MAE 275 - Homework 5

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May 9, 2015

1 Defining the System

The lateral linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta v, \Delta p, \Delta r, \Delta\varphi, \Delta\psi$, as

$$A = \begin{bmatrix} Y_v & Y_p & [Y_r - u_0] & g\cos\theta_0 & 0 \\ L'_v & L'_p & L'_r & 0 & 0 \\ N'_v & N'_p & N'_r & 0 & 0 \\ 0 & 1 & \tan\theta_0 & 0 & 0 \\ 0 & 0 & \sec\theta_0 & 0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} Y_{\delta_r} & Y_{\delta_a} \\ L'_{\delta_r} & L'_{\delta_a} \\ N'_{\delta_r} & N'_{\delta_a} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1/u_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

with $x = [\Delta v, \Delta p, \Delta r, \Delta\varphi, \Delta\psi]$ and $u = [\Delta\delta_r, \Delta\delta_a]$.
Plugging in the data for the DC-8 aircraft in Flight Condition 8002 from Appendix A of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -1.0080e-1 & 0 & -4.6820e+2 & +3.2200e+1 & 0 \\ -5.7881e-3 & -1.2320e+0 & +3.9700e-1 & 0 & 0 \\ +2.7787e-3 & -3.4600e-2 & -2.5700e-1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.3480e+1 & 0 \\ +3.9200e-1 & -1.6200e+0 \\ -8.6400e-1 & -1.8750e-2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} +2.1358e-3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

2 Designing the Controllers

Two controllers were designed. The first controller, Gc_ϕ was designed as

```
−360.94 (s+1.413) (s+0.07616)
——————————————————————————————
       s (s+100) (s+3)
```

This controller was determined using loop-shaping principles such that it had a roll-attitude bandwidth of ∼3 rad/sec (-3dB criterion) and a minimum overshoot in step response.

The second controller, Gc_r was designed as

```
21.969 (s−1.86)
————————————————
(s+10) (s+8.199)
```

This controller was determined using loop-shaping principles such that it had a bandwidth of ∼1 rad/sec (-3dB criterion), giving a factor of approximately three between r-loop and ϕ-loop bandwidths.

Additionally, both controllers have:
more poles than zeros (are strictly proper compensators)
gain margins of at least 12dB
phase margins of at least 40 deg

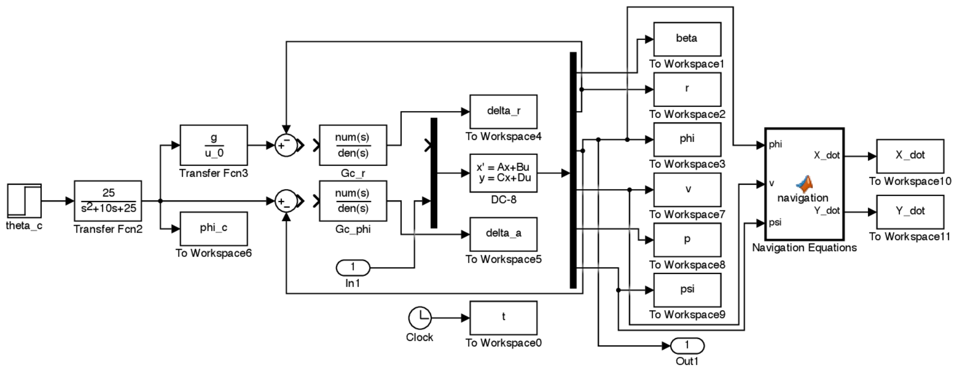

Figure 1: Simulink Diagram for First Controller Design

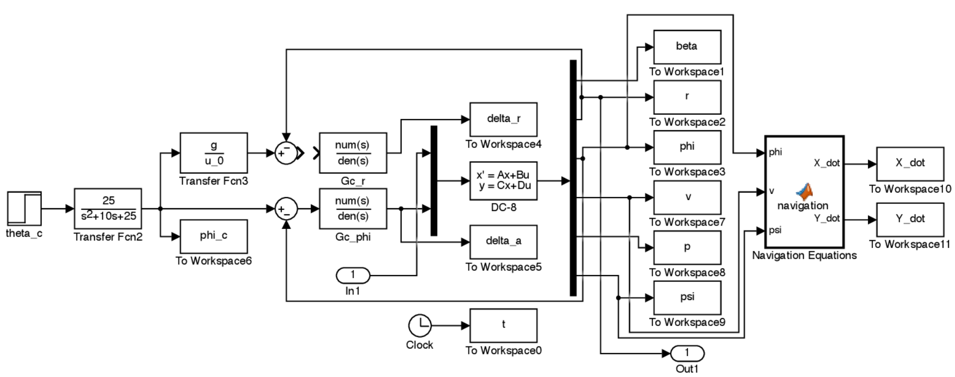

Figure 2: Simulink Diagram for Second Controller Design

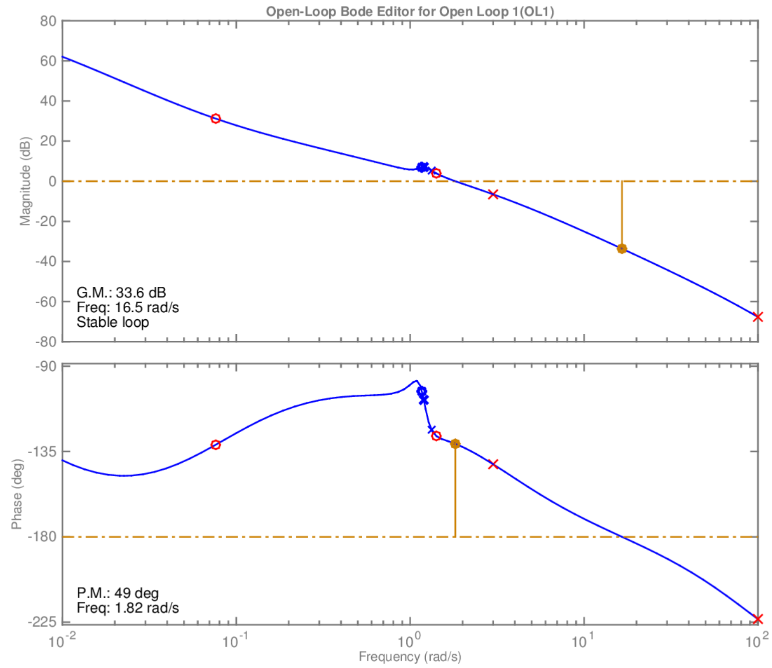

Figure 3: ϕ-loop open-loop Bode with G.M. of 34 dB and P.M. of 49 deg

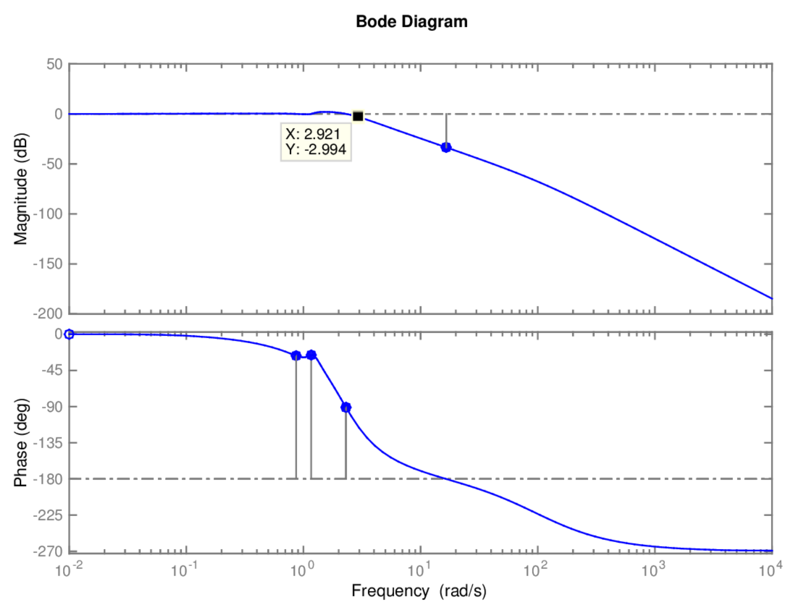

Figure 4: ϕ-loop closed-loop Bode with bandwidth of 3 rad/s (3dB criterion)

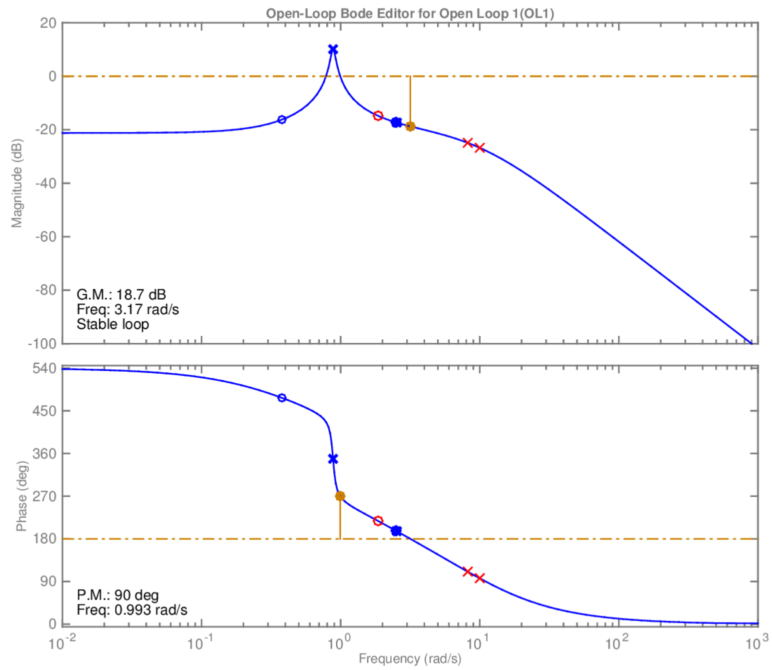

Figure 5: r-loop open-loop Bode with G.M. of 18 dB and P.M. of 90 deg

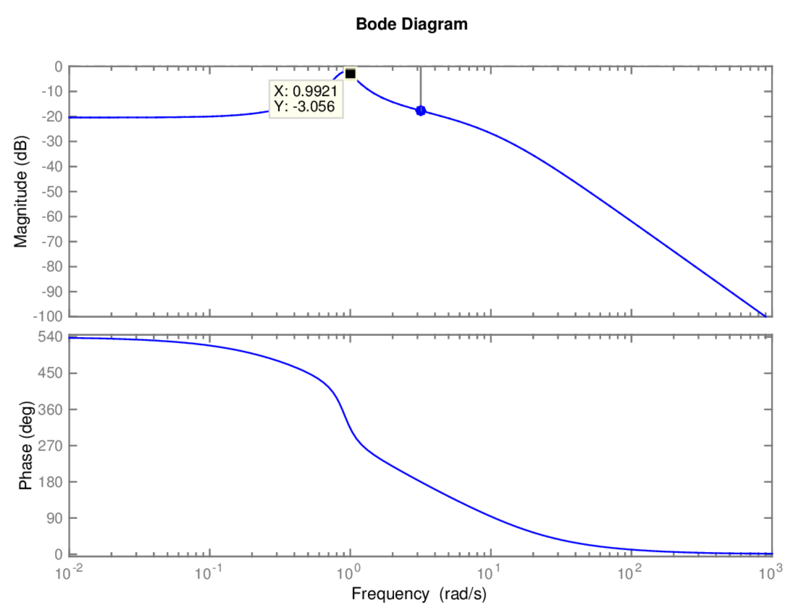

Figure 6: r-loop closed-loop Bode with bandwidth of 1 rad/s (3dB criterion)

3 Final Simulink Diagram

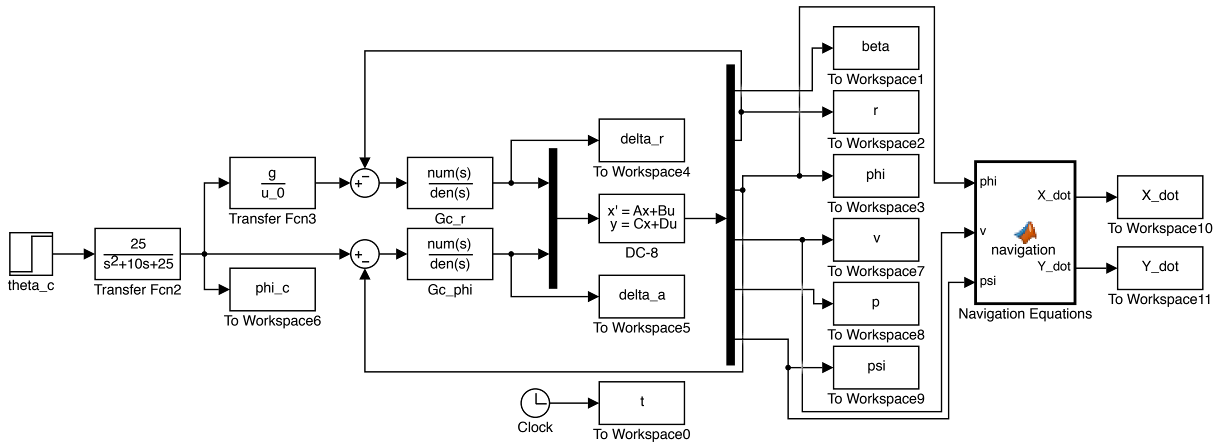

Figure 7: Simulink Diagram

Where the navigation function is defined as

```
function [X_dot, Y_dot] = navigation(phi, v, psi)
%#codegen

theta = 0;
U = 468.2;
V = v;
W = 0;

X_dot = U * (cos(psi) * cos(theta)) + ...
        V * (cos(psi) * sin(theta) * sin(phi) - sin(psi) * cos(phi)) + ...
        W * (cos(psi) * sin(theta) * cos(phi) + sin(psi) * sin(phi));

Y_dot = U * (sin(psi) * cos(theta)) + ...
        V * (sin(psi) * sin(theta) * sin(phi) - cos(psi) * cos(phi)) + ...
        W * (sin(psi) * sin(theta) * cos(phi) + cos(psi) * sin(phi));
```

4 Results

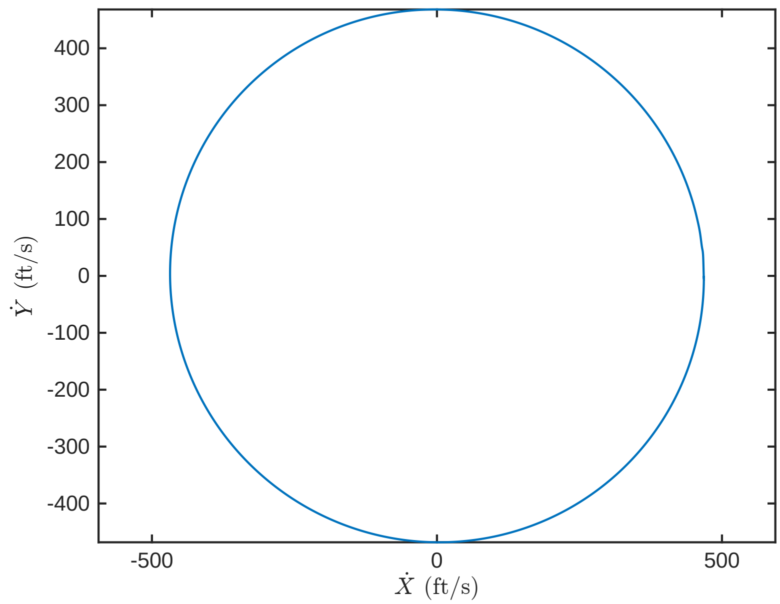

Figure 8: Demonstration of turn-coordination from ~ 267 second simulation

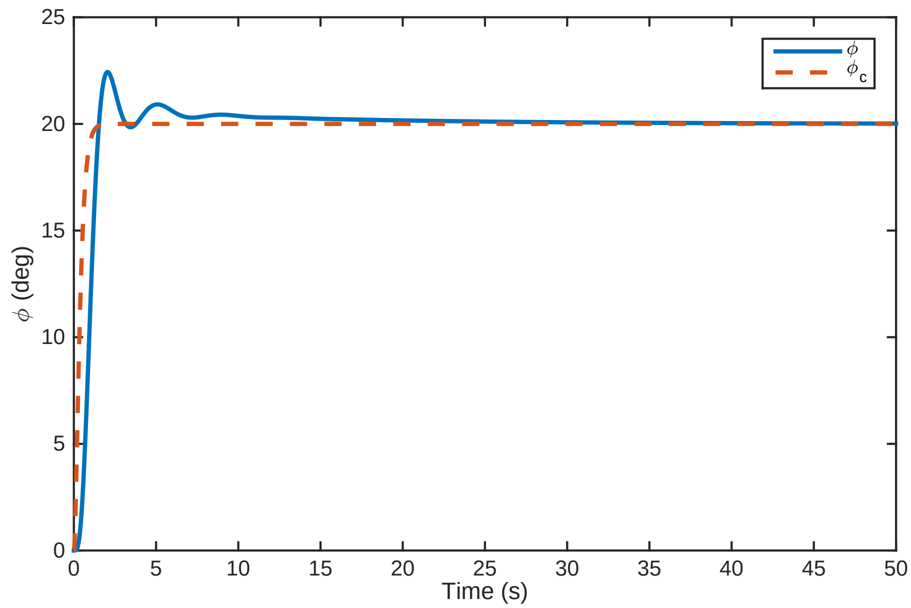

Figure 9: ϕ response to ϕ_c, showing $\sim 10\%$ overshoot

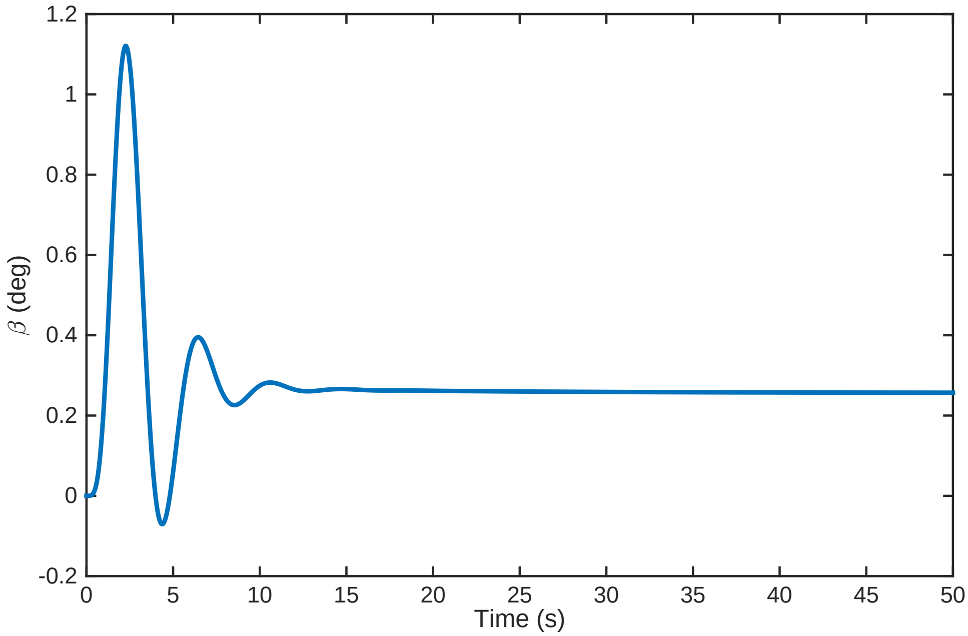

Figure 10: β Response

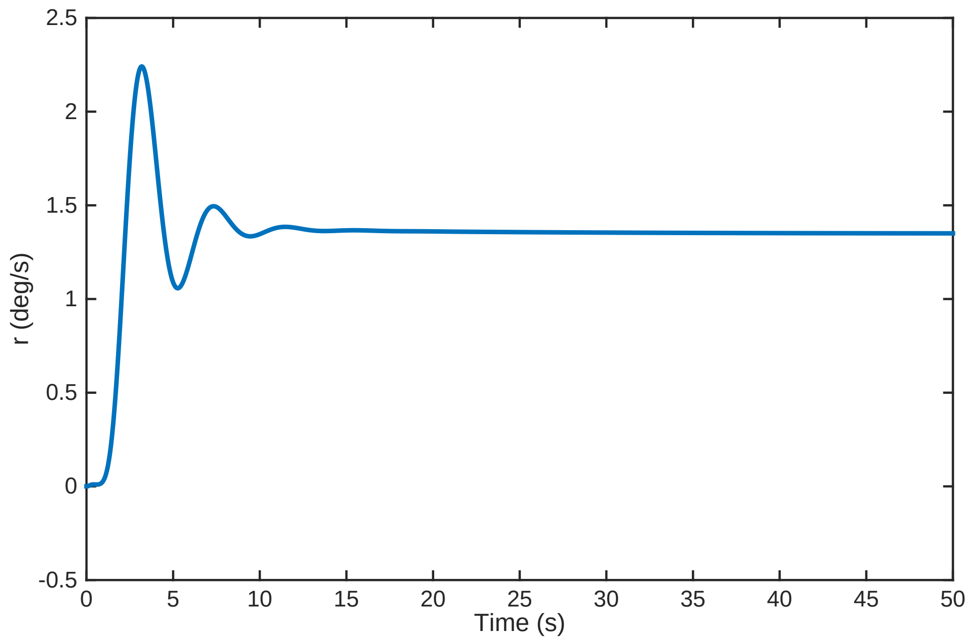

Figure 11: r Response

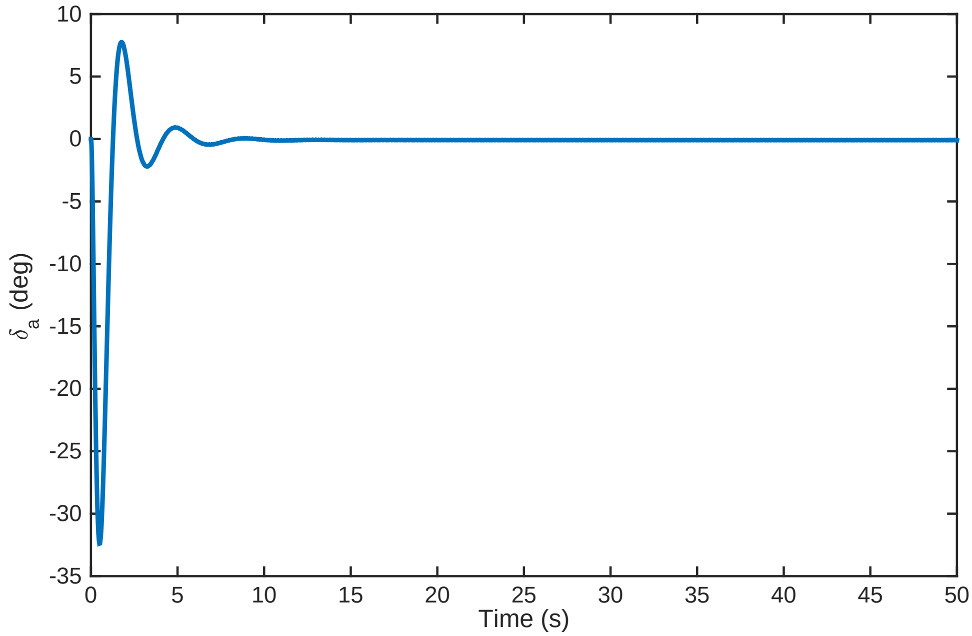

Figure 12: δ_a Input

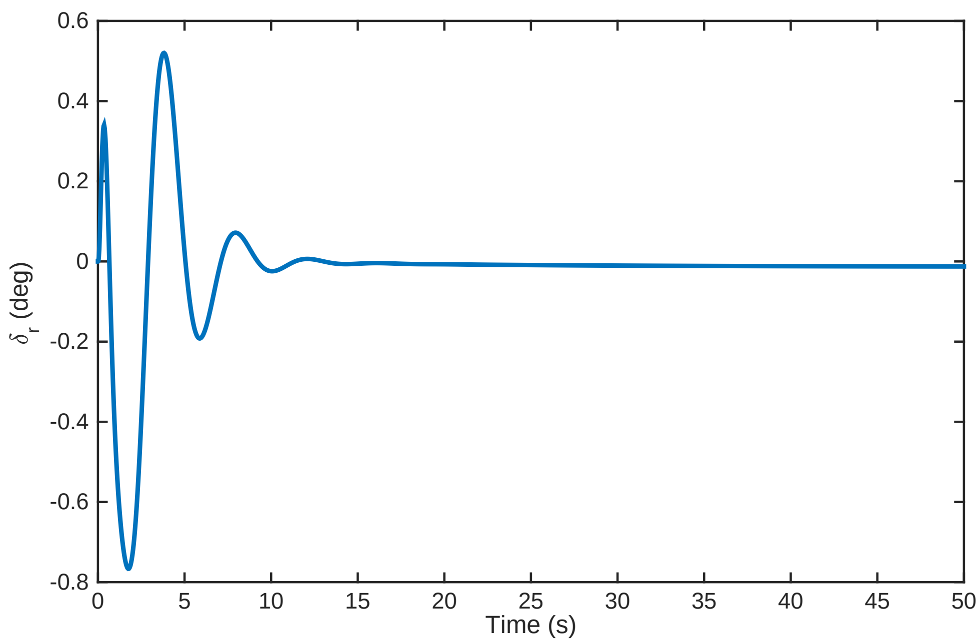

Figure 13: δ_r Input