Problem 1.

Orbital Transfer Review: remind yourself: (discuss/justify decisions)

All three of the transfers below can be accomplished with a Hohmann transfer. The amount of ΔV required for a Hohmann transfer is

$$
\begin{aligned}
\Delta v_1 &= \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}}-1\right),\\
\Delta v_2 &= \sqrt{\frac{\mu}{r_2}}\left(1-\sqrt{\frac{2r_1}{r_1+r_2}}\right),\\
\Delta v_{total} &= \Delta v_1 + \Delta v_2.
\end{aligned}
$$

Additionally, part (b) requires an inclination change. The ΔV budget for a inclination change for a circular orbit is

$$\Delta v_i = 2v\sin\left(\frac{\Delta i}{2}\right).$$

The assumption of a circular orbit should be fine here, as both HST and ISS orbits have very low eccentricity.

(a) Determine the ΔV required to move from a 200km coplanar parking orbit to the HST orbit

$$
\begin{aligned}
r_1 &= 200km + 6,371km\\
r_2 &= 569km + 6,371km\\
\Delta v_{total} &= \Delta v_{Hohmann}\\
&= 0.210km/s.
\end{aligned}
$$

(b) Determine the ΔV required to move from the ISS orbit to the HST orbit

$$
\begin{aligned}
r_{ISS} &= 414.1km + 6,371km\\
r_{HST} &= 569km + 6,371km\\
i_{ISS} &= 0.9014rad\\
i_{HST} &= 0.4969rad\\
v_{HST} &= 7.59km/s
\end{aligned}
$$

$$
\begin{aligned}
\Delta v_{total} &= \Delta v_{Hohmann} + \Delta v_{PlaneChange}\\
&= 0.086 + 3.049\\
&= 3.135km/s,
\end{aligned}
$$

where we've again assumed a circular orbit. The plane change should take place after the Hohmann transfer, as the orbital velocity is lower at HST orbit, leading to a smaller required ΔV.

(c) Determine the ΔV required to deorbit from the HST orbit (must choose your de-orbit orbital params)

$$\begin{aligned} r_{deorbit} &= 100km + 6,371km \\ r_{HST} &= 569km + 6,371km \\ \Delta v &= \sqrt{\frac{\mu}{r_1}}\left(\sqrt{\frac{2r_2}{r_1+r_2}} - 1\right), \\ &= 0.136km/s, \end{aligned}$$

where a height of $100km$ should be sufficient to cause the vehicle to deorbit rapidly.

Problem 2.

Eclipse durations (text 5.3.2): an important design aspect of your solar array system is the relative durations of eclipse and insolation. Using the algorithm given in section 5.3.2,

(a) Compute the eclipse period for ISS, for HST, and for a typical GPS satellite (choose one)

I ran the algorithm in section 5.3.2 over one complete orbit at 0.1 degree increments, and for sun positions of one complete year at day increments. The mean and standard error of the mean was calculated for eclipse period, as was the total time in eclipse and the total number of eclipses over the one year period. These results are presented in Table 1.

Satellite	Mean (min)	SEM (min)	Total (hrs)	Count
HST	33.97	0.10	414.45	732
ISS	34.99	0.09	426.98	732
GPS	46.09	2.55	90.64	118

Table 1: Eclipse statistics for the HST, ISS, and a GPS satellite (Navstar 43)

Problem 3.

Lets say we lose control of your spacecraft after it has undocked from HST, but before it has de-orbited.

(a) Estimate the orbital lifetime of your spacecraft (text 5.3.4) following loss of communications: assume HST circular orbit, average solar activity.

(b) Would it make any difference to the decay timescale whether your spacecraft was tumbling or not?

Problem 4.

Geostationary orbits (text 5.6):

(a) Using the linearized solution to Keplers equation given in eqns 5.27-5.29, plot ground-track fluctuations as longitude vs latitude. Describe the results.

(b) Define deadband and control limit-cycle in the context of GEO station keeping.

(c) Consider a GEO satellite with nominal longitude of -100deg, and an onboard propellant system capable of providing a total ΔV of 200m/s. For a maximum longitudinal error magnitude of 0.22deg, for how long can the satellite station-keep?

Problem 5.

Two spacecraft in elliptical Earth orbit with the orbital parameters as follows. Compute the relative position and velocity vectors.

(a) h= 52,059 km^2/s, e=0.0257240, i=60deg, Ω=40deg, ω=30deg, θ=40deg

(b) h= 52,362 km^2/s, e=0.0072696, i=50deg, Ω=40deg, ω=120deg, θ=40deg

Problem 6.

Fly-around relative trajectories: for the lost EVA toolbox example considered in lecture, generate the relative motion plot for 1 orbital period, given initial conditions of:

(a) Release relative velocity = (-0.1, 0, 0) m/s (prolate cycloid)

(b) Release relative velocity = (0, 0, 0.1) m/s (ellipse)

(c) Release relative velocity = (-0.1, 0, 0.1) m/s (initially 45deg backwards and up; describe subsequent motion)

(d) For a and b, plot the trajectory with and without the nt$\ll$1 assumption. Discuss.

(e) How about a release relative velocity = (0, 0.1, 0) m/s? Would you see the toolbox again or not?

Problem 7.

For your HST re-boost spacecraft, assume:

- Launch: drop-off circular orbit at 200km, in-plane with HST, 65deg phase angle behind HST - Phasing: 4-orbit phasing to point S1, 30km behind and 10km below HST
- Homing: Hohmann S1 to co-orbit waiting point S2, 1km behind HST
- Closing: Cycloid close waiting point S3, 200m behind HST

(a) compute the required ΔV and elapsed time for each phase, and for the total rendezvous to S3

(b) compute the view-angle to HST, measured from the orbit-tangent (for sensor acquisition)

(c) plot the total quantitative relative motion (like Walter Fig. 8.26)