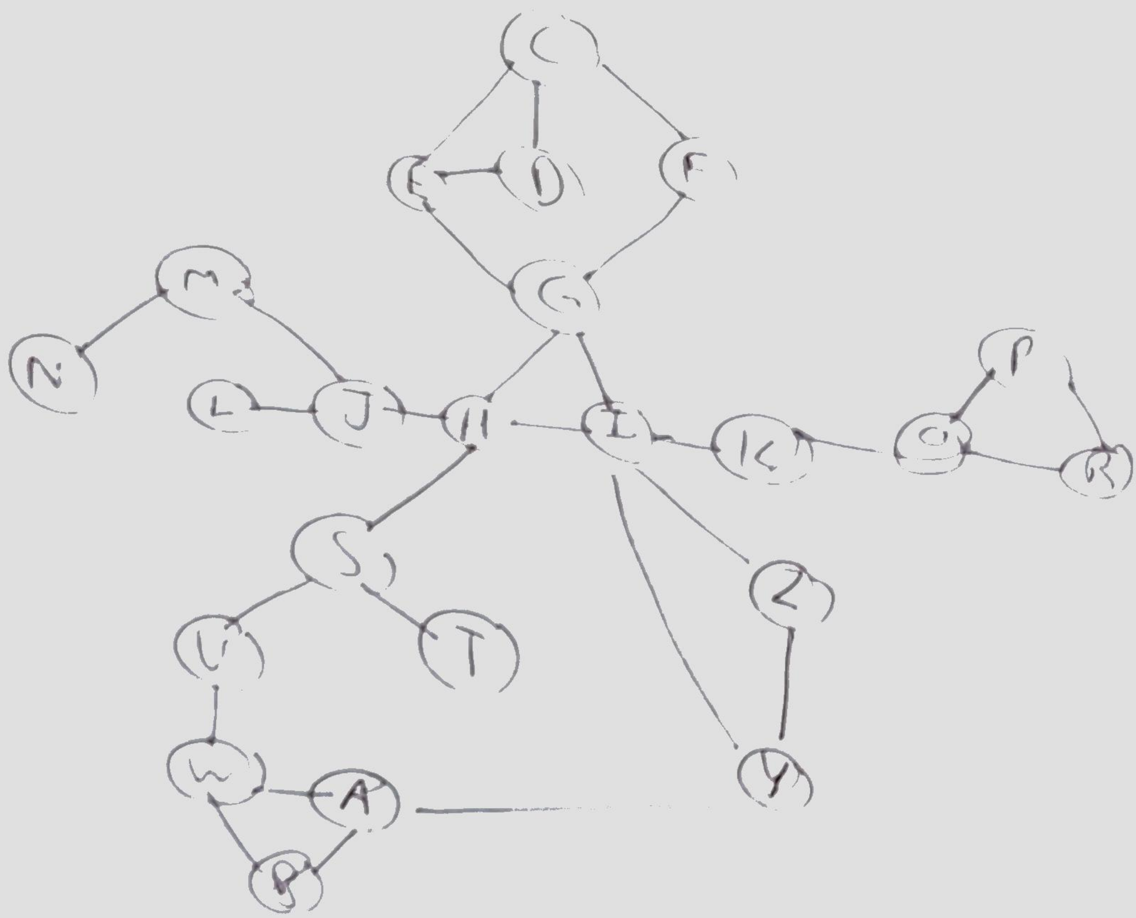
Question (1) kv2154

=) Original tree



Question (1) {kuzisid

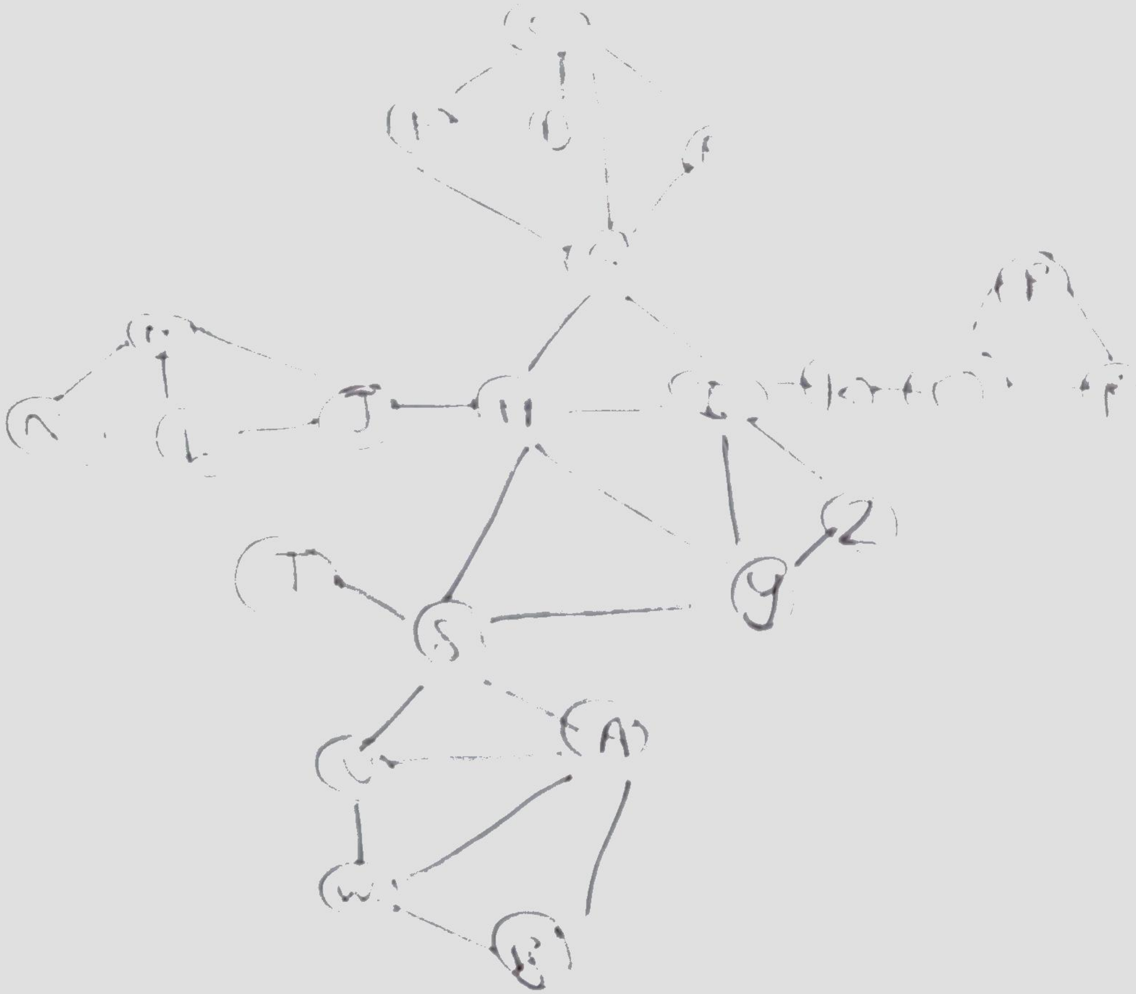
=> Moralization



[illegible] [illegible]

(3)

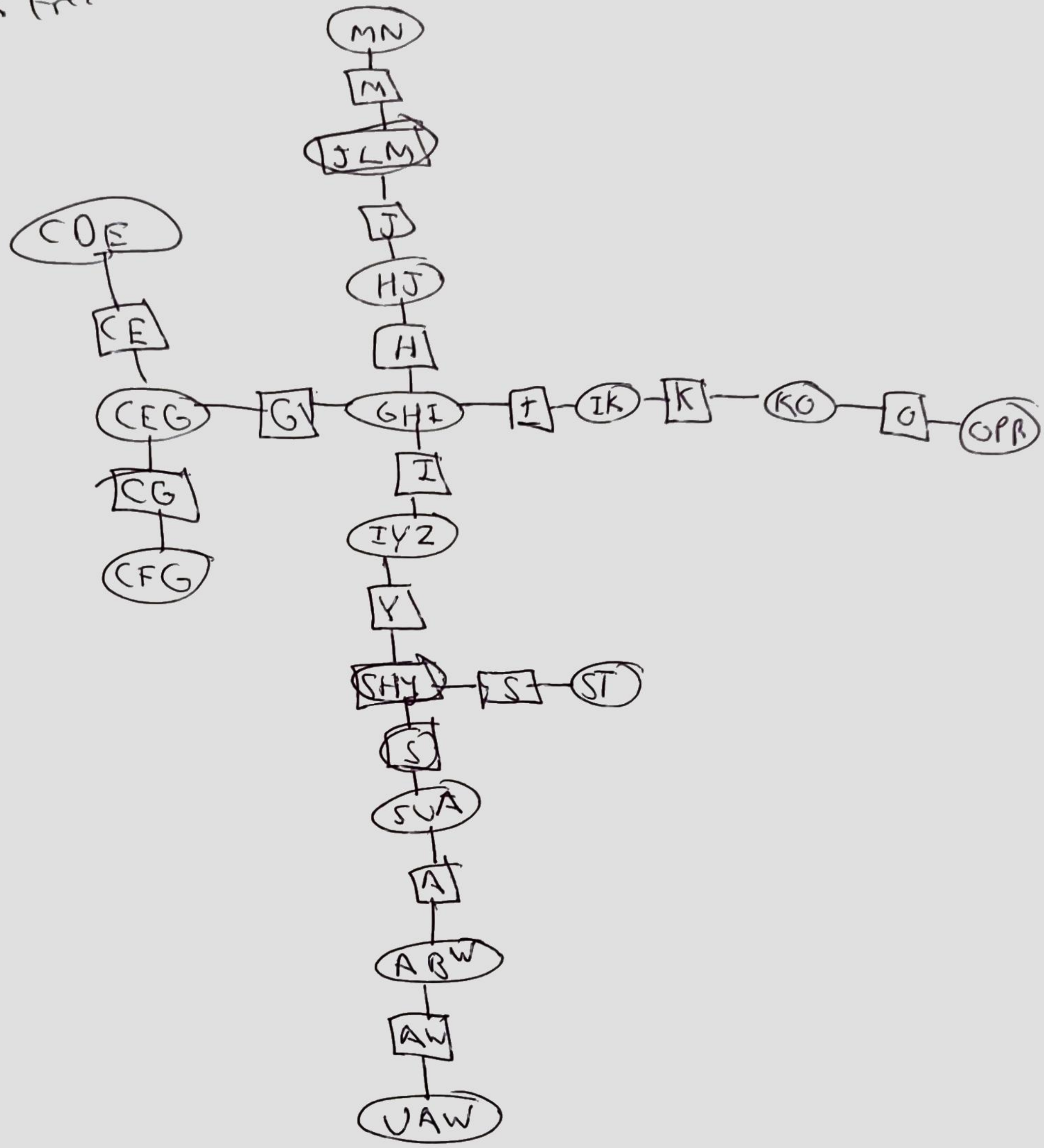
=) [illegible]



[illegible] CDE, CEG, C[illegible], CHE, MN, MLJ, JH, IK
K[illegible], CPR, IZY, SHY, ST, SUA, A[illegible], UA[illegible]

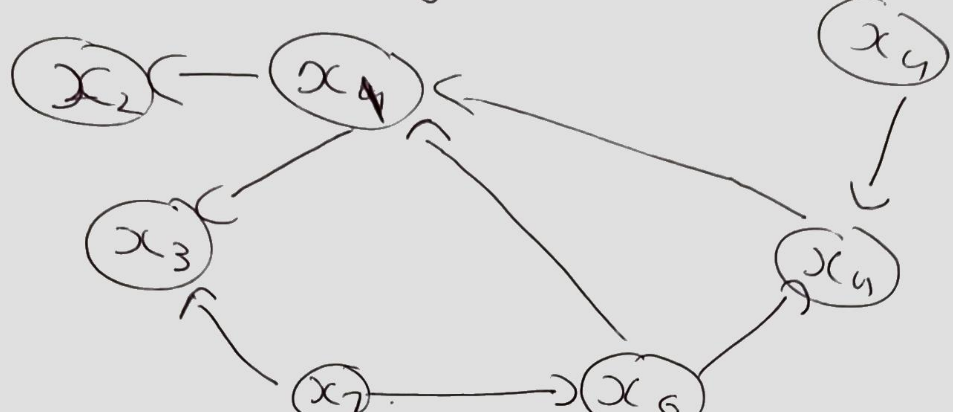
Question (1) Kv21545

⇒ Junction tree



Question (2) $\boxed{kv2154}$ (1)

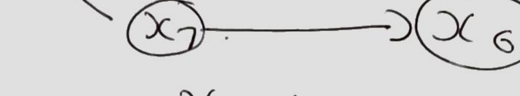
Given the bayesian Network



$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \prod_{i=1}^{n} P(x_i \mid Pa_i)$$
$$= \prod_{i=1}^{n} P(x_i \mid \pi_i) = \prod_{i=1}^{n} P(x_i \mid \pi_i)$$

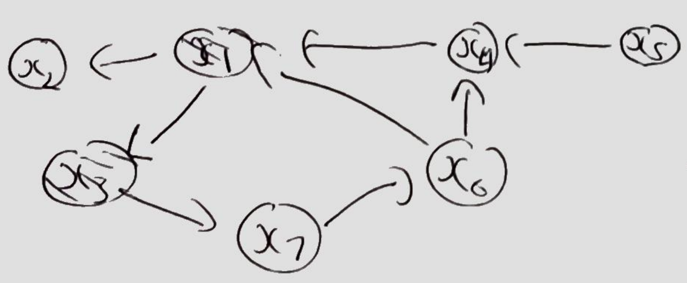
$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = P(x_1 \mid x_4 x_6) \cdot P(x_2 \mid x_1) \cdot$$
$$P(x_3 \mid x_1, x_7) \cdot P(x_4 \mid x_5 x_6) \cdot P(x_5) \cdot P(x_6 / x_7) P(x_7)$$

(a) x_2 and x_6 are independent – $\boxed{\text{False}}$



x_1 is not shaded and ball passes through 'x_1'

(B) x_2 and x_6 are conditionally independent given x_1, x_3 and x_5 – $\boxed{\text{True}}$



$\rightarrow x_6 \rightarrow x_1 \rightarrow x_2$

$\rightarrow x_6 \rightarrow x_4 \rightarrow x_1 \rightarrow x_2$

$\rightarrow x_6 \rightarrow x_7 \rightarrow x_3 \rightarrow x_1 \rightarrow x_2$

Question (1) kv21541

kv21541 (2)

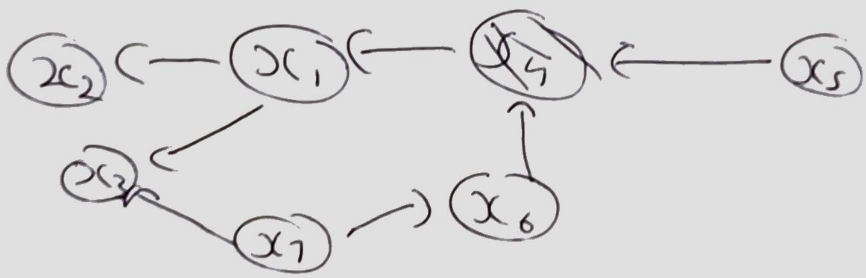
-> When chosen $x_4 \to x_1 \to x_2$
Ball passes from x_1

-> when chosen $x_6 \to x_4 \to x_1 \to x_2$
Ball moves from x_4 to x_2 passing through x_1

-> when chosen $x_6 \to x_7 \to x_3 \to x_1 \to x_2$

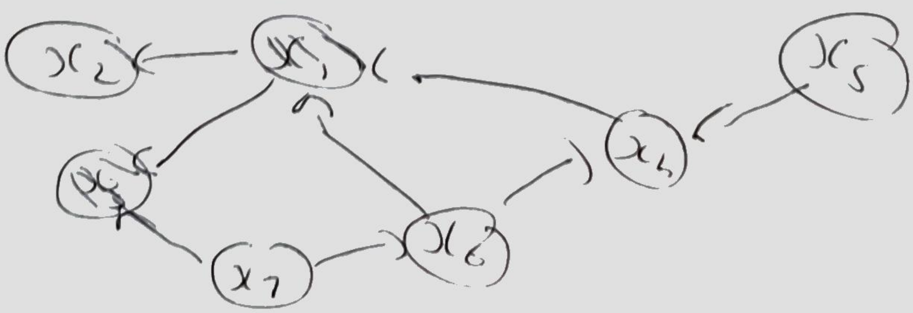
x_7 to x_1 from x_3
x_6 to x_7 as x_3, x_7 and x_6 and middle node is not shaded.

(C) x_1 and x_7 are conditionally independent given x_4 - $\boxed{\text{False}}$



-> x_7, x_6 and x_1 form markov chain and middle node is not sadded here.

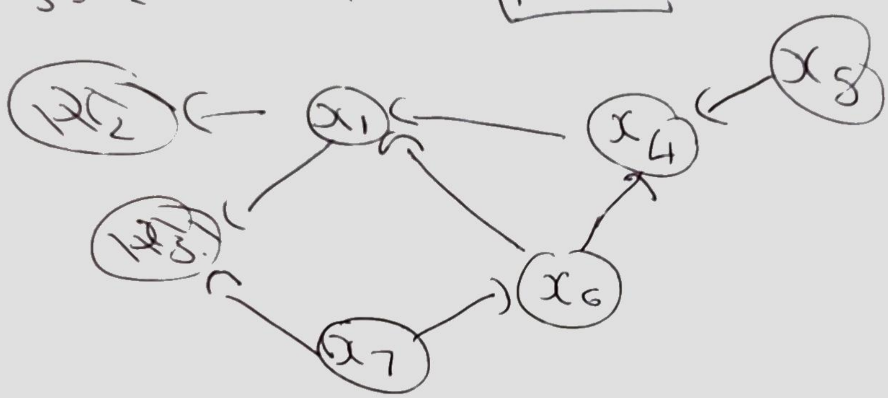
(D) x_5 and x_2 are conditionally independent given x_1 and x_3 - $\boxed{\text{True}}$



Question (2) $\boxed{KV2154}$ (3)

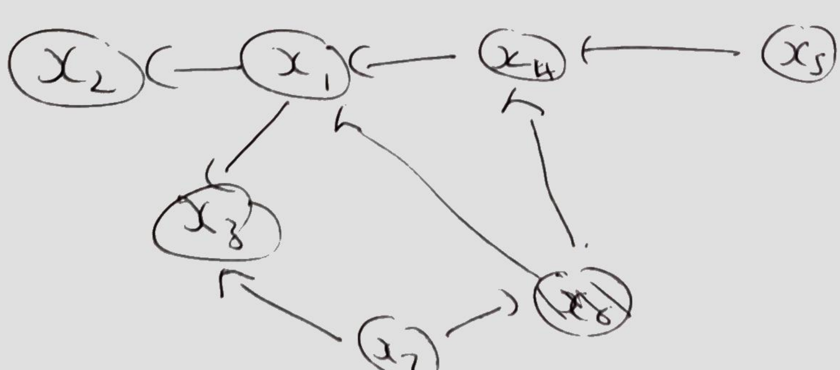
→ $x_4 \to x_1 \to x_2$ Markov chain with middle node shaded
→ $x_5 \to x_4 \to x_6$ middle node not shaded
→ Ball does not pass $x_5 \to x_4 \to x_6 \to x_7 \to x_3 \to x_1 \to x_2$

(E) x_5 and x_1 are conditionally independent given x_3, x_2 and x_4 — $\boxed{\text{False}}$



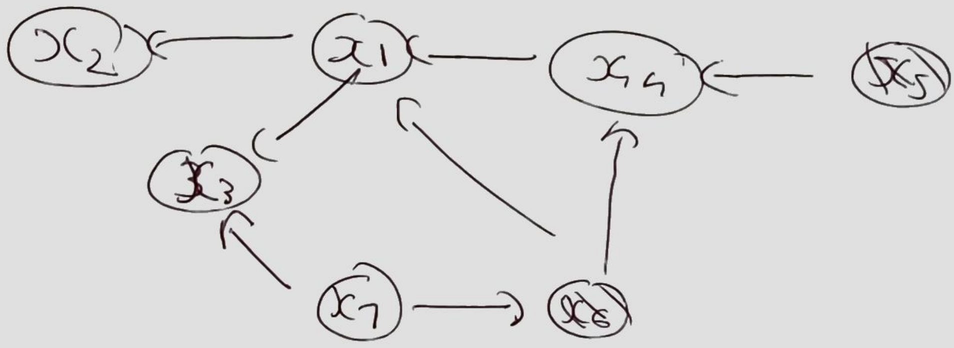
→ $x_5 \to x_6$ passes
→ $x_6 \to x_1$ passes and middle node not shaded

(F) x_4 and x_3 are conditionally independent given x_6
$\boxed{\text{False}}$



→ x_4 to x_3 from x_1 forms Markov chain with middle node shaded kv2154 (5)

(G) x_2 and x_7 are conditionally independent given x_5 and x_6 - $\boxed{\text{True}}$

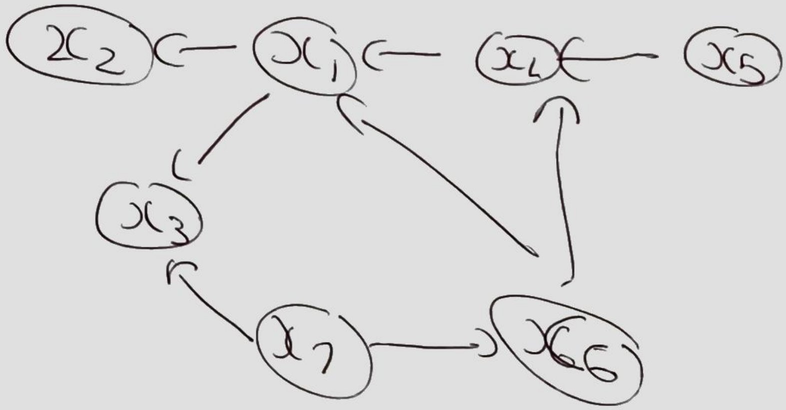


-> x_5, x_6 : shaded

-> Doesn't pass $x_2 \to x_1 \to x_3 \to x_7$ middle node not shadde.

-> $x_2 \to x_6 \to x_1$ form markov chain and middle node shaded

(H) x_5 and x_3 are conditionally independent given x_6 and x_7 - $\boxed{\text{False}}$



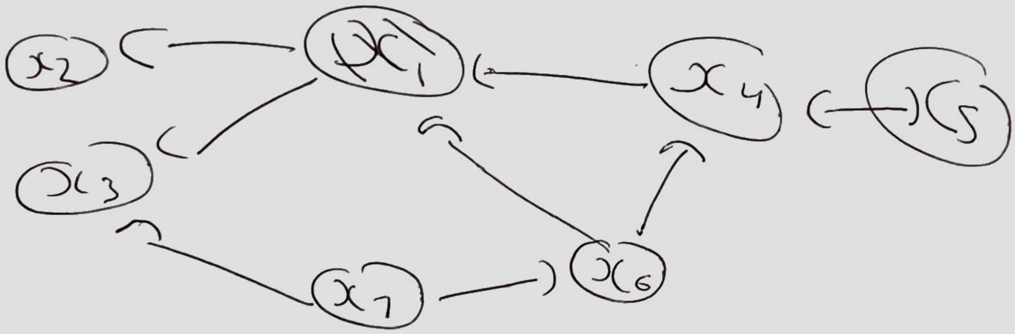
-> $x_5 \to x_4 \to x_1 \to x_3$ form 2 markov chains

Question 2 $\boxed{\text{KU2154}}$

I) x_5 and x_2 are independent – $\boxed{\text{False}}$

→ No nodes shaded

J) x_2 and x_4 are conditionally independent given x_1 – $\boxed{\text{True}}$



→ $x_2 \to x_1 \to x_4$ markov chain with middle node shaded

→ $x_2 \to x_1 \to x_3$:- Does not pass

→ $x_2 \to x_1 \to x_6 \to x_4$:- Does not pass

→ $x_2 \to x_1 \to x_6$ → Markov chain with middle node shaded

Question 3 $\boxed{kv2154}$

Given,

$$\pi = P(q_0) = \begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\pi = P(q_0) = [\,1/3,\ 2/3\,]$$

$$a^T = P(q_t \mid q_{t-1}) = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix} \end{array}$$

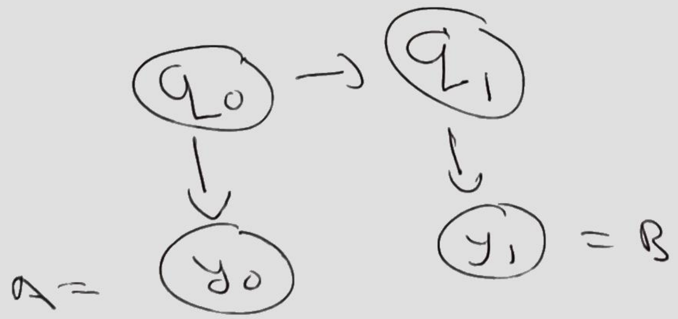
$$\eta^T = P(y_t \mid q_t) = \begin{array}{c|c} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix} \end{array}$$

To find,

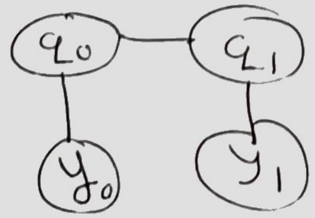
(i) likelihood of $P(y)$ using HMM

(ii) Individual marginals of States where $P(q_0 \mid y)$ and $P(q_1 \mid y)$

Step 1 :- 2 State HMM



Step 2 :- Converting to junction tree



Question (3) [kv2154] (1)

Given,

$$\pi = P(q_0) = \begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\pi = P(q_0) = [\, 1/3 ,\ 2/3 \,]$$

$$a^T = P(q_t \mid q_{t-1}) = \begin{array}{c|cc} & 1 & 2 \\ \hline 1 & 1/8 & 1/2 \\ 2 & 7/8 & 1/2 \end{array}$$

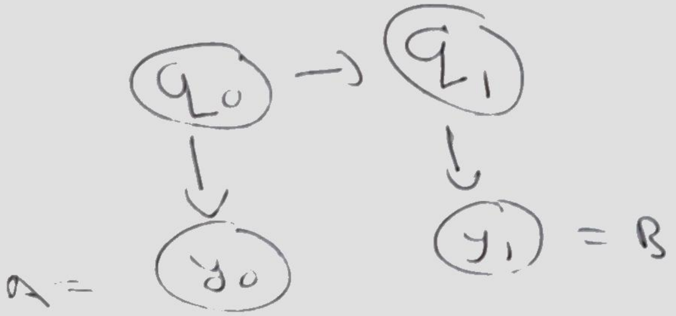
$$n^T = P(y_t \mid q_t) = \begin{array}{c|cc} & A & 2 \\ \hline A & 1/4 & 3/4 \\ B & 3/4 & 1/4 \end{array}$$

To find,

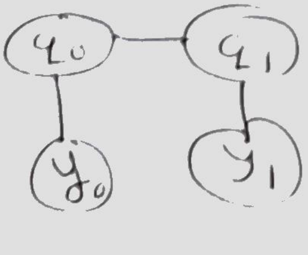
(i) likelihood of $P(y)$ using HMM

(ii) Individual marginals of States where $P(q_0 \mid y)$ and $P(q_1 \mid y)$

Step 1: 2 State HMM



Step 2 :- Converting to junction tree



Question 8 $\boxed{KV2154}$ 2)

Implementing moralization and triangulation here,

$\boxed{q_0 y_0}$ — $\boxed{y_0}$ — $q_0 q_1$ — $\boxed{q_1}$ — $q_1 y_1$

$\Rightarrow$ $q_0 y_0$ — $\boxed{q_0}$ —

$q_0 = 1$	1/2	1/4
$q_F = 2$	1/2	1/6

$\Rightarrow$ — $q_0 q_1$ — $\boxed{q_1}$ —

	$q_1 = 1$	2
$q_0 = 1$	1/8	1/8
$q_1 = 2$	1/2	1/2

1	2
1	1

$\Rightarrow$ .... — $q_1 y_1$

	$y_1 =$ A	B
$q_0 = 1$	1/4	3/4
$q_1 = 2$	3/4	1/4

Question 8 $\boxed{\text{ku2154}}$ 8

$q_0 y_0$ —— $\boxed{q_0}$ —— $q_0 q_1$

y_0	A	B
$q_0 = 1$	1/12	
$q_0 = 2$	1/2	

q_0 1 .. 2

1	1

$q_1 = 1 \ldots 2$

1/8	1/8
1/2	1/2

$\Rightarrow$ —— $\boxed{q_1}$ ...... $q_1 y_1$

1	1

	A	B
$q_1 = 1$		3/4
$q_2 = 2$		1/4

$\Rightarrow$ $q_0 y_0$ —— $\boxed{f_0}$

$q_0 = 1$	1/2	
$q_0 = 2$	1/2	

$q_0 = 1, \ldots 2$

1	1

Question 8 $\boxed{\text{KU2154}}$

=) $q_0 q_1 \xleftarrow{\quad} \boxed{q_1}$

1/8	7/8
1/2	1/2

$q_1 = 1 \ldots 2$

1	1

=) $q \ldots y_1$

	A	B
$q_1 = 1$		3/4
$q_1 = 2$		1/4

=) $q_0 q_1$

	1	2
$q_0 = 1$	1/8	7/8
$q_0 = 2$	1/2	1/2

q_1

1	2
3/4	1/4

=) $q_1 y_1$

	A	B
$q_1 = 1$		3/4
$q_1 = 2$		1/4

Question 8 $\boxed{\text{KV2154}}$ 5

$\therefore$ likelihood $p(y)$

$$\begin{bmatrix} \frac{1}{12} \times \frac{1}{8} , & \frac{1}{12} \times \frac{7}{8} \\ \frac{1}{2} \times \frac{1}{2} , & \frac{1}{2} \times \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1/128 & 7/384 \\ 3/16 & 1/16 \end{bmatrix}$$

$$\therefore p(y) = \left[\frac{1}{128} + \frac{7}{384} + \frac{3}{16} + \frac{1}{16} \right]$$

$$\boxed{p(y) = \frac{53}{192}}$$

Question 4 KV21541

To Solve k-means clustering algorithm.

given that,

$C_1 = (-4, -5)$, $C_2 = (5, 4)$

Given 2-D Dataset

A(-3, -1) B(-1, -3) C(-2, -6) D(-5, -7), E(3, 1)
F(2, 3), G(3, 6), H(8, 1)

=) Initializing Value of centroids

$C_1 = (-4, -5)$, $C_2 = (5, 4)$

=) objects - centroid distance.

In this step, we calculate the euclidean distance between each object of the 2D dataset and the centroid

Iteration 0:- when using C_1 : (-4, -5)

$$A = \sqrt{(-3+4)^2 + (-1+5)^2} = 4.12$$

$$B = \sqrt{(-1+4)^2 + (-3+5)^2} = 3.60$$

$$C = \sqrt{(-2+4)^2 + (-6+5)^2} = 2.23$$

$$D = \sqrt{(-5+4)^2 + (-7+5)^2} = 2.23$$

$$E = \sqrt{(3+4)^2 + (1+5)^2} = 9.21$$

$$F = \sqrt{(2+4)^2 + (3+5)^2} = 10$$

Question (4) $\boxed{KV21541}$

$G = \sqrt{(3+4)^2 + (6+5)^2} = 13.03$

$H = \sqrt{(8+4)^2 + (1+5)^2} = 13.41$

When $C_2 = (5,4)$

$A = \sqrt{(-3-5)^2 + (-1-4)^2} = 9.43$

$B = \sqrt{(-1-5)^2 + (-3-4)^2} = 9.21$

$C = \sqrt{(-2-5)^2 + (-6-4)^2} = 12.20$

$D = \sqrt{(-5-5)^2 + (-7-5)^2} = 15.62$

$E = \sqrt{(3-5)^2 + (1-4)^2} = 3.60$

$F = \sqrt{(2-5)^2 + (3-4)^2} = 3.16$

$G = \sqrt{(3-5)^2 + (6-4)^2} = 2.82$

$H = \sqrt{(8-5)^2 + (1-4)^2} = 4.24$

Distance matrix can be given as.

$$D_0 = \begin{array}{cccccccccc} & A & B & C & D & E & F & G & H & \\ \left[\begin{array}{c} \\ \\ \end{array}\right. & \begin{array}{c} 4.12 \\ 9.43 \end{array} & \begin{array}{c} 3.60 \\ 9.21 \end{array} & \begin{array}{c} 2.23 \\ 12.20 \end{array} & \begin{array}{c} 2.23 \\ 15.62 \end{array} & \begin{array}{c} 9.21 \\ 3.20 \end{array} & \begin{array}{c} 10 \\ 3.16 \end{array} & \begin{array}{c} 13.03 \\ 2.82 \end{array} & \begin{array}{c} 13.41 \\ 4.24 \end{array} & \left.\begin{array}{c} \\ \\ \end{array}\right] \begin{array}{c} C_1 \\ C_2 \end{array} \end{array}$$

Question (4) $\boxed{\text{KV2154}}$ (8)

$\Rightarrow$ Step 3 :- Object clustering

We have to assign each object basing upont the minimum distance

Then

$$G^{0} = \begin{array}{c} \\ \end{array}\begin{bmatrix} A & B & C & D & E & F & G & H \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{array}{l} \\ \rightarrow \text{group 1} \\ \rightarrow \text{group 2} \end{array}$$

$\Rightarrow$ Step 4 :- Updating Centroids

$$C_1 = \left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4} \right)$$

$$= \left(\frac{-11}{4}, \frac{-17}{4} \right) = (-2.75, -4.25)$$

$$C_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right) = \left(4, \frac{11}{4} \right) = (4, 2.75)$$

Iteration 1 :- Using C_1

$$A = \sqrt{(-3+2.75)^2 + (-1+4.25)^2} = 3.25$$

$$B = \sqrt{(-1+2.75)^2 + (-3+4.25)^2} = 2.15$$

$$C = \sqrt{(-2+2.75)^2 + (-6+4.25)^2} = 1.90$$

$$D = \sqrt{(-5+2.75)^2 + (-7+4.25)^2} = 3.55$$

$$E = \sqrt{(3+2.75)^2 + (1+4.25)^2} = 7.78$$

$\boxed{\text{KV2154}}$ 4

$F = \sqrt{(2+2.25)^2 + (3+4.25)^2} = 8.66$

$G = \sqrt{(3+2.75)^2 + (6+4.25)^2} = 11.75$

$H = \sqrt{(5+2.75)^2 + (4+4.25)^2} = 11.31$

when using $C_2 = (4, 2.75)$

$A = \sqrt{(-3-4)^2 + (-1-2.75)^2} = 7.94$

$B = \sqrt{(-1-4)^2 + (-3-2.75)^2} = 7.61$

$C = \sqrt{(-2-4)^2 + (-6-2.75)^2} = 10.60$

$D = \sqrt{(-5-4)^2 + (-7-2.75)^2} = 13.26$

$E = \sqrt{(3-4)^2 + (1-2.75)^2} = 2.01$

$F = \sqrt{(2-4)^2 + (3-2.75)^2} = 2.01$

$G = \sqrt{(3-4)^2 + (6-2.75)^2} = 3.40$

$H = \sqrt{(5-4)^2 + (4-2.75)^2} = 1.60$

Distance matrix

$$\begin{array}{cccccccc} A & B & C & D & E & F & G & H \\ \end{array}$$
$$\begin{bmatrix} 3.25 & 2.15 & 1.90 & 3.55 & 7.78 & 8.66 & 11.75 & 11.31 \\ 7.94 & 7.61 & 10.60 & 13.26 & 2.01 & 2.01 & 3.40 & 1.60 \end{bmatrix}$$

clustering

$$\begin{array}{cccccccc} A & B & C & D & E & F & G & H \\ \end{array}$$
$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{array}{l} \rightarrow \text{group 1} \\ \rightarrow \text{group 2} \end{array}$$

Question 4 KV2154

8

⇒ Updated centroids

$$c_1 = \left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4} \right) = \left(\frac{-11}{4}, \frac{-17}{4} \right)$$

$$= (-2.75, -4.25)$$

$$c_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right) = (4, 2.75)$$

Question (5) | kv2154 |

Jensen's Inequality :- To, for non-negative real numbers $x_1, x_2, \ldots, x_n$

$$\frac{x_1 + x_2 + \ldots + x_n}{n} \geqslant \sqrt[n]{x_1 \cdot x_2 \cdot \cdots x_n}$$

This is of the form that arithmetic mean greater than the geometric mean. As 'log' function is an increasing function we apply 'log' on both sides to proceed with jensen's inequality.

i.e. $\log\left(\frac{x_1 + x_2 + \ldots + x_n}{n}\right) \geqslant \frac{1}{n} \log(x_1 \cdot x_2 \cdots x_n)$

$$= \frac{\log x_1 + \ldots \log x_n}{n}$$

ie; $\log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geqslant \frac{1}{n}\log(x_1 \ldots x_n)$

$\Rightarrow \log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geqslant \frac{1}{n}\log\left(\prod_{i=1}^{n} x_i\right)$

or,

$$\log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geqslant \log\left(\prod_{i=1}^{n} x_i\right)^{1/n}$$

Then,

$$\frac{\sum_{i=1}^{n} x_i}{n} \geqslant \sqrt[n]{\prod_{i=1}^{n} x_i}$$

$$\therefore \frac{x_1 + x_2 + \ldots + x_n}{n} \geqslant \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

Problem (5) Kv2154 (22)

When we use $E[x]$ as mean of x_i's.

Then,

$$\log(E[x]) \geq E[\log(x)]$$

Hence, it is proved using concavity of log given by Jensen's inequality.

Problem (6) kv2154

$\Rightarrow$ Final layer size $= 10 \times 12 \times 9$

Maxpoling with

region size $= 3 \times 3$

Stride $= 2 \times 2$

$$\frac{x - 3}{2} + 1 = 12 \qquad \frac{y - 3}{2} + 1 = 9$$

$$x = 25 \qquad y = 19$$

$\Rightarrow$ ReLU doesnot change the size

$\Rightarrow$ Convolutional layer $\therefore$ - $8 \rightarrow 10$, 2×2, 2×2

$$\frac{x - 2}{2} + 1 = 25 \qquad \frac{y - 1}{2} + 1 = 19$$

$$x = 50 \qquad y = 38$$

$\Rightarrow 8 \times 50 \times 38$

$\Rightarrow$ maxpooling with

region : 3×3.

Stride : 2×2

$$\frac{x - 3}{2} + 1 = 50 \qquad \frac{y - 3}{2} + 1 = 38$$

$$x = 101 \qquad y = 77$$

$\Rightarrow 8 \times 101 \times 77$

$\Rightarrow$ ReLU doesnot change the size

Problem ⓑ $\boxed{\text{KV2154}}$ ②

$\Rightarrow$ Convolutional layer :- $1 \to 8,\ 2\times 2,\ 3\times 2$

~~Region~~

$$\frac{x-2}{3}+1 = 101 \qquad \frac{y-2}{2}+1 = 77$$

$$x = 302 \qquad y = 154$$

$\therefore$ The size of the input is $1 \times 302 \times 154$

$$\boxed{\begin{aligned} \therefore\ x &= 302 \\ y &= 154 \end{aligned}}$$