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0.1 Introduction

We're going to play with a shoulder roll locked SSRMS.

0.2 Finite Kinematic Analysis

0.2.1 Denavit-Hartenberg Parameters

i	θ_i	α_i	a_i	d_i
1	θ_1	90	0	d_1
2	θ_2	90	0	d_2
3	θ_3	0	a_3	d_3
4	θ_4	0	a_4	0
5	θ_5	90	0	0
6	θ_6	90	0	d_6
7	θ_7	90	0	d_7

Table 1: The Denavit-Hartenberg parameters for the SSRMS. These parameters are the joint angle, θ, the link twist angle, α, the link length, a, and the joint offset, d.

0.2.2 Inverse Kinematics Solution

In general we can define

$$
\begin{aligned}
T_{07} &= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= T_{01}T_{12}T_{23}T_{34}T_{45}T_{56}T_{67}
\end{aligned}
$$

Premultiplying both sides by T_{01}^{-1} yields,

$$T_{01}^{-1}T_{07} = T_{12}T_{23}T_{34}T_{45}T_{56}T_{67}$$

Equating each element (i, j) on both the left and right hand sides yields:

$$
\begin{aligned}
&(1,1) \quad n_x c_1 + n_y s_1 &&= (s_2 s_6 + c_2 c_6 c_{345})\, c_7 + s_7 s_{345} c_2 \\
&(1,2) \quad o_x c_1 + o_y s_1 &&= -s_2 c_6 + s_6 c_2 c_{345} \\
&(1,3) \quad a_x c_1 + a_y s_1 &&= (s_2 s_6 + c_2 c_6 c_{345})\, s_7 - s_{345} c_2 c_7 \\
&(1,4) \quad p_x c_1 + p_y s_1 &&= a_3 c_2 c_3 + a_4 c_2 c_{34} + d_3 s_2 + d_6 s_{345} c_2 - d_7 s_2 c_6 + d_7 s_6 c_2 c_{345} \\
&(2,1) \quad n_z &&= (s_2 c_6 c_{345} - s_6 c_2)\, c_7 + s_2 s_7 s_{345} \\
&(2,2) \quad o_z &&= s_2 s_6 c_{345} + c_2 c_6 \\
&(2,3) \quad a_z &&= (s_2 c_6 c_{345} - s_6 c_2)\, s_7 - s_2 s_{345} c_7 \\
&(2,4) \quad -d_1 + p_z &&= a_3 s_2 c_3 + a_4 s_2 c_{34} - d_3 c_2 + d_6 s_2 s_{345} + d_7 s_2 s_6 c_{345} + d_7 c_2 c_6 \\
&(3,1) \quad n_x s_1 - n_y c_1 &&= -s_7 c_{345} + s_{345} c_6 c_7 \\
&(3,2) \quad o_x s_1 - o_y c_1 &&= s_6 s_{345} \\
&(3,3) \quad a_x s_1 - a_y c_1 &&= s_7 s_{345} c_6 + c_7 c_{345} \\
&(3,4) \quad p_x s_1 - p_y c_1 &&= a_3 s_3 + a_4 s_{34} + d_2 - d_6 c_{345} + d_7 s_6 s_{345} \\
&(4,1) \quad 0 &&= 0 \\
&(4,2) \quad 0 &&= 0 \\
&(4,3) \quad 0 &&= 0 \\
&(4,4) \quad 1 &&= 1
\end{aligned}
$$

where I have defined $s_i = \sin i, c_i = \cos i, s_{ij} = \sin i + j, c_{ij} = \cos i + j, s_{ijk} = \sin i + j + k$ and $c_{ijk} = \cos i + j + k$.