MAE 275 - Homework 2

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1 Problem 1

We can define the longitudinal and lateral linearized aircraft equations of motion. The longitudinal equations can be expressed as

$$\begin{aligned}
\Delta\dot{u} =& X_u\Delta u + X_w\Delta w - g\cos\theta_0\Delta\theta + \sum_{i=1}^{n} X_{\delta_i}\Delta\delta_i \\
\Delta\dot{w} =& \frac{Z_u}{1-Z_{\dot{w}}}\Delta u + \frac{Z_w}{1-Z_{\dot{w}}}\Delta w + \frac{Z_q+u_0}{1-Z_{\dot{w}}}\Delta q - \frac{g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta + \frac{1}{1-Z_{\dot{w}}}\sum_{i=1}^{n} Z_{\delta_i}\Delta\delta_i \\
\Delta\dot{q} =& \left[M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}}\Delta u\right] + \left[M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}}\Delta w\right] + \left[M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}}\Delta q\right] \\
&- \left[\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta\right] + \frac{M_{\dot{w}}}{1-Z_{\dot{w}}}\sum_{i=1}^{n} Z_{\delta_i}\Delta\delta_i + \sum_{i=1}^{n} M_{\delta_i}\Delta\delta_i \\
\Delta\dot{\theta} =& \Delta q \\
\Delta\dot{h} =& -\Delta w + u_0\Delta\theta
\end{aligned} \tag{1}$$

or in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix}
X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\
\frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{Z_q+u_0}{1-Z_{\dot{w}}} & \frac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\
M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & u_0 & 0
\end{bmatrix}$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{bmatrix}$$

The lateral equations can be expressed as

$$
\begin{aligned}
\Delta\dot{v} =& Y_v\Delta v + Y_p\Delta p + [Y_r - u_0]\,\Delta r + g\cos\theta_0\Delta\varphi + \sum_{i=1}^{n} Y_{\delta_i}\Delta\delta_i \\
\Delta\dot{p} =& L'_v\Delta v + L'_p\Delta p + L'_r\Delta r + \sum_{i=1}^{n} L'_{\delta_i}\Delta\delta_i \\
\Delta\dot{r} =& N'_v\Delta v + N'_p\Delta p + N'_r\Delta r + \sum_{i=1}^{n} N'_{\delta_i}\Delta\delta_i \\
\Delta\dot{\varphi} =& \Delta p + r\tan\theta_0\Delta r \\
\Delta\dot{\psi} =& r\sec\theta_0\Delta r
\end{aligned}
\tag{2}
$$

or in state space form, with state variables $\Delta v, \Delta p, \Delta r, \Delta\varphi(\text{roll}), \Delta\psi$, as

$$
A = \begin{bmatrix}
Y_v & Y_p & [Y_r - u_0] & g\cos\theta_0 & 0 \\
L'_v & L'_p & L'_r & 0 & 0 \\
N'_v & N'_p & N'_r & 0 & 0 \\
0 & 1 & \tan\theta_0 & 0 & 0 \\
0 & 0 & \sec\theta_0 & 0 & 0
\end{bmatrix}
$$

Plugging in the appropriate data

$$
A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{bmatrix}
$$

2 Problem 2

2.1 Longitudinal

The following MATLAB command is called to identify the characteristc roots and eigenvector elements

```
[v, d] = eig(A);
```

resulting in two complex pairs

$$d_1 = X \pm iY$$
$$d_2 = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$
$$v_2 = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The longitudinal A matrix from above is used, along with B, C, and D matrices defined as

```
B = [0; 0; 0; 0; 0];

C = [[1, 0, 0, 0, 0];
     [0, 1/u_0, 0, 0, 0];
     [0, 0, 1, 0, 0];
     [0, 0, 0, 1, 0];
     [0, 0, 0, 0, 1]];

D = [0; 0; 0; 0; 0];
```

finally, the initial state can be defined and the initial command can be run by

```
i1 = real(v(:,1));
initial(A, B, C, D, i1, 5)
```

and results in the following figures

2.2 Lateral

The lateral eigenvalues are identified as

$$d_1 = X \pm iY$$
$$\cdots$$
$$d_n = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$
$$\dots$$
$$v_n = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The lateral A matrix from above is used, along with B, C, and D matrices defined as

```
B = [0; 0; 0; 0; 0];

C = [[1/u_0, 0, 0, 0, 0];
     [0, 1, 0, 0, 0];
     [0, 0, 1, 0, 0];
     [0, 0, 0, 1, 0];
     [0, 0, 0, 0, 1]];

D = [0; 0; 0; 0; 0];
```

Exciting each of these modes with the appropriate eigenvector results in

3 Problem 3

3.1 Longitudinal

A control input to the elevators, $\Delta\delta_e$ can be found by solving Equation 1, resulting in a B matrix,

$$B = \begin{bmatrix} X_{\delta_e} \\ \dfrac{Z_{\delta_e}}{1 - Z_{\dot{w}}} \\ \dfrac{M_{\dot{w}} Z_{\delta_e}}{1 - Z_{\dot{w}} + M_{\delta_e}} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

After defining the C and D matrices, we can find the resulting transfer function with the following commands

```
C=[0 0 0 1 0];
D = 0;

[n, d] = ss2tf(A, B, C, D);
minreal(zpk(tf(n,d)))
```

which results in

```
some shit
```

which compare quite well with the values found on page A-5,

```
some other shit
```

3.2 Lateral

A control input to the aileron, $\Delta\varphi_e$ can be found by solving Equation 2, resulting in a B matrix,

$$B = \begin{bmatrix} Y_{\delta_a} \\ L'_{\delta_a} \\ N'_{\delta_a} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Defining the C and D matrices the same as above, we can again find the resulting transfer function

```
some shit
```

which compare quite well with the values found on page A-5,

```
some other shit
```

4 Problem 4

The response to a step control input of $\Delta\delta_e = 5/57.3$ rad can be for $\Delta\theta$ and Δh. Two responses were seen, a phugoid
and a short-period response
Similarly, the response to a step control input of $\Delta\delta_a = 5/57.3$ rad can be for $\Delta\varphi$ and Δr