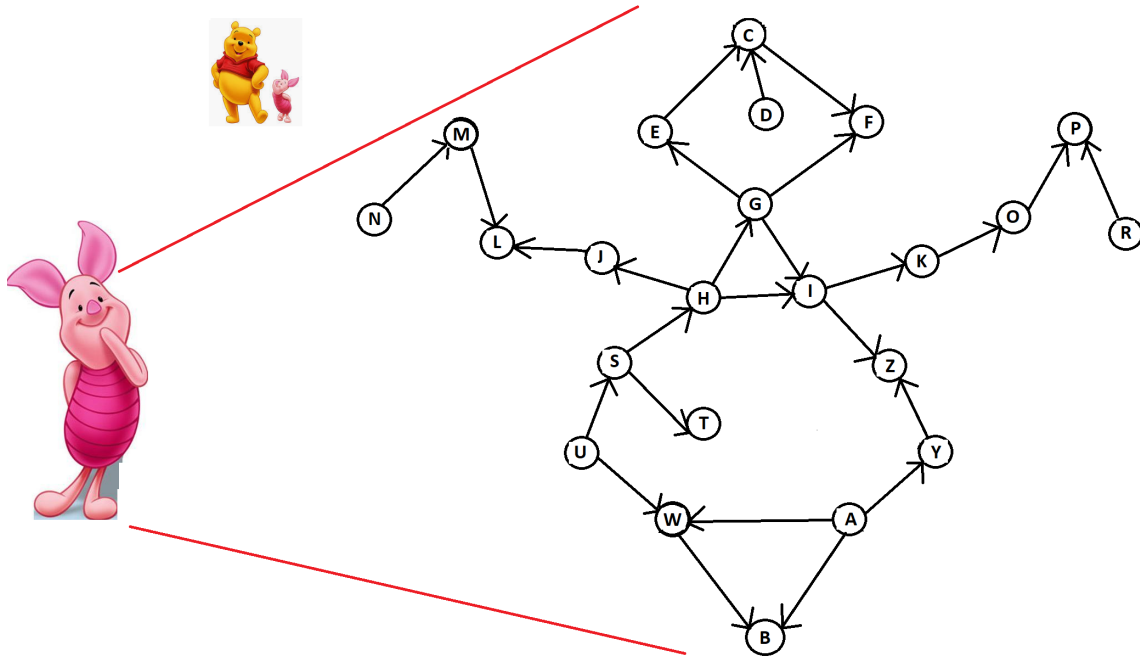


Final exam

Introduction to Machine Learning
Fall 2021
Instructor: Anna Choromanska

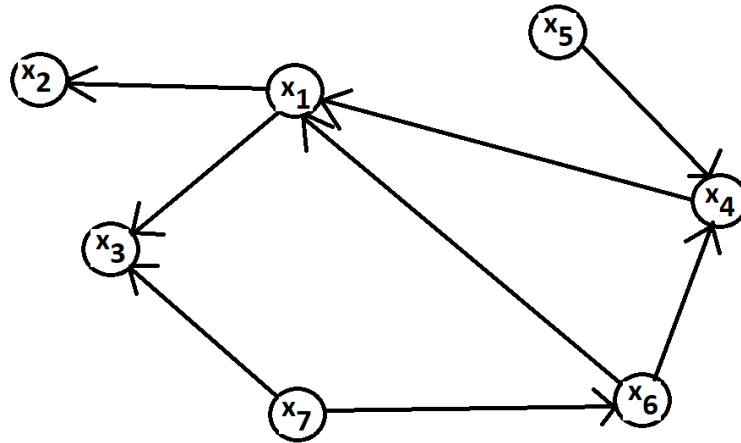
Problem 1 (100 points)

Piglet is looking for Winnie the Pooh in the forest but he can't find his friend. Piglet decided to perform the junction-tree algorithm to obtain cyber representation of his friend and post his digital photo online. Help him out by designing a junction-tree from the graph below which Piglet should use for Winnie the Pooh. Show ALL steps of creating the junction tree (including the table for the Kruskal algorithm).



Problem 2 (60 points)

Consider the Bayesian network below with binary variables x_1, x_2, \dots, x_7 .



Write out the factorization of the probability distribution $p(x_1, \dots, x_7)$ implied by this directed graph. (10 points) Then, using the Bayes ball algorithm, indicate for each statement below if it is True or False and justify your answers (50 points)

- (a) x_2 and x_6 are independent.
- (b) x_2 and x_6 are conditionally independent given x_1, x_3 , and x_5 .
- (c) x_1 and x_7 are conditionally independent given x_4 .
- (d) x_5 and x_2 are conditionally independent given x_1 and x_3 .
- (e) x_5 and x_1 are conditionally independent given x_3, x_2 , and x_4 .
- (f) x_4 and x_3 are conditionally independent given x_6 .
- (g) x_2 and x_7 are conditionally independent given x_5 and x_6 .
- (h) x_3 and x_5 are conditionally independent given x_6 and x_7 .
- (i) x_5 and x_2 are independent.
- (j) x_2 and x_4 are conditionally independent given x_1 .

Problem 3 (100 points)

You are given the parameters of a 2-state HMM. You observed the input sequence AB (from a 2-symbol alphabet A or B). In other words, you observe two symbols from your finite state machine, A and then B. Using the junction tree algorithm, evaluate the likelihood of this data $p(y)$ given your HMM and its parameters. Also compute (for decoding) the individual marginals of the states after the evidence from this sequence is observed: $p(q_0|y)$ and $p(q_1|y)$. The parameters for the HMM are provided below. They are the initial state prior $p(q_0)$, the state transition matrix given by $p(q_t|q_{t-1})$, and the emission matrix $p(y_t|q_t)$, respectively.

$$\pi = p(q_0) = \begin{array}{cc} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{bmatrix} 1/3 & 2/3 \end{bmatrix} \end{array}$$

$$a^T = p(q_t | q_{t-1}) = \begin{array}{cc} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} 1 \\ 2 \end{matrix} & \begin{bmatrix} 1/8 & 1/2 \\ 7/8 & 1/2 \end{bmatrix} \end{array} \quad \eta^T = p(y_t | q_t) = \begin{array}{cc} & \begin{matrix} 1 & 2 \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} 1/4 & 3/4 \\ 3/4 & 1/4 \end{bmatrix} \end{array}$$

Problem 4 (40 points)

Show the first two iterations (after the initialization) of the k -means clustering algorithm (show centers and assignments of data points to clusters) for the following 2D data set: $(-3, -1), (-1, -3), (-2, -6), (-5, -7), (3, 1), (2, 3), (3, 6), (8, 1)$. Assume the number of centers is equal to 2 and the centers are initialized to $(-4, -5)$ and $(5, 4)$.

Problem 5 (50 points)

Prove (using Jensen's inequality) that for non-negative real numbers x_1, x_2, \dots, x_n the following holds:

$$\frac{x_1 + x_2 + \dots + x_n}{n} \geq \sqrt[n]{x_1 \cdot x_2 \cdot \dots \cdot x_n}.$$

Problem 6 (50 points)

Consider the fragment of the convolutional architecture given below:

- Input image: $1 \times x \times y$
- Convolutional layer: $\underbrace{1 \rightarrow 8}_{\text{number of input and output channels}}, \underbrace{2 \times 2}_{\text{filter size}}, \underbrace{3 \times 2}_{\text{stride}}$
- ReLU
- MaxPooling: $\underbrace{3 \times 3}_{\text{region size}}, \underbrace{2 \times 2}_{\text{stride}}$
- Convolutional layer: $8 \rightarrow 10, 2 \times 2, 2 \times 2$
- ReLU
- MaxPooling: $3 \times 3, 2 \times 2$
- Flattening (3D to 1D):
 $\underbrace{10 \times 12 \times 9}_{\text{number of feature maps} \times \text{size of the feature map (12} \times 9)}} \rightarrow 1080$

What is the size of the input (in other words what is x and y)?