MAE 275 - Midterm

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1 Defining the System

The state-space system can be defined,

$$\begin{aligned} \dot{\vec{x}} &= A\vec{x} + B\vec{u} \\ \vec{y} &= C\vec{x} + D\vec{u} \end{aligned} \tag{1}$$

where the linearized longitudinal aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & -\dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} X_{\delta_e} & X_{\delta_T} & -X_u \\ \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} & \dfrac{Z_{\delta_T}}{1-Z_{\dot{w}}} & \dfrac{-Z_u}{1-Z_{\dot{w}}} \\ M_{\delta_e} + \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} & M_{\delta_T} + \dfrac{M_{\dot{w}}Z_{\delta_T}}{1-Z_{\dot{w}}} & -M_u - \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{u_0} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & u_0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Using the longitudinal equations of motion for C-5A for level flight ($u_0 = 246$ ft/s) at sea level, the resultant system is

$$\vec{x} = \begin{bmatrix} u \\ w \\ q \\ \theta \\ h \end{bmatrix}, \qquad \vec{u} = \begin{bmatrix} \delta_e \\ \delta_T \\ u_g \end{bmatrix}, \qquad \vec{y} = \begin{bmatrix} u \\ \alpha \\ h \\ \dot{h} \\ \theta \end{bmatrix}$$

$$\dot{\vec{x}} = \begin{bmatrix} -0.0214 & +0.0957 & 0 & -32.2 & 0 \\ -0.231 & -0.634 & +246 & 0 & 0 \\ +1.964 \times 10^{-4} & -8.895 \times 10^{-4} & -0.8275 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & +246 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} +0.45 & +0.554 \times 10^{-4} & +0.0214 \\ -9.53 & -0.193 \times 10^{-5} & +0.231 \\ -0.6795 & +1.4571 \times 10^{-7} & -1.9642 \times 10^{-4} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{u}$$

$$\vec{y} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.0041 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 246 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \vec{x} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \vec{u}$$

The loops are sequentially closed in the order: $\theta \to \delta_e$, $u \to \delta_T$, $\dot{h} \to \theta_e$. The following page lists the linearized transfer functions used to design the compensators, along with the transfer function of each compensator. The gain and phase margins and bandwidth are also listed for the compensated loop.