

nn2233

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MACHINE LEARNING FINAL EXAM

Question 1:

[illegible]

1:43



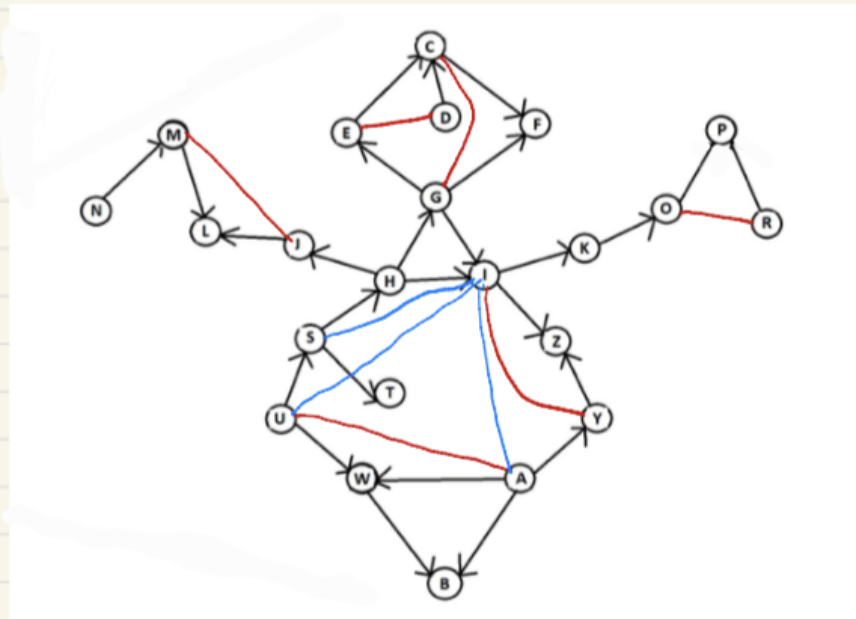
◀ Search

Done

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Problem 1:

MN, MLJ, JH, GHI, GCE, GCF, CDE, IK, KO, OPR,
 HSI, TS, USI, UIA, UWA, IAY, IZY, WAB

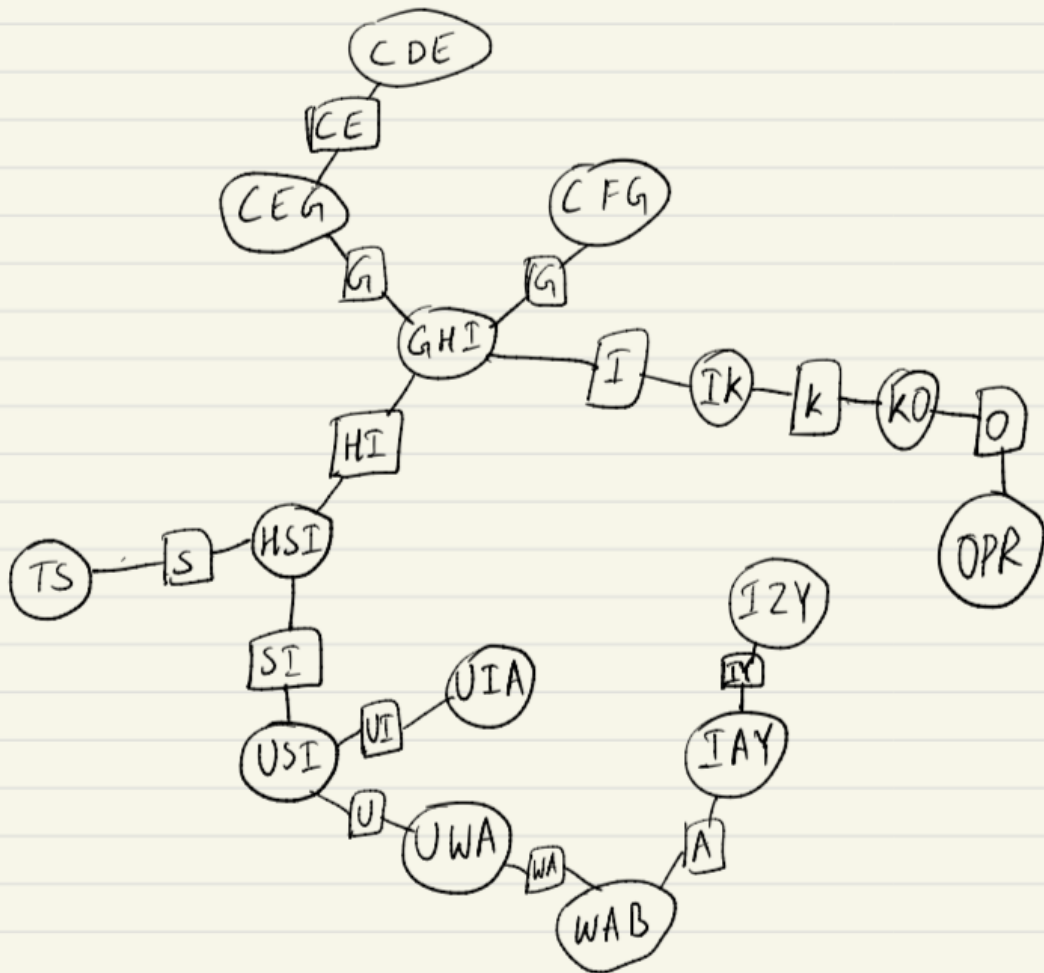
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Done

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2:

a-False

b-True

c-False

d-True

e-False

f-False

g-True

h-False

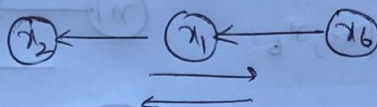
i-False

j-True

$$2. \quad P(x_1, x_2, \dots, x_7) = P(x_1 | x_4, x_6) P(x_2 | x_1) P(x_3 | x_1, x_7) \\ P(x_4 | x_5, x_6) P(x_5) P(x_6 | x_7) P(x_7)$$

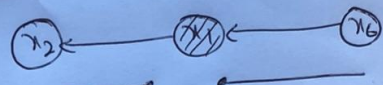
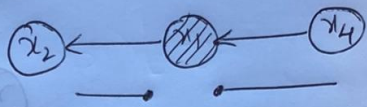
a) False

$x_2 \not\perp x_6$



b) True

$x_2 \perp x_6 | x_1, x_3, x_5$



c) False

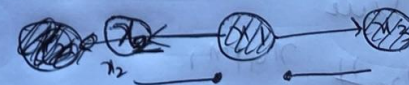
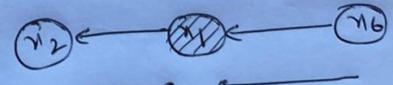
$x_1 \not\perp x_7 | x_4$



d) True

~~$x_5 \not\perp$~~

$x_5 \perp x_2 | x_1, x_3$



e) False

$x_5 \not\perp x_1, x_3, x_2, x_4$



$x_5 - x_4 - x_6 - x_1$

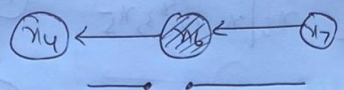
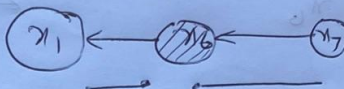
f) False

$\gamma_4 \# \gamma_3 | \gamma_6$



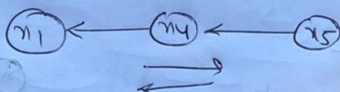
g) True

$\gamma_2 \perp \gamma_7 | \gamma_5, \gamma_6$



h) False

$\gamma_3 \# \gamma_5 | \gamma_6, \gamma_7$



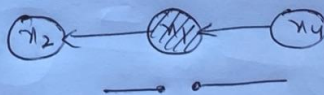
i) False

$\gamma_5 \# \gamma_2$



j) True

$\gamma_2 \perp \gamma_4 | \gamma_1$



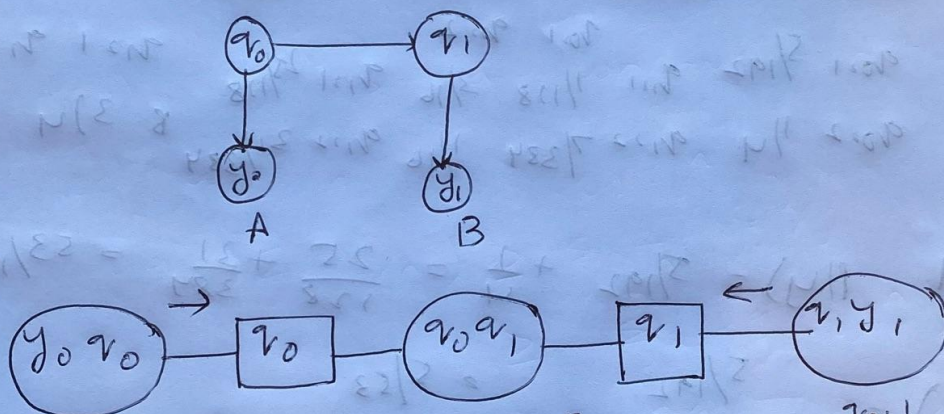
Question 3:

3.

$$\pi = P(q_0) = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

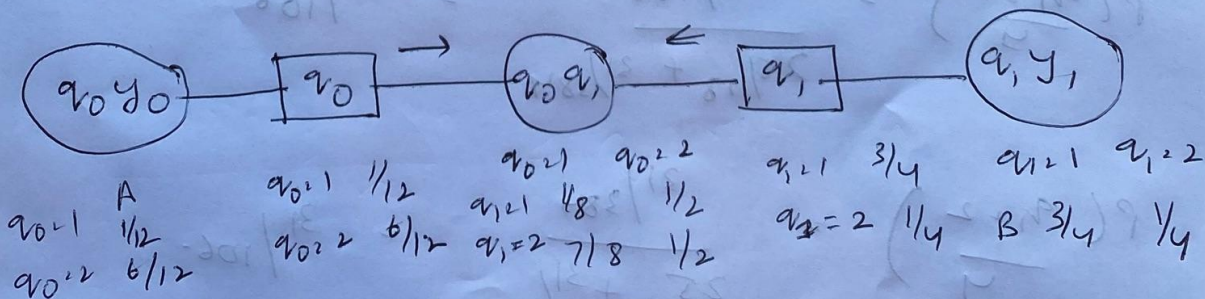
$$a^T = P(q_t | q_{t-1}) = \begin{bmatrix} \frac{1}{8} & \frac{1}{2} \\ \frac{7}{8} & \frac{1}{2} \end{bmatrix}$$

$$\eta^T = P(y_t | q_t) = \begin{matrix} & \begin{matrix} 1 \\ 2 \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} \frac{1}{4} & \frac{3}{4} \\ \frac{3}{4} & \frac{1}{4} \end{bmatrix} \end{matrix}$$



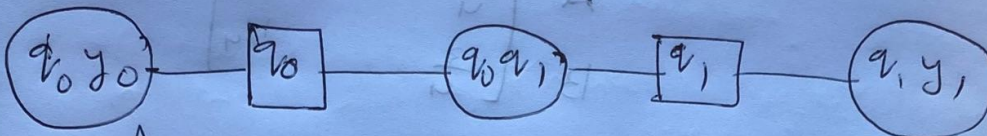
| | | | | | | | | |
|-----------|----------------|----------------|---------------|---------------|---------------|---------------|---------------|---------------|
| | | | $q_{0,1}$ | $q_{0,2}$ | | $q_{1,1}$ | $q_{1,2}$ | |
| $q_{0,1}$ | A | B | $q_{0,1}$ | $q_{0,2}$ | $q_{1,1}$ | A | $q_{1,2}$ | B |
| | $\frac{1}{12}$ | $\frac{3}{12}$ | $\frac{1}{8}$ | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ |
| $q_{0,2}$ | $\frac{6}{12}$ | $\frac{2}{12}$ | $\frac{7}{8}$ | $\frac{1}{2}$ | $\frac{3}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{1}{4}$ |

We know the output sequence i.e. $AB \Rightarrow y_0 = A, y_1 = B$





$$\begin{array}{ccccccc}
 & A & & q_{0,1} & q_{0,2} & & \\
 q_{0,1} & 1/12 & q_{0,1} & 1/12 & q_{1,1} & 1/128 & 3/16 \\
 q_{0,2} & 6/12 & q_{0,2} & 6/12 & q_{1,2} & 7/384 & 1/16
 \end{array}$$



$$\begin{array}{ccccccc}
 & A & & q_{0,1} & q_{0,2} & & \\
 q_{0,1} & 1/12 & q_{0,1} & 5/192 & q_{1,1} & 1/128 & 3/16 \\
 q_{0,2} & 6/12 & q_{0,2} & 1/4 & q_{1,2} & 7/384 & 1/16
 \end{array}$$

Likelihood $P(y) = \frac{5}{192} + \frac{1}{4} = \frac{25}{128} + \frac{31}{384} = \frac{53}{192}$

$$P\left(\frac{q_{0,1}}{y}\right) = \frac{5/192}{5/192 + 1/4} = 5/53$$

$$P\left(\frac{q_{0,2}}{y}\right) = \frac{1/4}{5/192 + 1/4} = 48/53$$

$$P\left(\frac{q_{1,1}}{y}\right) = \frac{25/128}{25/128 + 31/384} = 75/106$$

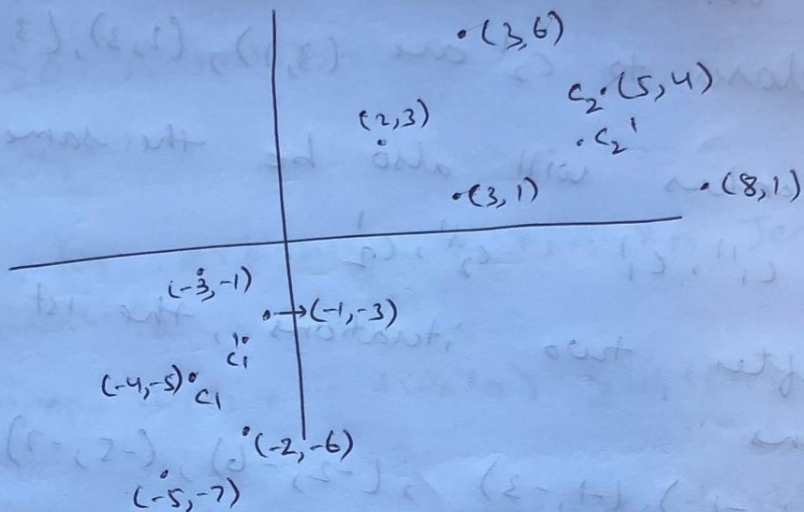
$$P\left(\frac{q_{1,2}}{y}\right) = \frac{31/384}{25/128 + 31/384} = 31/106$$

Question 4:

4.

$$c_1 = (-4, -5)$$

$$c_2 = (5, 4)$$



Points closer or nearer to c_1 are

$$(-3, -1), (-1, -3), (-2, -6), (-5, -7)$$

Points closer & nearer to c_2 are

$$(3, 1), (2, 3), (3, 6), (8, 1)$$

Iteration 1:-

Centroid of first set of points

$$= \left(\frac{-3-1-2-5}{4}, \frac{-1-3-6-7}{4} \right)$$

$$= \left(-11/4, -17/4 \right)$$

Centroid of second set of points

$$= \left(\frac{3+2+3+8}{4}, \frac{1+3+6+1}{4} \right)$$

$$= \left(16/4, 11/4 \right)$$

So the new centers are c_1' , c_2'

$$c_1' = \left(-11/4, -17/4 \right) \quad c_2' = \left(16/4, 11/4 \right)$$

$$= (-2.75, -4.25)$$

$$= (4, 2.75)$$

Iteration 2:-

Points closer to c_1^1 are $(-3, -1), (-1, -3), (-2, -6), (-5, -7)$

Points closer to c_2^1 are $(3, 1), (2, 3), (3, 6), (8, 1)$

New centers will also be the same

$c_1'', c_1^1 \quad c_2'', c_2^1$

So after two iterations, the 1st cluster contains

$(-3, -1), (-1, -3), (-2, -6), (-5, -7)$.

Second cluster contains

$(3, 1), (2, 3), (3, 6), (8, 1)$.

Question 5:

5. Let $f(x)$ be a concave function and
 let w_1, w_2, \dots, w_n be weights with
 $w_j \geq 0, j = 1, 2, \dots, n$
 $\sum_{j=1}^n w_j = 1$

Then, for arbitrary x_1, x_2, \dots, x_n Jensen's
 inequality states:

$$f(w_1 x_1 + w_2 x_2 + \dots + w_n x_n) \geq w_1 f(x_1) + w_2 f(x_2) + \dots + w_n f(x_n)$$

$f(x) = \log x$ is a concave function

Let $w_j = 1/n, j = 1, 2, \dots, n$

$$\begin{aligned} \log\left(\sum_j x_j/n\right) &\geq \sum_j \frac{1}{n} \log x_j \\ &\geq \sum_j (\log x_j)^{1/n} \\ &\geq \log\left(\prod_j x_j^{1/n}\right) \end{aligned}$$

Taking antilog

$$\sum_j x_j/n \geq \left(\prod_j x_j\right)^{1/n}$$

$$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \times x_2 \times \dots \times x_n}$$

Question 6:

$$x=302$$

$$y=154$$

6. Final layer size = $10 \times 12 \times 9$
 Maxpooling with $3 \times 3, 2 \times 2$

$$\frac{a_1 - 3}{2} + 1 = 12$$

$$\frac{b_1 - 3}{2} + 1 = 9$$

$$\frac{a_1 - 3}{2} = 11$$

$$\frac{b_1 - 3}{2} = 8$$

$$a_1 - 3 = 22$$

$$b_1 - 3 = 16$$

$$a_1 = 25$$

$$b_1 = 19$$

\Rightarrow Input size before maxpooling = $10 \times 25 \times 19$

ReLU does not change the size

convolution layer: $8 \rightarrow 10, 2 \times 2, 2 \times 2$

$$\frac{a_2 - 2}{2} + 1 = 25$$

$$\frac{b - 2}{2} + 1 = 19$$

$$\frac{a_2 - 2}{2} = 24$$

$$\frac{b - 2}{2} = 18$$

$$a_2 - 2 = 48$$

$$b - 2 = 36$$

$$a_2 = 50$$

$$b = 38$$

$$a = 50$$

$$\Rightarrow 8 \times 50 \times 38$$

Maxpooling with $3 \times 3, 2 \times 2$

$$\frac{a_0 - 3}{2} + 1 = 50$$

$$\frac{b_0 - 3}{2} + 1 = 38$$

$$\frac{a_0 - 3}{2} = 49$$

$$\frac{b_0 - 3}{2} = 37$$

$$a_0 - 3 = 98$$

$$b_0 - 3 = 74$$

$$a_0 = 101$$

$$b_0 = 77$$

$$\Rightarrow 8 \times 101 \times 77$$

ReLU does not change size

Convolution layer: $1 \rightarrow 8, 2 \times 2, 3 \times 2$

$$\frac{x-2}{3} + 1 = 101$$

$$\frac{x-2}{3} = 100$$

$$x-2 = 300$$

$$x = 302$$

$$\frac{y-2}{2} + 1 = 77$$

$$\frac{y-2}{2} = 76$$

$$y-2 = 152$$

$$y = 154$$

The size of the input is $1 \times 302 \times 154$
 $x = 302, y = 154$.