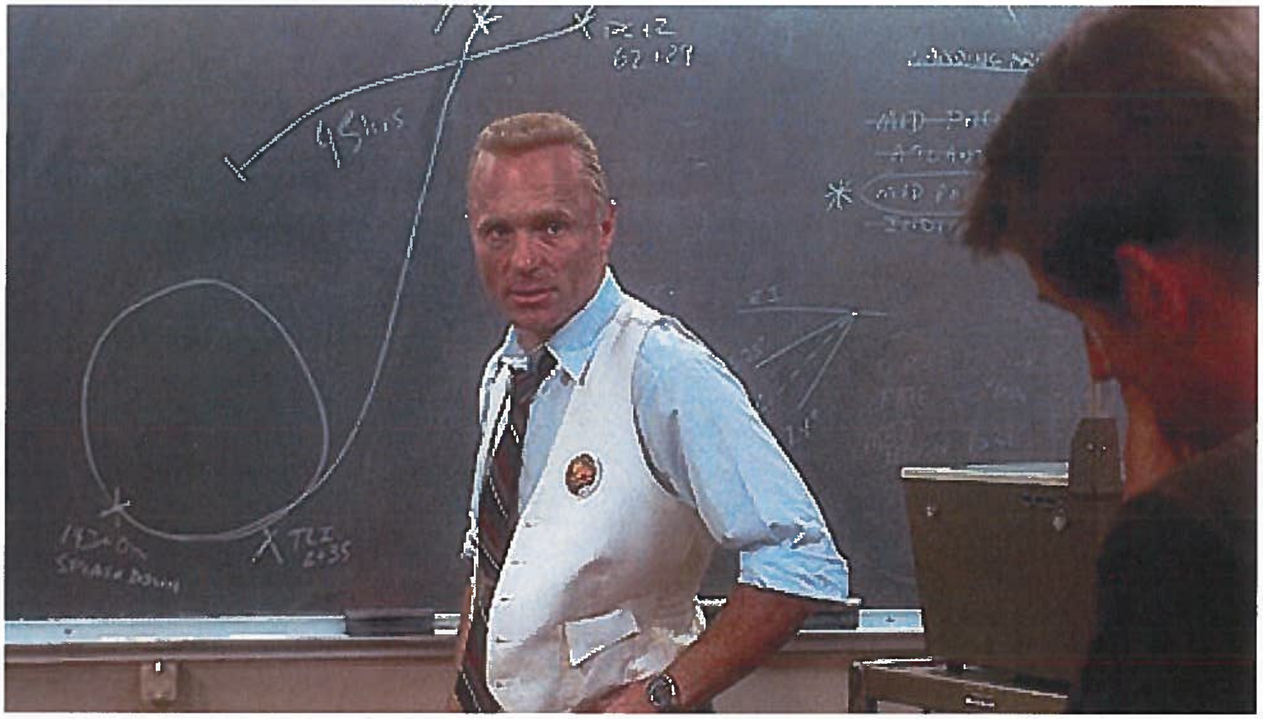
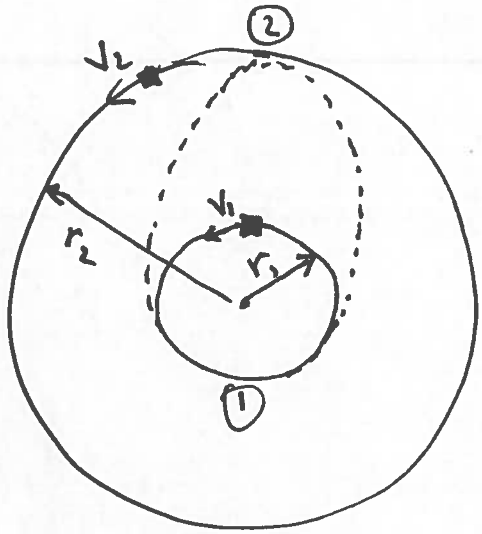
Mission Analysis



Review: Hohmann Transfers (Keplerian Orbits)

- Most energy-efficient 2-impulse maneuver to "transfer" ~~from~~ between coplanar circular orbits.



text fig 5.2

Steps

$\boxed{1}$ start with velocity on original circular orbit 1:

$$v_1 = \sqrt{\frac{\mu}{r_1}}$$

$\boxed{2}$ what is velocity at perigee of xfer ellipse?

Vis-Viva (energy) eqn for ellipse:

$$\frac{v^2}{2} - \frac{\mu}{r} = -\frac{\mu}{2a} \quad \text{or} \quad v = \sqrt{2\mu}\sqrt{\frac{1}{r} - \frac{1}{2a}}$$

At perigee, $r = r_1$ and $2a = r_1 + r_2$

$$\therefore \quad v_p = \sqrt{2\mu}\sqrt{\frac{1}{r_1} - \frac{1}{r_1+r_2}}$$

$\boxed{3}$ $$\Delta v_{1 \to \text{xfer}} = v_p - v_1 = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\left(\frac{2r_2}{r_1+r_2}\right)} - 1\right]$$ Text 5.1

$\boxed{4}$ Similar to get Δv between apogee of ellipse and v_2

$$\Delta v_{\text{xfer} \to 2} = \sqrt{\frac{\mu}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1+r_2}}\right]$$ 5.2

$\boxed{\text{Continuous Thrust Transfers}}$

(Curtis sec 6.10)

ΔV over long period t

(ion propulsion
gas leak)

drag force → like HW!

- Eqn of motion:

$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \frac{\vec{F}}{m}$$

← continuous thrust/mass term

$$= -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{V}}{V}$$

Thrust ↗ ↖ unit vector

Defn of ISP: $\dfrac{dm}{dt} = -\dfrac{T}{I_{sp}\, g_0}$

T = thrust
I_{sp} = engine specific impulse
g_0 = sea-level grav.

- Numerical orbit propagation:

$\dot{\vec{y}} = \vec{f}(t, \vec{y})$ set of 1st order ODE's

where

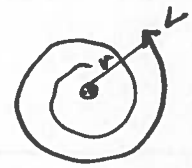
$$\vec{y} = \begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} ; \quad \dot{\vec{y}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} ; \quad \vec{f} = \begin{bmatrix} y_4 \\ y_5 \\ y_6 \\ -\mu \dfrac{y_1}{r^3} + \dfrac{T}{m}\dfrac{y_4}{v} \\ -\mu \dfrac{y_2}{r^3} + \dfrac{T}{m}\dfrac{y_5}{v} \\ -\mu \dfrac{y_3}{y^3} + \dfrac{T}{m}\dfrac{y_6}{v} \\ -\dfrac{T}{I_{sp}\, g_0} \end{bmatrix}$$

Curtis 6.30

- Vector algebra gives relevant version of energy eqn:

$$\frac{d}{dt}\underbrace{\left(\frac{V^2}{2} - \frac{\mu}{r}\right)}_{\substack{KE \quad PE \\ E}} = \frac{T}{m}V \quad \left(\begin{array}{c}\text{energy}\\ \text{increases}\end{array}\right)$$

valid at any point in the outward spiral



for low thrusts, orbit is always nearly circular, so let $V = \sqrt{\mu/r}$ and separate vars:

$$\frac{d(\mu/r)}{\sqrt{\mu/r}} = -2\frac{T}{m}dt$$

$$m = \underset{\substack{\uparrow \\ \text{initial vehicle} \\ \text{mass}}}{m_0} - \underset{\substack{\uparrow \\ \text{propellant mass} \\ \text{expenditure rate}}}{\dot{m}_e}\, t$$

$$\therefore \quad \frac{d(\mu/r)}{\sqrt{\mu/r}} = -2\frac{T}{(m_0 - \dot{m}_e t)}dt$$

- Integrate and let $r = r_0$ at $t = 0$:

Also let $\dot{m}_e = \frac{T}{I_{sp}\, g_0}$ $\quad I_{sp}$ = specific impulse of engine, g_0 = sea-level grav

so $$\sqrt{\frac{\mu}{r}} - \sqrt{\frac{\mu}{r_0}} = I_{sp}\, g_0 \ln\left(1 - \frac{T}{m_0 g_0 I_{sp}}t\right)$$

- Solve to get

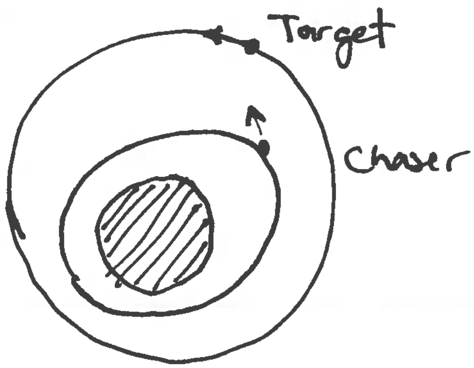
$$r(t) = \mu\left[\sqrt{\frac{\mu}{r_0}} + I_{sp}\, g_0 \ln\left(1 - \frac{T}{m_0 g_0 I_{sp}}t\right)\right]^{-2}$$

radius for elapsed time, but not state vector!

$$t(r) = \frac{m_0 g_0 I_{sp}}{T}\left[1 - e^{\left(\frac{1}{I_{sp} g_0}\right)\left(\sqrt{\frac{\mu}{r}} - \sqrt{\frac{\mu}{r_0}}\right)}\right]$$

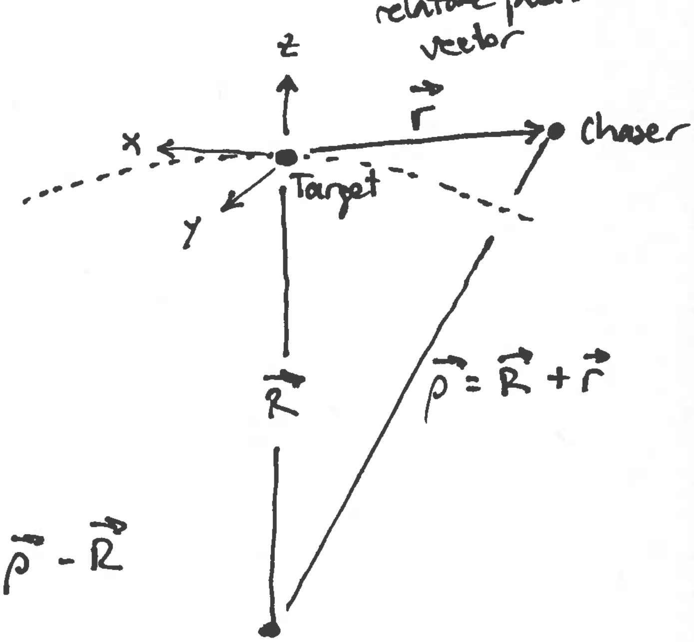
Relative Motion

(Walter Chap 8, Curtis Chap 7)



- Describe motion of chaser relative to target vehicle
- Linearize eqns of motion for small changes in relative distance and velocity
- Then convert to inertial ref. frame

Equation of Relative Motion:



Target: $\ddot{\vec{R}} = -\dfrac{\mu}{R^3}\vec{R}$

chaser: $\ddot{\vec{\rho}} = -\dfrac{\mu}{\rho^3}\vec{\rho}$

relative position vector: $\vec{r} = \vec{\rho} - \vec{R}$

$= x\vec{u}_x + y\vec{u}_y + z\vec{u}_z$

unit vector

in the target-centered non-inertial reference frame

Our goal is the equation of motion of the relative position vector $\vec{r}$

- $\vec{r} = \vec{\rho} - \vec{R}$

need to express $\vec{\rho}$ in terms of $\vec{r}$ and $\vec{R}$

- From chaser eqn of motion $\ddot{\vec{\rho}} = -\frac{\mu}{\rho^3}\vec{\rho}$.

expand $1/\rho^3$ term as

$$\frac{1}{\rho^3} = \left[\frac{1}{\sqrt{x^2+y^2+(z+R)^2}}\right]^3$$

where x, y, z is the position of the chaser relative to the target (or $\vec{r}$)

- Let $r \ll R \rightarrow x, y \ll (z+R)$

$$\therefore \quad \frac{1}{\rho^3} \cong \frac{1}{(z+R)^3} = \frac{1}{R^3}\frac{1}{(1+z/R)^3} = \frac{1}{R^3}\left(1-3\frac{z}{R}\right)$$

- So $\ddot{\vec{\rho}}$ can now be expressed in terms of $\vec{r}(x,y,z)$ and R as

$$\ddot{\vec{\rho}} = -\frac{\mu}{\rho^3}\vec{\rho} \cong -\frac{\mu}{R^3}(R+r)\left(1-3\frac{z}{R}\right)$$

$$\cong -\frac{\mu}{R^3}\left(\vec{R}+\vec{r}-3z\vec{u}_z - \cancel{3\frac{z}{R}\vec{r}}^{\,r \ll R}\right)$$

- Now we can combine

$$\vec{\rho} = \vec{R} + \vec{r} \quad \text{defin of } \vec{\rho}$$

and

$$\underbrace{\ddot{\vec{\rho}}}_{-\frac{\mu}{R^3}(\vec{R}+\vec{r}-3z\vec{u}_z)} = \underbrace{\ddot{\vec{R}}}_{-\frac{\mu}{R^3}\vec{R}} + \ddot{\vec{r}}$$

$$\therefore \quad \ddot{\vec{r}} = \ddot{\vec{\rho}} - \ddot{\vec{R}}$$

or

$$\boxed{\ddot{\vec{r}} = -\frac{\mu}{R^3}\left(\vec{r} - 3z\vec{u}_z\right)} \quad \text{for } x, y, z, r \ll R \qquad 8.5.1$$

Eqn of motion of chaser in target frame

- But target frame is a rotating frame, so (ω)
familiar extra terms for any vector in a rotating frame:

$$\ddot{\vec{r}}_{rel} = \ddot{\vec{r}}_i + \underbrace{2\vec{\omega} \times \dot{\vec{r}}_i}_{\text{Coriolis force}} + \underbrace{\vec{\omega} \times (\vec{\omega} \times \vec{r})}_{\text{centrifugal force}} + \dot{\vec{\omega}} \times \vec{r}$$

$$= (\ddot{x} + 2\omega\dot{z} - \omega^2 x + \dot{\omega} z)\vec{u}_x$$

$$+ \ddot{y}\,\vec{u}_y$$

$$+ (\ddot{z} - 2\omega\dot{x} - \omega^2 z - \dot{\omega} z)\vec{u}_z$$

where $r = (x, y, z)$ = distance from chaser to target
ω = orbital frequency

- Plug our expression for $\ddot{\vec{r}}_{rel}$ into our eqn of motion

$$\ddot{\vec{r}} = -\frac{\mu}{R^3}\left(\vec{r} - 3z\,\vec{u}_z\right)$$

to get

$$\ddot{\vec{r}} = \begin{bmatrix} \ddot{x} + 2\omega\dot{z} - \omega^2 x + \dot{\omega} z \\ \ddot{y} \\ \ddot{z} - 2\omega\dot{x} - \omega^2 z - \dot{\omega} x \end{bmatrix} = -\frac{\mu}{R^3}\begin{bmatrix} x \\ y \\ z - 3z \end{bmatrix}$$

- Each vector eqn holds individually:

$$\boxed{\begin{aligned} &\ddot{x} + 2\omega\dot{z} - \left(\omega^2 - \frac{\mu}{R^3}\right)x + \dot{\omega} z = 0 \\ &\ddot{y} + \frac{\mu}{R^3}\,y = 0 \\ &\ddot{z} - 2\omega\dot{x} - \left(\omega^2 + 2\frac{\mu}{R^3}\right)z \,\cancel{+}\!- \dot{\omega} x = 0 \end{aligned}}$$

7.5.3

for $x, y, z, r \ll R$

Equations of motion of a chaser, with coordinates of (x, y, z) relative to its target, and where target ref frame rotates with instantaneous orbital frequency compared to an inertial frame of $\omega(t)$.

- Note $\ddot{x}$ and $\ddot{z}$ eqns are coupled, but not $\ddot{y}$.

- Where are we??

Have used the standard eqns of 2-body motion and a rotating reference frame relative to inertial to derive eqns of motion of a chaser relative to a target.

But we haven't really said anything about <u>orbits</u> yet!

- To close ~~solve~~ eqns 8.5.3, we need to know R (target) and ω and $\dot{\omega}$.

Consider Keplerian elliptical orbits:

$$\omega = \dot{\theta} = \frac{\sqrt{\mu a(1-e^2)}}{R}$$

$$\dot{\omega} = -\frac{2\mu a e}{R^4}\sqrt{1-e^2}\,\sin E$$

$$R(t) = a\left[1 - e\cos E(t)\right]$$

where $t\sqrt{\frac{\mu}{a^3}} = E(t) - e\sin E(t)$ (Kepler's Equation)

so at time t, and a, e, μ known for elliptical orbit,

solve for $E \rightarrow R \rightarrow \dot{\omega}, \omega \longrightarrow$ numerically solve 8.5.3 for $(x,y,z), (\dot{x},\dot{y},\dot{z}), (\ddot{x},\ddot{y},\ddot{z})$ at time t

Circular Target Orbit - Hills Equations

- Recall that drag tends to circularize elliptic orbits!
- ISS : $e < 0.001$
- Circular Earth orbits very common
- $$\ddot{\vec{R}} = -\frac{\mu}{R^3}\vec{R}$$

$$\text{Circ: } n \equiv \frac{2\pi}{\tau} = \sqrt{\frac{\mu}{R^3}} = \text{constant} = \omega$$

and $\dot{\omega} = 0$

- Eqns of relative motion 8.5.3 become:

$$\boxed{\begin{aligned} \ddot{x} + 2n\dot{z} &= 0 \\ \ddot{y} + n^2 y &= 0 \\ \ddot{z} - 2n\dot{x} - 3n^2 z &= 0 \end{aligned}} \qquad 8.5.5$$

Hills Equations

(Circular target orbit)

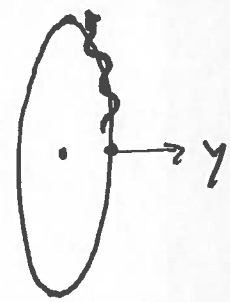
Solving Hill's Equations

Solution to Hill's gives motion of chaser $r(t) = [x(t), y(t), z(t)]$ for a target in a circular orbit.

- IC's at time $= 0$: $\underbrace{x_0, y_0, z_0 \; ; \; \dot{x}_0, \dot{y}_0, \dot{z}_0}_{\text{initial SV of chaser}}$

- Note that y eqn is decoupled from others:

  Harmonic oscillator in y - across orbital plane



$$\ddot{y} + n y = 0 \quad \text{(undamped system)}$$

Classical solution for $y(t=0) = y_0$:

$$y(t) = y_0 \cos nt + \frac{\dot{y}_0}{n} \sin nt \quad \left(\text{recall } n \equiv \frac{2\pi}{\tau} = \omega\right)$$ y ✓

- Integrate x eqn once, with IC's:

$$\ddot{x} + 2n\dot{z} = 0$$

$$\downarrow$$

$$\dot{x} = -2nz + 2nz_0 + \dot{x}_0$$

- Plug into z-eqn:

$$\ddot{z} = 2n\dot{x} + 3n^2 z = -n^2 \left[\overbrace{z - \left(4z_0 + 2\frac{\dot{x}_0}{n}\right)}\right]$$ another harmonic oscillator

soln: $$z - \left(4z_0 + 2\frac{\dot{x}_0}{n}\right) = \frac{\dot{z}_0}{n} \sin nt - \left(3z_0 + 2\frac{\dot{x}_0}{n}\right) \cos nt$$

or $$z(t) = 4z_0 + 2\frac{\dot{x}_0}{n} + \frac{\dot{z}_0}{n} \sin nt - \left(3z_0 + 2\frac{\dot{x}_0}{n}\right) \cos nt$$ z ✓

still need $x(t)$ ...

- Back to x-equation, already integrated once:

$$\dot{x} = -2n\underset{\substack{\uparrow \\ \text{plug in our} \\ \text{expression for } z(t)}}{z} + 2n z_0 + \dot{x}_0$$

$$\dot{x} = -\left(6 z_0 + 3\frac{\dot{x}_0}{n}\right)n - 2\dot{z}_0 \sin nt + \left(6n z_0 + 4\frac{\dot{x}_0}{n}\right)\cos nt$$

Integrate once more to get:

$$x(t) = x_0 - 2\frac{\dot{z}_0}{n} - \left(6 z_0 + 3\frac{\dot{x}_0}{n}\right)nt + 2\frac{\dot{z}_0}{n}\cos nt + \left(6 z_0 + 4\frac{\dot{x}_0}{n}\right)\sin nt$$ x ✓

- Summarize: Solution of Hill's Equations:

8.5.6

$$\underbrace{\begin{bmatrix} x \\ z \\ \dot{x} \\ \dot{z} \end{bmatrix}}_{\substack{\text{final SV} \\ \text{at time}=t}} = \underbrace{\begin{bmatrix} 1 & 6(\sin nt - nt) & (4\sin nt)/n - 3t & 2(\cos nt - 1)/n \\ 0 & 4 - 3\cos nt & 2(1-\cos nt)/n & (\sin nt)/n \\ 0 & 6n(\cos nt - 1) & 4\cos nt - 3 & -2\sin nt \\ 0 & 3n\sin nt & 2\sin nt & \cos nt \end{bmatrix}}_{\text{"time evolution matrix"}} \underbrace{\begin{bmatrix} x_0 \\ z_0 \\ \dot{x}_0 \\ \dot{z}_0 \end{bmatrix}}_{\substack{\text{initial SV} \\ \text{at time}=t_0}}$$

8.5.7

$$\begin{bmatrix} y \\ \dot{y} \end{bmatrix} = \begin{bmatrix} \cos nt & (\sin nt)/n \\ -n\sin nt & \cos nt \end{bmatrix} \begin{bmatrix} y_0 \\ \dot{y}_0 \end{bmatrix}$$

Example: Hill's Equations for Rendezvous

- Recall $\vec{r}$ is the separation vector between spacecraft

- Terminology: $\vec{V}_{0-}$ = velocity vector of chaser just before impulsive thrust at t_0

  $\vec{V}_0$ = just after t_0

  $\vec{V}$ = after elapsed time t

- So initial condition at $t = t_0$: (before thrust changes $\vec{V}_0$)

  rel. position: $\vec{r}_0 = (x_0, y_0, z_0)$

  $\vec{V}_{0-} = (\dot{x}_{0-}, \dot{y}_{0-}, \dot{z}_{0-})$

- What should initial velocity of chaser be to meet target at location $\vec{r}(x, y, z) = (0, 0, 0)$ after elapsed time t?

- Set $x=y=z=0$ in eqns 8.5.6 and 8.5.7 and solve for velocity after thrust impulse $\vec{V}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0)$

$$\dot{x}_0 = \frac{-2x_0 n(1-\cos nt) + z_0 n(4\sin nt - 3nt\cos nt)}{3nt\sin nt - 8(1-\cos nt)}$$

$$\dot{y}_0 = -\frac{y_0 nt}{\tan nt}$$ 8.5.8

$$\dot{z}_0 = \frac{x_0 n\sin nt - z_0 n[6nt\sin nt - 14(1-\cos nt)]}{3nt\sin nt - 8(1-\cos nt)}$$

- ΔV required to achieve this motion:

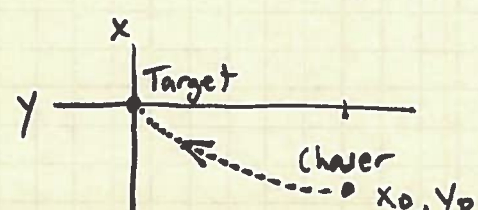
$$\Delta\vec{V} = \vec{V}_0 - \vec{V}_{0-}$$

known initial velocity before any action

$$\therefore \quad |\Delta\vec{V}| = \sqrt{(\dot{x}_0 - \dot{x}_{0-})^2 + (\dot{y}_0 - \dot{y}_{0-})^2 + (\dot{z}_0 - \dot{z}_{0-})^2}$$ 8.5.9

- Apply Hill's Equations to rendezvous problem :

  - Nearly-circular orbits
  - Null out of plane motion to make it a 2-D (coplanar) problem (z, $\dot{z}$)
  - Choose initial position of chaser x_0, y_0
  - Choose desired rendezvous time t
  - Solve unforced Hill's Equations for required initial rates, $\dot{x}_0$ and $\dot{y}_0$
  - Use rocket thrust to achieve $\dot{x}_0$ and $\dot{y}_0$ rates at starting point x_0, y_0.



Hill's Eqns for Relative Motion over $\boxed{\text{Short times}}$

- Let $t \to 0$ for eqns 8.58 to get:

$$\dot{x}_0 = -\frac{x_0}{t} - n z_0 - \frac{1}{6} n^2 x_0 t + O(n^3 z_0 t^2)$$

$$\dot{z}_0 = -\frac{z_0}{t} + n x_0 - \frac{5}{6} n^2 z_0 t + O(n^3 x_0 t^2)$$

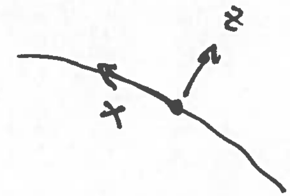
$$\dot{y}_0 = -\frac{y_0}{t} + \frac{1}{3} n^2 y_0 t + O(n^4 y_0 t^3)$$

- Simpler yet: for $\boxed{(nt)^2 \ll 1 \quad (\text{or } t \lesssim 0.02\tau)}$

$$\boxed{\begin{aligned} \dot{x}_0 &= -\frac{x_0}{t} - n z_0 \\ \dot{z}_0 &= -\frac{z_0}{t} + n x_0 \\ \dot{y}_0 &= -\frac{y_0}{t} \end{aligned}} \qquad 8.5.10$$

Flyaround Trajectory #1 : Prolate Cycloid

- Lost toolbox from ISS EVA :



Assume retrograde release

$$\vec{v}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0) = (-0.1, 0, 0)\ \frac{m}{s}$$

$$\vec{r}_0 = (x_0, y_0, z_0) = (0, 0, 0) \quad \text{released } \underline{\text{at}} \text{ "target"}$$

- Backwards rendezvous : from 8.5.6 and IC's

$$x(t) = \frac{v_0}{n}(4 \sin nt - 3nt) \cong v_0 t$$

$$y(t) = 0$$

$$z(t) = 2\frac{v_0}{n}(1 - \cos nt) \quad \cong \underbrace{n v_0 t^2}_{\text{for } nt \ll 1}$$ 8.5.11

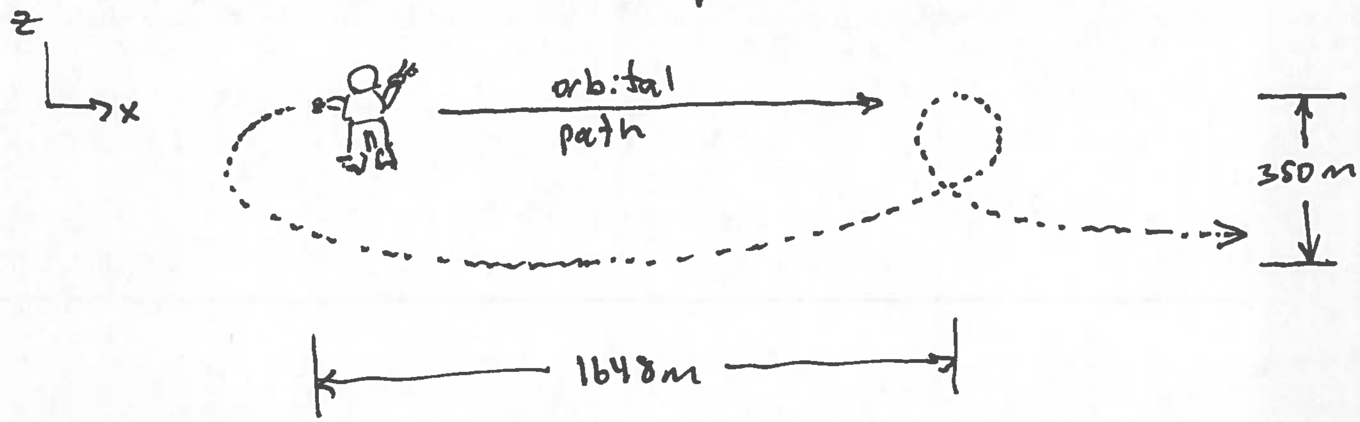
$$\dot{x}(t) = v_0(4 \cos nt - 3)$$

$$\dot{y}(t) = 0$$

$$\dot{z}(t) = 2v_0 \sin nt$$

- Relative motion of lost toolbox: prolate cycloid

"RelMo" plot



- Can we make sense of this apparent motion from eqns 8.5.11?

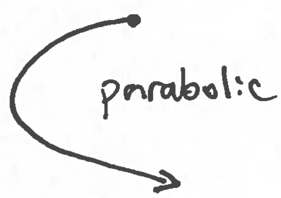
Yes! (HW)

- Note that geometric motion in the x-z plane is initially parabolic:

$$z \cong n v_0 t^2 = \frac{n x^2}{v_0}$$

good only for $nt \ll 1$

early part of trajectory



- Also note that this is the first burn in a Hohmann xfer to a lower orbit!

Prolate Cycloid Trajectories: Relative Motion

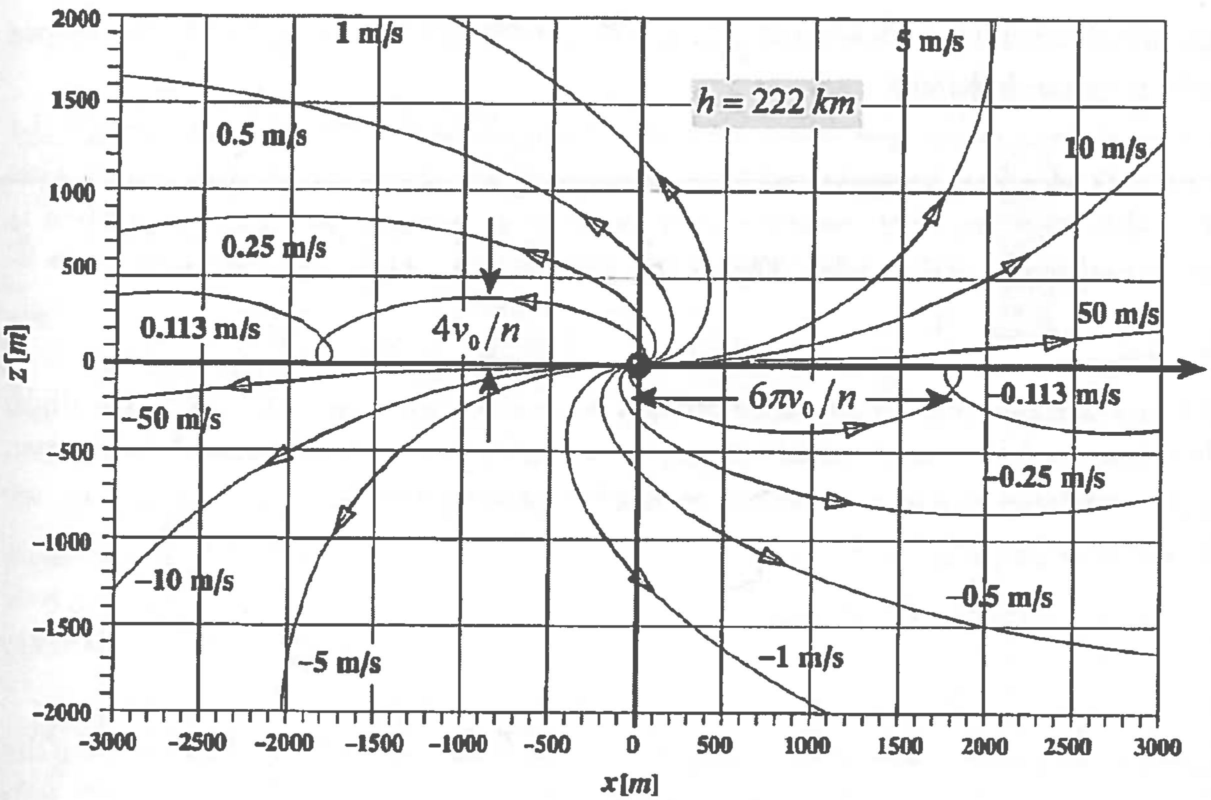


Figure 8.22 Shown are for different v_0 in x direction the trajectories (prolate cycloid) of the object moving relative to a reference point (center dot, which itself moves on an orbit at altitude $h = 222$ km to the right (bold arrow)).

Prolate Cycloid Trajectories: Absolute Motion

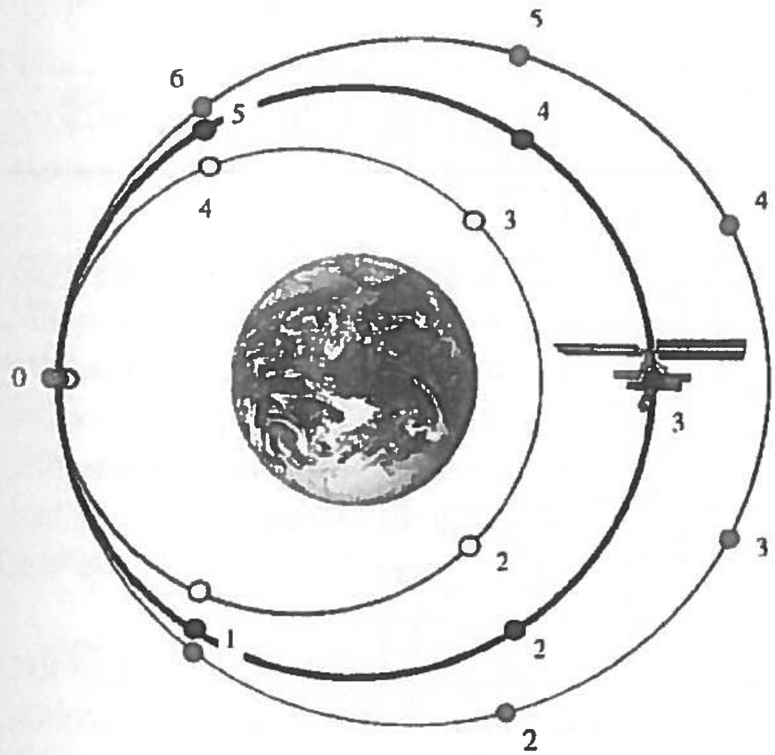


Figure 8.23 A schematic sketch of the prolate cycloid motion as viewed from the inertial reference frame of Earth. The smaller ellipse is for an object with smaller velocity and the larger ellipse with a larger velocity at point 0. The numbered points give the positions on each orbit after constant time intervals.

Flyaround Trajectory #2 : Ellipse

- Same lost toolbox, different release velocity vector:

  Assume radial release (away from Earth)

$$\vec{V}_0 = (\dot{x}_0, \dot{y}_0, \dot{z}_0) = (0, 0, 0.1) \ \frac{m}{s}$$

$$\vec{r}_0 = (x, y, z) = (0, 0, 0)$$

- From 8.5.6

$$x(t) = 2\frac{V_0}{n}(\cos nt - 1) \quad \cong -nV_0 t^2$$

$$y(t) = 0$$ 8.5.6

$$z(t) = \frac{V_0}{n}\sin nt \quad \underbrace{\cong V_0 t}_{\text{for } nt \ll 1}$$

- Combine to get trajectory in x-z plane:

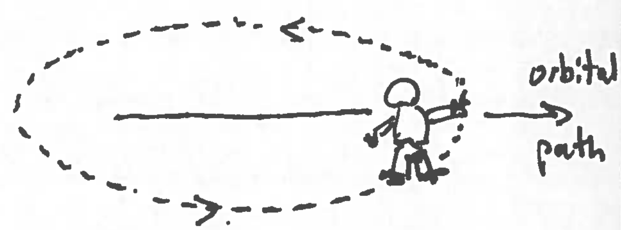
  It's an ellipse:

$$\frac{(x + 2V_0/n)^2}{(2V_0/n)^2} + \frac{z^2}{(V_0/n)^2} = 1$$

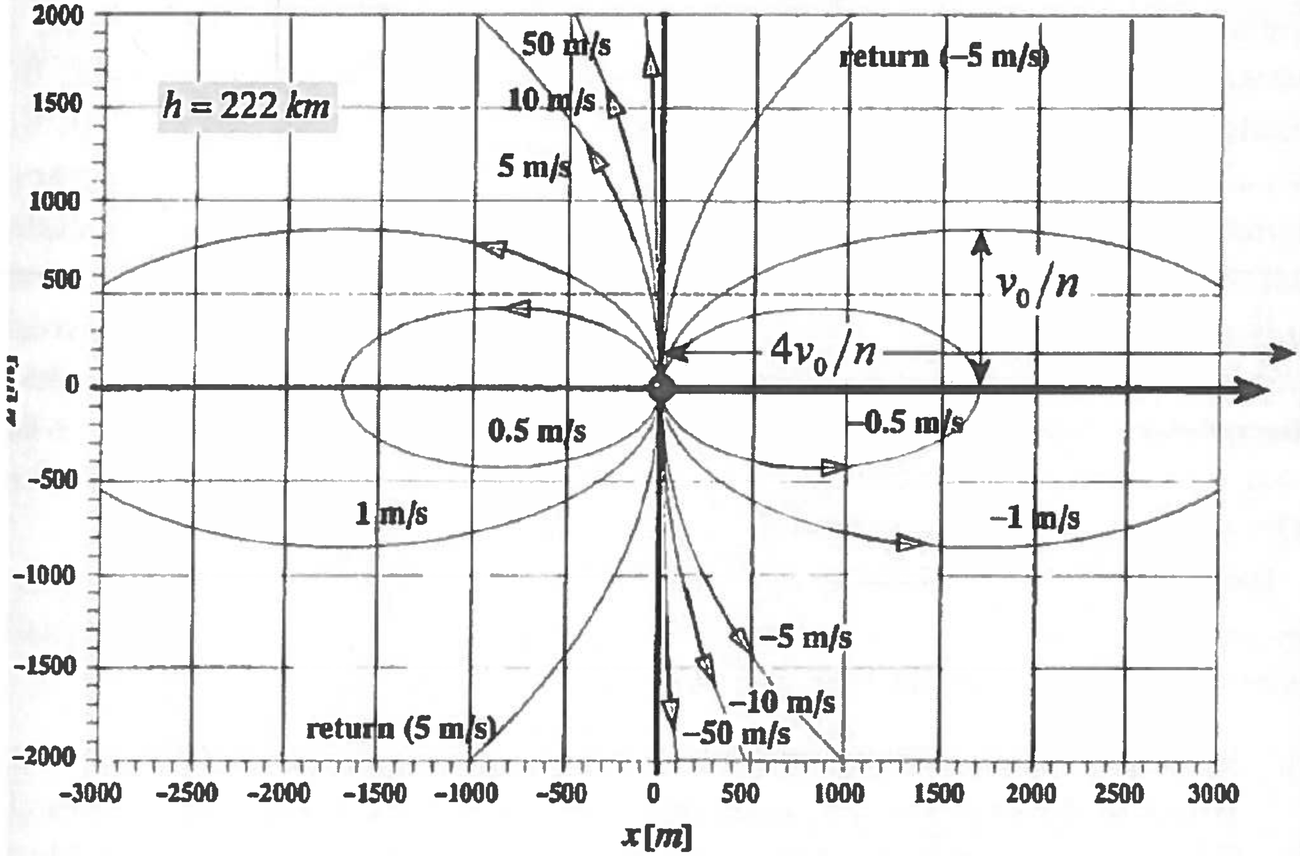
semi-major axis $a = 2V_0/n$

minor $b = V_0/n$

ellipticity $e = \sqrt{\frac{1-b^2}{a^2}}$



Elliptical Trajectories: Relative Motion



igure 8.25 Shown are the trajectories (ellipses) of the object moving relative to a reference point :enter dot, which itself moves on an orbit at altitude $h = 222$ km to the right (bold arrow)) for ifferent v_0 in z direction.

Orbital Rendezvous Operations

- Complex, difficult, high-stakes, essential
- Rendezvous: astrodynamics of relative navigation
- Docking: fly all the way into physical contact
- Capture/Berthing: capture by robotic arm
  maneuver to berth
- Mission Phases

  1. Launch
  2. Phasing
  3. Homing
  4. Closing
  5. Final approach
  6. Docking or Capture

Relative Motion for Rendezvous

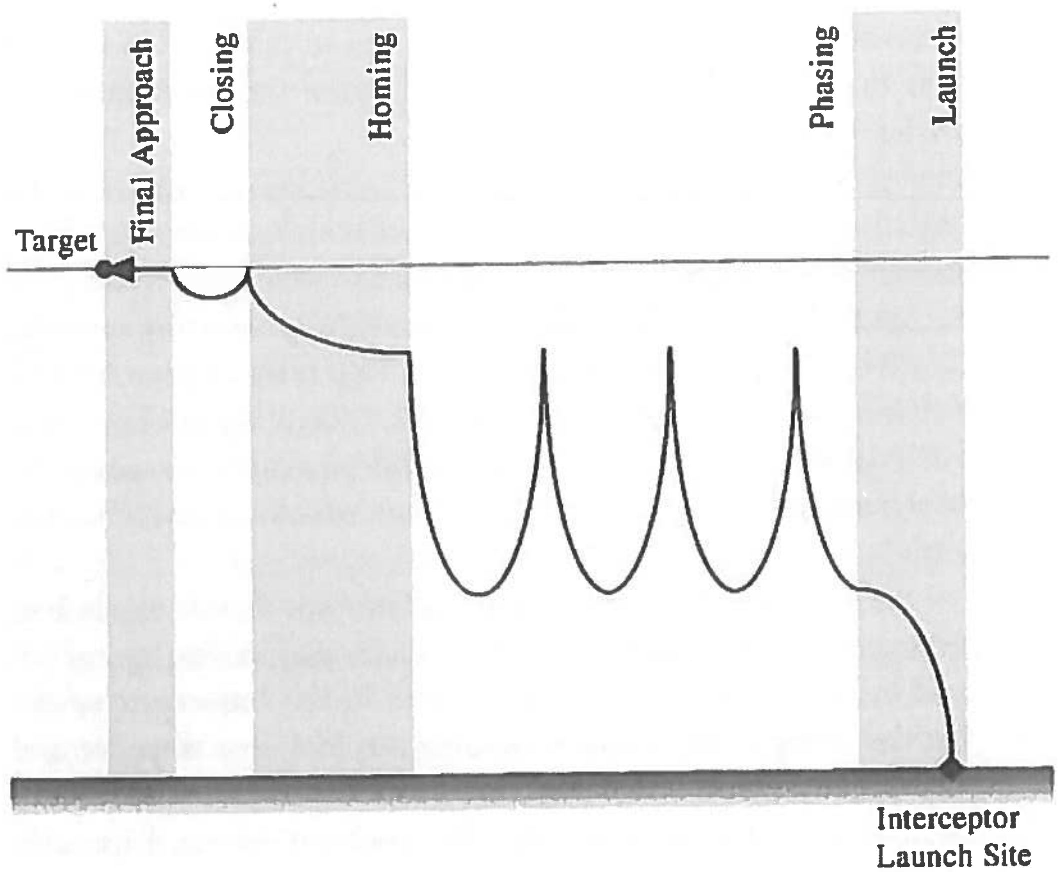


Figure 8.26 Sketch of a typical R&D mission profile consisting of launch phase, phasing maneuvers, homing, and close range rendezvous (closing and final approach) including docking.

① Launch Phase :

- Injection of chaser into orbital plane of target and achieve stable orbital conditions (usually a near-circular orbit).

- Required launch azimuth :

from geometry (Fig 8.27) and Napier's Rules

$$\cos i = \cos\beta \sin(180° - \phi) = \cos\beta \sin\phi$$

$$\left.\begin{aligned} \phi_1 &= \arcsin\frac{\cos i}{\cos\beta} \quad \text{(ascending pass)} \\ \phi_2 &= 180° - \phi_1 \quad \text{(descending pass)} \end{aligned}\right\} \text{2 opportunities per day, theoretically}$$ 8.6.1

where i = inclination of target orbit

β = launch site lattitude

ϕ = chaser launch azimuth (from North)

- Note: since $\cos i = \cos\beta \sin\phi$

  and $|\sin\phi| \le 1$

  then $\cos i \le \cos\beta \Rightarrow \boxed{i \ge \beta}$ ☆ ~~Highest~~ Lowest orbit inclination is launchsite lattitude!

Launch Geometry

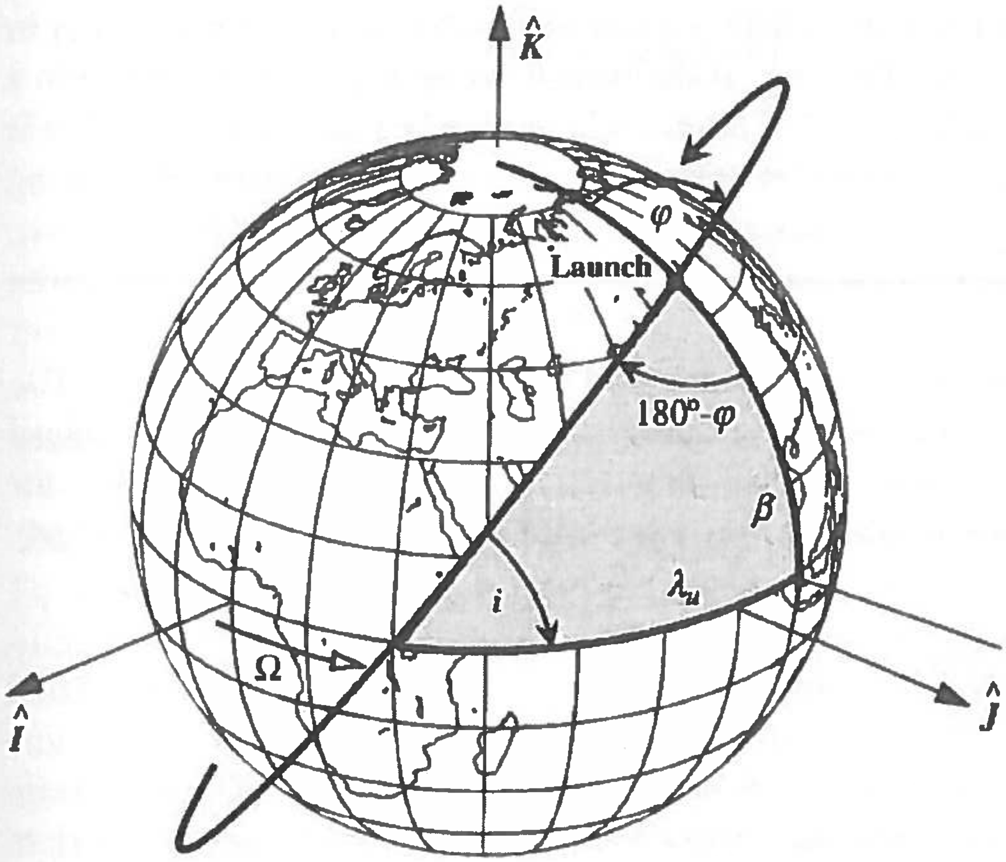
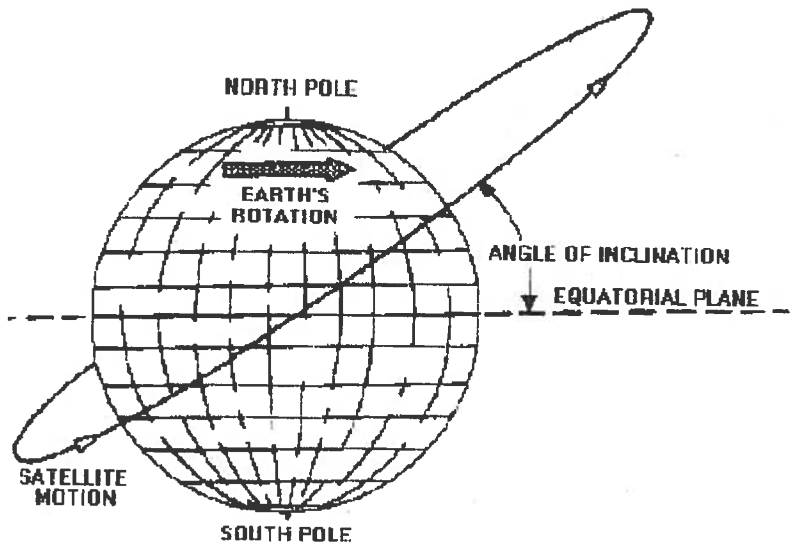
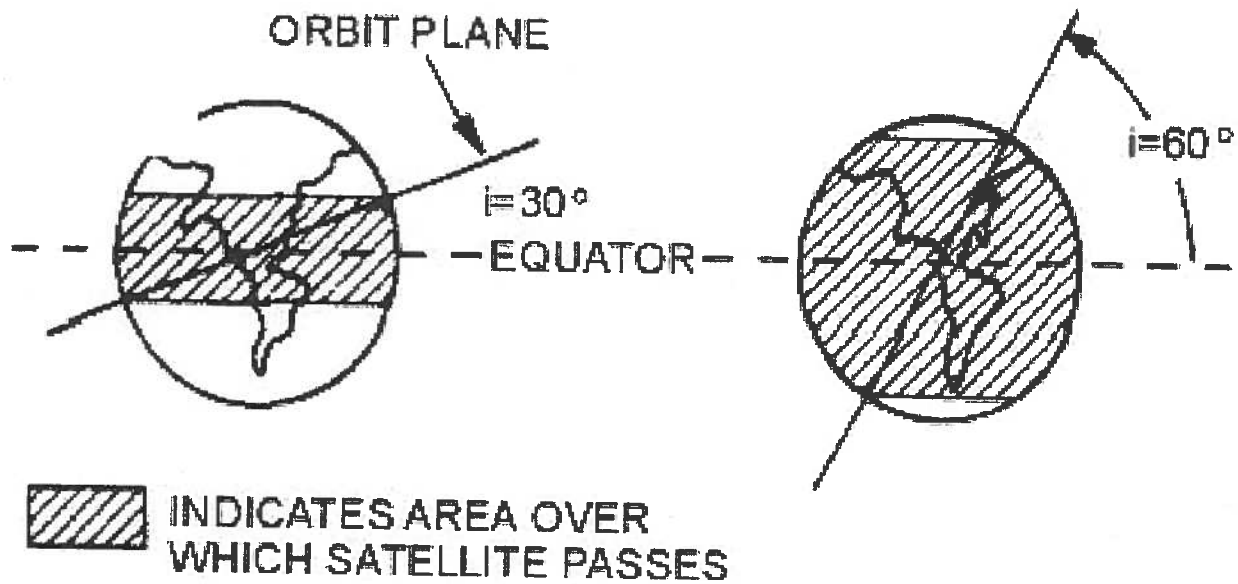
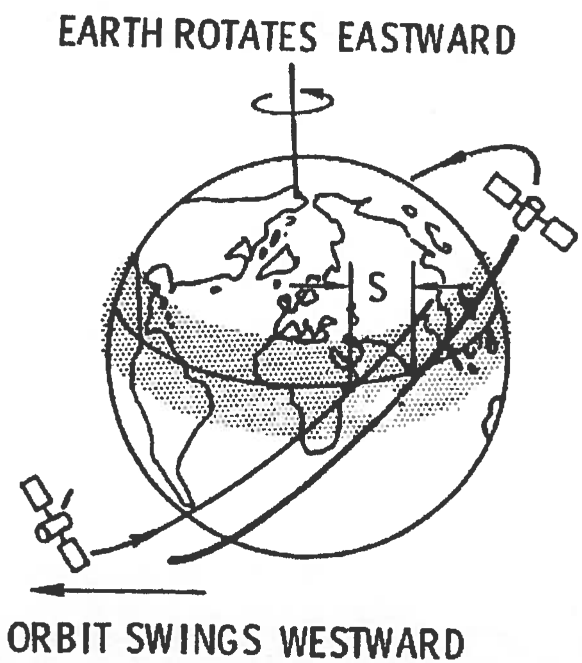


Figure 8.27 Launch window trigonometry. Illustrated are the target orbit with RAAN Ω and inclination i, launch site latitude β, launch azimuth φ, and auxiliary angle λ_u.

REVIEW:
Launching Into Plane







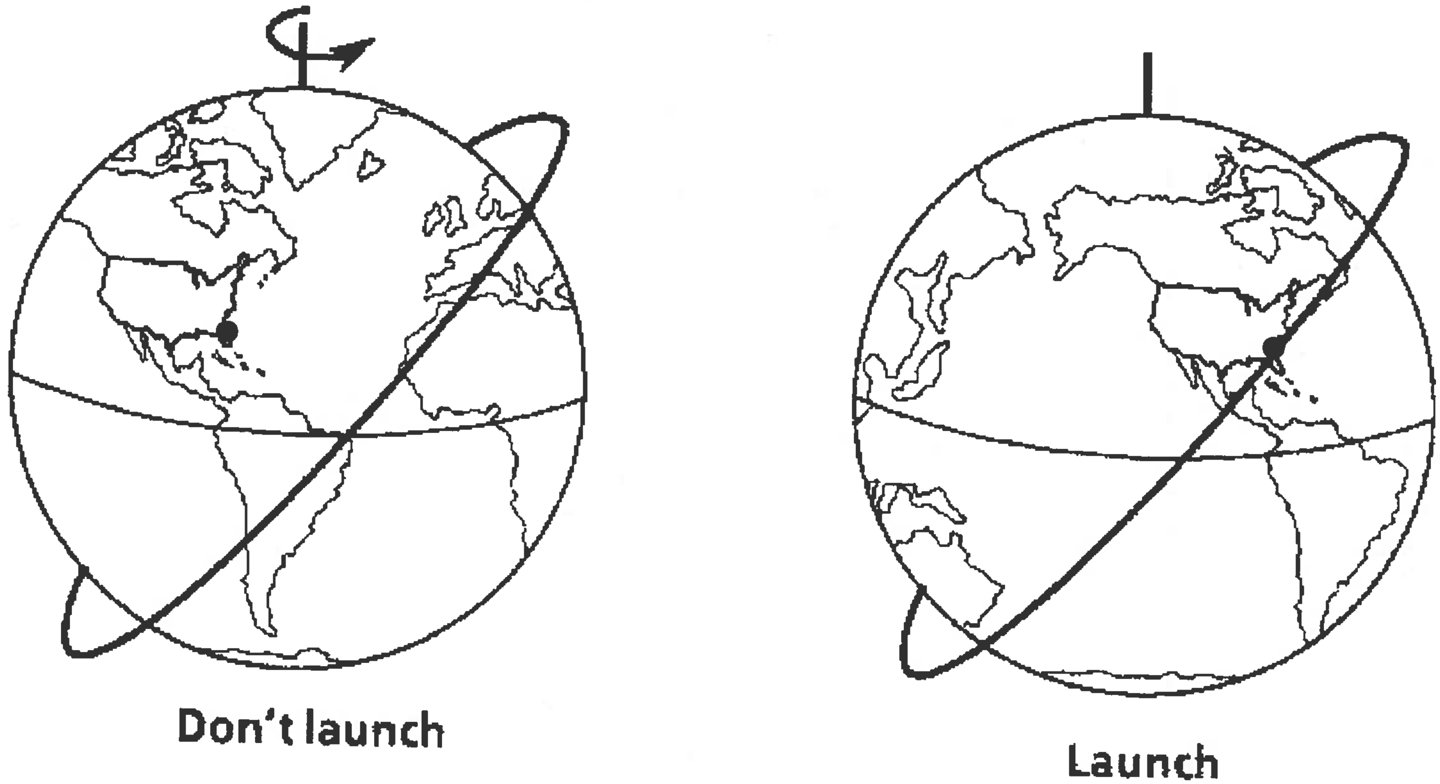
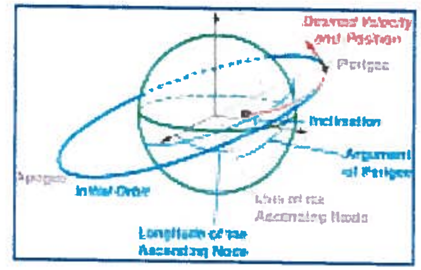
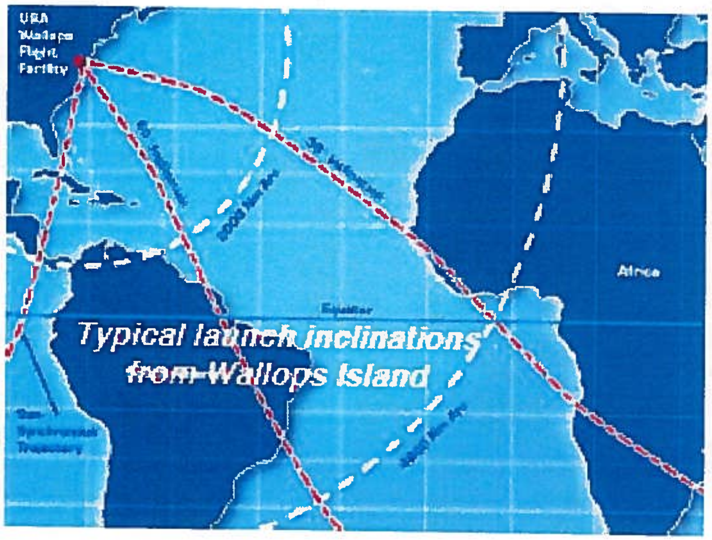
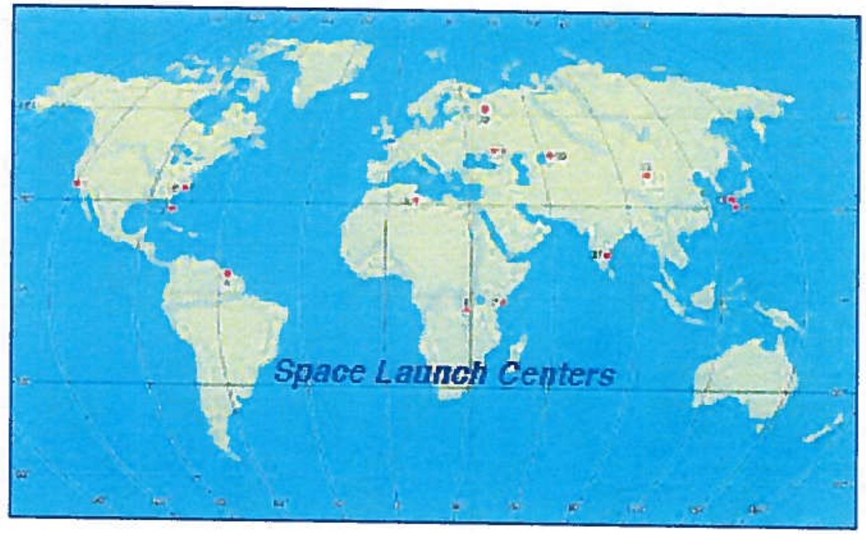


Figure 2: Launching into target orbital plane.

Effect of Launch Latitude on Orbital Parameters







- Launch latitude establishes <u>minimum</u> orbital inclination (without "dogleg" maneuver)
- Time of launch establishes line of nodes
- Argument of perigee established by
  - Launch trajectory
  - On-orbit adjustment

Orbit Inclination

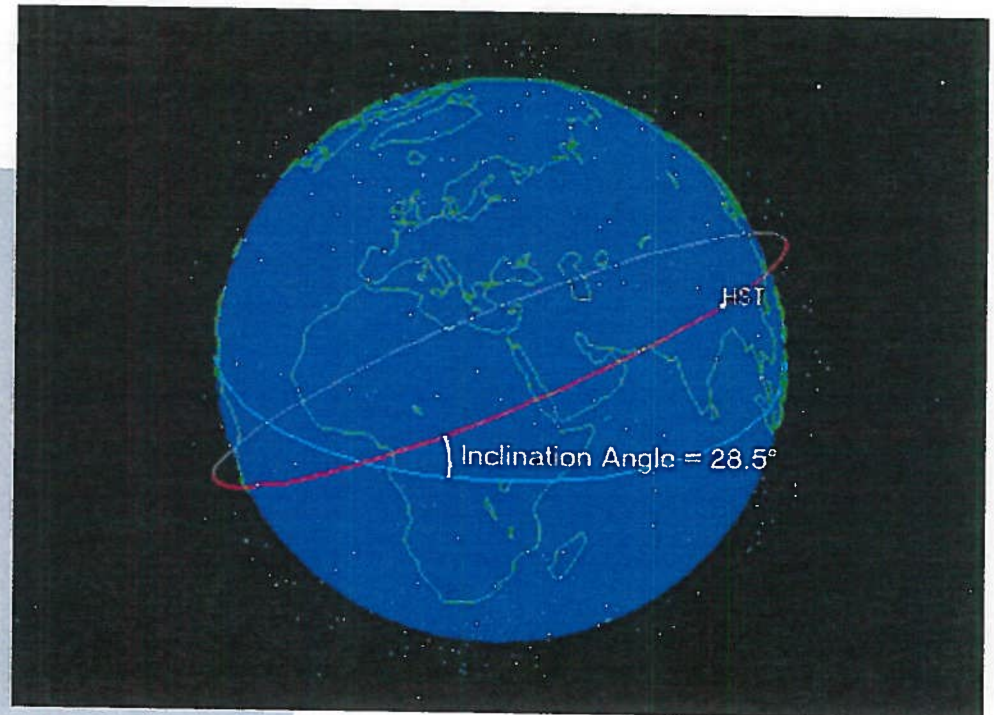
$$\boxed{\cos i = \cos \phi \sin \beta}$$

where i = orbit inclination

ϕ = launch site latitude

β = launch azimuth

(initial launch direction angle, from North)



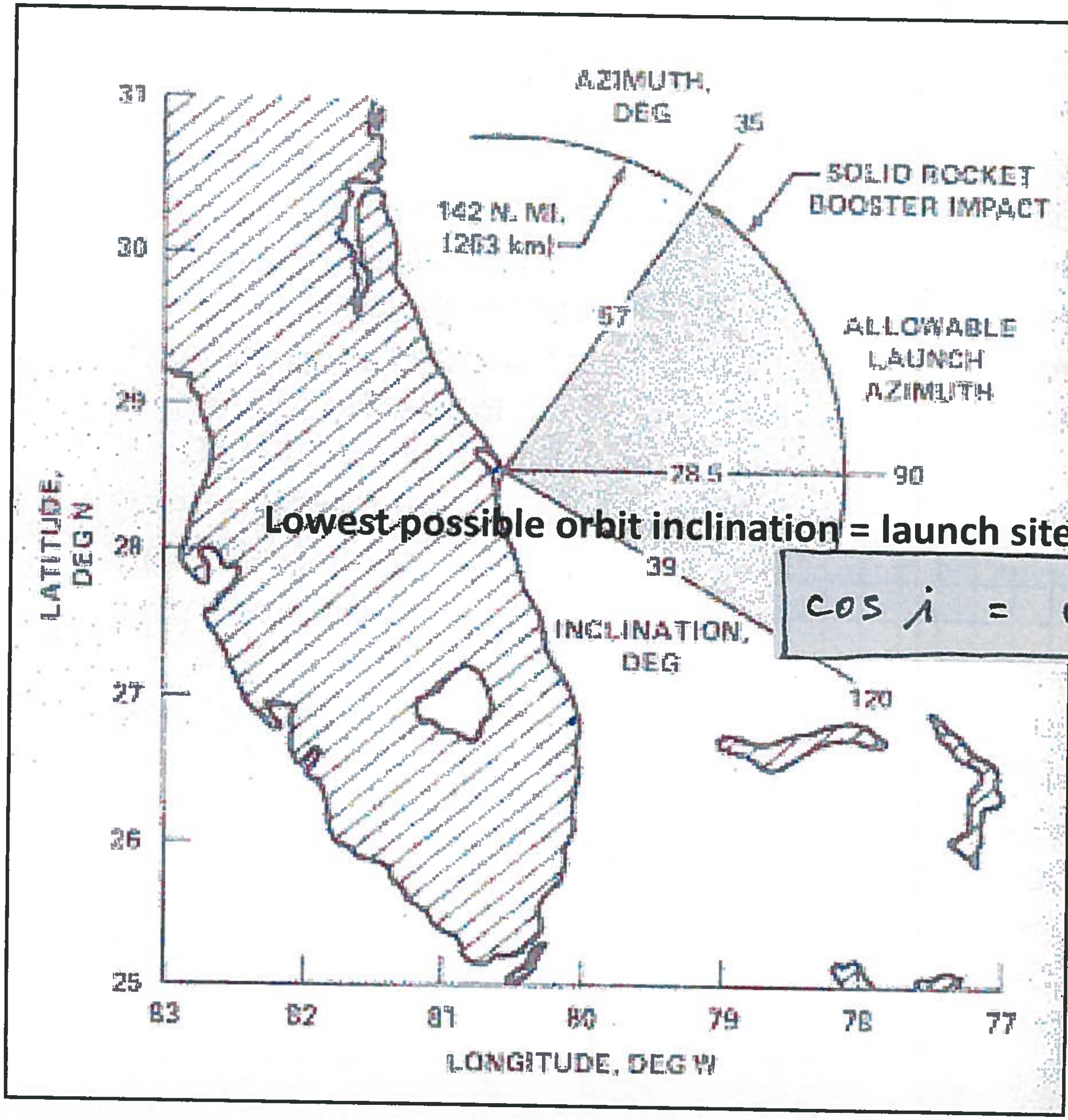
Example: Hubble Space Telescope

$i = 28.5°$

$\phi = 28.5°$ Cape Canaveral

$$\therefore \beta = \arcsin\left(\frac{\cos i}{\cos \phi}\right) = 90° \text{ (East)}$$

Orbit Inclination



Lowest possible orbit inclination = launch site lattitude!

$$\boxed{\cos i = \cos\phi \sin\beta}$$

② Phasing

- The chase has begun. Assume:

  Target orbit circular (launch + circ. burn)

  Chaser orbit co-planar with target orbit
  nearly circular

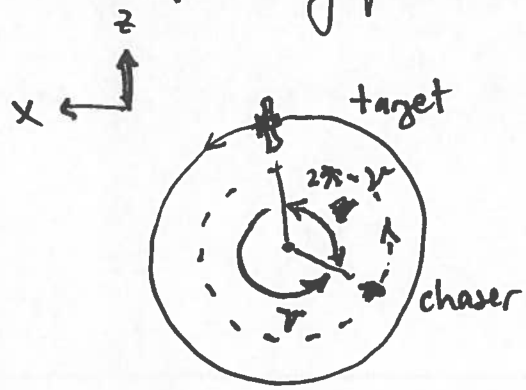
- which kind of navigation?

  absolute: both target + chaser SV's tracked relative to inertial ref frame (onboard GPS, IMU's, ground radar, etc.) Used until chaser sensors can track target

  relative: Chaser onboard sensors measure $\vec{r}, \dot{\vec{r}}$ to target (and sometimes vice-versa). Often combined w/absolute nav "solution" via dynamic Kalman filters

- On-board sensors: cameras (vis and IR)
  radar
  Lidar (scan)
  laser (single-beam)
  star-tracker
  prismatic retroreflectors
  computer-vision landmarks
  ⋮

- The rendezvous phasing problem:



must "catch up"

$\nu \equiv$ phase angle between spacecraft (in flight dir)

$= \Delta\theta$ diff in true anomaly

< 0 for chasing from behind

- If chaser is in "smaller" orbit, it closes on target:

$$a_1 < a_T \implies n_1 = \underbrace{\sqrt{\frac{\mu}{a_1^3}}}_{\text{mean orbital frequency } \frac{2\pi}{T}} > n_T$$

so chaser catches up

a_1: semi-major axis of initial chaser orbit

a_T: target orbit

- How much is phase angle ν reduced per orbit?

$$\Delta\nu = \Delta n\, T = \left(\frac{dn}{da}\Delta a\right) T = -\frac{3}{2}\frac{n}{a}\Delta a\, T$$

(T: target period; a: target)

or $\boxed{\Delta\nu\Big|_{\text{per orbit}} = -3\pi\frac{\Delta a}{a}}$ where $\Delta a = a_T - a_1 > 1$

so phase angle $\Delta\nu$ decreases

- Closing distance per orbit: $\Delta x = \nabla\nu\, a = -3\pi\Delta a$

- Phasing example : ISS rendezvous from below:

ISS $h_T \cong 350\ km$

$a_T = R_E + h_T$ (circ)

chaser $h_1 = 200\ km$

$a_1 = R_E + h_1$

$\Delta a = a_T - a_1 = 150\ km$

$$\Delta \nu = -3\pi \frac{\Delta a}{a_T} = -3\pi \frac{150}{350 + 6371} \text{rad} \frac{360 \text{deg}}{2\pi \text{rad}} = 12 \frac{\text{deg}}{\text{orbit}}$$

worst case: $360°$ behind $\Rightarrow \dfrac{360°}{12°/\text{orbit}} = 30$ orbits

$= 45$ hrs

~ 2 days

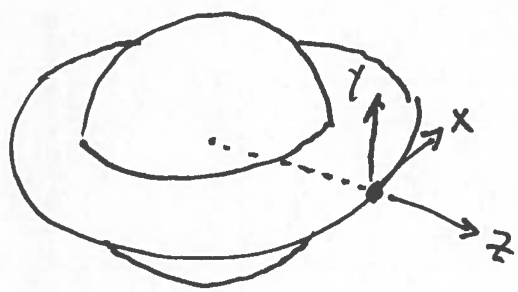
3) Homing Phase (fig 8.26)

- Reduce ellipticity to give relative nav sensors a relatively stable range and aimpoint

- Approx 50km in trail (many dependencies)

- LVLH reference system (fig 8.30):

  "Local Vertical Local Horizontal" (topocentric)



± y : out of plane

+ x : V-bar

− z : R-bar

"V-bar approach": approach from target's +X (from in front)

"R-bar approach": approach from target's −Z (from below)

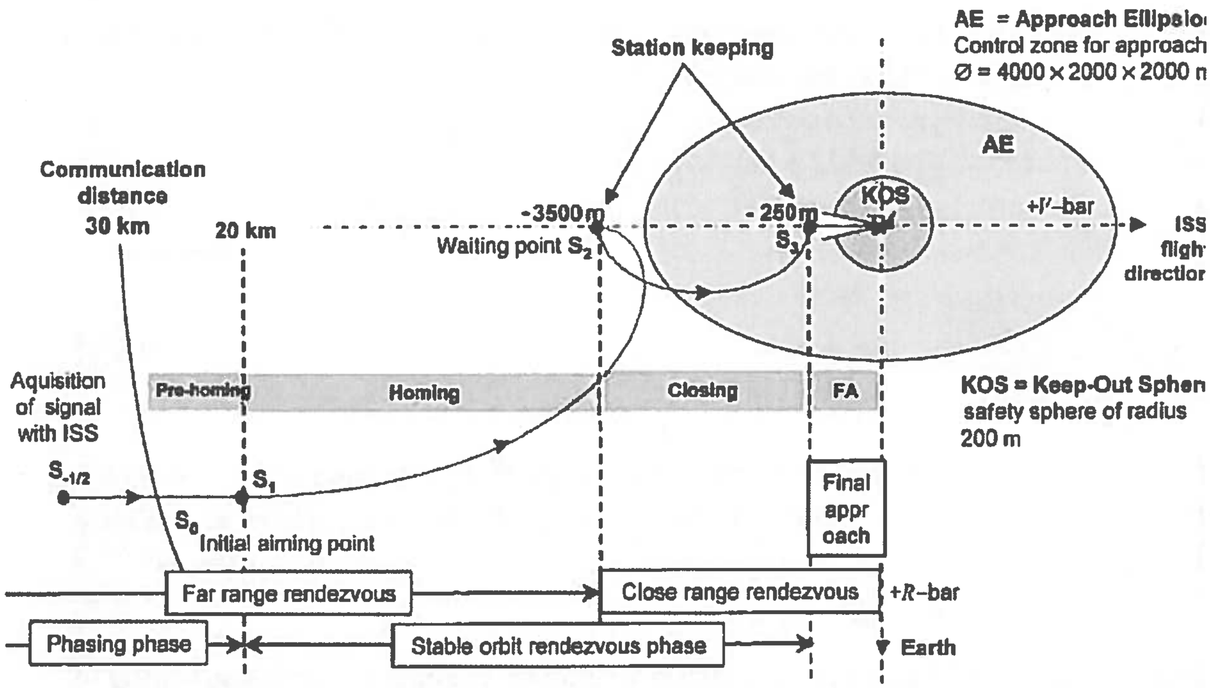


Figure 8.29 Homing, closing, and final approach profile and phases for ISS rendezvous. ISS safi approach procedures require station-keeping points S on the V-bar, an approach corridor, and a Keep-Out Sphere around ISS that approaching spacecraft must use.

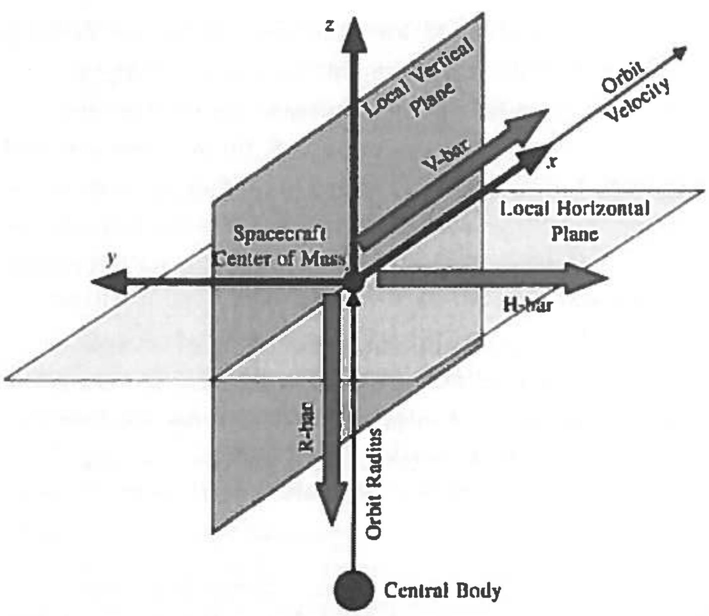


Figure 8.30 Local Vertical Local Horizontal reference system: + V-bar (+ x-axis) is in the directioi of the spacecraft's velocity vector, + R-bar (+ z-axis) is in the direction of the negative radius vectoi and + H-bar ($-y$-axis) completes the right-handed system.