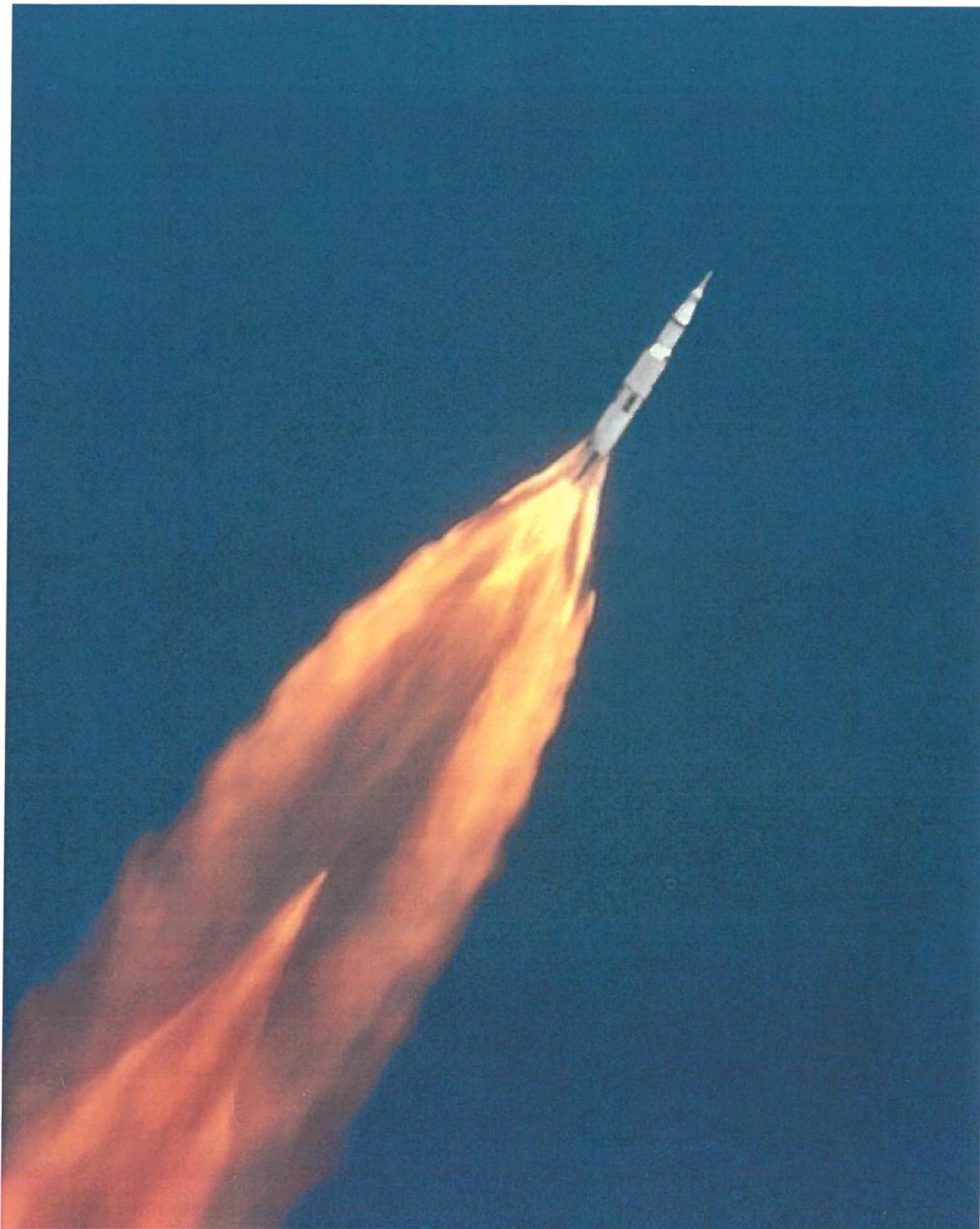
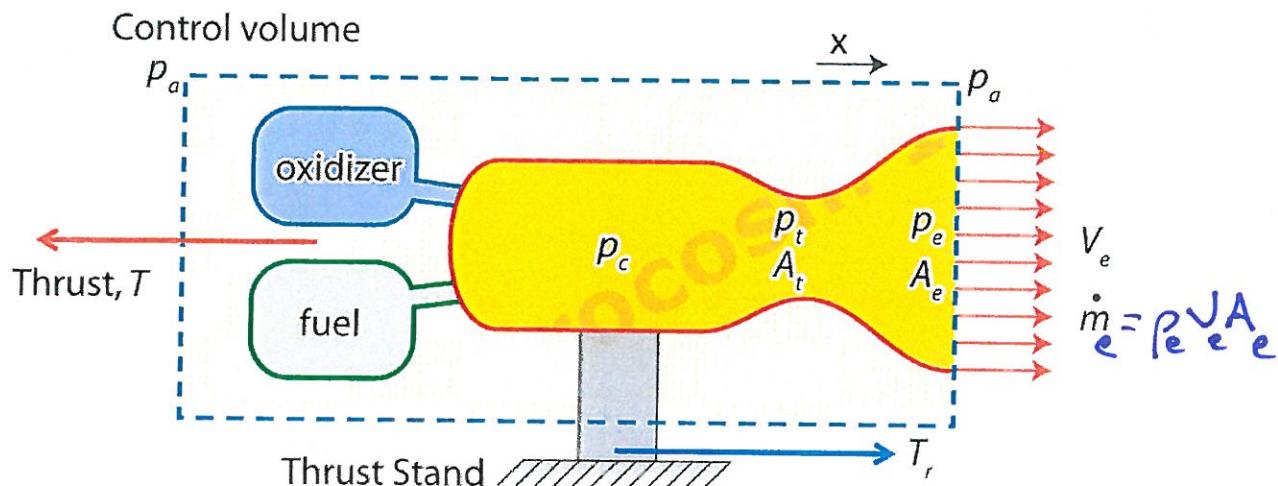


## Propulsion Systems



## Text 18.1: Basic Rocket Equations



(the structure holding the rocket in place feels a reaction force equal in magnitude to the thrust but in the opposite direction)

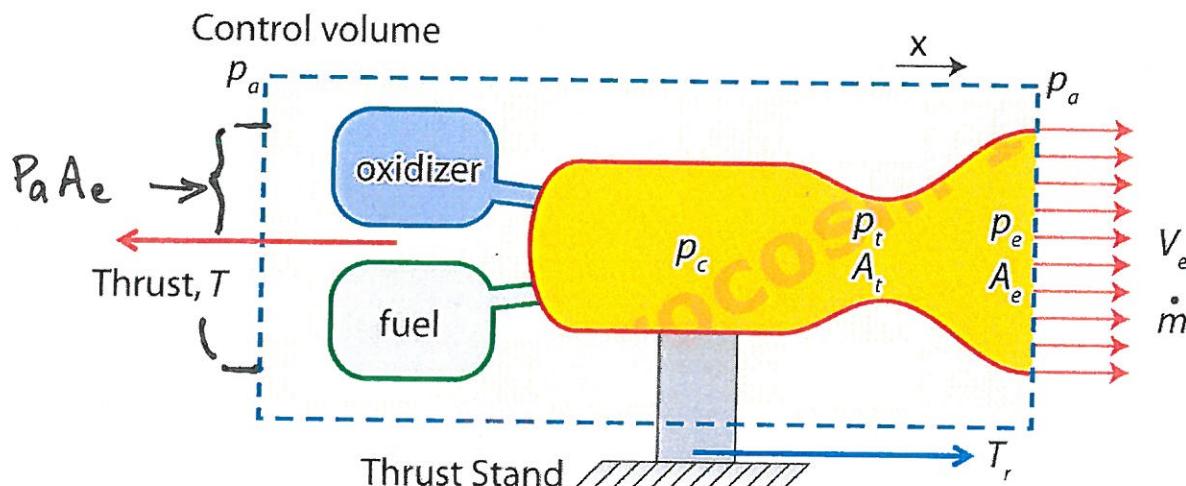
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- Rocket engines produce thrust,  $T$ .  
We'd like to understand which variables influence  $T$ .
- Newton's 2<sup>nd</sup> Law :  $\sum \vec{F} = \frac{d}{dt} (\vec{mV})$   
for a set of mass particles
- Transform to Control Volume using Reynolds Transport Theorem:

$$\sum \vec{F} = \frac{d}{dt} (\vec{mV}) = \underbrace{\frac{d}{dt} \left( \int_{cv} \vec{V} \rho dV \right)}_{=0 \text{ steady}} + \underbrace{\int_{cs} \vec{V} \rho (\vec{V} \cdot \vec{n}) dA}_{\text{discrete, 1D outlet}}$$

$$= (\dot{m} \vec{V})_{out} - (\dot{m} \vec{V})_{in} \rightarrow 0$$

## Text 18.1: Basic Rocket Equations



(the structure holding the rocket in place feels a reaction force equal in magnitude to the thrust but in the opposite direction)

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- Continue  $x$ -momentum balance :

$$\sum F_x = (\dot{m} V_x)_{\text{out}} \quad \text{where } \dot{m} = \text{exhaust mass-flow rate}$$

$$= p_e V_e A_e$$

$$T_r + p_a A_e - p_e A_e = \dot{m} V_e$$

$$T_r = \dot{m} V_e + A_e (p_e - p_a) \quad \text{acting in } +x \text{ direction}$$

or

$$T = |T_r| = \dot{m} V_e + A_e (p_e - p_a) \quad \text{acting in } -x \text{ direction}$$

momentum thrust      pressure thrust

note :  $V_e$  = velocity of exhaust relative to rocket

$p_a \Rightarrow 0$  outside atmosphere

generally,  
 $T_{\text{press}} \ll T_{\text{mom}}$

## Text 18.1: Basic Rocket Equations

- So Thrust  $T = \dot{m} V_e + A_e (P_e - P_a)$

How to maximize  $T$ ?

- Exit flow is a supersonic nozzle, so

could increase  $A_e \rightarrow$  increase  $V_e$

but decrease  $P_e$

(don't want  $(P_e - P_a) < 0$ )

So large  $A_e$  OK in vacuum (2nd stage)

Also, larger nozzle  $\rightarrow$  weight!

- For fixed geometry and  $\dot{m}$ , could increase  $V_e$  by increasing chamber temp and pressure by changing propellant properties/mixture.

But higher  $P_c$  and  $T_c$  require more strength (weight) and more cooling (cost, complexity, efficiency reduced).

- Could increase  $\dot{m}$ , but go through fuel more quickly.

## Text 18.1: Basic Rocket Equations

### Measures of rocket performance

- "Effective exhaust velocity"

$$V_{eq} \equiv \frac{T}{\dot{m}} = V_e + \left( \frac{P_e - P_a}{\dot{m}} \right) A_e$$

One measure of how efficiently the engine converts propellant mass to thrust

- "Characteristic exhaust velocity"

$$C^* \equiv \frac{P_e A_t}{\dot{m}}$$

$P_e$  = chamber pressure

$A_t$  = throat area

$\dot{m}$  = prop. mass flow rate

A measure of combustion efficiency.

Independent of nozzle design.

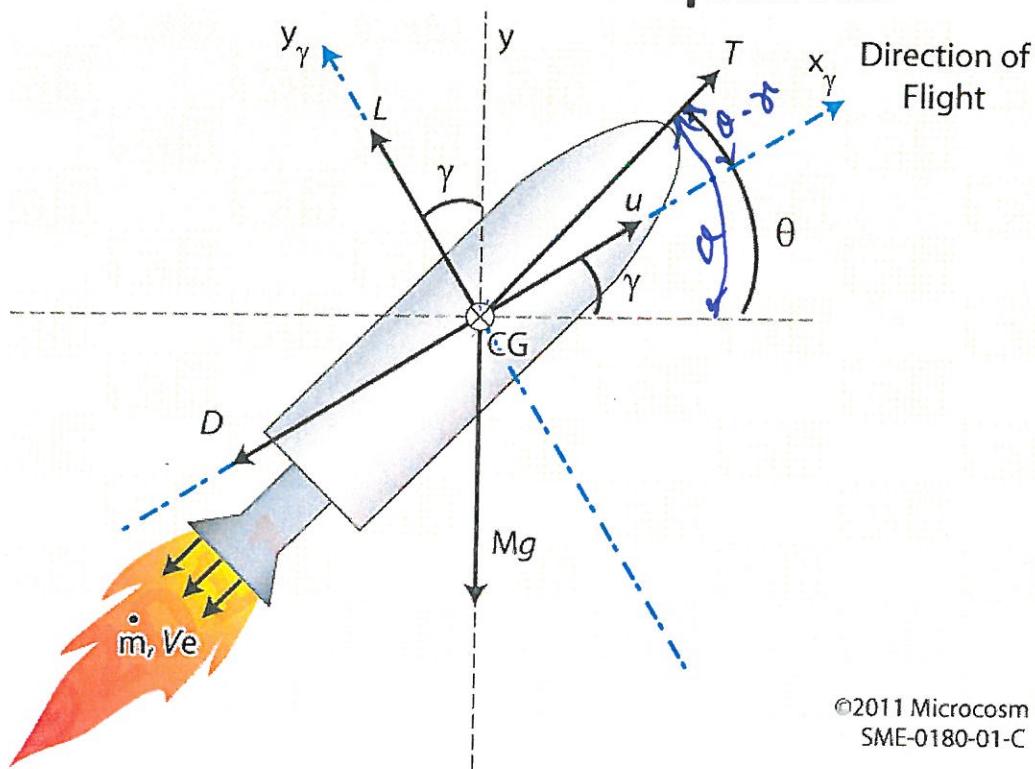
Depends mostly on the chemistry of the particular propellants.

$C^*$  experiment → from ground experiments

~ 90-98%

$C^*$  theory → from thermochemistry theory

## Text 18.1: Basic Rocket Equations

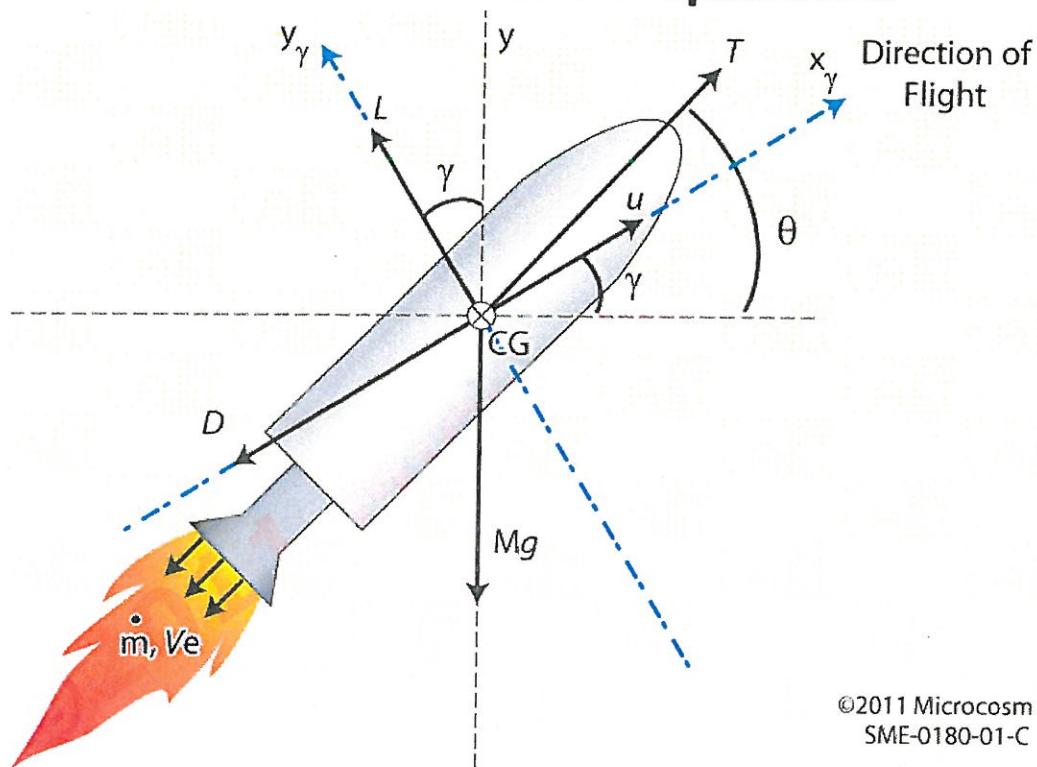


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### Derive the Ideal Rocket Equation

- Most general case:  $T$ : thrust (acting along rocket axis)  
 $Mg$ : weight of rocket (+ propellant on board)  
 $D$ : atmospheric drag  
 $L$ : atmospheric lift
- Note that direction of flight  $\neq$  direction of thrust
- All variables are dynamic functions of time, so force/momentum balance is instantaneous
- For real rockets launching from Earth, want  $\frac{T}{mg} \approx 1.3$

## Text 18.1: Basic Rocket Equations



- Newton's 2<sup>nd</sup> Law in direction of flight ( $\gamma$ ):

$$\sum F_\gamma = M \frac{dv}{dt} = \underbrace{T \cos(\theta - \gamma)}_{\substack{\text{thrust in} \\ \text{direction of flight}}} - \underbrace{D}_{\text{drag}} - \underbrace{Mg \sin \gamma}_{\text{weight component}}$$

(note lift term is  $\perp$  to direction of flight.)

$$\text{where } T = \underbrace{\dot{m}V_e}_{\substack{\text{momentum} \\ \text{thrust}}} + \underbrace{A_e (P_e - P_a)}_{\substack{\text{pressure} \\ \text{thrust}}}$$

## Text 18.1: Basic Rocket Equations

- Calculate velocity ( $V_f$ ) attained by a rocket after a given time, integrate momentum eqn from an initial time ( $t_0$ ) to a final time ( $t_f$ ):

$$\frac{dV}{dt} = \frac{T}{M} - \frac{T}{M} [1 - \cos(\theta - \gamma)] - \frac{D}{M} - g \sin \delta t$$

$$\int_{V_0}^{V_f} dV = \int_{t_0}^{t_f} \frac{T}{M} dt - \int_{t_0}^{t_f} \frac{T}{M} [1 - \cos(\theta - \gamma)] dt - \int_{t_0}^{t_f} \frac{D}{M} dt - \int_{t_0}^{t_f} g \sin \delta t dt$$

or:

$$\Delta V = \underbrace{\Delta V_{\text{prop}}}_{\substack{\text{ideal velocity} \\ \text{gain due} \\ \text{to rocket thrust}}} - \underbrace{\Delta V_{\text{steering}}}_{\substack{\text{thrust losses due to angle of attack,} \\ \text{drag, and gravity}}} - \underbrace{\Delta V_{\text{drag}}}_{\downarrow} - \underbrace{\Delta V_{\text{gravity}}}_{\substack{\text{lanes to} \\ \approx 1300 - 1700 m/s for LED}}}$$

$\approx 1300 - 1700 \text{ m/s}$  for LED

## Text 18.1: Basic Rocket Equations

- Simplify  $\Delta V$  equation:

Substitute  $V_{eq} \equiv \frac{T}{m}$

and realize  $\underbrace{-\frac{dM}{dt}}_{\text{rate at which rocket loses mass}} = \dot{m}$

$\dot{m}$  mass flow rate leaving rocket as exhaust

Also, let  $\theta = \gamma$

neglect drag  $D$

reflect gravity

So now,

$$\Delta V = \int_{t_0}^{t_f} \frac{T}{M} dt = 0 - 0 - 0$$

$$= V_{eq} \int_{t_0}^{t_f} \frac{\dot{m}}{M} dt$$

$$= V_{eq} \int_{M_0}^{M_f} \frac{dM}{M}$$

## Text 18.1: Basic Rocket Equations

$$\Delta V = V_{eg} \int_{M_0}^{M_f} \frac{1}{M} dM$$

$$\begin{aligned}\Delta V &= -V_{eg} \ln\left(\frac{M_f}{M_0}\right) = V_{eg} \ln\left(\frac{M_0}{M_f}\right) \\ &= V_{eg} \ln\left(\frac{M_0}{M_0 - M_p}\right)\end{aligned}$$

Ideal  
Rocket  
Equation

- where  $V_{eg} \equiv \frac{T}{m} = V_e + \left(\frac{P_e - P_a}{m}\right) A_e$

$M_0$  = initial mass of rocket+propellant, at  $t_0$

$M_f$  = final =  $M_0 - M_p$ , at  $t_f$

$M_p$  = mass of propellant burned between  $t_0$  and  $t_f$

- recall assumptions:

weight = negligible (far from large body)

drag = negligible (out of atmosphere)

- Define mass ratio =  $\frac{M_f}{M_0}$

## Text 18.1: Basic Rocket Equations

Pause - Where are we?

- For changing orbits, we framed our equations in terms of  $\Delta V$  required.

Now we can calculate how much  $\Delta V$  our rocket can give us, in terms of mass properties and engine performance.

- $\Delta V = V_{eg} \ln \left( \frac{M_0}{M_f} \right) = V_{eg} \ln \left( \frac{M_0}{M_0 - M_p} \right)$

where  $V_{eg} = \frac{I}{\dot{m}}$

So for example, if rocket is

$$50\% \text{ propellant}, \Delta V_{\text{avail}} = 0.69 V_{eg}$$

$$90\% \text{ propellant}, \Delta V_{\text{avail}} = 2.30 V_{eg}$$

## Text 18.1: Basic Rocket Equations

"Specific Impulse" of a Rocket or Engine (like mpg)

- Total Impulse defined as :

$$I \equiv \int_0^T T dt \quad \text{for } T = f(t) \text{ (non-constant)}$$

= proportional to energy provided by engine(s)

=  $T t_{\text{burn}}$  for constant  $T$  over burn time  $t_{\text{burn}}$

- Specific Impulse defined as :  $\frac{\text{Total Impulse}}{\text{weight of propellants}}$

$$I_{sp} \equiv \frac{\int_0^T T dt}{g_0 \int_0^T m dt} \quad (m = \text{rate of propellant burn})$$

for  $T = \text{constant}$  and  $m = \text{constant}$

$$I_{sp} = \frac{T}{m g_0} = \frac{V_{eg}}{g_0}$$

where

$$V_{eg} \equiv V_e + \left( \frac{P_e - P_a}{m} \right) A_e \\ = T/m$$

where  $T = \text{constant thrust}$

$m = \text{rate of burn of propellants} = \text{constant}$

$$g_0 = \text{grav. constant at Earth surface} \\ = 9.8 \text{ m/s}^2$$

## Text 18.1: Basic Rocket Equations

### More on Specific Impulse

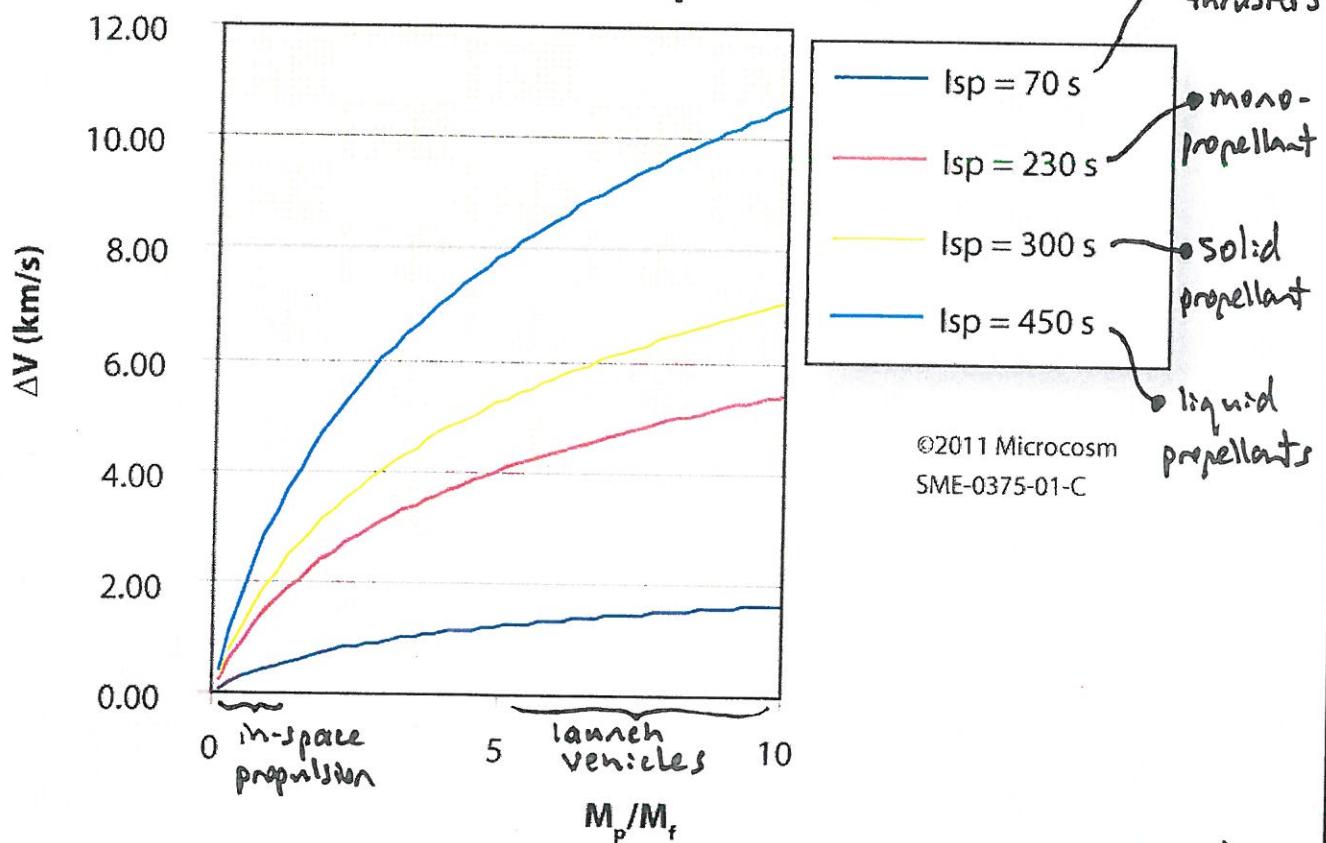
- I<sub>sp</sub> is a measure of how efficiently we produce thrust. The higher the I<sub>sp</sub>, the less propellant used to achieve a given ΔV.
- For launch propulsion, thrust T is somewhat more important than I<sub>sp</sub>, because we must lift up through atmosphere against gravity.
- For in-space propulsion, I<sub>sp</sub> is most important to minimize the propellant required to achieve the required orbit/velocity.
- Substitute I<sub>sp</sub> equation into Rocket Eqn:

$$\Delta V = V_{ef} \ln\left(\frac{M_0}{M_f}\right) = I_{sp} g_0 \ln\left(\frac{M_0}{M_f}\right)$$

so  $\Delta V \propto I_{sp}$  → improvements in I<sub>sp</sub> give large effect on ΔV

various engine types

## Text 18.1: Basic Rocket Equations



- Alternate forms of Rocket Equation in terms of propellant mass:

$$M_p = M_f \left( e^{\frac{\Delta V}{V_{eg}}} - 1 \right) = M_f \left[ e^{\left( \frac{\Delta V}{Isp g_0} \right)} - 1 \right] \quad (\text{plotted above})$$

$$M_p = M_0 \left( 1 - e^{-\frac{\Delta V}{V_{eg}}} \right) = M_0 \left[ 1 - e^{\left( \frac{-\Delta V}{Isp g_0} \right)} \right]$$

18.1

## Text 18.2: Staging

Propulsion Function	Typical $\Delta V$ and Other Requirements
<i>Orbit Transfer to GEO (orbit insertion)</i>	
Perigee Burn	2,400 m/s
Apogee Burn	1,500 (low inclination) to 1800 m/s (high inclination)
<i>Initial Spin Up</i>	1–60 rpm
<i>LEO to Higher Orbit Raising <math>\Delta V</math></i>	
Drag makeup $\Delta V$	60–500 m/s
Controlled Re-entry	120–150 m/s
<i>Acceleration to Escape Velocity from LEO Parking Orbit</i>	3,600–4,000 m/s into a heliocentric orbit
<i>Orbit Maintenance</i>	
Despin	60 to 0 rpm
Spin control	$\pm 1$ to $\pm 5$ rpm
Orbit correction $\Delta V$	15 to 75 m/s per year
East-West stationkeeping $\Delta V$ at GEO	3 to 6 m/s per year
North-South stationkeeping $\Delta V$ at GEO	45 to 55 m/s per year
Survivability or evasive maneuvers (highly variable) $\Delta V$	150 to 4,600 m/s
<i>Attitude Control</i>	
Acquisition of Sun, Earth, Star	3–10% of total propellant mass low total impulse, typically <5,000 N-s, 1K to 10K pulses, 0.01 to 5.0 sec pulse width
On-orbit normal mode control with 3-axis stabilization, limit cycle	100K to 200K pulses, minimum impulse bit of 0.01 N-s, 0.01 to 0.25 sec pulse width
Precession control (spinners only)	low total impulse, typically <7,000 N-s, 1K to 10K pulses, 0.02 to 0.2 sec pulse width
Momentum management (wheel unloading)	5 to 10 pulse trains every few days, 0.02 to 0.10 s pulse width
3-axis control during $\Delta V$	on-off pulsing, 10K to 100K pulses, 0.05 to 0.20 s pulse width

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Table 18-2. Typical Functions and Requirements for Upper Stages and In-Space Propulsion

18.1

# Text 18.3: Chemical Propulsion Systems

Propulsion System	Orbit Insertion		Orbit Maintenance and Maneuvering	Attitude Control	Typical Range of $I_{sp}$ (s)
	Perigee	Apogee			
Cold Gas			X	X	45–73
Solid	X	X			290–304
Liquid					
Monopropellant			X	X	200–235
Bipropellant	X	X	X	X	274–467
Electric		X	X	X	500–3,000

Table 18-3. Overview of Common Applications for Different Propulsion Systems.

## Real-World $I_{sp}$ :

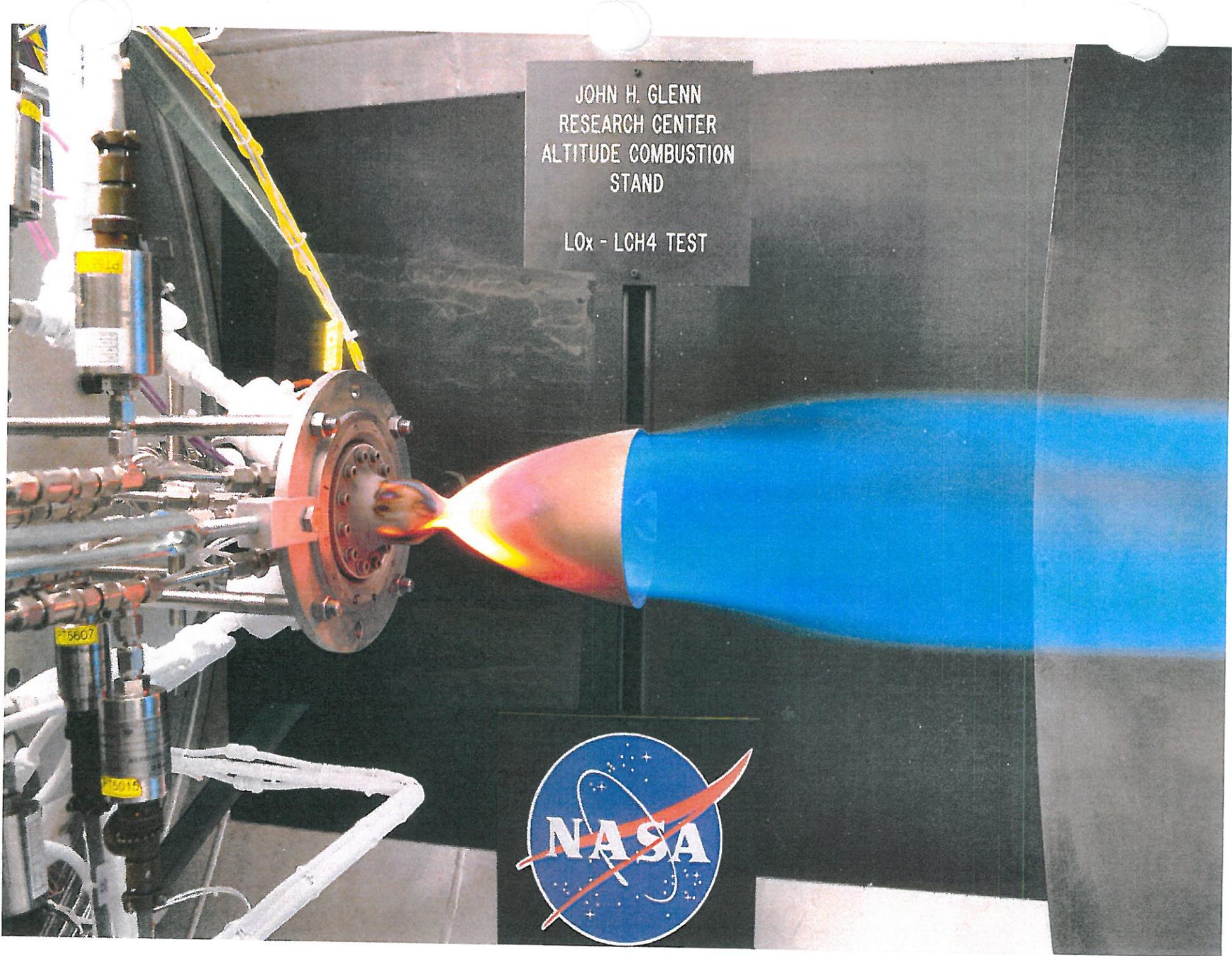
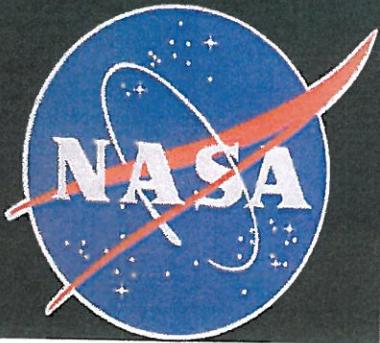
$$\begin{aligned}
 I_{sp} &= \frac{T}{\dot{m} g_0} \\
 &= \frac{\underbrace{m V_e + A_e (P_e - P_a)}_{\text{pressure thrust}}}{\dot{m} g_0} \rightarrow \text{increases with altitude until } P_a = 0 \text{ (vac.)}
 \end{aligned}$$

$$\therefore I_{sp} \underset{\text{vacuum}}{>} I_{sp} \underset{\text{Sealevel}}{}$$

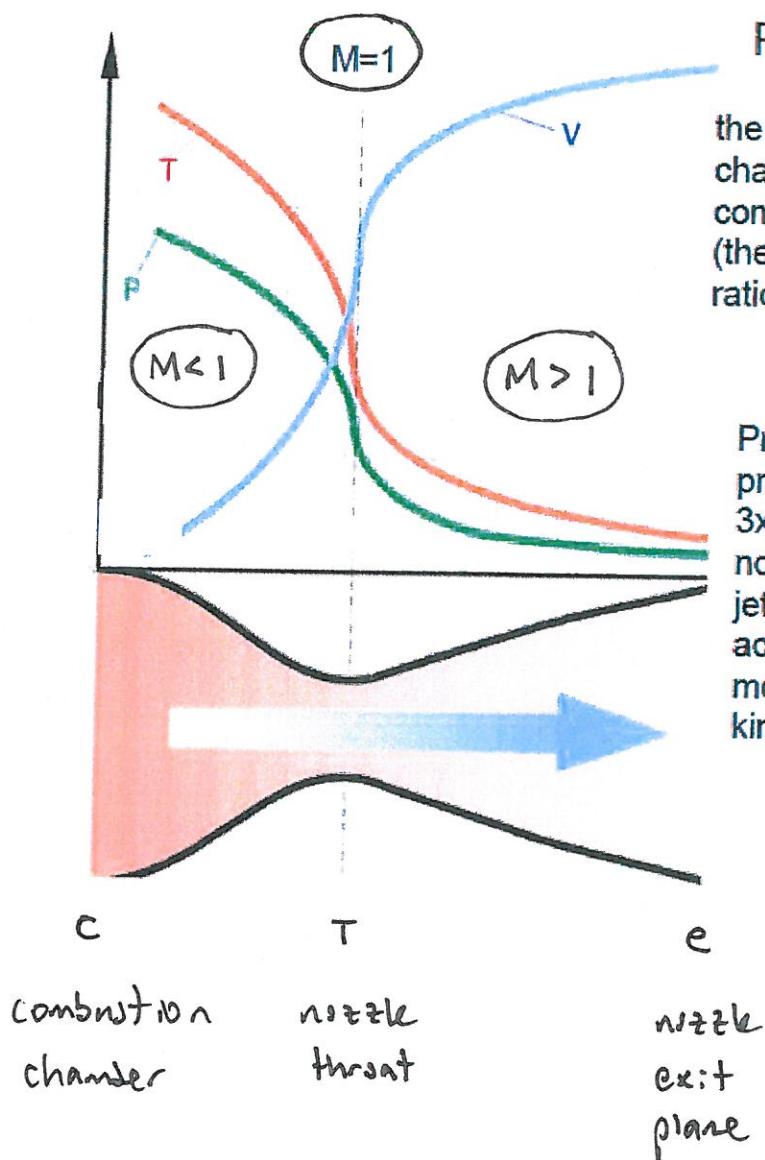
A rocket gets more efficient as it leaves atmosphere!  
(nothing to do with drag)

JOHN H. GLENN  
RESEARCH CENTER  
ALTITUDE COMBUSTION  
STAND

LOx - LCH<sub>4</sub> TEST



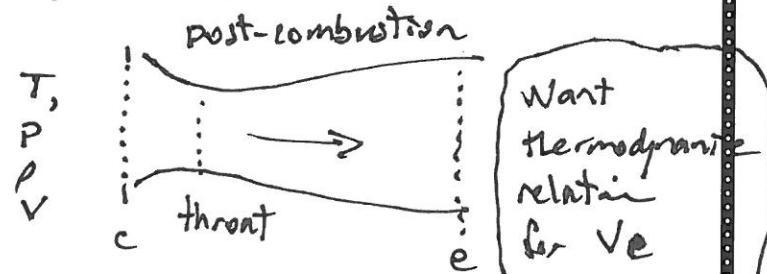
## Text 18.1: Basic Rocket Equations



### Rocket nozzles

the hot gas produced in the combustion chamber is permitted to escape from the combustion chamber through an opening (the "throat"), within a high expansion-ratio 'de Laval nozzle'.

Provided sufficient pressure is provided to the nozzle (about 2.5-3x above ambient pressure) the nozzle *chokes* and a supersonic jet is formed, dramatically accelerating the gas, converting most of the thermal energy into kinetic energy.

Nozzle Performance

- Assumptions :
  - combustion products constant composition & homogeneous
  - Perfect gas  $P = \rho \frac{R_0}{w} T$ ;  $R_0 = \text{univ. gas constant}$   
 $w = \text{molecular weight}$
  - constant  $C_p, C_T$
  - 1-D, steady, isentropic flow, adiabatic
- Cons. Mass :  $\dot{m} = \rho V A$
- Cons. Energy :  $\underbrace{\frac{1}{2} V^2}_{\text{kinetic}} + \underbrace{C_p T}_{\text{internal}} = \text{constant}$
- Adiabatic relations :
 
$$\frac{P}{\rho^{\gamma}} = \text{constant} ; \frac{T}{T_c} = \left( \frac{\rho}{\rho_c} \right)^{\gamma-1} = \left( \frac{P}{P_c} \right)^{\frac{\gamma-1}{\gamma}}$$
- Want an expression for  $V_e$ , to see influences on throat.

Start by maximizing  $m$  through nozzle, so need expression for  $m$  ...

- Energy eqn from C :

$$\cancel{\frac{1}{2} \rho v_c^2 + C_p T_c} = \frac{1}{2} V^2 + C_p T$$

↑  
 $\left(\frac{\dot{m}}{\rho A}\right)^2$

$$\left(\frac{\dot{m}}{\rho A}\right)^2 = C_p (T_c - T)$$

$$\frac{\dot{m}}{A} = \left[ \rho C_p (T_c - T) \right]^{\frac{1}{2}}$$

$T_c \left(1 - \frac{T}{T_c}\right)$   
 $T_c \left(1 - \frac{P}{P_c}^{\frac{x-1}{x}}\right)$

$$\frac{P}{P^*} = \frac{P_c}{P_c^*}$$

$$\rho^* = \frac{P}{P_c} P_c^*$$

$$\rho = \left[ \frac{P}{P_c} P_c^* \right]^{+\frac{1}{x}}$$

- Collect :

$$\frac{\dot{m}}{A} = \left\{ \frac{2x}{r-1} P_c P_c^* \left( \frac{P}{P_c} \right)^{\frac{2}{r}} \left[ 1 - \left( \frac{P}{P_c} \right)^{\frac{r-1}{r}} \right] \right\}^{\frac{1}{2}}$$

recall that  $\dot{m}_{\max}$  when throat velocity is sonic :  $V_t = a$

- Choked nozzle : sonic throat conditions

$$\frac{P_t}{P_c} = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_t}{T_c} = \frac{2}{\gamma+1}$$

- Plug into 6.1b to get

$$\left. \frac{\dot{m}}{A} \right|_{\text{max}} = (\gamma P_t \rho_t)^{\frac{1}{2}} = (\rho_t V_t)$$

where  $V_t$  = "critical" (sonic) throat velocity

$$= a_t$$

$$= \left( \gamma \frac{P_t}{\rho_t} \right)^{\frac{1}{2}} = \left( \gamma \frac{R_0 T_t}{w} \right)^{\frac{1}{2}}$$

- So  $\dot{m}$  determined solely by throat conditions :

$$\boxed{\dot{m} = \sqrt{\gamma} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \frac{P_c A_t}{R_0 T_c / w}}$$

6.17

nozzle mass-flow rate in terms of throat area  
and combustion chamber  $P$  and  $T$

- Expression for  $V_e$ :

Energy:  $\frac{1}{2} V_e^2 + C_p T_e = \frac{1}{2} V_e^2 + C_p T_c$

$V_e = 0$

$$V_e = \sqrt{2 C_p (T_c - T_e)}$$

$$V_e = \sqrt{\frac{2 \pi R_0 T_e}{(\gamma - 1) w} \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

b.20

- To maximize  $V_e$  (and therefore thrust):

- increase  $\frac{P_e}{P_c}$
- increase  $T_c$
- decrease  $w$  molecular weight

- Common shorthands : "characteristic velocity"  $c^*$

$$\dot{m} = \rho_c A_t / c^*$$

where  $c^* = \sqrt{\frac{R_e T}{\gamma}} / \left[ \sqrt{\gamma} \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right]$

6.18

- "Characteristic thrust coefficient" :  $C_F^0$

$$V_e = c^* C_F^0$$

where

$$C_F^0 = \sqrt{\left[ \gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \right] \frac{2\gamma}{\gamma-1} \left[ 1 - \left( \frac{P_e}{\rho_e} \right)^{\frac{\gamma-1}{\gamma}} \right]}$$

6.21

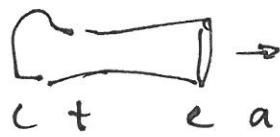
- Exit-to-throat area ratio :

continuity  
↓

$$\frac{A_e}{A_t} = \frac{\rho_e V_e}{\rho_t V_t} = \frac{\gamma \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \left( \frac{\rho_e}{\rho_t} \right)^{\frac{1}{\gamma}}}{C_F^0}$$

6.22

## Nozzle Design - Real World



- Thrust eqn:

$$F = \dot{m} V_e + A_e (P_e - P_a)$$

- $A_e$  for max thrust?

$(P_e = \text{const})$

$$\delta F = \dot{m} \delta V_e + \delta A_e (P_e - P_a) + A_e \delta P_e$$

for constant  $\dot{m}$

Momentum:

$$\dot{m} \delta V_e + A_e \delta P_e = 0$$

Solve for

$$\frac{\delta F}{\delta A_e} = \dot{m} \frac{\delta V_e}{\delta A_e} + (P_e - P_a) + A_e \frac{\delta P_e}{\delta A_e}$$

$$= (P_e - P_a) + \frac{1}{\delta A_e} (\dot{m} \delta V_e + A_e \delta P_e)$$

$\cancel{\dot{m} \delta V_e + A_e \delta P_e}$  D.M.m.

So max thrust when  $P_e = P_a$

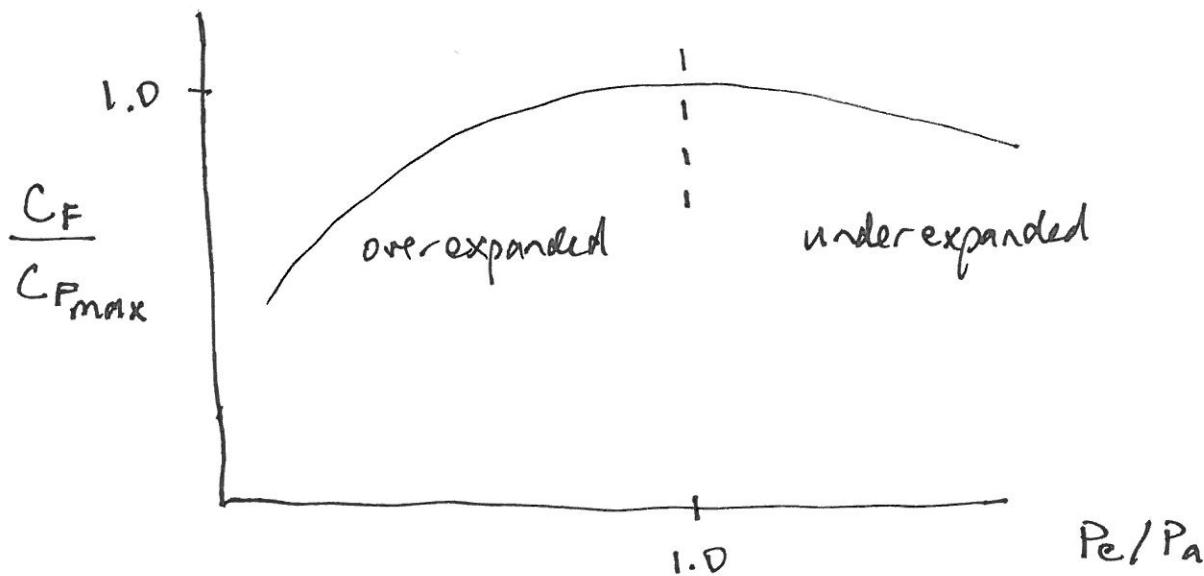
↑  
ideal expansion  
aft of nozzle

- Effect of non-ideal expansion :

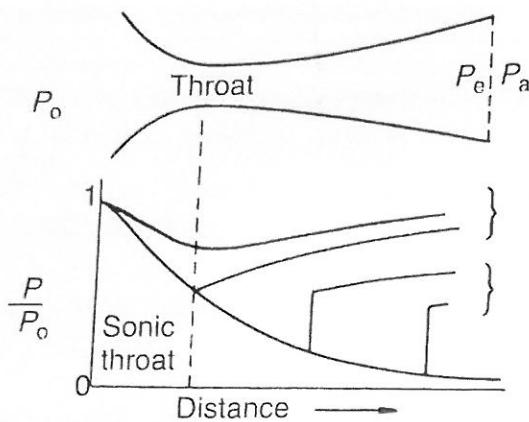
define thrust coefficient

$$C_F = \frac{F}{P_e A_T} = C_F^0 + \frac{A_e}{A_T} \left( \frac{P_e}{P_a} - \frac{P_a}{P_e} \right)$$

6.23



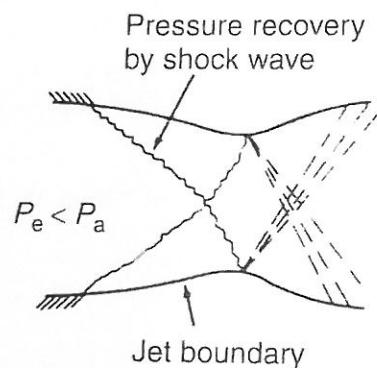
- somewhat worse to overexpand
- adaptive geometry nozzles : complex, thrust-efficient  
Aerospike engine



Subsonic (isentropic) flow

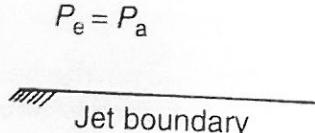
Normal shock waves in divergent section

$P_e = P_a$  Ideally expanded supersonic flow

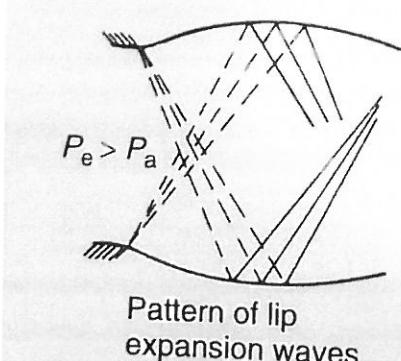


#### Over expanded

(Shock waves separate wall boundary layers and reduce expansion ratio, viscous losses enhanced-characteristic of high ambient pressure, sea-level or test-bed operation.)



#### Ideal, fully expanded jet



#### Under-expanded

(Incomplete nozzle expansion-characteristic of low ambient pressure, space vacuum operation.)

**Figure 6.6** Nozzle flows: non-ideal expansion

18.1

## Text 18.2: Staging

- Recall

$$\begin{aligned}
 I_{sp} &= \frac{T}{\dot{m} g_0} \\
 &= \frac{\text{momentum}}{\dot{m} V_e + A_e (\bar{P}_e - P_a)} \\
 &\quad \xrightarrow{\text{generally much less than momentum thrust}} \\
 &\approx \frac{V_e}{g_0}
 \end{aligned}$$

- From previous page:

$$I_{sp} \approx \frac{1}{g_0} \sqrt{2 \frac{R_u}{MW} \left( \frac{k}{k-1} \right) T_c \left[ 1 - \left( \frac{P_e}{P_c} \right)^{\frac{k-1}{k}} \right]}$$

(same as text eqn 18-17, but without pressure thrust term)

$R_u$  = universal gas constant = 8.314  $\frac{\text{J}}{\text{mol K}}$

MW = exhaust molecular weight

$T_c$  = combustion temp

$k = C_p / C_v$

$P_e$  = nozzle exit pressure

$P_c$  = combustion pressure

For high  $I_{sp}$ , want:

low exhaust MW

high combustion temp

high press. ratio  $\frac{P_e}{P_c}$

# Rocket Engine Efficiency: ISP Considerations

- Chemical propellants:  $175 < \text{ISP} < 400$
- For high ISP:
  - high exhaust-gas temp: large heat of combustion
  - Low molecular-density combustion products: H, C, O, F, Al, Be, Li, etc
- Propellant density:
  - lower density = higher tank volume and mass
- Coolant properties
- Safety & handling:
  - Corrosive, flammable, toxic
  - All three!
- Availability, storage
- Environmental concerns

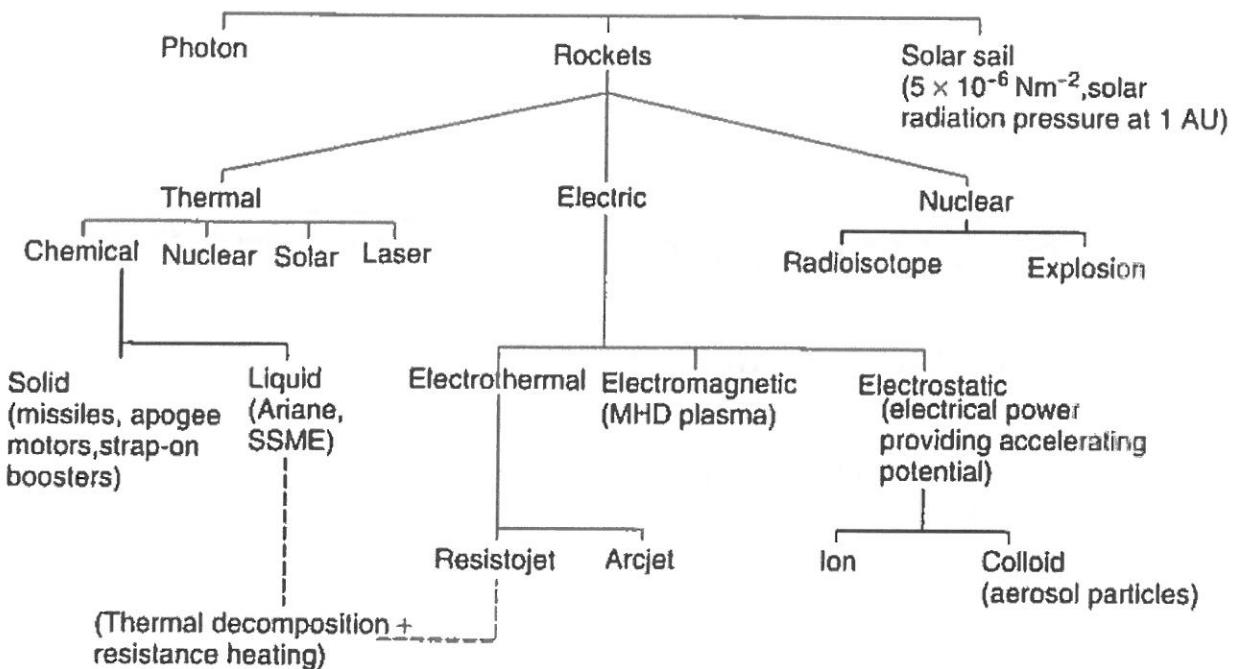


Figure 6.1 Propulsion systems classification

## Propellants

- Exothermic heat ~~loss~~ release in nozzle gas flow
  - specific energy content
  - rate of heat release
  - storage, handling
  - plumbing
  - failure modes
  - toxicity (before and after combustion)
  - demands on system mat'l properties

## Liquid Propellants

- bi-propellant : 2 liquids
  - and - combust : hypergolic
  - monopropellant : exothermic decomposition
- requires pumping system, tanks, plumbing
  - throttling
  - stop/start
- May require cryogenic storage - time limit
- Recall : want high  $T_c$  and low  $\bar{W}$  for max  $V_e$
- Common : LOX/LH<sub>2</sub> and LOX/RP1 (kerosene) - Large thrust  
MMH/N<sub>2</sub>O<sub>4</sub> - Lower thrust

**Table 6.1** Liquid propellants

Fuel	Oxidizer	Molecular weight of products	Combustion temperature $T_c$ (K)	Ideal specific impulse (s)	Mean density kg/m <sup>3</sup>
H <sub>2</sub> (hydrogen)	O <sub>2</sub> (oxygen)	10	2980	390	280
	F <sub>2</sub> (fluorine)	12.8	4117	410	460
Kerosine	O <sub>2</sub>	23.4	3687	301	1020
	F <sub>2</sub>	23.9	3917	320	1230
	RFNA (red fuming nitric acid)	25.7	3156	268	1355
	N <sub>2</sub> O <sub>4</sub> (nitrogen tetroxide)	26.2	3460	276	1260
	H <sub>2</sub> O <sub>2</sub> (hydrogen peroxide)	22.2	3008	278	1362
N <sub>2</sub> H <sub>4</sub> (hydrazine)	O <sub>2</sub>	19.4	3410	313	1070
	HNO <sub>3</sub> (nitric acid)*	20	2967	278	1310
UDMH (CH <sub>3</sub> ) <sub>2</sub> NNH <sub>2</sub> (unsymmetrical dimethyl hydrazine)	O <sub>2</sub>	21.5	3623	310	970
	HNO <sub>3</sub> *	23.7	3222	276	1220
* hypergolic					
<i>Monopropellants</i>					
N <sub>2</sub> H <sub>4</sub>		10.3	966	199	1011
H <sub>2</sub> O <sub>2</sub>		22.7	1267	165	1422

\*Note: All quoted values are for  $p_c = 7$  MPa with an ideal expansion to  $p_e = 0.1$  MPa. Higher chamber pressures admit increases in  $I_{SP}$ —for example, at 20 MPa, LOX/LH<sub>2</sub> yields a specific impulse of ~460 s.

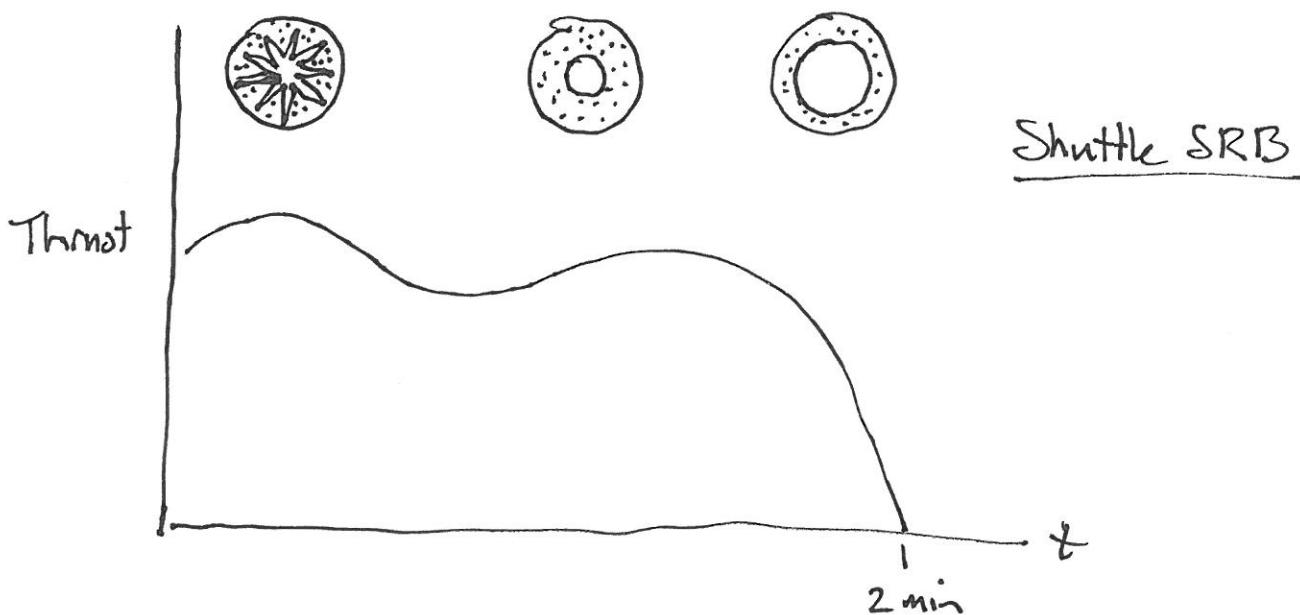
# Solid Propellants

- Solid Propellants :
  - Composite: fuel & oxidizer mixed & solidified with binder
  - Most (up to 80%) of the volume is oxidizer
  - Often used as low-altitude add-ons for liquid rockets
- Solid Propellants Pros:
  - Simple, few moving parts, no propellant feed systems
  - compact (higher density than some liquid props)
  - transportable, safe, low-cost
  - Scalable – massive thrust possible (SRB 3Mlbf)
- Solid Propellants Cons:
  - Entire stage is the combustion chamber – structural penalties & segment seals (shuttle Challenger)
  - Case heats during burn – more structural penalties
  - Propellant itself must load-bearing within the case (rocket body)
  - No stop or restart, no throttling (until recently)
  - Lower efficiency than liquid (lower exhaust velocities)
  - Thrust-vectoring requires complex gimbal seals

## Solid Propellants

- Lower ISP than liquid
- Throttle, stop/start almost impossible
- Poured or extruded, then cured - rubberlike
- Thrust-history tailoring:

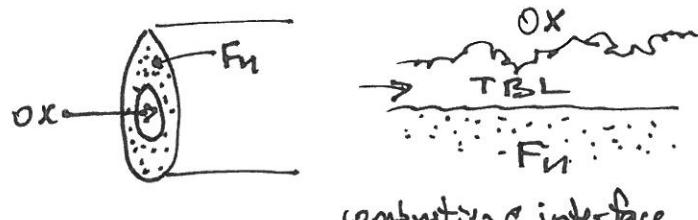
grain-pattern evolves via combustion:



## Shuttle SRB

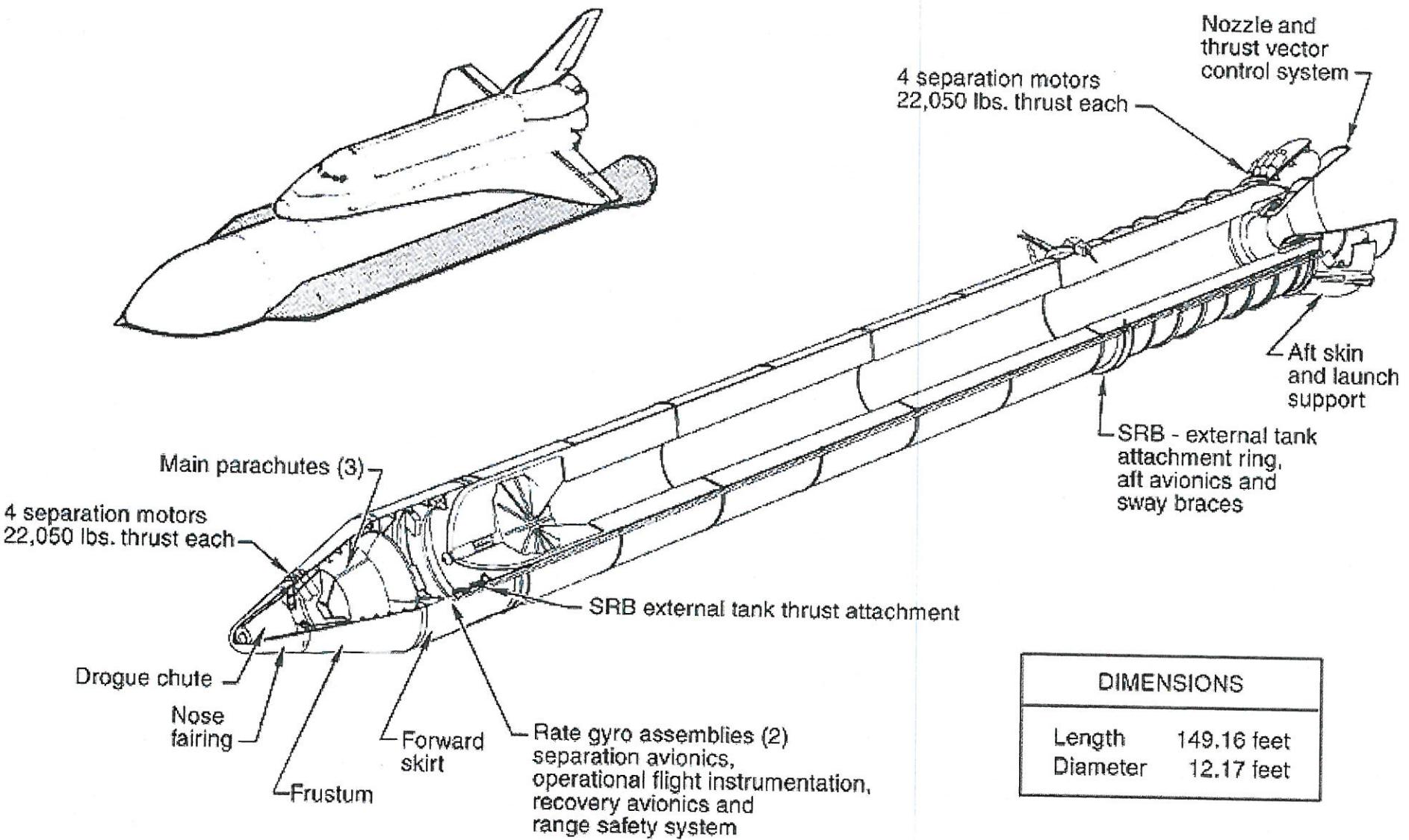
## Hybrid Propellants

- Liquid ox, solid fuel
- Throttle, stop/start
- $\frac{1}{2}$  plumbing, storage complexity of liquid bi-propellants
- Fluid-solid interface combustion limits ISP



combustion interface

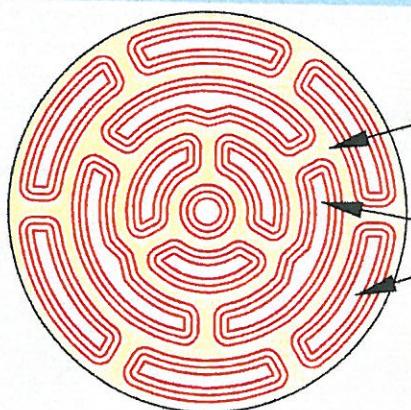
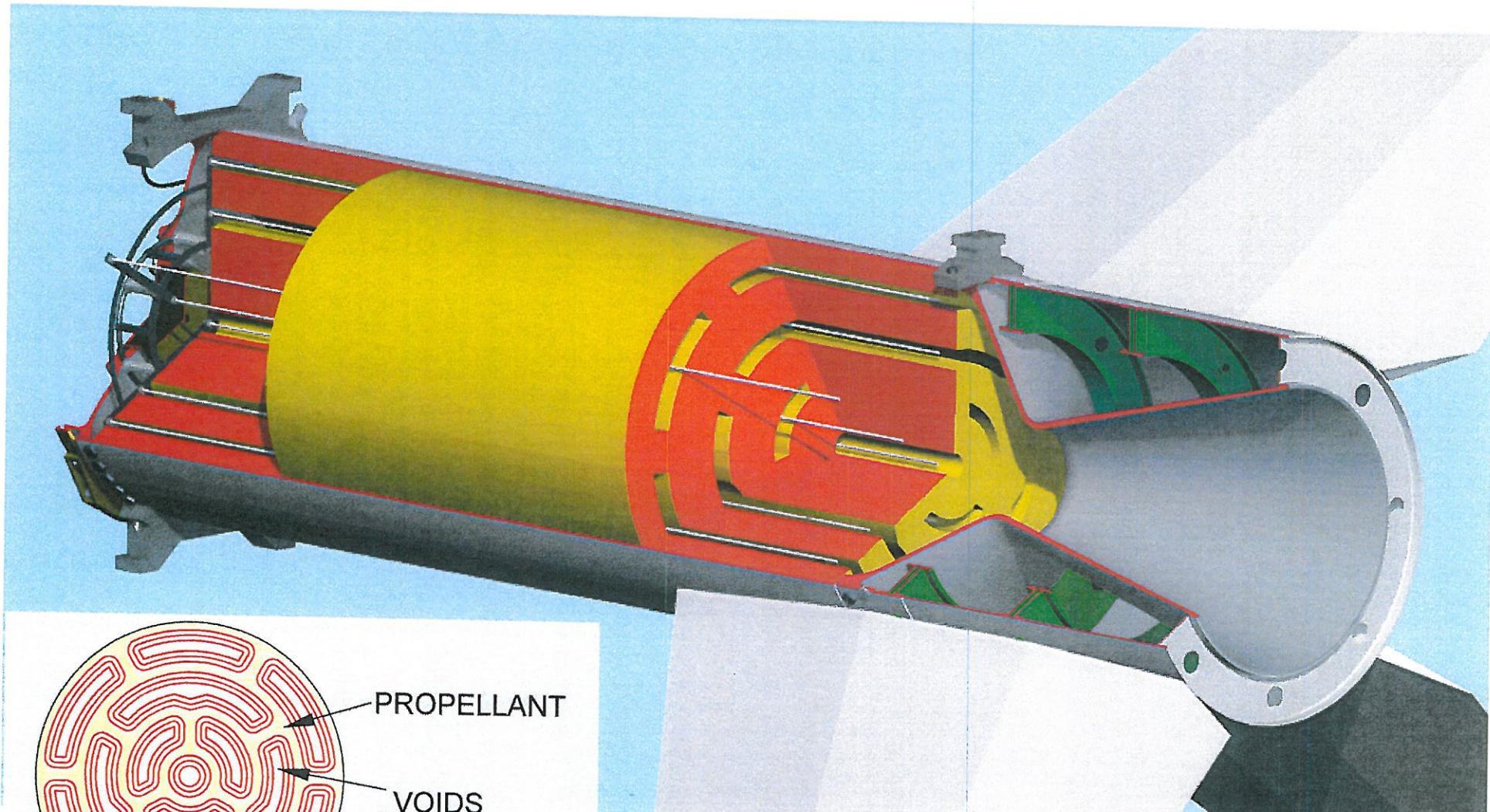
# Space Shuttle Solid Rocket Boosters



# Solid Rocket Grain Patterns

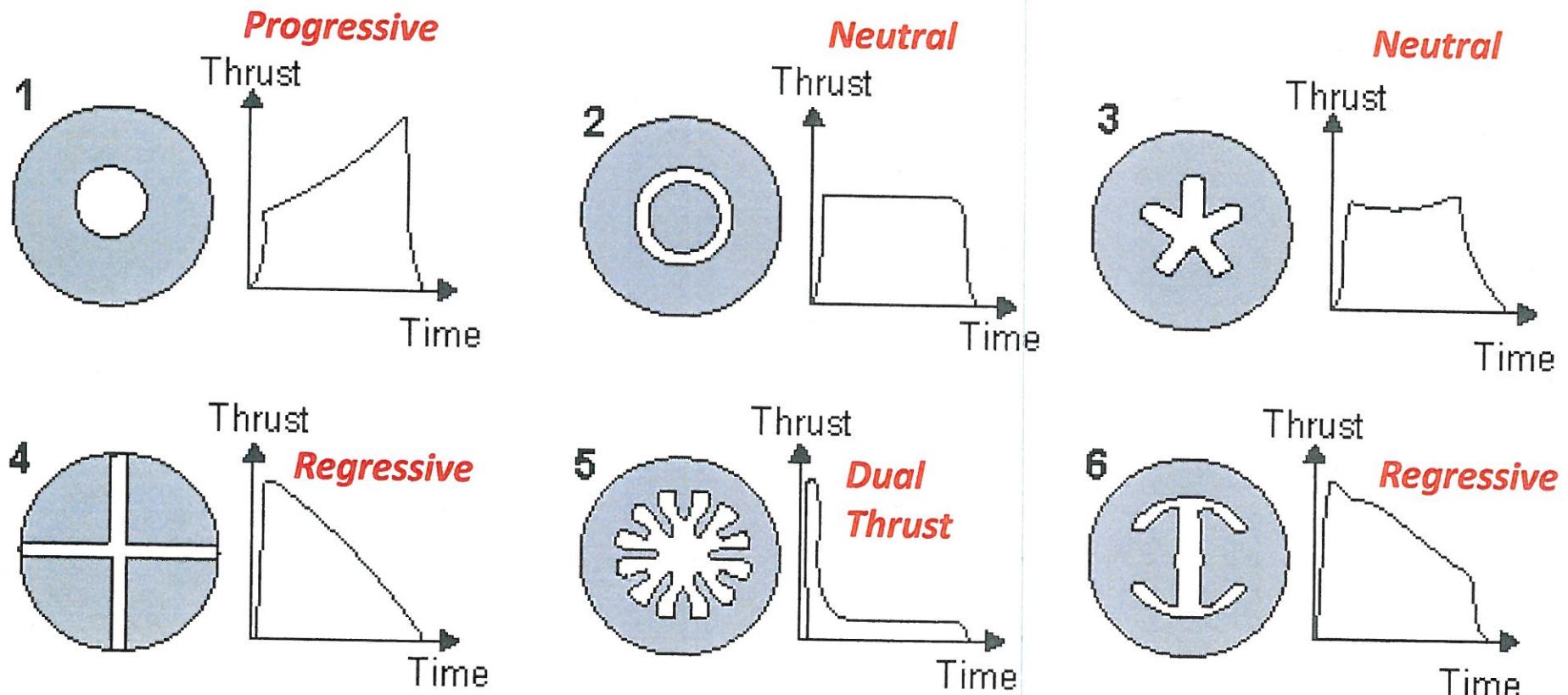


# Solid Rocket Grain Patterns

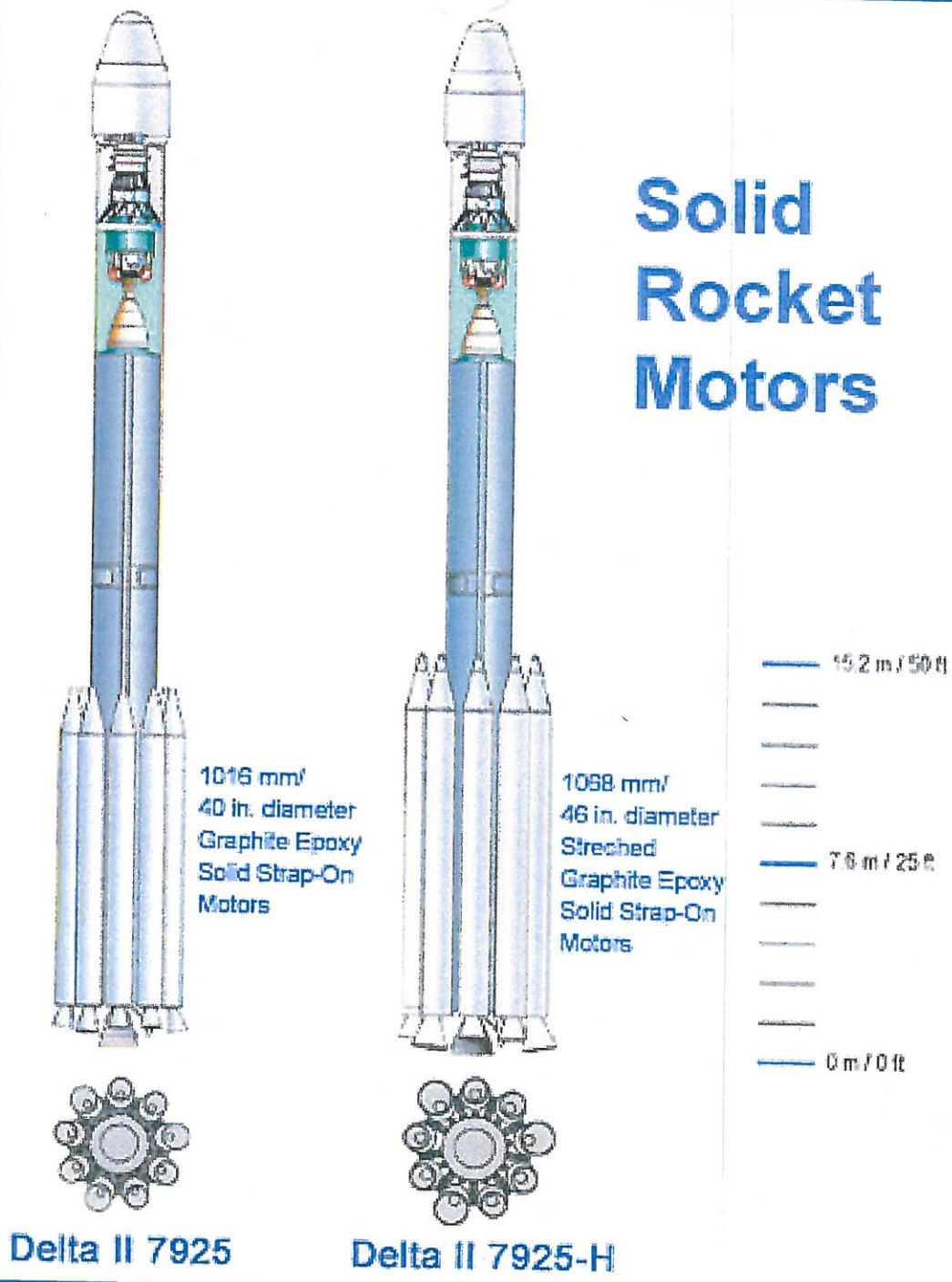


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# Solid Rocket Grain Patterns



# Solid Rocket Motors



Delta II 7925

Delta II 7925-H

## Open-Cycle Design:

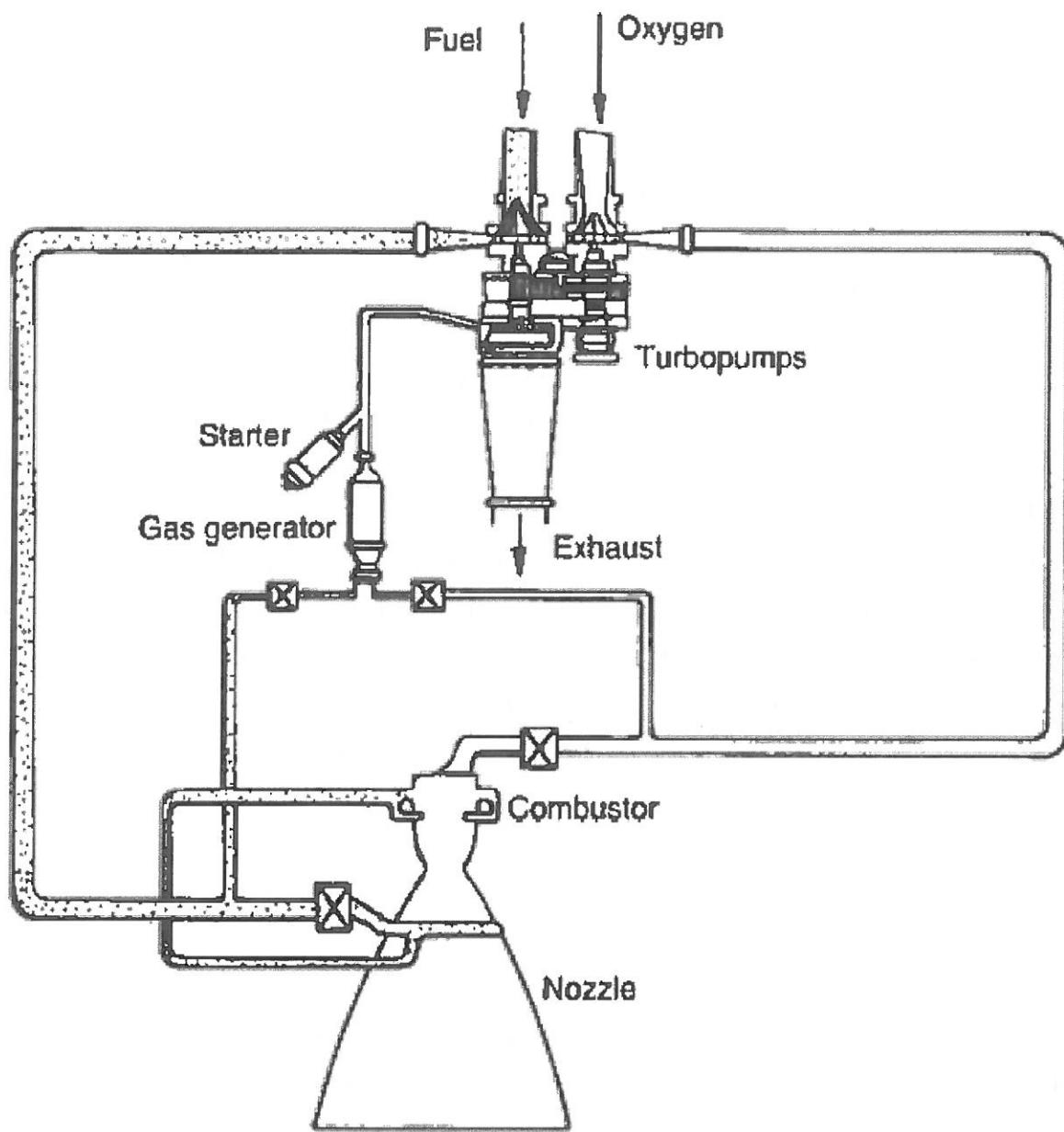
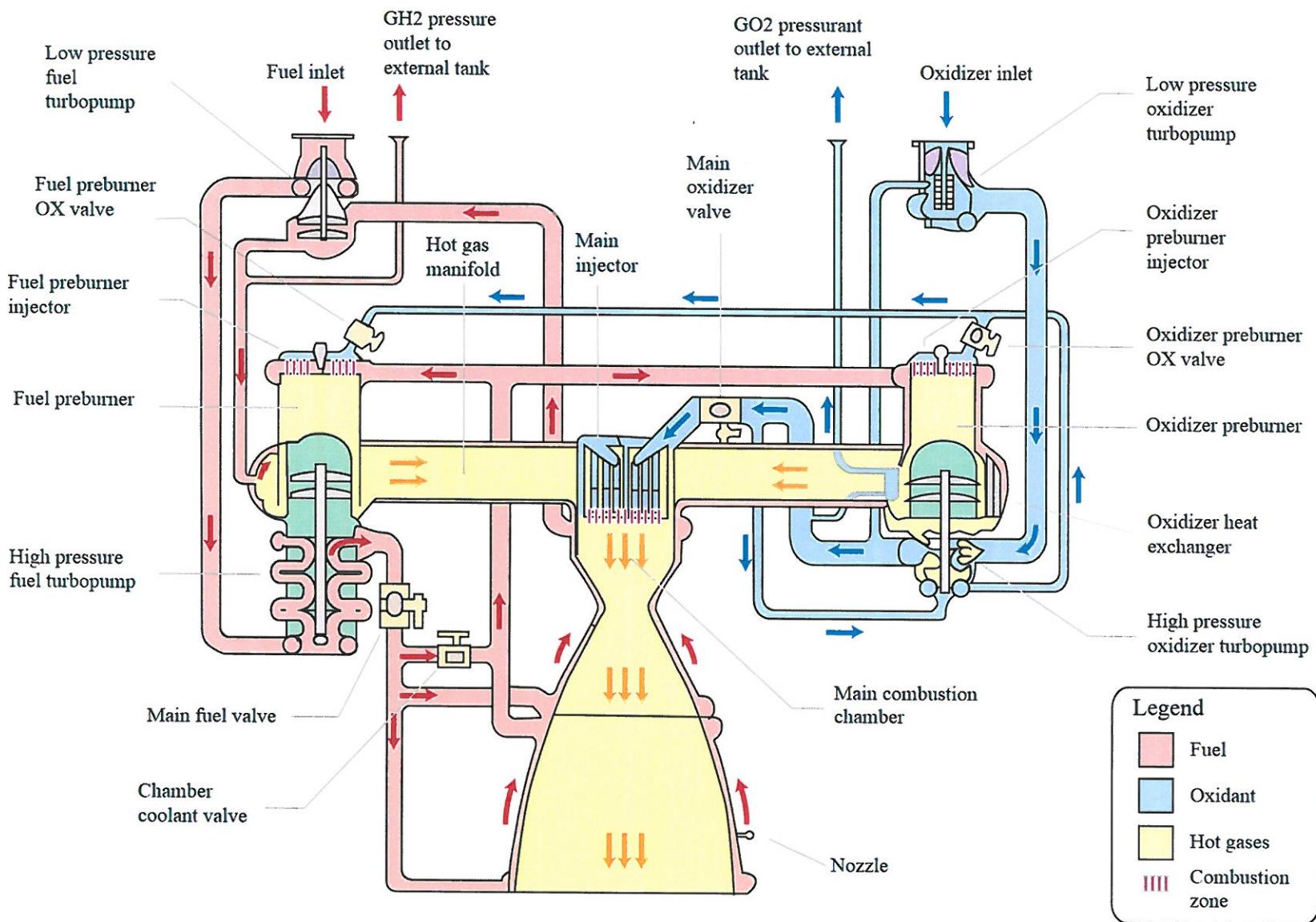


Figure 6.10 Schematic of a liquid rocket motor

## Closed Cycle Design: Shuttle Main Engine



## Closed (SSME) vs Open (Vulcain):

**Table 6.2 Illustrative comparison of closed- and open-cycle engines**

	SSME	Vulcain (Ariane 5)
<b>Thrust (kN):</b>		
Vacuum	2090	1390
Sea level	1700	960
<b>Specific impulse (s):</b>		
Vacuum	455	432
Sea Level	363	310
<b>Mixture ratio</b>		
(stoichiometric 8:1 $2\text{H}_2 + \text{O}_2 \rightarrow 2\text{H}_2\text{O}$ )	6:1	5.3:1
<b>Chamber pressure (bar)</b>	207	108
<b>Nozzle area ratio</b>	77	45
<b>Flowrates (kg/s)</b>	468 (engine) 248 (pre-combustor)	270 10 (gas generator)
<b>Pump discharge pressure (bar)</b>	309 (LOX) 426 (LH <sub>2</sub> )	125 (LOX) 150 (LH <sub>2</sub> )
<b>Burn time (s)</b>	480	540
<b>Mass (kg)</b>	3022	1650

## Propellant Management:

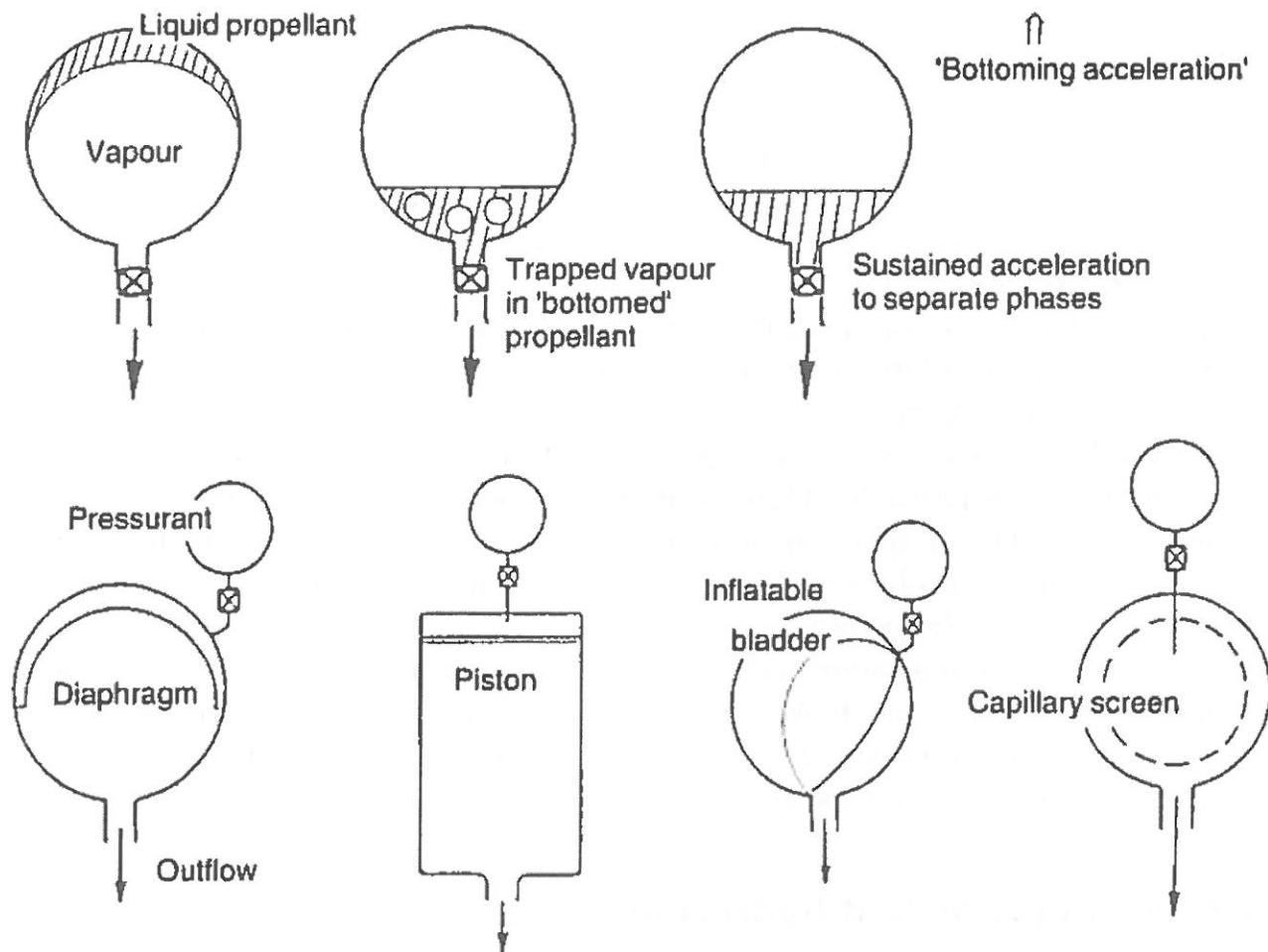
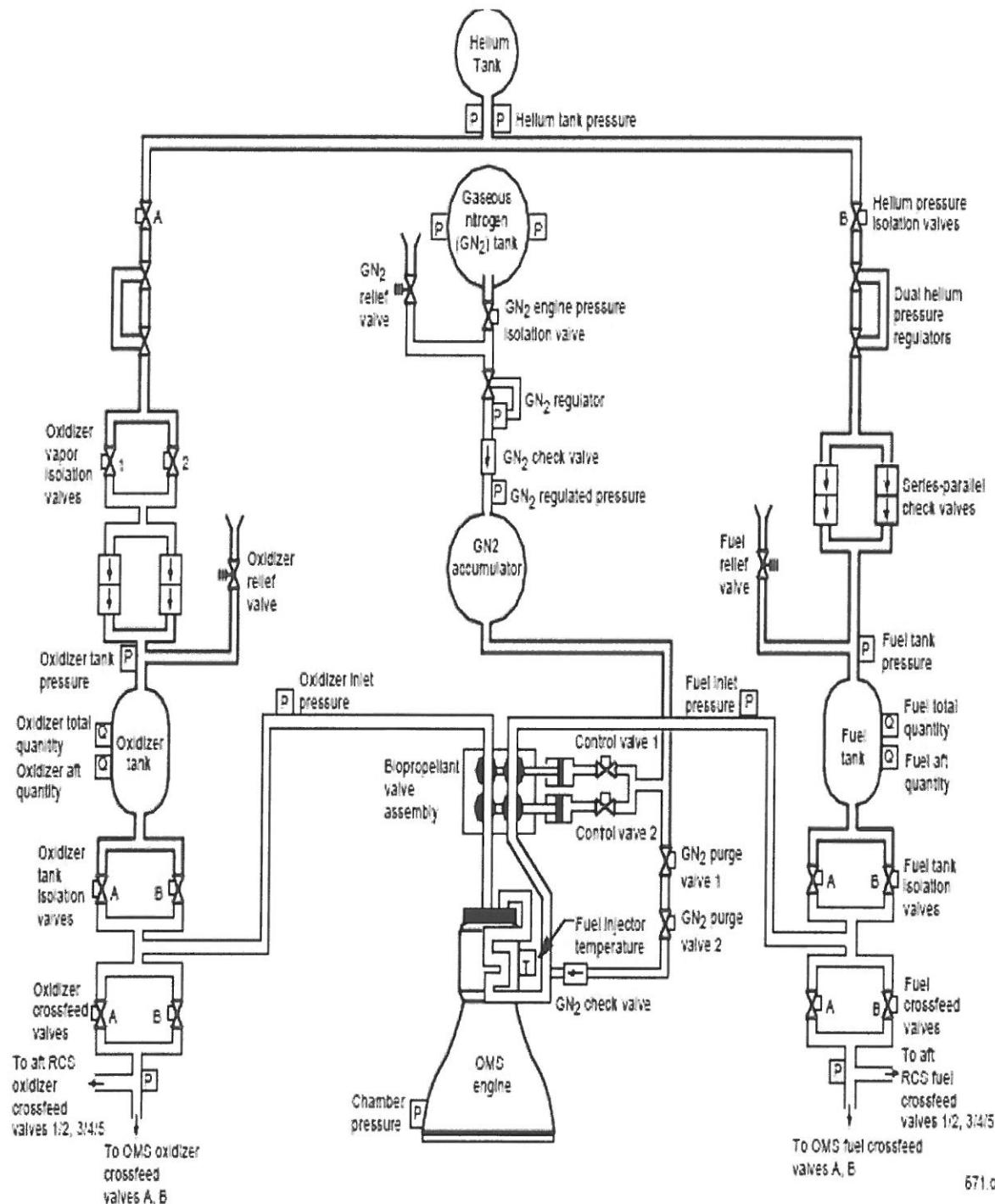


Figure 6.17 Illustrative propellant storage and delivery systems

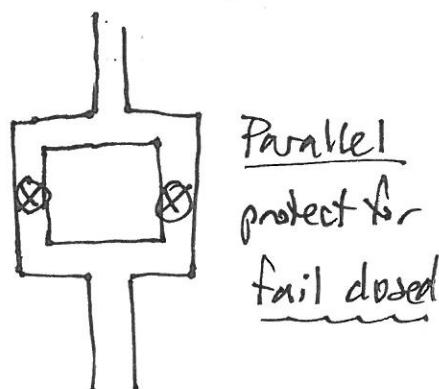
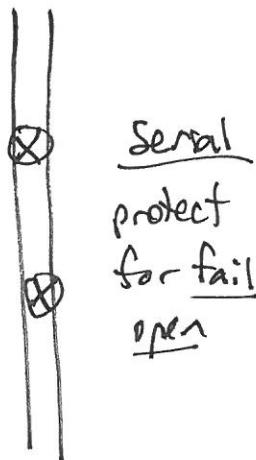
## Propellant Management: Shuttle Orbital Maneuvering System (OMS) Engine ( $T = 6\text{Klbf}$ )



Orbital Maneuvering System Pressurization and Propellant Feed System  
for One Engine (other Engine Identical)

## Biflop Engine Design

- Open vs closed
  - ↓  
SSME
  - ↓  
F9?
- ~~Fig~~ Table b-2 : SSME vs Vulcan
- Prop management
  - DMS diagram
- Parallel vs Serial valve design



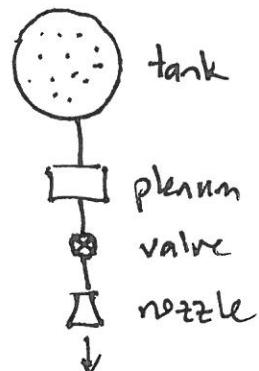
- Fig b-17

## Spacecraft Propulsion

- Tasks:
  - final orbit acquisition after launcher drop-off
  - orbit maintenance, station keeping
  - relative maneuvering
  - attitude control
  - reboost
  - de-orbit

### Cold Gas

- pressurized inert gas:  $\text{N}_2$ , Ar, freon, hydrocarbon (propane, etc)
- Liquid or gas form storage
- Exhaust plume  $\rightarrow$  sensors?
- $T = 10^{-4} \text{ N} - 10^{-2} \text{ N}$
- $\pm \text{SP} \sim 50 \text{ sec}$  (low efficiency)
- CubeSats, SAFER, etc



## Monopropellant Hydrazine

- $N_2H_4 \xrightarrow{\text{thermal}} N_2, NH_3, H_2 + \text{Heat}$   
 $\xrightarrow{\text{catalytic}}$
- ISP 200-250 s
- Liquid-form storage, with inert pressurant ( $N_2$  or He)
- $T \sim 10 N$

## Bi-Propellant ; MMH + N<sub>2</sub>O<sub>4</sub> (hypergolic)

- ISP  $\sim 300 s$

(figs 6.20, 6.21)

## Mono-propellant System

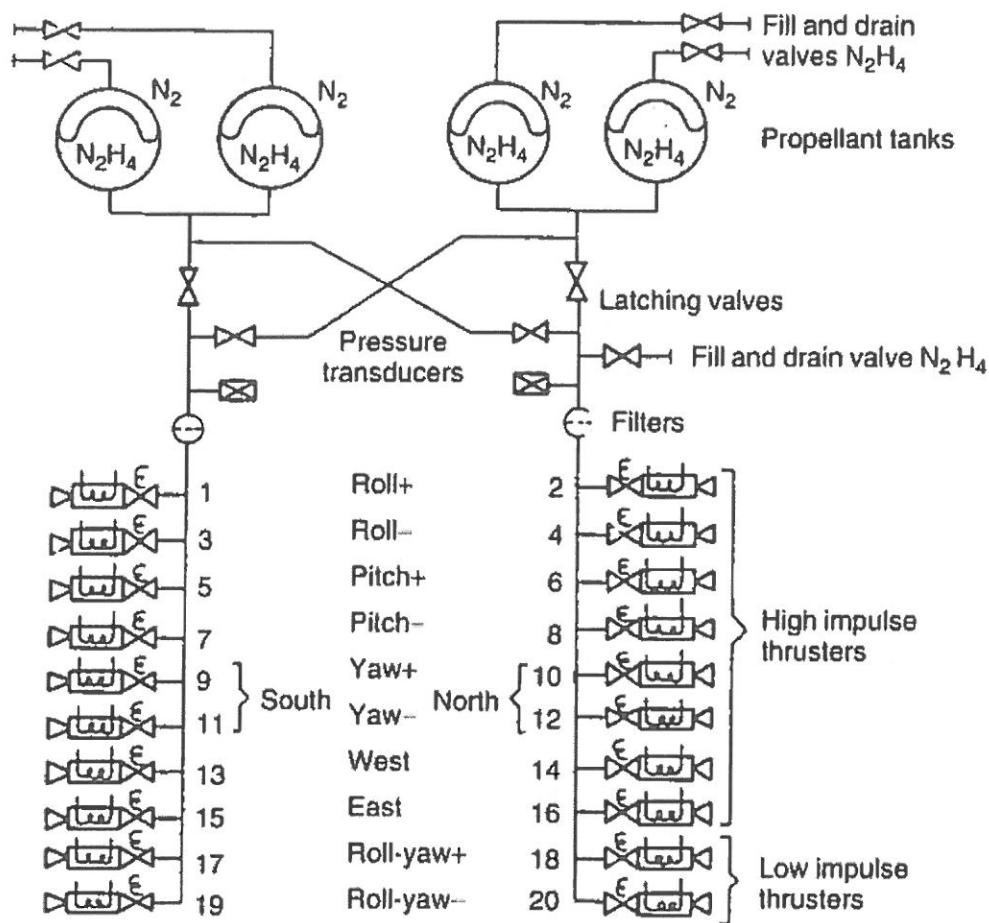
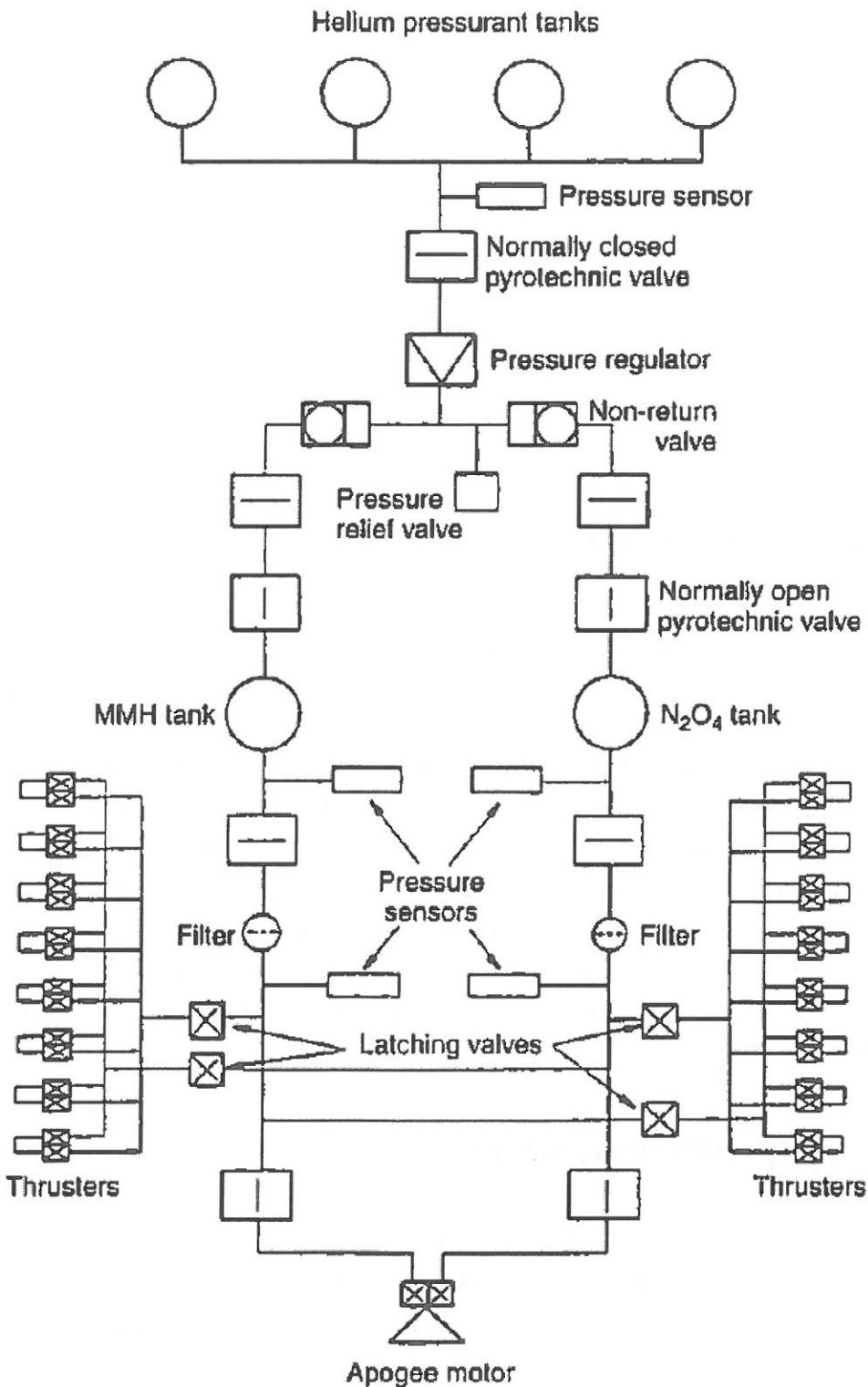


Figure 6.20 Illustrative spacecraft propulsion system using monopropellant thrusters

## Bi-propellant Hypergolic System



**Figure 6.21** Schematic of a typical bi-propellant propulsion system

## Electric Propulsion

- Let  $M_w$  = powerplant mass

$M_e$  = expellant mass

$M_p$  = payload mass

- Power-plant output:

$$W = \frac{1}{2} \dot{m} V_e^2 \quad (\text{power})$$

↑  
expellant

- Assume power-plant output prop. to mass:

$$M_w = dW$$

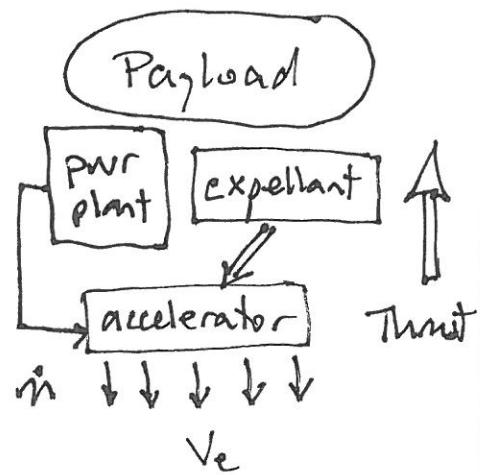
↑ "inverse specific power"

- Assume  $\dot{m}$  constant throughout burn time  $t_b$ :

$$\dot{m} = \frac{M_e}{t_b}$$

- Define total rocket mass

$$M_0 = M_w + M_e + M_p$$



- Solve for  $M_e$ :

$$M_e = \dot{m} t_b = \frac{2w}{V_e^2} t_b = \frac{2 M_w}{V_e^2} \frac{t_b}{\alpha}$$

$$= \frac{M_0 - M_p}{1 + \left( V_e^2 / \frac{2 t_b}{\alpha} \right)}$$
6.28

- Similarly,

$$M_w = \frac{M_0 - M_p}{1 + \left( \frac{2 t_b}{\alpha} / V_e^2 \right)}$$
6.29

- Use rocket equation to get  $\Delta V$ :

$$\Delta V = V_e \ln \left( \frac{\overrightarrow{m_{initial}}}{m_{final}} \right)$$

$$\boxed{\Delta V = V_e \ln \left[ \frac{1 + (V_e N_c)^2}{\frac{M_p}{M_0} + (V_e/V_c)^2} \right]}$$
6.30

(for 6.24)

Note that there is an optimum  $V_e$

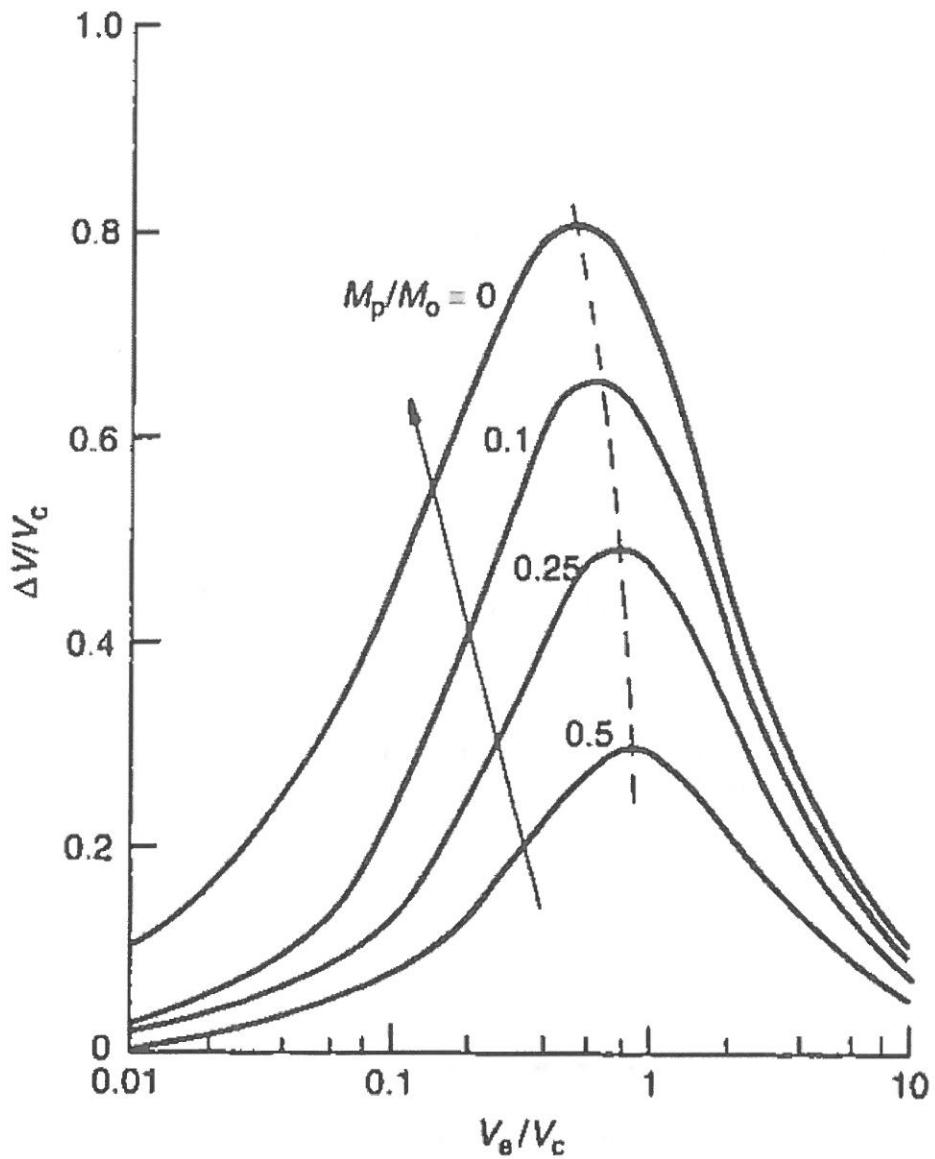


Figure 6.24 Separately powered electric rocket performance

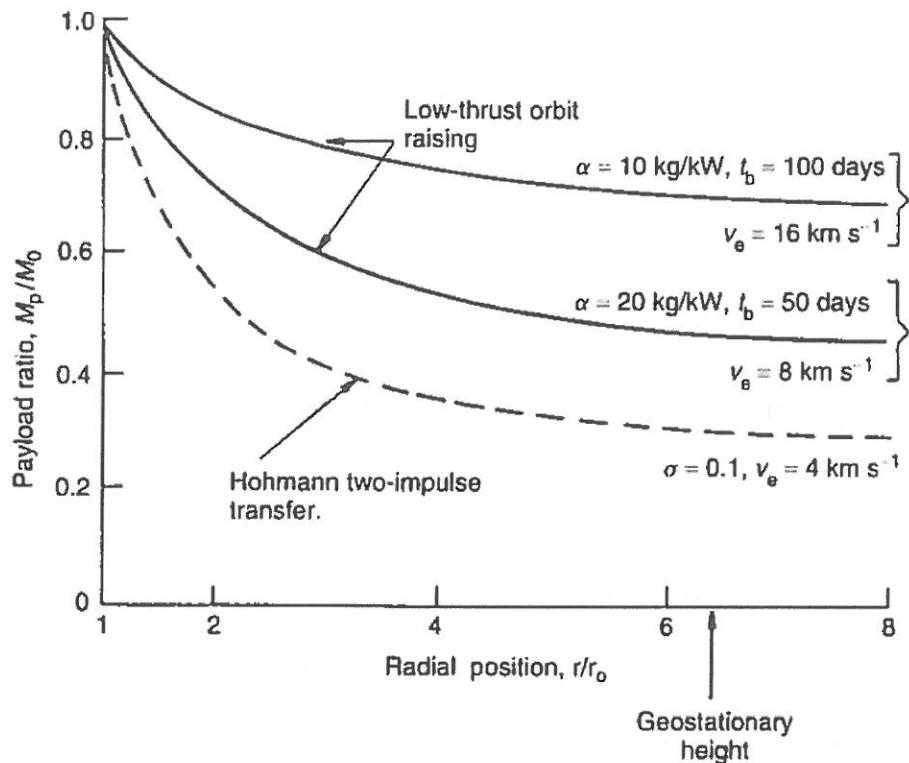


Figure 6.25 Comparative performance: chemical impulse versus low-thrust orbital transfer

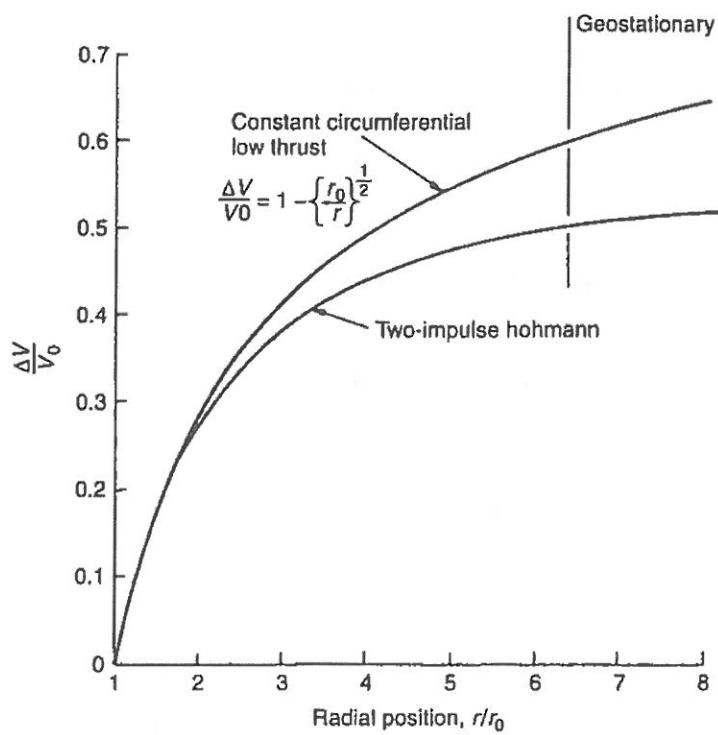


Figure 6.26 Comparative  $\Delta V$  requirement for transfer between circular orbits  $r_0$  to  $r$