Problem 1.

Consider the robot manipulator shown below.

(a) Determine the D-H parameters for the robot and the D-H transformation for each joint.

(b) Derive the kinematic equations for the coordinates of a point at the tip of the last link (XYZ) in terms of the joint variables.

(c) Determine the inverse kinematic solution.

	1	2	3
a_i	0	0	r
α_i	0	180^o	0
θ_i	θ	0	180^o
s_i	0	h	0

$$\underline{\underline{T}}_{12} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \underline{\underline{T}}_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}, \underline{\underline{T}}_{34} = \begin{bmatrix} -1 & 0 & 0 & r \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{x}_1 = \underline{\underline{T}}_{12}\underline{\underline{T}}_{23}\underline{\underline{T}}_{34}\underline{x}_4$$

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\cos\theta & -\sin\theta & 0 & r\cos\theta \\ -\sin\theta & \cos\theta & 0 & r\sin\theta \\ 0 & 0 & 1 & -h \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -X\cos\theta - Y\sin\theta + r\cos\theta \\ -X\sin\theta + Y\cos\theta + r\sin\theta \\ Z - h \\ 1 \end{bmatrix}$$

Which gives us three equations. The third row is immediately solvable,

$$\boxed{Z = h}$$

Setting rows one and two to zero, and then squaring yields

$$(-X + r)^2 \cos^2\theta = Y^2 \sin^2\theta$$
$$(-X + r)^2 \sin^2\theta = Y^2 \cos^2\theta$$

Adding these and taking advantage of trignometric identities

$$
\begin{aligned}
(-X+r)^2\left(\cos^2\theta+\sin^2\theta\right) &= Y^2\left(\cos^2\theta+\sin^2\theta\right)\\
Y &= -X+r
\end{aligned}
$$

Substituting this back in row 2 yields

$$
\begin{aligned}
0 &= -X\sin\theta+(-X+r)\cos\theta+r\sin\theta\\
0 &= -X\left(\sin\theta+\cos\theta\right)+r\left(\cos\theta+\sin\theta\right)\\
&\boxed{X=r}
\end{aligned}
$$

Finally, plugging this result into row 1 yields

$$
\begin{aligned}
0 &= -r\cos\theta-Y\sin\theta+r\cos\theta\\
Y &= \frac{1}{\sin\theta}\\
&\boxed{Y=\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}}
\end{aligned}
$$