Review of Chapter 1 in ILA (6th Edition) / Gilbert Strang

1. Multiplying $A\boldsymbol{x}$ by rows or by columns

$$A\boldsymbol{x} = \begin{bmatrix} \text{row } 1 \\ \vdots \\ \text{row } m \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \end{bmatrix} = \begin{bmatrix} \text{row } 1 \cdot \boldsymbol{x} \\ \vdots \\ \text{row } m \cdot \boldsymbol{x} \end{bmatrix} \quad \text{Dot products}$$

$$A\boldsymbol{x} = \begin{bmatrix} \boldsymbol{a}_1 & \cdot\cdot & \boldsymbol{a}_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1\boldsymbol{a}_1 + \cdots + x_n\boldsymbol{a}_n \quad \begin{array}{c}\text{Combination} \\ \text{of columns}\end{array}$$

Column space of $A =$ **all combinations of columns = all** $A\boldsymbol{x}$

2. Multiplying $\boldsymbol{AB} = (m \times n)\,(n \times p) = m \times p$ (4 ways)

Key idea: Column j of $AB = A$ times Column j of B

Row i of $AB =$ Row i of A times B

Associative Law $(\boldsymbol{AB})\boldsymbol{Z} = \boldsymbol{A}(\boldsymbol{BZ})$

Row operations using A Then column operations using Z.

Column operations using Z Then row operations using A.

3. Factorization of $\boldsymbol{A}$ **Every matrix $\boldsymbol{A = CR}$** $= (m \times r)\,(r \times n)$

$\boldsymbol{C}$ contains the first r independent columns of A

$\boldsymbol{R}$ combines those columns to give all columns of A

Column space of $\boldsymbol{A}$ = Column space of $\boldsymbol{C}$

Row space of $\boldsymbol{A}$ = Row space of $\boldsymbol{R}$

$$\boxed{\boldsymbol{r} = \textbf{Column rank} \text{ of } A = \textbf{Row rank} \text{ of } A}$$

4. Rank 1 matrix $A =$ (One column in C) (One row in R)

$$\boldsymbol{A} = \begin{bmatrix} 1 & 4 & 6 \\ 2 & 8 & 12 \\ 3 & 12 & 18 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 \end{bmatrix} = \boldsymbol{CR}$$

Column space = line Row space = line Rank $r = 1$

All columns are parallel $\Leftrightarrow$ All rows are parallel

5. Dot products + Lengths + Angles: Vectors $\boldsymbol{v}$ and $\boldsymbol{w}$

$\boldsymbol{v} \cdot \boldsymbol{w} = v_1w_1 + v_2w_2 + \cdots + v_nw_n$

$\boldsymbol{v} \cdot \boldsymbol{v} = ||\boldsymbol{v}||^2 = (\text{Length of vector } \boldsymbol{v})^2 = v_1^2 + \cdots + v_n^2$

$\boldsymbol{v} \cdot \boldsymbol{w} = ||\boldsymbol{v}||\,||\boldsymbol{w}|| \cos\theta =$ Law of Cosines on page 14

$|\boldsymbol{v} \cdot \boldsymbol{w}| = |\boldsymbol{v}^{\mathrm{T}}\boldsymbol{w}| \le ||\boldsymbol{v}||\,||\boldsymbol{w}||$ **Schwarz inequality**

$||\boldsymbol{v} + \boldsymbol{w}|| \le ||\boldsymbol{v}|| + ||\boldsymbol{w}||$ **Triangle inequality**

Chapter 2 Square $\boldsymbol{n}$ by $\boldsymbol{n}$ matrices with rank $\boldsymbol{n}$