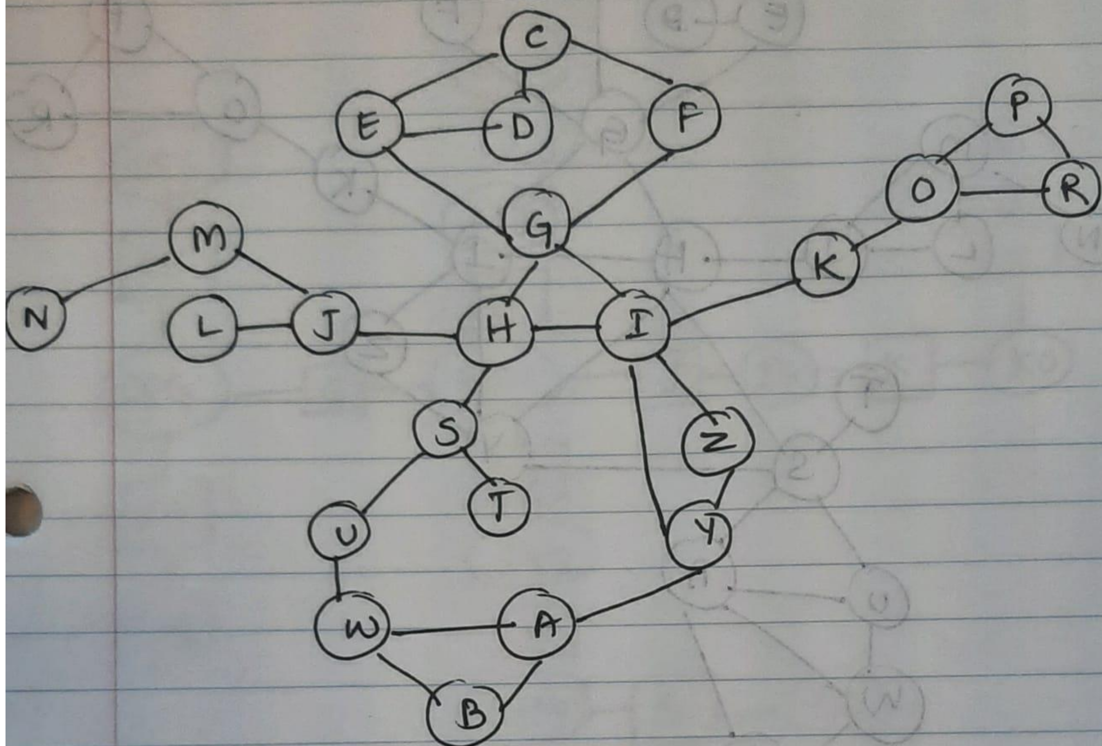
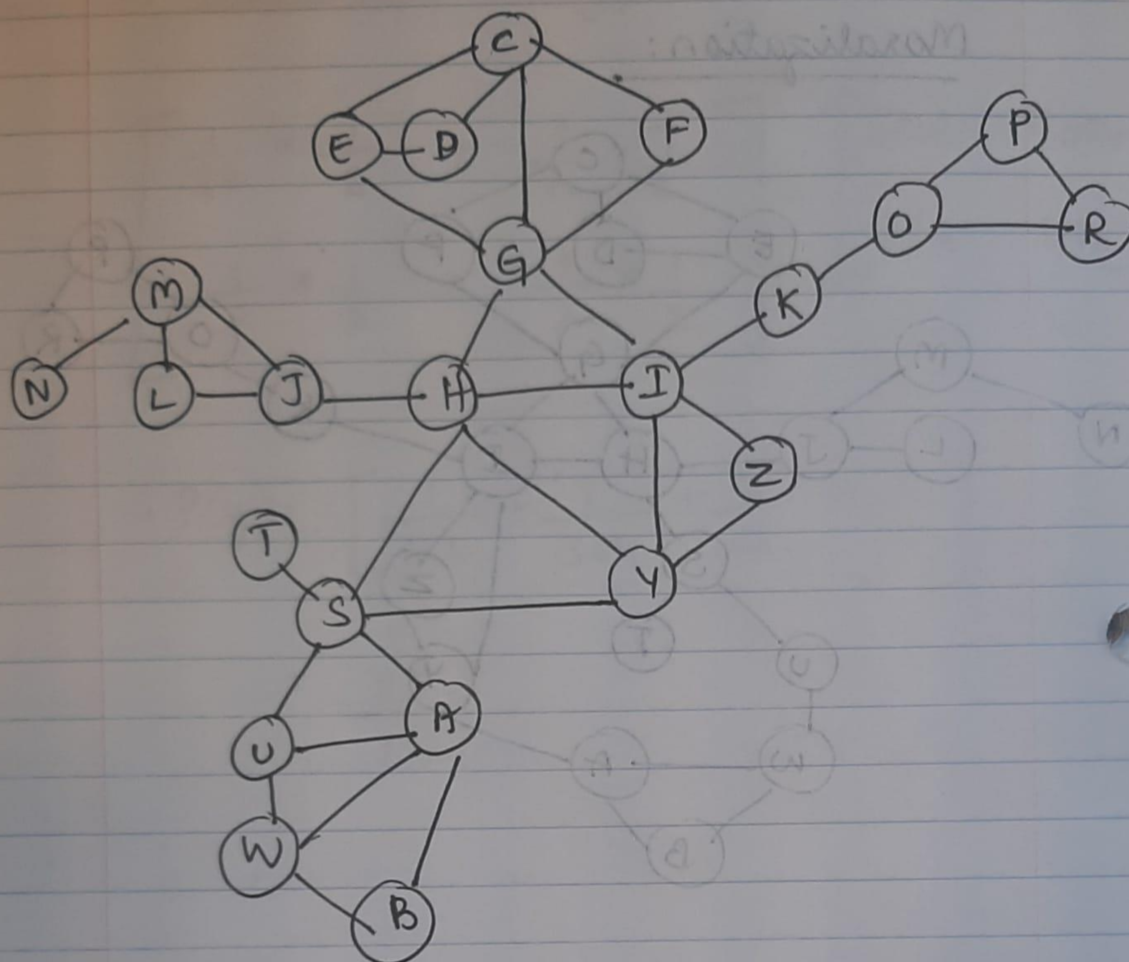
N19774260.

Question-1:

Moralization:

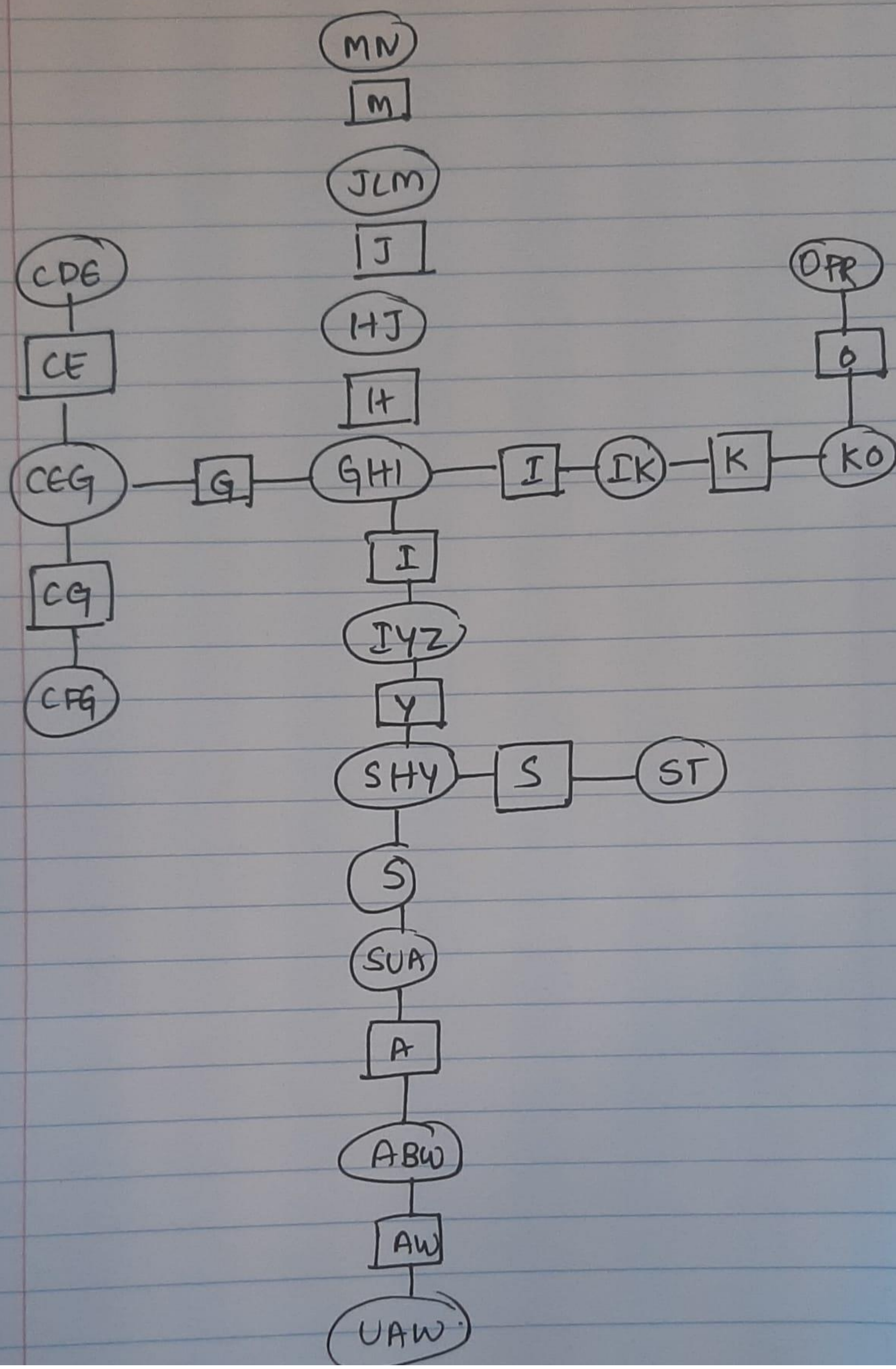


Triangulation:



Cliques: CDE, CEG, CFG, GHI, MN, MLJ, JH, IK, KO, OPR, IZY, SHY, ST, SUA, ABW, UAW.

Junction Tree:



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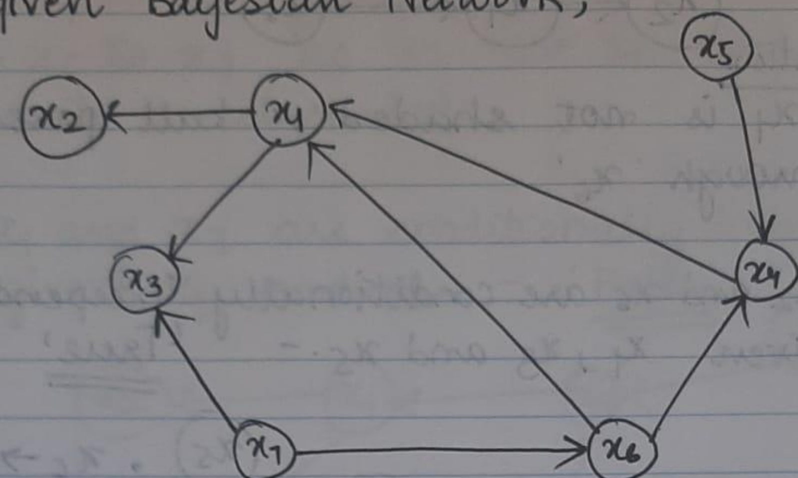
Question 2:

Given Bayesian Network,



Factorization of probability distribution can be given as,

$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = \prod_{i=1}^{n} P(x_i \mid Pa_i)$$

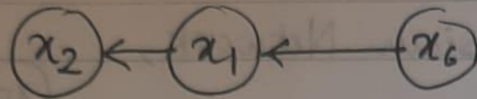
$$= \prod_{i=1}^{n} P(x_i \mid \pi_i) = \prod_{i=1}^{n} P(x_i \mid \pi_i)$$

$$P(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = P(x_1 \mid x_4 x_6) \cdot$$

$$P(x_2 \mid x_1)\, P(x_3 \mid x_1 x_7)\, P(x_4 \mid x_5 x_6)\, P(x_5)$$

$$P(x_6 \mid x_7)\, P(x_7)$$

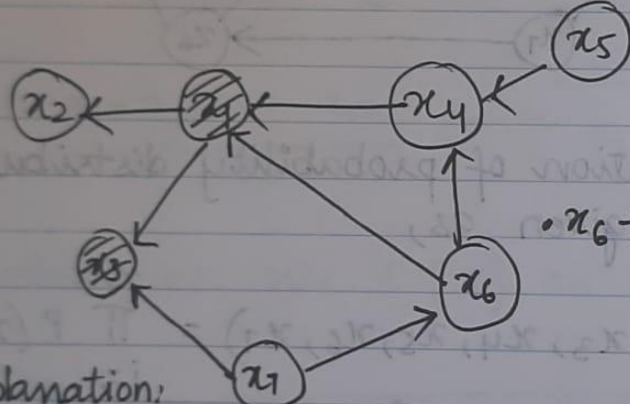
(a) x_2 and x_6 are independent - 'False'



Explanation:

x_1 is not shaded & ball passes through 'x_6'

(b) x_2 and x_6 are conditionally independent given x_1, x_3 and x_5. - '<u>True</u>'



- $x_6 \to x_1 \to x_2$
- $x_6 \to x_4 \to x_1 \to x_2$
- $x_6 \to x_7 \to x_3 \to x_1 \to x_2$

Explanation:

- When chosen $x_6 \to x_1 \to x_2$.

  Ball passes from x_1 (not shaded ∵ Markov chain).

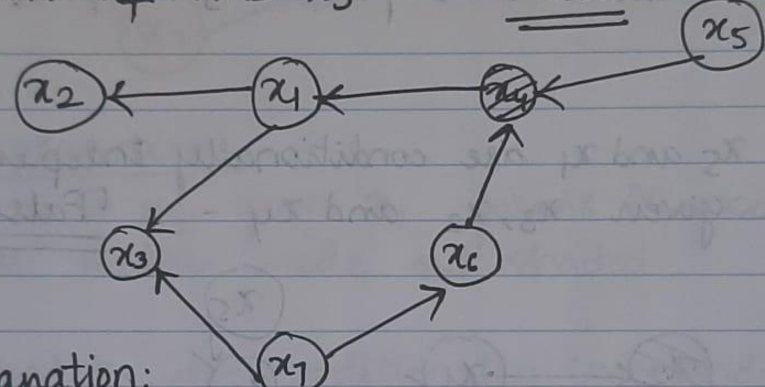
- When chosen $x_6 \to x_4 \to x_1 \to x_2$

  Ball moves from x_4 to x_2 passing through x_1

- When chosen $x_6 \to x_7 \to x_3 \to x_1 \to x_2$

→ x_7 to x_1 from x_3
→ x_6 to x_7 as x_3, x_7 & x_6 and middle node is not shaded.

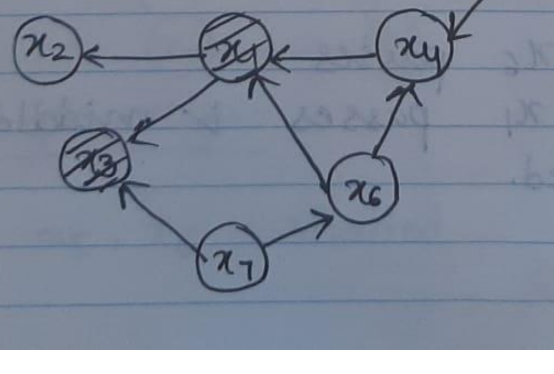
(c) x_1 and x_7 are conditionally independent given x_4 ~~and x_3~~. - 'False'



Explanation:

- x_7, x_6 and x_4 form Markov chain and middle node is not shaded here.

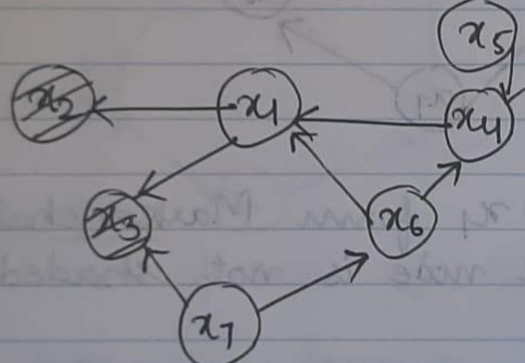
(d) x_5 and x_2 are conditionally independent given x_1 and x_3 - 'True' - ~~'False'~~



Explanation:

- $x_4 \to x_1 \to x_2$ Markov chain with middle node shaded
- $x_5 \to x_4 \to x_6$ middle node not shaded
- Ball doesnot pass $x_5 \to x_4 \to x_6 \to x_7 \to x_3 \to x_1 \to x_2$.

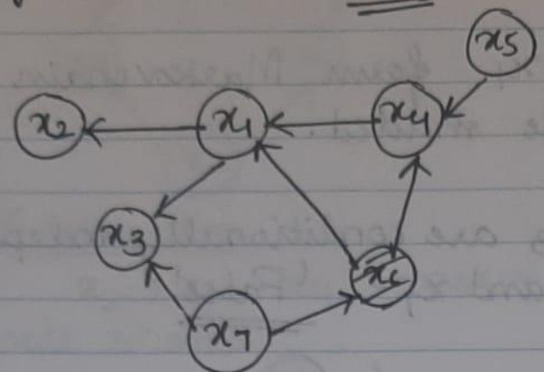
(e) x_5 and x_1 are conditionally independent given x_3, x_2 and x_4 - 'False'



Explanation:

- $x_5 \to x_6$ passes
- $x_6 \to x_1$ passes & middle node not shaded.

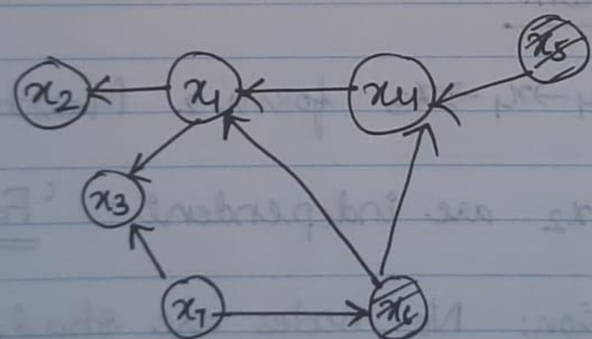
(f) x_4 and x_3 are conditionally independent given x_6 - '**False**'



Explanation:

- x_4 to x_3 from x_1 forms Markov chain with middle node not shaded.

(g) x_2 and x_7 are conditionally independent given x_5 and x_6 - '**True**'



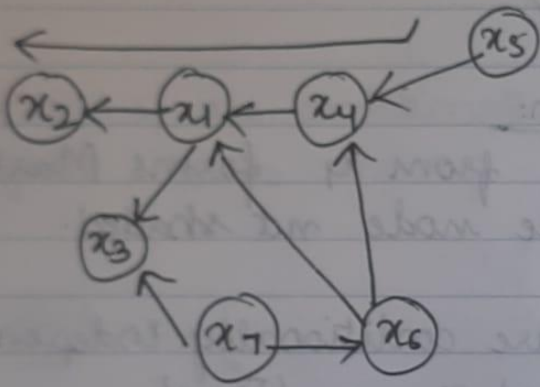
Explanation:

- x_5, x_6 : shaded

• Doesn't pass $x_2 \to x_1 \to x_3 \to x_7$
middle node not shaded.

• $x_2 \to x_6 \to x_1$ form Markov chain & middle node shaded.

(h) x_5 and x_3 are conditionally independent given x_6 and x_7 - '<u>False</u>'



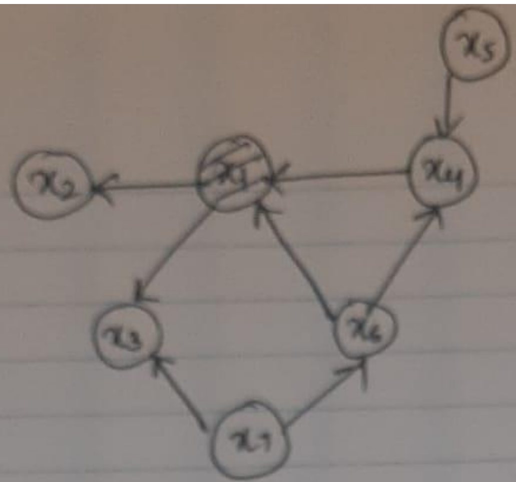
<u>Explanation:</u>

• $x_5 \to x_4 \to x_1 \to x_3$ form 2 Markov chains

(i) x_5 and x_2 are independent - '<u>False</u>'

<u>Explanation:</u> No nodes are shaded.

(j) x_2 and x_4 are conditionally independent given x_1 - '<u>True</u>'



Explanation:

- $x_2 \to x_1 \to x_4$ markov chain with middle node shaded.

- $x_2 \to x_1 \to x_3$: doesnot pass

- $x_2 \to x_1 \to x_6 \to x_4$: doesnot pass

- $x_2 \to x_1 \to x_6$: Markov chain with middle node shaded.

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Question-3

Given,

$$\pi = p(q_0) = \begin{array}{cc} 1 & 2 \\ \end{array}$$
$$\pi = p(q_0) = \left[\begin{array}{cc} 1/3 & 2/3 \end{array}\right]$$

$$a^T = p(q_t | q_{t-1}) = \begin{array}{c|cc} & 1 & 2 \\ 1 & 1/8 & 1/2 \\ 2 & 7/8 & 1/2 \end{array}$$

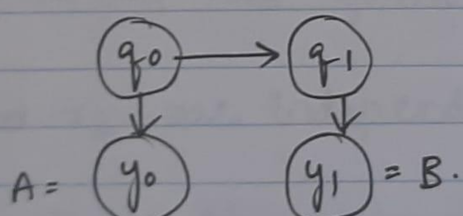
$$n^T = p(y_t | q_t) = \begin{array}{c|cc} & A & B \\ A & 1/4 & 3/4 \\ B & 3/4 & 1/4 \end{array}$$

To find:

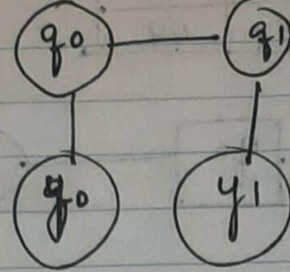
(i) likelihood of $p(y)$ using HMM

(ii) Individual marginals of states where $p(q_0/y)$ and $p(q_1/y)$.

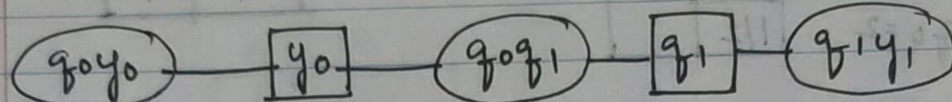
Step 1: 2 state HMM

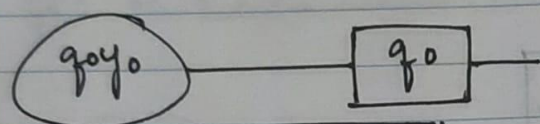


Step 2: Converting to junction tree



Implementing moralization & triangulation here.



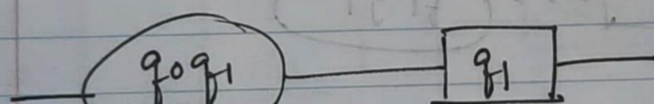


$q_0 = 1$
$q_1 = 2$

1/2	1/4
1/2	1/6

- $q_1 y_1$: emission matrix
- $q_0 q_1$: transition matrix

→



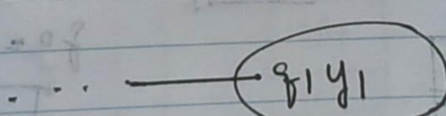
$q_1 =$ 1 2

$q_0 = 1$
$q_1 = 2$

1/8	1/8
1/2	1/2

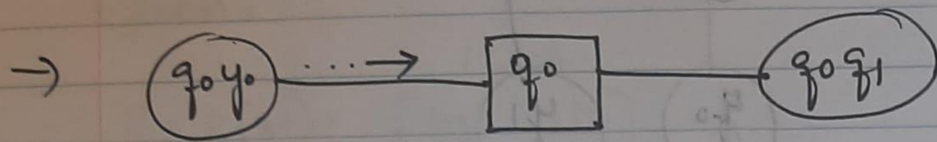
1 2

1	1



$y_1 =$ A B

$q_0=1$	1/4	3/4
$q_1=2$	3/4	1/4



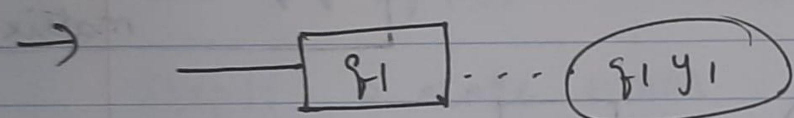
y_0	A	B
$q_0=1$	1/12	
$q_0=2$	1/2	

q_0 1 . . 2

1	1

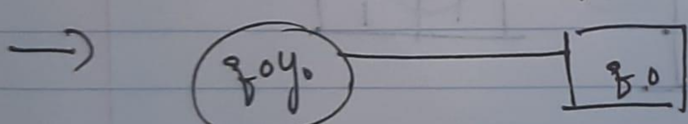
q_1 = 1 . . 2

1/8	1/8
1/2	1/2



1	1

	A	B
$q_1=1$		3/4
$q_1=2$		1/4



$q_0=1$	1/2	
$q_0=2$	1/2	

q_0 = 1 . . . 2

1	1

→ $q_0 q_1$ ← $\boxed{q_1}$

	1	2
	1/8	7/8
	1/2	1/2

$q_1 = 1..2$

1	1

→ $q \ldots y_1$

	A	B
$q_1 = 1$		3/4
$q_1 = 2$		1/4

→ $q_0 q_1$

	1	2
$q_0 = 1$	1/8	7/8
$q_0 = 2$	1/2	1/2

q_1

1	2
3/4	1/4

→ $q_1 y_1$

	A	B
$q_1 = 1$		3/4
$q_1 = 2$		1/4

$\therefore$ likelihood $P(y)$:

$$\begin{bmatrix} \frac{1}{12} \times \frac{1}{8} , & \frac{1}{12} \times \frac{7}{8} \\ \frac{1}{2} \times \frac{1}{2} , & \frac{1}{2} \times \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1/128 & 7/384 \\ 3/16 & 1/16 \end{bmatrix}$$

$$\therefore P(y) = \left[\frac{1}{128} + \frac{7}{384} + \frac{3}{16} + \frac{1}{16} \right]$$

$$= \frac{53}{192}.$$

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Question-4

To solve K-means clustering algorithm.
Given that,

$C_1 = (-4,-5)$, $C_2 = (5,4)$

Given 2-D dataset,

A $(-3,-1)$ B $(-1,-3)$ C $(-2,-6)$ D $(-5,-7)$ E $(3,1)$
F $(2,3)$ G $(3,6)$ H $(8,1)$

Step 1: Initializing value of centroids

i.e; $C_1 = (-4,-5)$, $C_2 = (5,4)$

Step2: objects - centroid distance.

In this step, we calculate the Euclidean distance between each object of the 2D dataset and the cluster centroid.

Iteration 0: When using C_1: $(-4,-5)$

$A = \sqrt{(-3+4)^2 + (-1+5)^2} = 4.12$

$B = \sqrt{(-1+4)^2 + (-3+5)^2} = 3.60$

$C = \sqrt{(-2+4)^2 + (-6+5)^2} = 2.23$

$D = \sqrt{(-5+4)^2 + (-7+5)^2} = 2.23$

$E = \sqrt{(3+4)^2 + (1+5)^2} = 9.21$

$F = \sqrt{(2+4)^2 + (3+5)^2} = 10$

$G = \sqrt{(3+4)^2 + (6+5)^2} = 13.03$

$H = \sqrt{(8+4)^2 + (1+5)^2} = 13.41$

When using $C_2 = (5,4)$

$A = \sqrt{(-3-5)^2 + (-1-4)^2} = 9.43$

$B = \sqrt{(-1-5)^2 + (-3-4)^2} = 9.21$

$C = \sqrt{(-2-5)^2 + (-6-4)^2} = 12.20$

$D = \sqrt{(-5-5)^2 + (-7-5)^2} = 15.62$

$E = \sqrt{(3-5)^2 + (1-4)^2} = 3.60$

$$F = \sqrt{(2-5)^2 + (3-4)^2} = 3.16$$

$$G = \sqrt{(3-5)^2 + (6-4)^2} = 2.82$$

$$H = \sqrt{(8-5)^2 + (1-4)^2} = 4.24$$

Distance matrix can be given as,

$$D_0 = \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 4.12 & 3.60 & 2.23 & 2.23 & 9.21 & 10 & 13.03 & 13.41 \\ 9.43 & 9.21 & 12.20 & 15.62 & 3.60 & 3.16 & 2.82 & 4.24 \end{bmatrix} \end{array} \begin{array}{c} C_1 \\ C_2 \end{array}$$

C_1 is considered as group 1

C_2 is considered as group 2

Step 3: Object clustering:

We have to assign each object basing upon the minimum distance.

A, B, C, D belong to group 1

E, F, G, H belong to group 2

Then,

$$G^0 = \begin{array}{c} \begin{array}{cccccccc} A & B & C & D & E & F & G & H \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \text{group 1} \\ \text{group 2.} \end{array}$$

Step 4: updating centroids

$$C_1 = \left(\frac{(-3-1-2-5)}{4}, \frac{-1-3-6-7}{4} \right) = \left(\frac{-11}{4}, \frac{-17}{4} \right)$$
$$= (-2.75, -4.25)$$

$$C_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right) = \left(4, \frac{11}{4} \right)$$
$$= (4, 2.75)$$

Iteration 1: Using C_1

$$A = \sqrt{(-3+2.75)^2 + (-1+4.25)^2} = 3.25$$

$$B = \sqrt{(-1+2.75)^2 + (-3+4.25)^2} = 2.15$$

$$C = \sqrt{(-2+2.75)^2 + (-6+4.25)^2} = 1.90$$

$$D = \sqrt{(-5+2.75)^2 + (-7+4.25)^2} = 3.55$$

$$E = \sqrt{(3+2.75)^2 + (1+4.25)^2} = 7.78$$

$$F = \sqrt{(2+2.75)^2 + (3+4.25)^2} = 8.66$$

$$G = \sqrt{(3+2.75)^2 + (6+4.25)^2} = 11.75$$

$$H = \sqrt{(5+2.75)^2 + (4+4.25)^2} = 11.31$$

When using $C_2 = (4, 2.75)$

$A = \sqrt{(-3-4)^2 + (-1-2.75)^2} = 7.94$

$B = \sqrt{(-1-4)^2 + (-3-2.75)^2} = 7.61$

$C = \sqrt{(-2-4)^2 + (-6-2.75)^2} = 10.60$

$D = \sqrt{(-5-4)^2 + (-7-2.75)^2} = 13.26$

$E = \sqrt{(3-4)^2 + (1-2.75)^2} = 2.01$

$F = \sqrt{(2-4)^2 + (3-2.75)^2} = 2.01$

$G = \sqrt{(3-4)^2 + (6-2.75)^2} = 3.40$

$H = \sqrt{(5-4)^2 + (4-2.75)} = 1.60$

Distance matrix

$$\begin{array}{cccccccc} \multicolumn{4}{c}{\overbrace{\hspace{8em}}^{\text{group1}}} & \multicolumn{4}{c}{\overbrace{\hspace{8em}}^{\text{group 2.}}} \\ A & B & C & D & E & F & G & H \end{array}$$

$$\begin{bmatrix} 3.25 & 2.15 & 1.90 & 3.55 & 7.78 & 8.66 & 11.75 & 11.31 \\ 7.94 & 7.61 & 10.60 & 13.26 & 2.01 & 2.01 & 3.40 & 1.60 \end{bmatrix}$$

→ __updated centroid:__

$G = \cancel{\left(\frac{3.25+2.15+1.9+3.55}{4}, \frac{7.94+7.61+10.6+13.26}{4}\right)}$ $\left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4}\right)$

$= \left(\frac{-11}{4}, \frac{-17}{4}\right)$

$C_2 = \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4}\right) = (4, 2.75)$

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Question-5

Jensen's inequality:

To prove: for non-negative real numbers $x_1, x_2, \cdots, x_n$

$$\frac{x_1+x_2+\cdots+x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \cdots x_n}$$

This is of the form that Arithmetic mean greater than the Geometric mean.

As 'log' function is an increasing function we apply 'log' on both sides to proceed with Jensen's inequality.

i.e; $\log\left(\frac{x_1+x_2+\cdots+x_n}{n}\right) \geq \frac{1}{n}\log(x_1 \cdot x_2 \cdots x_n)$

$$= \frac{\log x_1 + \cdots + \log x_n}{n}.$$

i.e; $\log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geq \frac{1}{n}\log(x_1 \cdots x_n)$

$$\Rightarrow \log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geq \frac{1}{n} \log\left(\prod_{i=1}^{n} x_i\right)$$

(or)

$$\log\left(\frac{1}{n}\sum_{i=1}^{n} x_i\right) \geq \log\left(\prod_{i=1}^{n} x_i\right)^{1/n}$$

Then,

$$\frac{\sum_{i=1}^{n} x_i}{n} \geq \sqrt[n]{\prod_{i=1}^{n} x_i}$$

$$\therefore \frac{x_1 + x_2 + \cdots x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdots x_n}$$

When we use $E[x]$ as mean of x_i's.

Then,

$$\boxed{\log(E[x]) \geq E[\log(x)]}$$

Hence, it is proved using concavity of log given by 'Jensen's inequality'.

Question 6.)

Convolution layer:

$8 \rightarrow 10, \quad 2\times2, \quad 2\times2$

$\Rightarrow \left(\frac{x}{2}, \frac{y-1}{2}\right)$

$\therefore$ $\left(\frac{x}{2}, \frac{y-1}{2}\right)$ is feature map here.

Max Pooling:

$3\times3, \ 2\times2.$

$3(101-2)+2$

$\left(\frac{x-6+4}{4}\right)\left(\frac{y-3}{4}\right) = 302 \times 154$

Flattening: $10\times12\times9 \rightarrow 1080$

$\left(\frac{x-2}{4}\right)$

$\Big[\because \frac{x-2}{4} = 12$

$x-2 = 48$

$x = 50$

$\frac{y-3}{4} = 9$

$\Rightarrow y = 154 \Big]$

ReLU: No effect