MAE 275 - Homework 4

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{Z_q+u_0}{1-Z_{\dot{w}}} & -\frac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u+\frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w+\frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q+\frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} -X_u & -X_w & 0 & X_{\delta_e} \\ \frac{-Z_u}{1-Z_{\dot{w}}} & \frac{-Z_w}{1-Z_{\dot{w}}} & \frac{-Z_q}{1-Z_{\dot{w}}} & \frac{Z_{\delta_e}}{1-Z_{\dot{w}}} \\ -M_u-\frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & -M_w-\frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & -M_q-\frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & \frac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}}+M_{\delta_e} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{u_0} & 0 & 0 & 0 \\ \frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{Z_q+u_0}{1-Z_{\dot{w}}} & -\frac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Z_{\delta_e} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Where it is assumed that

$$\alpha \approx \frac{\Delta w}{u_0}$$
$$a_z = \dot{w} - u_0 q \approx \dot{w} + Z_{\delta_e}\delta_e$$

Plugging in the data for the A4-D aircraft in Flight Condition 5 from Appendix A of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -1.2660e-2 & -5.8800e-3 & 0 & -3.2200e+1 & 0 \\ -1.0104e-1 & -8.1668e-1 & +6.3298e+2 & 0 & 0 \\ -3.4382e-4 & -1.9546e-2 & -1.4219e+0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +6.3400e+2 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.2660e-2 & +5.8800e-3 & 0 & 0 \\ +1.0104e-1 & +8.1668e-1 & 0 & -5.6828e+1 \\ +3.4382e-4 & +1.9546e-2 & +1.4219e+0 & -1.9388e+1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & +1.5773e-3 & 0 & 0 & 0 \\ -1.0104e-1 & -8.1668e-1 & 6.3298e+2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5.6920e+1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

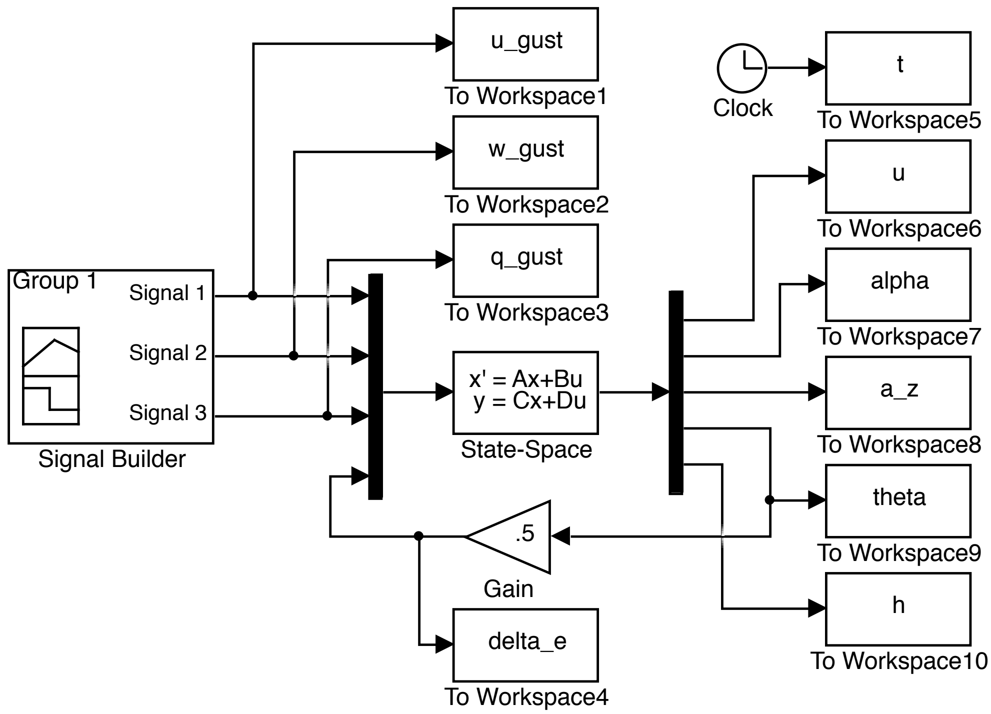
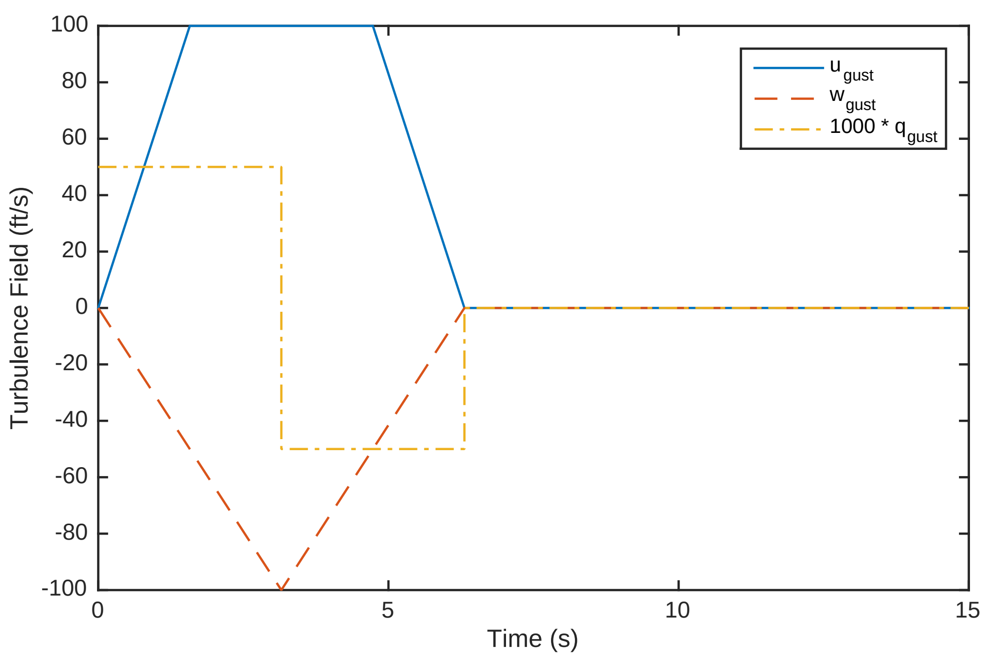
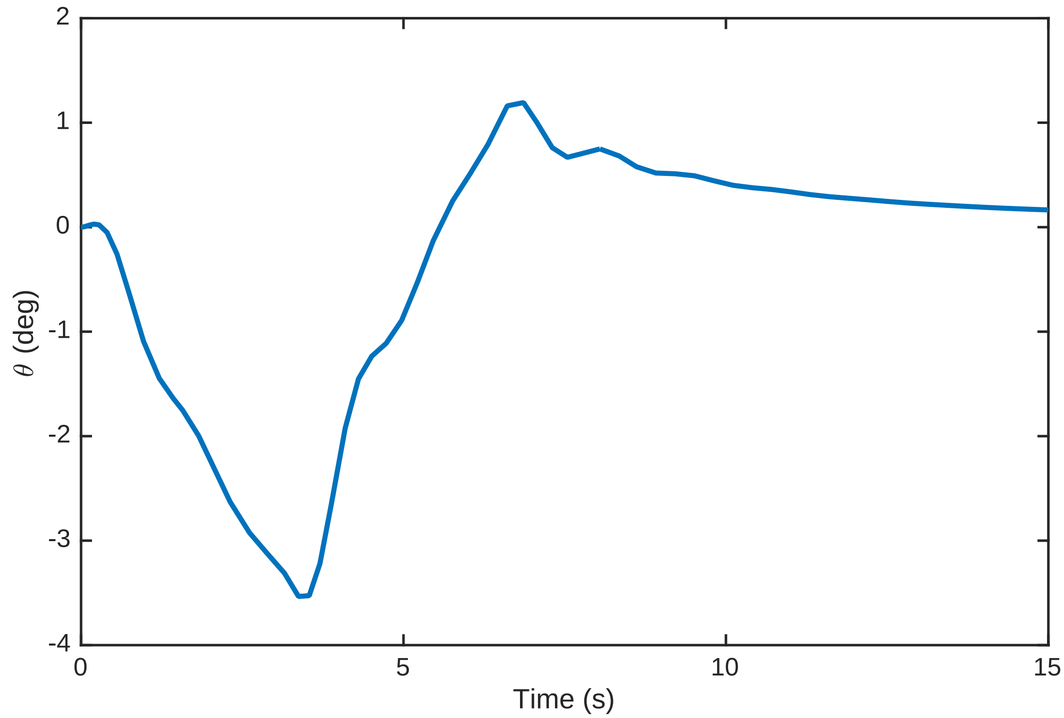
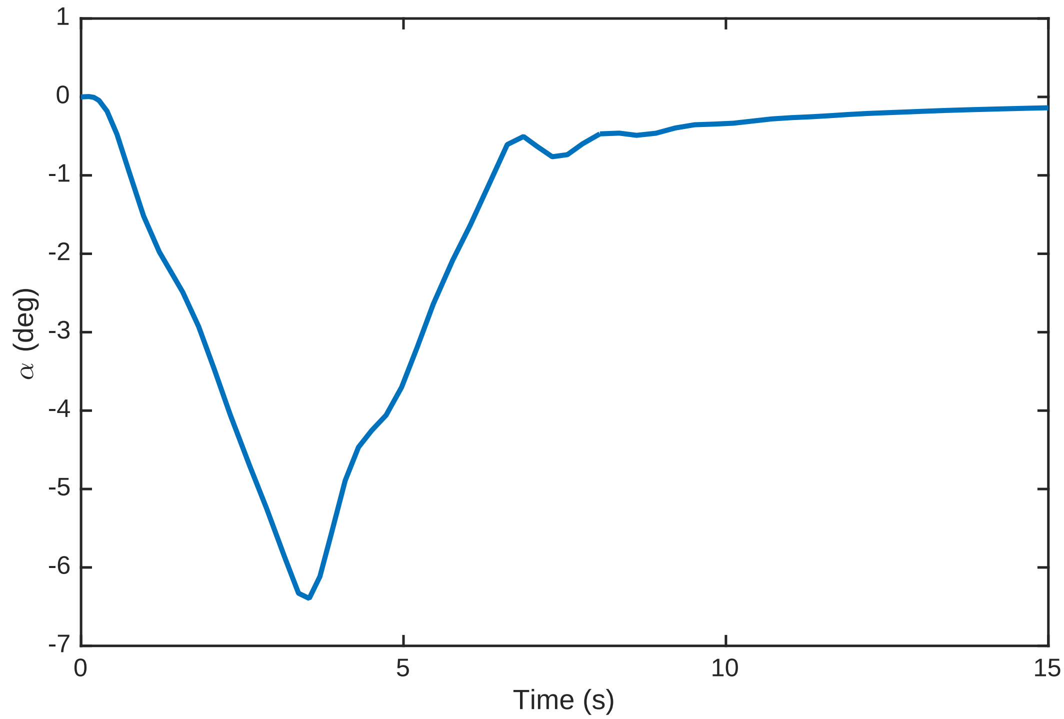


Figure 1: Simulink Diagram

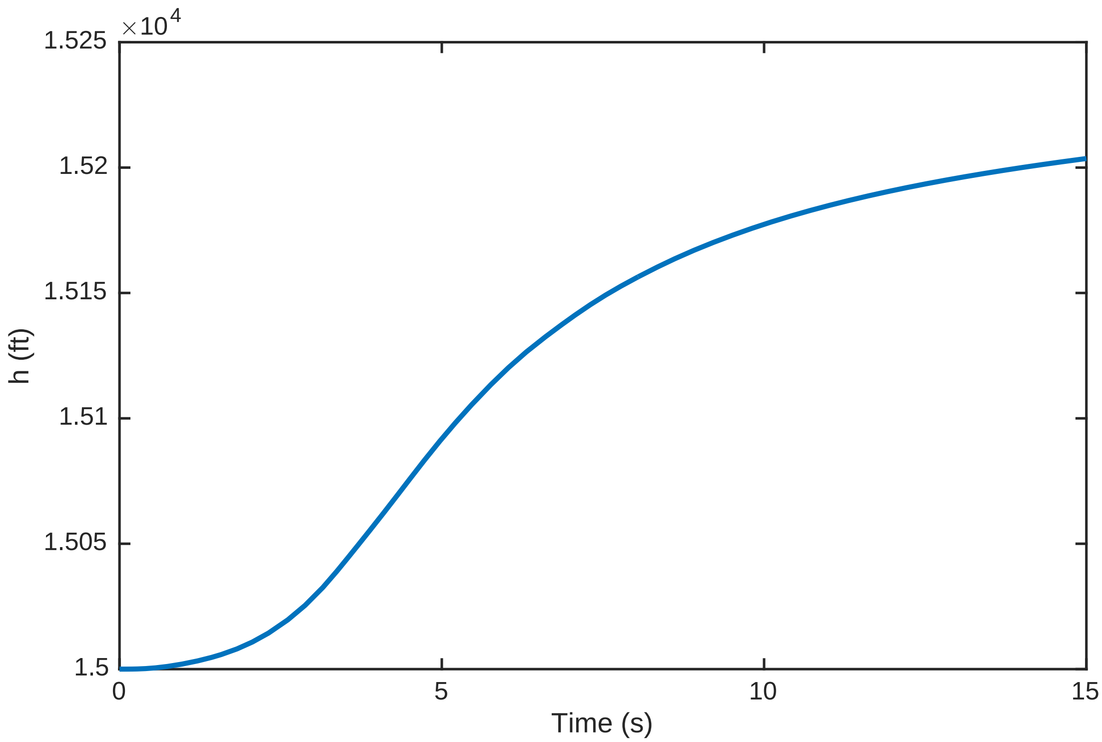


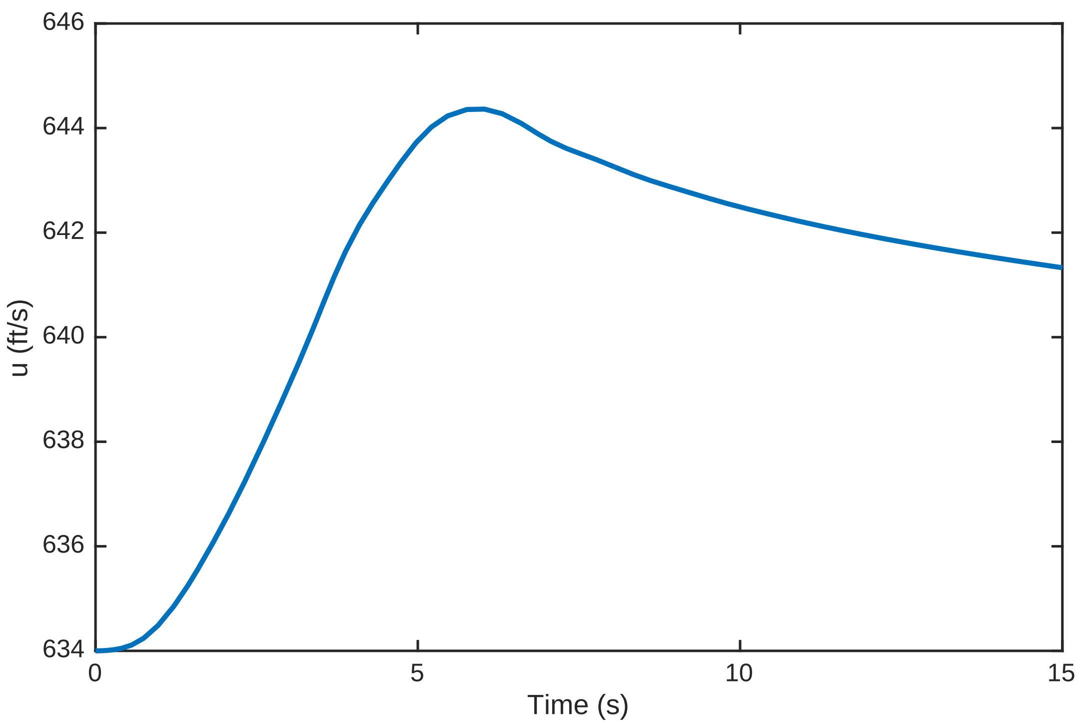


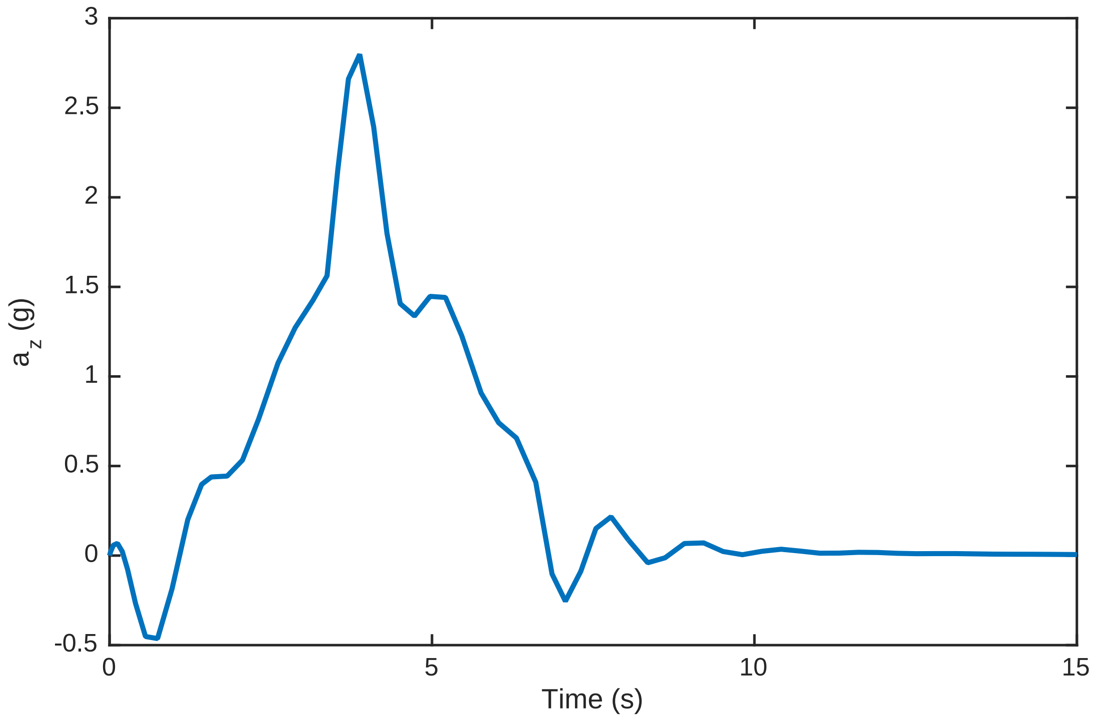
2
1
0
-1
-2
-3
-4
θ (deg)
0
5
10
15
Time (s)



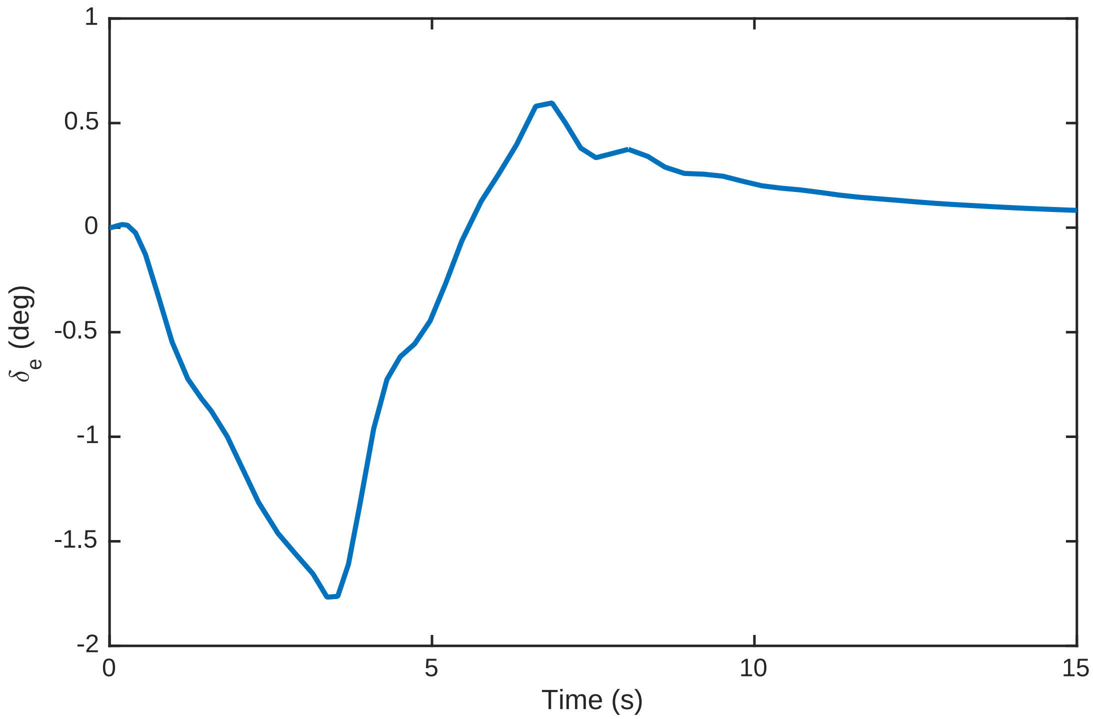
1
0
-1
-2
-3
-4
-5
-6
-7
α (deg)
0
5
10
15
Time (s)


×10^4
1.525
1.52
1.515
1.51
1.505
1.5
h (ft)
0
5
10
15
Time (s)


646
644
642
640
638
636
634
u (ft/s)
0
5
10
15
Time (s)



3
2.5
2
1.5
1
0.5
0
-0.5
a_z (g)
0
5
10
15
Time (s)



1
0.5
0
-0.5
-1
-1.5
-2
δ_e (deg)
0
5
10
15
Time (s)