LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 7: EXERCISES.

1. Problem 1

If a 3 by 3 matrix has $\det A = -1$, find:
(a) $\det(2A)$
(b) $\det(-A)$
(c) $\det(A^2)$
(d) $\det(A^{-1})$.
Hint: you can use that $\det(AB) = \det A \det B$ for any 3 by 3 matrix B. In part (d) use that $\det(I) = \det(A \cdot A^{-1}) = \det A \det(A^{-1})$.

2. Problem 2

Compute determinants and inverses of these matrices:

(a) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

Hint: to compute A^{-1}, use the formula $A^{-1} = \frac{C^T}{\det A}$, where C is the matrix of 2 by 2 cofactors of A.

3. Problem 3

Find the determinants of a rank one matrix A and a skew-symmetric matrix B.

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 3 \\ -1 & 0 & 4 \\ -3 & -4 & 0 \end{bmatrix}.$$

4. Problem 4

If all the cofactors are zero, how do you know that A has no inverse? If none of the cofactors are zero, is A sure to be invertible?

5. Problem 5*

Suppose $\det A = 1$ and you know all the cofactors of A. How can you find A?