# MAE 275 - Midterm

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## 1 Defining the System

The state-space system can be defined,

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} + D\vec{u}$$

where the linearized longitudinal aircraft equations of motion can be expressed in state space form.

$$\vec{x} = \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix}, \qquad \vec{u} = \begin{bmatrix} \delta_e \\ \delta_T \\ u_g \end{bmatrix}, \qquad \vec{y} = \begin{bmatrix} u \\ \alpha \\ h \\ \dot{h} \\ \theta \end{bmatrix}$$

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & -\dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} & -X_u \\ \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \dfrac{Z_{\delta_T}}{1-Z_{\dot{w}}} & \dfrac{-Z_u}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} & M_{\delta_T} + \dfrac{M_{\dot{w}}Z_{\delta_T}}{1-Z_{\dot{w}}} & -M_u - \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{u_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & u_0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Using the longitudinal equations of motion for C-5A for level flight ($u_0 = 246$ ft/s) at sea level, the resultant system is

$$\dot{\vec{x}} = \begin{bmatrix} -0.0214 & +0.0957 & 0 & -32.2 & 0 \\ -0.231 & -0.634 & +246 & 0 & 0 \\ +1.964\times10^{-4} & -8.895\times10^{-4} & -0.8275 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +246 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} +0.45 & +0.554\times10^{-4} & +0.0214 \\ -9.53 & -0.193\times10^{-5} & +0.231 \\ -0.6795 & +1.4571\times10^{-7} & -1.9642\times10^{-4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{u}$$

$$\vec{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.0041 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 246 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{u}$$

The loops are sequentially closed in the order: $\theta \to \delta_e$, $u \to \delta_T$, $\dot{h} \to \theta_e$. The following pages lists the linearized transfer functions used to design the compensators, along with the transfer function of each compensator. The gain and phase margins and bandwidth are also listed for the compensated loop.

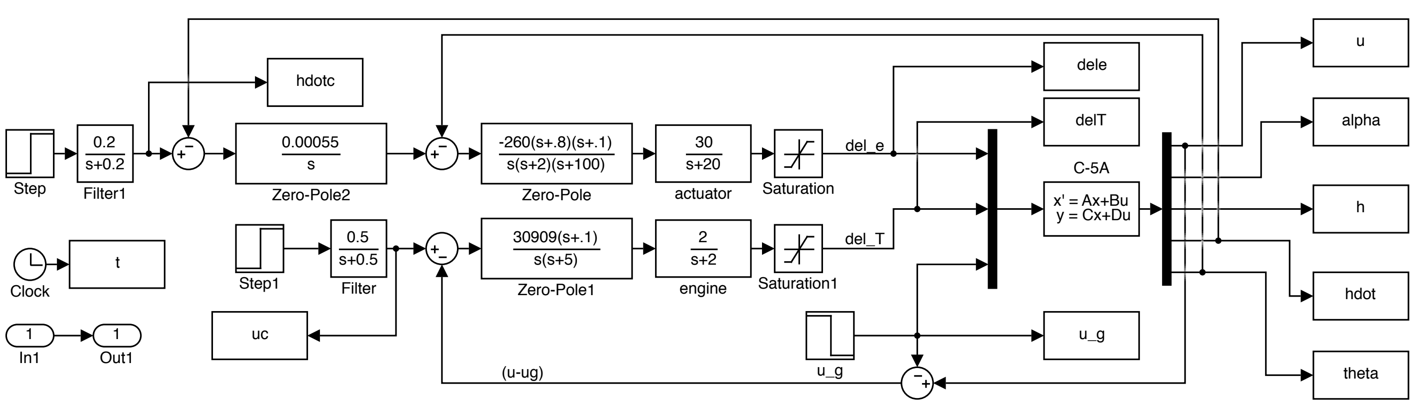


Figure 1: Final Simulink Model

## 2 Compensators

$$\frac{\theta}{\delta_e} = \frac{-20.387(s+0.5819)(s+0.06093)}{(s+20)(s^2+0.02027s+0.01411)(s^2+1.463s+0.7531)}$$

$$\boxed{Gc_\theta = -260 \times \frac{(s+0.8)(s+0.1)}{s(s+2)(s+100)}}$$

$$G_m = 22.4\,\text{dB at } 5.64\,\text{rad/s}$$
$$P_m = 57.9^\circ \text{ at } 1.18\,\text{rad/s}$$
$$\omega_{BW} = 1.98\,\text{rad/s (3dB criterion)}$$

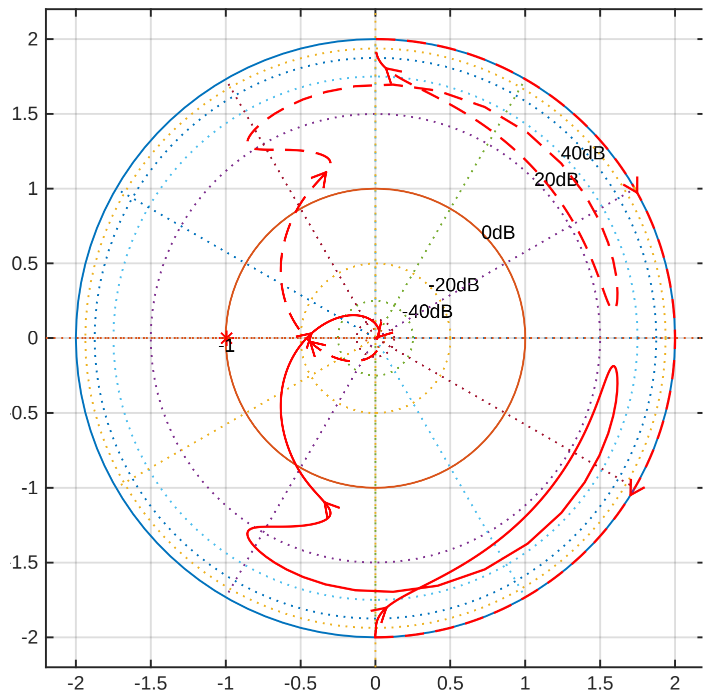


Figure 2: Nyquist Diagram for $\frac{\theta}{\delta_e}$

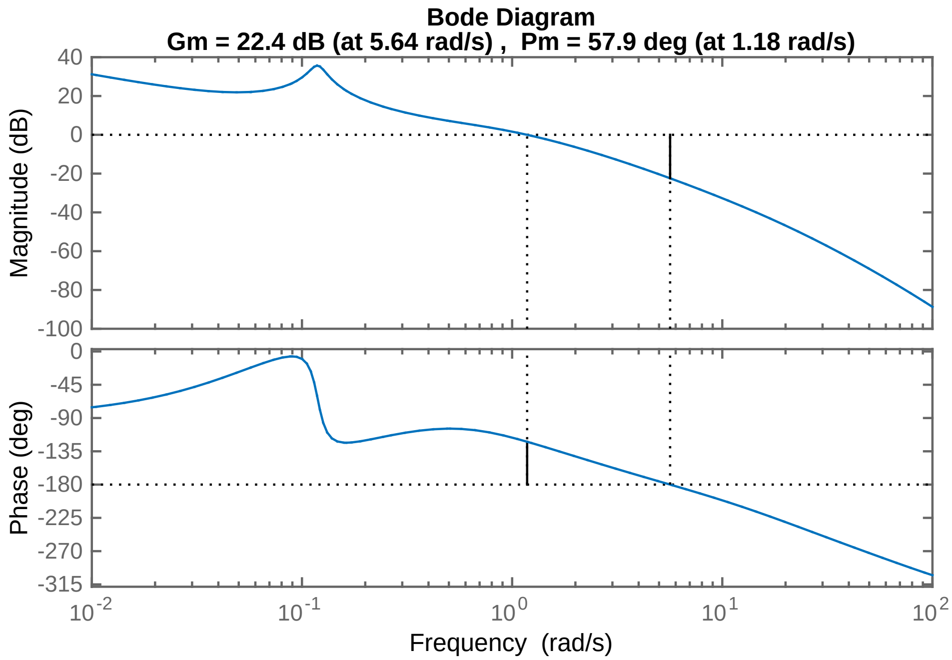


Figure 3: Open-loop Compensated Bode Diagram for $\frac{\theta}{\delta_e}$

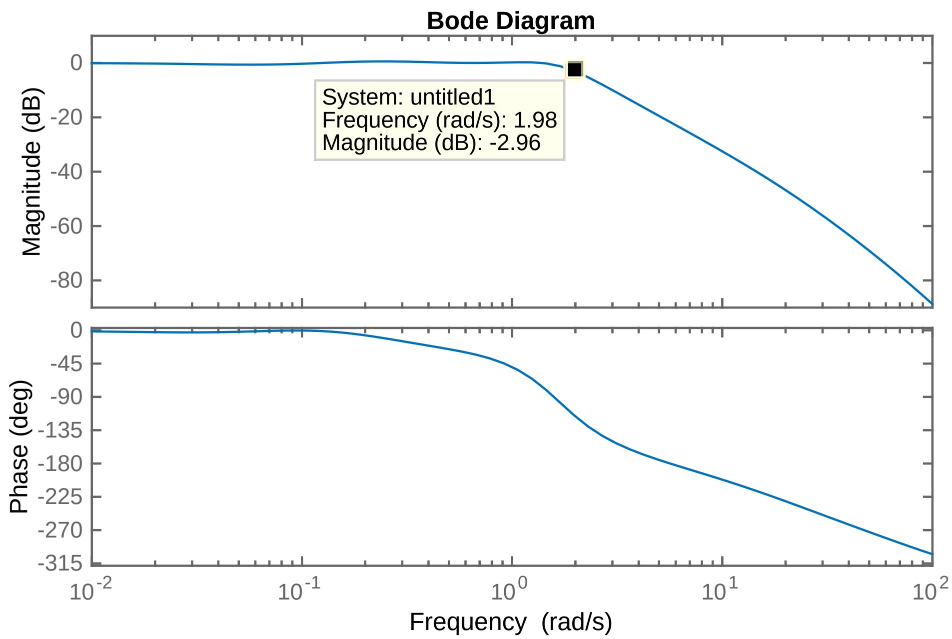


Figure 4: Close-loop Compensated Bode Diagram for $\frac{\theta}{\delta_e}$

$$\frac{u}{\delta_T} = \frac{0.0001108(s+1.093)(s+0.3076)(s+0.1381)(s^2+1.745s+2.785)}{(s+2)(s+1.078)(s+0.0452)(s^2+0.4706s+0.05552)(s^2+1.716s+2.754)}$$

$$\boxed{Gc_{\dot{h}} = 30909 \times \frac{(s+0.1)}{s(s+5)}}$$

$$G_m = 25.7\,\text{dB at}\,3.08\,\text{rad/s}$$
$$P_m = 72.3^\circ\,\text{at}\,0.351\,\text{rad/s}$$
$$\omega_{BW} = 0.499\,\text{rad/s (3dB criterion)}$$

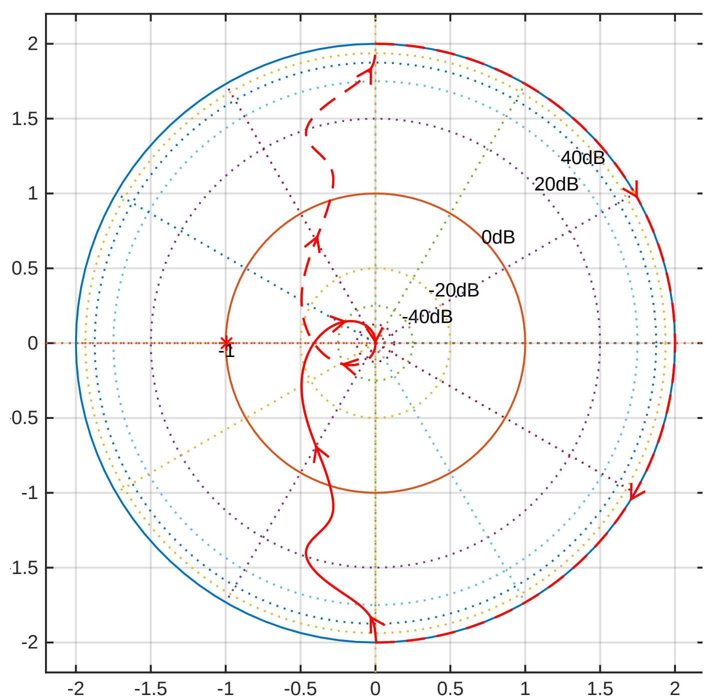


Figure 5: Nyquist Diagram for $\dfrac{u}{\delta_T}$

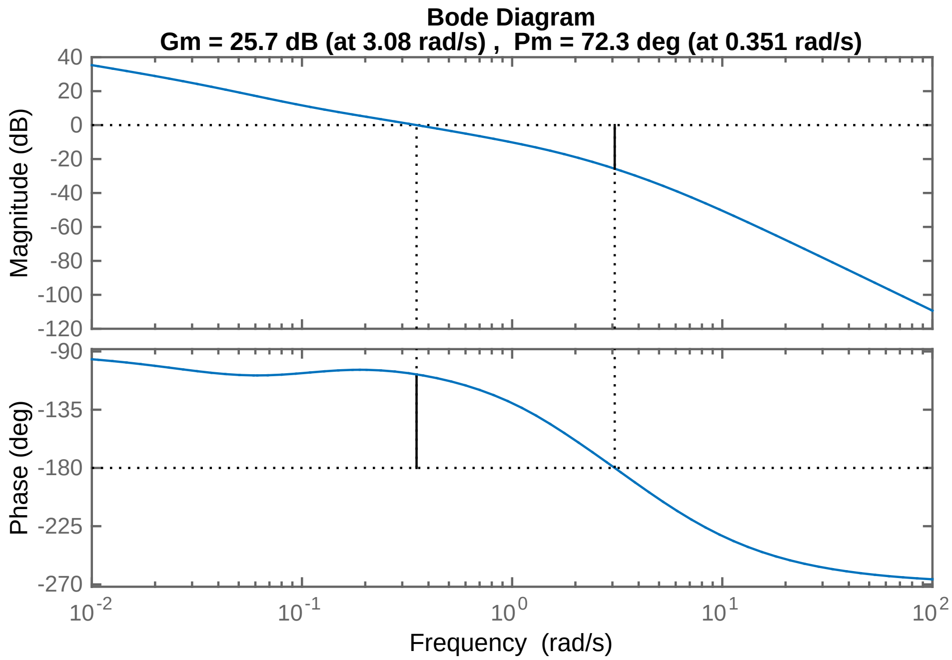


Figure 6: Open-loop Compensated Bode Diagram for $\frac{u}{\delta_T}$

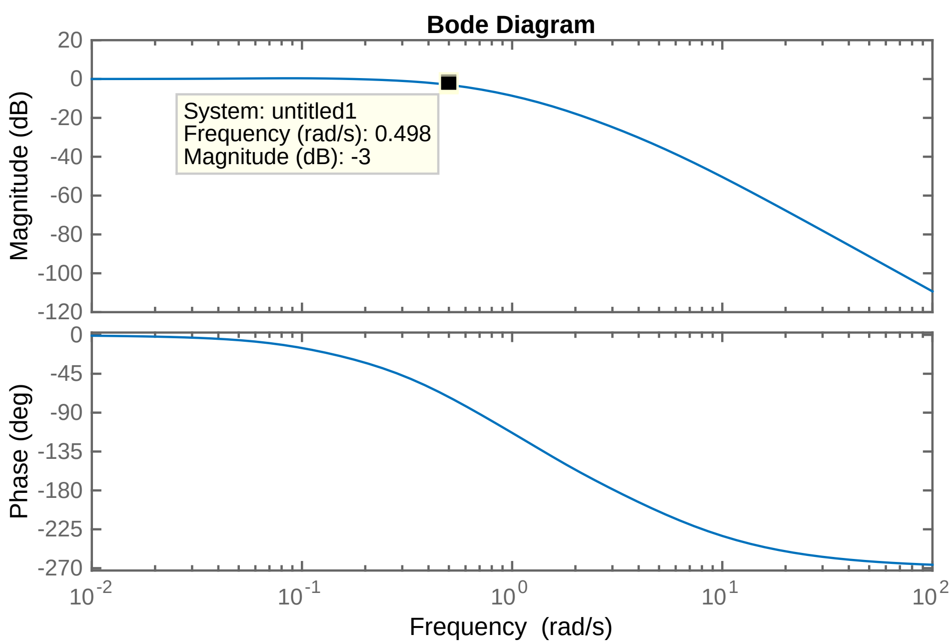


Figure 7: Close-loop Compensated Bode Diagram for $\frac{u}{\delta_T}$

$$\frac{\dot{h}}{\theta_e} = \frac{-74334(s+3.752)(s+1.35)(s+0.8)(s+0.2827)(s+0.1736)(s+0.1)(s-2.901)}{(s+99.99)(s+20.18)(s+1.371)(s+0.9979)(s^2+0.2143s+0.01295)(s^2+0.8171s+0.1759)(s^2+1.707s+2.74)}$$

$$\boxed{Gc_u = 0.00055 \times \frac{1}{s}}$$

$$G_m = 19.8\,\mathrm{dB}\ \text{at}\ 0.785\,\mathrm{rad/s}$$
$$P_m = 68.1^\circ\ \text{at}\ 0.132\,\mathrm{rad/s}$$
$$\omega_{BW} = 0.202\,\mathrm{rad/s}\ \text{(3dB criterion)}$$

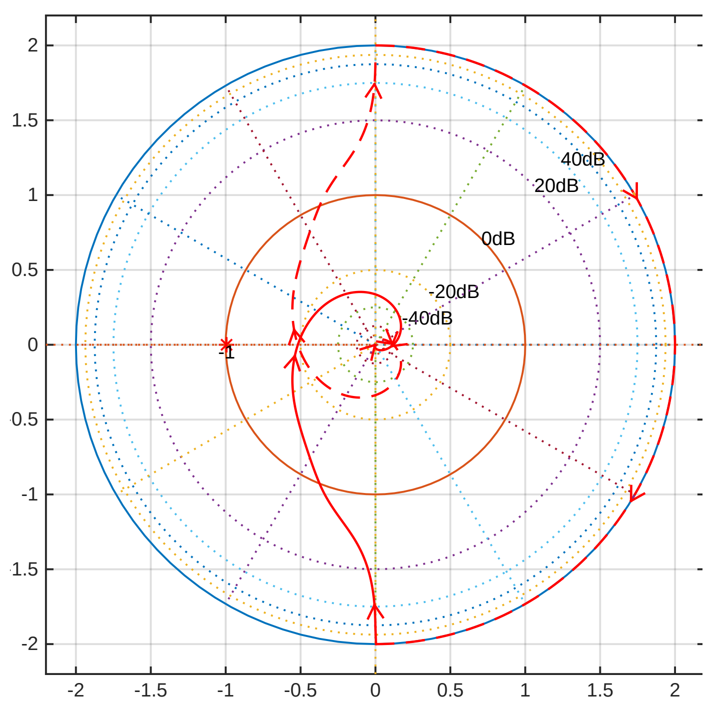


Figure 8: Nyquist Diagram for $\frac{\dot{h}}{\theta_e}$

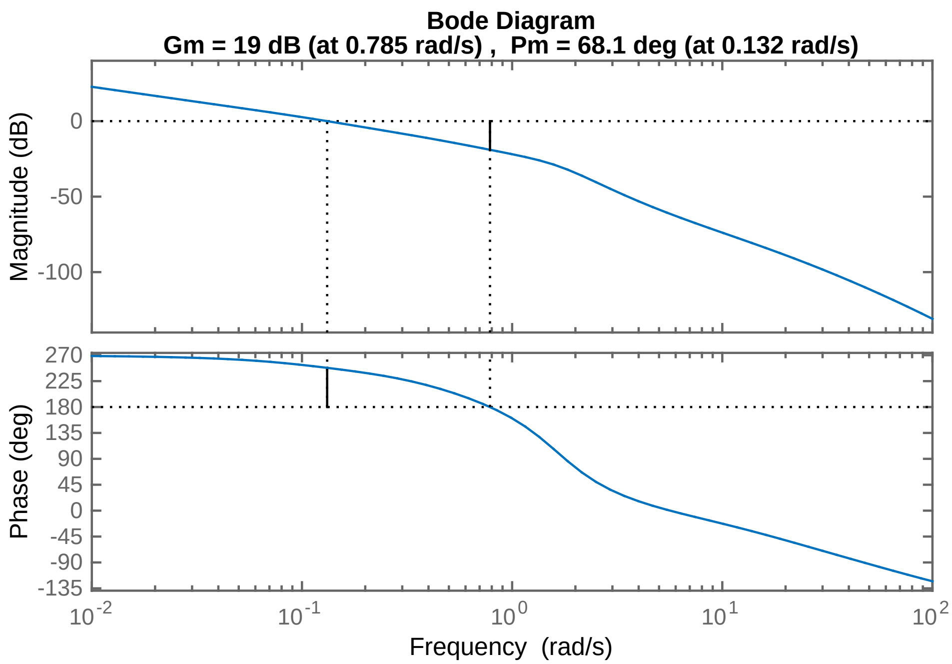


Figure 9: Open-loop Compensated Bode Diagram for $\dfrac{\dot{h}}{\theta_e}$

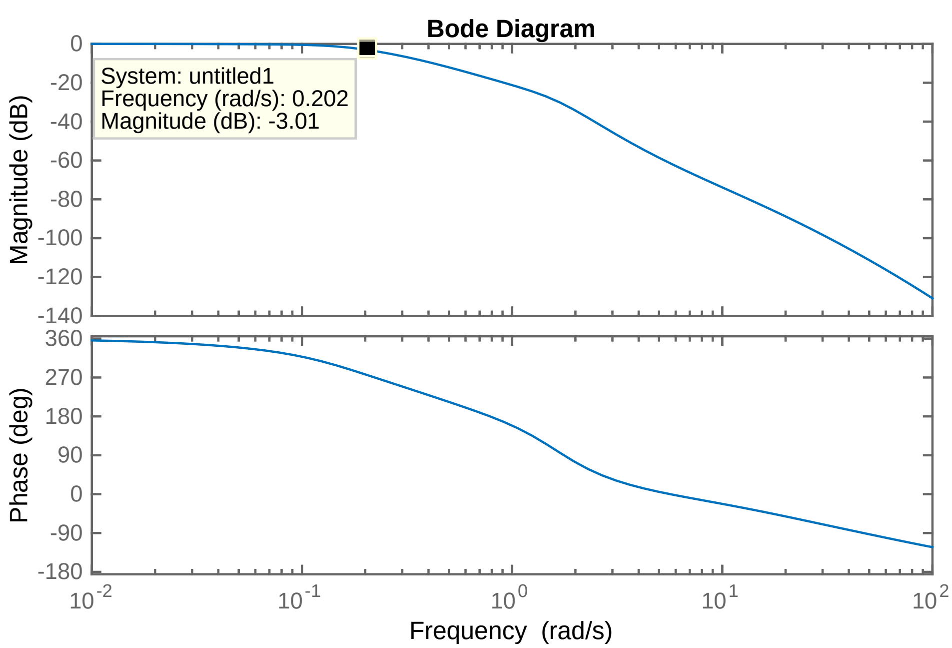


Figure 10: Close-loop Compensated Bode Diagram for $\dfrac{\dot{h}}{\theta_e}$

# 3 Response to Inputs

Two initial conditions were investigated: 1) a filtered step altitude-rate command of 20 ft/sec and a filtered step airspeed command of 20 ft/sec, applied simultaneously; 2) a step $u_g$ of 20 ft/sec (a tail wind), without the altitude and airspeed commands. The aircraft reaches steady-state in $\sim$ 50 sec for both scenarios.

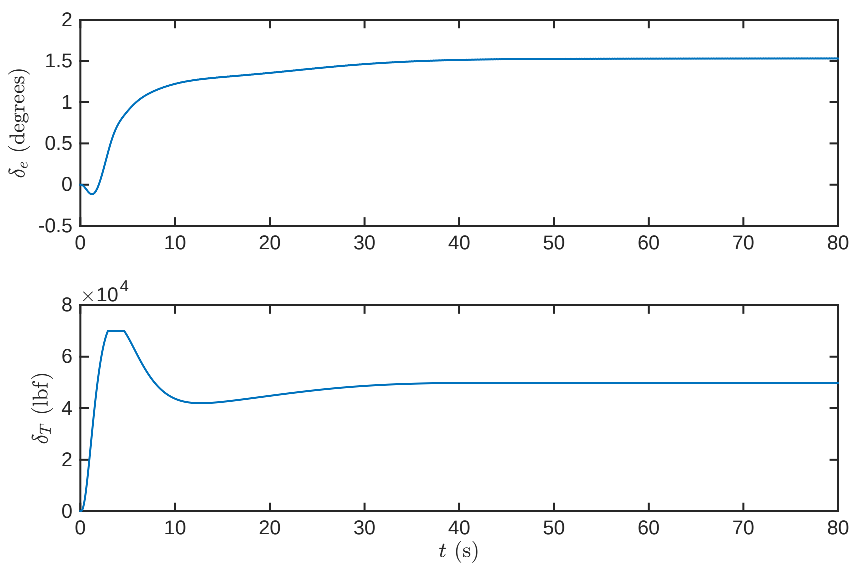


Figure 11: Commanded Inputs for Scenario 1

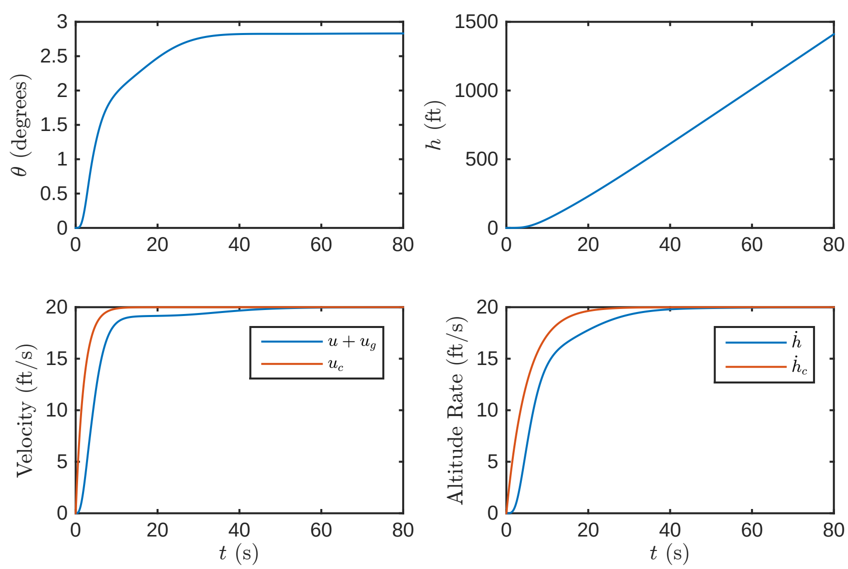


Figure 12: Outputs for Scenario 1

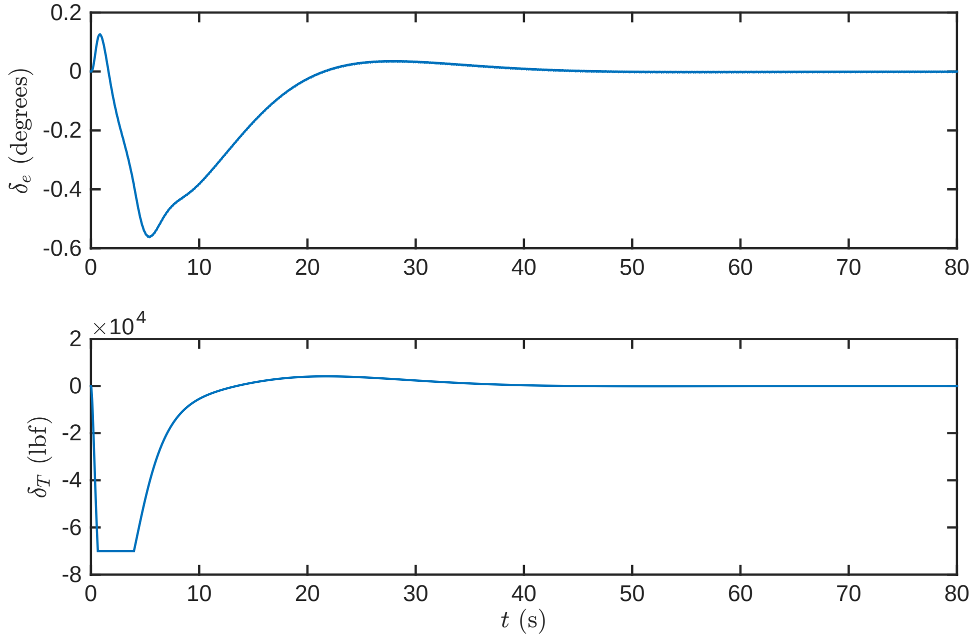


Figure 13: Commanded Inputs for Scenario 2

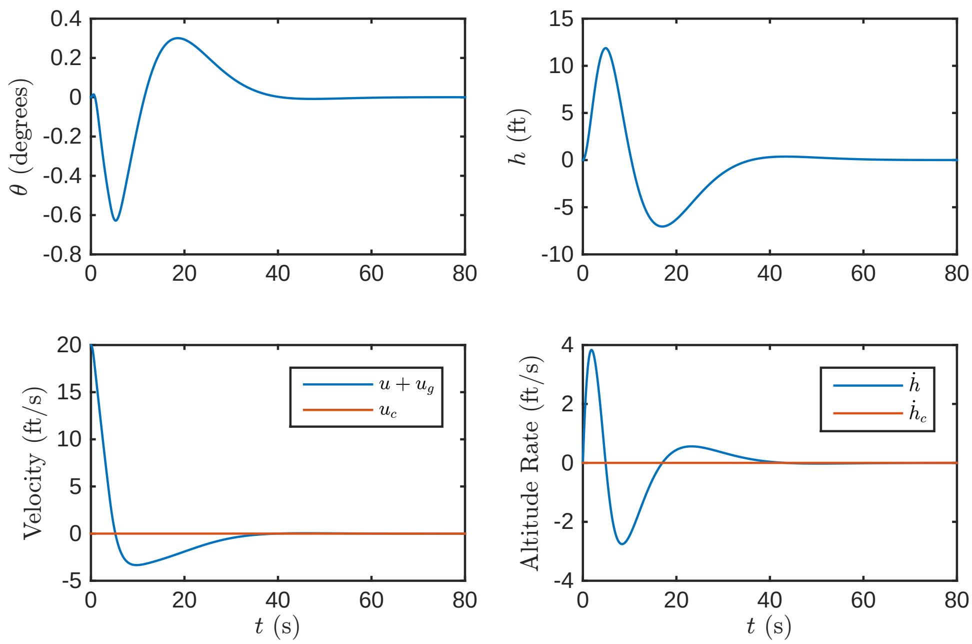


Figure 14: Outputs for Scenario 2