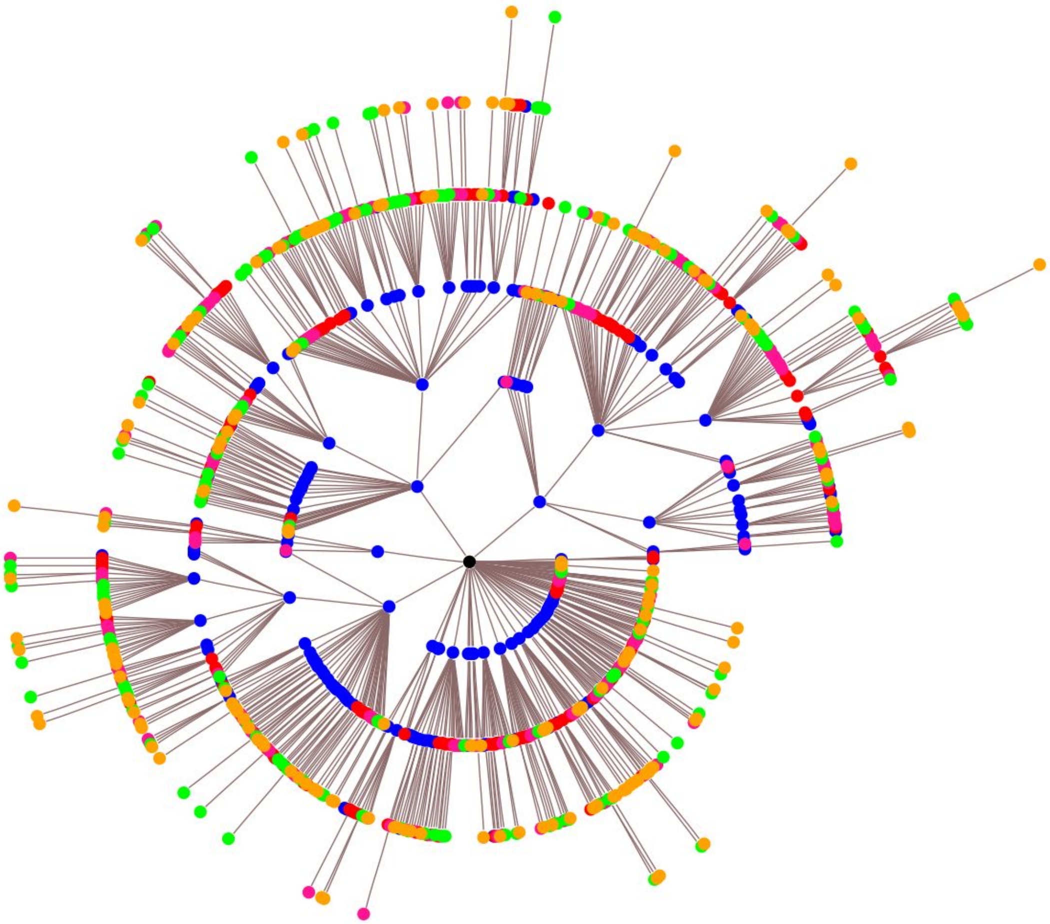
ECS 253 / MAE 253, Lecture 5

April 12, 2016



"Internet measurement and Optimization approaches to network growth"

Homework 1
Assignment

- HW1: To be completed by all
  (adjacency matrix, rate equations)
  Due Thurs April 21

- HW1a) Project pitch, due this FRIDAY April 15, 5pm via smartsite.

- HW1b) Advanced: Erdős-Rényi random graphs
  Due Thurs April 21

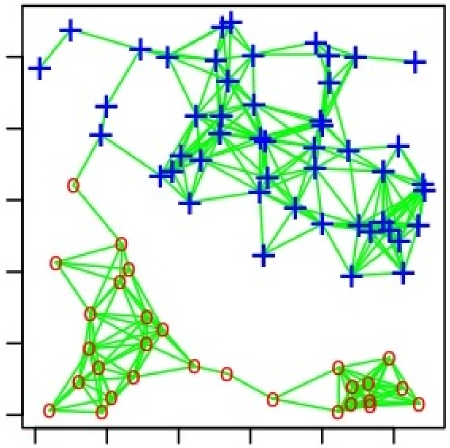
Project pitch

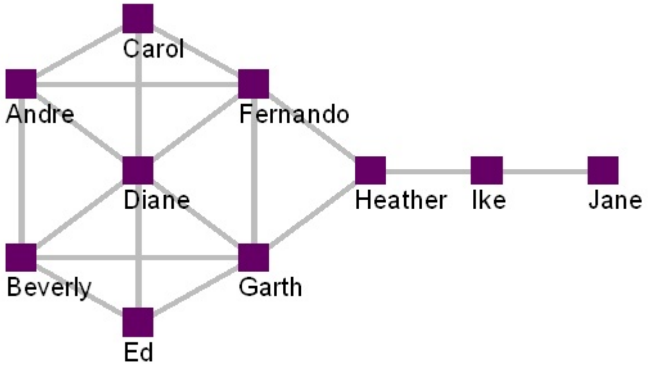
- 1-2 paragraphs (∼400 words) describing your idea.
  Submitted via Smartsite and shared with the class.
- This is a pitch, it is not binding. It is your chance to recruit team mates.
- Skill sets to merge: System / Application / Method / Data
- We have a lot of ideas (See smartsite):

Last time

- **"Error and attack tolerance of complex networks"**
  Random networks with power law degree distribution show:
  – Fragility to degree-targeted removal
  – Robustness to random node removal
  (This is in the context of keeping the full network connected.)

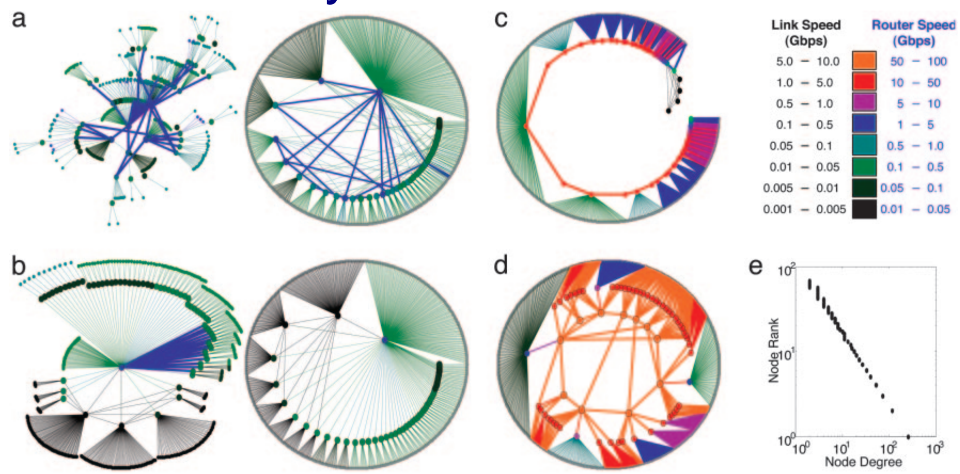
- **Important nodes beyond degree**
  – Betweeness centrality (shortest paths)
  (Are their local ways to detect this?)
  – Boundary spanners / peripheral players / weak-ties





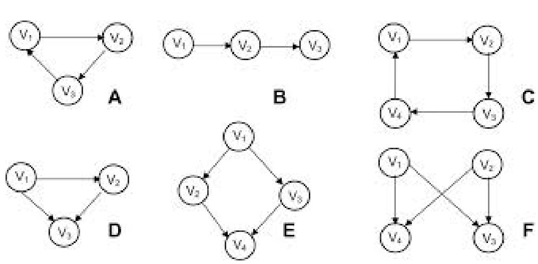
Last time (cont.): Structure beyond degree distribution

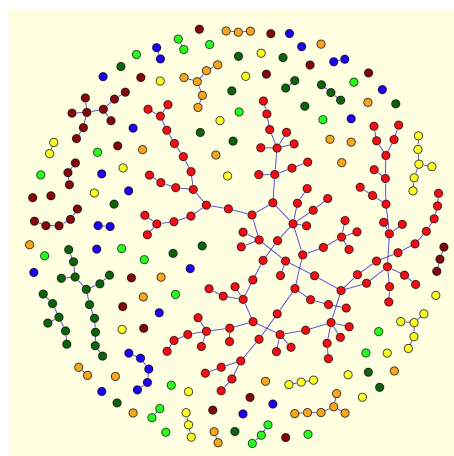
- Power law degree distribution actually a weak constraint on network structure:

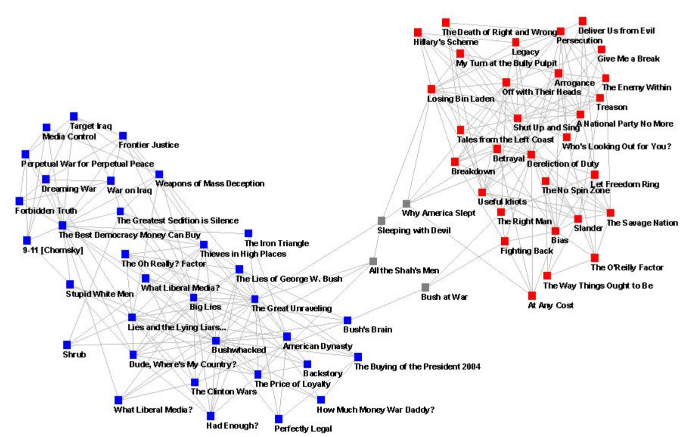


- Additional properties include:

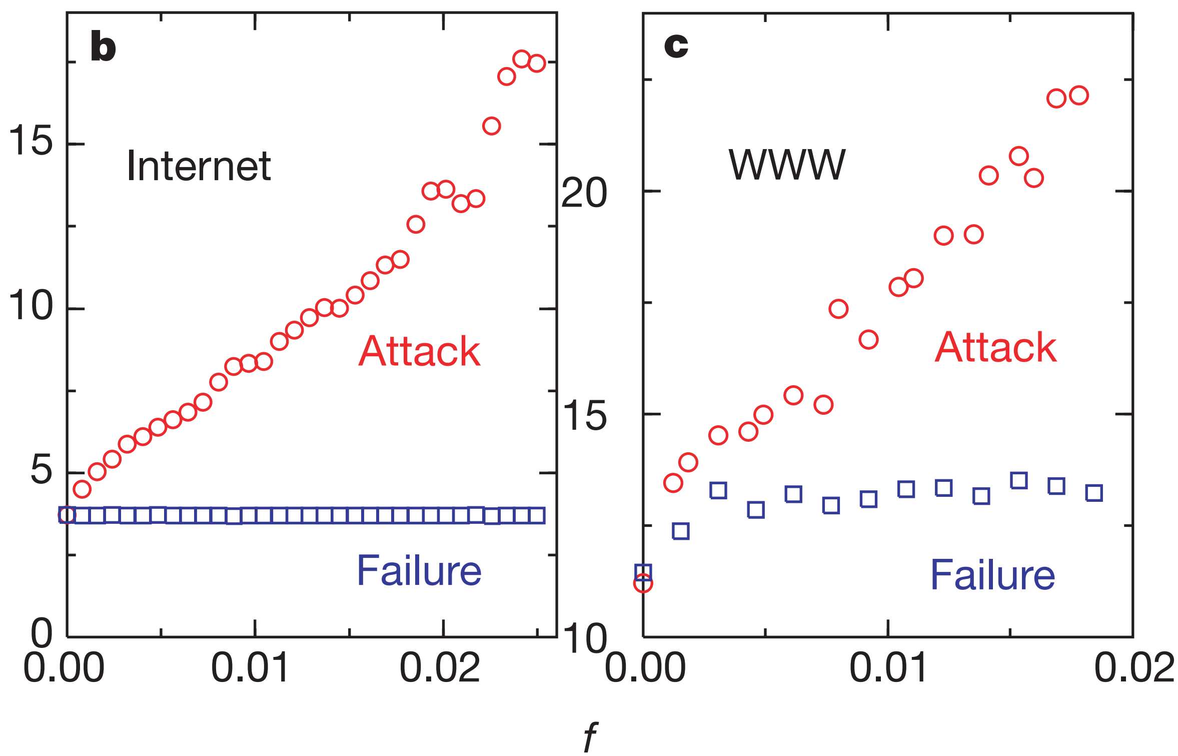
Motifs | Components | Communities





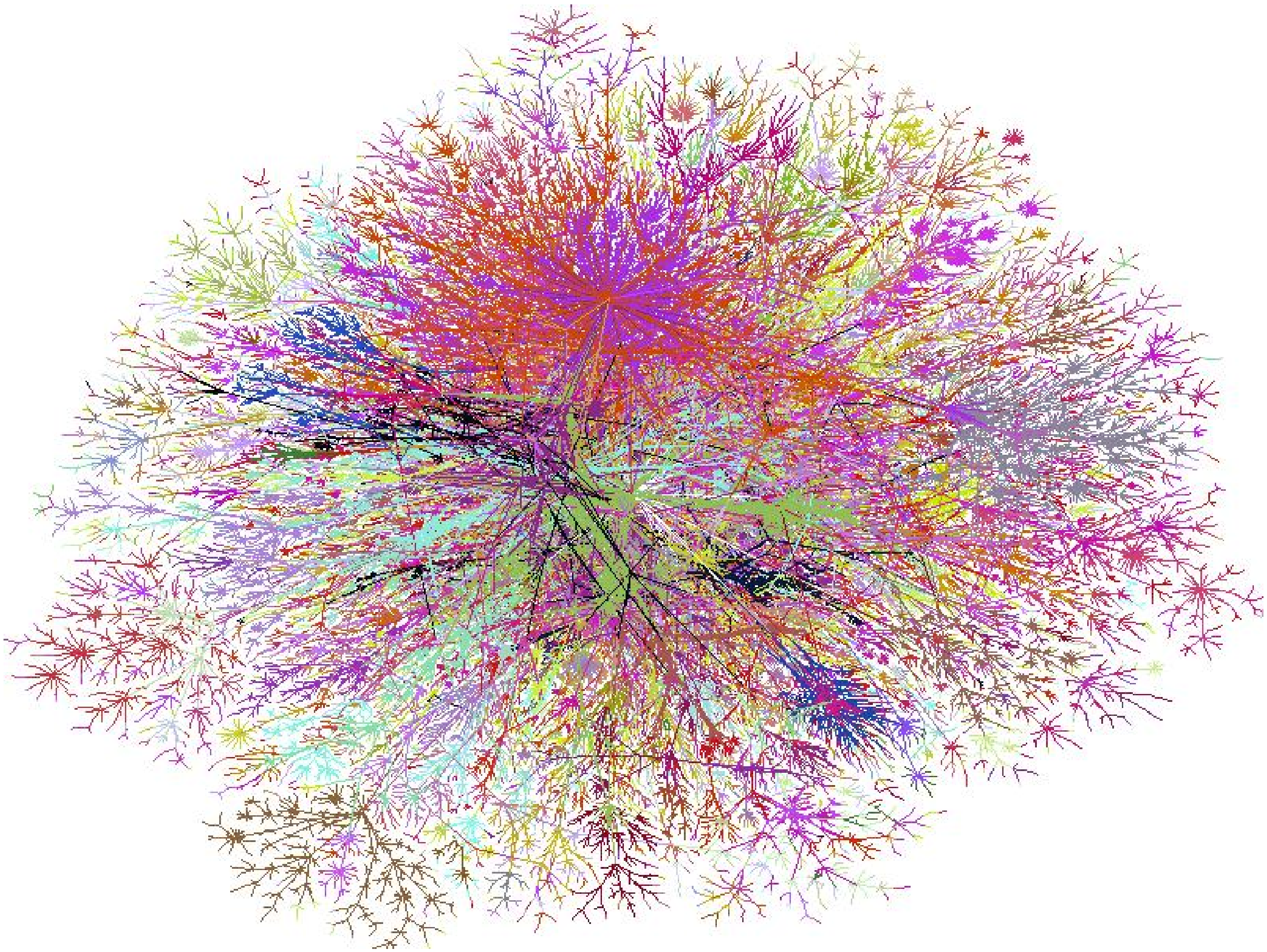


Power law random graph:
Robust to random failure, vulnerable to targeting attack



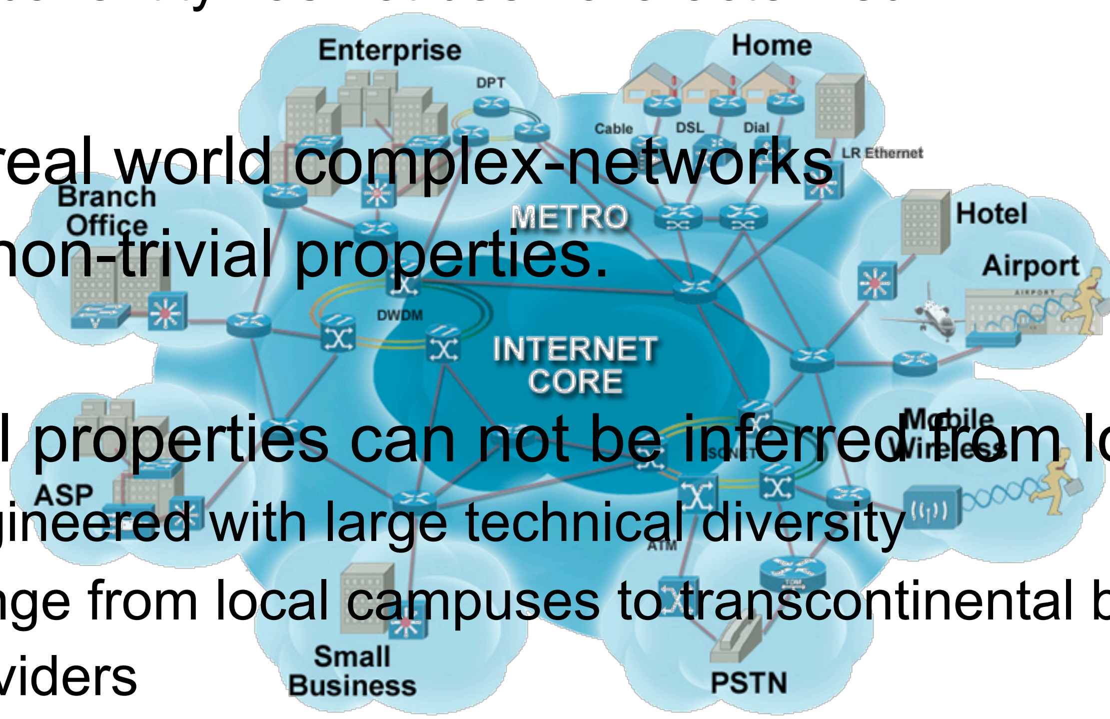
Why did Albert, Jeong and Barabasi find that their sample of the internet topology was vulnerable to degree targeted attack?

What is the Internet?



Internet

- Web of interconnected networks
  - Grows with no central authority
  - Autonomous Systems optimize local communication efficiency
  - The building blocks are engineered and studied in depth
  - Global entity has not been characterized

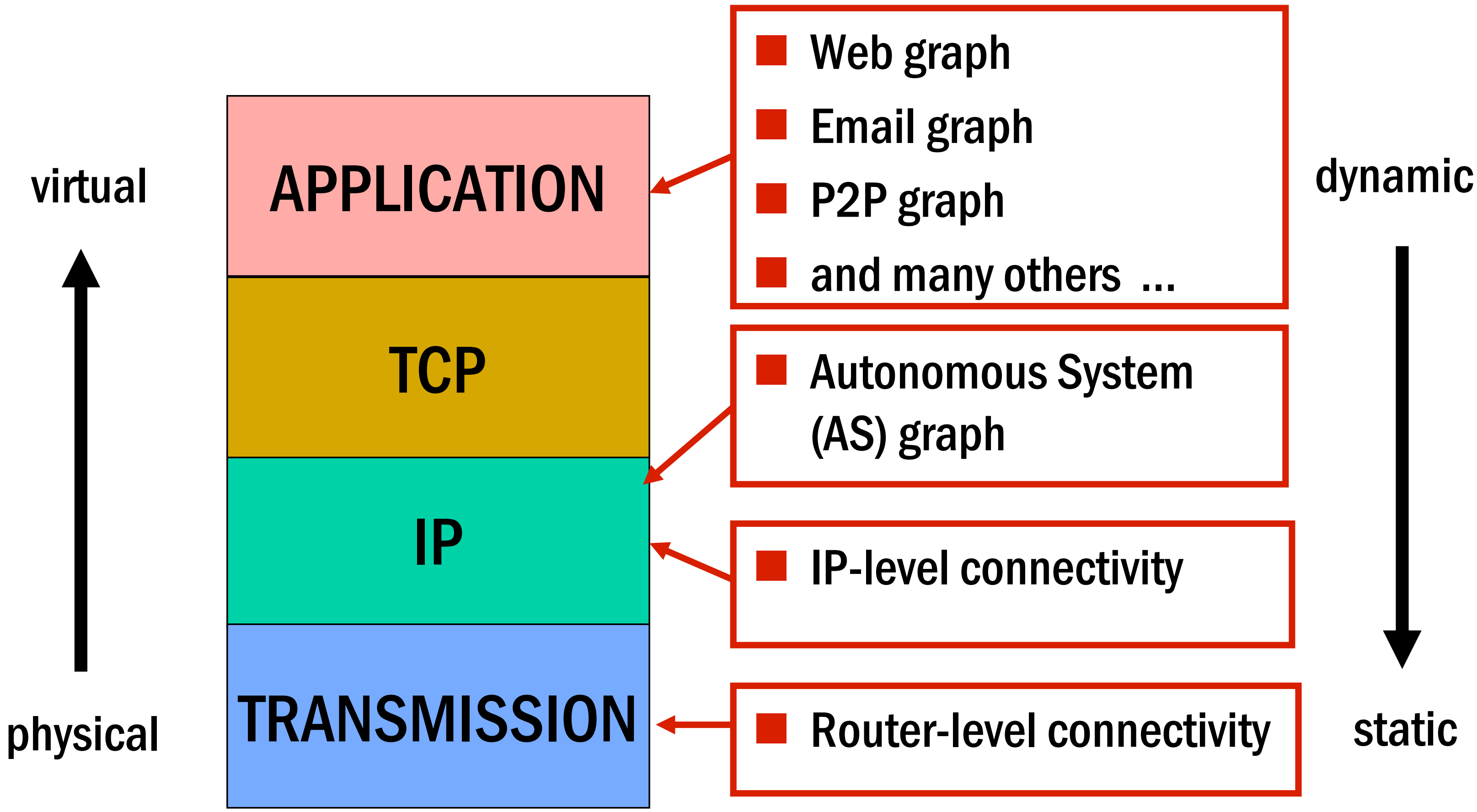


- Most real world complex-networks have non-trivial properties.

- Global properties can not be inferred from local ones
  - Engineered with large technical diversity
  - Range from local campuses to transcontinental backbone providers

Power Laws in the Internet?
Definition of "node" depends on level of representation

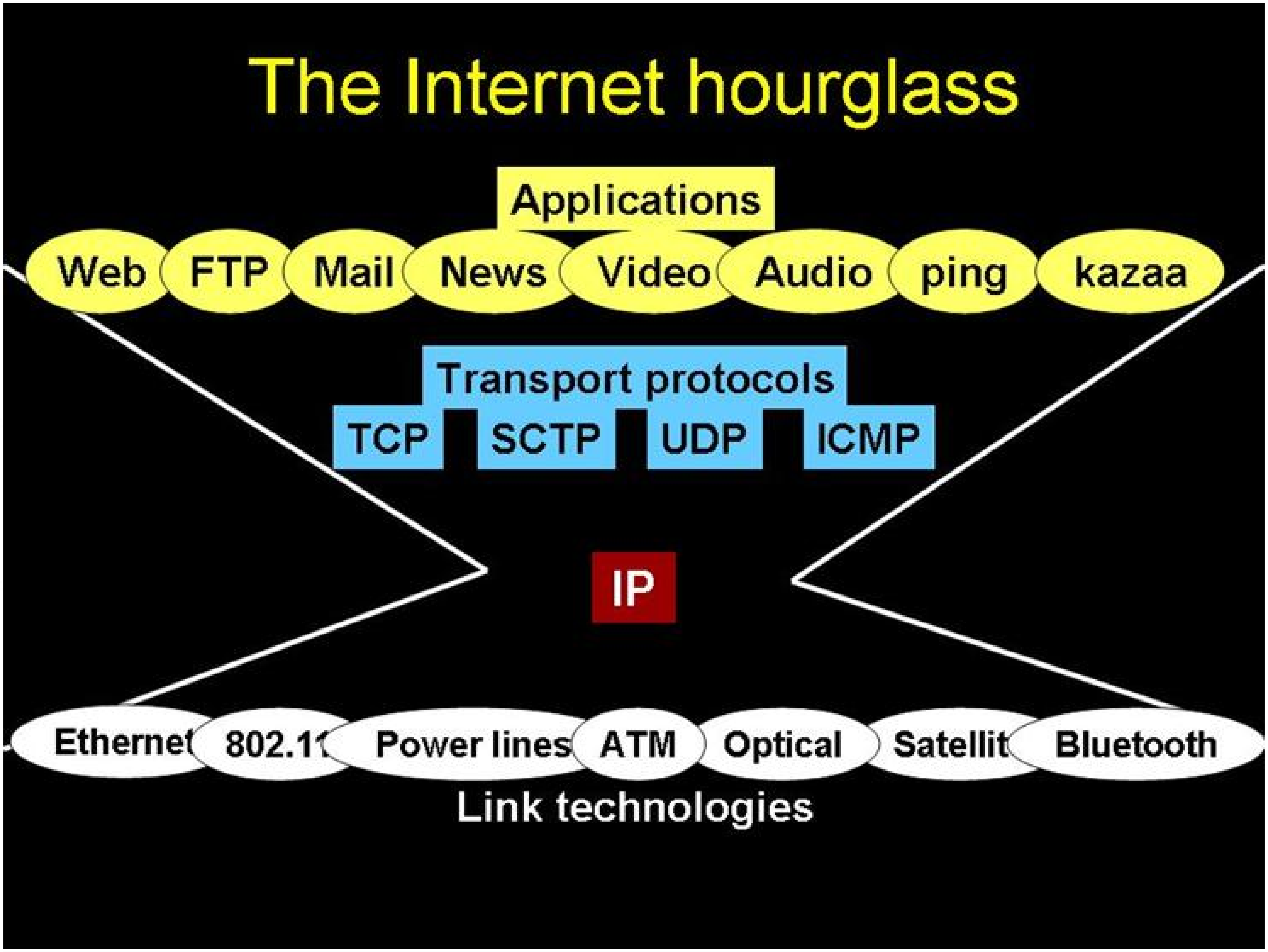
Internet connectivity structures are different at each layer



AS
(BGP routes)

TraceRoute

Router level



(picture from David Alderson)

TCP / IP

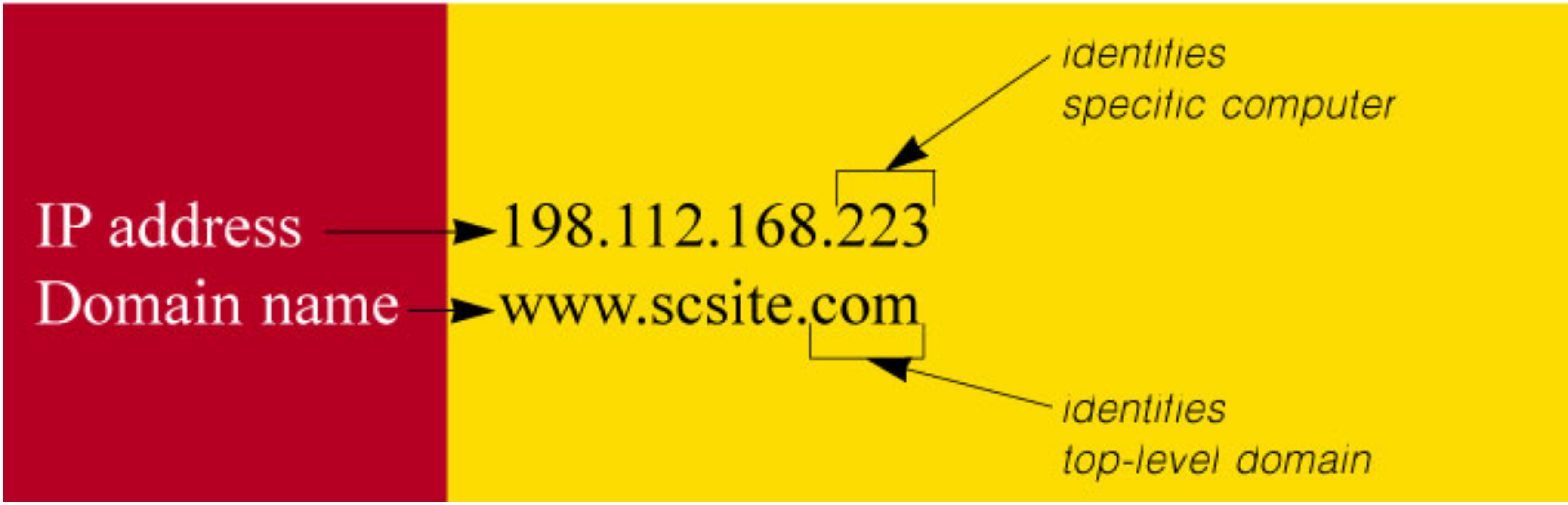
- The TCP protocol: a collection of rules for formatting, ordering, and error-checking data sent across a network.

- In 1974, Vincent Cerf and Robert Kahn developed the Transmission Control Protocol (TCP) which was further split into the Internet Protocol (IP) and TCP in 1978.

- In 1982, DoD adopted TCP/IP as the standard protocol in the Internet.

- IP address: a unique 4-byte number to identify each machine

Internet Infrastructure
The IP address



Common top domain names in the US: .com, .mil, .edu, .org

Outside of the US, the top-level domain identifies the country:
uk (England), fr (France), cn (China), …

Two computers can have the same high level name if they are not on the same domain

Internet Infrastructure The Transmission Control Protocol

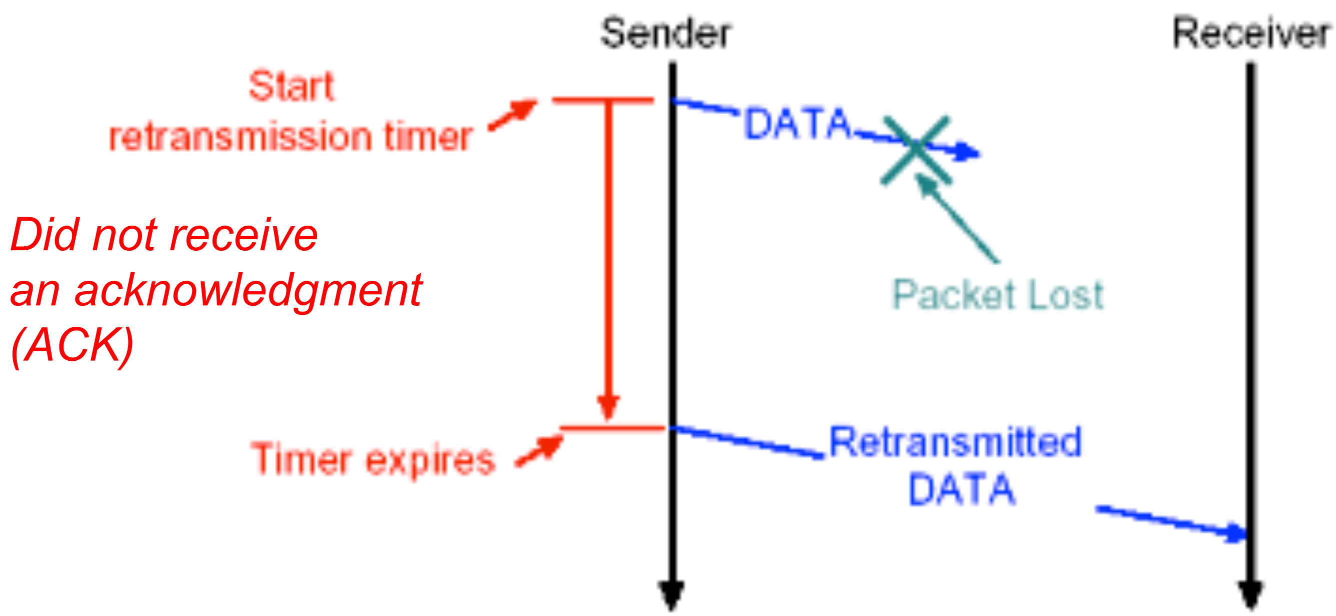
Structure of a TCP/IP packet

See also: http://en.wikipedia.org/wiki/Transmission_Control_Protocol

Bit offset	Bits 0 - 7	8–15	16–23	24–31
0	Source address *Computer sending the packet*			
32				
64				
96				
128	Destination address *Destination computer*			
160				
192				
224				
256	TCP length *Length of the packet*			
288	Zeros			Next header
320	Source port		Destination port	
352	Sequence number			
384	Acknowledgement number			
416	Data offset / Reserved	Flags	Window	
448	Checksum		Urgent pointer	
480	Options (optional)			
480/512+	*Checksum for integrity* Data			

Internet Infrastructure
The Transmission Control Protocol

How does the sender know it needs to retransmit:



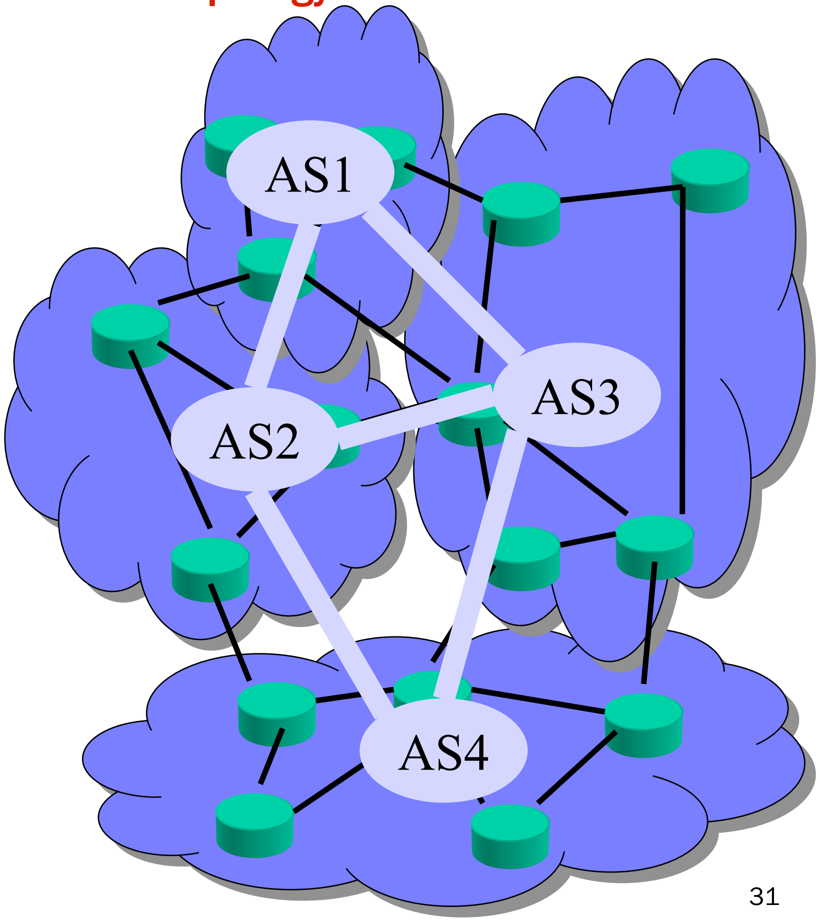
- TCP a *decentralized* protocol with non-linear ramp-up and random restart.

Autonomous system

A collection of connected Internet Protocol (IP) routing prefixes under the control of one or more network operators that presents a common, clearly defined routing policy to the Internet

AS-Level Topology

- Nodes = (sets of) entire networks (Autonomous Systems or ASes)
- Links = peering relationships between ASes
- Really a map of economic or business relationships, not of physical connectivity



Internet Measurements

- The Internet is man-made, so why do we need to measure it?
  - Because we still don't really understand it
    - Sometimes things go wrong
    - Malicious users
  - Measurement for network operations
    - Detecting and diagnosing problems
    - What-if analysis of future changes
  - Measurement for scientific discovery
    - Creating accurate models that represent reality
    - Identifying new features and phenomena

How to measure the structure of the Internet?

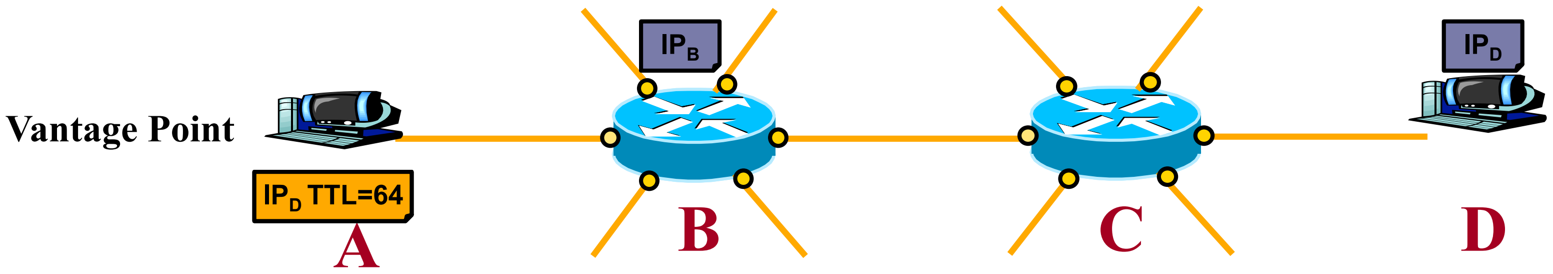
- Traceroute (router level)   see: unix traceroute command
- BGP tables (AS level)
- "Whois" data (AS level)

Repositories / public resources (mostly AS level)

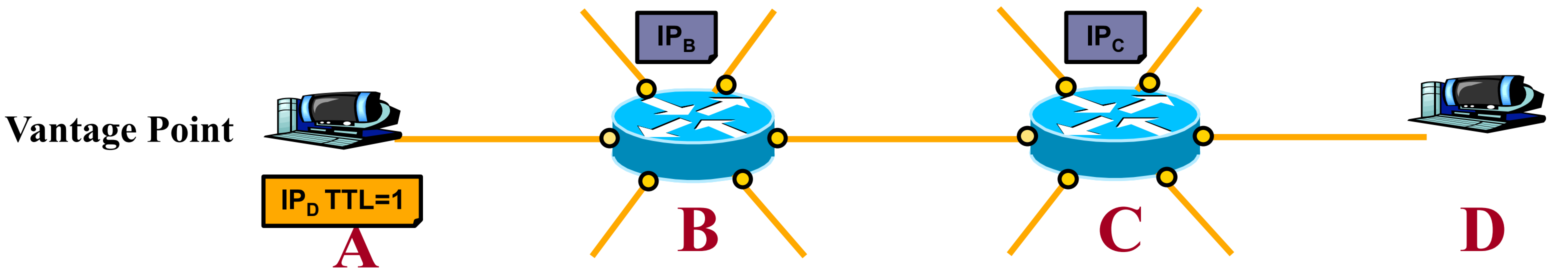
- University of Oregon Route Views Project
  http://www.routeviews.org/
- CAIDA (Cooperative Association for Internet Data Analysis, UCSD)
  http://www.caida.org/home/

Internet Topology Measurements
Probing

- Direct probing

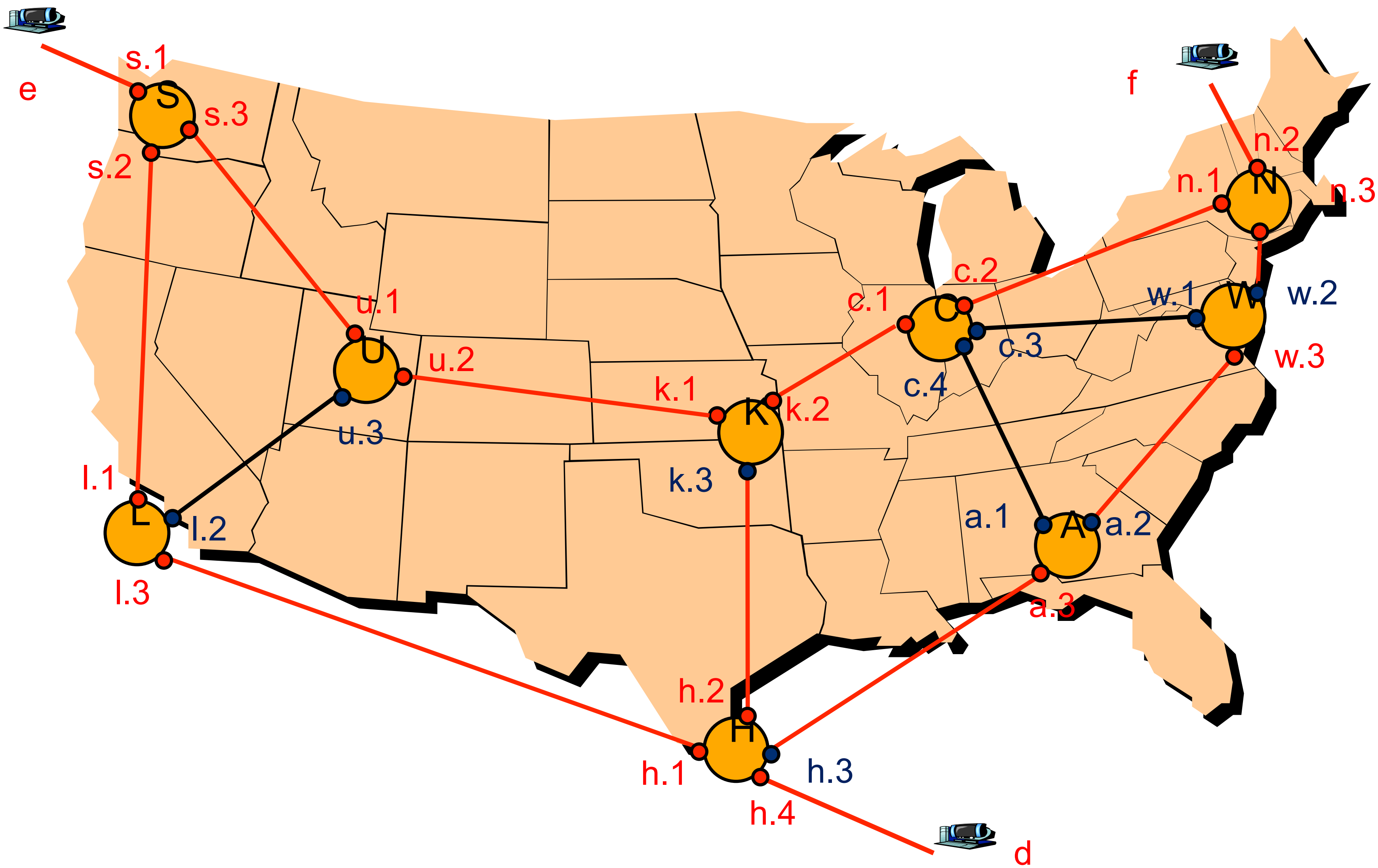


- Indirect probing



http://www.caida.org/publications/animations/active_monitoring/traceroute.mpg

Internet Topology Measurement:
Background



Problems: Traceroute

– Lakhina, Byers, Crovella, Xie, *INFOCOM*, 2003.

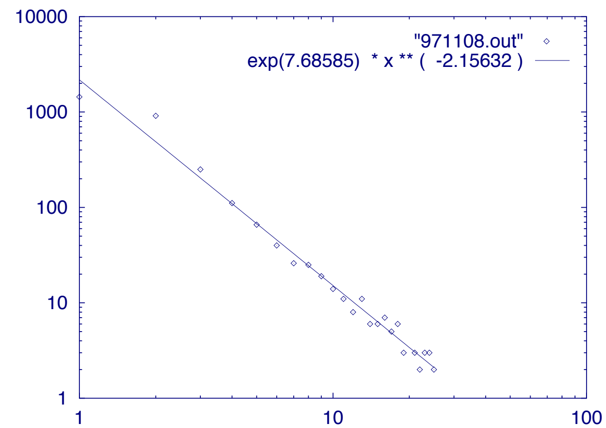
– Achlioptas, Clauset, Kempe, Moore, *STOC*, 2005.

– Achlioptas, Clauset, Kempe, Moore, *J. of ACM*, 56 (4), 2009.

- Build approximately single-source, all-destinations, shortest-path trees. (Union of traceroute samples.)

  – Faloutsos^3 *SIGCOMM*, 1999.

  – Albert, Jeong, Barabasi, *Nature*, 2000.



(a) Int-11-97

- Sampling bias

  – Nodes close to root sampled more accurately

  – High degree nodes sampled more accurately than low degree. (Follow an edge at random, k times as likely to lead to node of degree k than degree 1.)

Traceroute sampling bias

- Lakhina, et al *INFOCOM*, 2003: Show empirically that Erdős-Rényi random graphs (Poisson dist) appear to have power law degree distribution.

- Petermann and De Los Rios [2004] and Clauset and Moore [2005]: Even if a power law, the exponent γ is underestimated.

- Achlioptas et all 2005 and 2009: Rigorous proof of bias and consequences.
  – Poisson degree dist
  – d-regular random graphs (all nodes have degree d).

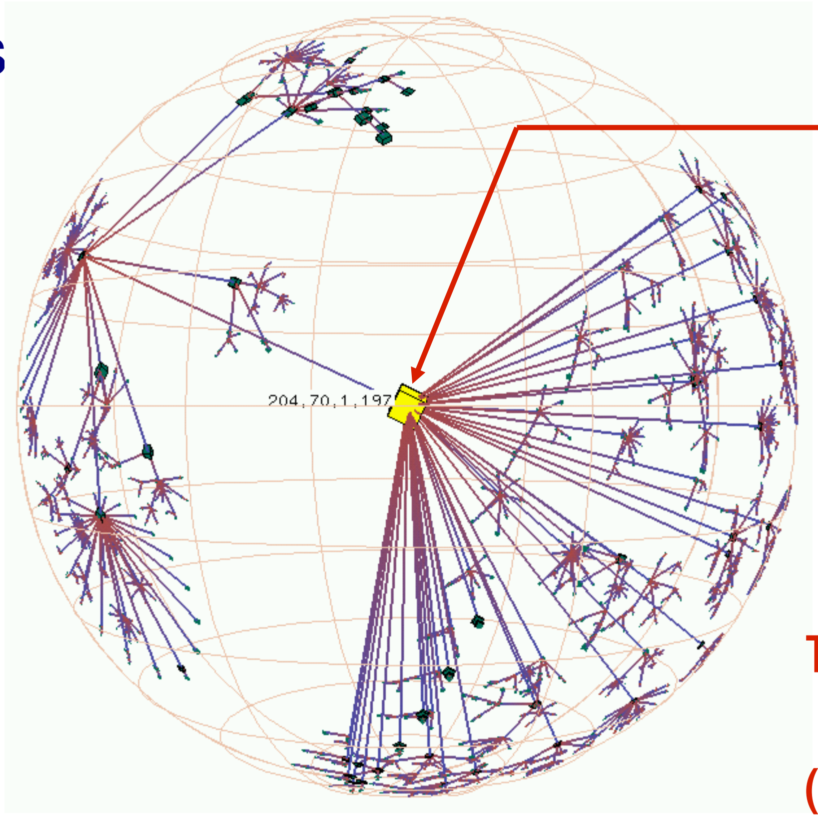
- Recommendation: Traceroute sampling over the union of a very large number of sources more accurate.

Problems: AS level topology

AS level connections inferred from BGP routing tables.

- AS level does not reflect physical connectivity (geographically distant routers can appear as one AS).

- Hidden subgraphs



- www.savvis.net
- managed IP and hosting company
- founded 1995
- offering "private IP with ATM at core"

This "node" is an entire network! (not just a router)

http://www.caida.org/tools/measurement/skitter/

(Picture from Willinger presentation)

The Internet?

- Michalis Faloutsos, Petros Faloutsos, Christos Faloutsos, "On power-law relationships of the Internet topology", ACM SIGCOMM Computer Communication Review Volume 29 , Issue 4 Oct. 1999.

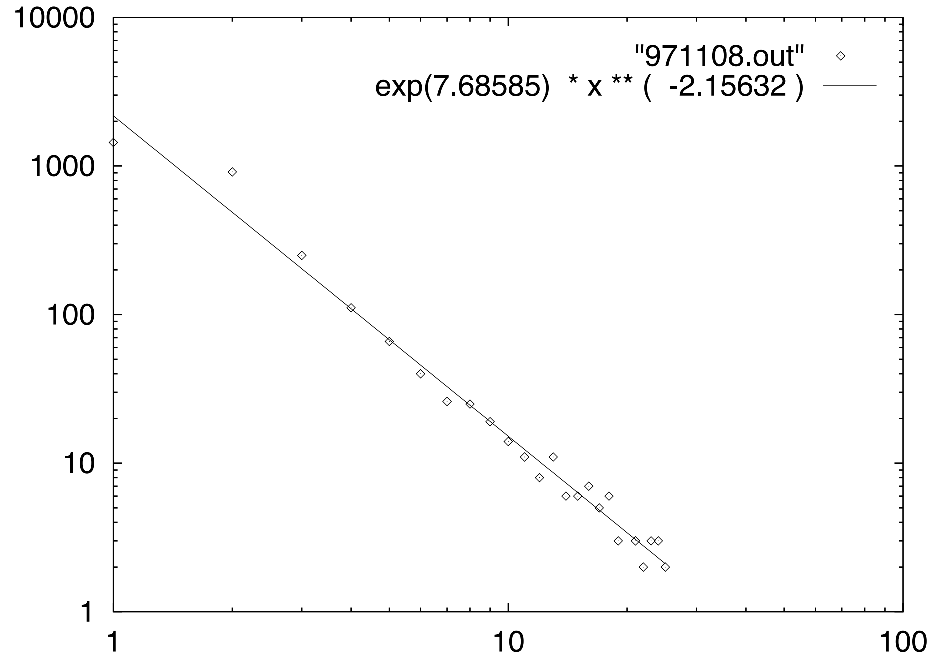
- Only one order of magnitude (even exponential can look power law in a short regime).

- $\gamma \approx 2.1$

- (over 3100 cites)

- Only 6000 nodes

- Consequences for topology generators



(a) Int-11-97

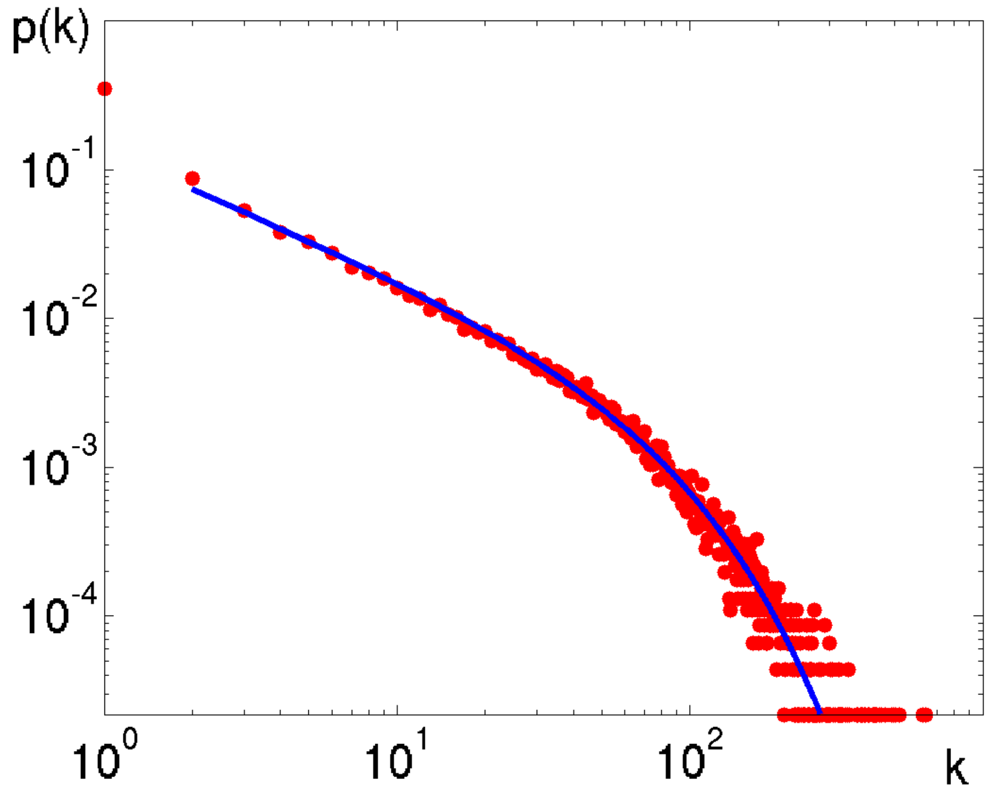
Can there be real Power Laws in data?

- in the WWW .... sure.
- in a social network ... possible.
- in earthquake magnitude ... yes, but to some cutoff.
- in the Internet?

Why power laws cannot continue: Finite size effects, resource limitations, physical geometric (Internet) vs virtual geometry-free (WWW)....

The "Who-is-Who" network in Budapest

(Analysis by Balázs Szendröi and Gábor Csányi)

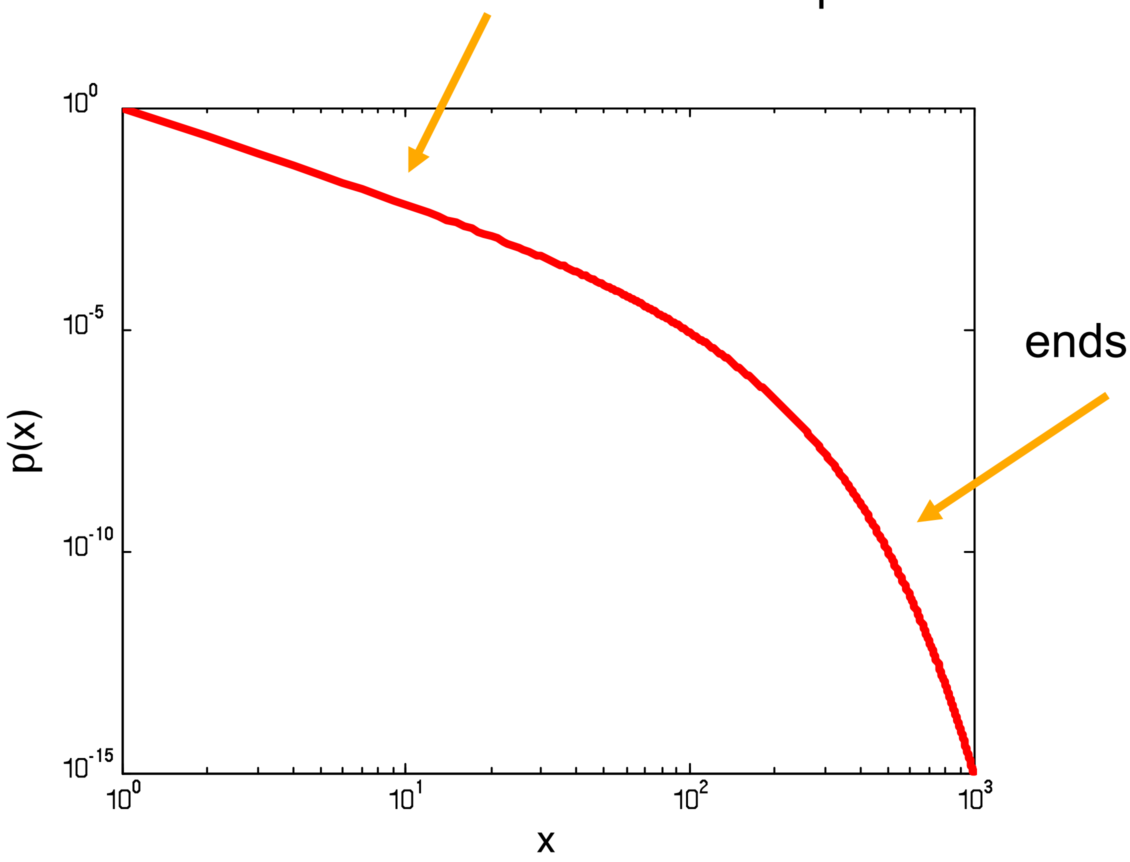


Bayesian curve fitting $\rightarrow p(k) = ck^{-\gamma}e^{-\alpha k}$

Another common distribution: power-law with an exponential cutoff

- $p(x) \sim x^{-a} e^{-x/\kappa}$

starts out as a power law



ends up as an exponential

but could also be a lognormal or double exponential…

“Power law” $\to$ power law with exponential tail

Ubiquitous empirical measurements:

System with: $p(x) \sim x^{-B} \exp(-x/C)$	B	C
Full protein-interaction map of *Drosophila*	1.20	0.038
High-confidence protein-interaction map of *Drosophila*	1.26	0.27
Gene-flow/hydridization network of plants as function of spatial distance	0.75	10^5 m
Earthquake magnitude	1.35 - 1.7	$\sim 10^{21}$ Nm
Avalanche size of ferromagnetic materials	1.2 - 1.4	$L^{1.4}$
ArXiv co-author network	1.3	53
MEDLINE co-author network	2.1	~ 5800
PNAS paper citation network	0.49	4.21

(Saturation and PA often put in apriori to explain)

Known Mechanisms for Power Laws

- Phase transitions (singularities)
- Random multiplicative processes (fragmentation)
- Combination of exponentials (e.g. word frequencies)
- **Preferential attachment / Proportional attachment** (Polya 1923, Yule 1925, Zipf 1949, Simon 1955, Price 1976, Barabási and Albert 1999)

Attractiveness is proportional to size:

$$\boxed{\frac{ds}{dt} \propto s}$$

- Add in **saturation** [Amaral 2000, Börner 2004], get PA with exponential decay .

An alternate view, Mandelbrot, 1953: optimization

(Information theory of the statistical structure of language)

- **Goal:** Optimize information conveyed for unit transmission cost (what probability distribution over words gives most info?)

- Consider an alphabet of d characters, with n distinct words

- Order all possible words by length (A,B,C,....AA,BB,CC....)

- "Cost" of j-th word, $C_j \sim \log_d j$

- Ave information per word: $H = -\sum p_j \log p_j$

- Ave cost per word: $C = \sum p_j C_j$

- Minimize: $\frac{d}{dp_j}\left(\frac{C}{H}\right) \implies p_j \sim j^{-\alpha}$

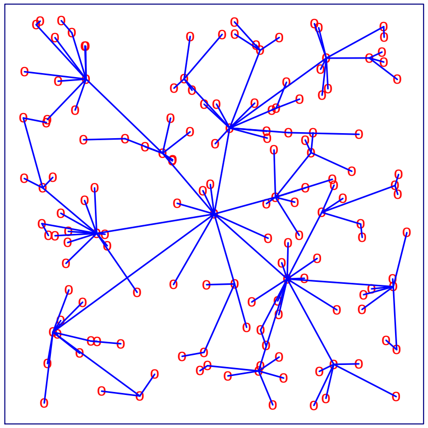
Optimization versus Preferential Attachment origin of power laws

Mandelbrot and Simon's heated public exchange

- A series of six letters between 1959-61 in *Information and Control*.
- Optimization on hold for many years, but recently resurfaced:
- Calson and Doyle, HOT, 1999
- Fabrikant, Koutsoupias, and Papadimitriou, 2002
- Solé, 2002

FKP (Fabrikant, Koutsoupias, and Papadimitriou, 2002)
A local optimization model of internet growth

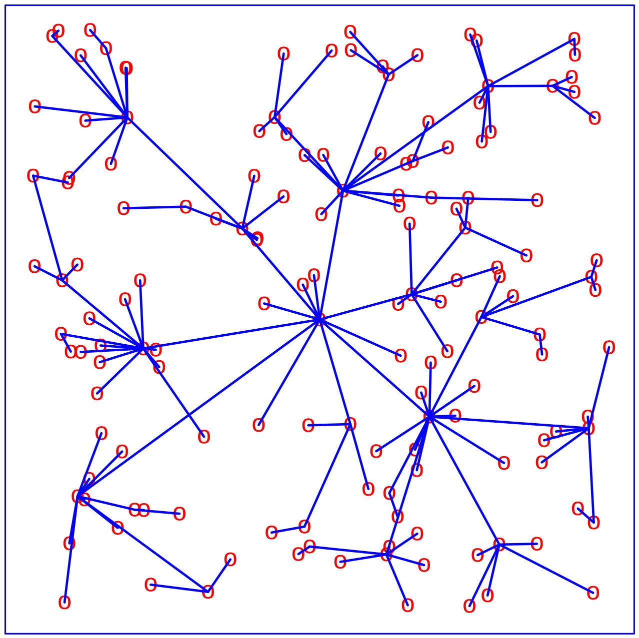
- Nodes arriving sequentially at random in a unit square.
- Upon arrival, node i connects to an already existing node j that minimizes "cost": $$\alpha d_{ij} + h_j$$
- d_{ij} is Euclidean distance between i and j.
  h_j is the hop distance from j to the root node.
- i.e., connect to the closest node that has good network performance



FKP cont

- αd_{ij} introduces a *scale*. The first node to arrive an uninhabited area collects all the subsequent arrivals.

- Eventually get hubs-and-leaf structure.



Tempered Preferential Attachment

[D'Souza, Borgs, Chayes, Berger, Kleinberg, *PNAS* 2007.]

[Berger, Borgs, Chayes, D'Souza, Kleinberg, *ICALP* 2004.]

[Berger, Borgs, Chayes, D'Souza, Kleinberg, *CPC*, 2005.]

- **Optimization**
  Like FKP, start with linear tradeoffs, but consider a scale-free metric. (Plus will result in local model.) Gives rise to:

$\rightarrow$ **PA**

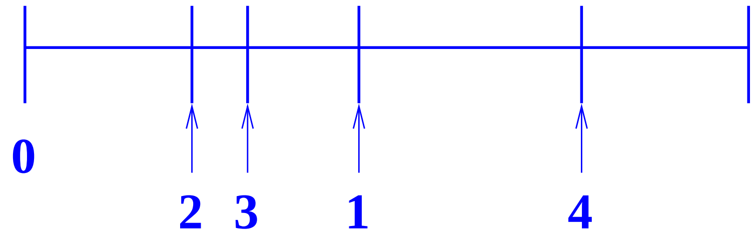
$\rightarrow$ **Saturation**

$\rightarrow$ **Viability**

(Not all children have equal fertility, not all spin-offs equally fit, etc).

Competition-Induced Preferential Attachment

Consider points arriving sequentially, uniformly at random along the unit line:



Each incoming node, t, attaches to an existing node j (where $j < t$), which minimizes the function:

$$F_{tj} = \min_j \left[\alpha_{tj} d_{tj} + h_j\right]$$

Where $\alpha_{tj} = \alpha \rho_{tj} = \alpha n_{tj} / d_{tj}$.

The "cost" becomes: $\boxed{F_{tj} = \min_j \left[\alpha n_{tj} + h_j\right]}$

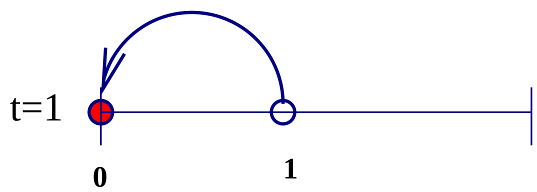
$$\boxed{F_{tj} = \min_j \left[\alpha n_{tj} + h_j\right]}$$

- $\alpha_{tj} = \alpha \rho_{tj}$ local density, e.g. real estate in Manhattan.
- Reduces to n_{tj} — number of points in the interval between t and j
- “Transit domains” — captures realistic aspects of Internet costs (i.e. AS/ISP-transit requires BGP and peering).
- Like FKP, tradeoff intial connection cost versus usage cost.
- Note cases $\alpha = 0$ and $\alpha > 1$.

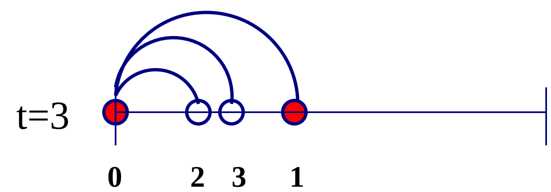
The process on the line (for $1/3 < \alpha < 1/2$)

"Border Toll Optimization Problem" (BTOP)

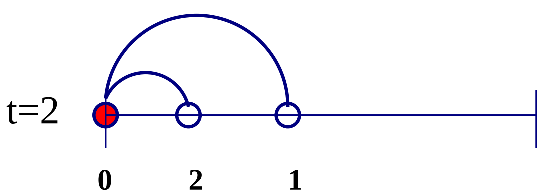
$$\boxed{F_{tj} = \min_j \left[\alpha n_{tj} + h_j\right]}$$



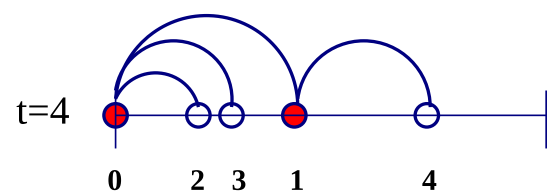
F(10) = 0



F(30) = α
F(32) = 1
F(31) = 1



F(20) = 0
F(21) = 1

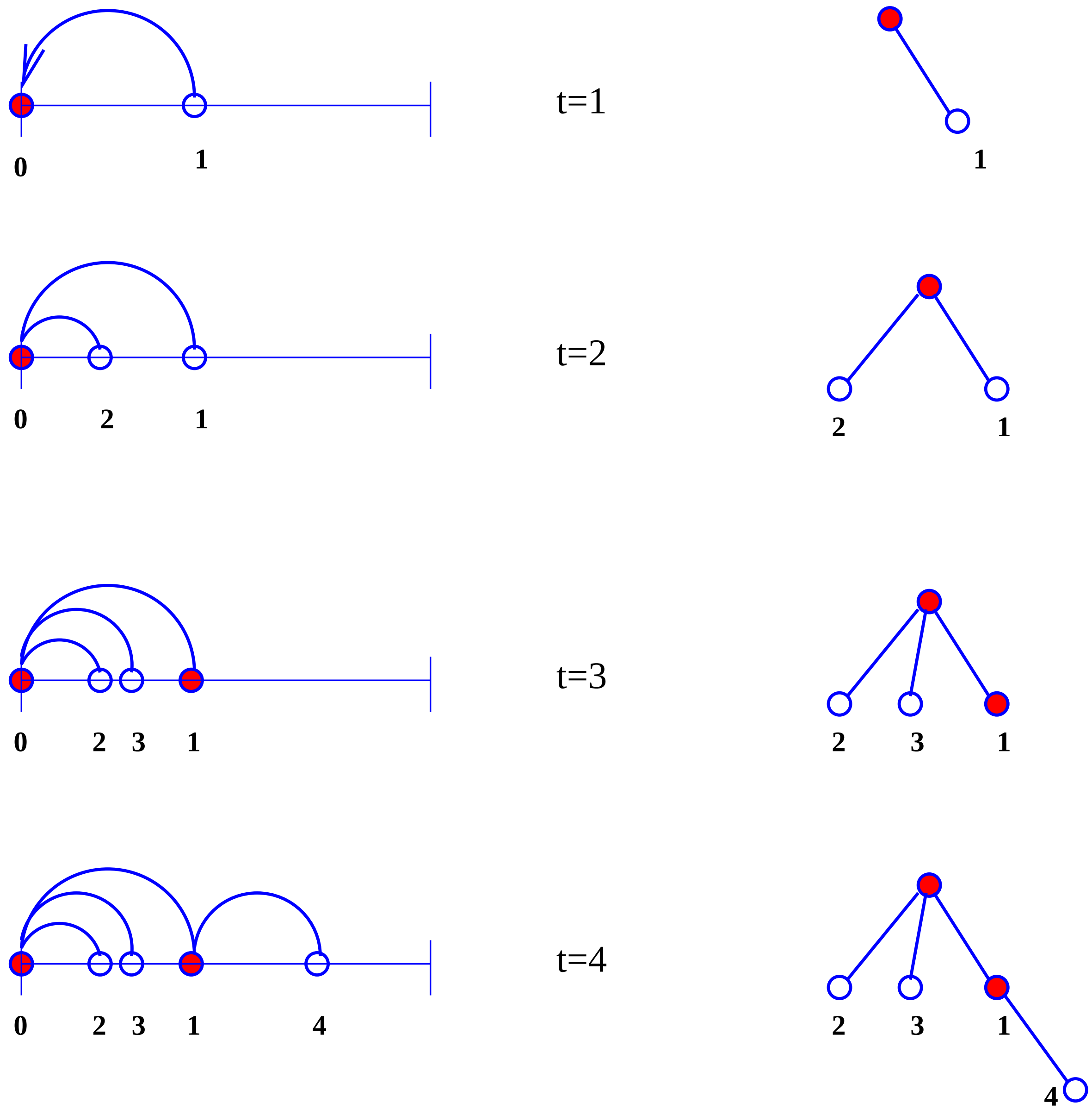


F(40) = 3 α
F(42) = 1 + 2 α
F(43) = 1 + α
F(41) = 1

(A **local** model – connect either to closest node, or its parent.)

Mapping onto a tree

(equal in distribution to the line)



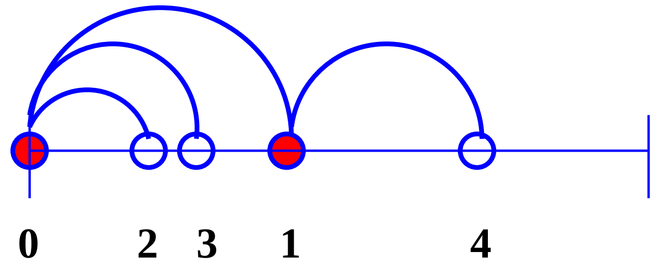
From line to tree

Integrating out the dependence on interval length from the conditional probability:

$$
\begin{aligned}
Pr\left[x_{t+1} \in I_k \,|\pi(t)\right] &= \int Pr\left[x_{t+1} \in I_k \,|\pi(t), \vec{s}(t)\right] dP(\vec{s}(t)) \\
&= \int s_k(t) dP(\vec{s}(t)) = \frac{1}{t+1},
\end{aligned}
$$

i.e., The probability to land in the k-th interval is uniform over all intervals.

Preferential attachment with a cutoff



Let $d_j(t)$ equal the degree of fertile node j at time t.

The number of intervals contributing to j's fertility is $\max(d_j(t), A)$.

Probability node $(t+1)$ attaches to node j is:

$$\boxed{Pr(t+1 \to j) = \max(d_j(t), A)/(t+1).}$$

The process on degree sequence

Let $N_0(t) \equiv$ number of infertile vertices.

Let $N_k(t) \equiv$ number of fertile vertices of degree k (for $1 \leq k < A$).

Let $N_A(t) \equiv$ number of fertile vertices of degree $k \geq A$ (i.e. $N_A(t) = \sum_{k=A}^{\infty} N_k(t)$ "the tail")

In terms of $p_k(t)$:

$$
\begin{aligned}
p_1(t+1)(t+1) - p_1(t)(t) &= Ap_A(t) - p_1(t) \\
p_k(t+1)(t+1) - p_k(t)(t) &= (k-1)p_{k-1}(t) - kp_k(t), \quad 1 < k < A \\
p_A(t+1)(t+1) - p_A(t)(t) &= (A-1)p_{A-1}(t).
\end{aligned}
$$

Proposition 1 (Convergence of expectations to stationary distribution): $p_k(t) \to p_k$.

$$
\begin{aligned}
p_1 &= Ap_A - p_1 \\
p_k &= (k-1)p_{k-1} - kp_k, \qquad 1 < k < A \\
p_A &= (A-1)p_{A-1}.
\end{aligned}
$$

Proposition (2): (Concentration) (i.e., How big are the fluctuations about $n_k(t)$?) Requires second-moment method.

Recursion relation

$$p_k = (k-1)p_{k-1}(t) - kp_k(t), \qquad 1 < k < A.$$

Implies

$$\boxed{p_k = \prod_{i=2}^{k} \left(\frac{i-1}{i+1}\right) p_1, \qquad 1 < k < A.}$$

Power law for $1 < k < A$

$$\frac{p_k}{p_1} = \prod_{i=2}^{k} \left(\frac{i-1}{i+1} \right) = \frac{2}{k(k+1)}$$

$$\sim \; c\, k^{-2}$$

Exponential decay for $k > A$

Recursion relation: $p_k = A\left(p_{k-1} - p_k\right), \qquad k \geq A.$

Implies

$$\boxed{p_k = \left(\frac{A}{A+1}\right)^{k-A} p_A, \qquad k \geq A.}$$

$$p_k = \left(1 - \frac{1}{A+1}\right)^{k-A} p_A = \left[\left(1 - \frac{1}{A+1}\right)^{A+1}\right]^{(k-A)/(A+1)} p_A$$

$$\sim \exp\left[-(k-A)/(A+1)\right] p_A.$$

Generalizing: from A to (A_1, A_2)

Let the “viability threshold” A_1 differ from the “attractiveness saturation” A_2.

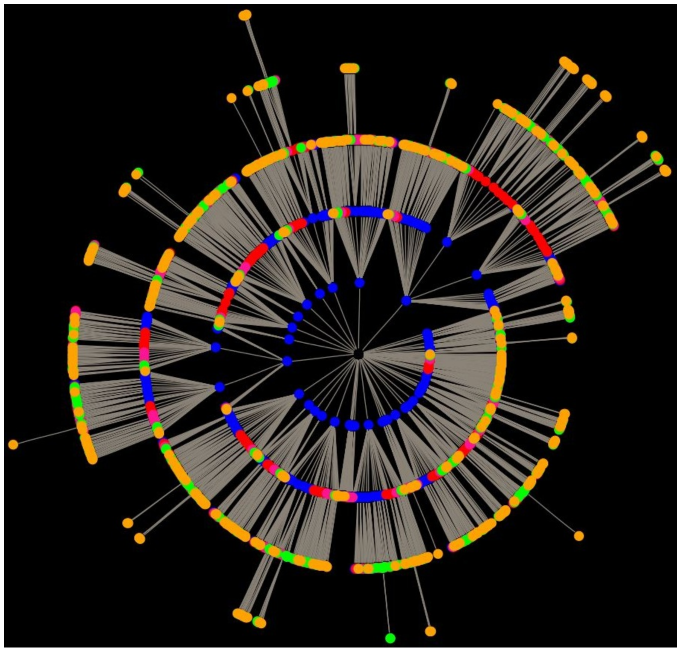
Consider the Markov matrix describing the evolution of the degree sequence (A1 not equal A2)

Limits: $A_1 = 1, A_2 = \infty$, Preferential Attachment

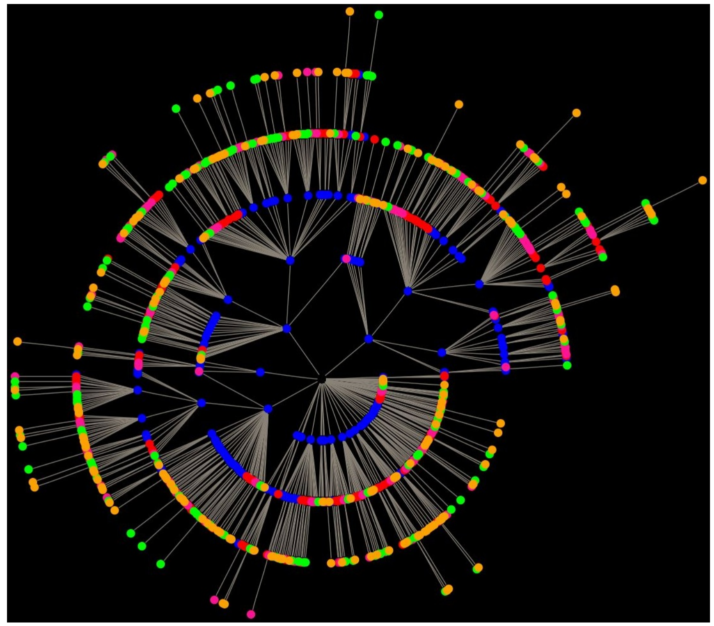
$A_1 = 1, A_2$ finite, PA with a cutoff

$A_1 = A_2 = 1$, uniform.

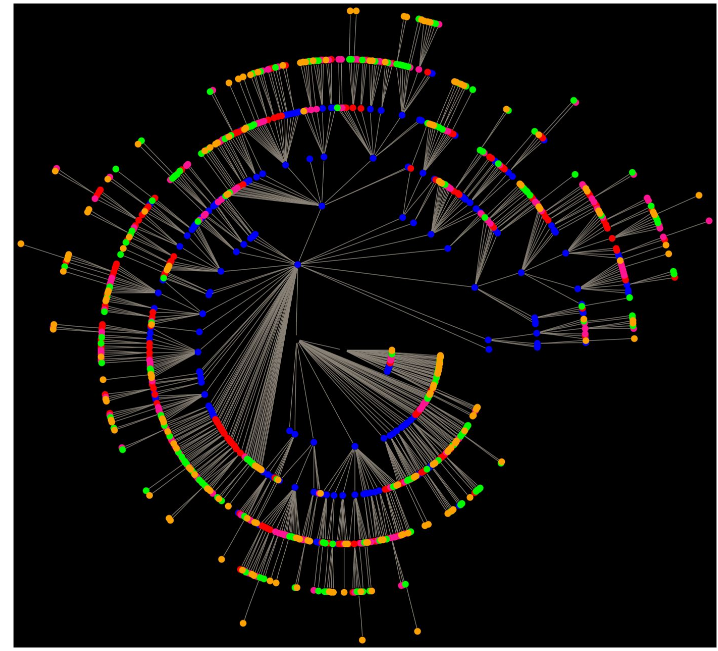
Example range of TPA graphs



$A_1 = 10, A_2 = 25$



$A_1 = 3, A_2 = 20$



$A_1 = 3, A_2 = 20$, (3-roots)

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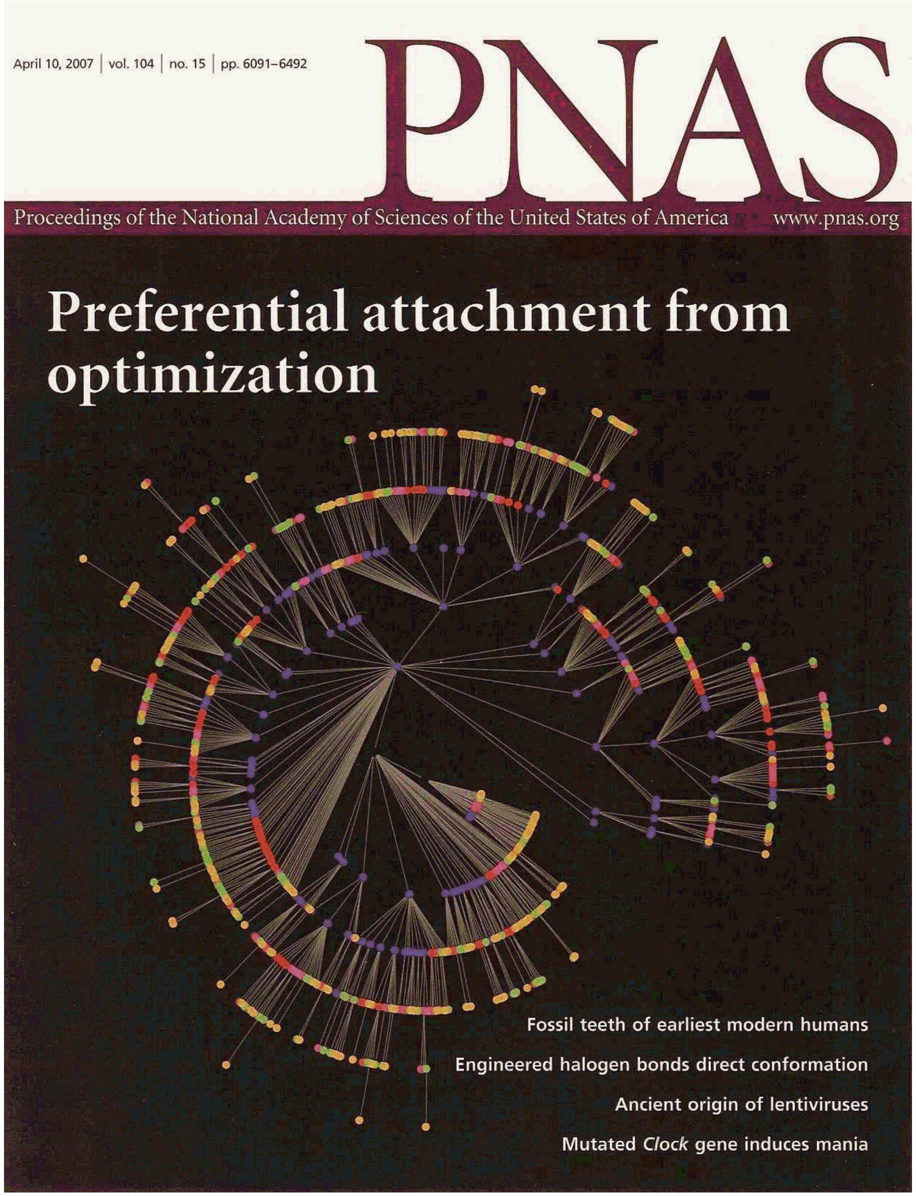
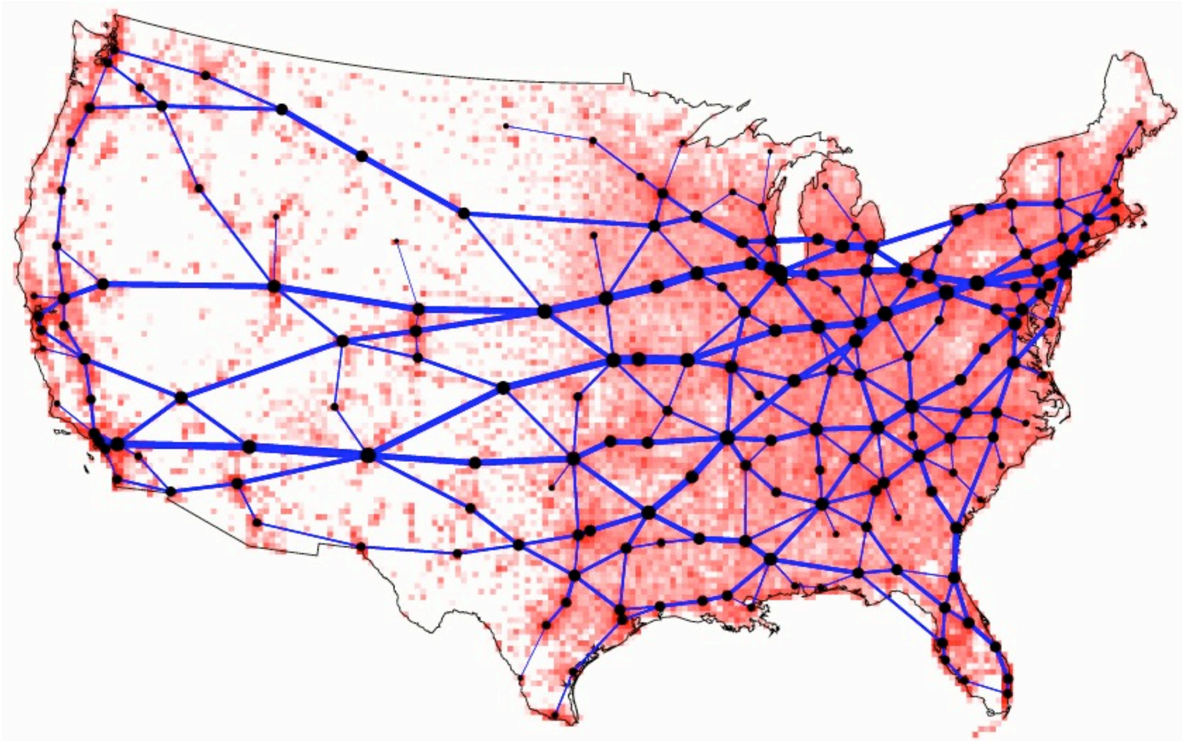


Linear optimization and transportation networks (Applying the "FKP" ideas)

- M. T. Gastner, M.E.J. Newman, "The spatial structure of networks", cond-mat/0407680, 2004.
- M. T. Gastner, M.E.J. Newman, "Shape and efficiency in spatial distribution networks", *Journal of Statistical Mechanics*, 2006.
- M. T. Gastner, M.E.J. Newman, "Optimal design of spatial distribution networks", *Physical Review E*, 74, 016117, 2006.

Optimal networks of optimally located facilities

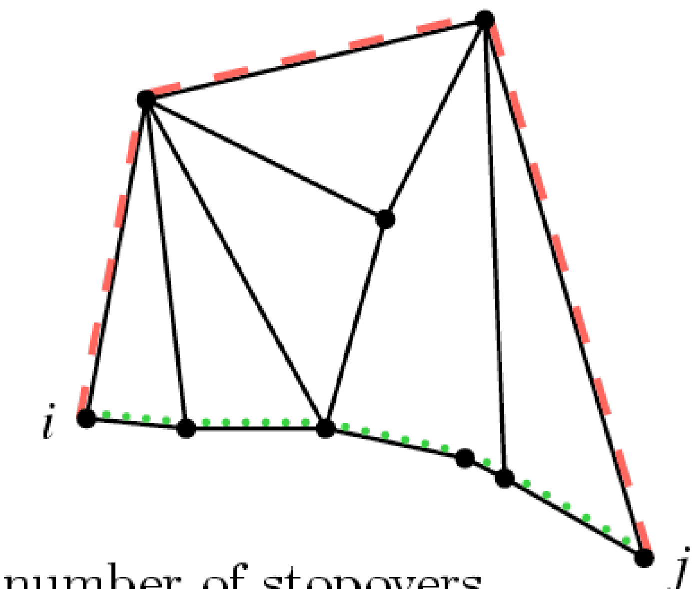
The optimal network design problem then consists of two parts.
First, we distribute p facilities on the map by solving the p-median problem.
Then we find the network minimizing the total cost C.



Different routing strategies

There is another complicating factor. We have assumed that travel costs are proportional to geometric distances.

In some networks, users may not choose the geometrically shortest path, especially if it has many edges.



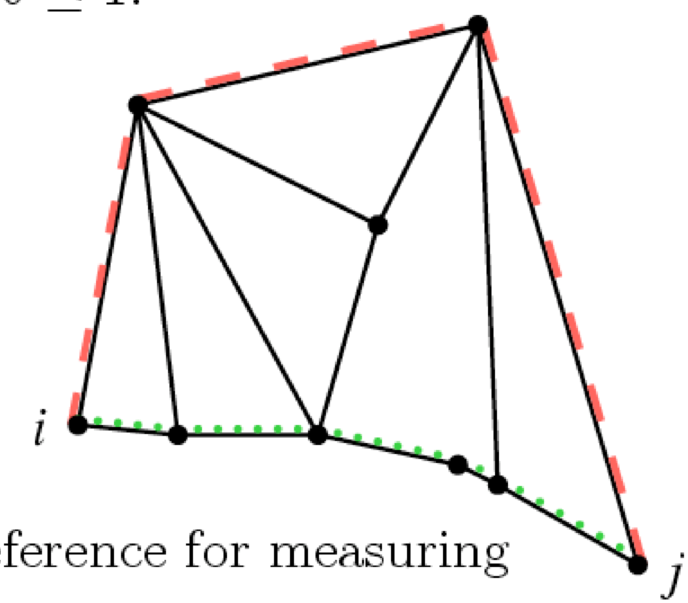
Examples:

- Airline passengers want to limit the number of stopovers.
- Internet packets arrive more quickly and reliably if the number of routers along the way is small.

Different routing strategies

We can account for such situations by using a more flexible notion of distance. We assign to each pair of adjacent vertices an effective length (travel cost)

$$\tilde{l}_{ij} = (1-\delta)l_{ij} + \delta, \quad 0 \le \delta \le 1.$$

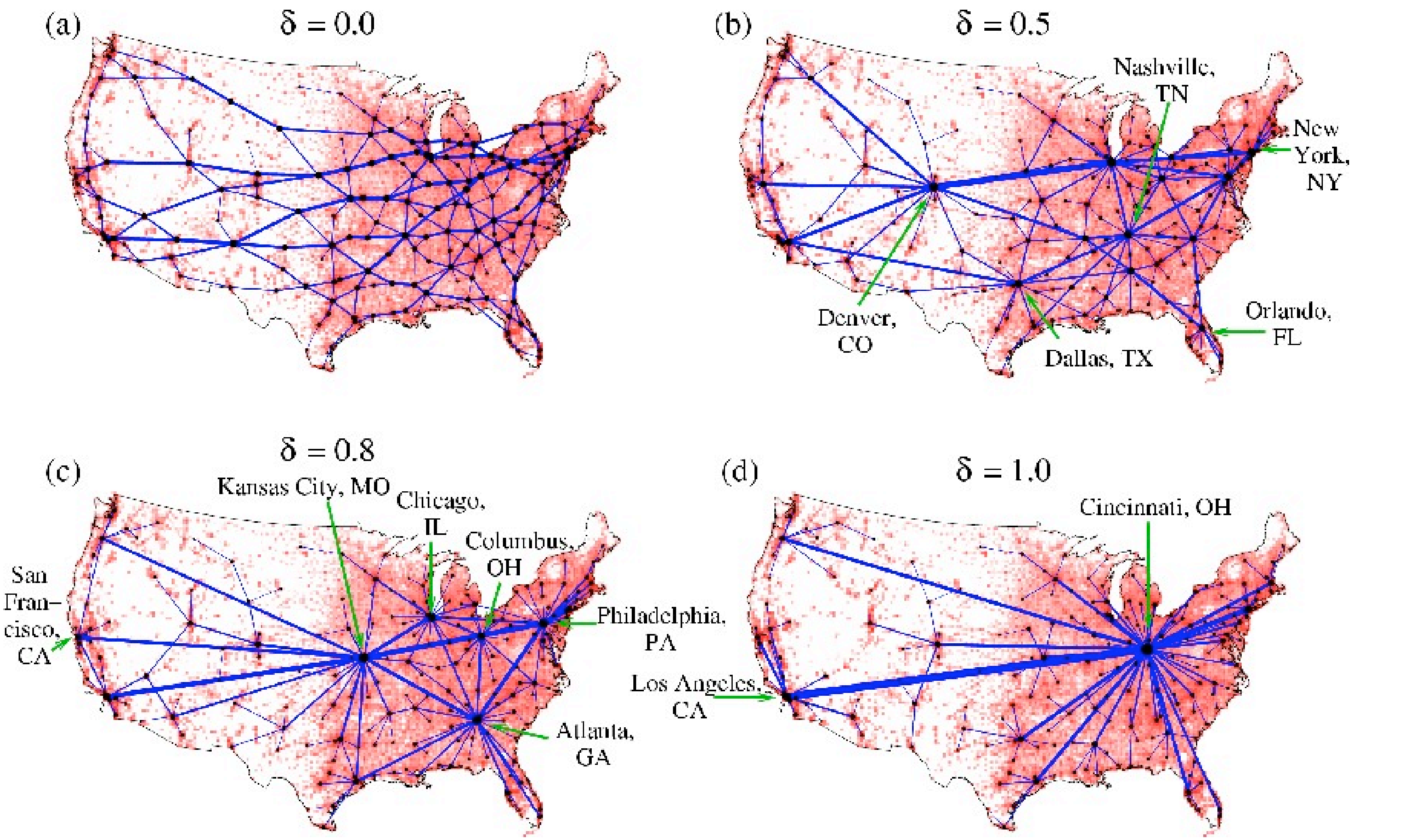


The travel cost is then $Z = \sum_{i<j} w_{ij}\tilde{l}_{ij}$.

The parameter δ determines the user's preference for measuring distance in terms of kilometers or edges:

- $\delta = 0$: geometric distance
- $\delta = 1$: number of edges (graph distance)

Different routing strategies



Summary

- Internet measurement :
  - Traceroute sampling (router level)
  - Peering agreements/ routing tables (AS level)

- Optimization approaches to network growth :
  - FKP (leads to hubs and leaves)
  - TPA (Pref Attachment with saturation, fertility / viability)
  - Gastner/Newman: FKP approach to transport networks.