LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 4: EXERCISES.

My name is Vasily Krylov. If you have any questions or comments, please feel free to ask me by email (krvas@mit.edu) or during my office hours (Thursday 5 p.m. - 7 p.m., Room 2-361).

1. Problem 1

Find bases and dimensions for the four subspaces ($C(A^T), N(A), C(A), N(A^T)$) associated with the following matrix A:

(a)

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}.$$

(b)

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 5 & 8 \end{bmatrix}.$$

2. Problem 2

(a) If a 7 by 9 matrix has rank 5, what are the dimensions of the four subspaces?

(b) If a 3 by 4 matrix has rank 3, what are the column space ($C(A)$) and the left nullspace ($N(A^T)$)?

3. Problem 3

For which numbers c and d do this matrix have rank **2**:

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix}?$$

Hint: consider four cases ($c \neq 0 \neq d$, $c = 0 \neq d$, $d = 0 \neq c$, $c = 0 = d$). Use echelon form together with the fact that the rank of a matrix written in echelon form is equal to the number of nonzero rows.

4. Problem 4

Suppose A is a symmetric matrix ($A = A^T$).

If $Ax = 0$, $Az = 5z$, which subspaces contain these "eigenvectors" x and z? Show that x and z are perpendicular.

Hint: use that $C(A^T)$ is perpendicular to $N(A)$.