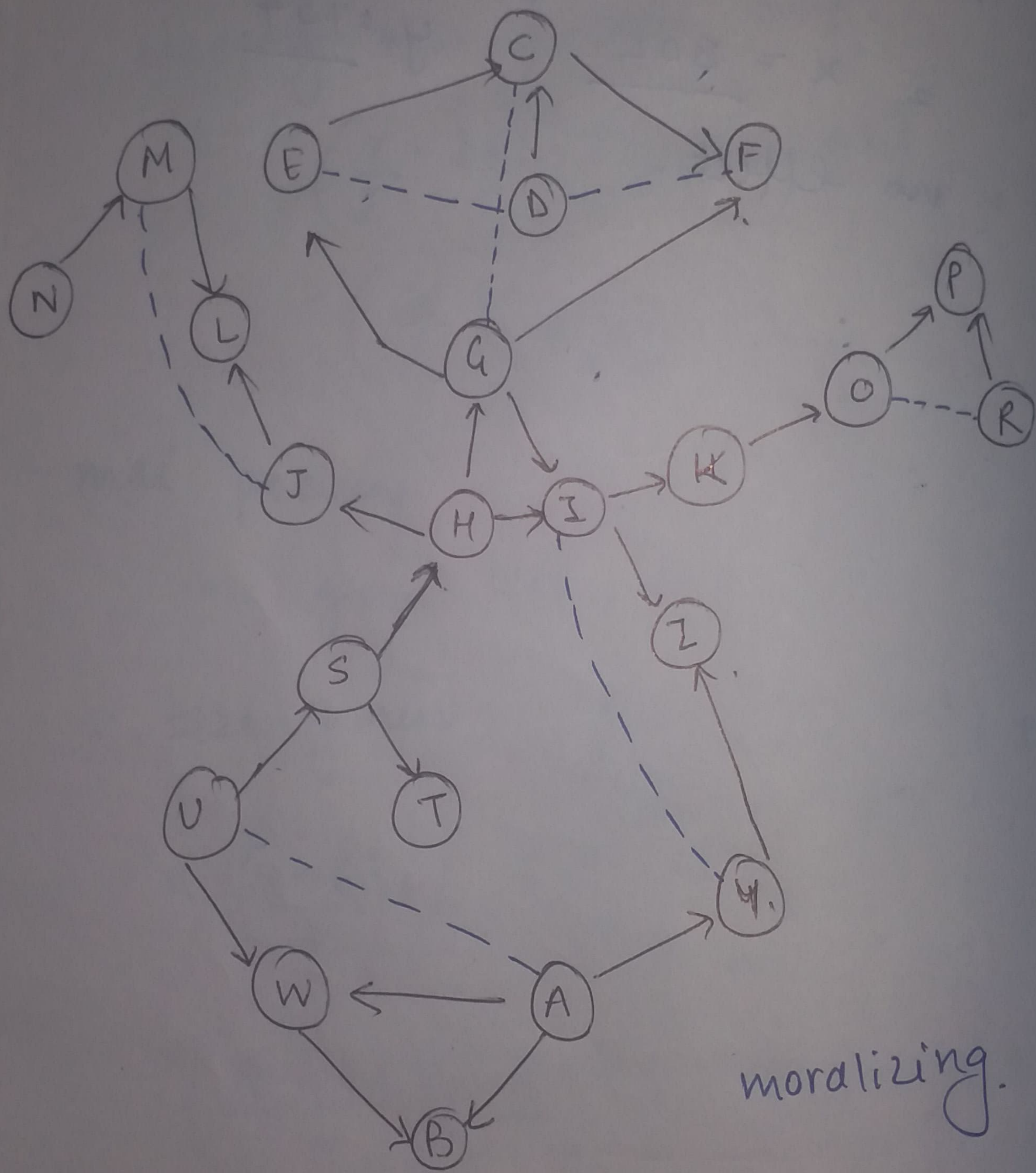
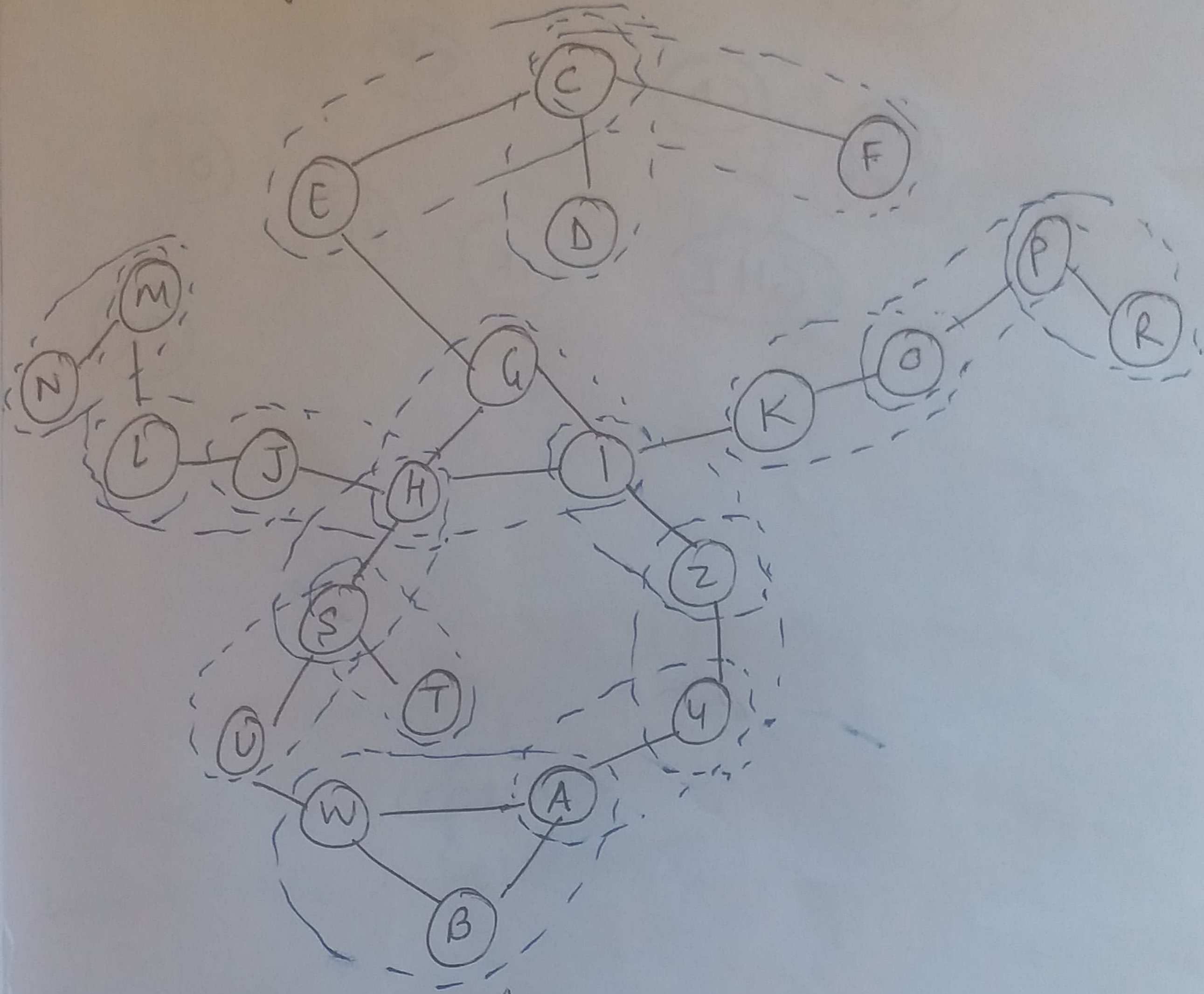
Problem-1 :



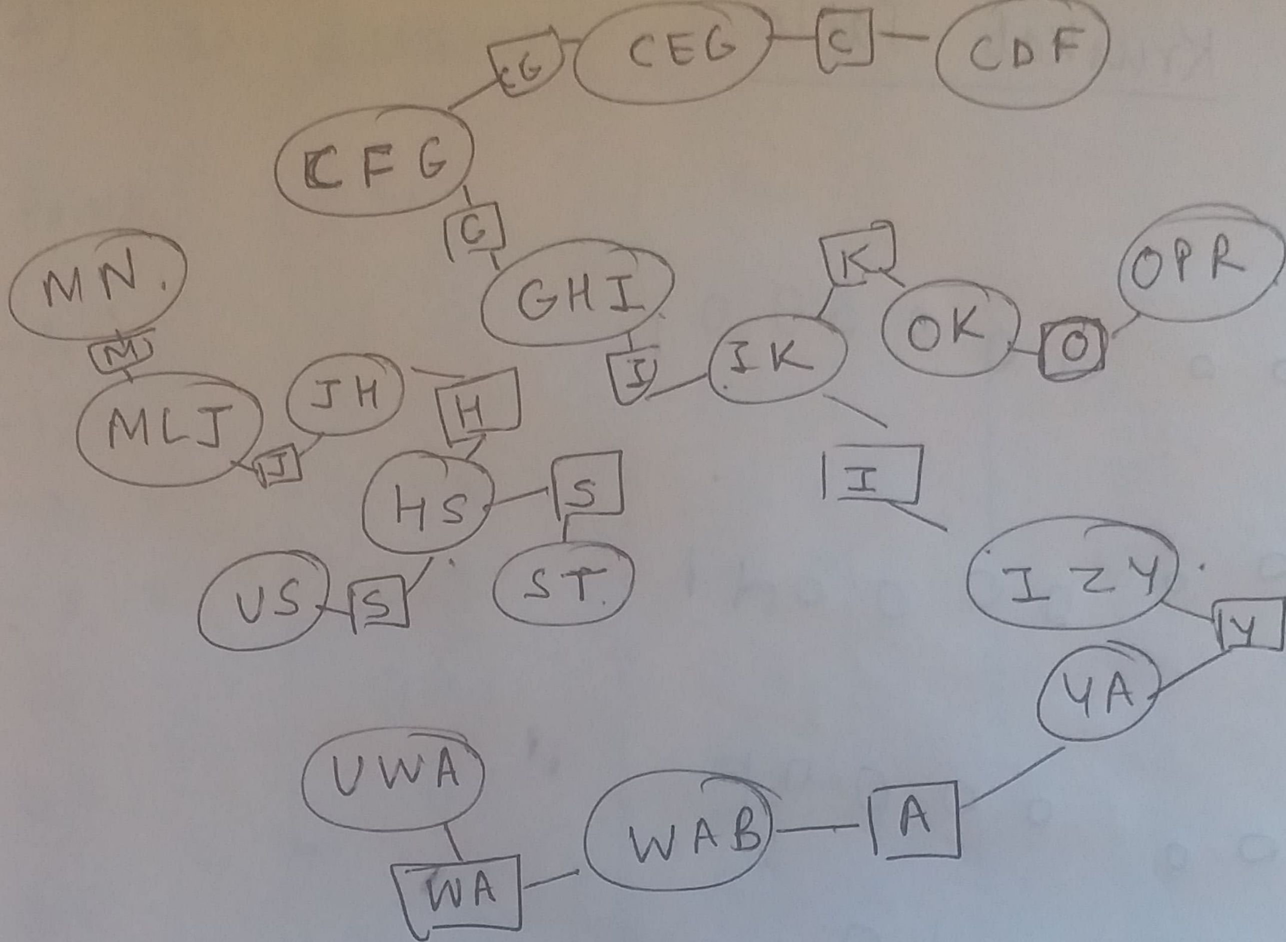
moralizing.

moralising



undirected graph

clique tree



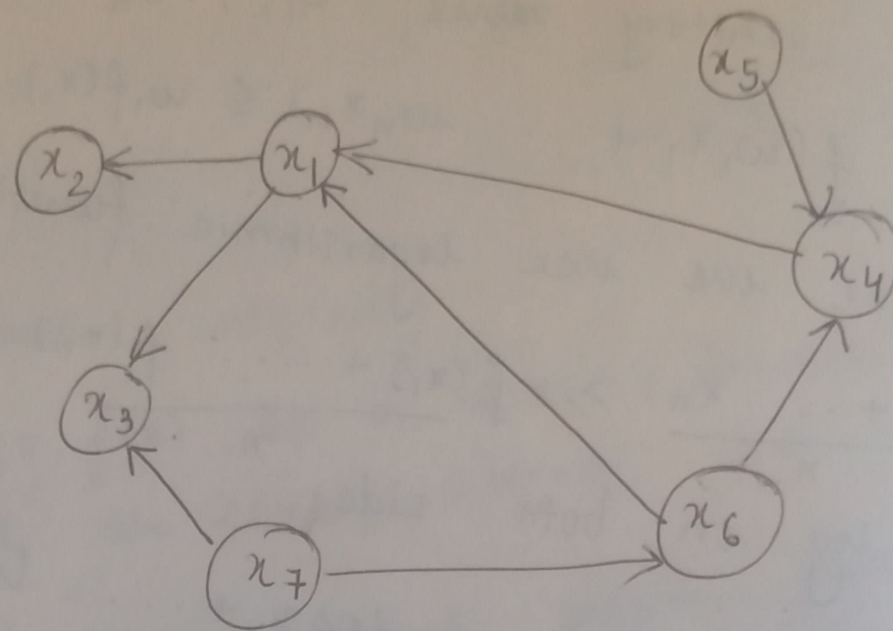
Adding separators hence the following junction tree.

~~Kruskal Table~~ :

Kruskal Table

	CFG	CEG	CDF	GHI	OPR	IZY	WAB	UWA	MJL.
CFG	—	2	1	1.	0	0	0	0	0
CEG	[illegible]	—	1	1	0	0	0	0	0
CDF			—	0	0	0	0	0	0
GHI				—	0	1	0	0	0
OPR					—	0	0	0	0
IZY						—	0	0	0
WAB							—	2	0
WUA								—	0
MJL									—

Problem-2 :



The joint proababiliity distribution of the graph is

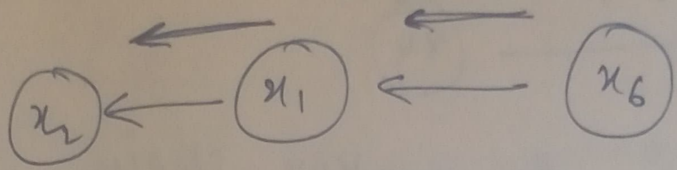
$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7)$$

$$= P(x_5) \cdot P(x_1 | x_4, x_6) \cdot P(x_2 | x_3) \cdot P(x_3 | x_1 x_7) \cdot P(x_4 | x_5, x_6) \cdot P(x_5 | \; P(x_6 | x_7) \cdot P(x_7).$$

(a) x_2 & x_6 are independent:

~~Answer~~: False

hence the ball can pass through.

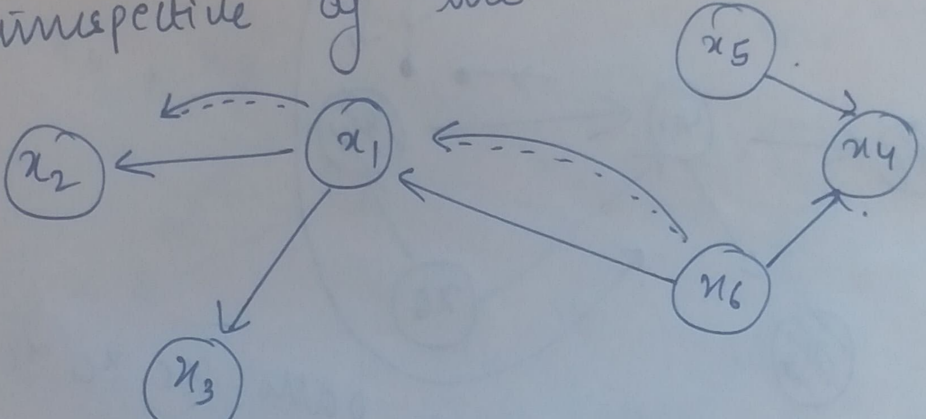


forms a markhov chain

(b). x_2 & x_6 are conditionally independent given x_1, x_3, x_5.

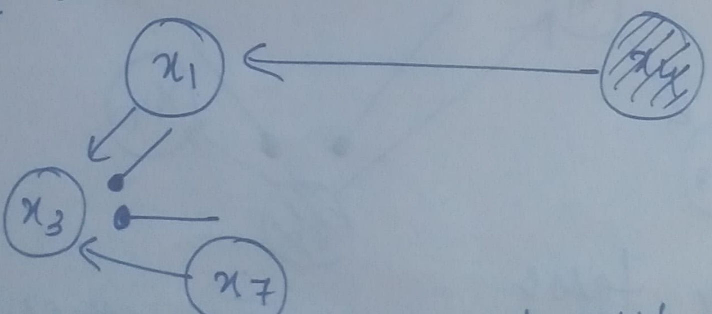
~~Answer~~: True

since the ball passes through irrespective of the nodes connected.



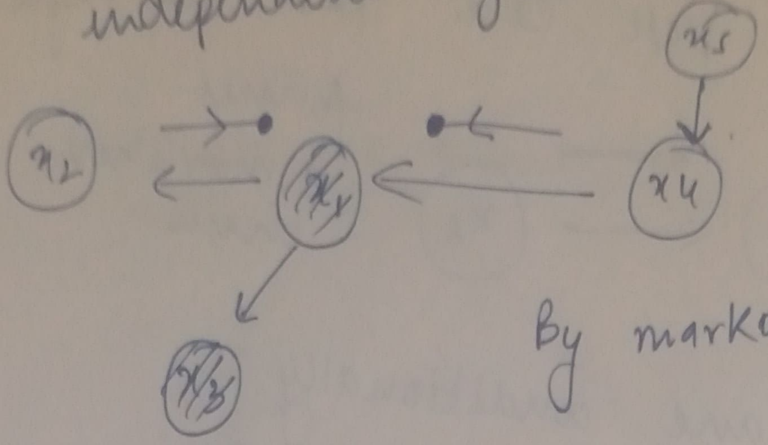
(c). x_1 & x_7 are conditionally independent given x_4.

x_1, x_6, x_7 is simple markhov chain



False they are always independent

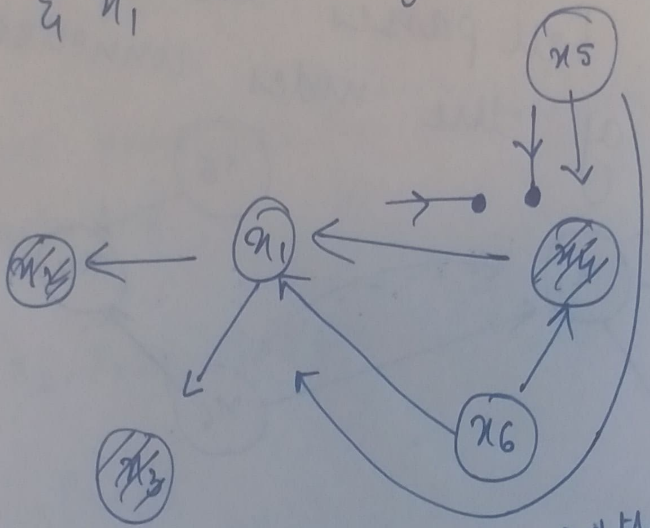
(d) x_5 & x_2 are conditionally independent given x_1 & x_3.



By markov chain

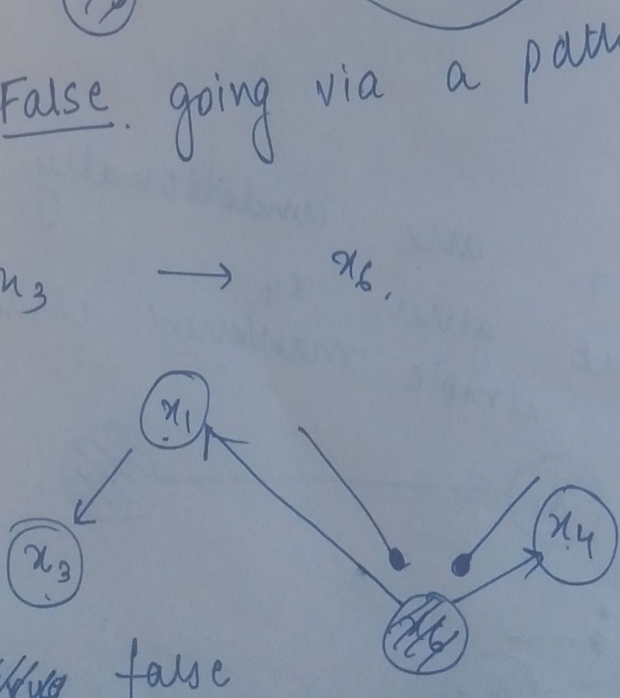
~~False~~. True

(e). x_5 & x_1 → given x_3, x_2, x_4.



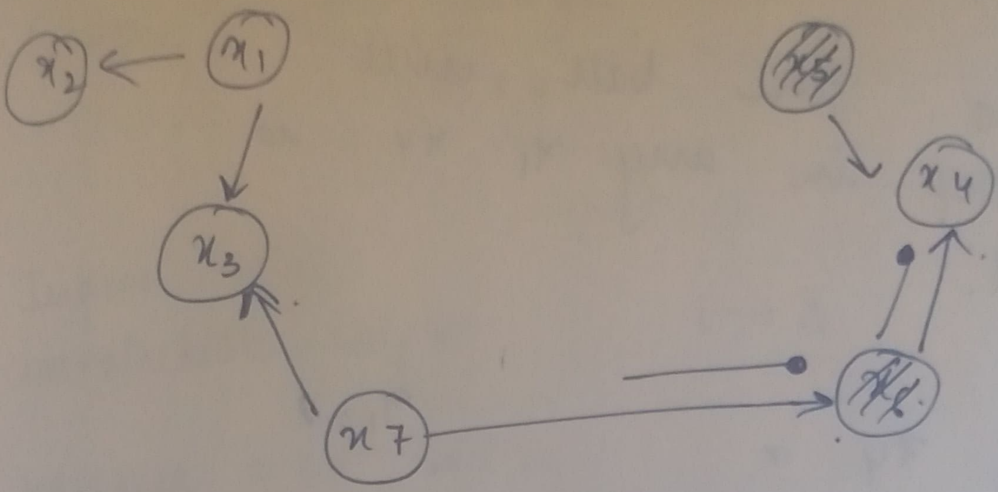
~~True~~. False. going via a path x_5, x_6, x_1

(f) x_4, x_3 → x_6.



~~False~~ ~~true~~. false
the ball passes ~~through~~ stops.
markhow chain.

(g) x_2 , x_7 x_5 & x_6



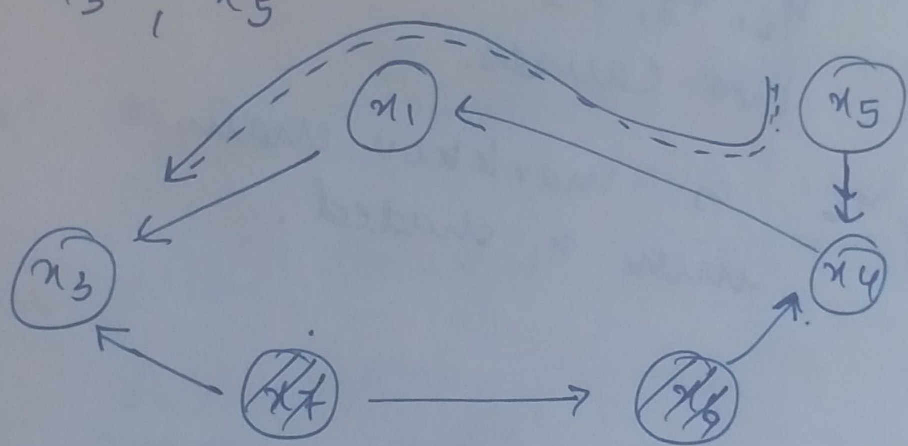
True:

x_1, x_3, x_7 two causes

x_1, x_6, x_7 markhov chain

x_4 x_6 x_7 markhov chain

(h) x_3 , x_5 x_6 , x_7



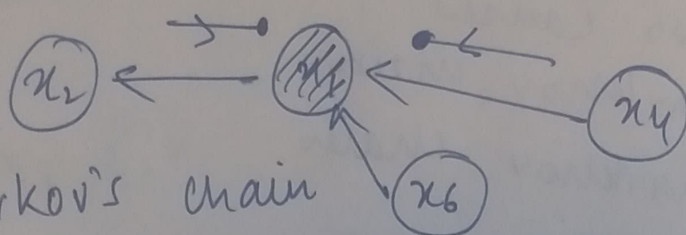
False.

markhov chain

(i). x_5 & x_2 are independent

~~True~~ False as the ball will always stop in any x_1 x_4 is absorbed.

(j). x_2 & x_4 r x_1



markov's chain

True. x_2, x_3, x_1

two causes.

x_2, x_1, x_6 is markhov chain with x_1 shaded.

Problem-3:

Given that

$$\pi = p(q_0) = \begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$$

$$a^T = p(q_t \mid q_{t-1}) = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix} \end{array}$$

$$\eta^T = p(y_t \mid q_t) = \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix} \end{array}$$

According to the 2 states given:

$$\begin{tikzcd} q_0 \arrow[r] \arrow[d] & q_1 \arrow[d] \\ y_0 & y_1 \end{tikzcd}$$

$y_0 = A$ $\quad y_1 = B.$

the initialised junction tree.

$$\boxed{q_0 y_0} — \boxed{q_0} — \boxed{q_0 q_1} — \boxed{q_1} — \boxed{q_1 y_1}$$

	$y_0 = A$	$y_0 = B$		
$y_0 = 1$	1/4	3/4	1/8	3/8
$y_0 = 2$	3/4	1/4.	1/2	1/6

$\psi(q_0, y_0) = p(q_0 \propto p(y_0 | q_0)$.

	$y_0 = A$	$y_0 = B$
$q_0 =$	1/8	3/8
	1/2	1/6

$\psi(q_1, y_1) = p(y_1 | q_1)$.

	$y_1 = A$	$y_1 = B$
$q_0 =$	1/4	3/4
$q_0 =$	3/4	1/4.

initialise the separators to 1.

1/8	3/8
1/2	1/6

1	1

1/8	1/48
1/2	1/12

1/4	3/4
3/4	1/4.

1/8	
1/2	

1/8	1/2

3/4	1/4

	3/4
	1/4

After messaging passing

$\phi^{**}(q_0) = \phi^{**} q_1) = p(y_1 y_2)$.

$\frac{10}{256} + \frac{1}{4} = \frac{51}{236} + \frac{23}{236} = \frac{37}{128} = 0.289$

$p(q_0 = 1) = \frac{10/256}{\frac{10}{256} + 1/4}$

$= p(q_0 = 2) = \frac{1/4}{\frac{10}{256} + 1/4}$

$\frac{5}{37}$, $\frac{32}{37}$

$P(q_1 = 1/y) = \dfrac{51/256}{\frac{51}{256} + \frac{23}{256}}$. $P(q_1 = 1/y)$.

$= \dfrac{\frac{23}{256}}{\frac{51}{236} + \frac{23}{256}}$

$= \frac{51}{74}$. $\Rightarrow$ $\frac{23}{74}$.

Ans

(4). 2D dataset given.

Points	C_1:	C_2:	clusters
(-3, -1)	4.12	9.43	C_1
-1, -3	3.6	9.2	C_1
-2, -6	2.23	12.2	C_1
-5, -7.	2.23.	14.86.	C_1.
3, 1	9.21	3.6	C_2
2, 3	10	3.16	C_2
3, 6	13.03	2.82	C_2
8, 1	13.41	4.2	C_2

new dataset

$C_1 - x = \frac{(-3-1-2-5)}{4} = -2.75$

$C_1 \quad y = \frac{(-1-3-6-7)}{4} = -4.25$

$C_2 \quad x = \frac{(3+2+3+8)}{4} = \cancel{2.7}\ 4.$

$y = \frac{(1+3+6+1)}{4} = 2.75$

C_1 $(-2.75, -4.25)$, $C_2 \rightarrow (4, 2.75)$

<u>Iteration 2</u> : (After finding C_1 & C_2).

Point	C_1	C_2	clusters
-3, -1	3.2	7.9	C_1
-1, -3	2.15	7.6	C_1
-2, -6	1.9	10.6	C_1
-5, -7.	3.5	13.2	C_1
3, 1	7.7	2.01	C_2
2, 3	8.66	2.91	C_2
3, 6	11.75	3.4	C_2
8, 1	11.96	4.3	C_2.

$C_1 = (-2.75, -4.25)$ $C_2 = (4, 2.75)$

Ans

Problem-5:

To prove: $\frac{x_1 + x_2 + \dots x_n}{n} \geq \sqrt[n]{x_1 . x_2 \dots x_n}$

By jensens inequality
let $y = f(x)$ be a convex function defined on some interval in R.

An important bound from jensen for convex f: $f(E\{x\}) \leq E\{f(x)\}$

$\therefore$ This is like the AM-GM inequality proving for non-negative numbers.

$\therefore$ let $x_1 \dots x_n$ be positive real numbers; then

$$\frac{x_1 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \dots x_n}$$

with equality if ? and only if $x_1 = \dots = x_n$ (hence all real numbers should be equal).

Jensen's Inequality states that for a convex function $f(x)$

$w_i > 0$; $\Sigma w_i = 1$

if we arbitary value $x_1 \dots x_n$

then $f(w_1 x_1 + \dots w_n x_n) \leq w_1 f(x_1) \dots w_n f(x_n)$

To prove we use logarithmic functions

$$\therefore f\frac{(x_1 + \dots x_n)}{n} \geq \frac{f(x_1) + \dots f(x_n)}{n}.$$

taking log on both sides.

$$\log\frac{(x_1 + \dots x_n)}{n} \geq \frac{\log x_1 + \dots \log x_n}{n}$$

By log property $\log a \times b = \log a + \log b$.

by exponentiating

$$\log\frac{(x_1 + \dots x_n)}{n} \geq \frac{1}{n}\left[\log(x_1) + \dots \log x_n)\right].$$

$$\log(\text{Avg } M) \geq \log(GM).$$

$$= \log\frac{(x_1 + \dots x_n)}{n} \geq \frac{1}{n}\log(x_1 \cdot x_2 \cdot x_3 \dots x_n)$$

$$\therefore = \frac{x_1 + x_2 \dots x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \dots x_n}$$

hence proved

Problem - 6 :

Given Input image :

$1 \times x \times y$

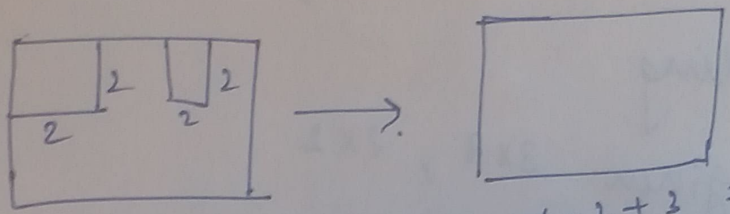
Input size

convolution layer $1 \rightarrow 8$ $\quad \underbrace{2\times2}, \underbrace{3\times 2}$

kernel size 2×2

stride 3×2

$y-2+2=y$



$x-2+3 = x+1$

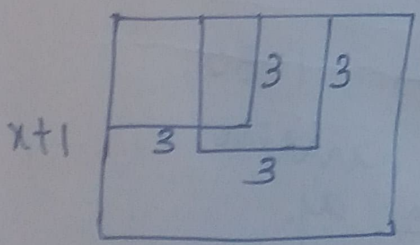
size of new feature map $\underline{(x+1).y}$

$= 2(12-1)+3$

$2(9-1)+3 = \underline{25\times19}$

Given max pooling

3×3 region , 2×2 stride



by subtracting & formulating

we get new feature map :

$$\left(\frac{x-2+2}{2}\right)\left(\frac{y-3+2}{2}\right) = \left(\frac{x}{2}, \frac{y-1}{2}\right)$$

convolutional layer:

$8 \to 10, \quad 2 \times 2, \quad 2 \times 2.$

$2(25-1)+2$

$2(19-1)+2$

$= 50 \times 38.$

$\therefore$ size of new feature map.

$$\left(\frac{x}{2} - 2 + 2\right)\left(\frac{y-1}{2} - 2 + 2\right)$$

$$= \left(\frac{x}{2}\right)\left(\frac{y-1}{2}\right)$$

$=$

$\therefore$ max pooling

given 3×3, 2×2

$\therefore$ size of new: $$\left(\frac{\frac{x}{2} - 3}{2}\right)\left(\frac{\frac{y-1}{2} - 3}{2}\right) + 1$$

$$= \frac{\left(\frac{x-6}{2}\right) + 1}{2} \quad \left(\frac{\frac{y-1}{2} - 2}{2}\right) + 1$$

$$= \left(\frac{x - 6 + 4}{4}\right)\left(\frac{y-3}{4}\right).$$

$3(101-2)+2$

$2(77-1)+2$

$= 302 \times 154.$

Flattening:

$10 \times 12 \times 9. \longrightarrow 1080.$

By comparing it with the new size of feature map.

$$\therefore \left(\frac{x-2}{4}\right)$$

$\frac{x-2}{4} = 12$ $\qquad$ $\frac{y-3}{4} = 9$

$x - 2 = \cancel{48}$ $\qquad$ $y - 3 = \cancel{36}$

$x = \cancel{50}$ $\qquad$ $y = \cancel{39}.$

$\therefore\ x = \underline{302}$, $y = \underline{154}.$

RELU = no effect.