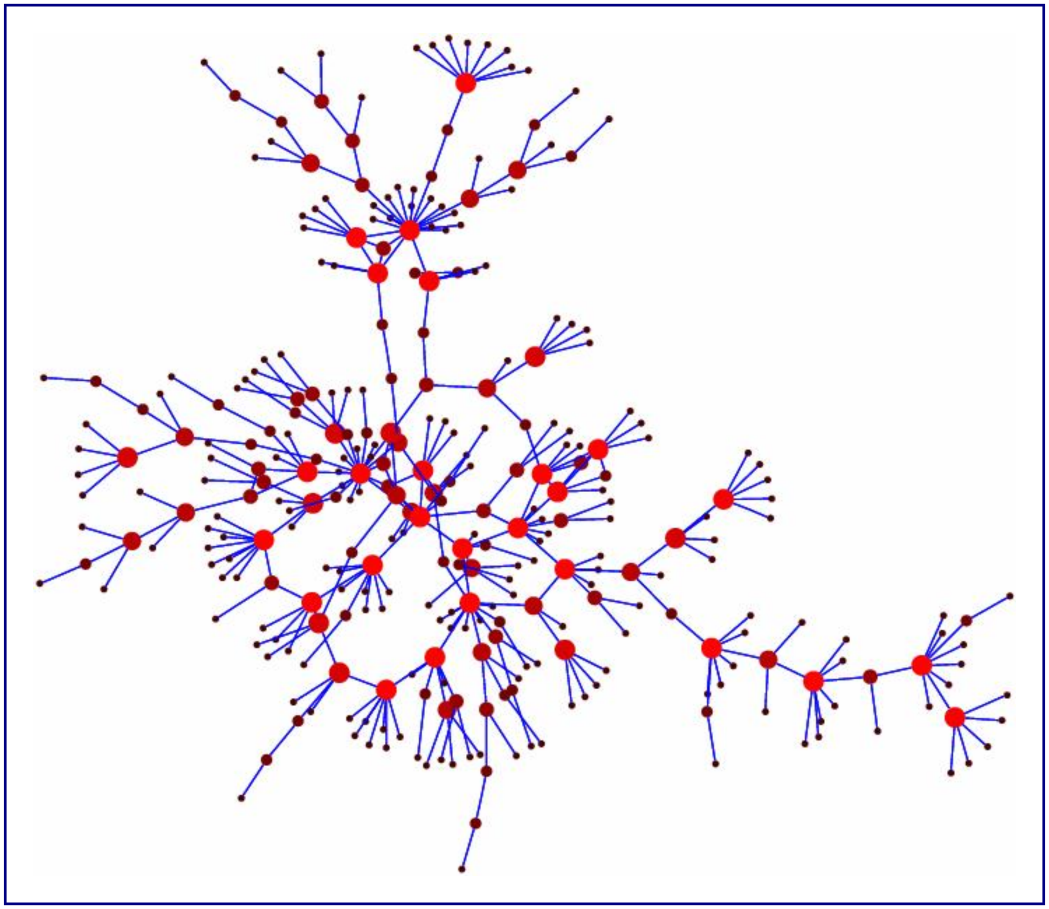
ECS 253 / MAE 253, Lecture 13

May 10, 2016



I. "Games on networks"

II. "Diffusion, Cascades and Influence"

Summary of spatial flows and games

- Optimal location of facilities to maximize access for all.
- Designing "optimal" spatial networks
  (collection/distribution networks – subways, power lines, road networks, airline networks).
- Details of flows on actual networks make all the difference!
  - Users act according to Nash
  - Braess paradox
    (removing edges may improve a network's performance!)
  - The "Price of Anarchy"
    (cost of worst Nash eqm / cost of system optimal)

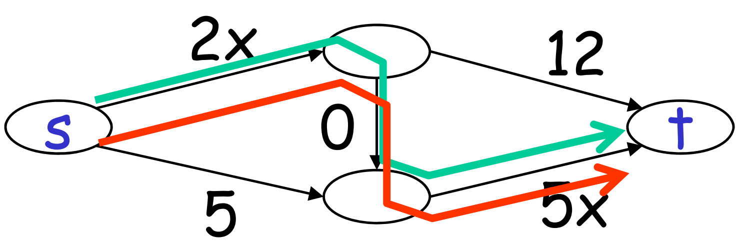
“The price of anarchy”

E. Koutsoupias, C. H. Papadimitriou
“Worst-case equilibria,” STACS 99.

Cost of worst case Nash equilibrium / cost of system optimal solution.

The Price of Anarchy

Nash Equilibrium:



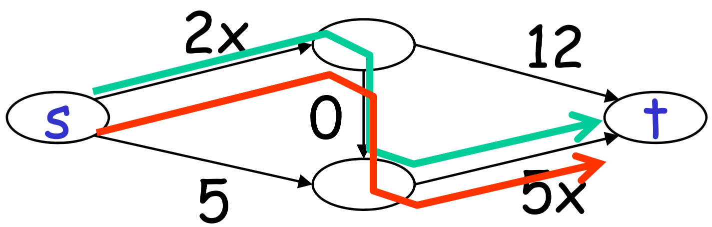
cost = 14+14 = 28

- Assume two units of flow from s to t.
- Both follow the user optimal path.

(From Roughgarden Barbados Talk, http://theory.stanford.edu/ tim/)

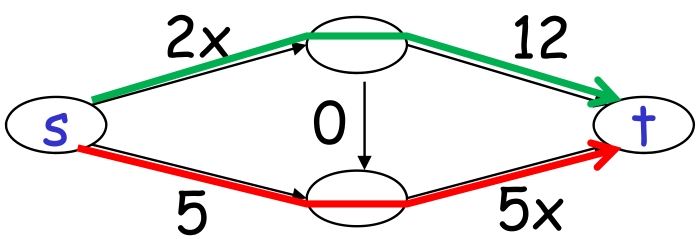
The Price of Anarchy

Nash Equilibrium:



cost = 14+14 = 28

To Minimize Cost:



cost = 14+10 = 24

Price of anarchy = 28/24 = 7/6.

- if multiple equilibria exist, look at the *worst* one

(From Roughgarden Barbados Talk, http://theory.stanford.edu/ tim/)

Selfish routing and the POA on the Internet

T. Roughgarden and E. Tardos, How Bad is Selfish Routing?, FOCS '00/JACM '02

- Routing in the Internet is *decentralized*: Each router makes a decision, so path dynamically decided as packet passed on.
- Cost of an edge $c(e)$, may be constant (infinite capacity) or depend on the load.
- *"Shortest path"* routing (really lowest $\sum c(e)$ routing) typically implemented.
- This is equivalent to "selfish routing" (each router chooses best option available to it).
- **Resulting POA = 2!**

Braess and the POA for Internet traffic

Greg Valiant, Tim Roughgarden, Eva Tardos
"Braess's paradox in large random graphs", Proceedings of the 7th ACM conference on Electronic commerce, 2006.

- Removing edges from a network with "selfish routing" can decrease the latency incurred by traffic in an equilibrium flow.

- With high probability, (as the number of vertices goes to infinity), there is a traffic rate and a set of edges whose removal improves the latency of traffic in an equilibrium flow by a constant factor.

- Braess paradox found in random networks often (not just "classic" 4-node construction).

The Nash Inequality ratio

Sam Johnson and R.D., *Internet Mathematics*, 2015.

- How unfair is a particular Nash equilibrium to the individual players?
- NIR = the maximal ratio between the highest and lowest costs incurred to individual agents in a Nash equilibrium strategy
- Contrary to common expectations, efficiency (system optimal) does not necessarily come at the expense of increased inequality.

Algorithmic game theory

- Since we know users act according to Nash, can we design algorithms (mechanisms) that bring Nash and System Optimal as close together as possible?

- Typically we think of players who interact via a network, or who's connectivity is described by a network of interactions.

  – Multiplayer games for users connected in a network or interacting via a network.

  – Designing algorithms with desirable Nash equilibrium.

  – Computing equilibrium when agents connected in a network.

mechanism design
(or *inverse* game theory)

- agents have utilities – but these utilities are known *only to them*
- game designer prefers certain outcomes *depending on players' utilities*
- designed game (mechanism) has designer's goals as dominating strategies

(Papadimitriou, "Algorithms, Games, and the Internet" presented at STOC/ICALP 2001.)

Some traditional games:

e.g.

matching pennies

1,-1	-1,1
-1,1	1,-1

prisoner's dilemma

3,3	0,4
4,0	1,1

chicken

0,0	0,1
1,0	-1,-1

auction

	1 … n
1 . . n	$0,\ v-y$ $u-x,\ 0$

focs 2001

7

(Papadimitriou, "Algorithms, Games, and the Internet" presented at STOC/ICALP 2001.)

Mechanism design example:

e.g., Vickrey auction

- sealed-highest-bid auction encourages gaming and speculation
- Vickrey auction: Highest bidder wins,

  pays second-highest bid

Theorem: Vickrey auction is a truthful mechanism.

Theorem: It maximizes social benefit *and* auctioneer expected revenue.

(Papadimitriou, “Algorithms, Games, and the Internet” presented at STOC/ICALP 2001.)

(Modified) Vickrey auctions in real life – Google AdWords, and Yahoo's ad sales

- Bidding on a "keyword" so that your advertisement is displayed when a search user enters in this keyword
- You can safely bid the maximum price you think is fair, and if you win, you actually pay less!
- Mechanism design
  – Incentivizes users to bid what they think is fair (reveal their true utilities)
  – Keeps more people bidding

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  - The “Price of Anarchy” (cost of worst Nash eqm / cost of system optimal)
- Mechanism design / algorithmic game theory

II. Diffusion and cascades in networks
(Nodes in one of two states)

- Viruses (human and computer)
  – contact processes
  – epidemic thresholds

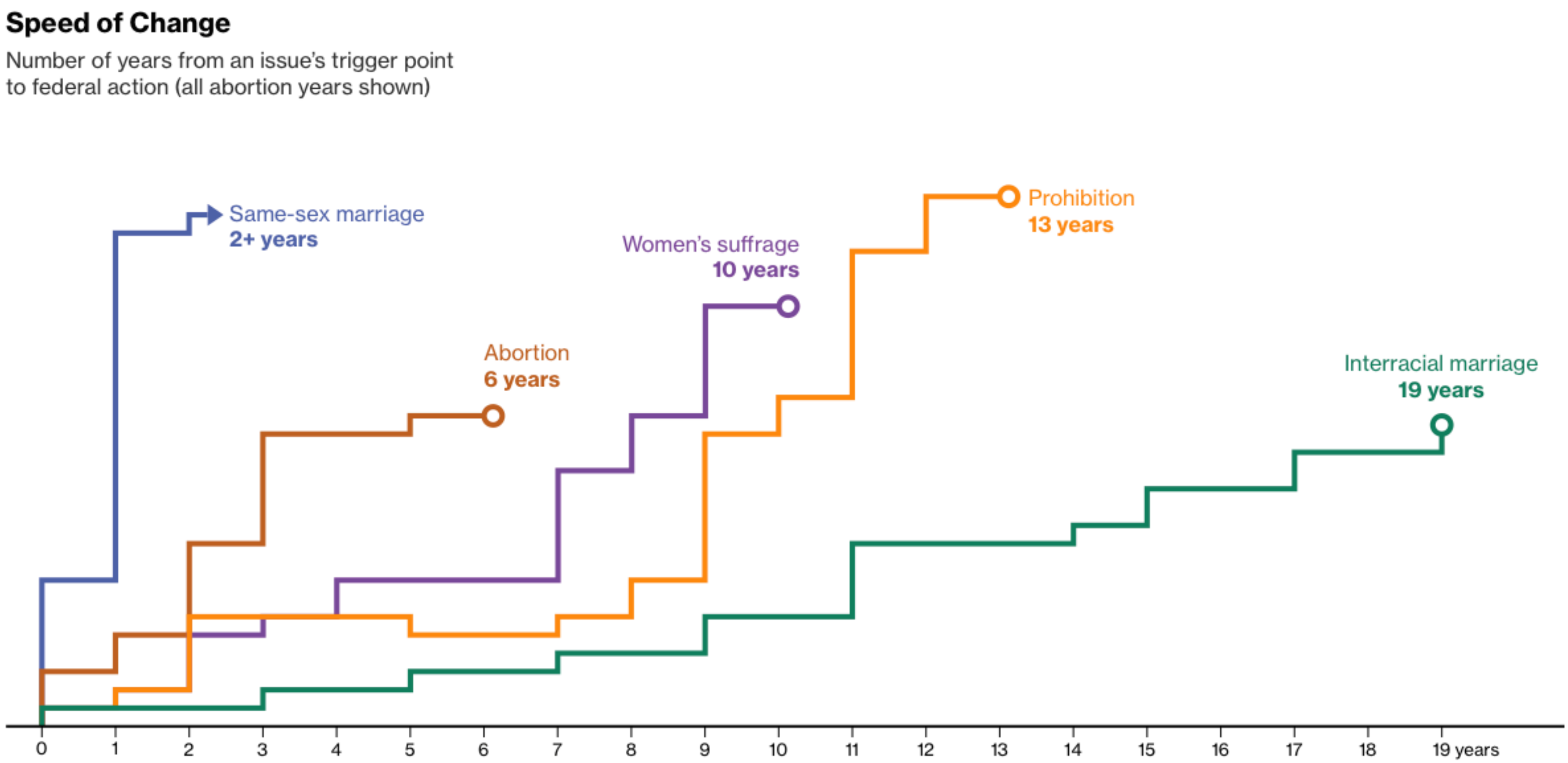
- Adoption of new technologies
  – Winner take all
  – Benefit of first to market
  – Benefit of second to market

- Political or social beliefs and societal norms

A long history of study, now trying to add impact of underlying network structure.

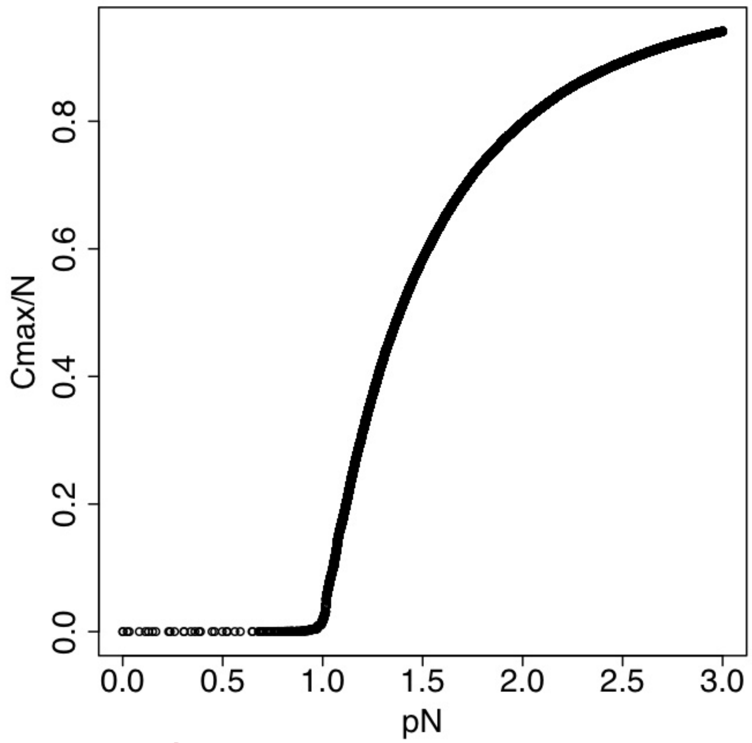
What drives social change?

Accelerating pace of social change

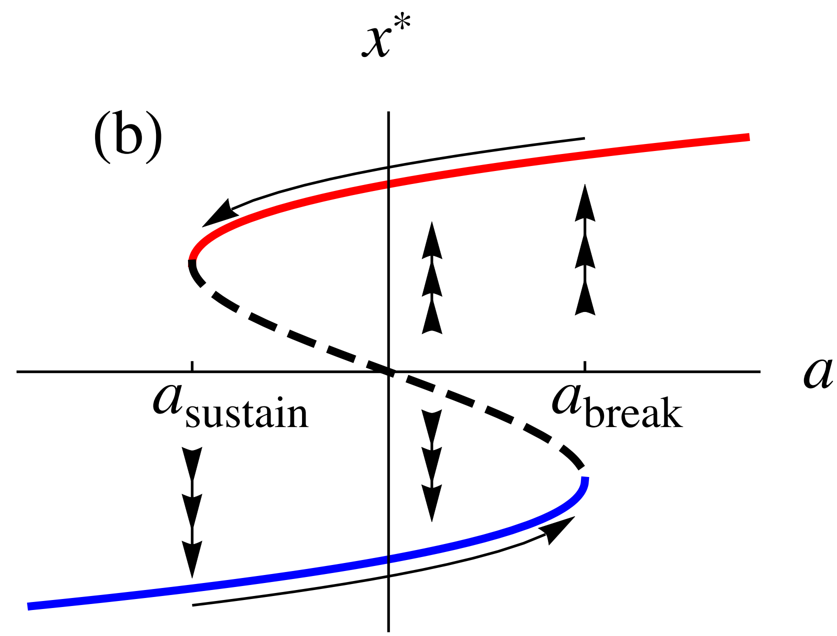


Bloomberg, April 26, 2015.

Collective phenomena: Phase transitions



Smooth transition



Cusp bifurcation/catastrophe

- Percolation
- Contact processes
- Epidemic spreading

- $\frac{dx}{dt} = -x^3 + x + a.$
- Abrupt shift as slow-time parameter varies
  - e.g., Vinyl records vs digital music

Phase transitions depend on the underlying details

- **The network structure**
  - Degree distribution (variation in connectivity)
  - Modular structure
- **The model of human behavior**
  - Simple contact process / percolation / epidemic spreading
    - Thresholds (critical mass) versus diminishing returns
    - Influential versus susceptible individuals
  - Voter models
  - Opinion dynamics / consensus
    - The role of zealots
  - Strategic interactions / Nash equilibrium (decentralized solutions)

Simplest model of human behavior:

Binary opinion dynamics

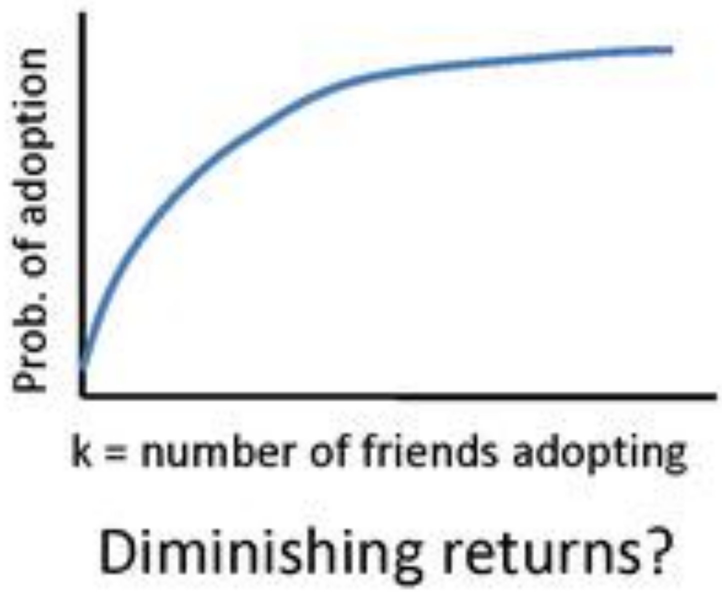
Each individual can be in one of two states $\{-1, +1\}$

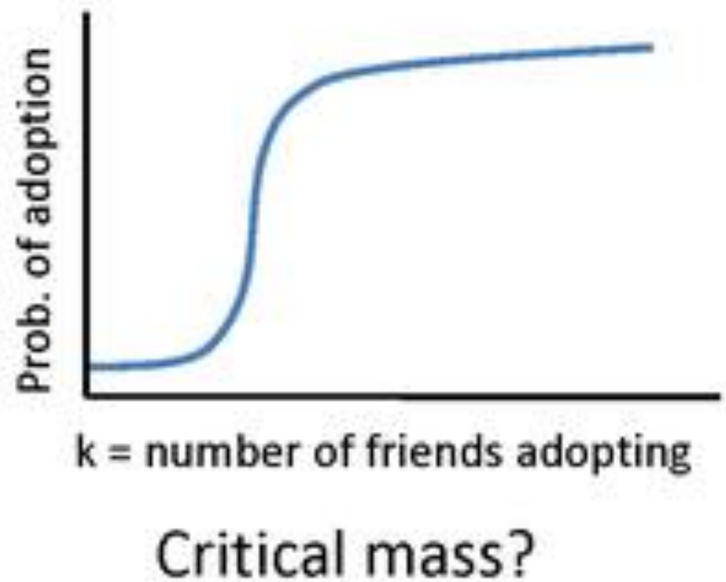
- "Infected" or "healthy" (relevant to both human and computer networks)
- Holding opinion "A" or "B"
- Adopting new product, or sticking with status quo
- Many other choices....

But what causes opinion to change?

Q1: Diminishing returns versus thresholds

Why would an individual person change their mind?





Kleinberg, Leskovec, Kempe
e.g., *KDD* 2003.

"Hill climbing" / best response
Algorithms for influential seed nodes

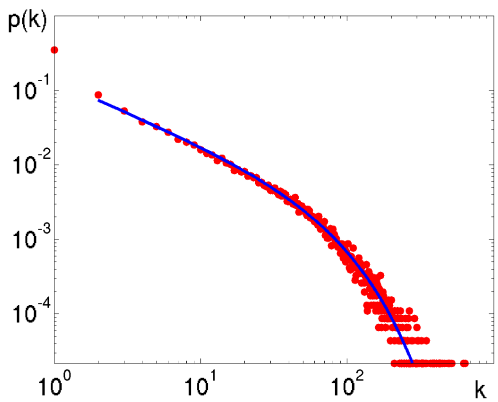
Watts, Dodds
e.g. *PNAS* 2002.

Percolation & generating functions
Susceptibles vs influentials/mavens
(Depends on active vs passive influence.)

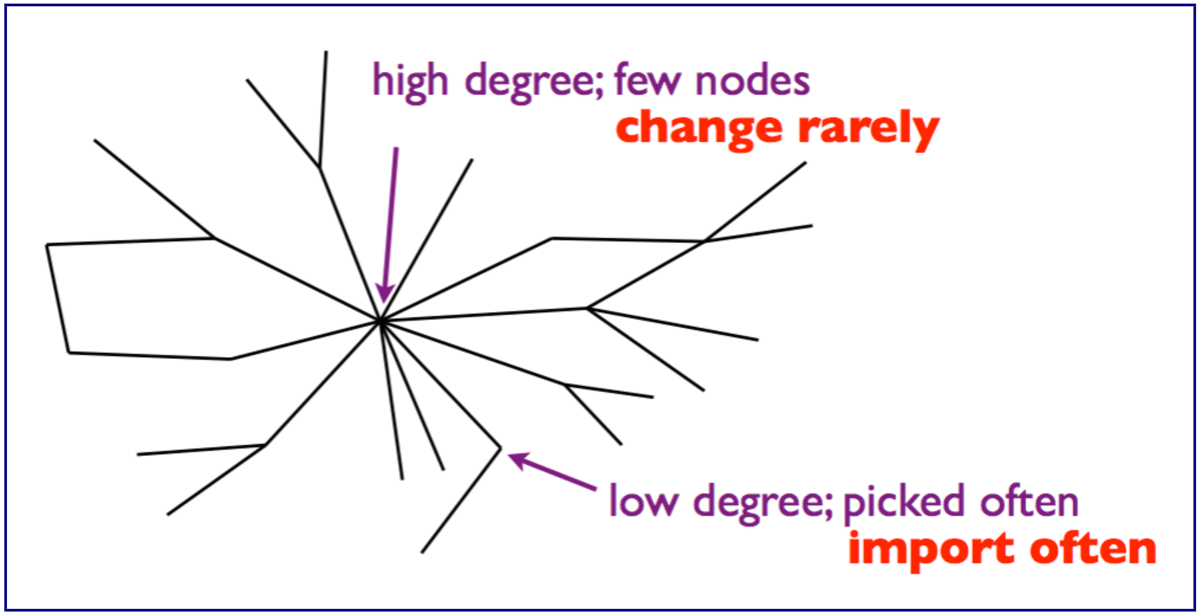
Q2: The Voter model, “Tell me what to think”

V. Sood, S. Redner, *Phys. Rev. Lett.* 94, 2005.

- At each time step in the process, pick a node at random.
- That node picks a random neighbor, and adopts the opinion of the neighbor.
- Ultimately, only one opinion prevails. The high degree nodes (hubs) win.



Degree distribution

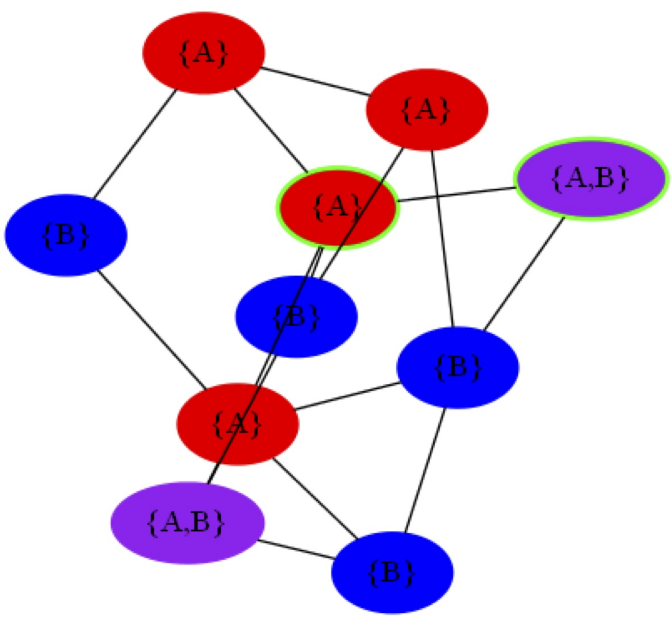


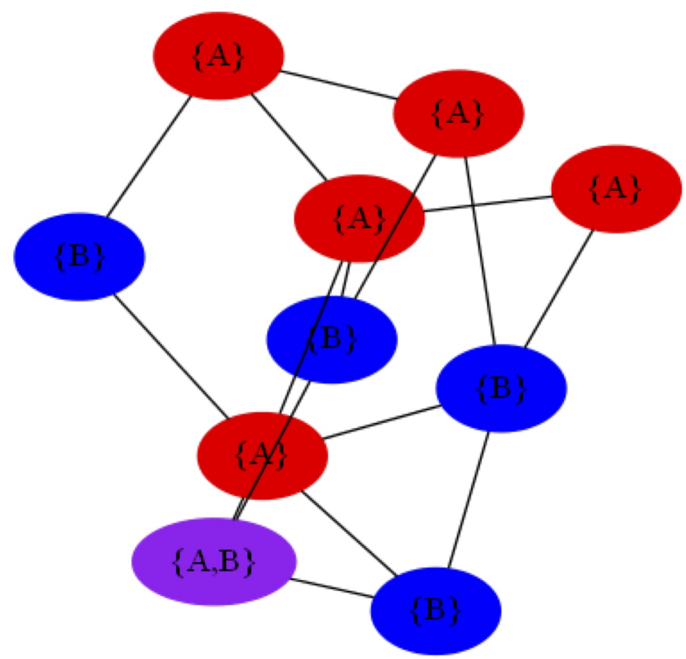
(Invasion percolation process yields the opposite: leaf nodes propagate opinons.)

Q3: "The Naming Game" / open minded individuals

Steels, *Art. Life* 1995; Barrat *et al.*, *Chaos* 2007; Baronchelli *et al.*, *Int. J. Mod. Phys.* 2008.

- Originally introduced for linguistic convergence. Two opinions, A and B.
- And each individual can hold A, B, or $\{A, B\}$.
- At each time-step, choose a node at random and a neighbor at random and exchange opinions.
- If disagree, add the opinion to your set. If any overlap, reach consensus.





Diffusion, Cascade behaviors, and influential nodes

Part I: Ensemble models

Generating functions / Master equations / giant components

- Contact processes / more similar to biological epidemic spreading
- Heterogeneity due to node degree (not due to different node preferences)
- Epidemic spreading
- Opinion dynamics
- Social nets: Watts PNAS (threshold model; no global cascade region)

Diffusion, Cascade behaviors, and influential nodes
Part II: Contact processes with individual node preferences

- Long history of empirical / qualitative study in the social sciences (Peyton Young, Granovetter, Martin Nowak ...; diffusion of innovation; societal norms)

- Recent theorems: "network coordination games" (bigger payout if connected nodes in the same state) (Kleinberg, Kempe, Tardos, Dodds, Watts, Domingos)

- Finding the influential set of nodes, or the k most influential. Often NP-hard and not amenable to approximation algorithms

- Key distinction:
  – thresholds of activation (leads to unpredictable behaviors)
  – diminishing returns (submodular functions nicer)

Diffusion, Cascade behaviors, and influential nodes

Part III: Markov chains and mixing times

- New game-theoretic approaches (general coordination games)
  – Results in an Ising model.
  – Montanari and Saberi PNAS 2010.

- Techniques from Parts I and II suggest:
  – Innovations spread quickly in highly connected networks.
  – Long-range links benefit spreading.
  – High-degree nodes quite influential (enhance spreading).

- Techniques from Part III suggest:
  – Innovations spread quickly in locally connected networks.
  – Local spatial coordination enhances spreading (having a spatial metric; graph embeddable in small dimension).
  – High-degree nodes slow down spreading.