LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 9: EXERCISES.

1. Problem 1

For the complex number $z = 1 - i$, find $\overline{z}$ and $r = |z|$ and the angle θ.

2. Problem 2

Find the eigenvalues and eigenvectors of the Hermitian matrix

$$S = \begin{bmatrix} 1 & 1+i \\ 1-i & 2 \end{bmatrix}$$

(This is a problem from PSet 7, so I will not post the complete solution)

3. Problem 3

If $\overline{Q}^T Q = 1$ (unitary matrix = complex orthogonal) and $Q\mathbf{x} = \lambda \mathbf{x}$, show that $|\lambda| = 1$.
Hint: look at $|Q\mathbf{x}|^2 = Q\mathbf{x} \cdot Q\mathbf{x} = (\overline{Qx})^T Q\mathbf{x}$.
(This is a problem from PSet 7, so I will not post the complete solution)

4. Problem 4

(a) Verify Euler's great formula $e^{i\theta} = \cos\theta + i\sin\theta$ using these first terms for

$$e^{i\theta} \text{ is aproximately } 1 + i\theta + \frac{1}{2}(i\theta)^2 + \frac{1}{6}(i\theta)^3,$$

$$\cos\theta \text{ is aproximately } 1 - \frac{1}{2}\theta^2, \ \sin\theta \text{ is aproximately } \theta - \frac{1}{6}\theta^3.$$

(b) Find $\cos 2\theta$ and $\sin 2\theta$ using Euler's great formula and $(e^{i\theta})(e^{i\theta}) = (e^{2i\theta})$.

5. Problem 5

(a) Find the matrix F_3 with orthogonal columns = eigenvectors of

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

(b) Write P as $F_3 \Lambda F_3^{-1}$ for some diagonal matrix Λ.

6. Problem 6

If $w = e^{2\pi i/64}$ then w^2 and $\sqrt{w}$ are among the — and — roots of 1.