MIT 18.06 Practice Exam 1, Spring 2023
Strang and Horning

Your name: ______________________________
(*printed*)

Student ID: ______________________________

Recitation: ______________________________

Problem 1 (6+6+10+8=30 points):

Record your answers in the alloted spaces. You may use the rest of this page and the following for your calculations. Consider the matrix $A = LPU$ given by:

$$A = \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 0 & 1 \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{P} \underbrace{\begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{U}.$$

(a) The matrices L (______), P (______), and U (______) are invertible. (Write True or False next to each).

(b) Write A^{-1} in terms of L^{-1}, P^{-1}, and U^{-1} (without computing any numbers):

$$A^{-1} =$$

(c) What right-hand-side vector b should one choose so that $Ax = b$ has solution $x =$ (**first** column of A^{-1})?

$$b = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

(d) Compute x, the first column of A^{-1}:

$$x = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}.$$

(blank page for your work if you need it)

Problem 2 (16+4+4+12=36 points):

Record your answers in the alloted spaces. You may use the rest of this page and the following for your calculations.

(a) Compute the factorization $A = LU$ of the matrix

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \\ 4 & 2 & 1 & 0 \\ 8 & 4 & 2 & 1 \end{pmatrix} = LU.$$

$$L = \begin{pmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{pmatrix} \qquad U = \begin{pmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{pmatrix}$$

(b) Put an X next to the correct answer. The matrix A is

(i) invertible ______

(ii) not invertible ______

(c) The rank of A is ______.

(d) Solve $Ly = b$ for the vector $b = \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix}^T$, and then solve $Ux = y$, so that $Ax = LUx = Ly = b$.

$$y = \begin{pmatrix} \quad \\ \\ \\ \\ \end{pmatrix} \qquad x = \begin{pmatrix} \quad \\ \\ \\ \\ \end{pmatrix}$$

(blank page for your work if you need it)

Problem 3 (10+4+4+10+6=34 points):

Record your answers in the alloted spaces. You may use the rest of this page and the following for your calculations.

(a) Compute a new factorization of the matrix

$$A = \begin{pmatrix} 3 & 2 & 1 & 5 & 2 \\ 2 & 2 & 0 & 4 & 0 \\ 1 & 0 & 1 & 1 & 2 \end{pmatrix}.$$

Enter the linearly independent rows of A (in order from top to bottom) into the factor R_{new} and choose the columns of C_{new} so that $A = C_{new}R_{new}$:

$$C_{new} = \begin{pmatrix} \quad & \quad \\ & \\ & \end{pmatrix} \qquad R_{new} = \begin{pmatrix} \quad & \quad \\ & \\ & \end{pmatrix}$$

(b) Put an X next to the correct answer. The column space of A is

(i) a line ______

(ii) a plane ______

(iii) the whole 3D space ______

(iv) none of the above ______

(c) Put an X next to the correct answer. The row space of A is

(i) a line ______

(ii) a plane ______

(iii) a 3D subspace ______

(iv) none of the above ______

(d) Use $A = C_{new}R_{new}$ to compute Ax for the vector $x = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix}^T$ in two steps:

$$R_{new}x = \begin{pmatrix} \quad \\ \\ \end{pmatrix} \qquad Ax = C_{new}(R_{new}x) = \begin{pmatrix} \quad \\ \\ \end{pmatrix}$$

(e) If we multiply the "dot-product" way, $y = R_{new}x$ requires ______ dot product(s) bewteen 5×1 vectors and $Ax = C_{new}y$ requires 3 dot product(s) between ______ $\times 1$ vectors.

(blank page for your work if you need it)