MAE 275 - Final

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & -\dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} & -X_u & -X_w & 0 \\ \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \dfrac{Z_{\delta_T}}{1-Z_{\dot{w}}} & \dfrac{-Z_u}{1-Z_{\dot{w}}} & \dfrac{-Z_w}{1-Z_{\dot{w}}} & \dfrac{-Z_q}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} & M_{\delta_T} + \dfrac{M_{\dot{w}}Z_{\delta_T}}{1-Z_{\dot{w}}} & -M_u - \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & -M_w - \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & -M_q - \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} \\ 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Plugging in the data for the A-7E aircraft in a landing approach to an aircraft carrier yields

$$A = \begin{bmatrix} -5.4534e-2 & +6.4327e-2 & 0 & -3.2200e+1 & 0 \\ -2.8695e-1 & -5.2887e-1 & +2.1800e+2 & 0 & 0 \\ -8.2071e-5 & -7.8112e-3 & -3.9053e-1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +2.1800e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +7.3284e-1 & +1.3170e-3 & +5.4534e-2 & -6.4327e-2 & 0 \\ -1.4714e+1 & -2.5000e-4 & +2.8695e-1 & +5.2887e-1 & 0 \\ -2.1846e+0 & +4.0722e-6 & +8.2071e-5 & +7.8112e-3 & +3.9053e-01 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

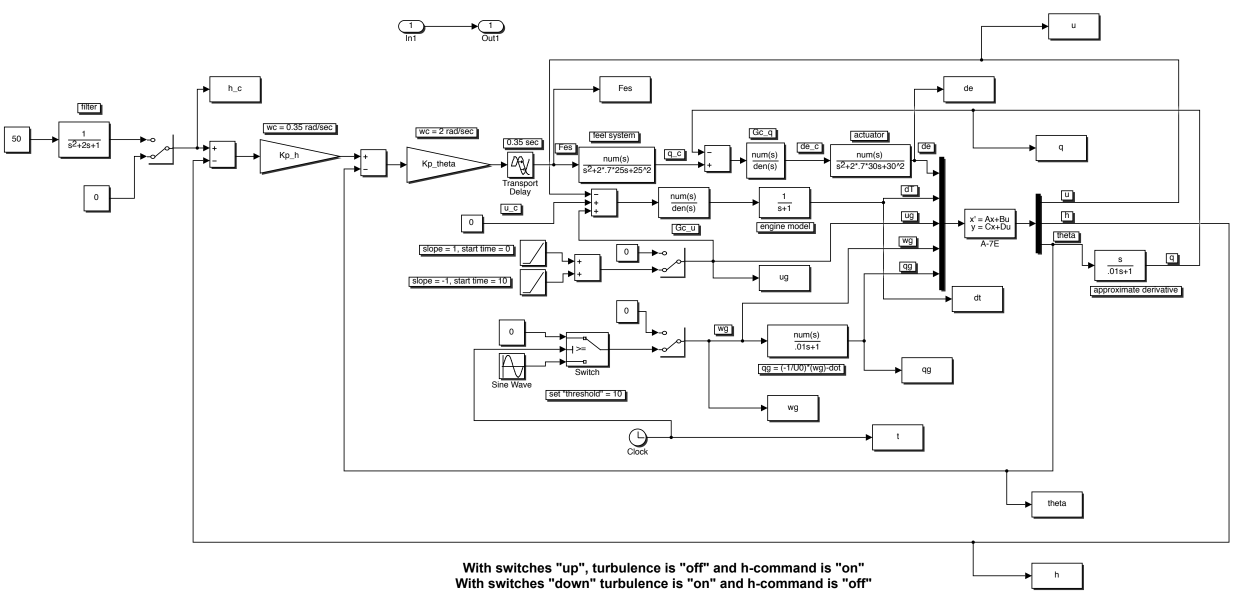

Figure 1: Final Simulink Diagram

2 Controller Design

Two controller were designed as part of a Stability and Command Augmentation System (SCAS) system. These controllers were designed to control the pitch-rate and airspeed loops. The resultant controllers were:

$$\frac{q}{\delta_e} = \frac{-218.46s(s+0.4291)(s+0.1018)}{(s+100)(s^2+0.03945s+0.03717)(s^2+0.9345s+1.904)}$$

$$\boxed{GC_q = -2 \times \frac{(s^2+0.04s+0.04)(s^2+s+1.25)}{s(s+0.1)^3}}$$

$$\begin{gathered} G_m = Inf\,\mathrm{dB} \\ P_m = 86.7^\circ \text{ at } 5.06\,\mathrm{rad/s} \\ \omega_{BW} = 5\,\mathrm{rad/s}\ \text{(3dB criterion)} \\ Unstable \end{gathered}$$

$$\left.\frac{u}{\delta_T}\right|_{q\rightarrow\delta_e} = \frac{0.001317(s+5.486)(s+0.5165)}{(s+5.459)(s+1)(s+0.4842)(s+0.1018)} * \frac{(s^2+0.03252s+0.04026)(s^2+1.196s+1.248)(s^2+36.57s+643.2)}{(s^2+0.03952s+0.04002)(s^2+1.203s+1.256)(s^2+36.58s+643.6)}$$

$$\boxed{GC_u = 335 \times \frac{(s+0.1)}{s(s+1.5)}}$$

$$\begin{gathered} G_m = 18.3\,\mathrm{dB} \text{ at } 1.21\,\mathrm{rad/s} \\ P_m = 64.5^\circ \text{ at } 0.286\,\mathrm{rad/s} \\ \omega_{BW} = .497\,\mathrm{rad/s}\ \text{(3dB criterion)} \\ Stable \end{gathered}$$

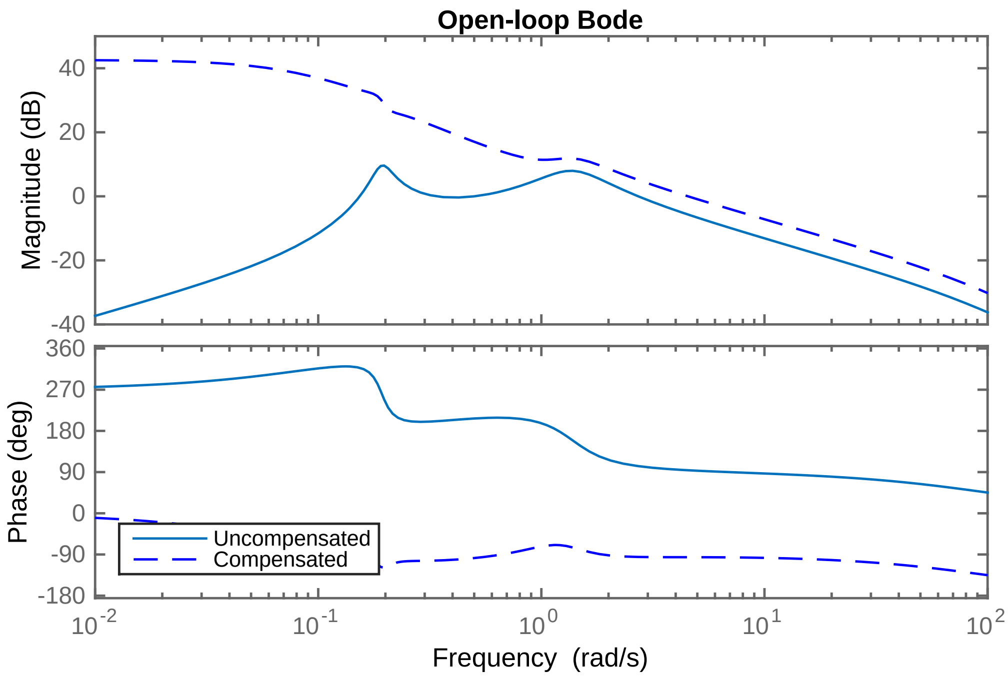

Figure 2: Open-loop Bode for Gc_q

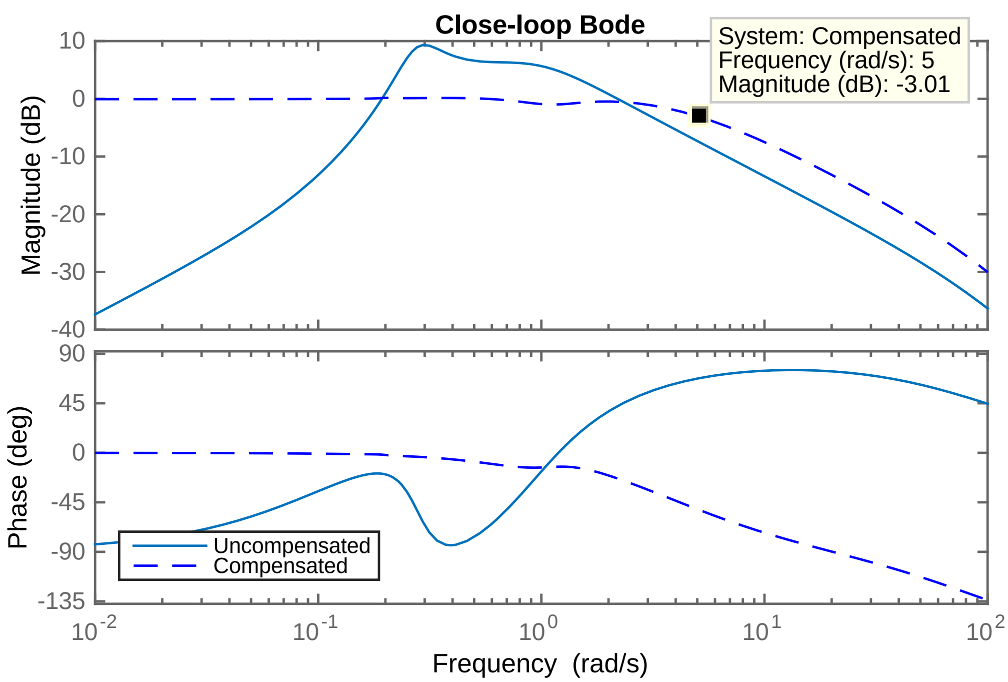

Figure 3: Close-loop Bode for Gc_q

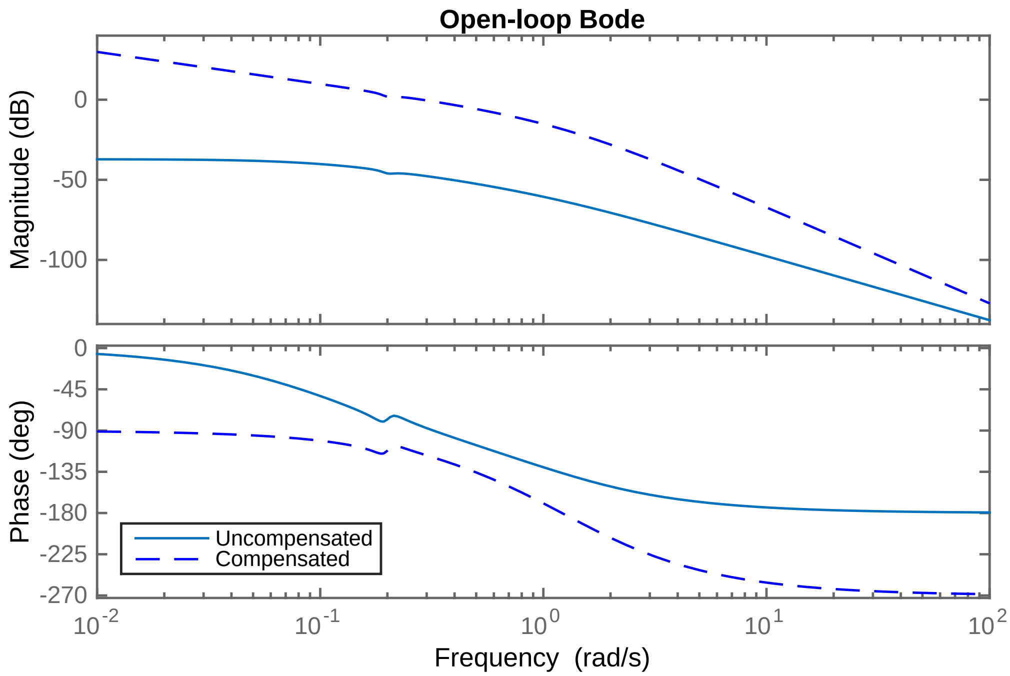

Figure 4: Open-loop Bode for Gc_u

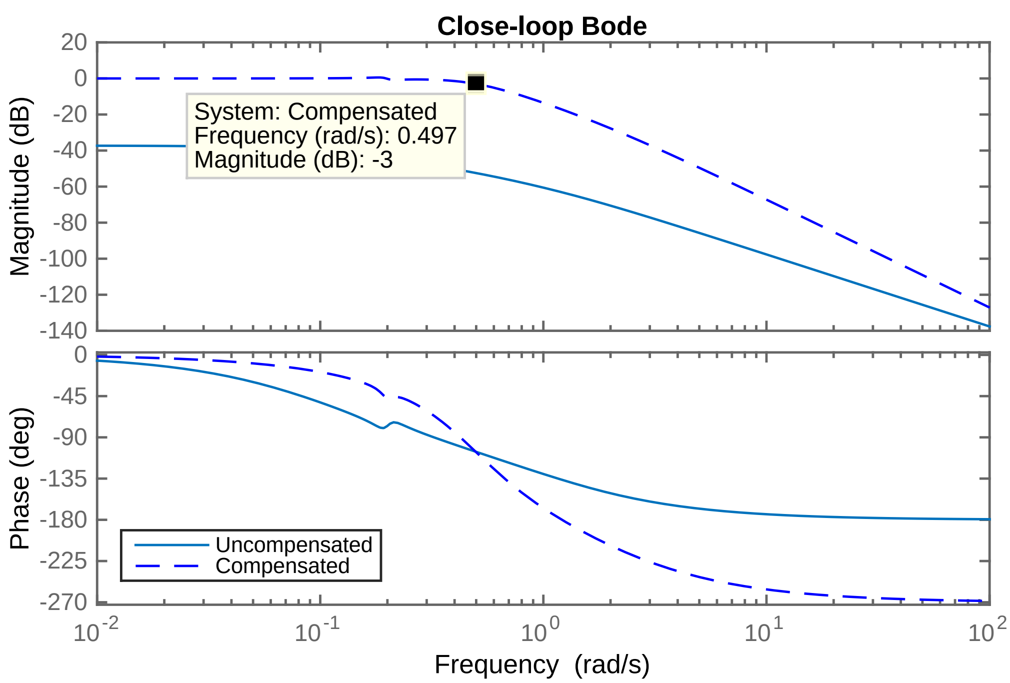

Figure 5: Close-loop Bode for Gc_u

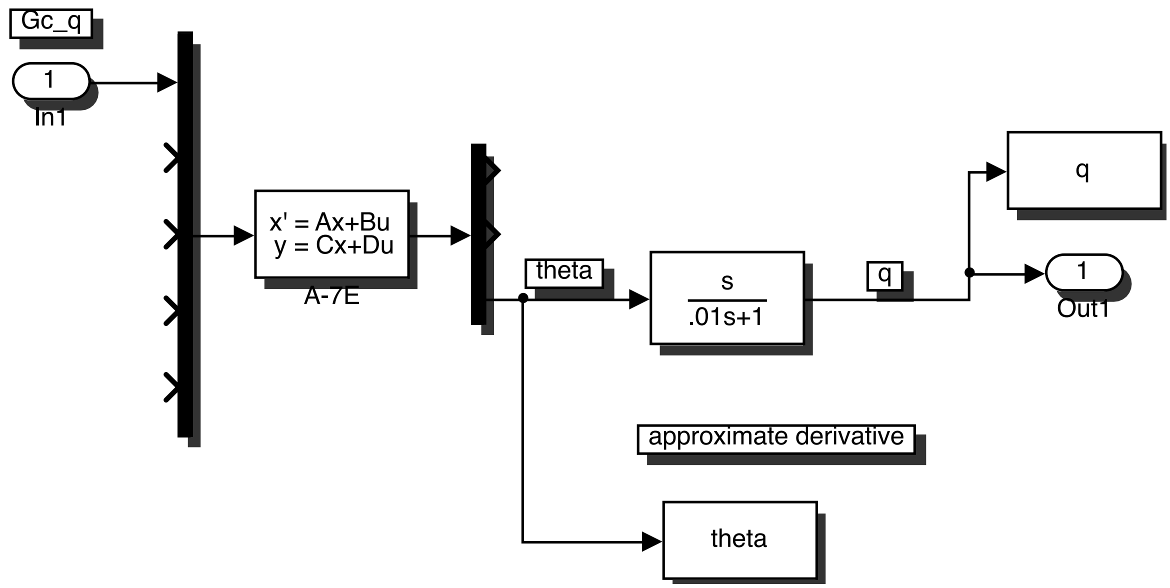

Figure 6: Simulink Diagram used to design the Pitch-rate loop

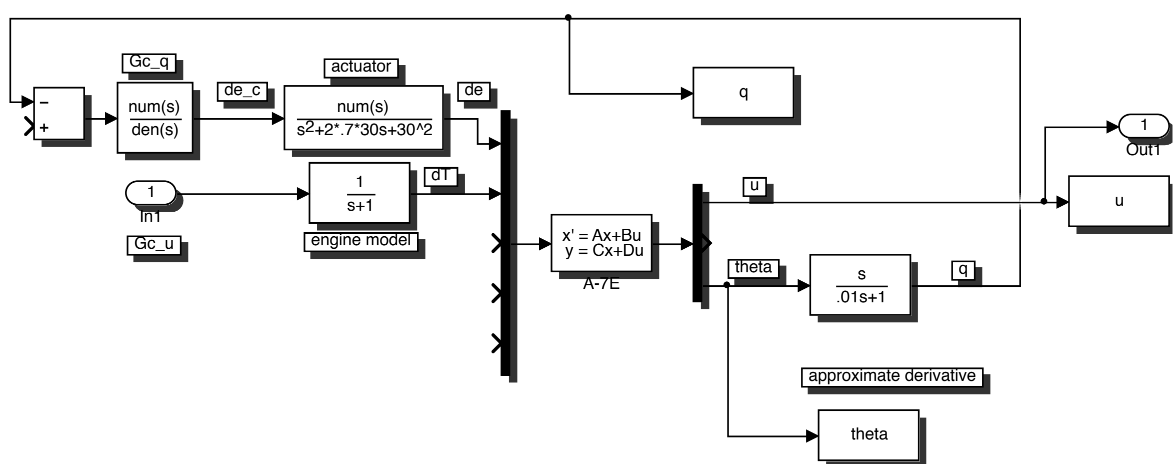

Figure 7: Simulink Diagram used to design the Airspeed loop

3 Human Pilot Model

In addition to the SCAS controllers, a pair of pilot models were used to emulate a pilot's control of altitude (through pitch attitude). The pilot models are $Y_{p_\theta} = K_\theta e^{-0.35s}$ 1/sec; $Y_{p_h} = K_h$ rad/ft. K_θ was chosen to give a 2 rad/sec crossover frequency in the θ-loop and K_h to give a 0.35 rad/sec crossover frequency in the h-loop. Two gains were chosen

$$\boxed{K_\theta = 2.04}\boxed{K_h = 0.0024}$$

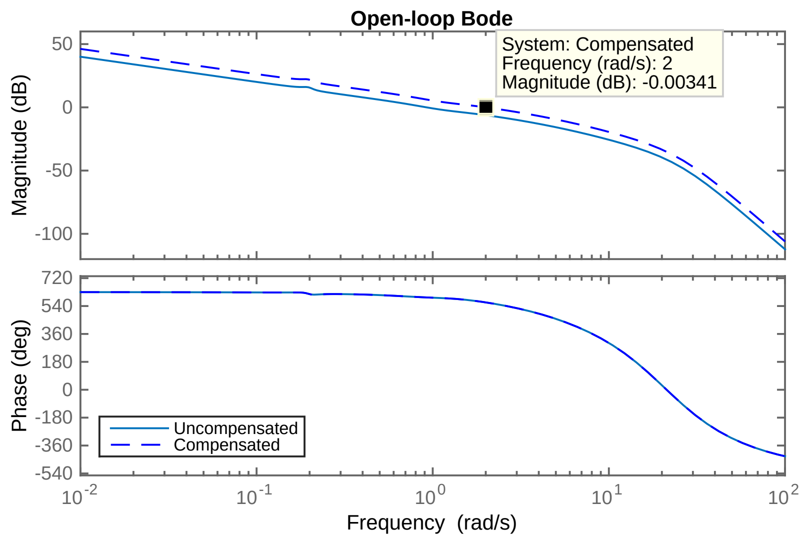

Figure 8: Open-loop Bode for Gc_u

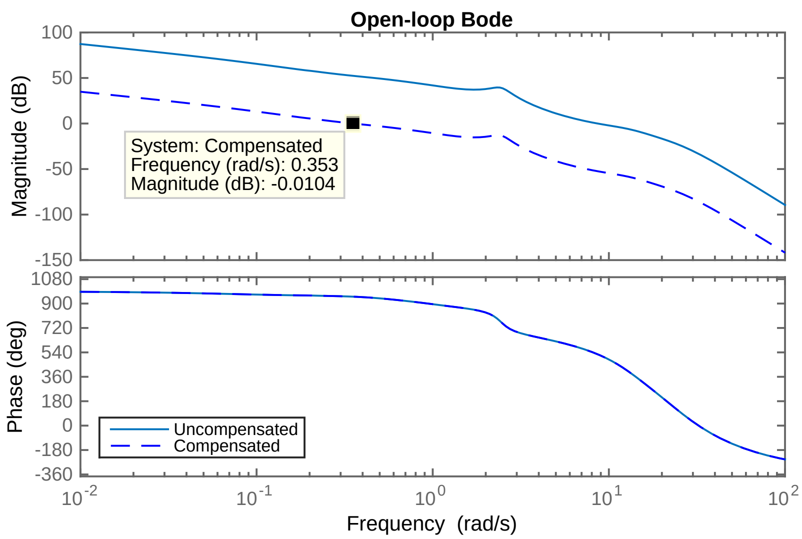

Figure 9: Open-loop Bode for Gc_u

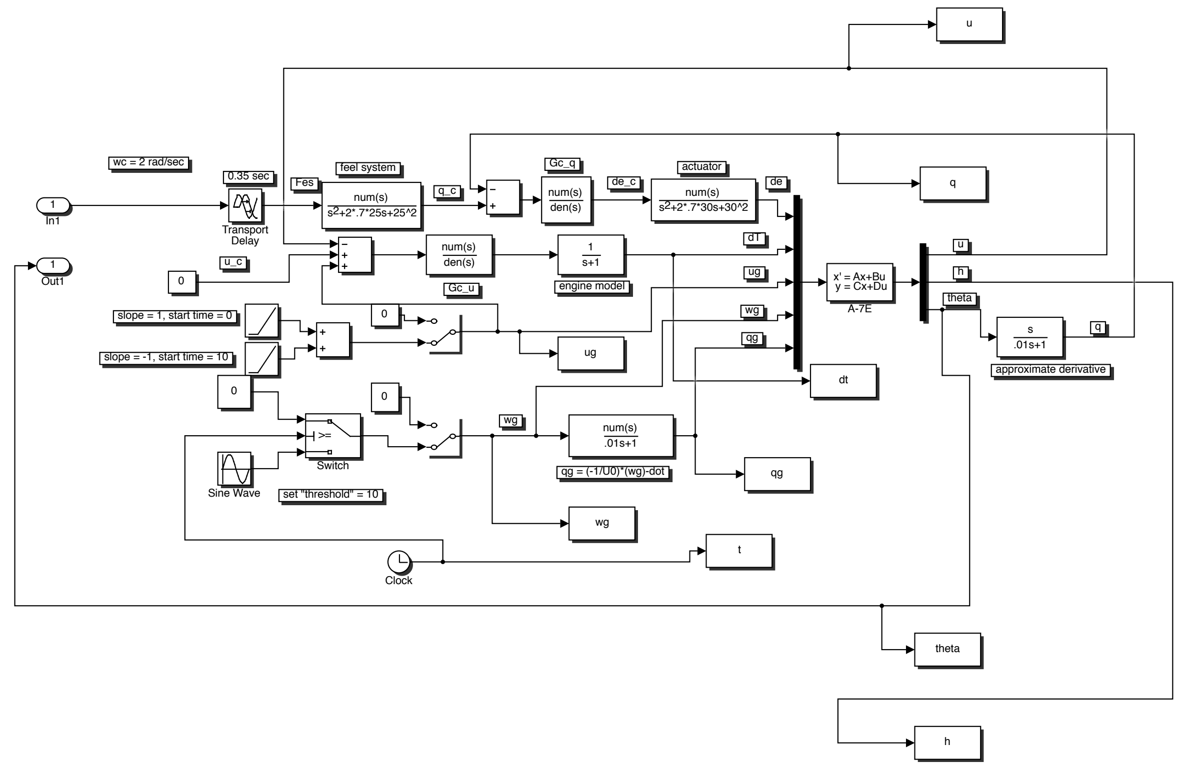

Figure 10: Simulink Diagram used to choose K_θ

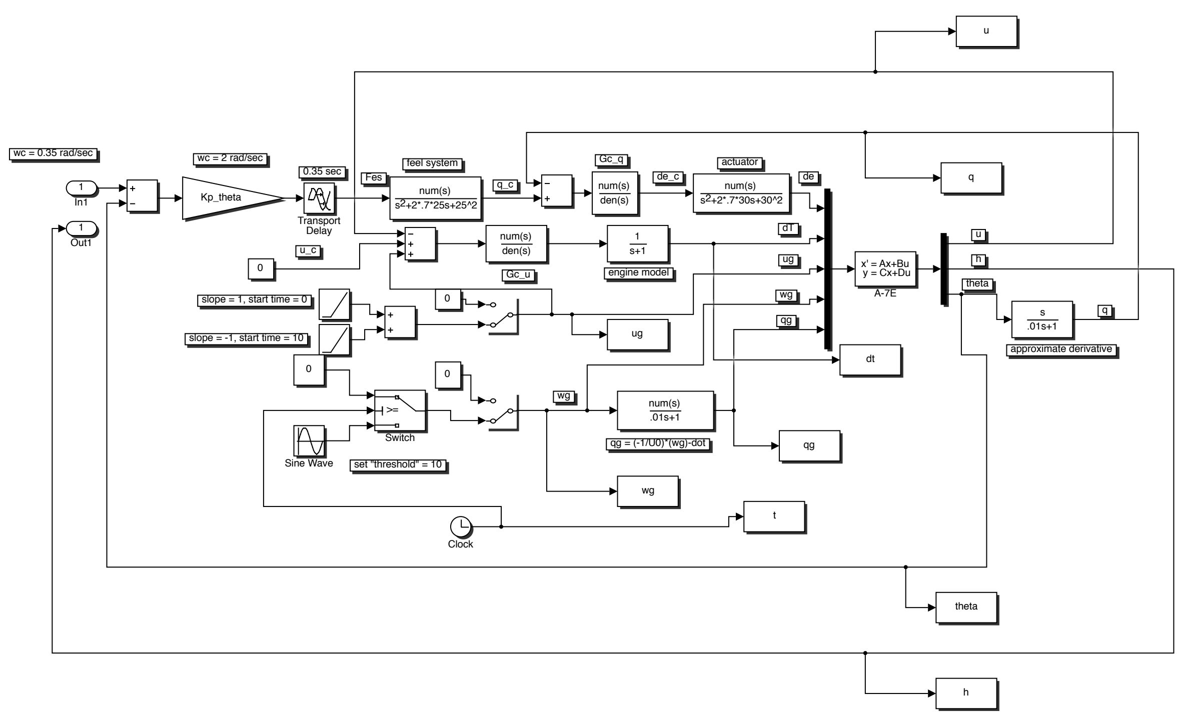

Figure 11: Simulink Diagram used to choose K_h

4 Simulation Results

4.1 Step Command h_c

4.2 Burble Response

5 Handling Qualities

The handling qualities of the pitch-rate SCAS can be estimated using the Bandwidth/Phase-Delay boundaries explained in the handout. The bandwidth is defined as the lesser of $w_{BW_{gain}}$ and $w_{BW_{phase}}$, which is 3.09 rad/s. The phase delay, τ_p is defined

$$\tau_p = \frac{\Delta\Phi 2w_{180}}{57.3(2w_{180})} = \frac{244 - 180}{57.3(12.8)} = 0.09\text{s}$$

These values suggest Level 1 handling qualities.

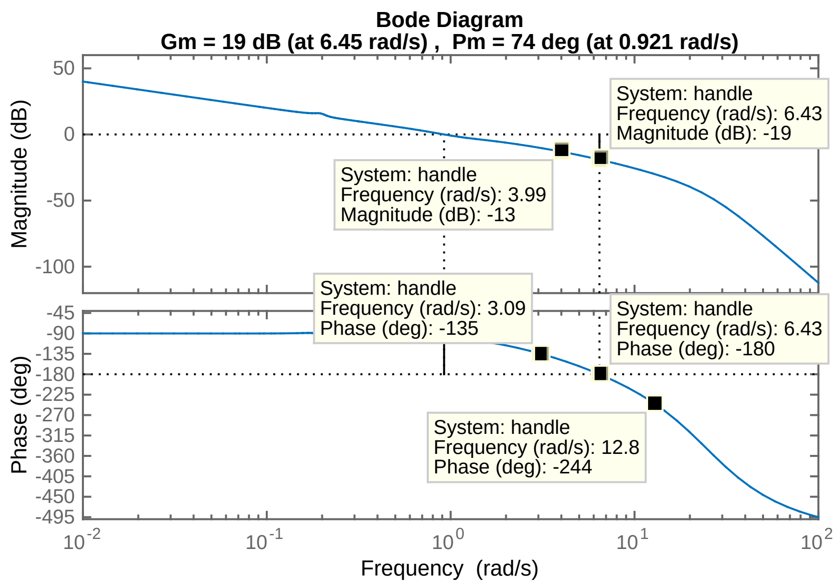

Figure 12: $|\frac{\theta}{F_{es}}|$ bode with relevant points selected

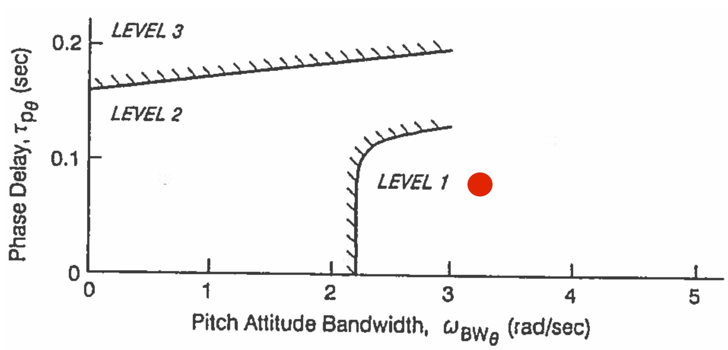

Figure 13: Handling qualities diagram with location marked