

# MAE 275 - Final

John Karasinski

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## 1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables  $\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta h$ , as

$$A = \begin{bmatrix} \frac{X_u}{Z_u} & \frac{X_w}{Z_w} & 0 & -g \cos(\theta_0) & 0 \\ \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{Z_q + u_0}{1 - Z_{\dot{w}}} & -\frac{g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\ M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}}(Z_q + u_0)}{1 - Z_{\dot{w}}} & -\frac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} \frac{X_{\delta_e}}{Z_{\delta_e}} & \frac{X_{\delta_T}}{Z_{\delta_T}} & \frac{-X_u}{-Z_u} & \frac{-X_w}{-Z_w} & \frac{0}{-Z_q} \\ \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} \\ M_{\delta_e} + \frac{M_{\dot{w}} Z_{\delta_e}}{1 - Z_{\dot{w}}} & M_{\delta_T} + \frac{M_{\dot{w}} Z_{\delta_T}}{1 - Z_{\dot{w}}} & -M_u - \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & -M_w - \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & -M_q - \frac{M_{\dot{w}}(Z_q + u_0)}{1 - Z_{\dot{w}}} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Plugging in the data for the A-7E aircraft in a landing approach to an aircraft carrier yields

$$A = \begin{bmatrix} -5.4534e-2 & +6.4327e-2 & 0 & -3.2200e+1 & 0 \\ -2.8695e-1 & -5.2887e-1 & +2.1800e+2 & 0 & 0 \\ -8.2071e-5 & -7.8112e-3 & -3.9053e-1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +2.1800e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +7.3284e-1 & +1.3170e-3 & +5.4534e-2 & -6.4327e-2 & 0 \\ -1.4714e+1 & -2.5000e-4 & +2.8695e-1 & +5.2887e-1 & 0 \\ -2.1846e+0 & +4.0722e-6 & +8.2071e-5 & +7.8112e-3 & +3.9053e-01 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

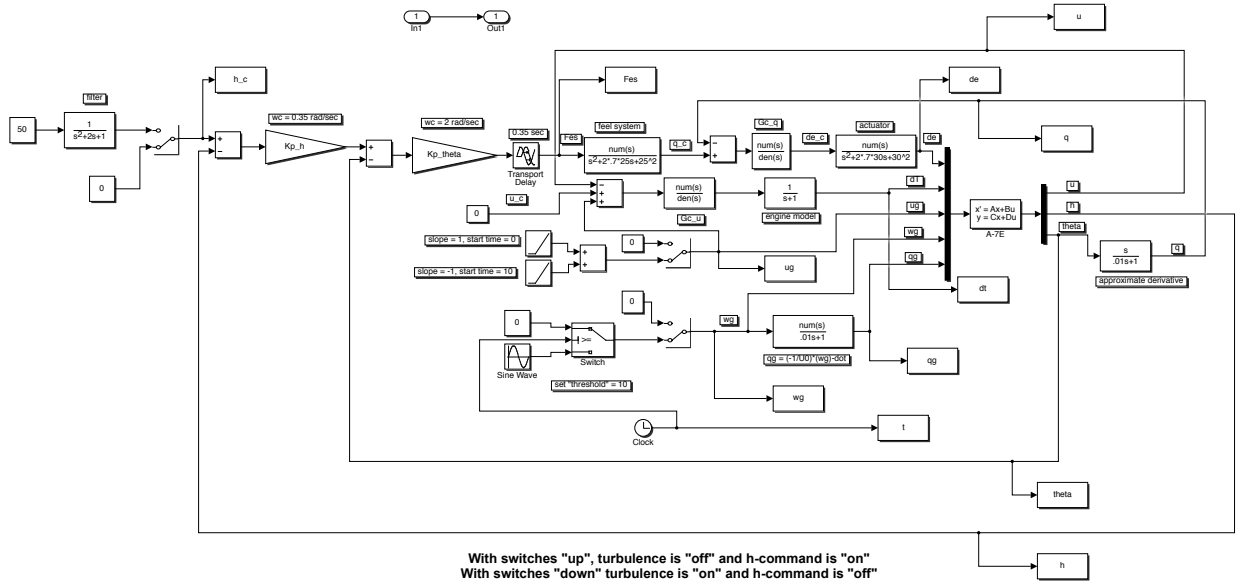


Figure 1: Final Simulink Diagram

## 2 Controller Design

Two controllers were designed as part of a Stability and Command Augmentation System (SCAS). These controllers were designed to control the pitch-rate and airspeed loops. The resultant controllers were:

$$\frac{q}{\delta_e} = \frac{-218.46s(s + 0.4291)(s + 0.1018)}{(s + 100)(s^2 + 0.03945s + 0.03717)(s^2 + 0.9345s + 1.904)}$$

$$GC_q = -2 \times \frac{(s^2 + 0.04s + 0.04)(s^2 + s + 1.25)}{s(s + 0.1)^3}$$

$$G_m = Inf \text{ dB}$$

$$P_m = 86.7^\circ \text{ at } 5.06 \text{ rad/s}$$

$$\omega_{BW} = 5 \text{ rad/s } (-3\text{dB criterion})$$

*Unstable*

$$\begin{aligned} \left. \frac{u}{\delta_T} \right|_{q \rightarrow \delta_e} &= \frac{0.001317(s + 5.486)(s + 0.5165)}{(s + 5.459)(s + 1)(s + 0.4842)(s + 0.1018)} \\ &* \frac{(s^2 + 0.03252s + 0.04026)(s^2 + 1.196s + 1.248)(s^2 + 36.57s + 643.2)}{(s^2 + 0.03952s + 0.04002)(s^2 + 1.203s + 1.256)(s^2 + 36.58s + 643.6)} \end{aligned}$$

$$GC_u = 335 \times \frac{(s + 0.1)}{s(s + 1.5)}$$

$$G_m = 18.3 \text{ dB at } 1.21 \text{ rad/s}$$

$$P_m = 64.5^\circ \text{ at } 0.286 \text{ rad/s}$$

$$\omega_{BW} = .497 \text{ rad/s } (-3\text{dB criterion})$$

*Stable*

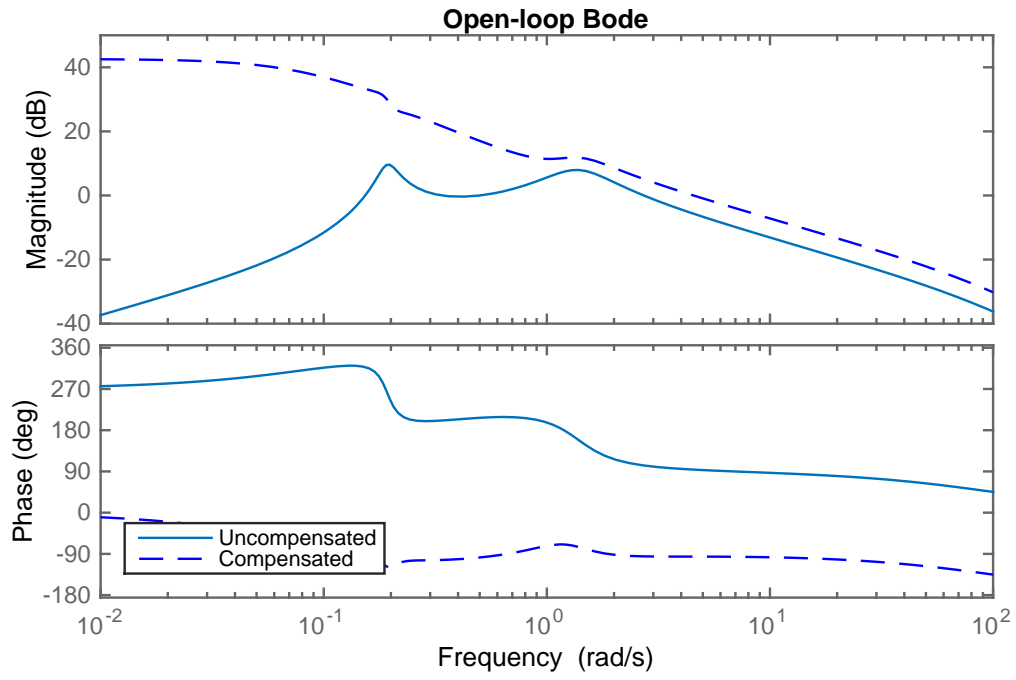


Figure 2: Open-loop Bode for  $Gc_q$

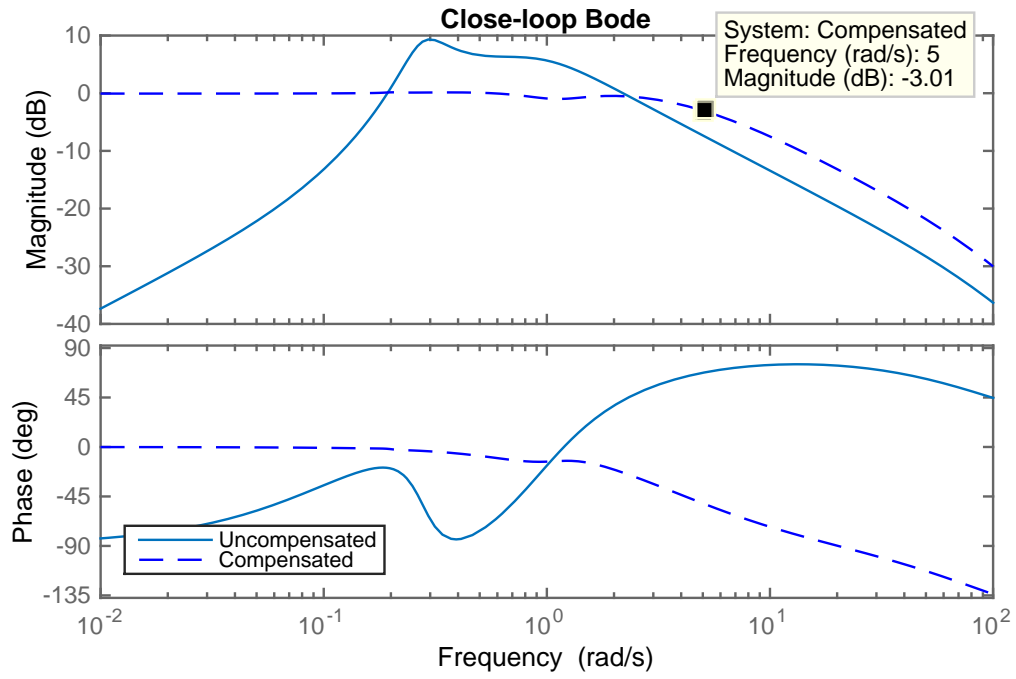


Figure 3: Close-loop Bode for  $Gc_q$

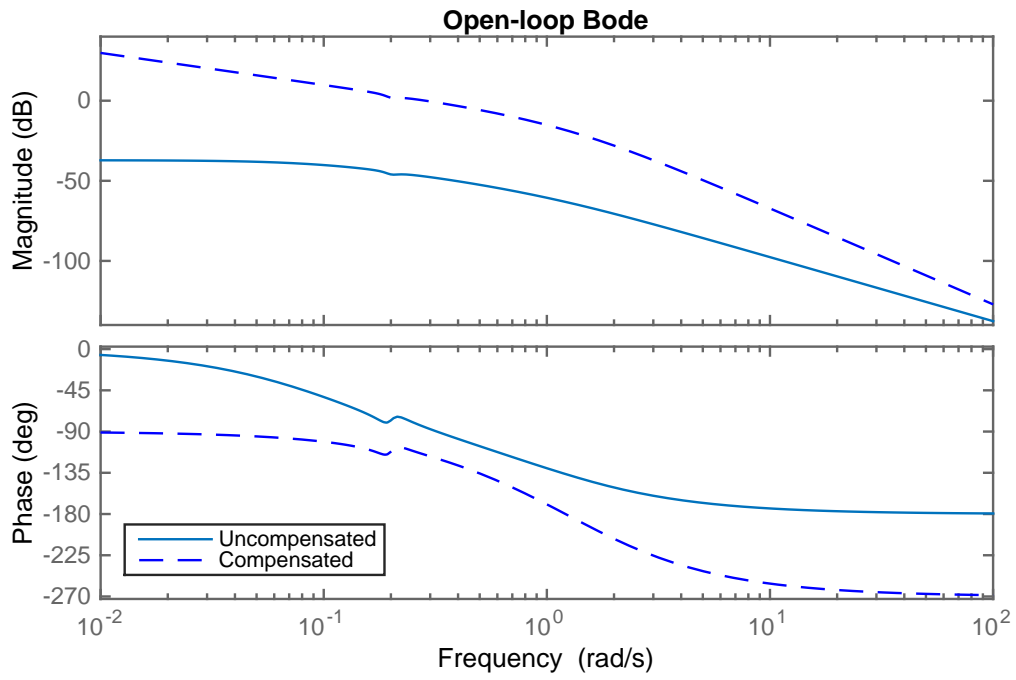


Figure 4: Open-loop Bode for  $Gc_u$

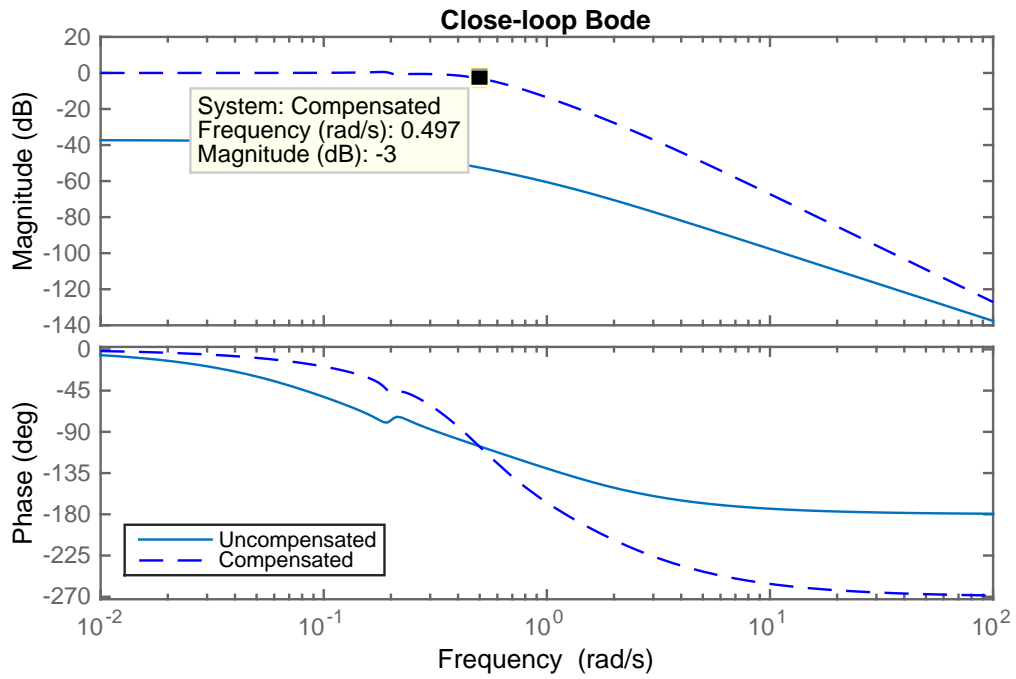


Figure 5: Close-loop Bode for  $Gc_u$

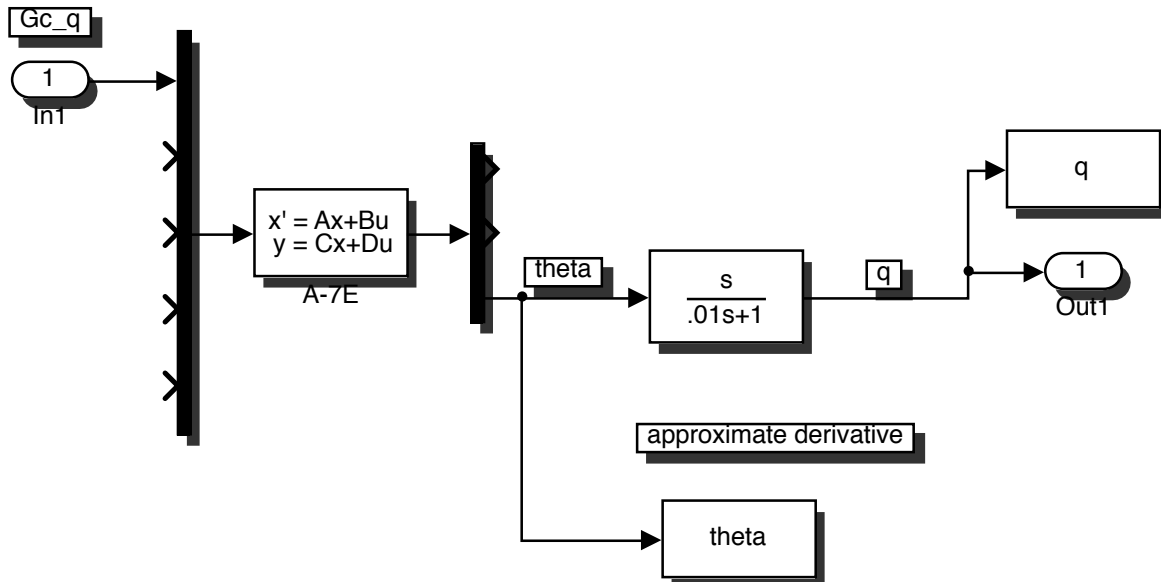


Figure 6: Simulink Diagram used to design the Pitch-rate loop

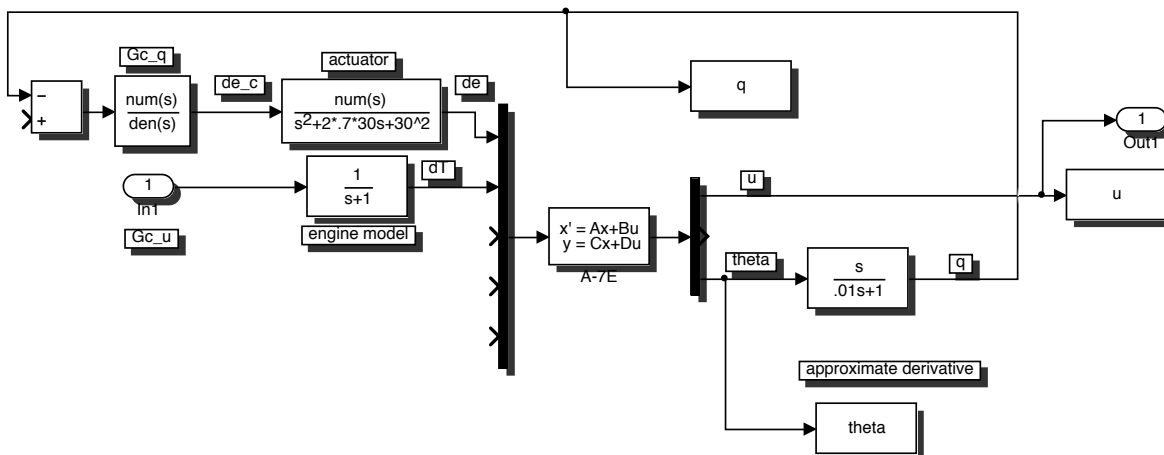


Figure 7: Simulink Diagram used to design the Airspeed loop

### 3 Human Pilot Model

In addition to the SCAS controllers, a pair of pilot models were used to emulate a pilot's control of altitude (through pitch attitude). The pilot models are  $Y_{p_\theta} = K_\theta e^{-0.35s}$  1/sec;  $Y_{p_h} = K_h$  rad/ft.  $K_\theta$  was chosen to give a 2 rad/sec crossover frequency in the  $\theta$ -loop and  $K_h$  to give a 0.35 rad/sec crossover frequency in the h-loop. Two gains were chosen:

$$K_\theta = 2.04 \quad K_h = 0.0024$$

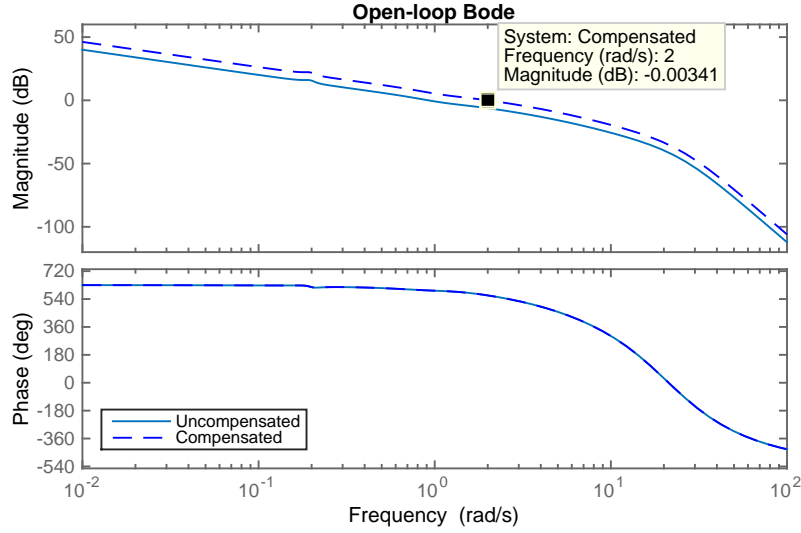


Figure 8: Open-loop Bode for  $Gc_u$

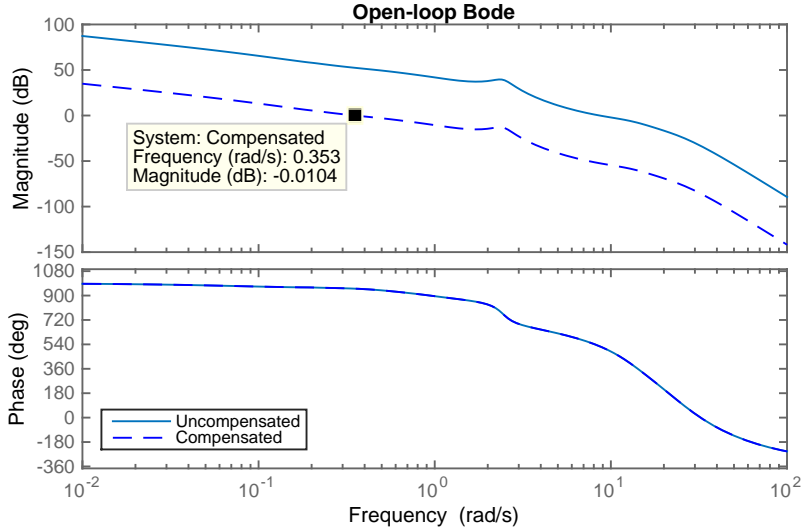


Figure 9: Open-loop Bode for  $Gc_u$

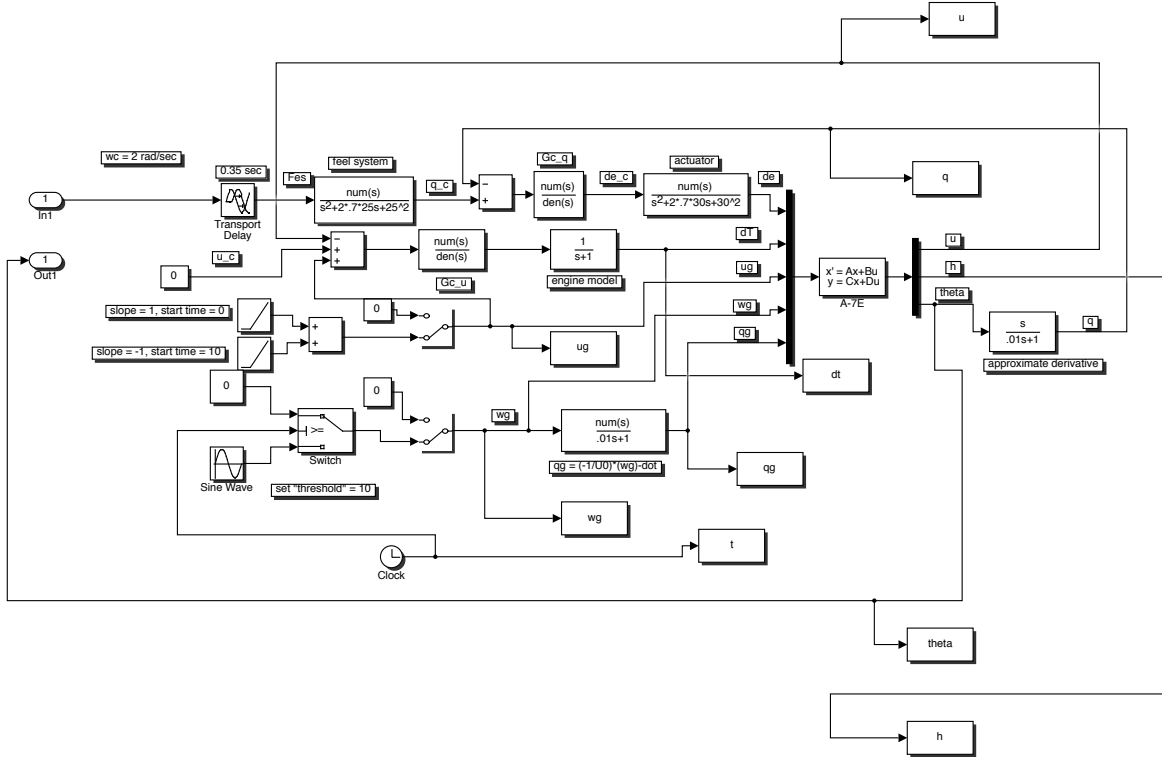


Figure 10: Simulink Diagram used to choose  $K_\theta$

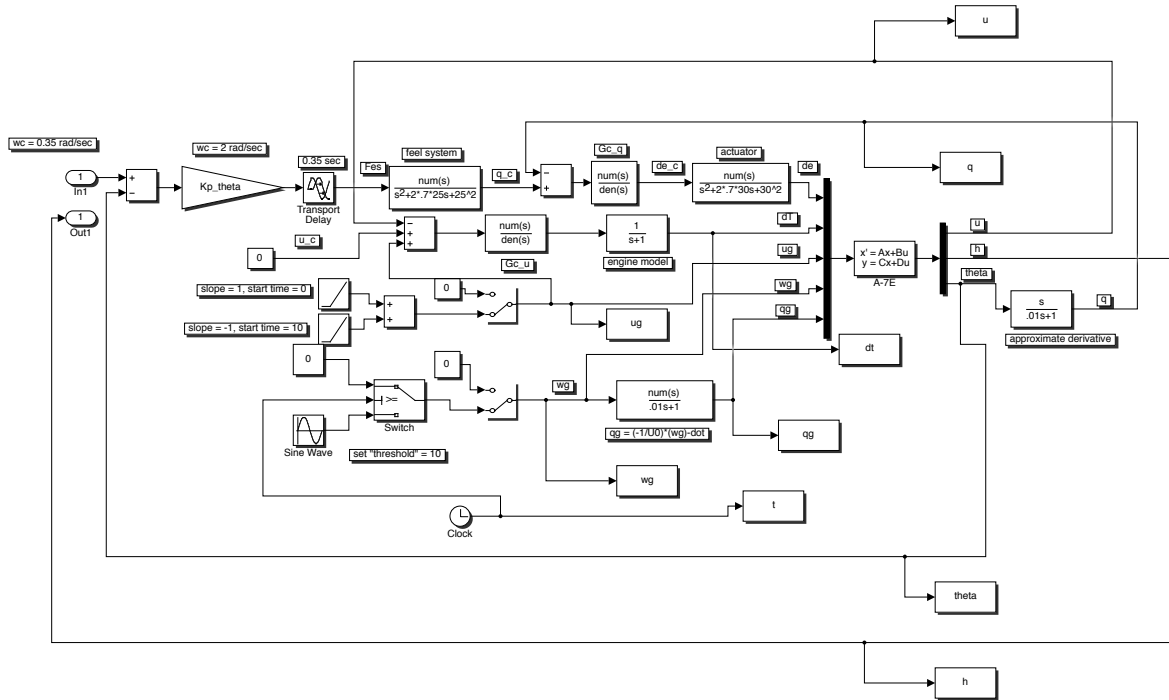


Figure 11: Simulink Diagram used to choose  $K_h$



## 4 Simulation Results

### 4.1 Step Command $h_c$

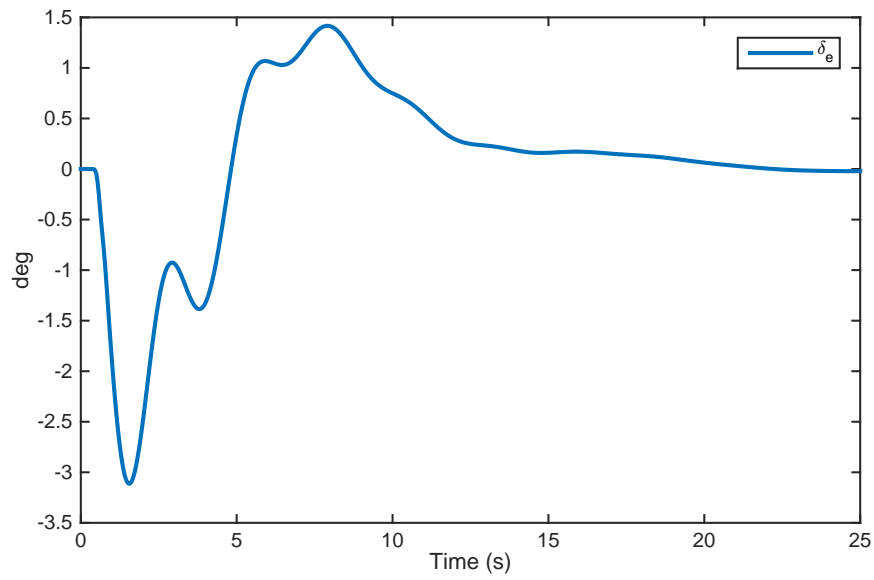


Figure 12:  $\delta_e$  Response to Step Command

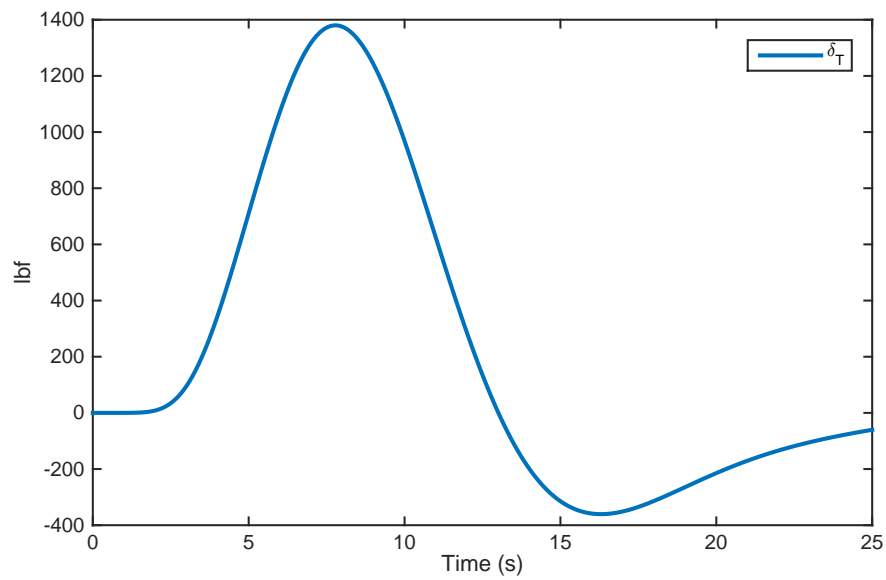


Figure 13:  $\delta_T$  Response to Step Command

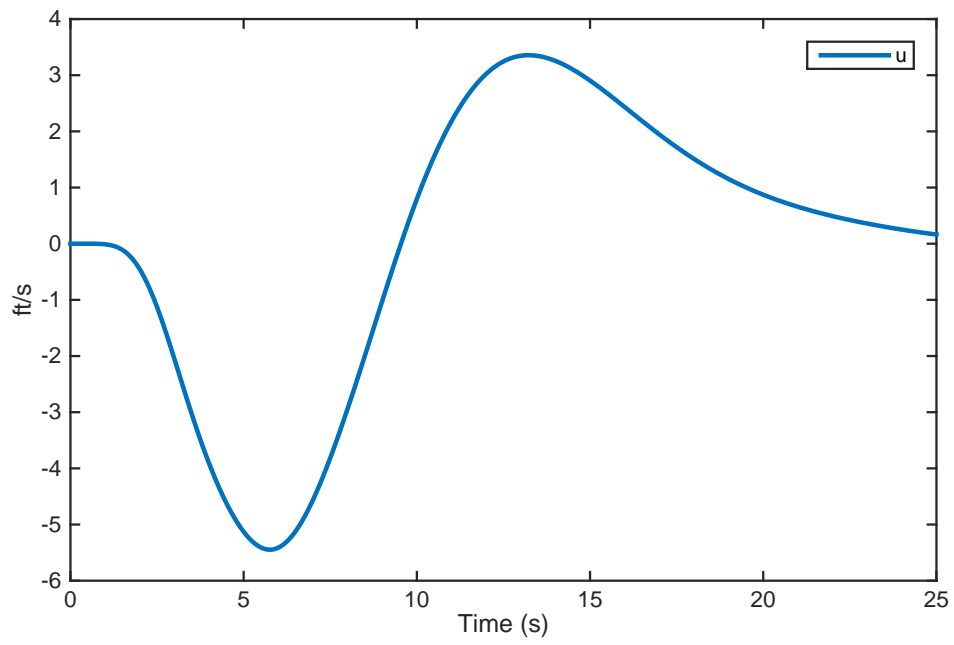


Figure 14:  $u$  Response to Step Command

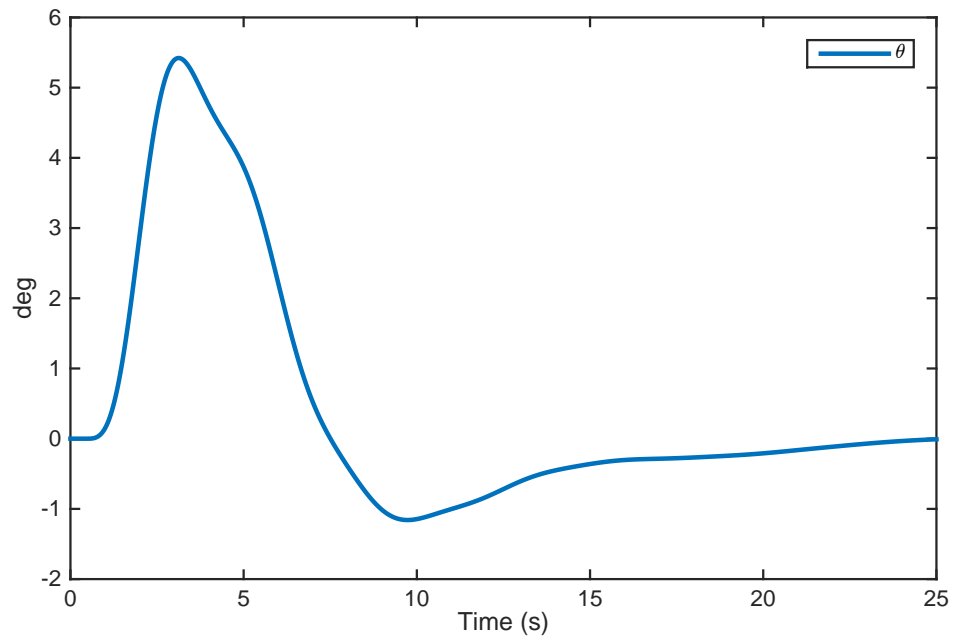


Figure 15:  $\theta$  Response to Step Command

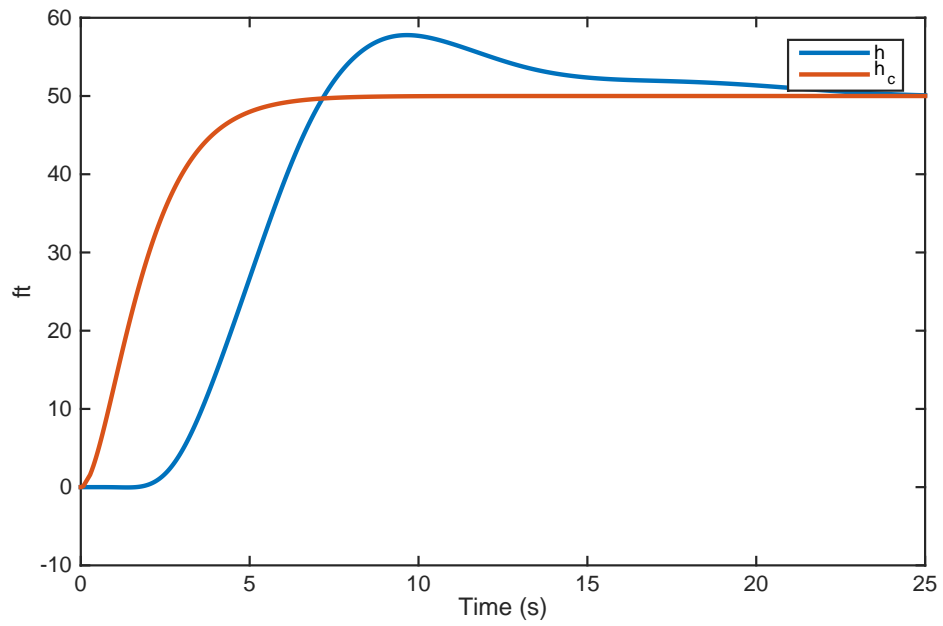


Figure 16:  $h$  Response to Step Command

## 4.2 Burble Response

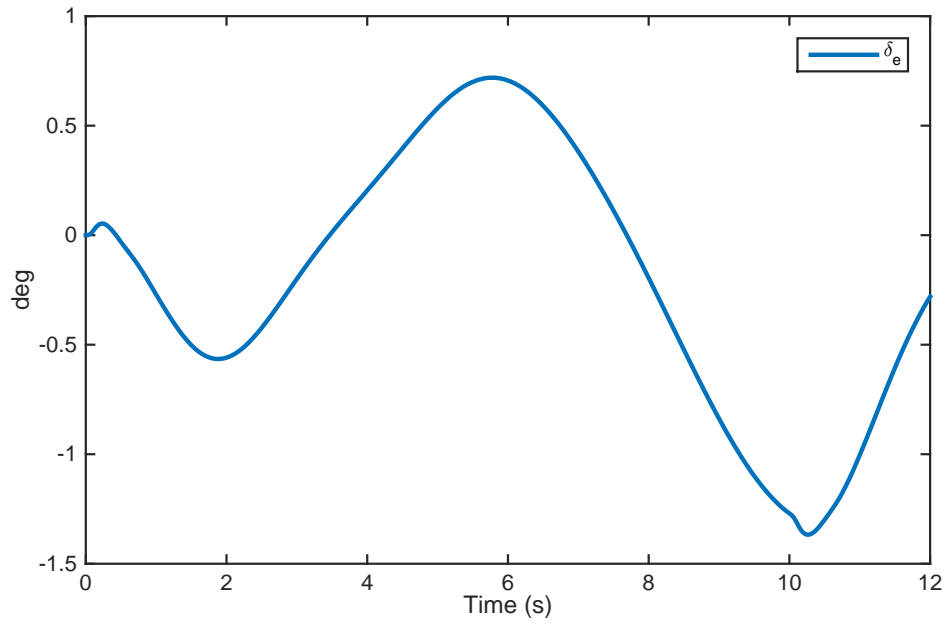


Figure 17:  $\delta_e$  Response to Burble

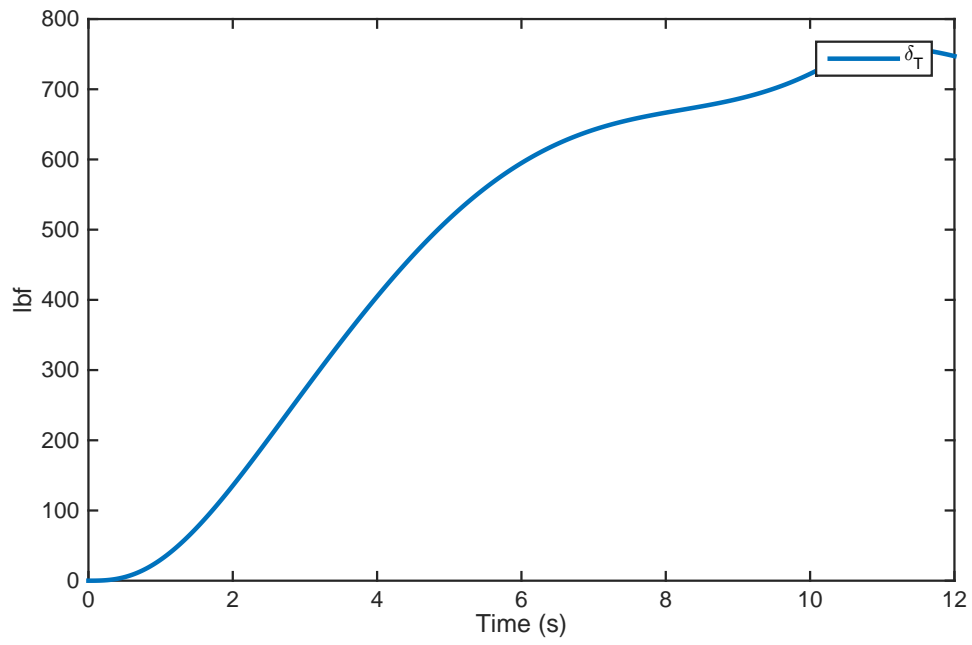


Figure 18:  $\delta_T$  Response to Burble

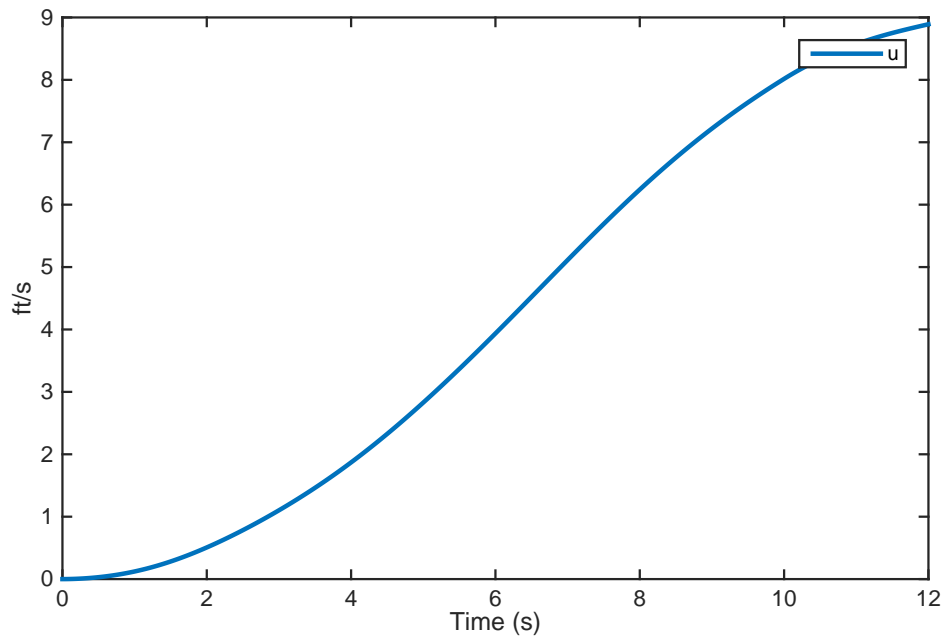


Figure 19:  $u$  Response to Burble

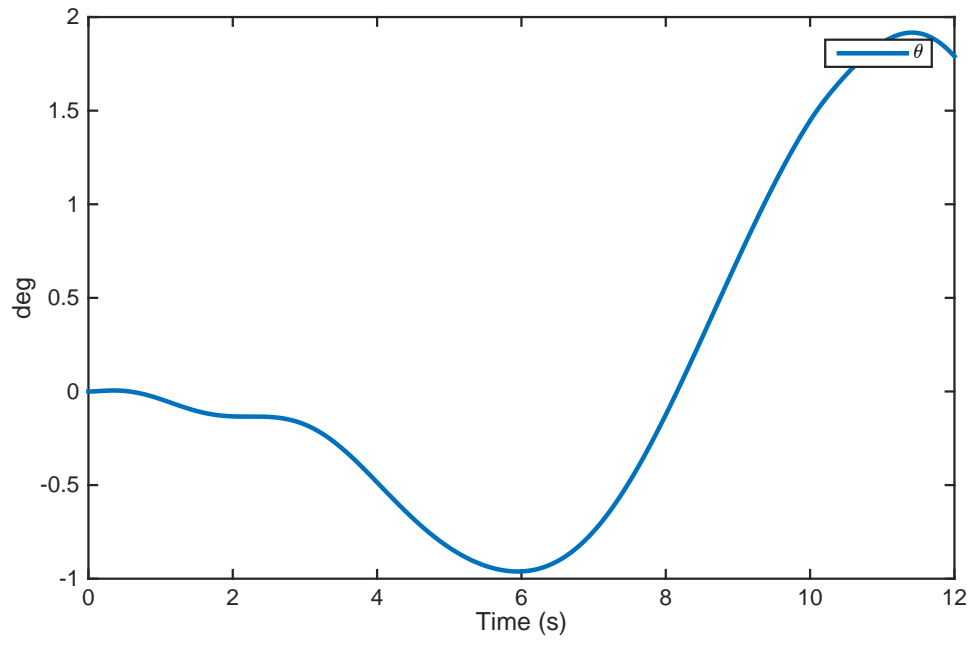


Figure 20:  $\theta$  Response to Burble

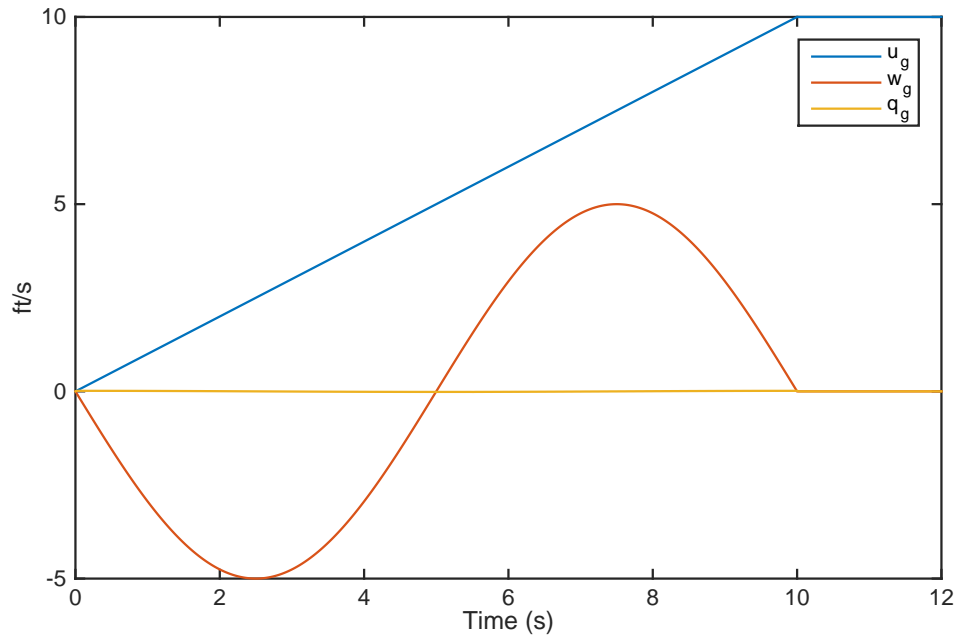


Figure 21:  $u_g, w_g, q_g$  Burble Inputs

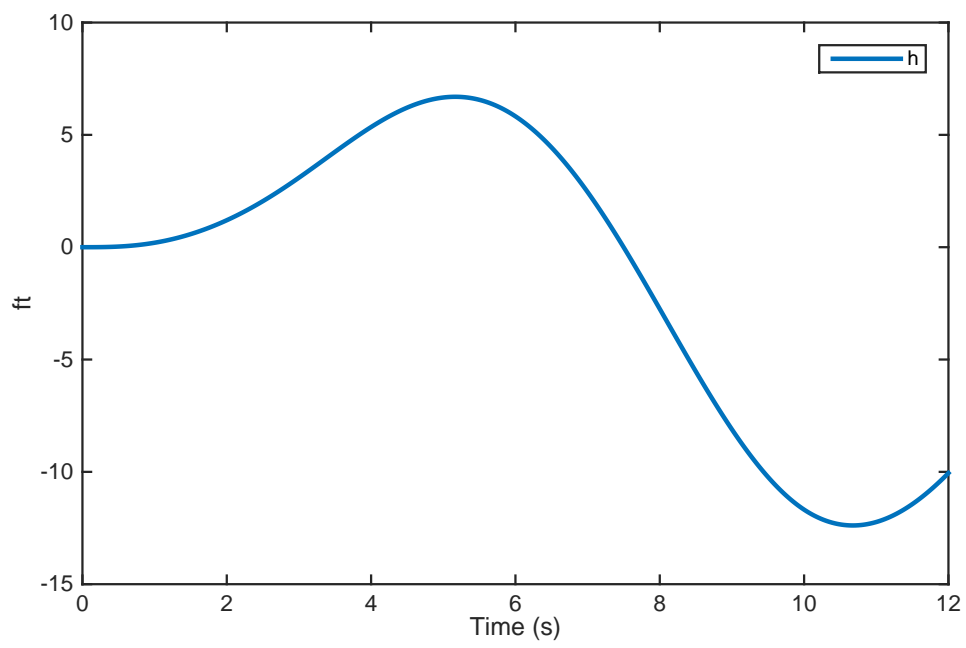


Figure 22:  $h$  Response to Burble

## 5 Handling Qualities

The handling qualities of the pitch-rate SCAS can be estimated using the Bandwidth/Phase-Delay boundaries explained in the handout. The bandwidth is defined as the lesser of  $w_{BW_{gain}}$  and  $w_{BW_{phase}}$ , which is 3.09 rad/s. The phase delay,  $\tau_p$  is defined

$$\tau_p = \frac{\Delta\Phi 2w_{180}}{57.3(2w_{180})} = \frac{244 - 180}{57.3(12.8)} = 0.09s$$

These values suggest Level 1 handling qualities.

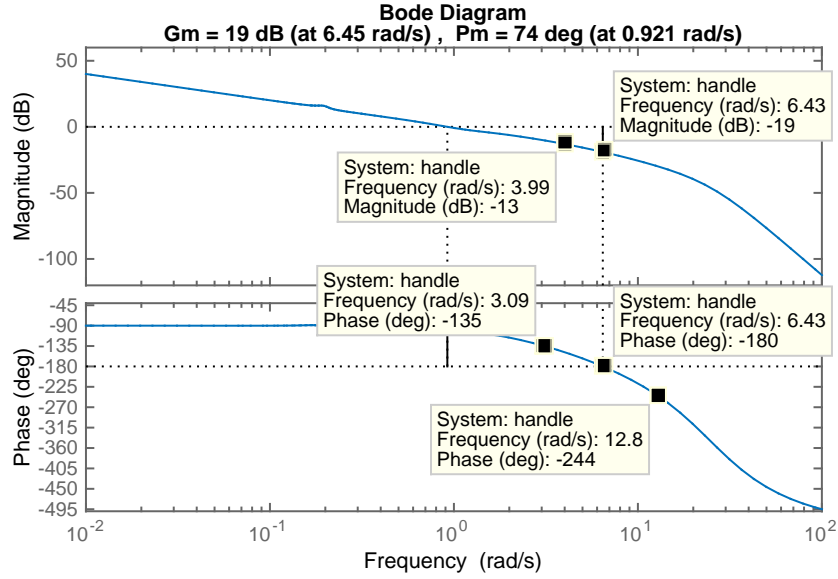


Figure 23:  $|\frac{\theta}{F_{es}}|$  bode with relevant points selected

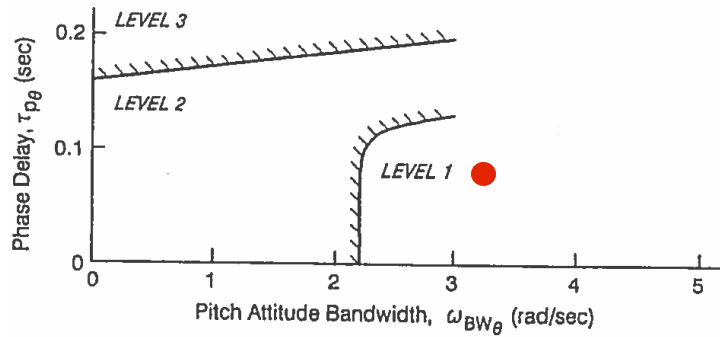


Figure 24: Handling qualities diagram with location marked