Community Structure

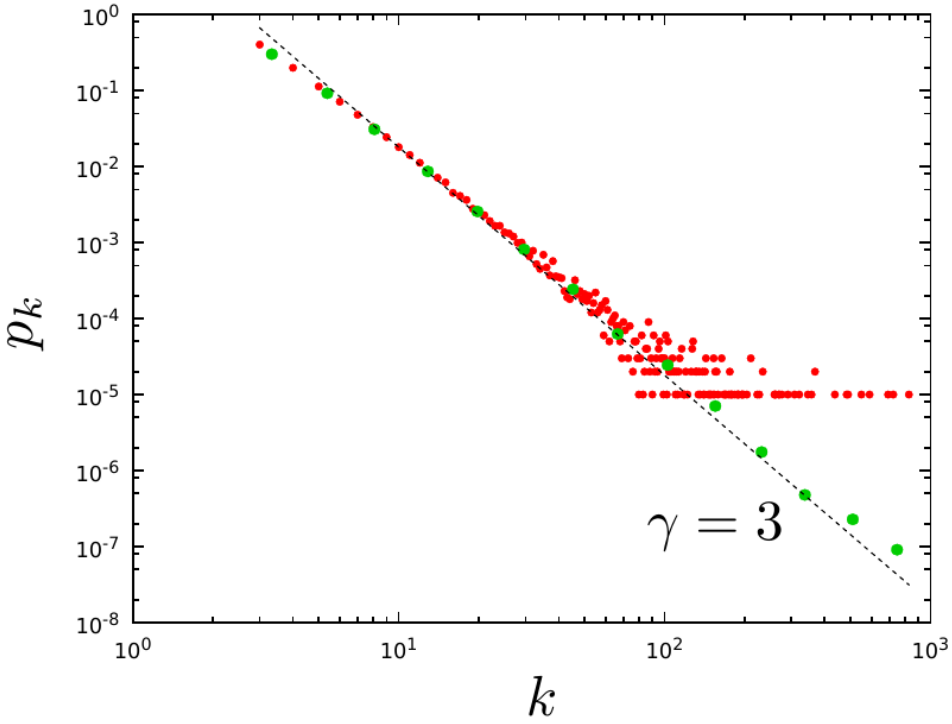
Network Theory and Applications

ECS 253 / MAE 253

Spring 2016

Communities

- So far:
  - Properties of nodes: degree, centralities, triangles
  - Macroscopic properties: degree distribution, resilience

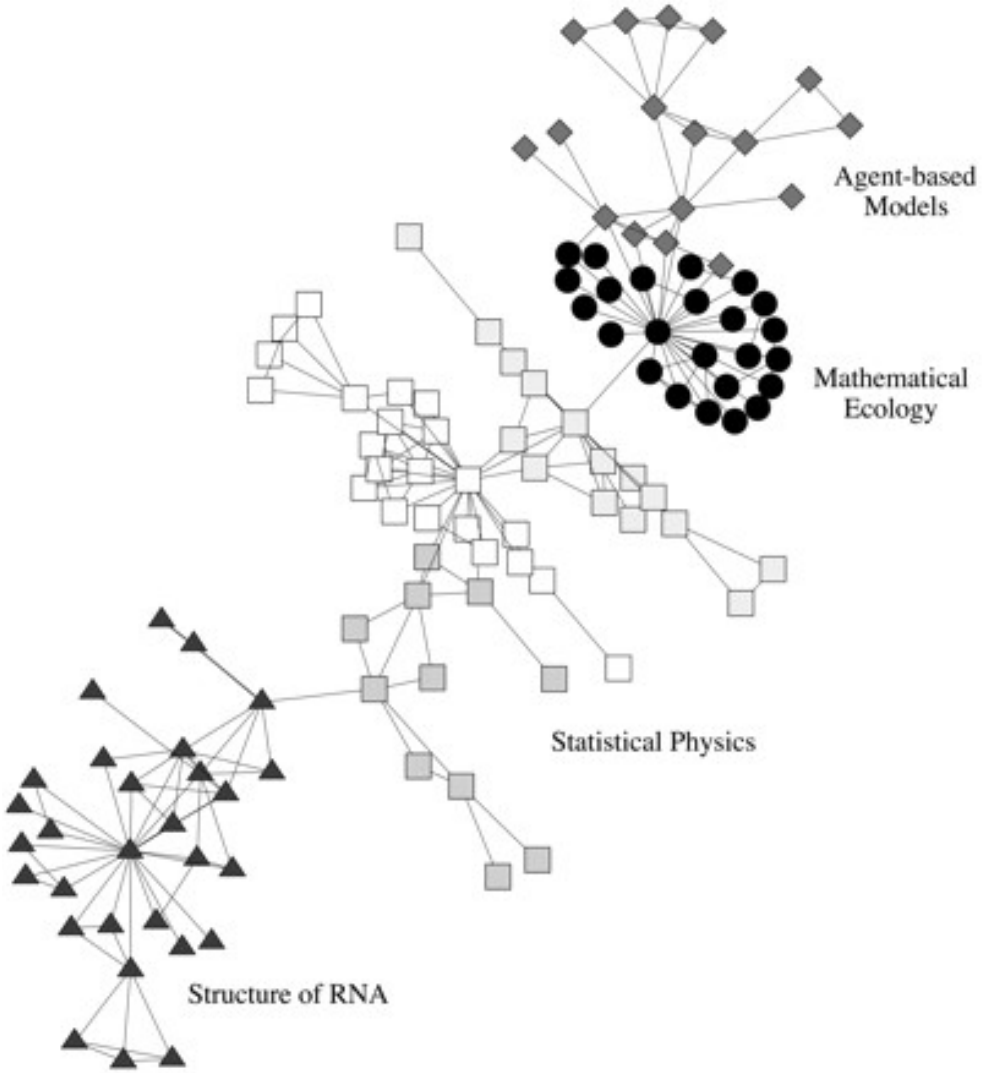


- How about large-scale organization?
  - Core-periphery, hierarchy,...
  - Communities

- What are communities? Why is it interesting? → Examples.

Scientific collaboration at SFI

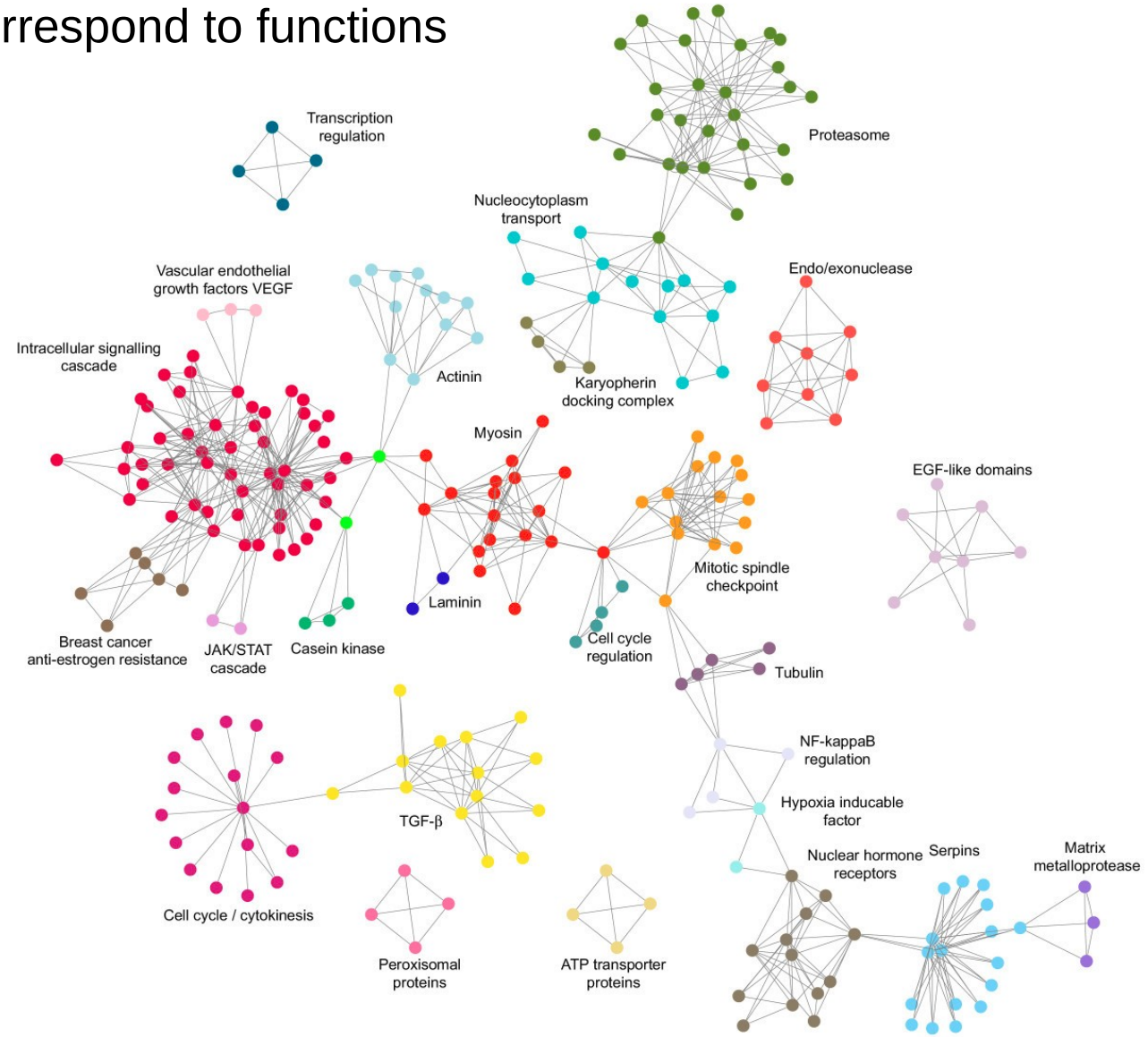
- Link (A – B) : A and B coauthored a paper
- Node classification by role according to position in community



PF Jonsson et al., *BMC Bioinf.* 7:2 (2006).

Rat protein-protein interaction network

- Link (A – B) : A and B proteins physically interact
- Modules correspond to functions



PF Jonsson et al., *BMC Bioinf.* 7:2 (2006).

Basics

- What is a community?
  - Intuitively: densely connected subnetworks
- Why is it interesting?
  - Nodes that participate in same function/nodes with similar attributes form groups and these groups are represented in network structure.
- Challenges:
  - No single clear definition. → Many competing options.
  - Large networks, different features. → Even more algorithms.
  - Which one to choose?
  - How to evaluate performance of method?
- History
  - Social sciences, computer science
  - Early 2000s, network science.

Outline

- How to identify communites?
- How to asses the quality of a community division?
- How asses the quality of a method?

- A lot of competing methods and measures

  → Here: selection that shows the development of the field
- And some guidelines to navigate the field

Method 1:
Hierarchical Clustering

Hierarchical clustering

- Traditional method used by social scientists.

1) Each node in its own community.
2) Calculate a distance between pairs of communities.
3) Join closest pair.
4) Go to step 2.

Hierarchical clustering: distance

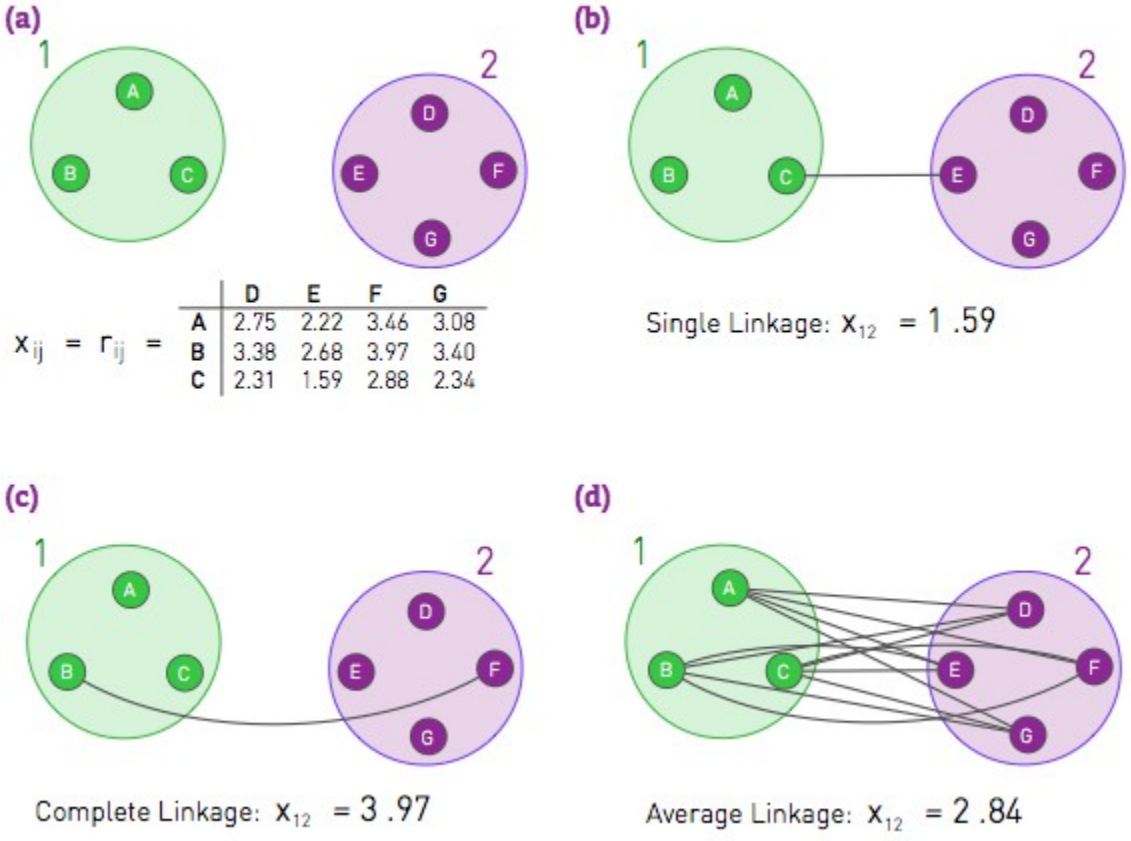
- Node distance should be low if nodes are in a community.
- Popular choice:

#common neighbors

$$\sigma_{ij} = \frac{n_{ij}}{\sqrt{k_i k_j}}$$

degree

- Distance between communities with more than one node:

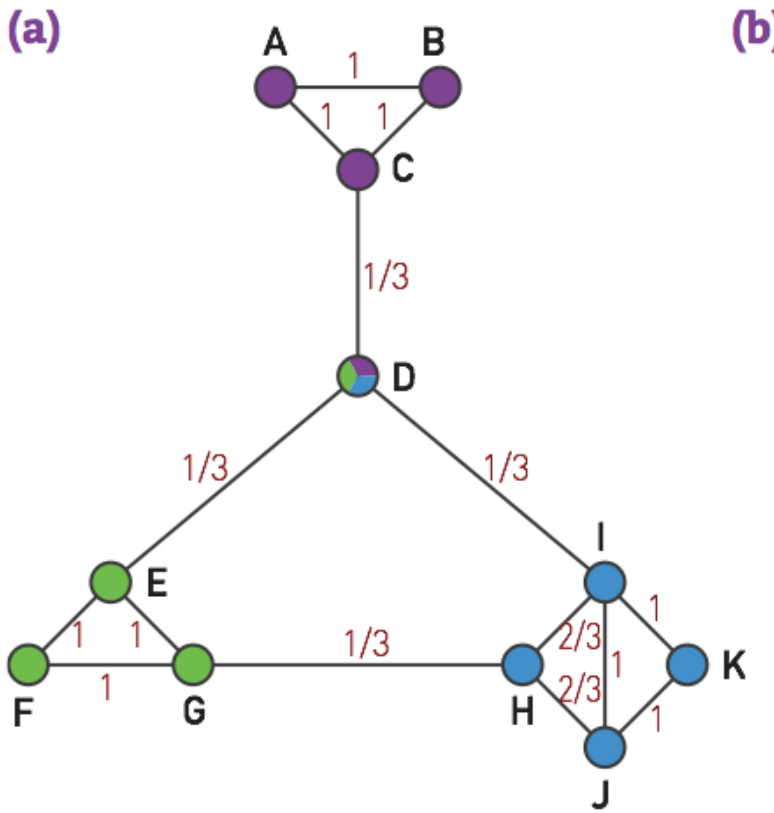


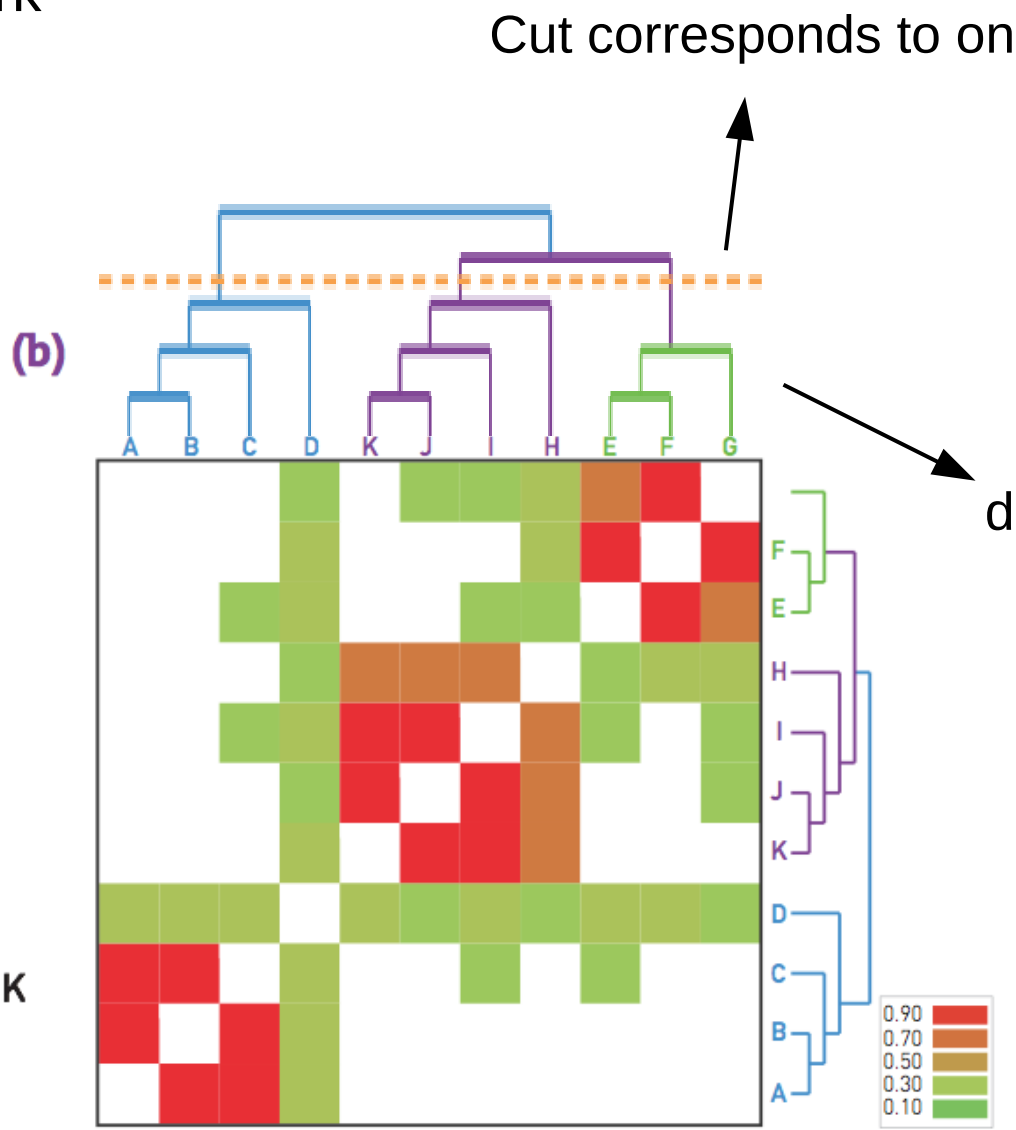
$x_{ij} = r_{ij} =$

	D	E	F	G
A	2.75	2.22	3.46	3.08
B	3.38	2.68	3.97	3.40
C	2.31	1.59	2.88	2.34

Hierarchical clustering: result

- Example artificial network





Cut corresponds to one partition

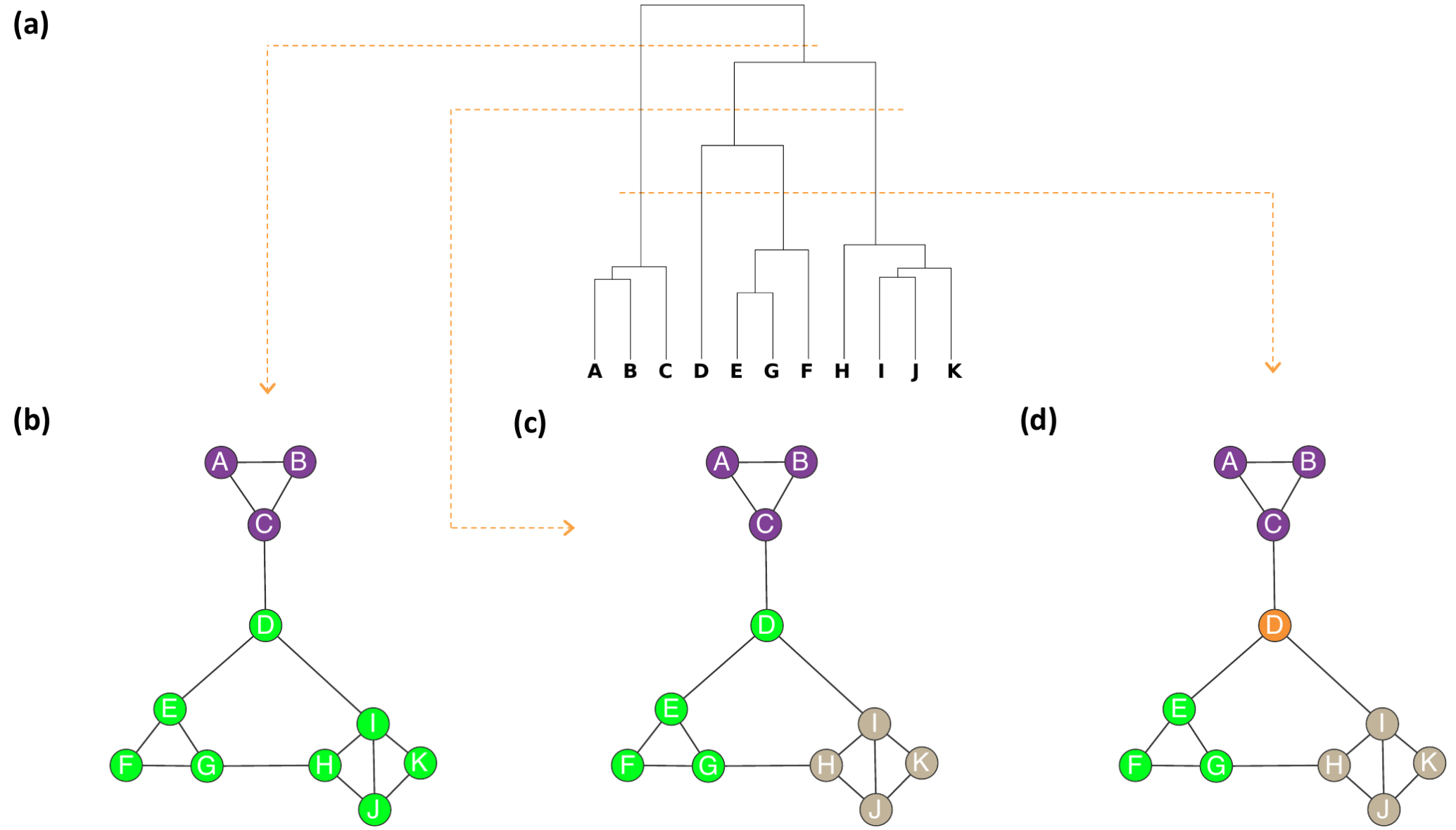
dendrogram

A.-L. Barabási, *Network Science: Communities*.

E. Ravasz et al., *Science* 297 (2002).

Hierarchical clustering: result

- Dendrogram



A.-L. Barabási, *Network Science: Communities*.

Hierarchical clustering: issues

- Advantages:
  - Easy to understand
  - Easy to implement
- Disadvantages:
  - Slow(ish), number of steps to evaluate: ~N^2 or ~N^3, depending on linkage
  - Tends to group together those nodes with the strongest connections but leave out those with weaker connections → divisions consist of a few dense cores surrounded by a periphery of unattached nodes
  - (Results depend on distance and linkage)
- Open question: where to cut the dendrogram?

M.E.J. Newman, *Nature Phys.*, 8:25-31 (2012)

Method 2:
Betweenness based division

Betweenness based division

- Alternative method: instead of agglomerating communities, breaking one large into smaller ones

1) Start from one large community
2) Caluclate a centrality measure for each link
3) Remove link with highest centrality
4) Go to step 2

- Centralities: **betweenness for** edge *i*

$$B(i) = \sum_{s \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

#shorest path between s and t traversing i

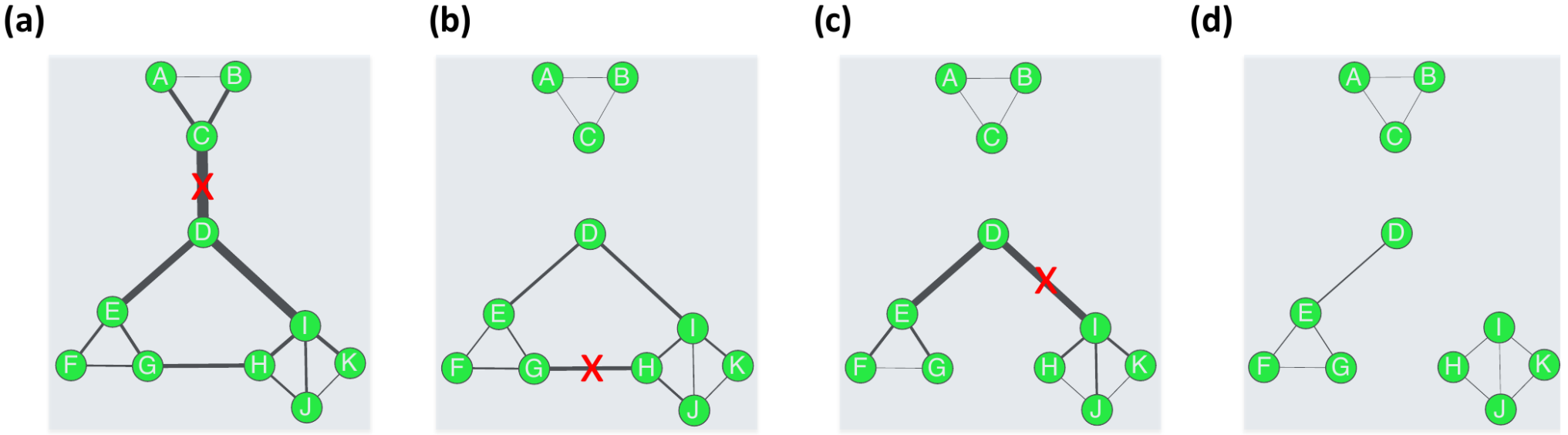
#shorest path between s and t

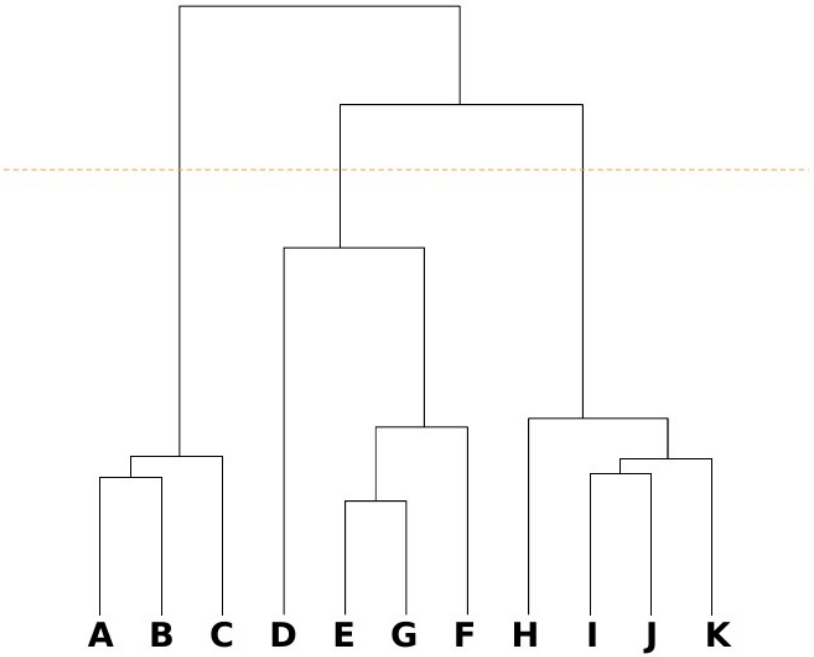
- Other: random walk, ...

M. Girvan et al., *PNAS*, 99:12 (2002)

Betweenness based division

- Our little example:





A.-L. Barabási, *Network Science: Communities*.

Betweenness based division

- Advantages:
  - Easy to understand
  - Easy to implement
  - Perhaps less decisions have to be made
- Disadvantages:
  - Slow, number of steps to evaluate: ~NL^2 or faster if we don't recalculate the betweenness in each step
  - (Results depend on centrality)
- **Still open question: where to cut the dendrogram?**

M.E.J. Newman, *Nature Phys.*, 8:25-31 (2012)

Quality of community division

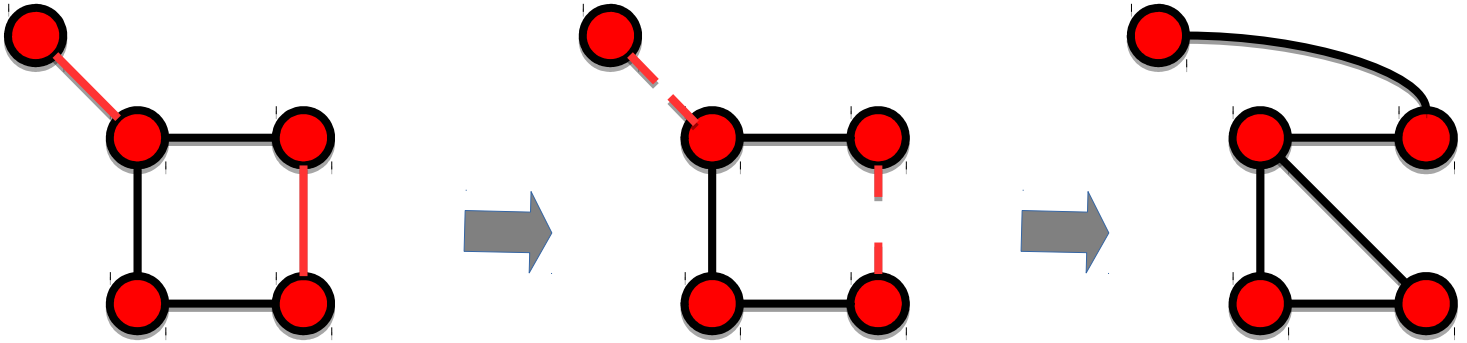
Modularity

- Again: many existing measures.
- Naïve: fraction of links that are inside communities → best division is one large community
- Instead: fraction of links inside communities compared to what you would expect by chance
- Entire network one community: even if links are randomly placed, all links are inside the community
- What is by chance?

M. Girvan et al., *PNAS*, 99:12 (2002)

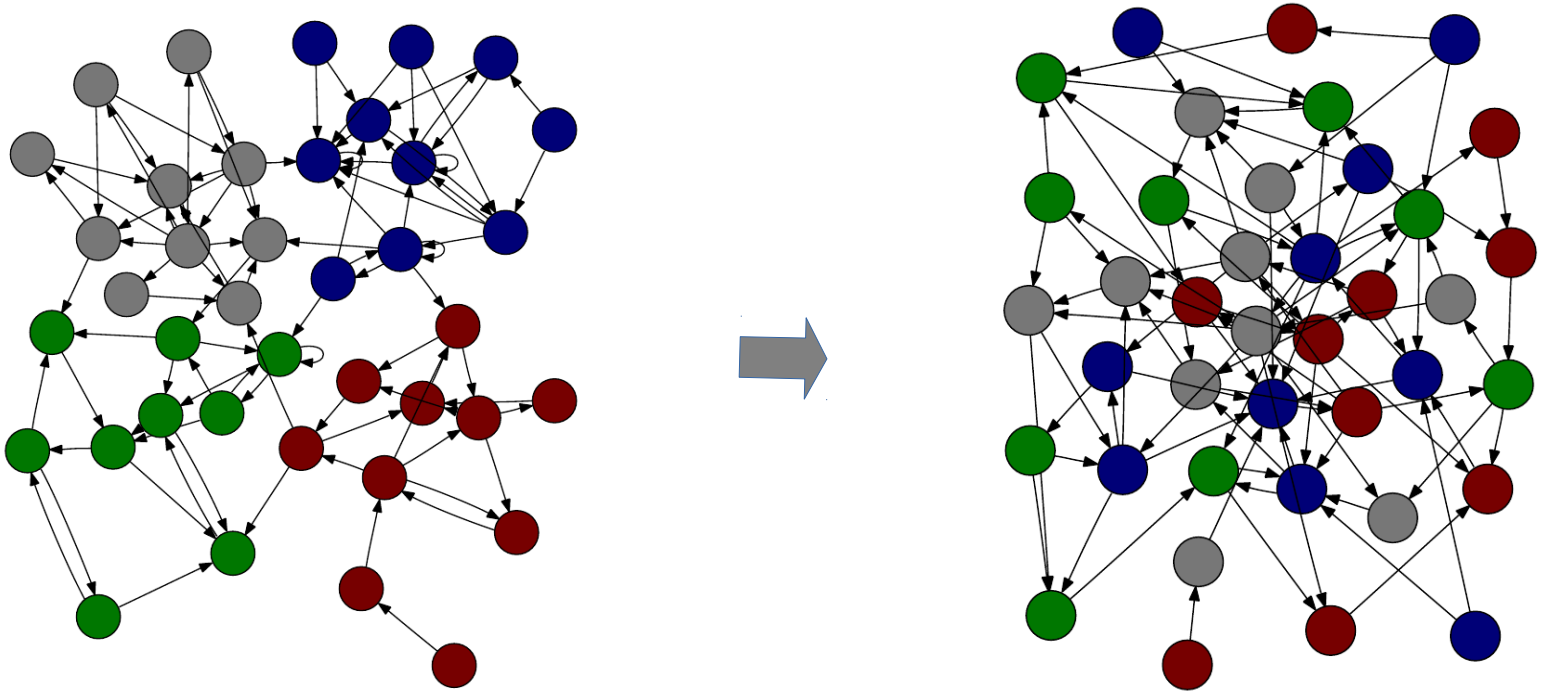
What is by chance?

- Degree preserved randomization



Original

What we compare to



Modularity

- Modularity:

$$M = Q = \frac{1}{2m} \sum_{ij} \left[A_{ij} - \frac{k_i k_j}{2m} \right] \delta_{s_i, s_j}$$

Real link

Probability of link in randomized version

- m: #links in the network
- A_{ij}: adjacency matrix, 1 if i and j are connected, 0 of not
- k_i: degree of node I
- δ_{s1s2}: 1 if in the same community, 0 if not
- High M → good division

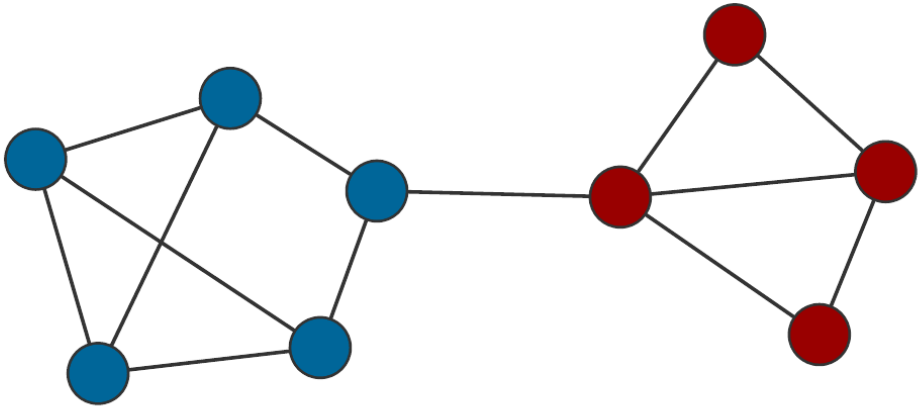
M. Girvan et al., *PNAS*, 99:12 (2002)

Modularity

(a)

Optimal Partition

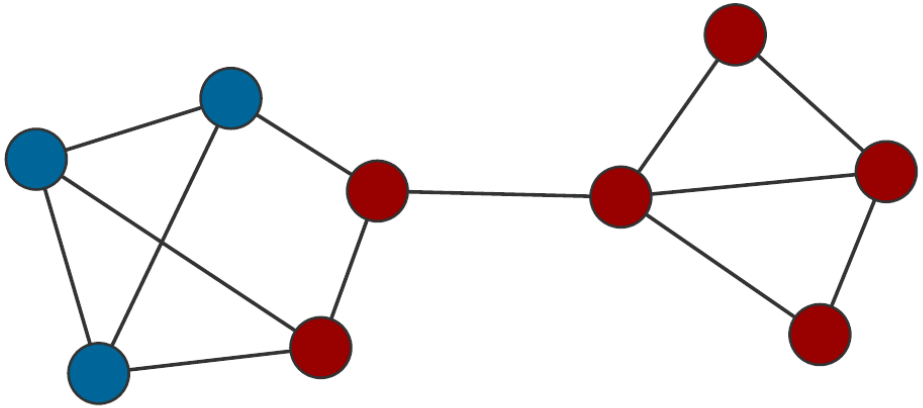
$M = 0.41$



(b)

Suboptimal Partition

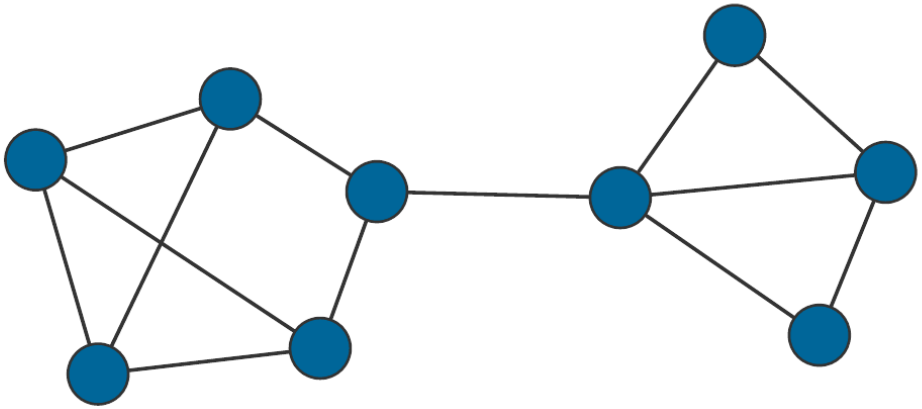
$M = 0.22$



(c)

Single Community

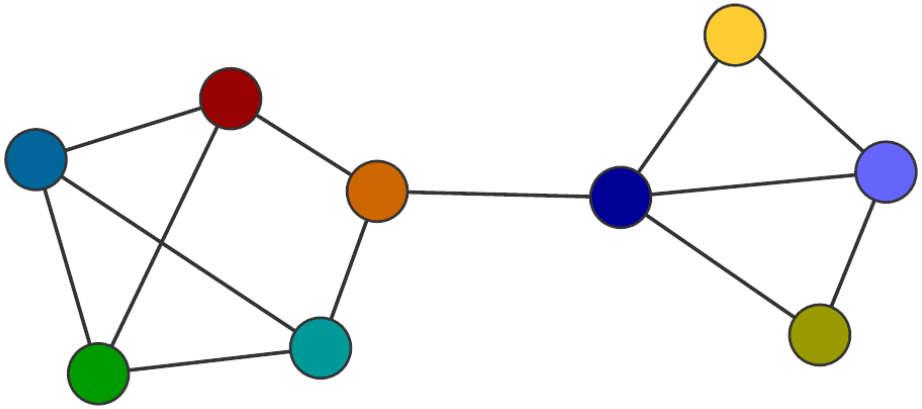
$M = 0$



(d)

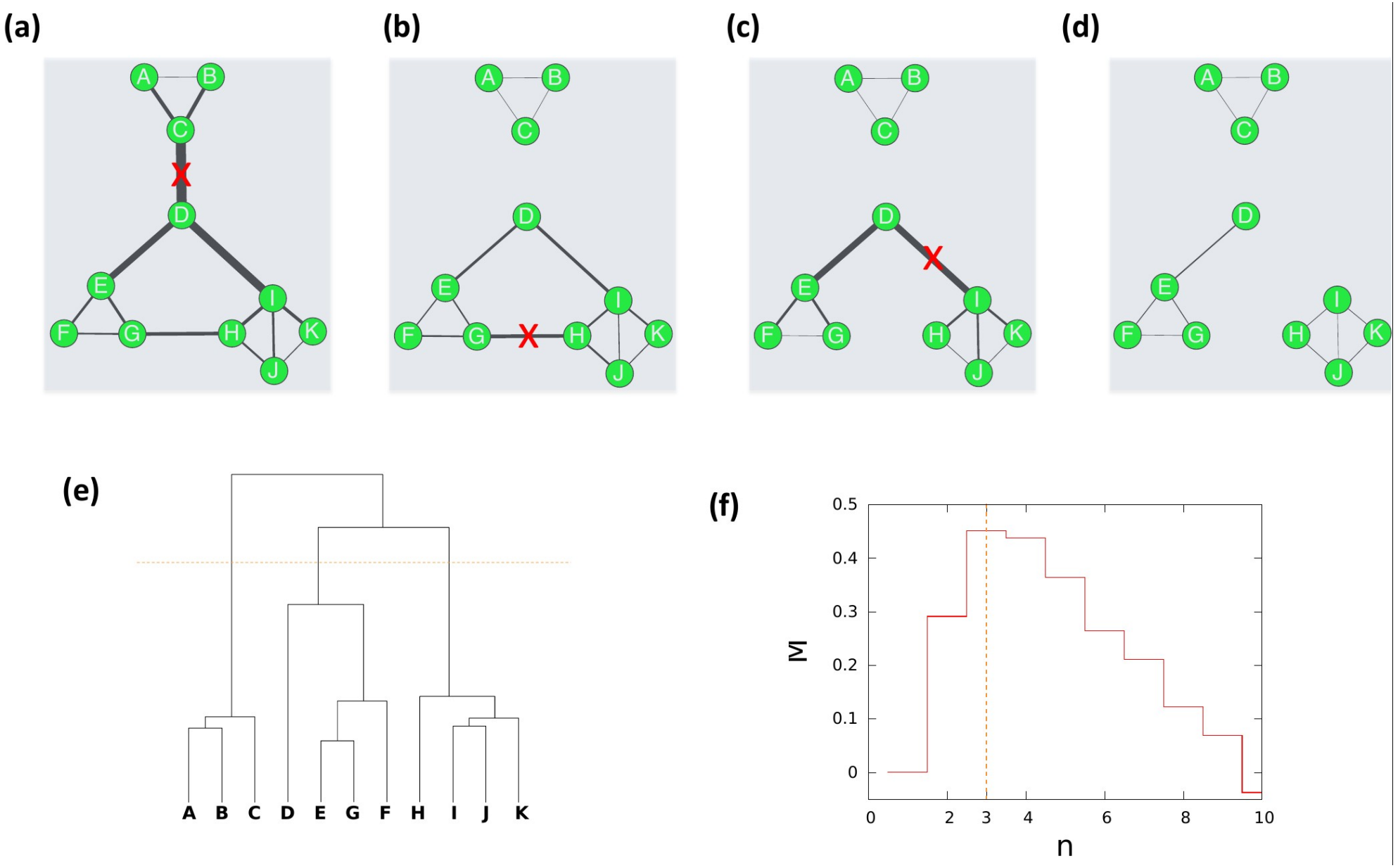
Negative Modularity

$M = -0.12$



Modularity

- Where to cut dendrogram? At maximum Q!



M. Girvan et al., *PNAS*, 99:12 (2002)

Method 3:
Direct optimization of Q

Direct optimization of modularity

- Exact maximum of Q → NP-complete (exponentially increasingly difficult with N)
- Approximation methods: a lot to choose from
- Louvain method
  - Fast, ~N
  - Typically preforms well on tests
- Two steps applied iteratively:

1) Find local maximum.

2) Coarse grain network.

V.D. Blondel et al, *J. Stat. Mech*. P10008 (2008).

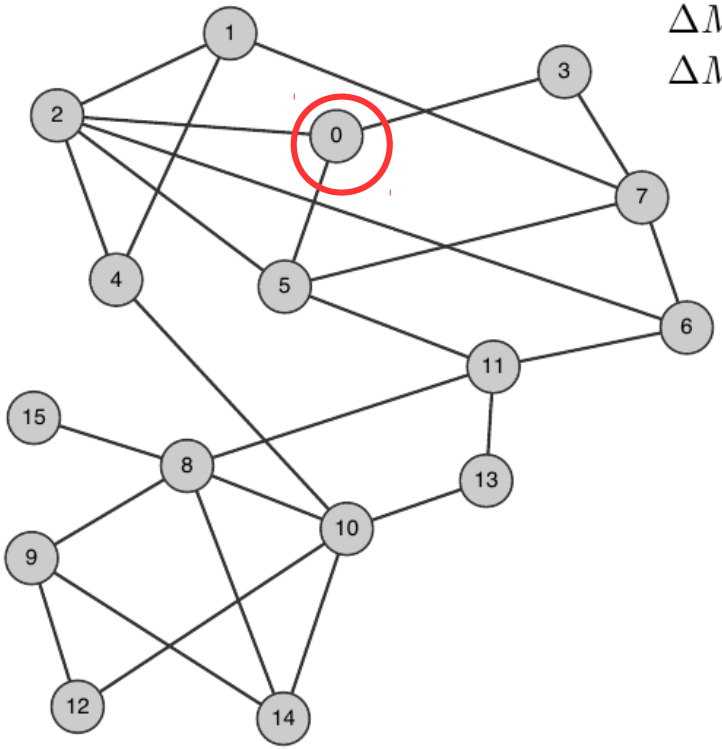
Louvain method: Step 1

1) For node i, calculate ΔQ for each neighbor j if node i is removed from its community and placed in the commnuity of j.Coarse grain network.

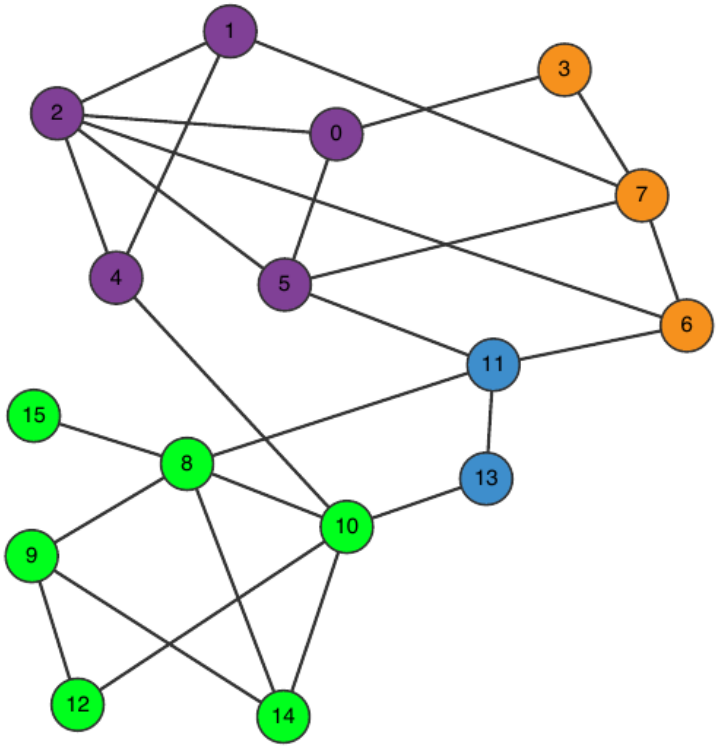
2) Move i to community that maximizes ΔQ, if ΔQ>0.

3) Repeat while Q increases.

$$\Delta M_{0,2} = 0.023$$
$$\Delta M_{0,3} = 0.032$$
$$\Delta M_{0,4} = 0.026$$
$$\Delta M_{0,5} = 0.026$$



Step I

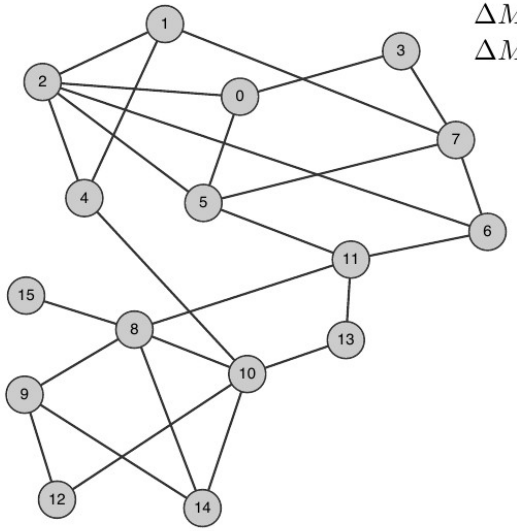


V.D. Blondel et al, *J. Stat. Mech*. P10008 (2008).

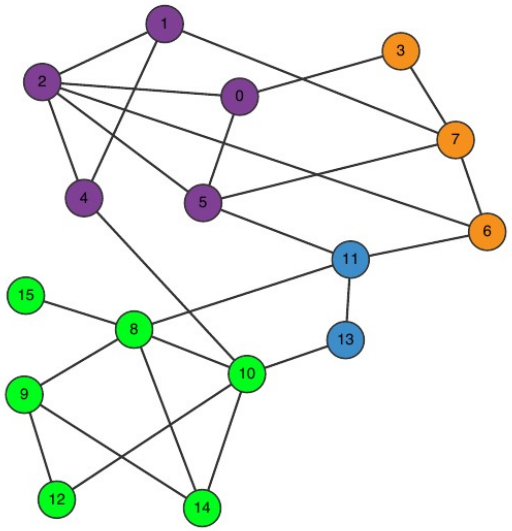
Louvain method

1st Pass

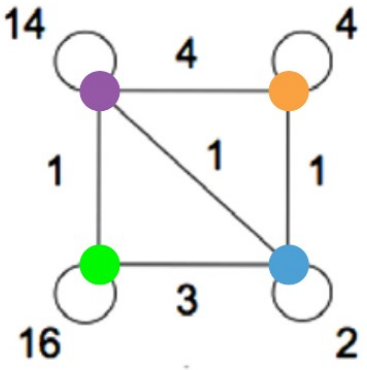
$$\Delta M_{0,2} = 0.023$$
$$\Delta M_{0,3} = 0.032$$
$$\Delta M_{0,4} = 0.026$$
$$\Delta M_{0,5} = 0.026$$



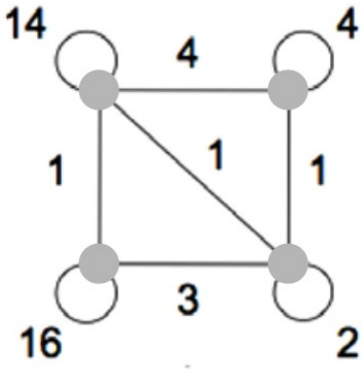
Step I



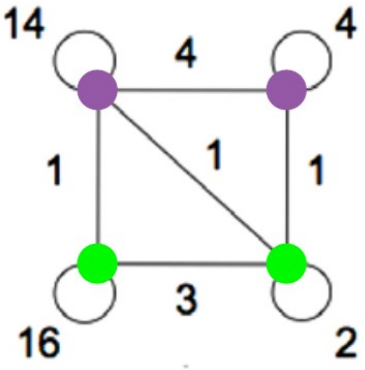
Step II



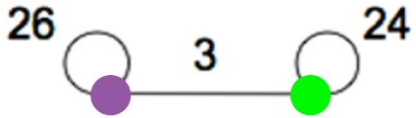
2nd Pass



Step I



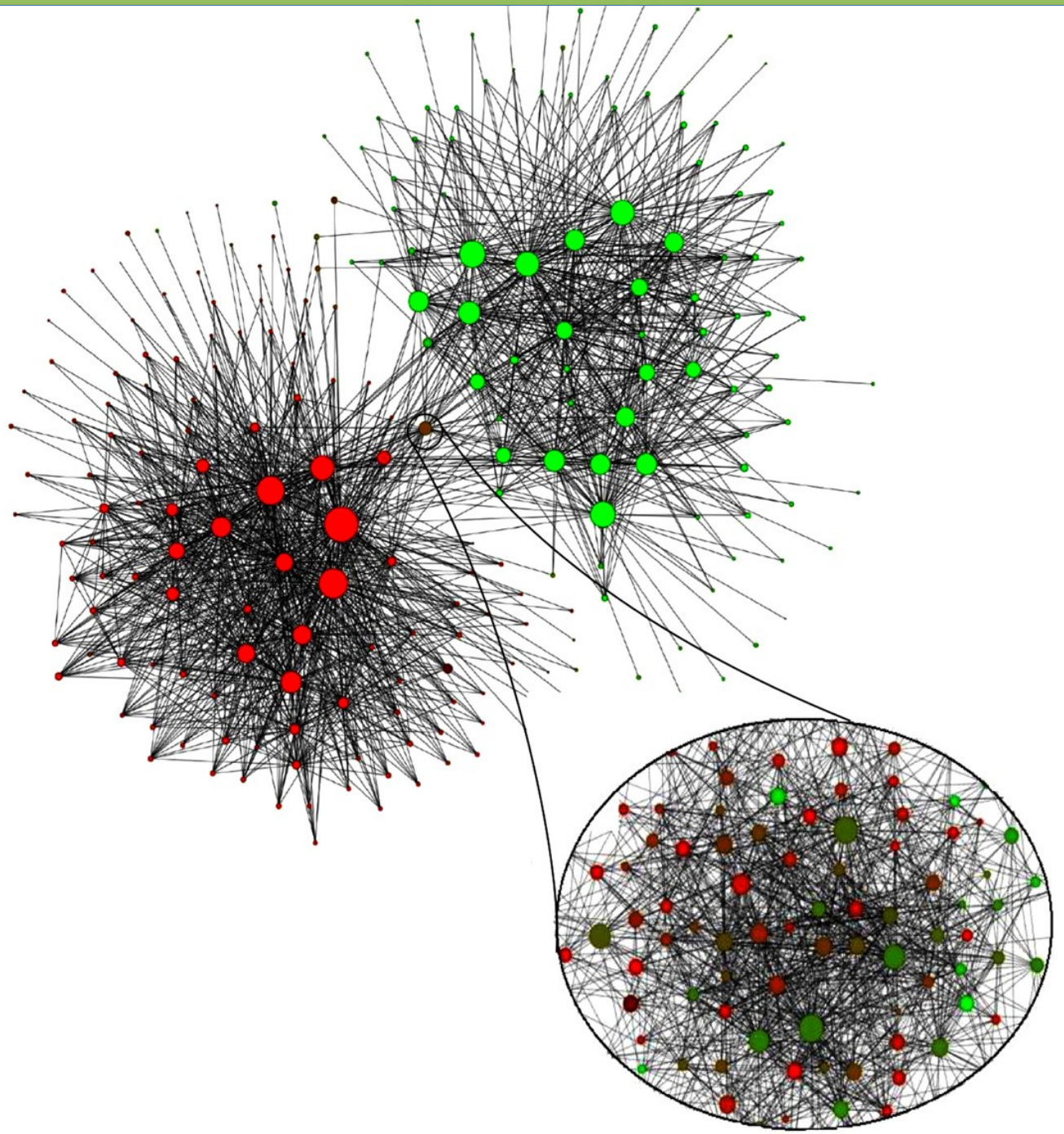
Step II



V.D. Blondel et al, *J. Stat. Mech*. P10008 (2008).

Application: Belgium phone call network

- Link (A – B) : A and B talk frequently on the phone
- Phone calls of ~2 million customers



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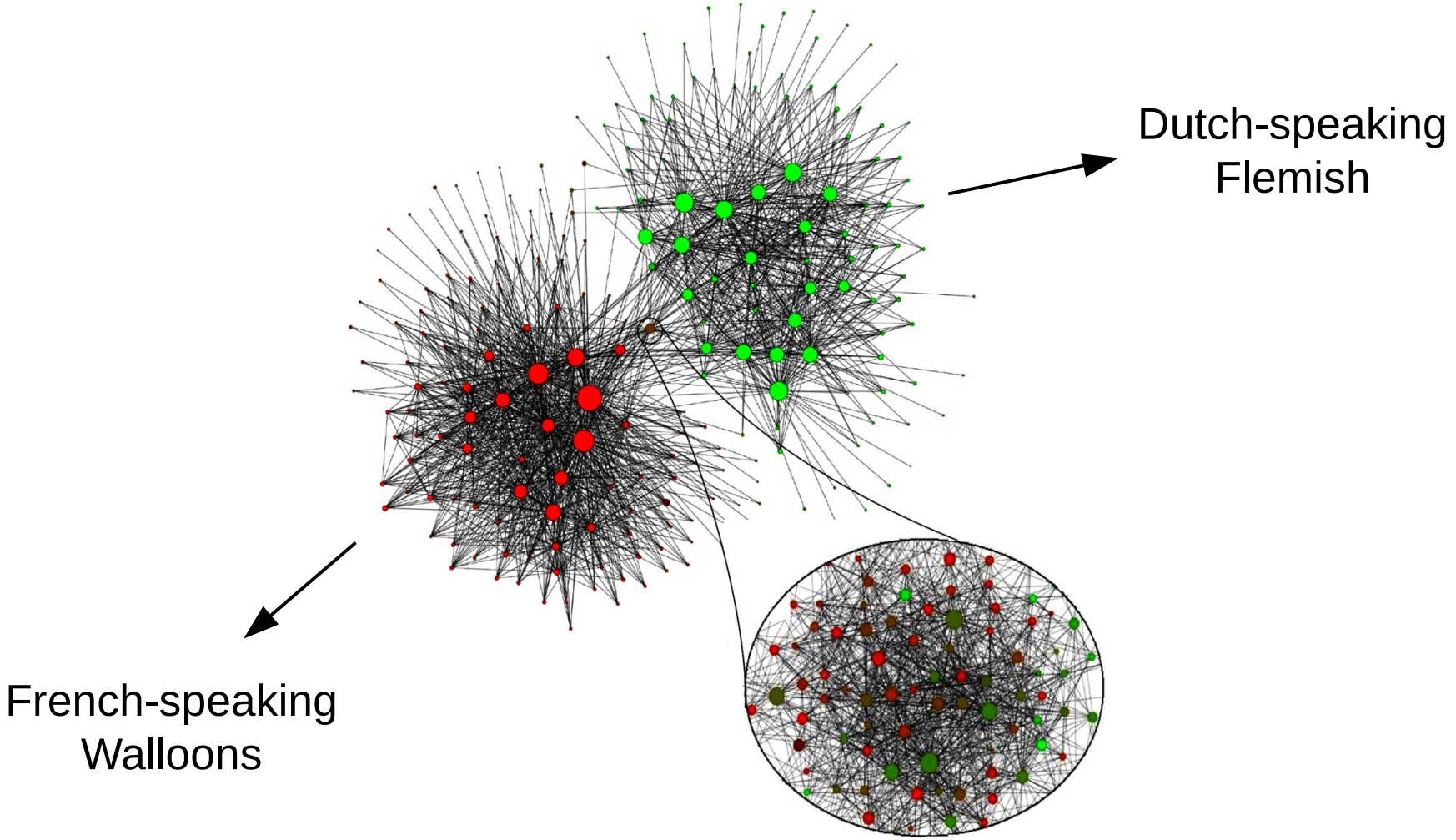


V.D. Blondel et al, *J. Stat. Mech*. P10008 (2008).

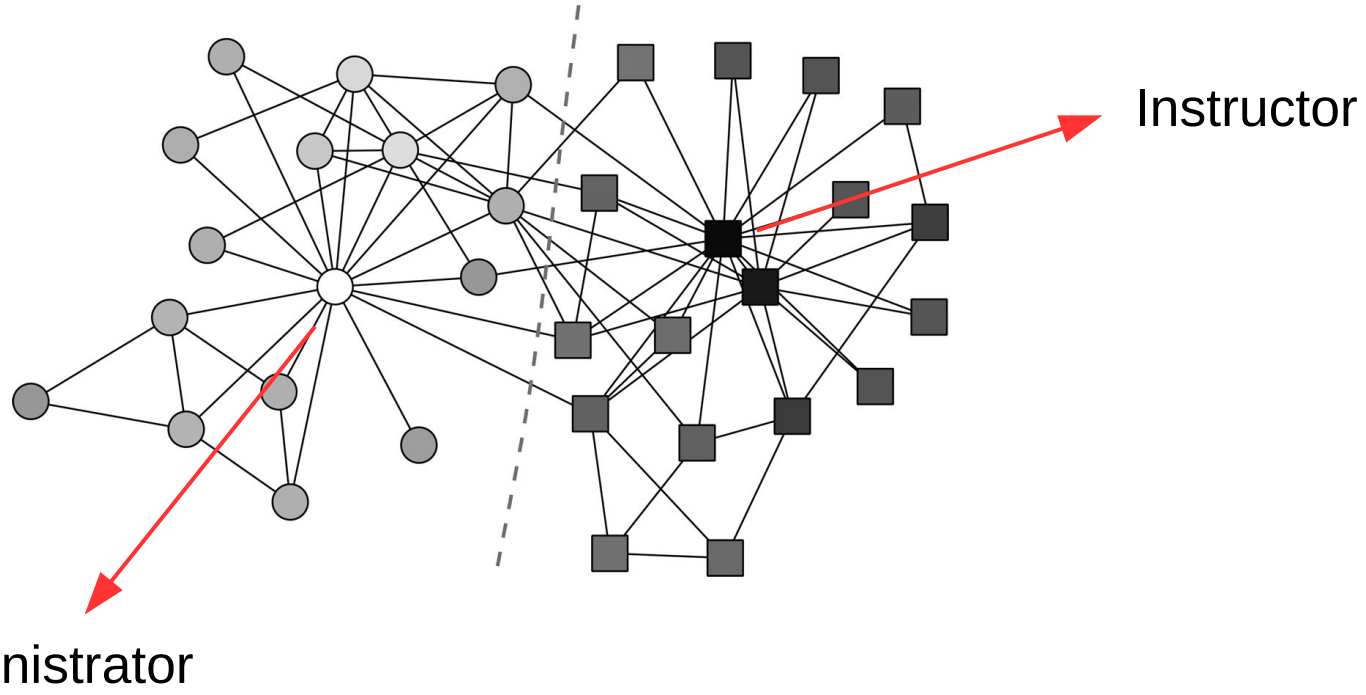
Comparing methods

Comparing methods

- We need a network where we know the true community.
- Option I: Real networks with known ground truth
- Option II: Model networks with built-in communites

Comparing methods

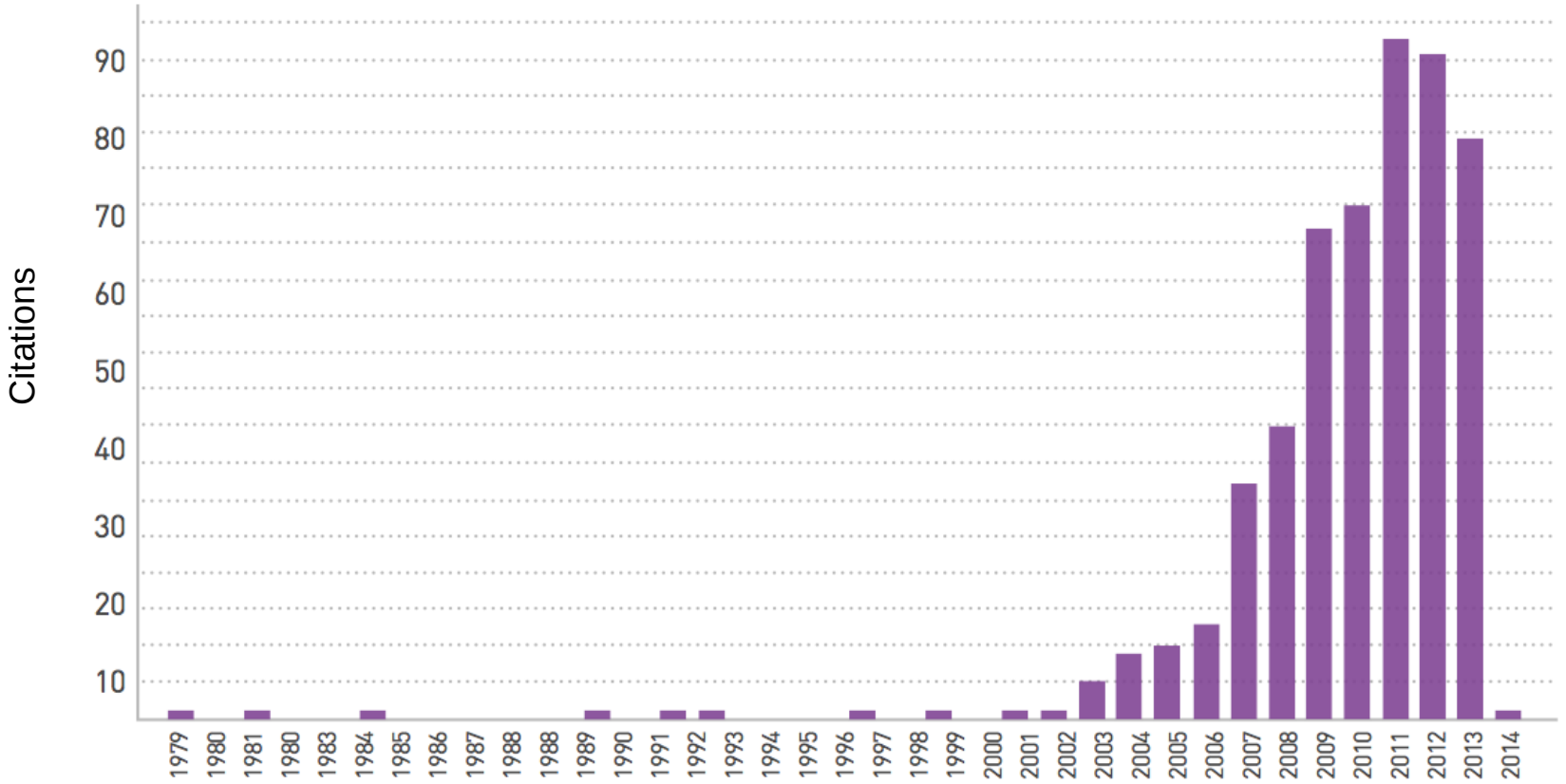
- We need a network where we know the true community.
- Option I: Real networks
- Zachary Karate Club



M. Girvan et al., *PNAS*, 99:12 (2002)
W.W. Zachary, *J. Anthropol. Res.* 33:452-473 (1977).

Zachary Karate Club

- Link (A – B) : A and B friends
- Club split into two
- Data collected before split



A.-L. Barabási, *Network Science: Communities*.

W.W. Zachary, *J. Anthropol. Res.* 33:452-473 (1977).

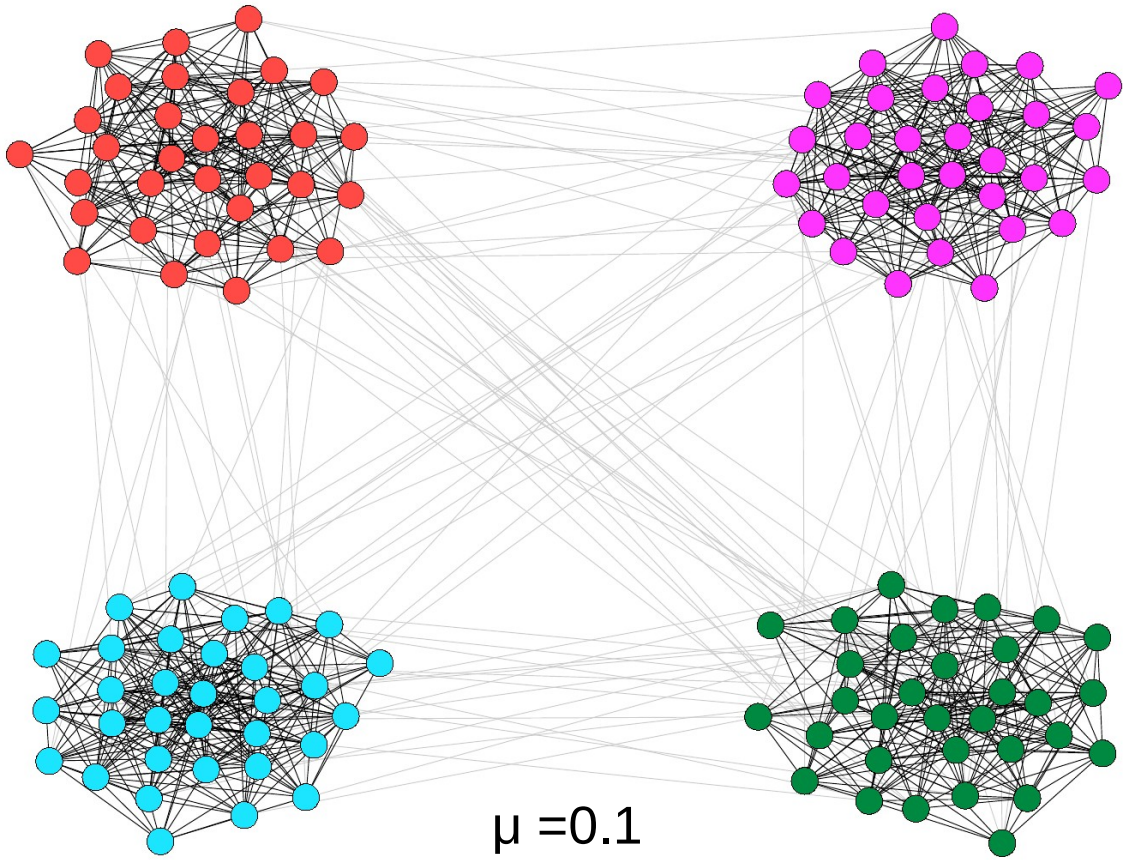
Zachary Karate Club Club

First presenter to mention the ZKC at a conference gets the trophy.

http://networkkarate.tumblr.com/

Artificial benchmarks

- Girvan-Newman benchmark
- N=128 node divided into 4 groups, <k>=16
- p_{in} = prob. that two nodes in the same group are connected
- p_{out} = prob. that two nodes in different groups are connected (not independent)
- μ = fraction of external links = $3p_{out}/(p_{in}+3p_{out})$
- No communities:
  $p_{in}=p_{out}$ or μ =0.75
- Not realistic

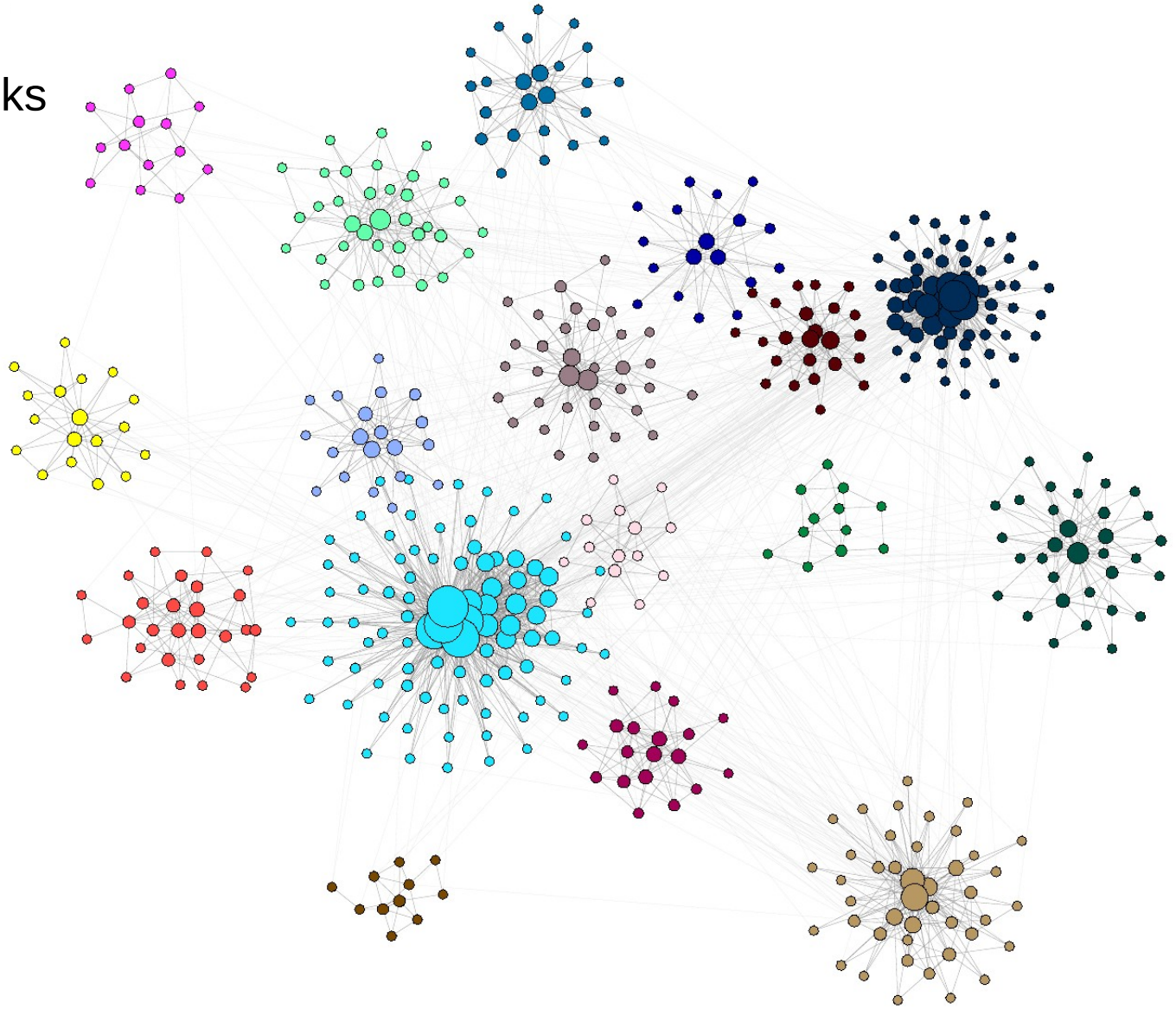


μ =0.1

M. Girvan et al., *PNAS*, 99:12 (2002)

Artificial benchmarks

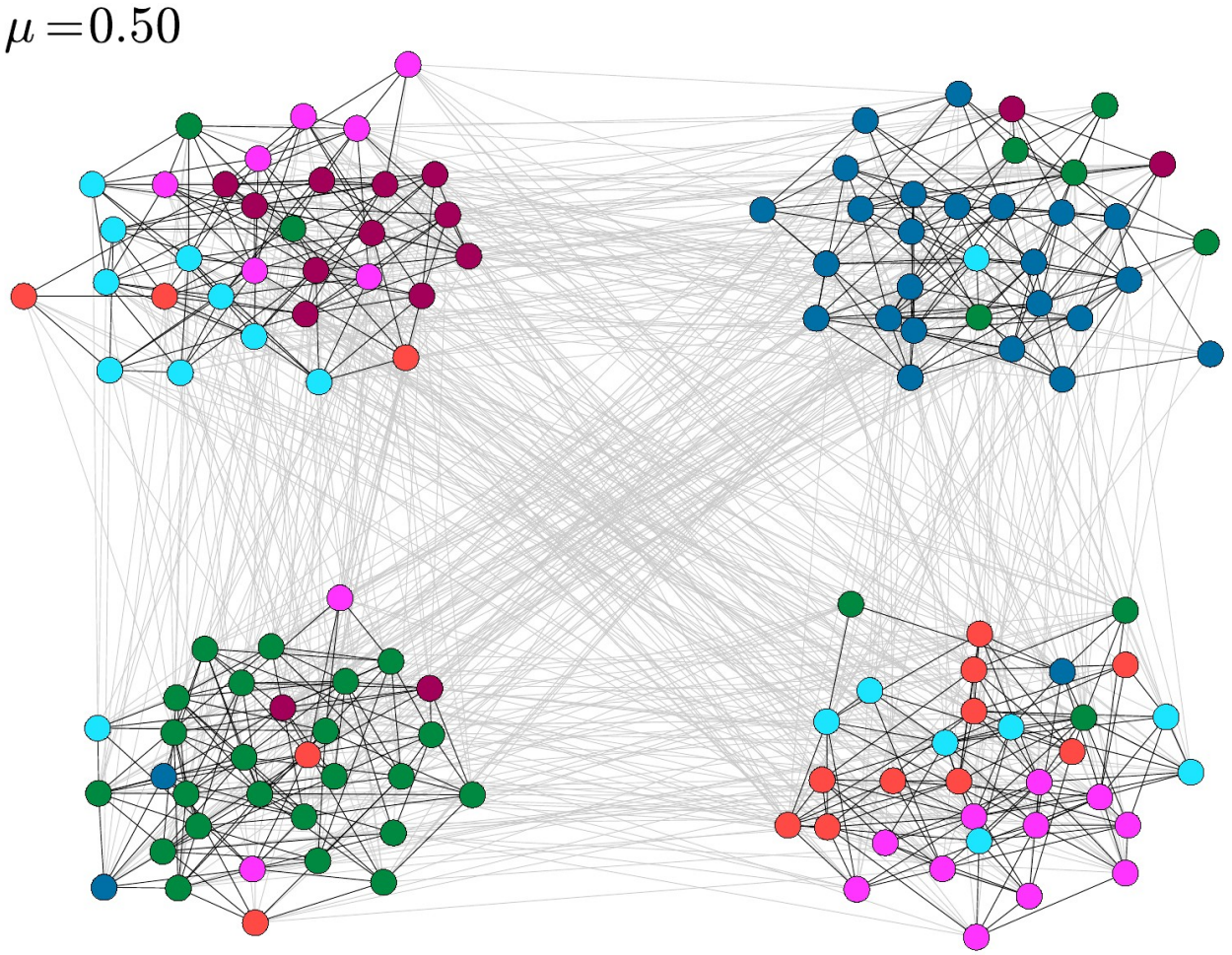
- Lancichinetti–Fortunato–Radicchi (LFR) benchmark
- N nodes, N_c communities
- μ = fraction of external links
- Power-law degree distribution
- Power-law community size distribution



A. Lancichinetti et al., *PRE*, 78:046110 (2008)

Comparing community divisions

- We know what we should get.
- How to systematically compare what we found?
- Again, a lot of possibilities
- Our choice now: Normalized mutual information



Normalized mutual information

Information theory approach: if two partitions are similar, one needs very little information to infer one partition given the other. We can use **the mutual information**

$$I\left(\{C_1\},\{C_2\}\right)=\sum_{C_1}\sum_{C_2}p\left(C_1,C_2\right)\log\frac{p\left(C_1,C_2\right)}{p\left(C_1\right)p\left(C_2\right)}$$

Joint probability that a randomly chosen node belongs to community C_1 in the first partition and C_2 in the second

$$p\left(C_1,C_2\right)=\frac{\text{how many nodes that are in } C_1 \text{ are also in } C_2}{\text{sum over all possible pairs } C_1 \text{ and } C_2}=\frac{N_{C_1C_2}}{\sum_{C_1,C_2}N_{C_1C_2}}$$

Probability that a randomly chosen node belongs to community C_1

$$p(C_1)=\frac{\text{how many nodes belong to } C_1}{\text{sum over all partitions}}=\frac{N_{C_1}}{\sum_C N_C}$$

Normalization by average Shannon entropy:

$$I_n\left(\{C_1\},\{C_2\}\right)=\frac{2I\left(\{C_1\},\{C_2\}\right)}{H\left(\{C_1\}\right)+H\left(\{C_2\}\right)}$$

L. Danon et al, *JStat.* P09008 (2005)
S. Fortunato, *Phys. Rep.* 486 (2010)

Normalized mutual information

- In summary:

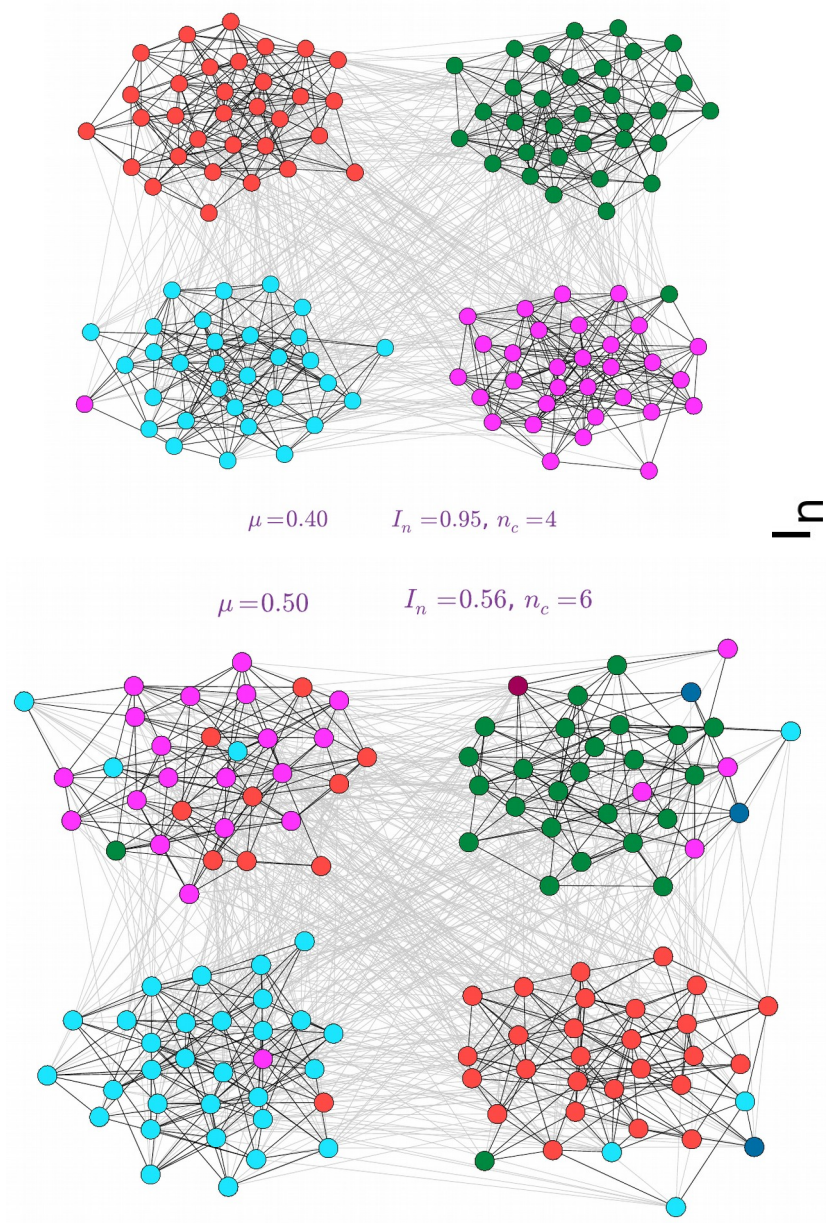
$$I_n\left(\{C_1\},\{C_2\}\right) = \frac{2I\left(\{C_1\},\{C_2\}\right)}{H\left(\{C_1\}\right) + H\left(\{C_2\}\right)}$$

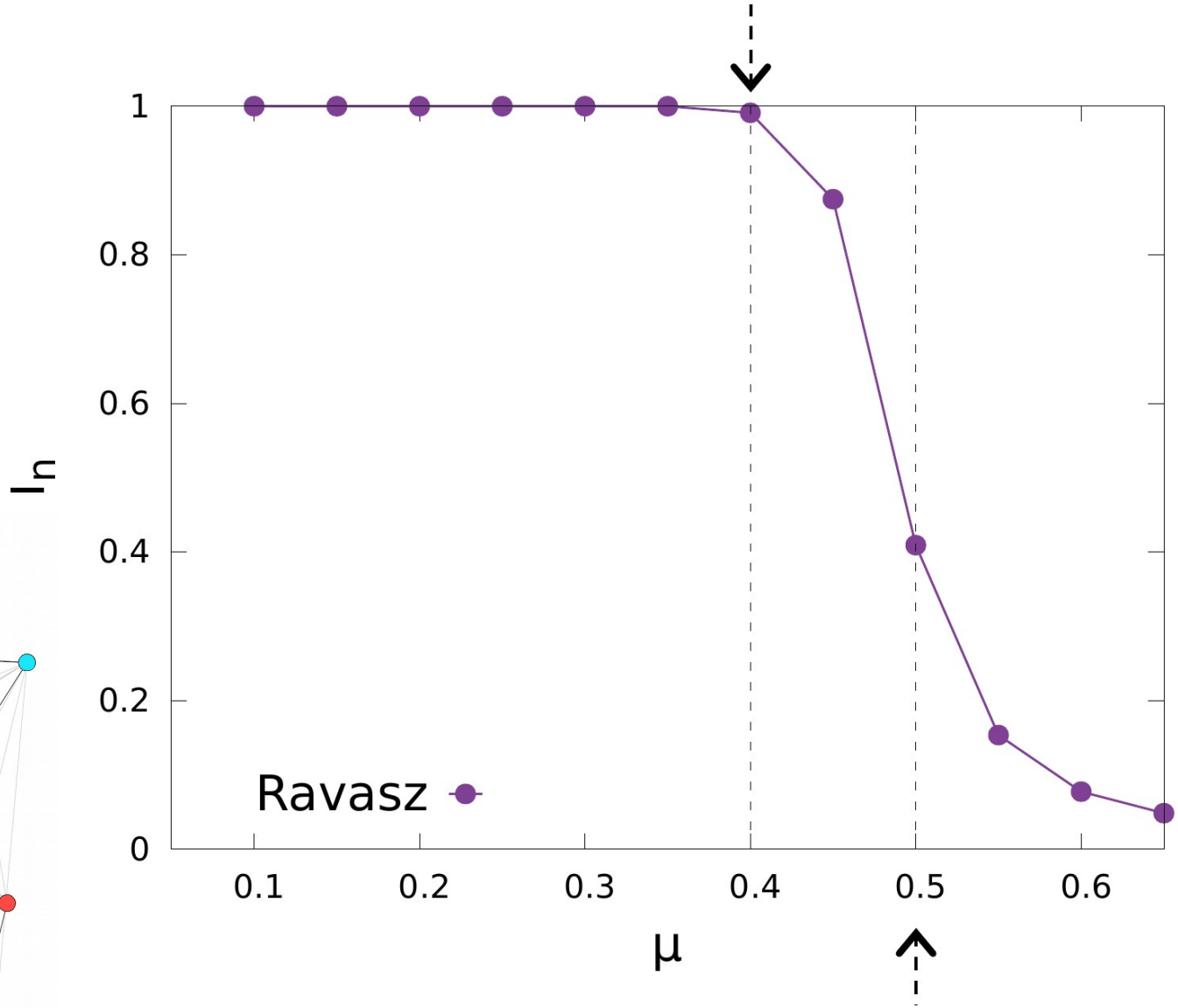
- it quantifies the "amount of information" (in units such as bits) obtained about one random variable, through the other random variable (wiki)
- $I_n = 1 \rightarrow$ same division
- $I_n = 0 \rightarrow$ two divisions independent from each other

L. Danon et al, *JStat.* P09008 (2005)
S. Fortunato, *Phys. Rep.* 486 (2010)

Benchmarks and NMI in action

- NG benchmark, hierarchical clustering

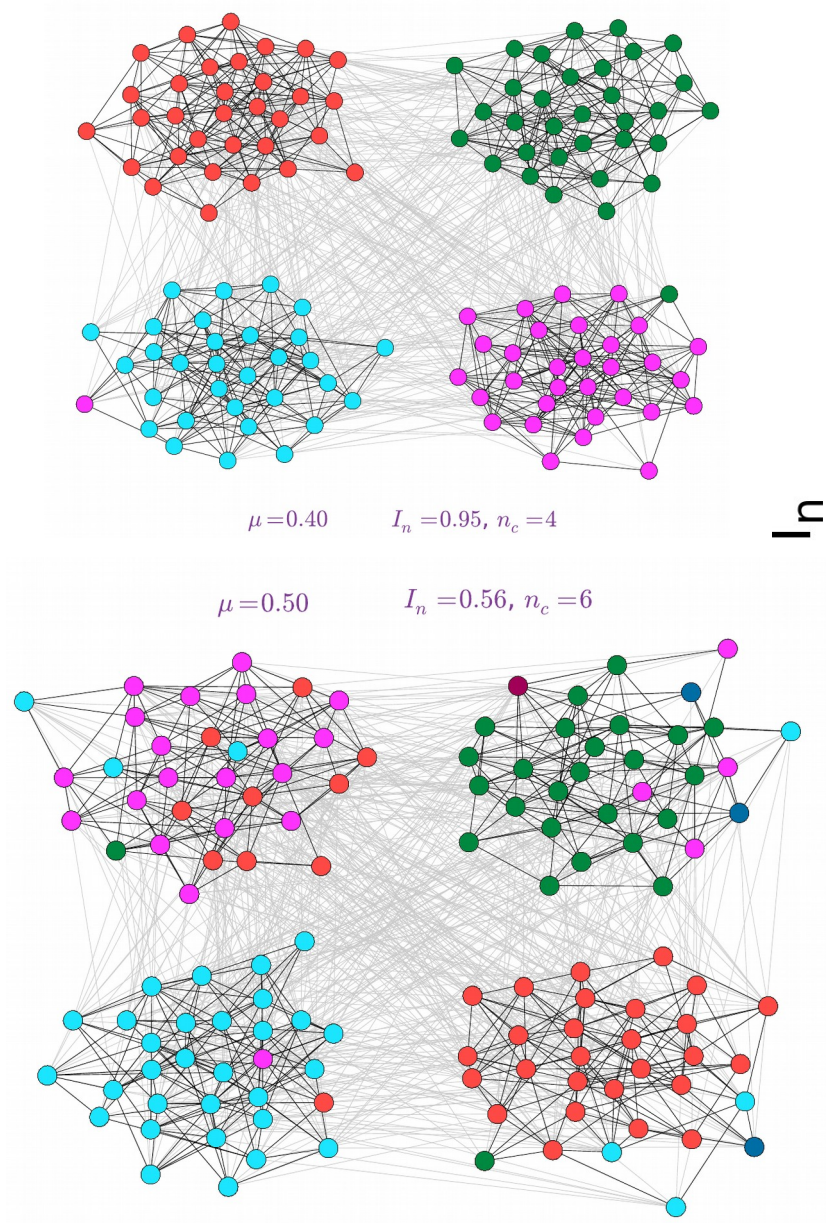


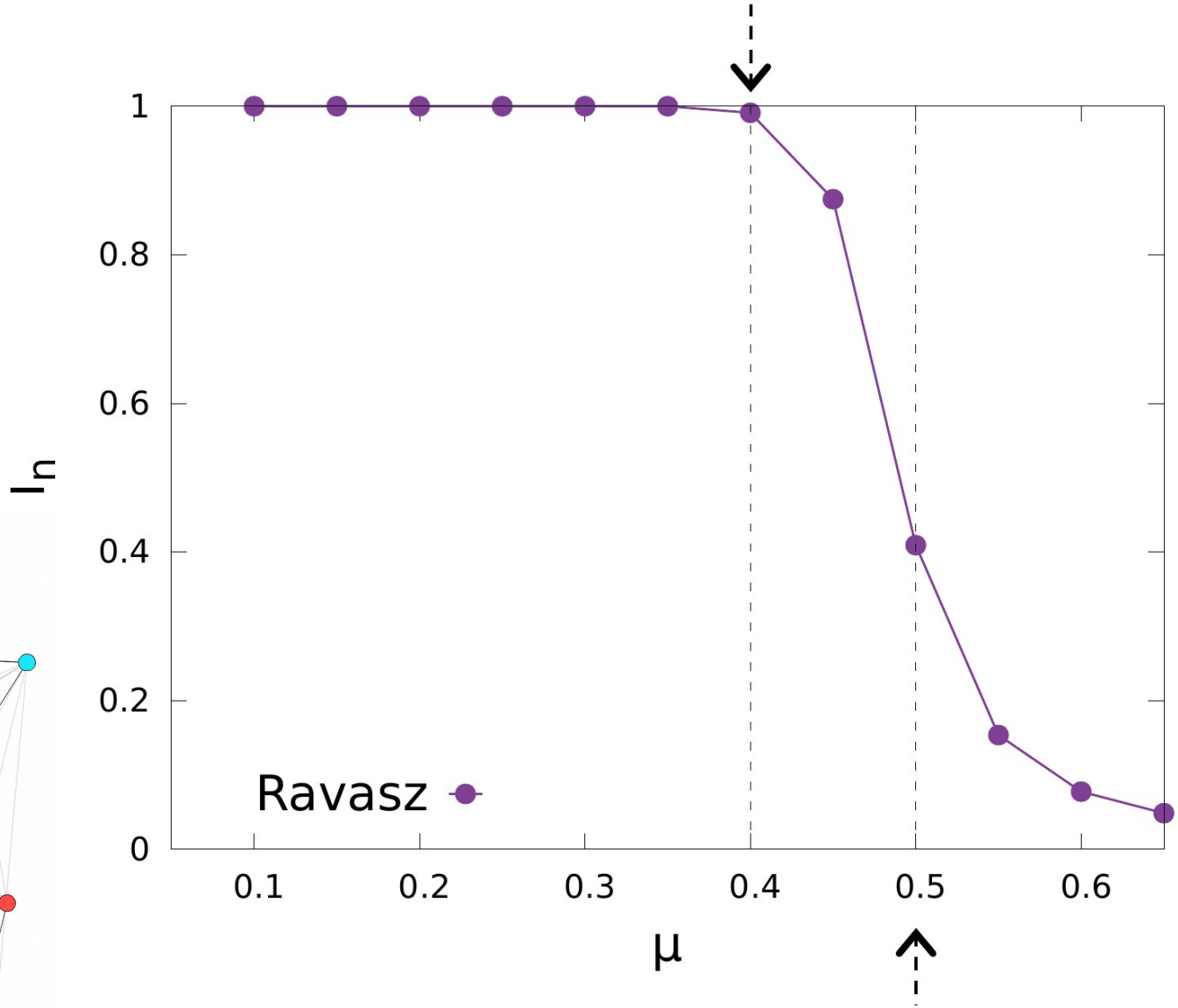


A.-L. Barabási, *Network Science: Communities*.

Benchmarks and NMI in action

- NG benchmark, hierarchical clustering

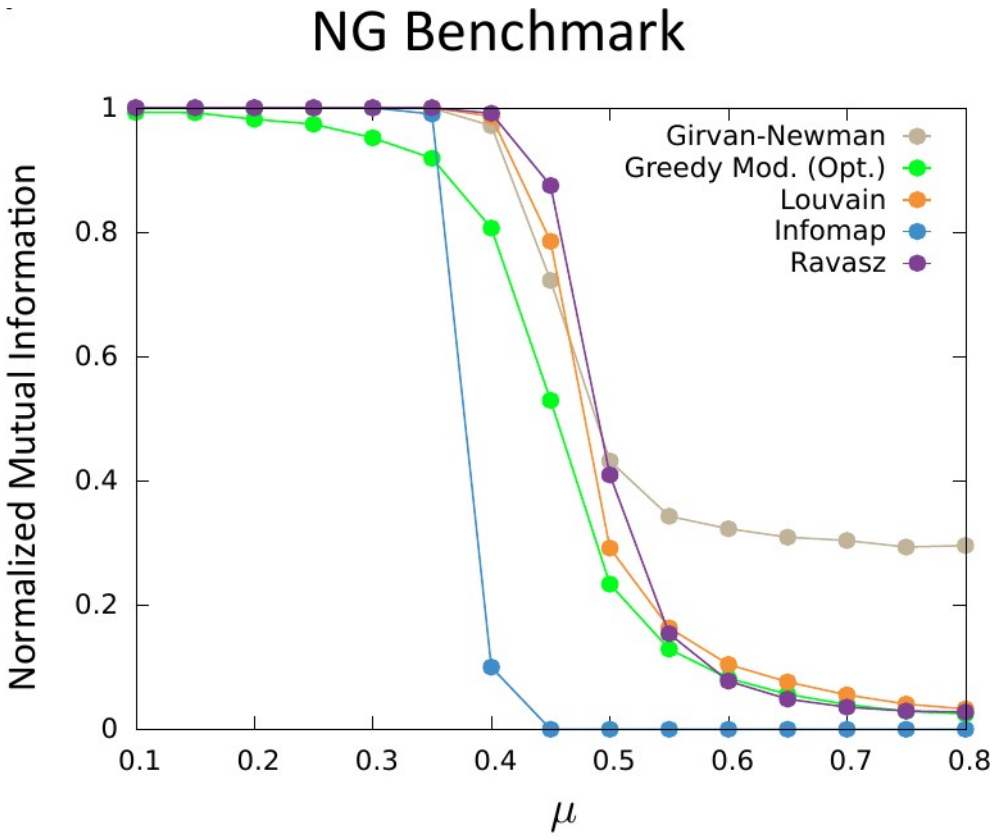


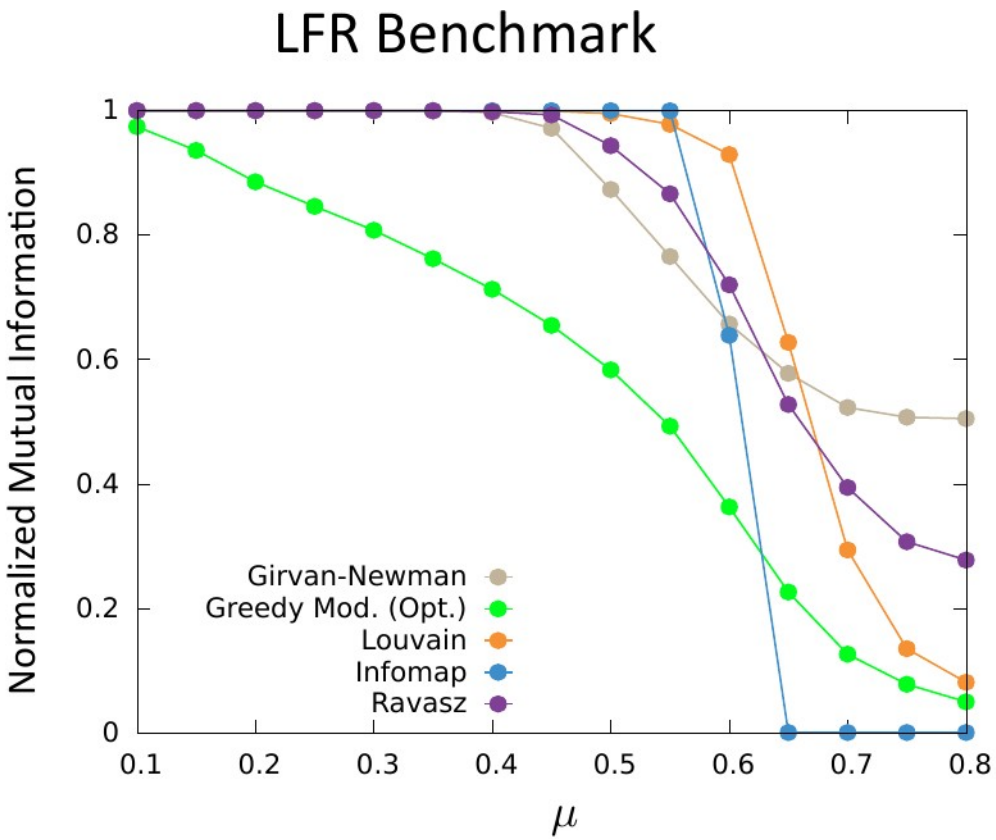


A.-L. Barabási, *Network Science: Communities*.

Benchmarks and NMI in action

- Purple: Hierarchical; Orange: Louvain; Gray: Betweenness





LFR parameters: N = 1000; degree exp. = 2; max degree = 50; comm. size exponent = 1, min comm. size = 20, max = 100

A.-L. Barabási, *Network Science: Communities*.

Short list of other methods

- Many other methods to find communities:
  - Local: instead of finding global division, find the community a given node belongs to
  - Spectral: based on spectrum of graph Laplacian
  - Dynamical: Potts-model, oscillators, random walks
  - Stochastic block models: find best fit using bechmark-like model

How to choose method?

- What is the best method?

  → No clear answer.

- Better question: What is the method that fits my needs?
  - Network features: Size? Directed? Weighted? Bipartite?
  - What do we expect to find? Overlapping communities? Size of the groups?

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  → No clear answer.

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  - What do we expect to find? Overlapping communities? Size of the groups?

How to choose method?

	Name	Overlap	Dir	Weight	Dyn	NoPar	MDim	Incr	Multip	Complexity	BESn	BESm	Year	Ref
Feature Distance	Evolutionary*				✓			✓		$\mathcal{O}(n^2)$	5k	?	2006	[21]
	MSN-BD			✓					✓	$\mathcal{O}(n^2ck)$	6k	3M	2006	[22]
	SocDim	✓		✓			✓			$\mathcal{O}(n^2 \log n)*$	80k	6M	2009	[23]
	PMM			✓			✓			$\mathcal{O}(mn^2)$	15k	27M	2009	[24]
	MRGC		✓		✓		✓		✓	$\mathcal{O}(mD)$	40k	?	2007	[25]
	Infinite Relational					✓	✓			$\mathcal{O}(n^{2c}D)$	160	?	2006	[26]
	Find-Tribes				✓				✓	$\mathcal{O}(mnK^2)$	26k	100k	2007	[27]
	AutoPart		✓			✓		✓		$\mathcal{O}(mk^2)$	75k	500k	2004	[28]
	Timefall				✓	✓			✓	$\mathcal{O}(mk)$	7.5M	53M	2008	[29]
	Context-specific Cluster Tree					✓			✓	$\mathcal{O}(mk)$	37k	367k	2008	[30]
IntDensity	Modularity	✓	✓	✓			✓	✓	✓	$\mathcal{O}(mk \log n)$	118M	1B	2004	[18]
	MetaFac				✓		✓			$\mathcal{O}(mnD)$	?	2M	2009	[31]
	Variational Bayes		✓			✓				$\mathcal{O}(mk)$	115	613	2008	[32]
	$LA \rightarrow IS^2*$	✓	✓							$\mathcal{O}(mk+n)$	16k	?	2005	[33]
	Local Density		✓			✓		✓		$\mathcal{O}(nK \log n)$	108k	330k	2005	[34]
Bridge	Edge Betweenness		✓	✓						$\mathcal{O}(m^2n)$	271	1k	2002	[4]
	CONGO*	✓		✓						$\mathcal{O}(n \log n)$	30k	116k	2008	[35]
	L-Shell	✓						✓		$\mathcal{O}(n^3)$	77	254	2005	[36]
	Internal-External Degree	✓								$\mathcal{O}(n^2 \log n)$	775k	4.7M	2009	[37]
Diffusion	Label Propagation			✓		✓		✓		$\mathcal{O}(m+n)$	374k	30M	2007	[38]
	Node Colouring				✓				✓	$\mathcal{O}(ntk^2)$	2k	?	2007	[39]
	Kirchhoff	✓		✓						$\mathcal{O}(m+n)$	115	613	2004	[40]
	Communication Dynamic	✓	✓		✓			✓		$\mathcal{O}(mnt)$	160k	530k	2008	[41]
	GuruMine		✓		✓					$\mathcal{O}(TAn^2)$	217k	212k	2008	[9]
	DegreeDiscountIC		✓							$\mathcal{O}(k \log n + m)$	37k	230k	2009	[42]
	MMSB	✓	✓							$\mathcal{O}(nk)$	871	2k	2007	[43]
Close	Walktrap			✓						$\mathcal{O}(mn^2)$	160k	1.8M	2006	[44]
	DOCS	✓								?	325k	1M	2009	[45]
	Infomap		✓	✓		✓				$\mathcal{O}(m \log^2 n)$	6k	6M	2008	[46]
Structure	K-Clique	✓								$\mathcal{O}(m^{\frac{\ln m}{10}})$	20k	127k	2005	[3]
	S-Plexes Enumeration	✓								$\mathcal{O}(kmn)$	?	?	2009	[47]
	Bi-Clique	✓							✓	$\mathcal{O}(m^2)$	200k	500k	2008	[48]
	EAGLE	✓	✓	✓						$\mathcal{O}(3^{\frac{n}{3}})$	16k	31k	2009	[49]
Link	Link modularity	✓		✓					✓	$\mathcal{O}(2mk \log n)$	20k	127k	2009	[50]
	HLC*	✓		✓					✓	$\mathcal{O}(n\bar{K}^2)$	885k	5.5M	2010	[51]
	Link Maximum Likelihood	✓								$\mathcal{O}(mk)$	4.8M	42M	2011	[52]
NoD	Hybrid*	✓	✓	✓		✓				$\mathcal{O}(nkK)$	325k	1.5M	2010	[53]
	Multi-relational Regression			✓			✓			?	?	?	2005	[54]
	Hierarchical Bayes									$\mathcal{O}(n^2)$	1k	4k	2008	[55]
	Expectation Maximization		✓							?	112	?	2007	[7]

M. Coscia et al, *Statistical Analysis and Data Mining,* 4(5):512546 (2011)

Literature

- Where to start reading?

1) Newman, M. E., Communities, modules and large-scale structure in networks. Nature Physics, 8(1):25-31 (2012).
   → Short, big picture

2) Fortunato, S., Community detection in graphs. Physics reports, 486(3):75-174 (2010).
   → >100 pages, complete at the time, good for looking up methods

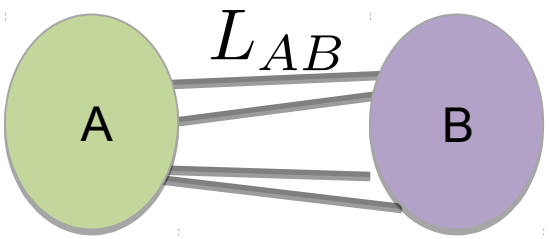
3) Coscia, M., Giannotti, F., & Pedreschi, D., A classification for community discovery methods in complex networks. Statistical Analysis and Data Mining, 4(5):512-546 (2011).
   → shorter, compares a lot of methods

4) A.-L. Barabási, Network Science, Chapter 9
   http://barabasi.com/networksciencebook/
   Appears in print in May.
   → Lot of figures of the lecture are from here. Easy to read, tells a detailed story, but does not cover everything.

Extra time:
Problems with modularity

Resolution limit

- Should we merge two communities?



$$\Delta M_A = \frac{1}{2L}\sum_{i,j\in A} A_{ij} - \frac{k_i k_j}{2L} = \frac{2k_A^{\text{int}}}{2L} - \frac{1}{2L}\sum_{i,j\in A}\frac{k_i k_j}{2L}$$

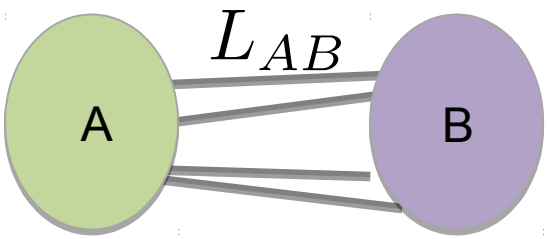
$$\begin{aligned}
\Delta M_{AB} = &- M_A - M_B + M_{AB} = \\
&= -\left(\frac{k_A^{\text{int}}}{L} - \sum_{i,j\in A}\frac{k_i k_j}{(2L)^2})\right) - \left(\frac{k_B^{\text{int}}}{L} - \sum_{i,j\in B}\frac{k_i k_j}{(2L)^2})\right) + + \\
&+ \left(\frac{k_A^{\text{int}} + k_A^{\text{int}} + L_{AB}}{L} - \sum_{i,j\in A}\frac{k_i k_j}{(2L)^2} - \sum_{i,j\in B}\frac{k_i k_j}{(2L)^2} - 2\sum_{i\in A, j\in B}\frac{k_i k_j}{(2L)^2}\right) = \\
&= \frac{L_{AB}}{L} - \frac{k_A k_B}{2L^2}
\end{aligned}$$

S. Fortunato. *PNAS*, 104(1):36-41 (2006)

A.-L. Barabási, *Network Science: Communities*.

Resolution limit

- Should we merge two communities?



$$\Delta M_{AB} = \frac{L_{AB}}{L} - \frac{k_A k_B}{2L^2},$$

k_A and k_B total degree in A and B

If $\frac{k_A k_B}{2L} < 1$ and $L_{AB} \geq 1$ ➡ $\Delta M_{AB} > 0$ We merge A and B to maximize modularity.

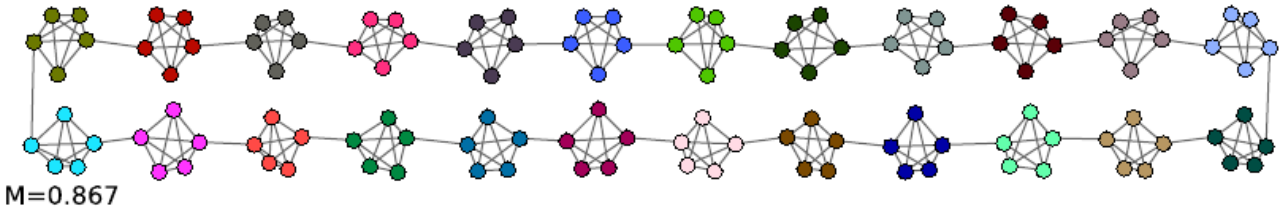
Assuming $k_A \sim k_B = k$ ➡ $k \leq \sqrt{2L}$

Modularity has a resolution limit, as it cannot detect communities smaller than this size.

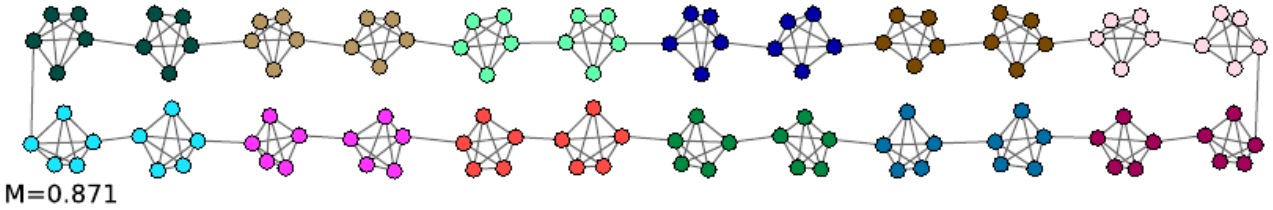
S. Fortunato. *PNAS*, 104(1):36-41 (2006)
A.-L. Barabási, *Network Science: Communities*.

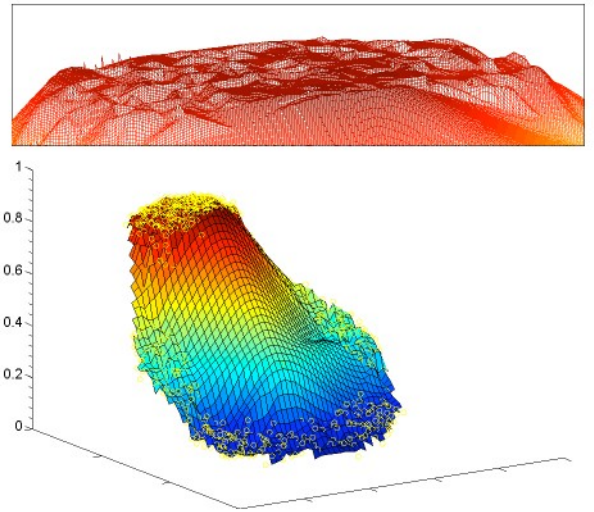
Is the maximum unique?

- Should we merge two communities?
- Intuitive:

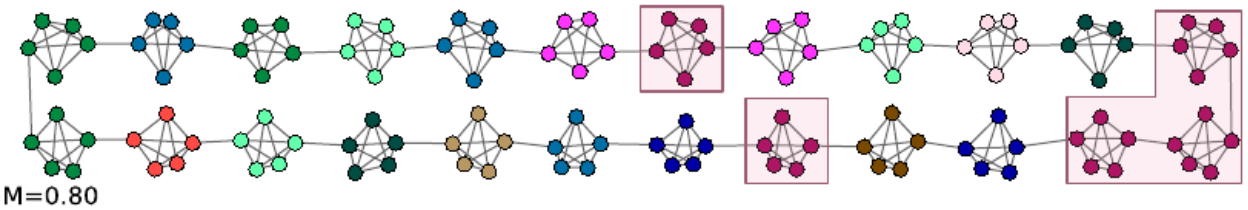


- Global maximum:





- Random:



Good et al., *PRE*, 81:046106 (2010)

A.-L. Barabási, *Network Science: Communities*.

Even more time:
Link communities

Link communities

→ Nodes tend to belong to multiple communities

→ Links tend to be specific, capturing the nature of the relationship between two nodes.

Social networks, a link may indicate:

- they are in the same family
- they work together
- they share a hobby.

Biological networks:
each interaction of a protein is responsible for a different function, uniquely defining the protein's role in the cell

⇒ Define a hierarchical algorithm based on <u>similarity of links</u>

Ahn, Bragow and Lehmann, *Nature* 466 (2010).
A.-L. Barabási, *Network Science: Communities*.

Link communities

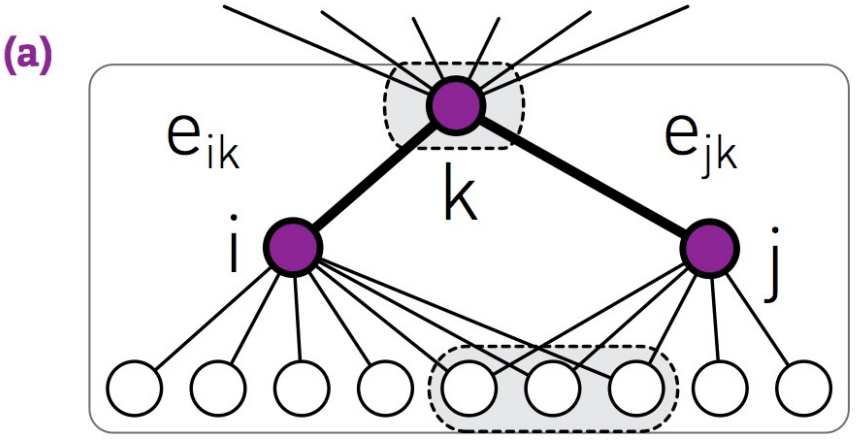
1. **Define link similarity**

$$S(e_{ik}, e_{jk}) = \frac{|n_+(i) \cap n_+(j)|}{|n_+(i) \cup n_+(j)|}$$

$n_+(i)$: the list of the neighbors of node i, including itself.
S measures the relative number of common neighbors i and j have.

$$\left|n_+(i) \cap n_+(j)\right| = 4$$
$$\left|n_+(i) \cup n_+(j)\right| = 12$$

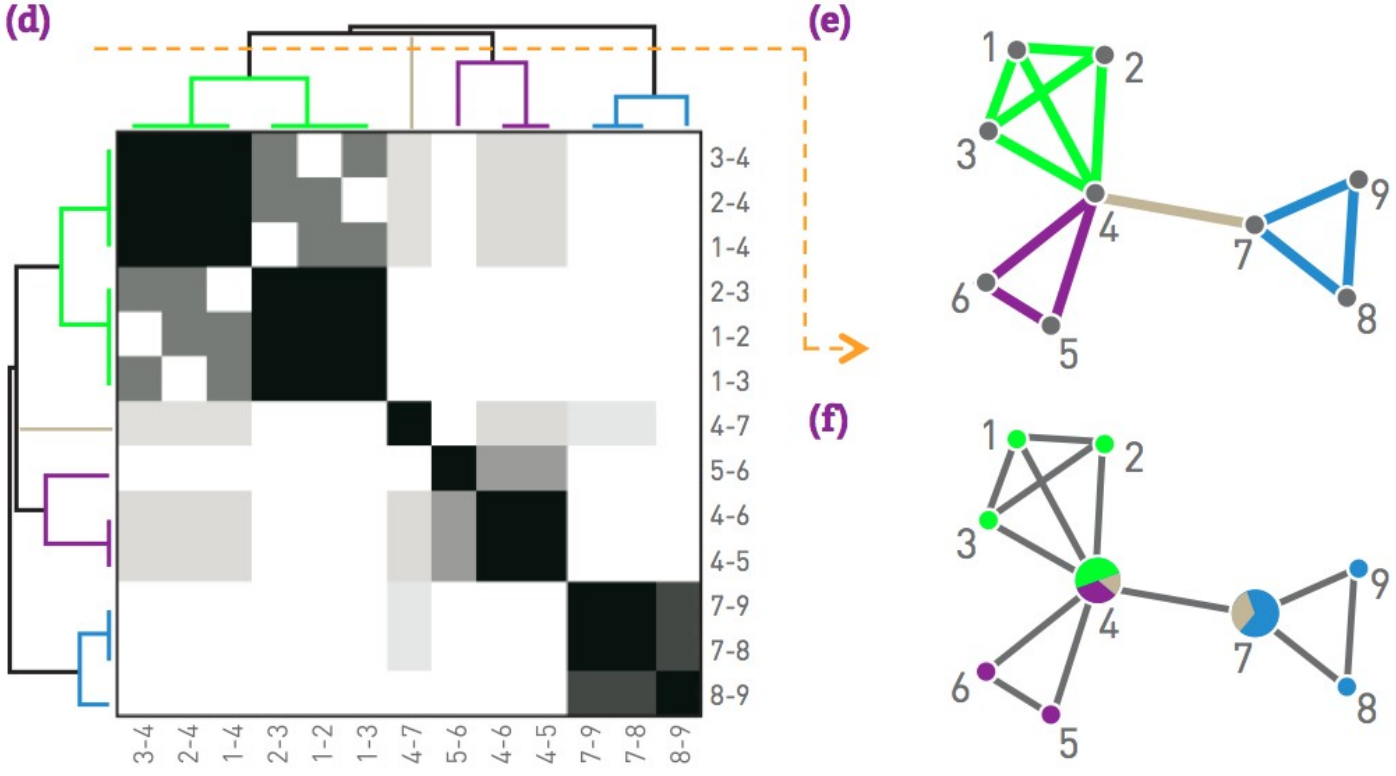
$$S\left(e_{ik}, e_{jk}\right) = \frac{1}{3}$$



Ahn, Bragow and Lehmann, *Nature* 466 (2010).
A.-L. Barabási, *Network Science: Communities*.

Link communities

2. **Apply hierarchical clustering (agglomerative, single linkage)**



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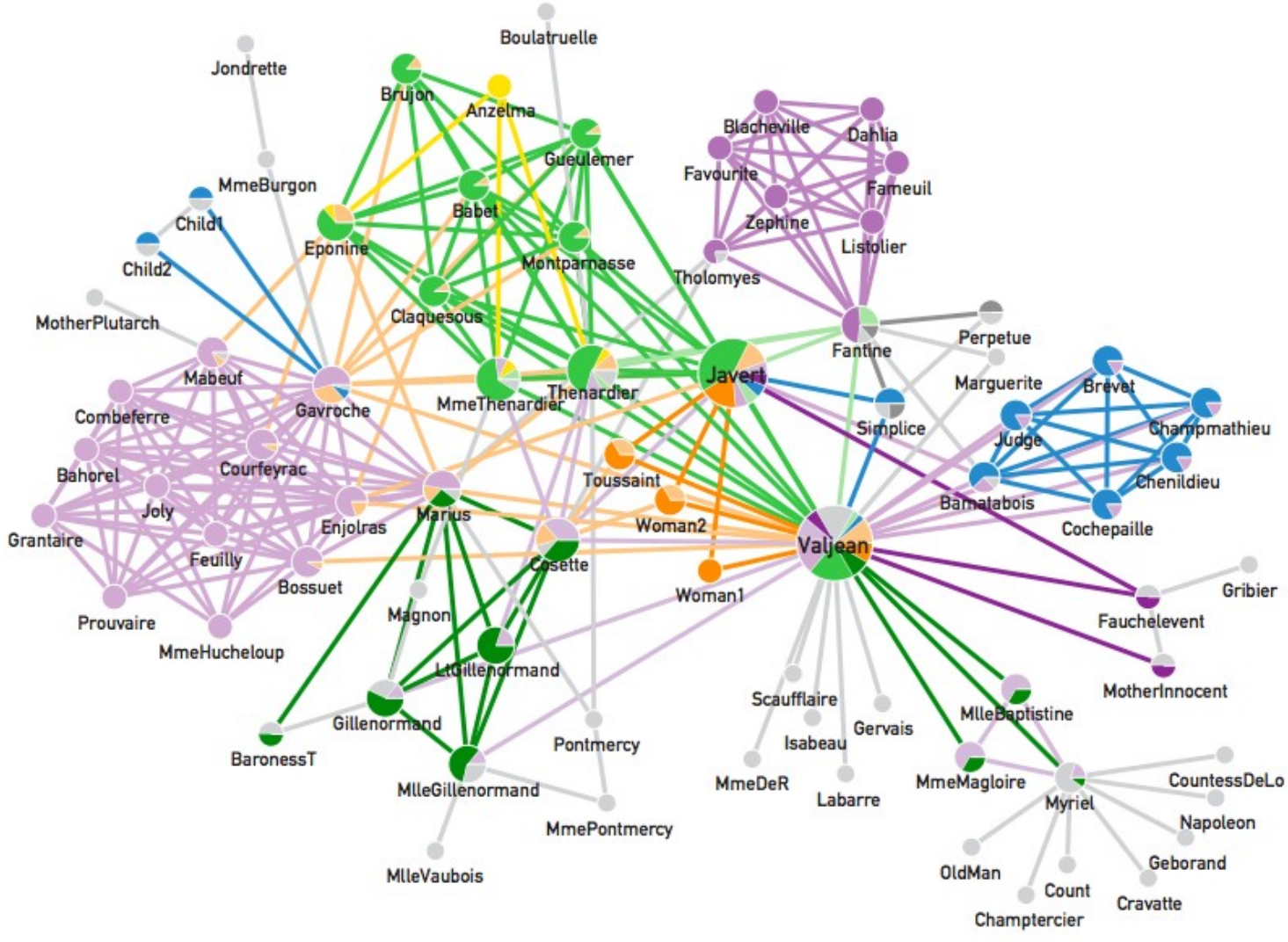
A.-L. Barabási, *Network Science: Communities*.

Link communities



The network of characters in Victor Hugo's 1862 novel *Les Miserables*. Two characters are connected if they interact directly with each other in the story. The link colors indicate the clusters, grey nodes corresponding to single-link clusters. Each node is depicted as a pie-chart, illustrating its membership in multiple communities. Not surprisingly, the main character, Jean Valjean, has the most diverse community membership

Ahn, Bragow and Lehmann, *Nature* 466 (2010).
A.-L. Barabási, *Network Science: Communities*.