

# MAE 275 - Homework 2

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## 1 Problem 1

We can define the longitudinal and lateral linearized aircraft equations of motion. The longitudinal equations can be expressed as

$$\begin{aligned}
 \Delta \dot{u} &= X_u \Delta u + X_w \Delta w - g \cos \theta_0 \Delta \theta + \sum_{i=1}^n X_{\delta_i} \Delta \delta_i \\
 \Delta \dot{w} &= \frac{Z_u}{1 - Z_{\dot{w}}} \Delta u + \frac{Z_w}{1 - Z_{\dot{w}}} \Delta w + \frac{Z_q + u_0}{1 - Z_{\dot{w}}} \Delta q - \frac{g \sin \theta_0}{1 - Z_{\dot{w}}} \Delta \theta + \frac{1}{1 - Z_{\dot{w}}} \sum_{i=1}^n Z_{\delta_i} \Delta \delta_i \\
 \Delta \dot{q} &= \left[ M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} \Delta u \right] + \left[ M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} \Delta w \right] + \left[ M_q + \frac{M_{\dot{w}} (Z_q + u_0)}{1 - Z_{\dot{w}}} \Delta q \right] \\
 &\quad - \left[ \frac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} \Delta \theta \right] + \frac{M_{\dot{w}}}{1 - Z_{\dot{w}}} \sum_{i=1}^n Z_{\delta_i} \Delta \delta_i + \sum_{i=1}^n M_{\delta_i} \Delta \delta_i \\
 \Delta \dot{\theta} &= \Delta q \\
 \Delta \dot{h} &= -\Delta w + u_0 \Delta \theta
 \end{aligned} \tag{1}$$

or in state space form, with state variables  $\Delta u, \Delta w, \Delta q, \Delta \theta, \Delta h$ , as

$$A = \begin{bmatrix} \frac{X_u}{1 - Z_{\dot{w}}} & \frac{X_w}{1 - Z_{\dot{w}}} & 0 & -\frac{g \cos(\theta_0)}{1 - Z_{\dot{w}}} & 0 \\ M_u + \frac{M_{\dot{w}} Z_u}{1 - Z_{\dot{w}}} & M_w + \frac{M_{\dot{w}} Z_w}{1 - Z_{\dot{w}}} & M_q + \frac{M_{\dot{w}} (Z_q + u_0)}{1 - Z_{\dot{w}}} & -\frac{M_{\dot{w}} g \sin \theta_0}{1 - Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Plugging in the data for the F-89 aircraft (Flight Condition 8901) on pages A3-A5 in the Appendix of **Aircraft Dynamics and Automatic Control** yields

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

The lateral equations can be expressed as

$$\begin{aligned}
\Delta \dot{v} &= Y_v \Delta v + Y_p \Delta p + [Y_r - u_0] \Delta r + g \cos \theta_0 \Delta \varphi + \sum_{i=1}^n Y_{\delta_i} \Delta \delta_i \\
\Delta \dot{p} &= L'_v \Delta v + L'_p \Delta p + L'_r \Delta r + \sum_{i=1}^n L'_{\delta_i} \Delta \delta_i \\
\Delta \dot{r} &= N'_v \Delta v + N'_p \Delta p + N'_r \Delta r + \sum_{i=1}^n N'_{\delta_i} \Delta \delta_i \\
\Delta \dot{\varphi} &= \Delta p + r \tan \theta_0 \Delta r \\
\Delta \dot{\psi} &= r \sec \theta_0 \Delta r
\end{aligned} \tag{2}$$

or in state space form, with state variables  $\Delta v, \Delta p, \Delta r, \Delta \varphi(\text{roll}), \Delta \psi$ , as

$$A = \begin{bmatrix} Y_v & Y_p & [Y_r - u_0] & g \cos \theta_0 & 0 \\ L'_v & L'_p & L'_r & 0 & 0 \\ N'_v & N'_p & N'_r & 0 & 0 \\ 0 & 1 & \tan \theta_0 & 0 & 0 \\ 0 & 0 & \sec \theta_0 & 0 & 0 \end{bmatrix}$$

Plugging in the appropriate data

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}$$

## 2 Problem 2

### 2.1 Longitudinal

The following MATLAB command is called to identify the characteristic roots and eigenvector elements

```
1 [v, d] = eig(A);
```

resulting in two complex pairs

$$d_1 = X \pm iY$$

$$d_2 = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$

$$v_2 = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The longitudinal A matrix from above is used, along with B, C, and D matrices defined as

```
1 B = [0; 0; 0; 0; 0];
2
3 C = [[1, 0, 0, 0, 0];
4      [0, 1/u_0, 0, 0, 0];
5      [0, 0, 1, 0, 0];
6      [0, 0, 0, 1, 0];
7      [0, 0, 0, 0, 1]];
8
9 D = [0; 0; 0; 0; 0];
```

finally, the initial state can be defined and the initial command can be run by

```
1 i1 = real(v(:,1));
2 initial(A, B, C, D, i1, 5)
```

and results in the following figures

### 2.2 Lateral

The lateral eigenvalues are identified as

$$d_1 = X \pm iY$$

...

$$d_n = X \pm iY$$

and their associated eigenvectors

$$v_1 = [1, 2, 3, 4, 5]$$

...

$$v_n = [1, 2, 3, 4, 5]$$

Before exciting these modes, the rest of the state space system must be defined. The lateral A matrix from above is used, along with B, C, and D matrices defined as

```

1 B = [0; 0; 0; 0; 0];
2
3 C = [[1/u_0, 0, 0, 0, 0];
4       [0, 1, 0, 0, 0];
5       [0, 0, 1, 0, 0];
6       [0, 0, 0, 1, 0];
7       [0, 0, 0, 0, 1]];
8
9 D = [0; 0; 0; 0; 0];

```

Exciting each of these modes with the appropriate eigenvector results in

### 3 Problem 3

#### 3.1 Longitudinal

A control input to the elevators,  $\Delta\delta_e$  can be found by solving Equation 1, resulting in a B matrix,

$$B = \begin{bmatrix} \frac{X_{\delta_e}}{Z_{\delta_e}} \\ \frac{1 - Z_{\dot{w}}}{M_{\dot{w}} Z_{\delta_e}} \\ \frac{1 - Z_{\dot{w}} + M_{\delta_e}}{0} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

After defining the C and D matrices, we can find the resulting transfer function with the following commands

```
1 C=[0 0 0 1 0];
2 D = 0;
3
4 [n, d] = ss2tf(A, B, C, D);
5 minreal(zpk(tf(n,d)))
```

which results in

```
1 some shit
```

which compare quite well with the values found on page A-5,

```
1 some other shit
```

#### 3.2 Lateral

A control input to the aileron,  $\Delta\varphi_e$  can be found by solving Equation 2, resulting in a B matrix,

$$B = \begin{bmatrix} Y_{\delta_a} \\ L'_{\delta_a} \\ N'_{\delta_a} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$$

Defining the C and D matrices the same as above, we can again find the resulting transfer function

```
1 some shit
```

which compare quite well with the values found on page A-5,

```
1 some other shit
```

## 4 Problem 4