MAE 275 - Homework 6

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1 Defining the System

The state-space system can be defined,

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}$$
$$\vec{y} = C\vec{x} + D\vec{u}$$

where the linearized lateral aircraft equations of motion can be expressed in state space form.

$$\vec{x} = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix}, \qquad \vec{u} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}, \qquad \vec{y} = \begin{bmatrix} p \\ \beta \end{bmatrix}$$

Using the lateral equations of motion for F-89 at flight condition 8901, the resultant system is

$$\dot{\vec{x}} = \begin{bmatrix} -8.2900e-2 & 0 & -6.6000e+2 & +3.2200e+1 & 0 \\ -6.8939e-3 & -1.7000e+0 & +1.7200e-1 & 0 & 0 \\ +5.1212e-3 & -6.5400e-2 & -8.9300e-2 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & +7.6500e+0 \\ +2.7300e+1 & +5.7600e-1 \\ +3.9300e-1 & -1.3600e+0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}$$

$$\vec{y} = \begin{bmatrix} 0 & +1 & 0 & 0 & 0 \\ +1.5152e-3 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}$$

Coupling numerators can be used to decide which loops to close in which order, and appropriate compensators can then be designed.

2 Coupling Numerators

The coupling numerators can be derived using the notes in the assignment and the HARV handout. They are

$$\boxed{\frac{p}{v_1} = \frac{27.3s(s^2+0.1747s+3.453)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}}$$

$$\frac{p}{v_2} = \frac{0.576s(s-2.885)(s+2.56)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\frac{\beta}{v_1} = \frac{-0.393(s-6.282)(s+0.04952)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\frac{\beta}{v_2} = \frac{0.011591(s+117.4)(s+1.753)(s-0.003733)}{(s+1.781)(s-0.001359)(s^2+0.09275s+3.529)}$$

$$\left.\frac{p}{v_1}\right|_{\beta\to v_2} = \frac{27.3s(s+118.1)}{(s+117.4)(s+1.753)(s-0.003733)}$$

$$\boxed{\left.\frac{\beta}{v_2}\right|_{p\to v_1} = \frac{0.011591(s+118.1)}{(s^2+0.1747s+3.453)}}$$

$$\left.\frac{p}{v_2}\right|_{\beta\to v_1} = \frac{-0.80517s(s+118.1)}{(s-6.282)(s+0.04952)}$$

$$\left.\frac{\beta}{v_1}\right|_{p\to v_2} = \frac{0.54936(s+118.1)}{(s-2.885)(s+2.56)}$$

From these transfer functions we can:

1. rule out controlling p with v_2 first, as there is a non-minimum phase zero (s-2.885) in the transfer function that would limit the crossover frequency to values significantly below 2.885
2. rule out $\left.\frac{p}{v_2}\right|_{\beta\to v_1}$ and $\left.\frac{\beta}{v_1}\right|_{p\to v_2}$ due to the closed-loop unstable poles that would be produced

This leaves only one viable option: to first close p with v_1, then close β with v_2 with the $p - v_1$ loop closed.

3 Compensators

Two compensators were designed:

$$GC_p = \frac{0.18(s+2)}{s}$$
$$GC_\beta = \frac{3.72(s^2+.2s+3.5)}{(.05*s^2+s)}$$

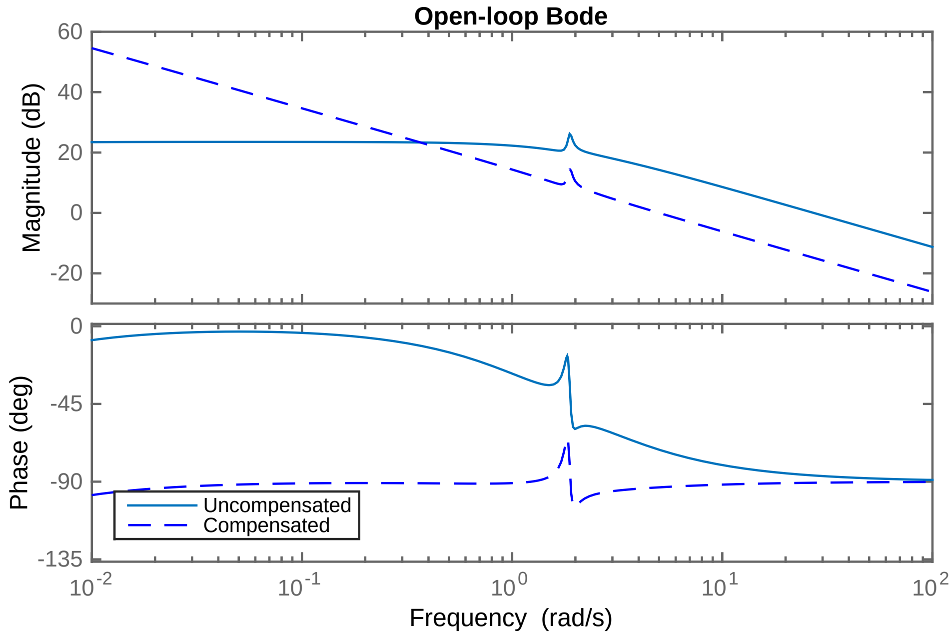

Figure 1: Open-loop Bode Plot for p loop

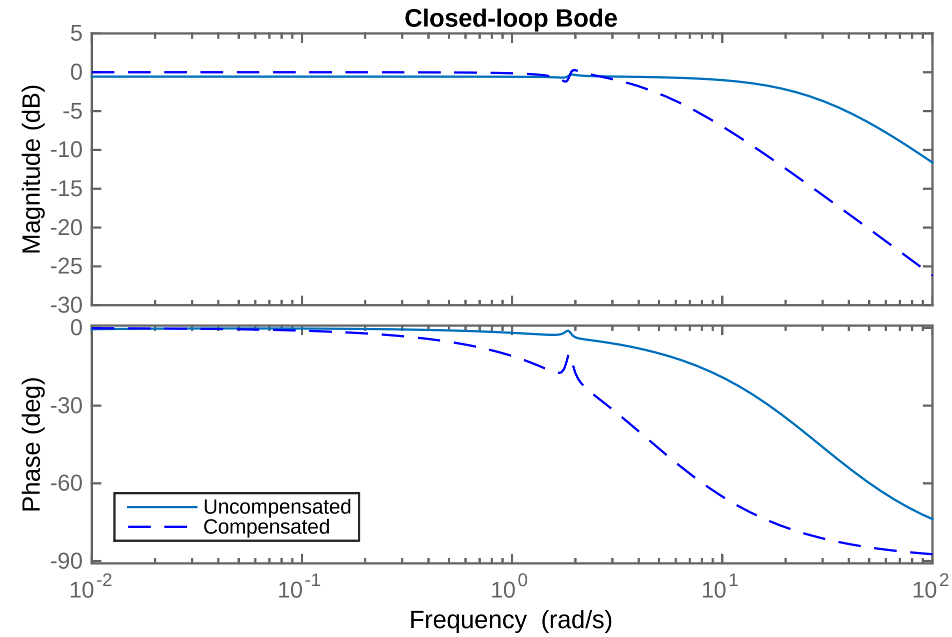

Figure 2: Close-loop Bode Plot for p loop

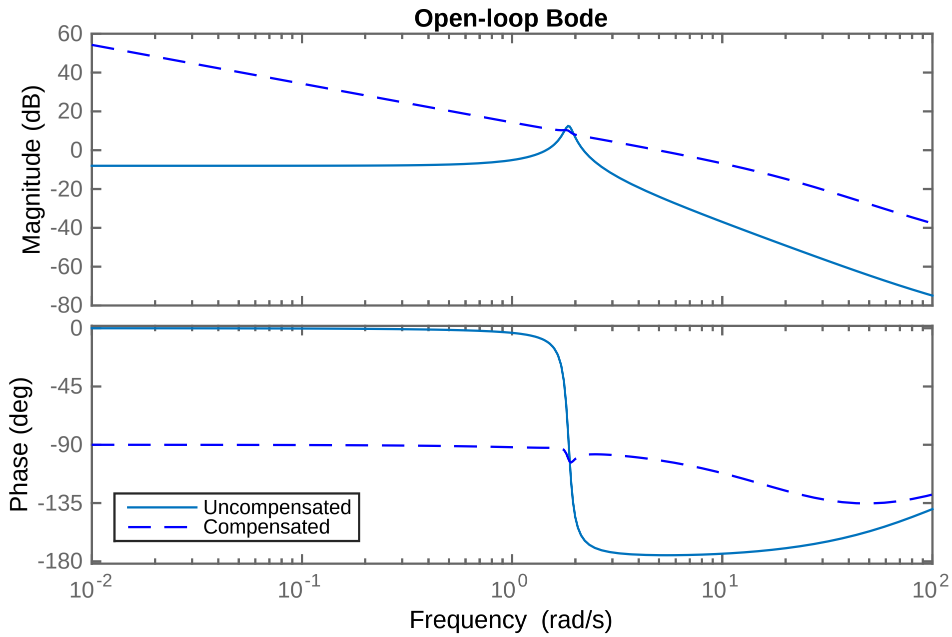

Figure 3: Open-loop Bode Plot for β loop

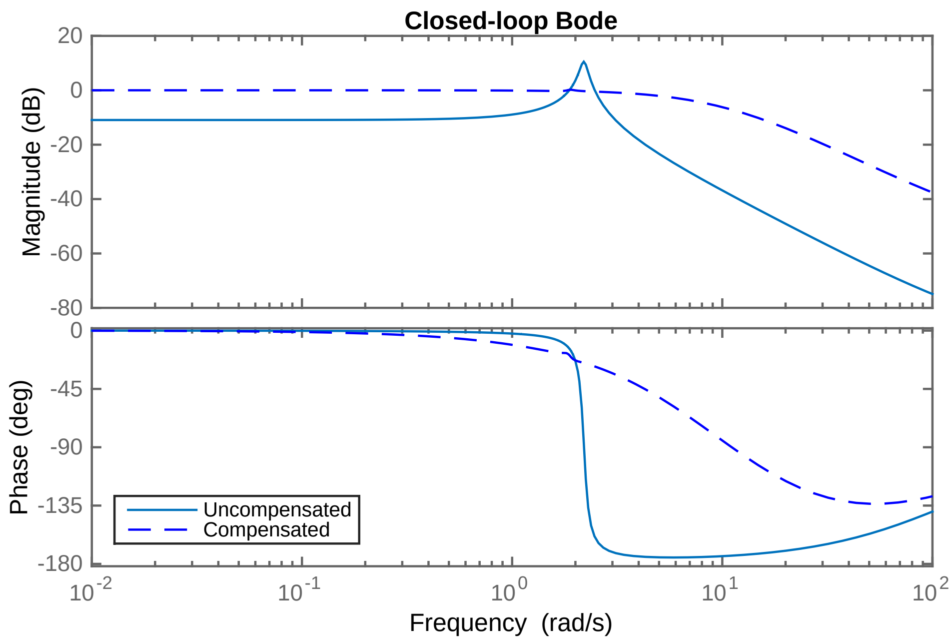

Figure 4: Close-loop Bode Plot for β loop

4 Response to Command Inputs

Two initial conditions were investigated (both commands were filtered with a filter of $\frac{25}{(s^2+10s+25)}$):

1. a ± 5 deg/sec doublet with each of the two pulses lasting 2 sec for the p-loop with no command for the β-loop
2. a $+5$ deg/sec command for the β-loop with no command for the p-loop

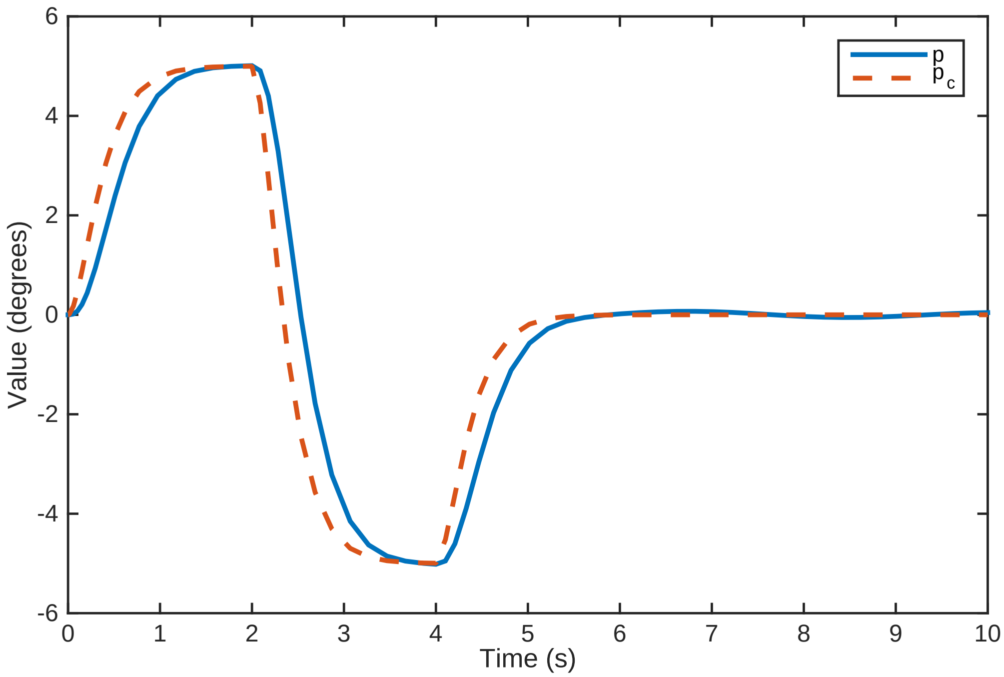

Figure 5: p Response for Scenario 1

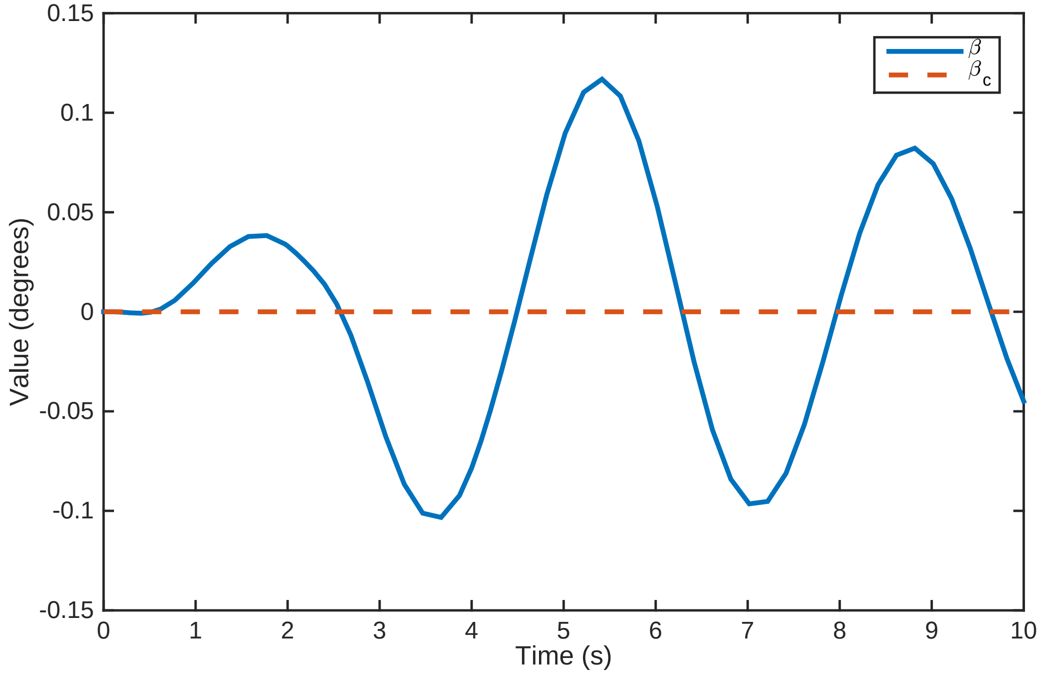

Figure 6: β Response for Scenario 1

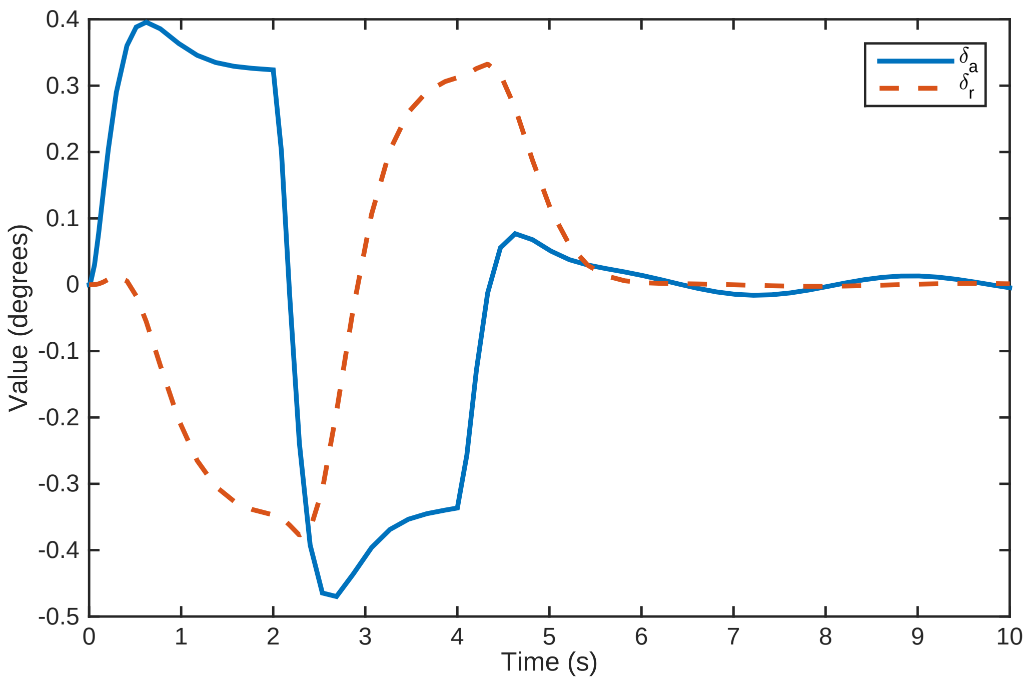

Figure 7: Command Inputs for Scenario 1

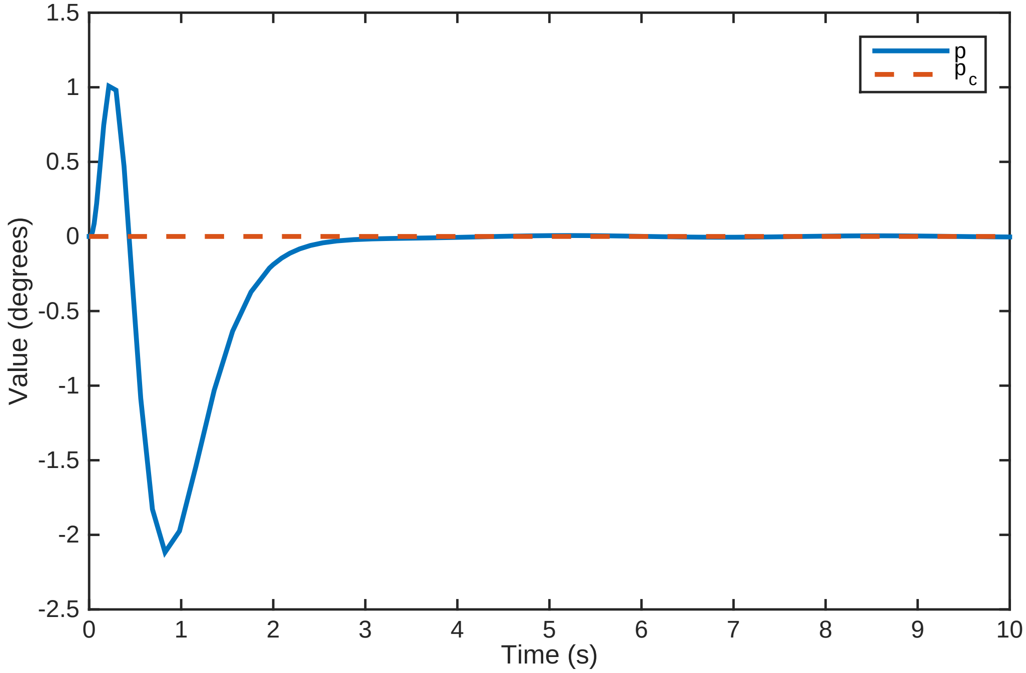

Figure 8: p Response for Scenario 2

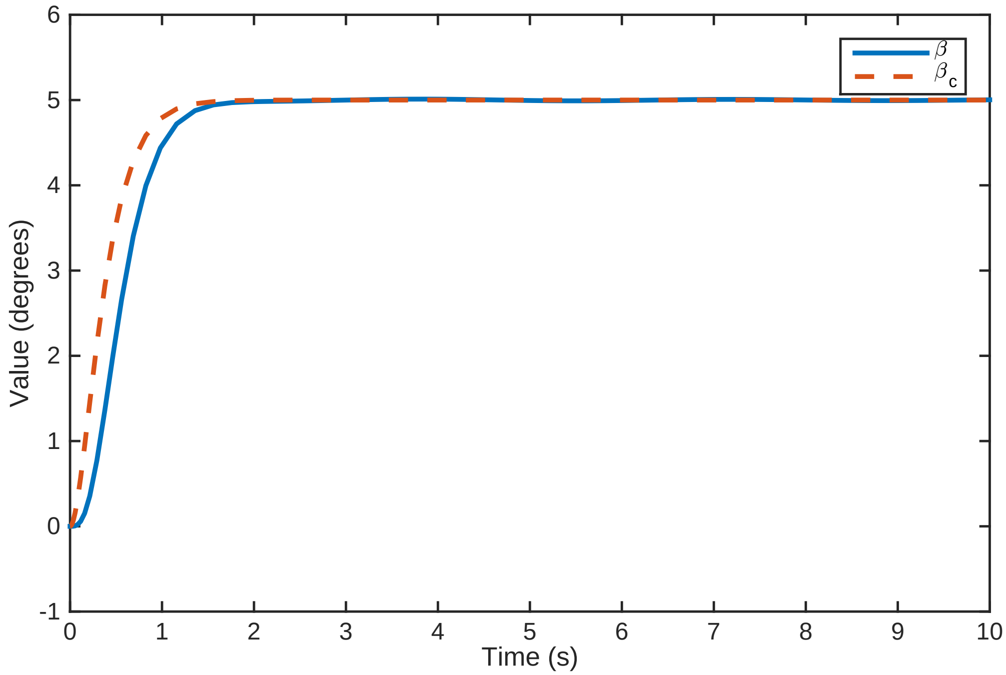

Figure 9: β Response for Scenario 2

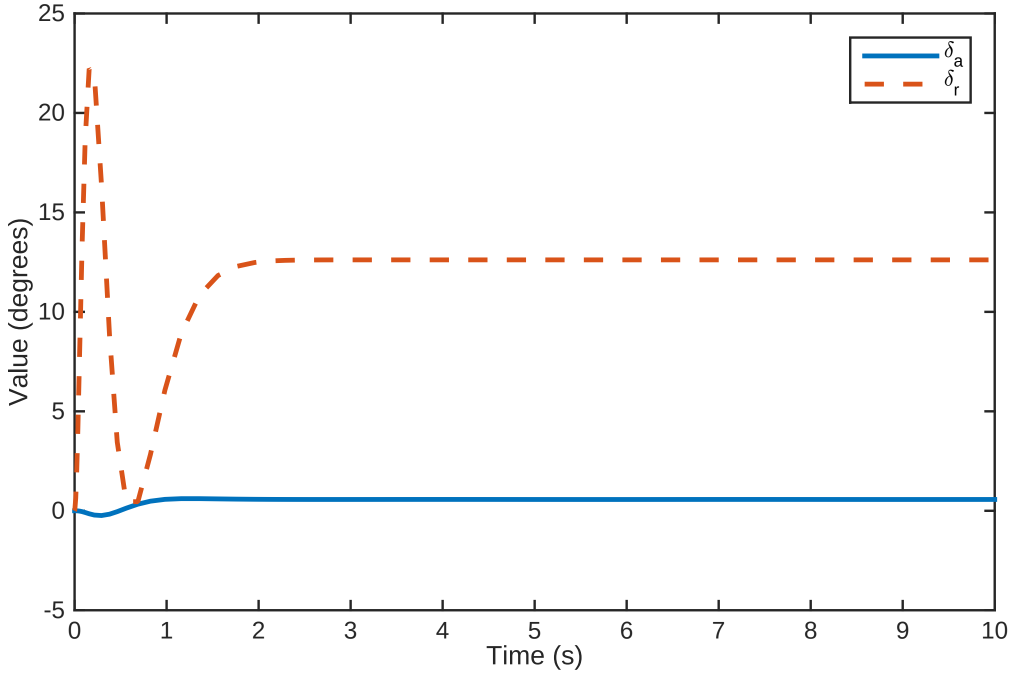

Figure 10: Command Inputs for Scenario 2