LINEAR ALGEBRA. VASILY KRYLOV. RECITATION 11: EXERCISES.

1. Problem 1

Find CR decomposition for a matrix:

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 7 \\ 1 & 3 & 5 \end{bmatrix}.$$

2. Problem 2

A and B are symmetric across the diagonal. Find their triple factorizations LDU and say how U is related to L for these symmetric matrices:

$$A = \begin{bmatrix} 2 & 4 \\ 4 & 11 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{bmatrix}.$$

Remark 2.1. *Recall that LDU factorization is the factorization into lower triangular matrix L with 1's on the diagonal, diagonal matrix D and upper triangular matrix U with 1's on the diagonal.*

3. Problem 3

Find the height C of the best *horizontal line* to fit $\mathbf{b} = (0, 8, 8, 20)$. An exact fit would solve the unsolvable equations $C = 0, C = 8, C = 8, C = 20$. Find the 4 by 1 matrix A in these equations and solve $A^T A\hat{x} = A^T\mathbf{b}$. Draw the horizontal line at height $\hat{x} = C$ and the four errors in $\mathbf{e}$.

4. Problem 4

(a) Find orthonormal vectors $\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3$ such that $\mathbf{q}_1$, $\mathbf{q}_2$ span the column space of

$$\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

(b) Which of the four fundamental spaces contains $\mathbf{q}_3$?
(c) Solve $A\mathbf{x} = (1, 2, 7)$ by least squares.