MAE 275 - Homework 2

John Karasinski

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1 Problem 1

We can define the longitudinal and lateral linearized aircraft equations of motion. The longitudinal equations can be expressed as

$$\begin{aligned}
\Delta\dot{u} =& X_u\Delta u + X_w\Delta w - g\cos\theta_0\Delta\theta + \sum_{i=1}^{n} X_{\delta_i}\Delta\delta_i \\
\Delta\dot{w} =& \frac{Z_u}{1-Z_{\dot{w}}}\Delta u + \frac{Z_w}{1-Z_{\dot{w}}}\Delta w + \frac{Z_q+u_0}{1-Z_{\dot{w}}}\Delta q - \frac{g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta + \frac{1}{1-Z_{\dot{w}}}\sum_{i=1}^{n} Z_{\delta_i}\Delta\delta_i \\
\Delta\dot{q} =& \left[M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}}\Delta u\right] + \left[M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}}\Delta w\right] + \left[M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}}\Delta q\right] \\
&- \left[\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta\right] + \frac{M_{\dot{w}}}{1-Z_{\dot{w}}}\sum_{i=1}^{n} Z_{\delta_i}\Delta\delta_i + \sum_{i=1}^{n} M_{\delta_i}\Delta\delta_i \\
\Delta\dot{\theta} =& \Delta q \\
\Delta\dot{h} =& -\Delta w + u_0\Delta\theta
\end{aligned} \tag{1}$$

or in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{Z_q+u_0}{1-Z_{\dot{w}}} & \frac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -9.7000e-03 & +1.6000e-03 & 0 & -3.2200e+01 & 0 \\ -9.5500e-02 & -1.4300e+00 & +6.6000e+02 & 0 & 0 \\ +1.2415e-04 & -2.1641e-02 & -2.7780e+00 & 0 & 0 \\ 0 & 0 & +1.0000e+00 & 0 & 0 \\ 0 & -1.0000e+00 & 0 & +6.6000e+02 & 0 \end{bmatrix}$$

The lateral equations can be expressed as

$$
\begin{aligned}
\Delta\dot{v} =& Y_v\Delta v + Y_p\Delta p + \left[Y_r - u_0\right]\Delta r + g\cos\theta_0\Delta\varphi + \sum_{i=1}^{n} Y_{\delta_i}\Delta\delta_i \\
\Delta\dot{p} =& L'_v\Delta v + L'_p\Delta p + L'_r\Delta r + \sum_{i=1}^{n} L'_{\delta_i}\Delta\delta_i \\
\Delta\dot{r} =& N'_v\Delta v + N'_p\Delta p + N'_r\Delta r + \sum_{i=1}^{n} N'_{\delta_i}\Delta\delta_i \\
\Delta\dot{\varphi} =& \Delta p + r\tan\theta_0\Delta r \\
\Delta\dot{\psi} =& r\sec\theta_0\Delta r
\end{aligned}
\tag{2}
$$

or in state space form, with state variables $\Delta v, \Delta p, \Delta r, \Delta\varphi, \Delta\psi$, as

$$
A = \begin{bmatrix}
Y_v & Y_p & [Y_r - u_0] & g\cos\theta_0 & 0 \\
L'_v & L'_p & L'_r & 0 & 0 \\
N'_v & N'_p & N'_r & 0 & 0 \\
0 & 1 & \tan\theta_0 & 0 & 0 \\
0 & 0 & \sec\theta_0 & 0 & 0
\end{bmatrix}
$$

Plugging in the appropriate data

$$
A = \begin{bmatrix}
-8.2900e-02 & 0 & -6.6000e+02 & +3.2200e+01 & 0 \\
-6.8939e-03 & -1.7000e+00 & +1.7200e-01 & 0 & 0 \\
+5.1212e-03 & -6.5400e-02 & -8.9300e-02 & 0 & 0 \\
0 & +1.0000e+00 & 0 & 0 & 0 \\
0 & 0 & +1.0000e+00 & 0 & 0
\end{bmatrix}
$$

2 Problem 2

2.1 Longitudinal

The following MATLAB command is called to identify the characteristic roots and eigenvector elements

```
[v, d] = eig(A);
```

resulting in two complex pairs of eigenvalues

$$
\begin{aligned}
d_1 =& -2.1043e+0 \pm i3.7184e+0 \text{ (Short-Period Mode)} \\
d_2 =& -4.5114e-3 \pm i6.2756e-2 \text{ (Phugoid Mode)}
\end{aligned}
$$

and their associated eigenvectors

$$
\begin{aligned}
v_1 = [&-7.3823e-3 \mp i6.3366e-3, \\
&-9.9688e-1, \\
&\ \ 1.0175e-3 \mp i5.6172e-3, \\
&-1.2615e-3 \pm i4.4029e-4, \\
&\ \ 4.0253e-2 \mp i6.6963e-2] \\
v_2 = [&4.8201e-2 \pm i7.1255e-3, \\
&-4.8242e-4 \mp i8.0362e-5, \\
&\ \ 5.9403e-6 \pm i8.1160e-7, \\
&\ \ 6.0964e-6 \mp i9.5094e-5, \\
&-9.9881e-1].
\end{aligned}
$$

Before exciting these modes, the rest of the state space system must be defined. The longitudinal A matrix from above is used, along with B, C, and D matrices defined as

```
B = [0; 0; 0; 0; 0];

C = [[1, 0, 0, 0, 0];
     [0, 1/u_0, 0, 0, 0];
     [0, 0, 1, 0, 0];
     [0, 0, 0, 1, 0];
     [0, 0, 0, 0, 1]];

D = [0; 0; 0; 0; 0];
```

Finally, the initial state can be defined and the initial command can be run by

```
i1 = real(v(:,1));
initial(A, B, C, D, i1, 5)
```

and results in the following figures

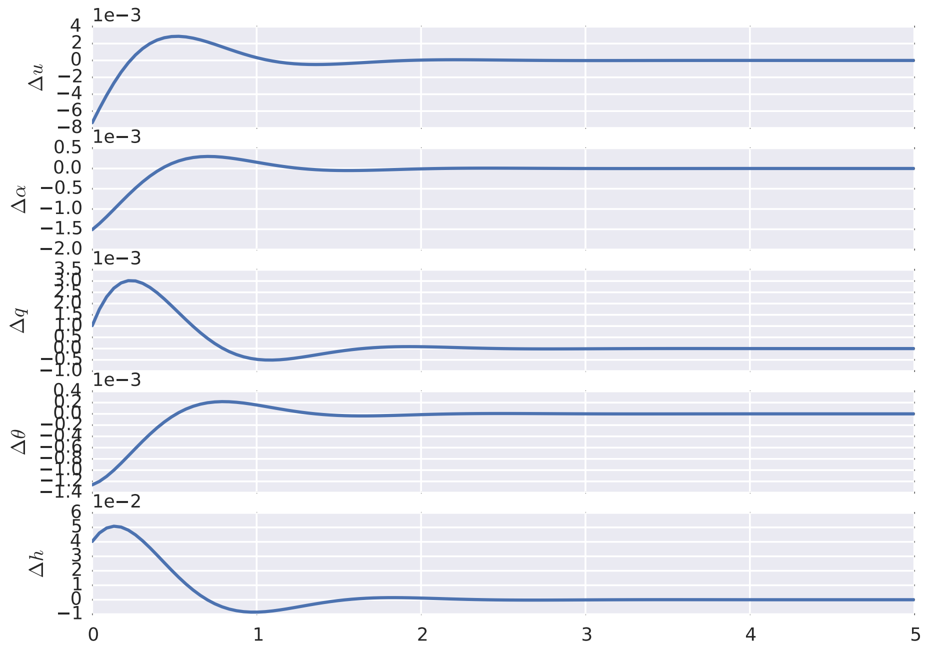

Figure 1: Initial response to eigenvector v_1

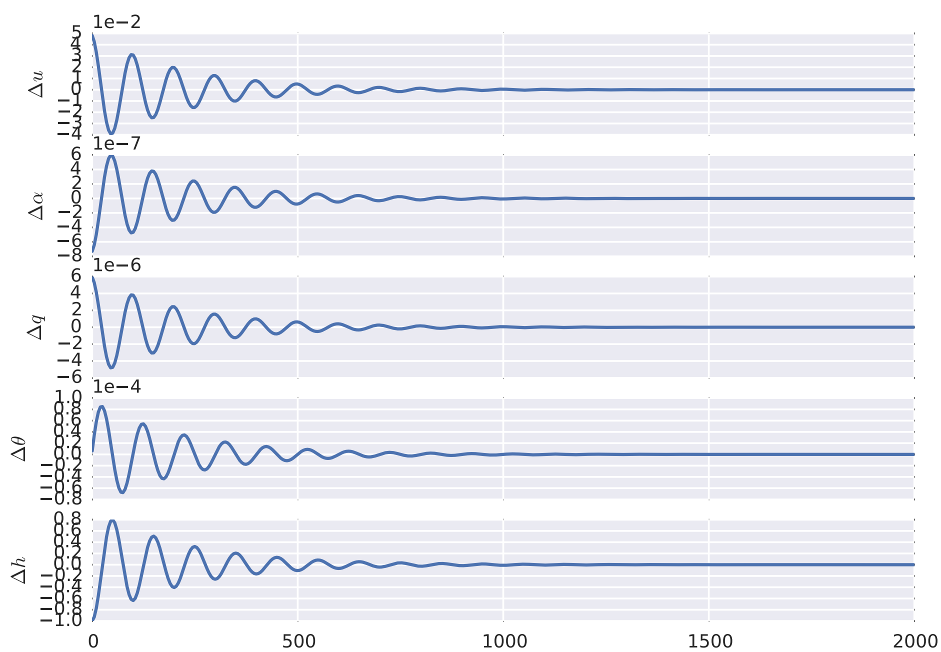

Figure 2: Initial response to eigenvector v_2

2.2 Lateral

The lateral eigenvalues are identified as

$$
\begin{aligned}
d_1 &= -1.7808e+0 \text{ (Roll Convergence Mode)}\\
d_2 &= +1.3590e-3 \text{ (Slightly Unstable Spiral Mode)}\\
d_3 &= -4.6373e-2 \pm i1.8779e+0 \text{ (Dutch Roll Mode)}
\end{aligned}
$$

and their associated eigenvectors

$$
\begin{aligned}
v_1 = [&-9.9531e-1,\\
&-8.4376e-2,\\
&-2.4891e-4,\\
&+4.7381e-2,\\
&+1.3977e-4]\\
v_2 = [&-2.4525e-2,\\
&-3.7916e-5,\\
&-1.3581e-3,\\
&-2.7900e-2,\\
&-9.9931e-1]\\
v_3 = &+9.9999e-1,\\
&-1.9651e-3 \pm i1.9409e-3,\\
&-3.6905e-6 \mp i2.7955e-3,\\
&+1.0587e-3 \pm i1.0203e-3,\\
&-1.4877e-3 \pm i3.8701e-5]
\end{aligned}
$$

Before exciting these modes, the rest of the state space system must be defined. The lateral A matrix from above is used, along with B, C, and D matrices defined as

```
B = [0; 0; 0; 0; 0];

C = [[1/u_0, 0, 0, 0, 0];
     [0, 1, 0, 0, 0];
     [0, 0, 1, 0, 0];
     [0, 0, 0, 1, 0];
     [0, 0, 0, 0, 1]];

D = [0; 0; 0; 0; 0];
```

Exciting each of these modes with the appropriate eigenvector results in

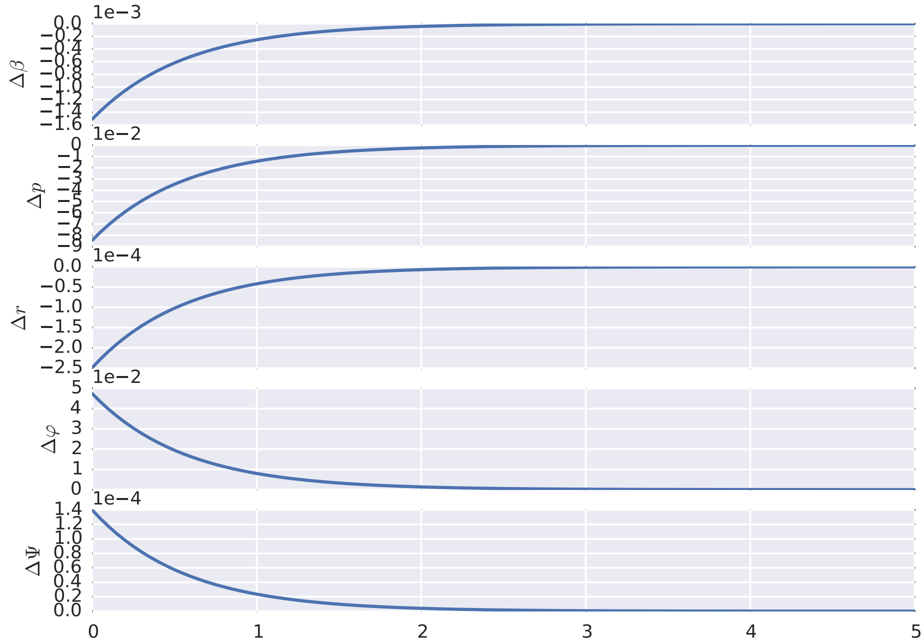

Figure 3: Initial response to eigenvector v_1

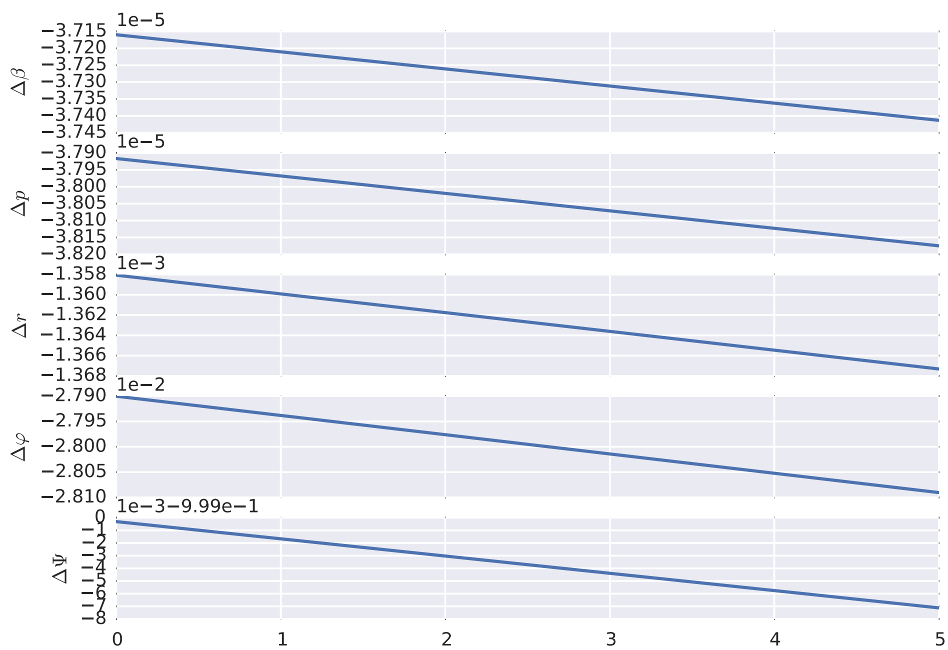

Figure 4: Initial response to eigenvector v_2

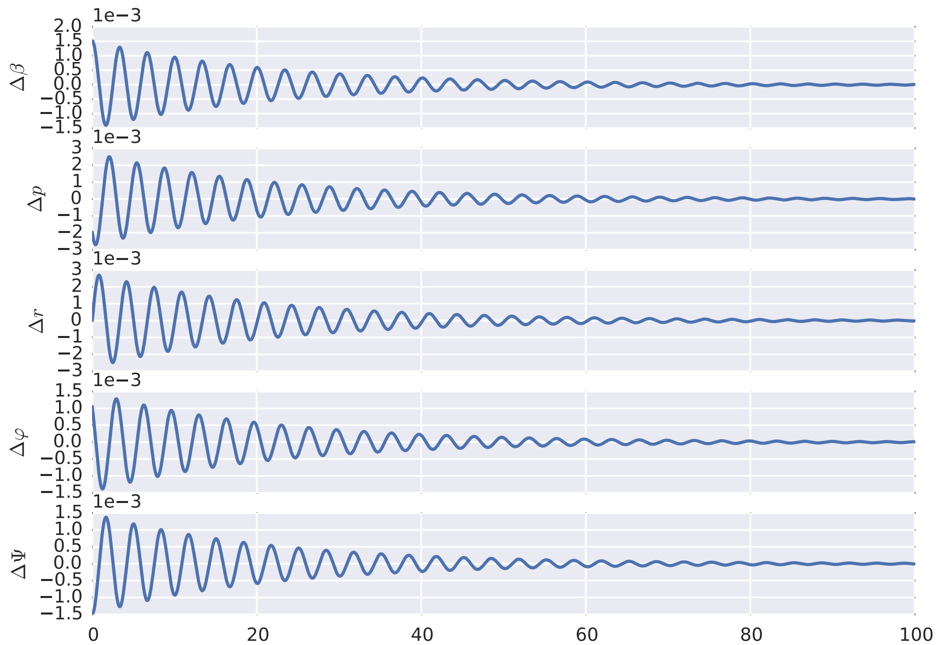

Figure 5: Initial response to eigenvector v_3

3 Problem 3

3.1 Longitudinal

A control input to the elevators, $\Delta\delta_e$, can be found by solving Equation 1, resulting in a B matrix,

$$B = \begin{bmatrix} X_{\delta_e} \\ \dfrac{Z_{\delta_e}}{1 - Z_{\dot{w}}} \\ \dfrac{M_{\dot{w}} Z_{\delta_e}}{1 - Z_{\dot{w}}} + M_{\delta_e} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -6.9800e+1 \\ -2.6009e+1 \\ 0 \\ 0 \end{bmatrix}$$

After defining the C and D matrices, we can find the resulting transfer function with the following commands

```
C=[0 0 0 1 0];
D = 0;

[n, d] = ss2tf(A, B, C, D);
minreal(zpk(tf(n,d)))
```

which results in

```
             -26.009 (s+1.372) (s+0.009813)
  ----------------------------------------------------
  (s^2 + 0.009023s + 0.003959) (s^2 + 4.209s + 18.25)
```

yielding

$$\begin{aligned} A_\theta &= -26.009 \\ 1/T_{\theta_1} &= +0.009813 \\ 1/T_{\theta_2} &= +1.372, \end{aligned}$$

which compare quite well with the values found on page A-5,

$$\begin{aligned} A_\theta &= -26.1 \\ 1/T_{\theta_1} &= +0.0098 \\ 1/T_{\theta_2} &= +1.372 \end{aligned}$$

3.2 Lateral

A control input to the aileron, $\Delta\delta_a$, can be found by solving Equation 2, resulting in a B matrix,

$$B = \begin{bmatrix} Y_{\delta_a} \\ L'_{\delta_a} \\ N'_{\delta_a} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 2.7300e+01 \\ 3.9500e-01 \\ 0 \\ 0 \end{bmatrix}$$

Defining the C and D matrices the same as above, we can find the resulting transfer function

```
          27.3 (s^2 + 0.1747s + 3.453)
 ------------------------------------------------
 (s+1.781) (s-0.001359) (s^2 + 0.09275s + 3.529)
```

yielding

$$
\begin{aligned}
A_\theta &= +\,27.3 \\
\zeta_\varphi &= +\,4.7007e-02 \\
\omega_\varphi &= -\,1.8582e+00,
\end{aligned}
$$

which compare quite well with the values found on page A-5,

$$
\begin{aligned}
A_\theta &= +\,27.3 \\
\zeta_\varphi &= +\,0.047 \\
\omega_\varphi &= -\,1.86
\end{aligned}
$$

4 Problem 4

The response to a step control input of $\Delta\delta_e = 5/57.3$ rad can be for $\Delta\theta$ and Δh. Two responses were seen, a phugoid,

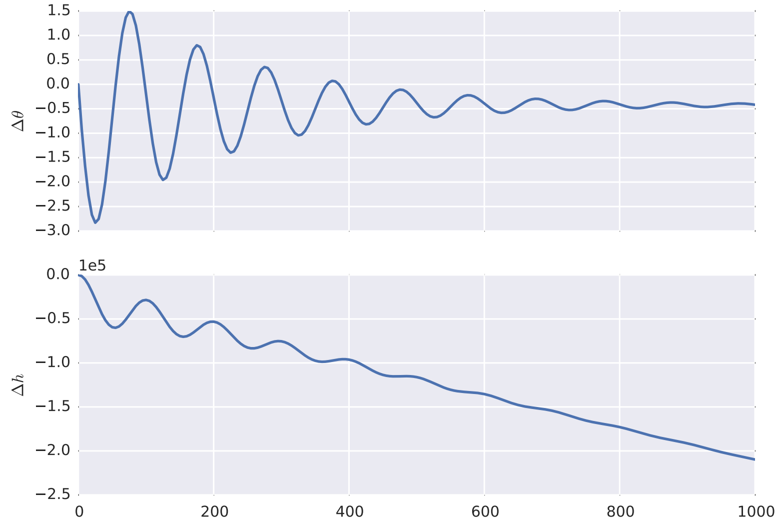

Figure 6: Phugoid response to step input

and a short-period response which manifests as a small oscillation in $\Delta\theta$

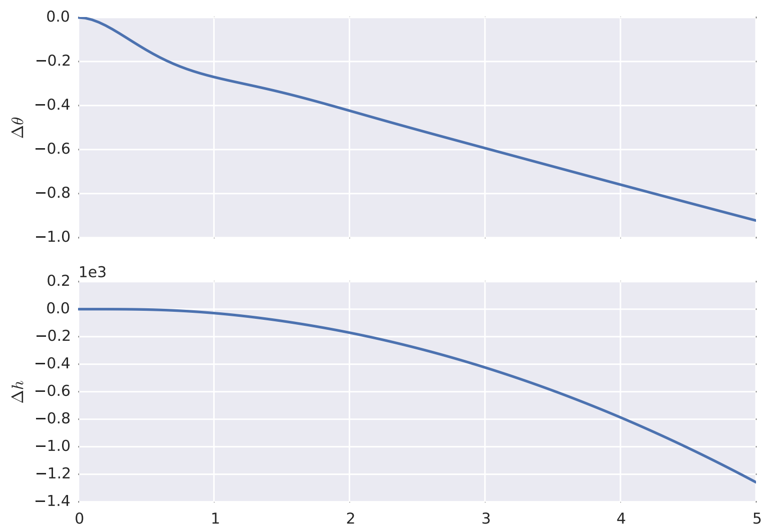

Figure 7: Short-period response to step input

Similarly, the response to a step control input of $\Delta\delta_a = 5/57.3$ rad can be seen for $\Delta\varphi$ and Δr

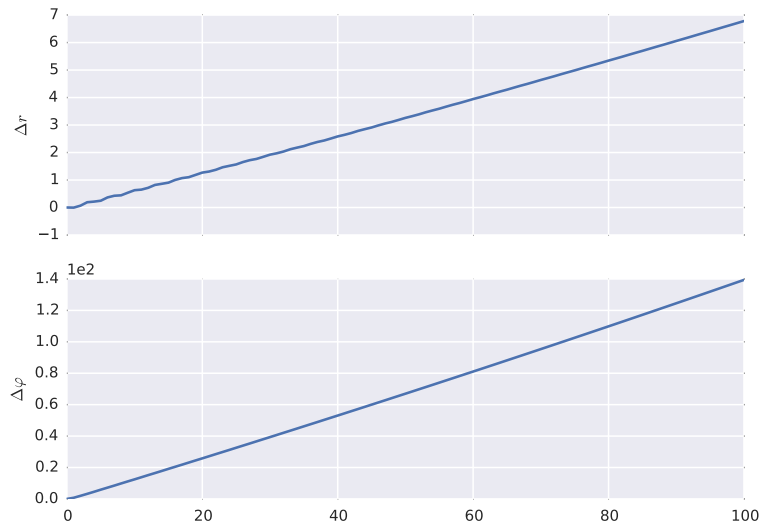

Figure 8: Step response to a step input in $\Delta\delta_a$

$$
\begin{aligned}
h &= 20000 \\
M &= 0.638 \\
g &= 32.2 \\
\theta_0 &= 0 \\
u_0 &= 660
\end{aligned}
$$

Figure 9: Basic Simulation Parameters

$$
\begin{aligned}
X_u &= -0.0097 \\
X_w &= 0.0016 \\
X_{\delta_e} &= 0 \\
\\
Z_u &= -0.0955 \\
Z_{\dot{w}} &= 0 \\
Z_w &= -1.43 \\
Z_{\delta_e} &= -69.8 \\
\\
M_u &= 0 \\
M_{\dot{w}} &= -0.0013 \\
M_w &= -0.0235 \\
M_q &= -1.92 \\
M_{\delta_e} &= -26.1 \\
\\
Z_q &= 0
\end{aligned}
$$

$$
\begin{aligned}
Y_v &= -0.0829 \\
Y_{\delta_a} &= 0 \\
\\
L'_p &= -1.70 \\
L'_r &= 0.172 \\
L'_{\delta_a} &= 27.3 \\
L'_\beta &= -4.55 \\
L'_v &= \frac{L'_\beta}{u_0} = -0.0069 \\
\\
N'_p &= -0.0654 \\
N'_r &= -0.0893 \\
N'_{\delta_a} &= 0.395 \\
N'_\beta &= 3.38 \\
N'_v &= \frac{N'_\beta}{u_0} = 0.0051 \\
\\
Y_p &= 0 \\
Y_r &= 0
\end{aligned}
$$

Figure 10: Longitudinal and Lateral Simulation Parameters