MAE 275 - Homework 3

John Karasinski

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta, \Delta h$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) & 0 \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & \dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ M_u + \dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w + \dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q + \dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & u_0 & 0 \end{bmatrix}$$

Plugging in the data for the VZ-4 “Doak” aircraft in Appendix A of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -1.3700e-1 & 0 & 0 & -3.2200e+1 & 0 \\ 0 & -1.3700e-1 & 0 & 0 & 0 \\ +1.3600e-2 & 0 & -4.5200e-2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}$$

Matrices B, C, and D can also be formed

$$B = \begin{bmatrix} X_{\delta_e} \\ \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} \\ \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}} + M_{\delta_e} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1.08 \\ 1.00 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 \end{bmatrix}$$

2 System Properties

We can find the resulting transfer functions, $\frac{q}{\delta e}(s)$ and $\frac{\theta}{\delta e}(s)$, with the following commands

```
C = eye(5);
D = [0; 0; 0; 0; 0];

[n, d] = ss2tf(A, B, C, D);
minreal(zpk(tf(n(3, :),d))) % q
minreal(zpk(tf(n(4, :),d))) % theta
```

which results in

```
             s (s+0.137)
  -----------------------------------
  (s+0.8223) (s^2 - 0.6401s + 0.5326)
```

and

```
              (s+0.137)
  -----------------------------------
  (s+0.8223) (s^2 - 0.6401s + 0.5326)
```

The following MATLAB command is called to identify the characteristic roots and eigenvector elements

```
[v, d] = eig(A);
```

resulting in two real eigenvalues and a complex pair of eigenvalues

$$\begin{aligned} d_1 &= -8.2230e-01 \\ d_2 &= -1.3700e-01 \\ d_3 &= +3.2005e-01 \pm i6.5583e-1 \end{aligned}$$

and their associated eigenvectors

$$\begin{aligned}
v_1 = [&-9.9962e-1,\\
&+0.0000e+0,\\
&+1.7494e-2,\\
&-2.1275e-2,\\
&+0.0000e+0],\\
v_2 = [&+0.0000e+0,\\
&+1.3573e-1,\\
&+0.0000e+0,\\
&+0.0000e+0,\\
&+9.9075e-1],\\
v_3 = [&+9.9953e-1,\\
&+0.0000e+0,\\
&+8.8107e-3 \mp i1.5820e-2,\\
&-1.4187e-2 \mp i2.0358e-2,\\
&+0.0000e+0]
\end{aligned}$$

3 Designing the Controller

The transfer function of the system, $VZ-4$, was identified as

```
          (s+1) (s+0.137)
-----------------------------------
(s+0.8223) (s^2 - 0.6401s + 0.5326)
```

A controller, G_C was designed as

```
10000 (s+0.8223) (s+0.7)^3
--------------------------
s^2 (s+500) (s+1) (s+0.137)
```

This controller was determined using loop-shaping priciples such that:

The closed-loop system is stable

The G_C has more poles than zeros

The closed-loop bandwidth (-3dB criterion) is at least 5 rad/sec (23.5 rad/sec)

~~The gain margin is at least 6dB~~ (-24 dB)

The phase margin is at least 40 deg (80 deg)

There is zero steady-state error to a step input θ_C

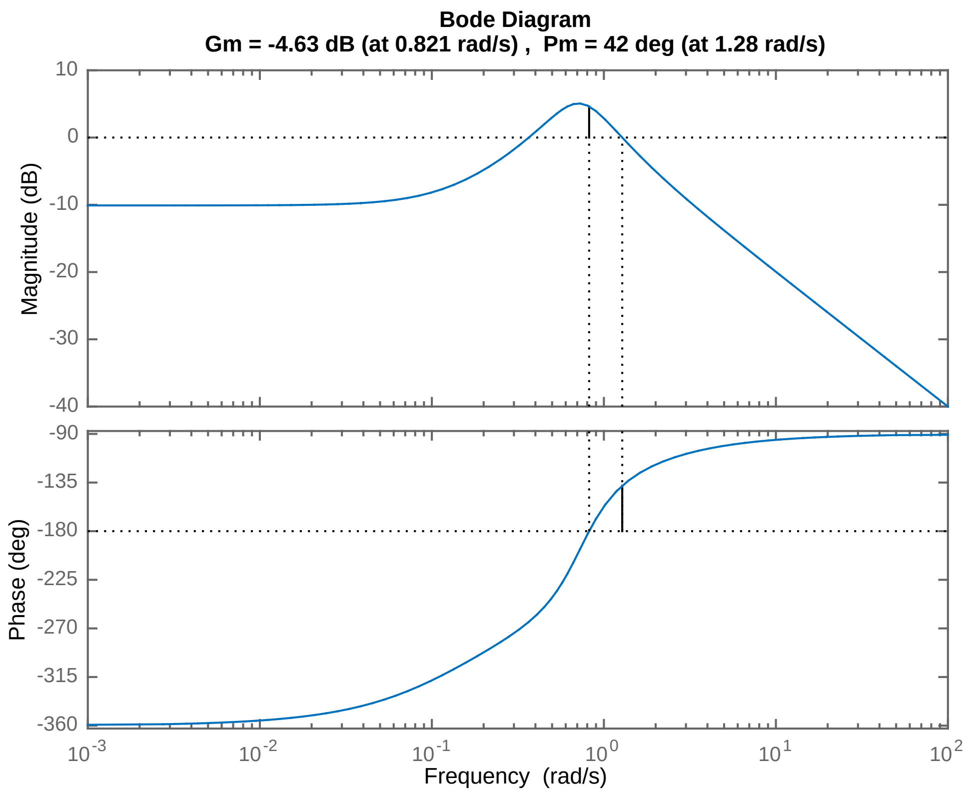

Figure 1: Open Loop Bode Diagram

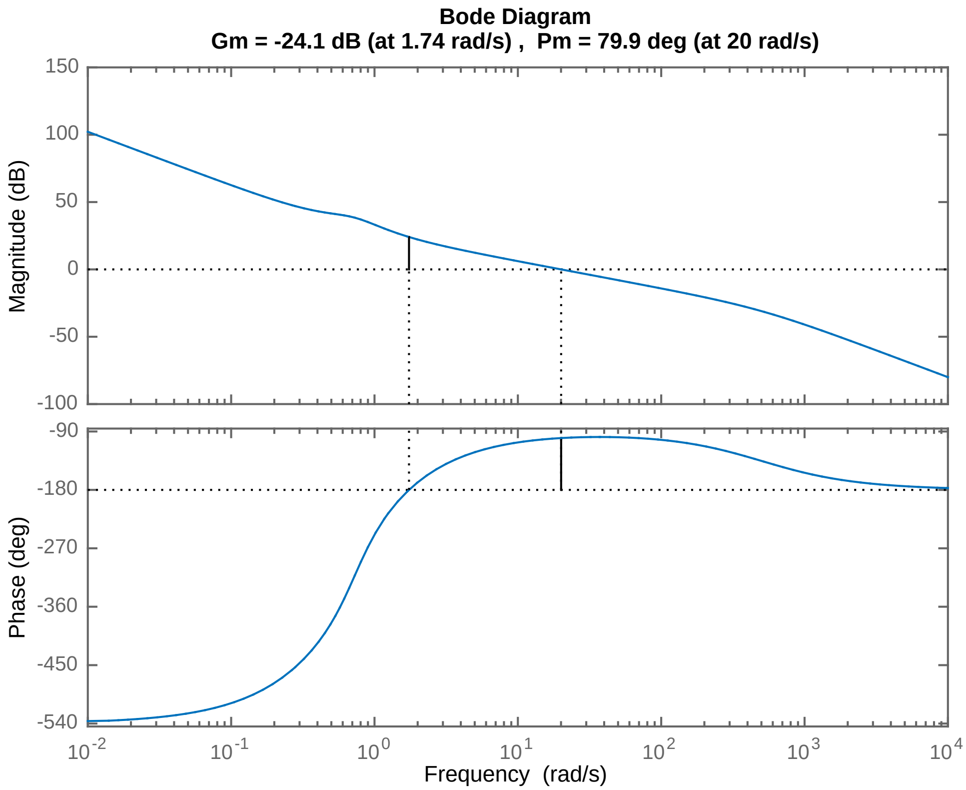

Figure 2: Compensated Open Loop Bode Diagram

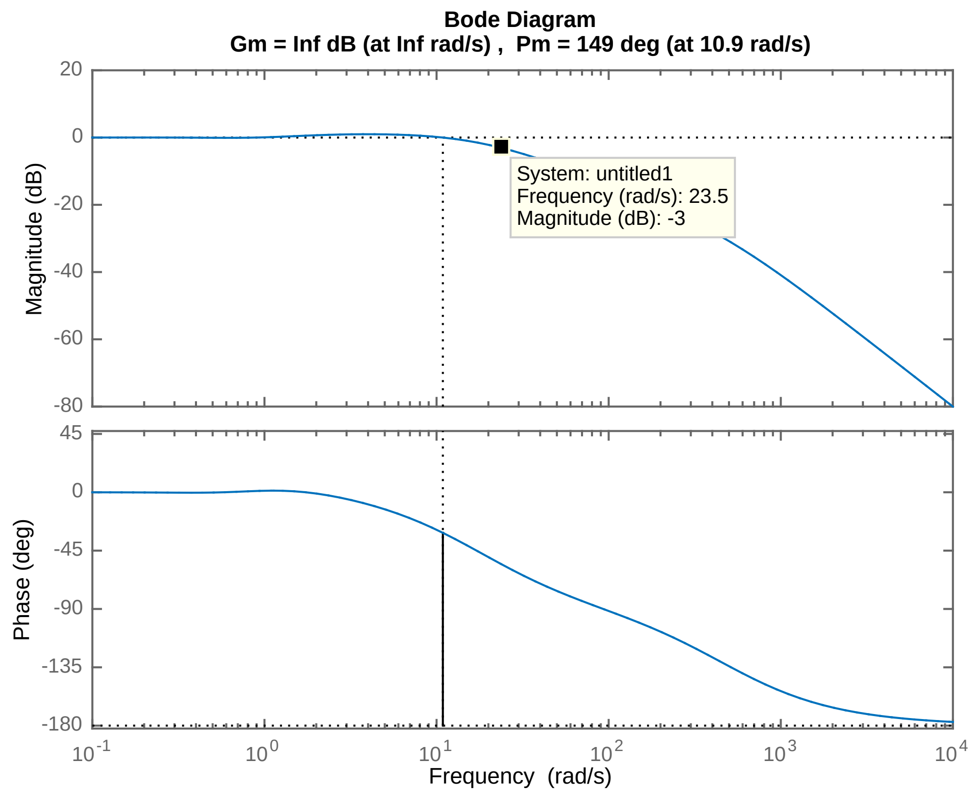

Figure 3: Compensated Closed Loop Bode Diagram

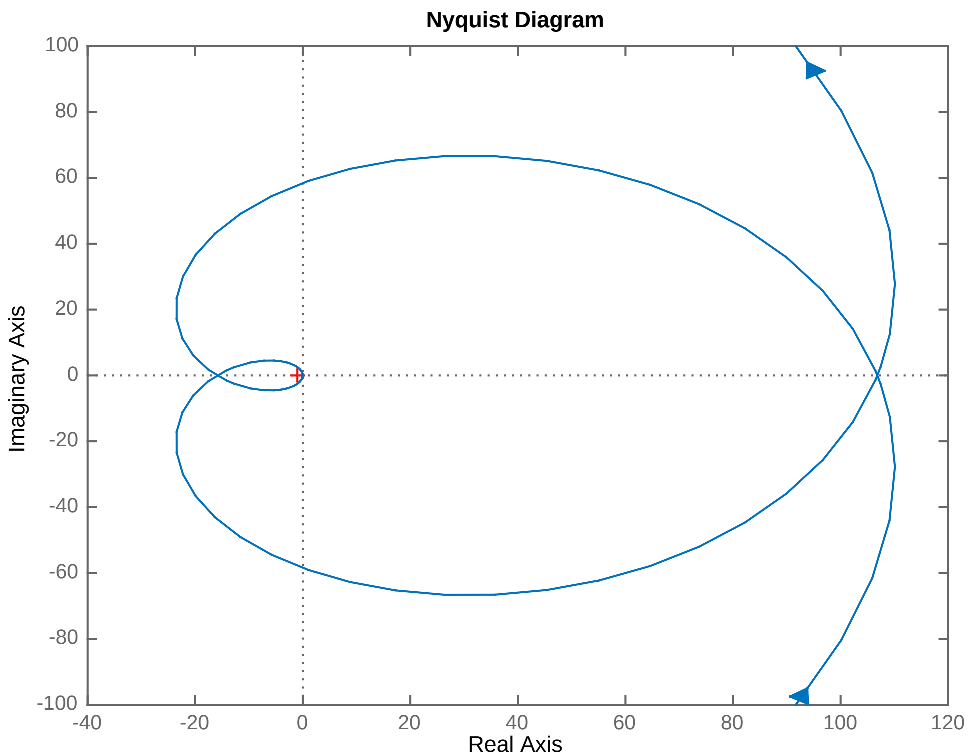

Figure 4: Nyquist Diagram

4 Simulink Diagram and Step Response

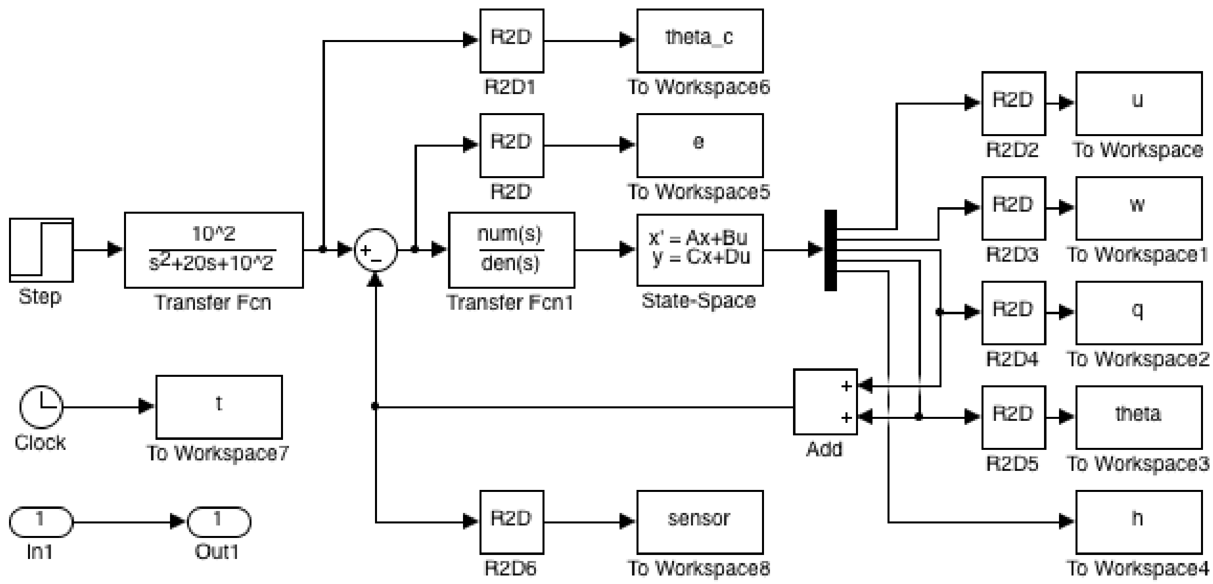

Figure 5: Simulink Diagram

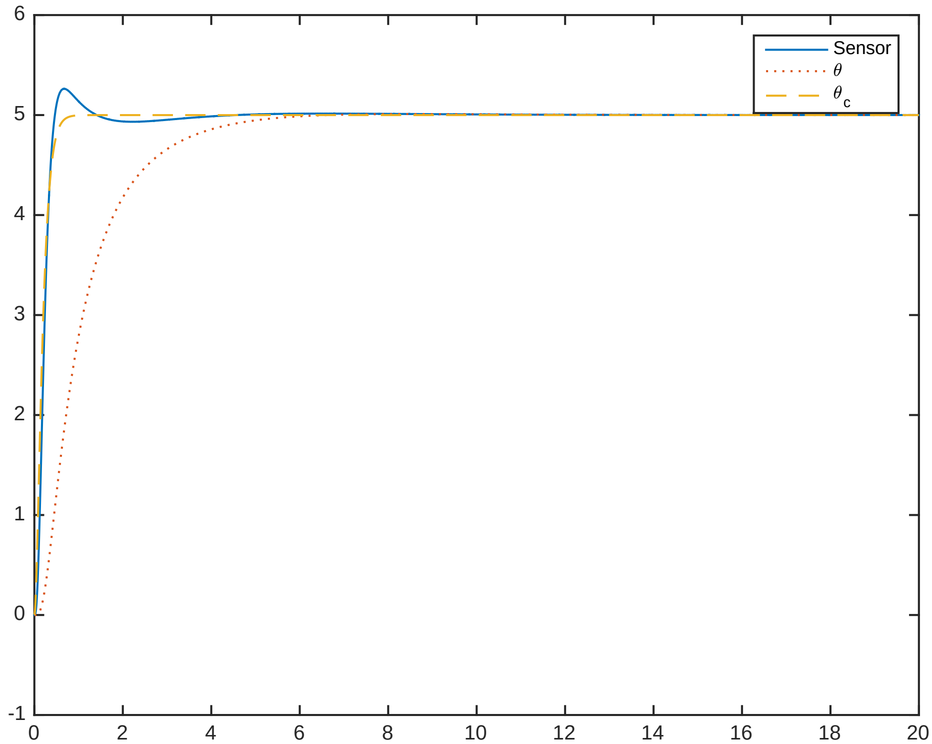

Figure 6: Step Response