MAE 275 - Homework 2

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1 Problem 1

We can define the longitudinal and lateral linearized aircraft equations of motion. The longitudinal equations can be expressed as

$$\Delta \dot{u} = X_u \Delta u + X_w \Delta w - g \cos \theta_0 \Delta \theta$$

$$\Delta \dot{w} = \frac{Z_u}{1 - Z_{\dot{w}}} \Delta u + \frac{Z_w}{1 - Z_{\dot{w}}} \Delta w + \frac{Z_q + u_0}{1 - Z_{\dot{w}}} \Delta q - \frac{g \sin \theta_0}{1 - Z_{\dot{w}}} \Delta \theta$$

$$\Delta \dot{q} = \left[M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} \Delta u \right] + \left[M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} \Delta w \right] + \left[M_q + \frac{M_{\dot{w}}(Z_q + u_0)}{1 - Z_{\dot{w}}} \Delta q \right] - \left[\frac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} \Delta \theta \right]$$

$$\Delta \dot{\theta} = \Delta q$$

$$\Delta \dot{h} = - \Delta w + u_0 \Delta \theta$$

or in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta h$, as

$$
A = \begin{bmatrix}
X_u & X_w & 0 & -g \cos(\theta_0) & 0 \\[2mm]
\dfrac{Z_u}{1 - Z_{\dot{w}}} & \dfrac{Z_w}{1 - Z_{\dot{w}}} & \dfrac{Z_q + u_0}{1 - Z_{\dot{w}}} & \dfrac{g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\[2mm]
M_u + \dfrac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q + u_0)}{1 - Z_{\dot{w}}} & -\dfrac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\[2mm]
0 & 0 & 1 & 0 & 0 \\[2mm]
0 & -1 & 0 & u_0 & 0
\end{bmatrix}
$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of `Aircraft Dynamics and Automatic Control` yields

$$
A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{bmatrix}
$$

The lateral equations can be expressed as

$$\Delta\dot{v} = Y_v\Delta v + Y_p\Delta p + [Y_r - u_0]\Delta r + g\cos\theta_0\Delta\varphi$$
$$\Delta\dot{p} = L'_v\Delta v + L'_p\Delta p + L'_r\Delta r$$
$$\Delta\dot{r} = N'_v\Delta v + N'_p\Delta p + N'_r\Delta r$$
$$\Delta\dot{\varphi} = \Delta p + r\tan\theta_0\Delta r$$
$$\Delta\dot{\psi} = r\sec\theta_0\Delta r$$

or in state space form, with state variables $\Delta v, \Delta p, \Delta r, \Delta\varphi(\text{roll}), \Delta\psi$, as

$$A = \begin{bmatrix} Y_v & Y_p & [Y_r - u_0] & g\cos\theta_0 & 0 \\ L'_v & L'_p & L'_r & 0 & 0 \\ N'_v & N'_p & N'_r & 0 & 0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 \\ 0 & 0 & \sec\theta_0 & 0 & 0 \end{bmatrix}$$

Plugging in the appropriate data

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

2   Problem 2

2.1   Longitudinal

The following MATLAB command is called to identify the characteristc roots and eigenvector elements

```
1  [v, d] = eig(A);
```

resulting in two complex pairs

$$d_1 = X \pm iY$$
$$d_2 = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$
$$v_2 = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The longitudinal A matrix from above is used, along with B, C, and D matrices defined as

```
1  B = [0; 0; 0; 0; 0];
2
3  C = [[1, 0, 0, 0, 0];
4       [0, 1/u_0, 0, 0, 0];
5       [0, 0, 1, 0, 0];
6       [0, 0, 0, 1, 0];
7       [0, 0, 0, 0, 1]];
8
9  D = [0; 0; 0; 0; 0];
```

finally, the initial state can be defined and the initial command can be run by

```
1  i1 = real(v(:,1));
2  initial(A, B, C, D, i1, 5)
```

and results in the following figures

2.2   Lateral

The lateral eigenvalues are identified as

$$d_1 = X \pm iY$$
$$\dots$$
$$d_n = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$
$$\dots$$
$$v_n = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The lateral A matrix from above is used, along with B, C, and D matrices defined as

```
1  B = [0; 0; 0; 0; 0];
2
3  C = [[1/u_0, 0, 0, 0, 0];
4       [0, 1, 0, 0, 0];
5       [0, 0, 1, 0, 0];
6       [0, 0, 0, 1, 0];
7       [0, 0, 0, 0, 1]];
8
9  D = [0; 0; 0; 0; 0];
```

Exciting each of these modes with the appropriate eigenvector results in

3   Problem 3

```matlab
%% Step Response Configuration
B = 5/57.3 * ...
    [Y_d_a;
     L_d_a_prime;
     N_d_a_prime;
     0;
     0];

C = [[0, 0, 1, 0, 0];
     [0, 0, 0, 1, 0]];

D = [0; 0];

%% Step Response
lat = ss(A, B, C, D);
step(lat, 100) % unstable

%%  Find Transfer Function
B = B/5;
C=[0 0 0 1 0];
D = 0;
[n, d] = ss2tf(A, B, C, D);
zpk(minreal(tf(n,d)))
```

4   Problem 4