Attitude of Spacecraft

MAE 243a – Spacecraft Engineering

Attitude of a craft free of disturbances

Starting with the equations from last time for a system aligned with the principle axis

$$I_{xx}\dot{\omega}_x - \left(I_{yy} - I_{zz}\right)\omega_y\omega_z = T_x = 0$$
$$I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_x\omega_z = T_y = 0$$
$$I_{zz}\dot{\omega}_z - \left(I_{xx} - I_{yy}\right)\omega_x\omega_y = T_z = 0$$

Thing of special cases where

$$I_{xx} = I_{yy} = I_{yy}$$

and

$$I_{xx} = I_{yy}$$

Are either or both of these practical assumptions for most spacecraft?

Spin with inertially asymmetric craft and no disturbances

$$I_t\dot{\omega}_x = (I_t - I_s)\omega_y\omega_z$$
$$I_t\dot{\omega}_y = (I_s - I_t)\omega_x\omega_z$$
$$I_s\dot{\omega}_z = 0$$

The angular velocity in the z axis affects the other two axes, yet the motion on the z axis remains independent of the other two

This system also has an analytic solution

$$\omega_x(t) = \omega_{x0}\cos\Omega t + \omega_{y0}\sin\Omega t$$
$$\omega_y(t) = \omega_{y0}\cos\Omega t - \omega_{x0}\sin\Omega t$$

Where

$$\Omega = \left(\frac{I_t - I_s}{I_t}\right)\omega_{z0}$$

Since $I_{zz}\dot{\omega}_z = 0$, ω_z=ω_{z0}

Introducing torques back in

$$I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y\omega_z = T_x$$
$$I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_x\omega_z = T_y$$
$$I_{zz}\dot{\omega}_z = T_z$$

Separating ω_x and ω_y leads to

$$I_{xx}(\ddot{\omega}_x + \Omega^2\omega_x) = \dot{T}_x - \Omega T_y$$

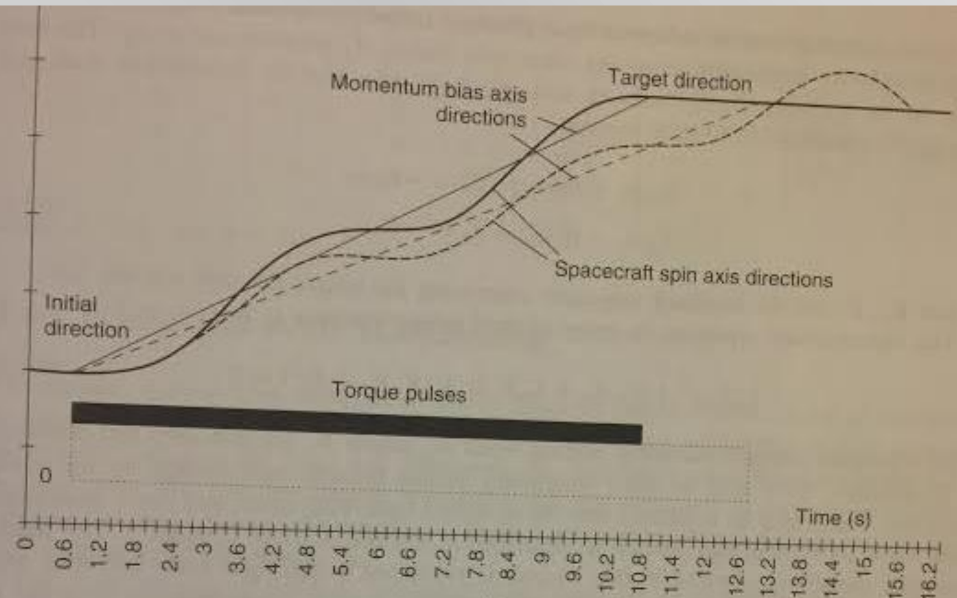
and

$$I_{yy}(\ddot{\omega}_y + \Omega^2\omega_y) = \dot{T}_y - \Omega T_x$$

To repoint a craft with a momentum bias

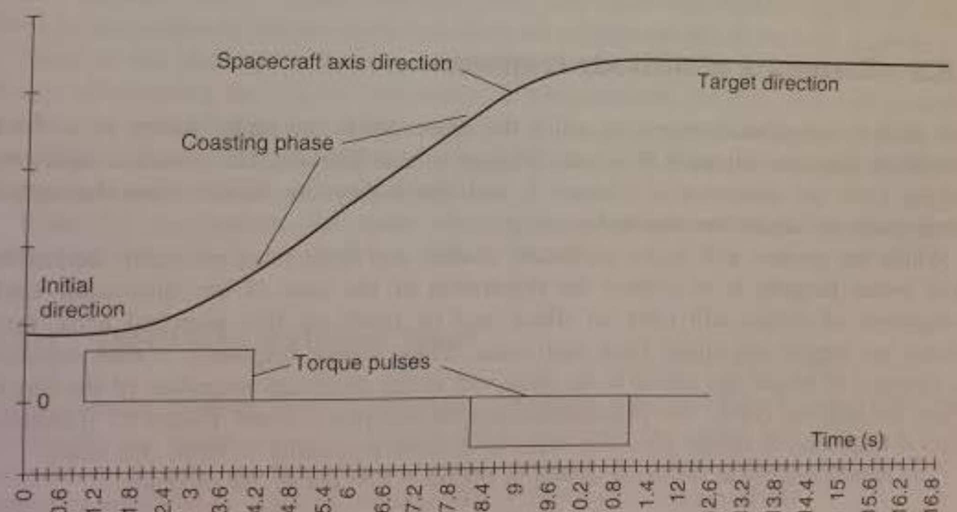
$$I_{xx}\dot{\Omega}_x + I_{zz}\omega_z\Omega_y = T_x$$
$$I_{yy}\dot{\Omega}_y + I_{zz}\omega_z\Omega_x = T_y$$
$$I_{zz}\dot{\omega}_z = T_z$$

Where Ω_x and Ω_y are the procession rates of the non spinning axis



Momentum bias axis directions
Target direction
Spacecraft spin axis directions
Initial direction
Torque pulses
0
Time (s)
0 0.6 1.2 1.8 2.4 3 3.6 4.2 4.8 5.4 6 6.6 7.2 7.8 8.4 9 9.6 10.2 10.8 11.4 12 12.6 13.2 13.8 14.4 15 15.6 16.2

(a)



Spacecraft axis direction
Target direction
Coasting phase
Initial direction
Torque pulses
0
Time (s)
0 0.6 1.2 1.8 2.4 3 3.6 4.2 4.8 5.4 6 6.6 7.2 7.8 8.4 9 9.6 10.2 10.8 11.4 12 12.6 13.2 13.8 14.4 15 15.6 16.2 16.8

(b)

Hybrid craft

For a craft with a spinning section with a rotational axis aligned with the z axis

$$I_{xx}\dot{\omega}_x - \left(I_{yy} - I_{zz}\right)\omega_y\omega_z + \omega_y H_z = T_x$$
$$I_{yy}\dot{\omega}_y - \left(I_{zz} - I_{xx}\right)\omega_x\omega_z - \omega_x H_z = T_y$$
$$I_{zz}\dot{\omega}_z - \left(I_{xx} - I_{yy}\right)\omega_x\omega_y + \dot{H}_z = T_z$$

This rotational element will also cause a nutation in the non-spinning axes with a frequency of

$$\Omega = \frac{H_z}{\sqrt{I_{xx}I_{yy}}}$$

This also applies to any other rotational elements on the spacecraft. The contributions of each element must be included:

$$\dot{H} = -\omega \times H + T$$
$$H = I\omega$$
$$T = \sum -\omega \times H_i + \dot{H}_i$$

Torques and Torquers

Table 9.1 Disturbance torques

External torques source	Height range over which it is potentially dominant
Aerodynamic	<about 500 km*
Magnetic	500–35 000 km
Gravity gradient	500–35 000 km
Solar radiation	>700 km*
Thrust misalignment	all heights
Internal torques source	
Mechanisms	
Fuel movement	
Astronaut movement	
Flexible appendages	
General mass movement	

*Values depend upon the level of solar activity.

Table 9.2 Types of torquer

Type	Advantages	Disadvantages
External types	Can control momentum build-up	
Gas jets	Insensitive to altitude Suit any orbit Can torque about any axis	Requires fuel On-off operation only Has minimum impulse Exhaust plume contaminants
Magnetic	No fuel required Torque magnitude is controllable	No torque about the local field direction Torque is altitude and latitude sensitive Can cause magnetic interference
Gravity gradient	No fuel or energy needed	No torque about the local vertical Low accuracy Low torque, altitude sensitive Libration mode needs damping
Solar radiation	No fuel required	Needs controllable panels Very low torque
Internal types	No fuel required Can store momentum Torque magnitude is controllable	Cannot control momentum build-up
Reaction wheels (RW) Momentum wheels (MWs)	Continuous, fine-pointing capability Provide momentum bias	Non-linearity at zero speed
Control moment gyroscope (CMG)	Suitable for three-axis control Provides momentum bias	Complicated Potential reliability problem

Table 19-5. Principal Internal Disturbance Torques. Spacecraft designers can minimize internal disturbances through careful planning and precise manufacturing, which may increase cost.

Disturbances	Effect on Vehicle	Typical Values
Uncertainty in Center of Gravity (cg)	Unbalanced torques during firing of couples thrusters Unwanted torques during translation thrusting	1–3 cm
Thruster Misalignment	Same as cg uncertainty	0.1–0.5 deg
Mismatch of Thruster Outputs	Similar to cg uncertainty	±5%
Reaction Wheel Friction and Electromotive Force (i.e., back EMF)	Resistance that opposes control torque effort. These torques are the limiting mechanism for wheels speed.	Roughly proportional to wheel speed, depending on model. At top speed, 100% of control torque (i.e., saturation)
Rotating Machinery (pumps, filter wheels)	Torques that perturb both stability and accuracy	Dependent on spacecraft design; may be compensated by counter-rotating elements
Liquid Slosh	Torques due to liquid dynamic pressure on tank walls, as well as changes in cg location.	Dependent on specific design; may be mitigated by bladders or baffles
Dynamics of Flexible Bodies	Oscillatory resonance at bending/twisting frequencies, limiting control bandwidth	Depends on spacecraft structure; flexible frequencies within the control bandwidth must be phase-stabilized, which may be undesirable.
Thermal Shocks ("snap") on Flexible Appendages	Attitude disturbances when entering/leaving umbra	Depends on spacecraft structure. Long inertia booms and large solar arrays can cause large disturbances.

Table 19-6. Attitude Control Methods and Their Capabilities. As requirements become tighter, more complex control systems become necessary.

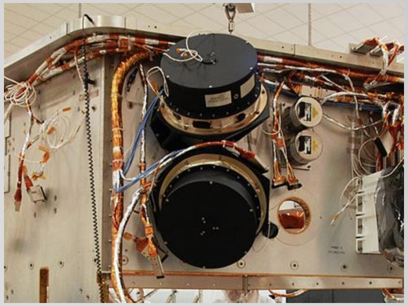
Type	Pointing Options	Attitude Maneuverability	Typical Accuracy	Lifetime Limits
Gravity-Gradient	Earth local vertical only	Very limited	±5 deg (2 axes)	None
Gravity-Gradient + Momentum Bias	Earth local vertical only	Very limited	±5 deg (3 axes)	Life of wheel bearings
Passive Magnetic	North/South only	Very limited	±5 deg (2 axes)	None
Rate-Damping + Target Vector Acquisition	Usually Sun (power) or Earth (communication)	Generally used as robust safe mode.	±5–15 deg (2 axes)	None
Pure Spin Stabilization	Inertially fixed any direction	Repoint with precession maneuvers; very slow with torquers, faster with thrusters	±0.1 deg to ±1 deg in 2 axes (proportional to spin rate)	Thruster propellant (if applies)*
Dual-Spin Stabilization	Limited only by articulation on despun platform	Same as above	Same as above for spun section. Despun dictated by payload reference and pointing	Thruster propellant (if applies)* Despun section bearings
Bias Momentum (1 wheel)	Local vertical pointing or inertial targets	Fast maneuvers possible around momentum vector Repoint of momentum vector as with spin stabilized	±0.1 deg to ±1 deg	Propellant (if applies)* Life of sensor and wheel bearings
Active Magnetic with Filtering	Any, but may drift over short periods	Slow (several orbits to slew); faster at lower altitudes	±1 deg to ±5 deg (depends on sensors)	Life of sensors
Zero Momentum (thruster only)	No constraints	No constraints High rates possible	±0.1 deg to 5 deg	Propellant
Zero momentum (3 wheels)	No constraints	No constraints	±0.0001 deg to ±1 deg (determined by sensors and processor)	Propellant (if applies)* Life of sensors and wheel bearing
Zero Momentum (CMG)	No constraints Short CMG life may require high redundancy	No constraints High rates possible	±0.001 deg to ±1 deg	Propellant (if applies)* Life sensors and CMG bearings

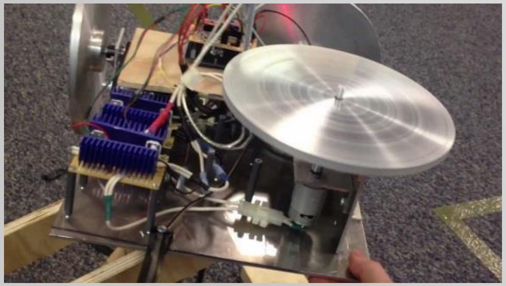
* Thrusters may be used for slewing and momentum dumping at all altitudes, but propellant usage may be high. Magnetic torquers may be used from LEO to GEO.

Table 19-6, Fig. 19-6◀, Eq. 19-8◀

Reaction wheels vs. control moment gyros

Reaction Wheel - Mainly causes momentum change through changing the speed of a rotor





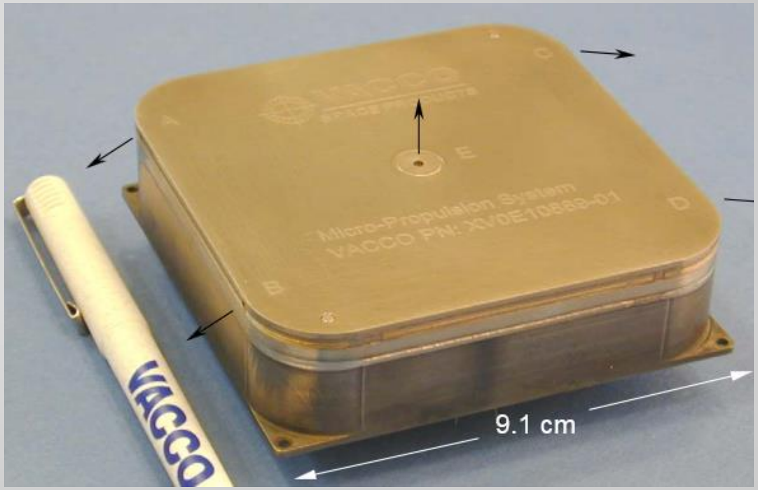
Control Moment Gyro – Mainly causes momentum change through changing the orientation of the rotation axis.

Thrusters

Torque created by thrusters is simple, just keep in mind they are usually arranged in a way that they can be fired in pairs to produce nearly pure force couples

$$\boldsymbol{T} = \boldsymbol{r} \times \boldsymbol{F}$$

Where r is the perpendicular distance from the CG and F is the force produced by the thruster



Magnetic Torquers

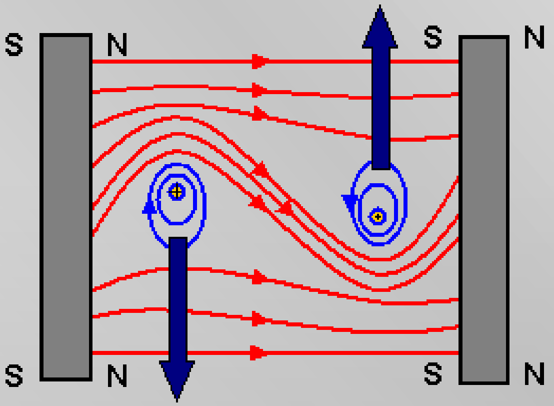
Magnetic torquers can be easy to predict as long as the have simple geometries
In the simplest for the torque can be represented as

$$T = m \times B$$

Where m is the magnetic moment and B is the local flux density
For a given environment or location on orbit the local flux density will be fairly constant but uncontrollable. Therefore control must be done through manipulation of the magnetic moment.

A common way to produce a magnetic moment is by running a current through a coil. Here the magnetic moment would be calculated by

$$m = nIAc$$



Here n is the number of coils, I as the current, A is the cross-sectional area of the coil, and c is the unit vector in the direction of the coil's axis.

Gravity Gradient

Any craft with an uneven mass distribution will have a gravity gradient
The gravitational force on an increment of mass dm can be found as

$$dF = \frac{\mu\, dm}{r^2}$$

If we were to sum the moments (about the center of mass) for each of the forces then we could find the torque produced on the craft due to the uneven mass distribution
These torques are found to be:

$$T_x = \left(\frac{3\mu}{2r^3}\right)\left(I_{zz} - I_{yy}\right)\sin 2\varphi\, cos^2\,\theta$$

$$T_y = \left(\frac{3\mu}{2r^3}\right)\left(I_{zz} - I_{xx}\right)\sin 2\theta\, cos\,\theta$$

$$T_z = \left(\frac{3\mu}{2r^3}\right)\left(I_{xx} - I_{yy}\right)\sin 2\theta\, sin\,\varphi$$

These torques will result in a conical pendulum motion with a motion defined by:

$$\Omega = \sqrt{\left[\left(\frac{3\mu}{r^3}\right)\left(1 - \frac{I_{zz}}{I_{xx}}\right)\right]}$$

Aerodynamic Drag

Aerodynamic forces can be calculated as

$$dF = 0.5\rho V^2 C_D(\hat{n}.\hat{V})(-\hat{V})dA$$

Where
ρ = atmospheric density
V = craft velocity relative to atmospheric gas
C_D = drag coefficient (usually taken to be 2.2)
$\hat{V}$ = unit vector aligned with velocity vector
$\hat{n}$ = unit vector normal to exposed surface
dA = incremental area

The integral is performed over the area where $\hat{n}.\hat{V} \geq 0$
The more complex the surface of the craft, the less accurate the estimate becomes
Works best for radially closed shapes

Radiation Pressure

Radiation pressure is largely due to solar radiation and albedo
An incremental force due to radiation pressure can be calculated as:

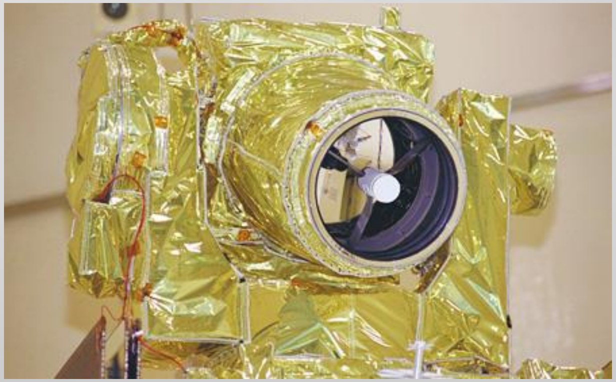
$$\boldsymbol{dF} = -\boldsymbol{P}\cos\boldsymbol{\theta}\,\boldsymbol{dA}\left[(\mathbf{1}-\boldsymbol{f_s})\hat{\boldsymbol{s}} + \mathbf{2}\left(\boldsymbol{f_s}\cos\boldsymbol{\theta} + \frac{\boldsymbol{f_d}}{\mathbf{3}}\right)\hat{\boldsymbol{n}}\right] \quad \text{for} \quad \hat{\boldsymbol{s}}.\hat{\boldsymbol{n}} \geq \mathbf{0}$$

Where
$\hat{s}$ = unit vector from spacecraft to sun
$\hat{n}$ = unit vector normal to exposed surface
P = mean momentum flux ($4.67\times10^{-6}\,Nm^{-2}$ solar flux at Earth)
$\theta = \cos^{-1}(\hat{s}.\hat{n})$ (incidence of radiation)
f_s = specular coefficient
f_d = diffuse coefficient

Each of these forces imparts a moment on the craft just like any other force
The resultant torque is found by integrating the effects of the forces on all exposed surfaces

Other Major Considerations

- **Mass movement**
  Anything within the payload that moves, crew movement, fluid pumping
- **Fuel Slosh**
  Different behavior than pumping
- **Impacts/Collisions**
  Less predictable unless accounting for collision during proximity operations
- **Instruments**
  Any moving instuments will have an adverse effect



Wheels and Saturation

Electronically powered reaction wheels and control moment gyros are limited by internal resistances. These are predominantly in the form of friction and back EMF. These loses are related to the rotational speed and therefore put a maximum rotation on control elements.

With the exception of saturation, electronic rotational control elements have huge advantages due to the fact that they run on an essentially renewable resource.

To desaturate wheels, external forces are usually used. This is in the form of thrusters or magnetic torquers for smaller applications.

Other types of more novel momentum dumps are also possible