LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 5: EXERCISES.

My name is Vasily Krylov. If you have any questions or comments, please feel free to ask me by email (krvas@mit.edu) or during my office hours (Thursday, 5 p.m. - 7 p.m., Room 2-361).

1. Problem 1

Find the projection matrix P onto the plane $2x - y + z = 0$.

Hint: find a matrix A such that $C(A)$ is the plane $2x - y + z = 0$ (columns of A should form a basis of the plane). Use the formula $P = A(A^T A)^{-1} A^T$.

Hint: use the formula $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

2. Problem 2

Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}$, solve the "normal equation" $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$ for $\mathbf{b} = (1, 1, 1)$.

Hint: you can either project $\mathbf{b}$ onto $C(A)$ to get $\mathbf{p}$ and then solve equation $A\hat{\mathbf{x}} = \mathbf{p}$ or you can just compute $(A^T A)^{-1}$ and get $\hat{\mathbf{x}} = (A^T A)^{-1} A^T\mathbf{b}$. You can also solve the equation $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$ using elimination! Please choose one of three options.

3. Problem 3

Consider points with coordinates $(t_1, b_1) = (0, 0)$, $(t_2, b_2) = (1, 8)$, $(t_3, b_3) = (3, 8)$, $(t_4, b_4) = (4, 20)$ (see Figure 4.8 at page 172 of the textbook).

(a) Find C, D defining the line $C + Dt$ that is closest to these four points. Do this by setting up and solving the normal equations:

$$A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$$

(recall that $\hat{\mathbf{x}} = (C, D)$ and $\mathbf{b} = (0, 8, 8, 20)$).

(b) For the best straight line $C + Dt$ (as in part (a)), find its four heights p_1, p_2, p_3, p_4 and four errors e_i. What is the minimum value $E = e_1^2 + e_2^2 + e_3^2 + e_4^2$?

Recall that if $\mathbf{p} = (p_1, p_2, p_3, p_4)$ then $\mathbf{p} = A\hat{\mathbf{x}}$ and if $\mathbf{e} = (e_1, e_2, e_3, e_4)$ then $\mathbf{e} = \mathbf{b} - \mathbf{p}$.

4. Problem 4

If $P^2 = P$ show that $(I - P)^2 = I - P$. When P projects onto the column space of A, $I - P$ projects onto the ——- .