

# MAE 275 - Homework 5

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## 1 Defining the System

The lateral linearized aircraft equations of motion can be expressed in state space form, with state variables  $\Delta v, \Delta p, \Delta r, \Delta \varphi, \Delta \psi$ , as

$$A = \begin{bmatrix} Y_v & Y_p & [Y_r - u_0] & g \cos \theta_0 & 0 \\ L'_v & L'_p & L'_r & 0 & 0 \\ N'_v & N'_p & N'_r & 0 & 0 \\ 0 & 1 & \tan \theta_0 & 0 & 0 \\ 0 & 0 & \sec \theta_0 & 0 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} Y_{\delta_r} & Y_{\delta_a} \\ L'_{\delta_r} & L'_{\delta_a} \\ N'_{\delta_r} & N'_{\delta_a} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 1/u_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

with  $x = [\Delta v, \Delta p, \Delta r, \Delta \varphi, \Delta \psi]$  and  $u = [\Delta \delta_r, \Delta \delta_a]$ . Plugging in the data for the DC-8 aircraft in Flight Condition 8002 from Appendix A of **Aircraft Dynamics and Automatic Control** yields

$$A = \begin{bmatrix} -1.0080e-1 & 0 & -4.6820e+2 & +3.2200e+1 & 0 \\ -5.7881e-3 & -1.2320e+0 & +3.9700e-1 & 0 & 0 \\ +2.7787e-3 & -3.4600e-2 & -2.5700e-1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.3480e + 1 & 0 \\ +3.9200e - 1 & -1.6200e + 0 \\ -8.6400e - 1 & -1.8750e - 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} +2.1358e - 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

## 2 Designing the Controllers

Two controllers were designed. The first controller,  $G_{c_\phi}$  was designed as

$$\frac{-360.94 (s+1.413) (s+0.07616)}{s (s+100) (s+3)}$$

This controller was determined using loop-shaping principles such that it had a roll-attitude bandwidth of  $\sim 3$  rad/sec (-3dB criterion) and a minimum overshoot in step response.

The second controller,  $G_{c_r}$  was designed as

$$\frac{21.969 (s-1.86)}{(s+10) (s+8.199)}$$

This controller was determined using loop-shaping principles such that it had a bandwidth of  $\sim 1$  rad/sec (-3dB criterion), giving a factor of approximately three between r-loop and  $\phi$ -loop bandwidths.

Additionally, both controllers have:

- more poles than zeros (are strictly proper compensators)
- gain margins of at least 12dB
- phase margins of at least 40 deg

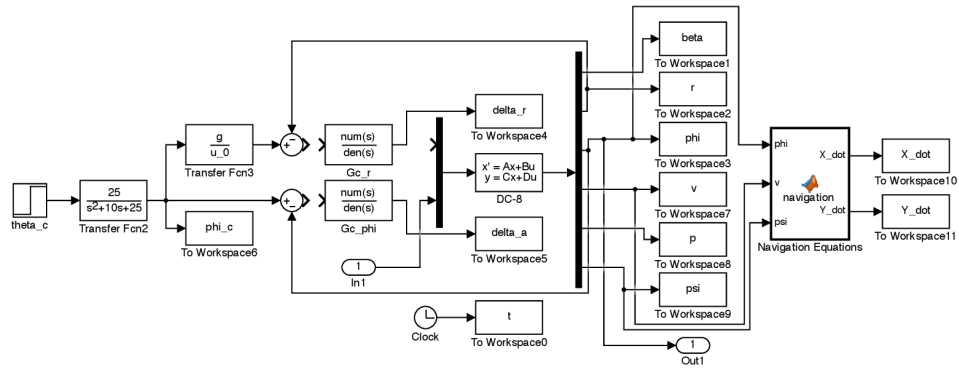


Figure 1: Simulink Diagram for First Controller Design

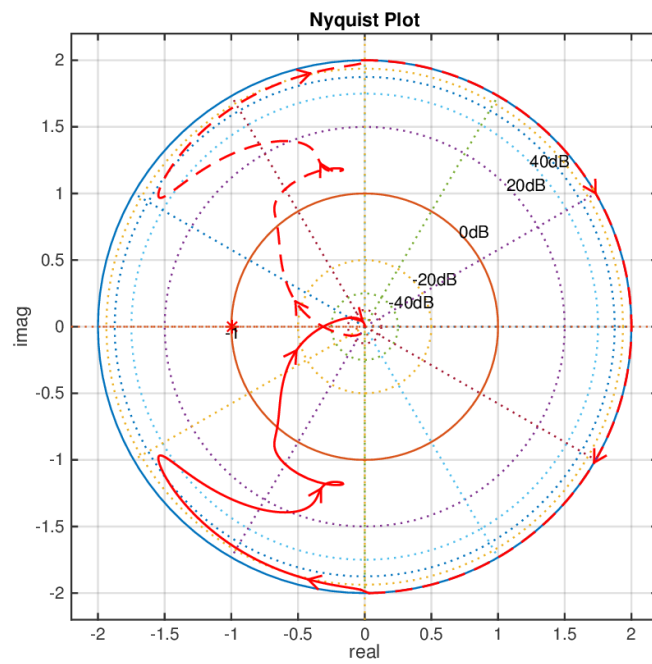


Figure 2: Nyquist Diagram for First Controller Design

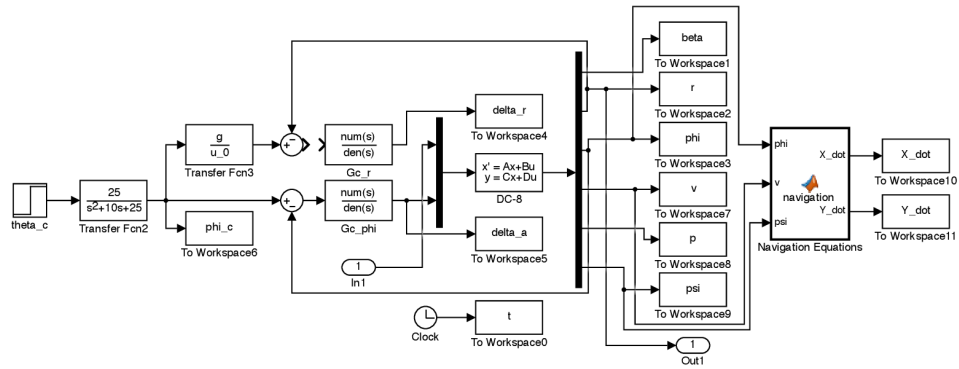


Figure 3: Simulink Diagram for Second Controller Design

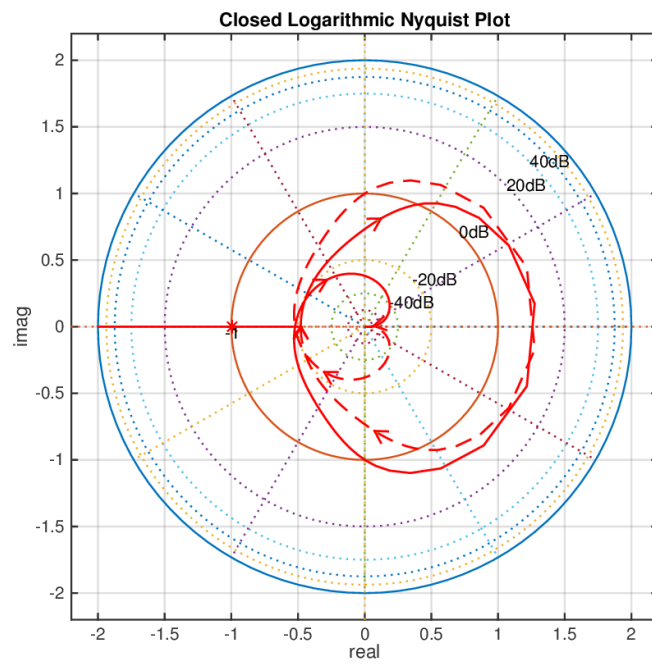


Figure 4: Nyquist Diagram for Second Controller Design



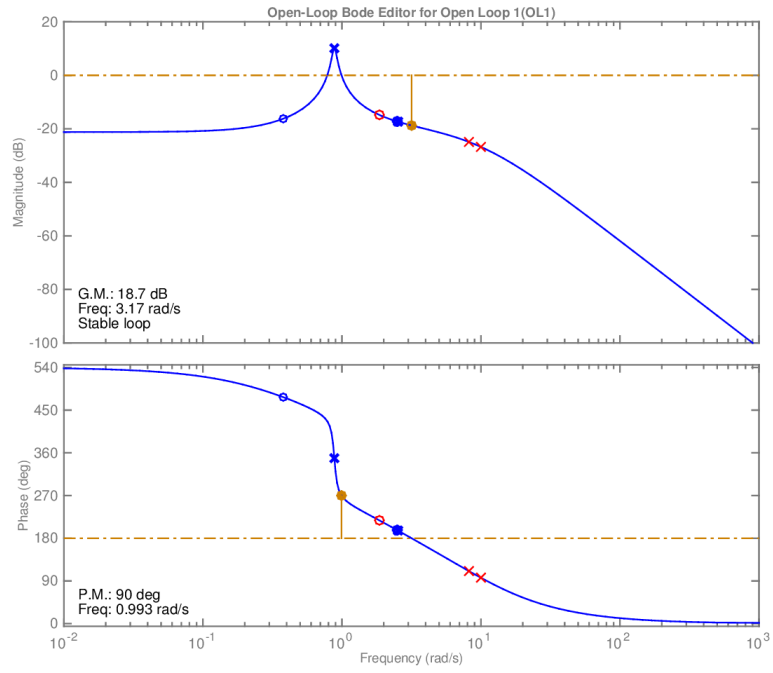


Figure 7:  $r$ -loop open-loop Bode with G.M. of 18 dB and P.M. of 90 deg

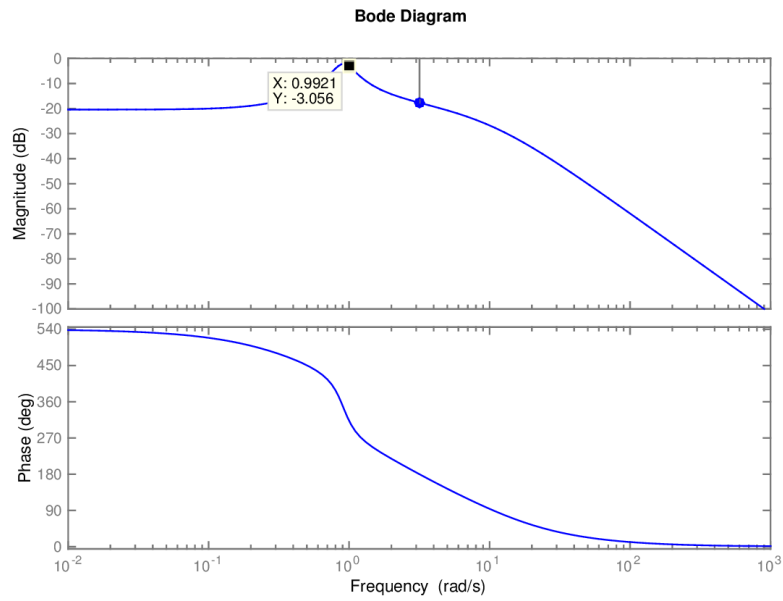


Figure 8:  $r$ -loop closed-loop Bode with bandwidth of 1 rad/s (3dB criterion)

### 3 Final Simulink Diagram

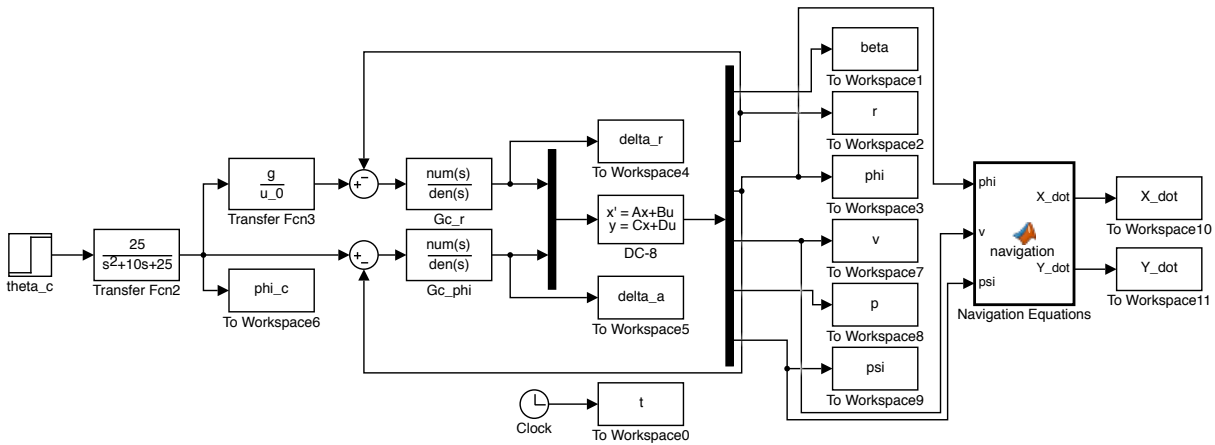


Figure 9: Final Simulink Diagram

Where the navigation function is defined as

```

1 function [X_dot, Y_dot] = navigation(phi, v, psi)
2 %#codegen
3
4 theta = 0;
5 U = 468.2;
6 V = v;
7 W = 0;
8
9 X_dot = U * (cos(psi) * cos(theta)) + ...
10         V * (cos(psi) * sin(theta) * sin(phi) - sin(psi) * cos(phi)) + ...
11         W * (cos(psi) * sin(theta) * cos(phi) + sin(psi) * sin(phi));
12
13 Y_dot = U * (sin(psi) * cos(theta)) + ...
14         V * (sin(psi) * sin(theta) * sin(phi) - cos(psi) * cos(phi)) + ...
15         W * (sin(psi) * sin(theta) * cos(phi) + cos(psi) * sin(phi));

```

## 4 Results

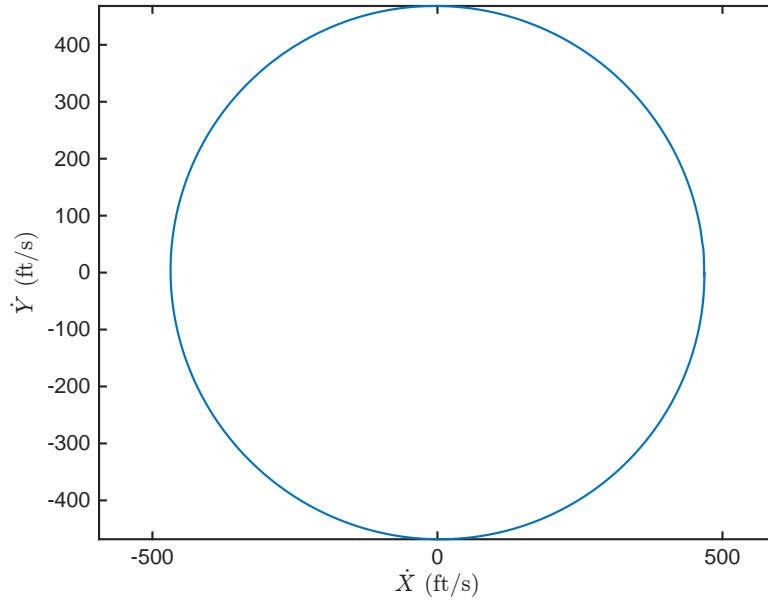


Figure 10: Demonstration of turn-coordination from  $\sim 267$  second simulation

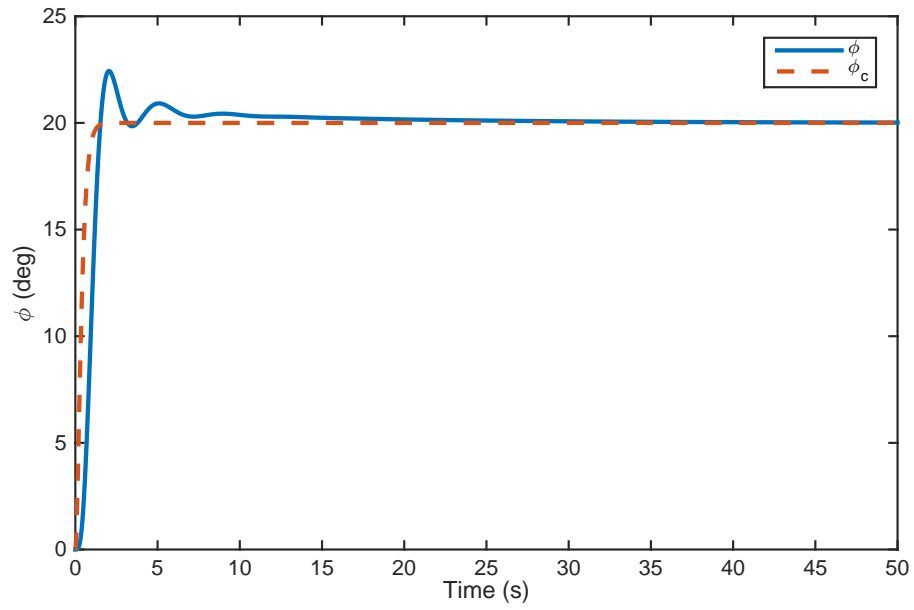


Figure 11:  $\phi$  response to  $\phi_c$ , showing  $\sim 10\%$  overshoot



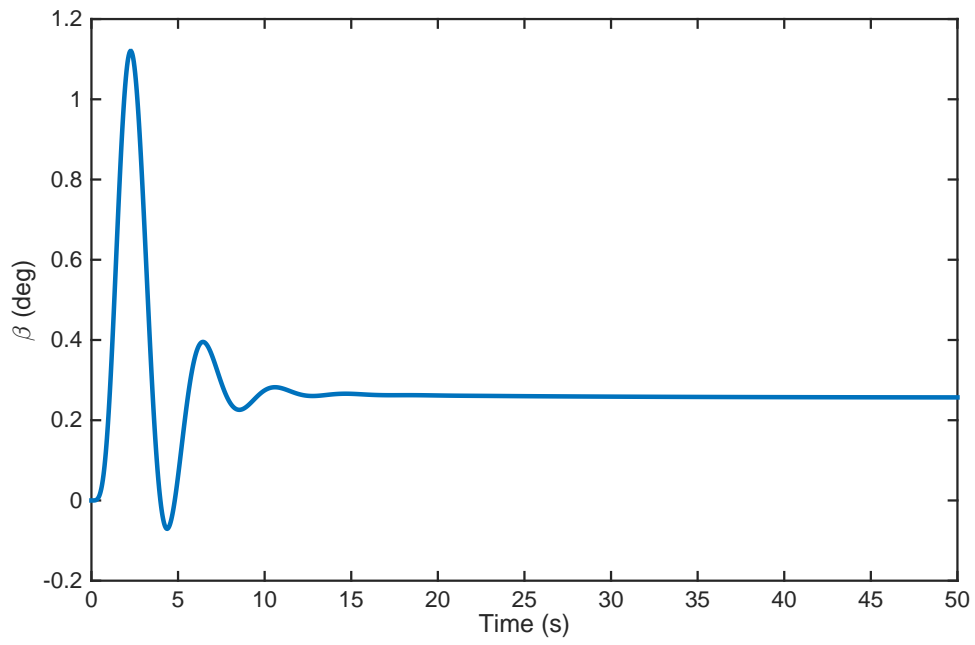


Figure 12:  $\beta$  Response

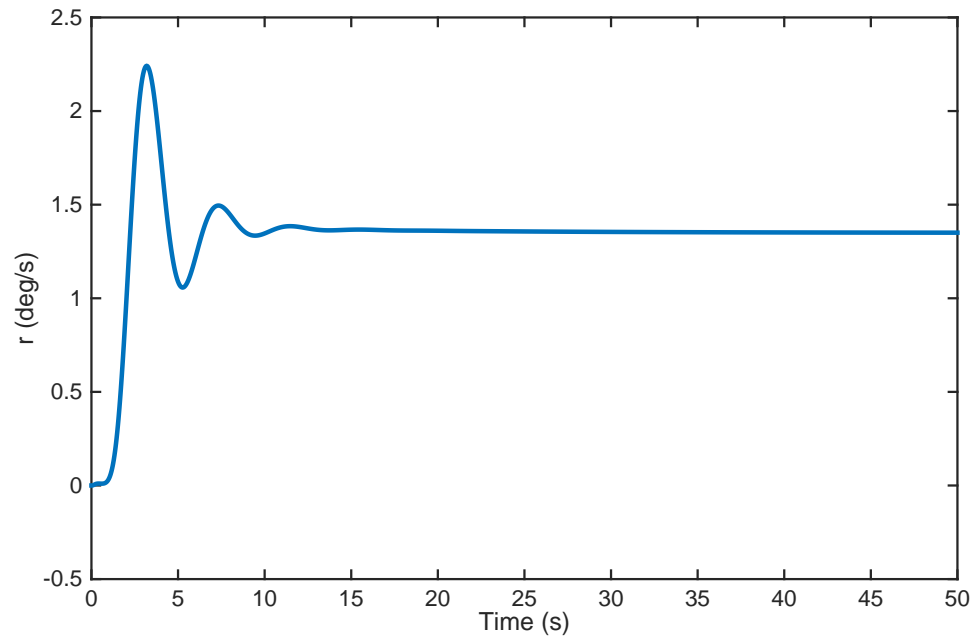


Figure 13:  $r$  Response

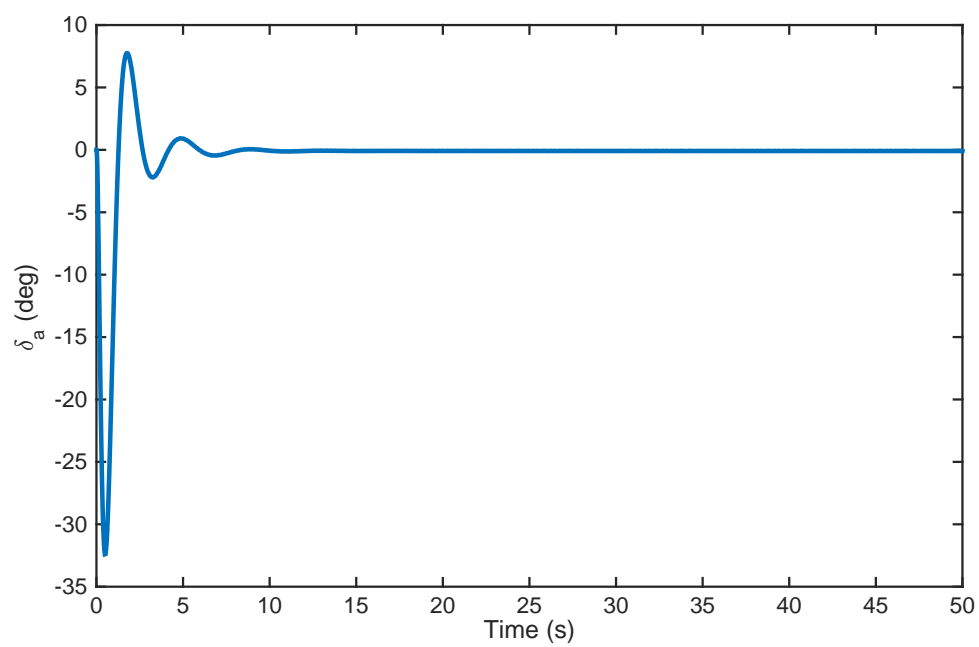


Figure 14:  $\delta_a$  Input

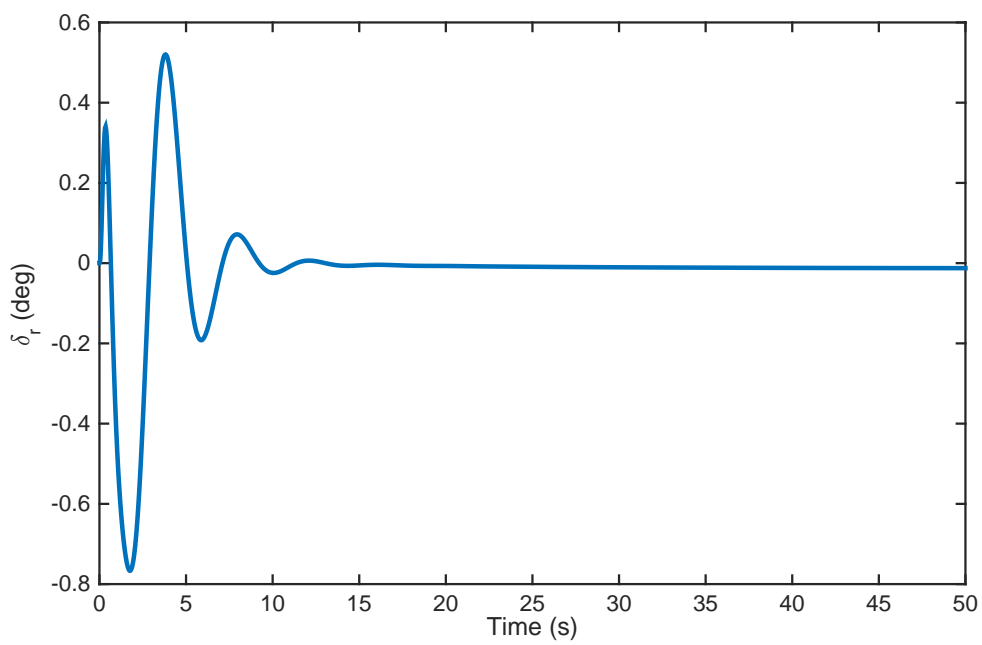


Figure 15:  $\delta_r$  Input

## 5 Handling Qualities

The handling qualities of the aircraft with the designed controllers can be estimated using the Bandwidth/Phase-Delay boundaries explained in the handout. The bandwidth is defined as  $w_{BW_{phase}} = 3.82$  rad/s. The phase delay,  $\tau_p$  is defined

$$\tau_p = \frac{\Delta\Phi 2w_{180}}{57.3(2w_{180})} = \frac{(-179.7) - (-193.3)}{57.3(2 * 32.54)} = 0.003s$$

These values suggest Level 1 handling qualities.

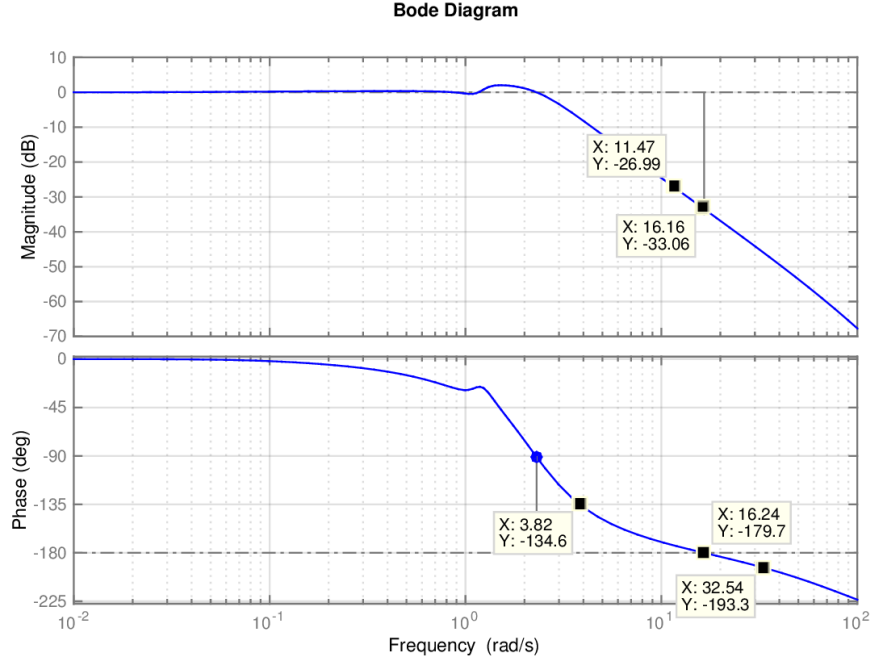


Figure 16: Closed-loop Bode with relevant points selected

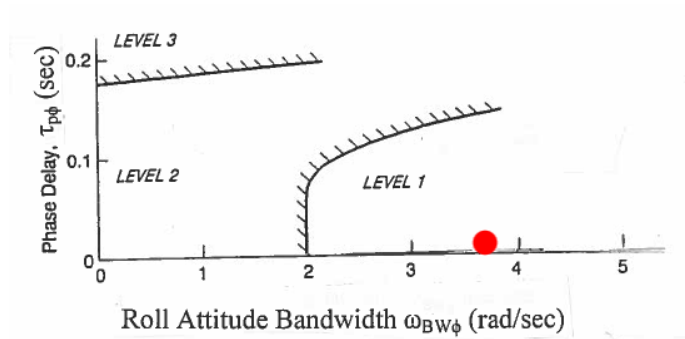


Figure 17: Handling qualities plot with location marked