MAE 275 - Final

John Karasinski

June 10, 2015

1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & -\dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} & -X_u & -X_w & 0 \\ \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \dfrac{Z_{\delta_T}}{1-Z_{\dot{w}}} & \dfrac{-Z_u}{1-Z_{\dot{w}}} & \dfrac{-Z_w}{1-Z_{\dot{w}}} & \dfrac{-Z_q}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} & M_{\delta_T} + \dfrac{M_{\dot{w}}Z_{\delta_T}}{1-Z_{\dot{w}}} & -M_u - \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & -M_w - \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & -M_q - \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Plugging in the data for the A-7E aircraft in a landing approach to an aircraft carrier yields

$$A = \begin{bmatrix} -5.4534e-2 & +6.4327e-2 & 0 & -3.2200e+1 & 0 \\ -2.8695e-1 & -5.2887e-1 & +2.1800e+2 & 0 & 0 \\ -8.2071e-5 & -7.8112e-3 & -3.9053e-1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +2.1800e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +7.3284e-1 & +1.3170e-3 & +5.4534e-2 & -6.4327e-2 & 0 \\ -1.4714e+1 & -2.5000e-4 & +2.8695e-1 & +5.2887e-1 & 0 \\ -2.1846e+0 & +4.0722e-6 & +8.2071e-5 & +7.8112e-3 & +3.9053e-01 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2 Handling Qualities

The handling qualities of the pitch-rate SCAS can be estimated using the Bandwidth/Phase-Delay boundaries explained in the handout. The bandwidth is defined as the lesser of $w_{BW_{gain}}$ and $w_{BW_{phase}}$, which is 3.09 rad/s. The phase delay, τ_p is defined

$$\tau_p = \frac{\Delta\Phi 2w_{180}}{57.3(2w_{180})} = \frac{244 - 180}{57.3(12.8)} = 0.09\text{s}$$

These values suggest Level 1 handling qualities.

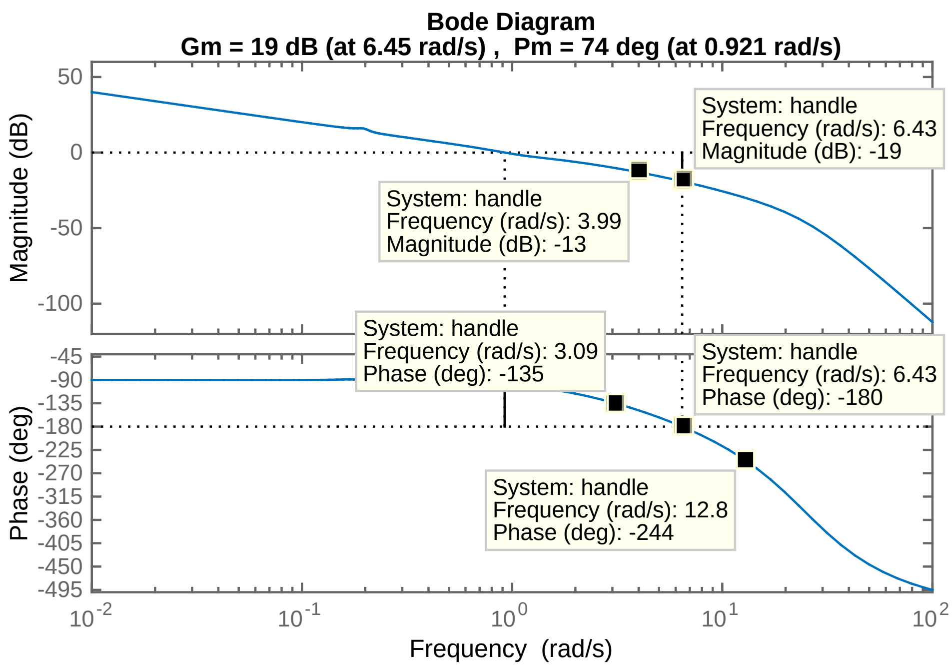

Figure 1: Closed-loop Bode with relevant points selected

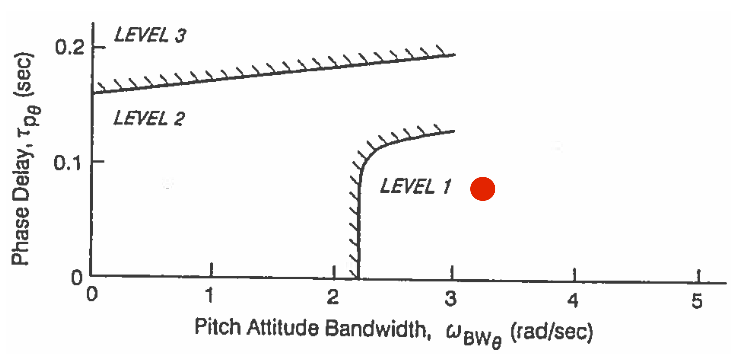

Figure 2: Handling qualities plot with location marked