

MAE 275 - Homework 6

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1 Defining the System

The state-space system can be defined,

$$\begin{aligned}\dot{\vec{x}} &= A\vec{x} + B\vec{u} \\ \vec{y} &= C\vec{x} + D\vec{u}\end{aligned}$$

where the linearized lateral aircraft equations of motion can be expressed in state space form.

$$\vec{x} = \begin{bmatrix} v \\ p \\ r \\ \phi \\ \psi \end{bmatrix}, \quad \vec{u} = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix}, \quad \vec{y} = \begin{bmatrix} p \\ \beta \end{bmatrix}$$

Using the lateral equations of motion for F-89 at flight condition 8901, the resultant system is

$$\begin{aligned}\dot{\vec{x}} &= \begin{bmatrix} -8.2900e-2 & 0 & -6.6000e+2 & +3.2200e+1 & 0 \\ -6.8939e-3 & -1.7000e+0 & +1.7200e-1 & 0 & 0 \\ +5.1212e-3 & -6.5400e-2 & -8.9300e-2 & 0 & 0 \\ 0 & +1 & 0 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & +7.6500e+0 \\ +2.7300e+1 & +5.7600e-1 \\ +3.9300e-1 & -1.3600e+0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u} \\ \vec{y} &= \begin{bmatrix} 0 & +1 & 0 & 0 & 0 \\ +1.5152e-3 & 0 & 0 & 0 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \vec{u}\end{aligned}$$

Coupling numerators can be used to determine input-output pairing and which loops to close in which order, and appropriate compensators can then be designed.

2 Coupling Numerators

The coupling numerators can be derived using the notes in the assignment and the HARV handout. They are

$$\boxed{\frac{p}{v_1} = \frac{27.3s(s^2 + 0.1747s + 3.453)}{(s + 1.781)(s - 0.001359)(s^2 + 0.09275s + 3.529)}}$$

$$\frac{p}{v_2} = \frac{0.576s(s - 2.885)(s + 2.56)}{(s + 1.781)(s - 0.001359)(s^2 + 0.09275s + 3.529)}$$

$$\frac{\beta}{v_1} = \frac{-0.393(s - 6.282)(s + 0.04952)}{(s + 1.781)(s - 0.001359)(s^2 + 0.09275s + 3.529)}$$

$$\frac{\beta}{v_2} = \frac{0.011591(s + 117.4)(s + 1.753)(s - 0.003733)}{(s + 1.781)(s - 0.001359)(s^2 + 0.09275s + 3.529)}$$

$$\left. \frac{p}{v_1} \right|_{\beta \rightarrow v_2} = \frac{27.3s(s + 118.1)}{(s + 117.4)(s + 1.753)(s - 0.003733)}$$

$$\boxed{\left. \frac{\beta}{v_2} \right|_{p \rightarrow v_1} = \frac{0.011591(s + 118.1)}{(s^2 + 0.1747s + 3.453)}}$$

$$\left. \frac{p}{v_2} \right|_{\beta \rightarrow v_1} = \frac{-0.80517s(s + 118.1)}{(s - 6.282)(s + 0.04952)}$$

$$\left. \frac{\beta}{v_1} \right|_{p \rightarrow v_2} = \frac{0.54936(s + 118.1)}{(s - 2.885)(s + 2.56)}$$

From these transfer functions we can:

1. rule out controlling p with v_2 first, as there is a non-minimum phase zero ($s-2.885$) in the transfer function that would limit the crossover frequency to values significantly below 2.885
2. rule out $\left. \frac{p}{v_2} \right|_{\beta \rightarrow v_1}$ and $\left. \frac{\beta}{v_1} \right|_{p \rightarrow v_2}$ due to the closed-loop unstable poles that would be produced

This leaves only one viable option: to first close p with v_1 , then close β with v_2 with the $p - v_1$ loop closed. As such, the ailerons are paired to the roll-rate, and the rudder is paired to the sideslip.

3 Compensators

Two compensators were designed. After both loops were closed, the reported gain and phase margins, and the crossover bandwidths were noted at

$$GC_p = 0.18 \times \frac{(s + 2)}{s}$$

$$G_m = Inf \text{ dB}$$

$$P_m = 86.7^\circ \text{ at } 5.06 \text{ rad/s}$$

$$\omega_{BW} = 5.06 \text{ rad/s}$$

$$GC_\beta = 74.4 \times \frac{(s^2 + .2s + 3.5)}{s(s + 20)}$$

$$G_m = Inf \text{ dB}$$

$$P_m = 74.4^\circ \text{ at } 4.96 \text{ rad/s}$$

$$\omega_{BW} = 4.96 \text{ rad/s}$$

Open and closed-loop bode plots for both compensators are listed on the following pages.

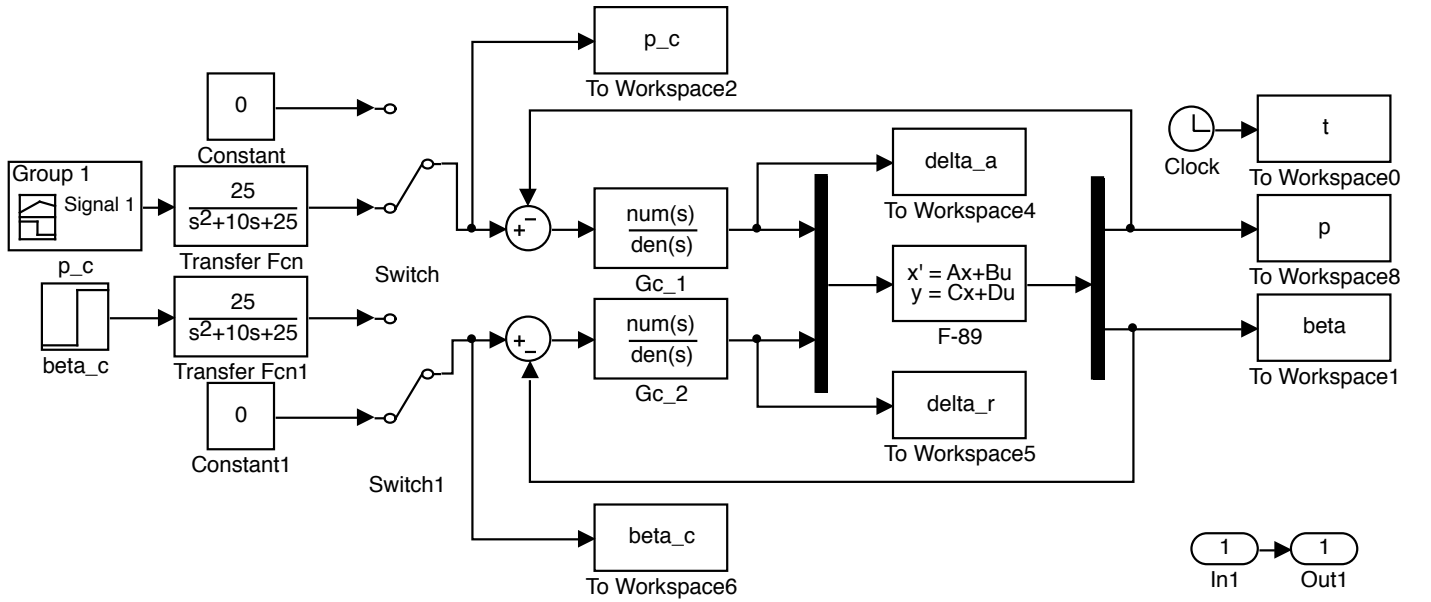


Figure 1: Final simulink diagram with both compensators

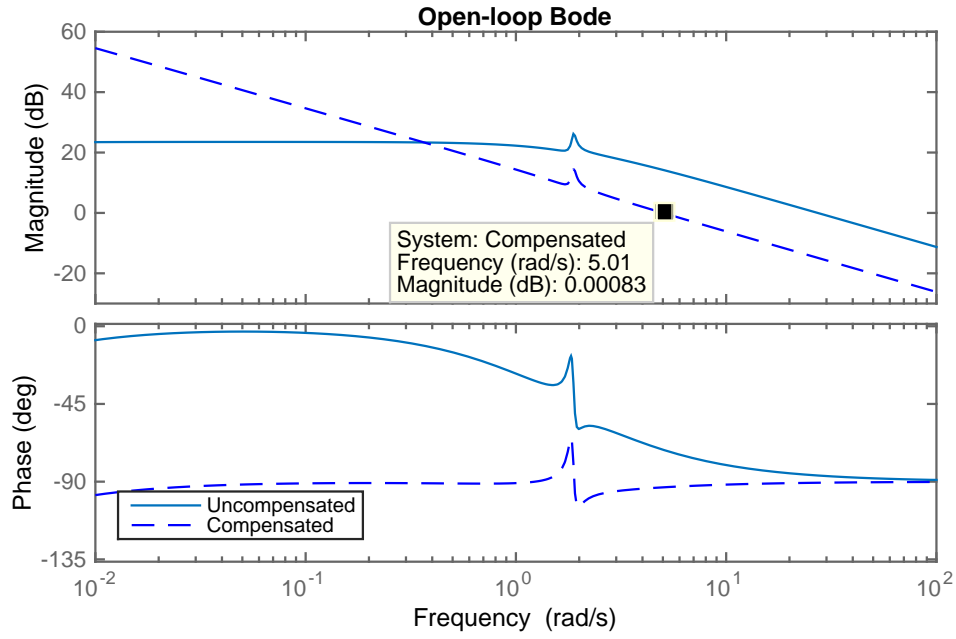


Figure 2: Open-loop Bode Plot for p loop

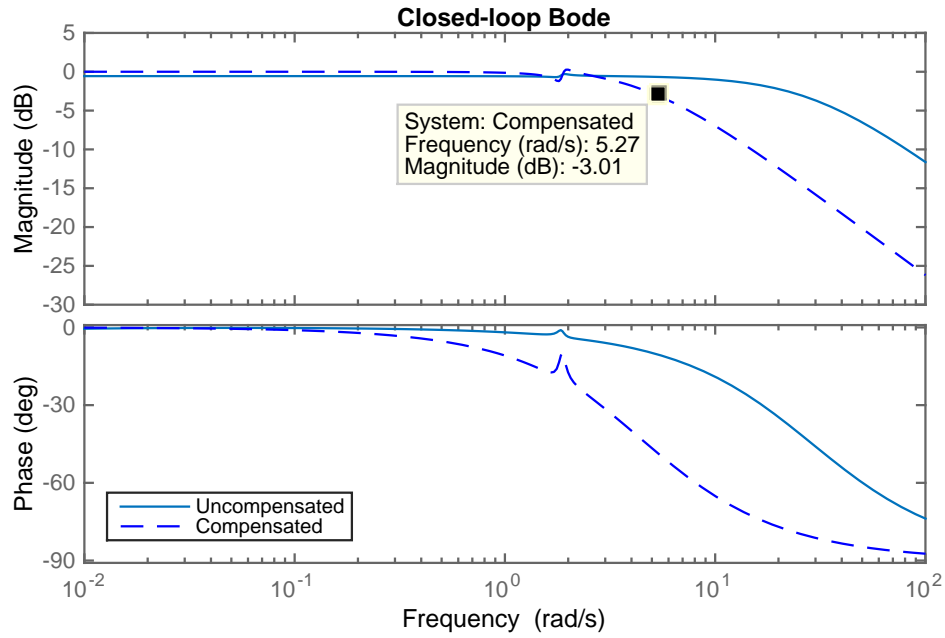


Figure 3: Close-loop Bode Plot for p loop

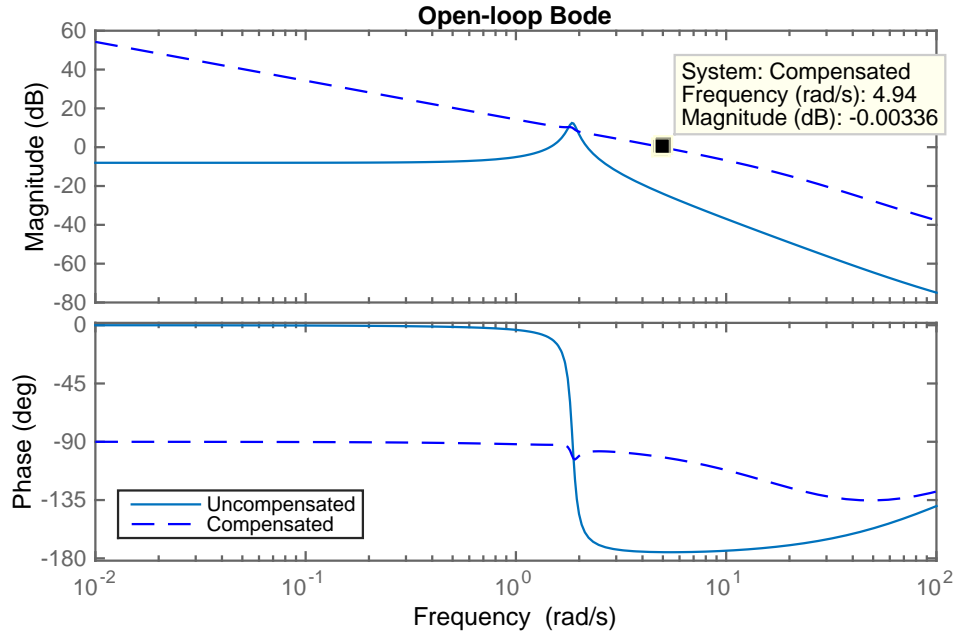


Figure 4: Open-loop Bode Plot for β loop

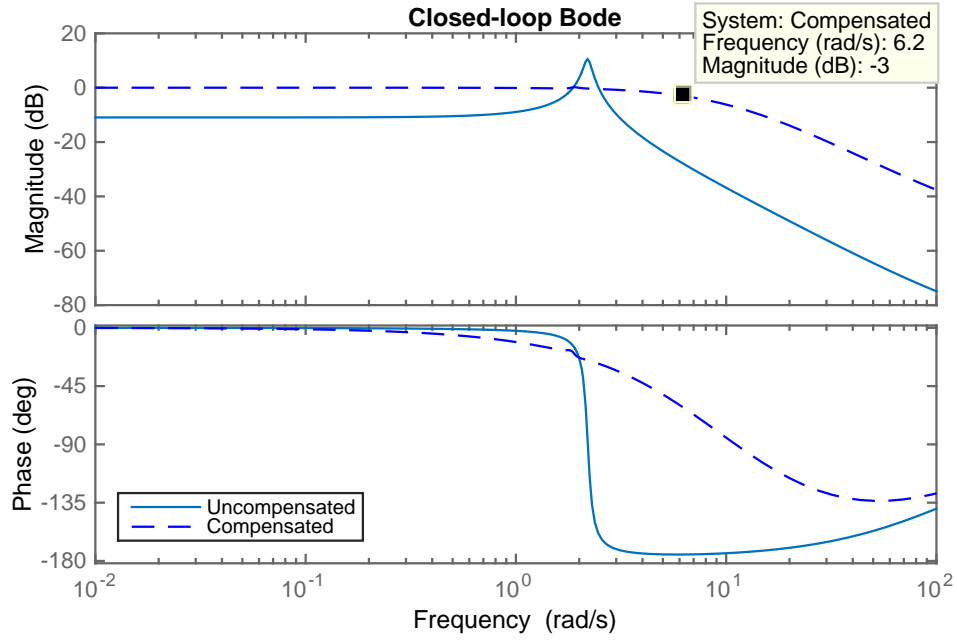


Figure 5: Close-loop Bode Plot for β loop

4 Response to Command Inputs

Two initial conditions were investigated (both commands were filtered with a filter of $\frac{25}{(s^2+10s+25)}$):

1. a ± 5 deg/sec doublet with each of the two pulses lasting 2 sec for the p -loop with no command for the β -loop
2. a $+5$ deg/sec command for the β -loop with no command for the p -loop

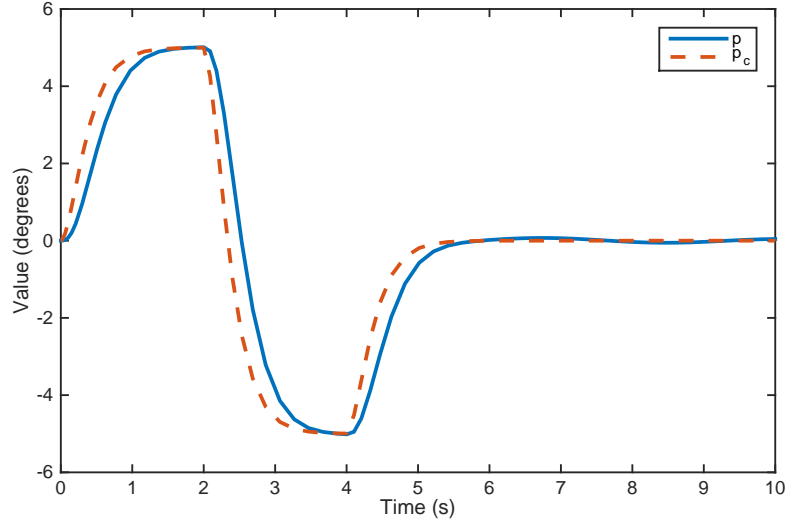


Figure 6: p Response for Scenario 1

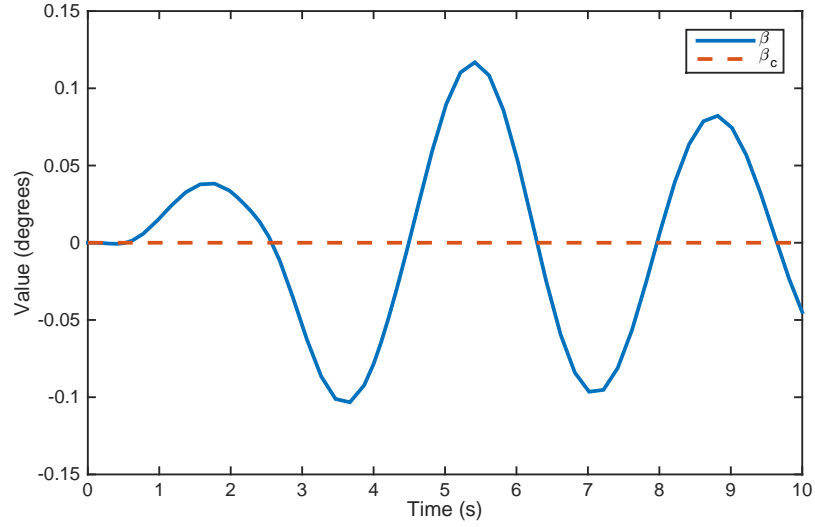


Figure 7: β Response for Scenario 1 (note the scale relative to the p response)

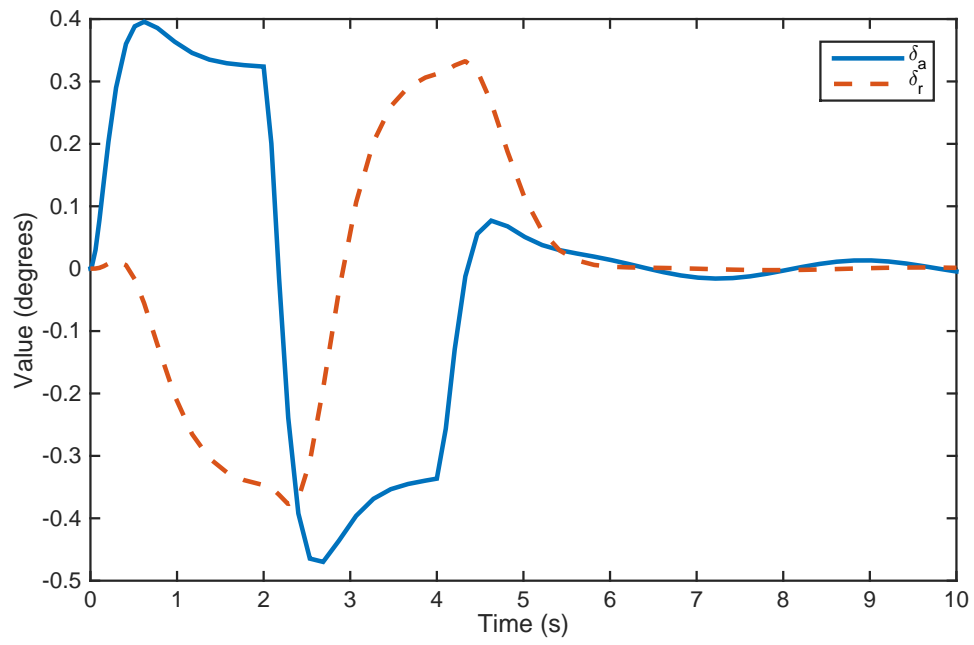


Figure 8: Command Inputs for Scenario 1

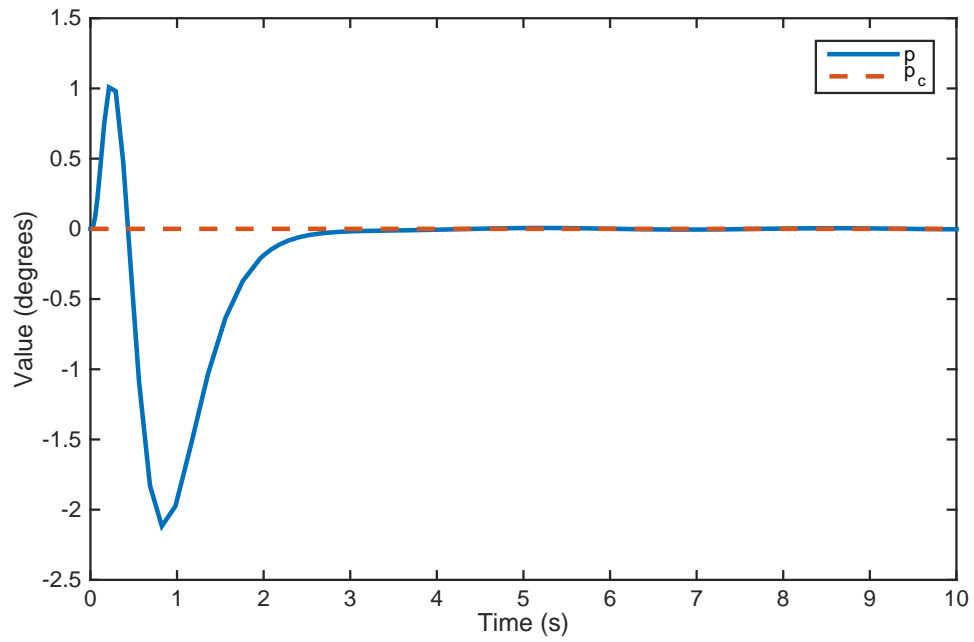


Figure 9: p Response for Scenario 2

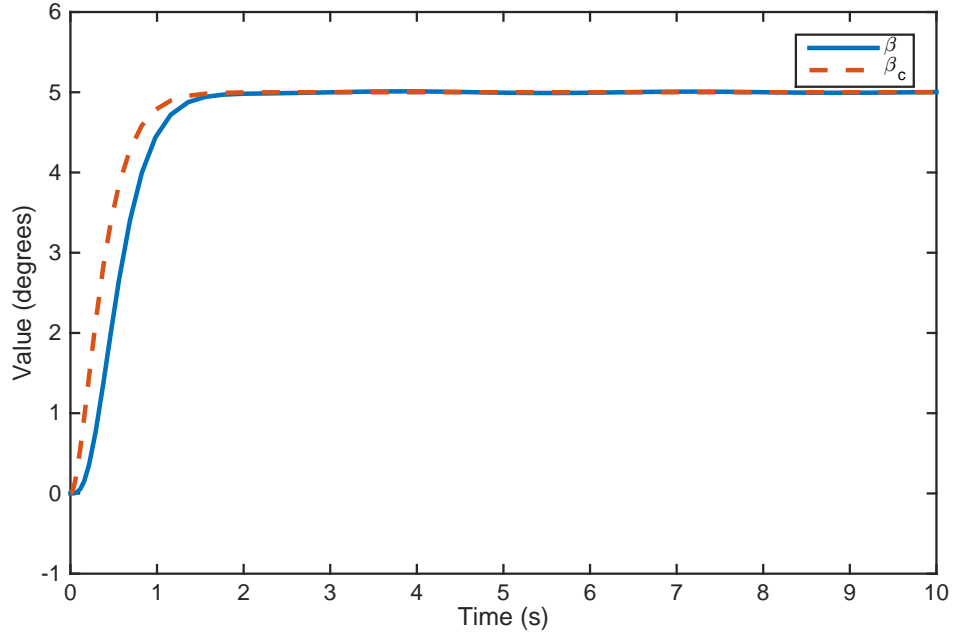


Figure 10: β Response for Scenario 2

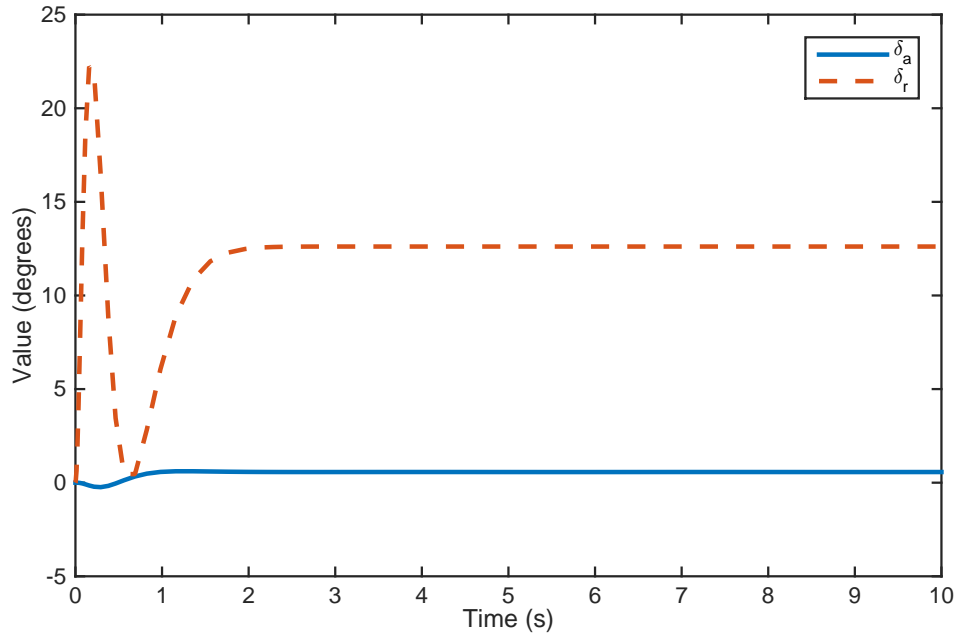


Figure 11: Command Inputs for Scenario 2