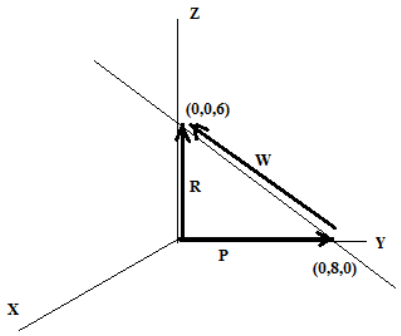
Hwk 2 Solutions

Prob. 1:

Part A: Consider the vectors shown: $\mathbf{R} = \mathbf{P} + \mathbf{W}$ so: $\mathbf{W} = \mathbf{R} - \mathbf{P} = 6\hat{\mathbf{k}} - 8\hat{\mathbf{j}}$



The unit vector along the screw axis is then: $\hat{w} = \dfrac{6\hat{\mathbf{k}} - 8\hat{\mathbf{j}}}{\sqrt{6^2 + 8^2}} = -0.8\hat{\mathbf{j}} + 0.6\hat{\mathbf{k}}$ or

$$\underline{\underline{\tilde{w}}} = \begin{pmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{pmatrix} = \begin{pmatrix} 0 & -0.6 & -0.8 \\ 0.6 & 0 & 0 \\ 0.8 & 0 & 0 \end{pmatrix} ; \; \underline{P} = (0,8,0)$$

$$\underline{\underline{\tilde{w}}}^2 = \begin{pmatrix} 0 & -0.6 & -0.8 \\ 0.6 & 0 & 0 \\ 0.8 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -0.6 & -0.8 \\ 0.6 & 0 & 0 \\ 0.8 & 0 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -0.36 & -0.48 \\ 0 & -.48 & -0.64 \end{pmatrix}$$

$\underline{\underline{\Theta}} = \underline{\underline{I}} + \underline{\underline{\tilde{w}}}\,\mathrm{Sin}\,\theta + \underline{\underline{\tilde{w}}}^2(1 - \mathrm{Cos}\,\theta)$ Substituting values:

$$\underline{\underline{\Theta}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -0.6 & -0.8 \\ 0.6 & 0 & 0 \\ 0.8 & 0 & 0 \end{pmatrix}\mathrm{Sin}\,20 + \begin{pmatrix} -1 & 0 & 0 \\ 0 & -0.36 & -0.48 \\ 0 & -.48 & -0.64 \end{pmatrix}(1 - \mathrm{Cos}\,20)$$

$$\underline{\underline{\Theta}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -0.6 & -0.8 \\ 0.6 & 0 & 0 \\ 0.8 & 0 & 0 \end{pmatrix}(0.342) + \begin{pmatrix} -1 & 0 & 0 \\ 0 & -0.36 & -0.48 \\ 0 & -.48 & -0.64 \end{pmatrix}(1 - 0.94)$$

$$\underline{\underline{\Theta}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & -0.2052 & -0.2736 \\ 0.2052 & 0 & 0 \\ 0.2736 & 0 & 0 \end{pmatrix} + \begin{pmatrix} -0.06 & 0 & 0 \\ 0 & -0.0216 & -0.0288 \\ 0 & -0.0288 & -0.0384 \end{pmatrix} = \begin{pmatrix} 0.94 & -0.2052 & -0.2736 \\ 0.2052 & 0.9784 & -0.0288 \\ 0.2736 & -0.0288 & 0.9616 \end{pmatrix}$$

$$\underline{d} = (\underline{\underline{I}} - \underline{\underline{\Theta}})\underline{P} + \phi \underline{w} \ ;$$

$$\underline{d} = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.94 & -0.2052 & -0.2736 \\ 0.2052 & 0.9784 & -0.0288 \\ 0.2736 & -0.0288 & 0.9616 \end{pmatrix} \right\} \begin{pmatrix} 0 \\ 8 \\ 0 \\ 1 \end{pmatrix} + (-6) \begin{pmatrix} 0 \\ -0.8 \\ 0.6 \\ 1 \end{pmatrix} = \begin{pmatrix} 1.6417 \\ 4.9737 \\ -3.3684 \\ 1 \end{pmatrix}$$

Finally

$$\underline{\underline{T}} = \begin{pmatrix} 0.94 & -0.2052 & -0.2736 & 1.6417 \\ 0.2052 & 0.9784 & -0.0288 & 4.9737 \\ 0.2736 & -0.0288 & 0.9616 & -3.3684 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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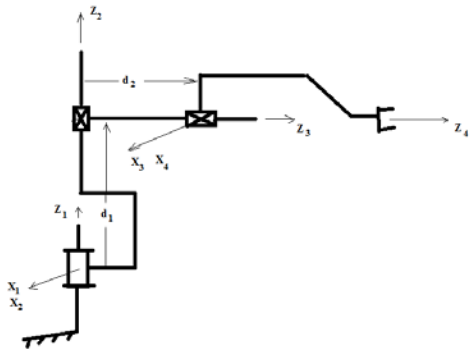

D-H	1	2	3
a	0	0	0
α	0	-90	0
θ	Θ1	0	0
S	0	d1	d2

$$A_1 = \begin{bmatrix} C_{\theta_1} & -S_{\theta_1} & 0 & 0 \\ S_{\theta_1} & C_{\theta_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix};\ A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$