MAE 275 - Homework 2

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1 Problem 1

We can define the longitudinal and lateral linearized aircraft equations of motion. The longitudinal equations can be expressed as

$$
\begin{aligned}
\Delta\dot{u} =& X_u\Delta u + X_w\Delta w - g\cos\theta_0\Delta\theta \\
\Delta\dot{w} =& \frac{Z_u}{1-Z_{\dot{w}}}\Delta u + \frac{Z_w}{1-Z_{\dot{w}}}\Delta w + \frac{Z_q+u_0}{1-Z_{\dot{w}}}\Delta q - \frac{g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta \\
\Delta\dot{q} =& \left[M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}}\Delta u\right] + \left[M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}}\Delta w\right] + \left[M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}}\Delta q\right] - \left[\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}}\Delta\theta\right] \\
\Delta\dot{\theta} =& \Delta q \\
\Delta\dot{h} =& -\Delta w + u_0\Delta\theta
\end{aligned}
$$

or in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$
A = \begin{bmatrix}
X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\
\frac{Z_u}{1-Z_{\dot{w}}} & \frac{Z_w}{1-Z_{\dot{w}}} & \frac{Z_q+u_0}{1-Z_{\dot{w}}} & \frac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\
M_u + \frac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\frac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & -1 & 0 & u_0 & 0
\end{bmatrix}
$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of "Aircraft Dynamics and Automatic Control" yields

$$
A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{bmatrix}
$$

The lateral equations can be expressed as

$$
\begin{aligned}
\Delta\dot{v} =& Y_v \Delta v + Y_p \Delta p + [Y_r - u_0]\, \Delta r + g\cos\theta_0 \Delta\varphi \\
\Delta\dot{p} =& L'_v \Delta v + L'_p \Delta p + L'_r \Delta r \\
\Delta\dot{r} =& N'_v \Delta v + N'_p \Delta p + N'_r \Delta r \\
\Delta\dot{\varphi} =& \Delta p + r\tan\theta_0 \Delta r \\
\Delta\dot{\psi} =& r\sec\theta_0 \Delta r
\end{aligned}
$$

or in state space form, with state variables $\Delta v, \Delta p, \Delta r, \Delta\varphi(\text{roll}), \Delta\psi$, as

$$
A = \begin{bmatrix}
Y_v & Y_p & [Y_r - u_0] & g\cos\theta_0 & 0 \\
L'_v & L'_p & L'_r & 0 & 0 \\
N'_v & N'_p & N'_r & 0 & 0 \\
0 & 1 & \tan\theta_0 & 0 & 0 \\
0 & 0 & \sec\theta_0 & 0 & 0
\end{bmatrix}
$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of "Aircraft Dynamics and Automatic Control" yields

$$
A = \begin{bmatrix}
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5 \\
1 & 2 & 3 & 4 & 5
\end{bmatrix}
$$