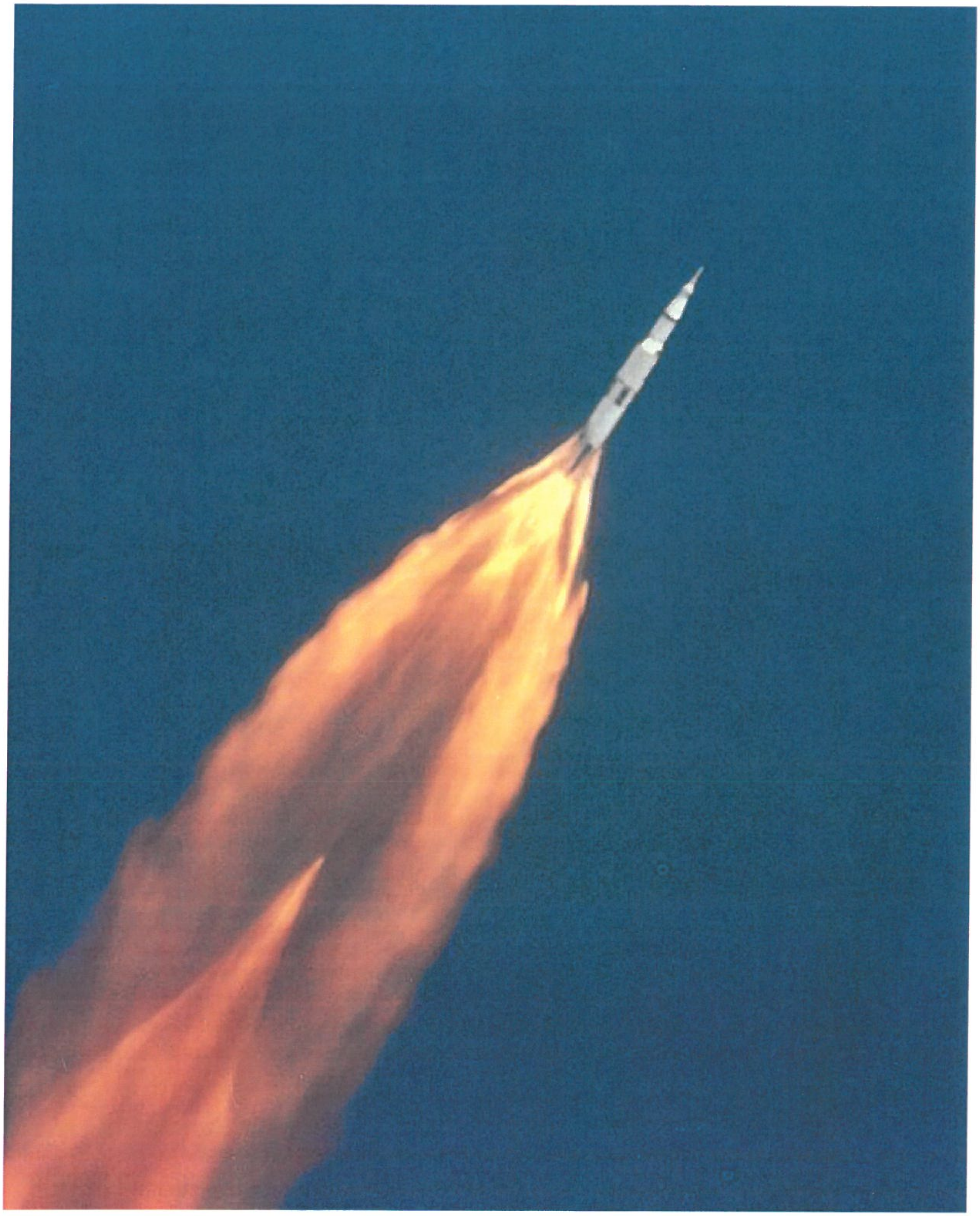
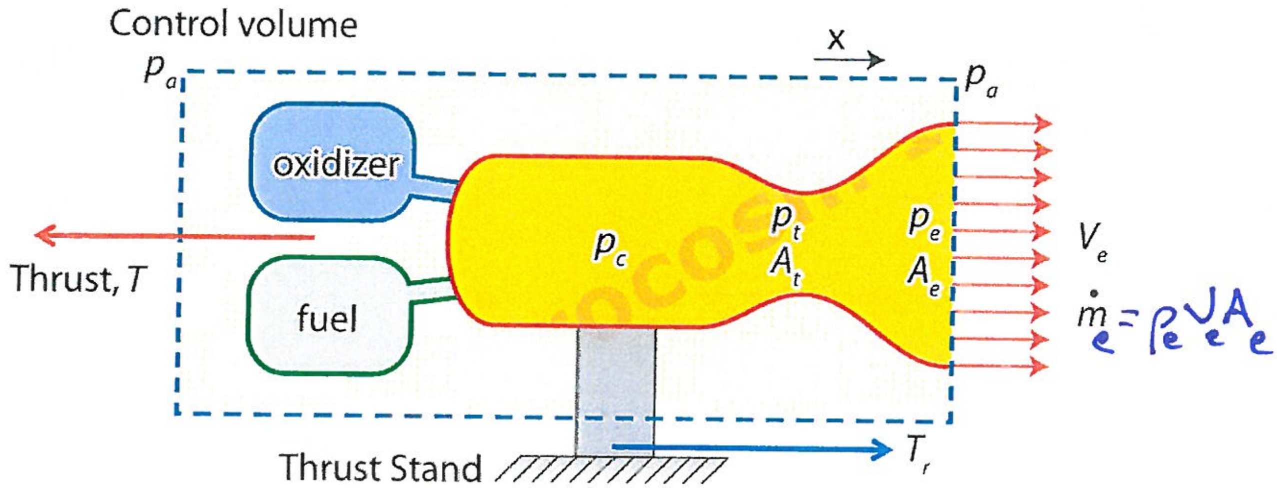
# Propulsion Systems



# Text 18.1: Basic Rocket Equations




(the structure holding the rocket in place feels a reaction force equal in magnitude to the thrust but in the opposite direction)



- Rocket engines produce thrust, T.

  We'd like to understand which variables influence T.

- Newton's 2nd Law: $\sum \vec{F} = \frac{d}{dt}(m\vec{V})$

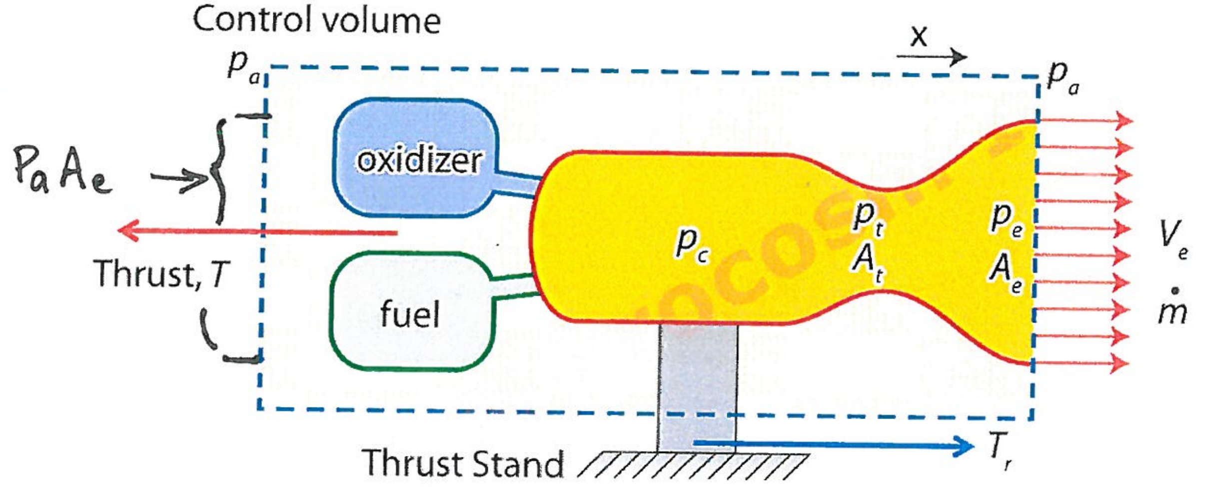
  for a set of mass particles

- Transform to Control Volume using Reynolds Transport Theorem:

$$\sum \vec{F} = \frac{d}{dt}(m\vec{V}) = \underbrace{\frac{d}{dt}\left(\int_{cv} \vec{V}\rho \, d\forall\right)}_{=0 \text{ steady}} + \underbrace{\int_{cs} \vec{V}\rho(\vec{V}\cdot\vec{n})\,dA}_{\text{discrete, 1D outlet}}$$

$$= (\dot{m}\vec{V})_{out} - \cancel{(\dot{m}\vec{V})_{in}}^{\to 0}$$

# Text 18.1: Basic Rocket Equations




(the structure holding the rocket in place feels a reaction force equal in magnitude to the thrust but in the opposite direction)



- Continue x-momentum balance:

$$\sum F_x = (\dot{m} V_x)_{out}$$ where $\dot{m}$ = exhaust mass-flow rate $= \rho_e V_e A_e$

$$T_r + P_a A_e - P_e A_e = \dot{m} V_e$$

$$T_r = \dot{m} V_e + A_e (P_e - P_a)$$ acting in +x direction

or $$\boxed{T = |T_r| = \dot{m} V_e + A_e (P_e - P_a)}$$ acting in −x direction

$$\underbrace{\dot{m} V_e}_{\text{momentum thrust}} \quad \underbrace{A_e (P_e - P_a)}_{\text{pressure thrust}}$$

generally, $T_{press} \ll T_{mom}$

<u>note</u>: $V_e$ = velocity of exhaust relative to rocket

$P_a \Rightarrow 0$ outside atmosphere

# Text 18.1: Basic Rocket Equations

- so Thrust $T = \dot{m} V_e + A_e (P_e - P_a)$

How to maximize T?

- Exit flow is a supersonic nozzle, so could increase $A_e$ → increase $V_e$

  but decrease $P_e$

  (don't want $(P_e - P_a) < 0$ )

  So large $A_e$ OK in vacuum ($2^{nd}$ stage)

  Also, larger nozzle → weight!

- For fixed geometry and $\dot{m}$, could increase $V_e$ by increasing chamber temp and pressure by changing propellant properties/mixture.

  But higher $P_c$ and $T_c$ require more strength (weight) and more cooling (cost, complexity, efficiency reduced).

- Could increase $\dot{m}$, but go through fuel more quickly.

# Text 18.1: Basic Rocket Equations

Measures of rocket performance

- "Effective exhaust velocity"

$$\boxed{V_{eq} \equiv \frac{T}{\dot{m}} = V_e + \left(\frac{P_e - P_a}{\dot{m}}\right) A_e}$$

One measure of how efficiently the engine converts propellant mass to thrust

- "Characteristic exhaust velocity"

$$\boxed{C^* \equiv \frac{P_c A_t}{\dot{m}}}$$

$P_c$ = chamber pressure

$A_t$ = throat area

$\dot{m}$ = prop. mass flow rate

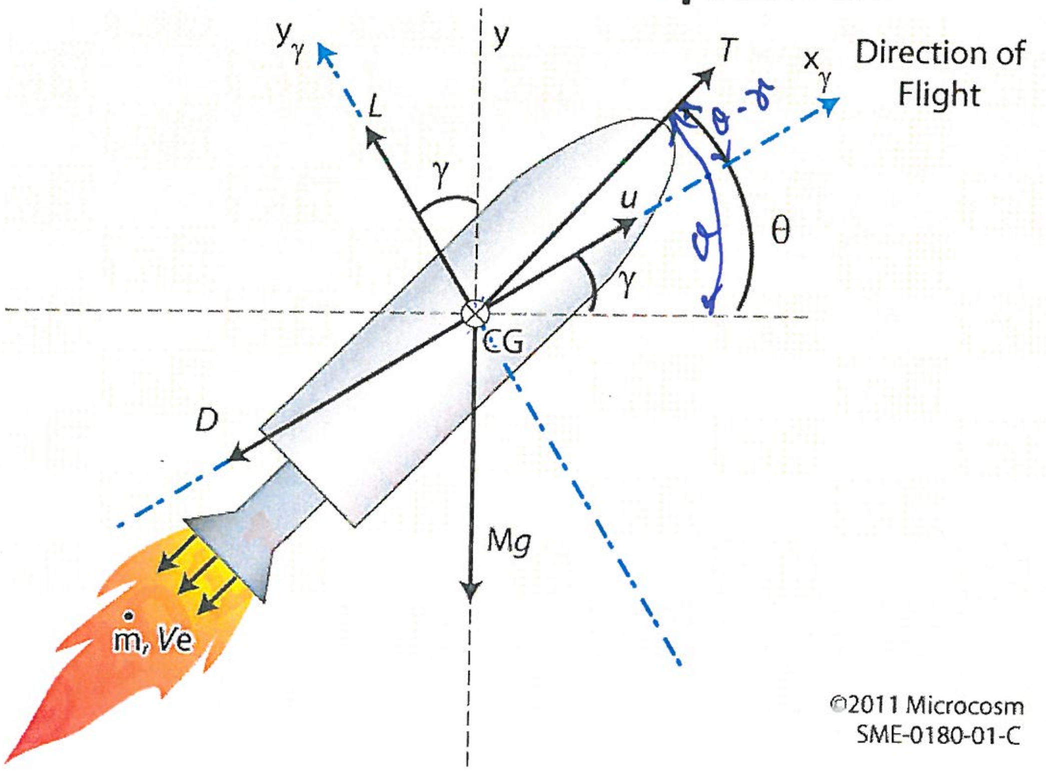
A measure of combustion efficiency.

Independent of nozzle design.

Depends mostly on the chemistry of the particular propellants.

$$\frac{C^*_{experiment} \rightarrow \text{from ground experiments}}{C^*_{theory} \longrightarrow \text{from thermochemistry theory}} \sim 90-98\%$$

# Text 18.1: Basic Rocket Equations




Derive the Ideal Rocket Equation

- Most general case: T: thrust (acting along rocket axis)

  Mg: weight of rocket (+ propellant on board)

  D: atmospheric drag

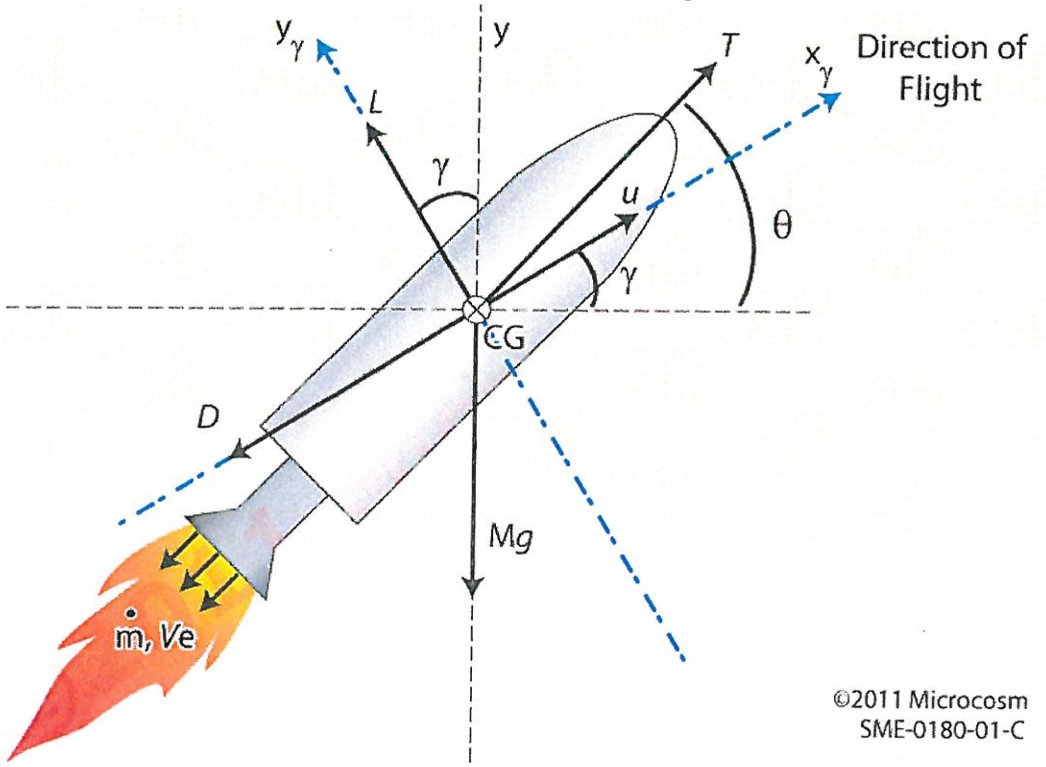
  L: atmospheric lift

- Note that direction of flight $\neq$ direction of thrust

- All variables are dynamic functions of time, so force/momentum balance is instantaneous

- For real rockets launching from Earth, want $\frac{T}{mg} \gtrsim 1.3$

# Text 18.1: Basic Rocket Equations






- Newton's $2^{nd}$ Law in direction of flight ($\gamma$):

$$\Sigma F_\gamma = M \frac{dV}{dt} = \underbrace{T\cos(\theta - \gamma)}_{\text{thrust in direction of flight}} - \underbrace{D}_{\text{drag}} - \underbrace{Mg\sin\gamma}_{\text{weight component}}$$

(note lift term is $\perp$ to direction of flight.)

where $T = \underbrace{\dot{m} V_e}_{\text{momentum thrust}} + \underbrace{A_e (P_e - P_a)}_{\text{pressure thrust}}$

# Text 18.1: Basic Rocket Equations

- Calculate velocity ($V_f$) attained by a rocket after a given time, integrate momentum eqn from an initial time ($t_0$) to a final time ($t_f$):

$$\frac{dV}{dt} = \frac{T}{M} - \frac{T}{M}\left[1-\cos(\theta-\gamma)\right] - \frac{D}{M} - g\sin\gamma$$

$$\int_{V_0}^{V_f} dV = \int_{t_0}^{t_f} \frac{T}{M}\,dt - \int_{t_0}^{t_f} \frac{T}{M}\left[1-\cos(\theta-\gamma)\right]dt - \int_{t_0}^{t_f} \frac{D}{M}\,dt - \int_{t_0}^{t_f} g\sin\gamma\,dt$$

or:

$$\Delta V = \underbrace{\Delta V_{prop}}_{\text{ideal velocity gain due to rocket thrust}} \underbrace{- \Delta V_{steering} - \Delta V_{drag} - \Delta V_{gravity}}_{\text{thrust losses due to angle of attack, drag, and gravity}}$$

↓

$\cong$ 1300 - 1700 m/s for launches to LEO

# Text 18.1: Basic Rocket Equations

- Simplify $\Delta V$ equation:

Substitute $V_{eq} \equiv \frac{T}{\dot{m}}$

and realize $\underbrace{-\frac{dM}{dt}}_{\text{rate at which rocket loses mass}} = \underbrace{\dot{m}}_{\text{mass flow rate leaving rocket as exhaust}}$

Also, let $\theta = \gamma$

neglect drag $D$

neglect gravity

So now,

$$\Delta V = \int_{t_0}^{t_f} \frac{T}{M}\, dt - 0 - 0 - 0$$

$$= V_{eq} \int_{t_0}^{t_f} \frac{\dot{m}}{M}\, dt$$

$$= V_{eq} \int_{M_0}^{M_f} \frac{dM}{M} \longrightarrow$$

# Text 18.1: Basic Rocket Equations

$$\Delta V = V_{eq} \int_{M_0}^{M_f} \frac{1}{M}\, dM$$

$$\boxed{\begin{aligned} \Delta V &= -V_{eq} \ln\left(\frac{M_f}{M_0}\right) = V_{eq} \ln\left(\frac{M_0}{M_f}\right) \\ &= V_{eq} \ln\left(\frac{M_0}{M_0 - M_p}\right) \end{aligned}}$$

<u>Ideal Rocket Equation</u>

- where $V_{eq} \equiv \frac{T}{\dot{m}} = V_e + \left(\frac{P_e - P_a}{\dot{m}}\right) A_e$

$M_0$ = initial mass of rocket + propellant, at $t_0$

$M_f$ = final $= M_0 - M_p$, at $t_f$

$M_p$ = mass of propellent burned between $t_0$ and $t_f$

- recall assumptions:

weight = negligable (far from large body)

drag = negligable (out of atmosphere)

- Define mass ratio $= \frac{M_f}{M_0}$

# Text 18.1: Basic Rocket Equations

Pause - Where are we?

- For changing orbits, we framed our equations in terms of $\Delta V$ required.

  Now we can calculate how much $\Delta V$ our rocket can give us, in terms of mass properties and engine performance.

- $$\Delta V = V_{eq} \ln\left(\frac{M_0}{M_f}\right) = V_{eq} \ln\left(\frac{M_0}{M_0 - M_p}\right)$$

  where $V_{eq} = \frac{T}{\dot{m}}$

  So for example, if rocket is

  50% propellant, $\Delta V_{avail} = 0.69\, V_{eq}$

  90% propellant, $\Delta V_{avail} = 2.30\, V_{eq}$

# Text 18.1: Basic Rocket Equations

"Specific Impulse" of a Rocket or Engine (like mpg)

- Total Impulse defined as:

$$I \equiv \int_0^t T\,dt \quad \text{for } T = f(t) \text{ (non-constant)}$$

= proportional to energy provided by engine(s)

$= T\,t_{burn}$ for constant T over burn time $t_{burn}$

- Specific Impulse defined as: $\frac{\text{Total Impulse}}{\text{weight of propellants}}$

$$I_{sp} \equiv \frac{\int_0^t T\,dt}{g_0 \int_0^t \dot{m}\,dt} \quad (\dot{m} = \text{rate of propellant burn})$$

for T = constant and $\dot{m}$ = constant

$$\boxed{I_{sp} = \frac{T}{\dot{m}\,g_0} = \frac{V_{eq}}{g_0}}$$

where

$$V_{eq} \equiv V_e + \left(\frac{P_e - P_a}{\dot{m}}\right) A_e$$

$= T/\dot{m}$

where T = constant thrust

$\dot{m}$ = rate of burn of propellants = constant

$g_0$ = grav constant at Earth surface
$= 9.8\ m/s^2$

# Text 18.1: Basic Rocket Equations

## More on Specific Impulse

- Isp is a measure of how efficiently we produce thrust. The higher the Isp, the less propellant used to achieve a given $\Delta V$.

- For launch propulsion, thrust T is somewhat more important than Isp, because we must lift up through atmosphere against gravity.

- For in-space propulsion, Isp is most important to minimize the propellant required to achieve the required orbit/velocity.

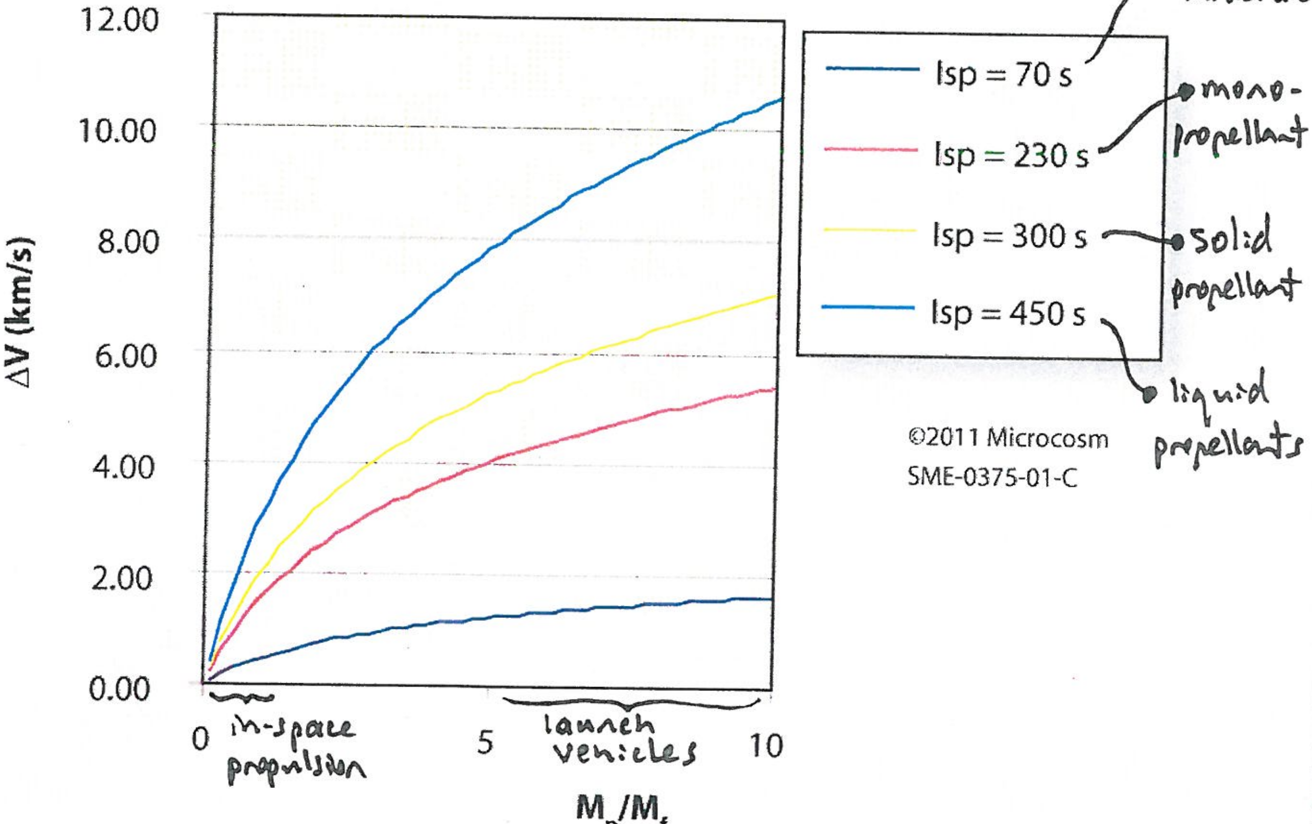
- Substitute Isp equation into Rocket Eqn:

$$\Delta V = V_{eq} \ln\left(\frac{M_0}{M_f}\right) = I_{sp}\, g_0 \ln\left(\frac{M_0}{M_f}\right)$$

So $\Delta V \propto I_{sp}$ → improvements in Isp give large effect on $\Delta V$

various engine types

# Text 18.1: Basic Rocket Equations




- Alternate forms of Rocket Equation in terms of propellant mass ($M_p$):

$$M_p = M_f\left(e^{\frac{\Delta V}{V_{eq}}} - 1\right) = M_f\left[e^{\left(\frac{\Delta V}{I_{sp}\, g_0}\right)} - 1\right] \quad \left(\text{plotted above}\right)$$

$$M_p = M_0\left(1 - e^{\frac{-\Delta V}{V_{eq}}}\right) = M_0\left[1 - e^{\left(\frac{-\Delta V}{I_{sp}\, g_0}\right)}\right]$$

18.1

# Text ~~18.2: Staging~~

| Propulsion Function | Typical $\Delta V$ and Other Requirements |
|---|---|
| ***Orbit Transfer to GEO (orbit insertion)*** | |
| Perigee Burn | 2,400 m/s |
| Apogee Burn | 1,500 (low inclination) to 1800 m/s (high inclination) |
| ***Initial Spin Up*** | 1–60 rpm |
| ***LEO to Higher Orbit Raising $\Delta V$*** | |
| Drag makeup $\Delta V$ | 60–500 m/s |
| Controlled Re-entry | 120–150 m/s |
| ***Acceleration to Escape Velocity from LEO Parking Orbit*** | 3,600–4,000 m/s into a heliocentric orbit |
| ***Orbit Maintenance*** | |
| Despin | 60 to 0 rpm |
| Spin control | ± 1 to ±5 rpm |
| Orbit correction $\Delta V$ | 15 to 75 m/s per year |
| East-West stationkeeping $\Delta V$ at GEO | 3 to 6 m/s per year |
| North-South stationkeeping $\Delta V$ at GEO | 45 to 55 m/s per year |
| Survivability or evasive maneuvers (highly variable) $\Delta V$ | 150 to 4,600 m/s |
| ***Attitude Control*** | 3–10% of total propellant mass |
| Acquisition of Sun, Earth, Star | low total impulse, typically <5,000 N-s, 1K to 10K pulses, 0.01 to 5.0 sec pulse width |
| On-orbit normal mode control with 3-axis stabilization, limit cycle | 100K to 200K pulses, minimum impulse bit of 0.01 N-s, 0.01 to 0.25 sec pulse width |
| Precession control (spinners only) | low total impulse, typically <7,000 N-s, 1K to 10K pulses, 0.02 to 0.2 sec pulse width |
| Momentum management (wheel unloading) | 5 to 10 pulse trains every few days, 0.02 to 0.10 s pulse width |
| 3-axis control during $\Delta V$ | on-off pulsing, 10K to 100K pulses, 0.05 to 0.20 s pulse width |



Table 18-2. Typical Functions and Requirements for Upper Stages and In-Space Propulsion

# Text ~~18.3~~ 18.1: Chemical Propulsion Systems

| ©2011 Microcosm Inc. Propulsion System | Orbit Insertion: Perigee | Orbit Insertion: Apogee | Orbit Maintenance and Maneuvering | Attitude Control | Typical Range of $I_{sp}$ (s) |
|---|---|---|---|---|---|
| ***Cold Gas*** | | | X | X | 45–73 |
| ***Solid*** | X | X | | | 290–304 |
| ***Liquid*** | | | | | |
| ***Monopropellant*** | | | X | X | 200–235 |
| ***Bipropellant*** | X | X | X | X | 274–467 |
| ***Electric*** | | X | X | X | 500–3,000 |

Table 18-3. Overview of Common Applications for Different Propulsion Systems.

- Real-World Isp :

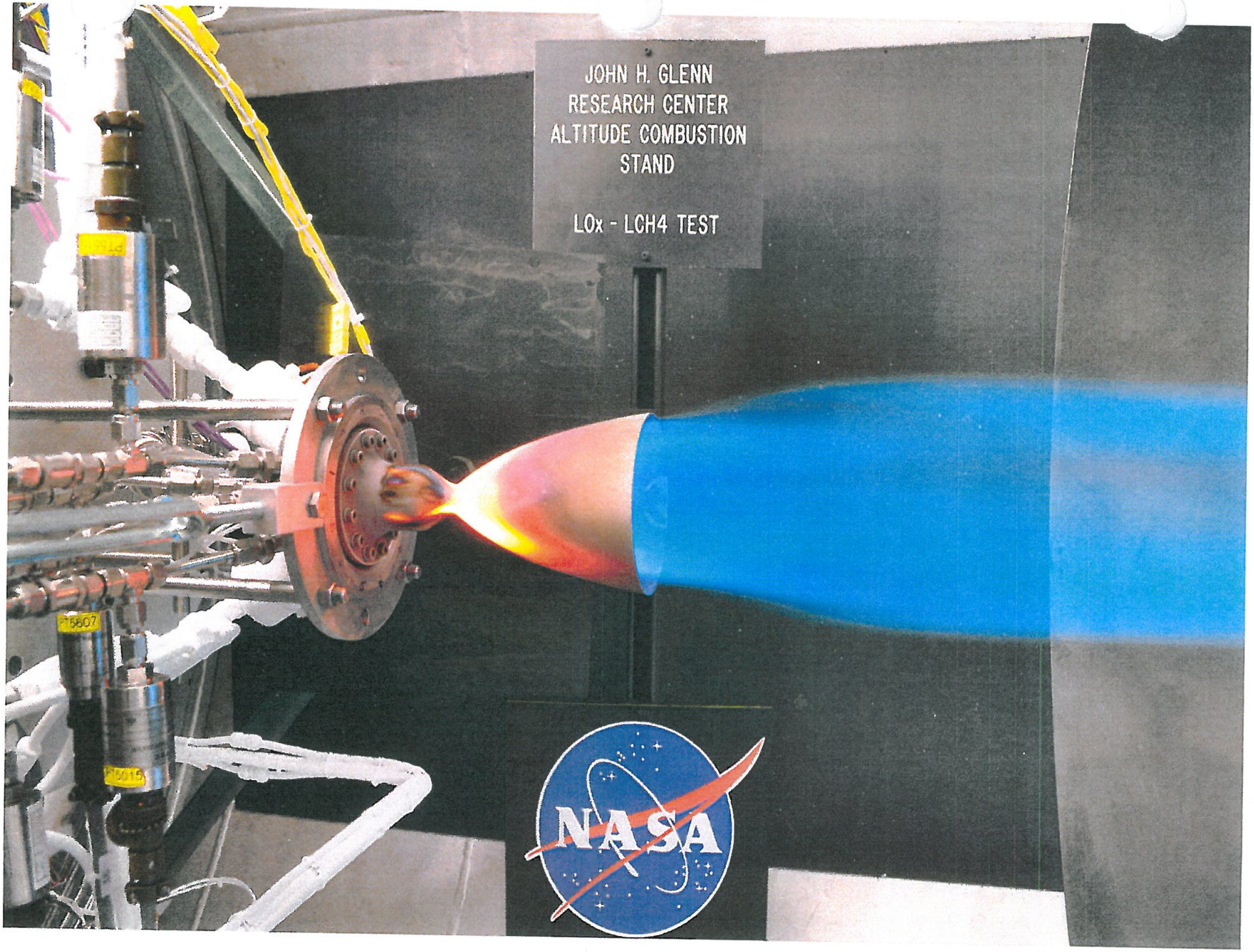
$$I_{sp} = \frac{T}{\dot{m} g_0}$$

$$= \frac{\dot{m} V_e + \overbrace{A_e (P_e - P_a)}^{\text{pressure thrust}}}{\dot{m} g_0}$$

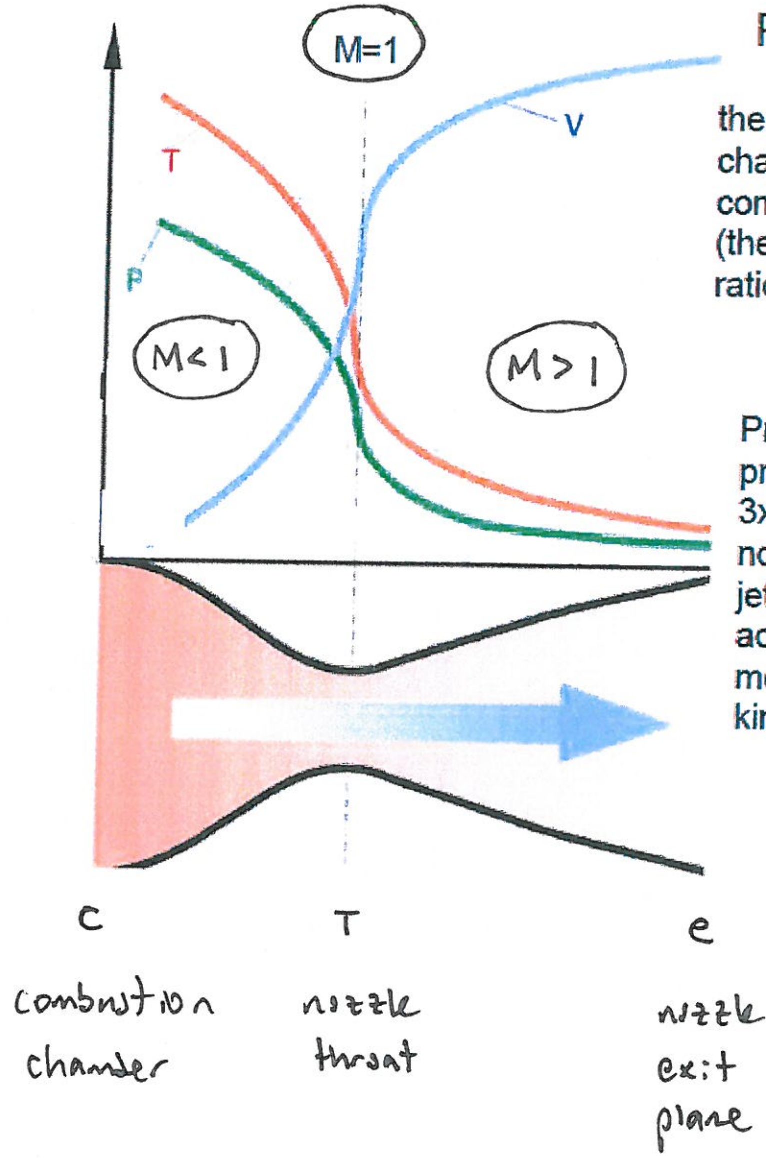
pressure thrust → increases with altitude until $P_a = 0$ (vac.)

$$\therefore \; I_{sp_{vacuum}} > I_{sp_{sealevel}}$$

A rocket gets more efficient as it leaves atmosphere!
(nothing to do with drag)


JOHN H. GLENN
RESEARCH CENTER
ALTITUDE COMBUSTION
STAND
LOx - LCH4 TEST
NASA

# Text 18.1: Basic Rocket Equations

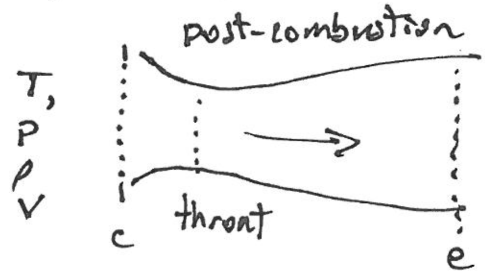



## Rocket nozzles

the hot gas produced in the combustion chamber is permitted to escape from the combustion chamber through an opening (the "throat"), within a high expansion-ratio 'de Laval nozzle'.

Provided sufficient pressure is provided to the nozzle (about 2.5-3x above ambient pressure) the nozzle *chokes* and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy.

## Nozzle Performance




Want thermodynamic relation for $V_e$

- Assumptions:
  - combustion products constant composition & homogeneous
  - Perfect gas $P = \rho \frac{R_0}{W} T$ ; $R_0$ = univ. gas constant, $W$ = molecular weight
  - constant $C_p$, $C_T$
  - 1-D, steady, isentropic flow, adiabatic

- Cons. Mass: $\dot{m} = \rho V A$

Cons. Energy: $\underbrace{\frac{1}{2} V^2}_{\text{kinetic}} + \underbrace{C_p T}_{\text{internal}} = \text{constant}$

- Adiabatic relations:

$$\frac{P}{\rho^{\gamma}} = \text{constant} \quad ; \quad \frac{T}{T_c} = \left(\frac{\rho}{\rho_c}\right)^{\gamma-1} = \left(\frac{P}{P_c}\right)^{\frac{\gamma-1}{\gamma}}$$

- Want an expression for $V_e$, to see influences on thrust.

Start by maximizing $\dot{m}$ through nozzle, so need expression for $\dot{m}$ ...

- Energy eqn from c :

$$\frac{1}{2}\cancel{V_c^2}^{\to 0} + C_p T_c = \frac{1}{2} V^2 + C_p T$$

$$\left(\frac{\dot{m}}{\rho A}\right)^2$$

$$\left(\frac{\dot{m}}{\rho A}\right)^2 = C_p \left(T_c - T\right)$$

$$\frac{\dot{m}}{A} = \left[\rho\, C_p \left(T_c - T\right)\right]^{1/2}$$

$$T_c\left(1 - \frac{T}{T_c}\right)$$

$$\frac{p}{\rho^\gamma} = \frac{p_c}{\rho_c^\gamma}$$

$$T_c\left(1 - \frac{p}{p_c}^{\frac{\gamma-1}{\gamma}}\right)$$

$$\rho^\gamma = \frac{p}{p_c}\rho_c^\gamma$$

$$\rho = \left[\frac{p}{p_c}\rho_c^\gamma\right]^{+\frac{1}{\gamma}}$$

- Collect :

$$\frac{\dot{m}}{A} = \left\{\frac{2\gamma}{\gamma-1} p_c \rho_c \left(\frac{p}{p_c}\right)^{\frac{2}{\gamma}}\left[1 - \left(\frac{p}{p_c}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}^{1/2} \quad 6.26$$

recall that $\dot{m}_{max}$ when throat velocity is <u>sonic</u> : $V_t = a$

- Choked nozzle : sonic throat conditions

$$\frac{P_t}{P_c} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_t}{T_c} = \frac{2}{\gamma+1}$$

- Plug into 6.16 to get

$$\left.\frac{\dot{m}}{A}\right|_{max} = \left(\gamma P_t \rho_t\right)^{1/2} = \left(\rho_t V_t\right)$$

where $V_t$ = "critical" (sonic) throat velocity

$$= a_t$$

$$= \left(\gamma \frac{P_t}{\rho_t}\right)^{1/2} = \left(\gamma \frac{R_0}{W} T_t\right)^{1/2}$$

- So $\dot{m}$ determined solely by throat conditions :

$$\boxed{\dot{m} = \sqrt{\gamma}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{P_c A_t}{\sqrt{R_0 T_c / W}}} \quad 6.17$$

nozzle mass-flow rate in terms of throat area and combustion chamber P and T

- Expression for $V_e$ :

Energy : $\frac{1}{2} V_e^2 + C_p T_e = \frac{1}{2} \cancel{V_c^2} + C_p T_c$ $\quad V_c \cong 0$

$$V_e = \sqrt{2 C_p (T_c - T_e)}$$

$$\boxed{V_e = \sqrt{\frac{2 \gamma R_0 T_c}{(\gamma - 1) W} \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma - 1}{\gamma}} \right]}}$$ 6.20

- To maximize $V_e$ (and therefore thrust):

  - increase $\frac{P_c}{P_e}$

  - increase $T_c$

  - decrease $W$ molecular weight

- Common shorthands : "characteristic velocity" $C^*$

$$\dot{m} = p_c A_t / C^*$$

$$\text{where } C^* = \sqrt{\frac{R_o T}{w}} \Big/ \left[ \sqrt{\gamma} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right]$$ 6.18

- "Characteristic thrust coefficient" : $C_F^o$

$$V_e = C^* C_F^o$$

where

$$C_F^o = \sqrt{ \left[ \gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{(\gamma-1)}} \right] \frac{2\gamma}{\gamma-1} \left[ 1 - \left( \frac{p_e}{p_c} \right)^{\frac{\gamma-1}{\gamma}} \right] }$$ 6.21

- Exit-to-throat area ratio :

$$\frac{A_e}{A_t} \overset{\text{continuity}}{=} \frac{\rho_t V_t}{\rho_e V_e} = \frac{\gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}} \left( \frac{p_c}{p_e} \right)^{\frac{1}{\gamma}}}{C_F^o}$$ 6.22

## Nozzle Design - Real World


- Thrust eqn:

$$F = \dot{m} V_e + A_e (P_e - P_a)$$

- $A_e$ for max thrust?

$(P_c = \text{const})$

$$\delta F = \dot{m}\,\delta V_e + \delta A_e (P_e - P_a) + A_e\,\delta P_e$$

for constant $\dot{m}$

Momentum:

$$\dot{m}\,\delta V_e + A_e\,\delta P_e = 0$$

solve for
$$\frac{\delta F}{\delta A_e} = \dot{m}\frac{\delta V_e}{\delta A_e} + (P_e - P_a) + A_e \frac{\delta P_e}{\delta A_e}$$

$$= (P_e - P_a) + \frac{1}{\delta A_e}\underbrace{(\dot{m}\,\delta V_e + A_e\,\delta P_e)}_{\to 0 \text{ Mom.}}$$

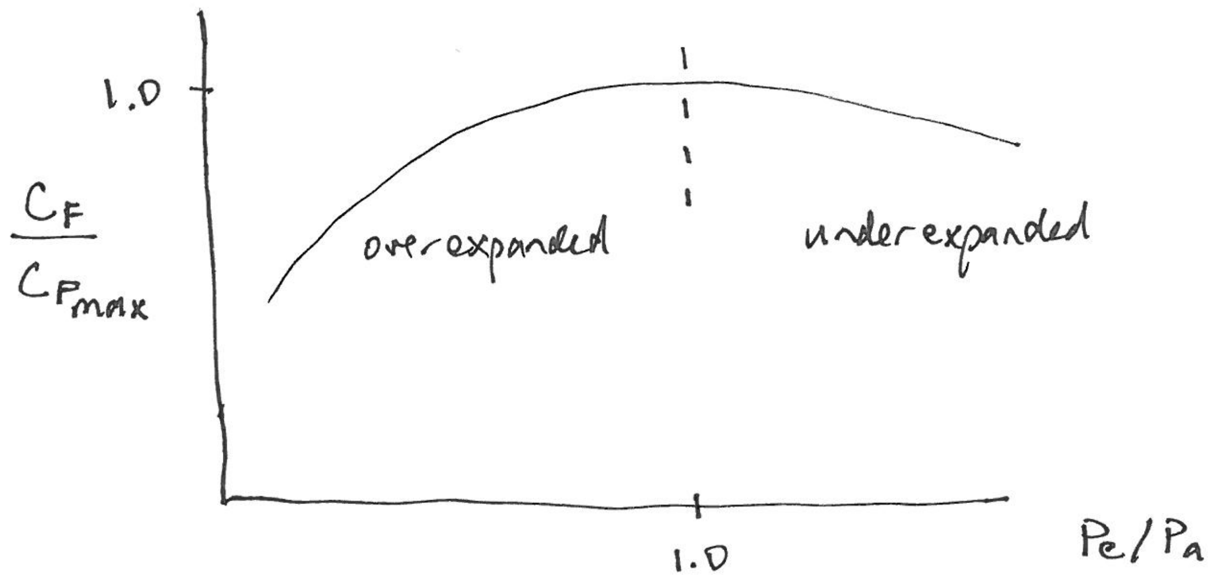
$\boxed{\text{So max thrust when } P_e = P_a}$

ideal expansion aft of nozzle

- Effect of non-ideal expansion:

define thrust coefficient

$$C_F = \frac{F}{P_c A_T} = C_F^0 + \frac{A_e}{A_t}\left(\frac{P_e}{P_c} - \frac{P_a}{P_c}\right) \quad 6.23$$




- somewhat worse to overexpand
- adaptive geometry nozzles: complex, thrust-efficient

  Aerospike engine

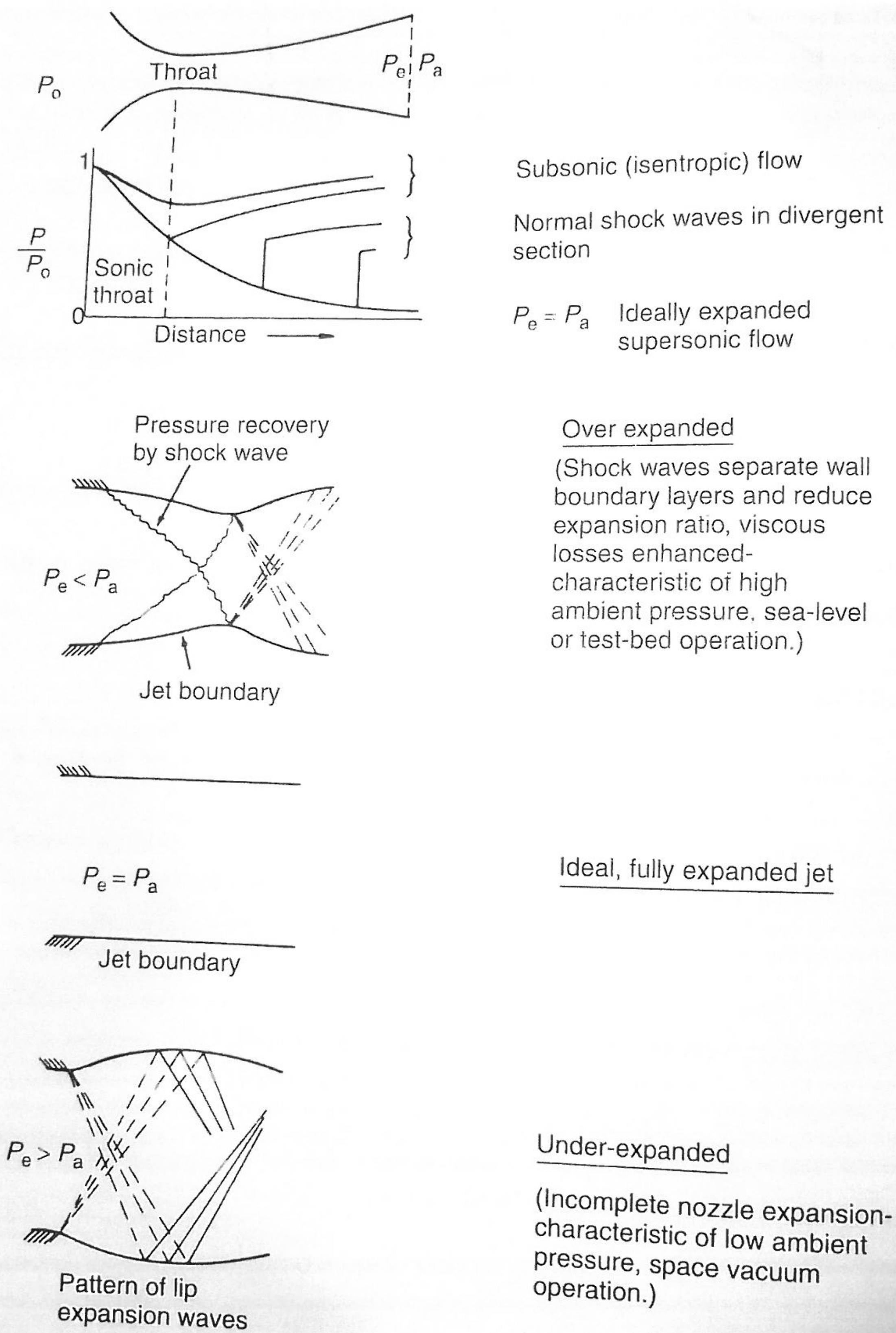



**Figure 6.6** Nozzle flows: non-ideal expansion

18.1

# Text ~~18.2: Staging~~

- Recall

$$I_{sp} = \frac{T}{\dot{m}\, g_0}$$

$$= \frac{\overbrace{\dot{m}\, V_e}^{\text{momentum}} + \cancel{A_e (P_e - P_a)}}{\dot{m}\, g_0}$$

generally much less than momentum thrust

$$\cong \frac{V_e}{g_0}$$

- From previous page:

$$\boxed{I_{sp} \cong \frac{1}{g_0} \sqrt{2 \frac{R_u}{MW} \left(\frac{k}{k-1}\right) T_c \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right]}}$$

(same as text eqn 18-17, but without pressure thrust term)

$R_u$ = universal gas constant $= 8.314 \frac{J}{mol\,K}$

$MW$ = exhaust molecular weight

$T_c$ = combustion temp

$k = C_p / C_v$

$P_e$ = nozzle exit pressure

$P_c$ = combustion pressure

<u>For high $I_{sp}$, want:</u>

low exhaust MW

high combustion temp

high press. ratio $\frac{P_c}{P_e}$

# Rocket Engine Efficiency: ISP Considerations

- Chemical propellants: $175 < ISP < 400$
- For high ISP:
  - high exhaust-gas temp: large heat of combustion
  - Low molecular-density combustion products: H, C, O, Fl, Al, Be, Li, etc
- Propellant density:
  - lower density = higher tank volume and mass
- Coolant properties
- Safety & handling:
  - Corrosive, flammable, toxic
  - All three!
- Availability, storage
- Environmental concerns

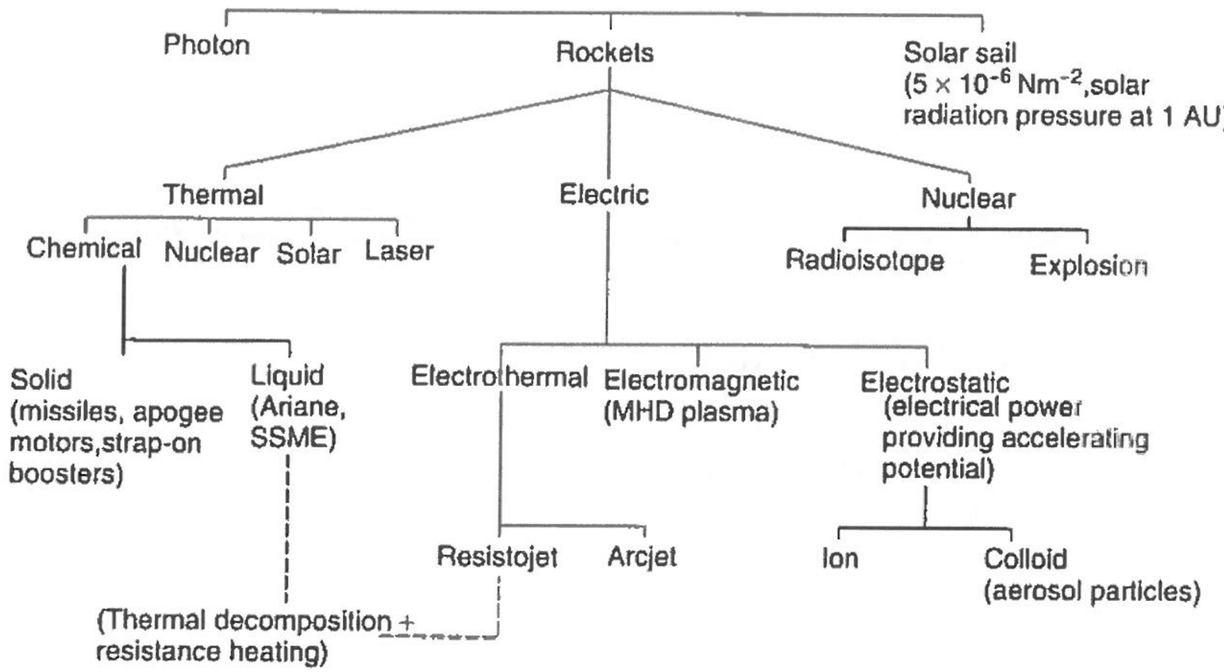



**Figure 6.1** Propulsion systems classification

# Propellants

- Exothermic heat ~~[illegible]~~ release in nozzle gas flow

    specific energy content
    rate of heat release
    storage, handling
    plumbing
    failure modes
    toxicity (before and after combustion)
    demands on system mat'l properties

## Liquid Propellants

- bi-propellant : 2 liquids

    amb-combust : hypergolic

    monopropellant : exothermic decomposition

- requires pumping system, tanks, plumbing

    throttling
    stop/start

- May require cryogenic storage – time limit

- Recall : want high $T_c$ and low $W$ for max $V_e$ 6.20

- Common : $LOX/LH_2$ and $LOX/RP1$ (kerosene) – Large thrust
    $MMH/N_2O_4$ – Lower thrust

**Table 6.1** Liquid propellants

| Fuel | Oxidizer | Molecular weight of products | Combustion temperature $T_c$ (K) | Ideal specific impulse (s) | Mean density $kg/m^3$ |
|---|---|---|---|---|---|
| $H_2$ (hydrogen) | $O_2$ (oxygen) | 10 | 2980 | 390 | 280 |
| | $F_2$ (fluorine) | 12.8 | 4117 | 410 | 460 |
| Kerosine | $O_2$ | 23.4 | 3687 | 301 | 1020 |
| | $F_2$ | 23.9 | 3917 | 320 | 1230 |
| | RFNA (red fuming nitric acid) | 25.7 | 3156 | 268 | 1355 |
| | $N_2O_4$ (nitrogen tetroxide) | 26.2 | 3460 | 276 | 1260 |
| | $H_2O_2$ (hydrogen peroxide) | 22.2 | 3008 | 278 | 1362 |
| $N_2H_4$ (hydrazine) | $O_2$ | 19.4 | 3410 | 313 | 1070 |
| | $HNO_3$ (nitric acid)* | 20 | 2967 | 278 | 1310 |
| UDMH | $O_2$ | 21.5 | 3623 | 310 | 970 |
| $(CH_3)_2NNH_2$ (unsymmetrical dimethyl hydrazine) | $HNO_3^*$ | 23.7 | 3222 | 276 | 1220 |
| | * hypergolic | | | | |
| *Monopropellants* | | | | | |
| $N_2H_4$ | | 10.3 | 966 | 199 | 1011 |
| $H_2O_2$ | | 22.7 | 1267 | 165 | 1422 |

*Note: All quoted values are for $p_c = 7$ MPa with an ideal expansion to $p_e = 0.1$ MPa. Higher chamber pressures admit increases in $I_{SP}$—for example, at 20 MPa, LOX/LH$_2$ yields a specific impulse of ~460 s.

# Solid Propellants

- Solid Propellants :
  - Composite: fuel & oxidizer mixed & solidified with binder
  - Most (up to 80%) of the volume is oxidizer
  - Often used as low-altitude add-ons for liquid rockets

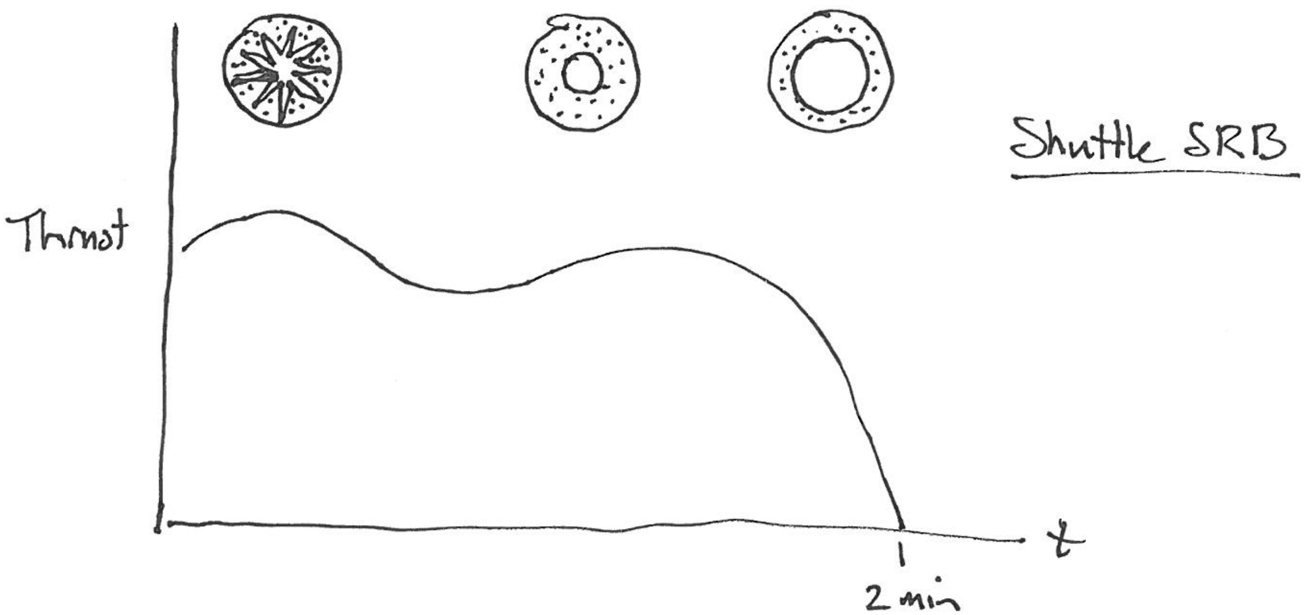
- Solid Propellants Pros:
  - Simple, few moving parts, no propellant feed systems
  - compact (higher density than some liquid props)
  - transportable, safe, low-cost
  - Scalable – massive thrust possible (SRB 3Mlbf)

- Solid Propellants Cons:
  - Entire stage is the combustion chamber – structural penalties & segment seals (shuttle Challenger)
  - Case heats during burn – more structural penalties
  - Propellant itself must load-bearing within the case (rocket body)
  - No stop or restart, no throttling (until recently)
  - Lower efficiency than liquid (lower exhaust velocities)
  - Thrust-vectoring requires complex gimbal seals

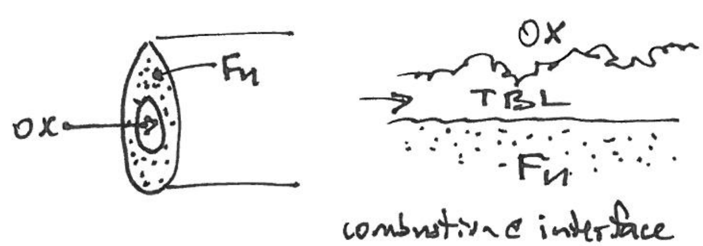
## Solid Propellants

- Lower ISP than liquid
- Throttle, stop/start almost impossible
- Poured or extruded, then cured - rubberlike
- Thrust-history tailoring:

grain-pattern evolves via combustion:

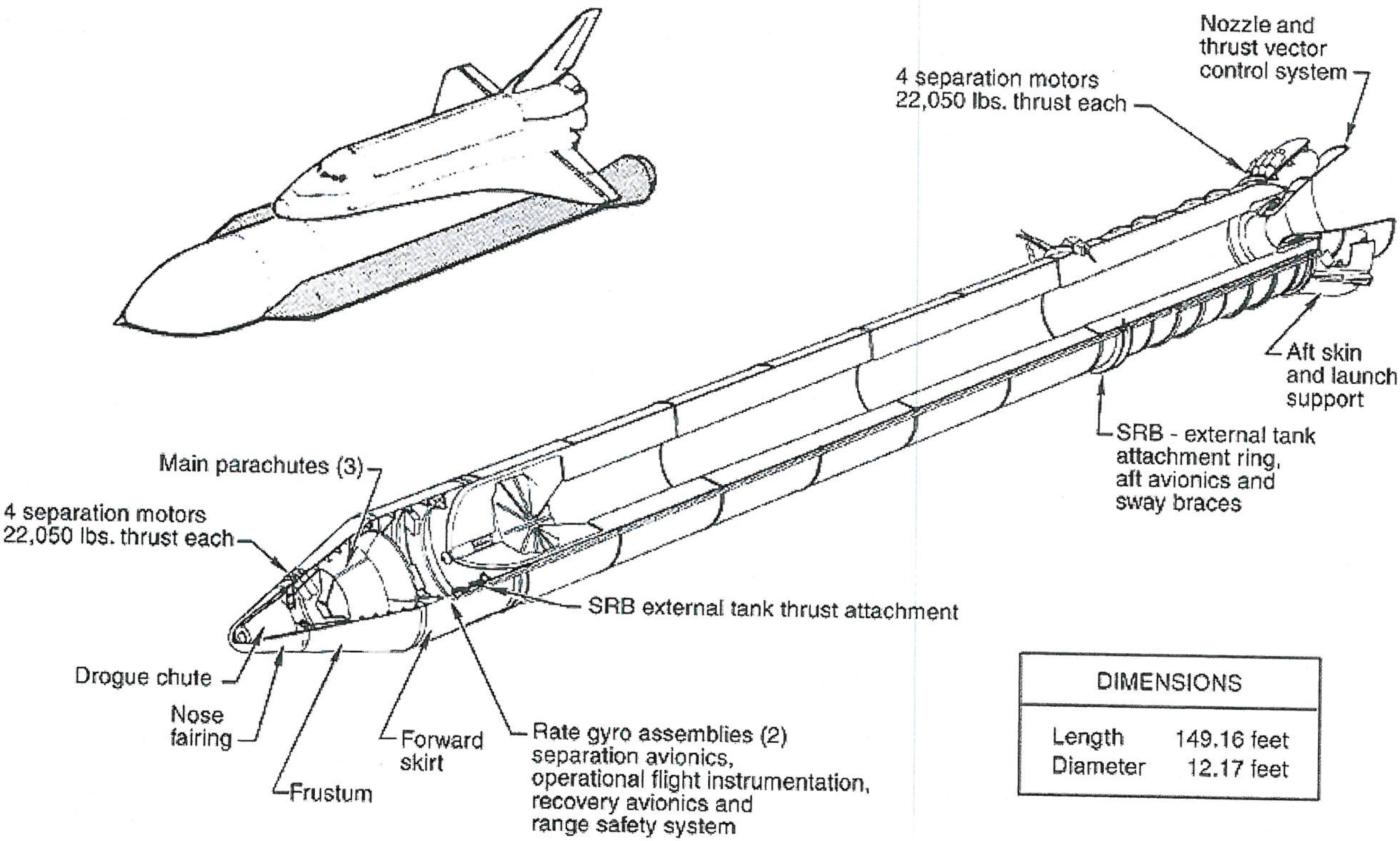



## Hybrid Propellants




- Liquid ox, solid fuel
- Throttle, stop/start
- ½ plumbing, storage complexity of liquid bi-propellants
- Fluid-solid interface combustion limits ISP

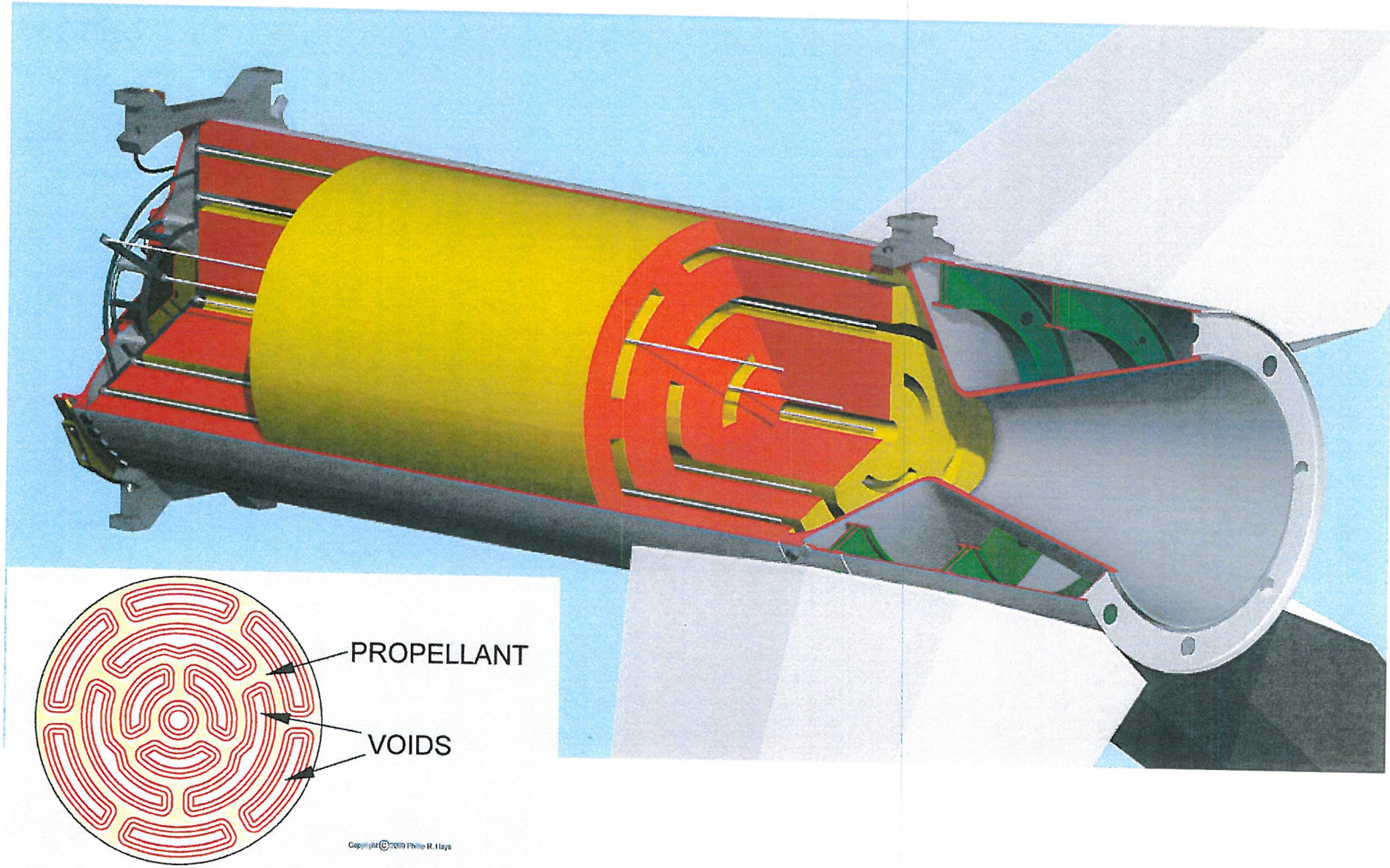
# Space Shuttle Solid Rocket Boosters



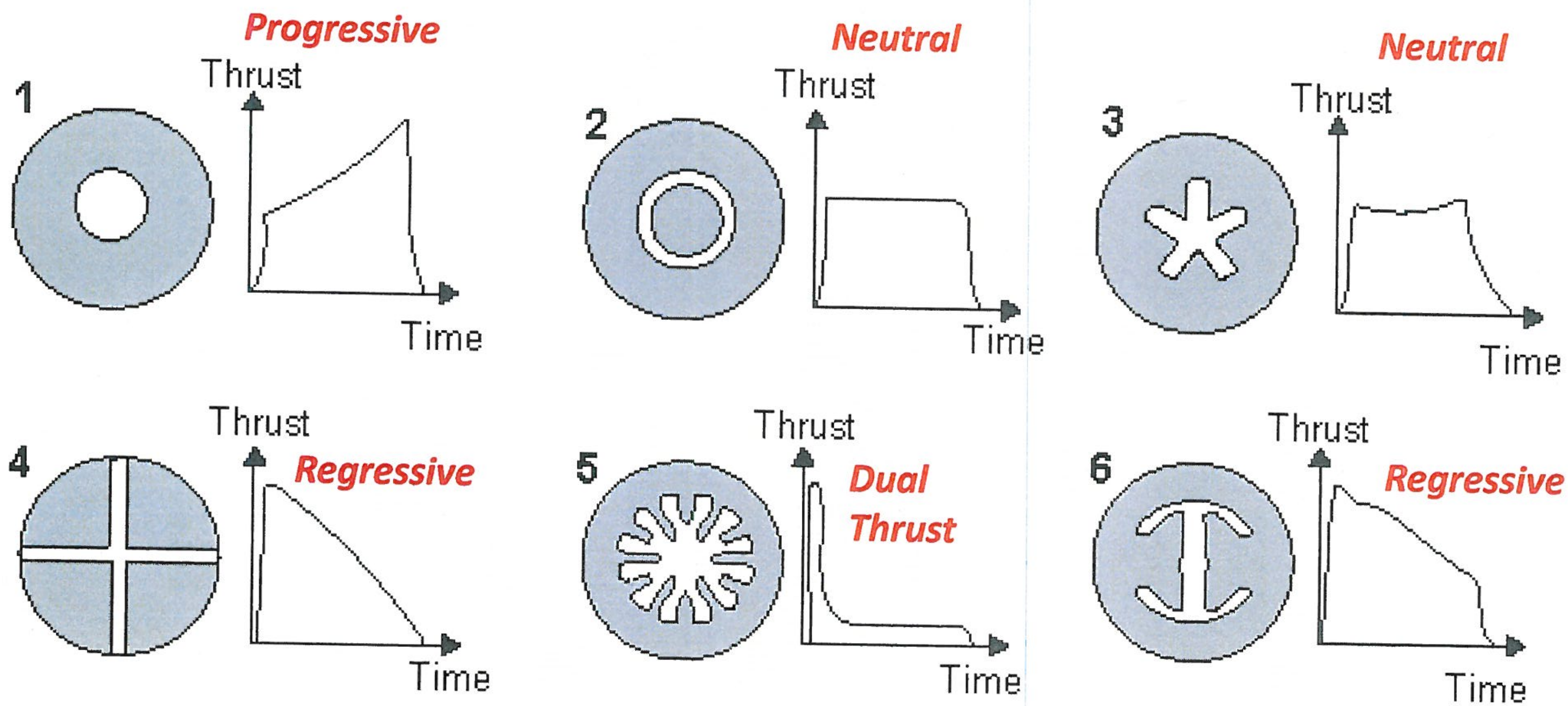

| DIMENSIONS | |
|---|---|
| Length | 149.16 feet |
| Diameter | 12.17 feet |

# Solid Rocket Grain Patterns

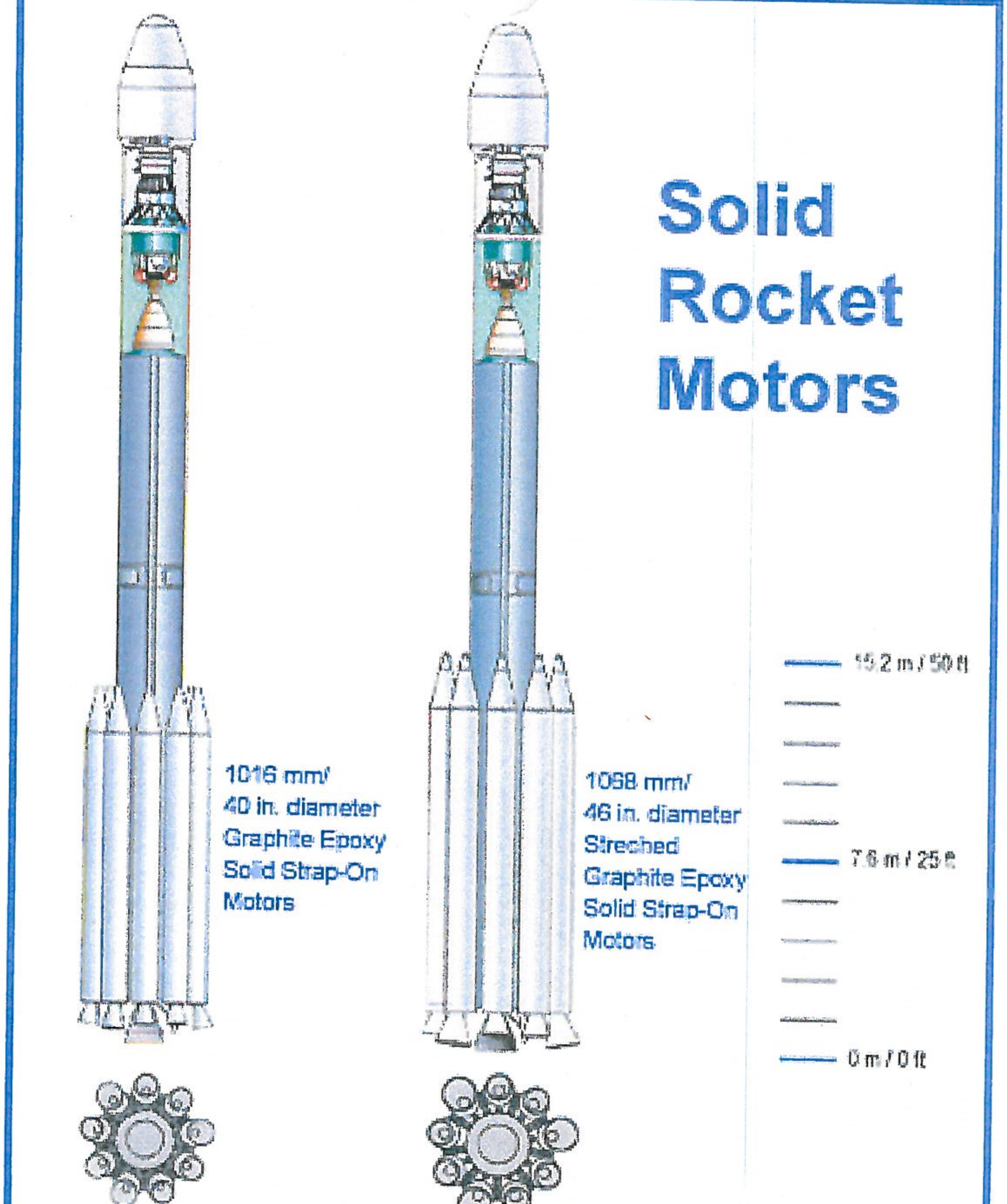
# Solid Rocket Grain Patterns



# Solid Rocket Grain Patterns



# Solid Rocket Motors



# Open-Cycle Design:

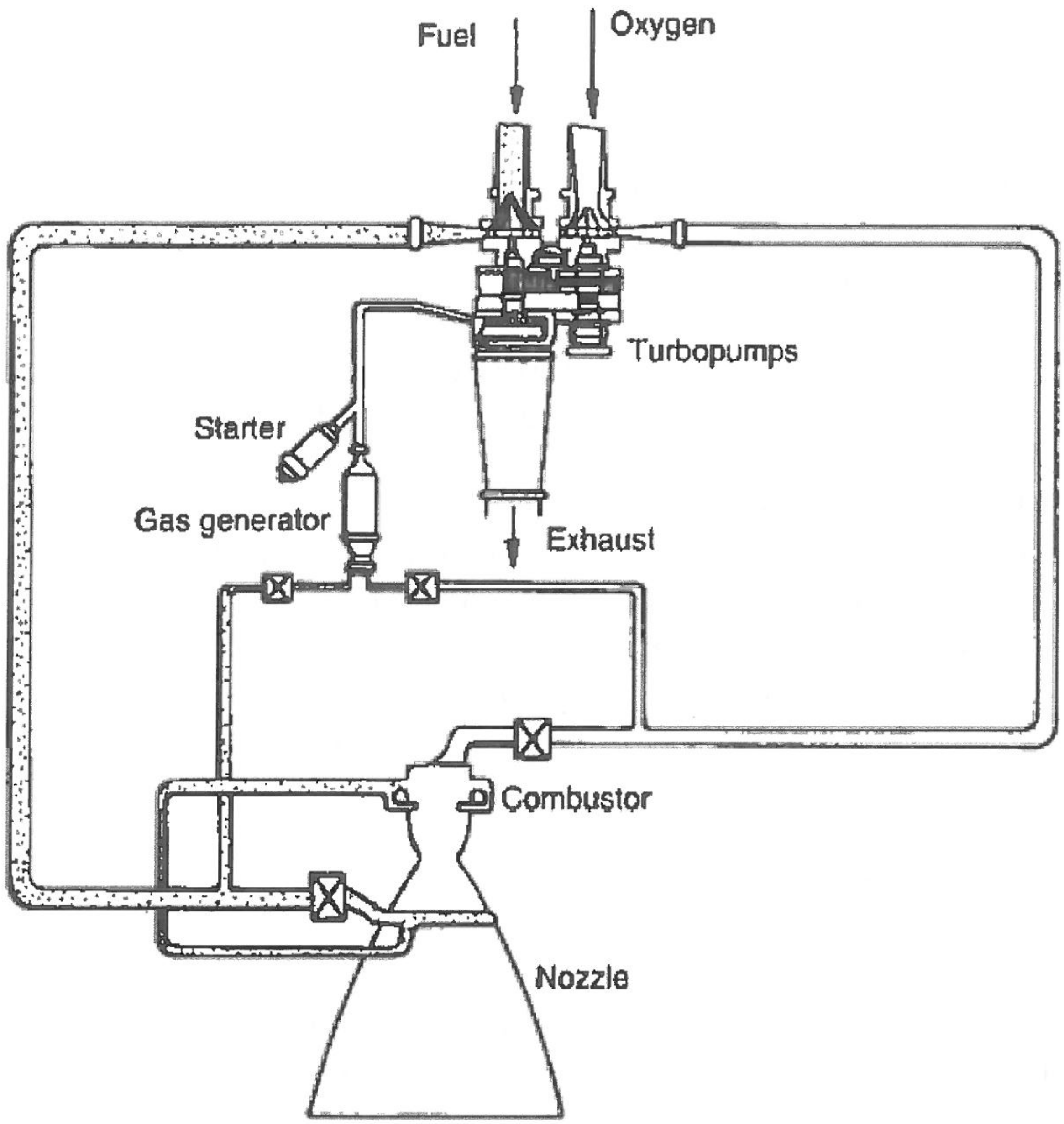
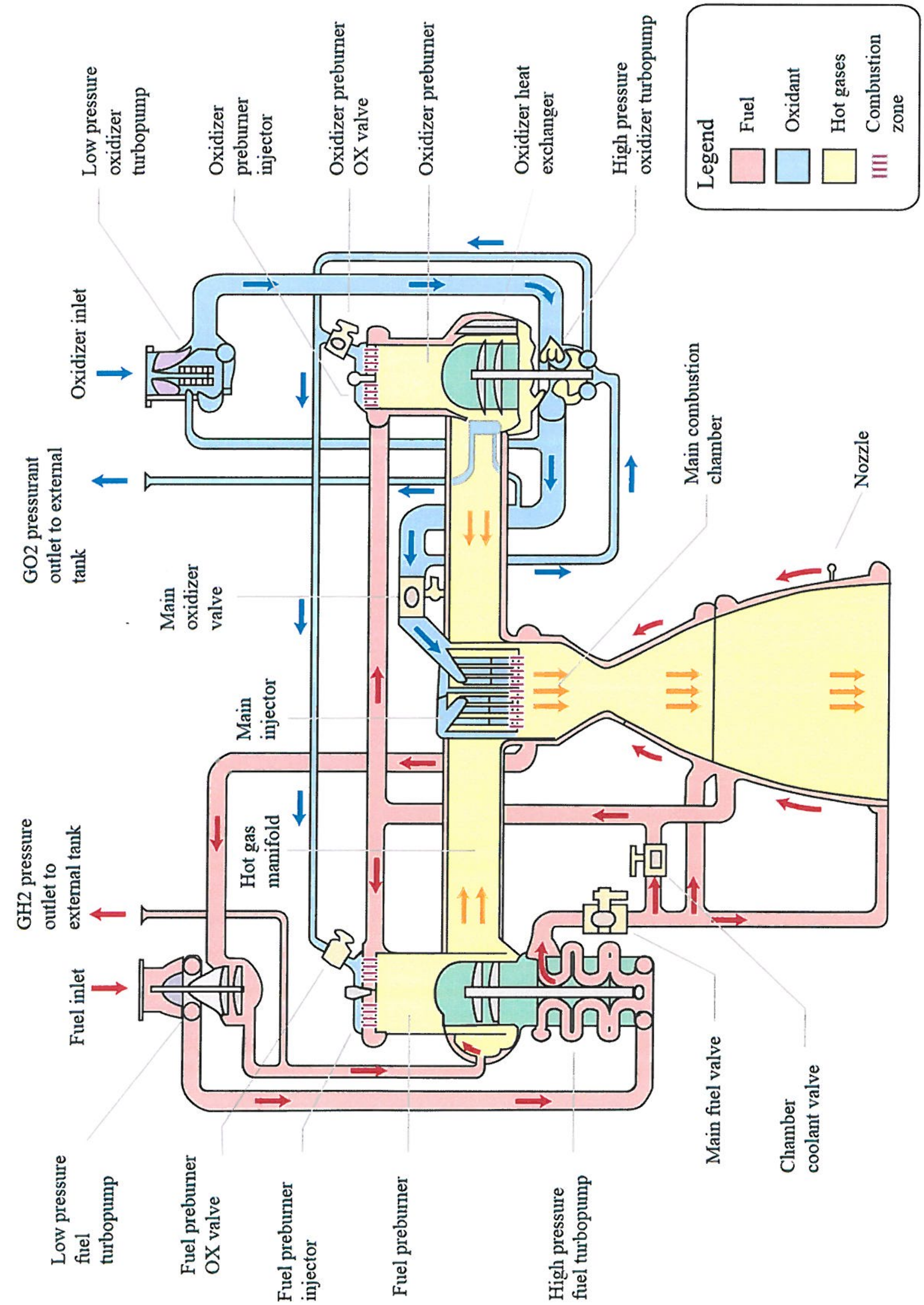



**Figure 6.10** Schematic of a liquid rocket motor

# Closed Cycle Design: Shuttle Main Engine



# Closed (SSME) vs Open (Vulcain):

**Table 6.2** Illustrative comparison of closed- and open-cycle engines

| | SSME | Vulcain (Ariane 5) |
|---|---|---|
| Thrust (kN): | | |
| Vacuum | 2090 | 1390 |
| Sea level | 1700 | 960 |
| Specific impulse (s): | | |
| Vacuum | 455 | 432 |
| Sea Level | 363 | 310 |
| Mixture ratio (stoichiometric 8:1 $2H_2 + O_2 \rightarrow 2H_2O$) | 6:1 | 5.3:1 |
| Chamber pressure (bar) | 207 | 108 |
| Nozzle area ratio | 77 | 45 |
| Flowrates (kg/s) | 468 (engine)<br>248 (pre-combustor) | 270<br>10 (gas generator) |
| Pump discharge pressure (bar) | 309 (LOX)<br>426 ($LH_2$) | 125 (LOX)<br>150 ($LH_2$) |
| Burn time (s) | 480 | 540 |
| Mass (kg) | 3022 | 1650 |

# Propellant Management:

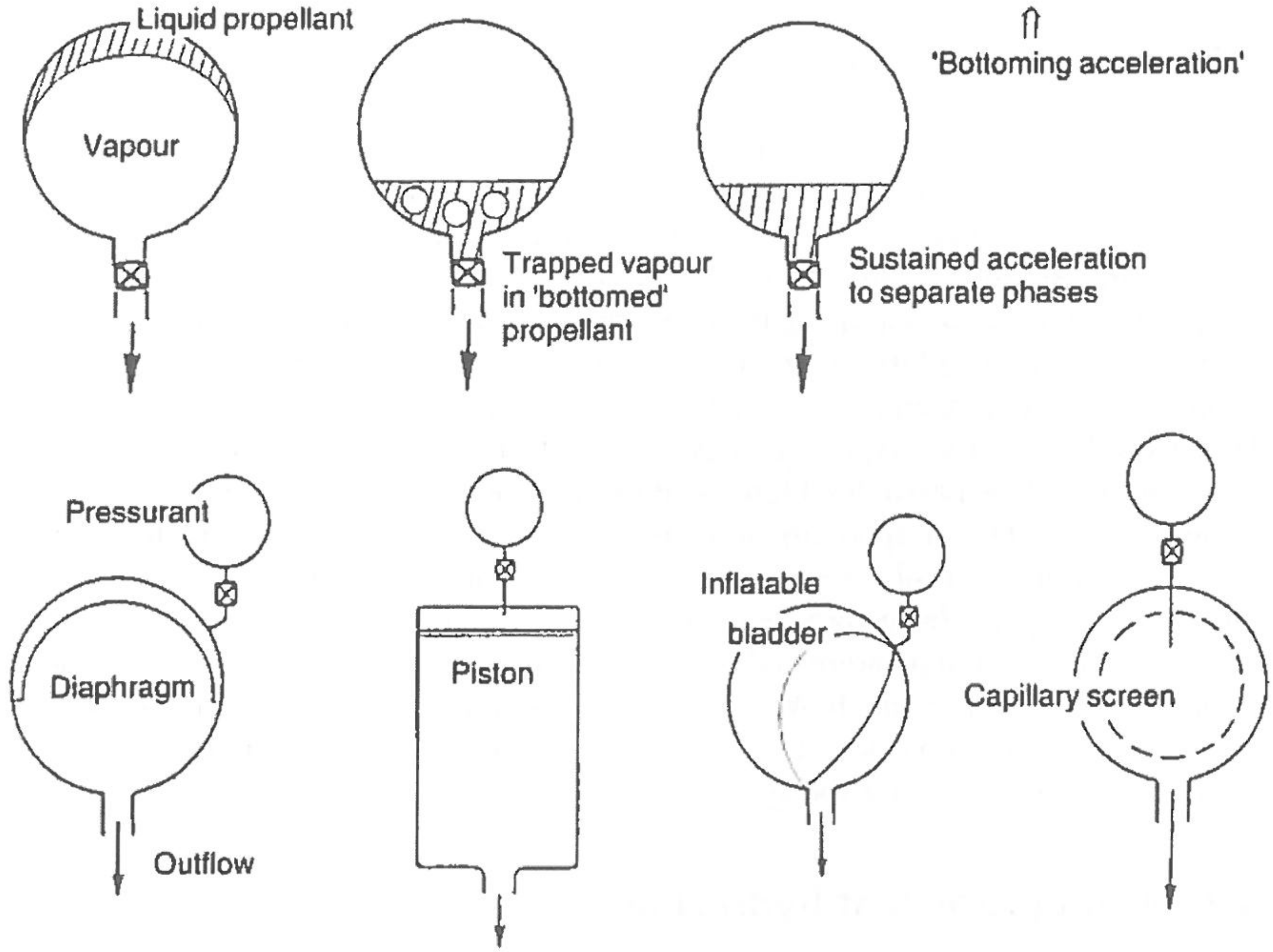
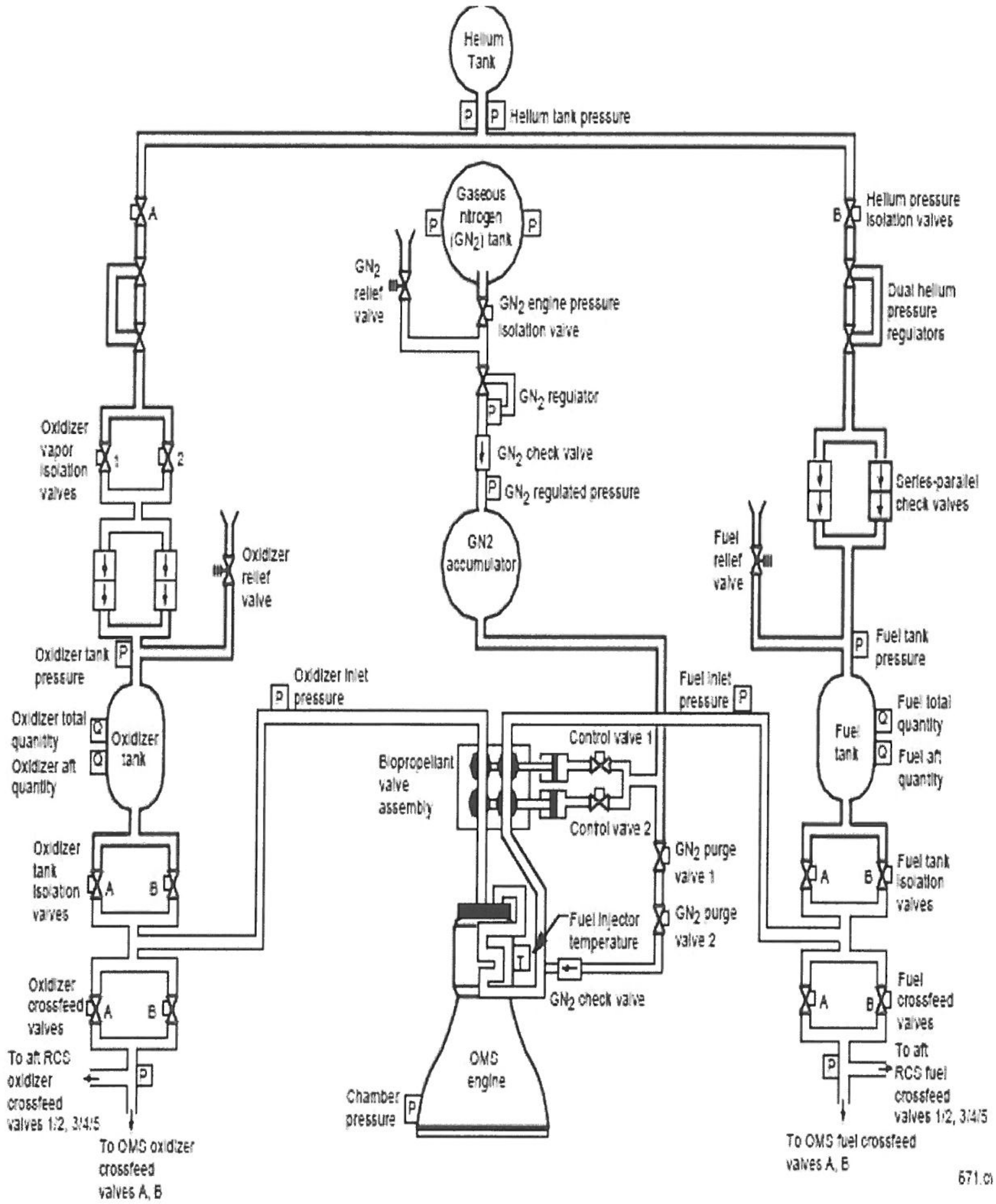



**Figure 6.17** Illustrative propellant storage and delivery systems

# Propellant Management: Shuttle Orbital Maneuvering System (OMS) Engine (T = 6Klbf)




Orbital Maneuvering System Pressurization and Propellant Feed System for One Engine (other Engine Identical)

## Biprop Engine Design

- Open vs closed

  ↓ SSME  ↓ FA ?

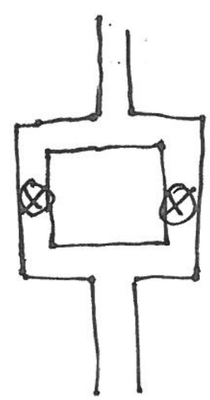
- ~~Fig~~ Table 6.2 : SSME vs Vulcain

- Prop management

  OMS diagram

- Parallel vs Serial valve design

Serial
protect for fail open



Parallel
protect for fail closed

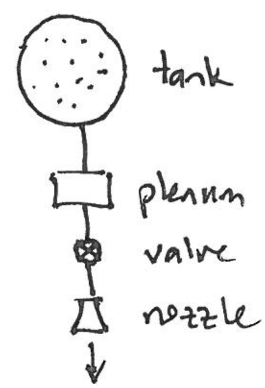
- Fig 6.17

# Spacecraft Propulsion

- Tasks: final orbit acquisition after launcher drop-off
  orbit maintenance, station keeping
  relative maneuvering
  attitude control
  reboost
  de-orbit

## Cold Gas

- pressurized inert gas: $N_2$, Ar, freon, hydrocarbon (propane, etc)
- Liquid or gas form storage
- Exhaust plume → sensors ?
- $T = 10^{-4}\,N - 10^{-2}\,N$
- $I_{SP} \sim 50$ sec (low efficiency)
- CubeSats, SAFER, etc



## Monopropellant Hydrazine

- $N_2H_4 \xRightarrow[\text{catalytic}]{\text{thermal}} N_2, NH_3, H_2$ + heat
- ISP 200-250 s
- Liquid-form storage, with inert pressurant ($N_2$ or He)
- $T \sim 10$ N

## Bi-Propellant ; MMH + $N_2O_4$ (hypergolic)

- ISP ~ 300 s

(figs 6.20, 6.21)

# Mono-propellant System

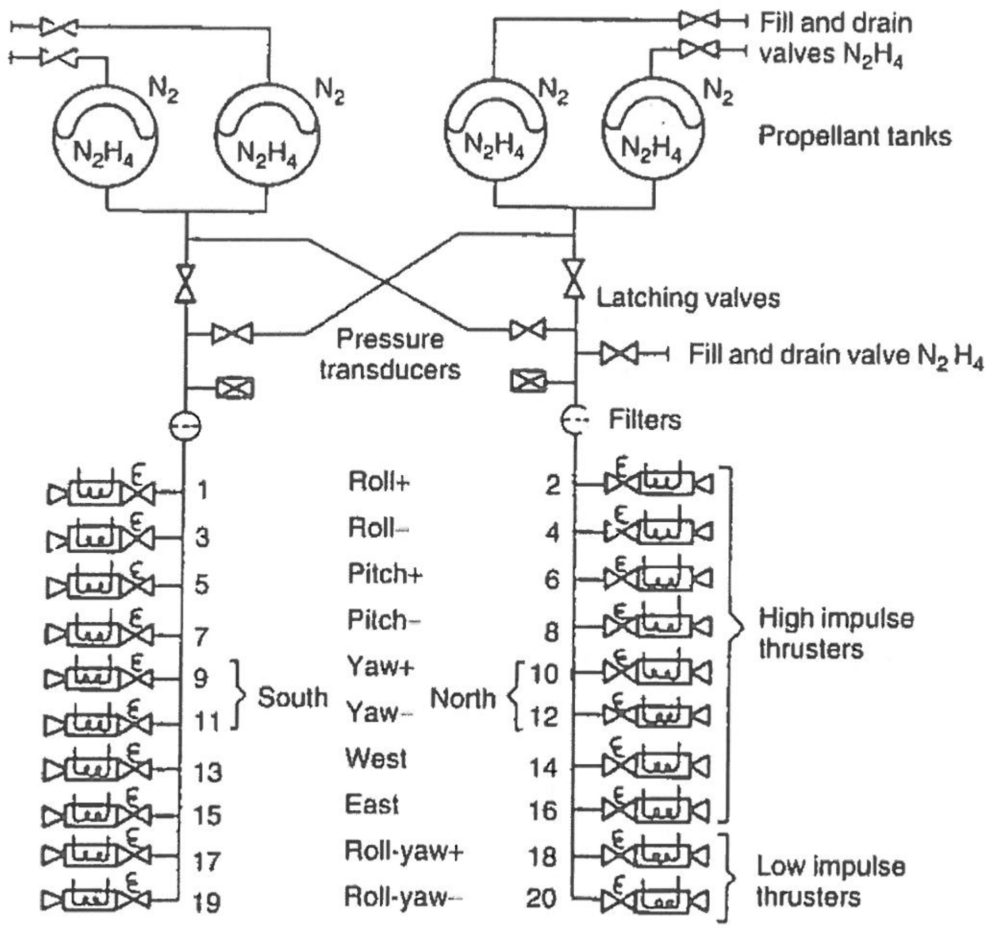



**Figure 6.20** Illustrative spacecraft propulsion system using monopropellant thrusters

# Bi-propellant Hypergolic System

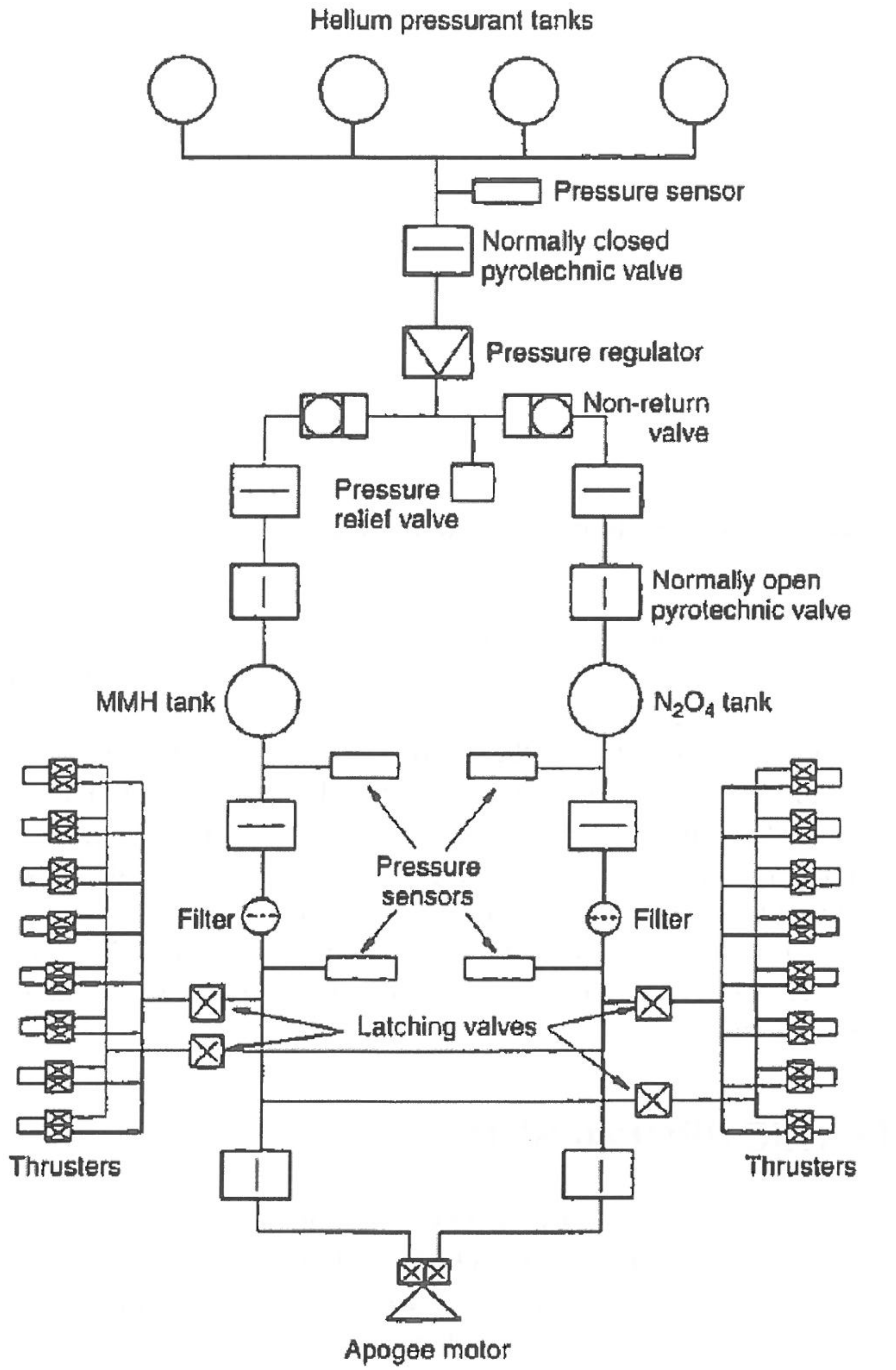
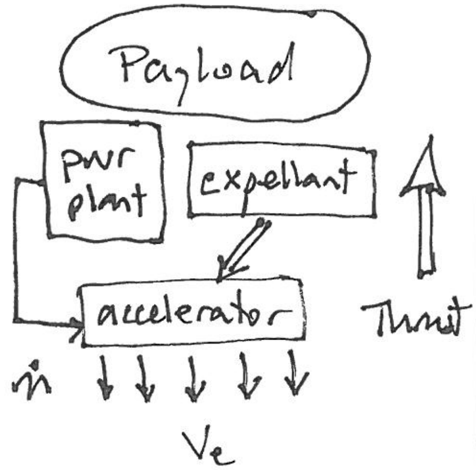



**Figure 6.21** Schematic of a typical bi-propellant propulsion system

# Electric Propulsion




- Let $M_W$ = powerplant mass

  $M_e$ = expellant mass

  $M_p$ = payload mass

- Power-plant output:

$$W = \frac{1}{2}\underbrace{\dot{m}}_{\text{expellent}} V_e^2 \quad \text{(power)}$$

- Assume power-plant output prop. to mass:

$$M_W = \underbrace{\alpha}_{\text{"inverse specific power"}} W$$

- Assume $\dot{m}$ constant throughout burn time $t_b$:

$$\dot{m} = \frac{M_e}{t_b}$$

- Define total rocket mass

$$M_0 = M_W + M_e + M_p$$

- Solve for $M_e$ :

$$M_e = \dot{m} t_b = \frac{2W}{V_e^2} t_b = \frac{2 M_w}{V_e^2} \frac{t_b}{\alpha}$$

$$= \frac{M_0 - M_p}{1 + \left(V_e^2 / \frac{2 t_b}{\alpha}\right)} \quad 6.28$$

- Similarly,

$$M_w = \frac{M_0 - M_p}{1 + \left(\frac{2 t_b}{\alpha} / V_e^2\right)} \quad 6.29$$

- Use rocket equation to get $\Delta V$ :

$$\Delta V = V_e \ln \left( \frac{M_{initial}}{M_{final}} \right)$$

$$\boxed{\Delta V = V_e \ln \left[ \frac{1 + (V_e/V_c)^2}{\frac{M_p}{M_0} + (V_e/V_c)^2} \right]} \quad 6.30$$

(Fig 6.24)

Note that there is an <u>optimum</u> $V_e$

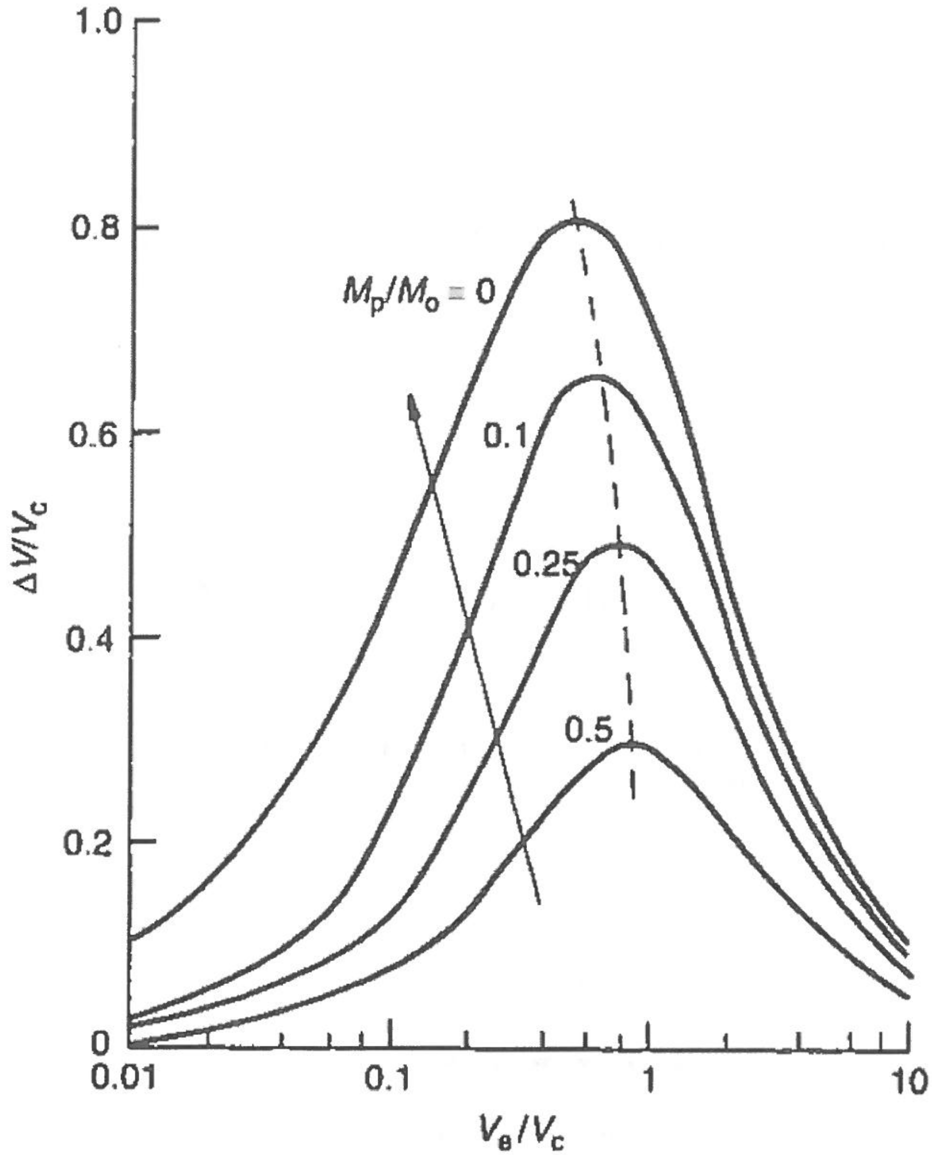



**Figure 6.24** Separately powered electric rocket performance

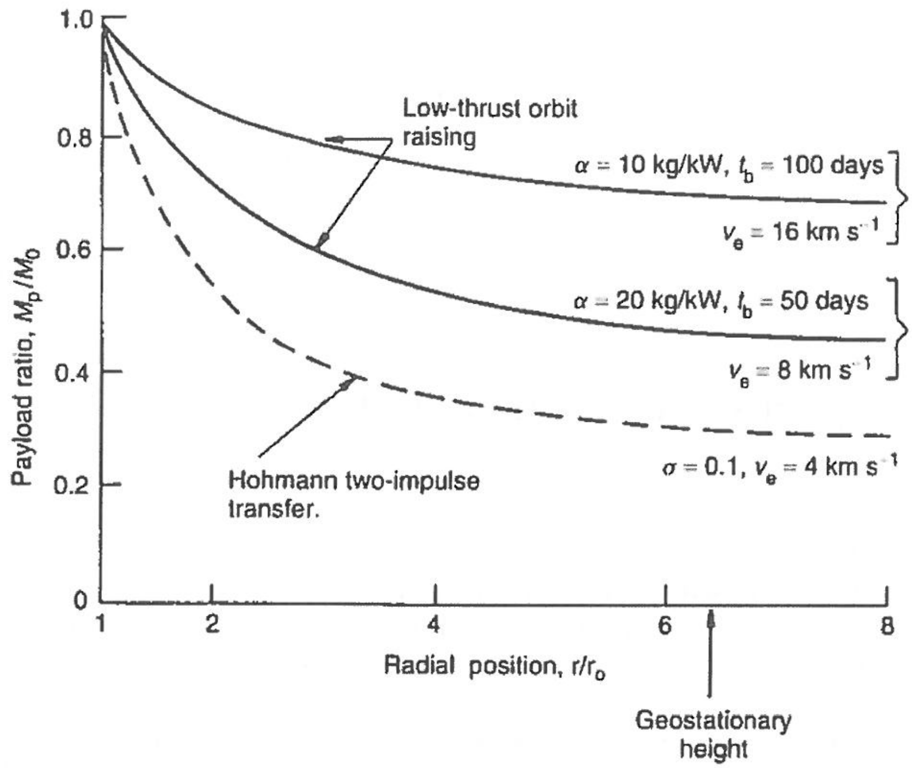



**Figure 6.25** Comparative performance: chemical impulse versus low-thrust orbital transfer

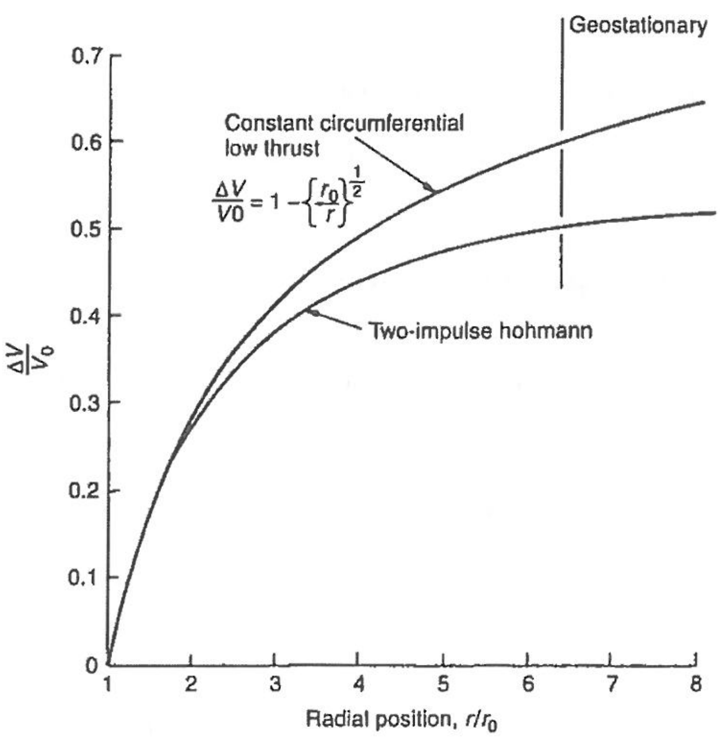



**Figure 6.26** Comparative $\Delta V$ requirement for transfer between circular orbits $r_0$ to $r$