Design of a Satellite Attitude Control System

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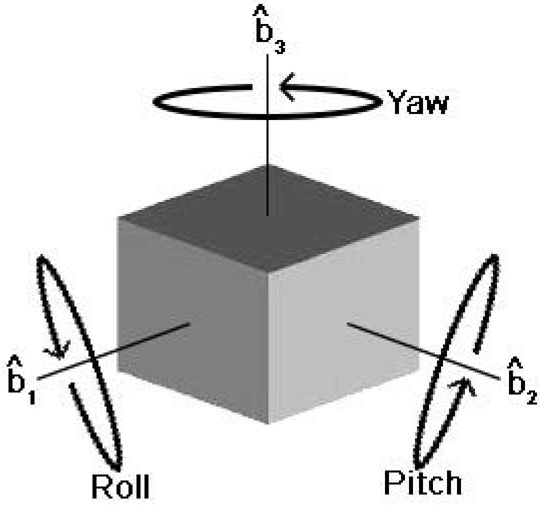
Problem

Maintain orientation and orbit of a satellite on mission in LEO

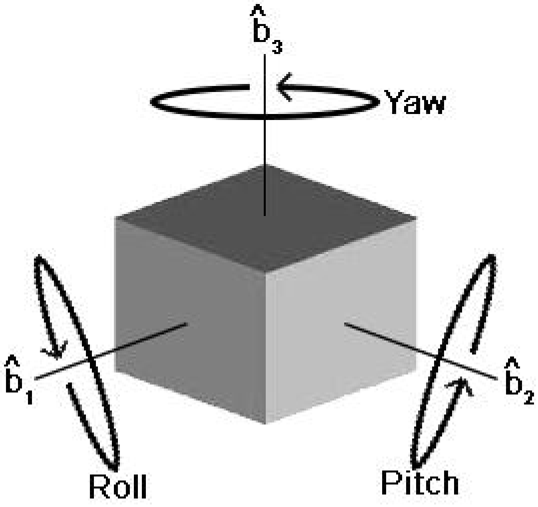
- rotational control *around* 3 body-centered axes
- translational control *along* 3 body-centered axes

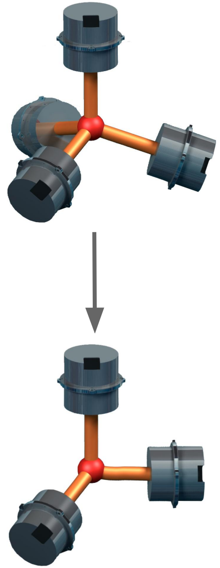


Simplifications

Maintain orientation and orbit of a satellite on mission in LEO

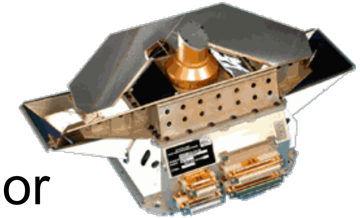
- rotational control *around* 3 body-centered axes
- ~~translational control *along* 3 body-centered axes~~

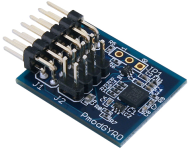




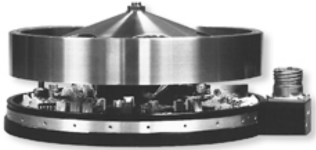
Sensors and Actuators

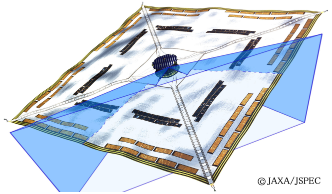
- External
  - Horizon sensor
  - Sun sensor
  - Magnetometer
  - Star tracker
- Internal
  - Gyroscope
  - Accelerometer





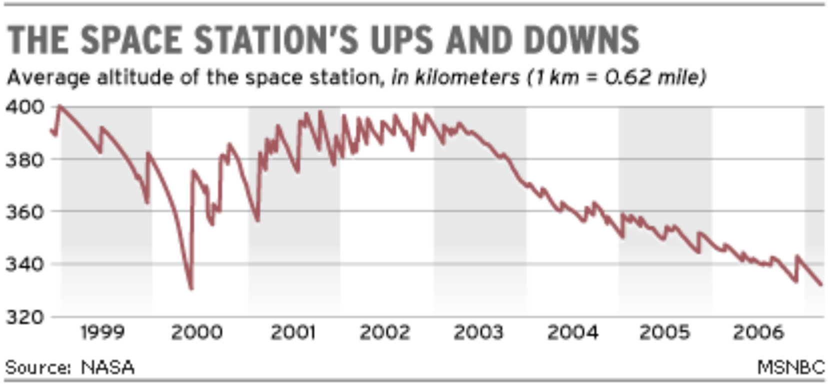
- Magnetic
- Mass-expulsion
- Momentum
- Pressure





Disturbances

- Aerodynamic drag
- Gravity gradient
- ~~Solar pressure~~
- ~~Magnetic fields~~
- ~~Micrometeorites~~
- ~~Space junk (debris)~~
- ~~Fluid sloshing~~
- ~~Vibration~~
- ~~Noise~~



Assumptions

General

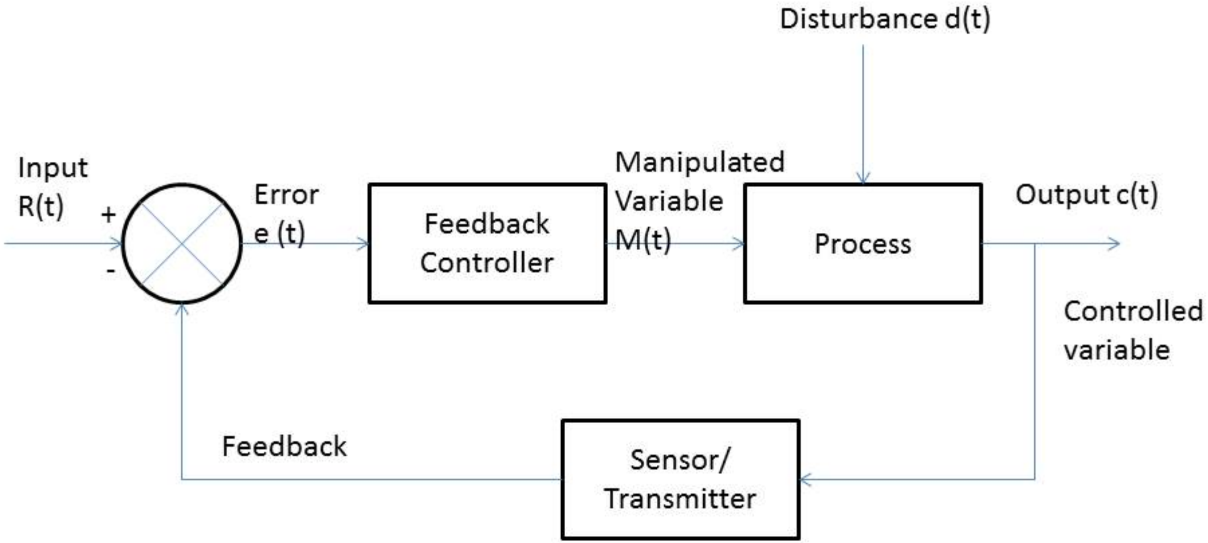
- Earth/environment
    - no gravitational or magnetic variations
    - uniform, sparse (<u>but present</u>) atmosphere at orbit altitude
    - insignificant solar pressure
- Satellite
    - constant, uniformly distributed mass (i.e. trivial gravity gradient)
    - no dipole moment
    - no internal noise
    - constant altitude orbit with zero eccentricity

Assumptions

Linear model

- small perturbations (e.g. angular)
- constant & trivial (i.e. diagonal) inertia matrix
- no saturation
- no singularities

General Model



Equations of Motion

$$M = \frac{{}^{N}d^{N}\vec{H}^{S}}{dt} = \frac{{}^{N}d^{N}\vec{H}^{B}}{dt} + \frac{{}^{N}d^{N}\vec{H}^{M}}{dt}$$

Equations of Motion

$$M = \frac{{}^{N}d^{N}\vec{H}^{S}}{dt} = \frac{{}^{N}d^{N}\vec{H}^{B}}{dt} + \frac{{}^{N}d^{N}\vec{H}^{M}}{dt}$$

By the derivative theorem

$$\dot{H}_B = \frac{{}^{B}dH^{B}}{dt} + {}^{N}\vec{\omega}^{B} \times {}^{N}\vec{H}^{B}$$

$$= [I_B]^{N}\dot{\omega}^{B} + S({}^{N}\vec{\omega}^{B})([I_B] \cdot {}^{N}\vec{\omega}^{B})$$

$${}^{N}\dot{H}^{M} = S({}^{N}\vec{\omega}^{B})(H_{ry})$$

Equations of Motion

$$M = \tau_D + \tau_C$$

1. Disturbance

    torque

$$\tau_D = \tau_{Da} + \tau_{Dg}$$

$$\tau_{Da} = \begin{bmatrix} L_\theta & L_\psi \\ M_\theta & M_\psi \\ N_\theta & N_\psi \end{bmatrix} \begin{bmatrix} \theta \\ \psi \end{bmatrix}$$

Equations of Motion

$$M = \tau_D + \tau_C$$

1. Disturbance torque

$$\tau_{Dg} = \tau_{gx} + \tau_{gy} + \tau_{gz}$$

$$\tau_{gx} = \frac{3}{2} n^2 (I_{zz} - I_{yy}) \cos^2(\theta) \sin(2\phi)$$

$$\tau_{gy} = \frac{3}{2} n^2 (I_{zz} - I_{xx}) \cos(\phi) \sin(2\theta)$$

$$\tau_{gz} = \frac{3}{2} n^2 (I_{xx} - I_{yy}) \sin(\phi) \sin^2(\theta)$$

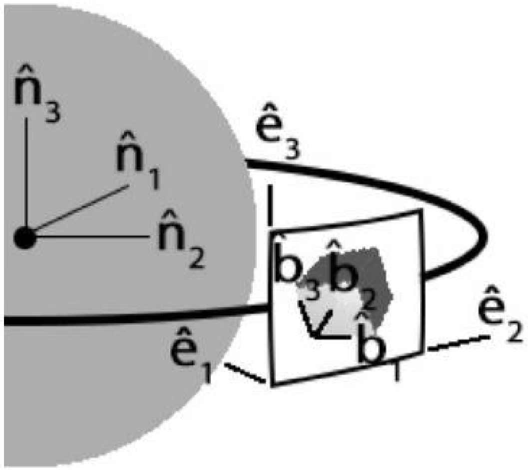
Equations of Motion

$$M = \tau_D + \tau_C$$

2. Control torque

$$^N\Omega^R = \Omega\hat{b}_1 + \Omega\hat{b}_2 + \Omega\hat{b}_3$$

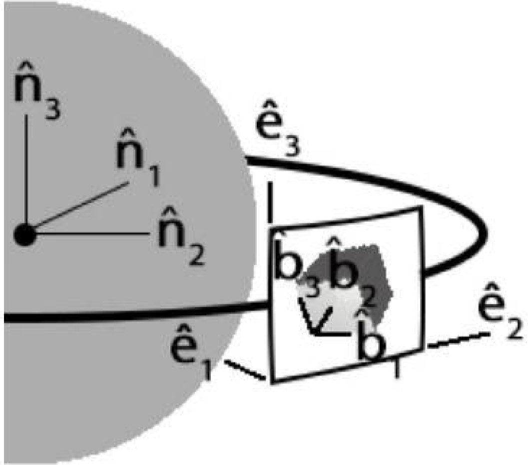
$$\tau_C = I_W\dot{\Omega}_1\hat{b}_1 + I_W\dot{\Omega}_2\hat{b}_2 + I_W\dot{\Omega}_3\hat{b}_3$$



$$ {}^{N}\vec{\omega}^{B} = p\hat{b}_1 + (-n+q)\hat{b}_2 + r\hat{b}_3 $$

$$ {}^{N}\vec{\omega}^{B} = -n\hat{e}_2 + \dot{\phi}\hat{b}_1 + \dot{\theta}\hat{b}_2 + \dot{\psi}\hat{b}_3 $$

$$ \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} roll \\ pitch \\ yaw \end{bmatrix} $$



$$^{N}\vec{\omega}^{B} = p\hat{b}_1 + (-n+q)\hat{b}_2 + r\hat{b}_3$$

$$^{N}\vec{\omega}^{B} = -n\hat{e}_2 + \dot{\phi}\hat{b}_1 + \dot{\theta}\hat{b}_2 + \dot{\psi}\hat{b}_3$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} roll \\ pitch \\ yaw \end{bmatrix}$$

$$I_B = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix}$$

$$n = \sqrt{\frac{\mu}{R^3}}$$

Nonlinear Model

$$\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\end{bmatrix} = \begin{bmatrix} c_\theta c_\psi & -c_\theta s_\psi & s_\theta \\ c_\phi s_\psi + c_\phi s_\psi s_\theta & c_\phi c_\psi - s_\psi s_\phi s_\theta & -c_\theta s_\psi \\ s_\phi s_\psi - c_\phi c_\psi s_\theta & s_\phi c_\psi + c_\phi s_\psi s_\theta & c_\theta c_\phi \end{bmatrix} \begin{bmatrix}\hat{b}_1\\ \hat{b}_2\\ \hat{b}_3\end{bmatrix}$$

$$\hat{e}_2 = (c_\phi s_\psi + c_\phi s_\psi s_\theta)\hat{b}_1 + (c_\phi c_\psi - s_\psi s_\phi s_\theta)\hat{b}_2 - c_\theta s_\psi \hat{b}_3$$

Nonlinear Model

$$\dot{\phi} = p + n((\cos(\phi)\sin(\psi) + \cos(\phi)\sin(\psi)\sin(\theta))$$

$$\dot{\theta} = -n + q + n(\cos(\phi)\cos(\psi) - \sin(\psi)\sin(\phi)\sin(\theta))$$

$$\dot{\psi} = r - n(\cos(\theta)\sin(\psi))$$

Nonlinear Model

$$[I_B]^N\dot{\omega}^B = \tau_D + \tau_C - S(^N\vec{\omega}^B)([I_B] \cdot^N \vec{\omega}^B + H_{ry}\hat{b}_2)$$

But if we expand this form, this equation becomes….

Nonlinear Model

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} =$$

$$\begin{bmatrix} r(I_{yx}p - I_{yy}(n-q) + I_{yz}r + H_{ry}) + (n-q)(I_{zx}p - I_{zy}(n-q) + I_{zz}r) \\ p(I_{zx}p - I_{zy}(n-q) + I_{zz}r) - r(I_{xx}p - I_{xy}(n-q) + I_{xz}r) \\ (-n+q)(I_{xx}p - I_{xy}(n-q) + I_{xz}r) - p(I_{yx}p - I_{yy}(n-q) + I_{yz}r + H_{ry}) \end{bmatrix}$$

Nonlinear Model

$$\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} =$$

$$\begin{bmatrix} r(I_{yx}p - I_{yy}(n-q) + I_{yz}r + H_{ry}) + (n-q)(I_{zx}p - I_{zy}(n-q) + I_{zz}r) \\ p(I_{zx}p - I_{zy}(n-q) + I_{zz}r) - r(I_{xx}p - I_{xy}(n-q) + I_{xz}r) \\ (-n+q)(I_{xx}p - I_{xy}(n-q) + I_{xz}r) - p(I_{yx}p - I_{yy}(n-q) + I_{yz}r + H_{ry}) \end{bmatrix}$$

$$+ \begin{bmatrix} L_\theta \theta + L_\psi \psi + \frac{3}{2} n^2 (I_{zz} - I_{yy}) \cos^2(\theta) \sin(2\phi) \\ M_\theta \theta + M_\psi \psi + \frac{3}{2} n^2 (I_{zz} - I_{yy}) \cos(\phi) \sin(2\theta) \\ N_\theta \theta + N_\psi \psi + \frac{3}{2} n^2 (I_{xx} - I_{yy}) \sin(\phi) \sin^2(\theta) \end{bmatrix} + \tau_C$$

Linearization

$$\dot{x} = A\vec{x} + B\vec{u}$$

$$\hat{e}_2 \approx \psi\hat{b}_1 + \hat{b}_2 - \phi\hat{b}_3$$

$$\vec{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix}$$

$$\vec{u} = \begin{bmatrix} I_W\dot{\Omega}_1 \\ I_W\dot{\Omega}_2 \\ I_W\dot{\Omega}_3 \end{bmatrix} \qquad I_B = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Linearization

$$\begin{bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\\ \dot{p}\\ \dot{q}\\ \dot{r}\end{bmatrix} = \begin{bmatrix} 0 & 0 & n & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -n & 0 & 0 & 0 & 0 & 1 \\ \frac{3n^2(I_{zz}-I_{yy})}{I_{xx}} & \frac{L_\theta}{I_{xx}} & \frac{L_\psi}{I_{xx}} & 0 & 0 & \frac{n(I_{zz}-I_{yy}+H_{ry})}{I_{xx}} \\ 0 & \frac{M_\theta+3n^2(I_{zz}-I_{xx})}{I_{yy}} & \frac{M_\psi}{I_{yy}} & 0 & 0 & 0 \\ 0 & \frac{N_\theta}{I_{zz}} & \frac{N_\psi}{I_{zz}} & \frac{n(I_{yy}-I_{xx})-H_{ry}}{I_{zz}} & & \end{bmatrix} \begin{bmatrix}\phi\\ \theta\\ \psi\\ p\\ q\\ r\end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{-1}{I_{xx}} & 0 & 0 \\ 0 & \frac{-1}{I_{yy}} & 0 \\ 0 & 0 & \frac{-1}{I_{zz}} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Linearization

$$
\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & n & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
-n & 0 & 0 & 0 & 0 & 1 \\
\frac{3n^2(I_{zz}-I_{yy})}{I_{xx}} & \frac{L_\theta}{I_{xx}} & \frac{L_\psi}{I_{xx}} & 0 & 0 & \frac{n(I_{zz}-I_{yy}+H_{ry})}{I_{xx}} \\
0 & \frac{M_\theta+3n^2(I_{zz}-I_{xx})}{I_{yy}} & \frac{M_\psi}{I_{yy}} & 0 & 0 & 0 \\
0 & \frac{N_\theta}{I_{zz}} & \frac{N_\psi}{I_{zz}} & \frac{n(I_{yy}-I_{xx})-H_{ry}}{I_{zz}} & &
\end{bmatrix}
\begin{bmatrix} \phi \\ \theta \\ \psi \\ p \\ q \\ r \end{bmatrix}
$$

$$
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\frac{-1}{I_{xx}} & 0 & 0 \\
0 & \frac{-1}{I_{yy}} & 0 \\
0 & 0 & \frac{-1}{I_{zz}}
\end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}
$$

$$
C = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

D matrix is a 3x3 zero matrix

Stability, Controllability, Observability

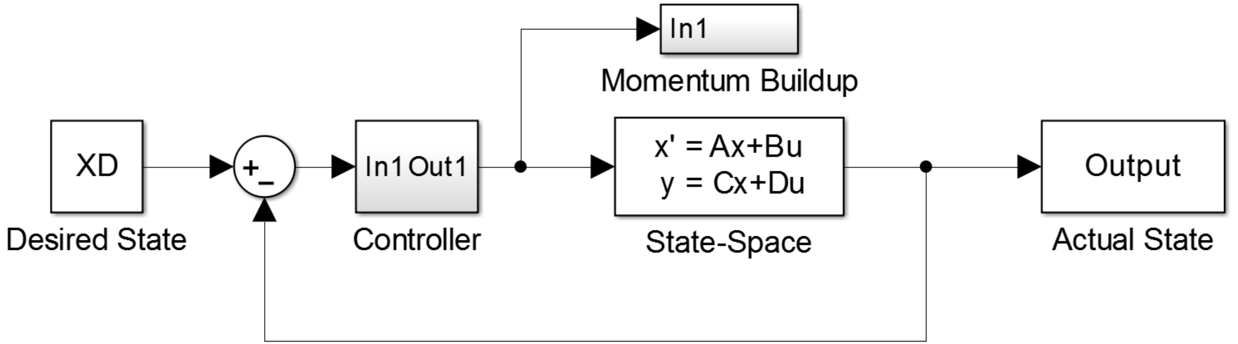
- Observable: yes
- Controllable: yes
- Stable: NO
    - Eigenvalues of A:

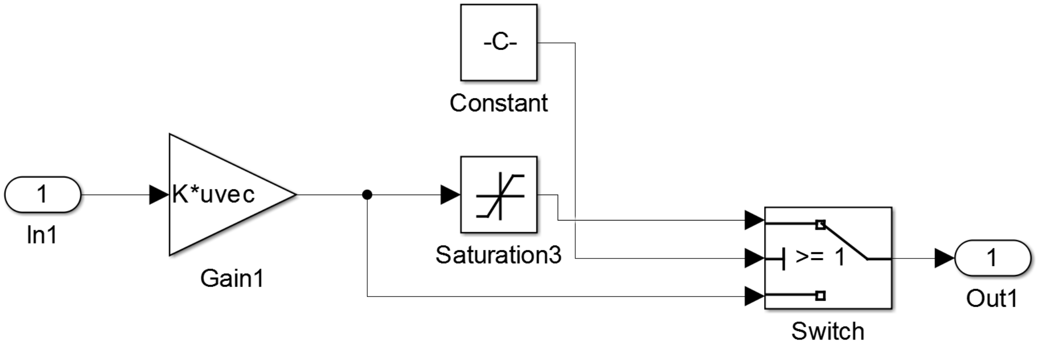
$$\lambda(A) = \begin{bmatrix} -4.07e-15 \pm 3.39e-1i \\ 2.97e-10 \pm 1.16e-3i \\ \pm 4.86e-4 \end{bmatrix}$$

LQR Controller

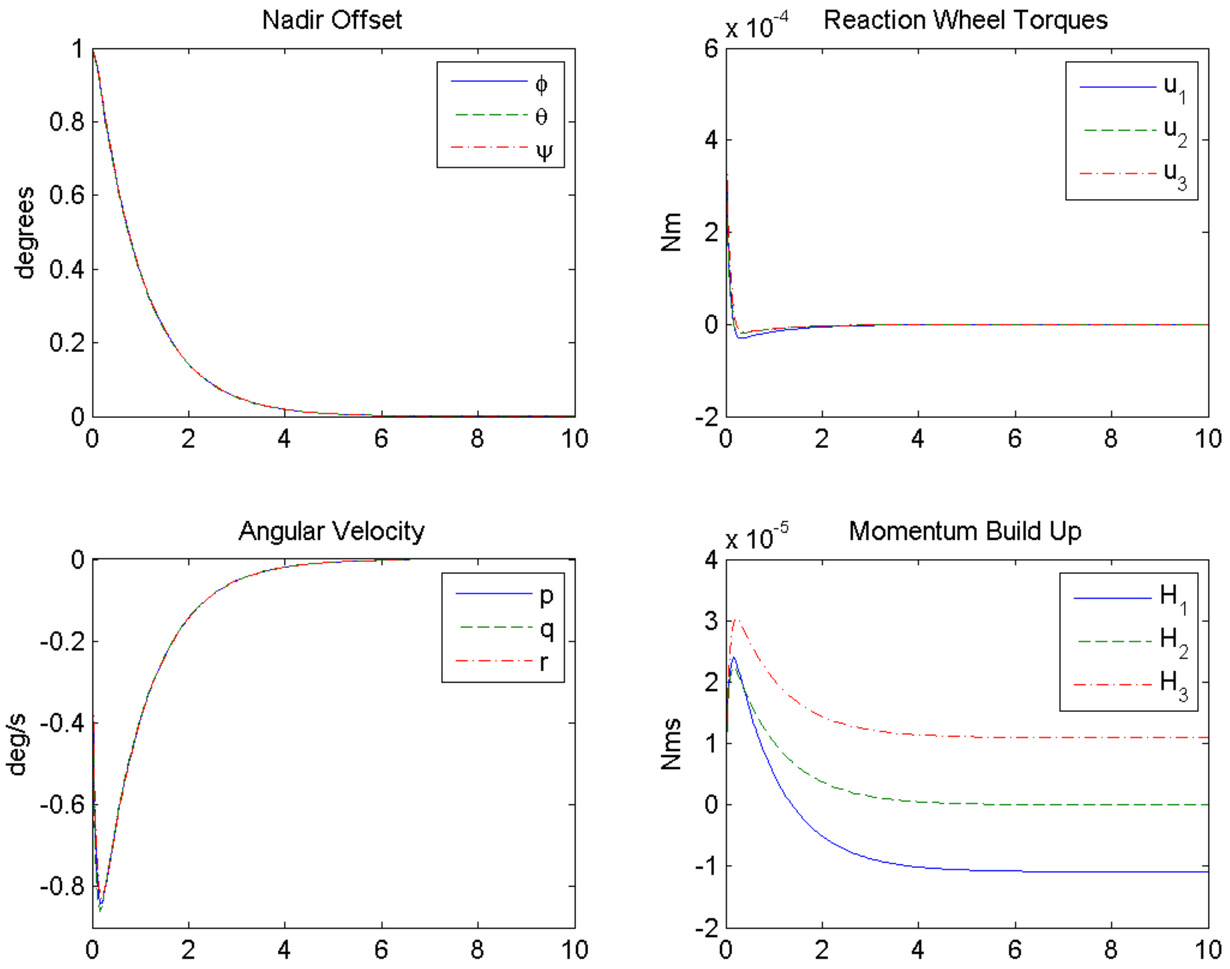
```
%This penalizes all states equally.
Q = 1E-3 * eye(6);

%This choice of R does not penalize any inputs.
R = eye(3);

% Controller
[K,S,e] = lqr(SYS,Q,R)
% Magic
```

Linear Model



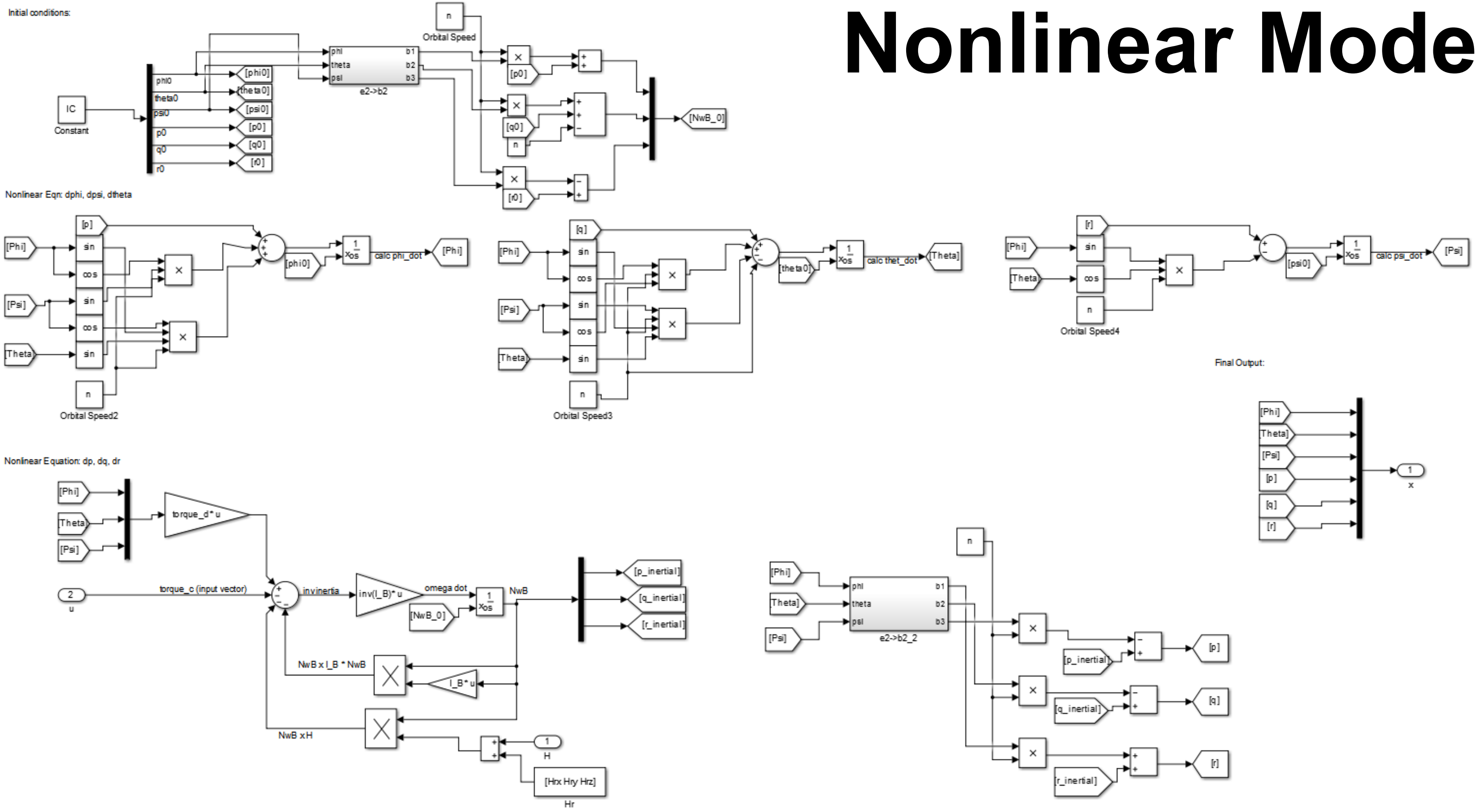


Linear

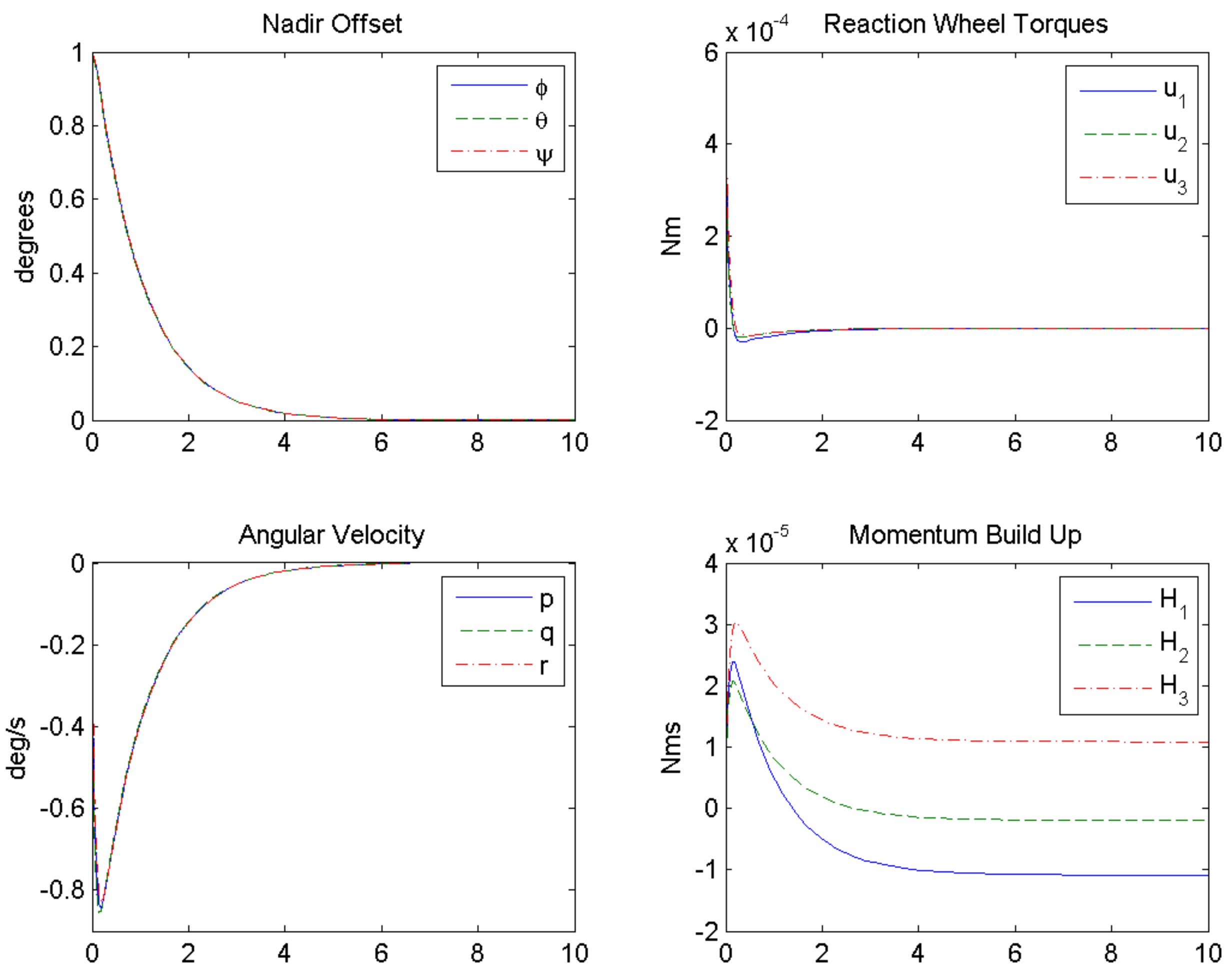


Nadir Offset
degrees
ϕ
θ
ψ
Reaction Wheel Torques
x 10^-4
Nm
u1
u2
u3
Angular Velocity
deg/s
p
q
r
Momentum Build Up
x 10^-5
Nms
H1
H2
H3

Nonlinear Model



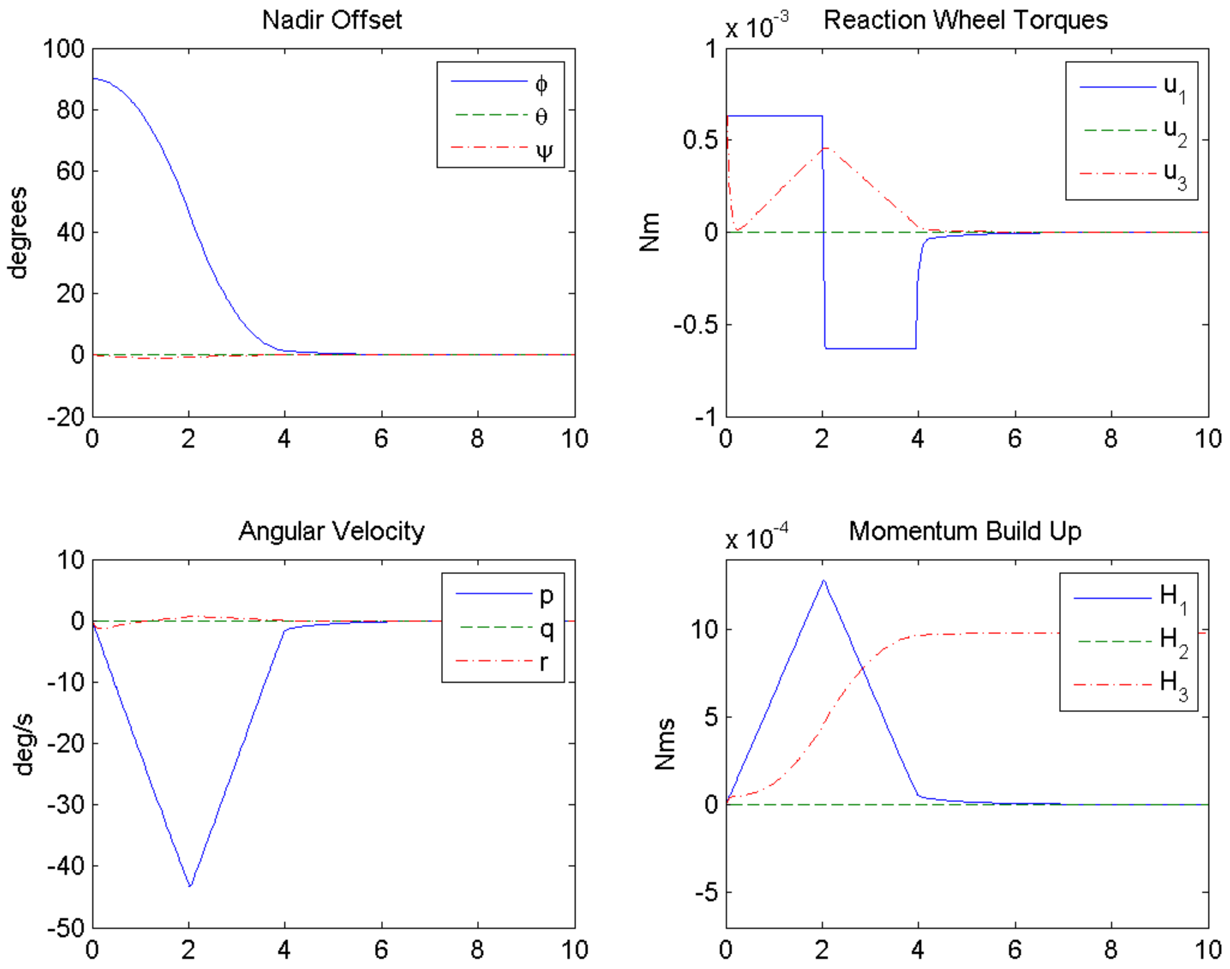
Non-linear



Nadir Offset
degrees
ϕ
θ
ψ
Reaction Wheel Torques
x 10^-4
Nm
u1
u2
u3
Angular Velocity
deg/s
p
q
r
Momentum Build Up
x 10^-5
Nms
H1
H2
H3

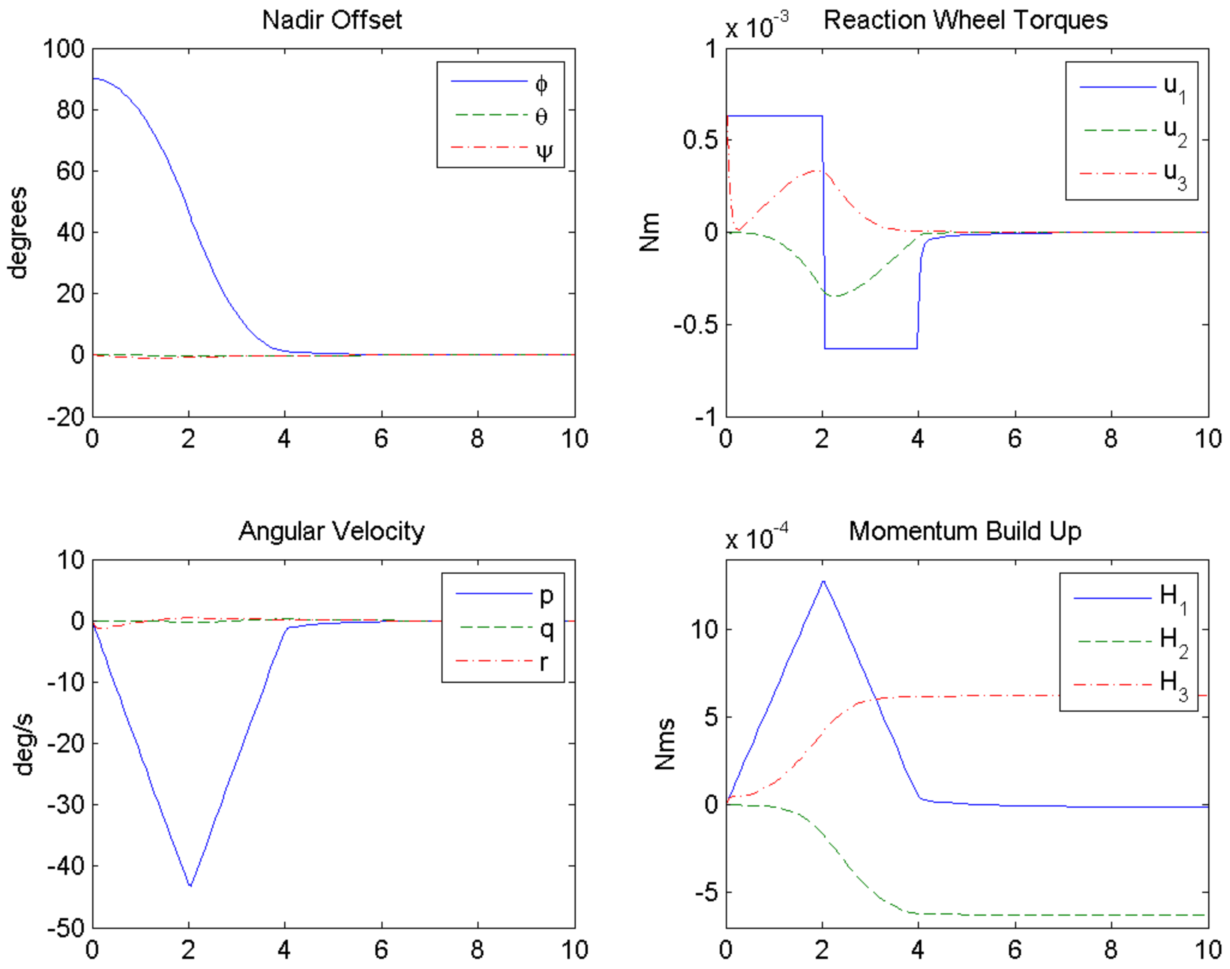
90 degree phi offset

Linear

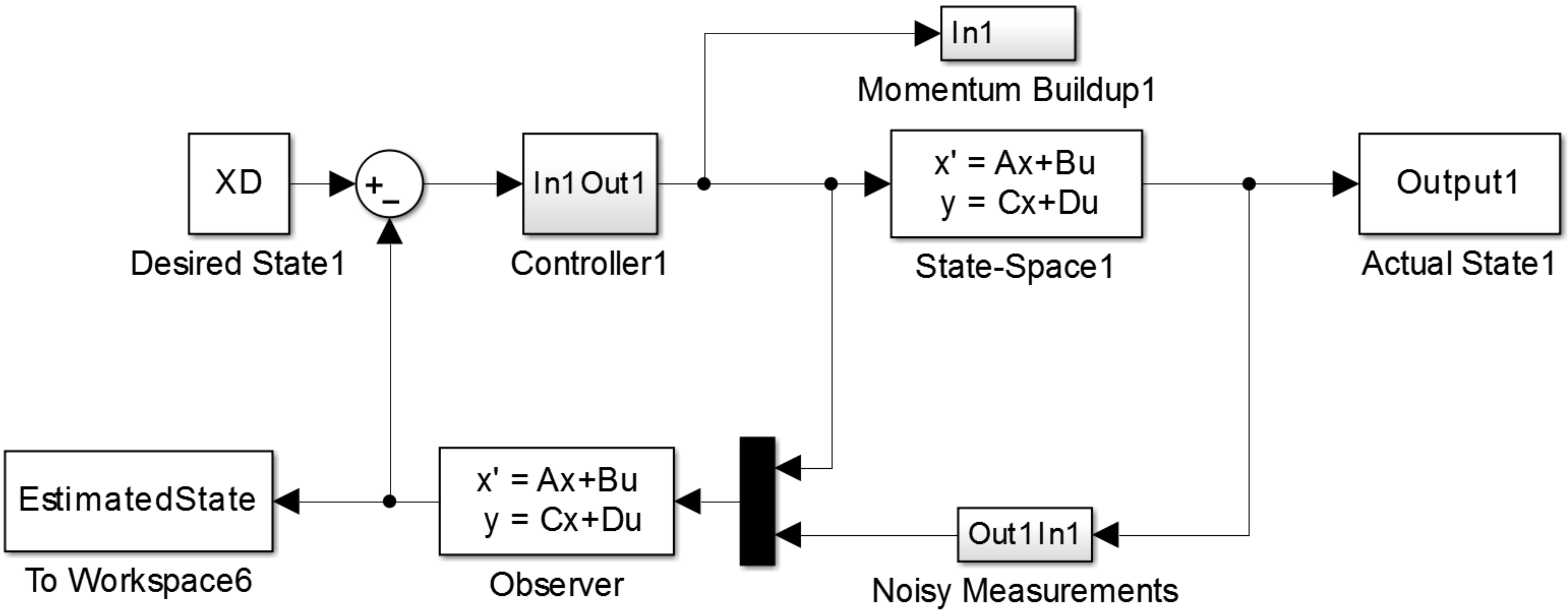


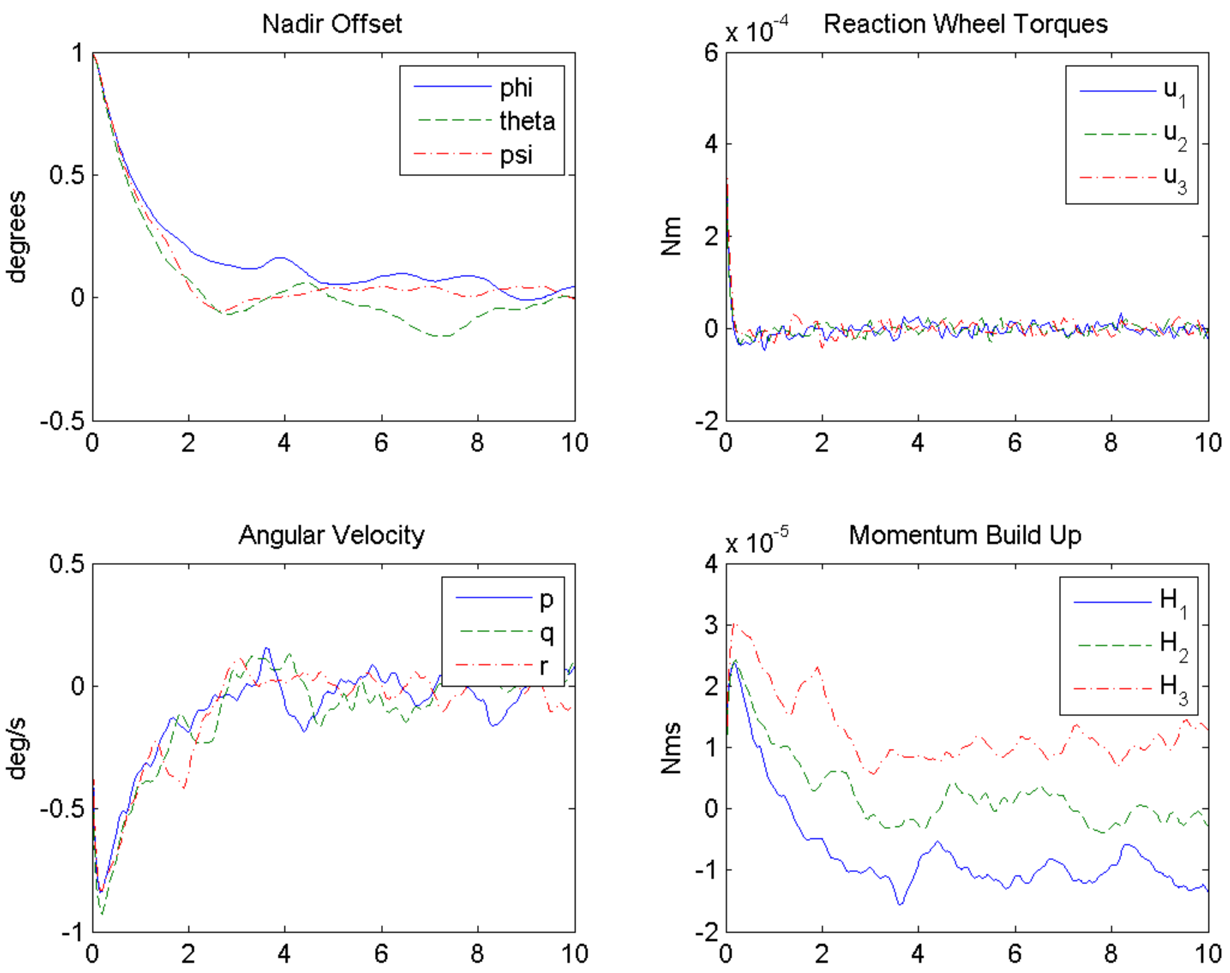
90 degree phi offset

Non-linear

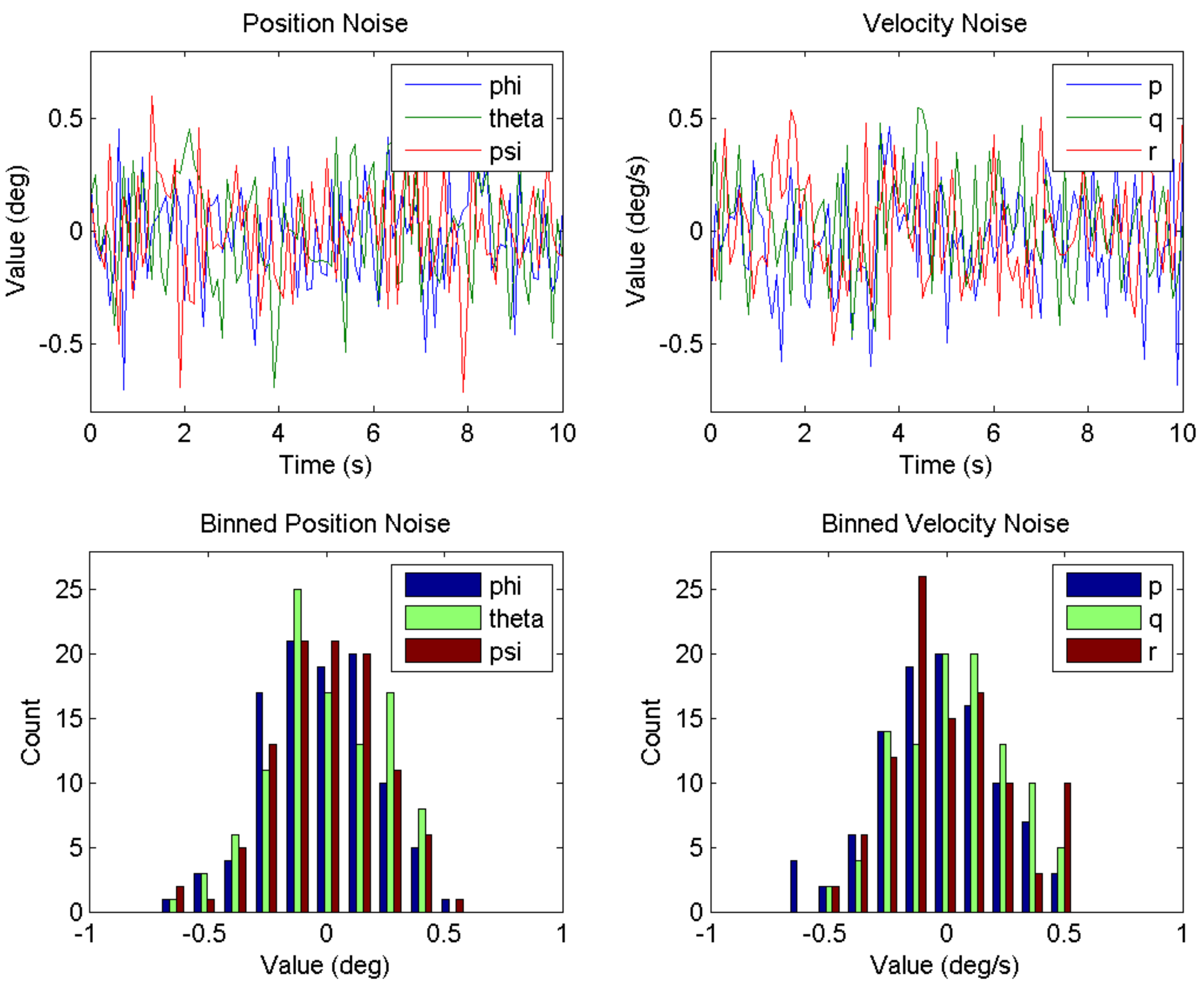


Observer Model





Nadir Offset
degrees
phi
theta
psi
Reaction Wheel Torques
x 10^-4
Nm
u1
u2
u3
Angular Velocity
deg/s
p
q
r
Momentum Build Up
x 10^-5
Nms
H1
H2
H3

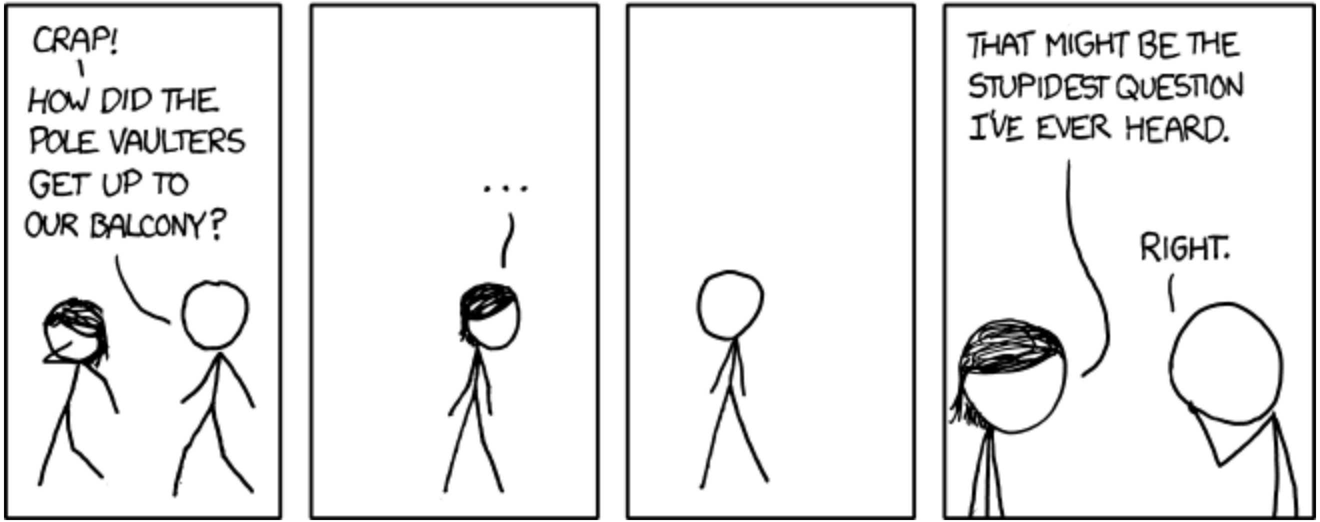


Position Noise
Value (deg)
Time (s)
phi
theta
psi
0.5
0
-0.5
0
2
4
6
8
10
Velocity Noise
Value (deg/s)
Time (s)
p
q
r
Binned Position Noise
Count
Value (deg)
25
20
15
10
5
-1
-0.5
1
Binned Velocity Noise
Value (deg/s)

Future Work

- Allow orbit to degrade based on aero effects
- Compensate for degradation with 1-DOF
- Saturation
- Noise filtration

Questions?


CRAP!
HOW DID THE POLE VAULTERS GET UP TO OUR BALCONY?
...
THAT MIGHT BE THE STUPIDEST QUESTION I'VE EVER HEARD.
RIGHT.

Backup Slides

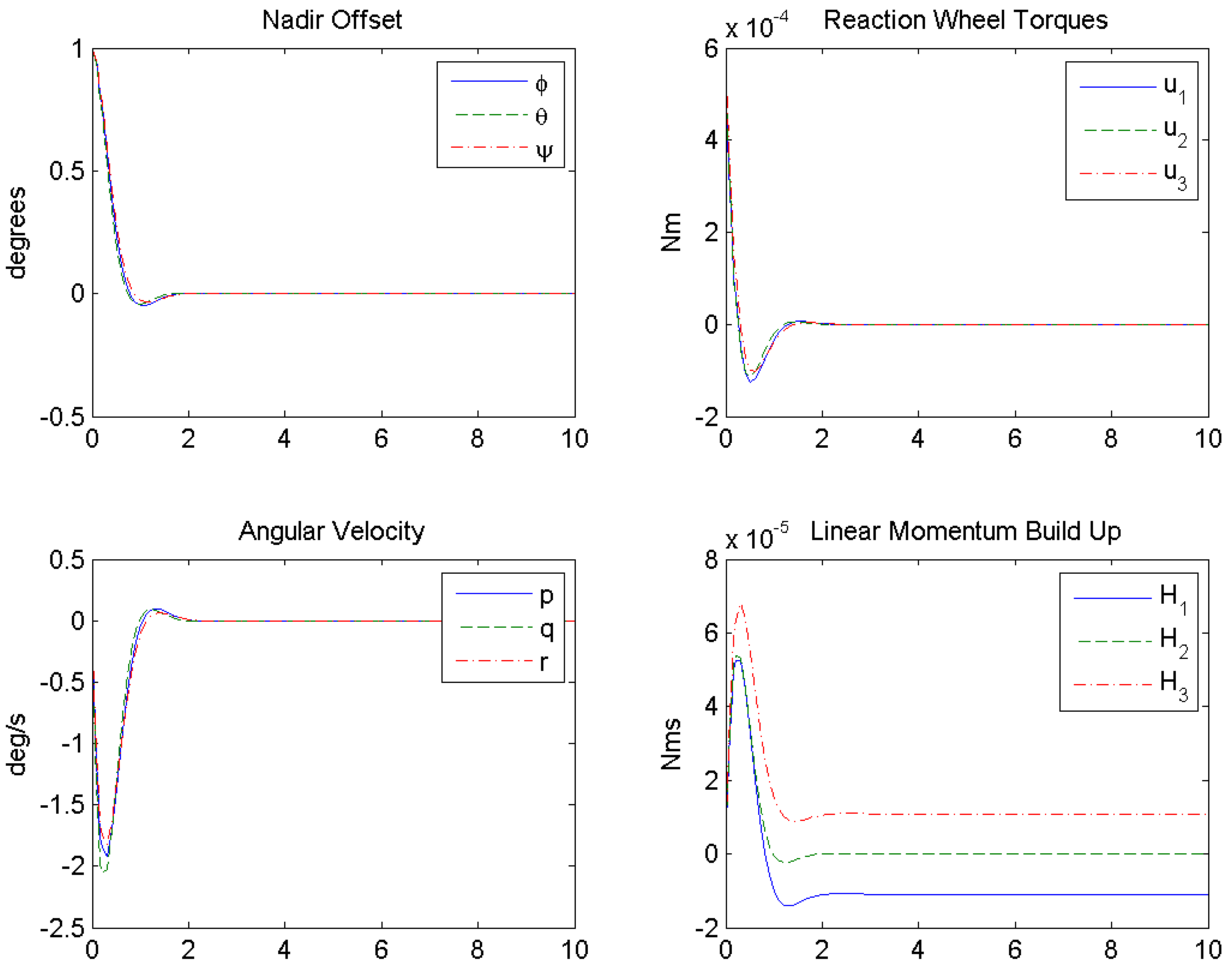
(Danger Zone)

Improved Controller

```
Q = 1E-3 *
   [1  0  0 0 0 0
    0  1  0 0 0 0
    0  0  1 0 0 0
    0  0  0 0 0 0
    0  0  0 0 0 0
    0  0  0 0 0 0];

R = eye(3);

% Improved Controller
[K,S,e] = lqr(SYS,Q,R)
```

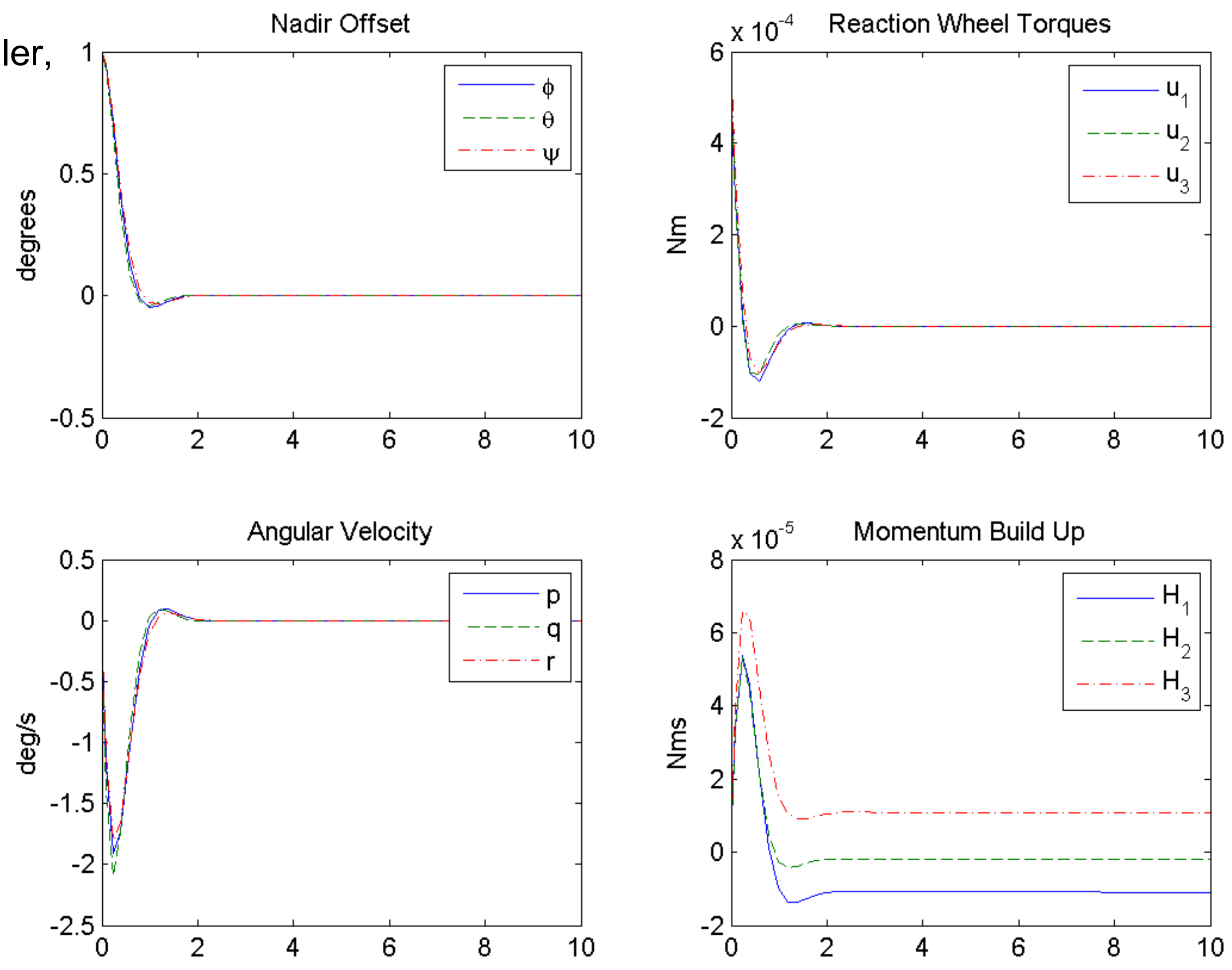
Improved Controller,

Linear

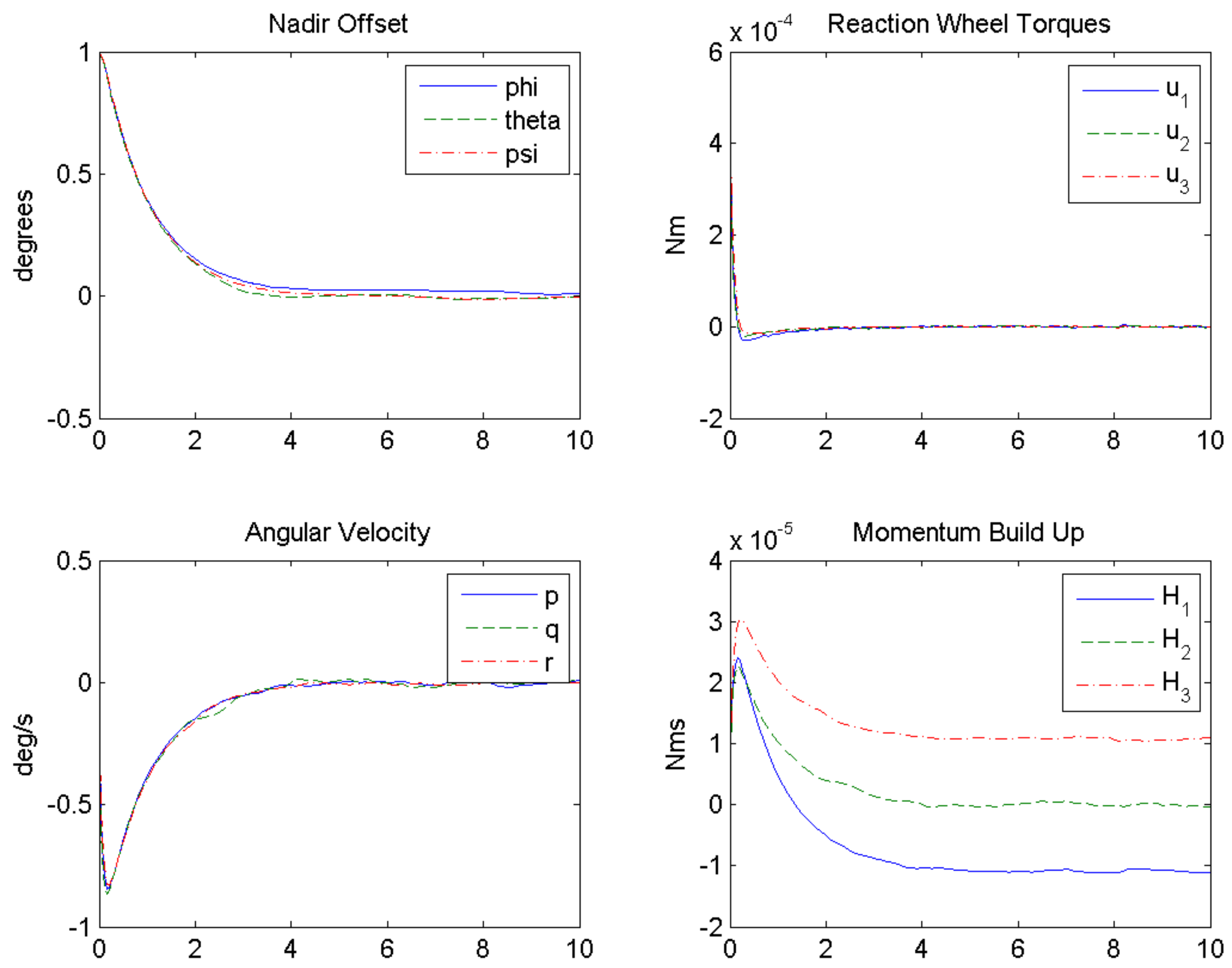


Improved Controller,

Non-linear



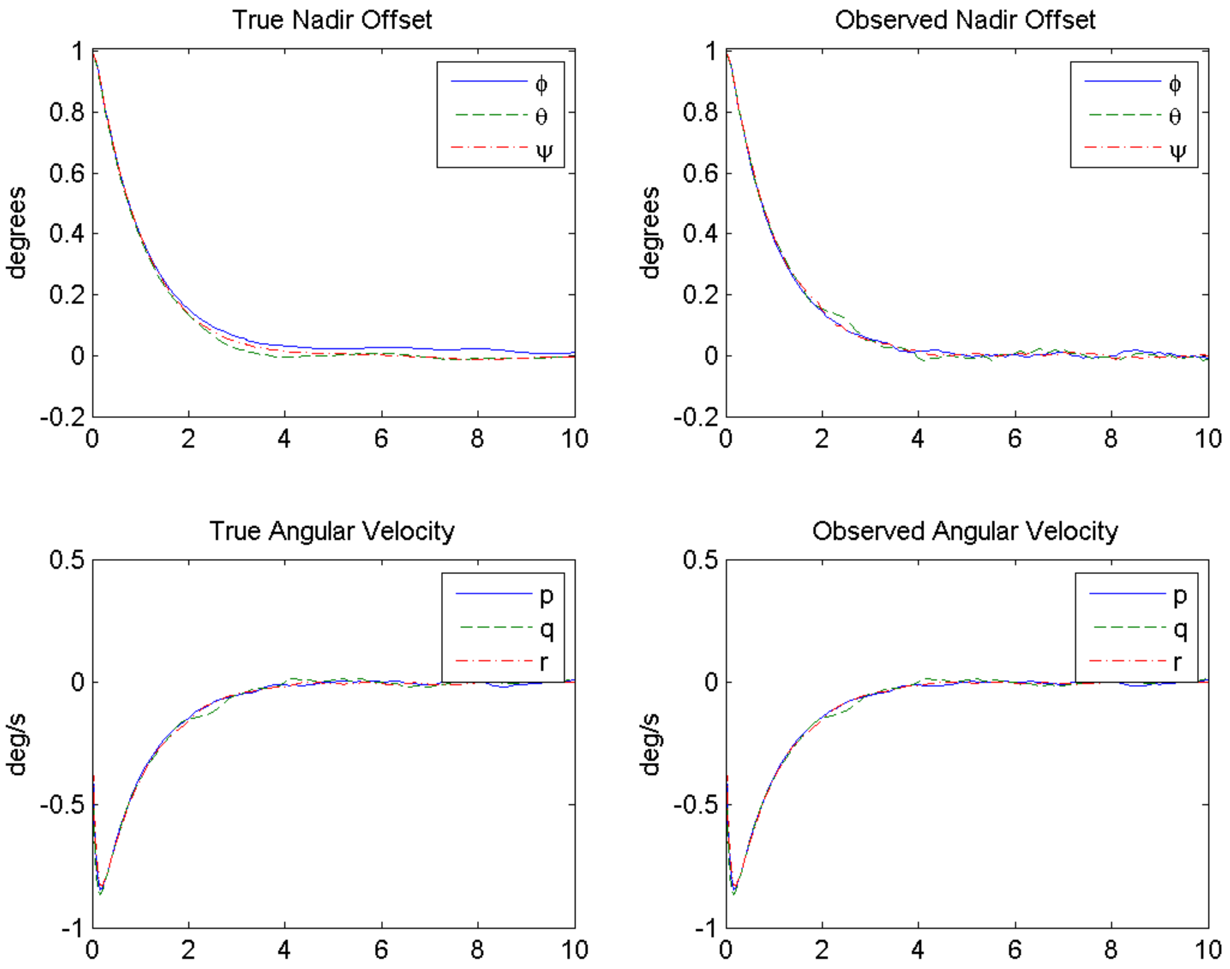
Improved Observer



Improved Observed State

vs

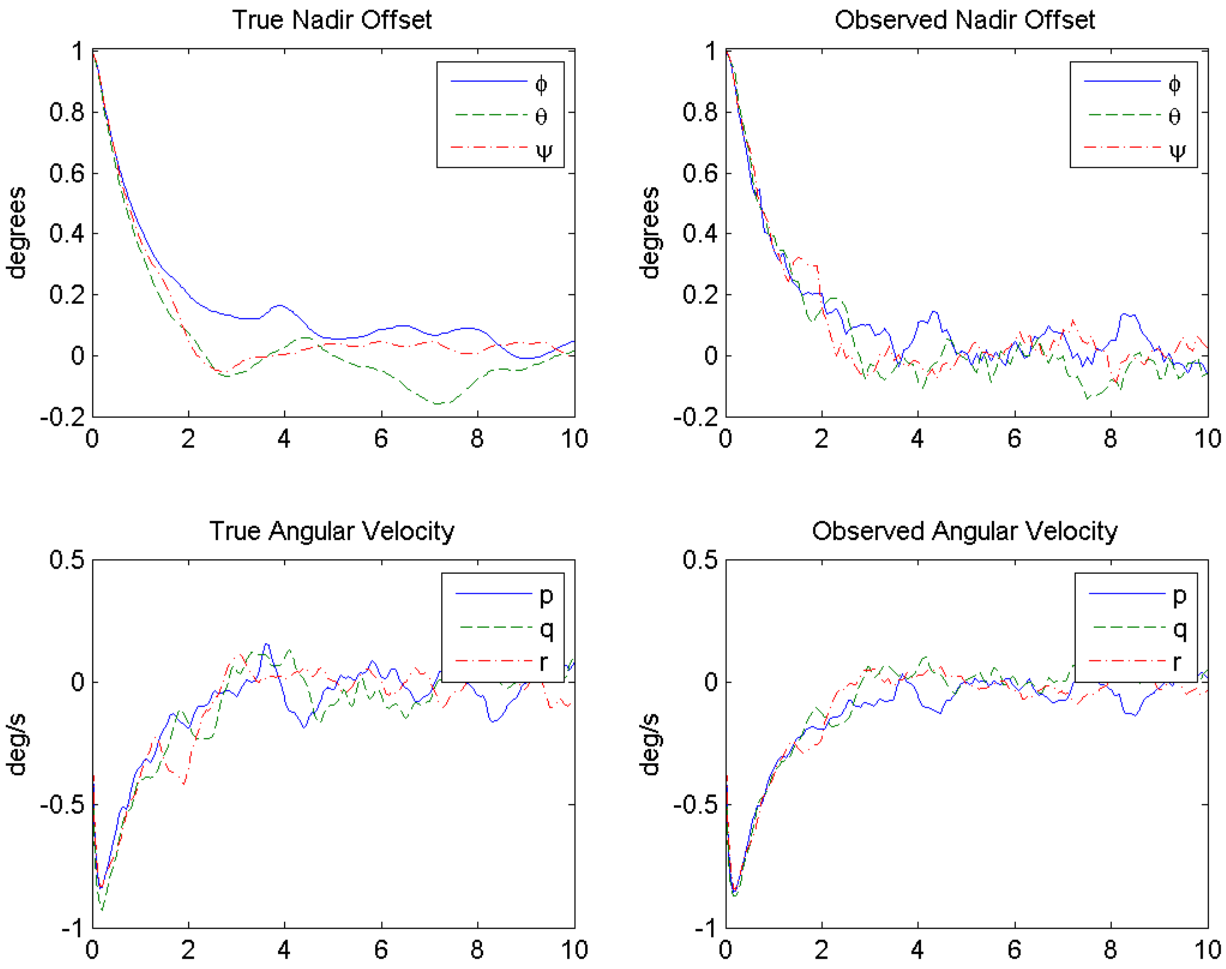
True State



Observed State

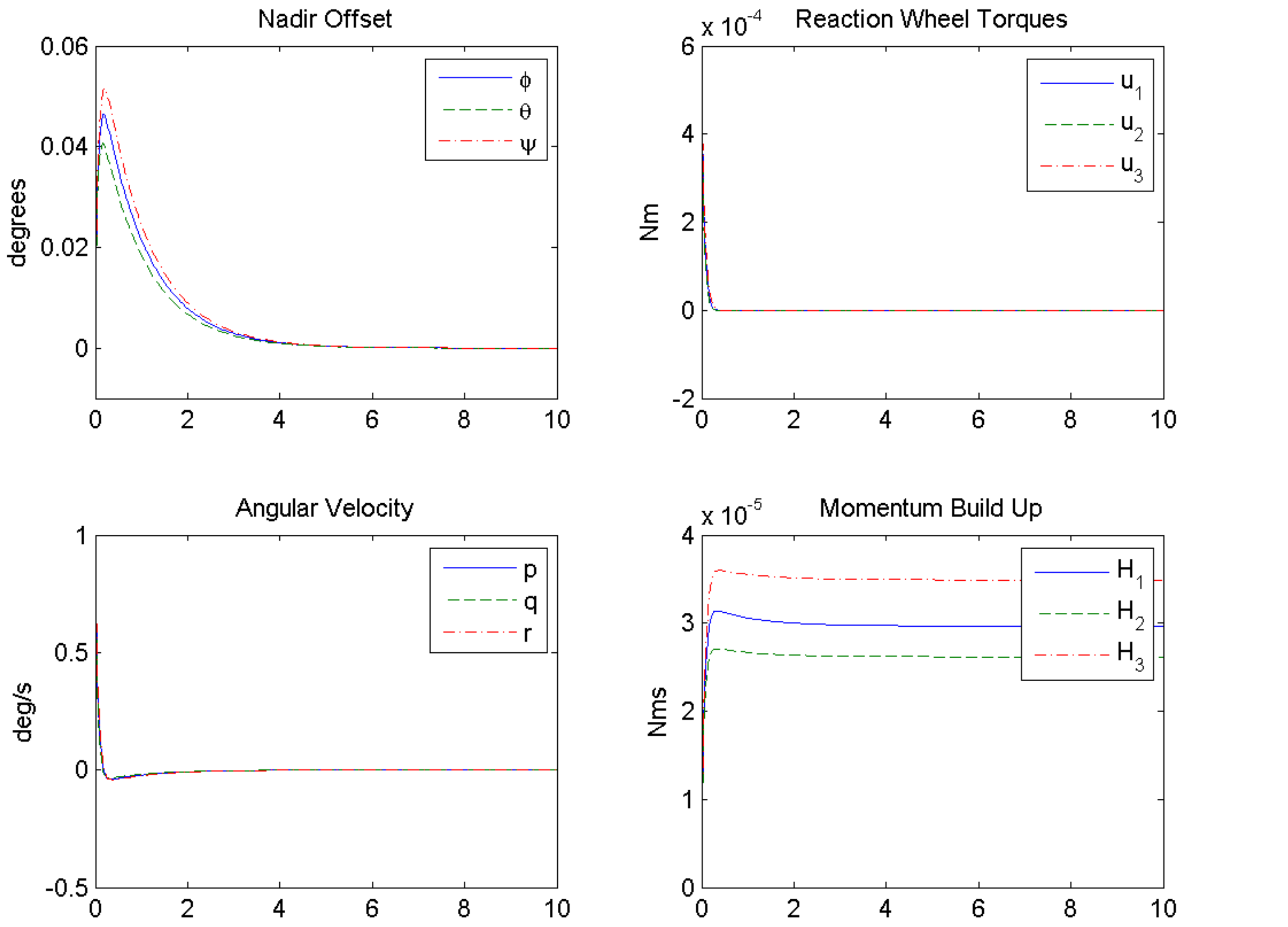
vs

True State



Initial Angular Velocity Perturbation

Linear



Initial Angular Velocity Perturbation

Non-linear