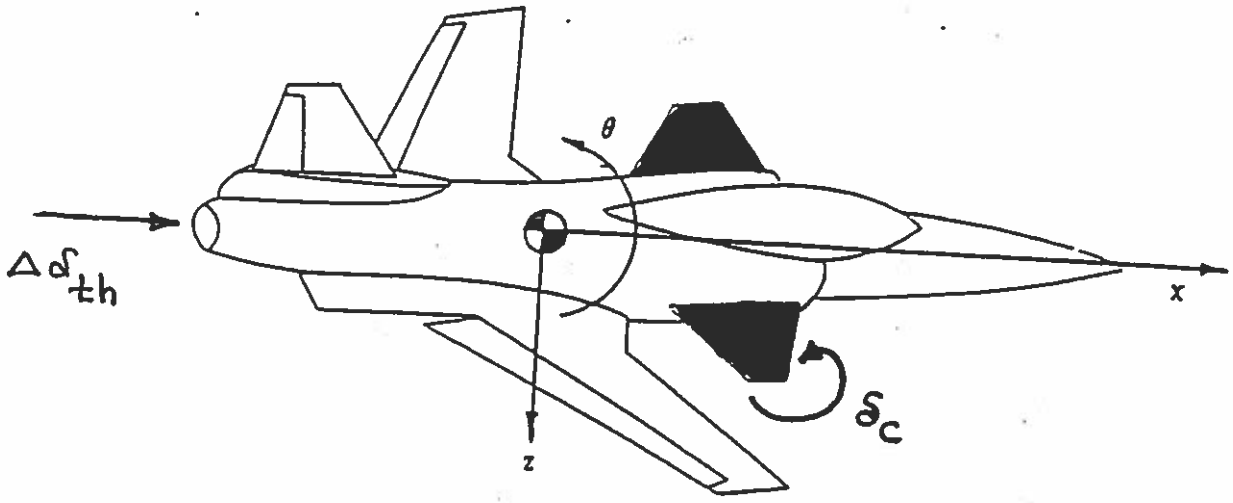
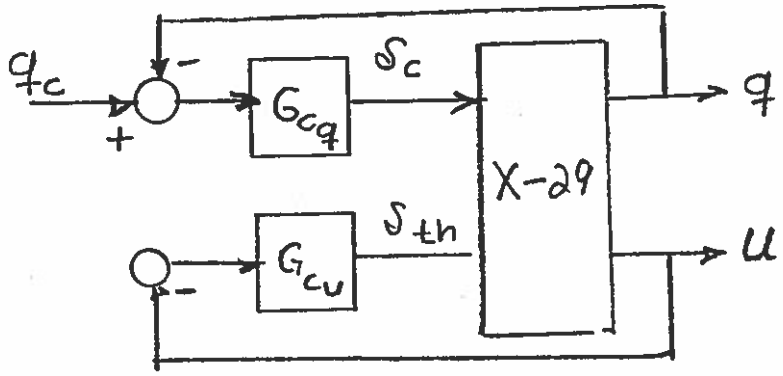
MAE-275

Use of "Regulated Variables"
to Eliminate Right Half Plane (RHP) Zeros Near Origin



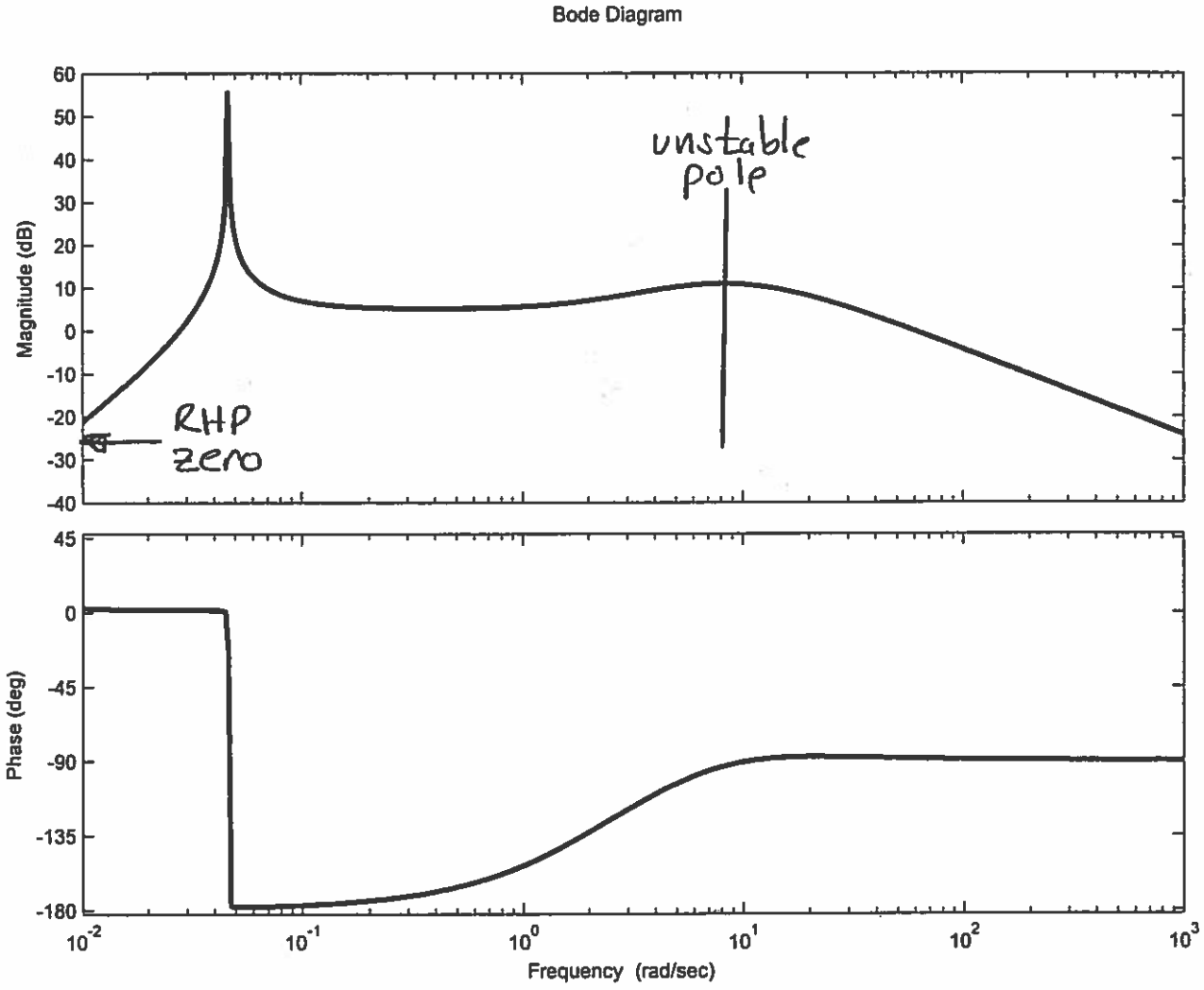


```
>> sys=ss(a,b,c,d);
>> sys=minreal(sys);
10 states removed.
>> [a,b,c,d]=ssdata(sys);
>> [num,den]=ss2tf(a,b,c,d);
>> g=tf(num,den)

Transfer function:
  61.33 s^3 + 136 s^2 - 0.05548 s - 2.776e-017
-----------------------------------------------
s^4 + 3.711 s^3 - 77.17 s^2 + 0.003764 s - 0.1648

>> zpk(g)

Zero/pole/gain:
          61.33 s (s+2.218) (s-0.0004078)
-----------------------------------------------
(s+10.83) (s-7.123) (s^2  + 5.391e-005s + 0.002135)
```

$= \frac{Y(s)}{U(s)}$

$= \frac{Y(s)}{U(s)}$

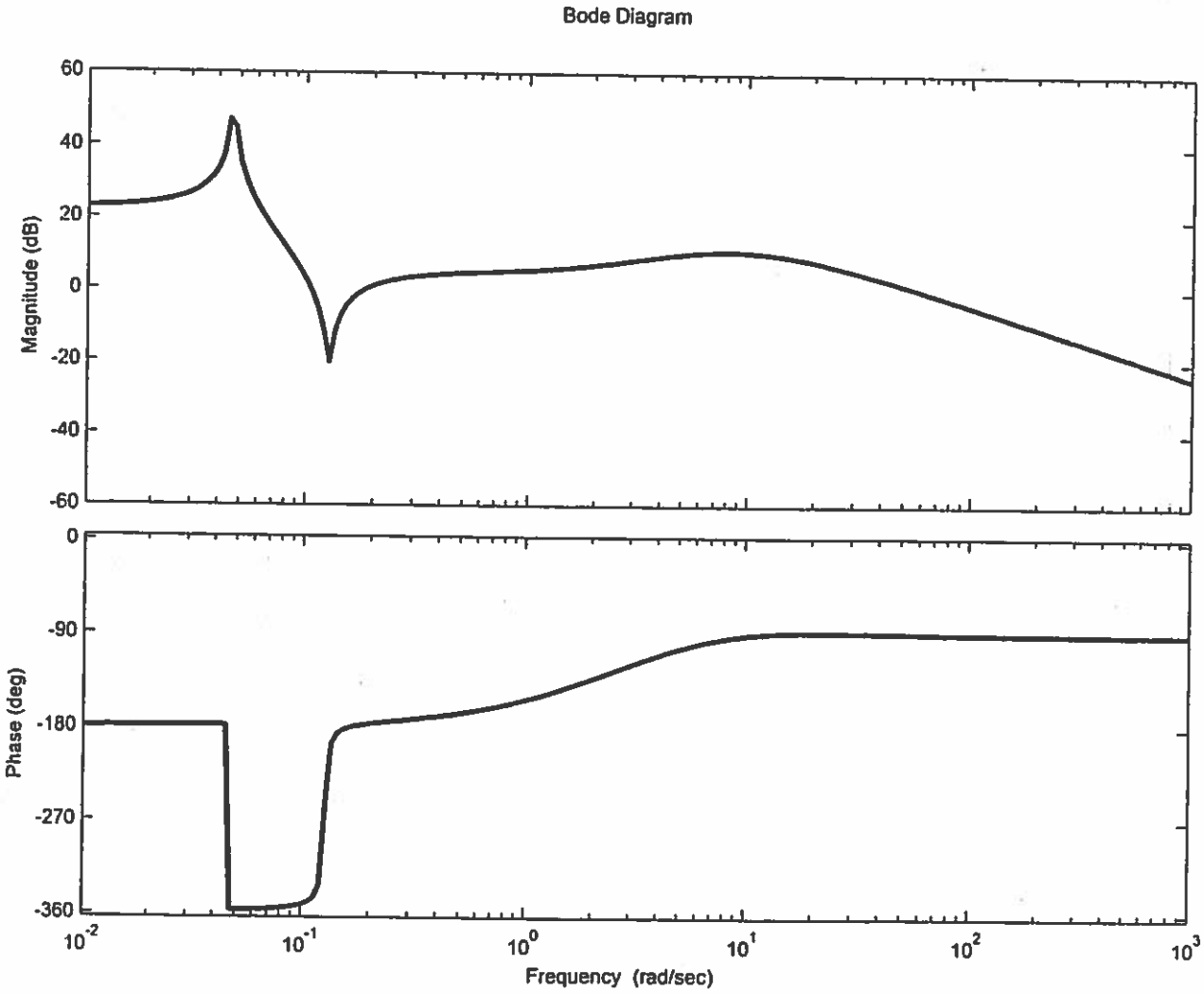
$\frac{q}{\delta_c}$



"regulated variable

$(q - .0005u)$

δ_c



```
>> sys=ss(a,b,c,d);
>> sys=minreal(sys);
10 states removed.
>> [a,b,c,d]=ssdata(sys);
>> [num,den]=ss2tf(a,b,c,d);
>> g=tf(num,den);
>> zpk(g)

Zero/pole/gain:
  61.3295 (s+2.219) (s^2  + 0.005935s + 0.01609)
-------------------------------------------------
(s+10.83) (s-7.123) (s^2  + 5.391e-005s + 0.002135)
```

no RHP zeros

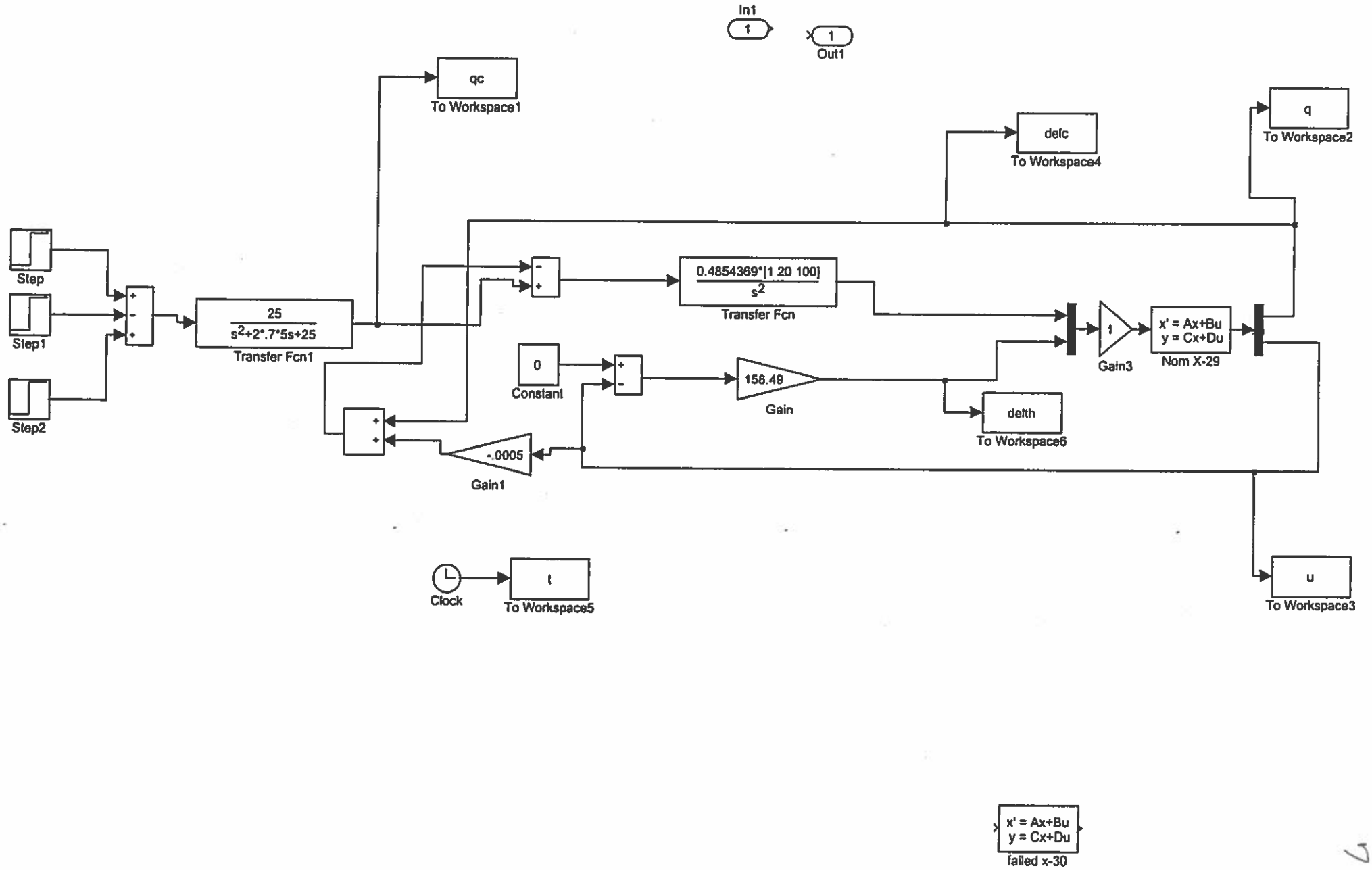
$$= \frac{q - .0005u}{\delta_c}$$

regulated variable = $q - .0005 \cdot u$

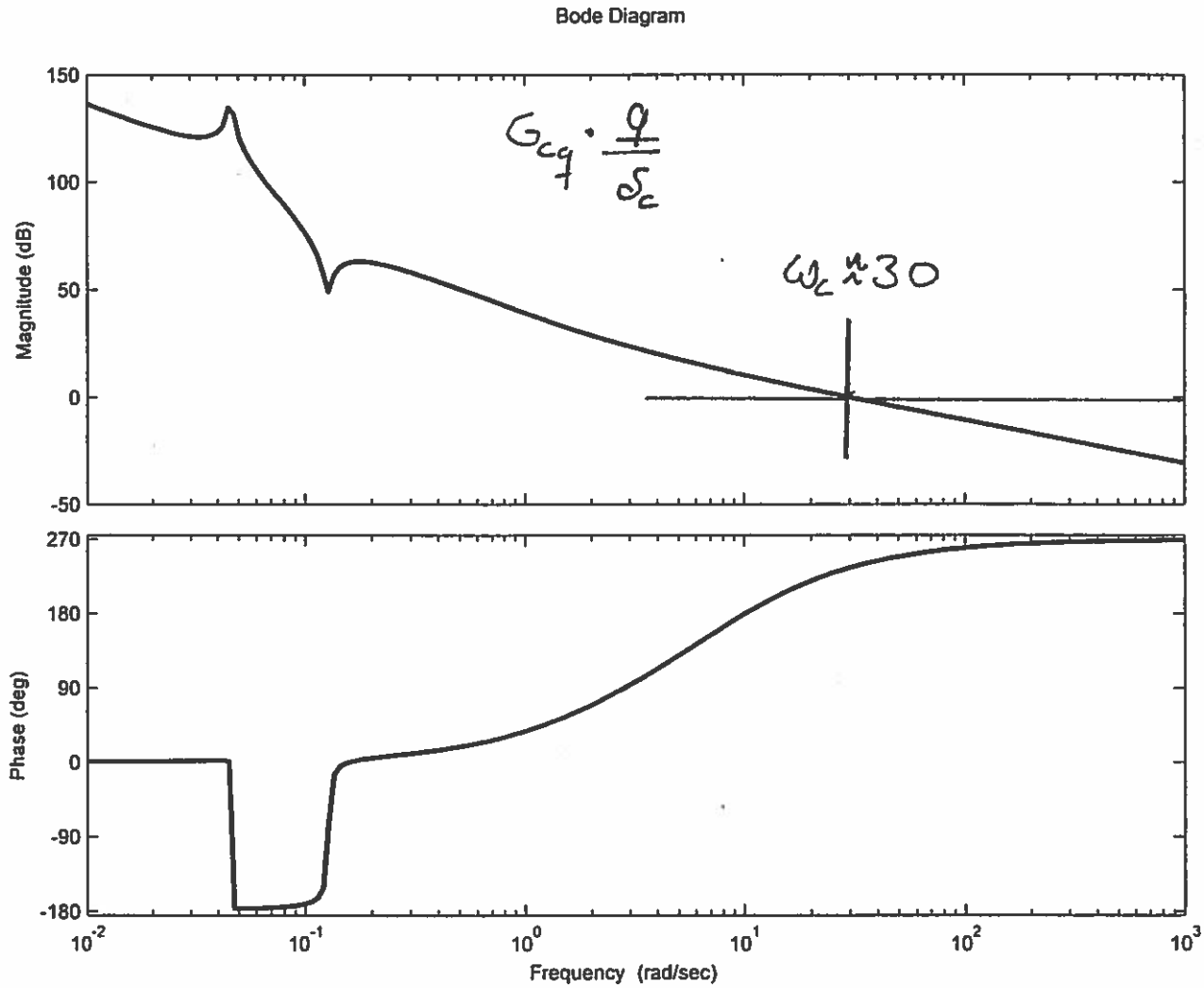
```
>> sys=ss(a,b,c,d);
>> sys=minreal(sys);
10 states removed.
>> [a,b,c,d]=ssdata(sys);
>> [num,den]=ss2tf(a,b,c,d);
>> g=tf(num,den);
>> zpk(g)

Zero/pole/gain:
3.5527e-015 (s+5.629e011) (s+10.94) (s+2.04)
-------------------------------------------
   (s+10.94) (s+2.04) (s+1) (s-0.0004078)
```

no rhp zeros!

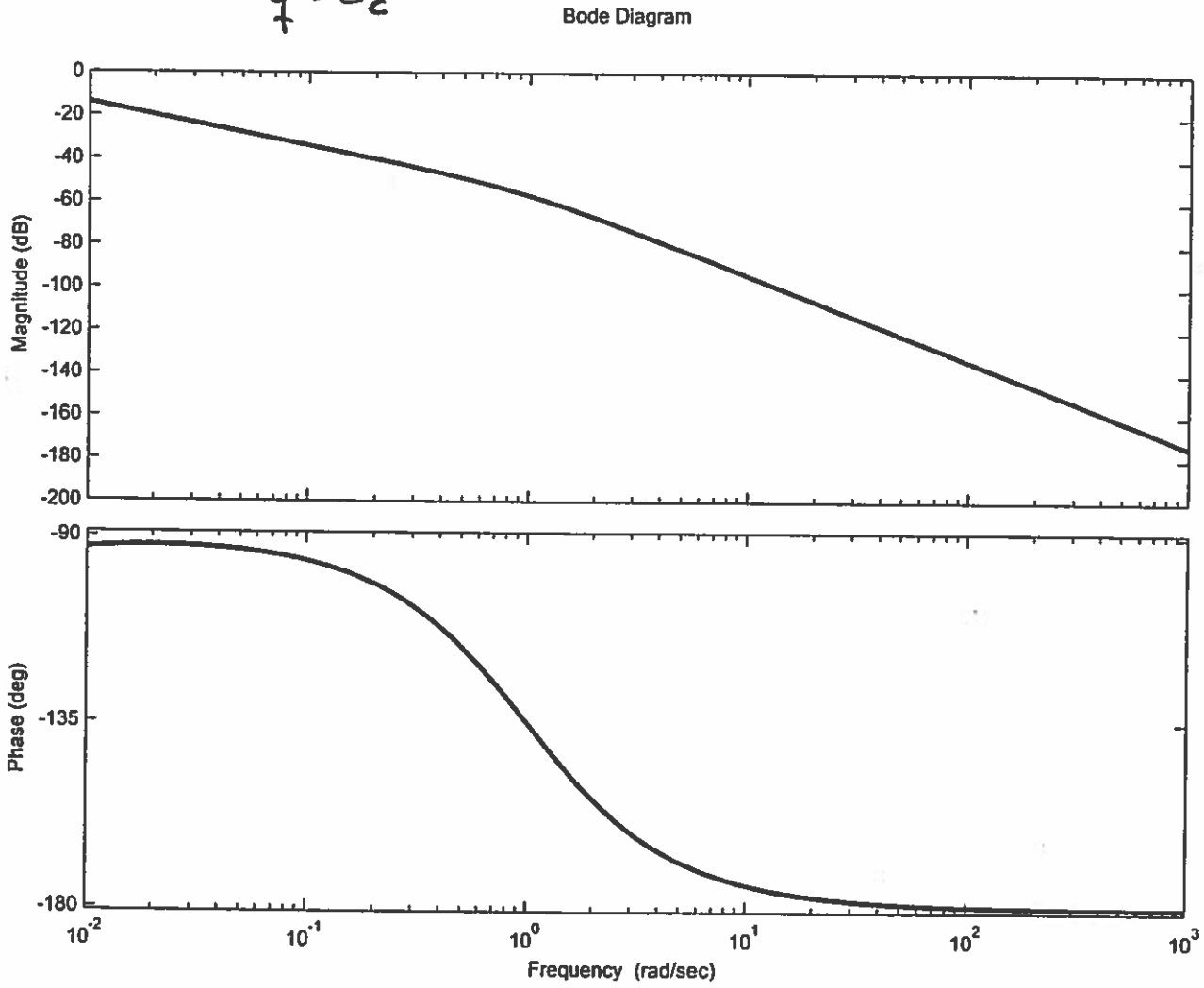
$$= \frac{u}{\delta_{th}}\Big|_{q-\delta_c}$$


In1
1
1
Out1
qc
To Workspace1
delc
To Workspace4
q
To Workspace2
Step
Step1
Step2
25
s2+2*.7*5s+25
Transfer Fcn1
0.4854369*[1 20 100]
s2
Transfer Fcn
0
Constant
158.49
Gain
delth
To Workspace6
1
Gain3
x' = Ax+Bu
y = Cx+Du
Nom X-29
-.0005
Gain1
Clock
t
To Workspace5
u
To Workspace3
x' = Ax+Bu
y = Cx+Du
failed x-30

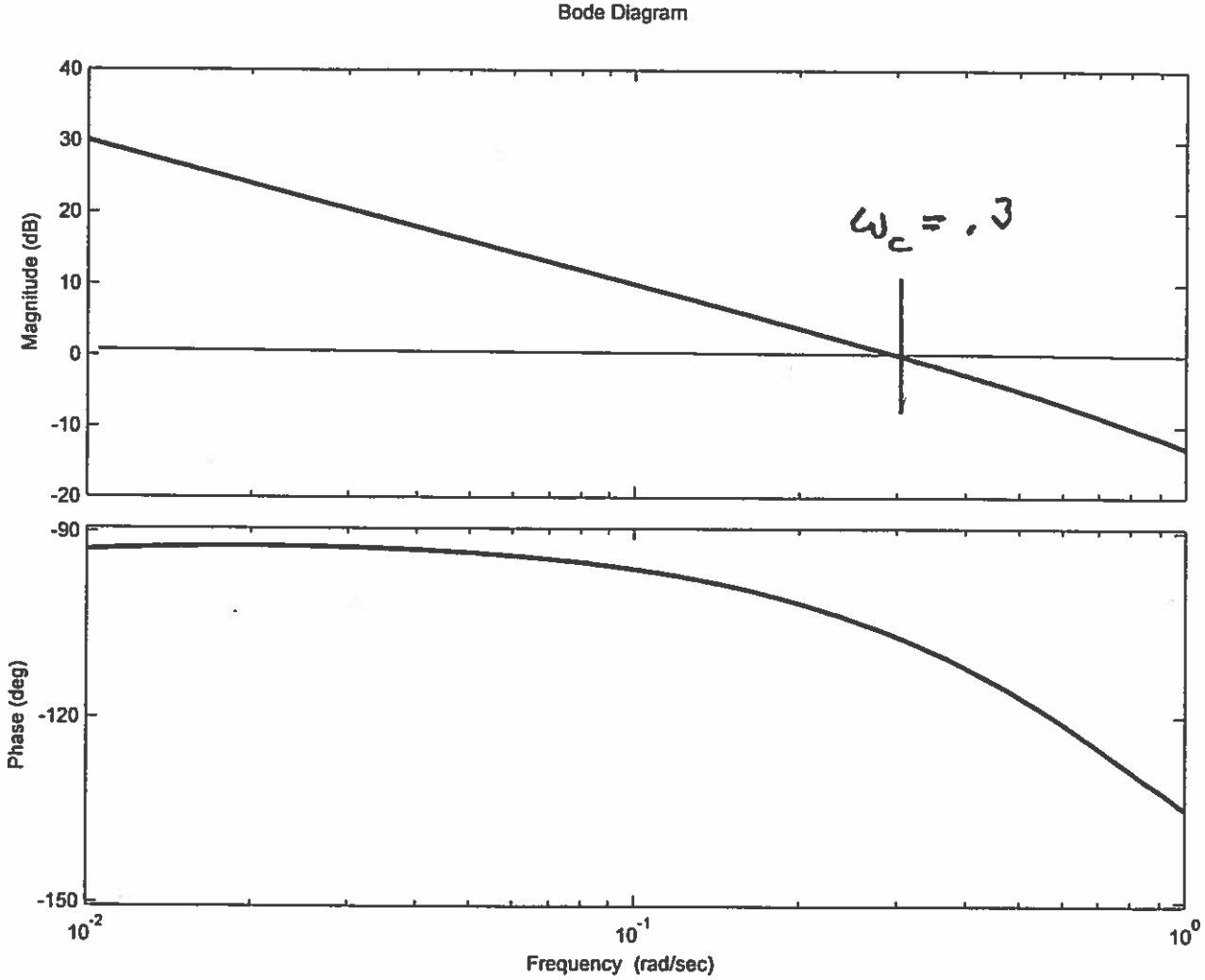
$$G_{cq} = \frac{0.49(s+10)^2}{s^2}$$

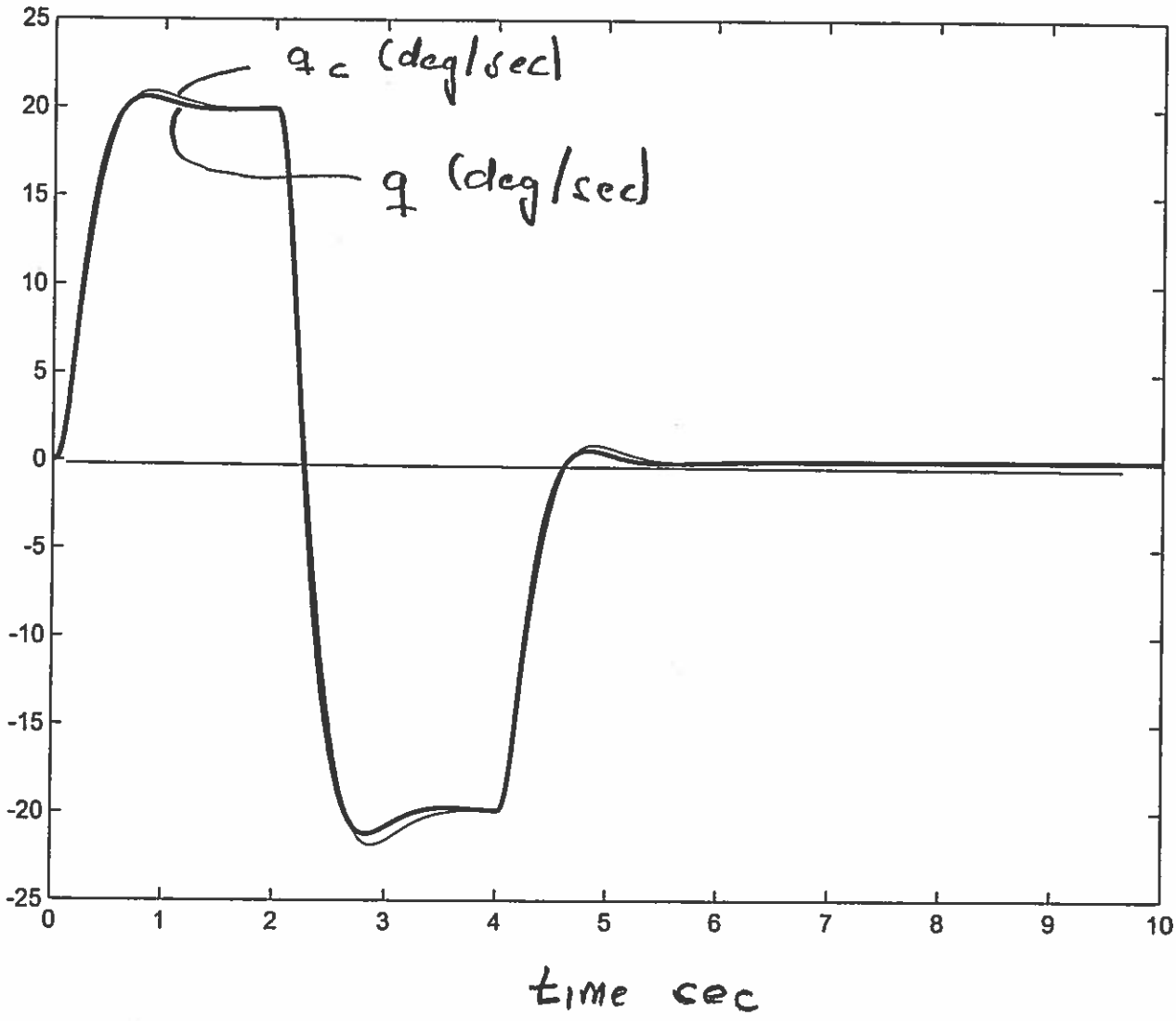


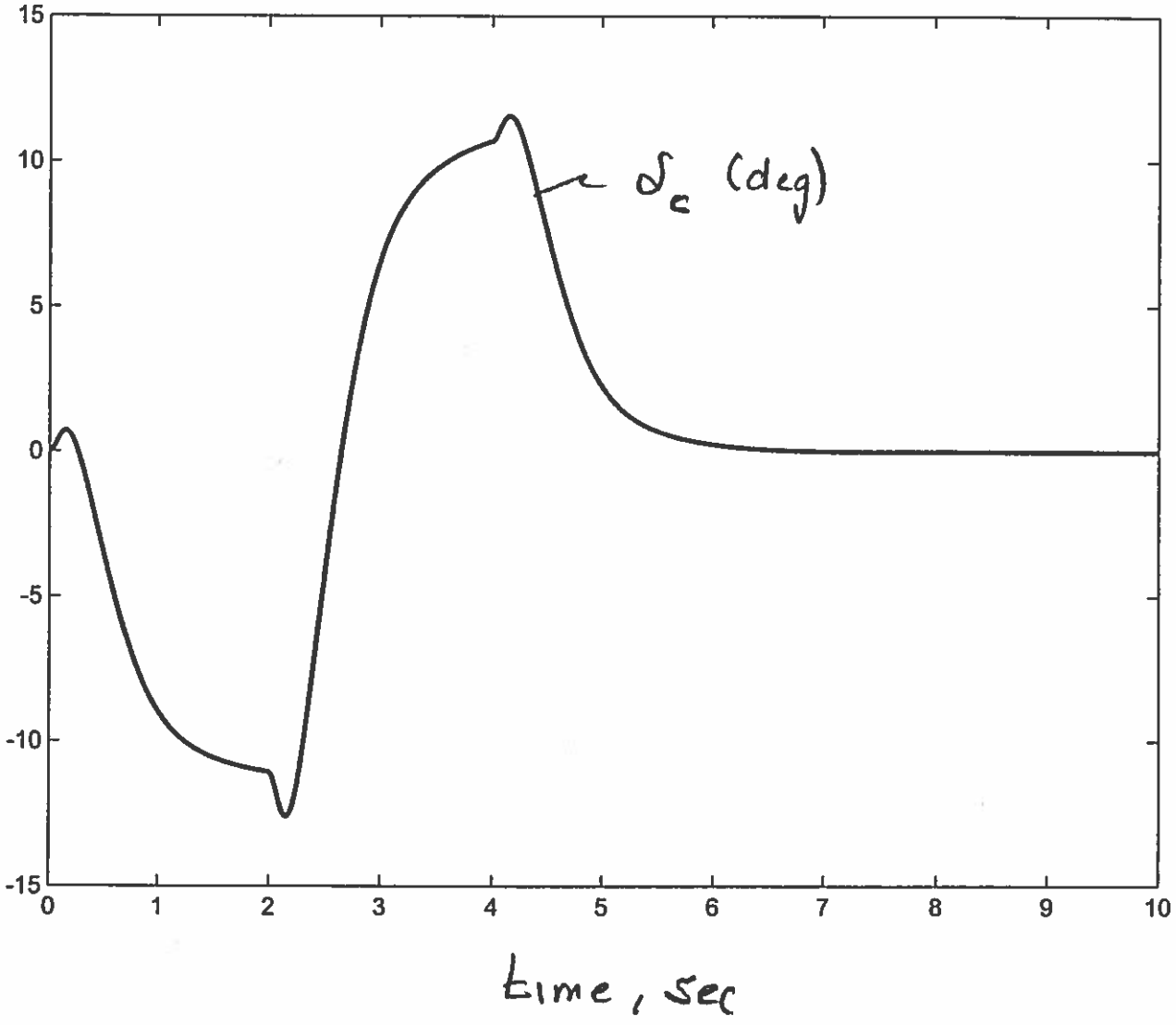
$$\left.\frac{q}{\delta_{th}}\right|_{q \to \delta_e}$$

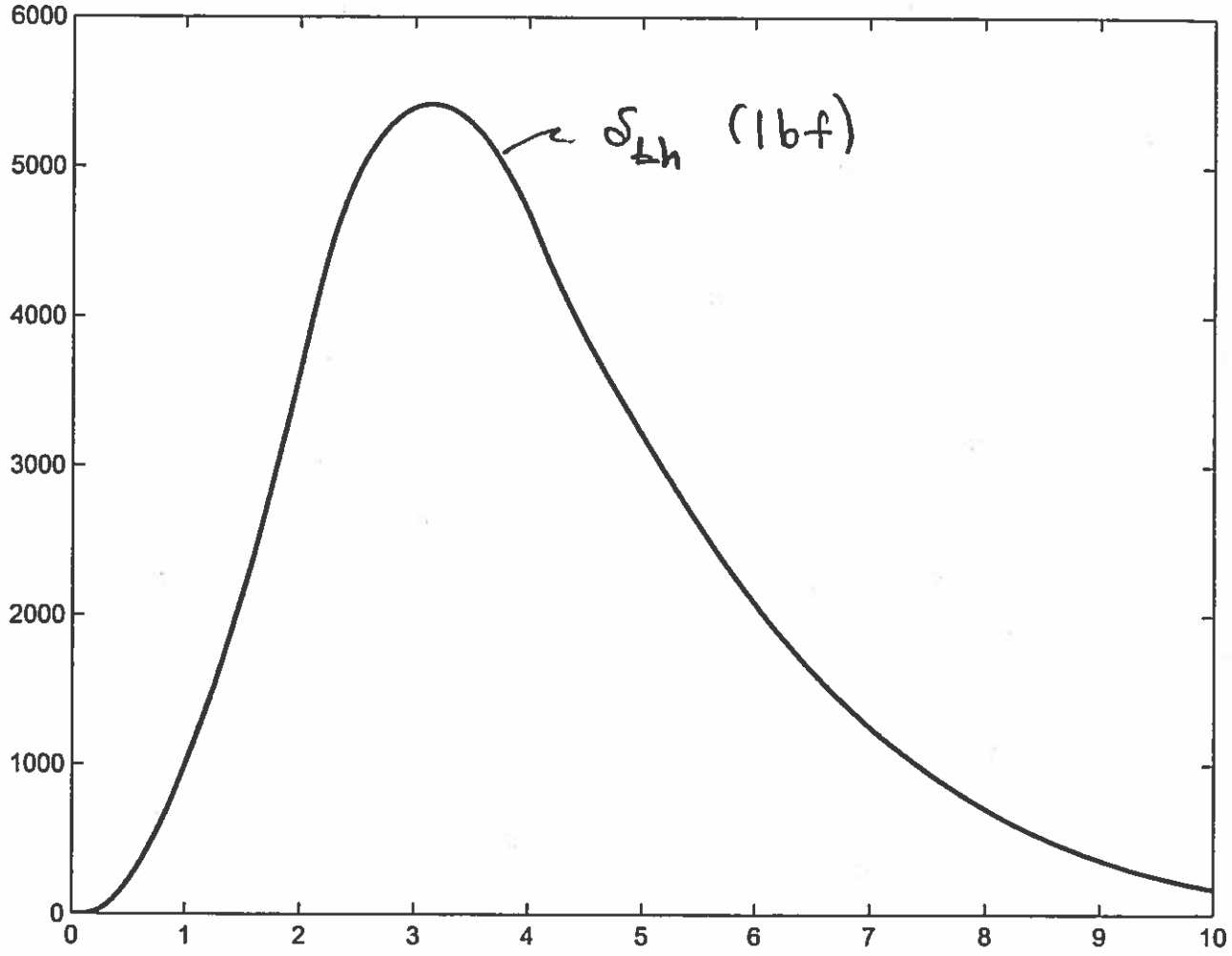


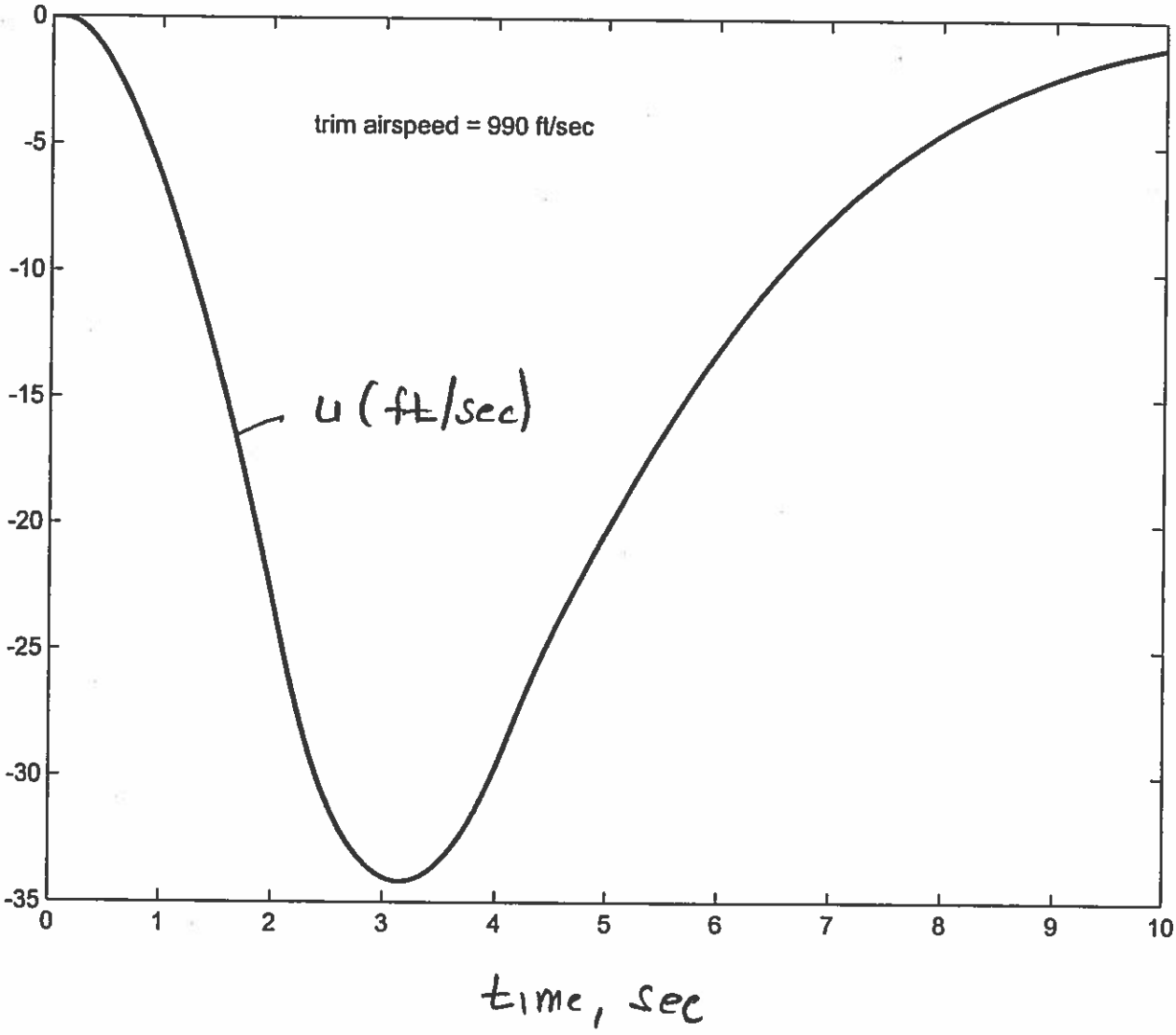
$$G_{cu} \cdot \left. \frac{u}{\delta_{th}} \right|_{q-\delta_c} \qquad G_{cu} = 158,49$$




25
20
15
10
5
0
-5
-10
-15
-20
-25
0
1
2
3
4
5
6
7
8
9
10
q_c (deg/sec)
q (deg/sec)
time sec

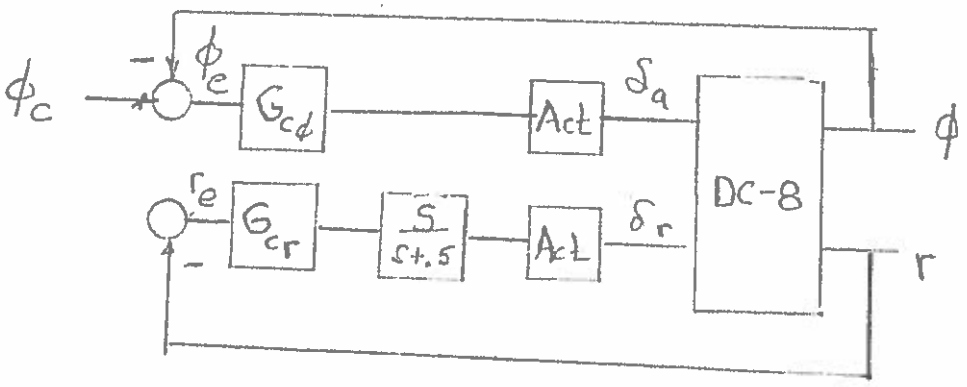

15
10
5
0
-5
-10
-15
0
1
2
3
4
5
6
7
8
9
10
δ_e (deg)
Time, sec


6000
5000
4000
3000
2000
1000
0
0
1
2
3
4
5
6
7
8
9
10
δ_{th} (1bf)


trim airspeed = 990 ft/sec
u (ft/sec)
0
-5
-10
-15
-20
-25
-30
-35
0
1
2
3
4
5
6
7
8
9
10
time, sec

MAE-275

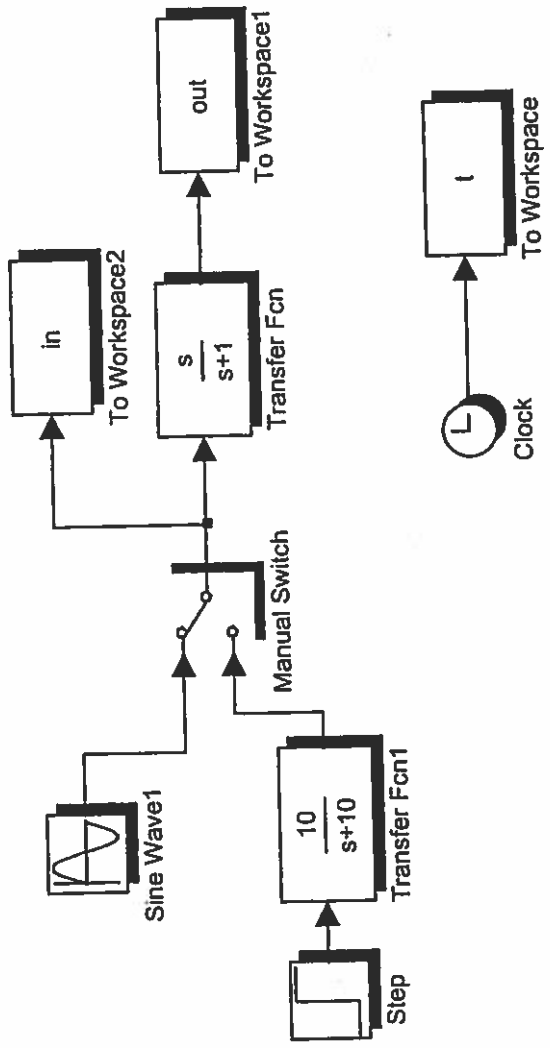
Design Example

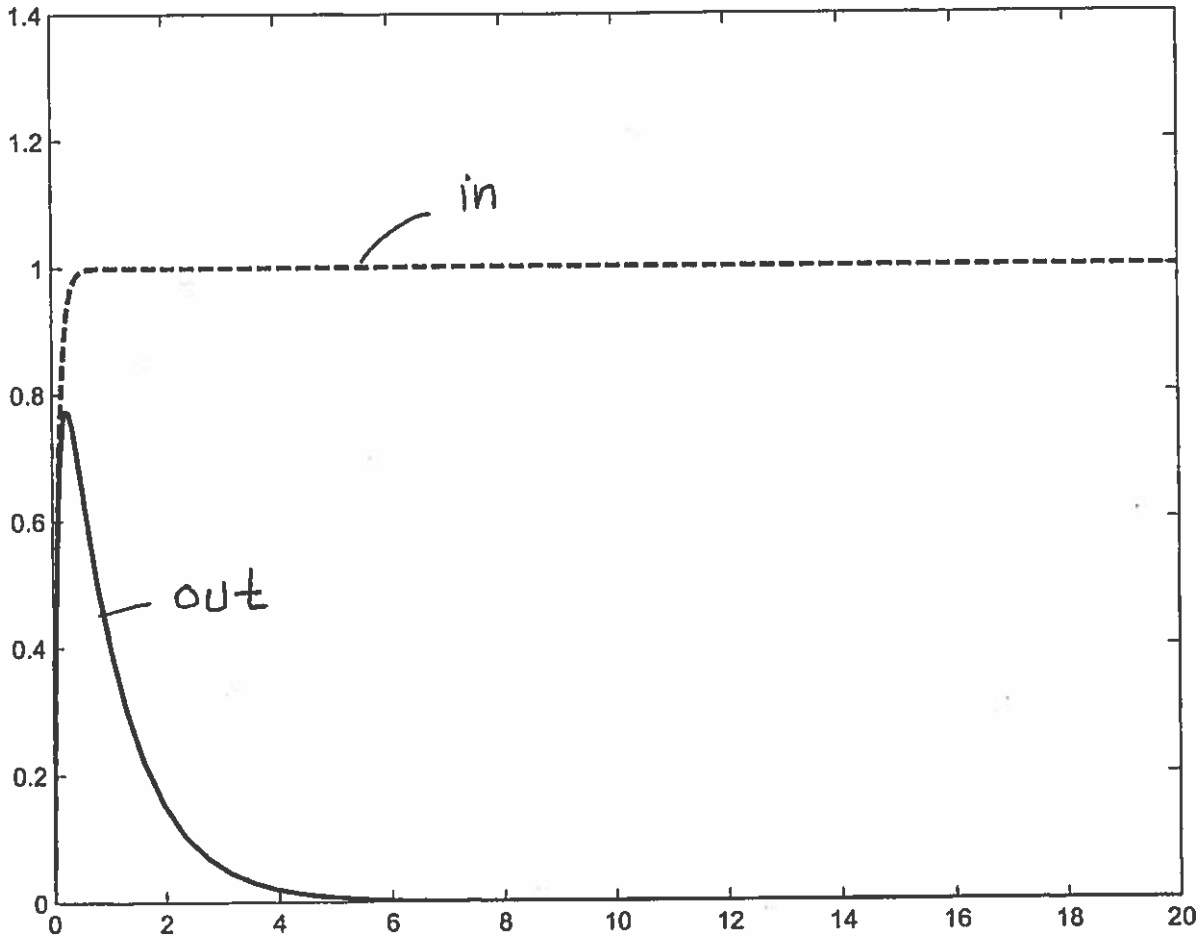


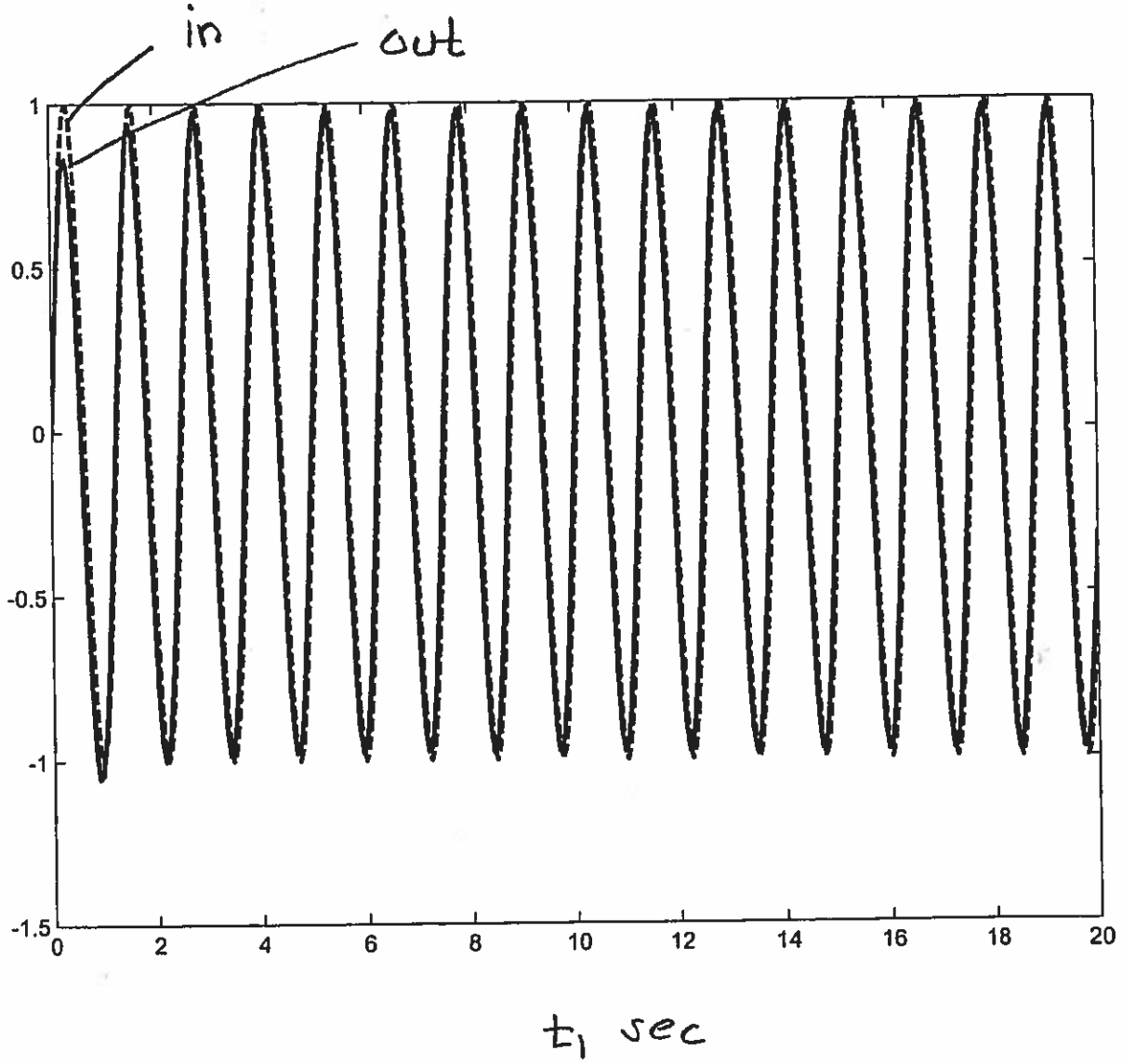
$\omega_{B_\phi} \approx 6$ rad/sec

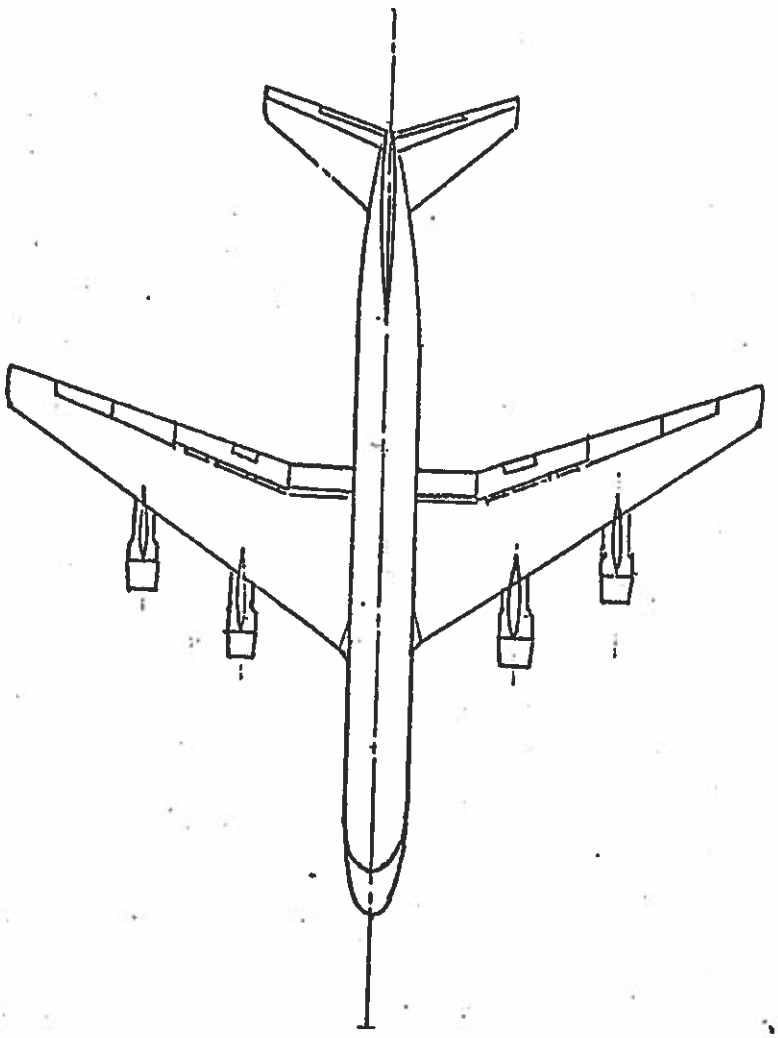
$\omega_{B_r} \approx 2$ rad/sec

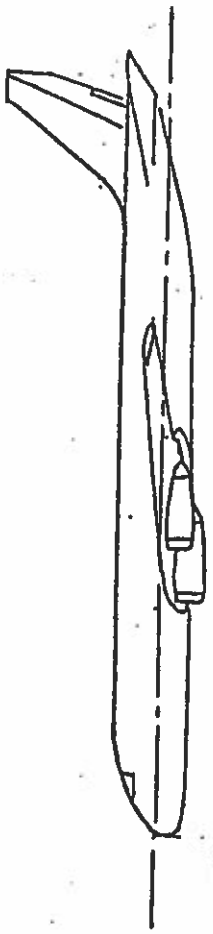
WASHOUT

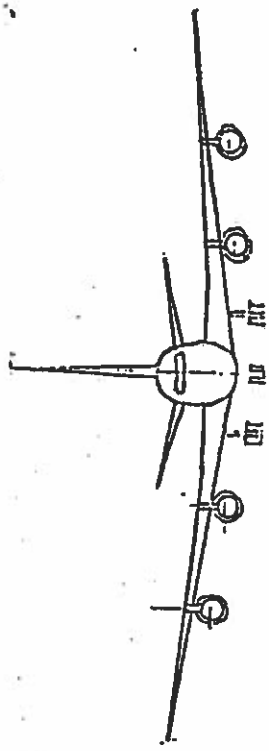

Sine Wave1
Step
10
s+10
Transfer Fcn1
Manual Switch
in
To Workspace2
s
s+1
Transfer Fcn
out
To Workspace1
Clock
t
To Workspace


1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
in
out


in
out
1
0.5
0
-0.5
-1
-1.5
0
2
4
6
8
10
12
14
16
18
20
t_1 sec







DC-8

TABLE A-5

A. GEOMETRICAL AND INERTIAL PARAMETERS FOR THE DC-8

Note: Data are for body-fixed stability axes

$S = 2600\ ft^2$, $b = 142.3\ ft$, $c = 23\ ft$, $\gamma_o = 0\ deg$

	FLIGHT CONDITION			
	8001 APPROACH	8002 HOLDING	8003 CRUISE	8004 V_{NE}
h (ft)	0	15,000	33,000	33,000
M (–)	0.219	0.443	0.84	0.88
a (ft/sec)	1117	1058	982	982
ρ (slugs/ft^3)	0.002378	0.001496	0.000795	0.000795
V_{T_o} (ft/sec)	243.5	468.2	824.2	863.46
$\bar{q} = \rho V^2/2$ (lb/ft^2)	71.02	163.97	270.0	296.36
W (lb)	190,000	190,000	230,000	230,000
m (slugs)	5900	5900	7143	7143
I_x (slug-ft^2)	3,090,000	3,110,000	3,770,000	3,770,000
I_y (slug-ft^2)	2,940,000	2,940,000	3,560,000	3,560,000
I_z (slug-ft^2)	5,580,000	5,880,000	7,130,000	7,130,000
I_{xz} (slug-ft^2)	28,000	–64,500	45,000	53,700
$x_{c.g.}/\bar{c}$	0.15	0.15	0.15	0.15
θ_o (deg)	0	0	0	0
U_o (ft/sec)	243.5	468.2	824.2	863.46
W_o (ft/sec)	0	0	0	0
δ_F (deg)	35	0	0	0

D. LATERAL DIMENSIONAL DERIVATIVES FOR THE DC-8

Note: Data are for body-fixed stability axes

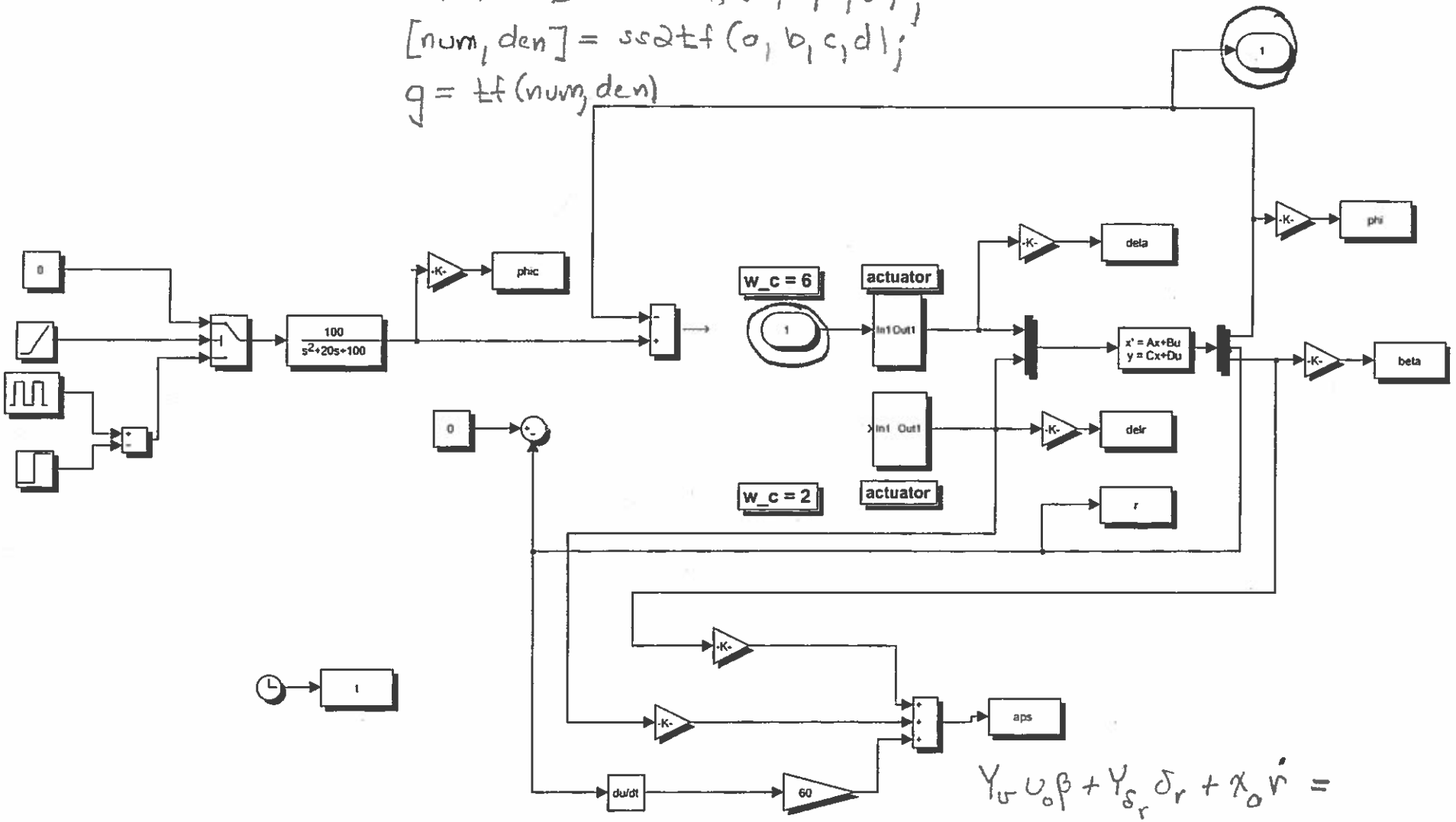
	FLIGHT CONDITION			
	8001	8002	8003	8004
h (ft)	0	15,000	33,000	33,000
M (–)	0.219	0.443	0.84	0.88
Y_v (1/sec)	–0.1113	–0.1008	–0.0868	–0.0931
Y_β [(ft/sec^2)/rad]	–27.1	–47.2	–71.5	–80.4
Y_{δ_a} [(ft/sec^2)/rad]	0	0	0	0
$Y_{\delta_a}^*$ [(1/sec)/rad]	0	0	0	0
Y_{δ_r} [(ft/sec^2)/rad]	5.79	13.48	18.33	20.12
$Y_{\delta_r}^*$ [(1/sec)/rad]	0.0238	0.0288	0.0222	0.0233
L_β (1/sec^2)	–1.335	–2.68	–4.43	–5.05
L_p (1/sec)	–0.95	–1.233	–1.18	–1.289
L_r (1/sec)	0.612	0.391	0.336	0.35
L_{δ_a} (1/sec^2)	–0.726	–1.62	–2.11	–2.3
L_{δ_r} (1/sec^2)	0.1848	0.374	0.559	0.63
L_β' (1/sec^2)	–1.328	–2.71	–4.41	–5.02
L_p' (1/sec)	–0.951	–1.232	–1.181	–1.29
L_r' (1/sec)	0.509	0.397	0.334	0.346
L_{δ_a}' (1/sec^2)	–0.726	–1.62	–2.11	–2.3
L_{δ_r}' (1/sec^2)	0.1813	0.392	0.549	0.612
N_β (1/sec^2)	0.763	1.271	2.17	2.47
N_p (1/sec)	–0.1192	–0.048	–0.01294	–0.00744
N_r (1/sec)	–0.268	–0.252	–0.23	–0.252
N_{δ_a} (1/sec^2)	–0.0496	–0.0365	–0.0519	–0.0615
N_{δ_r} (1/sec^2)	–0.39	–0.86	–1.168	–1.282
N_β' (1/sec^2)	0.757	1.301	2.14	2.43
N_p' (1/sec)	–0.124	–0.0346	–0.0204	–0.01715
N_r' (1/sec)	–0.265	–0.257	–0.228	–0.25
N_{δ_a}' (1/sec^2)	–0.0532	–0.01875	–0.0652	–0.0788
N_{δ_r}' (1/sec^2)	–0.389	–0.864	–0.01164	–1.277

```
» A

A =

   -0.1000         0 -468.2000   32.2000         0
   -0.0058   -1.2320    0.3970         0         0
    0.0028   -0.0346   -0.2570         0         0
         0    1.0000         0         0         0
         0         0    1.0000         0         0

» B

B =

         0   13.4800
   -1.6200    0.3920
   -0.0187   -0.8640
         0         0
         0         0

» C

C =

r
φ        0         0    1.0000         0         0
         0         0         0    1.0000         0
β   0.0021         0         0         0         0

» D

D =

     0     0
     0     0
     0     0

»
```

$$\underline{x} = \begin{Bmatrix} \Delta v \\ \Delta p \\ \Delta r \\ \Delta \phi \\ \Delta \psi \end{Bmatrix}$$

$$\underline{u} = \begin{Bmatrix} \Delta \delta_a \\ \Delta \Delta \delta_r \end{Bmatrix}$$

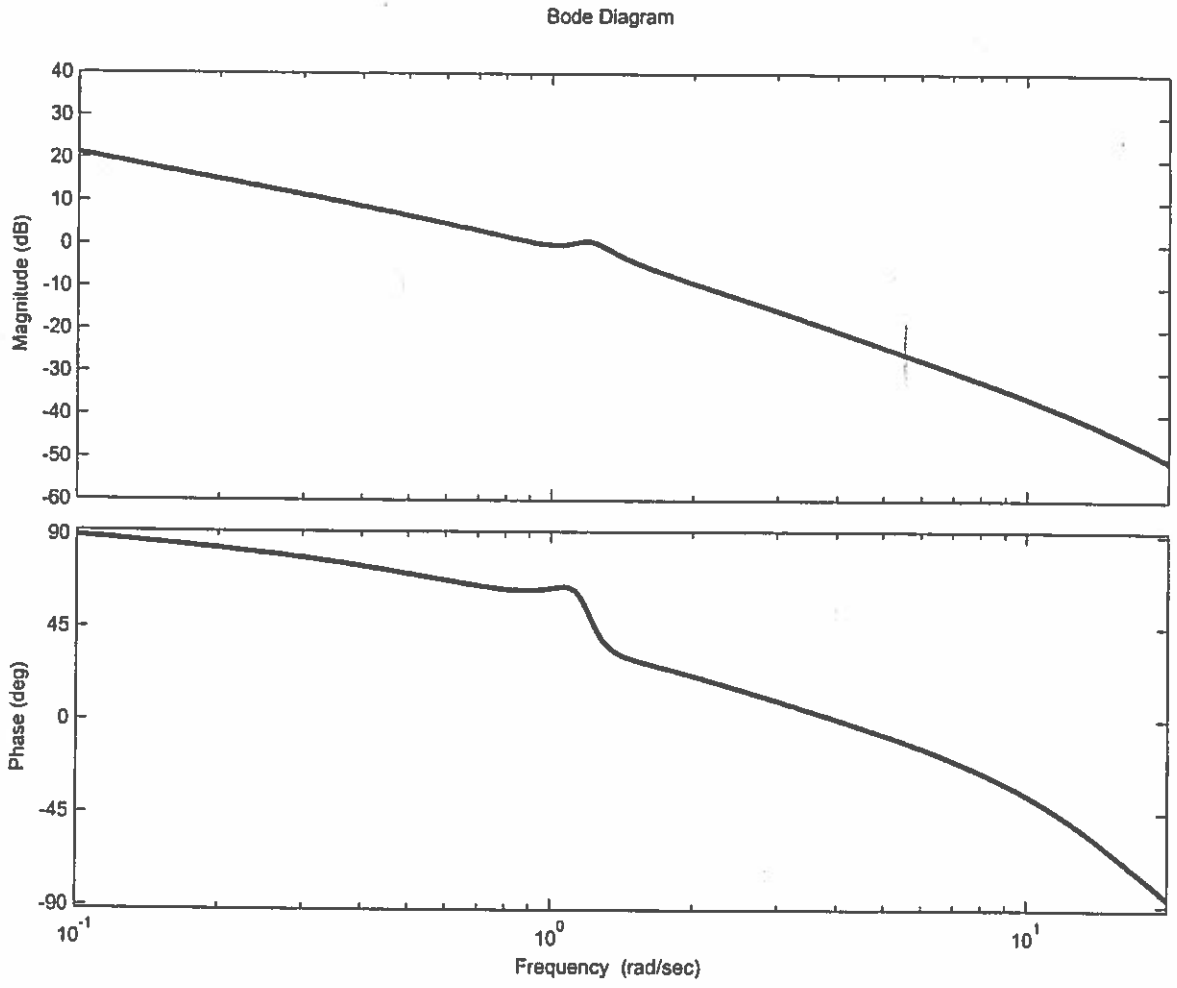
```
[a, b, c, d] = linmod('dc8');
[a, b, c, d] = minreal(a, b, c, d);
[num, den] = ss2tf(a, b, c, d);
g = tf(num, den)
```



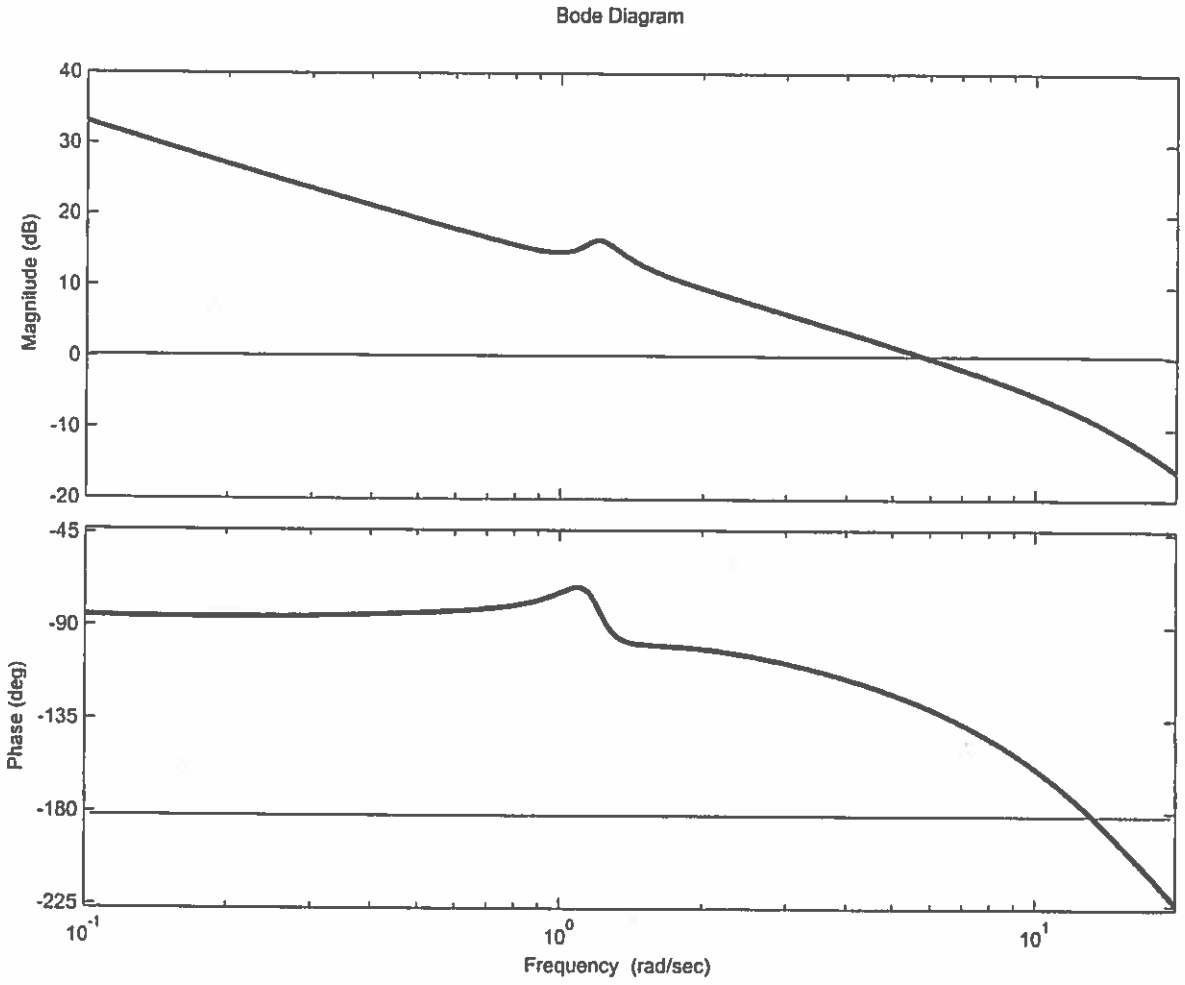
$Y_v v_o \beta + Y_{\delta_r} \delta_r + x_o \dot{r} =$

lateral accel @ pilot

$$\frac{\phi}{\delta_a}$$



$$\frac{\phi}{\phi_e}$$



```
>> zpk(phi_phie)

Zero/pole/gain:

8.5265e-014 (s+8.391e005) (s+1) (s^2  + 0.3615s + 1.359) (s^2  - 8.391e005s + 7.04e011)
-----------------------------------------------------------------------------------------
     (s+20) (s+1.329) (s+0.006784) (s^2  + 0.2533s + 1.433) (s^2  + 28.28s + 400)
```

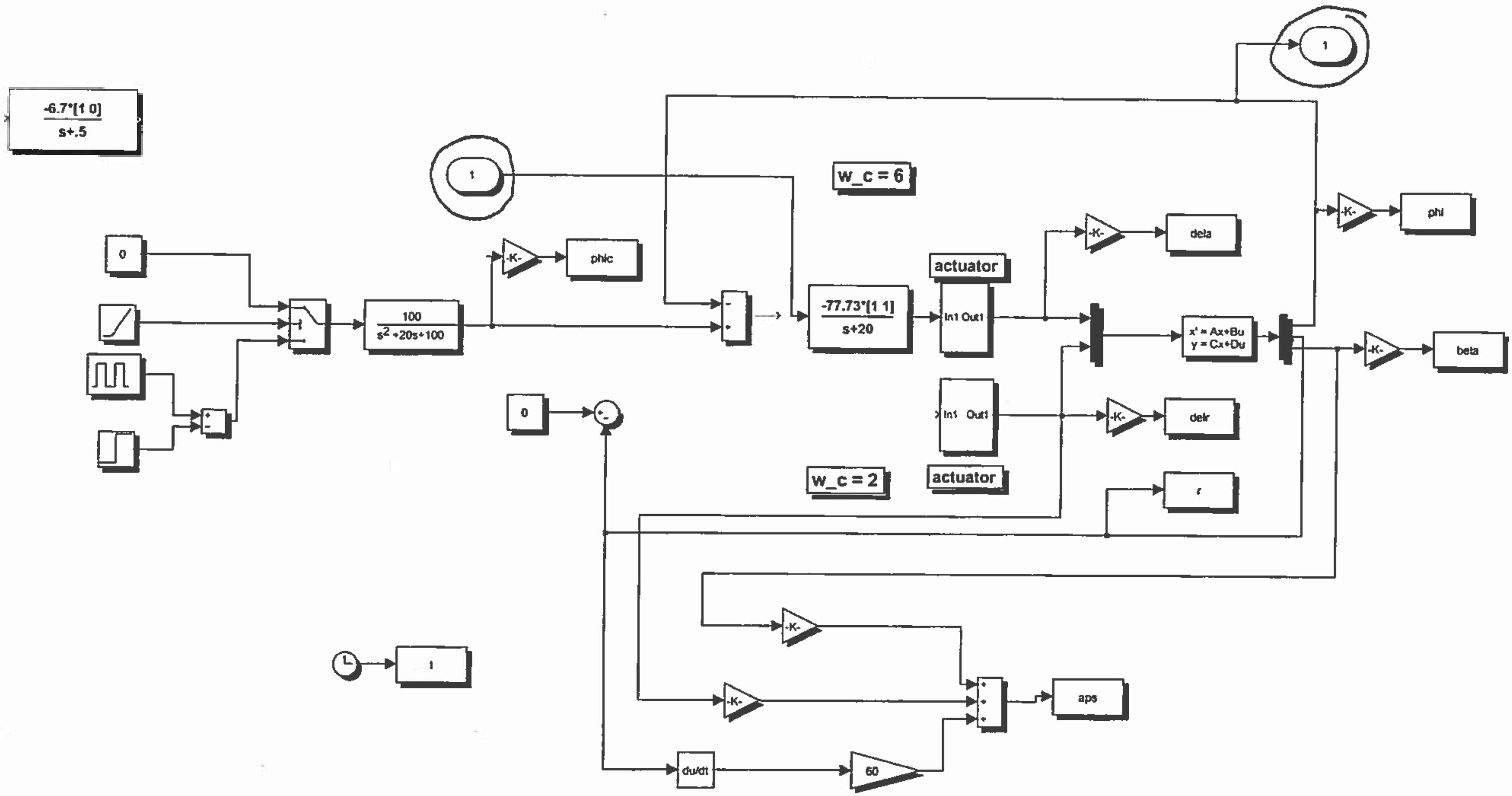
not real

$= \frac{\phi}{\phi_e}$

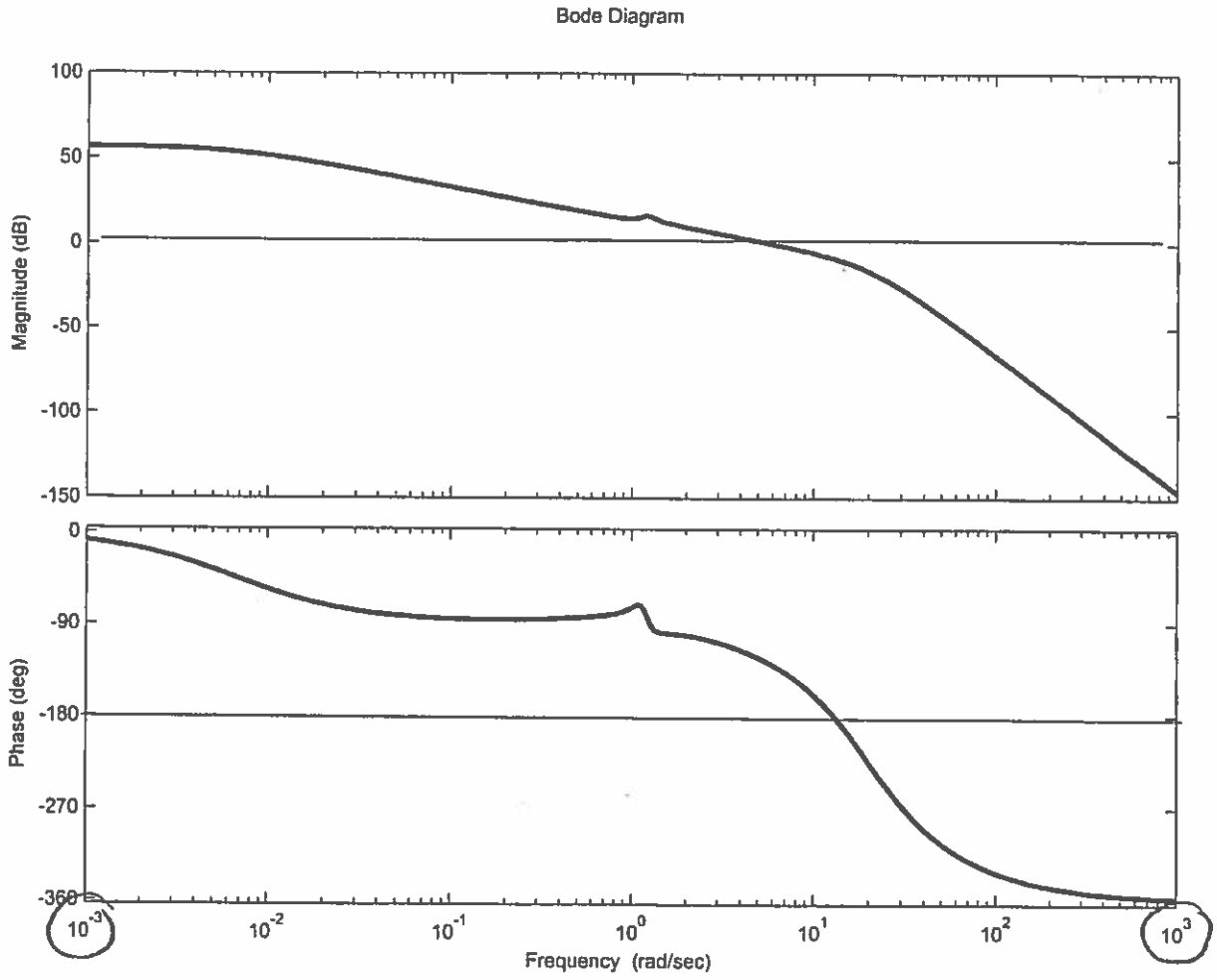
$n = 0$

$m = 0$

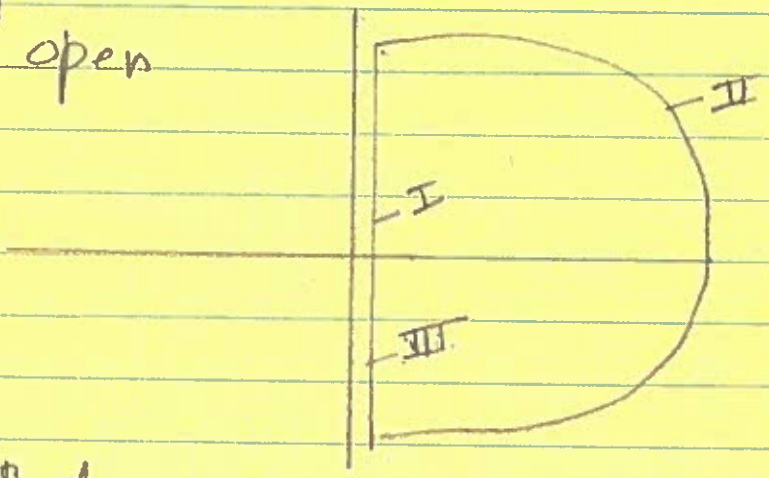
$p = 0$


-6.7*[1 0]
s+.5
0
100
s2 +20s+100
-K-
phic
1
0
w_c = 6
-77.73*[1 1]
s+20
actuator
In1 Out1
w_c = 2
x' = Ax+Bu
y = Cx+Du
dela
delr
r
phi
beta
aps
du/dt
60

$$\frac{\phi}{\phi_c}$$



ϕ-loop with r-loop open

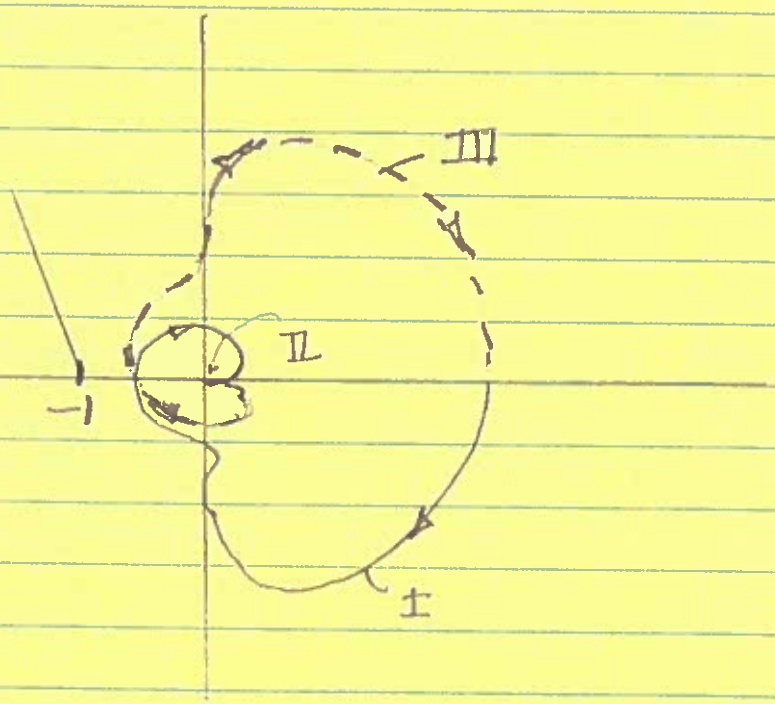


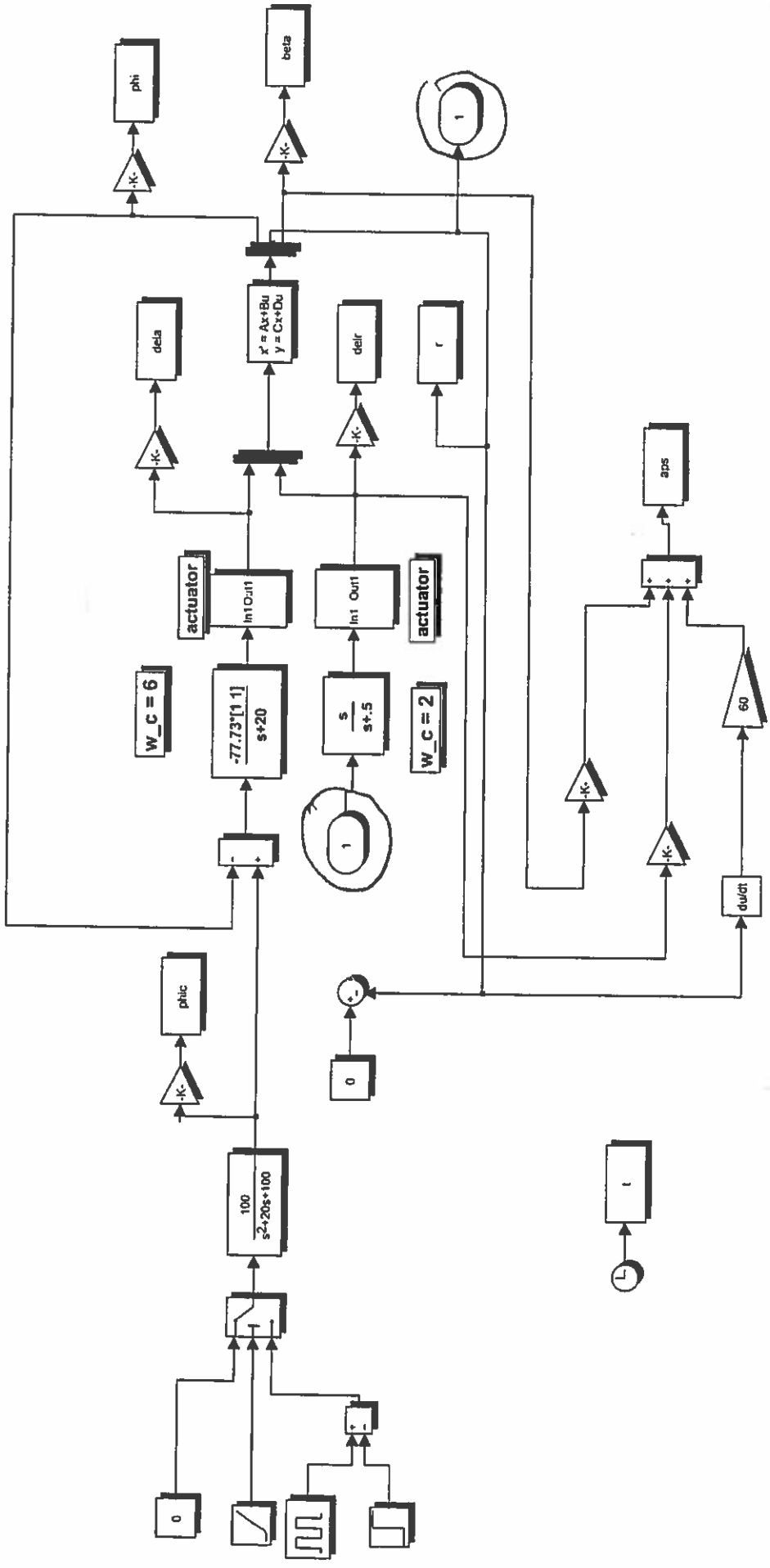
I from Bode

$N = Z - P$; $N = 0$

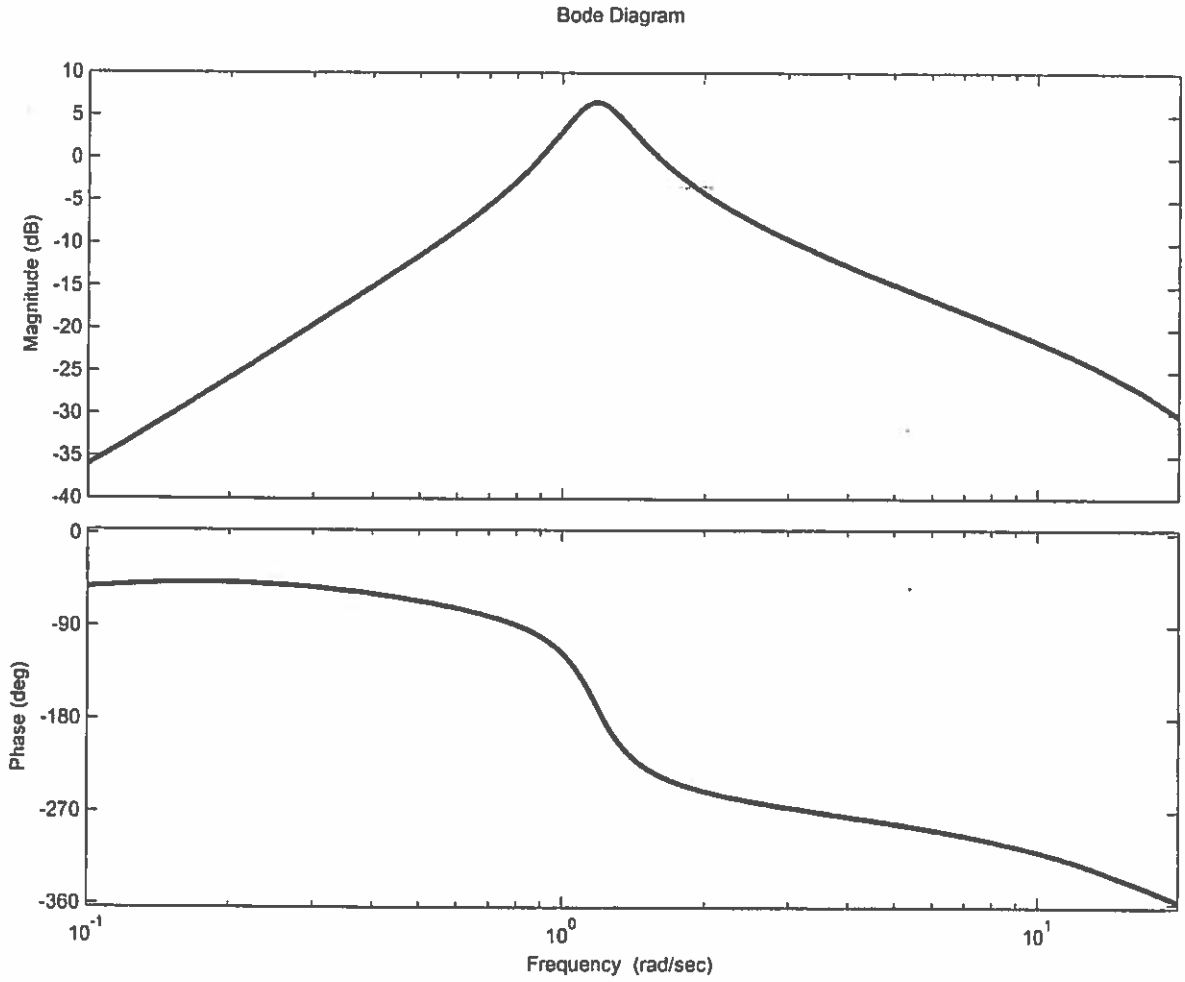
$P = 0$

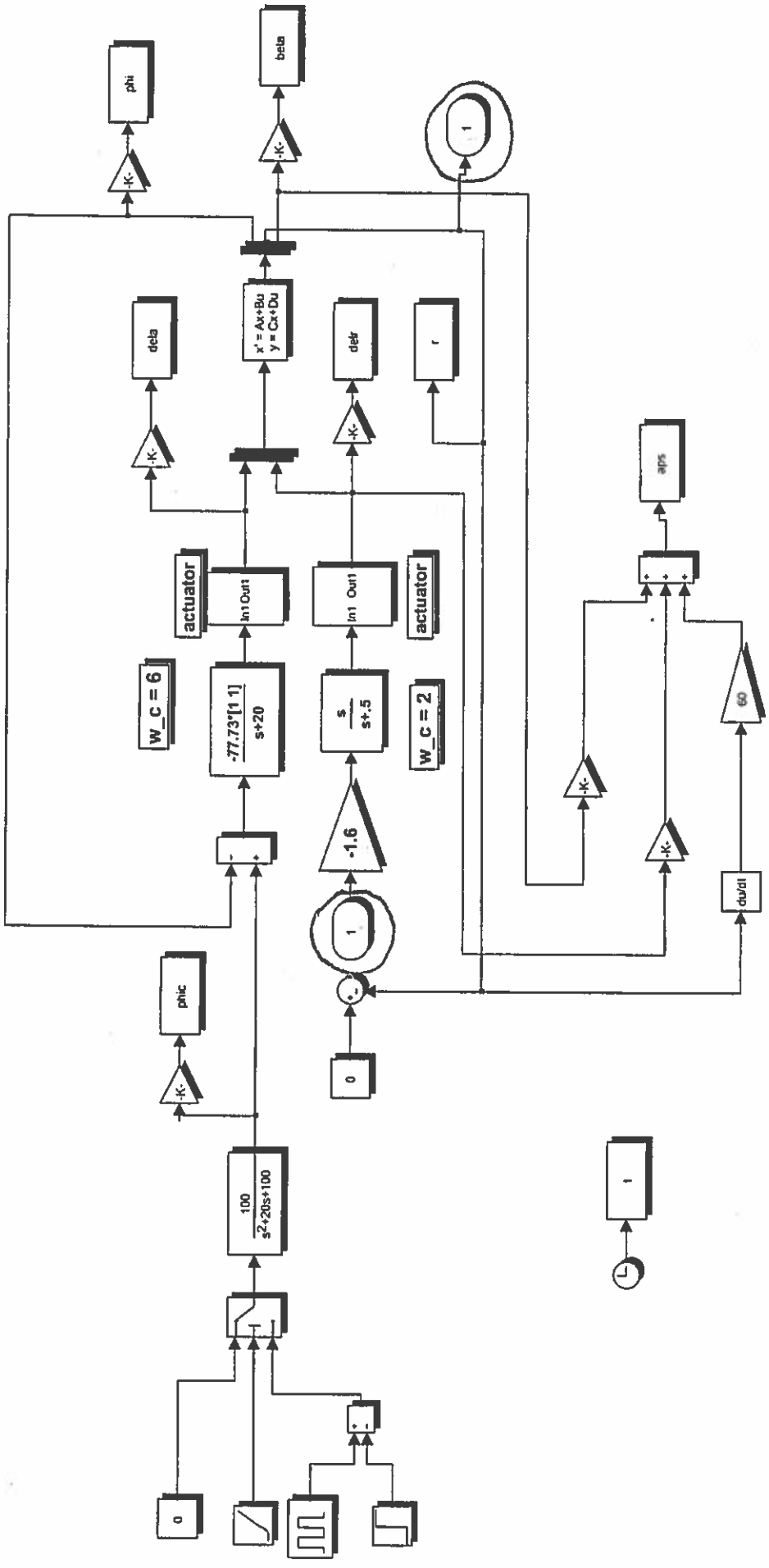
$\therefore Z = 0$




phi
beta
dela
x' = Ax+Bu
y = Cx+Du
delr
r
aps
actuator
In1 Out1
actuator
W_c = 6
-77.73*[1 1]
s+20
s
s+.5
W_c = 2
60
du/dt
phic
100
s^2+20s+100
0

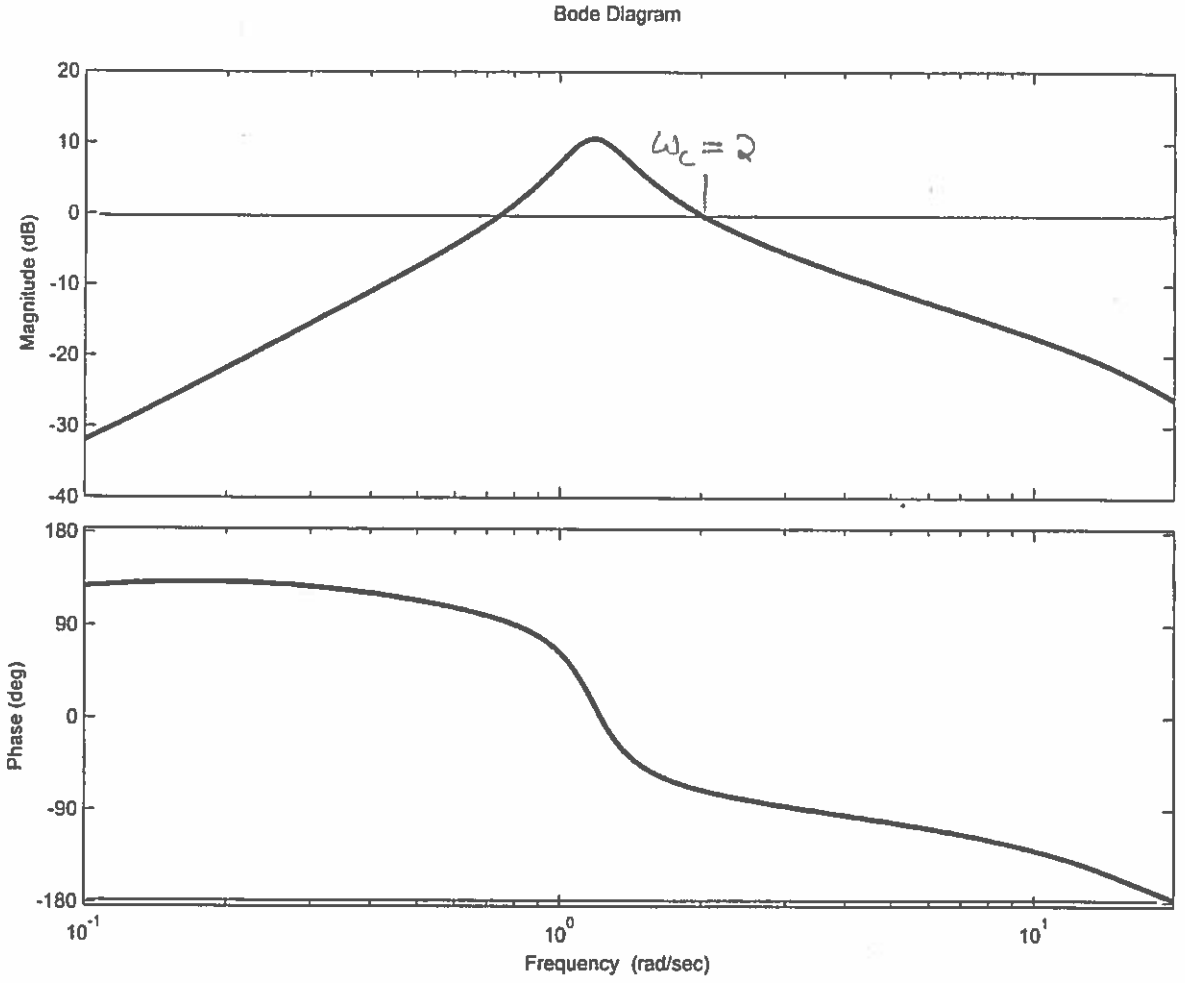
$$\left.\frac{r}{\delta_r}\right|_{\phi - \delta_a}$$




phi
beta
delta
delr
r
x' = Ax+Bu
y = Cx+Du
actuator
In1 Out1
W_c = 6
-77.73*[1 1]
s+20
s
s+.5
W_c = 2
-1.6
-K-
1
0
phic
100
s^2+20s+100
aps
dw/dt

$\frac{r}{r_c}$

$\phi - \delta_a$



```
>> zpk(r_re)

Zero/pole/gain:

-1.4211e-014 s (s+1.973e008) (s-1.973e008) (s+0.9351) (s+0.08107)

                          (s^2  + 7.924s + 93.43) (s^2  + 40.64s + 563.7)
-------------------------------------------------------------------------
(s+0.9442) (s+0.5) (s^2  + 0.3768s + 1.39) (s^2  + 7.955s + 92.87)

                          (s^2  + 40.59s + 562.4) (s^2  + 28.28s + 400)
```

$= \frac{C}{r_c}\Big|_{\Theta - \delta_q}$

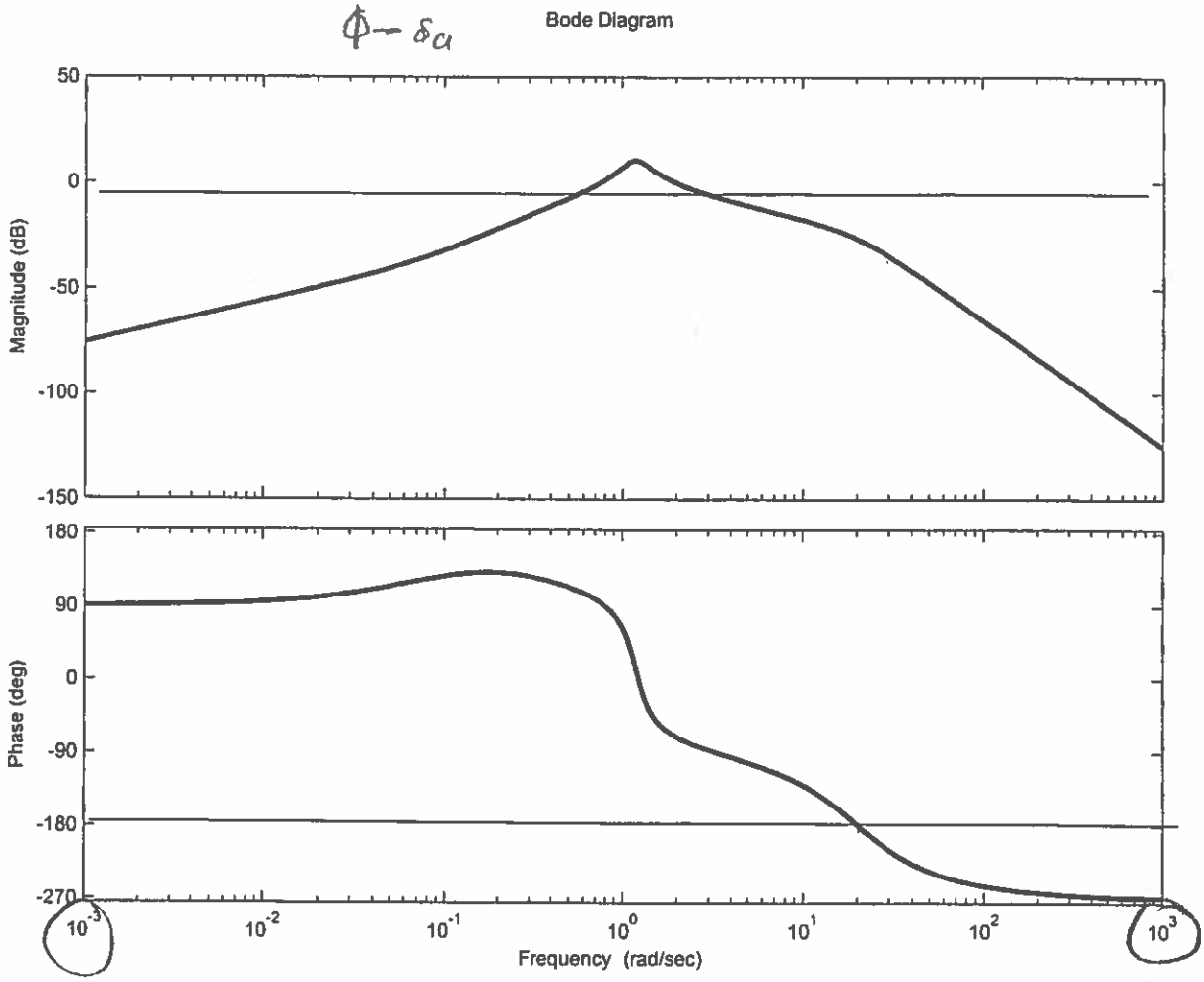
$n = 0$

$p = 1$

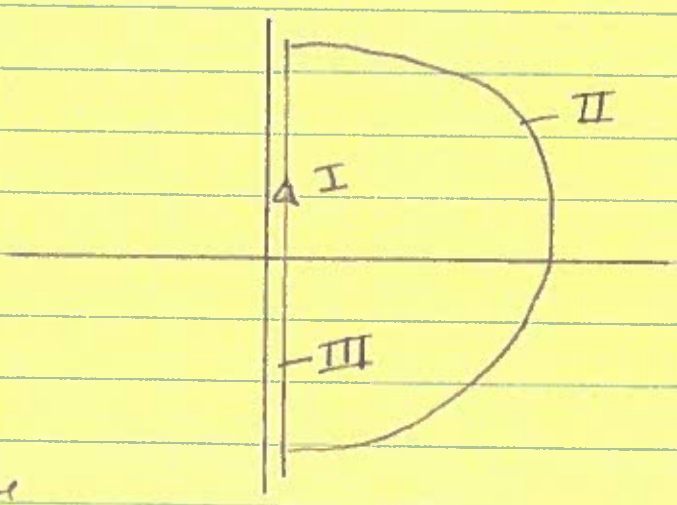
$m = 1$

$\frac{r}{r_e}$ |

$\phi - \delta_a$



r-loop with ϕ-loop closed

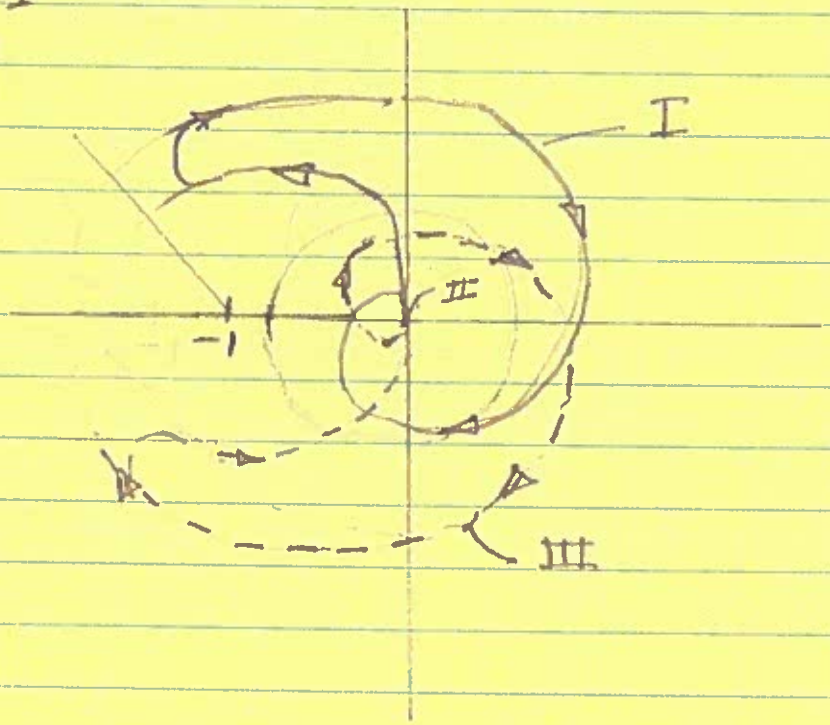


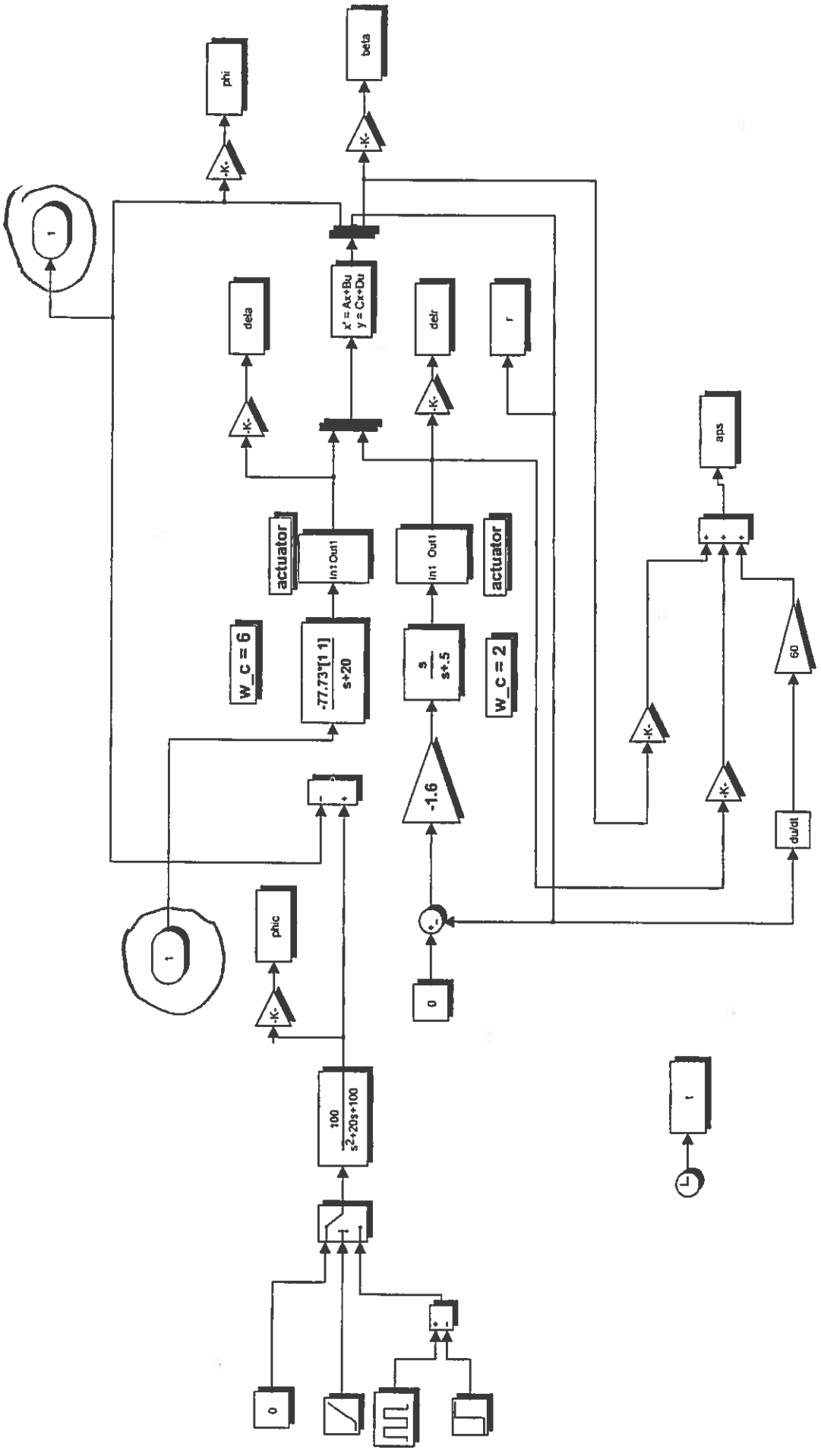
I: from Bode

$N = Z - P$

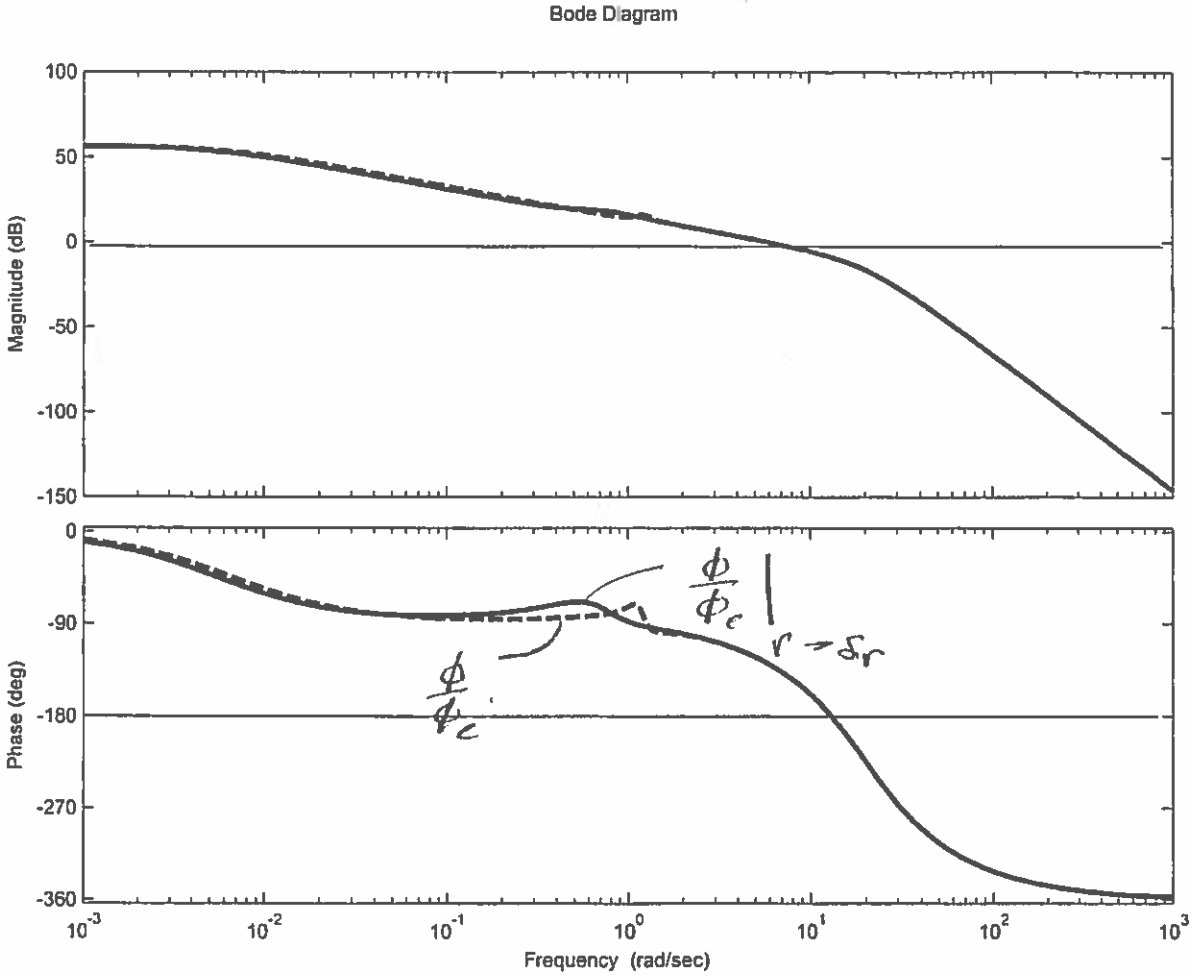
$P = 0$

$Z = 0$




phi
beta
-K-
1
x' = Ax+Bu
y = Cx+Du
dela
delr
r
actuator
In1 Out1
W_c = 6
-77.73*[1 1]
s+20
s
s+.5
W_c = 2
-1.6
aps
60
du/dt
phic
0
100
s^2+20s+100
1

$$\frac{\phi}{\phi_e} \quad \varepsilon \quad \frac{\phi}{\phi_e}\Big|_{r \to \delta_r}$$



```
>> zpk(gfinal)

Zero/pole/gain:

5.6843e-014 (s+9.605e005) (s+1.71) (s+1) (s^2 + 0.797s + 0.4449)

                    (s^2 + 26.63s + 357.3) (s^2 - 9.605e005s + 9.225e011)
-------------------------------------------------------------------------
(s+20) (s+1.713) (s+1.364) (s+0.005584) (s^2 + 0.6389s + 0.5537)

                              (s^2 + 26.65s + 357.6) (s^2 + 28.28s + 400)
```

$= \frac{\phi}{\phi_e}\Big|_{r \to \delta_r}$


```
>> zpk(phi_phie)

Zero/pole/gain:

8.5265e-014 (s+8.391e005) (s+1) (s^2 + 0.3615s + 1.359) (s^2 - 8.391e005s + 7.04e011)
-------------------------------------------------------------------------------------
     (s+20) (s+1.329) (s+0.006784) (s^2 + 0.2533s + 1.433) (s^2 + 28.28s + 400)
```

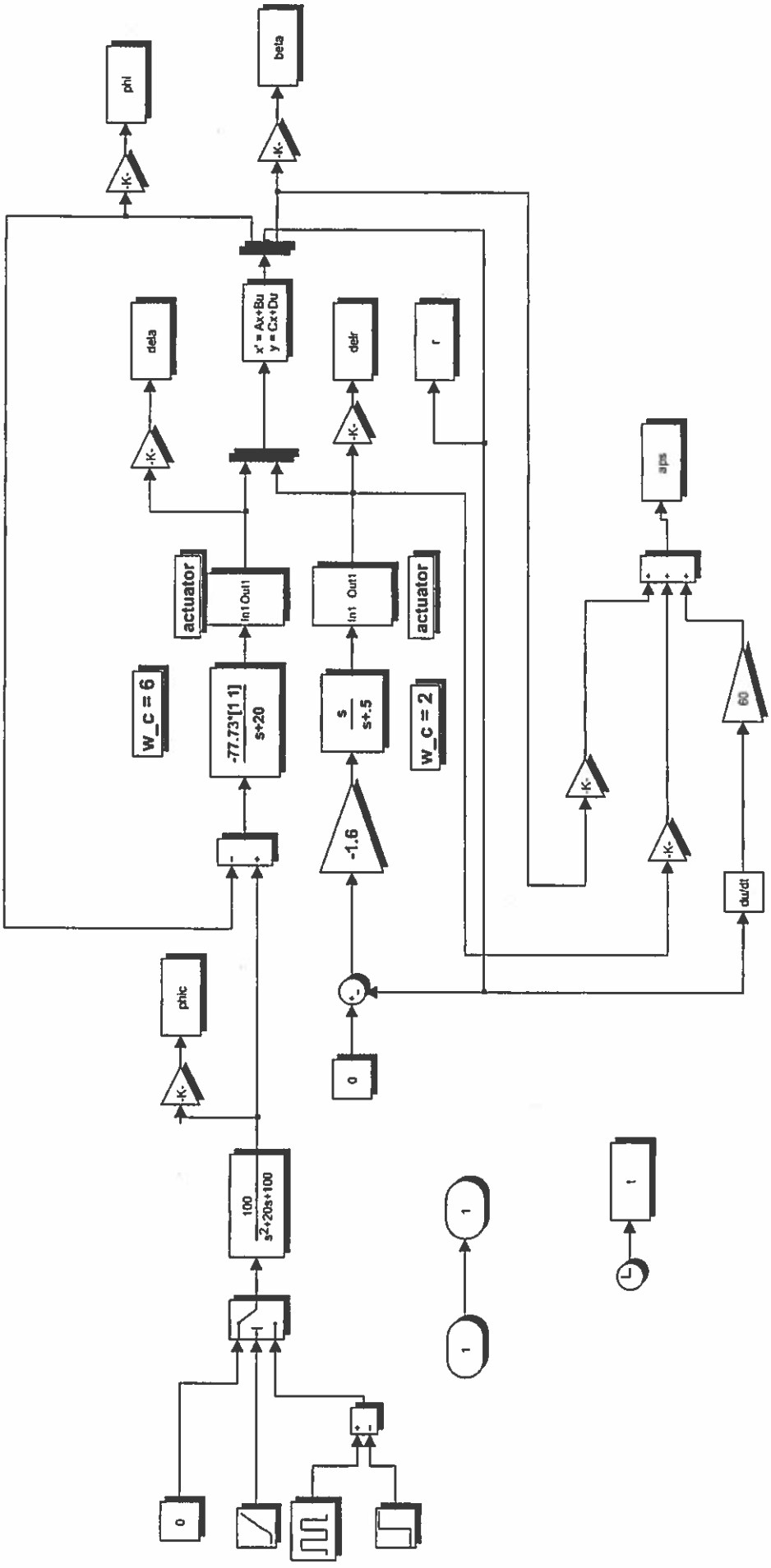
not real

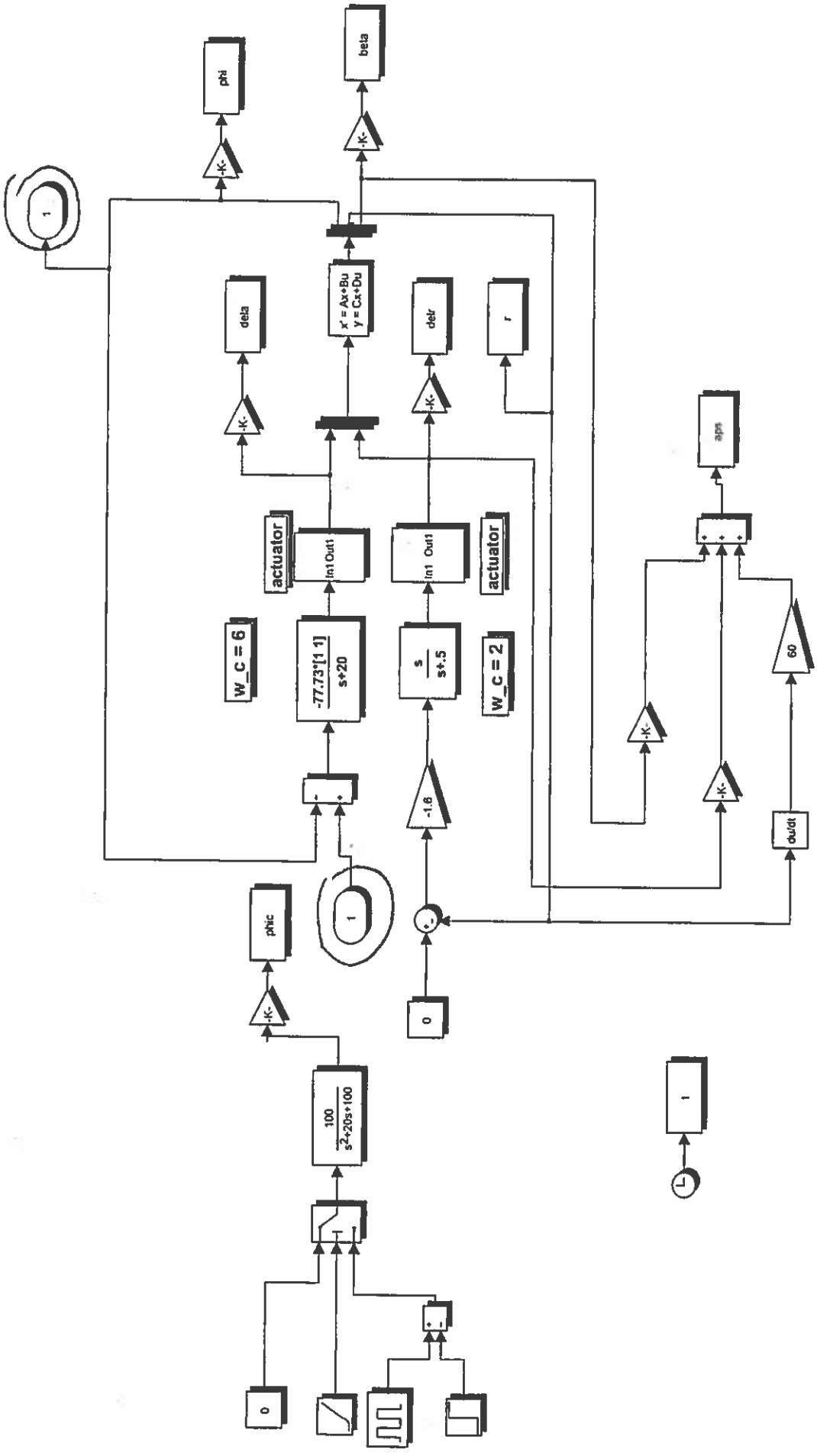
$= \frac{\phi}{\phi_e}$

$n = 0$

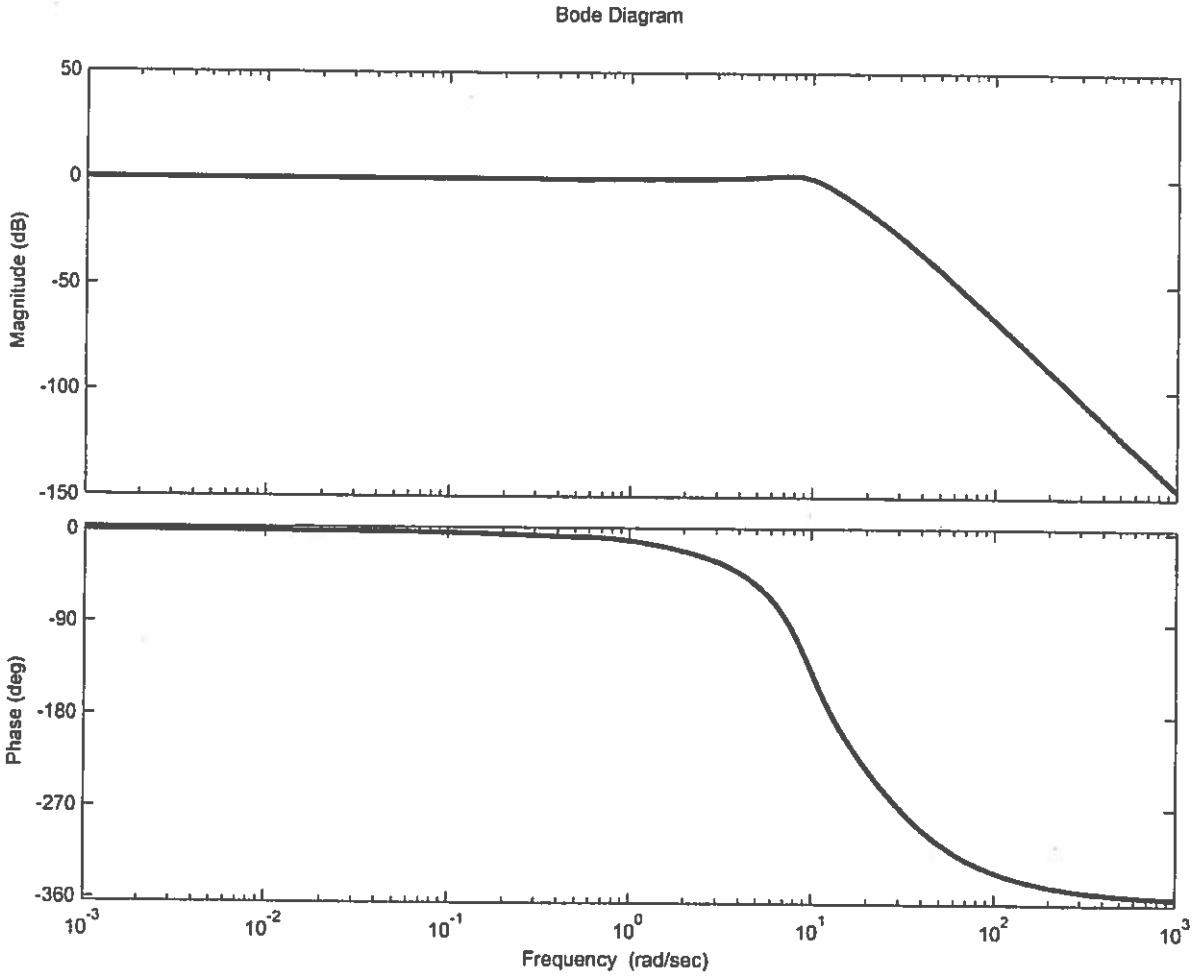
$m = 0$

$p = 0$

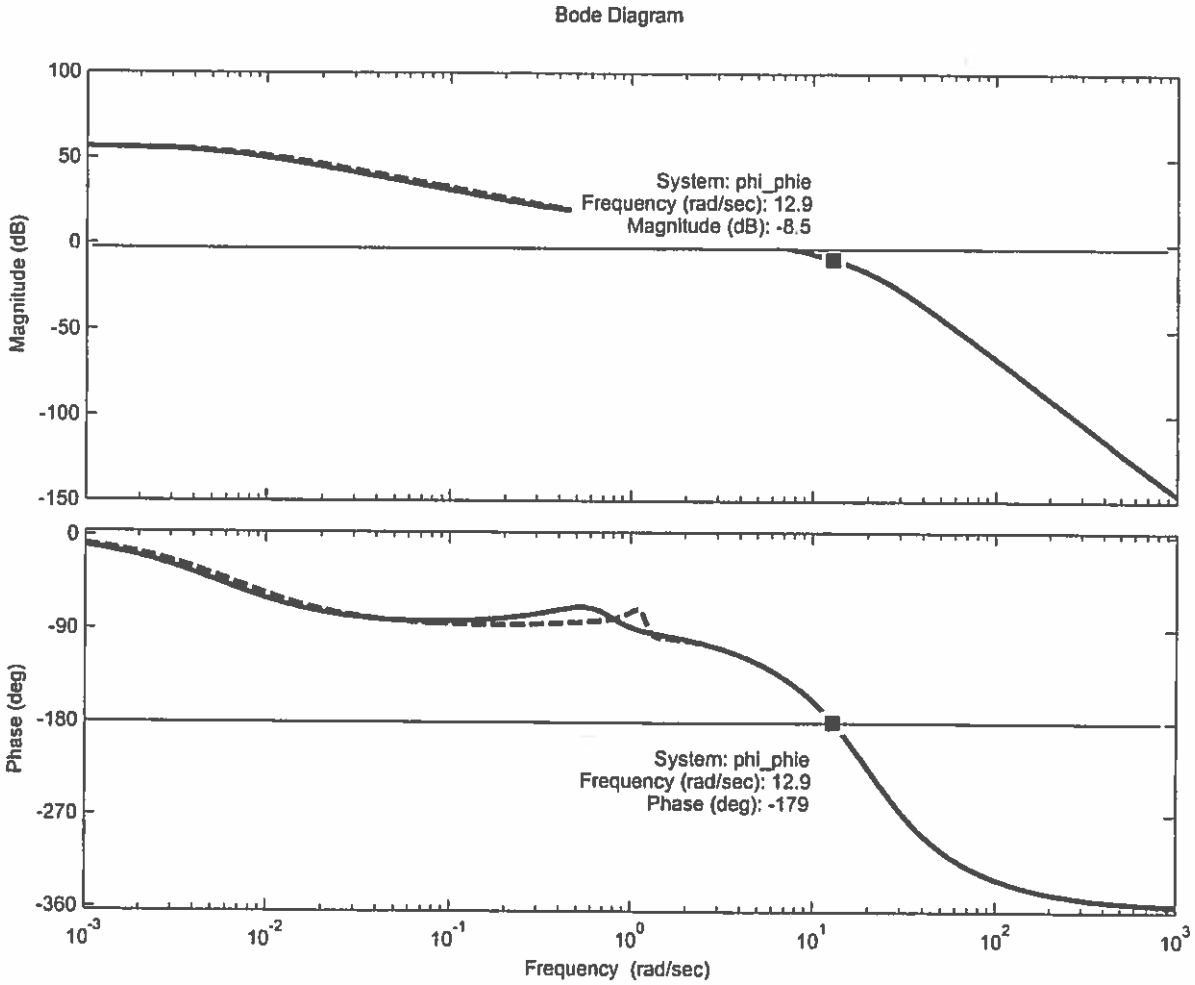

phi
beta
-K-
x' = Ax+Bu
y = Cx+Du
dela
delr
r
actuator
In1 Out1
W_c = 6
-77.73*[1 1]
s+20
s
s+.5
W_c = 2
-1.6
ay
60
du/dt
phic
0
100
s^2+20s+100
1
1
t
0


phi
beta
-K-
1
dela
x' = Ax+Bu
y = Cx+Du
delr
r
actuator
In1 Out1
W_c = 6
-77.73*[1 1]
s+20
s
s+.5
W_c = 2
-1.6
aps
60
du/dt
phic
0
100
s^2+20s+100

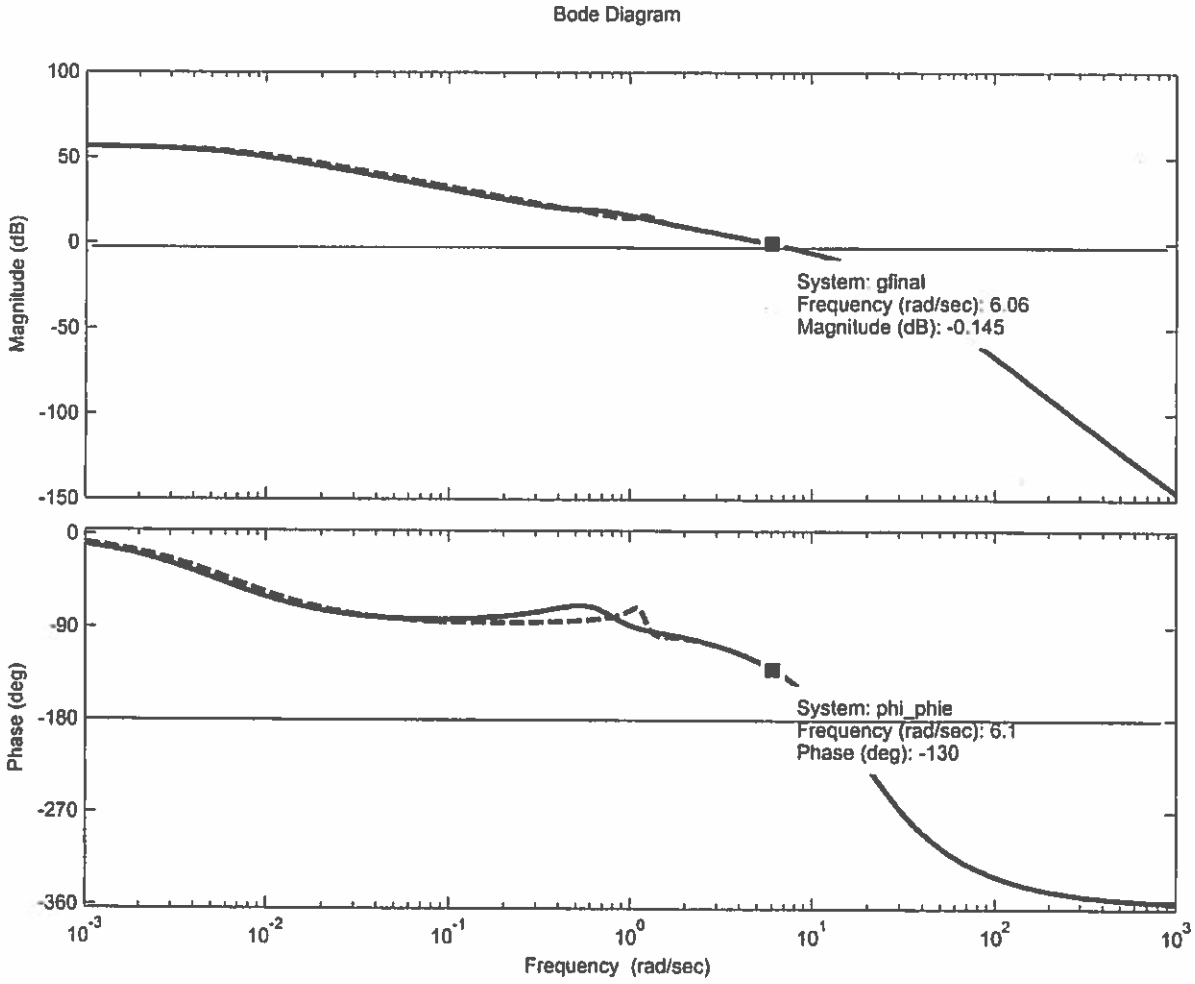
$$\left.\frac{\phi}{\phi_c}\right|_{r-\delta r}$$



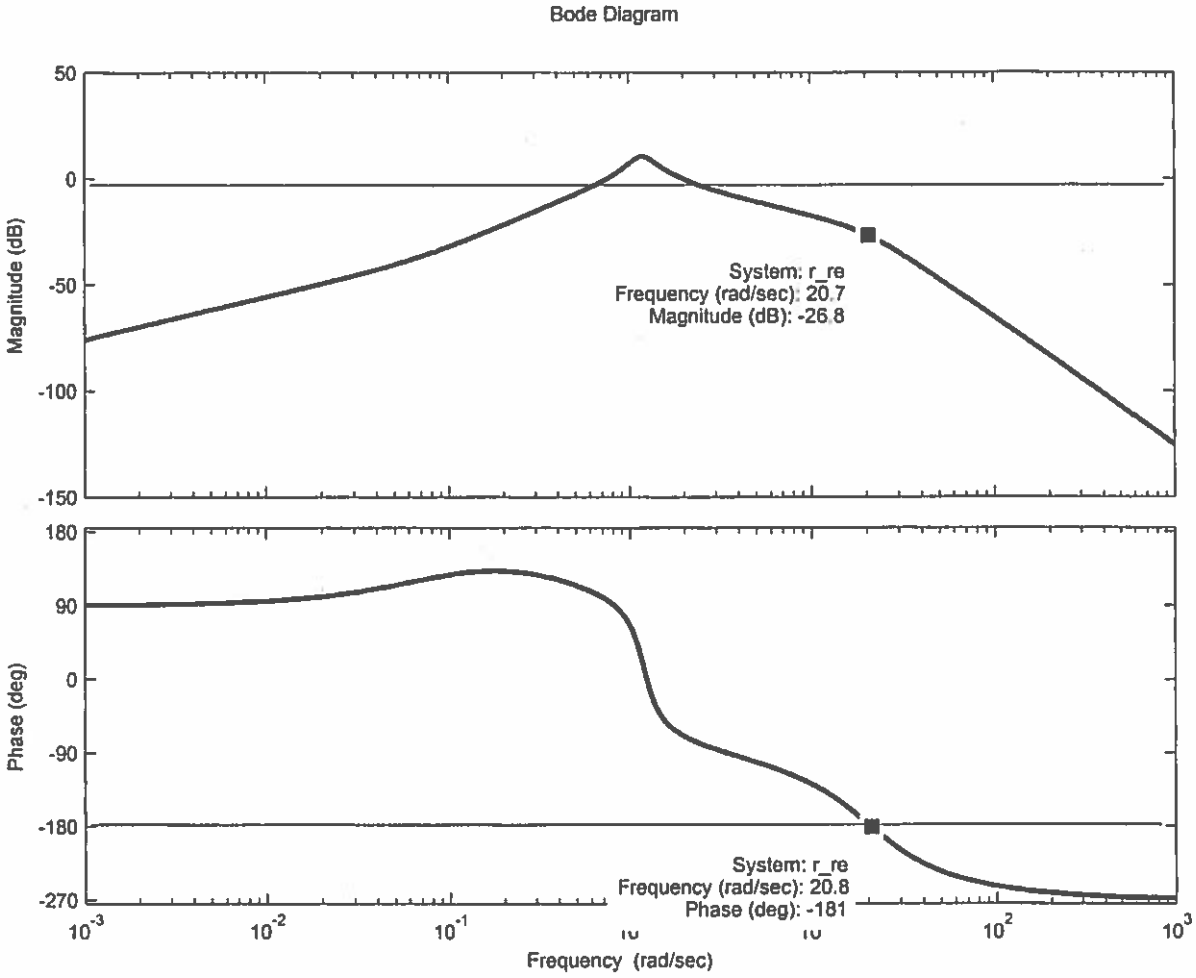
ϕ-loop gain margin = 8.5 dB



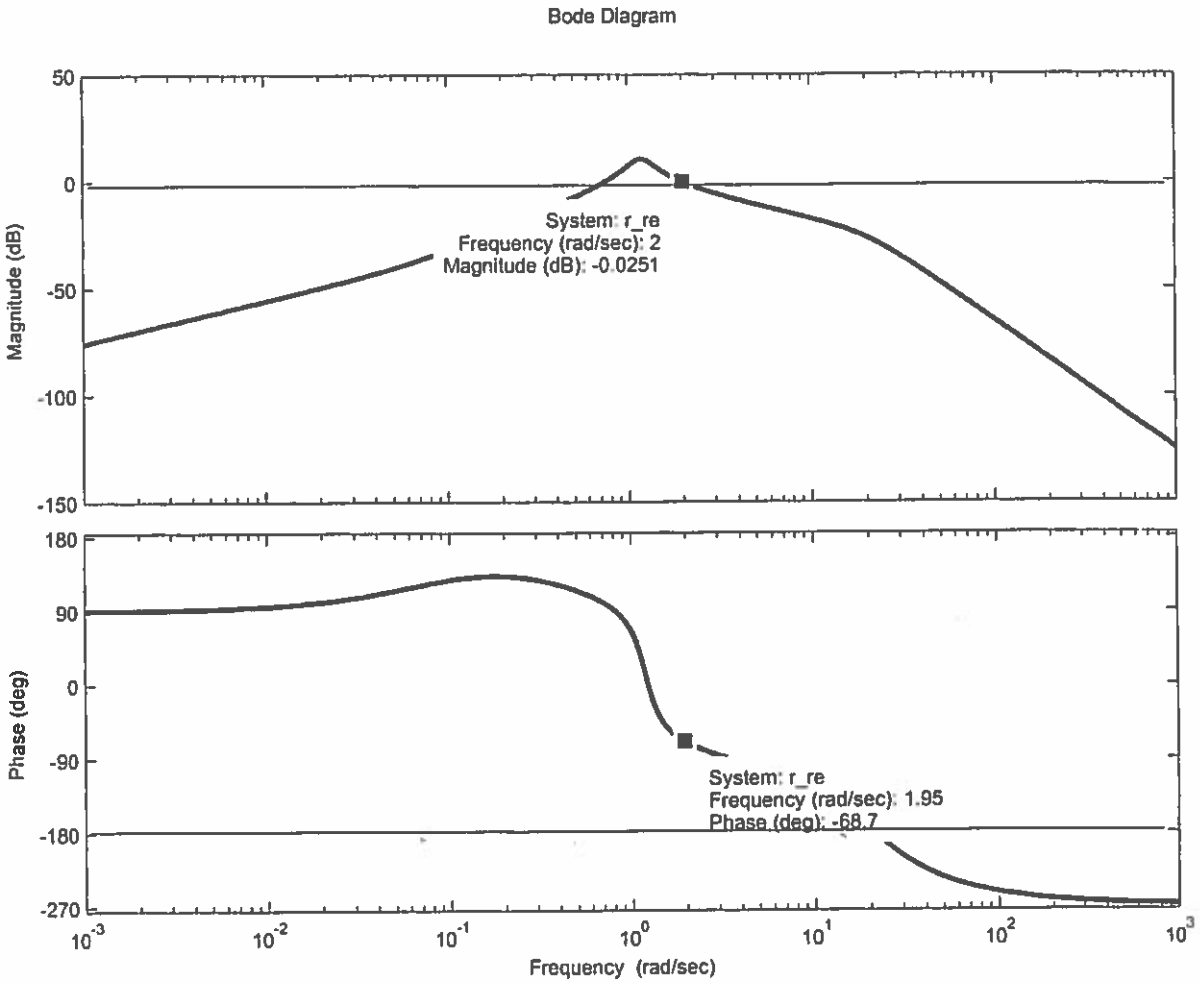
ϕ-loop phase-margin = 180 - 130 = 50°

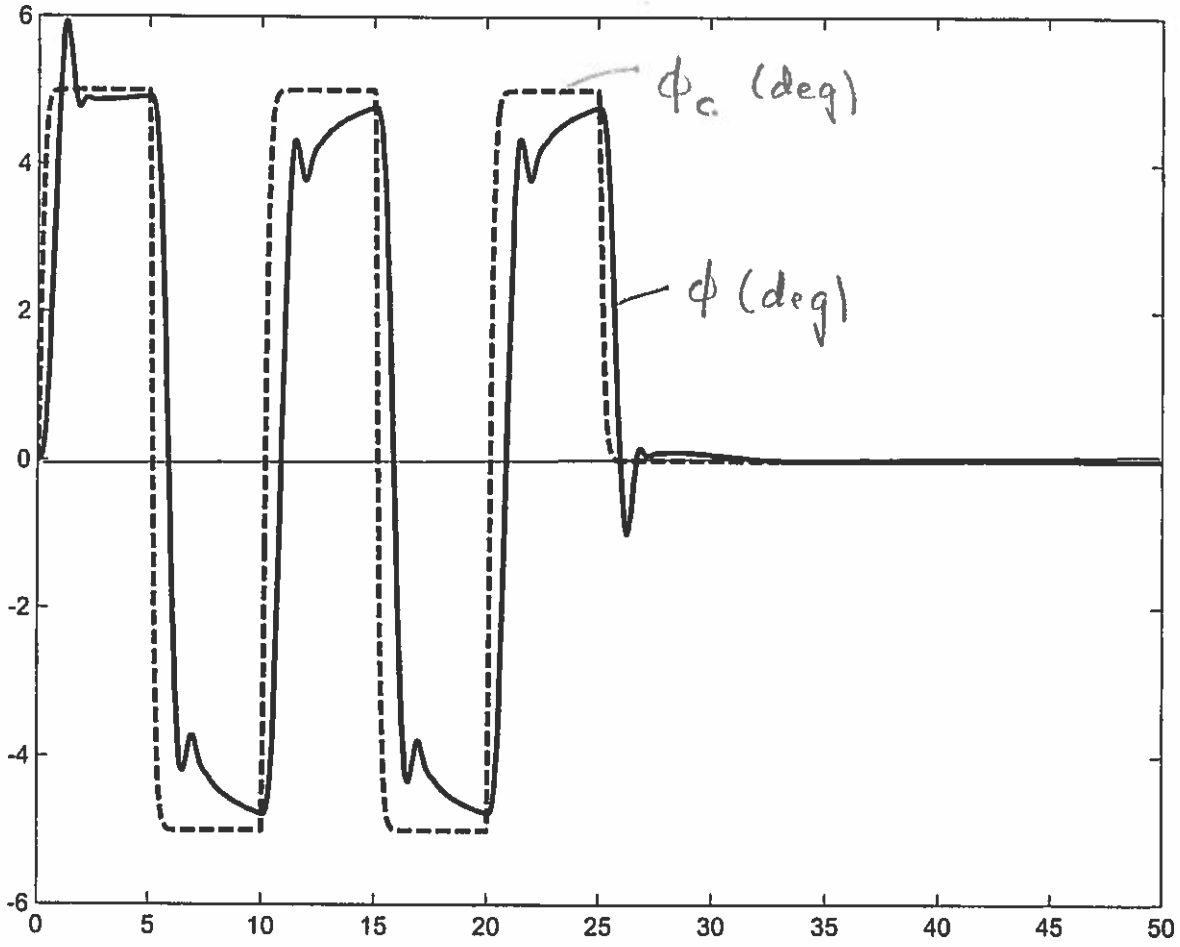


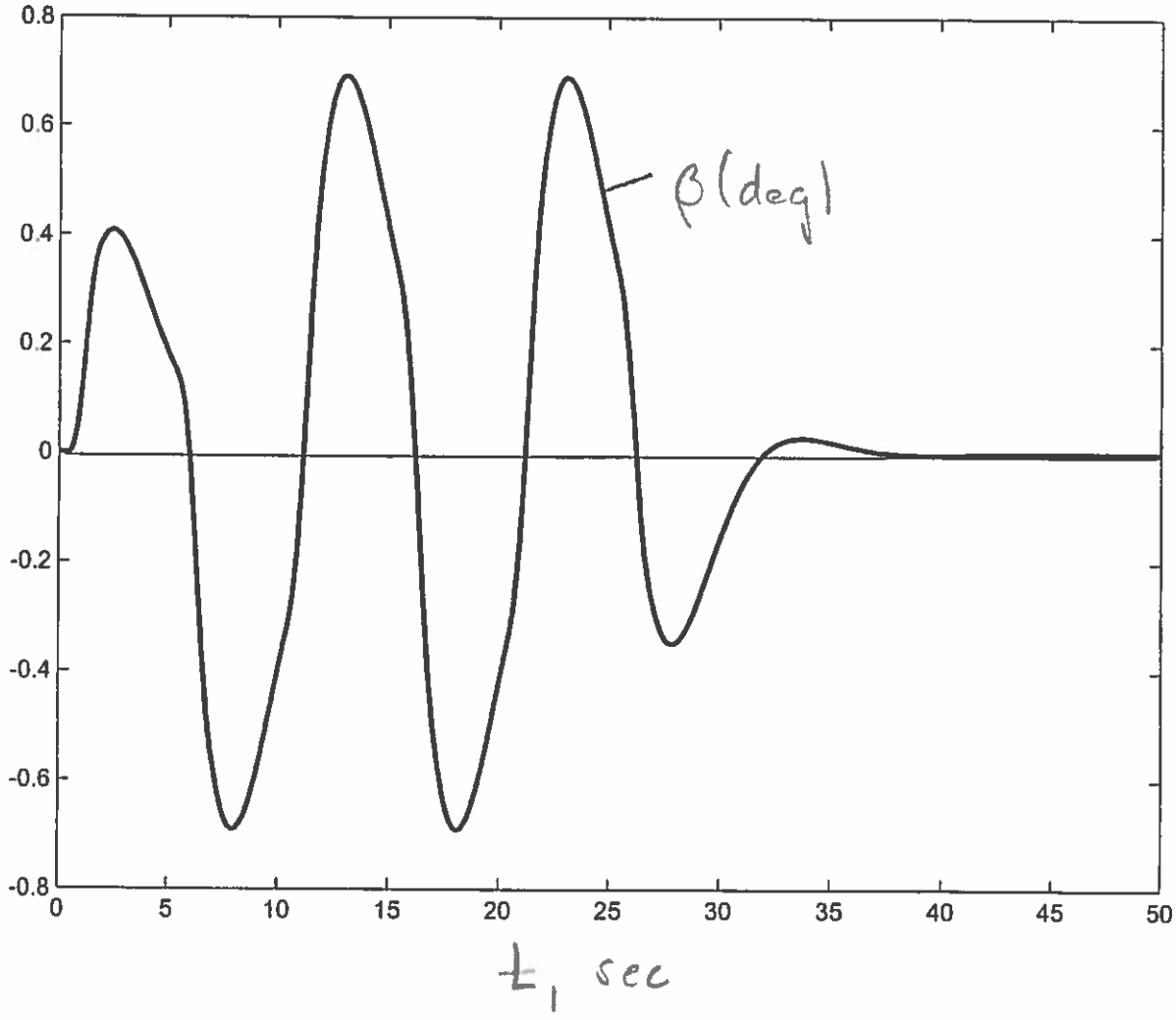
r-loop gain-margin = 26.8dB

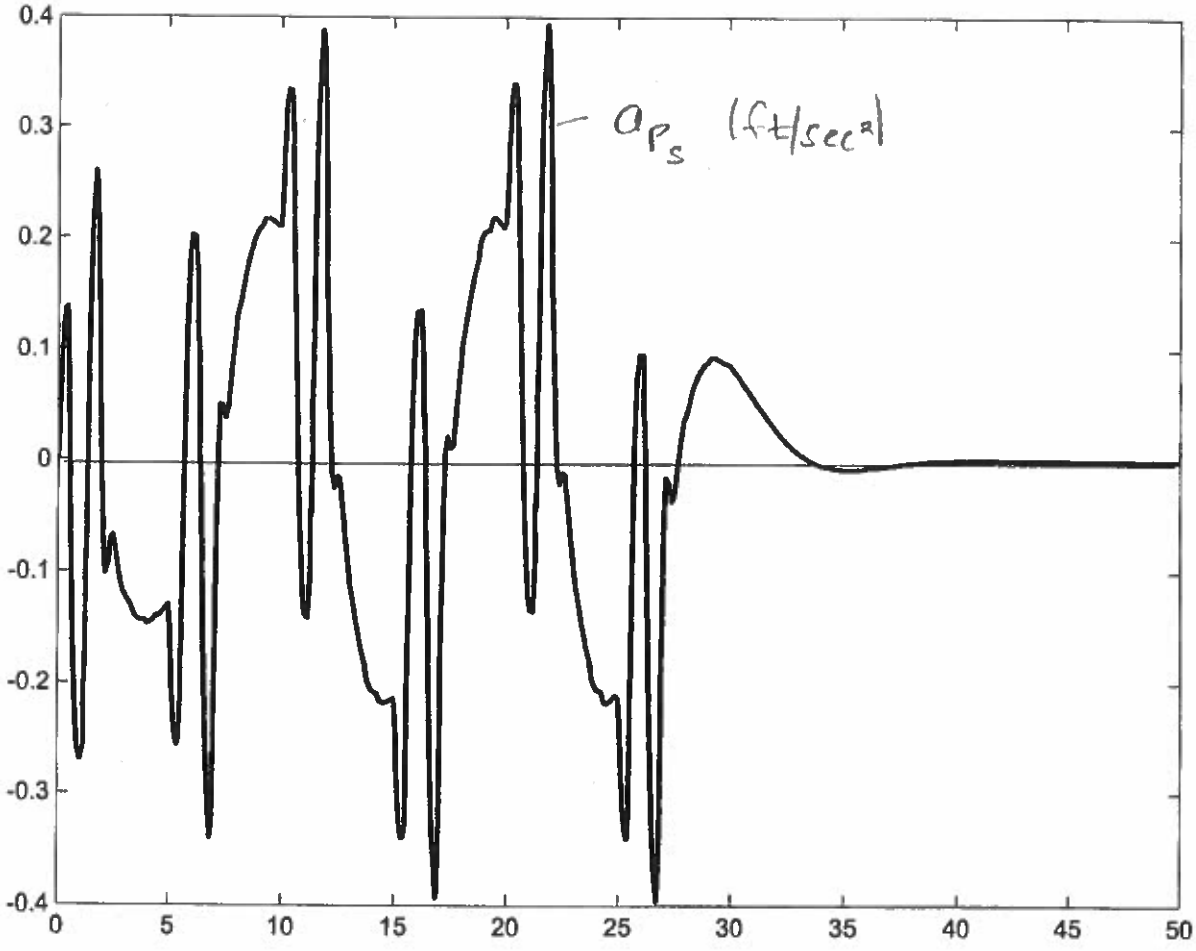


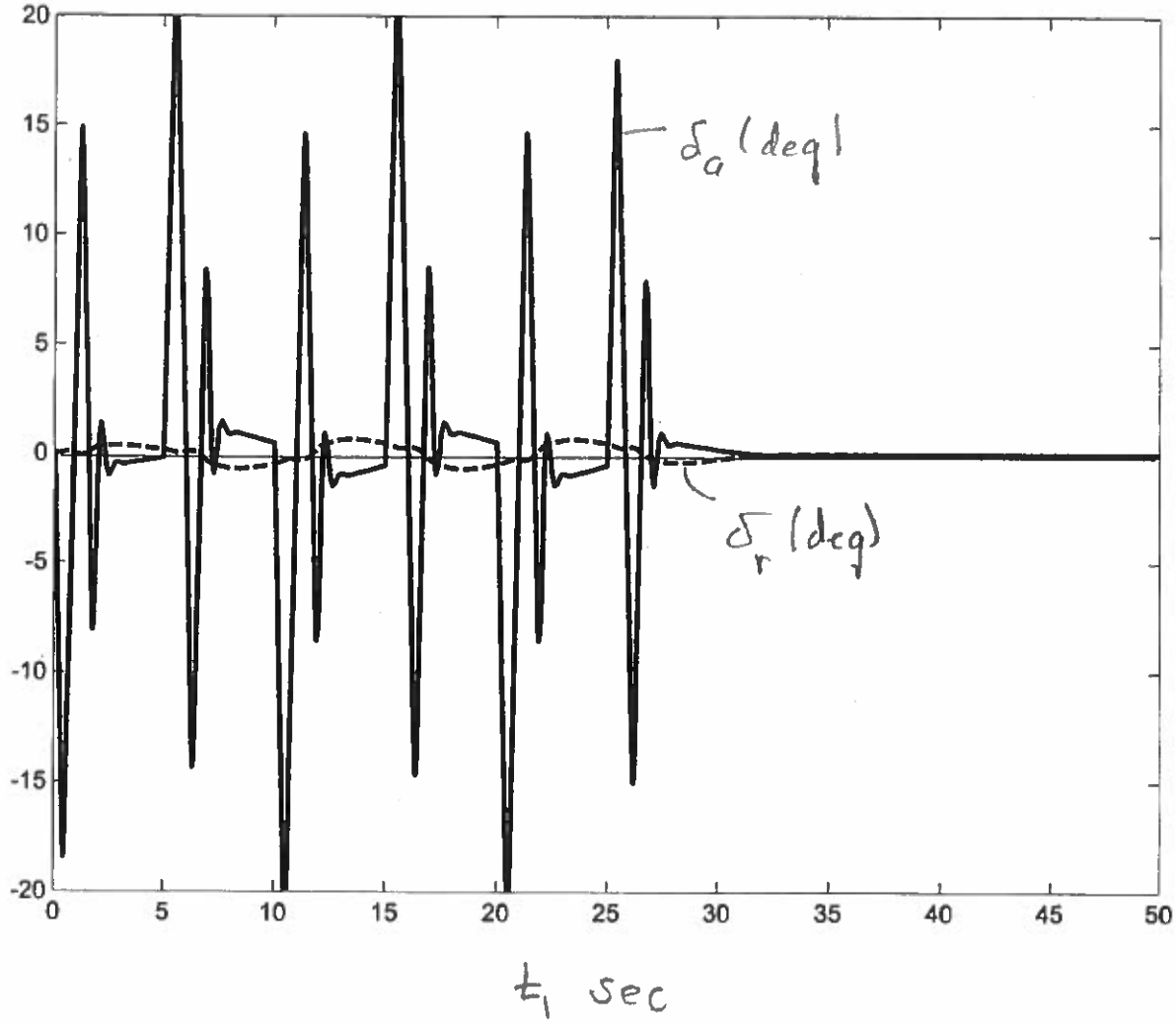
r-loop Phase margin = 111°




6
4
2
0
-2
-4
-6
0
5
10
15
20
25
30
35
40
45
50
ϕ_c (deg)
ϕ (deg)
t, sec


0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20
25
30
35
40
45
50
β(deg)
t, sec


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
0
5
10
15
20
25
30
35
40
45
50
a_{P_s} (ft/sec²)


20
15
10
5
0
-5
-10
-15
-20
0
5
10
15
20
25
30
35
40
45
50
δ_a (deg)
δ_r (deg)
t, sec