

Problem 1.

Orbital Transfer Review: remind yourself: (discuss/justify decisions)

All three of the transfers below can be accomplished with a Hohmann transfer. The amount of ΔV required for a Hohmann transfer is

$$\begin{aligned}\Delta v_1 &= \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right), \\ \Delta v_2 &= \sqrt{\frac{\mu}{r_2}} \left(1 - \sqrt{\frac{2r_1}{r_1 + r_2}} \right), \\ \Delta v_{total} &= \Delta v_1 + \Delta v_2.\end{aligned}$$

additionally, part (b) requires an inclination change. The ΔV budget for a inclination change for a circular orbit is

$$\Delta v_i = 2v \sin \left(\frac{\Delta i}{2} \right).$$

The assumption of a circular orbit should be fine here, as both HST and ISS orbits have very low eccentricity.

- (a) Determine the ΔV required to move from a 200km coplanar parking orbit to the HST orbit

$$\begin{aligned}r_1 &= 200km + 6,371km \\ r_2 &= 569km + 6,371km \\ \Delta v_{total} &= \Delta v_{Hohmann} \\ &= 0.21km/s.\end{aligned}$$

- (b) Determine the ΔV required to move from the ISS orbit to the HST orbit

$$\begin{aligned}r_{ISS} &= 414.1km + 6,371km \\ r_{HST} &= 569km + 6,371km \\ i_{ISS} &= 0.9014rad \\ i_{HST} &= 0.4969rad \\ v_{HST} &= 7.59km/s\end{aligned}$$

$$\begin{aligned}\Delta v_{total} &= \Delta v_{Hohmann} + \Delta v_{PlaneChange} \\ &= 0.086 + 3.049 \\ &= 3.135km/s,\end{aligned}$$

where we've again assumed a circular orbit. The plane change should take place after the Hohmann transfer, as the orbital velocity is lower at HST orbit, leading to a smaller required ΔV .

- (c) Determine the ΔV required to deorbit from the HST orbit (must choose your de-orbit orbital params)

$$r_{deorbit} = 100km + 6,371km$$

$$r_{HST} = 569km + 6,371km$$

$$\begin{aligned}\Delta v &= \sqrt{\frac{\mu}{r_1}} \left(\sqrt{\frac{2r_2}{r_1 + r_2}} - 1 \right), \\ &= 0.136km/s,\end{aligned}$$

where a height of $100km$ should be sufficient to cause the vehicle to deorbit rapidly.

Problem 2.

Eclipse durations (text 5.3.2): an important design aspect of your solar array system is the relative durations of eclipse and insolation. Using the algorithm given in section 5.3.2,

- (a) Compute the eclipse period for ISS, for HST, and for a typical GPS satellite (choose one)

Problem 3.

Lets say we lose control of your spacecraft after it has undocked from HST, but before it has de-orbited.

- (a) Estimate the orbital lifetime of your spacecraft (text 5.3.4) following loss of communications: assume HST circular orbit, average solar activity.
- (b) Would it make any difference to the decay timescale whether your spacecraft was tumbling or not?

Problem 4.

Geostationary orbits (text 5.6):

- (a) Using the linearized solution to Keplers equation given in eqns 5.27-5.29, plot ground-track fluctuations as longitude vs latitude. Describe the results.
- (b) Define deadband and control limit-cycle in the context of GEO station keeping.
- (c) Consider a GEO satellite with nominal longitude of -100deg, and an onboard propellant system capable of providing a total ΔV of 200m/s. For a maximum longitudinal error magnitude of 0.22deg, for how long can the satellite station-keep?

Problem 5.

Two spacecraft in elliptical Earth orbit with the orbital parameters as follows. Compute the relative position and velocity vectors.

- (a) $h = 52,059 \text{ km}^2/\text{s}$, $e = 0.0257240$, $i = 60^\circ$, $\Omega = 40^\circ$, $\omega = 30^\circ$, $\theta = 40^\circ$
- (b) $h = 52,362 \text{ km}^2/\text{s}$, $e = 0.0072696$, $i = 50^\circ$, $\Omega = 40^\circ$, $\omega = 120^\circ$, $\theta = 40^\circ$

Problem 6.

Fly-around relative trajectories: for the lost EVA toolbox example considered in lecture, generate the relative motion plot for 1 orbital period, given initial conditions of:

- (a) Release relative velocity = $(-0.1, 0, 0)$ m/s (prolate cycloid)
- (b) Release relative velocity = $(0, 0, 0.1)$ m/s (ellipse)
- (c) Release relative velocity = $(-0.1, 0, 0.1)$ m/s (initially 45deg backwards and up; describe subsequent motion)
- (d) For a and b, plot the trajectory with and without the $nt \ll 1$ assumption. Discuss.
- (e) How about a release relative velocity = $(0, 0.1, 0)$ m/s? Would you see the toolbox again or not?

Problem 7.

For your HST re-boost spacecraft, assume:

- Launch: drop-off circular orbit at 200km, in-plane with HST, 65deg phase angle behind HST - Phasing: 4-orbit phasing to point S1, 30km behind and 10km below HST
 - Homing: Hohmann S1 to co-orbit waiting point S2, 1km behind HST
 - Closing: Cycloid close waiting point S3, 200m behind HST
- (a) compute the required ΔV and elapsed time for each phase, and for the total rendezvous to S3
- (b) compute the view-angle to HST, measured from the orbit-tangent (for sensor acquisition)
- (c) plot the total quantitative relative motion (like Walter Fig. 8.26)