Eigenanalyses

For Some V/STOL (Vertical/Short Takeoff and Landing)

Aircraft

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Image Gallery

CL-84 Dynavert

Canadair's bold new design broke new ground with the CL-84. Its experimental "tilt-wing combined vertical take-off-and-landing with a low-speed, fixed-wing capability. Though only four CL-84's constructed, the design was considered a success.

Period: Postwar
Uses: Experimental
FirstFlight: 1965
Manufacturer: Canadair Ltd., Canada
WingSpan: 34 ft 8 in (10.6 m)
Length: 53 ft 7 1/2 in (16.3 m)
Height: 17 ft 1 1/2 in (5.2 m)
WeightEmpty: 8,775 lb (3,980 kg)
WeightGross: 14,500 lb (6,577 kg)
CruisingSpeed: 309 mph (497 km/h)
MaxSpeed: 321 mph (517 km/h)
RateClimb: 4,200 ft (1,280 m)/min

Challenges for the choppers - how rescuers face the floods
Mozambique has been flooding for three weeks due to the swollen rivers of Zimbabwe and South Africa. Thousands of people will have to be rescued - a task that falls onto the shoulders of helicopter pilots.

New airship design close to completion
Lockheed Martin is putting the final touches on a design for a new blimp-like airship. Once the actual ships are ready, they'll sit above the jet stream for months at a time.

Alaskan Airlines Flight 261 - trim tabs amiss?
Flight 261 went

V-22

CHANCE-VOUGHT/LTV XC-142A

The tilt-wing XC-142A was an experimental aircraft designed to investigate the operational suitability of vertical/short takeoff and landing (V/STOL) transports. Such an aircraft would permit rapid movement of troops and supplies into unprepared areas under all-weather conditions. An XC-142A first flew conventionally on Sept. 29, 1964 and on Jan. 11, 1965, completed its first transitional flight by taking off vertically, changing to forward flight, and finally landing vertically.

Tilting the wing and engines skyward permitted vertical takeoff like a helicopter and then the wing and engines were gradually tilted forward to provide the greater speed of a fixed-wing aircraft in forward flight. The engines were linked together so that a single engine could turn all four propellers and the tail rotor. In tests the XC-142A was flown from airspeeds of 35 mph backwards to 400 mph forward. XC-142As were tested extensively by the Army, Navy, Air Force and NASA.

The aircraft on display, the only remaining XC-142A, was one of five built. It was flown to the museum in 1970.

More XC-142 information...
- The XC-142 in hovering flight

SPECIFICATIONS
Span: 67 ft. 6 in.
Length: 58 ft. 2 in.
Height: 25 ft. 8 in.
Weight: 41,500 lbs. max.
Armament: None
Engines: Four General Electric T64s of 3,080 hp. each
Serial number: 65-5924
Other registrations: NASA 522

PERFORMANCE
Maximum speed: 400 mph.
Cruising speed: 235 mph.
Range: 820 miles
Service Ceiling: 25,000 ft.

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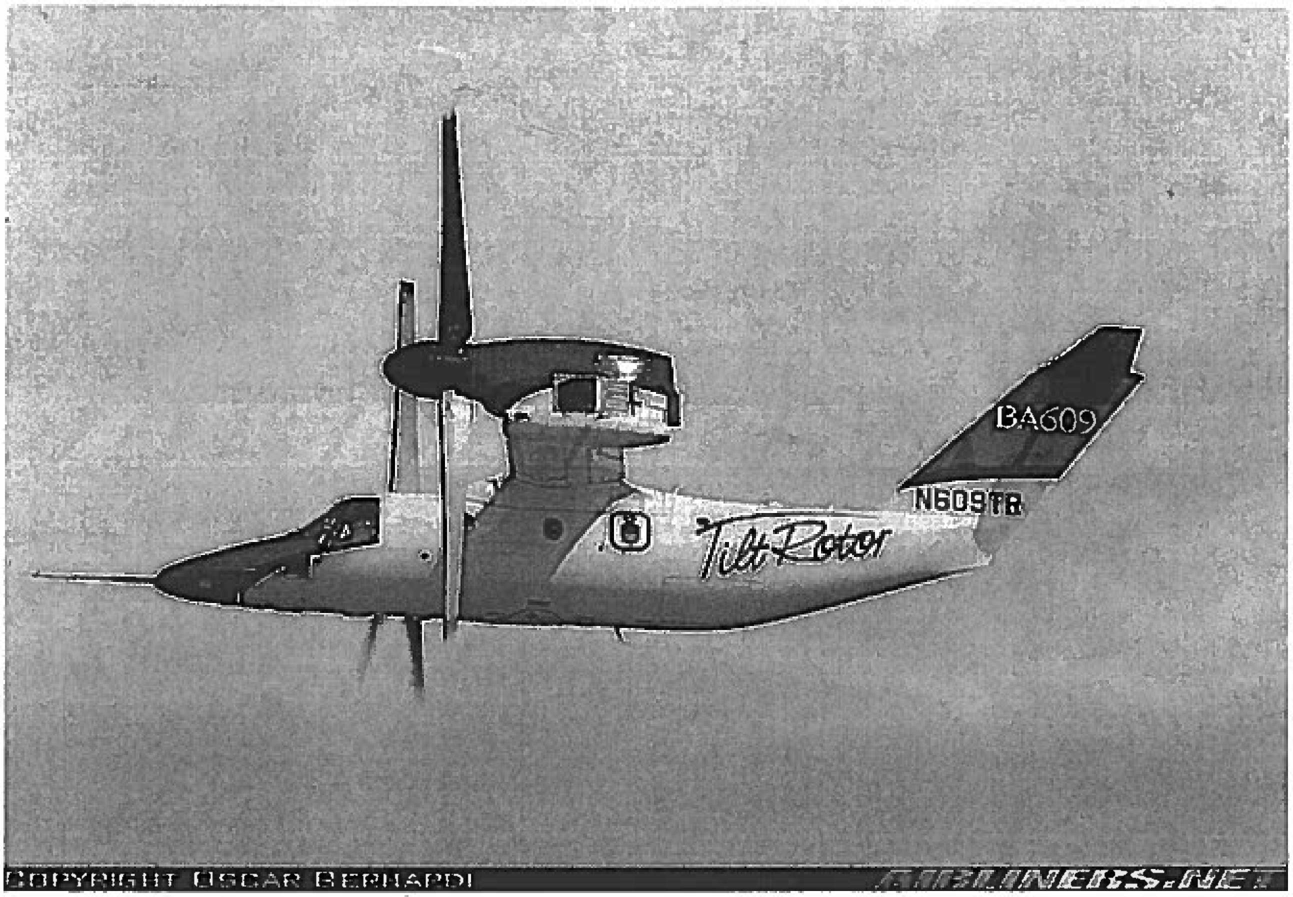
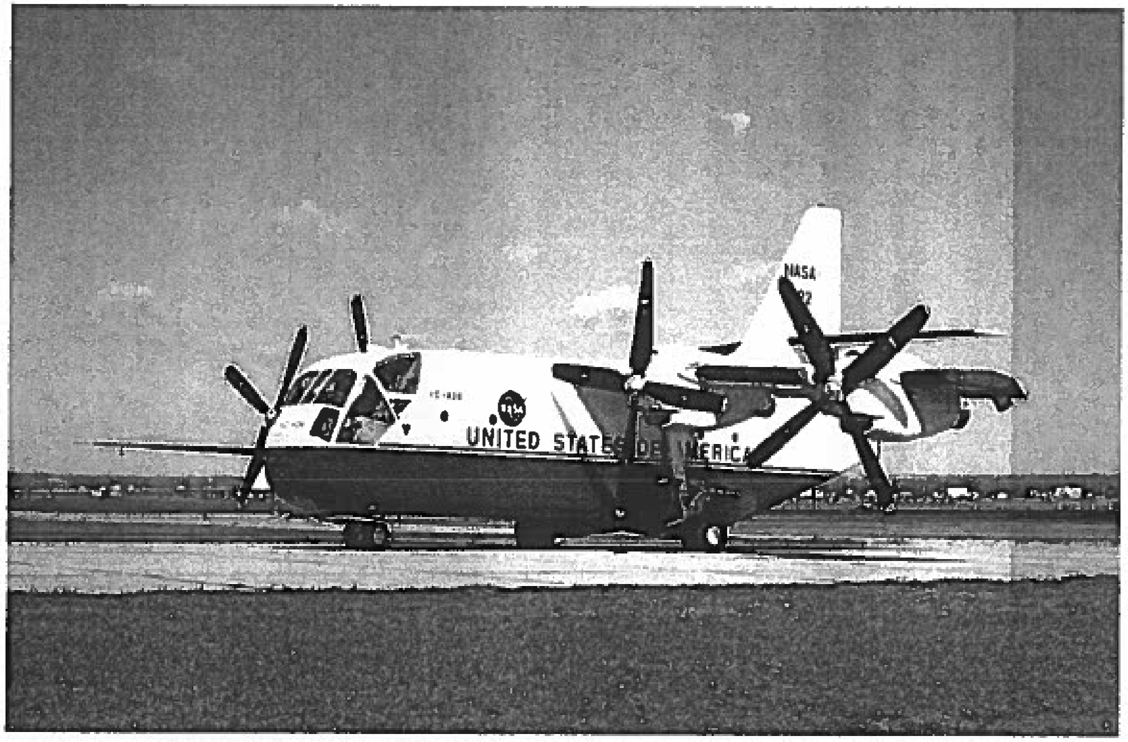
Research and Development Aircraft Main Page



Bell 609

	≈ 320 mph	
Maximum Cruise Speed	275 kt	509 km/hr
HOGE(ISA, MGW, AEO)	5,000 ft MSL	1,150 m
Service Ceiling (MCP) (All Engines Operating)	25,000 ft	7,622 m
O.E.I. (ISA, MGW)	12,800 ft MSL	3,866 m
Maximum Range * (no reserve)	1,000 nm	1,852 km

XC-142



Tilt Wing Prototype
NASA Langley Research Center 1/17/1969 Image # EL-2001-00399

YouTube XC-142

XC-142

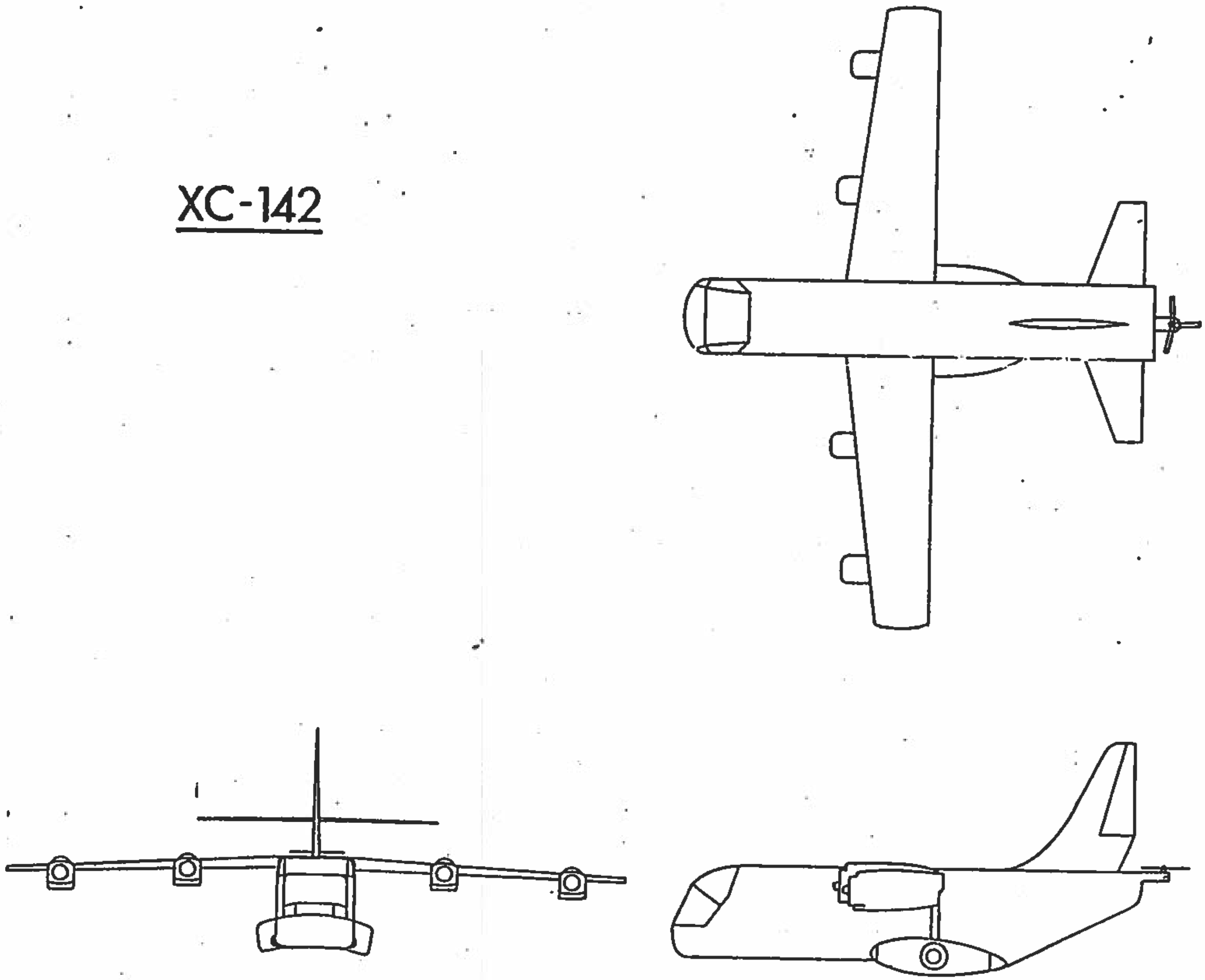


TABLE A-7

A. GEOMETRICAL AND INERTIAL PARAMETERS FOR THE XC-142

Note: Data are for body-fixed centerline axes

$S = 534\ ft^2$, $b = 67.5$ ft , $c = 8.07$ ft , $\gamma_o = 0$ deg

	FLIGHT CONDITION		
	1420 HOVER	1421 60 KTS	1422 120 KTS
h (ft)	0	0	0
M (–)	0	0.0906	0.1812
a (ft/sec)	1,117	1,117	1,117
ρ (slugs/ft^3)	0.002378	0.002378	0.002378
V_{T_o} (ft/sec)	1.0	101.28	202.56
$\bar{q} = \rho V^2/2$ (lb/ft^2)	0	12.2	48.8
W (lb)	37,474	37,474	37,474
m (slugs)	1,163.8	1,163.8	1,163.8
I_x (slug-ft^2)	173,000	173,000	173,000
I_y (slug-ft^2)	122,000	122,000	122,000
I_z (slug-ft^2)	267,000	267,000	267,000
I_{xz} (slug-ft^2)	7,000	7,000	7,000
$x_{c.g.}/\bar{c}$	0.20	0.20	0.20
i_w (deg)	90	14.5	1.25
θ_o (deg)	0	0	0
U_o (ft/sec)	1.0	101.28	202.56
W_o (ft/sec)	0	0	0

B. LONGITUDINAL DIMENSIONAL DERIVATIVES FOR THE XC-142

Note: Data are for body-fixed centerline axes; thrust corrections are included.

	FLIGHT CONDITION		
	1420 HOVER	1421 60 KTS	1422 120 KTS
h (ft)	0	0	0
M (–)	0	0.0906	0.1812
X_u (1/sec)	–0.21	–0.196	–0.22
X_w (1/sec)	0	0.035	0.060
†X_{δ_e} [(ft/sec^2)/in.]	0	0.124	0.120
‡X_{δ_T} [(ft/sec^2)/rad]	0	73	130.0
Z_u (1/sec)	0	–0.278	–0.15
$Z_{\dot{w}}$ (–)	0	0	0
Z_w (1/sec)	–0.065	–0.592	–0.85
†Z_{δ_e} [(ft/sec^2)/in.]	2.58	3.12	4.58
‡Z_{δ_T} [(ft/sec^2)/rad]	–119.0	–130	–97
M_u (1/sec-ft)	0.0073	0.0045	0.01
$M_{\dot{w}}$ (1/ft)	–0.00127	–0.00127	–0.00127
M_w (1/sec-ft)	0.0003	–0.0002	–0.0095
M_q (1/sec)	–0.085	–0.486	–0.89
†M_{δ_e} [(1/sec^2)/in.]	0.765	0.87	1.195
‡M_{δ_T} (1/sec^2-rad)	0.26	–3.71	–5.08

†δ_e – inches of scissors (horizontal tail and tail prop contributions included)

‡δ_e – radius of main prop blade angle (includes static governor effects)

E. LATERAL DIMENSIONAL DERIVATIVES FOR THE XC-142

Note: Data are for body-fixed centerline axes

	FLIGHT CONDITION		
	1420 HOVER	1421 60 KTS	1422 120 KTS
h (ft)	0	0	0
M (–)	0	0.0906	0.1812
Y_v (1/sec)	–0.015	–0.0945	–0.175
Y_β [(ft/sec^2)/rad]	–0.015	–9.58	–35.5
†Y_{δ_a} [(ft/sec^2)/in.]	—	—	—
†$Y_{\delta_a}^*$ [(1/sec)/in.]	—	—	—
‡Y_{δ_r} [(ft/sec^2)/in.]	0	0.248	0.94
‡$Y_{\delta_r}^*$ [(1/sec)/in.]	0	0.00245	0.00464
L_β (1/sec^2)	–0.0006	–0.724	–1.93
L_p (1/sec)	–0.235	–0.533	–0.85
L_r (1/sec)	–0.025	0.395	0.582
†L_{δ_a} [(1/sec^2)/in.]	–0.285	–0.1663	–0.192
‡L_{δ_r} [(1/sec^2)/in.]	0.0706	–0.081	0.0966
L'_β (1/sec^2)	–0.000616	–0.715	–1.91
L'_p (1/sec)	–0.235	–0.539	–0.855
L'_r (1/sec)	–0.0335	0.382	0.559
†L'_{δ_a} [(1/sec^2)/in.]	–0.285	–0.167	–0.193
‡L'_{δ_r} [(1/sec^2)/in.]	0.0622	–0.0871	0.0913
N_β (1/sec^2)	–0.00037	0.237	0.630
N_p (1/sec)	0	–0.123	–0.094
N_r (1/sec)	–0.21	–0.342	–0.58
†N_{δ_a} [(1/sec^2)/in.]	0	–0.0085	–0.0215
‡N_{δ_r} [(1/sec^2)/in.]	–0.21	–0.148	–0.134
N'_β (1/sec^2)	–0.000386	0.218	0.580
N'_p (1/sec)	–0.00617	–0.137	–0.116
N'_r (1/sec)	–0.211	–0.332	–0.565
†N'_{δ_a} [(1/sec^2)/in.]	–0.00748	–0.0129	–0.0266
‡N'_{δ_r} [(1/sec^2)/in.]	–0.208	–0.150	–0.132

† δ_a – inches of lateral stick (includes aileron and differential main prop blade angle) positive δ_a gives negative $\dot{p}$

‡ δ_r – inches of pedal (includes rudder, aileron, and differential main prop blade angle) positive δ_r gives negative $\dot{r}$

XC-142 @ HOVER

LONG

$$\Delta \dot{u} = -.21\,\Delta u + 0\,\Delta w + 0\,\Delta q - 32.2\,\Delta \theta$$

$$\Delta \dot{w} = 0\,\Delta u - .065\,\Delta w + 0\,\Delta q + 0\,\Delta \theta$$

$$\Delta \dot{q} = .0073\,\Delta u + .000383\,\Delta w - .085\,\Delta q + 0\,\Delta \theta$$

$$\Delta \dot{\theta} = \Delta q$$

LATERAL

$$\Delta \dot{v} = -.05\,\Delta v + 0\,\Delta p + 0\,\Delta r + 32.2\,\Delta \phi$$

$$\Delta \dot{p} = -.000616\,\Delta v - .236\,\Delta p - .0355\,\Delta r$$

$$\Delta \dot{r} = -.000286\,\Delta v - .00617\,\Delta p - .211\,\Delta r$$

$$\Delta \dot{\phi} = \Delta p$$

```
» [v,d]=eig(A)

v =

   Columns 1 through 3

Δu  9.9981e-001      -9.1452e-001 +4.0382e-001i  -9.1452e-001 -4.0382e-001i
Δw            0                   0                            0
Δq -1.1464e-002      -1.1675e-003 +1.1953e-002i  -1.1675e-003 -1.1953e-002i
Δθ  1.5886e-002       1.8662e-002 +9.7256e-003i   1.8662e-002 -9.7256e-003i

   Column 4

  -5.2436e-002
   9.9862e-001
  -1.5348e-005
   2.3612e-004


d =

   Columns 1 through 3

  -7.2164e-001                   0                            0
             0       2.1332e-001 +5.2936e-001i                0
             0                   0            2.1332e-001 -5.2936e-001i
             0                   0                            0

   Column 4

             0
             0
             0
  -6.5000e-002

»
```

EIGENVECTOR

Periodic Divergence

LONG

EIGENVECTORS FOR APERIODIC DIV. MODE

SINCE $U_0 = 0$, CANNOT NORMALIZE

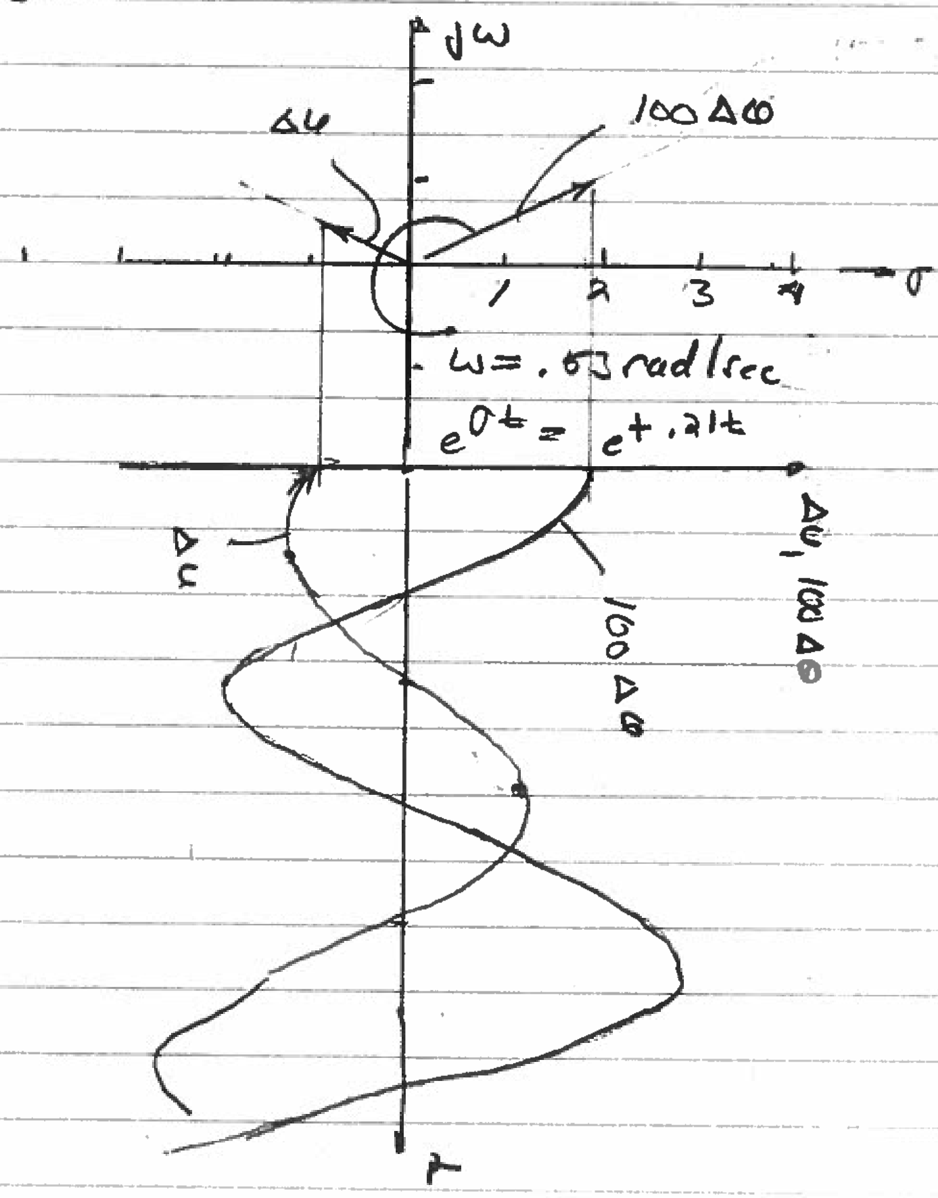
Δu: $-.916 + .404i$

Δw: 0

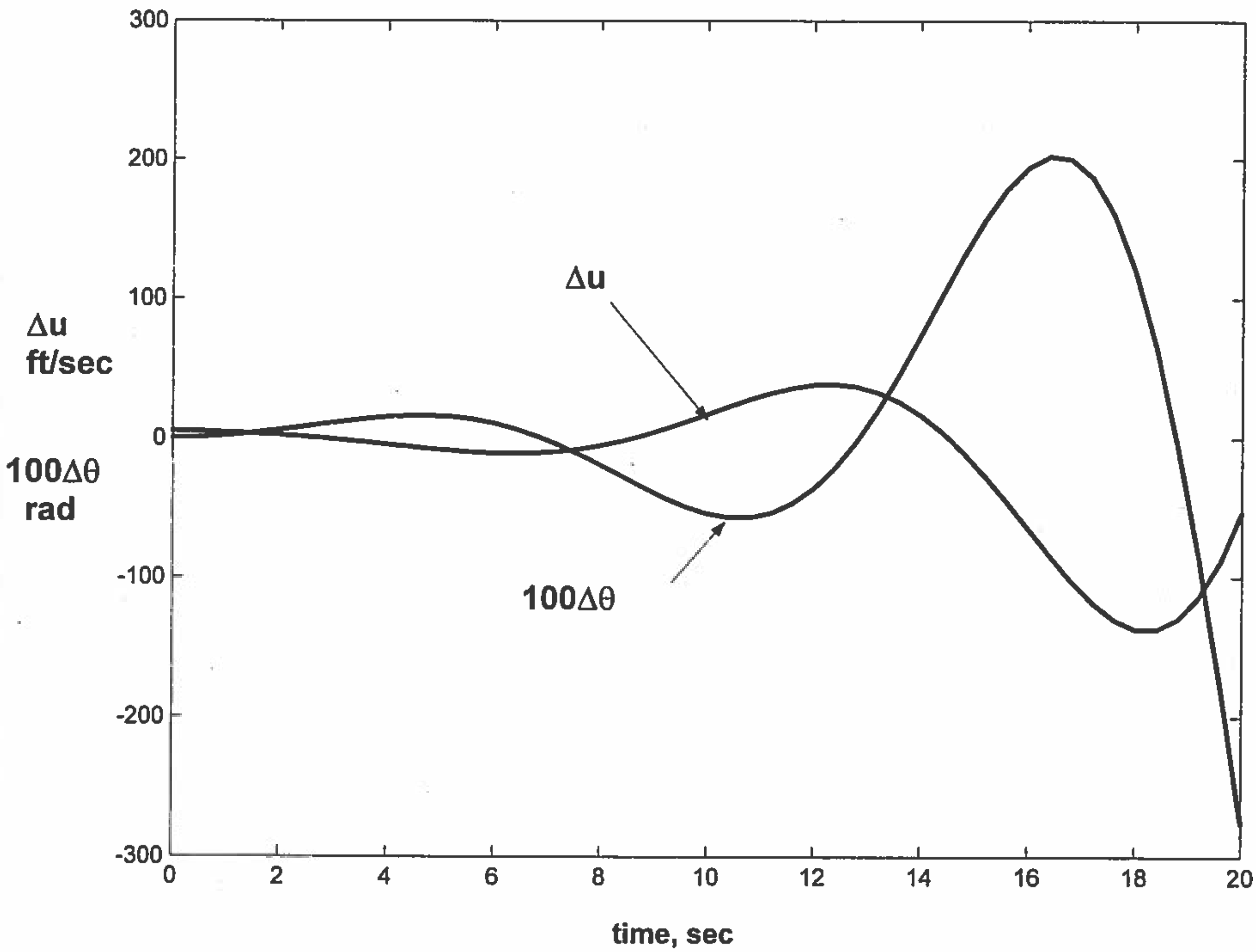
Δq: $-1.17 \cdot 10^{-3} + 1.2 \cdot 10^{-2} i$

$\Delta \theta$: $1.87 \cdot 10^{-2} + 9.73 \cdot 10^{-3} i$

Look @ Δu / $100\,\Delta\theta$



```
» load xc142long
» A

A =

 -2.1000e-001             0             0 -3.2200e+001
            0 -6.5000e-002             0             0
  7.3000e-003  3.8300e-004 -8.5000e-002             0
            0             0  1.0000e+000             0

» B

B =

     0
     0
     0
     0

» C

C =

     1     0     0     0
     0     0     0     1

» D

D =

     0
     0
```



300
200
100
0
-100
-200
-300
Δu
ft/sec
100Δθ
rad
Δu
100Δθ
0
2
4
6
8
10
12
14
16
18
20
time, sec

```
» [v,d]=eig(A)

v =

   Columns 1 through 3          EIGENVECTOR

Δu   8.3253e-001 +5.5393e-001i   8.3253e-001 -5.5393e-001i   9.9992e-001
Δp  -1.6059e-003 +1.6545e-005i  -1.6059e-003 -1.6545e-005i   4.5126e-003
Δr  -1.0862e-003 +8.1408e-005i  -1.0862e-003 -8.1408e-005i   2.3277e-003
Δφ  -1.8117e-003 +6.8970e-003i  -1.8117e-003 -6.8970e-003i  -1.1607e-002

   Column 4

   9.9979e-001
   1.0453e-003
  -1.9753e-002
  -5.4689e-003


d =

   Columns 1 through 3

   5.9457e-002 +2.1722e-001i              0                            0
             0               5.9457e-002 -2.1722e-001i                 0
             0                            0               -3.8878e-001
             0                            0                            0

   Column 4                 Periodic divergence

             0
             0
             0
 -1.9114e-001

»
```

LAT

EIGENVECTORS FOR APERIODIC DIV. MODE

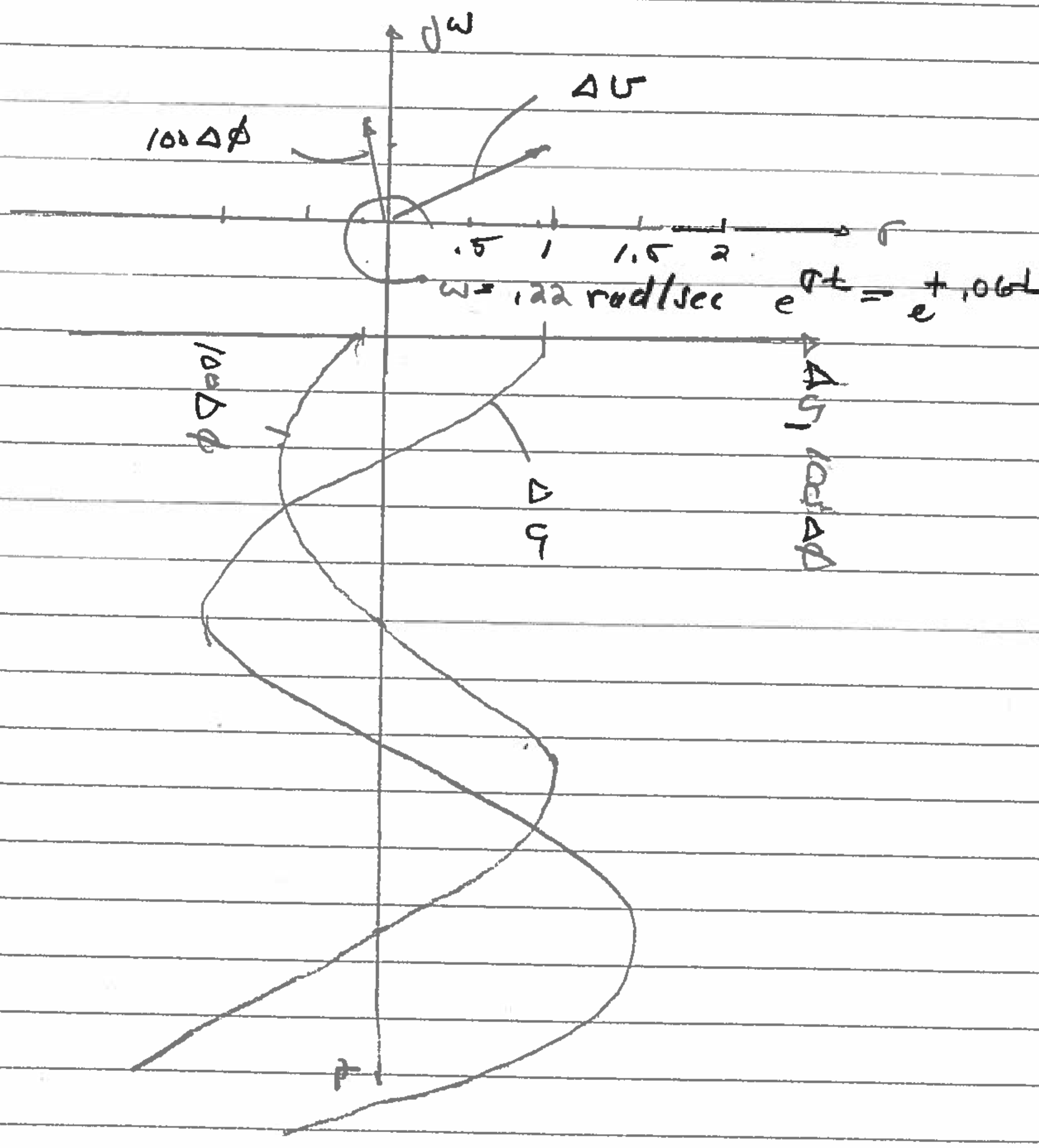
Δv : $.833 + .554i$

Δp : $-1.61 \cdot 10^{-3} + 1.65 \cdot 10^{-5} i$

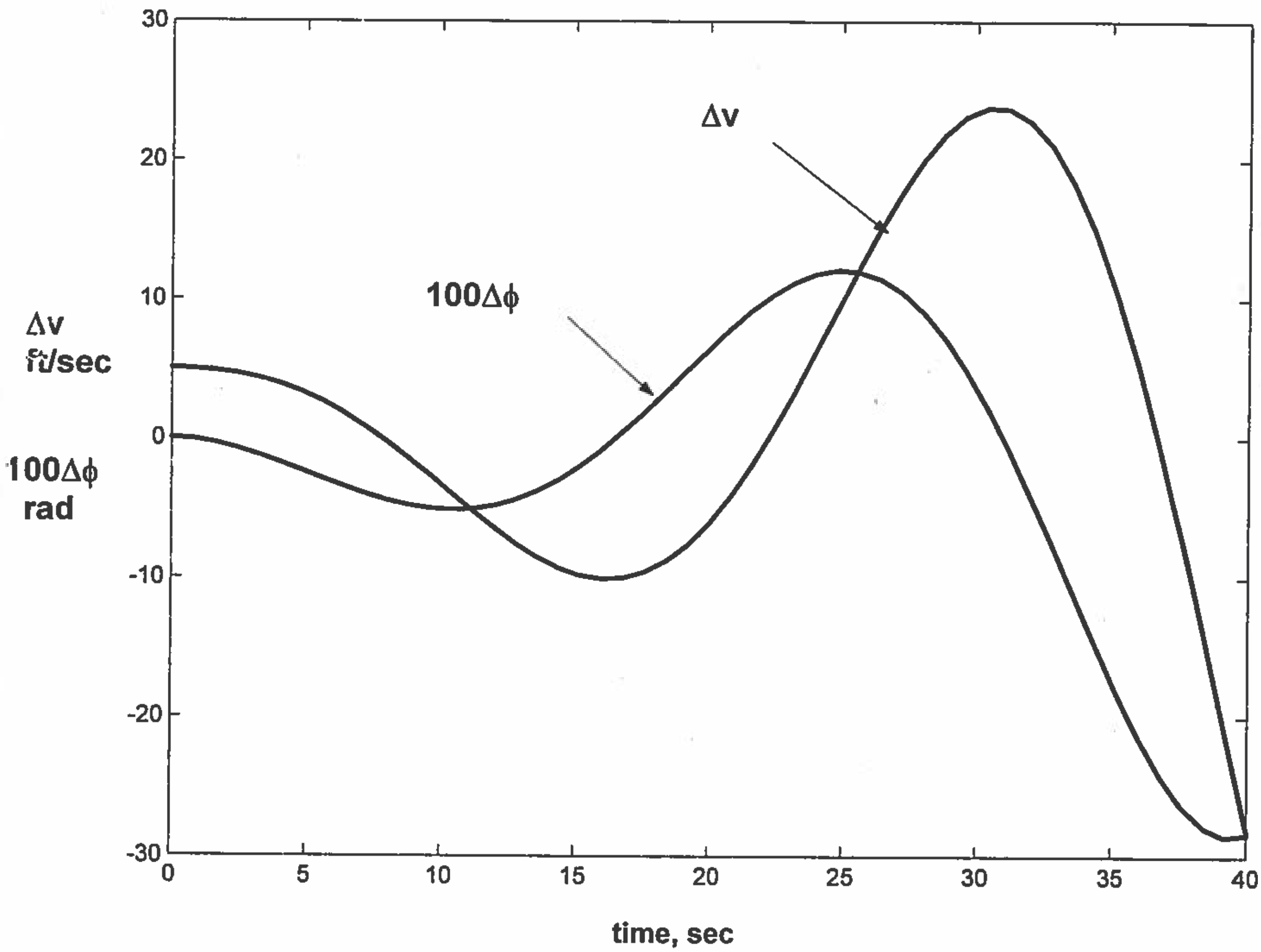
Δr : $-1.09 \cdot 10^{-3} + 8.14 \cdot 10^{-5} i$

$\Delta \phi$: $-1.81 \cdot 10^{-3} + 6.9 \cdot 10^{-3}$

Look @ Δv & $100\,\Delta\phi$



```
» load xc142lat
» A

A =

 -1.5000e-002             0             0   3.2200e+001
 -6.1600e-004 -2.3500e-001 -3.3500e-002             0
 -3.8600e-004 -6.1700e-003 -2.1100e-001             0
            0  1.0000e+000             0             0

» B

B =

     0
     0
     0
     0

» C

C =

     1     0     0     0
     0     0     1     0
     0     0     0     1

» D

D =

     0
     0
     0

»
```



30
20
10
0
-10
-20
-30
0
5
10
15
20
25
30
35
40
Δv
100Δφ
Δv
ft/sec
100Δφ
rad
time, sec

University of California, Davis

Dept. of Mechanical and Aerospace Engineering

MAE 275

Aspects of Rotorcraft Stability and Control

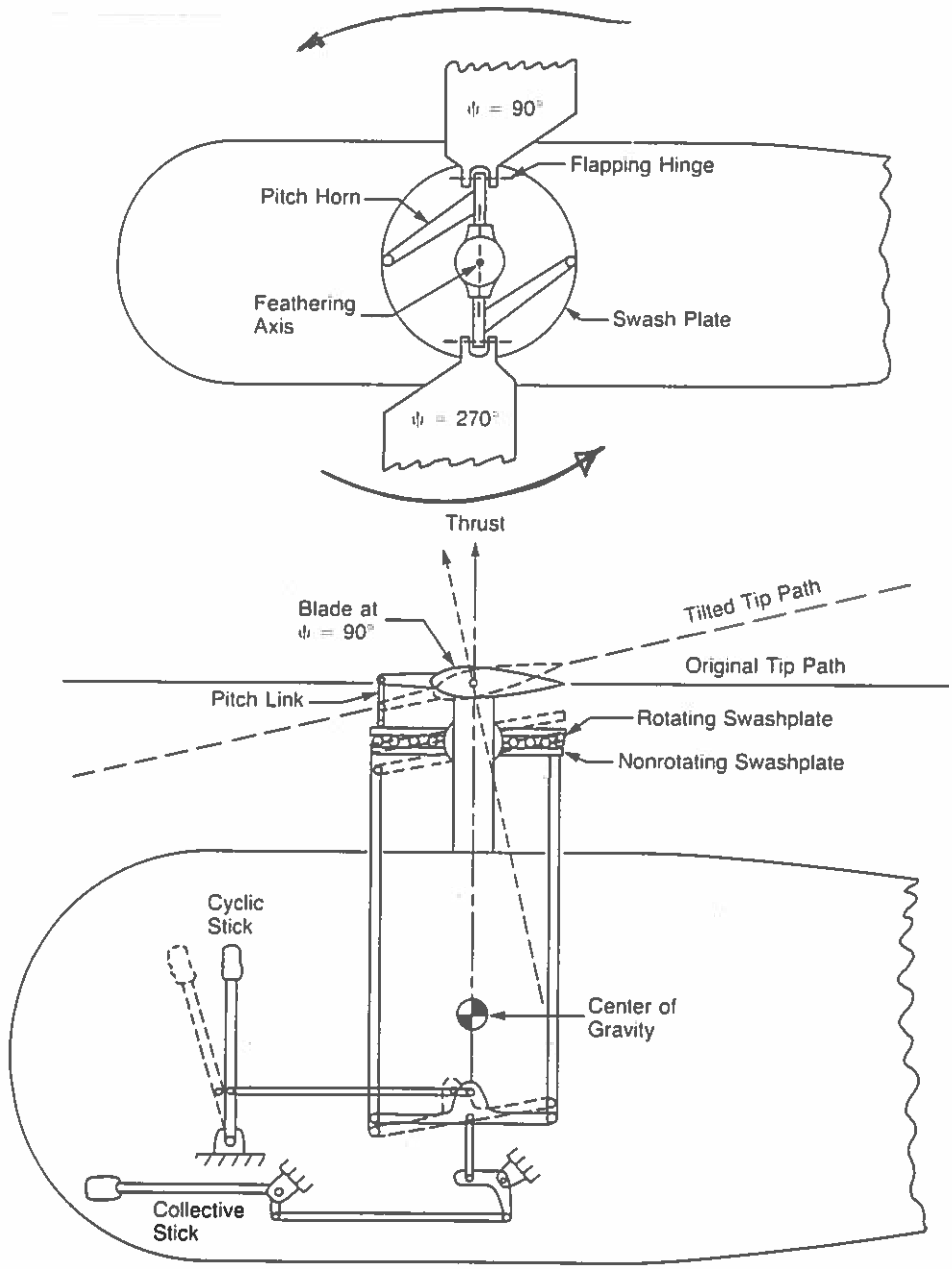
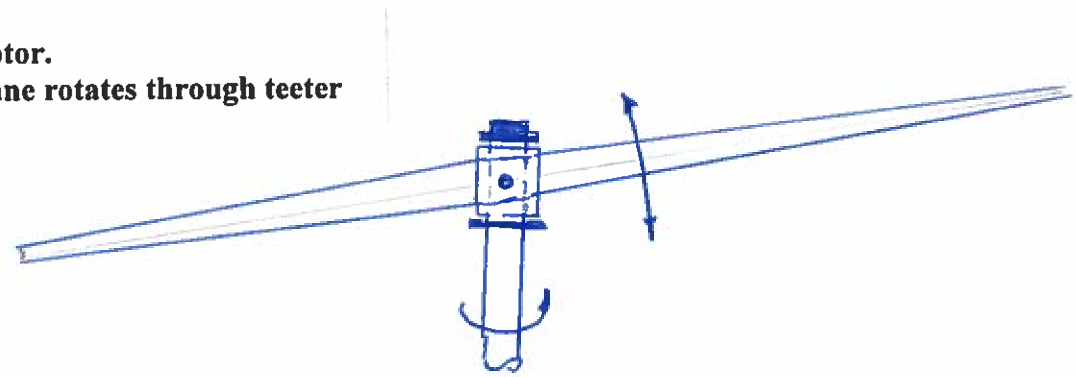
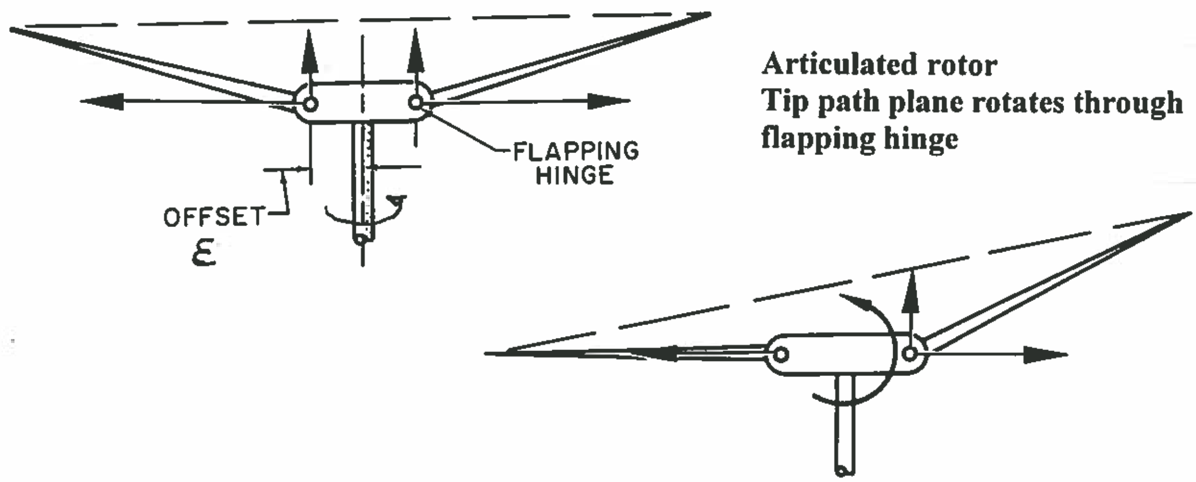


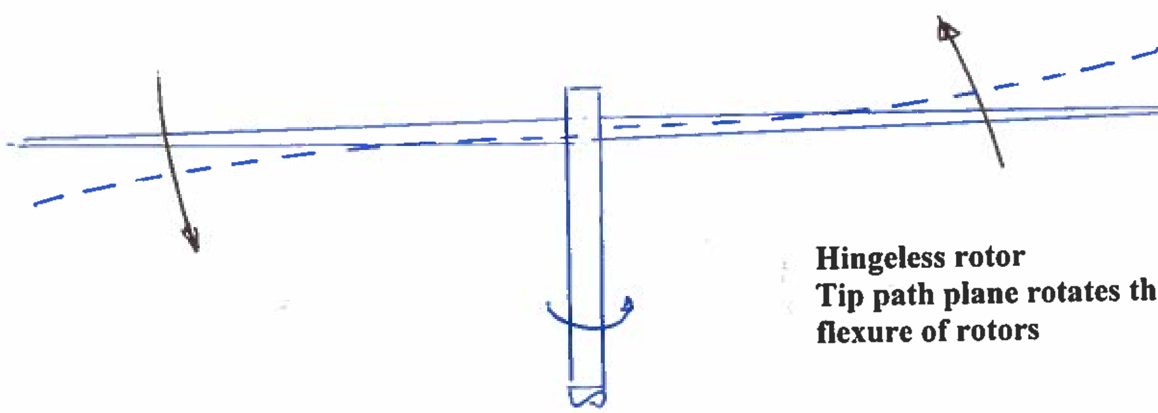
FIGURE 3.27 Schematic of Swashplate Control System

Teetering rotor.
Tip path plane rotates through teeter





Articulated rotor
Tip path plane rotates through flapping hinge



Hingeless rotor
Tip path plane rotates through flexure of rotors

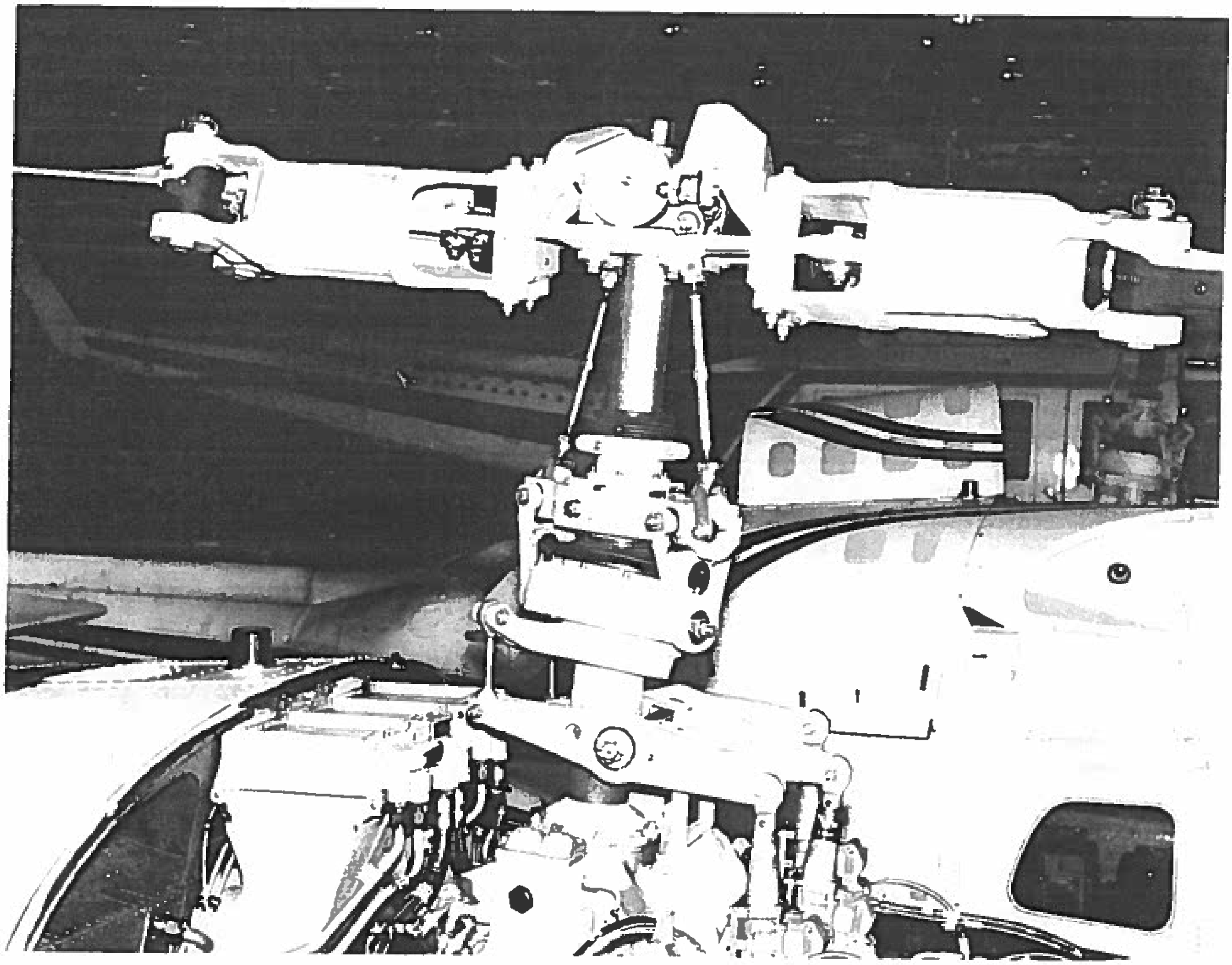


FIGURE 3.20 Teetering Rotor

TABLE IV-1

AH-1G DESCRIPTIVE DATA

MAIN ROTOR

- Blades 2
- Radius 6.706 m (22 ft)
- Chord 0.686 m (2.25 ft)
- Section 9.33% thickness, special symmetrical section
- Hub type Teetering
- Undersling 11.4 cm (4.5 in)
- Twist -10 deg
- Pitch flap coupling (δ_3) Zero
- Shaft tilt Zero
- Design rpm 314 to 324 (power on), 294 to 339 (power off)*
- Hub location FS 200, WL 152.76†
- Blade flapping inertia 1873.44 kg-m^2 (1381.8 slug-ft^2)

TAIL ROTOR

- Blades 2
- Radius 1.295 m (4.25 ft)
- Chord 0.214 m (0.701 ft)
- Twist Zero
- Gear ratio 5.123
- Hub location FS 520.7, WL 118.27, BL -14.85

WING

- Area 2.583 m^2 (27.8 ft^2)
- Aspect ratio 3.91
- Center of pressure location FS 192.0, BL 39.0, WL 62.0
- Incidence 14 deg
- Dihedral 3.5 deg

ELEVATOR (EACH SIDE, EXCLUDING FUSELAGE CARRY-THROUGH)

- Area 0.683 m^2 (7.35 ft^2)
- Aspect ratio 1.49
- Center of pressure location FS 398.5, BL $\pm$ 22.07, WL 56.0
- Incidence Variable

VERTICAL STABILIZER

- Area 1.728 m^2 (18.60 ft^2)
- Aspect ratio 1.56
- Center of pressure location FS 501.0, WL 84.0

---

* From Ref. 10.

† Manufacturer's fuselage reference system.

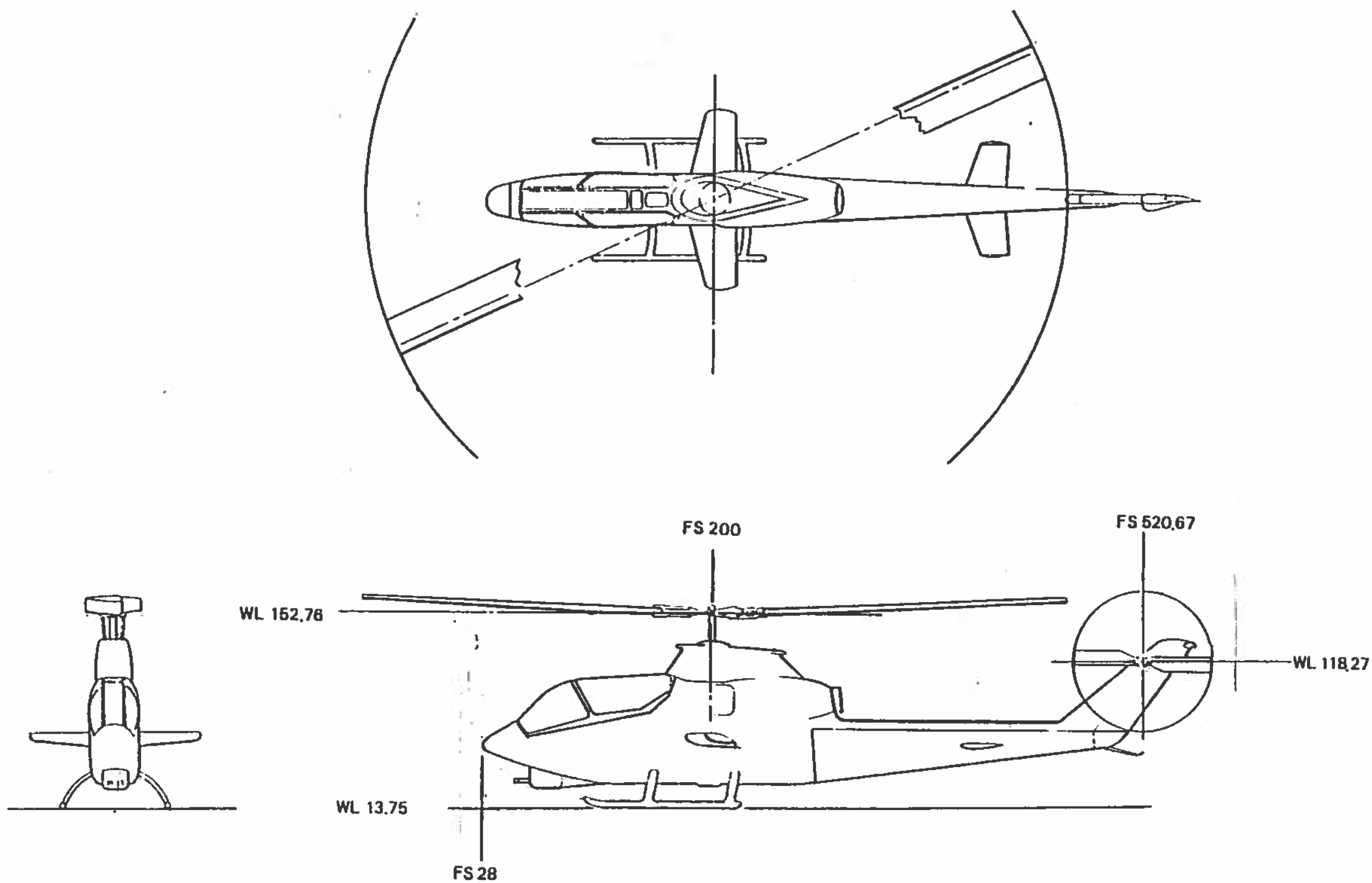


Figure IV-1. AH-1G General Arrangement

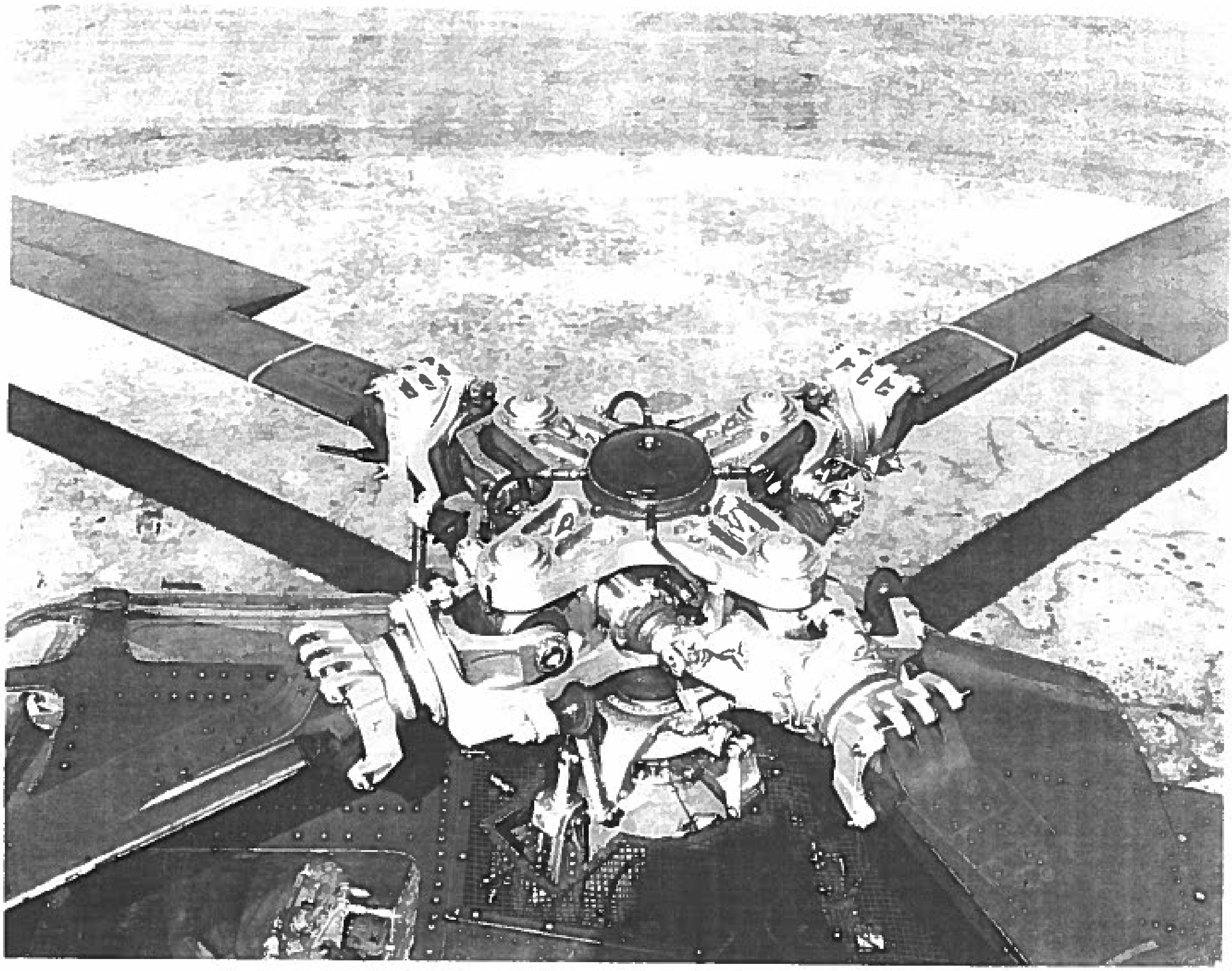


FIGURE 3.19 Fully Articulated Rotor

TABLE VI-1

CH-53D DESCRIPTIVE DATA

MAIN ROTOR

Blades 6

Radius 11.009 m (36.118 ft)

Chord 0.660 m (2.167 ft)

Section NACA 0011 Mod

Hub type Articulated

Twist -4.1 deg*

Pitch flap coupling (δ_3) Zero

Shaft tilt 5 deg forward

Design rpm 185 rpm = 100% N_r, max rpm = 125% N_r †

Hub location FS 336.413, WL 257‡

Blade flapping inertia 5486 kg-m^2 (4046 slug-ft^2)

TAIL ROTOR

Blades 4

Radius 2.44 m (8 ft)

Chord 0.391 m (1.284 ft)

Twist -8 deg*

RPM ratio 4.30

Hub location FS 870.25, WL 269., BL -33.‡

HORIZONTAL STABILIZER

Area 3.71 m^2 (40.0 ft^2)

Aspect ratio 2.59

Center of pressure location FS 846, BL 60.9, WL 290.0

Dihedral 5 deg

Incidence 3 deg

VERTICAL STABILIZER LOCATION

Area 3.252 m^2 (35.0 ft^2)

Aspect ratio 1.70

Center of pressure location FS 812.5, WL 228.9

---

* From Ref. 12.

† From Ref. 13.

‡ Manufacturer's fuselage reference system.

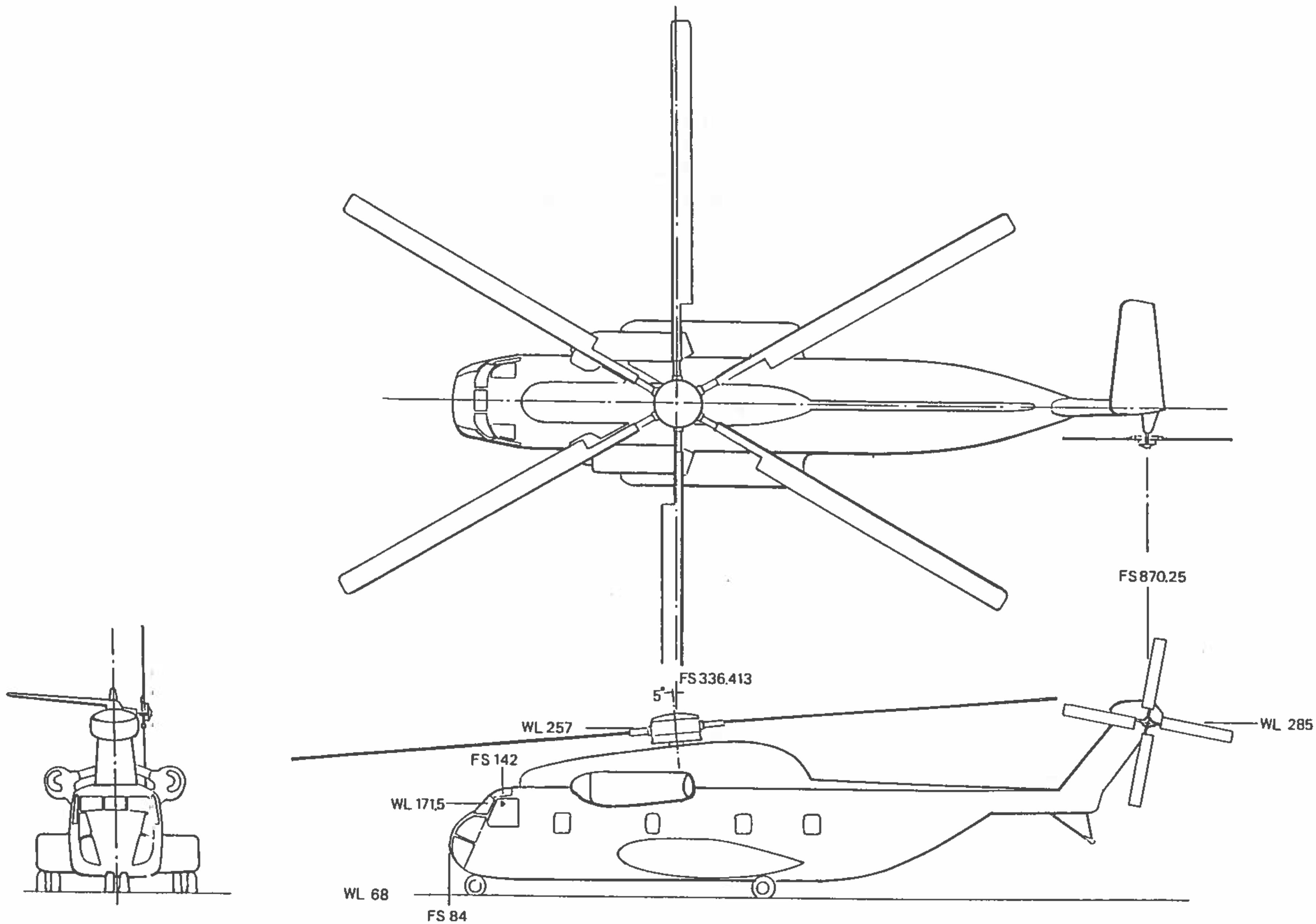


Figure VI-1. CH-53D General Arrangement

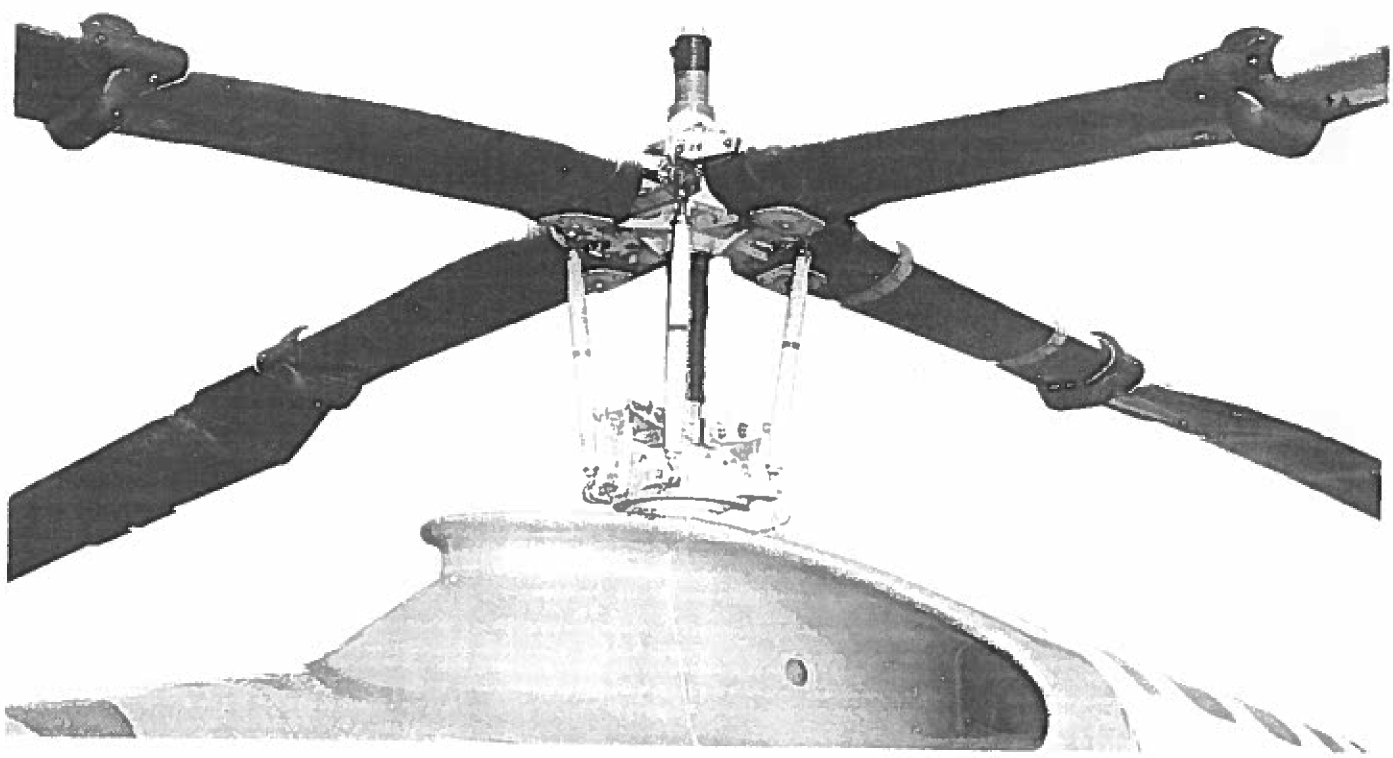


FIGURE 3.21 Hingeless Rotor

TABLE III-1

BO-105C DESCRIPTIVE DATA

MAIN ROTOR

Blades 4

Radius 4.91 m (16.11 ft)

Chord 0.27 m (0.886 ft)

Section NACA 23012 mod

Hub type Hingeless

Twist -8 deg linear

Shaft tilt 3 deg forward

Design rpm 403 to 433 (power on), 361 to 467 (power off)*

Hub location FS 98.44, WL 61.2†

Blade flapping inertia 219.50 kg-m^2 (161.9 slug-ft^2)†

TAIL ROTOR

Blades 2

Radius 0.95 m (3.12 ft)

Chord 0.18 m (0.59 ft)

Twist Zero

Gear ratio 5.24

Hub location FS 335, WL 68.7, BL -12.5†

HORIZONTAL STABILIZER

Area 0.809 m^2 (8.71 ft^2)†

Aspect ratio 8.09†

Quarter chord location FS 277.5, WL 25.84†

Dihedral Zero†

Incidence Zero†

---

* 424 rpm for tabulated data

† From Ref. 9.

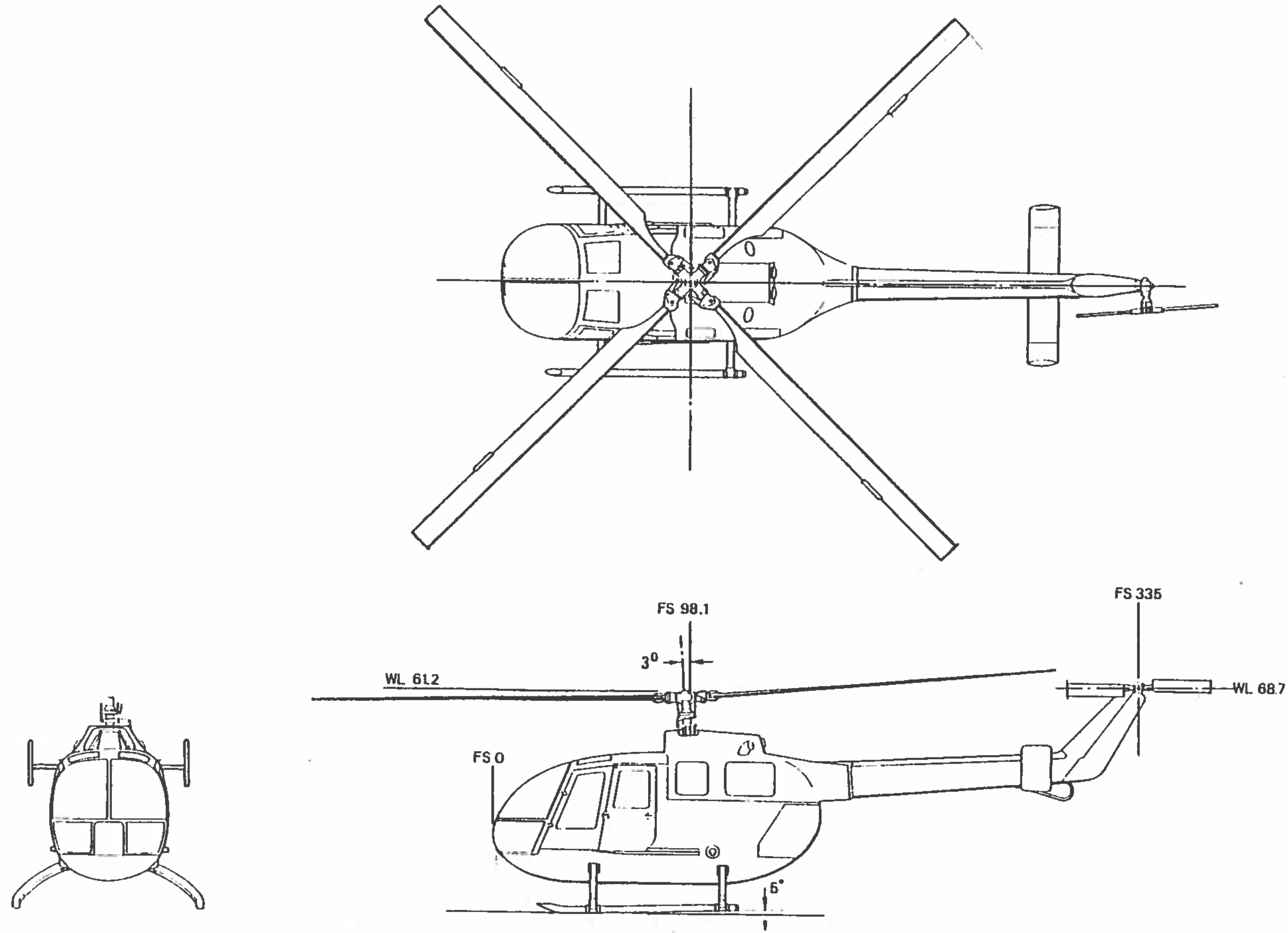


Figure III-1. BO-105C General Arrangement

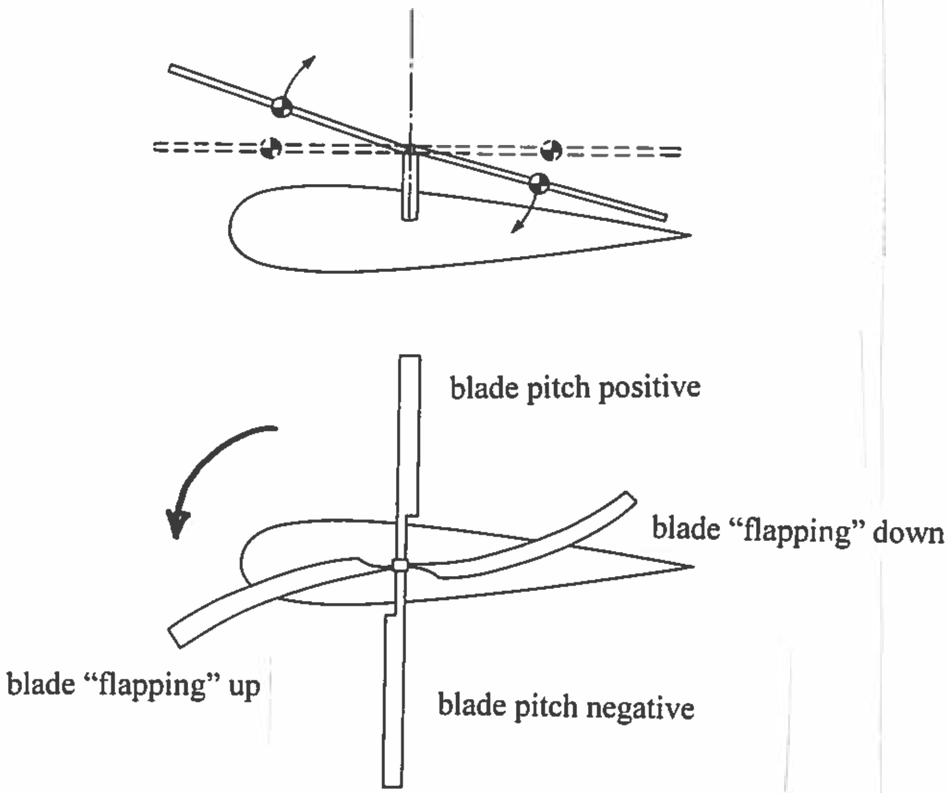


FIGURE 3.18 Inplane Blade Bending Due to Flapping Motion

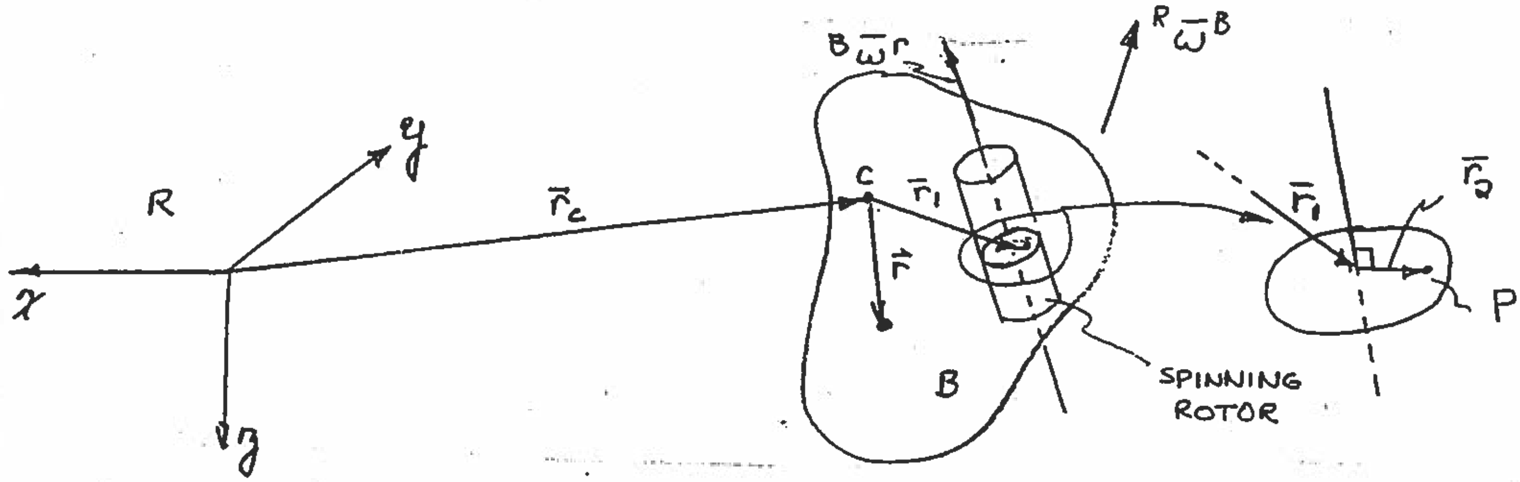
Reason for "lead-lag" rotor hinges

Dept. of Mechanical and Aeronautical Engineering

MAE - 275

Effect of Spinning Rotors on Calculation of Angular Momentum

Consider a rigid body B with a single spinning rotor "r". The angular velocity of the rigid portion of B as measured in the inertial frame R is ${}^R\bar{\omega}^B$



$\bar{r}$ = vector from center of mass of body + rotor to any point in body + rotor

Now, the angular momentum of any particle in the body is defined as

$$\delta\bar{h} = (\bar{r}\times\bar{v})\delta m = \bar{r}\times\left(\bar{v}_c + \frac{{}^R d\bar{r}}{dt}\right)\delta m$$

Summing over the entire body, including the spinning rotor, gives

$$\bar{h} = \Sigma\delta\bar{h} = \sum_{rigid}\delta\bar{h} + \sum_{rotor}\delta\bar{h}$$

$$= \sum_{rigid}\bar{r}\times\left(\bar{v}_c + \frac{{}^R d\bar{r}}{dt}\right)\delta m + \sum_{rotor}\bar{r}\times\left(\bar{v}_c + \frac{{}^R d\bar{r}}{dt}\right)\delta m$$

$$= \qquad '' \qquad + \sum_{rotor}(\bar{r}_1+\bar{r}_2)\times\left(\bar{v}_c + \frac{{}^R d}{dt}(\bar{r}_1+\bar{r}_2)\right)\delta m$$

The first summation above is take over the body B but not including the rotor. The second summation is taken over the rotor only. A point "p" is located in the rotor but no necessarily on the periphery. Vector $\bar{r}_2$ is drawn from (and perpendicular to) the axis of rotation to point "p". The vector $\bar{r}_1$ is drawn from the center of mass of the rigid body + rotor to the point where $\bar{r}_2$ intersects the axis of rotation. The equation above can be rewritten

$$\bar{h} = \sum_{rigid} \bar{r} \times \left(\bar{v}_c + \frac{^R d\bar{r}}{dt}\right)\delta m + \sum_{rotor} (\bar{r}_1 + \bar{r}_2) \times \left(\bar{v}_c + \underbrace{^R\bar{\omega}^B \times \bar{r}_1}_{\frac{^R d\bar{r}_1}{dt}} + \underbrace{\frac{^B d\bar{r}_2}{dt} + {^R\bar{\omega}^B} \times \bar{r}_2}_{\frac{^R d\bar{r}_2}{dt}}\right)\delta m$$

or

$$\bar{h} = \sum_{rigid} \bar{r} \times \left(\bar{v}_c + \frac{^R d\bar{r}}{dt}\right)\delta m + \sum_{rotor} (\bar{r}_1 + \bar{r}_2) \times \left(\bar{v}_c + {^R\bar{\omega}^B} \times (\bar{r}_1 + \bar{r}_2)\right)\delta m +$$

$$\sum_{rotor} (\bar{r}_1 + \bar{r}_2) \times \Big(\underbrace{^B\bar{\omega}^r \times \bar{r}_2}_{\frac{^B d\bar{r}_2}{dt}}\Big)\delta m$$

Now, only the last summation in the equation above involves rotor motion relative to the body ($^B\bar{\omega}^r$). In fact, one can see that the first two summations in the equation above define the angular momentum of the entire body + rotor as if the rotor were not moving. We can call this $\bar{h}_{rigid}$ where "rigid" refers to the $\bar{h}$ that was calculated assuming no spinning rotors. The last summation can be called $\bar{h}'$ which includes the effect of the rotor spinning relative to the body B. Thus

$$\bar{h} = \bar{h}_{rigid} + \bar{h}' = \bar{h}_{rigid} + \sum_{rotor} (\bar{r}_1 + \bar{r}_2) \times ({^B\bar{\omega}^r} \times \bar{r}_2)\delta m$$

Finally, it can be shown that the summation in the last equation can be further simplified by showing that

$$\sum_{rotor} (\bar{r}_1 + \bar{r}_2) \times ({^B\bar{\omega}^r} \times \bar{r}_2)\delta m = \sum_{rotor} \bar{r}_2 \times ({^B\bar{\omega}^r} \times \bar{r}_2)\delta m$$

This leads to the final equation

$$\bar{h} = \bar{h}_{rigid} + \bar{h}' = \bar{h}_{rigid} + \sum_{rotor} \bar{r}_2 \times ({^B\bar{\omega}^r} \times \bar{r}_2)\delta m$$

Note that $\bar{h}'$ can be easily determined once we know the geometry of the rotor and how fast it is spinning relative to the body B ($^B\bar{\omega}^r$).

$$\bar{h}' = \sum_{rotor} (\bar{r}_2 \cdot \bar{r}_2)\, {}^{B}\bar{\omega}^{r}\, \delta m - \sum_{rotor} \cancelto{0}{(\bar{r}_2 \cdot {}^{B}\bar{\omega}^{r})}\, \bar{r}_2\, \delta m$$

$$= {}^{B}\bar{\omega}^{r} \sum_{rotor} (\bar{r}_2 \cdot \bar{r}_2)\, \delta m$$

$$= {}^{B}\bar{\omega}^{r}\, I_r = ({}^{B}\omega^{r}_{x}\, \bar{i} + {}^{B}\omega^{r}_{y}\, \bar{j} + {}^{B}\omega^{r}_{z}\, \hat{k})\, I_r$$

where I_r is the moment of inertia of the rotor about it's axis of rotation and terms like ${}^{B}\omega^{r}_{x}$ represent the x, y, z, body-fixed axis components of ${}^{B}\bar{\omega}^{r}$

Now it's the time rate of change of $\bar{h}'$ that must be taken into account in our moment equations, i.e., recall that the external moment applied to a body is equal to the time rate of change of the angular momentum. Here

$$\frac{{}^{R}d\bar{h}'}{dt} = \cancelto{0}{\frac{{}^{B}d\bar{h}'}{dt}} + {}^{R}\bar{\omega}^{B} \times \bar{h}' = {}^{R}\bar{\omega}^{B} \times ({}^{B}\omega^{r}_{x}\, \bar{i} + {}^{B}\omega^{r}_{y}\, \bar{j} + {}^{B}\omega^{r}_{z}\, \hat{k})\, I_r$$

$$= (p\,\bar{i} + q\,\bar{j} + r\,\hat{k}) \times ({}^{B}\omega^{r}_{x}\, \bar{i} + {}^{B}\omega^{r}_{y}\, \bar{j} + {}^{B}\omega^{r}_{z}\, \hat{k})\, I_r$$

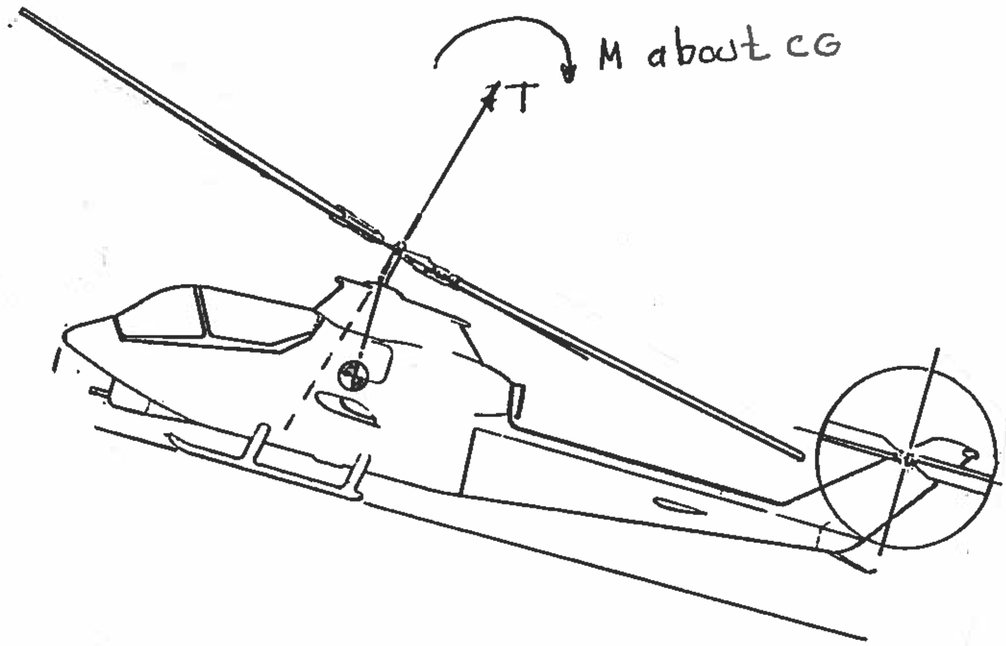
Finally, the following additional terms are added to the right hand sides of our linearized, stability axis moment equations:

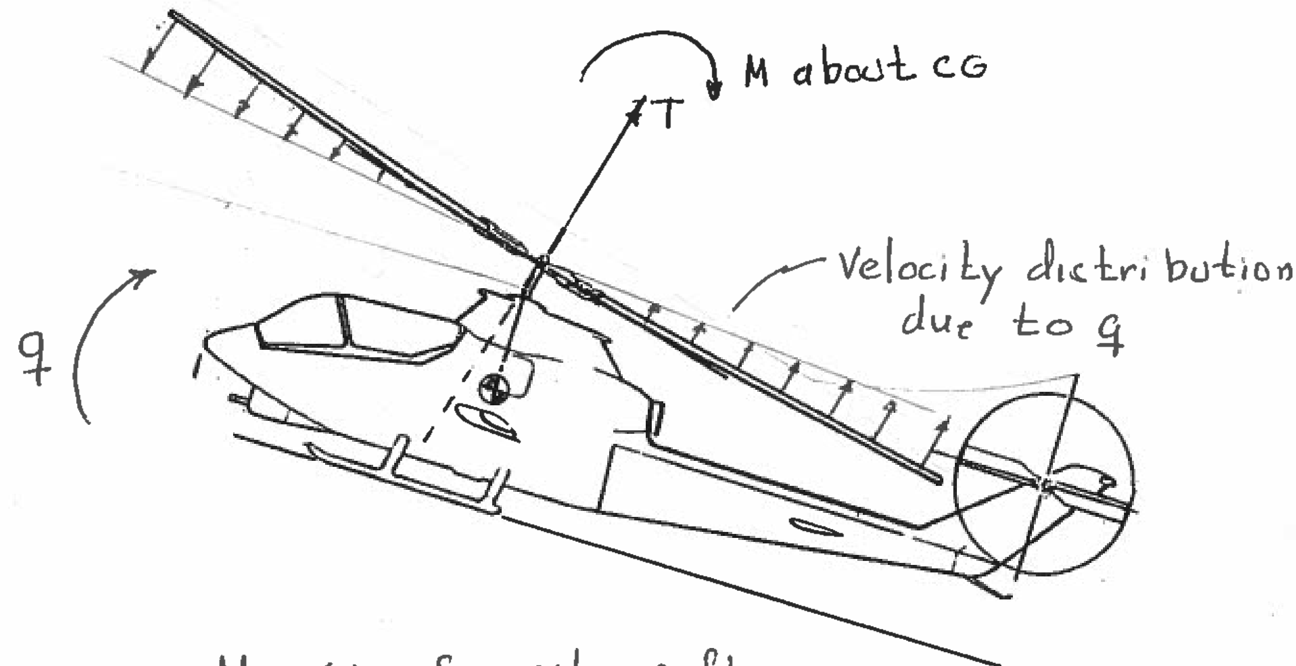
$$\dot{q} = [M_u + aM_{\dot{w}}Z_u](u-u_g) + [M_w + aM_{\dot{w}}Z_w](w-w_g) + [M_q + aM_{\dot{w}}(U_0+Z_q)](q-q_g) + [-aM_{\dot{w}}g\sin\theta_0]\theta + \sum_i [aM_{\dot{w}}Z_{\delta_i} + M_{\delta_i}]\delta_i - \left[r\,({}^{B}\omega^{r}_{x}) - p\,({}^{B}\omega^{r}_{z})\right](I_r/I_y)$$

$$\dot{p} = [L'_v + bL'_{\dot{v}}Y_v](v-v_g) + [L'_p + bL'_{\dot{v}}Y_p](p-p_g) + [L'_r + bL'_{\dot{v}}Y_r](r-r_g) - [L'_r + bL'_{\dot{v}}U_0]r + [bL'_{\dot{v}}g\cos\theta_0]\phi + \sum_j [L'_{\delta_j} + bL'_{\dot{v}}Y_{\delta_j}]\delta_j - \left[q\,({}^{B}\omega^{r}_{z}) - r\,({}^{B}\omega^{r}_{y})\right](I_r/I_x)$$

$$\dot{r} = [N'_v + bN'_{\dot{v}}Y_v](v-v_g) + [N'_p + bN'_{\dot{v}}Y_p](p-p_g) + +[N'_r + bN'_{\dot{v}}Y_r](r-r_g) - [N'_r + bN'_{\dot{v}}U_0]r + [bN'_{\dot{v}}g\cos\theta_0]\phi + \sum_j [N'_{\delta_j} + bN'_{\dot{v}}Y_{\delta_j}]\delta_j - \left[p\,({}^{B}\omega^{r}_{y}) - q\,({}^{B}\omega^{r}_{x})\right](I_r/I_z)$$

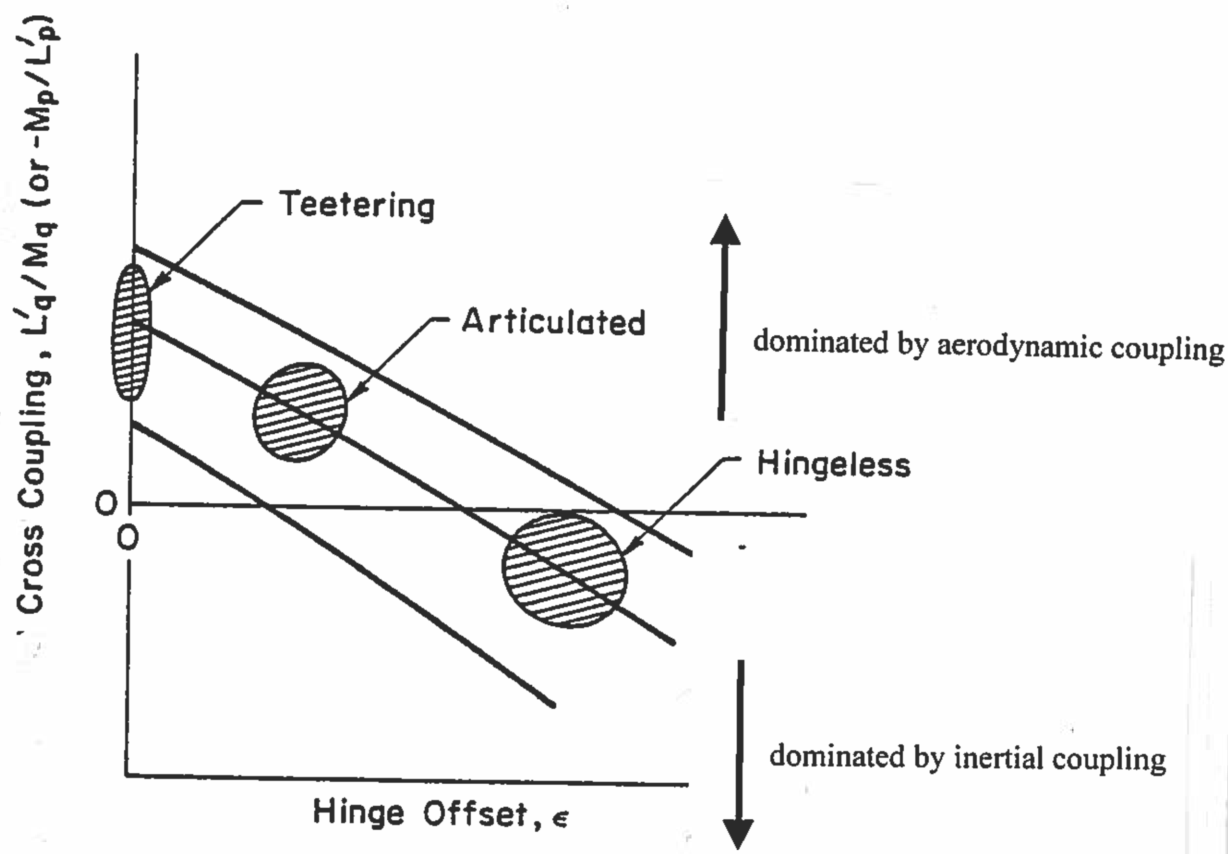
Note that the longitudinal and lateral-directional equations are no longer decoupled!


M about CG
T



$M_q < 0$ for rotorcraft

However due to velocity distribution on rotor due to q there is a negative aerodynamic L'_q



Cross Coupling, L'_q/M_q (or $-M_p/L'_p$)
Teetering
Articulated
Hingeless
O
O
Hinge Offset, ϵ
dominated by aerodynamic coupling
dominated by inertial coupling

TABLE I-4

US UNITS FOR NUMERICAL VALUES OF STABILITY AND CONTROL DERIVATIVES

CASE 4 0 KT LEVEL FLIGHT AT SEA LEVEL 2550 LB MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
Φ_o(deg)	Θ_o(deg)	Ψ_o(deg)	α_o(deg)	β_o(deg)	γ_o(deg)	θ_{MR}(deg)	B_{1_s}(deg)	A_{1_s}(deg)	θ_{TR}(deg)

XDOT	ZDOT	UO	VO	WO	VTO
$\dot{x}_o$(ft/sec)	$\dot{z}_o$(ft/sec)	U_o(ft/sec)	V_o(ft/sec)	W_o(ft/sec)	V_{T_o}(ft/sec)

	U	W	Q	V	P	R	DC	DB	DA	DP
X	$X_u(1/sec)$	$X_w(1/sec)$	$X_q(\frac{ft}{sec\text{-}rad})$	$X_v(1/sec)$	$X_p(\frac{ft}{sec\text{-}rad})$	$X_r(\frac{ft}{sec\text{-}rad})$	$X_{\delta_c}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_p}(\frac{ft}{sec^2\text{-}in})$
Z	$Z_u(1/sec)$	$Z_w(1/sec)$	$Z_q(\frac{ft}{sec\text{-}rad})$	$Z_v(1/sec)$	$Z_p(\frac{ft}{sec\text{-}rad})$	$Z_r(\frac{ft}{sec\text{-}rad})$	$Z_{\delta_c}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_p}(\frac{ft}{sec^2\text{-}in})$
M	$M_u(\frac{rad}{ft\text{-}sec})$	$M_w(\frac{rad}{ft\text{-}sec})$	$M_q(1/sec)$	$M_v(\frac{rad}{ft\text{-}sec})$	$M_p(1/sec)$	$M_r(1/sec)$	$M_{\delta_c}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_p}(\frac{rad}{sec^2\text{-}in})$
Y	$Y_u(1/sec)$	$Y_w(1/sec)$	$Y_q(\frac{ft}{sec\text{-}rad})$	$Y_v(1/sec)$	$Y_p(\frac{ft}{sec\text{-}rad})$	$Y_r(\frac{ft}{sec\text{-}rad})$	$Y_{\delta_c}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_p}(\frac{ft}{sec^2\text{-}in})$
L'	$L'_u(\frac{rad}{ft\text{-}sec})$	$L'_w(1/sec)$	$L'_q(1/sec)$	$L'_v(\frac{rad}{ft\text{-}sec})$	$L'_p(1/sec)$	$L'_r(1/sec)$	$L'_{\delta_c}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_p}(\frac{rad}{sec^2\text{-}in})$
N'	$N'_u(\frac{rad}{ft\text{-}sec})$	$N'_w(1/sec)$	$N'_q(1/sec)$	$N'_v(\frac{rad}{ft\text{-}sec})$	$N'_p(1/sec)$	$N'_r(1/sec)$	$N'_{\delta_c}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_p}(\frac{rad}{sec^2\text{-}in})$

AH-1G STABILITY AND CONTROL DERIVATIVES -- US UNITS

(BODY-FIXED FRL AXIS SYSTEM)

CASE 59   1 KT   LEVEL FLIGHT AT SEA LEVEL   8000 LB   MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-1.15	-0.73	0.00	-0.73	0.01	0.00	14.83	-0.76	-1.80	8.40

XDOT	ZDOT	UO	VO	WO	VTO
1.69	0.00	1.69	0.00	-0.02	1.69

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0165	-0.0155	0.3655	-0.0285	-1.8499	-0.1223	-0.2560	1.2425	-0.0130	-0.1453
Z	-0.1208	-0.3726	-0.0110	-0.0932	-0.1345	2.0756	-12.7606	0.1799	-0.0035	-0.0305
M	0.0005	-0.0033	$\boxed{-0.2345}$	0.0007	0.2305	0.0175	0.0076	-0.1462	0.0035	0.0322
Y	0.0173	-0.0051	-1.5941	-0.0552	-1.1026	0.7182	-0.5515	-0.1182	0.8095	1.2837
L'	0.0085	-0.0060	$\boxed{-1.0559}$	-0.0087	-0.7662	-0.0403	-0.0925	-0.0686	0.4709	0.1816
N'	-0.0012	-0.0047	-0.1161	0.0158	-0.3945	-0.5366	0.5795	-0.0095	0.0319	-0.8120

$L'_q / M_q = +4.5$

CH-53D STABILITY AND CONTROL DERIVATIVES -- US UNITS

(BODY-FIXED FRL AXIS SYSTEM)

CASE 182   0 KT   LEVEL FLIGHT   2000 FT   35000 LB   MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-3.14	5.64	-0.31	5.65	0.00	0.00	14.02	0.22	-0.89	19.56

XDOT	ZDOT	UO	VO	WO	VTO
0.00	0.00	0.00	0.00	0.00	0.00

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0917	0.0240	0.8700	0.0029	-2.8200	-0.3780	0.6293	1.5194	0.0974	-0.0076
Z	0.0168	-0.2980	0.2890	-0.1660	-0.3030	3.5900	-6.3839	0.1356	0.0031	-0.0009
M	0.0060	-0.0018	$\boxed{-0.4990}$	0.0020	0.1970	0.0063	0.0018	-0.1791	-0.0077	0.0017
Y	0.0030	-0.0025	-2.7500	-0.1450	-1.9200	1.1500	0.0977	-0.1810	0.9661	1.2210
L'	0.0027	-0.0003	$\boxed{-0.9370}$	-0.0310	-1.9000	0.2100	-0.0379	-0.0785	0.5154	0.2296
N'	-0.0008	0.0003	0.0870	0.0027	-0.1000	-0.3400	0.0831	-0.0047	0.0325	-0.3517

$L'_q / M_q = 1.87$

BO-105C STABILITY AND CONTROL DERIVATIVES -- US UNITS

(BODY-FIXED FRL AXIS SYSTEM)

CASE 29 0 KT LEVEL FLIGHT AT SEA LEVEL 4620 LB MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-2.97	2.64	0.00	2.63	-0.14	0.00	14.32	-0.42	-0.33	10.17

XDOT	ZDOT	UO	VO	WO	VTO
0.00	0.00	0.00	0.00	0.00	0.00

	U	W	Q	V	P	R	DC	DR	DA	DP
X	-0.0166	0.0124	1.6105	0.0004	-0.7260	-0.1192	0.4467	0.7894	-0.0045	-0.1202
Z	0.0100	-0.3317	0.1300	-0.0010	0.1473	1.8309	-9.8810	0.0429	-0.0160	-0.0131
M	0.0202	-0.0027	$\boxed{-3.3972}$	-0.0040	-0.8400	0.0439	-0.0805	-0.9727	0.1598	0.0577
Y	-0.0012	-0.0054	-0.4834	-0.0320	-1.7454	0.2050	-0.0489	0.0383	0.8014	-1.6381
L'	-0.0111	-0.0121	$\boxed{2.3000}$	-0.0632	-9.2439	-0.2240	-0.1275	0.4590	2.6446	-1.0126
N'	-0.0008	0.0018	-0.1215	0.0099	-0.0759	-0.3270	0.5651	-0.0045	0.0341	1.3931

$L'_q / M_q = -.667$

TABLE IV-4 CONTINUED

AH-1G STABILITY AND CONTROL DERIVATIVES -- US UNITS

(BODY-FIXED FRL AXIS SYSTEM)

CASE 59   1 KT   LEVEL FLIGHT AT SEA LEVEL   8000 LB   MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-1.15	$\boxed{-0.73}$	0.00	-0.73	0.01	0.00	14.83	-0.76	-1.80	8.40

XDOT	ZDOT	UO	VO	WO	VTO
1.69	0.00	1.69	0.00	-0.02	1.69

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0165	-0.0155	0.3655	-0.0285	-1.8499	-0.1223	-0.2560	1.2425	-0.0130	-0.1453
Z	-0.1208	-0.3726	-0.0110	-0.0932	-0.1345	2.0756	-12.7606	0.1799	-0.0035	-0.0305
M	0.0005	-0.0033	-0.2345	0.0007	0.2305	0.0175	0.0076	-0.1462	0.0035	0.0322
Y	0.0173	-0.0051	-1.5941	-0.0552	-1.1026	0.7182	-0.5515	-0.1182	0.8095	1.2837
L'	0.0085	-0.0060	-1.0559	-0.0087	-0.7662	-0.0403	-0.0925	-0.0686	0.4709	0.1816
N'	-0.0012	-0.0047	-0.1161	0.0158	-0.3945	-0.5366	0.5795	-0.0095	0.0319	-0.8120

X_u

Dept. of Mechanical and Aeronautical Engineering

EME – 275

Helicopter Equations of Motion

The equations of motion for a helicopter will follow the format of Ref. 1. Due to the inertial and aerodynamic coupling associated with the main rotor(s) the equations will not be considered as two separate groups. In state-space form the equations are

$$
\begin{Bmatrix} \dot{u} \\ \dot{w} \\ \dot{q} \\ \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\theta} \\ \dot{\phi} \\ \dot{\psi} \end{Bmatrix} =
\begin{bmatrix}
X_u & X_w & X_q & X_v & X_p & X_r & -g & 0 & 0 \\
Z_u & Z_w & Z_q + u_0 & Z_v & Z_p & Z_r & 0 & 0 & 0 \\
M_u & M_w & M_q & M_v & M_p & M_r & 0 & 0 & 0 \\
Y_u & Y_w & Y_q & Y_v & Y_p & Y_r - u_0 & 0 & g & 0 \\
L'_u & L'_w & L'_q & L'_v & L'_p & L'_r & 0 & 0 & 0 \\
N'_u & N'_w & N'_q & N'_v & N'_p & N'_r & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix} \cdot
\begin{Bmatrix} u \\ w \\ q \\ v \\ p \\ r \\ \theta \\ \phi \\ \psi \end{Bmatrix} +
\begin{bmatrix}
X_{\delta_C} & X_{\delta_B} & X_{\delta_A} & X_{\delta_P} \\
Z_{\delta_C} & Z_{\delta_B} & Z_{\delta_A} & Z_{\delta_P} \\
M_{\delta_C} & M_{\delta_B} & M_{\delta_A} & M_{\delta_P} \\
Y_{\delta_C} & Y_{\delta_B} & Y_{\delta_A} & Y_{\delta_P} \\
L'_{\delta_C} & L'_{\delta_B} & L'_{\delta_A} & L'_{\delta_P} \\
N'_{\delta_C} & N'_{\delta_B} & N'_{\delta_A} & N'_{\delta_P} \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \cdot
\begin{Bmatrix} \delta_C \\ \delta_B \\ \delta_A \\ \delta_P \end{Bmatrix}
$$

Here

δ_C = main rotor(s) collective input (changes thrust of main rotor(s))

δ_B = longitudinal cyclic input (similar to elevator)

δ_A = lateral cyclic input (similar to aileron)

δ_P = tail rotor collective input (similar to rudder)

The "primed" derivatives, e.g., L'_q are defined as previous in the fixed-wing equations of motion. Note that gust terms are absent. This is because we cannot include them as in the fixed-wing equations since some terms, like $L'q$, reflect both aerodynamic and inertia effects.

The format of the stability derivatives from Ref. 1 is shown on the following page. Here DC = δ_C, etc. An example of the derivatives for the AH-1G helicopter in near hover (a 1 kt forward velocity) is shown on the third page. Note that the stability derivatives are applied in a body-axis system that is NOT a stability axis system. That is, the x-axis is aligned along the "fuselage reference line", a line determined by the manufacturer. On page 3, note that the trim THETA = -0.73 deg. This means that the x-body axis is = -0.73 deg below the x stability axis. This is small enough to ignore in this case. In other cases, transformation of the stability derivatives between body and stability axes may be necessary.

Reference

[1] Heffley, R. K., Jewel, W. F., Lehman, J. M., and Van Winkle, R. A., "A Compilation and Analysis of Helicopter Handling Qualities Data," NASA CR-3144, March 1979.

TABLE I-4

US UNITS FOR NUMERICAL VALUES OF STABILITY AND CONTROL DERIVATIVES

CASE 4 | 0 KT | LEVEL FLIGHT AT SEA LEVEL | 2550 LB | MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
ϕ_o(deg)	θ_o(deg)	ψ_o(deg)	α_o(deg)	β_o(deg)	γ_o(deg)	θ_{MR}(deg)	B_{1_S}(deg)	A_{1_S}(deg)	θ_{TR}(deg)

XDOT	ZDOT	UO	VO	WO	VTO
$\dot{x}_o$(ft/sec)	$\dot{z}_o$(ft/sec)	U_o(ft/sec)	V_o(ft/sec)	W_o(ft/sec)	V_{T_o}(ft/sec)

	U	W	Q	V	P	R	DC	DB	DA	DP
X	$X_u(1/\text{sec})$	$X_w(1/\text{sec})$	$X_q(\frac{\text{ft}}{\text{sec-rad}})$	$X_v(1/\text{sec})$	$X_p(\frac{\text{ft}}{\text{sec-rad}})$	$X_r(\frac{\text{ft}}{\text{sec-rad}})$	$X_{\delta_c}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$X_{\delta_B}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$X_{\delta_A}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$X_{\delta_p}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$
Z	$Z_u(1/\text{sec})$	$Z_w(1/\text{sec})$	$Z_q(\frac{\text{ft}}{\text{sec-rad}})$	$Z_v(1/\text{sec})$	$Z_p(\frac{\text{ft}}{\text{sec-rad}})$	$Z_r(\frac{\text{ft}}{\text{sec-rad}})$	$Z_{\delta_c}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Z_{\delta_B}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Z_{\delta_A}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Z_{\delta_p}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$
M	$M_u(\frac{\text{rad}}{\text{ft-sec}})$	$M_w(\frac{\text{rad}}{\text{ft-sec}})$	$M_q(1/\text{sec})$	$M_v(\frac{\text{rad}}{\text{ft-sec}})$	$M_p(1/\text{sec})$	$M_r(1/\text{sec})$	$M_{\delta_c}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$M_{\delta_B}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$M_{\delta_A}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$M_{\delta_p}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$
Y	$Y_u(1/\text{sec})$	$Y_w(1/\text{sec})$	$Y_q(\frac{\text{ft}}{\text{sec-rad}})$	$Y_v(1/\text{sec})$	$Y_p(\frac{\text{ft}}{\text{sec-rad}})$	$Y_r(\frac{\text{ft}}{\text{sec-rad}})$	$Y_{\delta_c}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Y_{\delta_B}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Y_{\delta_A}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$	$Y_{\delta_p}(\frac{\text{ft}}{\text{sec}^2\text{-in}})$
L'	$L'_u(\frac{\text{rad}}{\text{ft-sec}})$	$L'_w(1/\text{sec})$	$L'_q(1/\text{sec})$	$L'_v(\frac{\text{rad}}{\text{ft-sec}})$	$L'_p(1/\text{sec})$	$L'_r(1/\text{sec})$	$L'_{\delta_c}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$L'_{\delta_B}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$L'_{\delta_A}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$L'_{\delta_p}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$
N'	$N'_u(\frac{\text{rad}}{\text{ft-sec}})$	$N'_w(1/\text{sec})$	$N'_q(1/\text{sec})$	$N'_v(\frac{\text{rad}}{\text{ft-sec}})$	$N'_p(1/\text{sec})$	$N'_r(1/\text{sec})$	$N'_{\delta_c}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$N'_{\delta_B}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$N'_{\delta_A}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$	$N'_{\delta_p}(\frac{\text{rad}}{\text{sec}^2\text{-in}})$

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STI Technical Report No. 1087-2

A Compilation and Analysis of Helicopter Handling Qualities Data

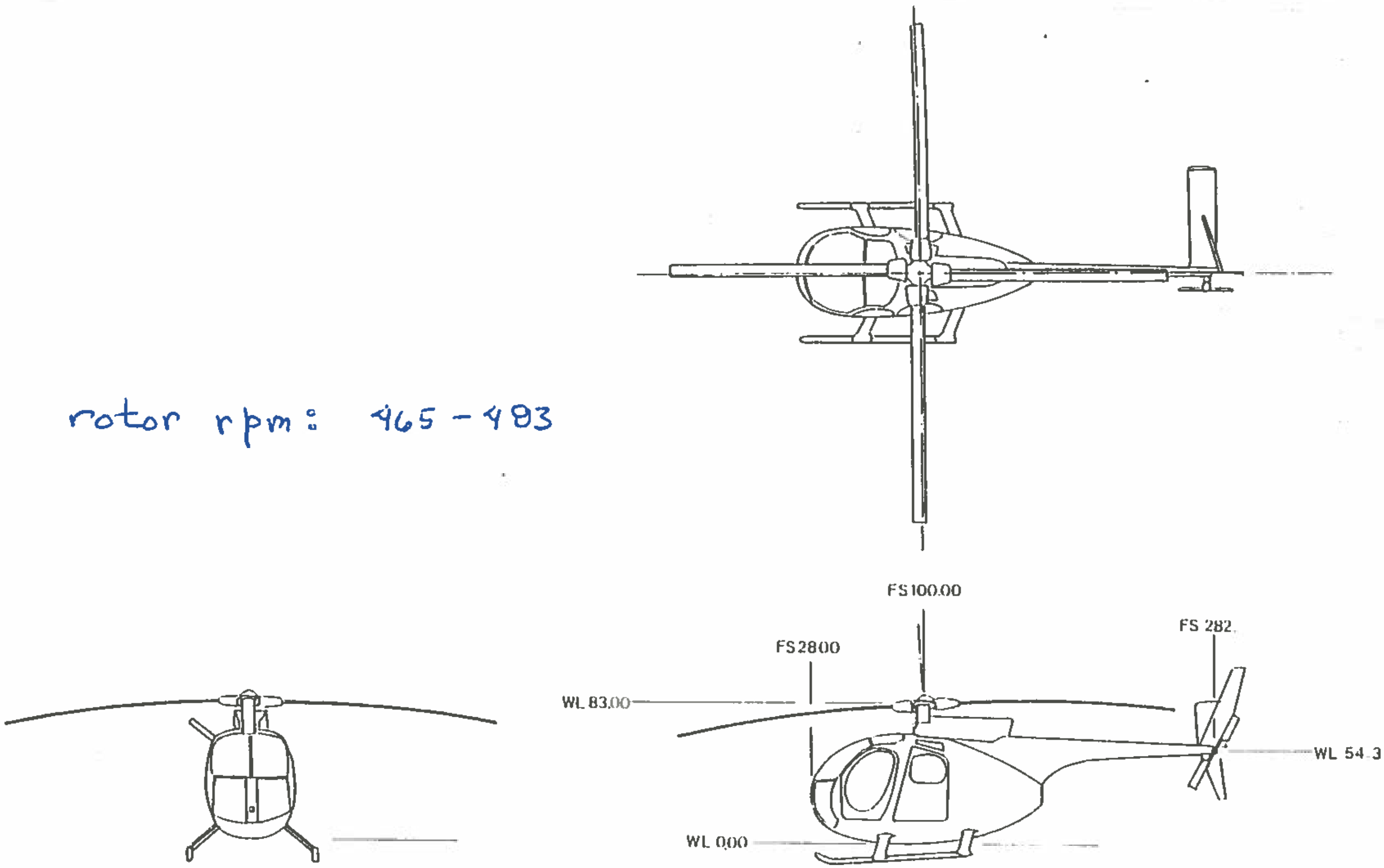
Volume One: Data Compilation
Volume Two: Data Analysis

Robert K. Heffley

March 1979

Contract NAS2-9344
National Aeronautics and Space Administration
Ames Research Center
Moffet Field, CA 94035

rotor rpm: 465 - 483



OH-6A

Figure II-1. General Arrangement

rotor rpm: 403-433

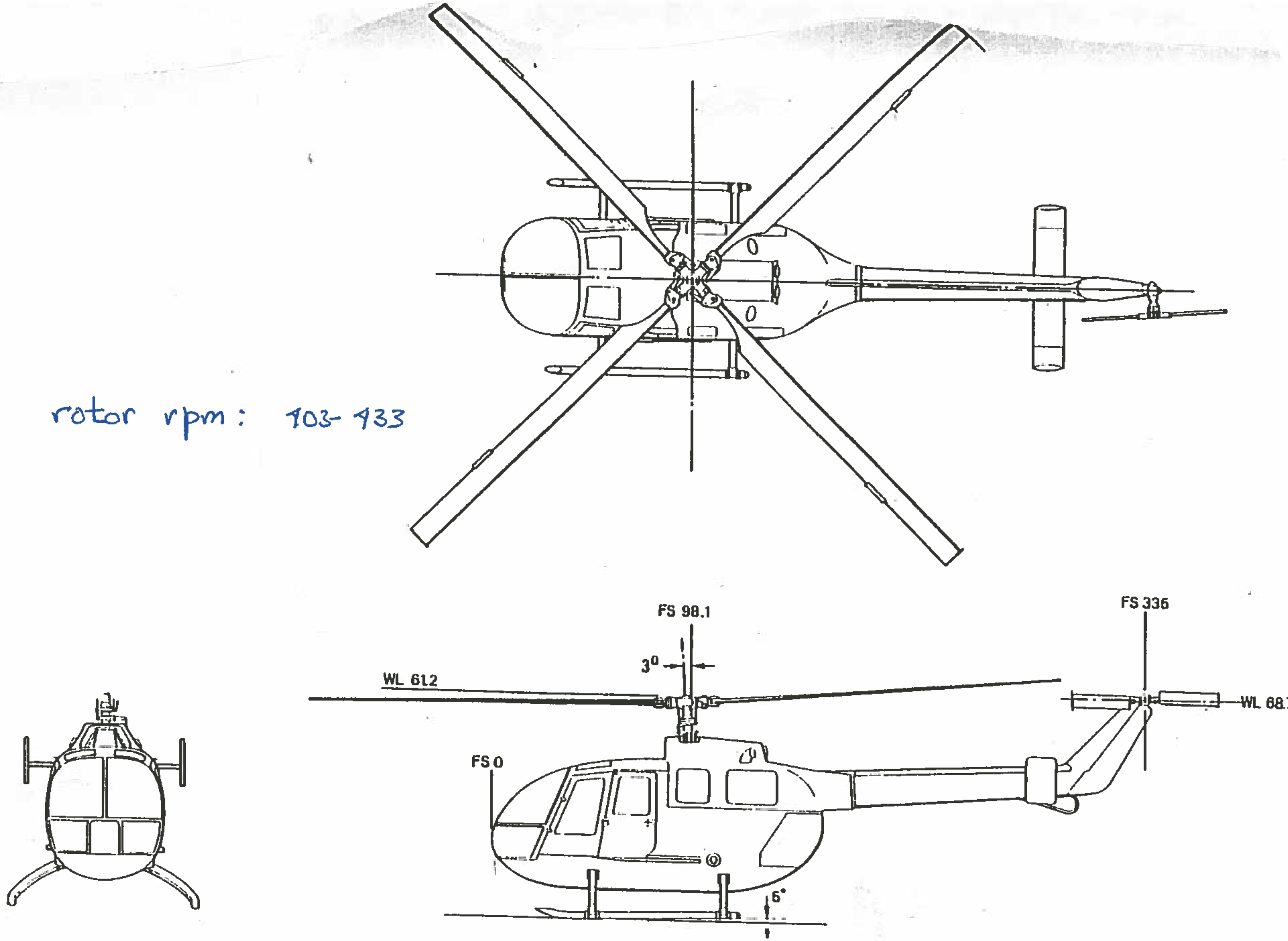


Figure III-1. BO-105C General Arrangement rotor rpm: 314-324

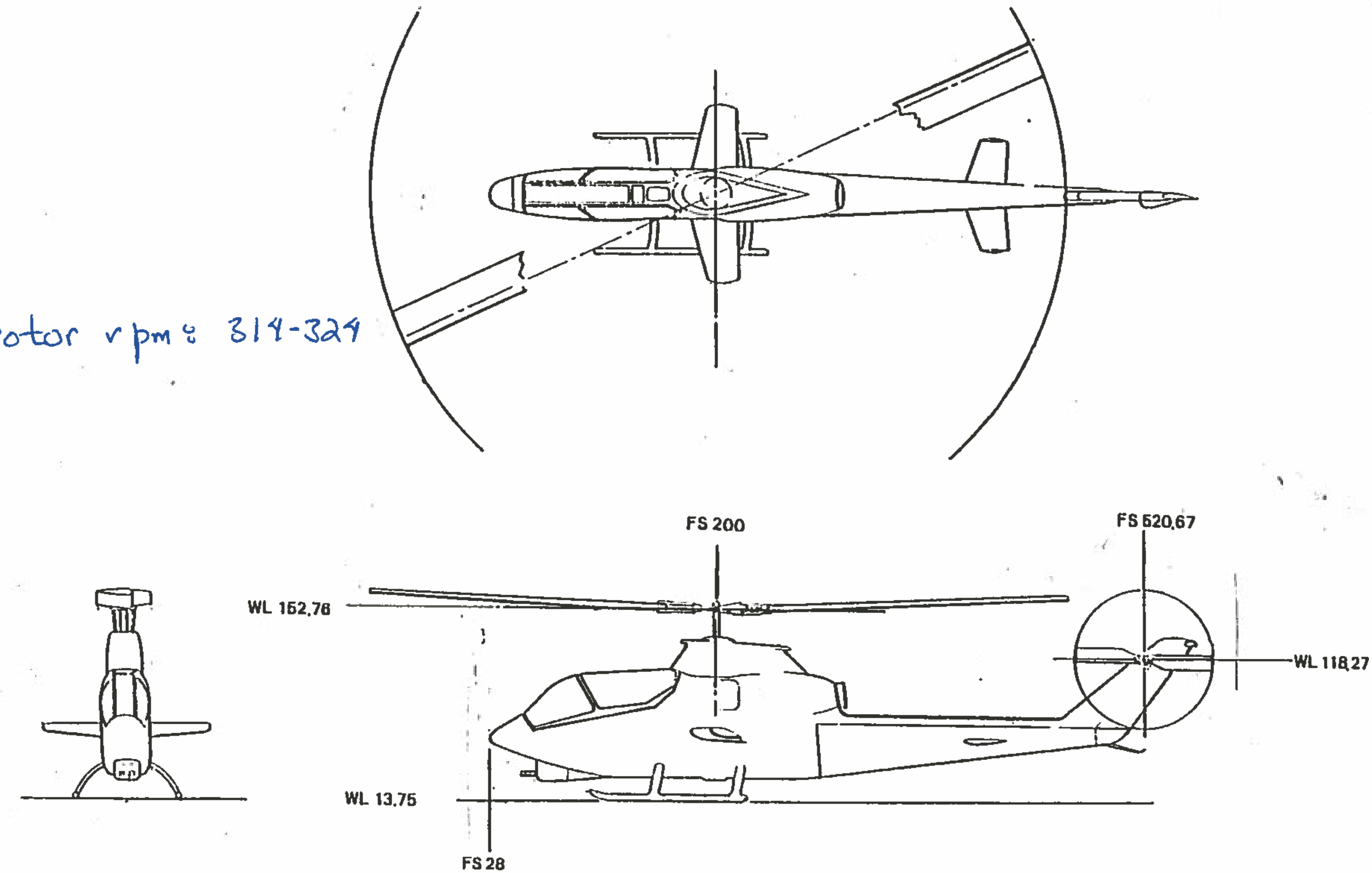
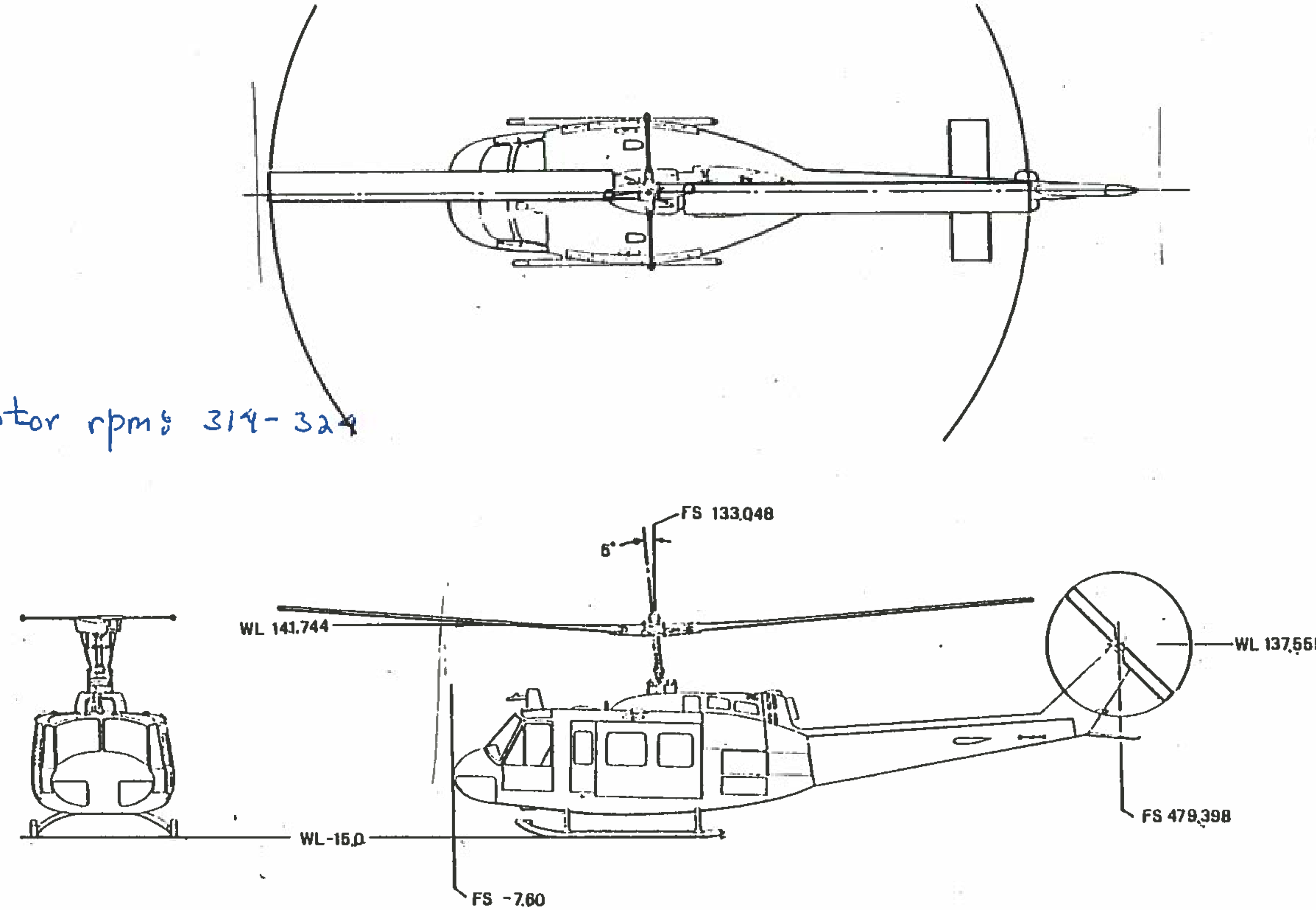


Figure IV-1. AH-1G General Arrangement



rotor rpm: 314-324

Figure V-1. UH-1H General Arrangement

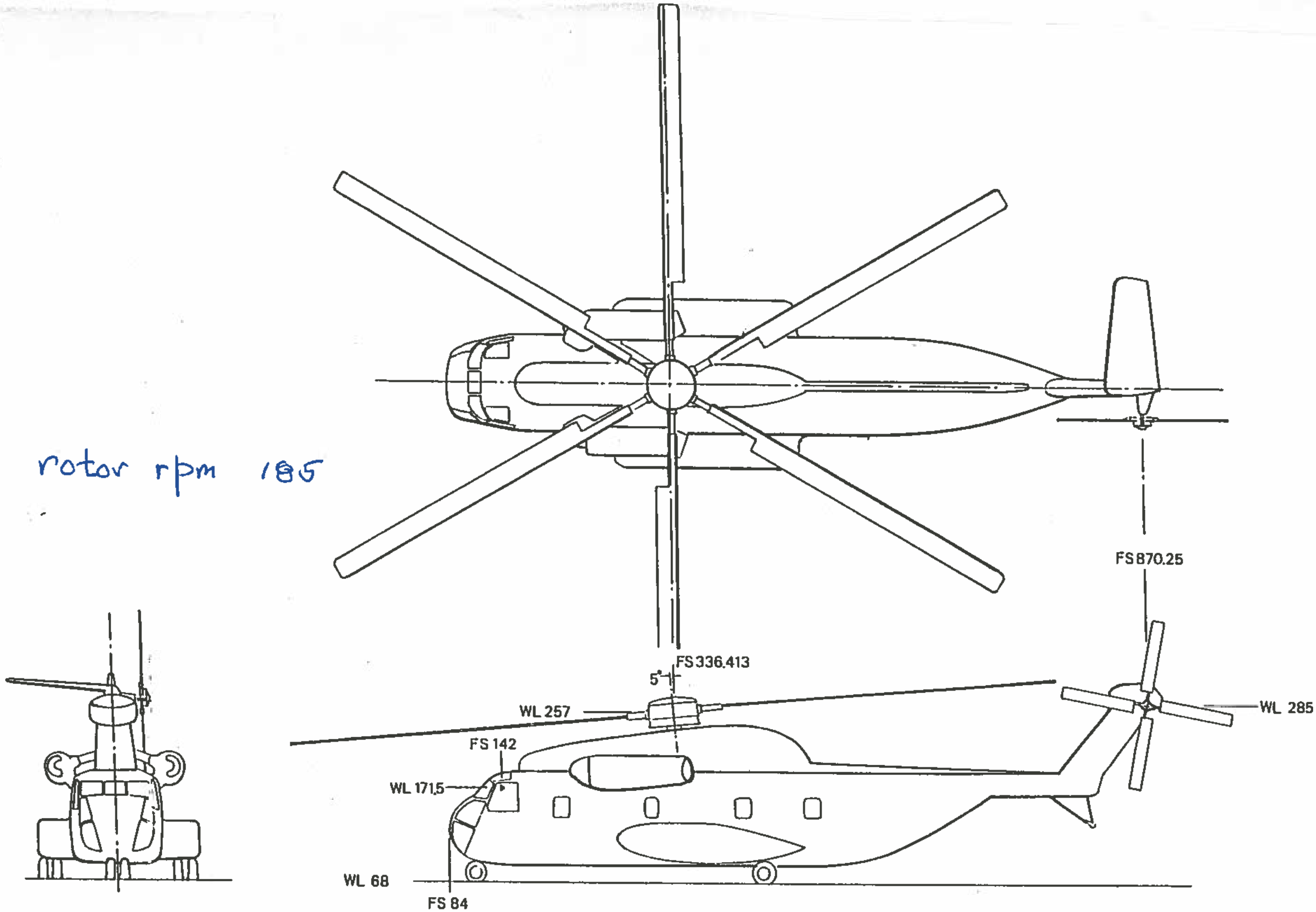


Figure VI-1. CH-53D General Arrangement

rotor rpm 185

TABLE I-4

US UNITS FOR NUMERICAL VALUES OF STABILITY AND CONTROL DERIVATIVES

CASE 4 | 0 KT | LEVEL FLIGHT AT SEA LEVEL | 2550 LB | MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
ϕ_o(deg)	θ_o(deg)	ψ_o(deg)	α_o(deg)	β_o(deg)	γ_o(deg)	θ_{MR}(deg)	B_{1_s}(deg)	A_{1_s}(deg)	θ_{TR}(deg)

XDOT	ZDOT	UO	VO	WO	VTO
$\dot{x}_o$(ft/sec)	$\dot{z}_o$(ft/sec)	U_o(ft/sec)	V_o(ft/sec)	W_o(ft/sec)	V_{T_o}(ft/sec)

	U	W	Q	V	P	R	DC	DB	DA	DP
X	$X_u(1/sec)$	$X_w(1/sec)$	$X_q(\frac{ft}{sec\text{-}rad})$	$X_v(1/sec)$	$X_p(\frac{ft}{sec\text{-}rad})$	$X_r(\frac{ft}{sec\text{-}rad})$	$X_{\delta_C}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$X_{\delta_P}(\frac{ft}{sec^2\text{-}in})$
Z	$Z_u(1/sec)$	$Z_w(1/sec)$	$Z_q(\frac{ft}{sec\text{-}rad})$	$Z_v(1/sec)$	$Z_p(\frac{ft}{sec\text{-}rad})$	$Z_r(\frac{ft}{sec\text{-}rad})$	$Z_{\delta_C}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$Z_{\delta_P}(\frac{ft}{sec^2\text{-}in})$
M	$M_u(\frac{rad}{ft\text{-}sec})$	$M_w(\frac{rad}{ft\text{-}sec})$	$M_q(1/sec)$	$M_v(\frac{rad}{ft\text{-}sec})$	$M_p(1/sec)$	$M_r(1/sec)$	$M_{\delta_C}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$M_{\delta_P}(\frac{rad}{sec^2\text{-}in})$
Y	$Y_u(1/sec)$	$Y_w(1/sec)$	$Y_q(\frac{ft}{sec\text{-}rad})$	$Y_v(1/sec)$	$Y_p(\frac{ft}{sec\text{-}rad})$	$Y_r(\frac{ft}{sec\text{-}rad})$	$Y_{\delta_C}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_B}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_A}(\frac{ft}{sec^2\text{-}in})$	$Y_{\delta_P}(\frac{ft}{sec^2\text{-}in})$
L'	$L'_u(\frac{rad}{ft\text{-}sec})$	$L'_w(1/sec)$	$L'_q(1/sec)$	$L'_v(\frac{rad}{ft\text{-}sec})$	$L'_p(1/sec)$	$L'_r(1/sec)$	$L'_{\delta_C}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$L'_{\delta_P}(\frac{rad}{sec^2\text{-}in})$
N'	$N'_u(\frac{rad}{ft\text{-}sec})$	$N'_w(1/sec)$	$N'_q(1/sec)$	$N'_v(\frac{rad}{ft\text{-}sec})$	$N'_p(1/sec)$	$N'_r(1/sec)$	$N'_{\delta_C}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_B}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_A}(\frac{rad}{sec^2\text{-}in})$	$N'_{\delta_P}(\frac{rad}{sec^2\text{-}in})$

TABLE IV-4 CONTINUED

AH-IG STABILITY AND CONTROL DERIVATIVES -- US UNITS

(BODY-FIXED FRL AXIS SYSTEM)

CASE 62 — 40 KT — LEVEL FLIGHT AT SEA LEVEL — 8000 LB — MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-0.71	-1.81	0.00	-1.81	0.02	0.00	12.77	-0.10	-1.93	3.49

XDOT	ZDOT	U0	V0	W0	VT0
67.51	0.00	67.48	0.03	-2.13	67.51

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0273	-0.0263	1.5624	-0.0084	-1.6517	-0.0980	-0.5425	1.3186	0.0156	-0.0750
Z	-0.1688	-0.7094	-0.2633	-0.0315	-0.9439	1.6536	-13.2839	1.8147	0.0528	-0.0696
M	0.0017	-0.0023	-0.2805	0.0007	0.1951	0.0149	0.0304	-0.1590	-0.0023	0.0151
Y	0.0081	0.0017	-1.5067	-0.0793	-1.9277	1.0265	-0.0712	0.0378	0.8501	1.4574
L'	0.0005	-0.0014	-0.7411	-0.0060	-1.2312	0.0250	0.0989	0.0327	0.4938	0.2938
N'	-0.0061	-0.0063	0.2837	0.0128	-0.4142	-0.7310	0.3660	0.0273	0.0295	-0.7841

CASE 63 — 60 KT — LEVEL FLIGHT AT SEA LEVEL — 8000 LB — MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-0.69	-2.25	0.00	-2.25	0.03	0.00	12.42	0.70	-1.54	2.39

XDOT	ZDOT	U0	V0	W0	VT0
101.27	0.00	101.19	0.05	-3.98	101.27

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0268	-0.0286	1.7170	-0.0046	-1.6068	-0.0668	-0.6761	1.3088	0.0163	-0.0506
Z	-0.1060	-0.8377	-1.4214	-0.0246	-1.4280	1.7699	-15.0092	2.9518	0.0733	-0.0848
M	0.0019	-0.0030	-0.3244	0.0000	0.1854	0.0199	0.0287	-0.1574	-0.0029	0.0152
Y	0.0006	-0.0003	-1.4114	-0.1019	-2.0445	1.3044	-0.0597	0.0196	0.8415	1.7390
L'	-0.0031	-0.0031	-0.6703	-0.0046	-1.2781	0.0375	0.0774	0.0227	0.4875	0.3409
N'	-0.0048	-0.0087	0.3029	0.0144	-0.3518	-0.8970	0.2937	0.0342	0.0268	-0.9430

CASE 64 — 80 KT — LEVEL FLIGHT AT SEA LEVEL — 8000 LB — MID CG

PHI	THETA	PSI	ALPHA	BETA	GAMMA	θMR	B1S	A1S	θTR
-0.78	-2.71	0.00	-2.71	0.04	0.00	12.55	1.77	-1.31	2.02

XDOT	ZDOT	U0	V0	W0	VT0
135.02	0.00	134.87	0.09	-6.38	135.02

	U	W	Q	V	P	R	DC	DB	DA	DP
X	-0.0296	-0.0261	1.7664	-0.0027	-1.5508	-0.0997	-0.6853	1.3064	0.0158	-0.1066
Z	-0.0731	-0.9243	-2.3562	-0.0213	-1.9744	1.7432	-16.4353	4.2224	0.0954	-0.1037
M	0.0022	-0.0042	-0.3656	-0.0004	0.1734	0.0251	0.0195	-0.1618	-0.0042	0.0238
Y	0.0022	-0.0051	-1.4209	-0.1236	-1.9600	1.5329	-0.0941	0.0627	0.8514	1.8191
L'	-0.0021	-0.0056	-0.6943	-0.0033	-1.2282	0.0351	0.0426	0.0494	0.4912	0.3369
N'	-0.0042	-0.0093	0.2347	0.0150	-0.3360	-1.0518	0.2546	0.0459	0.0232	-1.0179