University of California, Davis

Dept. of Mechanical and Aerospace Engineering

MAE 275

Homework Assignment 3

(solution)

Due: Thursday, April 30

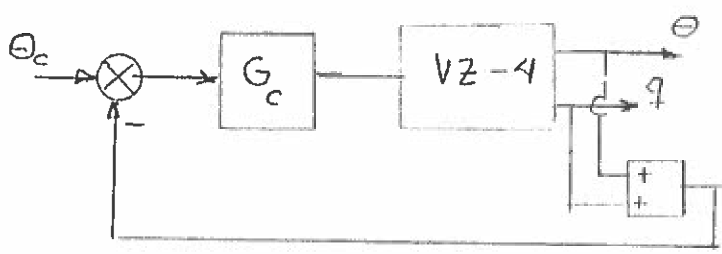
Consider the VZ-4 "Doak" vehicle in the hover flight condition given in Appendix A of the reader. Concentrate on the longitudinal motion.

1.) Determine the open-loop eigenvalues. How many modes of motion are there?

2.) Using MATLAB, determine the following transfer functions: $\frac{\theta}{\delta e}(s)$ and $\frac{q}{\delta e}(s)$

Note that, in hover, the action of the "elevator" is replaced by the force generated by the rear thrustor.

3.) Assume that an attitude gyro and a rate gyro are available to measure $\theta(t) + q(t)$. The control system below is to be designed and simulated, with the determination of G_c as the design objective.

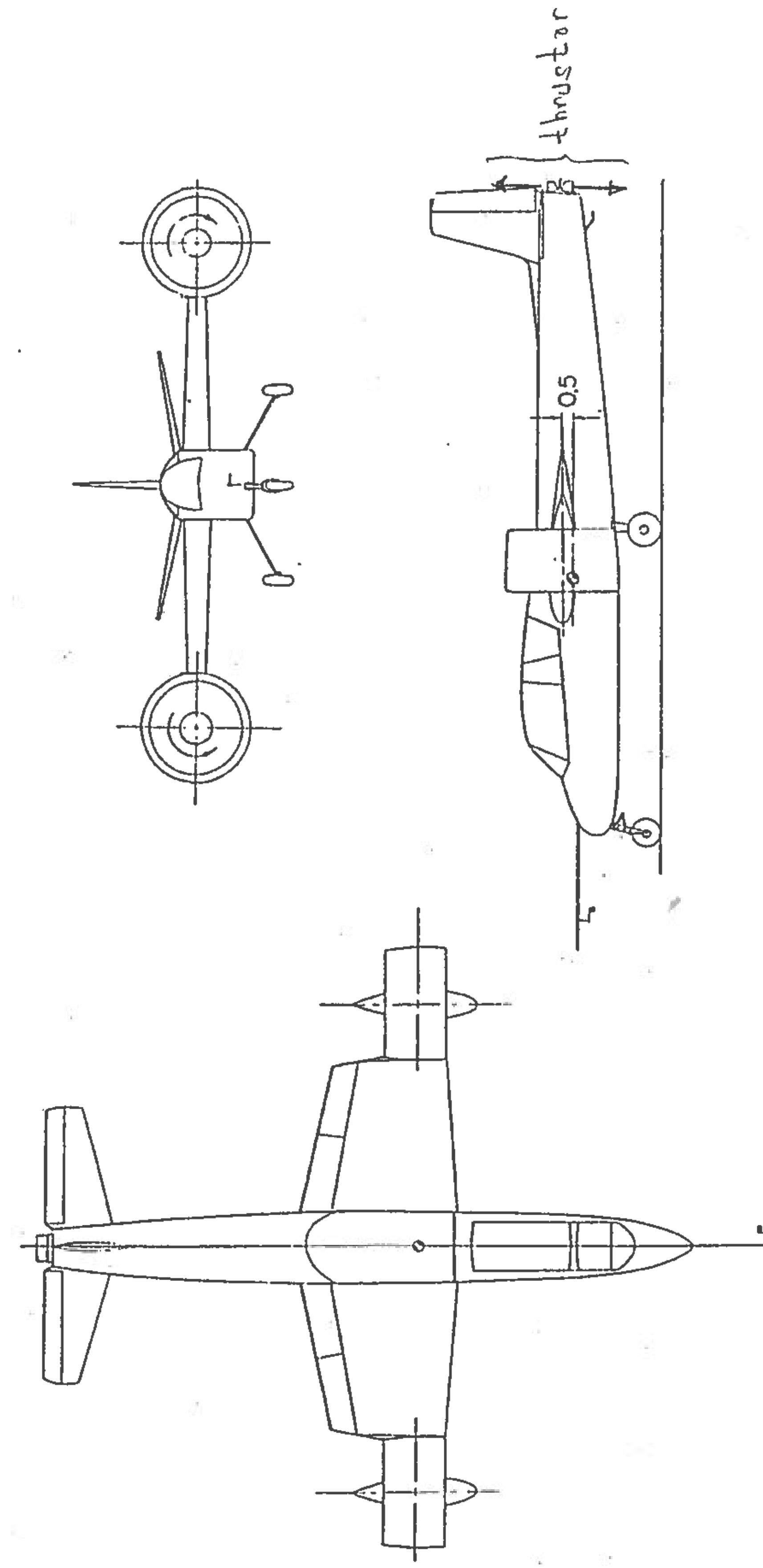


a.) Using loop-shaping principles, determine G_c such that:
- i.) The closed-loop system is stable
- ii.) The G_c has more poles than zeros.
- iii.) The closed-loop bandwidth (-3 dB criterion) is at least 5 rad/sec
- iv.) The gain and phase margins are at least 6 dB and 40 deg. Sketch the Nyquist diagram and show these margins (which can be determined form the open-loop Bode diagram).
- v.) There is zero stead-state error to a step input θ_c

4.) Create a Simulink simulation of your system and demonstrate the pitch attitude response to a pitch attitude command of 5 deg (5/57.3 rad). Filter your step command with a filter given by 10^2/(s^2+20s +10^2). Note that the pitch attitude command can be considered as coming from the pilot's control stick.

Your solution should include

1.) the A, B, C, D matrices of the vehicle with no feedback.
2.) Open-loop eigenvalues and determination of number of modes
3.) Appropriate open and closed-loop Bode plots with the compensated open-loop Bode diagram showing the gain and phase margins
4.) Nyquist plot sketch indicating the gain and phase margins
5.) G_c
6.) Simulink diagram
7.) Step response



VZ-4

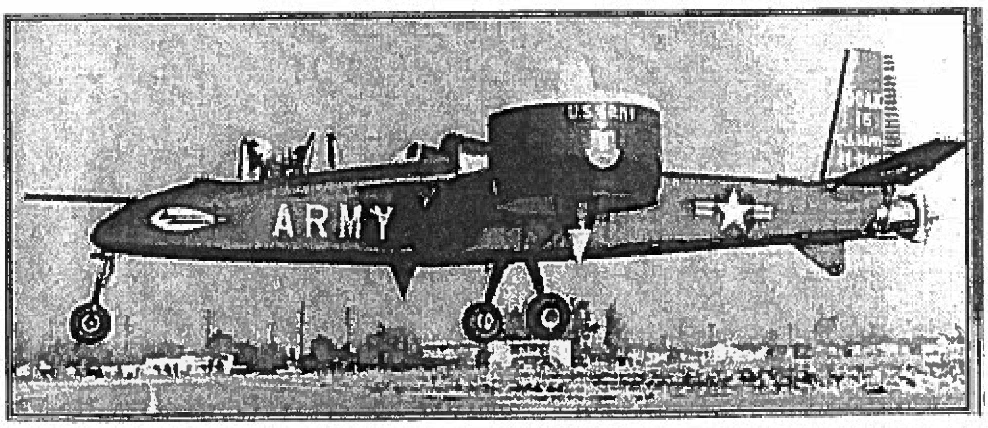

ARMY

TABLE A-9

A. LONGITUDINAL DERIVATIVES FOR THE DOAK VZ-4

U_o, ft/sec	0	58.8	73.0	126.6
X_u	-0.137	-0.130	-0.140	-0.210
X_w	0	-0.084	0.120	0.015
X_q	0	0	0	0
X_{δ_T}	0	0.342	0.442	0.914
X_{δ_e}	0	0	0	0
Z_u	0	-0.248	-0.285	-0.345
Z_w	-0.137	-0.526	-0.39	-0.718
Z_q	0	0	0	0
Z_{δ_T}	1.0	-0.940	-0.906	-0.406
Z_{δ_e}	1.08	1.00*	1.00*	1.00*
M_u	0.0136	0.0128	0.01205	0.0107
M_w	0	-0.032	-0.046	-0.082
$M_{\dot{w}}$	0	0	0	0
M_q	-0.0452	-0.858	-1.065	-1.839
M_{δ_T}	0	0	0	0
M_{δ_e}	1.0*	0.775	0.775	0.775
W, lb	3,100	3,100	3,100	3,100
I_y, slug-ft^2	1,790	1,790	1,790	1,790

*Normalized. (Note $M_{\delta_e}/Z_{\delta_e}$ changes with forward speed due to shift from jet to tail control. Values quoted are approximate.)

```
>> Along

Along =

   -0.1370         0         0  -32.2000
         0   -0.1370         0         0
    0.0136         0   -0.0452         0
         0         0    1.0000         0

>> Blong

Blong =

         0
    1.0800
    1.0000
         0

>> Clong

Clong =

     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1

>> Dlong

Dlong =

     0
     0
     0
     0

>> Clong1

Clong1 =

     0     0     1     1

>> Dlong1

Dlong1 =

     0
```

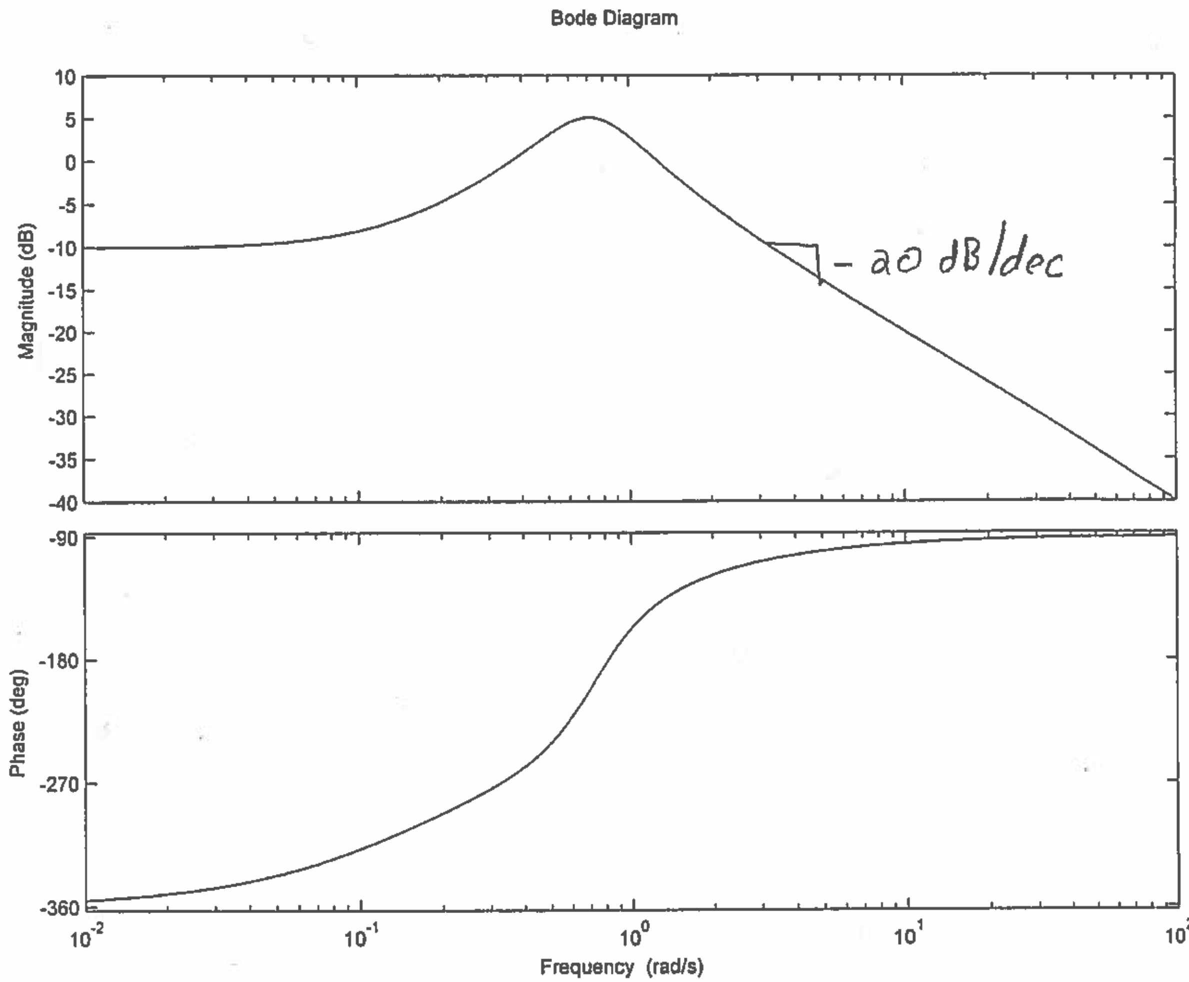
```
>> eig(Along)

ans =

  -0.8223
   0.3201 + 0.6558i
   0.3201 - 0.6558i
  -0.1370

>>
```

```
>> [num,den]=ss2tf(Along,Blong,Clong,Dlong);
>> thedele=tf(num(4,:),den);
>> thedele=minreal(thedele);
>> zpk(thedele)

Zero/pole/gain:
-5.5511e-016 (s-1.801e015) (s+0.137)
-----------------------------------
(s+0.8223) (s^2 - 0.6401s + 0.5326)

>> qdele=tf(num(3,:),den);
>> qdele=minreal(qdele);
>> zpk(qdele)

Zero/pole/gain:
           s (s+0.137)
-----------------------------------
(s+0.8223) (s^2 - 0.6401s + 0.5326)

>>
```

```
>> [num,den]=ss2tf(Along,Blong,Clong1,Dlong1);
>> thqde=tf(num,den);
>> thqde=minreal(thqde)

Transfer function:
        s^2 + 1.137 s + 0.137
---------------------------------------
s^3 + 0.1822 s^2 + 0.006192 s + 0.4379

>> zpk(thqde)

Zero/pole/gain:
          (s+1) (s+0.137)
------------------------------------
(s+0.8223) (s^2 - 0.6401s + 0.5326)

>>
```

```
>> wfreq=logspace(-2,2,200);
>> bode(thqde,wfreq)
```

$$\frac{\Delta\theta + \Delta q}{\Delta\delta_e}(s)$$



```
>> zpk(gcomp1)

Zero/pole/gain:
94.6 (s+0.9)^2
--------------
  s^2 (s+20)
```

= G_c

```
>> ol=gcomp1*thqde;
```

= open-loop with compensator

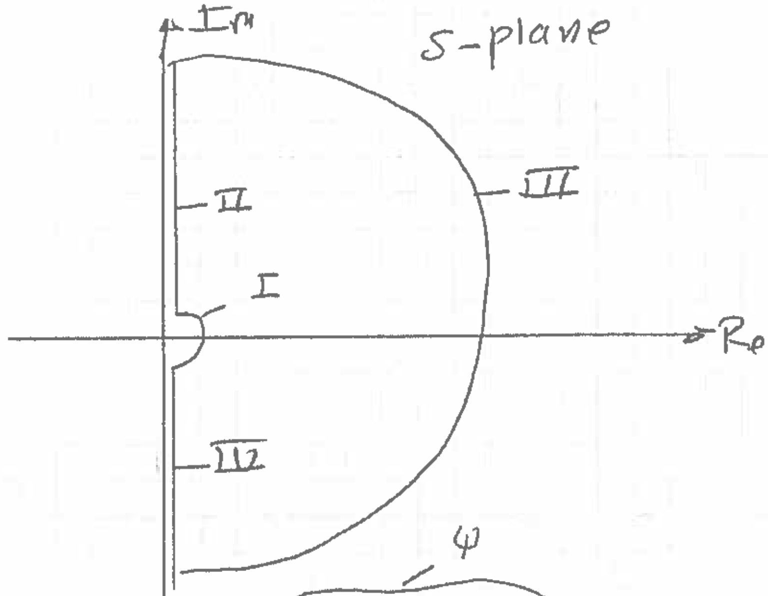
```
>> bode(ol,wfreq)

>> zpk(ol)

Zero/pole/gain:
        94.6 (s+1) (s+0.9)^2 (s+0.137)
-----------------------------------------------
s^2 (s+20) (s+0.8223) (s^2 - 0.6401s + 0.5326)
```

for Nyquist plot:

$n = 2$

$m = 2$

$p = 0$



ψ

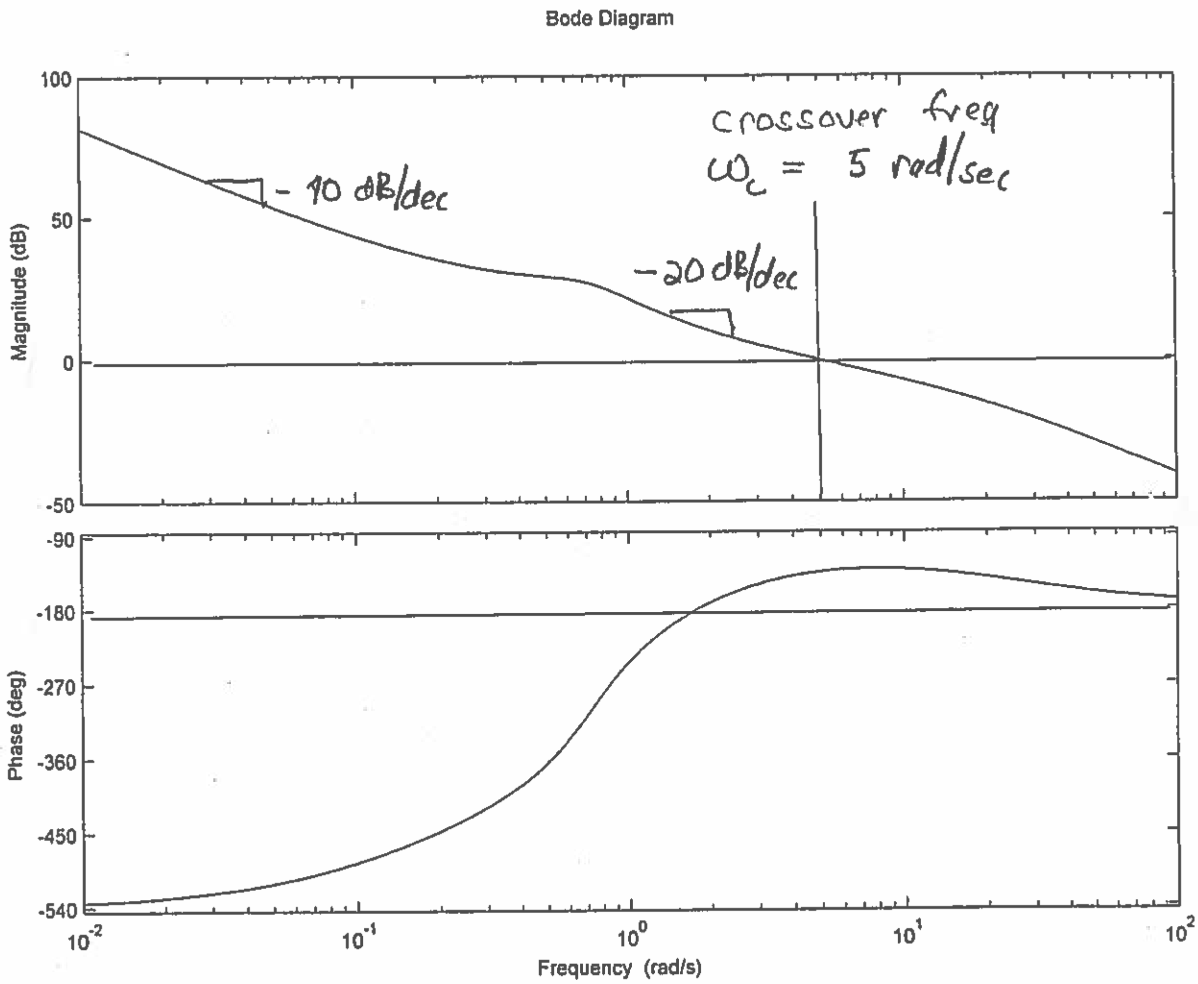
I: $\varepsilon^{-2} e^{-j(2\phi+2\pi)}$

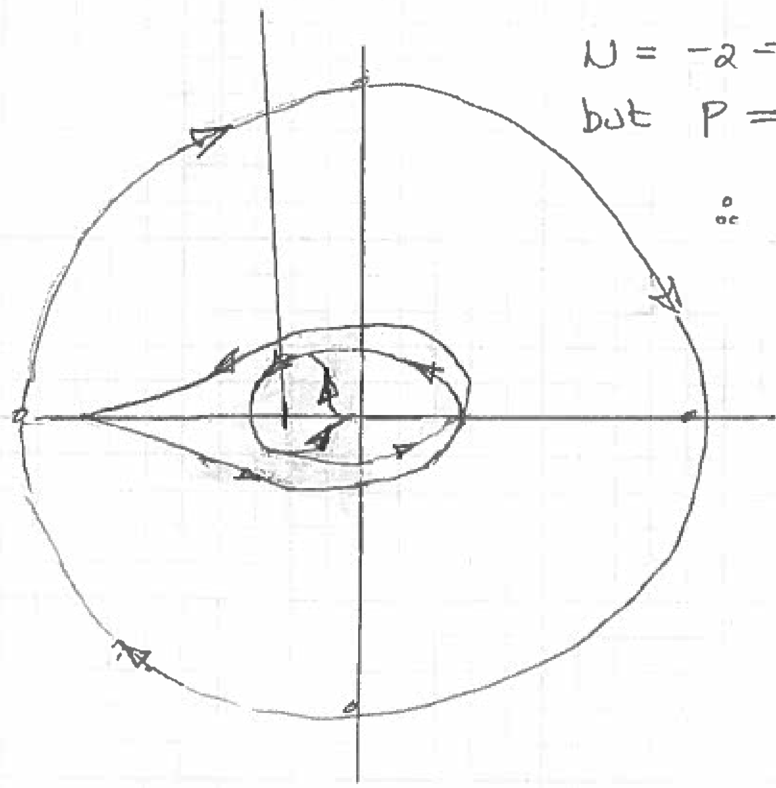
ϕ	ψ
$-\pi/2$	$-\pi$
$-\pi/4$	$-3\pi/2$
0	-2π
$\pi/4$	$-5\pi/2$
$\pi/2$	-3π

II & IV from Bode plot

III maps to origin

$$G_c * \frac{\Delta\theta + \Delta q}{\Delta\delta_e}(s)$$





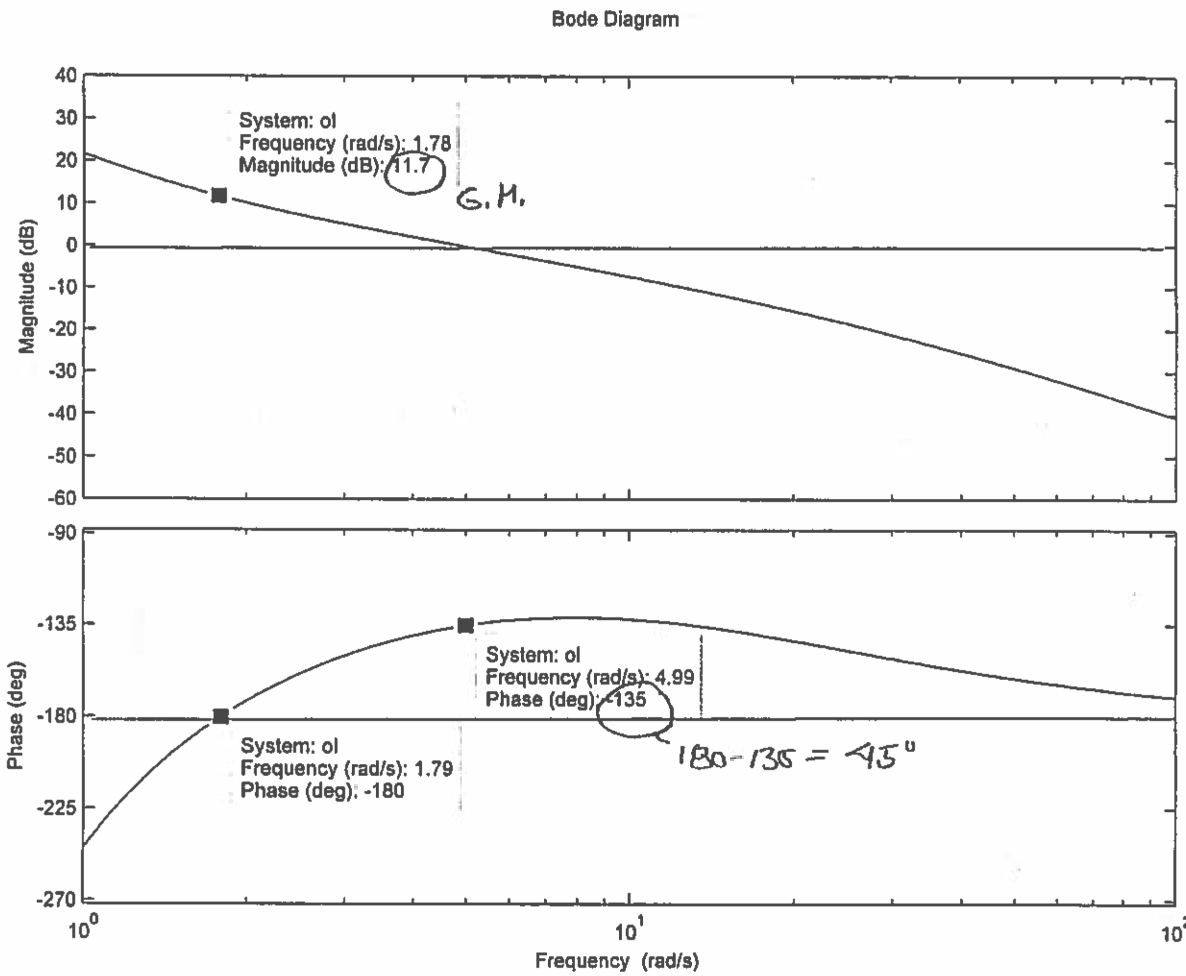
$N = -2 = Z - P$

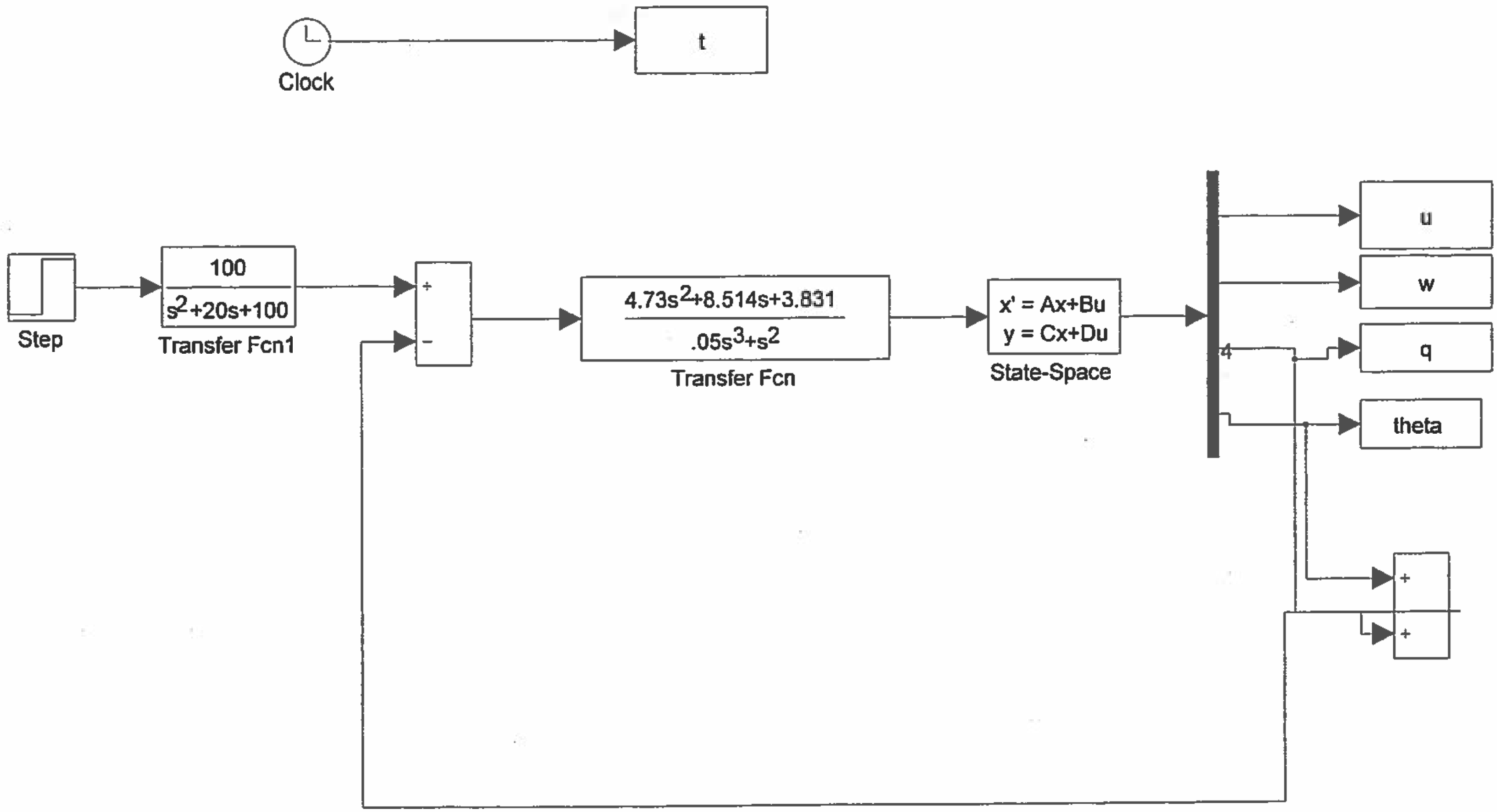
but $P = 2$

$\therefore \boxed{Z = 0}$

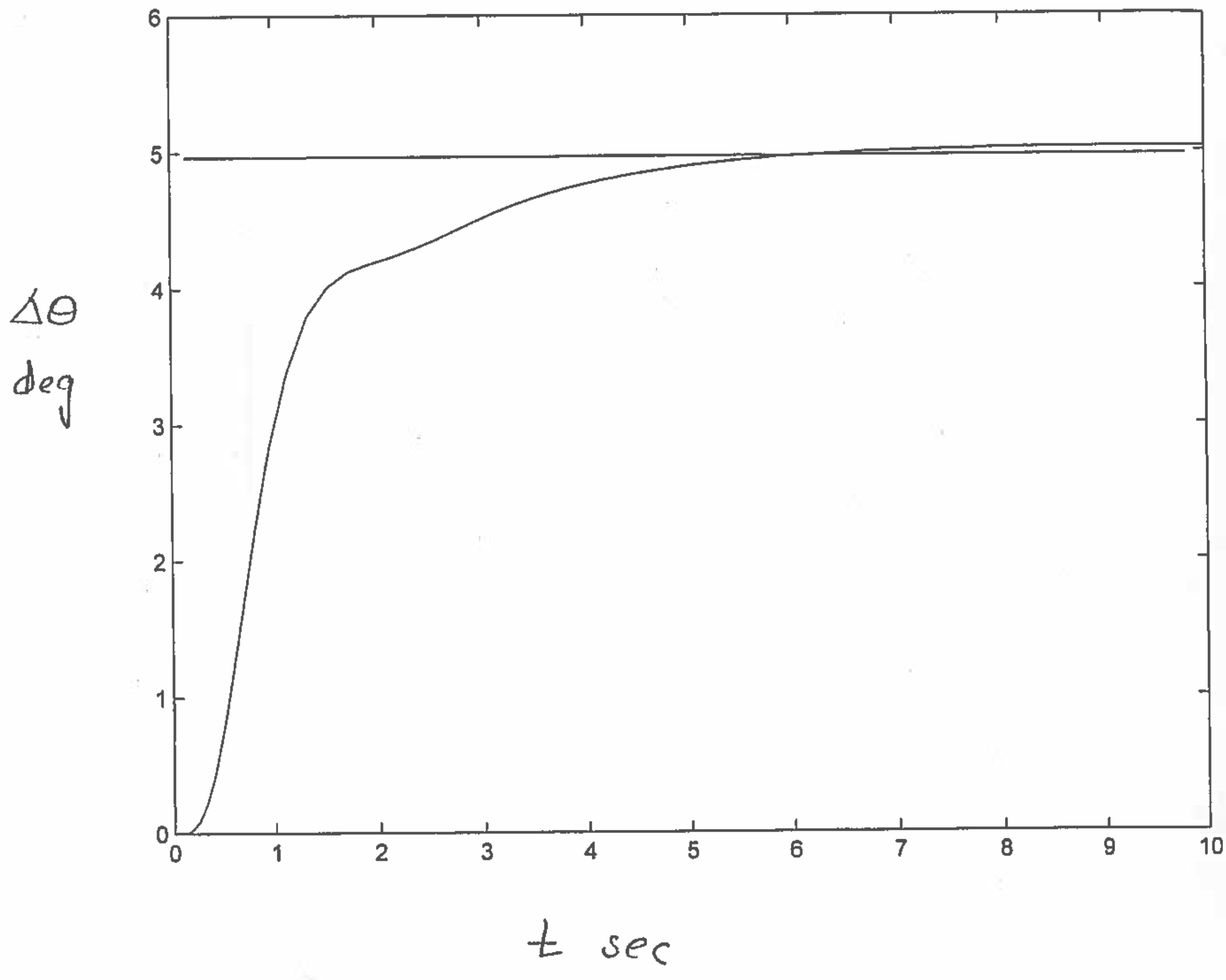
G.M. = 11.7 dB (reduction before unstable)

P.M. = 45°




Clock
t
Step
100
s2+20s+100
Transfer Fcn1
+
-
4.73s2+8.514s+3.831
.05s3+s2
Transfer Fcn
x' = Ax+Bu
y = Cx+Du
State-Space
4
u
w
q
theta
+
+

filtered step response



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MAE – 275

Homework Assignment 4

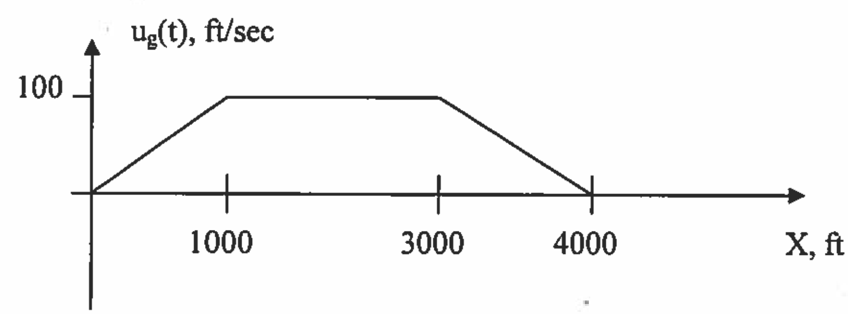
(solution)

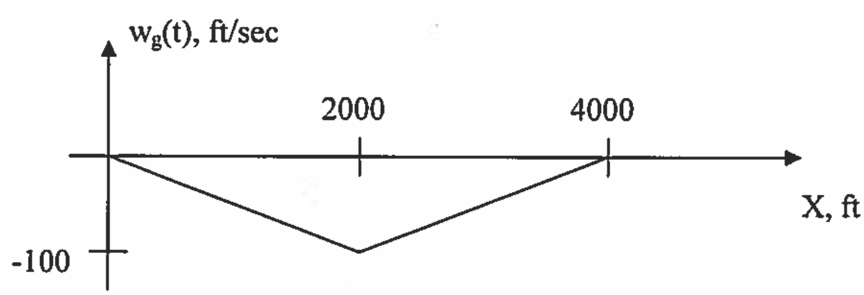
Due: Thursday, May 7

Using the aerodynamic data for longitudinal motion for the A4-D aircraft in Flight Condition 5 in McRuer, Ashkenas and Graham, determine the response of the aircraft to the turbulence field described below using Simulink. Assume only elevator control is used. The response variables of interest are (1) pitch attitude $\theta(t)$, (2) angle of attack $\alpha(t)$ (3) altitude deviation from trim h(t), (3) airspeed deviation from trim u(t), normal acceleration at the cg $a_z(t)$, and finally elevator angle $\delta_e(t)$. For an expression for $a_z(t)$, see p. 5-3 in McRuer, Ashkenas and Graham. For h(t), integrate $-\dot{Z}(t)$ where $\dot{Z}(t)$ is one of the "navigation" equations. In your model assume that a stabilization system is in operation in which the elevator is being driven by a signal proportional to pitch attitude as

$$\delta_e(t) = 0.5\theta(t)$$

Turbulence Field:



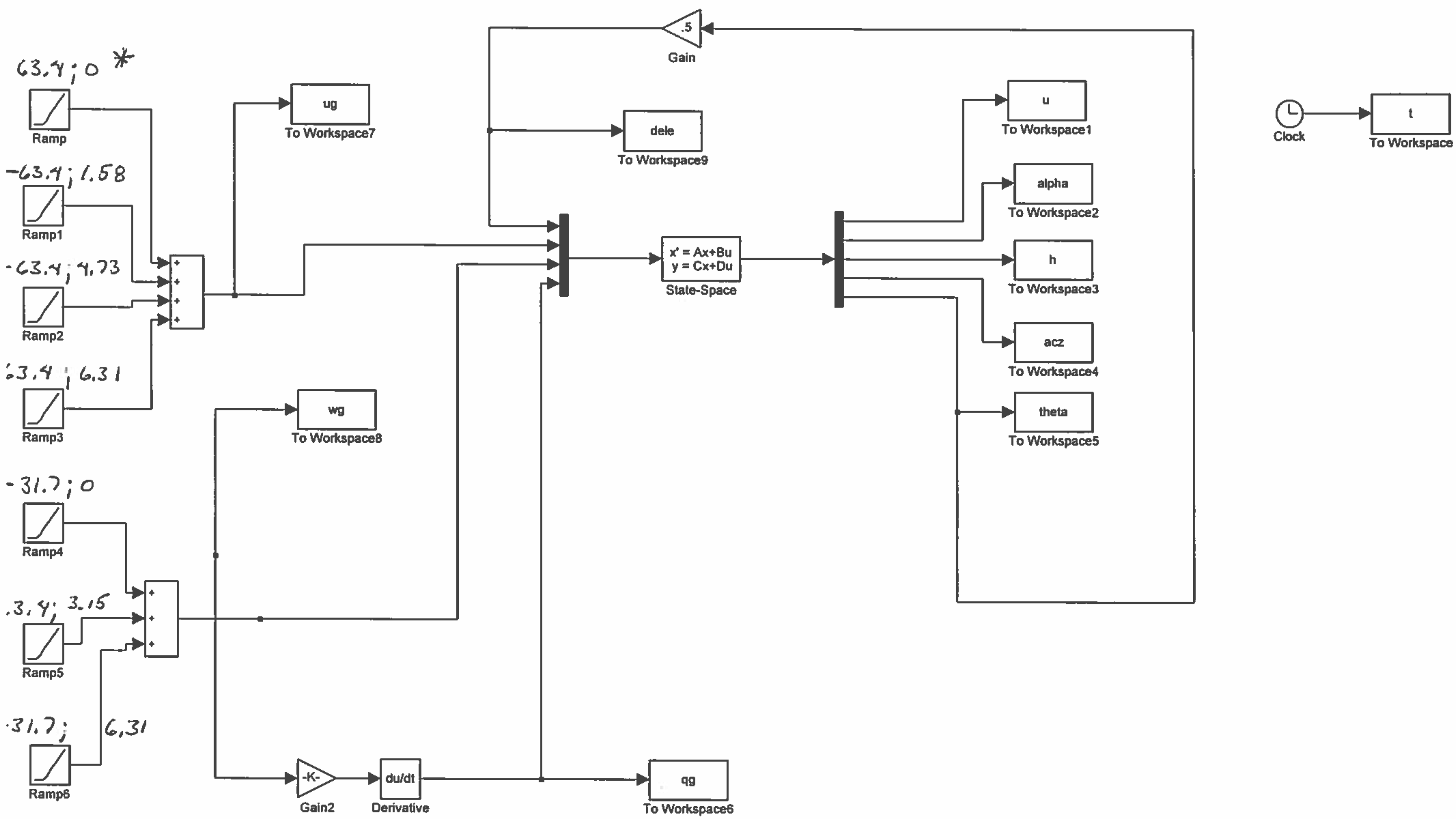


Plot your responses $0 \le t \le 15$ sec. Assume t = 0 when X = 0 above, where X is the distance along the linear flight path. Include your Simulink diagram in your solution.

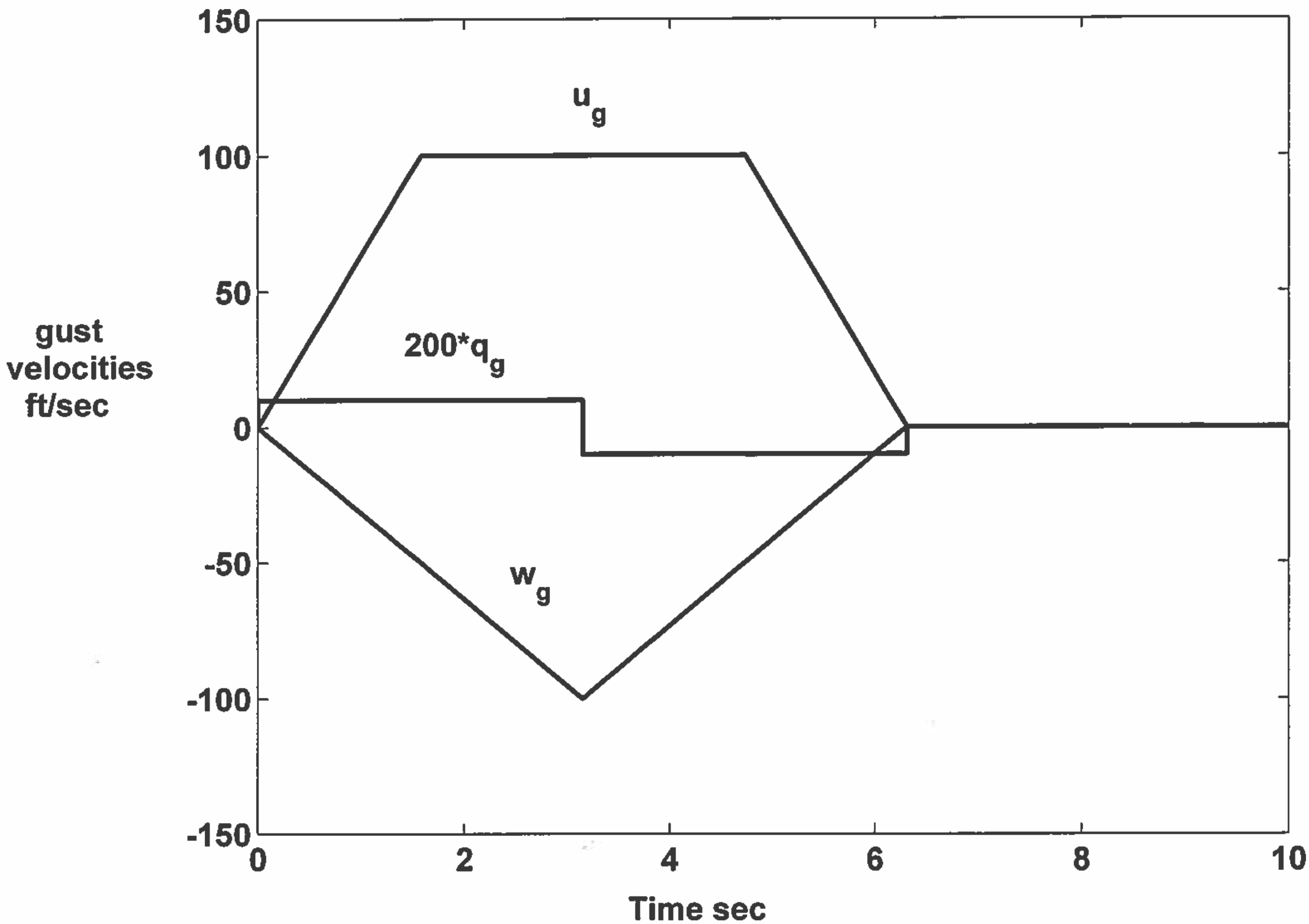
```
>> A4

A4 =

  -1.2700e-002 -5.9000e-003            0 -3.2200e+001            0
  -1.0100e-001 -8.1670e-001  6.3500e+002            0            0
  -3.0000e-004 -1.9500e-002 -1.4160e+000            0            0
             0            0  1.0000e+000            0            0
             0 -1.0000e+000            0  6.3400e+002            0

>> B4

B4 =

             0  1.2700e-002  5.9000e-003            0
  -5.6800e+001  1.0100e-001  8.1670e-001            0
  -1.9400e+001  3.0000e-004  1.9500e-002  1.4156e+000
             0            0            0            0
             0            0            0            0

>> C4

C4 =

   1.0000e+000            0            0            0            0
             0  1.5773e-003            0            0            0
             0            0            0            0  1.0000e+000
  -1.0100e-001 -8.1670e-001            0            0            0
             0            0            0  1.0000e+000            0

>> D4

D4 =

             0            0            0            0
             0            0            0            0
             0            0            0            0
  -5.6800e+001  1.0100e-001  8.1670e-001            0
```

0 0 0 0

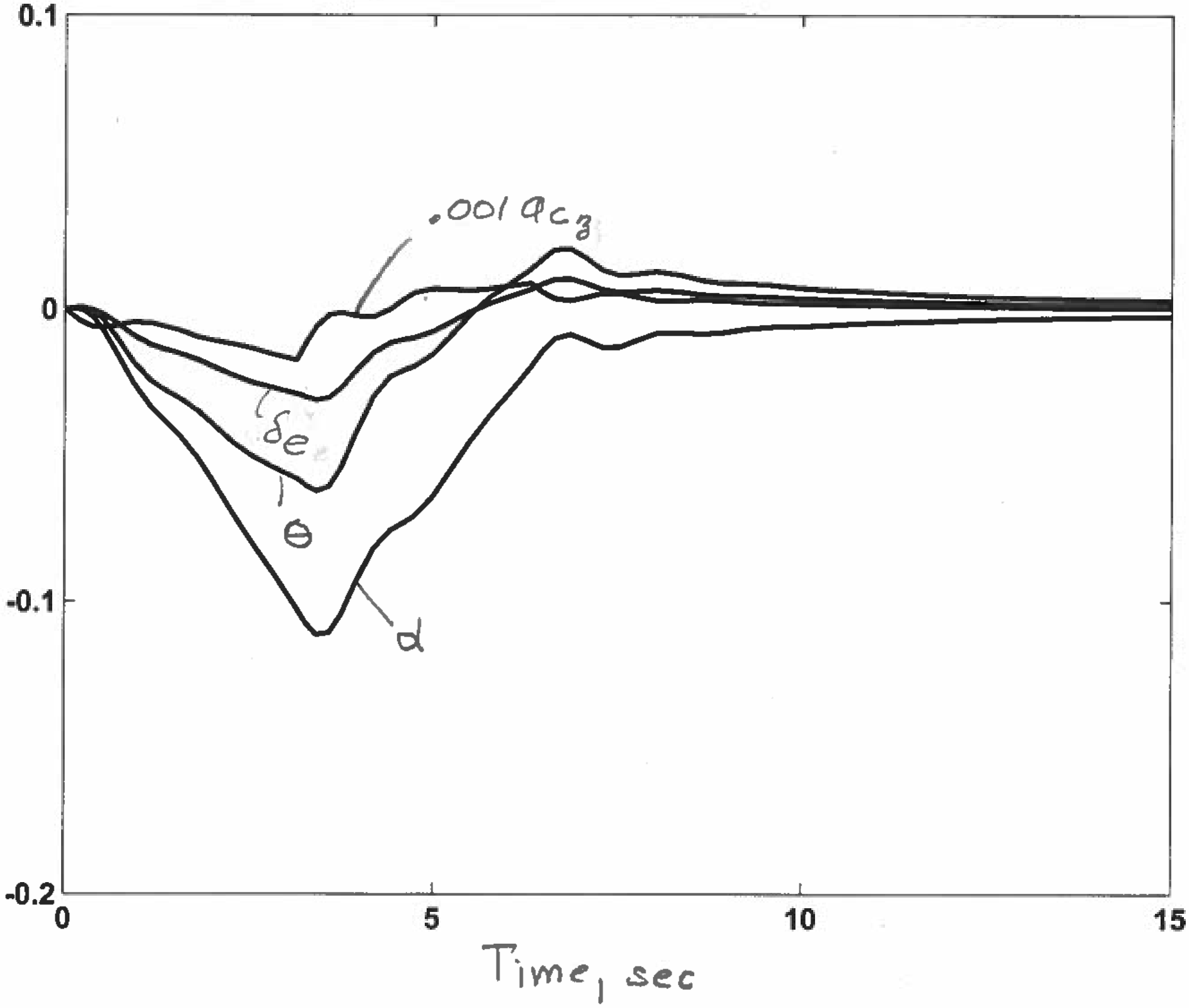
$$\{x\} = \begin{Bmatrix} u \\ w \\ q \\ \theta \\ h \end{Bmatrix} \qquad \{y\} = \begin{Bmatrix} u \\ \alpha \\ h \\ a_{cz} \\ \theta \end{Bmatrix} \qquad \text{"}u\text{"} = \begin{Bmatrix} \delta_e \\ u_g \\ w_g \\ q_g \end{Bmatrix}$$



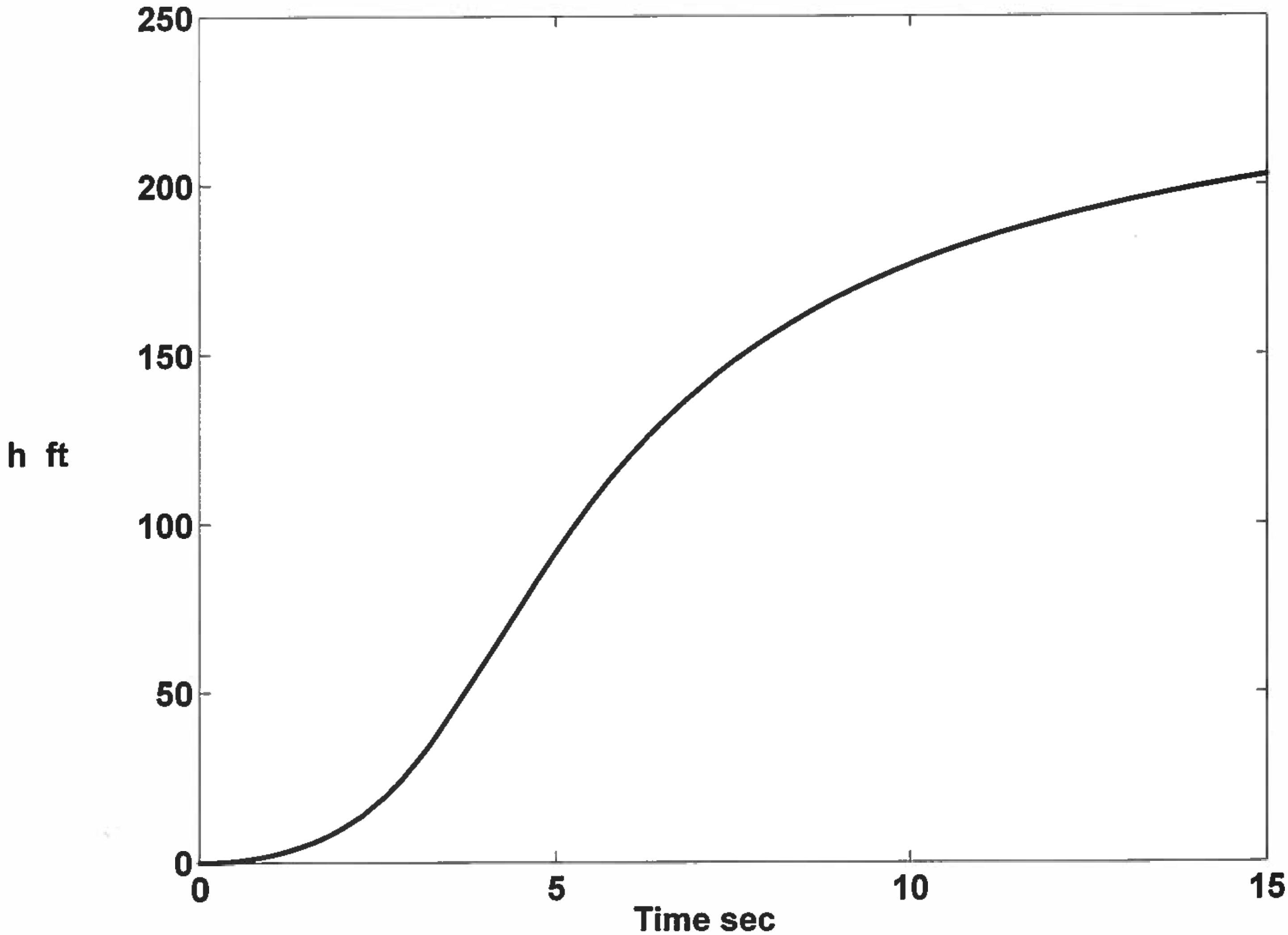
* slope; start time



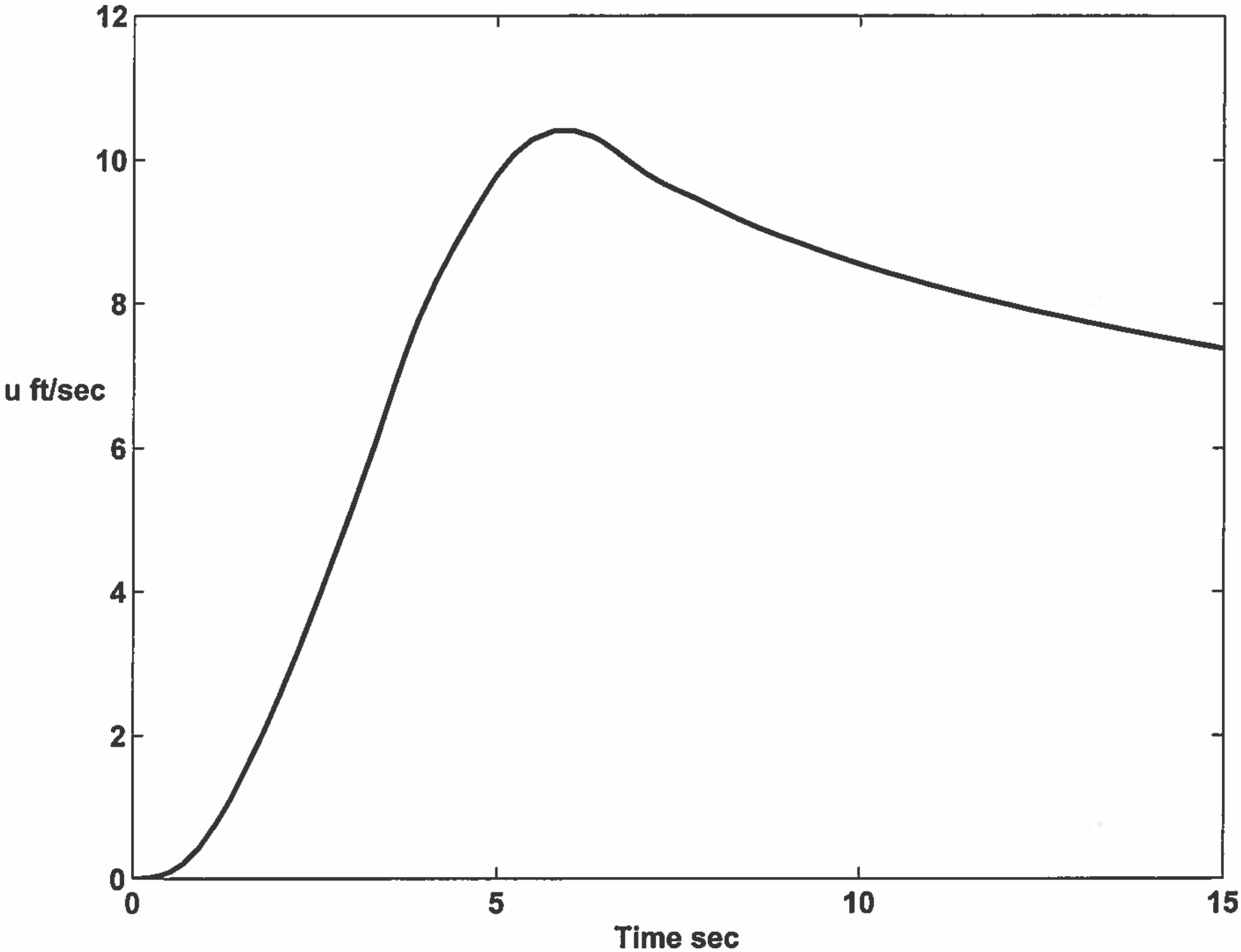
150
100
50
0
-50
-100
-150
gust
velocities
ft/sec
u_g
$200*q_g$
w_g
0
2
4
6
8
10
Time sec

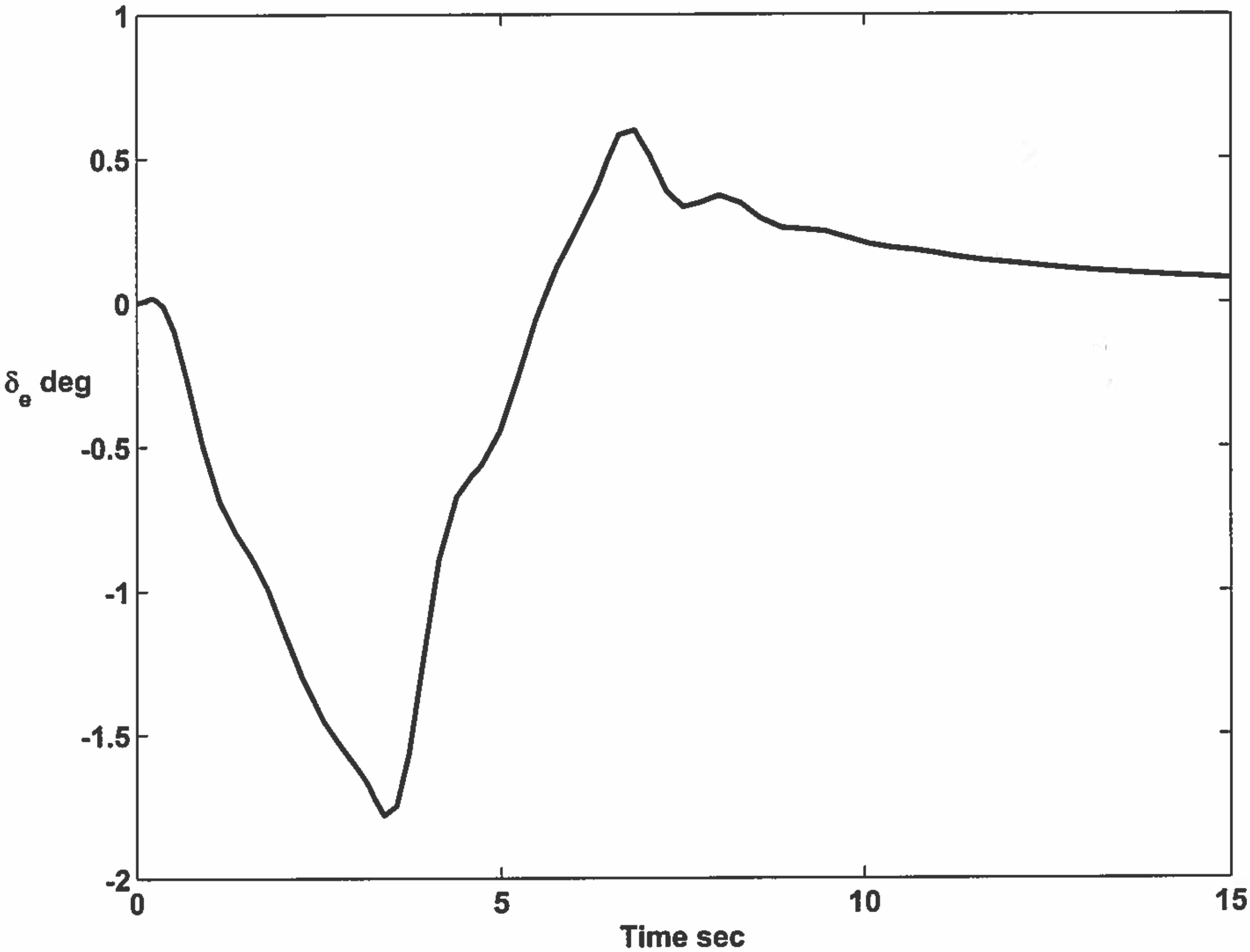


0.1
0
-0.1
-0.2
0
5
10
15
.001 a_{cz}
δ_e
θ
α
Time, sec



250
200
150
100
50
0
h ft
0
5
10
15
Time sec


12
10
8
u ft/sec
6
4
2
0
0
5
10
15
Time sec



1
0.5
0
-0.5
-1
-1.5
-2
δ_e deg
0
5
10
15
Time sec