MAE 275 - Homework 1

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1 Problem 1

Beginning with the integral equation from the handwritten notes

$$\overline{M}_0 + \overline{M}_{T_0} + \overline{M}_{IR_0} = \iiint\limits_{sys} \overline{r}^2 \dot{\overline{\Omega}} dm - \iiint\limits_{sys} (\overline{r} \cdot \dot{\overline{\Omega}}) \overline{r} dm - \iiint\limits_{sys} (\dot{\overline{\Omega}} \cdot \overline{r})(\overline{r} \times \dot{\overline{\Omega}}) dm$$

and defining

$$\begin{aligned}
\overline{r} &= x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}} \\
\overline{\Omega} &= P\hat{\mathbf{i}} + Q\hat{\mathbf{j}} + R\hat{\mathbf{k}} \\
\dot{\overline{\Omega}} &= \dot{P}\hat{\mathbf{i}} + \dot{Q}\hat{\mathbf{j}} + \dot{R}\hat{\mathbf{k}}
\end{aligned}$$

one can find the z components of the moment equation. Inserting these definitions and ignoring the x and y components

$$\begin{aligned}
N\hat{\mathbf{k}} = &+ \dot{R}\hat{\mathbf{k}} \iiint\limits_{sys} (x^2 + y^2 + z^2) dm \\
&- \iiint\limits_{sys} x\dot{P}z\hat{\mathbf{k}} dm - \iiint\limits_{sys} y\dot{Q}z\hat{\mathbf{k}} dm - \iiint\limits_{sys} z\dot{R}z\hat{\mathbf{k}} dm \\
&+ \iiint\limits_{sys} (Px + Qy + Rz)(Qx - Py)\hat{\mathbf{k}} dm
\end{aligned}$$

Simplifying and distributing terms

$$\begin{aligned}
N = &+ \dot{R} \iiint\limits_{sys} (x^2 + y^2 + z^2) dm - \dot{P} \iiint\limits_{sys} xz dm - \dot{Q} \iiint\limits_{sys} yz dm - \dot{R} \iiint\limits_{sys} z^2 dm \\
&+ \iiint\limits_{sys} (PQx^2 - P^2xy + Q^2xy - PQy^2 + RQxz - PRyz) dm
\end{aligned}$$

Adding and subtracting $PQ \iiint\limits_{sys} z^2 dm$ to the RHS

$$\begin{aligned} N = &+ \dot{R} \iiint\limits_{sys} (x^2 + y^2) dm - \dot{P} \iiint\limits_{sys} xzdm - \dot{Q} \iiint\limits_{sys} yzdm \\ &+ PQ \iiint\limits_{sys} (x^2 + z^2) dm - P^2 \iiint\limits_{sys} xydm + Q^2 \iiint\limits_{sys} xydm \\ &- PQ \iiint\limits_{sys} (y^2 + z^2) dm + RQ \iiint\limits_{sys} xzdm - PR \iiint\limits_{sys} yzdm \end{aligned}$$

Recognizing geometric terms

$$\begin{aligned} N = &+ \dot{R} I_z - \dot{P} I_{xz} - \dot{Q} I_{yz} \\ &+ PQI_y - P^2 I_{xy} + Q^2 I_{xy} \\ &- PQI_x + RQI_{xz} - PRI_{yz} \end{aligned}$$

Finally, assuming an XZ plane of symmetry ($I_{yz} = I_{xy} = 0$)

$$N = \dot{R} I_z - \dot{P} I_{xz} + PQ(I_y - I_x) + QRI_{xz}$$

2 Problem 2

Given the following z-force equation

$$Z = Z_T - Z_g = m(\dot{W} + PV - QU - g \cos(\theta_0) \cos(\phi_0))$$

it is possible to linearize this equation using a perturbation method. Adding a small disturbance to each term, so that

$$\begin{aligned} Z &= Z_0 + dZ \\ \dot{W} &= \dot{W}_0 + \dot{w} \\ P &= P_0 + p \\ V &= V_0 + v \\ Q &= Q_0 + q \\ U &= U_0 + u \end{aligned}$$

$$\begin{aligned} c(\theta_0 + \theta) c(\phi_0 + \phi) = &s\theta_0 s\phi_0 s\theta s\phi + c\theta_0 c\phi_0 c\theta c\phi - \\ &c\theta_0 s\phi_0 c\theta s\phi - s\theta_0 c\phi_0 s\theta c\phi \end{aligned} \tag{1}$$

The disturbances from the steady flight conditions are assumed to be small enough so that the sines and cosines of the disturbance angles are approximately the angles themselves and 1, respectively, and so that the products and squares of the disturbance quantities are negligible compared to the quantities themselves.

Applying this to Equation 1, we have

$$c(\theta_0 + \theta) c(\phi_0 + \phi) = c\theta_0 c\phi_0 - c\theta_0 s\phi_0 \phi - s\theta_0 c\phi_0 \theta$$

Plugging these terms into our z-force equation results in

$$
\begin{aligned}
Z_0 + dZ =& m[\dot{W}_0 + \dot{w} + (P_0 + p)(V_0 + v) - (Q_0 + q)(U_0 + u) - \\
& g(c\theta_0 c\phi_0 - c\theta_0 s\phi_0 \phi - s\theta_0 c\phi_0 \theta)]
\end{aligned}
$$

Multiplying the terms out results in

$$
\begin{aligned}
Z_0 + dZ = m[&\dot{W}_0 + \dot{w} + P_0 V_0 + P_0 v + V_0 p + pv \\
&- Q_0 U_0 - Q_0 u - U_0 q - qu - \\
&g(c\theta_0 c\phi_0 - c\theta_0 s\phi_0 \phi - s\theta_0 c\phi_0 \theta)]
\end{aligned}
$$

Subtracting Z_0 from both sides gives

$$dZ = m[\dot{w} + P_0 v + V_0 p + pv - Q_0 u - U_0 q - qu - g(-c\theta_0 s\phi_0 \phi - s\theta_0 c\phi_0 \theta)]$$

Simplifying some signs and removing products of disturbance quantities

$$dZ = m[\dot{w} + V_0 p + P_0 v - U_0 q - Q_0 u + (gc\theta_0 s\phi_0)\phi + (gs\theta_0 c\phi_0)\theta]$$