ECS 253 / MAE 253, Network Theory
Spring 2016
Problem Set # 2, Solutions

Problem 1: The Caley tree

(a) The vertices at exactly distance one from the center is the neighbors of the center node and since the center node has degree k, there are k nodes at exactly distance one from the center.

(b) For each node at distance one, there are k-1 nodes connected to besides the center node. Therefore there are k(k-1) nodes at distance 2.

(c) Let m_l denote the number of nodes at distance l. By the same argument in **(b)**, we have $m_l = (k-1)m_{l-1}$. Since $m_1 = k$, $m_l = k(k-1)^{l-1}$.

(d)

$$\begin{aligned} n(l) &= \sum_{i=1}^{l} m_i + 1 \\ &= \sum_{i=1}^{l} k(k-1)^{i-1} + 1 \\ &= 1 + k\frac{(k-1)^l - 1}{k-2} \\ &= \frac{k(k-1)^l - k}{k-2} + 1 \end{aligned} \tag{1}$$

(e) Consider a network with diameter d, the total number of nodes would be $n\left(\frac{d}{2}\right) = \frac{k(k-1)^{\frac{d}{2}}}{k-2} + 1$. Thus, $d = 2\log_{k-1}((n-1)(k-2)+k)/k$, when $n \gg k$, $d \sim \log_k n = \log n / \log k$

Problem 2: Finite size scaling

$$\begin{aligned} N(1 - P_{K_{MAX}}) &\approx 1 \\ 1 - \int_1^{K_{MAX}} p_k dk &\approx \frac{1}{N} \\ \int_1^{K_{MAX}} (\gamma - 1) k^{-\gamma} dk &\approx 1 - \frac{1}{N} \\ -k^{\gamma - 1}\Big|_{k=1}^{K_{MAX}} &\approx 1 - \frac{1}{N} \\ 1 - K_{MAX}^{-(\gamma-1)} &\approx 1 - N^{-1} \\ K_{MAX} &\approx N^{1/\gamma - 1} \end{aligned} \tag{2}$$

Substitute γ with 2,3,4 gives us: For $\gamma = 2$, $K_{MAX} = N$. For $\gamma = 3$, $K_{MAX} = N^{\frac{1}{2}}$. For $\gamma = 4$, $K_{MAX} = N^{\frac{1}{3}}$.