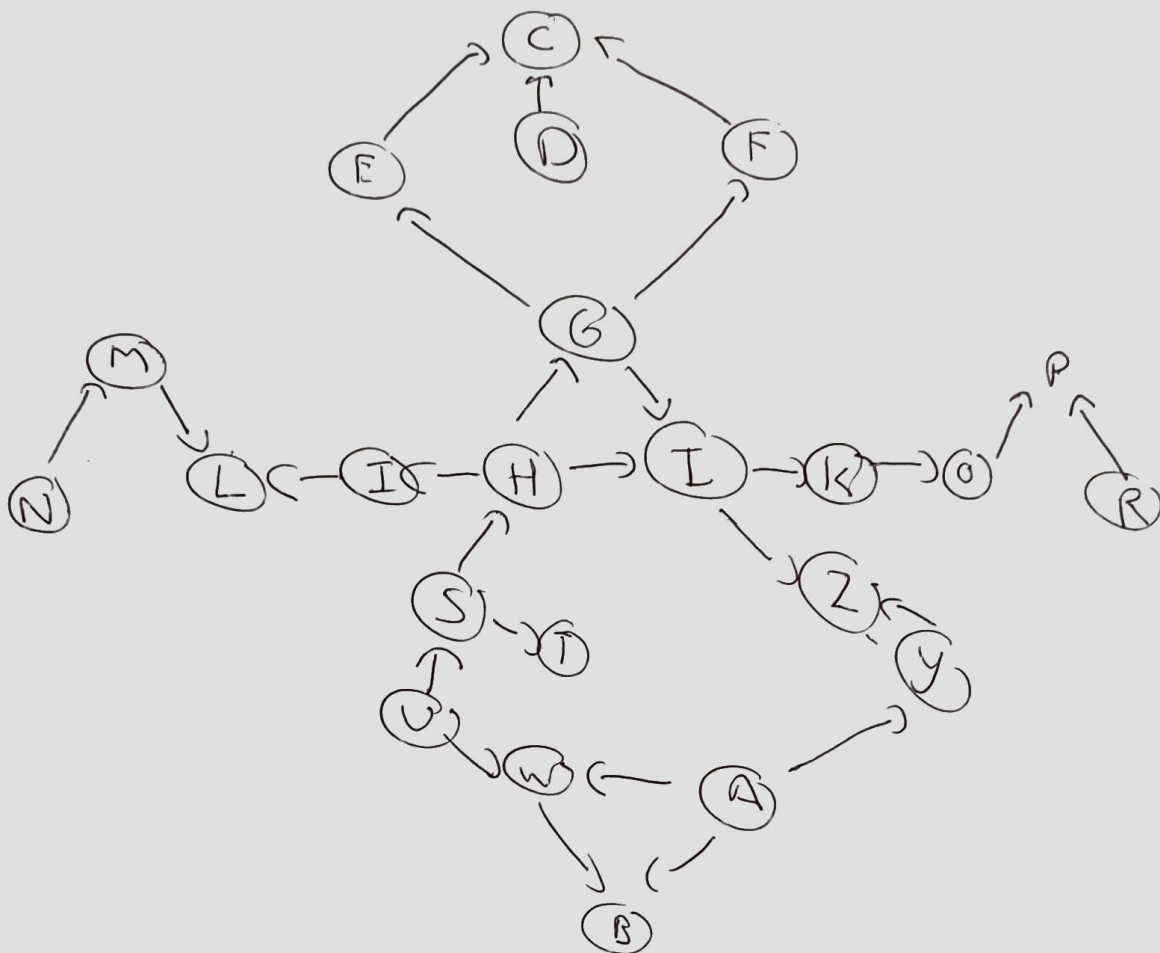


Question ①

1cv2154

①

=> original tree



(۲)

A handwritten graph diagram on a grid background. The graph consists of nodes represented by circles containing letters or numbers, connected by straight lines. The nodes are arranged in a roughly rectangular shape with a central horizontal path. The nodes are labeled as follows: A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. The graph is connected and contains several cycles. The central horizontal path consists of nodes J, H, I, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z. There are additional nodes connected to this path: A is connected to J; B is connected to J; C is connected to J; D is connected to J; E is connected to J; F is connected to J; G is connected to J; H is connected to J; I is connected to J; K is connected to J; L is connected to J; M is connected to J; N is connected to J; O is connected to J; P is connected to J; Q is connected to J; R is connected to J; S is connected to J; T is connected to J; U is connected to J; V is connected to J; W is connected to J; X is connected to J; Y is connected to J; Z is connected to J. There are also nodes connected to the central path: A is connected to J; B is connected to J; C is connected to J; D is connected to J; E is connected to J; F is connected to J; G is connected to J; H is connected to J; I is connected to J; K is connected to J; L is connected to J; M is connected to J; N is connected to J; O is connected to J; P is connected to J; Q is connected to J; R is connected to J; S is connected to J; T is connected to J; U is connected to J; V is connected to J; W is connected to J; X is connected to J; Y is connected to J; Z is connected to J.

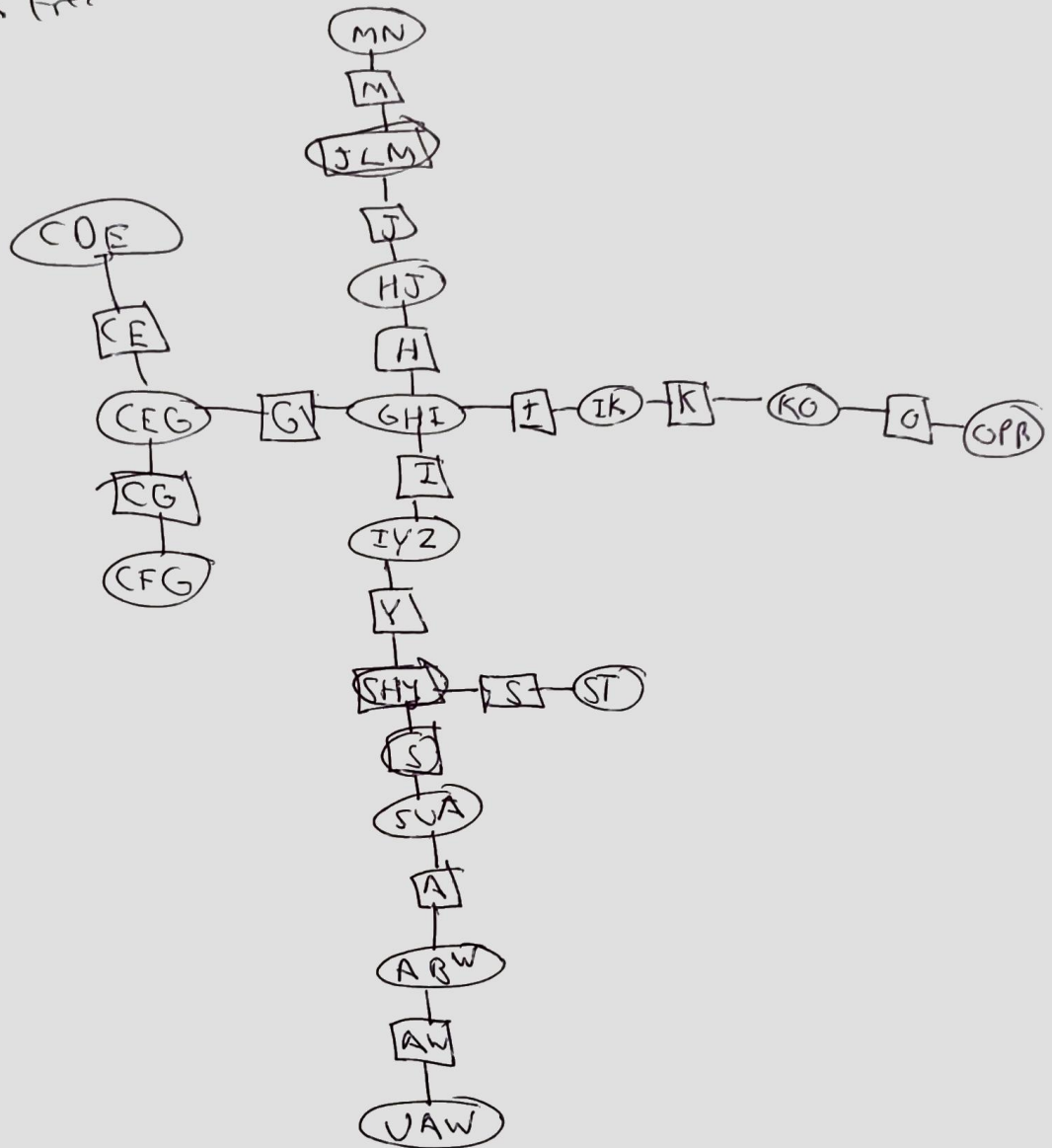
C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z, AA, AB, AC, AD, AE, AF, AG, AH, AI, AJ, AK, AL, AM, AN, AO, AP, AQ, AR, AS, AT, AU, AV, AW, AX, AY, AZ, BA, BB, BC, BD, BE, BF, BG, BH, BI, BJ, BK, BL, BM, BN, BO, BP, BQ, BR, BS, BT, BU, BV, BW, BX, BY, BZ, CA, CB, CC, CD, CE, CF, CG, CH, CI, CJ, CK, CL, CM, CN, CO, CP, CQ, CR, CS, CT, CU, CV, CW, CX, CY, CZ, DA, DB, DC, DD, DE, DF, DG, DH, DI, DJ, DK, DL, DM, DN, DO, DP, DQ, DR, DS, DT, DU, DV, DW, DX, DY, DZ, EA, EB, EC, ED, EE, EF, EG, EH, EI, EJ, EK, EL, EM, EN, EO, EP, EQ, ER, ES, ET, EU, EV, EW, EX, EY, EZ, FA, FB, FC, FD, FE, FF, FG, FH, FI, FJ, FK, FL, FM, FN, FO, FP, FQ, FR, FS, FT, FU, FV, FW, FX, FY, FZ, GA, GB, GC, GD, GE, GF, GG, GH, GI, GJ, GK, GL, GM, GN, GO, GP, GQ, GR, GS, GT, GU, GV, GW, GX, GY, GZ, HA, HB, HC, HD, HE, HF, HG, HH, HI, HJ, HK, HL, HM, HN, HO, HP, HQ, HR, HS, HT, HU, HV, HW, HX, HY, HZ, IA, IB, IC, ID, IE, IF, IG, IH, II, IJ, IK, IL, IM, IN, IO, IP, IQ, IR, IS, IT, IU, IV, IW, IX, IY, IZ, JA, JB, JC, JD, JE, JF, JG, JH, JI, JJ, JK, JL, JM, JN, JO, JP, JQ, JR, JS, JT, JU, JV, JW, JX, JY, JZ, KA, KB, KC, KD, KE, KF, KG, KH, KI, KJ, KL, KM, KN, KO, KP, KQ, KR, KS, KT, KU, KV, KW, KX, KY, KZ, LA, LB, LC, LD, LE, LF, LG, LH, LI, LJ, LK, LL, LM, LN, LO, LP, LQ, LR, LS, LT, LU, LV, LW, LX, LY, LZ, MA, MB, MC, MD, ME, MF, MG, MH, MI, MJ, MK, ML, MM, MN, MO, MP, MQ, MR, MS, MT, MU, MV, MW, MX, MY, MZ, NA, NB, NC, ND, NE, NF, NG, NH, NI, NJ, NK, NL, NM, NN, NO, NP, NQ, NR, NS, NT, NU, NV, NW, NX, NY, NZ, OA, OB, OC, OD, OE, OF, OG, OH, OI, OJ, OK, OL, OM, ON, OO, OP, OQ, OR, OS, OT, OU, OV, OW, OX, OY, OZ, PA, PB, PC, PD, PE, PF, PG, PH, PI, PJ, PK, PL, PM, PN, PO, PP, PQ, PR, PS, PT, PU, PV, PW, PX, PY, PZ, QA, QB, QC, QD, QE, QF, QG, QH, QI, QJ, QK, QL, QM, QN, QO, QP, QQ, QR, QS, QT, QU, QV, QW, QX, QY, QZ, RA, RB, RC, RD, RE, RF, RG, RH, RI, RJ, RK, RL, RM, RN, RO, RP, RQ, RR, RS, RT, RU, RV, RW, RX, RY, RZ, SA, SB, SC, SD, SE, SF, SG, SH, SI, SJ, SK, SL, SM, SN, SO, SP, SQ, SR, SS, ST, SU, SV, SW, SX, SY, SZ, TA, TB, TC, TD, TE, TF, TG, TH, TI, TJ, TK, TL, TM, TN, TO, TP, TQ, TR, TS, TT, TU, TV, TW, TX, TY, TZ, UA, UB, UC, UD, UE, UF, UG, UH, UI, UJ, UK, UL, UM, UN, UO, UP, UQ, UR, US, UT, UY, UV, UW, UX, UZ, VA, VB, VC, VD, VE, VF, VG, VH, VI, VJ, VK, VL, VM, VN, VO, VP, VQ, VR, VS, VT, VU, VV, VW, VX, VY, VZ, WA, WB, WC, WD, WE, WF, WG, WH, WI, WJ, WK, WL, WM, WN, WO, WP, WQ, WR, WS, WT, WU, WV, WW, WX, WY, WZ, XA, XB, XC, XD, XE, XF, XG, XH, XI, XJ, XK, XL, XM, XN, XO, XP, XQ, XR, XS, XT, XU, XV, XW, XX, XY, XZ, YA, YB, YC, YD, YE, YF, YG, YH, YI, YJ, YK, YL, YM, YN, YO, YP, YQ, YR, YS, YT, YU, YV, YW, YX, YY, YZ, ZA, ZB, ZC, ZD, ZE, ZF, ZG, ZH, ZI, ZJ, ZK, ZL, ZM, ZN, ZO, ZP, ZQ, ZR, ZS, ZT, ZU, ZV, ZW, ZX, ZY, ZZ.

Question (1)

KV21541

(4)

⇒ Junction tree

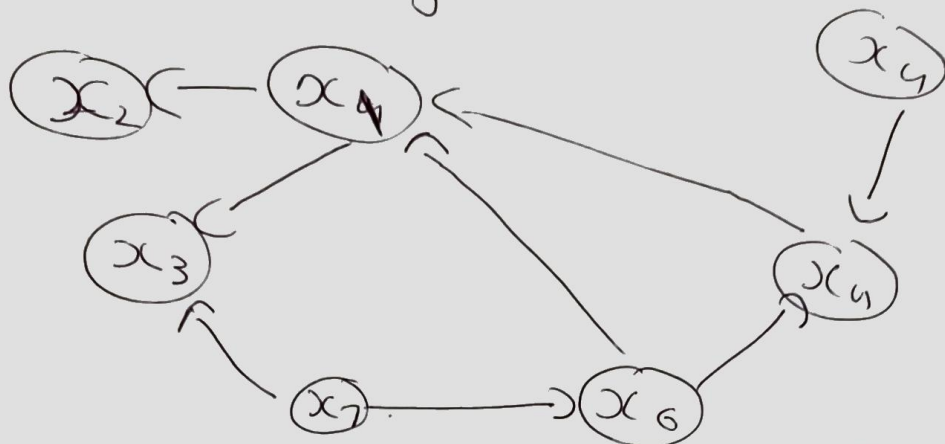


Question (2)

16/2154

1

Given the bayesian Network



$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \prod_{i=1}^n P(x_i | \pi_i)$$

$$= \prod_{i=1}^n P(x_i | \pi_i) = \prod_{i=1}^n P(x_i | \pi_i)$$

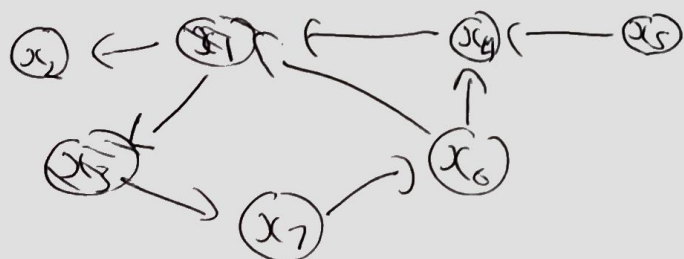
$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = P(x_1 | x_4, x_6) \cdot P(x_2 | x_1) \cdot P(x_3 | x_1, x_7) \cdot P(x_4 | x_5, x_6) \cdot P(x_5) \cdot P(x_6 | x_7) P(x_7)$$

(a) x_2 and x_6 are independent - False



x_1 is not shaded and ball passes through x_6

(b) x_2 and x_6 are conditionally independent given x_1, x_3 and x_5 - True



$\rightarrow x_6 \rightarrow x_1 \rightarrow x_2$
 $\rightarrow x_6 \rightarrow x_4 \rightarrow x_1 \rightarrow x_2$
 $\rightarrow x_6 \rightarrow x_7 \rightarrow x_3 \rightarrow x_1 \rightarrow x_2$

Question (1)

KV21541

KV21541

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→ When chosen $(x_6 \rightarrow x_1 \rightarrow x_2)$

Ball passes from x_1

→ When chosen $(x_6 \rightarrow x_4 \rightarrow x_1 \rightarrow x_2)$

Ball moves from x_4 to x_2 passing through x_1

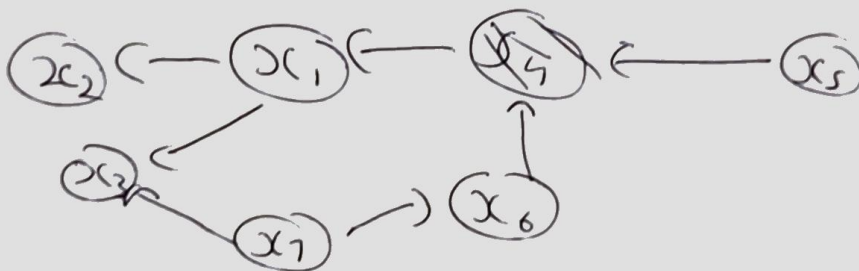
→ When chosen $(x_6 \rightarrow x_7 \rightarrow x_3 \rightarrow x_1 \rightarrow x_2)$

x_7 to x_1 from x_3

x_6 to x_7 as x_3 , x_7 and x_6 and middle node is not

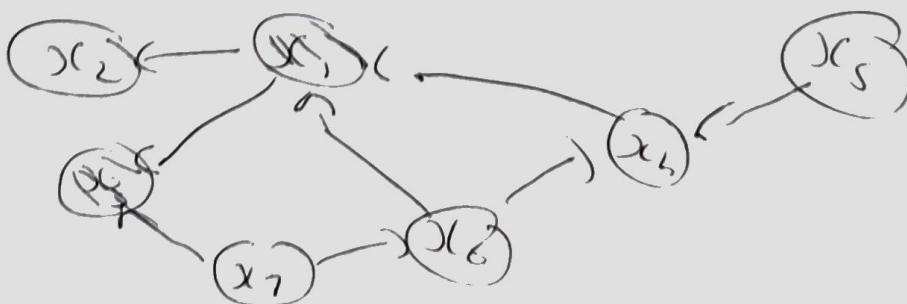
Shaded.

(C) x_1 and x_7 are conditionally independent given x_4 - **False**



→ x_7 , x_6 and x_1 forms Markov chain and middle node is not shaded here.

(D) x_5 and x_2 are conditionally independent given x_1 and x_3 - **True**



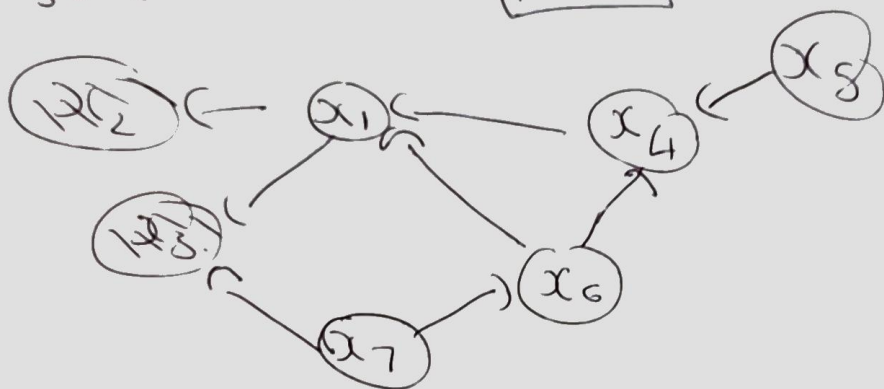
Question 2

KV2154

3

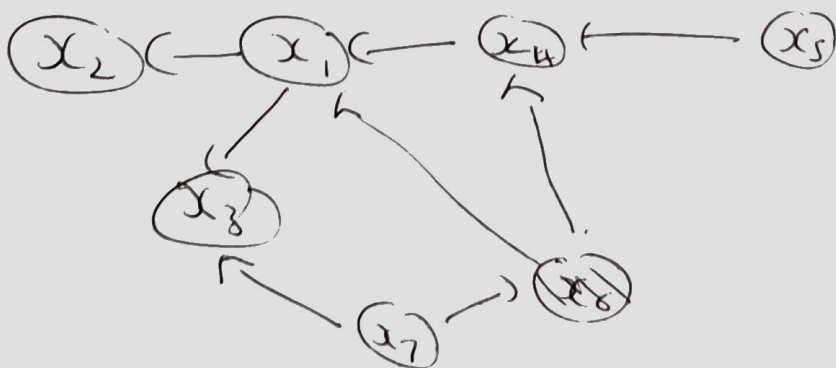
- $X_4 \rightarrow X_1 \rightarrow X_2$ Markov chain with middle node shaded
- $X_5 \rightarrow X_4 \rightarrow X_6$ middle node not shaded
- Ball does not pass $X_5 \rightarrow X_4 \rightarrow X_6 \rightarrow X_7 \rightarrow X_3 \rightarrow X_1 \rightarrow X_2$

(E) X_5 and X_1 are conditionally independent given X_3, X_2 and X_4 - **False**



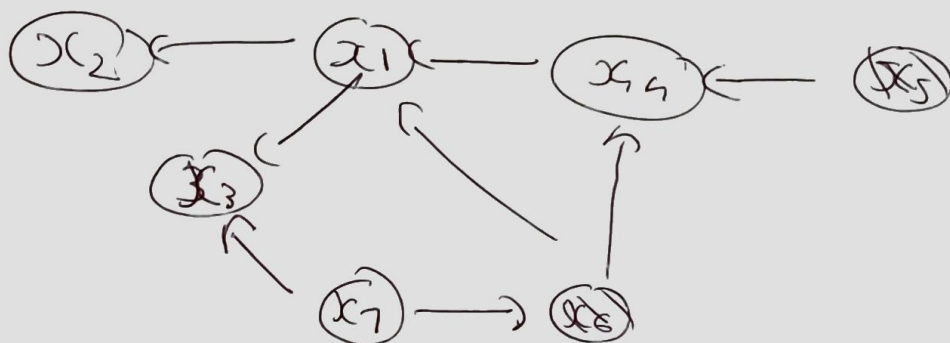
- $X_5 \rightarrow X_6$ Passes
- $X_6 \rightarrow X_1$ Passes and middle node not shaded

(F) X_4 and X_3 are conditionally independent given X_6 - **False**



- X_4 to X_3 from X_1 forms Markov chain with middle node shaded

(G) X_2 and X_7 are conditionally independent given X_5 and X_6 - **True**

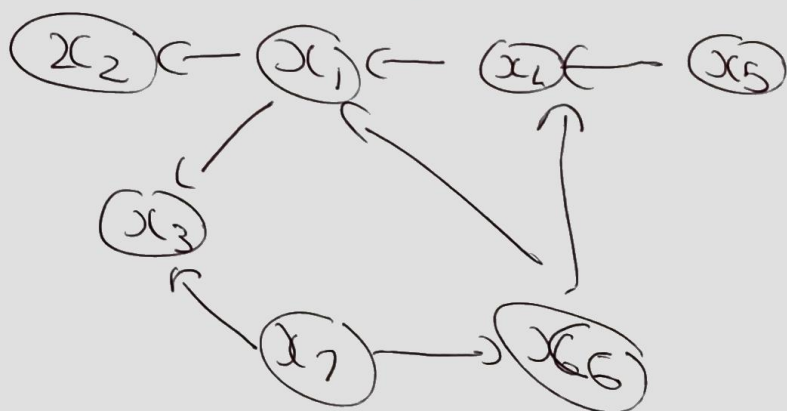


$\rightarrow X_5, X_6$: shaded

\rightarrow Doesn't pass $X_2 \rightarrow X_1 \rightarrow X_3 \rightarrow X_7$
middle node not shaded.

$\rightarrow X_2 \rightarrow X_6 \rightarrow X_1$ form Markov chain and middle node shaded

(H) X_5 and X_3 are conditionally independent given X_6 and X_7 - **False**



$\rightarrow X_5 \rightarrow X_4 \rightarrow X_1 \rightarrow X_3$ form 2 Markov chains

Question (2)

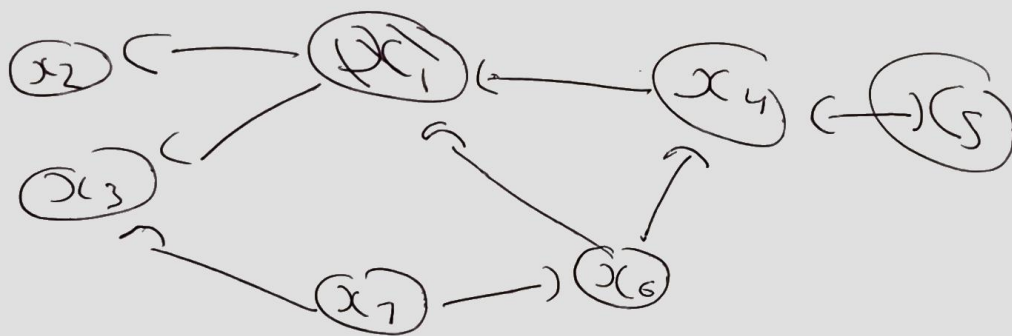
KV2154

5

(I) X_5 and X_2 are independent - False

→ No nodes shaded

(J) X_2 and X_4 are conditionally independent given X_1 - True



→ $X_2 \rightarrow X_1 \rightarrow X_4$ Markov chain with middle node shaded

→ $X_2 \rightarrow X_1 \rightarrow X_3$ - Does not pass

→ $X_2 \rightarrow X_1 \rightarrow X_6 \rightarrow X_4$ - Does not pass

→ $X_2 \rightarrow X_1 \rightarrow X_6$ - Markov chain with middle node shaded

Question 3)

1KV2154

1

Given,

$$\pi = P(q_0) = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$$

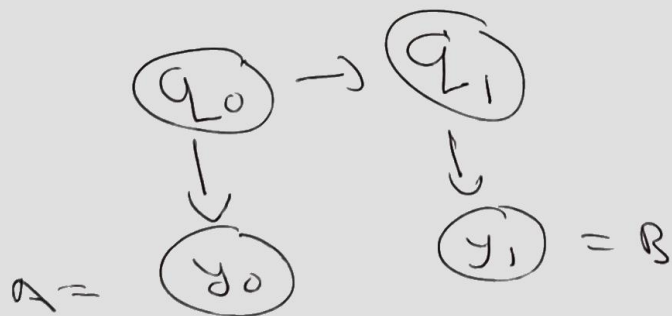
$$a^T = P(q_t | q_{t-1}) = \begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix}$$

$$b^T = P(y_t | q_t) = \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix}$$

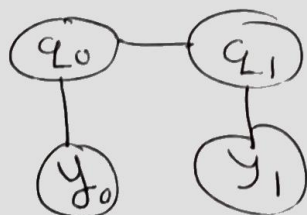
To find,

- (i) likelihood of $P(y)$ using HMM
- (ii) Individual marginals of States where $P(q_0 | y)$ and $P(q_1 | y)$

Step 1:- 2 state HMM



Step 2:- Converting to junction tree



Question (3)

[KV2154]

(1)

Given,

$$\pi = P(q_0) = \begin{bmatrix} 1/3 & 2/3 \end{bmatrix}$$

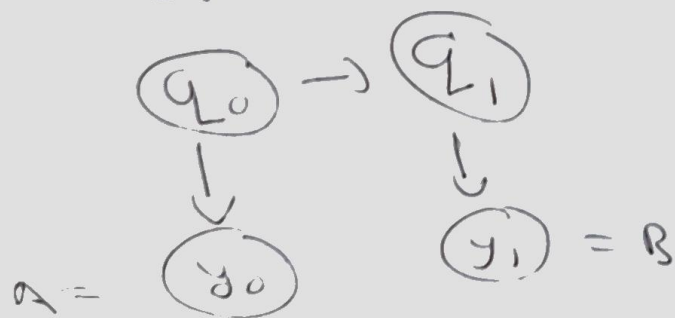
$$a^T = P(q_t | q_{t-1}) = \begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix}$$

$$n^T = P(y_t | q_t) = \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix}$$

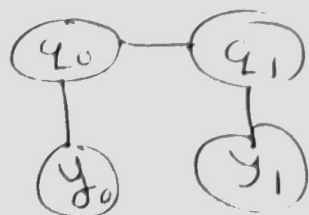
To find,

- (i) likelihood of $P(y)$ using HMM
- (ii) Individual marginals of states where $P(q_0 | y)$ and $P(q_1 | y)$

Step 1: 2 state HMM



Step 2: - Converting to junction tree

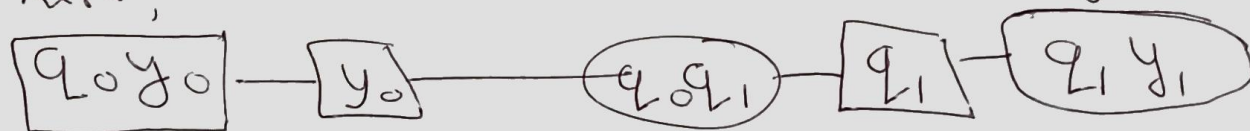


Question ③

KV2154

2)

Implementing moralization and triangulation here,



$q_0 = 1$
 $q_1 = 2$

$1/2$	$1/4$
$1/2$	$1/6$



$q_1 = 1$ 2
 $q_0 = 1$
 $q_1 = 2$

$1/8$	$1/8$
$1/2$	$1/2$

1 2

1	1
---	---



$y_1 =$ A B

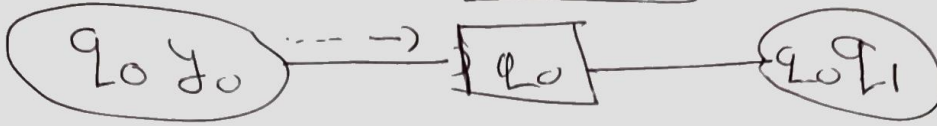
$q_0 = 1$
 $q_1 = 2$

$1/4$	$3/4$
$3/4$	$1/4$

Question 8

kv2154

8



y_0 A B

$q_0 = 1$	$1/12$	
$q_0 = 2$	$1/2$	

q_0 1 2

1	1
---	---

$q_1 = 1 \dots 2$

$1/8$	$1/8$
$1/2$	$1/2$



1	1
---	---

$q_1 = 1$
 $q_2 = 2$

A	B
	$3/4$
	$1/4$



$q_0 = 1$
 $q_0 = 2$

$1/2$	
$1/2$	

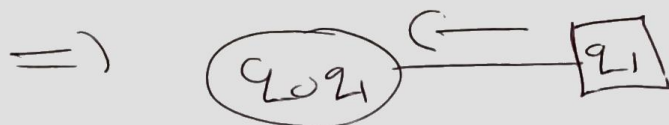
$q_0 = 1 \dots 2$

1	1
---	---

Question 3

KU2154

4



1/8	7/8
1/2	1/2

$q_1 = 1, \dots, 2$

1	1
---	---

\Rightarrow (q, \dots, y_1)

$q_1 = 1$
 $q_1 = 2$

A	B
	3/4
	1/4

$\Rightarrow q_0 q_1$

$q_0 = 1$
 $q_0 = 2$

1	2
1/8	7/8
1/2	1/2

q_1

1	2
3/4	1/4

$\Rightarrow (q, y_1)$

$q_1 = 1$
 $q_1 = 2$

A	B
	3/4
	1/4

Question (8)

KV2154

5

o.o like likelihood $P(y)$

$$\begin{bmatrix} \frac{1}{12} \times \frac{1}{8} & , & \frac{1}{12} \times \frac{7}{8} \\ \frac{1}{2} \times \frac{1}{2} & , & \frac{1}{2} \times \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1/128 & 7/384 \\ 3/16 & 1/16 \end{bmatrix}$$

$$\text{o.o } P(y) = \left[\frac{1}{128} + \frac{7}{384} + \frac{3}{16} + \frac{1}{16} \right]$$

$$P(y) = \frac{53}{192}$$

Question (4)

KV2154

1

To solve k-means clustering algorithm.

Given that,

$$C_1 = (-4, -5), \quad C_2 = (5, 4)$$

Given 2-D Dataset

$$A(-3, -1), B(-1, -3), C(-2, -6), D(-5, -7), E(3, 1) \\ F(2, 3), G(3, 6), H(8, 1)$$

\Rightarrow Initializing value of centroids

$$C_1 = (-4, -5), \quad C_2 = (5, 4)$$

\Rightarrow Objects - centroid distance.

In this step, we calculate the euclidean distance between each object of the 2D dataset and the centroid.

Iteration 0: - when using $C_1: (-4, -5)$

$$A = \sqrt{(-3+4)^2 + (-1+5)^2} = 4.12$$

$$B = \sqrt{(-1+4)^2 + (-3+5)^2} = 3.60$$

$$C = \sqrt{(-2+4)^2 + (-6+5)^2} = 2.23$$

$$D = \sqrt{(-5+4)^2 + (-7+5)^2} = 2.23$$

$$E = \sqrt{(3+4)^2 + (1+5)^2} = 9.21$$

$$F = \sqrt{(2+4)^2 + (3+5)^2} = 10$$

Question (4)

[Kv2154]

(2)

$$G = \sqrt{(3+4)^2 + (6+5)^2} = 13.03$$

$$H = \sqrt{(8+4)^2 + (1+5)^2} = 13.41$$

When $C_2 = (5, 4)$

$$A = \sqrt{(-3-5)^2 + (-1-4)^2} = 9.43$$

$$B = \sqrt{(-1-5)^2 + (-3-4)^2} = 9.21$$

$$C = \sqrt{(-2-5)^2 + (-6-4)^2} = 12.20$$

$$D = \sqrt{(-5-5)^2 + (-7-5)^2} = 15.62$$

$$E = \sqrt{(3-5)^2 + (1-4)^2} = 3.60$$

$$F = \sqrt{(2-5)^2 + (3-4)^2} = 3.16$$

$$G = \sqrt{(3-5)^2 + (6-4)^2} = 2.82$$

$$H = \sqrt{(8-5)^2 + (1-4)^2} = 4.24$$

Distance matrix can be given as.

$$D_0 = \begin{bmatrix} & A & B & C & D & E & F & G & H \\ A & & & & & & & & \\ B & 4.12 & & & & & & & \\ C & 3.60 & 2.23 & & & & & & \\ D & 2.23 & 2.23 & 9.21 & & & & & \\ E & 9.43 & 9.21 & 12.20 & 15.62 & & & & \\ F & & & & 3.20 & 3.16 & 2.82 & 4.24 \\ G & & & & & & & & \\ H & & & & & & & & \end{bmatrix} \begin{matrix} \\ \\ C_1 \\ C_2 \\ \\ \\ \\ \end{matrix}$$

Question (4)

KV2154

(3)

\Rightarrow Step 3:- Object clustering
We have to assign each object basing upon the minimum distance

Turn

$$G^0 = \begin{bmatrix} A & B & C & D & E & F & G & H \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{matrix} \rightarrow \text{group 1} \\ \rightarrow \text{group 2} \end{matrix}$$

\Rightarrow Step 4:- Updating Centroids

$$C_1 = \left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4} \right) = \left(-\frac{11}{4}, -\frac{17}{4} \right) = (-2.75, -4.25)$$

$$C_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right) = \left(4, \frac{11}{4} \right) = (4, 2.75)$$

Iteration 1:- Using C_1

$$A = \sqrt{(-3+2.75)^2 + (-1+4.25)^2} = 3.25$$

$$B = \sqrt{(-1+2.75)^2 + (-3+4.25)^2} = 2.15$$

$$C = \sqrt{(-2+2.75)^2 + (-6+4.25)^2} = 1.90$$

$$D = \sqrt{(-5+2.75)^2 + (-7+4.25)^2} = 3.55$$

$$E = \sqrt{(3+2.75)^2 + (1+4.25)^2} = 7.78$$

$$F = \sqrt{(2+2.25)^2 + (3+4.25)^2} = 8.66 \quad \boxed{kv2159} \textcircled{4}$$

$$G = \sqrt{(3+2.75)^2 + (6+4.25)^2} = 11.75$$

$$H = \sqrt{(5+2.75)^2 + (4+4.25)^2} = 11.31$$

when using $C_2 = (4, 2.75)$

$$A = \sqrt{(-3-4)^2 + (-1-2.75)^2} = 7.94$$

$$B = \sqrt{(-1-4)^2 + (-3-2.75)^2} = 7.61$$

$$C = \sqrt{(-2-4)^2 + (-6-2.75)^2} = 10.60$$

$$D = \sqrt{(-5-4)^2 + (-7-2.75)^2} = 13.26$$

$$E = \sqrt{(3-4)^2 + (1-2.75)^2} = 2.01$$

$$F = \sqrt{(2-4)^2 + (3-2.75)^2} = 2.01$$

$$G = \sqrt{(3-4)^2 + (6-2.75)^2} = 3.40$$

$$H = \sqrt{(5-4)^2 + (4-2.75)^2} = 1.60$$

Distance matrix

	A	B	C	D	E	F	G	H
A								
B	3.25							
C	7.94	7.61						
D	10.60	13.26	2.01	2.01	3.40	1.60		
E								
F								
G								
H								

clustering

$$\begin{bmatrix} A & B & C & D & E & F & G & H \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \text{group 1}$$

\rightarrow group 1

\rightarrow group 2

Question (4)

KV2154

(5)

⇒ Updated Centroids

$$C_1 = \left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4} \right) = \left(\frac{-11}{4}, \frac{-17}{4} \right) \\ = (-2.75, -4.25)$$

$$C_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right) = (4, 2.75)$$

Question ⑧

KV2154

①

Jensen's Inequality :- To, for non-negative real numbers x_1, x_2, \dots, x_n

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

This is of the form that arithmetic mean greater than the geometric mean. As 'log' function is an increasing function we apply 'log' on both sides to proceed with Jensen's inequality.

$$\text{i.e. } \log\left(\frac{x_1 + x_2 + \dots + x_n}{n}\right) \geq \frac{1}{n} \log(x_1 \cdot x_2 \cdot \dots \cdot x_n)$$

$$= \frac{\log x_1 + \dots + \log x_n}{n}$$

$$\text{i.e. } \log\left(\frac{1}{n} \sum_{i=1}^n x_i\right) \geq \frac{1}{n} \log(x_1 \cdot \dots \cdot x_n)$$

$$\Rightarrow \log\left(\frac{1}{n} \sum_{i=1}^n x_i\right) \geq \frac{1}{n} \log\left(\prod_{i=1}^n x_i\right)$$

or,

$$\log\left(\frac{1}{n} \sum_{i=1}^n x_i\right) \geq \log\left(\left(\prod_{i=1}^n x_i\right)^{1/n}\right)$$

Then,

$$\frac{\sum_{i=1}^n x_i}{n} \geq \sqrt[n]{\prod_{i=1}^n x_i}$$

$$\therefore \frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}$$

Problem ③

KV2154

②

When we use $E[x]$ as mean of x_i 's.

Then,

$$\log(E[x]) \geq E[\log(x)]$$

Hence, it is proved using concavity of \log given by Jensen's inequality.

Problem 6

kv2154

1

⇒ Final layer size = $10 \times 12 \times 9$

Maxpooling with

region size = 3×3

Stride = 2×2

$$\frac{x-3}{2} + 1 = 12$$

$$x = 25$$

$$\frac{y-3}{2} + 1 = 9$$

$$y = 19$$

⇒ ReLU does not change the size

⇒ Convolutional layer : $8 \rightarrow 10, 2 \times 2, 2 \times 2$

$$\frac{x-2}{2} + 1 = 25$$

$$x = 50$$

$$\Rightarrow 8 \times 50 \times 38$$

$$\frac{y-1}{2} + 1 = 19$$

$$y = 38$$

⇒ Maxpooling with

region: 3×3 .

Stride: 2×2

$$\frac{x-3}{2} + 1 = 50$$

$$x = 101$$

$$\frac{y-3}{2} + 1 = 38$$

$$y = 77$$

$$\Rightarrow 8 \times 101 \times 77$$

⇒ ReLU does not change the size

Problem ⑦

[KV2154]

②

\Rightarrow Convolutional layer :- $1 \rightarrow 8, 2 \times 2, 3 \times 2$

~~Region~~

$$\frac{x-2}{3} + 1 = 101$$

$$x = 302$$

$$\frac{y-2}{2} + 1 = 77$$

$$y = 154$$

\therefore The size of the input is $1 \times 302 \times 154$

$$\therefore x = 302$$

$$y = 154$$