ECE-GY 9143

Introduction to High Performance Machine Learning

Lecture 2   02/04/2023

Parijat Dube

ML Performance Optimization

Agenda

- Problem definition
- System vs. Algorithmic view
- Performance Optimization Methodology:
  - Measurement
  - Analysis
  - Optimization

System vs. Algorithmic view

ML Performance Factors

20% of the course

- **Algorithm Performance**
  - Training vs. Inference Performance
  - Training Algorithms Ex: Asynchronous vs. Synchronous Distributed SGD
- **Hyperparameters Performance**
  - ex: Learning rate, Network dimensions, Batch size, etc.

80% of the course

- **Implementation Performance**
  - Algorithms Implementation (vectorization, comm./comp. overlapping, parallelization, data access)
- **Framework Performance**
  - ML Frameworks: PyTorch, TensorFlow, Caffe, MXNET
- **Library Performance**
  - Math libraries (cuDNN), Communication Libraries (MPI, GLU)
- **Hardware Performance**
  - CPU, DRAM, GPU, HBM, Tensor Units, Disk/Filesystem, Network

System view: Bottom-Up (this course) ↑

Algorithmic view: Top-down ↓

A couple of examples

- Implementation Performance:
  - too many mallocs() in C (or *new* in C++): easily 10 – 100x slowdown

- Algorithmic Performance:
  - Search 1 element in 10 billion stored in an array
    - Linear search: O(n) – average: about 5 billions comparisons expected (*)
    - Binary search: O(log n) – average: about 32 comparisons expected (*)

(*) Assuming exactly one matching element exists and elements are uniformly distributed

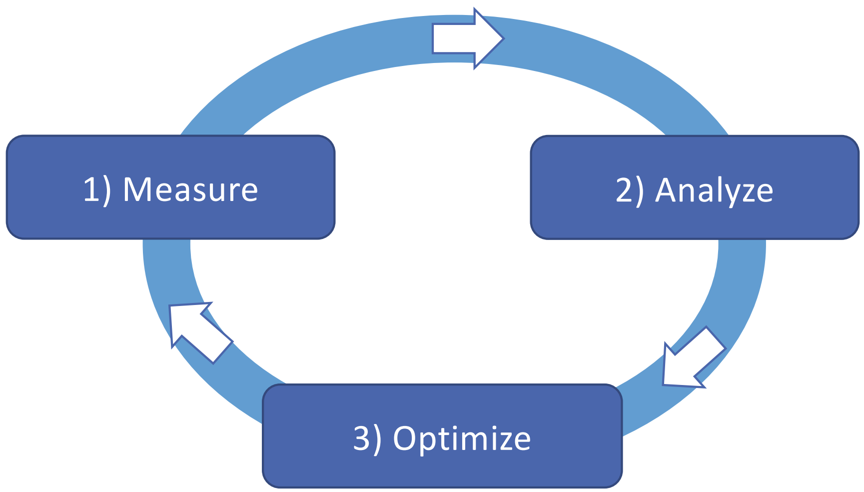
Software Performance Optimization

ML Performance Optimization Definition

- Software Performance Optimization for ML
  - Given:
    - A **system** (ex: NYU Compute node + PyTorch)
    - An **algorithm** (ex: Distributed SGD training) + **hyperparameters**
    - A **dataset** (ex: CIFAR100)
  - Obtain the **maximum** performance

Performance Optimization Methodology

- Execute workload
- Profiling
- Tracing
- Time measuring



- Understand hardware
- Understand software stack
- Understand data-movement
- Identify Critical Path
- Identify Bottleneck

- Implement code optimizations
- Change software configurations and parameters

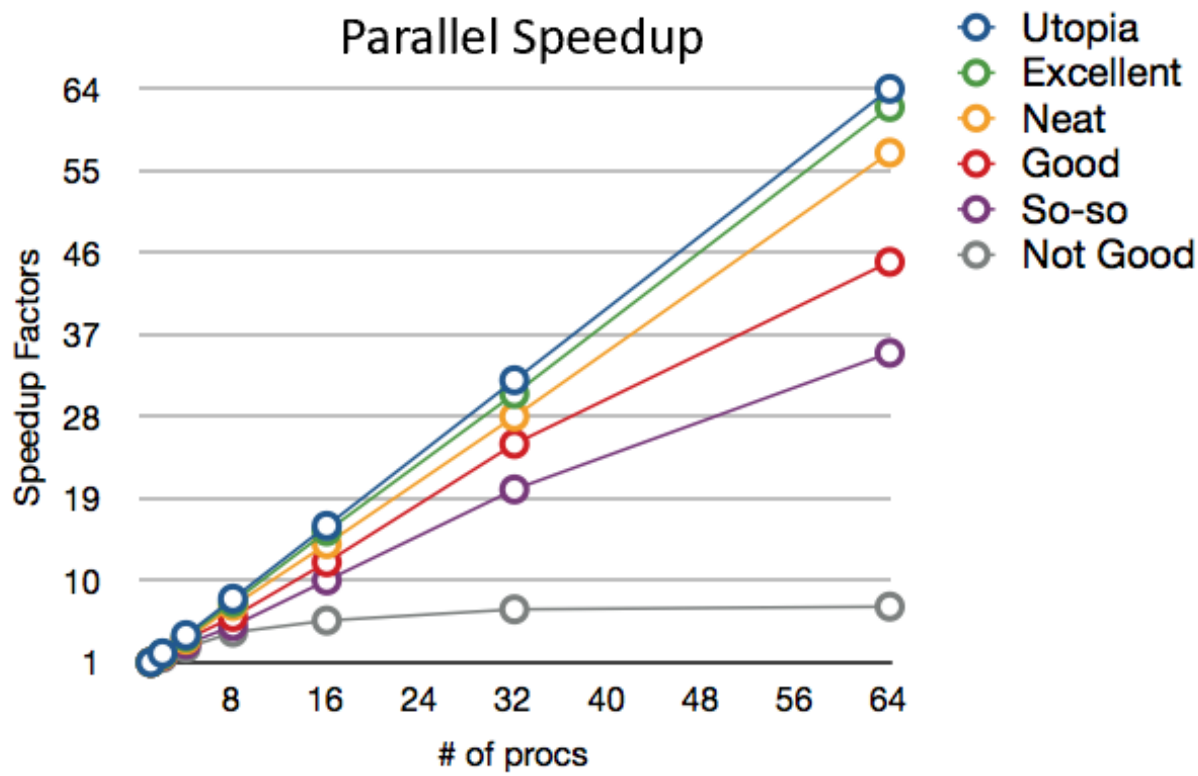
Performance optimization methodology (1): Measurement

What is performance?

- Basic metrics:
  - Execution time: t (for a single operation is called **latency**)
- Derived metrics:
  - Throughput: $\frac{\#\ operations}{t}$ or $\frac{\#\ programs}{t}$
  - FLOPS: $\frac{\#\ floating\_point\_operations}{t}$ (https://en.wikipedia.org/wiki/FLOPS)

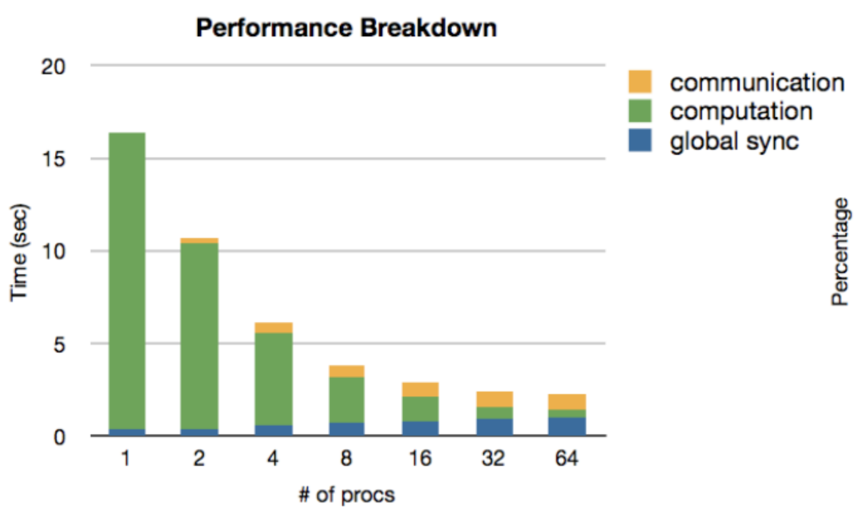
Speedup

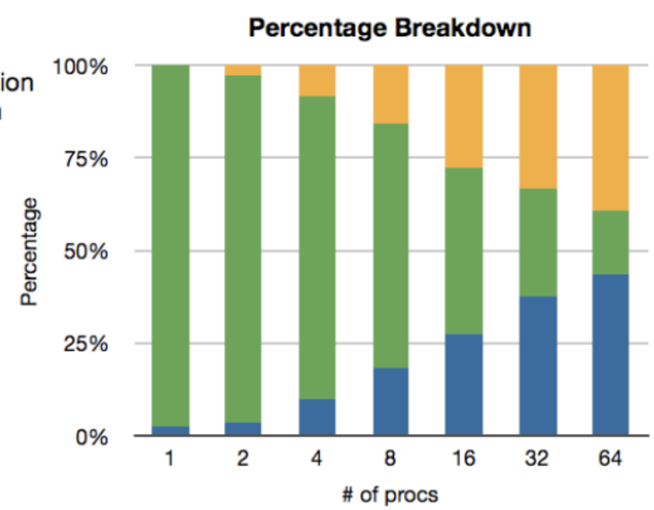
- Speedup of B w.r.t. A: $\frac{t_A}{t_B}$
- Parallel Speedup: $\frac{t_{serial}}{t_{parallel}}$
- Slowdown is inverse of Speedup



from: http://web.eecs.utk.edu/~huangj/hpc/hpc_intro.php

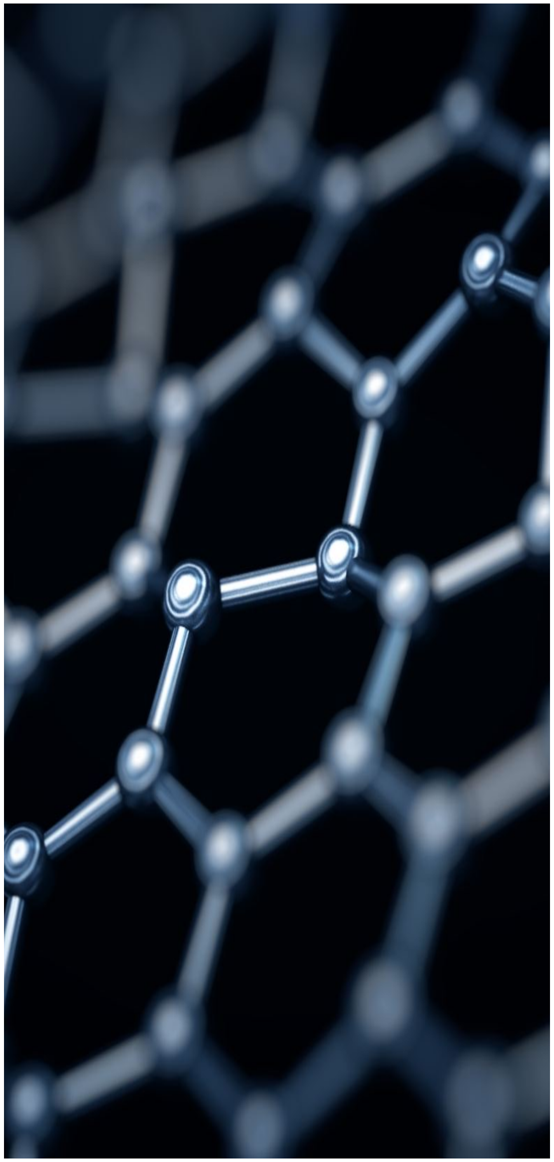
“Not Good” speedup





Scalability

- **Scaling Efficiency**
  - E = $\frac{t_{serial}}{t_{parallel} * p}$ <=1 p is the number of processes/threads/…
- **Strong Scaling**: Constant problem size while increasing p
  - How the solution time varies with the number of processors for a fixed total problem size.
  - Increasing synchronization cost, but fixed amount of work
- **Weak Scaling**: Increasing problem size proportional to p
  - Weak scaling is defined as how the solution time varies with the number of processors for a fixed problem size per processor.
  - Work per process is constant
  - Increasing synchronization cost, increasing work

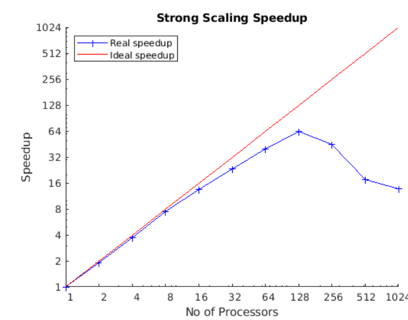


Weak vs. Strong Scaling

Assume Serial program that solves a problem size P in time T

E.g., Protein folding simulation in an hour

Weak Scaling	Weak scaling: run a larger problem or more problems within T • E.g., fold a bigger protein or more small ones in an hour
Strong Scaling	Strong scaling: run a problem faster than T • E.g. fold the same protein in a min



What Scaling?

- When my problem continues to increase in size, I can still solve the problem within the same amount of time by simply dedicating proportionally more resources at it.

- When my problem stays at the same size, I can solve the problem 10 times faster by dedicating 10 times more resources.

Computing Averages

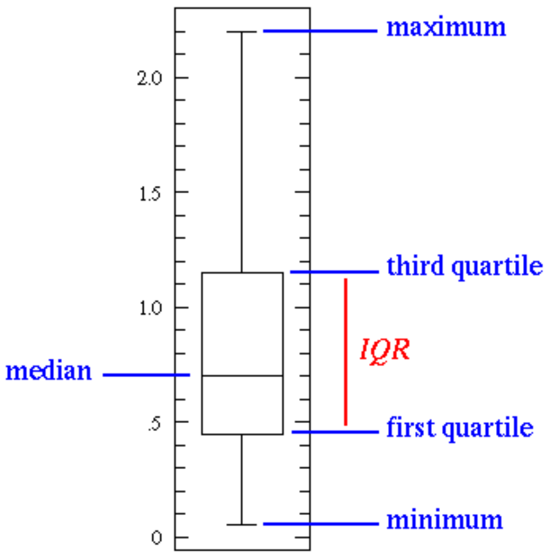
- Average Execution Time
  - Arithmetic mean: $\frac{1}{n}\sum_{i=1}^{n} t_i$
- Average Performance or Throughput
  - If t is held constant => Arithmetic mean
  - If *#operations* is held constant => Harmonic mean: $\frac{n}{\sum_{i=1}^{n} \frac{t_i}{\#operations}}$
- Average Speedup, Slowdown or any Ratio
  - Geometric mean: $\sqrt[n]{\prod_{i=1}^{n} speedup_i}$

Benchmarking Workloads

- Benchmarks in ascending order of complexity:
  1. Micro-kernels: test a specific processor feature
     Examples: Floating point, L1 Cache, L2 Cache,
  2. Micro-benchmark: small program from a programming assignment
     Examples: Merge sort in isolation
  3. Kernels: a specific algorithm in a real program
     Examples: Quicksort, Binary Search, DGEMM, DAXPY with context
  4. Synthetic Benchmarks: try to reproduce the workload of a class of applications
     Examples: Dhrystone, Linpack
  5. Real Applications: a real application used for a specific purpose
     Examples: Word, MySQL, NAMD (Molecular Dynamics)
  6. Real Workflows: a set of applications working together
     Example: CANDLE workflow

Measuring and Reporting Performance

- Reproducibility
  - Always include absolute execution time
  - Report relevant hardware and software info:
    - CPU, Memory, Network, Disk, etc.
    - Experiment configuration
    - Code, Pseudo code
    - Compiler ver., Compilation Flags, Libraries ver., OS ver.
- Accuracy
  - Repetitions: 5, 10, 100, … (depends on variability)
  - If high-variability results:
    - Try to understand why and reduce it
    - Include stddev, variance, max-min, inter-quartile range
    - Use box-plot for chart representation as shown in figure

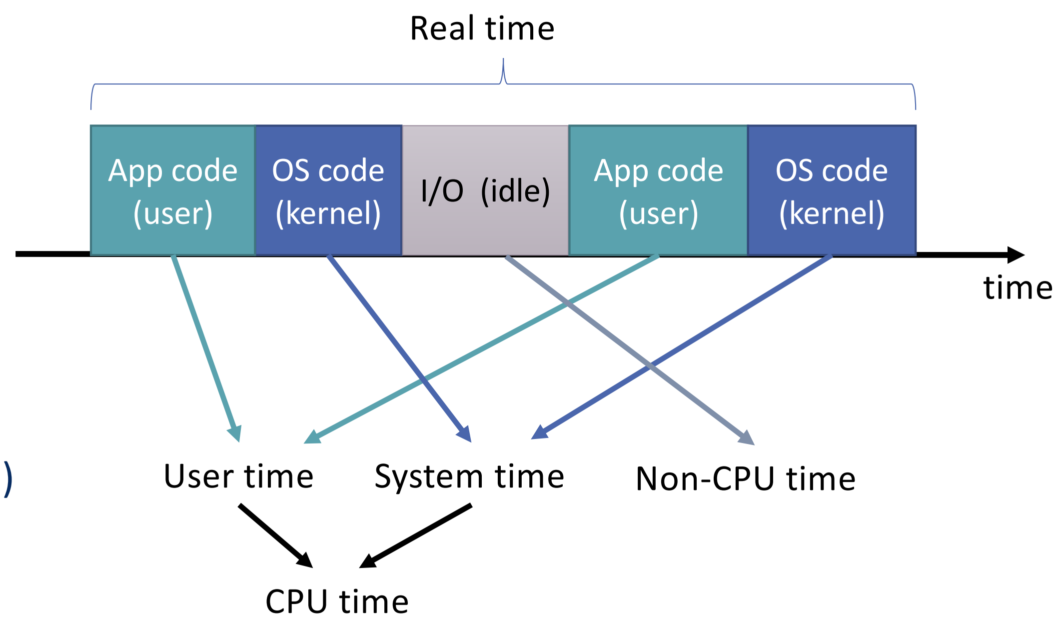


Basic and advanced measurement techniques

- Basic:
  - Time measurement
  - Application Throughput
  - Breakdown phases or iterations
- Advanced:
  - Profiling
  - Tracing

Time definitions

- **Real (or Wall Clock or Elapsed) Time** : actual elapsed time from a point in the past
- **CPU (or Process) Time**: time spent executing CPU instructions
    - **User Time** : time spent in user space
    - **System Time** : time spent in kernel space (OS)
- **Non-CPU Time**: time spent waiting (idle CPU) for: I/O, Virtualization, etc.



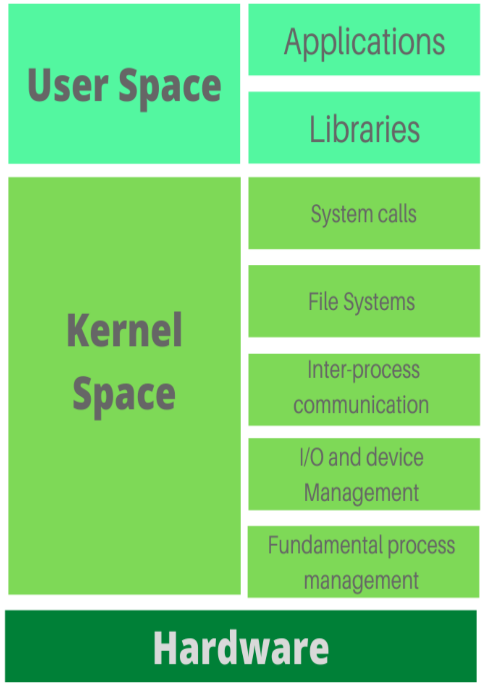
https://en.wikipedia.org/wiki/CPU_time

Time Measurement - Linux

- *time* command - Real, User and System times

```
$ time ./executable

real    0m1.057s
user    0m1.015s
sys     0m0.000s
```

- millisecond granularity, accuracy may vary betw
- real >= user + sys



Time measurement in C

- clock_gettime(CLOCK_MONOTONIC,..) - Real time
  - Nanosecond granularity - measuring in usec:

```
#include <time.h>
struct timespec start, end;

clock_gettime(CLOCK_MONOTONIC,&start);
<CODE TO MEASURE>
clock_gettime(CLOCK_MONOTONIC,&end);

double time_usec=(((double)end.tv_sec *1000000 + (double)end.tv_nsec/1000)
     - ((double)start.tv_sec *1000000 + (double)start.tv_nsec/1000));
printf("a=%d time: %.03lf\n", a, time_usec);
```

- http://btorpey.github.io/blog/2014/02/18/clock-sources-in-linux/

Execution Time measurement in Python

- Real Time:
  - granularity fractions of seconds – printing in seconds (Python 3.3)

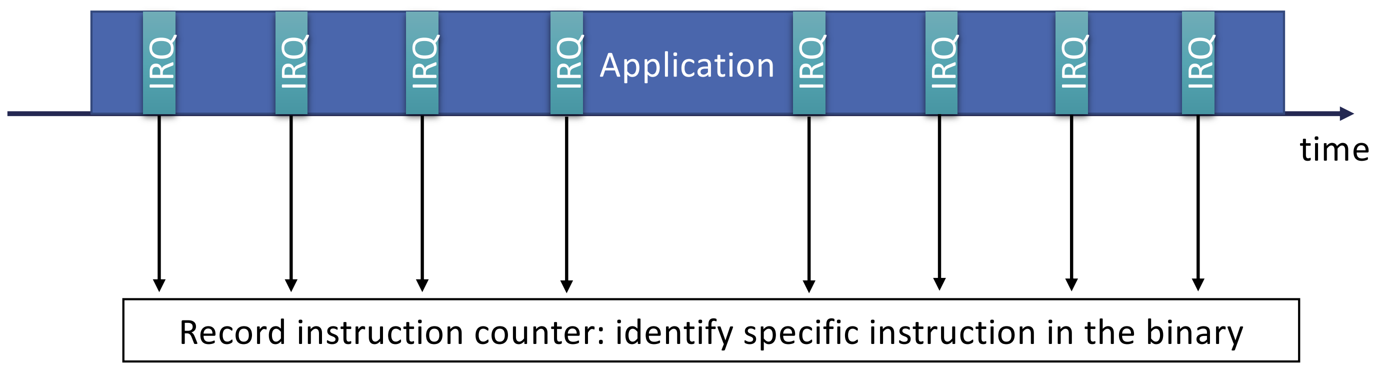
```
import time

start=time.monotonic()
<CODE TO MEASURE>
end=time.monotonic()

print("time: " + str(end-start))
```

- From Python 3.7: *time.monotonic_ns()* (granularity in nanoseconds)
- https://docs.python.org/3.7/library/time.html

Profiling

- Sampling:
  - Sample applications during execution to infer a statistical distribution: Example: approximate time spent in each instruction of the code
- Counting:
  - Count exact events
  - Software counters (implemented in kernel): count specific events
    - Example: count number of memory allocations (malloc())
  - **Hardware performance counters (aka Performance Counters)**
    - Counters maintained in registers
    - Examples: count number of L2 misses, Floating-point ops, Integer ops, number of branch mispredictions

Profiling - Sampling



- IRQ (interrupt request): interruption of the application to execute a different routine
- Profiling uses IRQs to register instruction counter and other metrics at regular intervals
- Relatively low-overhead, depending on IRQ frequency

Profiling – Sampling 1

```c
int
main() {
  long i,a=1;
  for ( i=0; i<1000000; i++)
    a += a*i;
  return a;
}
```

```
main  /mnt/nfs/nfsshare/user_homes/ufsecond/HPML/dummy
Percent │
        │
        │      Disassembly of section .text:
        │
        │      00000000004004cd <main>:
        │      main():
        │        push   %rbp
        │        mov    %rsp,%rbp
        │        movq   $0x1,-0x10(%rbp)
        │        movq   $0x0,-0x8(%rbp)
        │      ↓ jmp    2f
        │ 16:    mov    -0x8(%rbp),%rax
        │        lea    0x1(%rax),%rdx
 20.00  │        mov    -0x10(%rbp),%rax
 80.00  │        imul   %rdx,%rax
        │        mov    %rax,-0x10(%rbp)
        │        addq   $0x1,-0x8(%rbp)
        │ 2f:    cmpq   $0xf423f,-0x8(%rbp)
        │      ↑ jle    16
        │        mov    -0x10(%rbp),%rax
        │        pop    %rbp
        │      ← retq
```

- Example of Linux `perf annotate`
  - Annotated code showing time percentage
  - All the time associated with only 2 instructions ?
  - https://perf.wiki.kernel.org/index.php/Tutorial

Profiling – Sampling 2

```
int
main() {
  long i,a=1;
  for ( i=0; i<1000000000UL; i++)
    a += a*i;
  return a;
}
```

```
main  /mnt/nfs/nfsshare/user_homes/ufsecond/HPML/dummy
Percent│
       │
       │
       │      Disassembly of section .text:
       │
       │      00000000004004cd <main>:
       │      main():
       │        push   %rbp
       │        mov    %rsp,%rbp
       │        movq   $0x1,-0x10(%rbp)
       │        movq   $0x0,-0x8(%rbp)
       │      ↓ jmp    2f
  0.04 │16:     mov    -0x8(%rbp),%rax
  0.04 │        lea    0x1(%rax),%rdx
 75.86 │        mov    -0x10(%rbp),%rax
 11.99 │        imul   %rdx,%rax
  0.16 │        mov    %rax,-0x10(%rbp)
  0.04 │        addq   $0x1,-0x8(%rbp)
  0.56 │2f:     mov    -0x8(%rbp),%rax
       │        cmp    $0x3b9ac9ff,%rax
 11.31 │      ↑ jbe    16
       │        mov    -0x10(%rbp),%rax
       │        pop    %rbp
       │      ← retq
```

- Linux `perf annotate`
  - Annotated code showing time percentage
  - More samples => more realistic time association
  - https://perf.wiki.kernel.org/index.php/Tutorial

Profiling on a different system

Sampling 1

```
main  /root/a.out
Percent│
       │
       │
       │    Disassembly of section .text:
       │
       │    00000000000005fa <main>:
       │    main():
       │      push   %rbp
       │      mov    %rsp,%rbp
       │      movq   $0x1,-0x8(%rbp)
       │      movq   $0x0,-0x10(%rbp)
       │    ↓ jmp    2f
       │16:   mov    -0x10(%rbp),%rax
       │      lea    0x1(%rax),%rdx
       │      mov    -0x8(%rbp),%rax
  20.00│      imul   %rdx,%rax
  40.00│      mov    %rax,-0x8(%rbp)
  40.00│      addq   $0x1,-0x10(%rbp)
       │2f:   cmpq   $0xf423f,-0x10(%rbp)
       │    ↑ jle    16
       │      mov    -0x8(%rbp),%rax
       │      pop    %rbp
       │    ← retq
```

Sampling 2

```
main  /root/a.out
Percent│
       │
       │
       │    Disassembly of section .text:
       │
       │    00000000000005fa <main>:
       │    main():
       │      push   %rbp
       │      mov    %rsp,%rbp
       │      movq   $0x1,-0x8(%rbp)
       │      movq   $0x0,-0x10(%rbp)
       │    ↓ jmp    2f
  12.47│16:   mov    -0x10(%rbp),%rax
       │      lea    0x1(%rax),%rdx
       │      mov    -0x8(%rbp),%rax
  36.73│      imul   %rdx,%rax
  37.95│      mov    %rax,-0x8(%rbp)
  12.38│      addq   $0x1,-0x10(%rbp)
       │2f:   mov    -0x10(%rbp),%rax
   0.46│      cmp    $0x3b9ac9ff,%rax
       │    ↑ jbe    16
       │      mov    -0x8(%rbp),%rax
       │      pop    %rbp
       │    ← retq
```

Profiling Call Trees

```
extern int fa(unsigned size) {
    unsigned j,tmp=0;
    for (j=0;j<size;j++) {
       tmp+=j; tmp = tmp%5555555;
    }
}
extern int fsmall(unsigned size) {
    return fa(size);
}
extern int flarge(unsigned size) {
    return fa(size);
}
int main(void) {
    unsigned j, tmp;
    for (j=0;j<1000;j++) {
      tmp += fsmall(10);
      tmp += flarge(1000000);
    }
    return tmp;
}
```

Gprof, RHEL7.6

index	%time	self	children	called	name
[1]	100.0	0.00	65.56		main[1]
		0.00	32.78	1000/1000	fsmall[4]
		0.00	32.78	1000/1000	flarge[3]

- Gprof only samples the last stack entry
- Assembles call chains incrementally
- Assumes all calls to the same function F take the same time to derive call tree annotation!

https://ftp.gnu.org/old-gnu/Manuals/gprof-2.9.1/html_chapter/gprof_5.html

Tracing

```
int main(int argc, char **argv) {
    RECORD_TRACE_EVENT("after_main");
    struct timespec start, end;
    int i,a=1;
    clock_gettime(CLOCK_MONOTONIC,&start);
    RECORD_TRACE_EVENT("before_loop");
    for ( i=0; i < 1000000000; i++) {
        RECORD_TRACE_EVENT("in_loop");
        a += a*i;
    }
    RECORD_TRACE_EVENT("after_loop");
    clock_gettime(CLOCK_MONOTONIC,&end);
    RECORD_TRACE_EVENT("before_return");
    return 0;
}
```

```
              TRACE Example

 usec              event
[000012]          after_main
[000013]          before_loop
[000021]          in_loop
[000024]          in_loop
    …                 …
[012122]          in_loop
[012132]          after_loop
[012223]          before_return
```

- Explicit code instrumentation with tracing primitives
- Higher overhead than profiling
- Linux perf tracing can be applied to any code: Applications, Runtime, Kernel, Etc.
- Tracing utilities: strace (trace system calls made by an application), ftrace (trace execution flow of kernel functions)

strace

Strace monitors the system calls and signals of a specific program. It is helpful when you do not have the source code and would like to debug the execution of a program. strace provides you the execution sequence of a binary from start to end.

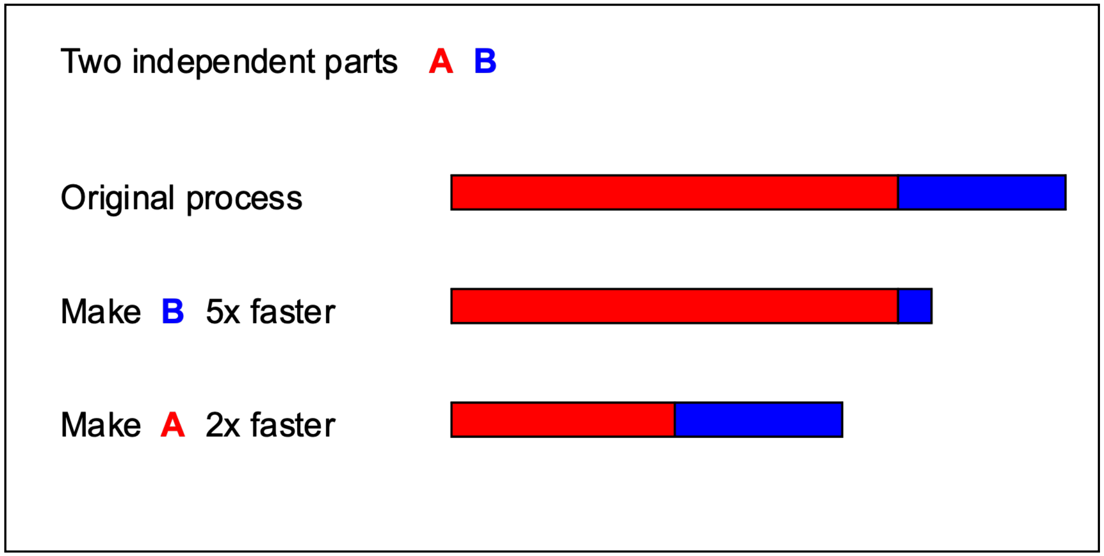
```
$ strace -e open ls
open("/etc/ld.so.cache", O_RDONLY)      = 3
open("/lib/libselinux.so.1", O_RDONLY)  = 3
open("/lib/librt.so.1", O_RDONLY)       = 3
open("/lib/libacl.so.1", O_RDONLY)      = 3
open("/lib/libc.so.6", O_RDONLY)        = 3
open("/lib/libdl.so.2", O_RDONLY)       = 3
open("/lib/libpthread.so.0", O_RDONLY)  = 3
open("/lib/libattr.so.1", O_RDONLY)     = 3
open("/proc/filesystems", O_RDONLY|O_LARGEFILE) = 3
open("/usr/lib/locale/locale-archive", O_RDONLY|O_LARGEFILE) = 3
open(".", O_RDONLY|O_NONBLOCK|O_LARGEFILE|O_DIRECTORY|O_CLOEXEC) = 3
Desktop  Documents  Downloads  examples.desktop  libflashplayer.so
Music  Pictures  Public  Templates  Ubuntu_OS  Videos
```

Performance optimization methodology (2): Analysis

Amdahl's Law

- $S(p, s) = \frac{1}{(1-p)+p/s}$
  - *S*: speedup of the entire application (or runtime, OS, etc.)
  - *p*: portion of the execution time that is spent in the code section before improvement (if time for *p* is high the section is called **critical section**)
  - *s*: speedup of the improved code section
- Overall speedup is limited by how much time the improved code takes compared to the rest

Amdhal's law effect example



From: https://en.wikipedia.org/wiki/Amdahl%27s_law

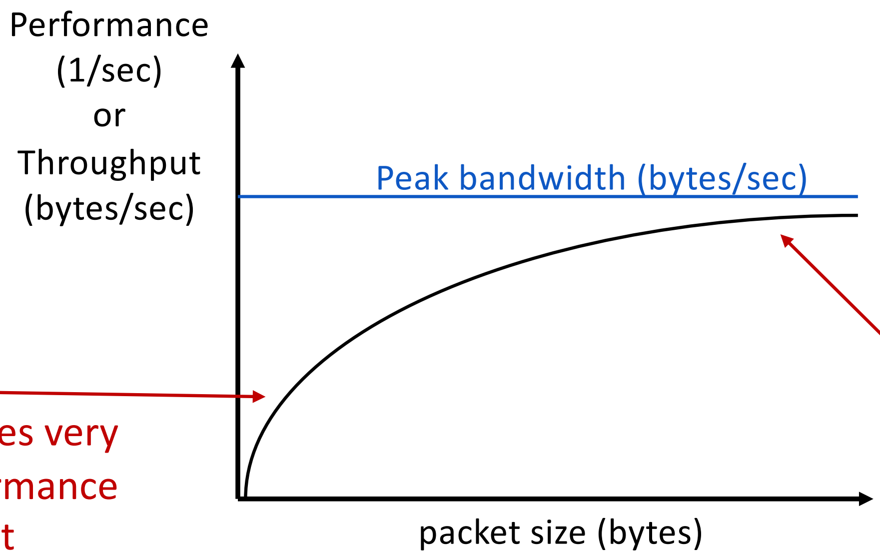
Performance Analysis Step

1. Identify **Critical Path**: the section of the program that accounts for most of the time (high value of Amdahl's p)
   - Critical Path characteristics: very slow to execute (high latency) and/or executed many times
   - Use output of performance measurement step (profiling, tracing, etc.)
   - Verify hypothesis of critical path: comment code and run again

2. Identify the **Bottleneck**: the **system resource** that affects the execution time of the critical path
   - Need to understand software/hardware architecture
   - Bottleneck type: **Data Movement** vs. **Computation**

Data Movement and Packet Size

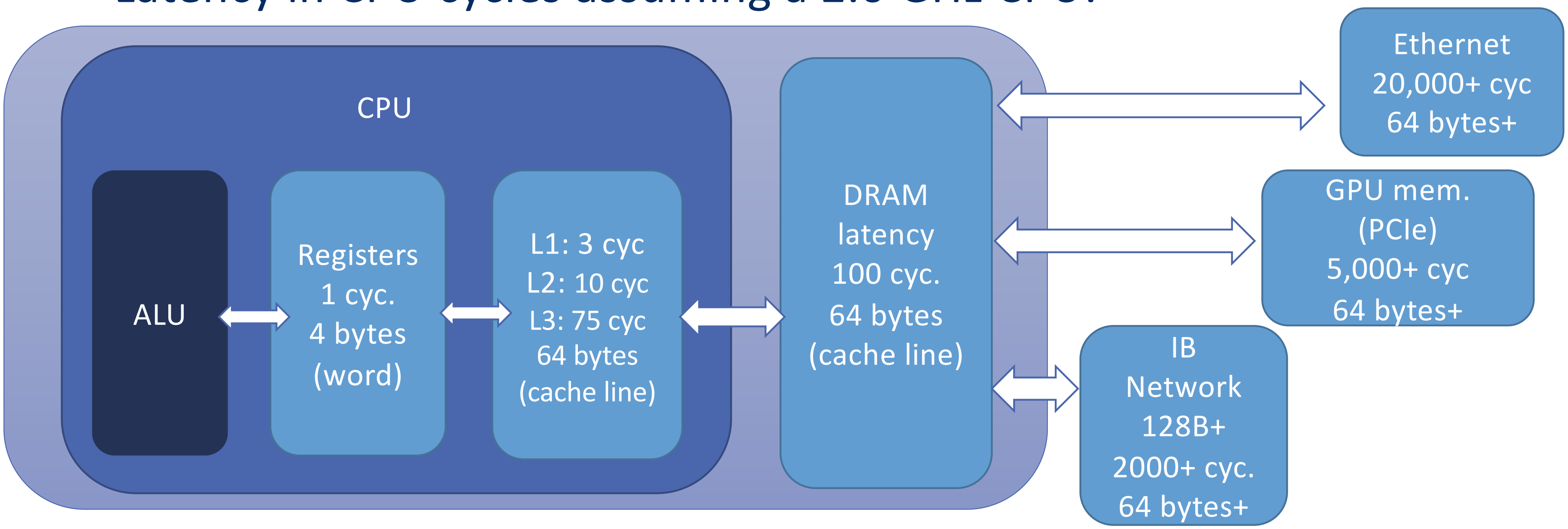
- True for any **Data Movement**: Network, PCIe, DRAM, etc.



- SMALL Packets:
  - **Low Latency** becomes very important for performance
  - Full Bandwidth is not reached
- LARGE Packets:
  - **High Bandwidth** becomes very important for performance
  - Latency is not relevant

Data movement Locality Principle - Latency

- Latency in CPU cycles assuming a 2.0 GHz CPU:

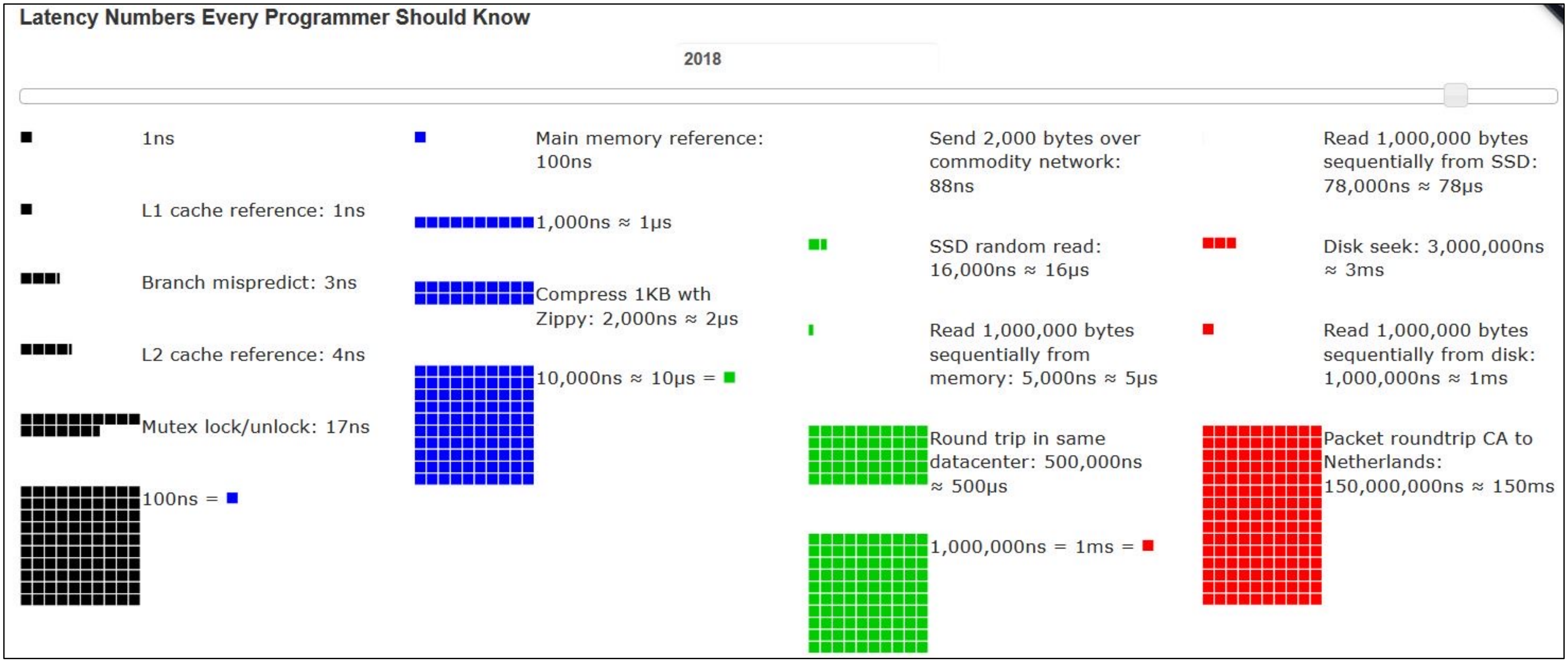


- small granularity
- low latency
- high bandwidth

- large granularity
- high latency
- low bandwidth

Latencies every programmer should know: link

Latency values over the years – very cool tool!

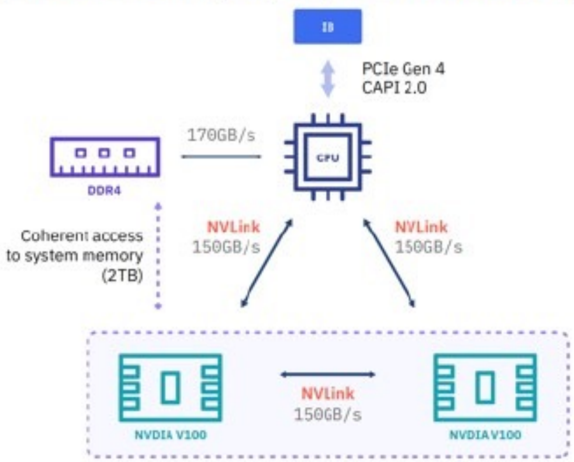


- https://people.eecs.berkeley.edu/~rcs/research/interactive_latency.html

Data Movement Locality Principle - Bandwidth

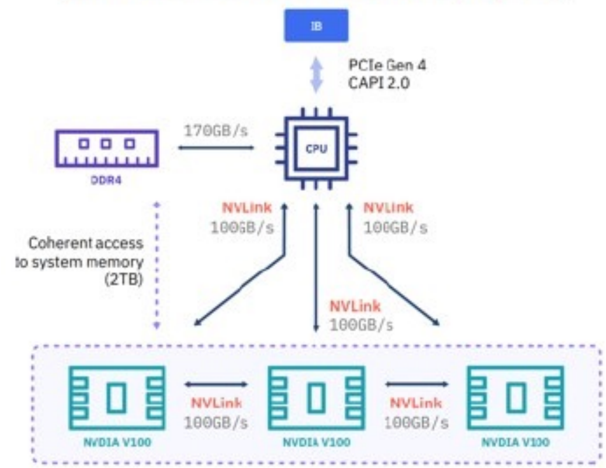
- IBM POWER9 + NVIDIA Volta GPU

4 GPUs - Air (4Q'17)/Water Cooled (2Q'18)



- Up to 4 GPUs, air/water cooled options
- 150GB/s of bandwidth from CPU-GPU

6 GPUs - Water Cooled (2Q'18)

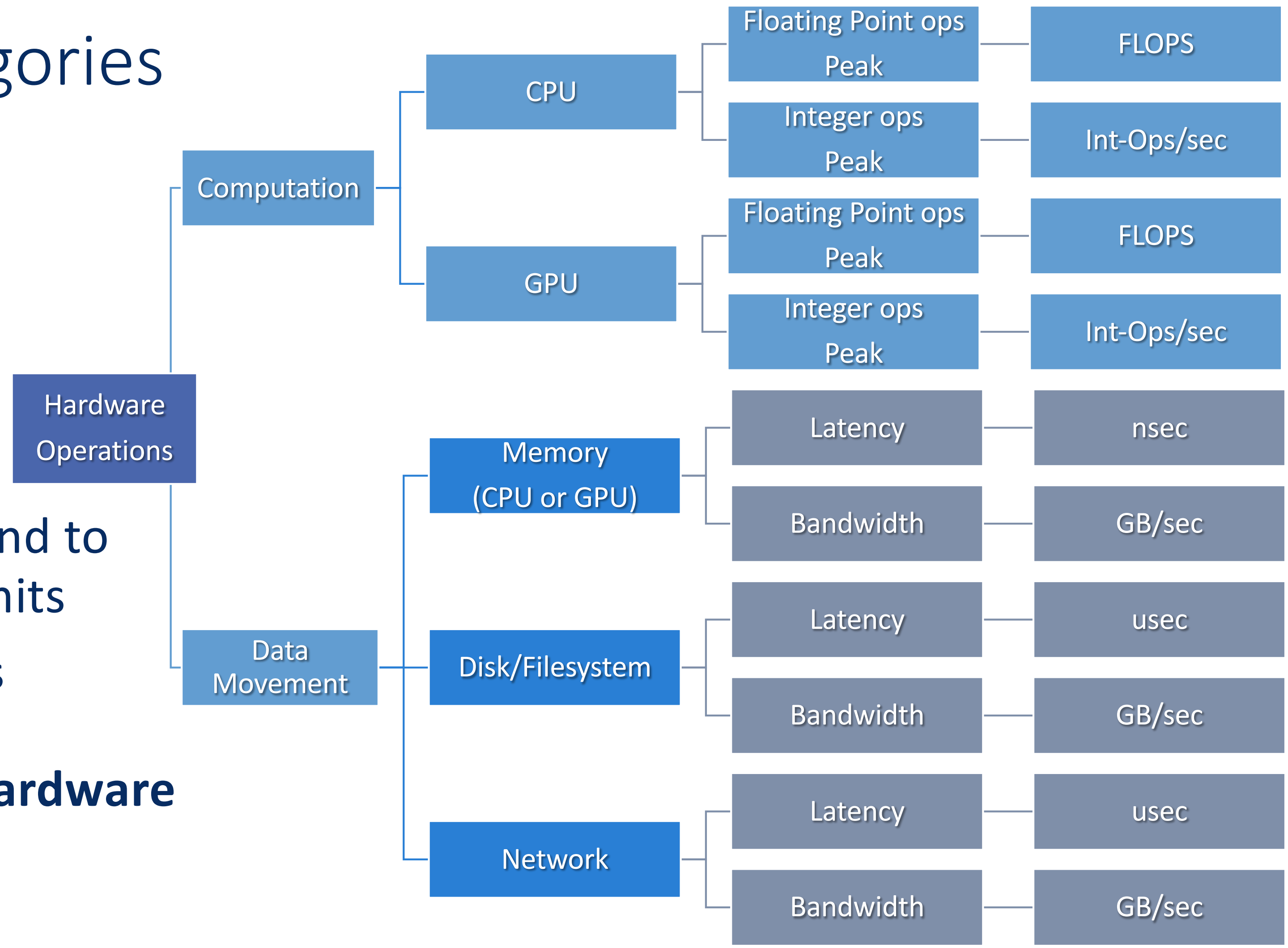


- Up to 6 GPUs, water cooled only
- 100 GB/s of bandwidth from CPU-GPU

- Coherent access to system memory
- PCIe Gen 4 and CAPI 2.0 to InfiniBand
- Water cooled options available in 2Q'18

- (Bi-directional bandwidth)

Bottleneck categories



- Bottlenecks correspond to specific Hardware Limits
- Performance Analysis Problem: **How far is performance from Hardware Limits?**

Performance Models Objectives

- Identify performance bottlenecks
- Determines Hardware Limits to Optimization
  - Determines how fare we are from hardware limits
  - Motivate algorithmic changes
- Project performance on future hardware or applications

Peak FLOPS

- Peak FLOPS depend on:
  - Compute unit architecture: CPU, GPU, TPU, FPGA etc.
  - #cores and #threads
  - Clock Frequency
  - Precision: DP (64 bit), SP (32 bit), HP (16 bit)
  - SIMD instructions in the cores: Intel AVX, IBM Altivec
- CPU formula for Peak FLOPS: $\#tot\_cores \cdot \frac{cycles}{seconds} \frac{FLOPs}{cycles}$

- see https://en.wikipedia.org/wiki/FLOPS

Performance Model – Constants (HW specs)

- Examples of HW Specs for the performance model
  - **CPU peak DP/SP FLOPS**: GFLOPS/s
  - **DRAM peak Bandwidth**: GB/s
  - **GPU peak DP FLOPS**: TFLOPS/s
  - **HBM peak Bandwidth**: TB/s
- How to obtain:
  - Vendor hardware specifications
  - Alternative: run micro-benchmarks for compute and memory (lower bounds than specs)

Performance Model - Variables

- Actual Experimental Measurements :
  - Computation (CPU/GPU) Performance: FLOPS
  - Memory throughput: GB/s or TB/s

- How to measure:
  - FLOPS: **hw performance counters** for FLOP divided by time
  - GB/s: **hw performance counters** for memory-ops divided by time

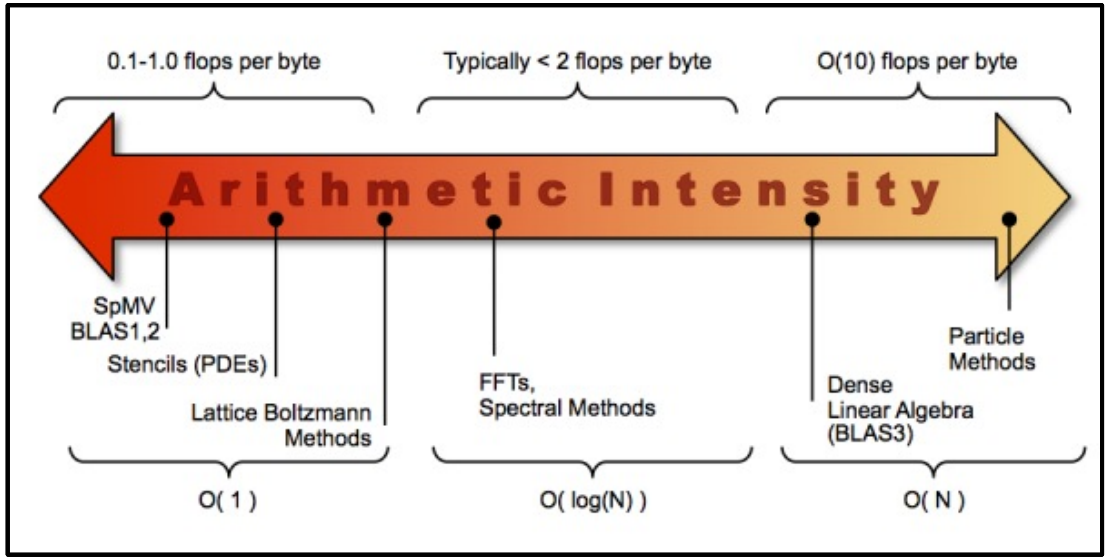
Roofline Performance Model (1)

- Throughput-based model
- Developed at
  DOE Lawrence Berkeley Labs
- Metrics:
  - Peak FLOPS
  - Memory Bandwidth: $\frac{data}{time}\left[\frac{bytes}{sec}\right]$
  - Arithmetic Intensity (program property):

    $\frac{\#arithmetic\ ops}{DRAM\ data}\left[\frac{FLOP}{bytes}\right]$

  (bytes as seen from DRAM)

    note: FLOP ≠ FLOPS



https://crd.lbl.gov/departments/computer-science/PAR/research/roofline

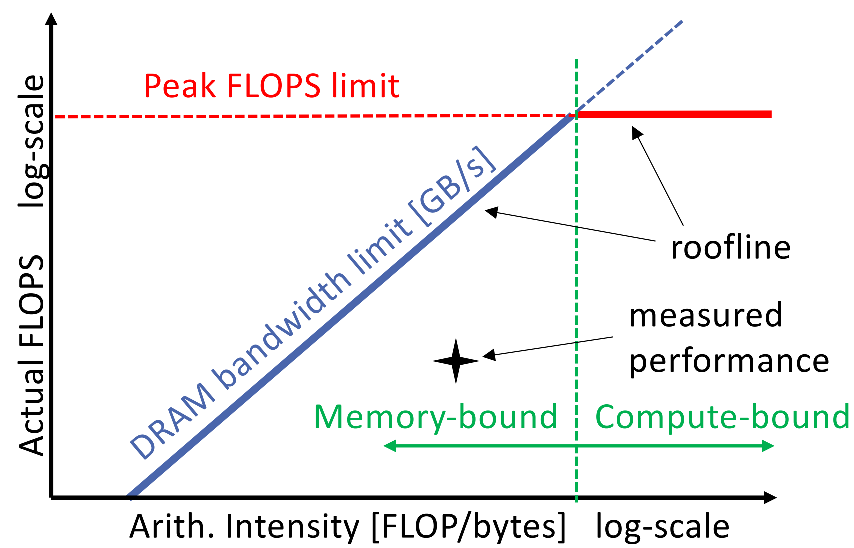
Arithmetic Intensity

- The ratio between the number of executed operations and the number of bytes transferred between the CPU and the memory is called arithmetic intensity.

- Smaller arithmetic intensity means a larger pressure on the memory subsystem, and conversely, larger arithmetic intensity means a larger pressure on the CPUs computational resources.

Roofline Performance Model

- Actual FLOPS are limited first by DRAM bandwidth (memory-bound) and then by CPU (or GPU) Peak FLOPS (compute-bound)
- Actual measured performance is below the roofline
- Depending on Arithmetic Intensity:
  - **memory-bound** code
  - **compute-bound** code



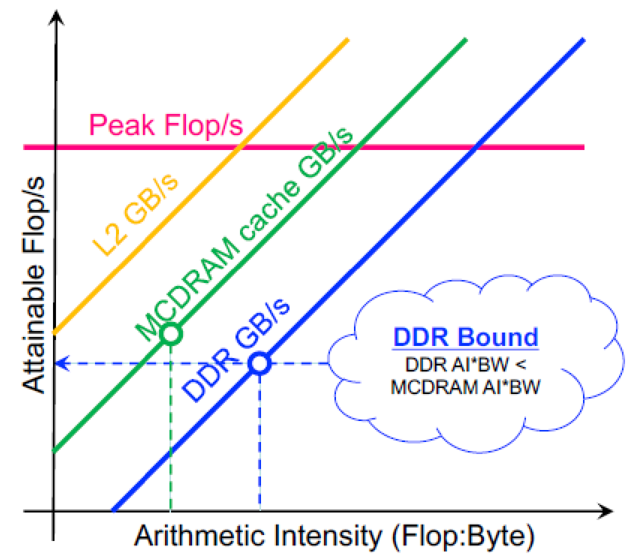
- **Log-log scale is used for clarity**

More complex Roofline Models

- We considered a basic **DRAM-only Roofline Model:**
  - Bytes as seen from DRAM access

- Not covered: Hierarchical Roof-line
  - for problems that fit in the cache we can add:
    - L1, L2, L3 bandwidth
  - Each Cache Level has its own A.I. (different bytes going through that level of the mem. Hierarchy)

- Also not covered: Cache-aware Roof-line
  - FLOP/bytes as seen from CORE
  - Different roof but same A.I.
  - Need to know from which level data is coming
  - http://www.inesc-id.pt/ficheiros/publicacoes/9068.pdf



Hierarchical Roofline
from: LBNL (SC17 Roofline Model Workshop slides)

crackle1.cims.nyu.edu compute node @NYU

- Intel Xeon E5630@2.53GHz performance:
  1. #cores: 4
  2. LLC (L3) size: 12MB
  3. Clock frequency: 2.53GHz
  4. **DRAM peak bandwidth**: 25.6 GB/s
  5. **CPU Peak FLOPS**: 81.3 DP GFLOPS – 162.56 SP GFLOPS

- DRAM peak bandwidth:
  - https://ark.intel.com/products/47924/Intel-Xeon-Processor-E5630-12M-Cache-2_53-GHz-5_86-GTs-Intel-QPI

- CPU peak FLOPS :
  - FLOPS = frequency * total_cores * FLOPS/cyc
  - https://en.wikipedia.org/wiki/FLOPS (architectures list - this is a *Sandy Bridge*)

$ ssh username@access.cims.nyu.edu
$ ssh crackle1.cims.nyu.edu
$ cat /proc/cpuinfo

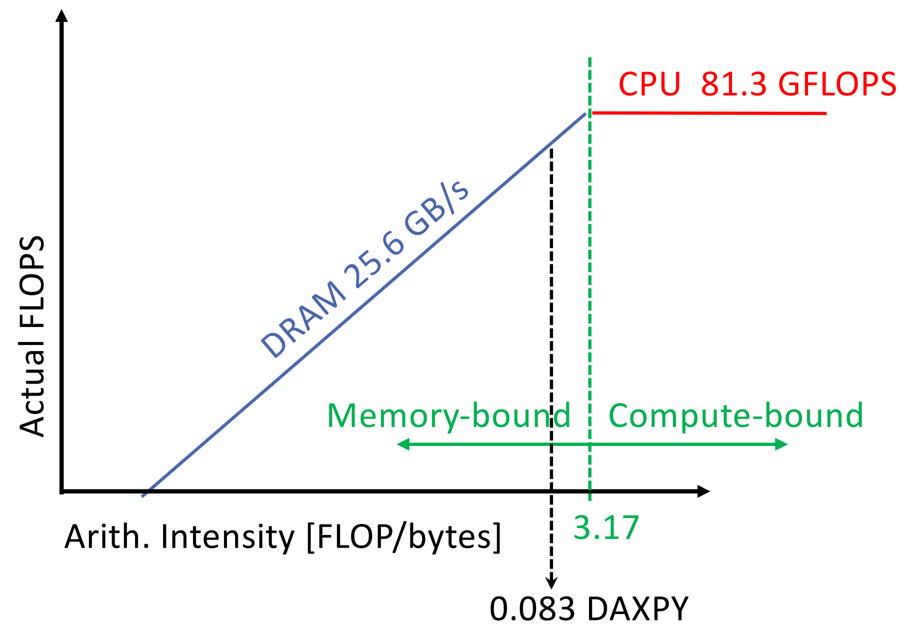
```
processor     : 0
vendor_id     : GenuineIntel
cpu family    : 6
model         : 44
model name    : Intel(R) Xeon(R) CPU E5630  @ 2.53GHz
stepping      : 2
microcode     : 0x15
cpu MHz       : 2527.014
cache size    : 12288 KB
physical id   : 0
siblings      : 8
core id       : 10
cpu cores     : 4
```

Roofline Model Example – crackle1

- *crackle1*:
  - CPU peak: 81.3 DP GFLOPS
  - DRAM peak BW: 25.6 GB/s
- DAXPY code:

```
for (i=0;i<N;i++) {
 Z[i]= A * (X[i] + Y[i])
}
```

- Y,A,X are 64 bit float (DP)
- DRAM and CPU cross at:
  - *81.3 GFLOPS /25.6 GB/s = 3.17 FLOP/byte*
  - CPU_peak/DRAM_BW
    - Where DP-bytes=8, SP-bytes=4, HP-bytes=2

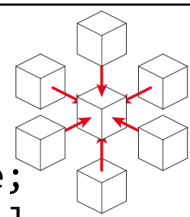


- A.I. = 2 FLOP/(3*8) bytes = 0.083 FLOP/byte
- Result: *0.083* < *3.17* => **Memory-bound** => how far for DRAM BW?

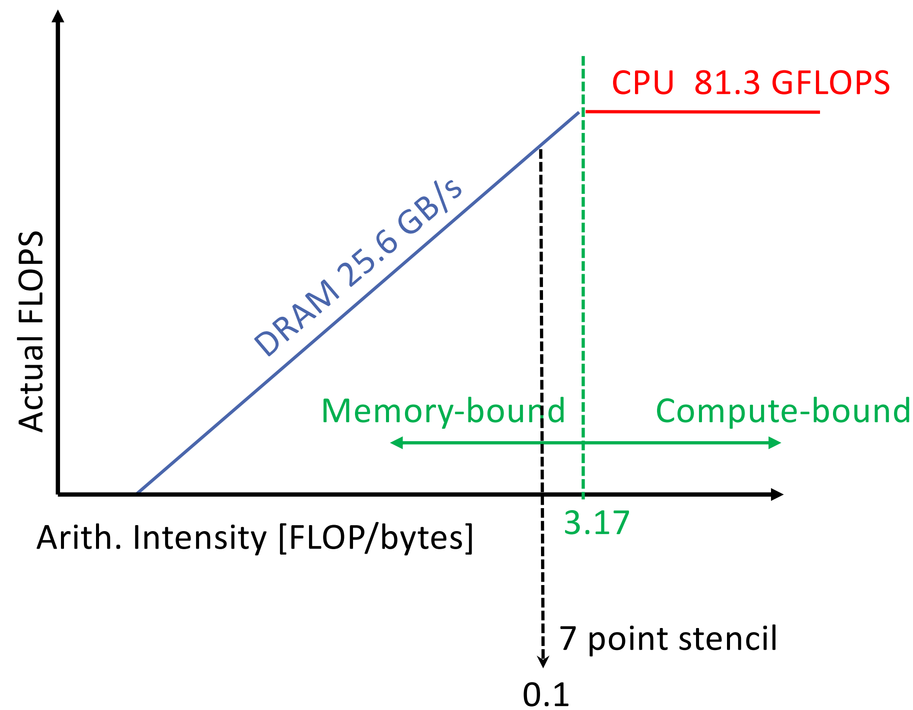
Roofline Model Example (2) – crackle1

- 7-Points Stencil code:

```
for(k=1;k<N;k++){
for(j=1;j<N;j++){
for(i=1;i<N;i++){
 int ijk = i+j*jStride+k*kStride;
 new[ijk] =-6.0*old[ijk         ]
               +old[ijk-1       ]
               +old[ijk+1       ]
               +old[ijk-jStride ]
               +old[ijk+jStride ]
               +old[ijk-kStride ]
               +old[ijk+kStride ];}}}
```

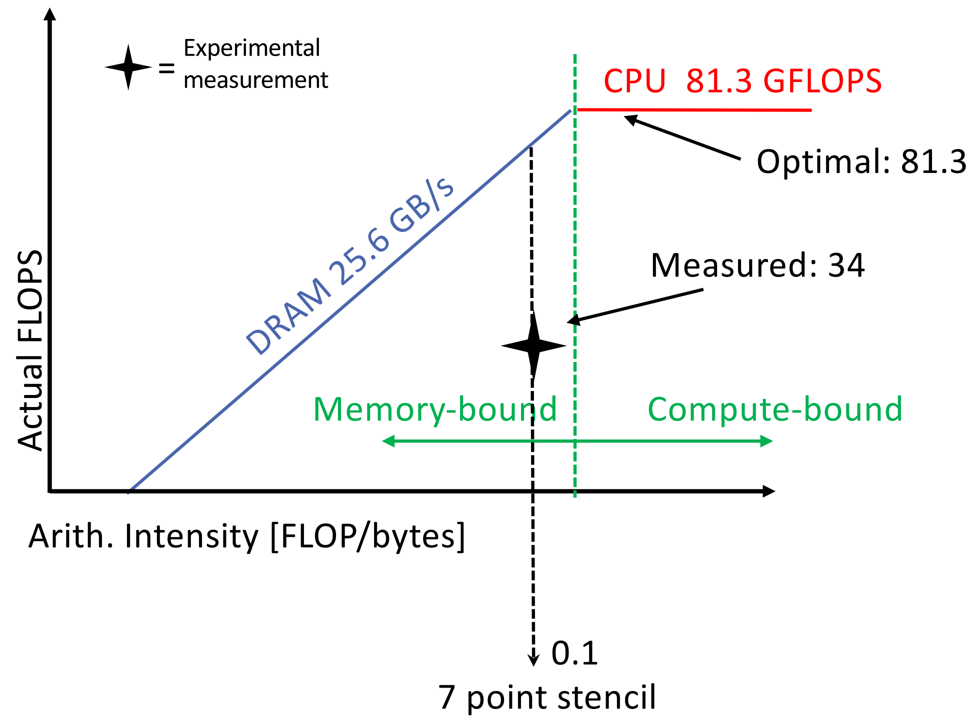


- 7 DP flops (new[] is 64bits)
- 8 memory references
- DRAM/CPU cross = 3.17 FLOP/byte
- AI = 7 FLOP/(8*8) bytes = 0.109 FLOP/byte
- Result: *0.109* < *3.17* => still **Memory Bound** => how to optimize?



Next steps - Optimization

1. Know the limitation from the model: CPU vs. DRAM
2. Measure actual performance:
   Example: 34 GFLOPS
3. Optimize to get close to max FLOPS! (81.3 GFLOPS)



Performance optimization methodology (3): Optimization

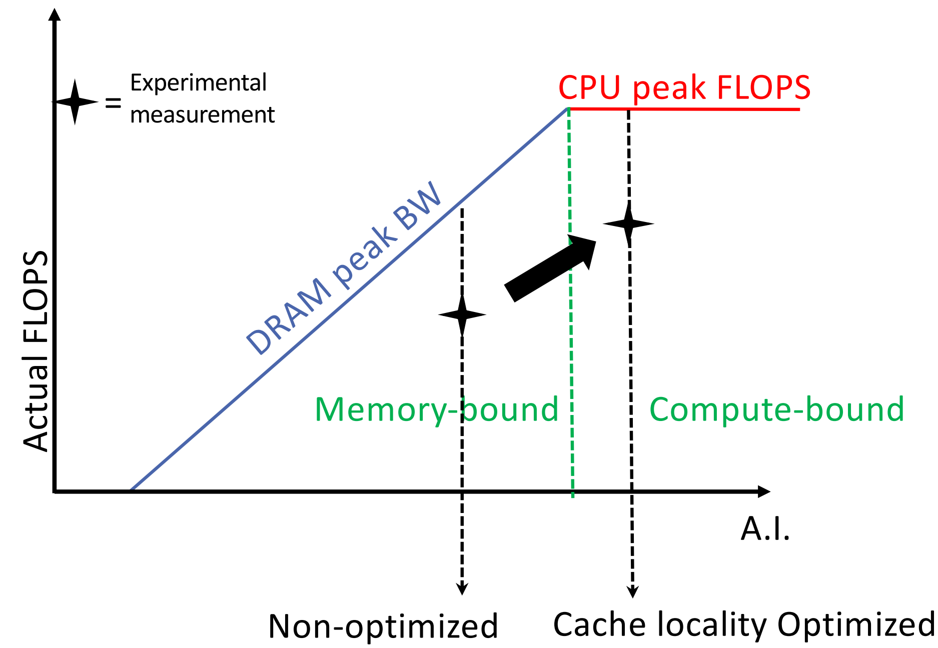
Two ways to Performance

- Reduce latency (do one operation faster):
  - Data access latency reduction

- Increase Parallelism (do more operations at the same time):
  - Vectorization
  - Instruction Level Parallelism
  - Thread Level Parallelism
  - Multi-core design
  - Computer Clusters

Optimization Example: Cache Blocking

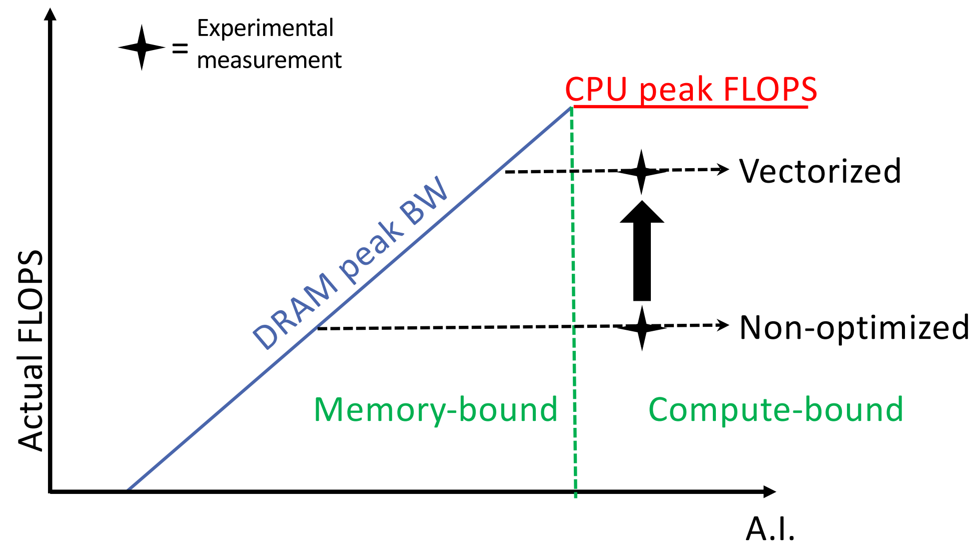
- Observation: Cache access latency is about 10x lower than DRAM and BW is much higher
- Optimization (cache blocking):
  - Divide program data structures in blocks of the **cache size**
  - Work on each block before switching to the next
  - Less DRAM bytes: **cache is filtering DRAM accesses**
  - A.I. [FLOPS/(DRAM bytes)] is higher
- Result:
  - Bottleneck moves: Code (may) become compute-bound with higher FLOPS!



https://www.intel.com/content/www/us/en/developer/articles/technical/cache-blocking-techniques.html

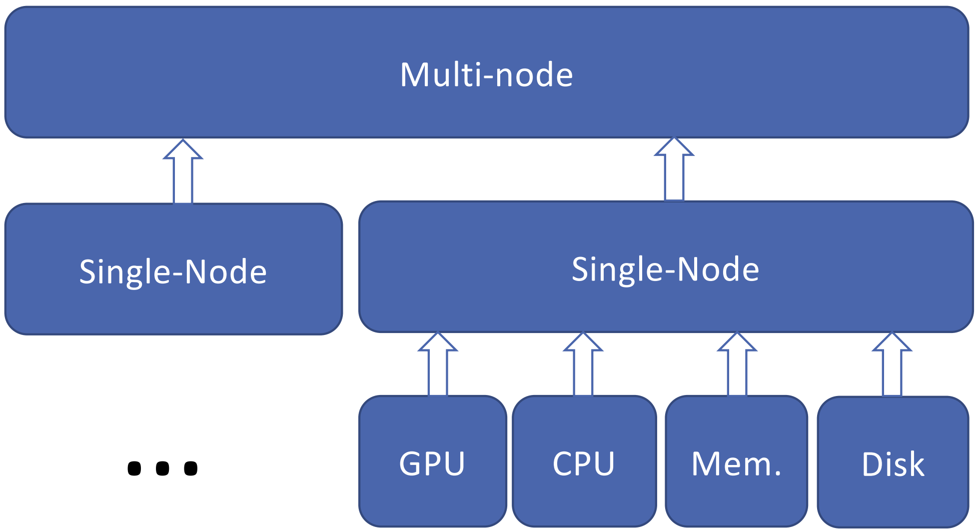
Optimization Example: Vectorization

- Observation: SIMD instructions execute multiple FLOP (2,4,8,16..) with 1 instruction => higher FLOPS
- Optimization (Vectorization):
  - Replace normal code with SIMD instructions
  - **Hint:** use math libraries like BLAS (CPU) or cuDNN (GPU) and they will do it for you!
- Result:
  - Code reaches higher FLOPS!



Hierarchical Perf. Optimization – (next lessons)

- Single Node optimizations:
  - CPU:
    - Vectorize/SIMD optimizations
    - CPU Cache/Memory optimizations
    - Multi-core scalability/parallelism
  - GPU:
    - SM Optimizations
    - SM Cache/Memory optimization
  - Disk and IO
- Multi-node optimizations:
  - Parallelism exposure
  - Domain decomposition
  - Load-balancing
  - Reduce synchronizations
  - Reduce collectives



Performance Optimization Methodology Recap

1) Measure
- 1.1) **Time** various sections of the code
- 1.2) Measure resource usage (**Perf Counters**)

2) Analyze
- 2.1) Identify **Critical Path** based on Time Measurement
- 2.2) Identify **Bottleneck** with **Roofline Model**

3) Optimize
- 3.1) Apply correct **Optimization Technique** based on bottleneck

Floating Point Errors

- Error: $E = |f(x)-F(x)|$
  - $F(x)$ is the correct result, $f(x)$ is the numerically computed result
- Relative error: $R = E/|F(x)|$
  - Floating point ‘roundoff’ relative error depends on number of bits in the mantissa!
- Cancellation
  - $C = A+B$ ➔ may result in $C==A$ for $B << A$
- Catastrophic cancellation
  - $C = B+A-A$ ➔ may result in 0 for $B<<A$, relative error is 1
  - $C = 1/(B+A-A)$!

Floating Point Error Example

- IEEE standard 754
  - FP32 1 bit sign + 8 bit exp + 23 bit mantissa, bias 127
  - FP64 1 bit sign + 11 bit exp + 52 bit msantissa, bias 1023
  - $(-1)^S * 1.M * 2^{\{E-bias\}}$

$$\frac{1}{3} \cong (-1)^0 * (1.3333333) * 2^{125-127} = 0.333333325$$

$$R = \left(\frac{1}{3} - 0.333333325\right) * 3 \cong 2.5 * 10^{\{-8)}$$

0111 1101b = 125

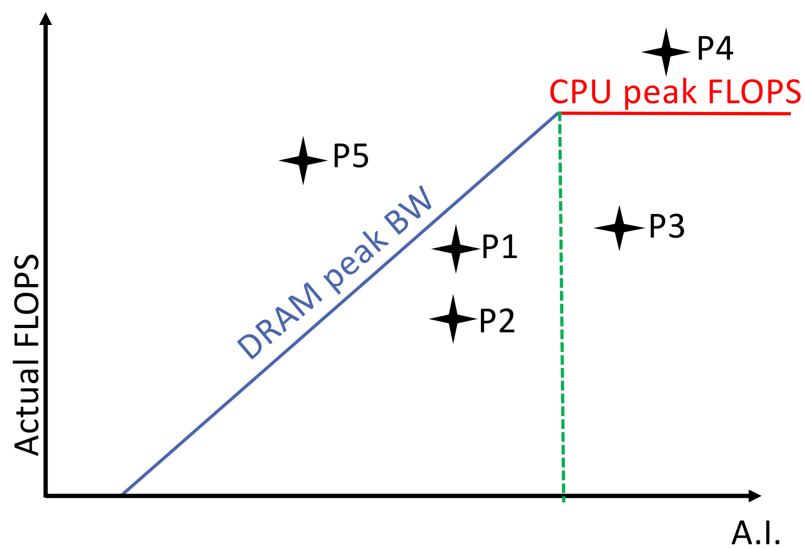
011 0010 1101 1100 1101 0101b = 3333333

Lesson Key Points

- ML Performance Factors
- Performance Optimization Methodology:
  1. Measurement: Metrics, Time/Resources and Techniques
  2. Analysis: Amdahl's Law, Bandwidth/Latency, Roofline Model
  3. Optimization (in relationship to Roofline model)

Acknowledgements

- The lecture material is prepared by Giacomo Domeniconi, Parijat Dube, Ulrich Finkler, and Alessandro Morari from IBM Research, USA.


P4
CPU peak FLOPS
P5
DRAM peak BW
P1
P3
P2
Actual FLOPS
A.I.