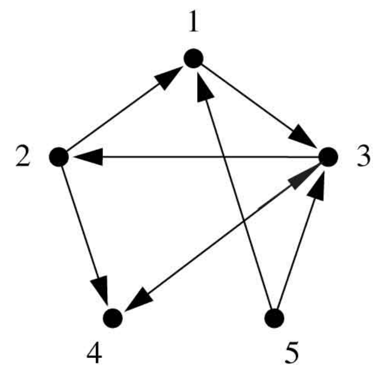
ECS 253 / MAE 253, Network Theory and Applications
Spring 2016
Common Problem Set # 1, Due April 21

Problem 1: Adjacency matrix



a) Consider the simple network shown above and write down its the adjacency matrix.

b) Consider a random walk on this network. What is the steady-state probability of finding the walker on each node?

c) What would be the steady-state probability of finding the walker on each node if the edges were instead *undirected*?

Problem 2: Rate equations: Network growth with uniform attachment

Consider a variant of the BA model that does not feature preferential attachment. We start with a single node at time $t = 1$. In each subsequent discrete time step, a new node is added with $m = 1$ links to existing nodes. The probability that a link arriving at time step $t + 1$ connects to any existing node i is uniformly distributions and independent of i:

$$\pi_i = \frac{1}{t}. \tag{1}$$

Let $n_{k,t}$ denote the expected number of nodes of degree k at time t. For the steps below, proceed as in lecture.

a) Write the rate equation for $n_{k,t+1}$ in terms of the $n_{j,t}$'s. (Note you will need to equations, one for $k = 1$ and one for $k > 1$.)

b) Converting from expected number of nodes to probabilities, $p_{k,t} = n_{k,t}/n_t$, rewrite the equations in part (a) in terms of the probabilities.

c) Assume steady-state, that $p_{k,t} = p_k$, and solve the recurrence relation to obtain p_k in terms of p_{k-1}.

d) Starting by solving for p_1 and recursing, derive the expression for the stationary degree distribution p_k.