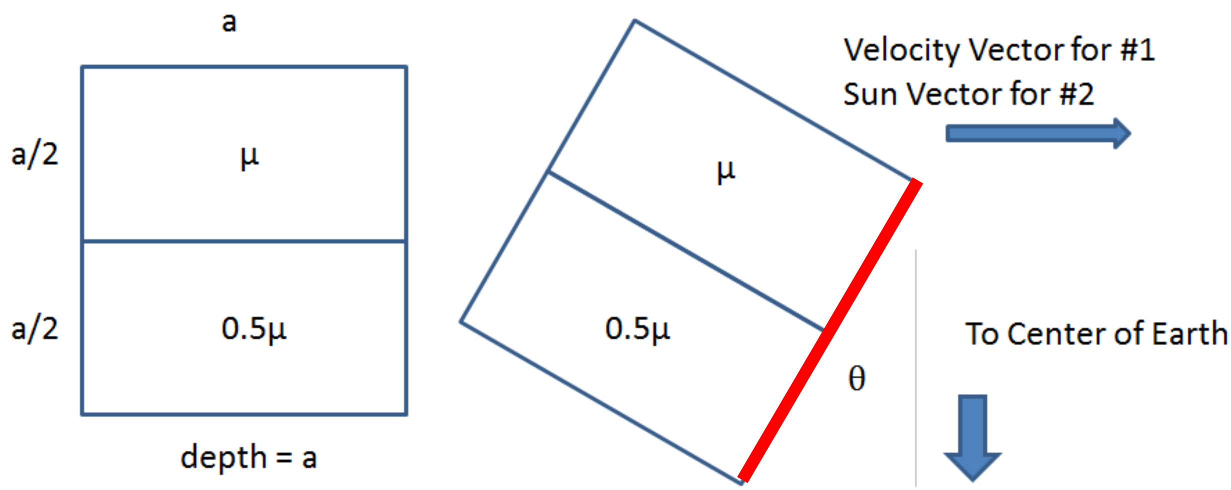
EAE243a/EME298 Winter 2016 – Homework #9 ***Due Thu 3/17/16 11:59pm***

Part 1 – Attitude Disturbance Analysis

Do calculations for the disturbances present for equatorial circular orbits at **200km** and **350km**

Consider a cube with two different halves. One half has twice the density (and therefore twice the mass) as the other. We will only be concerned with rotations about the axis out of the page.



a = 10cm μ = 1kg/1000cm^3 θ = 30°

1. **Drag**
   Find the disturbance torque on the cube about the mass center (use C_D = 2.2)

   $$\underline{T} = \sum \underline{r_i} \times \underline{F_i} \qquad \underline{F_i} = \frac{1}{2}\rho V^2 C_D(\hat{n}_i \cdot \hat{V})A_i(-\hat{V})$$

   ρ = gas density V = velocity
   A = exposed area C_D = drag coefficient
   n = normal vector for surface hats denote unit vectors

   This can be done in two parts, one for each exposed side
   Report the torque for each of the two altitudes

2. **Solar Pressure**
   Find the disturbance torque on the cube about the mass center
   Assume that the entire surface is covered in solar panels (f_s~0.21, f_d~0.1)

   $$\underline{T} = \sum \underline{r_i} \times \underline{F_i} \qquad \underline{F_i} = a_i\hat{s} + b_i\hat{n}_i$$

   $$a_i = -PA_i(1 - f_{s,i})cos\theta_i \qquad b_i = -2PA_i\left(f_{s,i}cos\theta_i + \frac{1}{3}f_{d,i}\right)cos\theta_i$$

   s = sun vector n = normal vector for surface
   P = mean momentum flux ~4.67x10^{-6}Nm^{-2} A = exposed area

f_s = specular coefficient

f_d = diffuse coefficient

θ_i = incidence angle

hats denote unit vectors

This can be done in two parts, one for each exposed side

Report the torque for each of the two altitudes

3. **Gravity Gradient**

Find the disturbance torque on the cube about the mass center

$$\underline{T} = \sum \underline{r_i} \times \underline{F_i} \qquad \underline{F_i} = \frac{\mu m_i}{r_i^2}(-\hat{r})$$

μ = Earth's gravitational constant $0.3986 \times 10^{15} m^3/s^2$

r = radius from center of Earth

This can be done in two parts, one for each side of homogenous density

Report the torque for each of the two altitudes

4. **Magnetic Dipole**

Find the disturbance torque on the cube about the mass center for two configurations

$$\underline{T} = \sum \underline{M_i} \times \underline{B} \qquad \underline{M_i} = nIA\hat{c}$$

n = number of coils (use 25)

I = current (use 0.1A)

A = cross section of the coil (use 25π cm^2)

c = vector along coils axis

$\underline{B}$ is out of the page if we assume an orbit with the rotation of the Earth

The coil is installed on the face indicated by the red line

Report the torque for each of the two altitudes

Part 2 – Reaction Wheel and Thruster Analysis

Assume a maximum constant on orbit disturbance torque of $1x10^{-5}$ Nm on a single axis
Size a reaction wheel to keep a craft pointed despite this disturbance. Assume a saturation speed of 6000rpm for the wheel. It should be capable of eliminating the maximum on orbit disturbance without desaturating for two weeks.

$$H_{storage} = \int \frac{dH}{dt} = \int T_{disturbance,max}\, dt = T_{disturbance,max} t$$
$$\int \frac{dH}{dt} = \int I_{wheel}\, \alpha\, dt = I_{wheel}\omega_{saturated}$$

This will give you the required inertia so treat the wheel as a disk and choose a suitable material, thickness, and diameter to achieve this inertia value.
With the reaction wheel sized we will now look at the thruster burn required to desaturate the wheel.
Assume the craft is of sufficient size to allow for two thrusters with a 1 meter separation to be arranged in such a way to produce a pure force couple (no net force, only a torque) about the center of mass.
Thrusters are commonly sized by their force output in vacuum. Assume we have two 1 N thrusters.
The torque produced by these thrusters is simple.

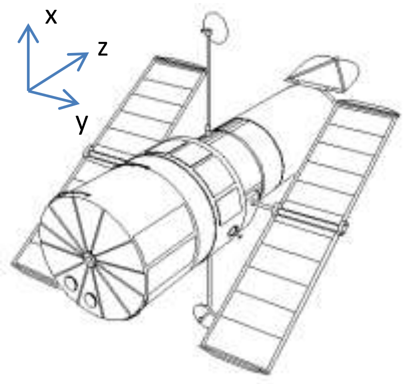
$$T = 0.5m * 1N + 0.5m * 1N = 1Nm$$

With this you can find how long the thrusters need to burn in order to desaturate the wheels using a similar method to the one used to size the wheels.

$$I_{wheel}\omega_{saturated} = \int I_{wheel}\, \alpha\, dt = \int \frac{dH}{dt}$$
$$\int \frac{dH}{dt} = \int T_{thrusters}\, dt = T_{thrusters} t$$

The only unknown is the time of the burn.
Report the wheel inertia, your wheel material/dimensions, and the burn time
Explain your choice of wheel material as well as dimensions (rod or disk)

Part 3 – Hubble Slew Problem



Either use published inertia values for Hubble or the estimates you made in the dynamics assignment.
Assume torquers are aligned with the principle axis.

$$I_{xx}\dot{\Omega}_x + H_{spin}\Omega_y = T_x$$
$$I_{yy}\dot{\Omega}_y + H_{spin}\Omega_x = T_y$$
$$H_{spin} = I_{wheel}\omega_{wheel}$$

Ω = precession rates

T = torque

ω = stabilizing spin

I = inertia

Assume our wheel has an inertia of 1 kg m^2 (estimate for 5cm thick steel disk with 0.2m radius) and is spinning with an angular velocity of 1500 RPM

1. **Two axis torques**
   Find the required variable torques (x and y) to constantly accelerate the angular precession by 0.1 deg/s^2 on the x axis while trying to keep the angular procession on the y-axis at a constant 0.1 deg/s.

   This means at t = 0:
   $\dot{\Omega}_x = 0.01\ deg/s^2$ $\quad$ $\dot{\Omega}_y = 0\ deg/s^2$ $\quad$ $\Omega_y = 0.1\ deg/s$ $\quad$ $\Omega_x = 0\ deg/s$

   And at t = 15sec
   $\dot{\Omega}_x = 0.01\ deg/s^2$ $\quad$ $\dot{\Omega}_y = 0\ deg/s^2$ $\quad$ $\Omega_y = 0.1\ deg/s$ $\quad$ $\Omega_x = 0.15\ deg/s$

   $\dot{\Omega}_x$ should be zero at t_{final} but just consider it to be 0.01 deg/s^2 for the full duration and only report the first 15 seconds.
   Report time histories of the torques on both the x and y axes

2. **Two axis torques**

   Find the required variable torque (x) to constantly accelerate the angular precession by 0.1 deg/s² on the x axis while applying no torque to the y axis.

   This means at t = 0:

   $\dot{\Omega}_x = 0.01\ deg/s^2$ $\quad \dot{\Omega}_y = 0\ deg/s^2$ $\quad \Omega_y = 0.1\ deg/s$ $\quad \Omega_x = 0\ deg/s$

   And at t = 15sec

   $\dot{\Omega}_x = 0.01\ deg/s^2$ $\quad \dot{\Omega}_y = \dot{\Omega}_{y,final}\ deg/s^2$ $\quad \Omega_y = \Omega_{y,final}\ deg/s$ $\quad \Omega_x = 0.15\ deg/s$

   $\dot{\Omega}_x$ should be zero at t_{final} but just consider it to be 0.01 deg/s² for the full duration and only report the first 15 seconds.

   Report time histories of the torque on the x axis and the precession rate in the y axis