Problem 1.

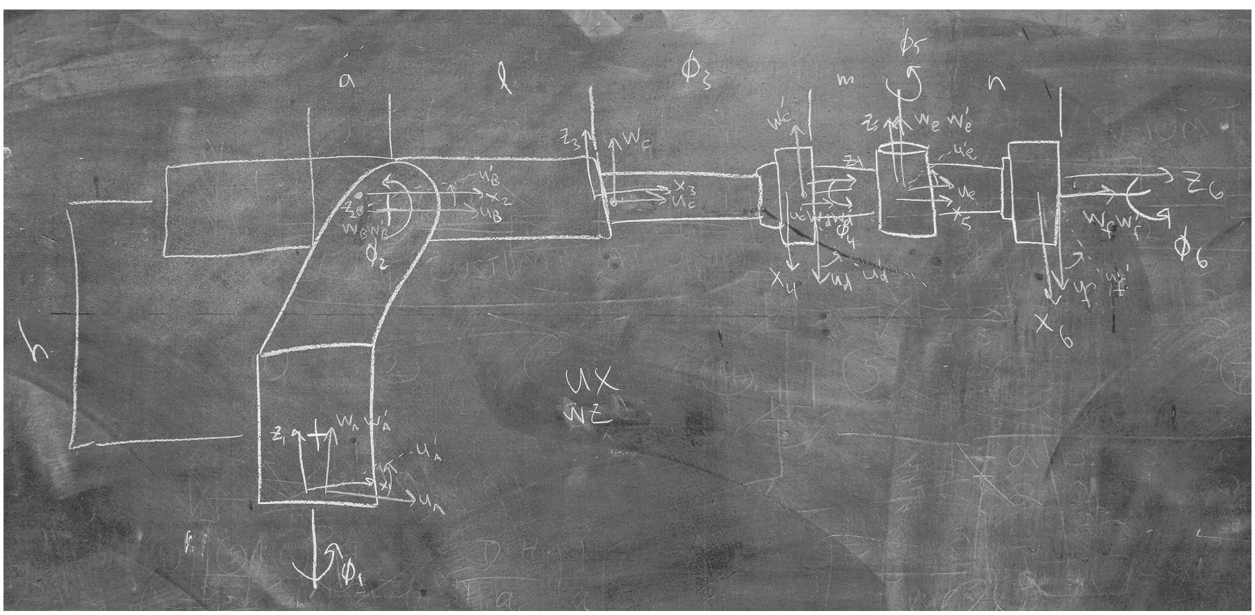
Consider a robot manipulator as show in Figure P5.6. The kinematic structure of this robotic arm is very similar to that of the Stanford manipulator studied in example 5.6 except that it has an offset (a) between the base and the shoulder (the first two) join axes. For this robotic arm, derive and solve the kinematic position equations using shape and joint matrices.

There are two types of joints in this system, five revolute joints and one prismatic joint. Joints 1, 2, 4, 5, and 6 are revolute joints and have the following transformation matrix

$$\Phi_h(\phi_h) = \begin{bmatrix} \cos\phi_h & -\sin\phi_h & 0 & 0 \\ \sin\phi_h & \cos\phi_h & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Join 3 is a prismatic joint and has the following transformation matrix

$$\Phi_h(\phi_h) = \begin{bmatrix} 1 & 0 & 0 & \phi_h \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Our shape matrices are obtained by inspection, and are

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{1A} = I, S_{2A} = \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 0 & 1 & -h \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{2B} = I, S_{3B} = \begin{bmatrix} 1 & 0 & 0 & -l \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{3C} = I, S_{4C} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{4D} = I, S_{5D} = \begin{bmatrix} 0 & 0 & 1 & -m \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{5E} = I, S_{6E} = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -n \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$S_{6F} = I, S_{7F} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{aligned} T_{i,i+1} &= S_{i,j}\Phi_j S_{i+1,j}^{-1} \\ T_{12} &= S_{1A}\Phi_A S_{2A}^{-1} \\ T_{23} &= S_{2B}\Phi_B S_{3B}^{-1} \\ T_{34} &= S_{3C}\Phi_C S_{4C}^{-1} \\ T_{45} &= S_{4D}\Phi_D S_{5D}^{-1} \\ T_{56} &= S_{5E}\Phi_E S_{6E}^{-1} \\ T_{67} &= S_{6F}\Phi_F S_{7F}^{-1} \end{aligned}$$

$$
\begin{aligned}
T_{12} &= S_{1A}\Phi_A S_{2A}^{-1} \\
&= I \begin{bmatrix} \cos\phi_1 & -\sin\phi_1 & 0 & 0 \\ \sin\phi_1 & \cos\phi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -a \\ 0 & 0 & 1 & -h \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \cos\phi_1 & 0 & \sin\phi_1 & a\cos\phi_1 \\ \sin\phi_1 & 0 & -\cos\phi_1 & a\sin\phi_1 \\ 0 & 1 & 0 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
T_{23} &= S_{2B}\Phi_B S_{3B}^{-1} \\
&= I \begin{bmatrix} \cos\phi_2 & -\sin\phi_2 & 0 & 0 \\ \sin\phi_2 & \cos\phi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -l \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} \cos\phi_2 & 0 & -\sin\phi_2 & l\cos\phi_2 \\ \sin\phi_2 & 0 & \cos\phi_2 & l\sin\phi_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
T_{34} &= S_{3C}\Phi_C S_{4C}^{-1} \\
&= I \begin{bmatrix} 1 & 0 & 0 & \phi_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} 0 & 0 & -1 & \phi_3 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
T_{45} &= S_{4D}\Phi_D S_{5D}^{-1} \\
&= I \begin{bmatrix} \cos\phi_4 & -\sin\phi_4 & 0 & 0 \\ \sin\phi_4 & \cos\phi_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & -m \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} 0 & -\sin\phi_4 & -\cos\phi_4 & 0 \\ 0 & \cos\phi_4 & -\sin\phi_4 & 0 \\ 1 & 0 & 0 & m \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
T_{56} &= S_{5E}\Phi_E S_{6E}^{-1} \\
&= I \begin{bmatrix} \cos\phi_5 & -\sin\phi_5 & 0 & 0 \\ \sin\phi_5 & \cos\phi_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -n \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} 0 & -\sin\phi_5 & \cos\phi_5 & n\cos\phi_5 \\ 0 & \cos\phi_5 & \sin\phi_5 & n\sin\phi_5 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
T_{67} &= S_{6F}\Phi_F S_{7F}^{-1} \\
&= I \begin{bmatrix} \cos\phi_6 & -\sin\phi_6 & 0 & 0 \\ \sin\phi_6 & \cos\phi_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} -\cos\phi_6 & \sin\phi_6 & 0 & 0 \\ -\sin\phi_6 & -\cos\phi_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

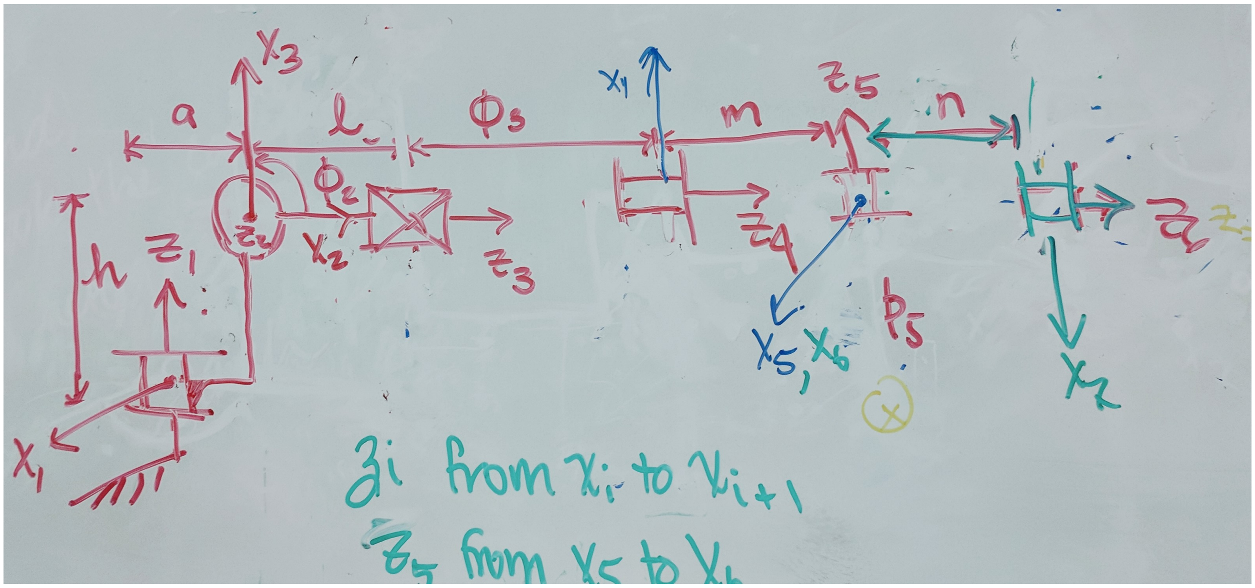
From here the kinematic position can be solved via

$$
\begin{aligned}
R_C &= T_{12}T_{23}T_{34}r_C^4 \\
\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} &= \begin{bmatrix} a\cos\phi_1 + l\cos\phi_1\cos\phi_2 + \phi_3\cos\phi_1\cos\phi_2 \\ a\sin\phi_1 + l\sin\phi_1\cos\phi_2 + \phi_3\sin\phi_1\cos\phi_2 \\ h + l\sin\phi_2 + \phi_3\sin\phi_2 \\ 1 \end{bmatrix}
\end{aligned}
$$

where $r_C = [0, 0, 0, 1]^T$.

Problem 2.

For the robot manipulator of problem 5.6, derive the kinematic position equations using Denavit-Hartenberg transformation matrices and find the solution to these equations using the partitioning method of section 5.7.



	1	2	3	4	5	6
a_i	a	0	0	0	0	0
α_i	90°	90°	0	90°	90°	0
s_i	h	0	$l+\phi_3$	m	n	0
θ_i	ϕ_1	ϕ_2	0	ϕ_4	ϕ_5	ϕ_6

$$T_1 = \begin{bmatrix} \cos\phi_1 & 0 & \sin\phi_1 & a\cos\phi_1 \\ \sin\phi_1 & 0 & -\cos\phi_1 & a\sin\phi_1 \\ 0 & 1 & 0 & h \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_2 = \begin{bmatrix} \cos\phi_2 & 0 & \sin\phi_2 & 0 \\ \sin\phi_2 & 0 & -\cos\phi_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l+\phi_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_4 = \begin{bmatrix} \cos\phi_4 & 0 & \sin\phi_4 & 0 \\ \sin\phi_4 & 0 & -\cos\phi_4 & 0 \\ 0 & 1 & 0 & m \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$T_5 = \begin{bmatrix} \cos\phi_5 & 0 & \sin\phi_5 & 0 \\ \sin\phi_5 & 0 & -\cos\phi_5 & 0 \\ 0 & 1 & 0 & n \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_6 = \begin{bmatrix} \cos\phi_6 & \sin\phi_6 & 0 & 0 \\ \sin\phi_6 & -\cos\phi_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23}^{-1} T_{12}^{-1} R_C = T_{34} r_C^4$$
$$\begin{bmatrix} X\cos\phi_1\cos\phi_2 + Y\cos\phi_2\sin\phi_1 + Z\sin\phi_2 - a\cos\phi_2 - h\sin\phi_2 \\ X\sin\phi_1 - Y\cos\phi_1 \\ X\cos\phi_1\sin\phi_2 + Y\sin\phi_1\sin\phi_2 - Z\cos\phi_2 - a\sin\phi_2 + h\cos\phi_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ l + \phi_3 \\ 1 \end{bmatrix}$$

The first angle can be solved for using the second equation

$$\begin{aligned} 0 &= X\sin\phi_1 - Y\cos\phi_1 \\ Y\cos\phi_1 &= X\sin\phi_1 \\ \tan\phi_1 &= \frac{Y}{X} \\ \phi_1 &= \tan^{-1}\frac{Y}{X} \end{aligned}$$

The second angle can be solved for using the first equation

$$\begin{aligned} 0 &= \cos\phi_2\left(X\cos\phi_1 + Y\sin\phi_1 - a\right) + \sin\phi_2\left(Z - h\right) \\ -\cos\phi_2\left(X\cos\phi_1 + Y\sin\phi_1 - a\right) &= \sin\phi_2\left(Z - h\right) \\ \tan\phi_2 &= \frac{-\left(Z-h\right)}{\left(X\cos\phi_1 + Y\sin\phi_1 - a\right)} \\ \phi_2 &= \tan^{-1}\left(\frac{h - Z}{X\cos\phi_1 + Y\sin\phi_1 - a}\right) \end{aligned}$$

Finally, the last angle can be solved for using the third equation, as all other variables are now known

$$\begin{aligned} l + \phi_3 &= \sin\phi_2\left(X\cos\phi_1 + Y\sin\phi_1 - a\right)\cos\phi_2\left(-Z + h\right) \\ \phi_3 &= \sin\phi_2\left(X\cos\phi_1 + Y\sin\phi_1 - a\right)\cos\phi_2\left(-Z + h\right) - l \end{aligned}$$