MIT 18.06 Exam 3, Spring 2023
Gilbert Strang & Andrew Horning

Your name: ______________________________________
(*printed*)

Student ID: ______________________________________

Recitation: ______________________________________

Problem 1 $(\mathbf{12 + 10 + 12 = 34}$ **points):**

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

1(a) Find the eigenvalues λ_1, λ_2 and eigenvectors $\boldsymbol{x}_1, \boldsymbol{x}_2$ of A:

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

$\lambda_1 =$	$\lambda_2 =$	$\boldsymbol{x}_1 =$	$\boldsymbol{x}_2 =$

1(b) Diagonalize $A = X\Lambda X^{-1}$ by finding those three matrices X and Λ and X^{-1}

$X =$	$\Lambda =$	$X^{-1} =$

1(c) Express the vector $\boldsymbol{u} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ as a combination of the eigenvectors x_1 and x_2 of A.

Then express the vector $A^4 u$ as a combination of those eigenvectors of A

$\boldsymbol{u} \ =$ $A^4\boldsymbol{u} \ =$

(blank page for your work if you need it)

Problem 2 $(10 + 10 = 20$ points$)$:

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

2(a) Find the determinant of this permutation matrix P and **explain your reasoning!**

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

determinant of P =

2(b) Find the cofactor of A_{11} and the determinant of A

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

Cofactor of A_{11} =

Determinant of A =

(blank page for your work if you need it)

Problem 3 $(12 + 12 = 24$ points$)$:

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

An n by n real matrix Q is an orthogonal matrix if $Q^{\mathrm{T}}Q = I$.

3(a) Show that the length of $x =$ the length of Qx for every real vector x

3(b) Show that the determinant of Q is 1 or -1

Problem 4 $(12 + 10 = 22$ points$)$:

Record your final answer in the allotted spaces. You may use the remaining space for your calculations.

4(a) What are the possible eigenvalues of a symmetric projection matrix P and why ?

4(b) If the vectors $\boldsymbol{q}_1, \boldsymbol{q}_2, \ldots, \boldsymbol{q}_n$ are orthonormal and $\boldsymbol{v} = c_1\boldsymbol{q}_1 + c_2\boldsymbol{q}_2 + \cdots + c_n\boldsymbol{q}_n$, find a formula for the last coefficient c_n.

$\boldsymbol{c_n} =$ **equals** ______________________________