LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 6: EXERCISES.

1. Problem 1

Are these pairs of vectors orthonormal or only orthogonal or only independent?

(a) $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}-1\\1\end{bmatrix}$

(b) $\begin{bmatrix}6\\8\end{bmatrix}$ and $\begin{bmatrix}4\\-3\end{bmatrix}$

(c) $\begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix}$ and $\begin{bmatrix}-\sin\theta\\\cos\theta\end{bmatrix}$.

2. Problem 2

Give an example of each of the following:

(a) A matrix Q that has orthonormal columns but $QQ^T \neq I$.

(b) Two orthogonal vectors that are not linearly independent.

(c) An orthonormal basis for $\mathbb{R}^3$, including the vector $q_1 = (1,1,1)/\sqrt{3}$.

3. Problem 3

Find two orthogonal vectors in the plane $x + y + 2z = 0$. Make them orthonormal.

4. Problem 4

Find orthogonal vectors A, B, C by Gram-Schmidt from a, b, c:

$$a = (1,-1,0,0), \quad b = (0,1,-1,0), \quad c = (0,0,1,-1).$$

A, B, C and a, b, c are bases for the vectors perpendicular to $d = (1,1,1,1)$.