University of California, Davis,
Dept. of Mechanical and Aerospace Engineering

Homework Assignment 6

Due: Tuesday, June 2

The state matrices for the lateral/directional control of the F-89 Aircraft at fligth condition 8901 are given below. The state variables are v,p,r,ϕ, and ψ. The controls are aileron δ_a, and rudder δ_r. The output variables are roll-rate p, and sideslip β.

```
>> Alat

Alat =

 -8.2900e-002            0 -6.6000e+002  3.2200e+001            0
 -6.8939e-003 -1.7000e+000  1.7200e-001            0            0
  5.1212e-003 -6.5400e-002 -8.9300e-002            0            0
            0  1.0000e+000            0            0            0
            0            0  1.0000e+000            0            0

>> Blat=[0 7.65;27.3 .576;.393 -1.36;0 0;0 0]

Blat =

            0  7.6500e+000
  2.7300e+001  5.7600e-001
  3.9300e-001 -1.3600e+000
            0            0
            0            0

>> Clat=[0 1 0 0 0;1/660 0 0 0 0]

Clat =

            0  1.0000e+000            0            0            0
  1.5152e-003            0            0            0            0

>> Dlat=[0 0;0 0]

Dlat =

     0     0
     0     0
```

The basic control architecture is shown below. It is desired to control p and β using aileron and rudder. The question is which control surface to use to control which output (input-output pairing).

1.) Using the coupling numerator approach, determine the appropriate input-output pairing. The accompanying hand-written notes along with the ICE design example should help. In this case, the control distribution matrix is just a 2x2 identity matrix.

2.) With the pairing problem solved, design appropriate compensators for each loop using the loop-shaping approach. The only design requirements are

a.) stable behavior

b.) Approximate 5 rad/s bandwidths in each loop.

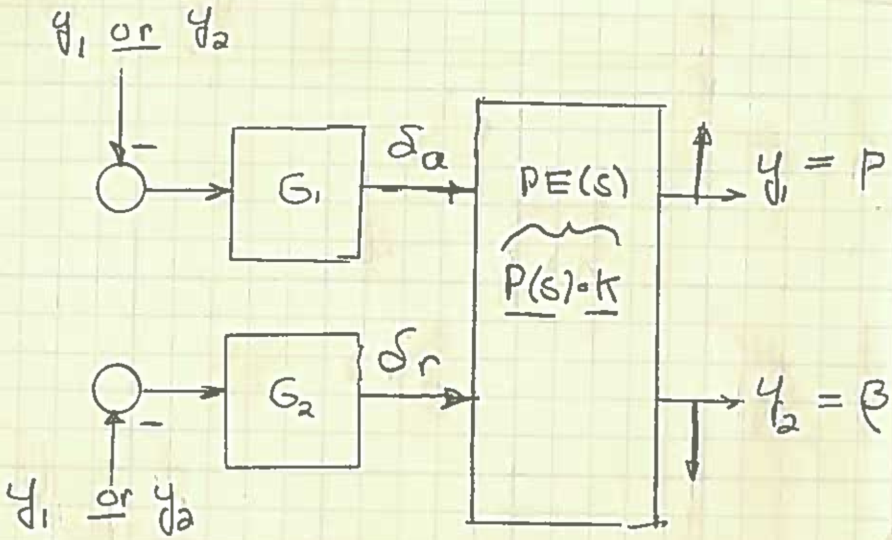
As part of your design, report the stability margins in each loop, with the second loop closed. Show all your Bode plots used in the design. NOTE: You may have to deal with the Dutch-Roll mode at some point. You may approximately cancel it, but do not do so exactly. Rather in your compensator numerator use the approximation factor $(s^2+0.2s+3.5)$

3.) Simulate your design using the following command inputs:

a.) for the p-loop use a ± 5 deg/sec (5/57.3 rad/sec) doublet with each of the two pulses lasting 2 sec. (no command for the β-loop but show the β response)

b.) for the β-loop us a step command of 5 deg (no command for the p-loop but show the p-response).

In each of these cases filter the commands with a filter $25/(s^2+10s+25)$.


y1 or y2
–
G1
δa
PE(s)
P(s)·K
y1 = p
G2
δr
–
y1 or y2
y2 = β

Using the Coupling Numerators handout, the following can be shown:

$$\left.\frac{y_1}{v_1}\right|_{y_2 \to v_2} = \frac{N_{v_1}^{y_1} + G_2 N_{v_1\ v_2}^{y_1\ y_2}}{\Delta + G_2 N_{v_2}^{y_2}}$$

and

$$\left.\frac{y_2}{v_2}\right|_{y_1 \to v_1} = \frac{N_{v_2}^{y_2} + G_1 N_{v_2\ v_1}^{y_2\ y_1}}{\Delta + G_1 N_{v_1}^{y_1}}$$

if $|G_2| >> 1$

$$\left.\frac{y_1}{v_1}\right|_{y_2 \to v_2} \cong \frac{N_{v_1, v_2}^{y_1, y_2}}{N_{v_2}^{y_2}}$$

if $|G_1| >> 1$

$$\left.\frac{y_2}{v_2}\right|_{y_1 \to v_1} \cong \frac{N_{v_2\ v_1}^{y_2\ y_1}}{N_{v_1}^{y_1}}$$

Now

$$\left.\frac{y_1}{v_2}\right|_{y_2 \to v_1} = \frac{N_{v_2}^{y_1} + G_1 N_{v_2\ v_1}^{y_1\ y_2}}{\Delta + G_1 N_{v_1}^{y_2}}$$

and

$$\left.\frac{y_2}{v_1}\right|_{y_1 \to v_2} = \frac{N_{v_1}^{y_2} + G_2 N_{v_1\ v_2}^{y_2\ y_1}}{\Delta + G_2 N_{v_2}^{y_1}}$$

if $|G_1| >> 1$

$$\left.\frac{y_1}{v_2}\right|_{y_2 \to v_1} \cong \frac{N_{v_2\ v_1}^{y_1\ y_2}}{N_{v_1}^{y_2}}$$

if $|G_2| >> 1$

$$\left.\frac{y_2}{v_1}\right|_{y_1 \to v_2} \cong \frac{N_{v_1\ v_2}^{y_2\ y_1}}{N_{v_2}^{y_1}}$$

Now. Let

$$(PE(s))^{-1} = \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}$$

Again referring to the Coupling Numerators handout:

$$N^{y_1 \, y_2}_{v_1 \, v_2} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$N^{y_2 \, y_1}_{v_2 \, v_1} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$N^{y_1 \, y_2}_{v_2 \, v_1} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$N^{y_2 \, y_1}_{v_1 \, v_2} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1$$

$$N^{y_2}_{v_2} = \begin{vmatrix} e_{11} & 0 \\ e_{21} & 1 \end{vmatrix} = e_{11}$$

$$N^{y_1}_{v_1} = \begin{vmatrix} 1 & e_{12} \\ 0 & e_{22} \end{vmatrix} = e_{22}$$

$$N^{y_2}_{v_1} = \begin{vmatrix} 0 & e_{12} \\ 1 & e_{22} \end{vmatrix} = -e_{12}$$

$$N^{y_1}_{v_2} = \begin{vmatrix} e_{11} & 1 \\ e_{21} & 0 \end{vmatrix} = -e_{21}$$

Finally

$$\left.\frac{y_1}{v_1}\right|_{y_2 \to v_2} = \frac{1}{e_{11}} = \frac{1}{(PE)^{-1}_{11}} \to \left\{ \frac{1}{\text{element } (1,1) \text{ in inverse of } PE(s)} \right.$$

$$\left.\frac{y_2}{v_2}\right|_{y_1 \to v_1} = \frac{1}{e_{22}} = \frac{1}{(PE)^{-1}_{22}}$$

$$\left.\frac{y_1}{v_2}\right|_{y_2 \to v_1} = \frac{-1}{-e_{12}} = \frac{1}{(PE)^{-1}_{12}}$$

$$\left.\frac{y_2}{v_1}\right|_{y_1 \to v_2} = \frac{-1}{-e_{21}} = \frac{1}{(PE)^{-1}_{21}}$$