MAE 275 - Homework 4

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & \dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} -X_u & -X_w & -X_q + X_{\dot{w}}u_0 & X_{\delta_e} \\ -Z_u & -Z_w & -Z_q + Z_{\dot{w}}u_0 & \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} \\ -M_u & -M_w & -M_q + M_{\dot{w}}u_0 & \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} + M_{\delta_e} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{u_0} & 0 & 0 & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \dfrac{-Z_u}{1-Z_{\dot{w}}} & \dfrac{-Z_w}{1-Z_{\dot{w}}} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Plugging in the data for the A4-D aircraft in Flight Condition 5 from Appendix A of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -1.2660e-2 & -5.8800e-3 & 0 & -3.2200e+1 & 0 \\ -1.0104e-1 & -8.1668e-1 & +6.3298e+2 & 0 & 0 \\ -3.4382e-4 & -1.9546e-2 & -1.4219e+0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +6.3400e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.2660e-2 & +5.8800e-3 & 0 & 0 \\ +1.0120e-1 & +8.1800e-1 & -1.0245e+0 & -5.6828e+1 \\ +4.0000e-4 & +2.0000e-2 & +7.1750e-1 & -1.9388e+1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & +1.5773e-3 & 0 & 0 & 0 \\ -1.0104e-1 & -8.1668e-1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1.0104e-1 & 8.1668e-1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

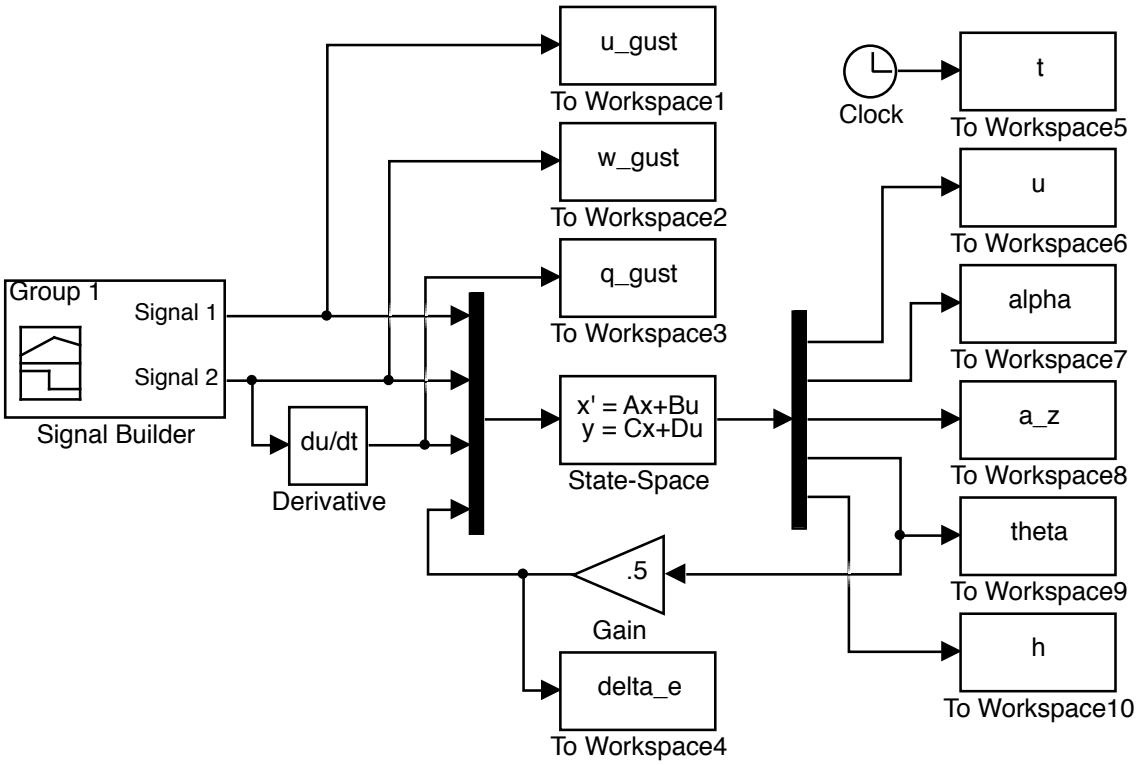

Figure 1: Simulink Diagram