Problem 1. Develop a very simple representation of the Hubble telescope.

```
#########################################################################################
#                                         Hubble                                        #
#     ^V2                                                                               #
#     |                          +----------------------+                               #
#  <--+                          |                      |                               #
#  V1                            |         SA1          |                               #
#                                |                      |                               #
#                                +-----------+----------+                               #
#                                            |                                          #
#                                            |    +-------+                             #
#                             +--------------++---+       |                             #
#                             |               |   |       |                             #
#                             |       1       | 2 |   3   |                             #
#                             |               |   |       |                             #
#                             |               |   |       |                             #
#                             +--------------++---+       |                             #
#                                            |    +-------+                             #
#                                            |                                          #
#                                +-----------+----------+                               #
#                                |                      |                               #
#                                |         SA1          |                               #
#                                |                      |                               #
#                                +----------------------+                               #
#                                                                                       #
#########################################################################################
#                                                                                       #
#     ^V3                                                                               #
#     |                                                                                 #
#  <--+                                           +-------+                             #
#  V1                         +---------------+---+       |                             #
#                             |               |   |       |                             #
#                             |       1       | 2 |   3   |                             #
#                             |  +-----------+----------+ |                             #
#                             |               |   |       |                             #
#                             +---------------+---+       |                             #
#                                                 +-------+                             #
#                                                                                       #
#                                                                                       #
#                                                                                       #
#########################################################################################
```

Diagram 1: Hubble ASCII Diagram. Solar panel distance from HST is exaggerated. One character is ≈ 20 inches.

We model both the body (3 sections) and the solar panels (2 sections) of the Hubble Space Telescope (HST). The body sections are connected as such: Section 1 is connected to Section 2, and Section 2 is connected to Section 3. The solar arrays are connected on Section 1, along the centerline, 20.75 inches V_1 away from the connection point with Section 2, and the near edge of the SA is 129 inches from center of Section 1. These sections are modeled with thin walled cylinders (TWC), solid cylinders (SC), and flat plates (FP). The rough

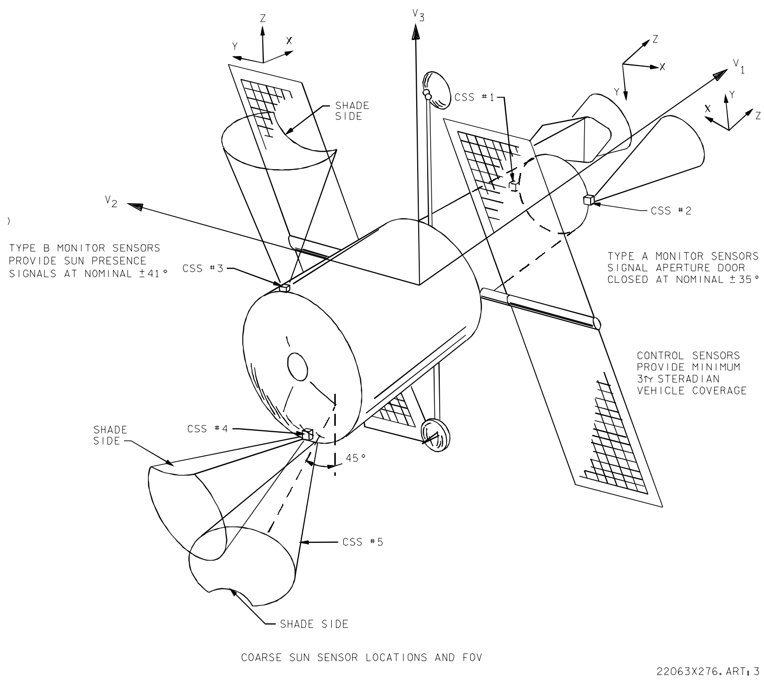

Figure 1: HST Axes Definition for V_1, V_2, V_3, CG is located at axis origin

layout of the sections is shown on the previous page, and the mass and length properties of each section are listed in Table 1.

Section	Model	V_1 (in)	V_2 (in)	Weight (lb)
Section 1				
Light Shield (LS)	-	153.2	120	-
Forward Shell (FS) (except OTA)	-	156.05	121.2	-
Total	TWC	270.75	121.2	2796
Section 2				
OTA Equipment Section	TWC	38.5	121.2	9033
Section 3				
SSM Equipment Section (SSM-ES)	-	61.25	168.16	10594
Aft Shroud (AS)	-	138.00	168.16	569
Total	SC	199.25	168.16	11163
Section 4				
Solar Arrays (SA)	FP	476.8[1]	113.5	735[2]

Table 1: [1]: This length can be fully rotated into V_3. [2]: Weight of both solar arrays. V_1 and V_2 indicate the measurements of the parts. All lengths taken from Hubble technical drawings [2]; all masses from [3].

Problem 2. Use this model as a basis to write a function(s) to determine the Mass Center and Inertia Matrix for any location.

Using the radial-center of the farthest tip of Section 1 as our zero point , the center of mass is located at $V = [327, 0, 0]$ inches. This makes since, as the model is symmetric about the V_2 and V_3 axes, and this value is on the boundary between Section 2 and Section 3, which is very close to where the reaction wheels are located.

The HST inertia matrix [1], was at one point measured as:

$$I = \begin{bmatrix} 36046 & -706 & 1491 \\ -706 & 86868 & 449 \\ 1491 & 449 & 93848 \end{bmatrix} kg \cdot m^2.$$

The Python script in Appendix gives the result of:

$$I = \begin{bmatrix} 35914 & 0 & 0 \\ 0 & 88215 & 0 \\ 0 & 0 & 113393 \end{bmatrix} kg \cdot m^2,$$

which has a relative error of:

$$I = \begin{bmatrix} 0 & 100 & 100 \\ 100 & -2 & 100 \\ 100 & 100 & -21 \end{bmatrix} \%.$$

Note that while our simple, 5 part model does a very good job of predicting the I_{V_1} and I_{V_2} components, the I_{V_3} component is not represented very well. This is likely due to leaving out the antenna booms, which should have the largest effect in the V_3 directions. It should also be noted that, due to the symmetric nature of our model, all of the off-axis terms are missing. For the rest of the analysis, the true values for the inertia matrix will be used.

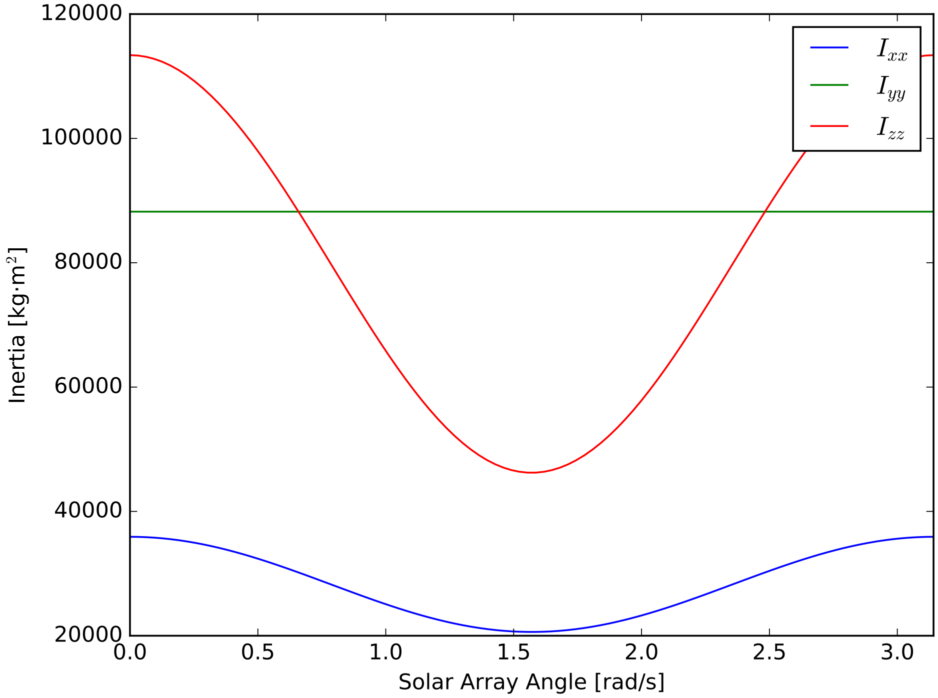

Figure 2: Inertias for principal axes for different solar array configurations

Problem 3. Write a function to find the current angular momentum relative to the mass center.

H increases linearly with ω. See Appendix for the code used to create this plot.

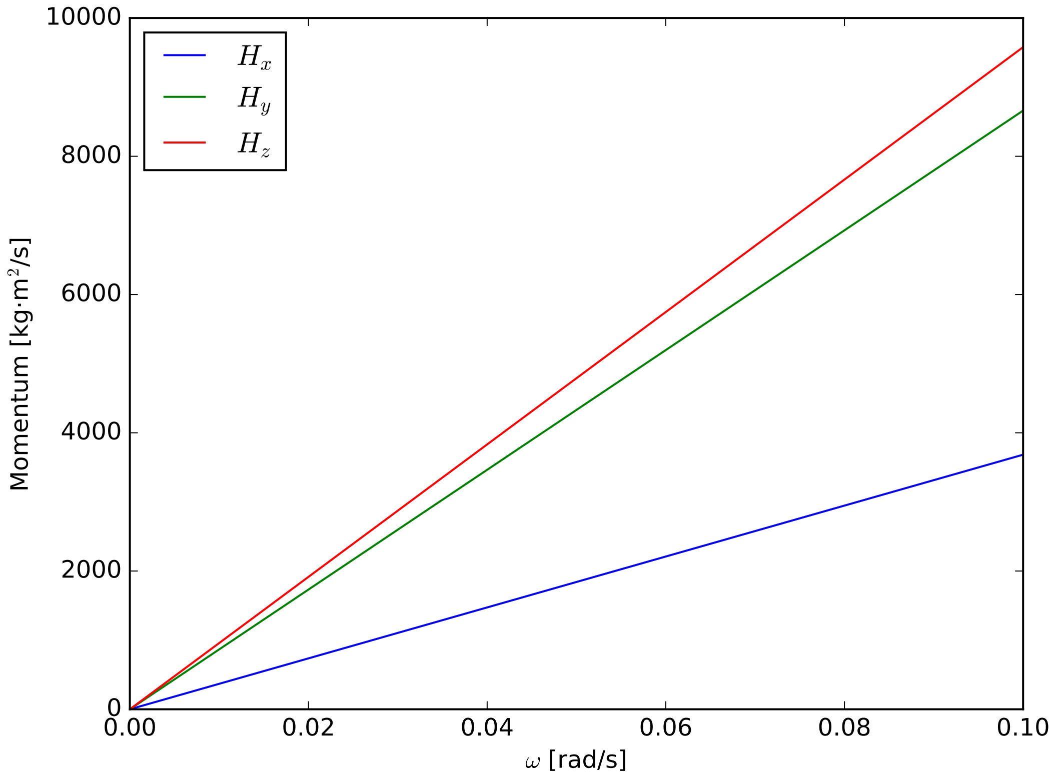

Figure 3: Effects of spin on each axis

Problem 4. Choose the optimal location for a torque producing system and explain why you think is the best location.

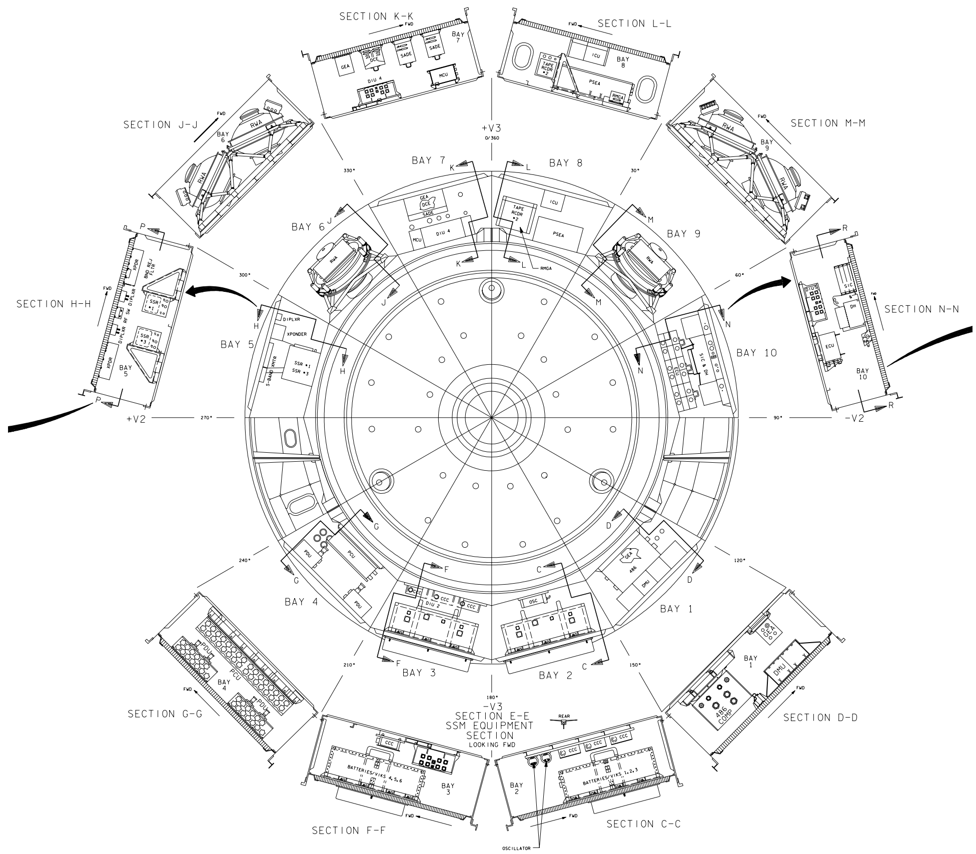

Figure 4: HST's reaction wheels are located in the SSM-ES in bays 6 and 9. The pairs were installed in at a 90^o offset to protect against failures [2].

The optimal location for a torque producing system is usually on the centerline, as near to the center of mass of the spacecraft as possible. Placing the torque producing system close the the spacecraft's center of mass minimizes the amount of undesirable resultant torque when the system is activated. In HST's case, the optimal location for a torque producing system is in the SSM-ES, see Fig. 4. The two wheels of a single SSM bay are canted off the V_2-V_3 plane by 20^o, one toward the $+V_1$ and one toward the -V_1. I think that this is likely the best location to place the system as it's the one that was actually used, and because the engineers that placed them the reaction wheels there had much greater access to information about HST than I do.

As we do not have to worry about reaction wheels failing for the purposes of this homework, I will instead simplify the system to a single location of three reaction wheels, one of which is aligned with each axis. I will place these three reaction wheels at in at the exact center of the SSM-ES along the V_1 axis, and at 60 inches along the V_3 axis. "Nominal dynamic torque range is 0.003 to 0.605 ft-lb (0.004 to 0.7 N·m), with a maximum wheel speed of ±3000 rpm” for each of the four reaction wheels [2].

Problem 5. Using your previous functions, write a program to find the resulting angular acceleration produced from a given torque.

Problem 6. Write what next steps you would take to develop a controller that keeps the craft pointed in a specific direction.

Bibliography

[1] Queen, S., “HRV GNC Peer Review, Flight Performance Analysis,” Tech. rep., NASA Goddard Space Flight Center, 2004.

[2] NASA, “Cargo Systems Manual (CSM): Hubble Space Telescope,” February 13, 2002

[3] Mattice, J., “Hubble Space Telescope Systems Engineering Case Study.”