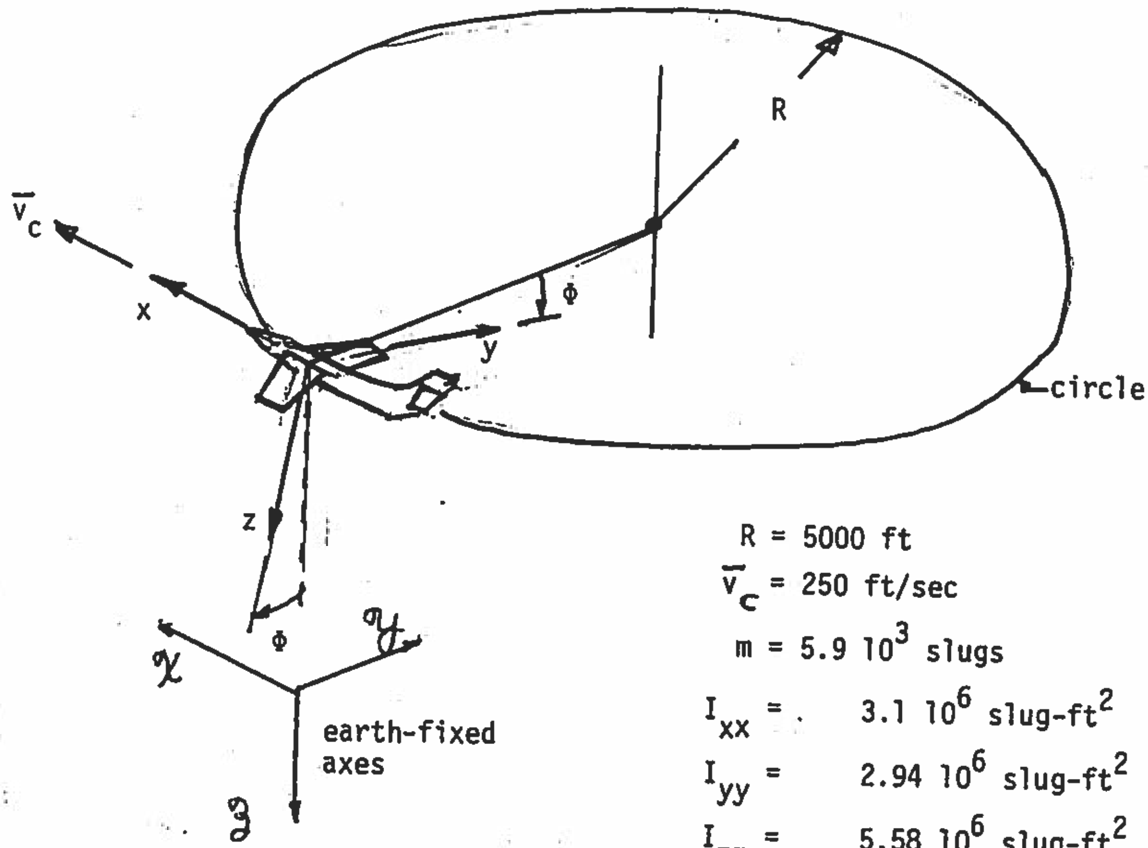
MAE - 275

Exercising the Nonlinear Equations of Motion



$R = 5000$ ft

$\overline{V}_c = 250$ ft/sec

$m = 5.9\ 10^3$ slugs

$I_{xx} = 3.1\ 10^6$ slug-ft^2

$I_{yy} = 2.94\ 10^6$ slug-ft^2

$I_{zz} = 5.58\ 10^6$ slug-ft^2

$I_{xz} = +2.8\ 10^4$ slug-ft^2

A large transport aircraft is in a "steady coordinated" turn with the conditions given above. If the steady-coordinated turn is defined as one in which $Y = 0$ and V_c is always tangent to the flight path, use the nonlinear equations of motion to find

X,Y,Z	P,Q,R
L,M,N	$\dot{\Psi}, \dot{\Theta}, \dot{\Phi}$
U,V,W	v_x, v_y, v_z

The Euler angles are measured with respect to the earth-fixed axis system shown above.

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The Equations Collected

(1) $X = m[\dot{U} + QW - RV + gs\Theta]$

(2) $Y = m[\dot{V} + RU - PW - gc\Theta s\Phi]$

(3) $Z = m[\dot{W} + PV - QU - gc\Theta c\Phi]$

(4) $L = \dot{P}I_X - \dot{R}I_{XZ} + QR(I_Z - I_Y) - PQI_{XZ}$

(5) $M = \dot{Q}I_Y + PR(I_X - I_Z) - R^2I_{XZ} + P^2I_{XZ}$

(6) $N = \dot{R}I_Z - \dot{P}I_{XZ} + PQ(I_Y - I_X) + QRI_{XZ}$

(7) $P = -\dot{\Psi}s\Theta + \dot{\Phi}$

(8) $Q = \dot{\Psi}c\Theta s\Phi + \dot{\Theta}c\Phi$

(9) $R = \dot{\Psi}c\Theta c\Phi - \dot{\Theta}s\Phi$

or

(10) $\dot{\Theta} - Qc\Phi - Rs\Phi$

(11) $\dot{\Phi} = P + Qs\Phi t\Theta + Rc\Phi t\Theta$ $t(\ ) = TAN(\ )$

(12) $\dot{\Psi} = (Qs\Phi + Rc\Phi)/c\Theta$

(13) $v_x = U(c\Psi c\Theta) + V(c\Psi s\Theta s\Phi - s\Psi c\Phi) + W(c\Psi s\Theta c\Phi + s\Psi s\Phi)$

(14) $v_y = U(s\Psi c\Theta) + V(s\Psi s\Theta s\Phi + c\Psi c\Phi) + W(s\Psi s\Theta c\Phi - c\Psi s\Phi)$

(15) $v_z = -U(s\Theta) + V(c\Theta s\Phi) + W(c\Theta c\Phi)$

With Euler angle definitions (Ψ - first)

$$\dot{\Psi} = \frac{|\bar{U}_c|}{R} \text{ rad/sec} = 0.05 \text{ rad/sec}$$

Also $Y = 0$ (problem statement)

$U = |\bar{U}_c|$ $\quad V = W = 0$

$\Theta = 0$ (Θ is angle x-body axis makes with horizontal!)

Eq (2) then gives

$$0 + mg \sin\Phi = m(RU)$$

but Eq (9) gives

$$R = \dot{\Psi}\cos\Phi$$

or $\sin\Phi = \dfrac{\dot{\Psi}\cos\Phi\, U}{g}$

or $\tan\Phi = \dfrac{\dot{\Psi} U}{g} = 0.388$

$$\Phi = 21.2^\circ$$

From Eq (1) $X = 0$

From Eq (3) $Z = -mg\cos\Phi + m(-QU)$

From Eq (8) $Q = \dot{\Psi}\sin\Phi = 0.182$ rad/sec

$\therefore$ $Z = -2{,}039 \cdot 10^6$ lbf

SINCE z-axis is $\perp$ to $\bar{U}_{o1}$ $\quad Z = -$ LIFT

From Eq (7) $P = 0$

From Eq (9) $R = \dot{\Psi}\cos\Phi - \cancel{\dot{\Theta}}^{\,0}\sin\Phi$

$\therefore$ $R = 0.0466$ rad/sec

$$\bar{\Omega} = P\vec{i} + Q\vec{j} + R\vec{k}$$

$$= 0\vec{i} + 0.182\vec{j} + 0.0446\vec{k}$$

$$\bar{M}_o = L\vec{i} + M\vec{j} + N\vec{k}$$

$$= QR(I_z - I_y)\vec{i} + (-R^2 I_{xz})\vec{j} + QR(I_{xz})\vec{k} \quad \Big\} \begin{matrix} \text{Eqs} \\ (4)-(6) \end{matrix}$$

$$\therefore \quad \bar{M}_o = \bar{M} = 2.239 \cdot 10^3 \vec{i} - 60.8\vec{j} + 23.7\vec{k}$$

$$v_x = 250\,c\psi = 250\cos(0.05t)$$

$$v_y = 250\,s\psi = 250\sin(0.05t)$$

$$v_z = 0$$