SOLUTION-5 :

To prove: $\dfrac{x_1 + \cdots + x_n}{n} \geq (x_1 \cdots x_n)^{1/n}$

Proof: $\dfrac{x_1 + \cdots + x_n}{n} \geq (x_1 \cdots x_n)^{1/n}$

taking log on both the sides, we get:

$$\Rightarrow \log\left(\frac{x_1 + \cdots + x_n}{n}\right) \geq \frac{1}{n} \log (x_1 \cdots x_n)$$

$$\Rightarrow \log\left(\frac{x_1 + \cdots + x_n}{n}\right) \geq \frac{\log x_1 + \cdots \log x_n}{n}$$

$$\Rightarrow \log(E[x]) \geq E[\log(x)] \quad (1)$$

($\because$ E denotes expectation)

Now, let's understand Jensen's inequality

If $p_1, \ldots, p_n$ are positive no which sum to 1 and f is a real continuous function that is convex, then

$$f\left(\sum_{i=1}^{n} p_i x_i\right) \leq \sum_{i=1}^{n} p_i f(x_i)$$

If f is concave, then the inequality reverses, giving

$$f\left(\sum_{i=1}^{n} p_i x_i\right) \geq \sum_{i=1}^{n} p_i f(x_i).$$

The special case, $p_i = 1/n$ with the concave function $\ln x$ gives

$$\ln\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geq \frac{1}{n}\sum_{i=1}^{n} \ln x_i .$$

this can be exponentiated to give AM-GM inequality

$$\boxed{\frac{x_1 + x_2 + \cdots x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n}}$$

(hence proved)

~~ALITER~~

Continuing with eq. 1, we can also prove:
Using concavity of log, by Jensen's inequality the inequality holds.

Sol-4

	X	Y
A	-3	-1
B	-1	-3
C	-2	-6
D	-5	-7
E	3	1
F	2	3
G	3	6
H	8	1

Initial centroids

$C_1 = -4, -5$, $C_2 = +5\ +4$

Iteration - I

Datapoint.	ED from $C_1(-4, -5)$	ED from C_2. $(+5\ +4)$	Clusters
(A) -3, -1	$\sqrt{(-3+4)^2 + (-1+5)^2} = 4.123$	$\sqrt{(-3-5)^2 + (-1-4)^2} = 9.43$	C_1
(B) -1, -3	$\sqrt{(-1+4)^2 + (-3+5)^2} = 3.6$	$\sqrt{(-1-5)^2 + (-3-4)^2} = 9.21$	C_1
(C) -2, -6	$\sqrt{(-2+4)^2 + (-6+5)^2} = 2.23$	$\sqrt{(-2-5)^2 + (-6-4)^2} = 12.2$	C_1
(D) -5, -7	$\sqrt{(-5+4)^2 + (-7+5)^2} = 2.23$	$\sqrt{(-5-5)^2 + (-7-4)^2} = 14.8$	C_1
(E) 3, 1	$\sqrt{(3+4)^2 + (1+5)^2} = 9.21$	$\sqrt{(3-5)^2 + (1-4)^2} = 3.6$	C_2
(F) 2, 3	$\sqrt{(2+4)^2 + (3+5)^2} = 10$	$\sqrt{(2-5)^2 + (3-4)^2} = 3.16$	C_2
(G) 3, 6	$\sqrt{(3+4)^2 + (6+5)^2} = 13.03$	$\sqrt{(3-5)^2 + (6-4)^2} = 2.82$	C_2
(H) 8, 1	$\sqrt{(8+4)^2 + (1+5)^2} = 13.4$	$\sqrt{(8-5)^2 + (1-4)^2} = 4.24$	C_2

Rough.

New centroids:

for C_1 : $-2.75, -4.25$.

C_2 : $4, 2.75$.

$\frac{-3-1-2-5}{4} = -2.75,$

$= \frac{-1-3-6-7}{4} = -4.25$

$\frac{16}{4} = 4, \frac{11}{4} = 2.75.$

Iteration II :

Datapoint	ED from $C_1 (-2.75, -4.25)$	ED from $C_2 (4, 2.75)$	Cluster
-3, -1	$\sqrt{(-3+2.75)^2 + (-1+4.25)^2} = 3.25$	$\sqrt{(-3-4)^2 + (-1-2.75)^2} = 7.94$	C_1
-1, -3	$\sqrt{(-1+2.75)^2 + (-3+4.25)^2} = 2.15$	$\sqrt{(-1-4)^2 + (-3-2.75)^2} = 7.61$	C_1
-2, -6	$\sqrt{(-2+2.75)^2 + (-6+4.25)^2} = 1.9$	$\sqrt{(-2-4)^2 + (-6-2.75)^2} = 10.6$	C_1
-5, -7	$\sqrt{(-5+2.75)^2 + (-7+4.25)^2} = 3.5$	$\sqrt{(-5-4)^2 + (-7-2.75)^2} = 13.2$	C_1
3, 1	$\sqrt{(3+2.75)^2 + (1+4.25)^2} = 7.78$	$\sqrt{(3-4)^2 + (1-2.75)^2} = 2.01$	C_2
2, 3	$\sqrt{(2+2.75)^2 + (3+4.25)^2} = 8.66$	$\sqrt{(3-4)^2 + (3-2.75)^2} = 2.01$	C_2
3, 6	$\sqrt{(3+2.75)^2 + (6+4.25)^2} = 11.75$	$\sqrt{(3-4)^2 + (6-2.75)^2} = 3.4$	C_2
8, 1	$\sqrt{(8+2.75)^2 + (1+4.25)^2} = 11.96$	$\sqrt{(8-4)^2 + (1-2.75)^2} = 4.36$	C_2

New Centroid after 2nd iteration

$C_1 = -2.75, -4.25$

$C_2 = 4, 2.75$

Soln-3:

$$\begin{tikzcd} q_0 \arrow[r] \arrow[d] & q_1 \arrow[d] \\ y_0 & y_1 \end{tikzcd}$$

$(y_0 = A)$ $\qquad$ $(y_1 = B)$.

$$\pi = p(q_0) = \begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\begin{bmatrix} 1/3 & 2/3 \end{bmatrix}.$$

$$\alpha^T = p(q_t \mid q_{t-1}) = \begin{array}{cc|c} 1 & 2 & \\ \hline \end{array}$$
$$\begin{array}{c} \\ \end{array}\begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix}\begin{array}{c} 1 \\ 2 \end{array}$$

$$\eta^T = p(y_t \mid q_t) = \begin{array}{cc} 1 & 2 \end{array}$$
$$\begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix}\begin{array}{c} A \\ B \end{array}$$

JTA Junction (Initialization):

$$\begin{tikzcd} (q_0, y_0) \arrow[r, dash] & \boxed{q_0} \arrow[r, dash] & (q_0, q_1) \arrow[r, dash] & \boxed{q_1} \arrow[r, dash] & (q_1, y_1) \end{tikzcd}$$

(q_0, y_0), $y_0 = A \ldots B$:

	A	B
$q_0 = 1$	1/12	1/4
$q_0 = 2$	1/2	1/6

q_0, $q_0 = 1 \ldots 2.$:

1	1

(q_0, q_1), $q_1 = 1 \ldots 2.$:

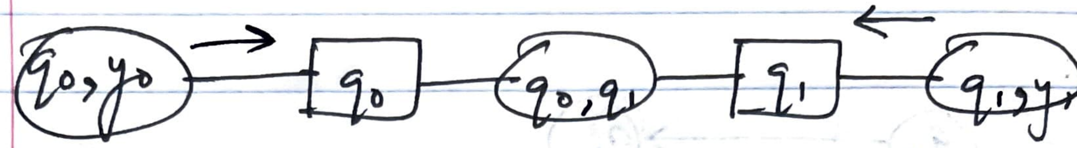
	$q_1 = 1$	$q_1 = 2$
$q_0 = 1$	1/8	7/8
$q_0 = 2$	1/2	1/2

q_1, $q_1 = 1 \ldots 2$:

1	1

(q_1, y_1), $y_1 = A \ldots B$:

	A	B
$q_1 = 1$	1/4	3/4
$q_1 = 2$	3/4	1/4



	$y_0 = A \ldots B$	
$q_0=1$	1/12	
$q_0=2$	1/2	

$q_0 = 1 \ldots 2$	
$\frac{1}{12}$	1/12

$q_1 = 1 \ldots 2$		
1/8	7/8	$q_0=1$
1/2	1/2	$q_0=2$

$y_1 = A \ldots B$	
3/4	1/4

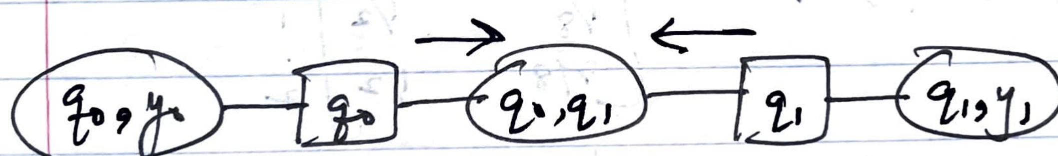
$q_1=1$, $q_1=2$

~~$\phi(q_0)$~~

...

3	

$$\phi^*(q_0) = \sum_B \varphi(y_0/q_0) \qquad \phi^*(q_1) = \sum_A \varphi(y_1/q_1)$$



	$y_0 = A \ldots B$
$q_0=1$	1/12
$q_0=2$	1/2

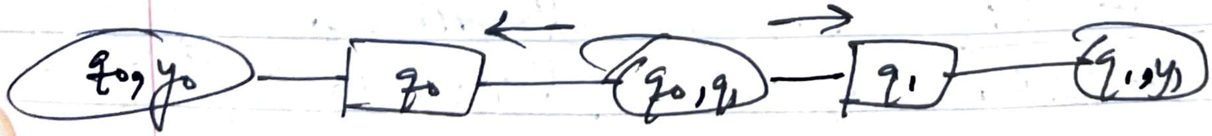
$q_0 = 1 \ldots 2$	
1/12	1/2

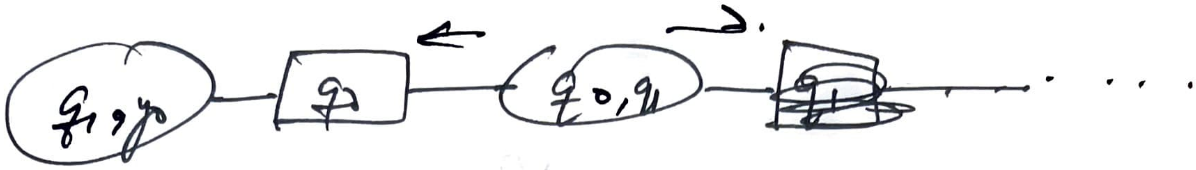
$q_1 = 1 \ldots 2$		
1/128	7/384	$q_0=1$
3/16	1/16	$q_0=2$

$q_1 = 1 \ldots 2$	
3/4	1/4

$y_1 = A \ldots B$	
3/4	$q_1=1$
1/4	$q_1=2$

$$\psi^*(q_1/q_0) = \frac{\phi^*(q_0)}{\phi(q_0)} \varphi(q_1/q_0) \frac{\phi^*(q_1)}{\phi(q_1)}.$$





$q_0 = 1$	$1/12$
$q_0 = 2$	$1/2$

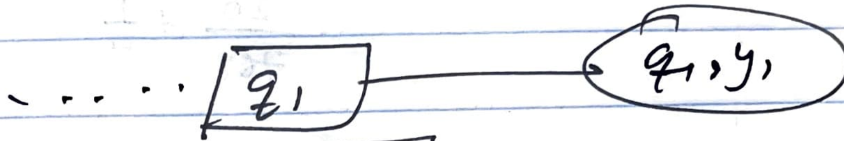
$y_0 = A \ldots B$

$\frac{20}{768}$	$\frac{1}{4}$

$q_1 = 1 \ldots 2$

$\frac{1}{128}$	$\frac{7}{384}$	$q_0 = 1$
$3/16$	$1/16$	$q_0 = 2$	....

$q_1 = 1 \ldots 2$



$\frac{25}{128}$	$\frac{31}{384}$

$q_1 = 1 \ldots 2$

$3/4$	$q_1 = 1$
$1/4$	$q_1 = 2$

$y_1 = A \ldots B$.

$$\phi^{**}(q_1) = \sum_{q_1} \psi^{*}(q_1 / q_1) \quad , \quad \phi^{**}(q_1)$$

$$= \sum_{q_0} \psi^{*}(q_0 / q_0)$$

$$\therefore P(y) = \left(\frac{20}{768} + \frac{1}{4} \right) = \left(\frac{1}{128} + \frac{7}{384} + \frac{3}{16} + \frac{1}{16} \right)$$

$$= \left(\frac{25}{128} + \frac{31}{384} \right)$$

$$\boxed{P(y) = \frac{212}{768}}$$

$$P(q_0 = 1/y) = \frac{\frac{20}{768}}{\frac{20}{768} + \frac{1}{4}} = \frac{20}{212} = \frac{5}{53}$$

$$P(q_0 = 2/y) = \frac{\frac{1}{4}}{\frac{20}{768} + \frac{1}{4}} = \frac{192}{212} = \frac{48}{53}$$

$$P(q_1 = 1/y) = \frac{\frac{25}{128}}{\frac{25}{128} + \frac{31}{384}} = \frac{150}{212} = \frac{75}{106}$$

$$P(q_1 = 2/y) = \frac{\frac{31}{384}}{\frac{25}{128} + \frac{31}{384}} = \frac{62}{212} = \underline{\frac{31}{106}},$$

Solution - 2.

a) False.

b) False

c) False.

d) True

e) ~~False~~ True

f) False.

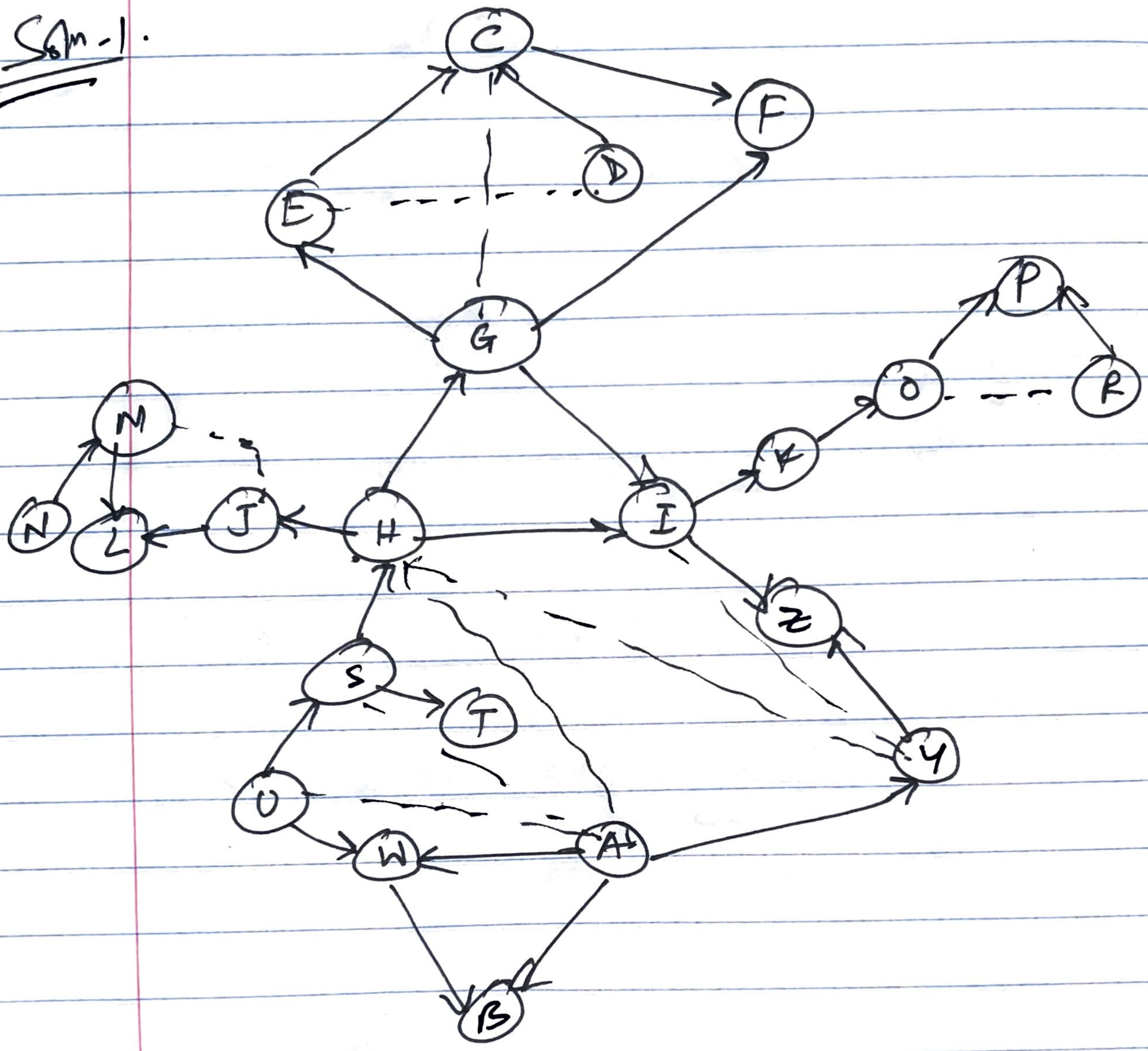
g) True.

h) False.

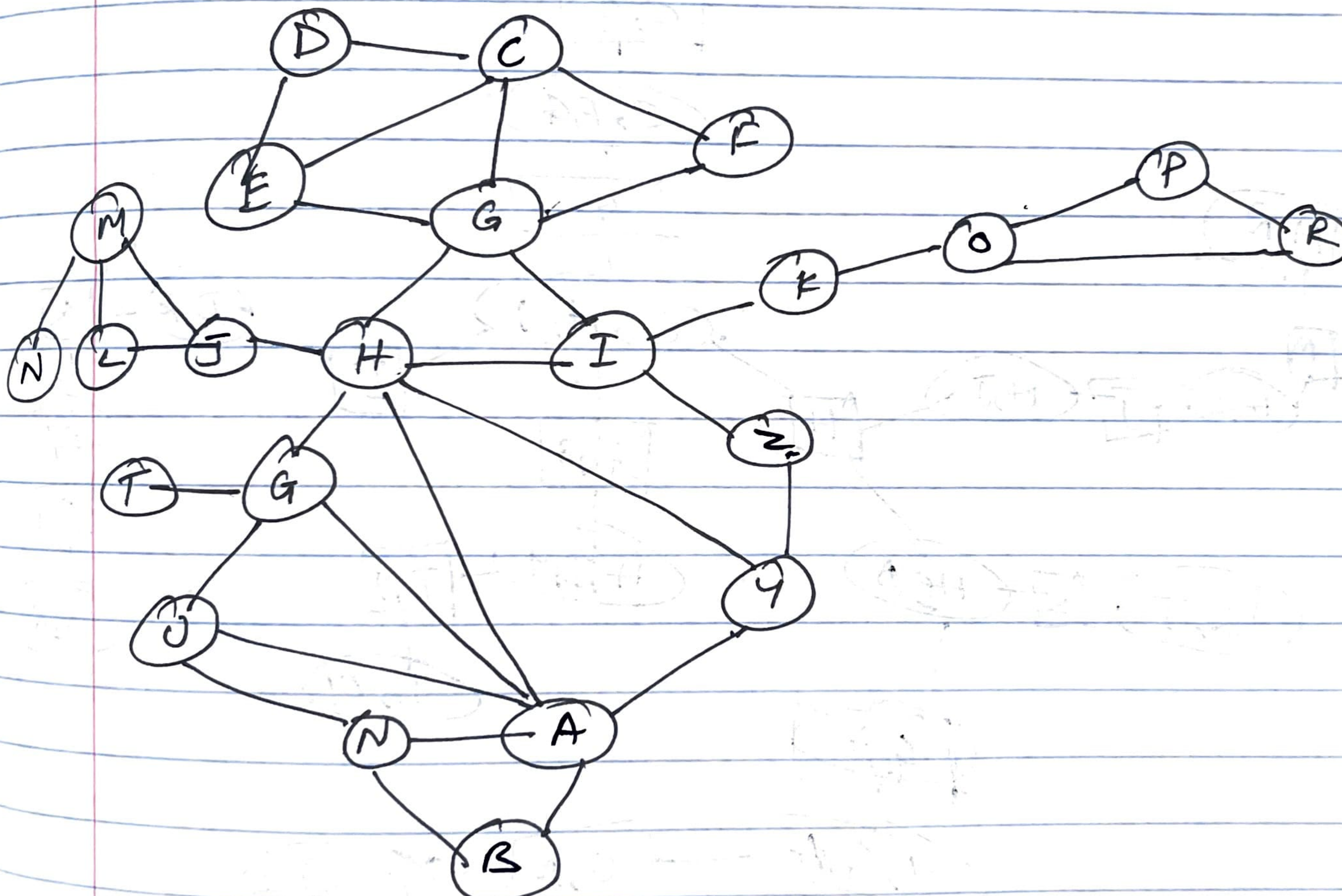
i) False.

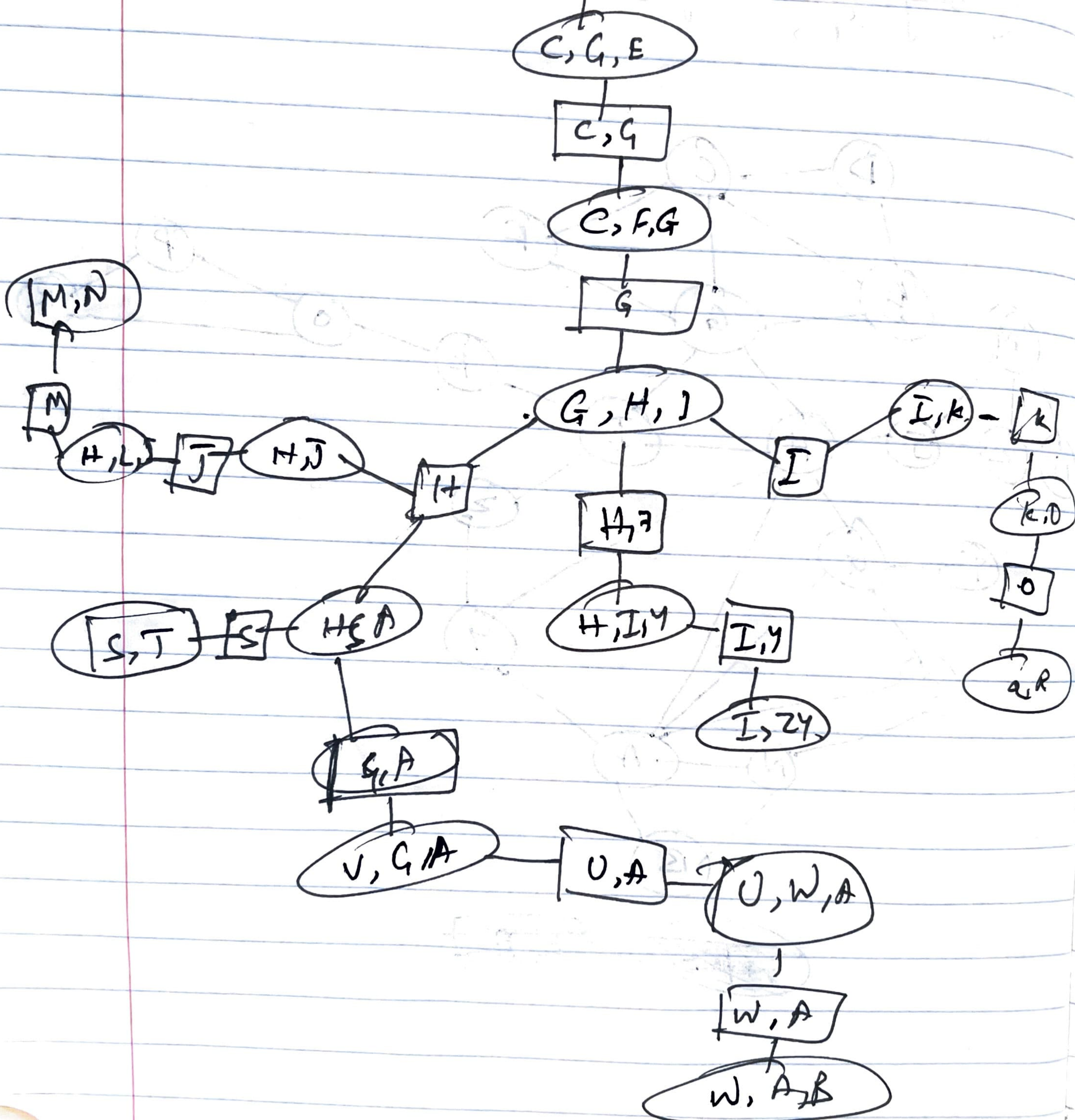
j) False.

Sam-1.



Creating moral graphs.
(joining parents of common children).





C, G, E
C, G
C, F, G
G
G, H, I
M, N
M
H, L
J
H, J
H
I, K
K
I
H, 7
K, O
O
S, T
S
H, S, A
H, I, Y
I, Y
Q, R
I, ZY
G, A
V, G, A
U, A
U, W, A
W, A
W, A, B

Sol. 6. Fragment of the Conv^n arch:

→ Input Image : $1 \times x \times y$.

Convolution layer:

$\underbrace{1 \longrightarrow 8}_{\text{no. of i/p \& o/p channels}}$, $\underbrace{2 \times 2}_{\text{filter size}}$, $\underbrace{3 \times 3}_{\text{Stride}}$

→ ReLU

→ Max pooling: $\underbrace{3 \times 3}_{\text{region size}}$, $\underbrace{2 \times 2}_{\text{stride}}$

→ Conv layer : $8 \longrightarrow 10$, 2×2, 2×2.

→ ReLU

→ Maxpooling : 3×3, 2×2

→ Flatting (3D to 1D) :

$\underbrace{10}_{\text{no. of feature maps}} \times \underbrace{12 \times 9}_{\text{size of feature map (12x9)}}$ → 1080

$2 \times 10 = 20$

$20 + 2 = 22$

$22 \times 2 = 44$

$44 - 3 + 2 = 41$.

$x \times y = \underline{41 \times 41}. =$ Input size