SSRMS

John Karasinski

March 16, 2018

Contents

1 Introduction

The Space Station Remote Manipulator System (SSRMS) was designed and built to construct the International Space Station (ISS) and grapple with visiting space vehicles. The SSRMS is a seven degree of freedom (DoF) manipulator consisting entirely of revolute joints. The arm is symmetric, consisting of a 3DoF (roll, yaw, pitch) “shoulder”, an ”elbow” pitch joint, and a 3DoF (pitch, yaw, roll) “wrist”. Due to this symmetric structure, the arm has the ability to “walk” along the station, greatly increasing it’s available working space. The arm can lock the wrist to a grapple fixture, then disconnect the shoulder (which becomes the new wrist) to walk along the station. See Figure 1a for an example graphic of arm, and Figure 1b for a picture of the arm grappling a visiting vehicle.

The SSRMS is operating from one of the two Robotic Work Stations (RWS) located in either the Cupola or the Destiny module on the ISS. While operating the arm from the RWS, astronauts commonly lock one of the shoulder joints, which allows for more predictable movement of the arm. The shoulder roll is the most commonly locked joint during training and operation of the arm [2]. For this reason, we will consider the shoulder roll joint (the first joint of the arm) to be locked at a fixed angle for the majority of this report.

2 Finite Kinematic Analysis

2.1 Denavit-Hartenberg Parameters

The Denavit-Hartenberg parameters form a minimal representation of the kinematic structure of a robotic arm. These four parameters are the joint angle, θ, the link twist angle, α, the link length, a, and the joint offset, d. These parameters are identified by inspection, and are based off the coordinates from and lengths defined in Figure 2. The resulting D-H parameters are presented in Table 1. The parameters are plugged into the generic D-H transformation matrix, see Equation 1. This equation transforms positions and rotations from the i^{th} to the $i+1^{th}$ coordinates frames.

$$T_{i,i+1} = \begin{bmatrix} \cos(\phi) & -\sin(\phi)\cos(\alpha) & \sin(\alpha)\sin(\phi) & a\cos(\phi) \\ \sin(\phi) & \cos(\alpha)\cos(\phi) & -\sin(\alpha)\cos(\phi) & a\sin(\phi) \\ 0 & \sin(\alpha) & \cos(\alpha) & d \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

i	θ_i	α_i	a_i	d_i
1	90	90	0	d_1
2	90	90	0	d_2
3	0	0	a_3	d_3
4	0	0	a_4	0
5	180	90	0	0
6	-90	90	0	d_6
7	180	90	0	d_7

Table 1: The Denavit-Hartenberg parameters for the SSRMS. These parameters are the joint angle, θ, the link twist angle, α, the link length, a, and the joint offset, d. These θ_is give the initial or "zero-displacement" configuration, but each θ_i is modeled as an individual variable below.

The resulting seven matrices are therefore

$$T_{01} = \begin{bmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & 0 \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{12} = \begin{bmatrix} \cos(\theta_2) & 0 & \sin(\theta_2) & 0 \\ \sin(\theta_2) & 0 & -\cos(\theta_2) & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23} = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & a_3\sin(\theta_3) \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{34} = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & a_4\cos(\theta_4) \\ \sin(\theta_4) & \cos(\theta_4) & 0 & a_4\sin(\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{45} = \begin{bmatrix} \cos(\theta_5) & 0 & \sin(\theta_5) & 0 \\ \sin(\theta_5) & 0 & -\cos(\theta_5) & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad T_{56} = \begin{bmatrix} \cos(\theta_6) & 0 & \sin(\theta_6) & 0 \\ \sin(\theta_6) & 0 & -\cos(\theta_6) & 0 \\ 0 & 1 & 0 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{67} = \begin{bmatrix} \cos(\theta_7) & 0 & \sin(\theta_7) & 0 \\ \sin(\theta_7) & 0 & -\cos(\theta_7) & 0 \\ 0 & 1 & 0 & d_7 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Once these seven matrices are defined, it is often desirable to be able to translate directly from the initial coordinate frame to the final end effector frame. This is easily found by multiplying the successive matrices together to form T_{07}, see Equation 2. Multiplying these matrices together yields

$$\begin{aligned}
T_{07} =& T_{01}T_{12}T_{23}T_{34}T_{45}T_{56}T_{67} \qquad (2)\\
T_{07}[1,1] =& \left(-s_1c_{345} + s_{345}c_1c_2\right)s_7 + \left(s_1s_{345}c_6 + s_2s_6c_1 + c_1c_2c_6c_{345}\right)c_7\\
T_{07}[1,2] =& s_1s_6s_{345} - s_2c_1c_6 + s_6c_1c_2c_{345}\\
T_{07}[1,3] =& \left(s_1c_{345} - s_{345}c_1c_2\right)c_7 + \left(s_1s_{345}c_6 + s_2s_6c_1 + c_1c_2c_6c_{345}\right)s_7\\
T_{07}[1,4] =& a_3s_1s_3 + a_3c_1c_2c_3 + a_4s_1s_{34} + a_4c_1c_2c_{34} + d_2s_1 + d_3s_2c_1 - d_6s_1c_{345} + d_6s_{345}c_1c_2\\
&+ d_7s_1s_6s_{345} - d_7s_2c_1c_6 + d_7s_6c_1c_2c_{345}\\
T_{07}[2,1] =& \left(s_1s_{345}c_2 + c_1c_{345}\right)s_7 + \left(s_1s_2s_6 + s_1c_2c_6c_{345} - s_{345}c_1c_6\right)c_7\\
T_{07}[2,2] =& - s_1s_2c_6 + s_1s_6c_2c_{345} - s_6s_{345}c_1\\
T_{07}[2,3] =& - \left(s_1s_{345}c_2 + c_1c_{345}\right)c_7 + \left(s_1s_2s_6 + s_1c_2c_6c_{345} - s_{345}c_1c_6\right)s_7\\
T_{07}[2,4] =& a_3s_1c_2c_3 - a_3s_3c_1 + a_4s_1c_2c_{34} - a_4s_{34}c_1 - d_2c_1 + d_3s_1s_2 + d_6s_1s_{345}c_2 + d_6c_1c_{345}\\
&- d_7s_1s_2c_6 + d_7s_1s_6c_2c_{345} - d_7s_6s_{345}c_1\\
T_{07}[3,1] =& \left(s_2c_6c_{345} - s_6c_2\right)c_7 + s_2s_7s_{345}\\
T_{07}[3,2] =& s_2s_6c_{345} + c_2c_6\\
T_{07}[3,3] =& \left(s_2c_6c_{345} - s_6c_2\right)s_7 - s_2s_{345}c_7\\
T_{07}[3,4] =& a_3s_2c_3 + a_4s_2c_{34} + d_1 - d_3c_2 + d_6s_2s_{345} + d_7s_2s_6c_{345} + d_7c_2c_6\\
T_{07}[4,1] =& 0\\
T_{07}[4,2] =& 0\\
T_{07}[4,3] =& 0\\
T_{07}[4,4] =& 1
\end{aligned}$$

2.2 Joint/Shape Matrices

We can similarly use joint and shape matrices to arrive at these T matrices. For simplicity of readability, we renumber the joints from $1-7$ to $A-G$. All of the joints of the SSRMS are revolute. A general revolute joint, h, can be modeled with the joint matrix of

$$\Phi_h\left(\phi_h\right) = \begin{bmatrix} \cos\phi_h & -\sin\phi_h & 0 & 0\\ \sin\phi_h & \cos\phi_h & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
\begin{aligned}
T_{i,i+1} &= S_{i,j} J_j S_{i+1,j}^{-1} \\
T_{12} &= S_{1A} J_A S_{2A}^{-1} \\
T_{23} &= S_{2B} J_B S_{3B}^{-1} \\
T_{34} &= S_{3C} J_C S_{4C}^{-1} \\
T_{45} &= S_{4D} J_D S_{5D}^{-1} \\
T_{56} &= S_{5E} J_E S_{6E}^{-1} \\
T_{67} &= S_{6F} J_F S_{7F}^{-1} \\
T_{78} &= S_{7G} J_G S_{8G}^{-1}
\end{aligned}
$$

For joints $J_A, J_B, J_C, J_D, J_E, J_F$, and J_G, we also define two shape matrices.

$$
T_{01} = \begin{bmatrix} 1 & 0 & 0 & x_0 \\ 0 & 1 & 0 & y_0 \\ 0 & 0 & 1 & z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad S_{1A} = I, S_{2A} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d_1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
S_{2B} = I, S_{3B} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d_2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad S_{3C} = I, S_{4C} = \begin{bmatrix} 1 & 0 & 0 & -a_3 \\ 0 & 0 & 1 & -d_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
S_{4D} = I, S_{5D} = \begin{bmatrix} 1 & 0 & 0 & -a_3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad S_{5E} = I, S_{6E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
S_{6F} = I, S_{7F} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d_6 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad S_{7G} = I, S_{8G} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -d_7 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

2.3 Inverse Kinematics Solution

In general we can define

$$
\begin{aligned}
T_{07} &= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
&= T_{01} T_{12} T_{23} T_{34} T_{45} T_{56} T_{67}
\end{aligned}
$$

Premultiplying both sides by T_{01}^{-1} yields,

$$
T_{01}^{-1} T_{07} = T_{12} T_{23} T_{34} T_{45} T_{56} T_{67}
$$

Equating each element (i, j) on both the left and right hand sides yields:

$$
\begin{align}
n_x c_1 + n_y s_1 &= (s_2 s_6 + c_2 c_6 c_{345}) c_7 + s_7 s_{345} c_2 \tag{3}\\
o_x c_1 + o_y s_1 &= -s_2 c_6 + s_6 c_2 c_{345} \tag{4}\\
a_x c_1 + a_y s_1 &= (s_2 s_6 + c_2 c_6 c_{345}) s_7 - s_{345} c_2 c_7 \tag{5}\\
p_x c_1 + p_y s_1 &= a_3 c_2 c_3 + a_4 c_2 c_{34} + d_3 s_2 + d_6 s_{345} c_2 - d_7 s_2 c_6 + d_7 s_6 c_2 c_{345} \tag{6}\\
n_z &= (s_2 c_6 c_{345} - s_6 c_2) c_7 + s_2 s_7 s_{345} \tag{7}\\
o_z &= s_2 s_6 c_{345} + c_2 c_6 \tag{8}\\
a_z &= (s_2 c_6 c_{345} - s_6 c_2) s_7 - s_2 s_{345} c_7 \tag{9}\\
-d_1 + p_z &= a_3 s_2 c_3 + a_4 s_2 c_{34} - d_3 c_2 + d_6 s_2 s_{345} + d_7 s_2 s_6 c_{345} + d_7 c_2 c_6 \tag{10}\\
n_x s_1 - n_y c_1 &= -s_7 c_{345} + s_{345} c_6 c_7 \tag{11}\\
o_x s_1 - o_y c_1 &= s_6 s_{345} \tag{12}\\
a_x s_1 - a_y c_1 &= s_7 s_{345} c_6 + c_7 c_{345} \tag{13}\\
p_x s_1 - p_y c_1 &= a_3 s_3 + a_4 s_{34} + d_2 - d_6 c_{345} + d_7 s_6 s_{345} \tag{14}\\
0 &= 0 \tag{15}\\
0 &= 0 \tag{16}\\
0 &= 0 \tag{17}\\
1 &= 1 \tag{18}
\end{align}
$$

where we have defined $s_i = \sin i, c_i = \cos i, s_{ij} = \sin(i+j), c_{ij} = \cos(i+j), s_{ijk} = \sin(i+j+k)$ and $c_{ijk} = \cos(i+j+k)$. Manipulating the equations, we take $(Eq.\ 3)\, s_2 - (Eq.\ 7)\, c_2$ and simplify, producing

$$\left(n_x c_1 + n_y s_1\right) s_2 - n_z c_2 = s_6 c_7 \tag{19}$$

Similarly, we can do $(Eq.\ 6)\, s_2 - (Eq.\ 10)\, c_2$ and simplify, which results in

$$\left(p_x c_1 + p_y s_1\right) s_2 - (-d_1 + p_z)\, c_2 = d_3 - c_6 d_7 \tag{20}$$

We can also subtract $(Eq.\ 8)\, c_2 - (Eq.\ 4)\, s_2$

$$o_z c_2 - \left(o_x c_1 + o_y s_1\right) s_2 = c_6 \tag{21}$$

Finally, we can also subtract $(Eq.\ 5)\, s_2 - (Eq.\ 9)\, c_2$

$$\left(a_x c_1 + a_y s_1\right) s_2 - a_z c_2 = s_6 s_7 \tag{22}$$

Rearranging Equations 20 and 21 to be equal to c_6 and equating the two yields

$$-d_3 = \Big((o_x d_7 - p_x)\, c_1 + \left(o_y d_7 - p_y\right) s_1\Big) s_2 + (-o_z d_7 - d_1 + p_z)\, c_2 \tag{23}$$

Locking the shoulder roll angle to a known angle, $\boxed{\theta_1 = \beta}$, we can solve for θ_2,

$$\boxed{\theta_2 = \text{SHOULDER} \cdot \text{acos}\left(\frac{d_3}{\sqrt{h_1^2 + q_1^2}}\right) + \text{atan2}(q_1, h_1)} \tag{24}$$

where

$$h_1 = (-o_z d_7 - d_1 + p_z) \tag{25}$$

$$q_1 = \Big((o_x d_7 - p_x)\, c_\beta + \big(o_y d_7 - p_y\big)\, s_\beta \Big) \tag{26}$$

With θ_1 and θ_2 now known, θ_6 can be solved using Equation 21,

$$\boxed{\theta_6 = \text{WRIST} \cdot \text{acos}\Big(o_z c_2 - \big(o_x c_1 + o_y s_1\big)\, s_2 \Big)} \tag{27}$$

And we can then combine Equations 19 and 22, yielding

$$\boxed{\theta_7 = \text{atan2}\left(\frac{\big(n_x c_1 + n_y s_1\big)\, s_2 - n_z c_2}{s_6}, \frac{\big(a_x c_1 + a_y s_1\big)\, s_2 - a_z c_2}{s_6} \right)} \tag{28}$$

With the shoulder and wrist joints resolved, we can now solve for the middle joints. We now take

$$\left(T_{12}^{-1}\right)(T_{17})\left(T_{67}^{-1}\right)\left(T_{56}^{-1}\right) = (T_{23})\,(T_{34})\,(T_{45})$$

Taking the left and right hand side $(1,4)$ and $(2,4)$ elements from the resulting matrix yields

$$\begin{aligned} a_3 c_3 + a_4 c_{34} = {} & d_6 \Big(a_z s_2 + c_2 \big(a_x c_1 + a_y s_1\big) \Big)\, c_7 \\ & - d_6 \Big(n_z s_2 + c_2 \big(n_x c_1 + n_y s_1\big) \Big)\, s_7 \\ & - d_7 \Big(o_z s_2 + c_2 \big(o_x c_1 + o_y s_1\big) \Big) \\ & + (-d_1 + p_z)\, s_2 + c_2 \big(p_x c_1 + p_y s_1\big) \end{aligned} \tag{29}$$

$$\begin{aligned} a_3 s_3 + a_4 s_{34} = {} & -d_2 + d_6 \big(a_x s_1 - a_y c_1\big)\, c_7 - d_6 \big(n_x s_1 - n_y c_1\big)\, s_7 \\ & - d_7 \big(o_x s_1 - o_y c_1\big) + p_x s_1 - p_y c_1 \end{aligned} \tag{30}$$

θ_4 is then solved by combining the above two equations, resulting in

$$\boxed{\theta_4 = \text{ELBOW} \cdot \text{acos}\left(\frac{X^2 + Y^2 - a_3^2 - a_4^2}{2 a_3 a_4} \right)} \tag{31}$$

where

$$\begin{aligned} X = {} & d_6 \left(\Big(a_z s_2 + c_2 \big(a_x c_1 + a_y s_1\big) \Big)\, c_7 - \Big(n_z s_2 + c_2 \big(n_x c_1 + n_y s_1\big) \Big)\, s_7 \right) \\ & - d_7 \Big(o_z s_2 + c_2 \big(o_x c_1 + o_y s_1\big) \Big) + (-d_1 + p_z)\, s_2 + c_2 \big(p_x c_1 + p_y s_1\big) \\ Y = {} & -d_2 + d_6 \big(a_x s_1 - a_y c_1\big)\, c_7 - d_6 \big(n_x s_1 - n_y c_1\big)\, s_7 - d_7 \big(o_x s_1 - o_y c_1\big) + p_x s_1 - p_y c_1 \end{aligned}$$

Substituting the solution into θ_4 and Equations 29 and 30 and combining yields

$$\boxed{\theta_3 = \text{atan2}\left(Y\left(a_3 + a_4 c_4\right) - X a_4 s_4, X\left(a_3 + a_4 c_4\right) + Y a_4 s_4\right)}$$

Subtracting $(Eq.\ 13)c_7$ and $(Eq.\ 11)s_7$ yields

$$c_{345} = \left(a_x s_1 - a_y c_1\right) c_7 - \left(n_x s_1 - n_y c_1\right) s_7$$

And from Equation 12 we have

$$s_{345} = \frac{o_x s_1 - o_y c_1}{s_6}$$

which we can combine to solve for the last joint

$$\theta_5 = (\theta_3 + \theta_4 + \theta_5) - (\theta_3 + \theta_4)$$
$$\boxed{\theta_5 = \text{atan2}\left(s_{345}, c_{345}\right) - (\theta_3 + \theta_4)}$$

2.4 Numerical Example

For practical purposes, the link length and offset values can be set to

$$a_3 = 2.30, a_4 = 2.30, d_1 = 0.65, d_2 = 0.30$$
$$d_3 = 0.90, d_6 = 0.30, d_7 = 0.65$$

As an example, plugging in these values and the initial angles given in Table 1 into Equation 2 yields

$$T_{07} = \begin{bmatrix} 0 & 0 & 1 & 0.6 \\ 1 & 0 & 0 & 0.9 \\ 0 & 1 & 0 & 5.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As another example, given the end effector pose

$$T_{07} = \begin{bmatrix} 0.8021 & 0.1217 & 0.5846 & 2.4790 \\ -0.5859 & 0.3495 & 0.7311 & -2.4734 \\ -0.1154 & 0.9290 & 0.3517 & -0.4927 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and locking the first joint variable $\theta_1 = \beta = 60°$, we can solve for the 8 possible configurations of the arm

θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_7
60.000	-20.268	64.074	79.722	-149.770	138.205	-77.426
60.000	-20.268	58.153	99.444	16.428	-138.205	102.573
60.000	-20.268	143.797	-79.722	-70.048	138.205	-77.426
60.000	-109.087	35.576	79.140	-119.938	49.659	-85.275
60.000	-20.268	157.598	-99.444	115.872	-138.205	102.573
60.000	-109.087	23.189	100.025	51.565	-49.659	94.724
60.000	-109.087	114.717	-79.140	-40.797	49.659	-85.275
60.000	-109.087	123.214	-100.025	151.590	-49.659	94.724

Table 2: The eight possible configurations for locking the first joint.

3 Differential Kinematic Analysis

3.1 Method 1: Kinematic Jacobian

Where $\hat{z}_i$ is taken from the last column of T_{1i}, and can be defined

$$T_{1i} = \begin{bmatrix} & \underline{\underline{\Theta}}_i & & \vdots & a_i \\ & \cdots & & & \cdots \\ 0 & 0 & 0 & \vdots & 0 \end{bmatrix}$$

$$\underline{\underline{\Theta}}_i = \begin{bmatrix} x_i & y_i & z_i \end{bmatrix}$$

$$\hat{z}_i = \left(\prod_{i=1}^{n} \underline{\underline{\Theta}}_i \right) z_i$$

and $\vec{r}_i$ is defined

$$\vec{r}_i = \sum_{i=1}^{n} \vec{a}_i$$

With these definitions, we can find the Jacobian via

$$\dot{\vec{P}} = \sum_{i=1}^{n} \left(\hat{z}_i \times \vec{r}_i\right) \dot{\theta}_i$$

$$\vec{w} = \sum_{i=1}^{n} \dot{\theta}_i \hat{z}_i$$

$$\underline{\underline{J}} \dot{q} = \begin{bmatrix} \underline{\dot{P}} \\ \underline{\vec{w}} \end{bmatrix}$$

$$\begin{bmatrix} \hat{z}_1 \times \vec{r}_1 & \hat{z}_2 \times \vec{r}_2 & \cdots & \hat{z}_7 \times \vec{r}_7 \\ \hat{z}_1 & \hat{z}_2 & \cdots & \hat{z}_7 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_7 \end{bmatrix} = \begin{bmatrix} \underline{\dot{P}}_{EE} \\ \underline{w}_{EE} \end{bmatrix}$$

$$
\begin{aligned}
J[1,1] &= -a_3 s_1 c_2 c_3 + a_3 s_3 c_1 - a_4 s_1 c_2 c_{34} + a_4 s_{34} c_1 + d_2 c_1 - d_3 s_1 s_2 \\
&\quad - d_6 s_1 s_{345} c_2 - d_6 c_1 c_{345} + d_7 s_1 s_2 c_6 - d_7 s_1 s_6 c_2 c_{345} + d_7 s_6 s_{345} c_1 \\
J[2,1] &= a_3 s_1 s_3 + a_3 c_1 c_2 c_3 + a_4 s_1 s_{34} + a_4 c_1 c_2 c_{34} + d_2 s_1 + d_3 s_2 c_1 \\
&\quad - d_6 s_1 c_{345} + d_6 s_{345} c_1 c_2 + d_7 s_1 s_6 s_{345} - d_7 s_2 c_1 c_6 + d_7 s_6 c_1 c_2 c_{345} \\
J[3,1] &= 0 \\
J[4,1] &= 0 \\
J[5,1] &= 0 \\
J[6,1] &= 1
\end{aligned}
$$

$$
\begin{aligned}
J[1,2] &= -\left(a_3 s_2 c_3 + a_4 s_2 c_{34} - d_3 c_2 + d_6 s_2 s_{345} + d_7 s_2 s_6 c_{345} + d_7 c_2 c_6\right) c_1 \\
J[2,2] &= -\left(a_3 s_2 c_3 + a_4 s_2 c_{34} - d_3 c_2 + d_6 s_2 s_{345} + d_7 s_2 s_6 c_{345} + d_7 c_2 c_6\right) s_1 \\
J[3,2] &= a_3 c_2 c_3 + a_4 c_2 c_{34} + d_3 s_2 + d_6 s_{345} c_2 - d_7 s_2 c_6 + d_7 s_6 c_2 c_{345} \\
J[4,2] &= s_1 \\
J[5,2] &= -c_1 \\
J[6,2] &= 0
\end{aligned}
$$

$$
\begin{aligned}
J[1,3] &= a_3 s_1 c_3 - a_3 s_3 c_1 c_2 + a_4 s_1 c_{34} - a_4 s_{34} c_1 c_2 + d_6 s_1 s_{345} \\
&\quad + d_6 c_1 c_2 c_{345} + d_7 s_1 s_6 c_{345} - d_7 s_6 s_{345} c_1 c_2 \\
J[2,3] &= -a_3 s_1 s_3 c_2 - a_3 c_1 c_3 - a_4 s_1 s_{34} c_2 - a_4 c_1 c_{34} + d_6 s_1 c_2 c_{345} \\
&\quad - d_6 s_{345} c_1 - d_7 s_1 s_6 s_{345} c_2 - d_7 s_6 c_1 c_{345} \\
J[3,3] &= \left(-a_3 s_3 - a_4 s_{34} + d_6 c_{345} - d_7 s_6 s_{345}\right) s_2 \\
J[4,3] &= s_2 c_1 \\
J[5,3] &= s_1 s_2 \\
J[6,3] &= -c_2
\end{aligned}
$$

$J[1,4] = a_4 s_1 c_{34} - a_4 s_{34} c_1 c_2 + d_6 s_1 s_{345} + d_6 c_1 c_2 c_{345} + d_7 s_1 s_6 c_{345} - d_7 s_6 s_{345} c_1 c_2$
$J[2,4] = -a_4 s_1 s_{34} c_2 - a_4 c_1 c_{34} + d_6 s_1 c_2 c_{345} - d_6 s_{345} c_1 - d_7 s_1 s_6 s_{345} c_2 - d_7 s_6 c_1 c_{345}$
$J[3,4] = (-a_4 s_{34} + d_6 c_{345} - d_7 s_6 s_{345})\, s_2$
$J[4,4] = s_2 c_1$
$J[5,4] = s_1 s_2$
$J[6,4] = -c_2$

$J[1,5] = d_6 s_1 s_{345} + d_6 c_1 c_2 c_{345} + d_7 s_1 s_6 c_{345} - d_7 s_6 s_{345} c_1 c_2$
$J[2,5] = d_6 s_1 c_2 c_{345} - d_6 s_{345} c_1 - d_7 s_1 s_6 s_{345} c_2 - d_7 s_6 c_1 c_{345}$
$J[3,5] = (d_6 c_{345} - d_7 s_6 s_{345})\, s_2$
$J[4,5] = s_2 c_1$
$J[5,5] = s_1 s_2$
$J[6,5] = -c_2$

$J[1,6] = d_7\,(s_1 s_{345} c_6 + s_2 s_6 c_1 + c_1 c_2 c_6 c_{345})$
$J[2,6] = d_7\,(s_1 s_2 s_6 + s_1 c_2 c_6 c_{345} - s_{345} c_1 c_6)$
$J[3,6] = d_7\,(s_2 c_6 c_{345} - s_6 c_2)$
$J[4,6] = -s_1 c_{345} + s_{345} c_1 c_2$
$J[5,6] = s_1 s_{345} c_2 + c_1 c_{345}$
$J[6,6] = s_2 s_{345}$

$J[1,7] = 0$
$J[2,7] = 0$
$J[3,7] = 0$
$J[4,7] = (s_1 s_{345} + c_1 c_2 c_{345})\, s_6 - s_2 c_1 c_6$
$J[5,7] = (s_1 c_2 c_{345} - s_{345} c_1)\, s_6 - s_1 s_2 c_6$
$J[6,7] = s_2 s_6 c_{345} + c_2 c_6$

3.2 Method 2: Geometric Jacobian

We first form our D_i matrices from

$$D_i = T_{0i} Q_i T_{0i}^{-1}$$

where, as all our joints are revolute,

$$Q = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Selecting elements from these D_i matrices, we form the Jacobian via

$$J = \begin{bmatrix} {}^0D_{14} & {}^1D_{14} & {}^2D_{14} & {}^3D_{14} & {}^4D_{14} & {}^5D_{14} & {}^6D_{14} \\ {}^0D_{24} & {}^1D_{24} & {}^2D_{24} & {}^3D_{24} & {}^4D_{24} & {}^5D_{24} & {}^6D_{24} \\ {}^0D_{34} & {}^1D_{34} & {}^2D_{34} & {}^3D_{34} & {}^4D_{34} & {}^5D_{34} & {}^6D_{34} \\ {}^0D_{32} & {}^1D_{32} & {}^2D_{32} & {}^3D_{32} & {}^4D_{32} & {}^5D_{32} & {}^6D_{32} \\ {}^0D_{13} & {}^1D_{13} & {}^2D_{13} & {}^3D_{13} & {}^4D_{13} & {}^5D_{13} & {}^6D_{13} \\ {}^0D_{21} & {}^1D_{21} & {}^2D_{21} & {}^3D_{21} & {}^4D_{21} & {}^5D_{21} & {}^6D_{21} \end{bmatrix}$$

Resulting in

$$J[1,1] = 0$$
$$J[2,1] = 0$$
$$J[3,1] = 0$$
$$J[4,1] = 0$$
$$J[5,1] = 0$$
$$J[6,1] = 1$$

$$J[1,2] = d_1 c_1$$
$$J[2,2] = d_1 s_1$$
$$J[3,2] = 0$$
$$J[4,2] = s_1$$
$$J[5,2] = -c_1$$
$$J[6,2] = 0$$

$$J[1,3] = -d_1 s_1 s_2 + d_2 c_1 c_2$$
$$J[2,3] = d_1 s_2 c_1 + d_2 s_1 c_2$$
$$J[3,3] = d_2 s_2$$
$$J[4,3] = s_2 c_1$$
$$J[5,3] = s_1 s_2$$
$$J[6,3] = -c_2$$

$$J[1,4] = -a_3 s_1 c_3 + a_3 s_3 c_1 c_2 - d_1 s_1 s_2 + d_2 c_1 c_2$$
$$J[2,4] = a_3 s_1 s_3 c_2 + a_3 c_1 c_3 + d_1 s_2 c_1 + d_2 s_1 c_2$$
$$J[3,4] = (a_3 s_3 + d_2)\, s_2$$
$$J[4,4] = s_2 c_1$$
$$J[5,4] = s_1 s_2$$
$$J[6,4] = -c_2$$

$$J[1,5] = -a_3 s_1 c_3 + a_3 s_3 c_1 c_2 - a_4 s_1 c_{34} + a_4 s_{34} c_1 c_2 - d_1 s_1 s_2 + d_2 c_1 c_2$$
$$J[2,5] = a_3 s_1 s_3 c_2 + a_3 c_1 c_3 + a_4 s_1 s_{34} c_2 + a_4 c_1 c_{34} + d_1 s_2 c_1 + d_2 s_1 c_2$$
$$J[3,5] = (a_3 s_3 + a_4 s_{34} + d_2)\, s_2$$
$$J[4,5] = s_2 c_1$$
$$J[5,5] = s_1 s_2$$
$$J[6,5] = -c_2$$

$$J[1,6] = -(d_1 c_2 - d_3)(s_1 s_{345} + c_1 c_2 c_{345}) - (a_3 c_{45} + a_4 c_5 + d_1 s_2 c_{345} + d_2 s_{345})\, s_2 c_1$$
$$J[2,6] = -(d_1 c_2 - d_3)(s_1 c_2 c_{345} - s_{345} c_1) - (a_3 c_{45} + a_4 c_5 + d_1 s_2 c_{345} + d_2 s_{345})\, s_1 s_2$$
$$J[3,6] = a_3 c_2 c_{45} + a_4 c_2 c_5 + d_2 s_{345} c_2 + d_3 s_2 c_{345}$$
$$J[4,6] = -s_1 c_{345} + s_{345} c_1 c_2$$
$$J[5,6] = s_1 s_{345} c_2 + c_1 c_{345}$$
$$J[6,6] = s_2 s_{345}$$

$$\begin{aligned}
J[1,7] &= \big((s_1 s_{345} + c_1 c_2 c_{345})\, c_6 + s_2 s_6 c_1\big)(a_3 s_{45} + a_4 s_5 + d_1 s_2 s_{345} - d_2 c_{345} + d_6) \\
&\quad + (s_1 c_{345} - s_{345} c_1 c_2)(a_3 c_6 c_{45} + a_4 c_5 c_6 + d_1 s_2 c_6 c_{345} - d_1 s_6 c_2 + d_2 s_{345} c_6 + d_3 s_6) \\
J[2,7] &= \big((s_1 c_2 c_{345} - s_{345} c_1)\, c_6 + s_1 s_2 s_6\big)(a_3 s_{45} + a_4 s_5 + d_1 s_2 s_{345} - d_2 c_{345} + d_6) \\
&\quad - (s_1 s_{345} c_2 + c_1 c_{345})(a_3 c_6 c_{45} + a_4 c_5 c_6 + d_1 s_2 c_6 c_{345} - d_1 s_6 c_2 + d_2 s_{345} c_6 + d_3 s_6) \\
J[3,7] &= (s_2 c_6 c_{345} - s_6 c_2)(a_3 s_{45} + a_4 s_5 + d_1 s_2 s_{345} - d_2 c_{345} + d_6) - \\
&\quad (a_3 c_6 c_{45} + a_4 c_5 c_6 + d_1 s_2 c_6 c_{345} - d_1 s_6 c_2 + d_2 s_{345} c_6 + d_3 s_6)\, s_2 s_{345} \\
J[4,7] &= s_1 s_6 s_{345} - s_2 c_1 c_6 + s_6 c_1 c_2 c_{345} \\
J[5,7] &= -s_1 s_2 c_6 + s_1 s_6 c_2 c_{345} - s_6 s_{345} c_1 \\
J[6,7] &= s_2 s_6 c_{345} + c_2 c_6
\end{aligned}$$

3.3 Velocity Equation

I should write this?

4 Conclusions

References

[1] C. Lee and M. Ziegler. Geometric approach in solving inverse kinematics of puma robots. *IEEE Transactions on Aerospace and Electronic Systems*, (6):695–706, 1984.

[2] S. Robinson and T. Virts. "private communication", 2018.

[3] W. Xu, Y. She, and Y. Xu. Analytical and semi-analytical inverse kinematics of ssrms-type manipulators with single joint locked failure. *Acta Astronautica*, 105(1):201–217, 2014.

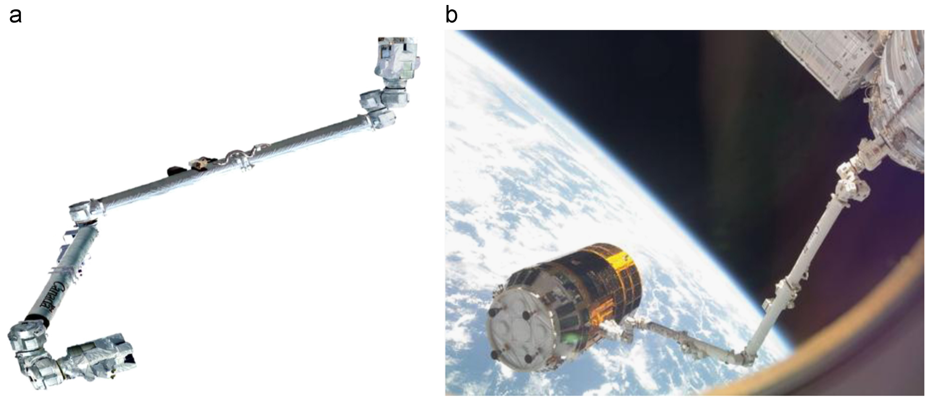

Figure 1: a. Render of the Space Station Remote Manipulator System (SSRMS). b. Image of the SSRMS grappling with a visiting vehicle.

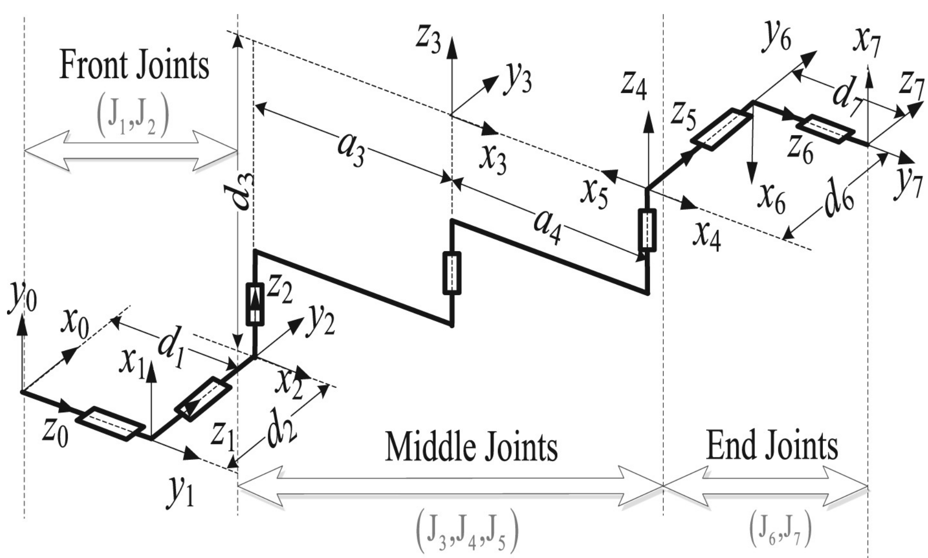

Figure 2: The Denavit-Hartenberg (D-H) parameters for the Space Station Remote Manipulator System (SSRMS).