MAE 275 - Homework 4

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1 Defining the System

The longitudinal linearized aircraft equations of motion can be expressed in state space form, with state variables $\Delta u, \Delta w, \Delta q, \Delta\theta$, as

$$A = \begin{bmatrix} X_u & X_w & 0 & -g\cos(\theta_0) \\ \dfrac{Z_u}{1-Z_{\dot{w}}} & \dfrac{Z_w}{1-Z_{\dot{w}}} & \dfrac{Z_q+u_0}{1-Z_{\dot{w}}} & \dfrac{g\sin\theta_0}{1-Z_{\dot{w}}} \\ M_u+\dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}} & M_w+\dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}} & M_q+\dfrac{M_{\dot{w}}(Z_q+u_0)}{1-Z_{\dot{w}}} & -\dfrac{M_{\dot{w}}g\sin\theta_0}{1-Z_{\dot{w}}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Relevant B, C, and D matrices can also be formed

$$B = \begin{bmatrix} -(X_u) & -(X_w) & X_{\delta_e} \\ -(\dfrac{Z_u}{1-Z_{\dot{w}}}) & -(\dfrac{Z_w}{1-Z_{\dot{w}}}) & \dfrac{Z_{\delta_e}}{1-Z_{\dot{w}}} \\ -(M_u+\dfrac{M_{\dot{w}}Z_u}{1-Z_{\dot{w}}}) & -(M_w+\dfrac{M_{\dot{w}}Z_w}{1-Z_{\dot{w}}}) & \dfrac{M_{\dot{w}}Z_{\delta_e}}{1-Z_{\dot{w}}}+M_{\delta_e} \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{u_0} & 0 & 0 \\ 0 & Z_w & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Z_{\delta_e} \\ 0 & 0 & 0 \end{bmatrix}$$

Where it is assumed that

$$\alpha \approx \frac{\Delta w}{u_0}$$
$$a_z \approx \Delta q Z_w$$

Plugging in the data for the A4-D aircraft in Flight Condition 5 from Appendix A of `Aircraft Dynamics and Automatic Control` yields

$$A = \begin{bmatrix} -1.2660e-2 & -5.8800e-3 & 0 & -3.2200e+1 \\ -1.0104e-1 & -8.1668e-1 & +6.3298e+2 & 0 \\ -3.5082e-4 & -1.9546e-2 & -1.4219e+0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} +1.2660e-2 & +5.8800e-3 & 0 \\ +1.0104e-1 & +8.1668e-1 & -5.6828e+1 \\ +3.4382e-4 & +1.9546e-2 & -1.9388e+1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & +1.5773e-03 & 0 & 0 \\ 0 & -8.1800e-01 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -5.6920e+01 \\ 0 & 0 & 0 \end{bmatrix}$$