# MIT 18.06 Exam 3, Spring 2023 Gilbert Strang & Andrew Horning

Your name: $(printed)$		
,		
Student ID:		
Recitation:		

### Problem 1 (12 + 10 + 12 = 34 points):

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

**1(a)** Find the eigenvalues  $\lambda_1, \lambda_2$  and eigenvectors  $\boldsymbol{x}_1, \boldsymbol{x}_2$  of A:

$$A = \left[ \begin{array}{cc} 2 & 3 \\ 1 & 4 \end{array} \right]$$

$$\lambda_1 = \qquad \qquad \lambda_2 = \qquad \qquad oldsymbol{x}_1 = \qquad \qquad oldsymbol{x}_2 = \qquad \qquad$$

**1(b)** Diagonalize  $A = X\Lambda X^{-1}$  by finding those three matrices X and  $\Lambda$  and  $X^{-1}$ 

 $X = \Lambda = X^{-1} =$ 

**1(c)** Express the vector  $\mathbf{u} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$  as a combination of the eigenvectors  $x_1$  and  $x_2$  of A.

Then express the vector  $A^4u$  as a combination of those eigenvectors of A

 $oldsymbol{u} = A^4 oldsymbol{u} =$ 

(blank page for your work if you need it)

# Problem 2 (10 + 10 = 20 points):

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

2(a) Find the determinant of this permutation matrix P and explain your reasoning!

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

determinant of $P =$	

2(b) Find the cofactor of  $A_{11}$  and the determinant of A

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Cofactor of  $A_{11} =$ 

Determinant of A =

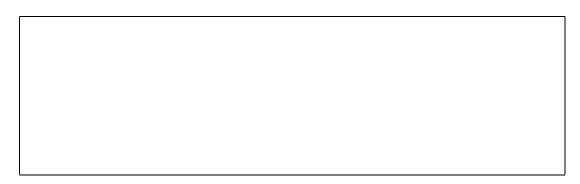
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#### Problem 3 (12 + 12 = 24 points):

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

An n by n real matrix Q is an orthogonal matrix if  $Q^{T}Q = I$ .

**3(a)** Show that the length of x =the length of Qx for every real vector x



**3(b)** Show that the determinant of Q is 1 or -1

## Problem 4 (12 + 10 = 22 points):

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

 $\mathbf{4(a)}$  What are the possible eigenvalues of a symmetric projection matrix P and why?



**4(b)** If the vectors  $\mathbf{q}_1, \mathbf{q}_2, \dots, \mathbf{q}_n$  are orthonormal and  $\mathbf{v} = c_1 \mathbf{q}_1 + c_2 \mathbf{q}_2 + \dots + c_n \mathbf{q}_n$ , find a formula for the last coefficient  $c_n$ .

 $c_n = {
m equals}$  \_\_\_\_\_\_