EME – 275

Principal Assumptions Invoked in Equation of Motion Development

Assumption 1. The airframe is assumed to be a rigid body.

In a rigid body the distances between any specified points in the body are fixed, so this assumption eliminates consideration of the forces acting between individual elements of mass. Consequently, the airframe motion can be described completely by a translation of the center of mass and by a rotation about this point.

Actual vehicles depart from the rigid-body assumptions in two ways — they are composed of several major elements which are required to move relative to one another, such as engines, rotors, or control devices; and incidental elastic deformations of the structure do occur, as in wing bending caused by air loads. Some of the changes required in the description of the aerodynamic forces due to such static deflection characteristics are illustrated later in this chapter (§4.9). Other changes, involving the dynamics of the structure* which can greatly increase the degrees of freedom to be considered in the equations of motion, are beyond the scope of the present treatment.

Assumption 2. The earth is assumed to be fixed in space.

The inertial frame of reference defined by this assumption, i.e., one which is fixed or moves at constant velocity relative to the earth, permits a description of vehicle motion which is accurate for relatively short term guidance and control analysis purposes. It does have practical limitations when very long term navigation or extra-atmospheric operations are of interest.

Assumption 3. The mass and mass distribution of the vehicle are assumed to be constant.

Actually there may be considerable differences in mass and its distribution throughout a mission as fuel is burned, stores expended, etc. The assumption is, nonetheless, ordinarily reasonable because the rates of change are relatively small and may be safely neglected for the time periods covered by most analyses.

Assumption 4. The XZ plane is assumed to be a plane of symmetry.

Assumption 4 is a very good approximation for most airborne vehicles. When it applies, we see from Fig. 4-8 that there is both a positive and a negative value of y for each value of x and z; consequently, $I_{yz} = \int yz\, dm = 0$ and $I_{xy} = \int xy\, dm = 0$.

Assumption 5

The disturbances from the steady flight conditions are assumed to be small enough so that the sines and cosines of the disturbance angles are approximately the angles themselves and 1, respectively, and so that the products and squares of the disturbance quantities are negligible in comparison with the quantities themselves.

Assumption 6

The steady lateral trim conditions are assumed to be $P_0 = R_0 = V_0 = \Phi_0 = 0$, and the longitudinal forces and moments <u>due to lateral perturbations about such trim conditions are assumed negligible</u>.

Assumption 6.1

Stability axes are being used, i.e., $W_0 = 0$.

Assumption 7. The aerodynamics are assumed to be quasi-steady.

Because of Assumption 7 all derivatives with respect to the rates of change of velocities are omitted with the exception of those with respect to $\dot{w}$ and $\dot{v}$, which are retained to account for the effect on the tail of the wing/body downwash and sidewash. This effect is present, as explained below, even when purely quasi-steady considerations apply.

Assumption 8

Variations of atmospheric properties, such as density or speed of sound, are considered negligible for the small altitude perturbations of usual interest.

Assumption 9

Effects associated with rotation of the vertical relative to inertial space will be assumed negligible; furthermore, the trim body-axis pitching velocity, Q_0, will be assumed zero.

Assumption 10. It is assumed that $X_{\dot{w}} = X_q = Z_{\dot{w}} = Z_q = 0$.

Perhaps the best general evidence in justification of Assumption 10 is that the derivatives named in it rarely appear in the technical literature concerned with aircraft dynamics. The inference here is that although individual investigators have evaluated the effects of these derivatives for a multitude of various airframe configurations, they have found them to be of only secondary importance. However, it must be remembered that if any of these derivatives are of actual importance for a particular airframe, this assumption may produce somewhat erroneous quantitative results for that airframe.

In general, any stability derivative may be neglected if it is first determined that the term containing the given derivative is small in comparison with other terms in the same equations. For the derivatives in question, comparing the term $X_{\dot{w}}sw$ with $X_w w$ shows that if the frequency range of interest extends as high as $|s| \doteq X_w/X_{\dot{w}}$, $X_{\dot{w}}$ can no longer be neglected a priori. Similarly, the upper frequency limit for the valid a priori neglect of X_q is $|s| \doteq |g/X_q|$. As Z_q occurs in the group $(Z_q + U_0)sw$, the appropriate criterion here is $|Z_q| \ll U_0$; and by grouping $\dot{w}$ terms together as $(s - Z_{\dot{w}}s)w$ the criterion for neglecting $Z_{\dot{w}}$ is $|Z_{\dot{w}}| \ll 1$.

Assumption 11. In the steady flight condition, the flight path of the airplane is assumed to be horizontal, $\gamma_0 = 0$.

Assumption 11 is introduced solely to simplify the mechanics of the analysis. When the flight path of an airplane is initially inclined to the horizontal, γ_0 must of course be included in the transfer functions.

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Assumption 12. It is assumed that $Y_{\dot{v}} = Y_p = Y_r = L'_{\dot{v}} = N'_{\dot{v}} = 0$.

Generally speaking, this is a good assumption for most configurations, especially when only control inputs are being considered. Notice, however, that for gust inputs the assumption eliminates completely the p_g term in the Y equation, and requires $|Y_{\dot{v}}s| \ll Y_v$ in addition to the more readily evaluated $|Y_{\dot{v}}| \ll 1$. Also, in the N equation $N'_r g/U_0$ is retained while $N'_{\dot{v}}$ is dropped, whereas they may both be of the same magnitude. In all cases the validity of the individual assumptions should be checked for the frequency range of interest, once the derivatives are known, by the process described on page 5-3 in connection with Assumption 10. Despite such reservations the approximations generally introduce only small errors, and are in accord with common flight control practice.