

MAE 275 - Homework 4

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta h$, as

$$A = \begin{bmatrix} \frac{X_u}{Z_u} & \frac{X_w}{Z_w} & 0 & -g \cos(\theta_0) & 0 \\ \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{Z_q + u_0}{1 - Z_{\dot{w}}} & \frac{g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\ M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}} (Z_q + u_0)}{1 - Z_{\dot{w}}} & -\frac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} \frac{-X_u}{-Z_u} & \frac{-X_w}{-Z_w} & 0 & \frac{X_{\delta_e}}{Z_{\delta_e}} \\ \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} & \frac{-Z_q}{1 - Z_{\dot{w}}} & \frac{1 - Z_{\dot{w}}}{1 - Z_{\dot{w}}} \\ -M_u - \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & -M_w - \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & -M_q - \frac{M_{\dot{w}} (Z_q + u_0)}{1 - Z_{\dot{w}}} & \frac{M_{\dot{w}} Z_{\delta_e}}{1 - Z_{\dot{w}}} + M_{\delta_e} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{Z_w} & 0 & 0 & 0 \\ 0 & \frac{u_0}{Z_w} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{\delta_e} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where it is assumed that

$$\alpha \approx \frac{\Delta w}{u_0}$$

$$a_z = \dot{w} - u_0 q \approx Z_\alpha \alpha + Z_{\delta_e} \delta_e$$

Plugging in the data for the A4-D aircraft in Flight Condition 5 from Appendix A of **Aircraft Dynamics and Automatic Control** yields

$$A = \begin{bmatrix} -1.2660e-2 & -5.8800e-3 & 0 & -3.2200e+1 & 0 \\ -1.0104e-1 & -8.1668e-1 & +6.3298e+2 & 0 & 0 \\ -3.4382e-4 & -1.9546e-2 & -1.4219e+0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +6.3400e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.2660e-2 & +5.8800e-3 & 0 & 0 \\ +1.0104e-1 & +8.1668e-1 & 0 & -5.6828e+1 \\ +3.4382e-4 & +1.9546e-2 & +1.4219e+0 & -1.9388e+1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & +1.5773e-3 & 0 & 0 & 0 \\ 0 & -1.2902e-3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5.6920e+1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

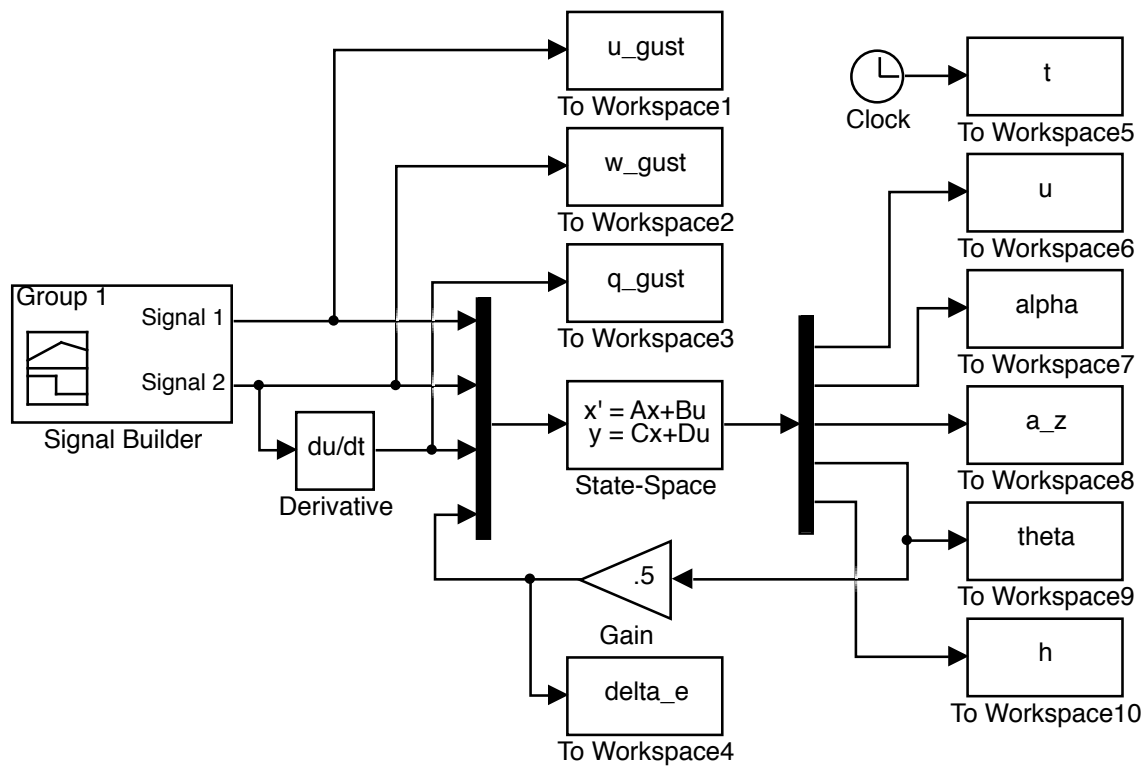


Figure 1: Simulink Diagram

