

**LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 3:
EXERCISES.**

My name is Vasily Krylov. If you have any questions or comments, please feel free to ask me by email (krvas@mit.edu) or during my office hours (**Thursday 5p.m. - 7 p.m. Room 2-361**).

1. PROBLEM 1

- (a) Does the set of all 3×3 matrices of rank 3 form a vector space?
- (b) Does the set of all 3×3 matrices of rank ≤ 2 form a vector space?

2. PROBLEM 2

Let \mathbf{M} be the vector space of all 2×2 matrices.

- (a) Describe a subspace of \mathbf{M} that contains $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ but not $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
- (b) If a subspace of \mathbf{M} does contain A and B , must it contain the identity matrix?

3. PROBLEM 3

For which right sides (find a condition on b_1, b_2, b_3) are these systems solvable?

- (a) $\begin{bmatrix} 1 & 4 & 2 \\ 2 & 8 & 4 \\ -1 & -4 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix},$
- (b) $\begin{bmatrix} 1 & 4 \\ 2 & 9 \\ -1 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$

4. PROBLEM 4

Construct a matrix whose column space contains $(1, 1, 0)$ and $(0, 1, 1)$ and whose nullspace contains $(1, 0, 1)$.