LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 1: EXERCISES

1. Problem 1

(a) Are columns of the matrix $\begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ linearly independent?

(b) How about columns of the matrix $\begin{bmatrix} 5 & 11 & 14 \\ 8 & 10 & 11 \\ 9 & 11 & 12 \end{bmatrix}$?

(c) How could you decide if the vectors $(1, 1, 1)$, $(-1, 1, -1)$ and (a, b, c) are linearly independent or dependent?

2. Problem 2

Describe geometrically (line, plane, or all $\mathbb{R}^3$) all linear combinentions of
(a) $(1, 2, 3)$ and $(3, 6, 9)$.
(b) $(2, 0, 0)$, $(0, 2, 2)$ and $(2, 2, 3)$.

3. Problem 3

True or false (give a reason if true or find a counterexample if false):
(a) If $\mathbf{u} = (1, 1)$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$ (i.e. $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w} = 0$), then $\mathbf{v}$ is parallel to $\mathbf{w}$ (i.e. $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent).
(b) If $\mathbf{u} = (1, 1, 1)$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$ (i.e. $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{w} = 0$), then $\mathbf{v}$ is parallel to $\mathbf{w}$ (i.e. $\mathbf{v}$ and $\mathbf{w}$ are linearly dependent).
(c) If $\mathbf{u}$ is perpendicular to $\mathbf{v}$ and $\mathbf{w}$, then $\mathbf{u}$ is perpendicular to $\mathbf{v} + 2\mathbf{w}$.
(d)* If $\mathbf{u}$ and $\mathbf{v}$ are perpendicular unit vectors then $\|\mathbf{u} - \mathbf{v}\|^2 = 2$.

4. Problem 4

(a) If three corners of a parallelogram are $(1, 1)$, $(4, 2)$ and $(1, 3)$, what are the possible fourth corners? Draw those parallelograms.
(b) If four corners of a unit cube are $(0, 0, 0)$, $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, what are the possibility for other four corners? Draw. Find coordinates of its center. How many edges this cube has? Describe one face and find the coordinates of its center. Describe all faces.

5. Problem 5*

If $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ has dependent rows, then it also has dependent columns.