MAE 275 - Homework 6

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1 Defining the System

The state-space system can be defined,

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} + D\vec{u}$$

where the linearized lateral aircraft equations of motion can be expressed in state space form.

$$\vec{x} = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix}, \qquad \vec{u} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}, \qquad \vec{y} = \begin{bmatrix} p \\ \beta \end{bmatrix}$$

Using the lateral equations of motion for F-89 at flight condition 8901, the resultant system is

$$\dot{\vec{x}} = \begin{bmatrix} -8.2900e-2 & 0 & -6.6000e+2 & +3.2200e+1 & 0 \\ -6.8939e-3 & -1.7000e+0 & +1.7200e-1 & 0 & 0 \\ +5.1212e-3 & -6.5400e-2 & -8.9300e-2 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & +7.6500e+0 \\ +2.7300e+1 & +5.7600e-1 \\ +3.9300e-1 & -1.3600e+0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}$$

$$\vec{y} = \begin{bmatrix} 0 & +1 & 0 & 0 & 0 \\ +1.5152e-3 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}$$

Coupling numerators can be used to determine input-output pairing and which loops to close in which order, and appropriate compensators can then be designed.

2 Coupling Numerators

The coupling numerators can be derived using the notes in the assignment and the ICE example in the coupling numerators handout. They are

$$\boxed{\frac{p}{\delta_a} = \frac{27.3s(s^2+0.1747s+3.453)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}}$$

$$\frac{p}{\delta_r} = \frac{0.576s(s-2.885)(s+2.56)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\frac{\beta}{\delta_a} = \frac{-0.393(s-6.282)(s+0.04952)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\frac{\beta}{\delta_r} = \frac{0.011591(s+117.4)(s+1.753)(s-0.003733)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\left.\frac{p}{\delta_a}\right|_{\beta\to\delta_r} = \frac{27.3s(s+118.1)}{(s+117.4)(s+1.753)(s-0.003733)}$$

$$\boxed{\left.\frac{\beta}{\delta_r}\right|_{p\to\delta_a} = \frac{0.011591(s+118.1)}{(s^2+0.1747s+3.453)}}$$

$$\left.\frac{p}{\delta_r}\right|_{\beta\to\delta_a} = \frac{0.54936(s+118.1)}{(s-2.885)(s+2.56)}$$

$$\left.\frac{\beta}{\delta_a}\right|_{p\to\delta_r} = \frac{-0.80517s(s+118.1)}{(s-6.282)(s+0.04952)}$$

From these transfer functions we can:

1. rule out controlling p with δ_r first, as there is a non-minimum phase zero (s-2.885) in the transfer function that would limit the crossover frequency to values significantly below 2.885
2. rule out $\left.\frac{p}{\delta_r}\right|_{\beta\to\delta_a}$ and $\left.\frac{\beta}{\delta_a}\right|_{p\to\delta_r}$ due to the closed-loop unstable poles that would be produced

This leaves only one viable option: to first close p with δ_a, then close β with δ_r with the $p-\delta_a$ loop closed. As such, the ailerons are paired to the roll-rate, and the rudder is paired to the sideslip.

3 Compensators

Two compensators were chosen. After both loops were closed, the reported gain and phase margins, and the crossover bandwidths were noted at

$$\frac{p}{\delta_a} = \frac{27.3s(s^2 + 0.1747s + 3.453)}{(s + 1.781)(s - 0.001359)(s^2 + 0.09275s + 3.529)}$$

$$\boxed{GC_p = 0.18 \times \frac{(s+2)}{s}}$$

$$\begin{aligned} G_m &= Inf\,\mathrm{dB} \\ P_m &= 86.7^\circ\ \mathrm{at}\ 5.06\,\mathrm{rad/s} \\ \omega_{BW} &= 5.06\,\mathrm{rad/s} \end{aligned}$$

$$\left.\frac{\beta}{\delta_r}\right|_{p\to\delta_a} = \frac{0.011591(s + 117.3)(s + 4.498)(s + 2.198)}{(s + 4.417)(s + 2.186)(s^2 + 0.1828s + 3.513)}$$

$$\boxed{GC_\beta = 74.4 \times \frac{(s^2 + .2s + 3.5)}{s(s+20)}}$$

$$\begin{aligned} G_m &= Inf\,\mathrm{dB} \\ P_m &= 74.4^\circ\ \mathrm{at}\ 4.96\,\mathrm{rad/s} \\ \omega_{BW} &= 4.96\,\mathrm{rad/s} \end{aligned}$$

Note that the actual value for $\left.\frac{\beta}{\delta_r}\right|_{p\to\delta_a}$ is very close to the value predicted by the coupling numerators approximation. The GC_β compensator approximately cancels the Dutch-Roll mode with a numerator of $(s^2 + .2s + 3.5)$. Open and closed-loop bode plots for both compensators are listed on the following pages.

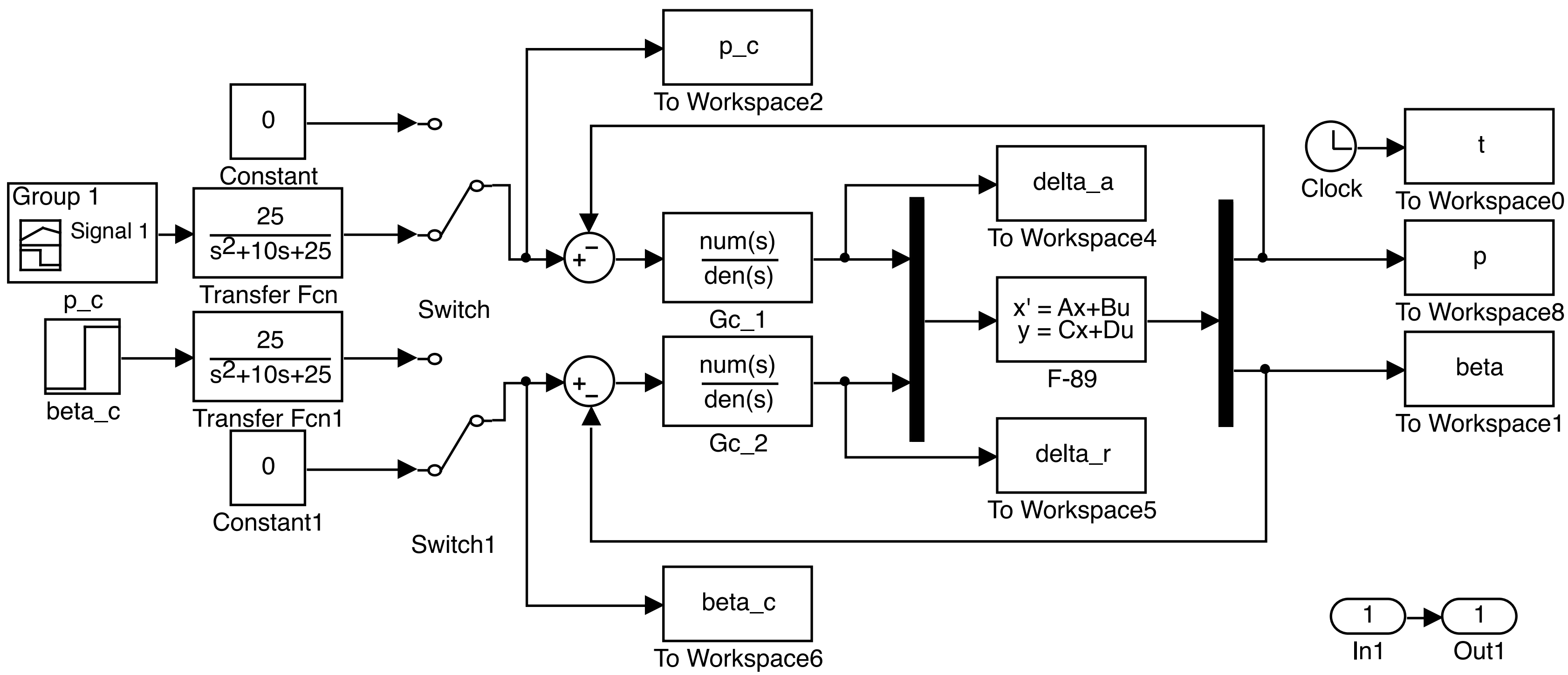

Figure 1: Final simulink diagram with both compensators

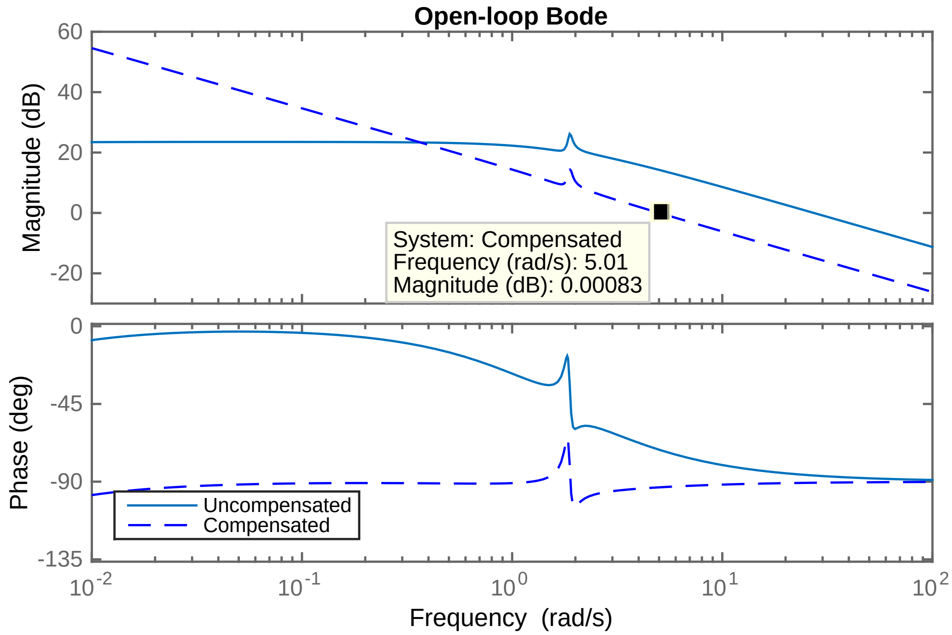

Figure 2: Open-loop Bode Plot for p loop

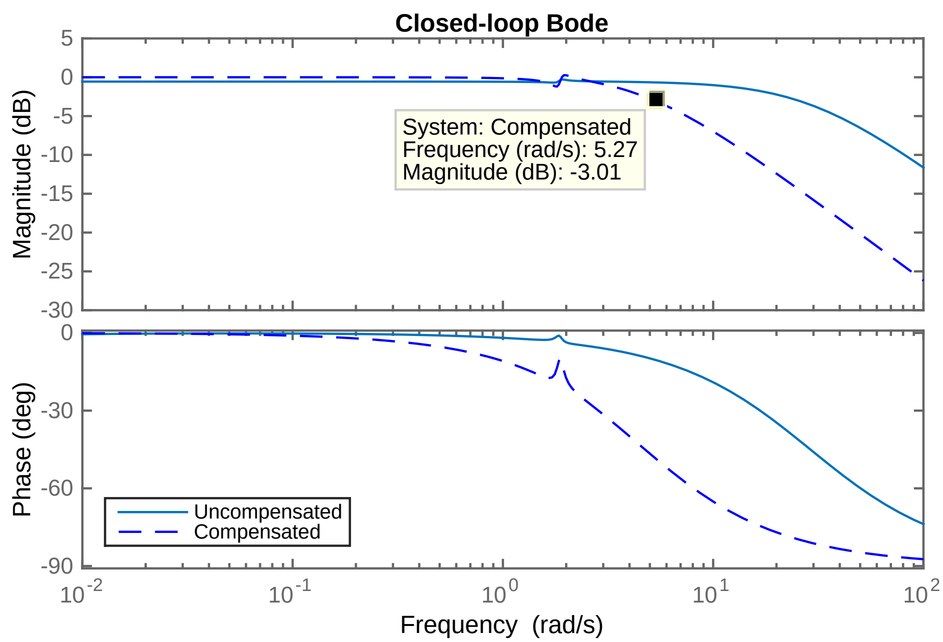

Figure 3: Close-loop Bode Plot for p loop

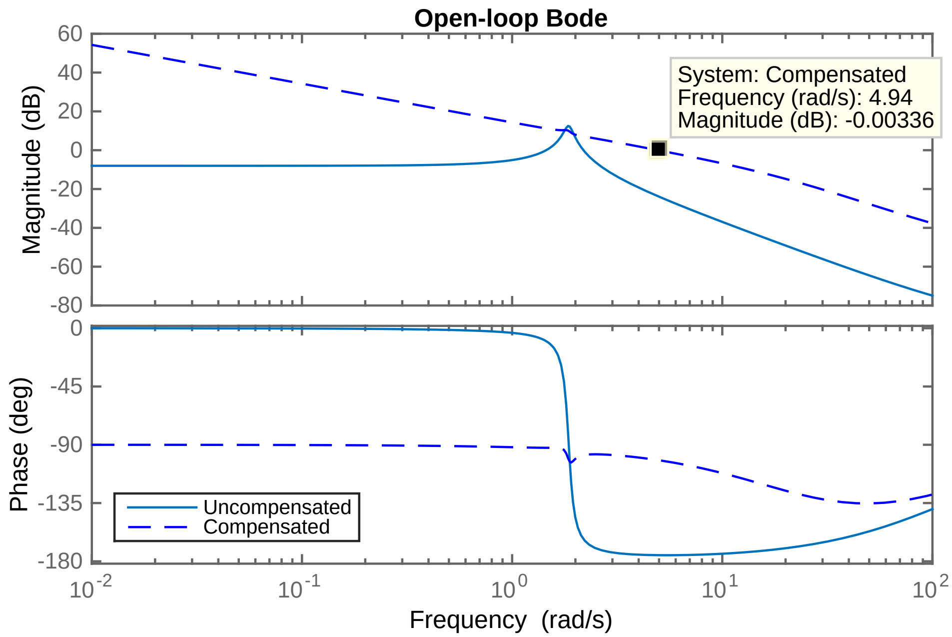

Figure 4: Open-loop Bode Plot for β loop

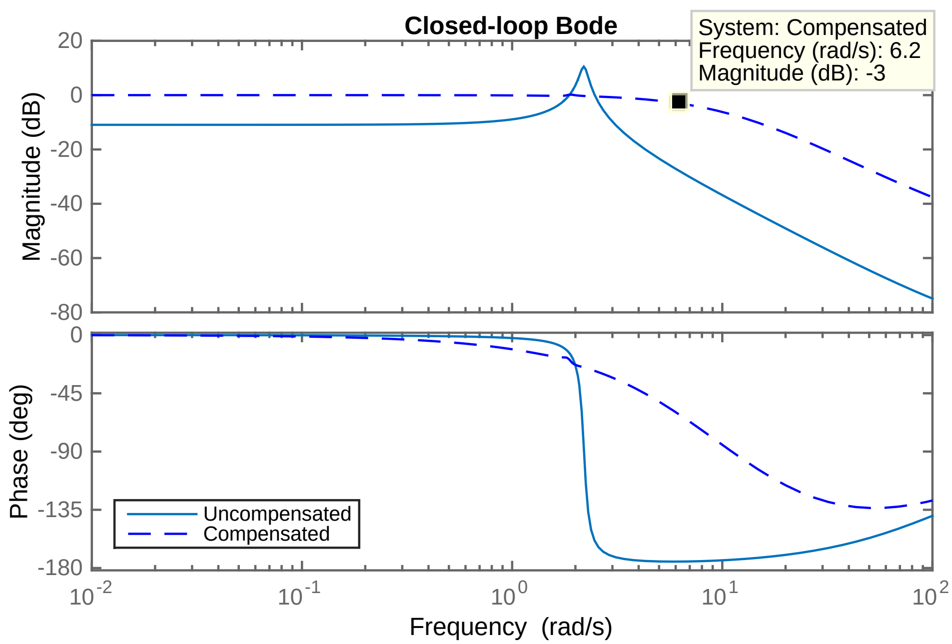

Figure 5: Close-loop Bode Plot for β loop

4 Response to Command Inputs

Two initial conditions were investigated (both commands were filtered with a filter of $\frac{25}{(s^2+10s+25)}$):

1. a ± 5 deg/sec doublet with each of the two pulses lasting 2 sec for the p-loop with no command for the β-loop
2. a $+5$ deg/sec command for the β-loop with no command for the p-loop

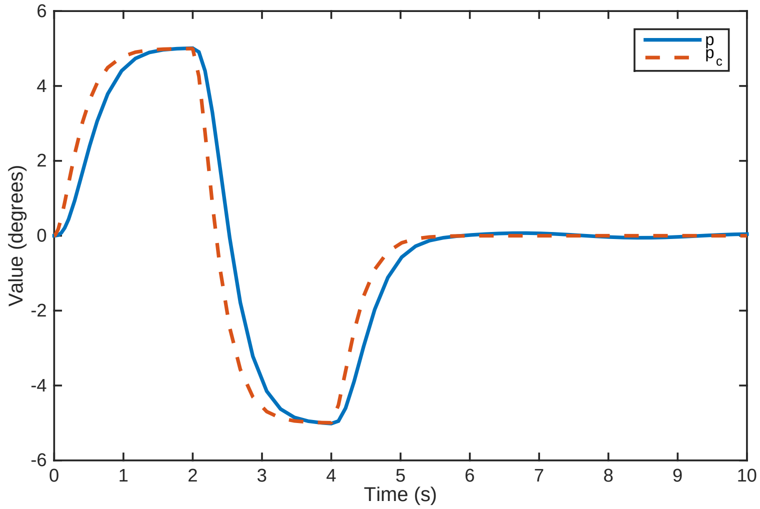

Figure 6: p Response for Scenario 1

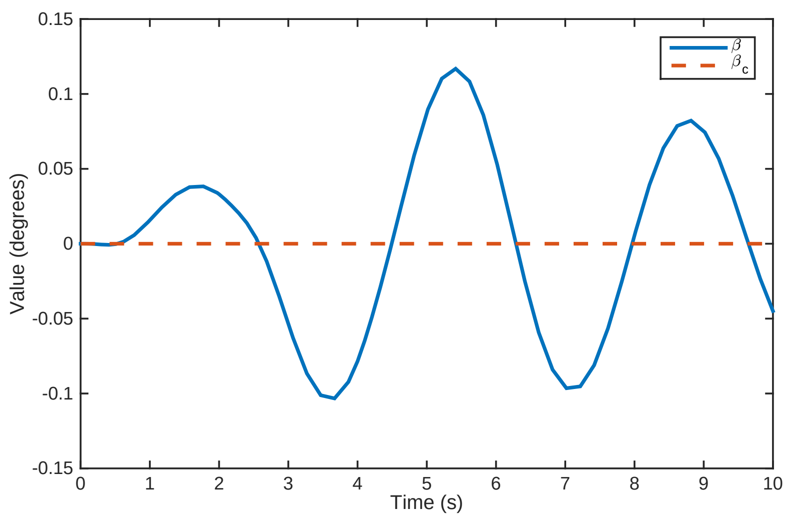

Figure 7: β Response for Scenario 1 (note the scale relative to the p response and decreasing magnitude of oscillation)

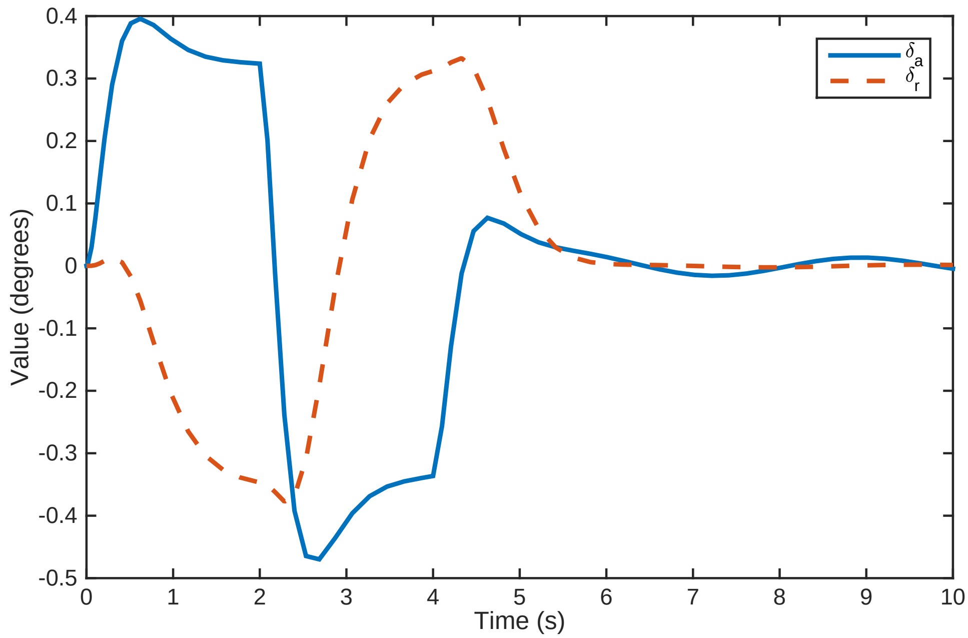

Figure 8: Command Inputs for Scenario 1

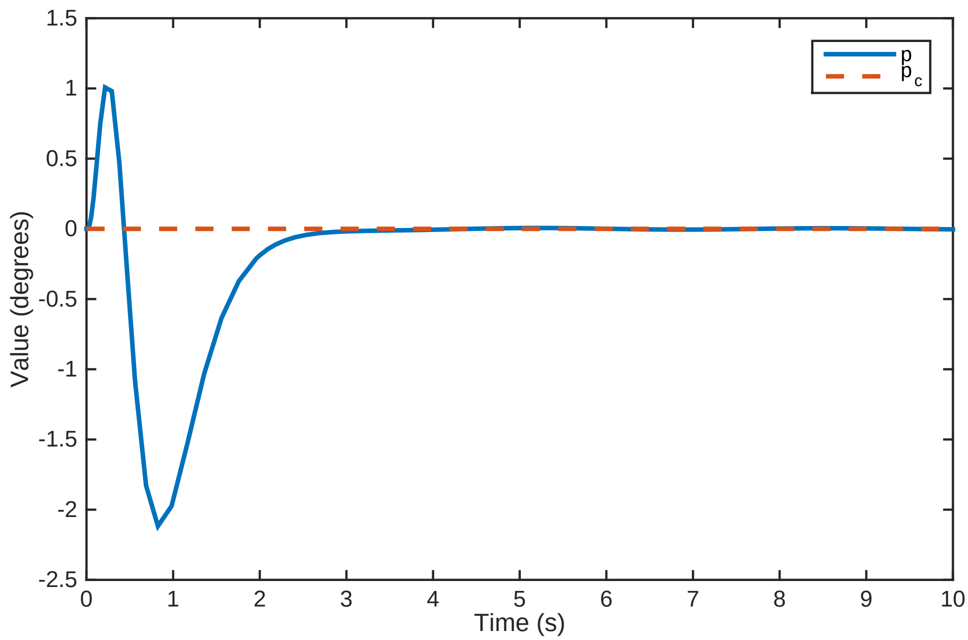

Figure 9: p Response for Scenario 2

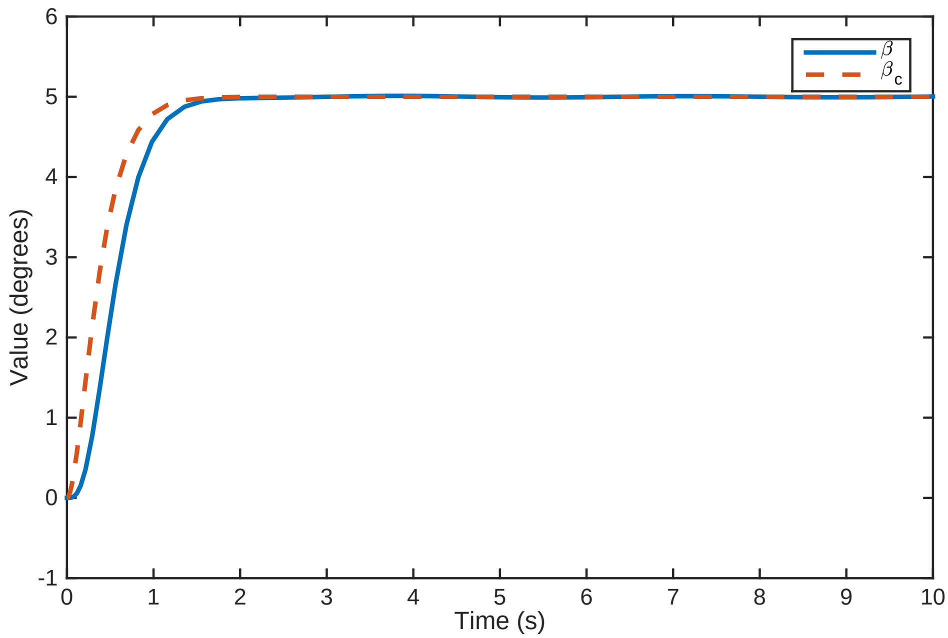

Figure 10: β Response for Scenario 2

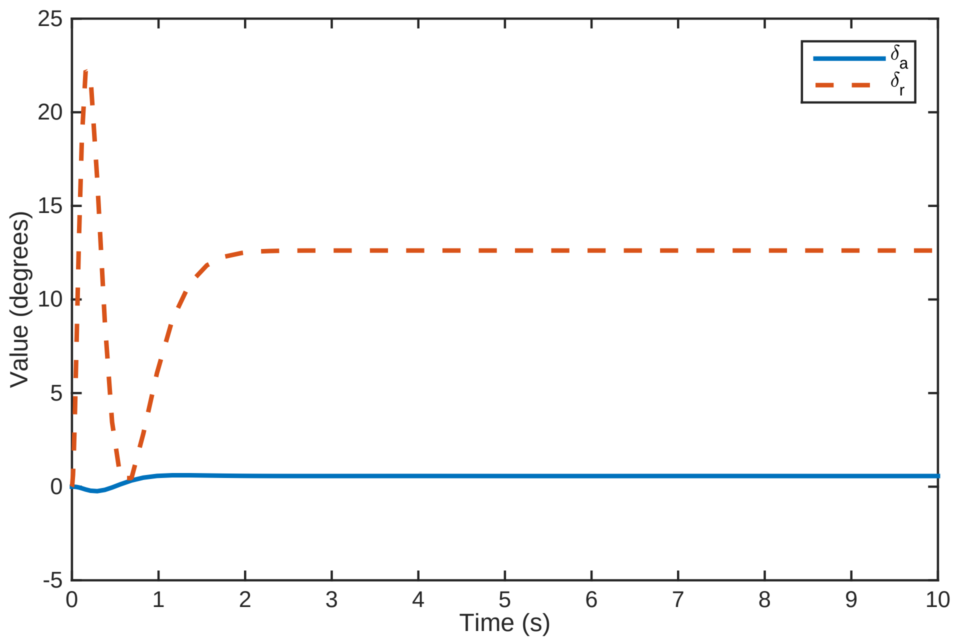

Figure 11: Command Inputs for Scenario 2