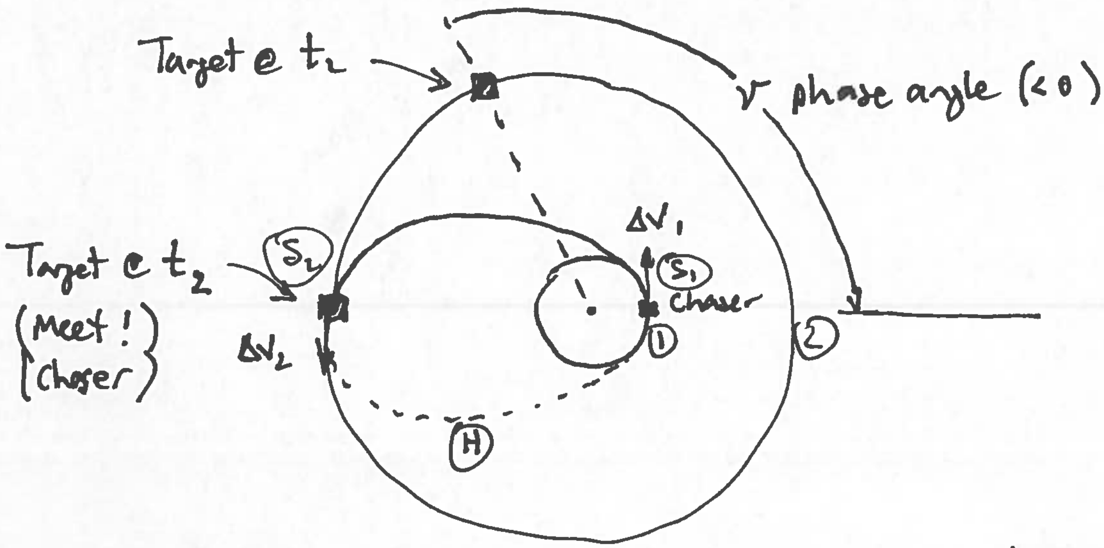
(Walter sec 8.6)

Homing Transfer Burn $S_1 \rightarrow S_2$ fig 8.29



Problem: Given geometry a_1 and a_2, and desired final phase angle, ν_f

what is the correct initial phase angle ν_i

and correct burn ΔV to reach S_2 from S_1?

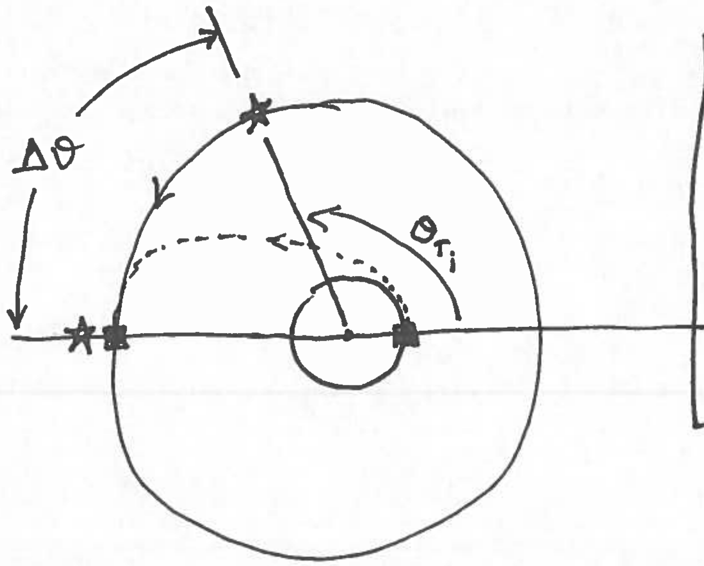
- Set up: recall $2a_H = a_1 + a_2 = 2a\left(1 + \frac{\Delta a}{2a}\right)$

(just geometry)

and $t_H = \pi\sqrt{\frac{a_H^3}{\mu}}$

(Hohmann ellipse xfer time for $\frac{1}{2}$ ellipse period)

• Geometry:



Phase Angle

$$\nu \equiv \theta_T - \theta_c$$

$$\nu_i = \theta_{T_i} - 0$$

$$\nu_f = \theta_{T_f} - 180$$

let $\Delta\theta \equiv \theta_{T_f} - \theta_{T_i}$

$$\nu_f - \nu_i = \theta_{T_f} - 180 - \theta_{T_i} - 0$$

$$= \Delta\theta - 180$$

• While chaser travels $\theta_c = 180°$, target travels $\Delta\theta$

$$\Delta\theta = n\, t_H \quad \text{where } n = \sqrt{\mu/a^3} = \frac{2\pi}{\tau} \quad \left(\begin{matrix}\text{angular}\\ \text{rate}\end{matrix}\right)$$

$$t_H = \pi\sqrt{\frac{a_H^3}{\mu}} \quad \left(\begin{matrix}\text{transfer time} = \\ \text{half ellipse period}\end{matrix}\right)$$

so $$\Delta\theta = \sqrt{\frac{\mu}{a^3}}\,\pi\sqrt{\frac{a_H^3}{\mu}} = 180° \left(\frac{a_H}{a}\right)^{3/2}$$

$$= 180° \left(1 + \frac{\Delta a}{2a}\right)^{3/2}$$

$$\cong 180° \left(1 + \frac{3}{4}\frac{\Delta a}{a}\right) \quad \text{neglecting higher order terms}$$

- Now that we have expression for $\Delta\theta$ in terms of orbit geometry, we also have an expression for the initial phase angle ν_i :

from prev. page,

$$\nu_i = \nu_f + (180 - \Delta\theta)$$

$$= \nu_f - 180^\circ \frac{3}{4} \frac{\Delta a}{a}$$

or $$\boxed{\nu_i = \nu_f - 135^\circ \frac{\Delta a}{a}}$$

This is the phase angle at which to execute kick-burn (impulsive) that takes chaser from S_1 to S_2

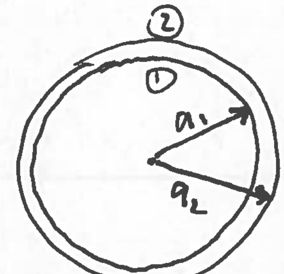
- How big should that kick-burn be?

What is correct ΔV?

(Walter sec. 8.6)

Homing Transfer Burn : $S_1 \rightarrow S_2$ fig 8.29

- Assume Hohmann xfer between adjacent, circular orbits



- One form of expression for Hohmann ΔV:

$$\Delta V = (V_1 - V_2)\left(\frac{\sqrt{a_1} + \sqrt{a_2}}{\sqrt{a_H}} - 1\right)$$

- let $\Delta a = a_1 - a_2$ (negative #)
- recall $2a_H = a_1 + a_2$

where V_1 = starting orbit veloc.
V_2 = final " "
a_1 = initial orbit radius
a_2 = final " "
a_H = semi-major axis of elliptical Hohmann xfer orbit

- For closely adjacent orbits, $a_1 \cong a_2$ and can be shown:

$$\left(\frac{\sqrt{a_1} + \sqrt{a_2}}{\sqrt{a_H}} - 1\right) \cong 1 - \frac{1}{16}\left(\frac{\overbrace{a_2 - a_1}^{\text{small}}}{a_1}\right)^2 \cong 1$$

- Thus

$$\boxed{\Delta V_{adj} \cong V_1 - V_2}$$

use to approximate Homing Transfer burn requirements.

- Can also express Hohman xfer ΔV for adjacent, circular orbits in terms of geometry alone:

- Recall for circular orbits $V = \sqrt{\mu/a}$

  So $\Delta V_{adj} = V_1 - V_2 = \sqrt{\frac{\mu}{a_1}} - \sqrt{\frac{\mu}{a_2}}$

- Express V_1 in terms of V_2:

$$V_1 = \underbrace{\sqrt{\frac{\mu}{a_1}}\sqrt{\frac{a_1}{a_2}}}_{V_2}\sqrt{\frac{a_2}{a_1}} = V_2\sqrt{\frac{a_2}{a_1}}$$

- $\Delta V_{adj} = V_1 - V_2 = V_2\left(\sqrt{\frac{a_2}{a_1}} - 1\right)$

  let $\Delta a \equiv a_1 - a_2 \ (<0)$

  so $a_1 = a_2 + \Delta a$

$$\therefore \Delta V_{adj} = V_2\left(\sqrt{\frac{a_2}{a_2+\Delta a}} - 1\right) = V_2\left(\sqrt{1 - \frac{\Delta a}{a_2}} - 1\right)$$

- Maclaurin Series:

$$f(x) = (1-x)^{1/2} - 1 \quad \text{and} \quad x = \frac{\Delta a}{a_2}$$

$$\cong f(0) + f'(0)x$$

$$= \left\{(1-0)^{1/2} - 1\right\} - \left[\frac{1}{2}(1-x)^{-\frac{1}{2}}\right\}x$$

$$= 0 \qquad\qquad - \frac{1}{2}x$$

recall $\Delta a < 0$
$\Delta a \equiv a_1 - a_2$

$$\boxed{\Delta V_{adj} \cong -\sqrt{\frac{\mu}{a_2}}\,\frac{1}{2}\,\frac{\Delta a}{a_2}}$$

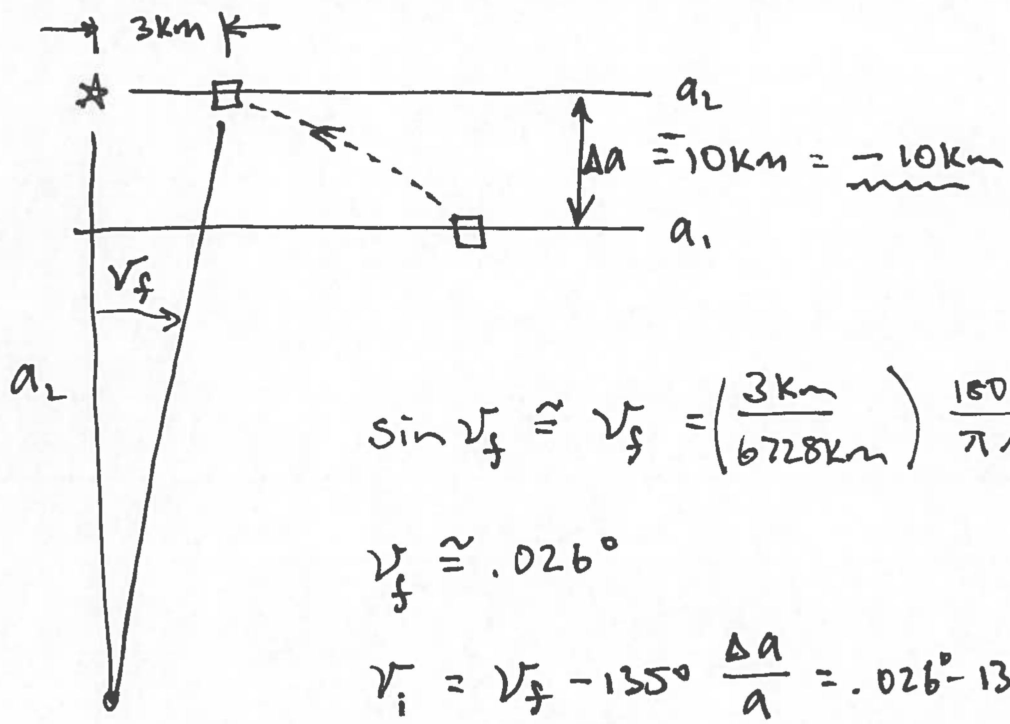
Example - Homing Maneuver

- Target orbit : $h = 350$ km $\Rightarrow a_2 = 6728$ km

  $\Delta a = -10$ km $(= a_1 - a_2)$

  So chaser starting 10 km below target

  Want to arrive via Homing Transfer at point S_2 which is 3 km behind target



$$\sin \nu_f \cong \nu_f = \left(\frac{3\text{ km}}{6728\text{ km}}\right)\frac{180°}{\pi \text{ rad}}$$

$$\nu_f \cong .026°$$

$$\nu_i = \nu_f - 135° \frac{\Delta a}{a} = .026° - 135° \frac{-10}{6728}$$

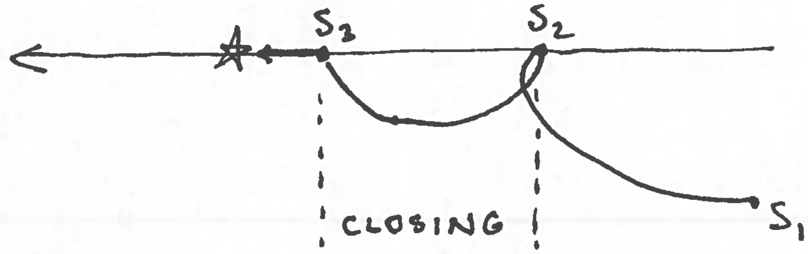
$$= \boxed{.227°} \Rightarrow \left(\frac{.227°}{360°}\right)(2\pi\ 6.728) $$

$$= \boxed{33.58 \text{ km behind}}$$

- $$\Delta V_{adj} = -\sqrt{\frac{\mu}{a_2}}\ \frac{1}{2}\ \frac{\Delta a}{a_2}$$

$$= -\sqrt{\frac{4E14\ m^3/s^2}{6.728\ E6\ m}}\ \frac{1}{2}\ \frac{-10}{6728} = \boxed{5.73\ \frac{m}{s}}$$

Closing Phase: $-V$ bar approach (from behind - simplest)



$S_1 \to S_2$: Homing to waiting point S_2 (station-keeping, sensors)

$S_2 \to S_3$: Closing to waiting point S_3 (station-keeping)

$S_3 \to$ Dock : Approach corridor

- Station-keeping for : sensor acquisition and checkout
  lighting
  relative attitude adjustments
  communication
  vehicle co-inspection

- Key objective: don't collide!

$S_2 \to S_3$: enter "approach ellipsoid"

$S_3 \to$ dock : w/in "approach corridor"
avoid "exclusion zone"
avoid "pluming" your target

-VBar Approach:

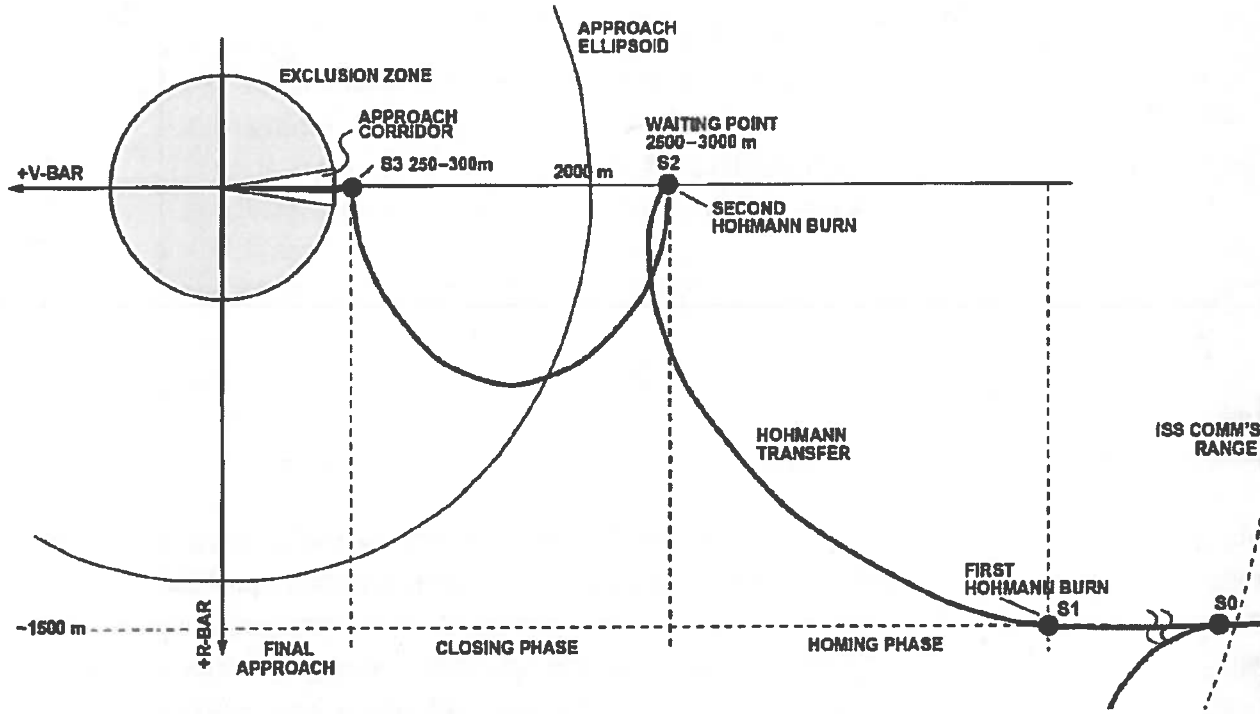


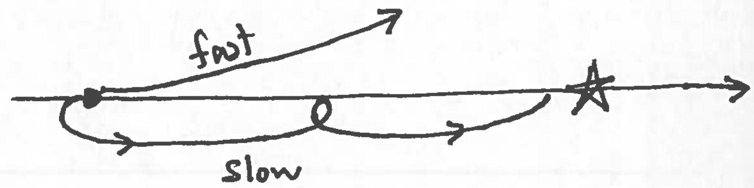
Figure 8.33 ISS stable orbit approach on −V-bar as typically adopted by an ATV rendezvous. First, a Hohmann transfer brings the interceptor to the waiting point S2. Then, it approaches the ISS on an elliptical trajectory to waiting point S3.

- Vbar Approach $S_2 \rightarrow S_3$

(fig 8.22)

- Options for approach trajectory:

① Prolate Cycloid:



tangential burn

($\therefore$ Hohmann)

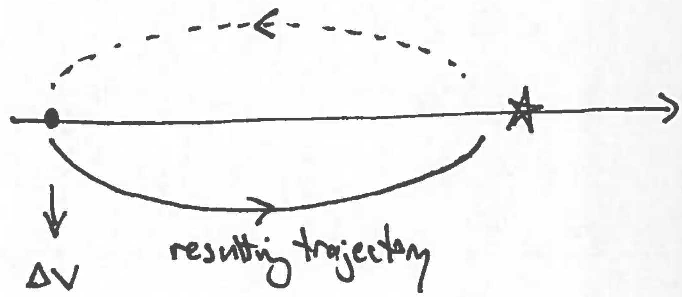
posigrade or retrograde

$\bar{\omega}_c > \bar{\omega}_T$ so always moving along Vbar

Safety: potential collision?

Low fuel cost if slow, cyclic

② Ellipse:



radial burn

"football"

returns to burn point

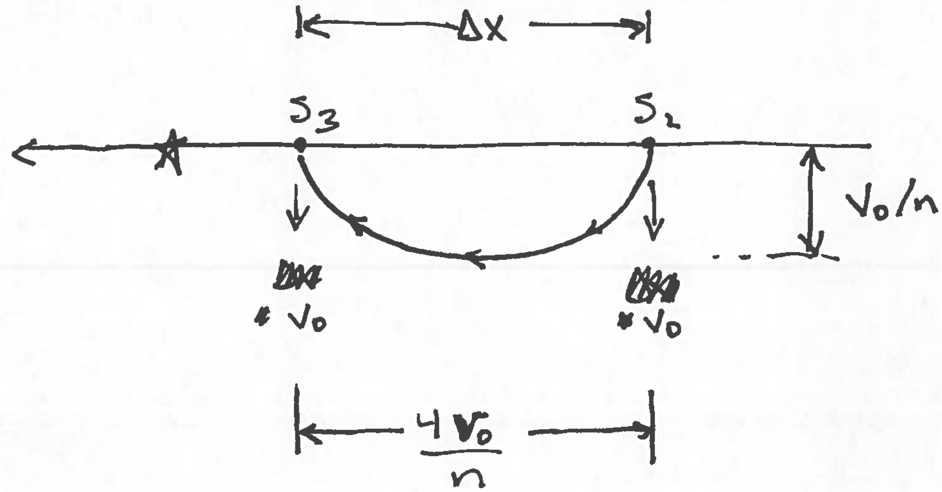
requires "null burn"

to stop on Vbar

can be safer - always comes back

ΔV Requirements for $S_2 \rightarrow S_3$ Closing

- Ellipse :



req'd total ΔV :

ΔX = distance from S_2 to S_3

$= 4 \frac{V_0}{n}$

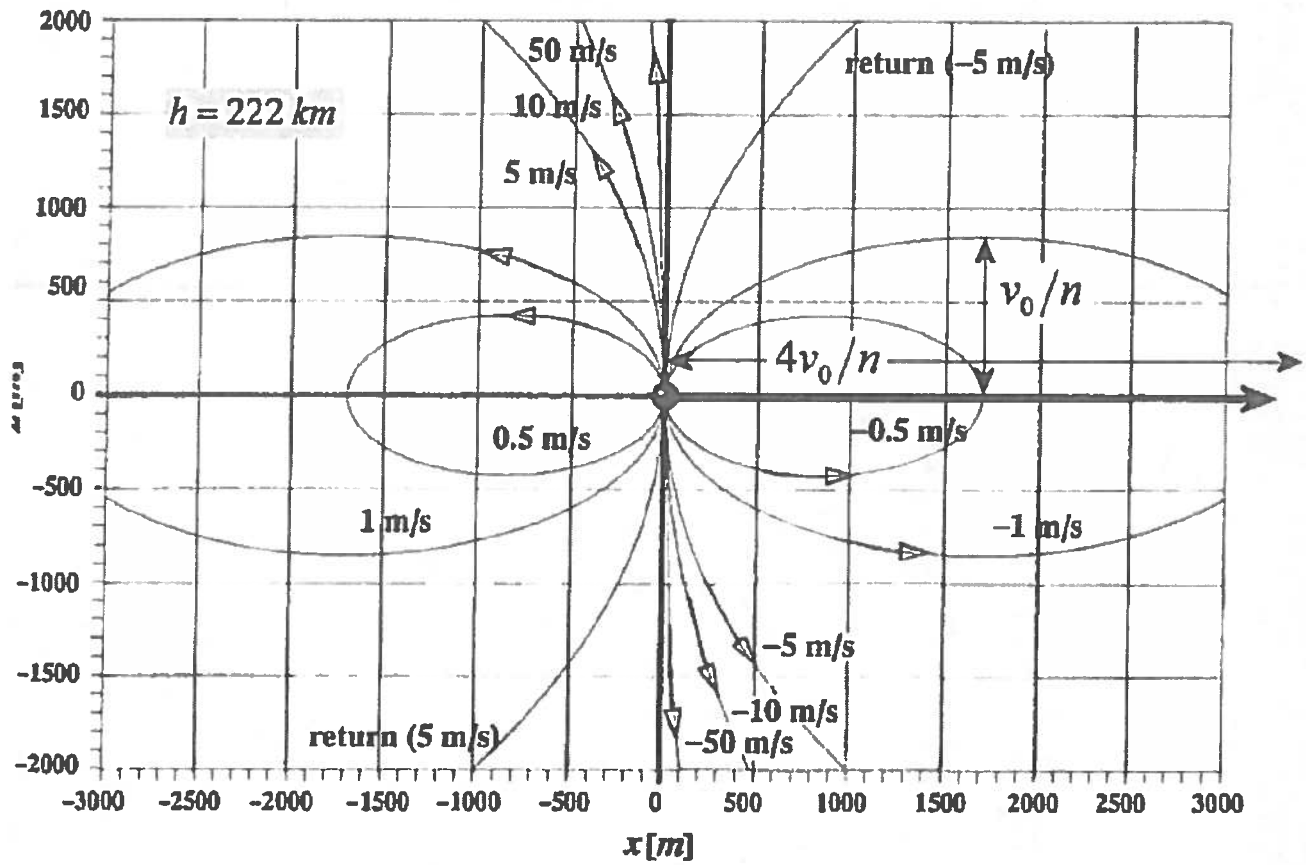
$n = \sqrt{\mu / a^3}$

$\therefore \quad \Delta X = 4 \frac{V_0}{\sqrt{\mu/a^3}} \quad \Rightarrow \quad V_0 = \frac{\Delta X}{4} \sqrt{\mu/a^3}$

ΔV required to initiate ellipse, (at S_2) and then stop on V-bar (at S_3) :

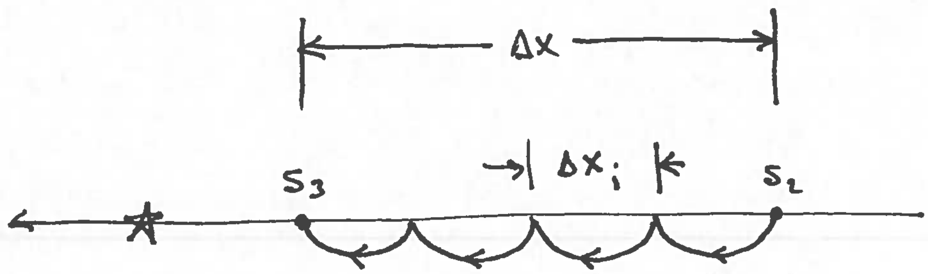
$$\boxed{\Delta V = 2V_0 = \frac{\Delta X}{2} \sqrt{\mu/a^3}}$$ Ellipse

Elliptical Trajectories: Relative Motion



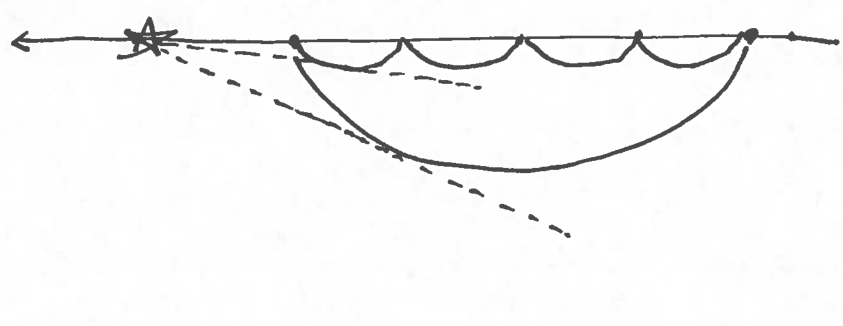
igure 8.25 Shown are the trajectories (ellipses) of the object moving relative to a reference point :enter dot, which itself moves on an orbit at altitude $h = 222$ km to the right (bold arrow)) for ifferent v_0 in z direction.

- Ellipse trajectory (con't): "hopping" approach



$$\Delta V = \frac{\sqrt{\mu/a^3}}{2} \sum_{i=1}^{N} \Delta x_i$$ no fuel cost for more hops

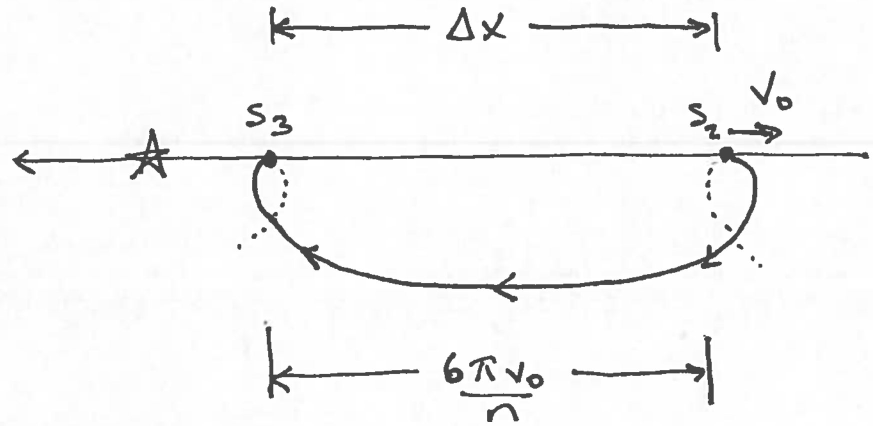
- more opportunities to station keep on V-bar
- slower approach (every step = 1 orbital period)
- narrower Field-of-View requirements for sensors



ΔV Requirements for $S_2 - S_3$ Closing

(Fig 8.22)

- Prolate Cycloid:



Δx = distance from S_2 to S_3

$$= \frac{6\pi V_0}{n}$$

$$n = \sqrt{\mu / a^3}$$

$$\therefore \quad \Delta x = \frac{6\pi V_0}{\sqrt{\mu/a^3}} \quad \Rightarrow \quad V_0 = \frac{\Delta x}{6\pi}\sqrt{\mu/a^3}$$

- ΔV required to initiate cycloid at S_2, then stop on Vbar at S_3

$$\boxed{\Delta V = 2 V_0 = \frac{\Delta x}{3\pi}\sqrt{\mu/a^3}} \quad \text{Cycloid}$$

Prolate Cycloid Trajectories: Relative Motion

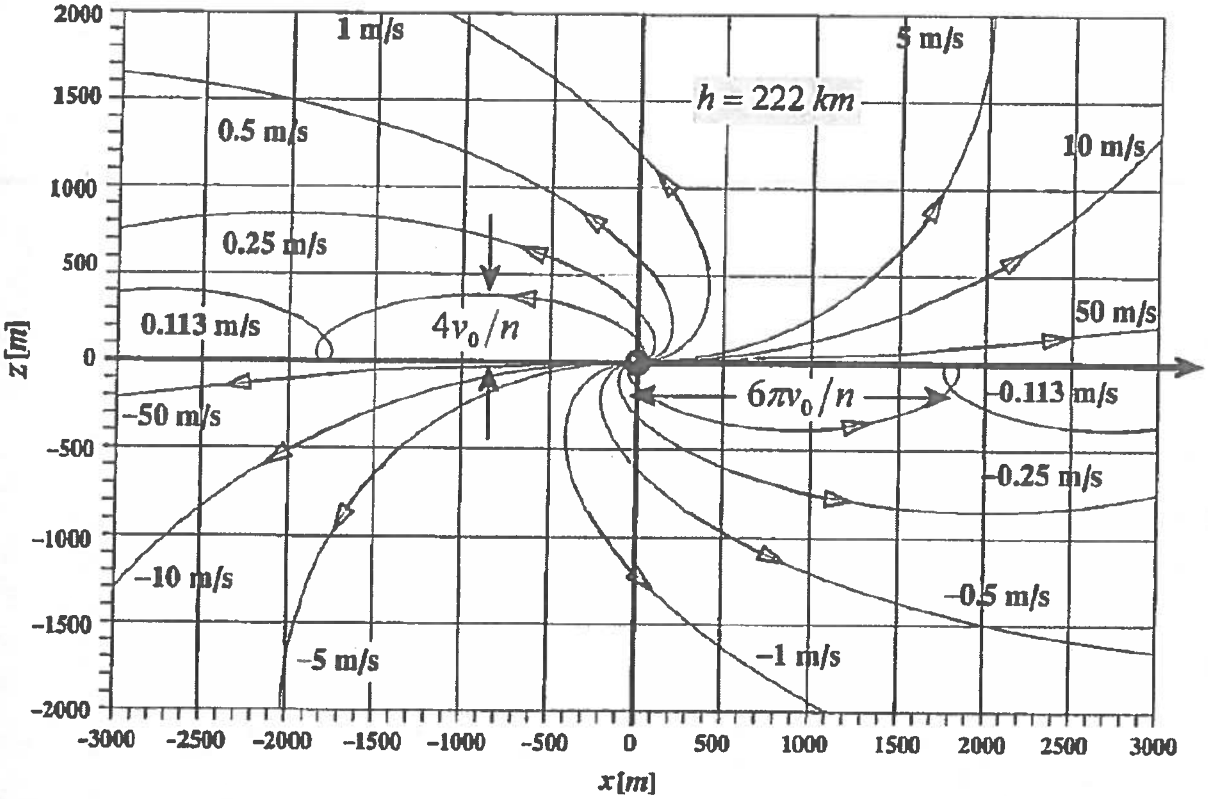


Figure 8.22 Shown are for different v_0 in x direction the trajectories (prolate cycloid) of the object moving relative to a reference point (center dot, which itself moves on an orbit at altitude $h = 222$ km to the right (bold arrow)).

- Fuel efficiency comparison:

$$\frac{\Delta V_{ellipse}}{\Delta V_{cycloid}} = \frac{\frac{\Delta x}{2}\sqrt{\mu/a^3}}{\frac{\Delta x}{3\pi}\sqrt{\mu/a^3}}$$

$$= \frac{3\pi}{2} \cong 4.7$$

So ellipse trajectory can require almost 5x the propellant as prolate cycloid trajectory.

+RBar Approach:

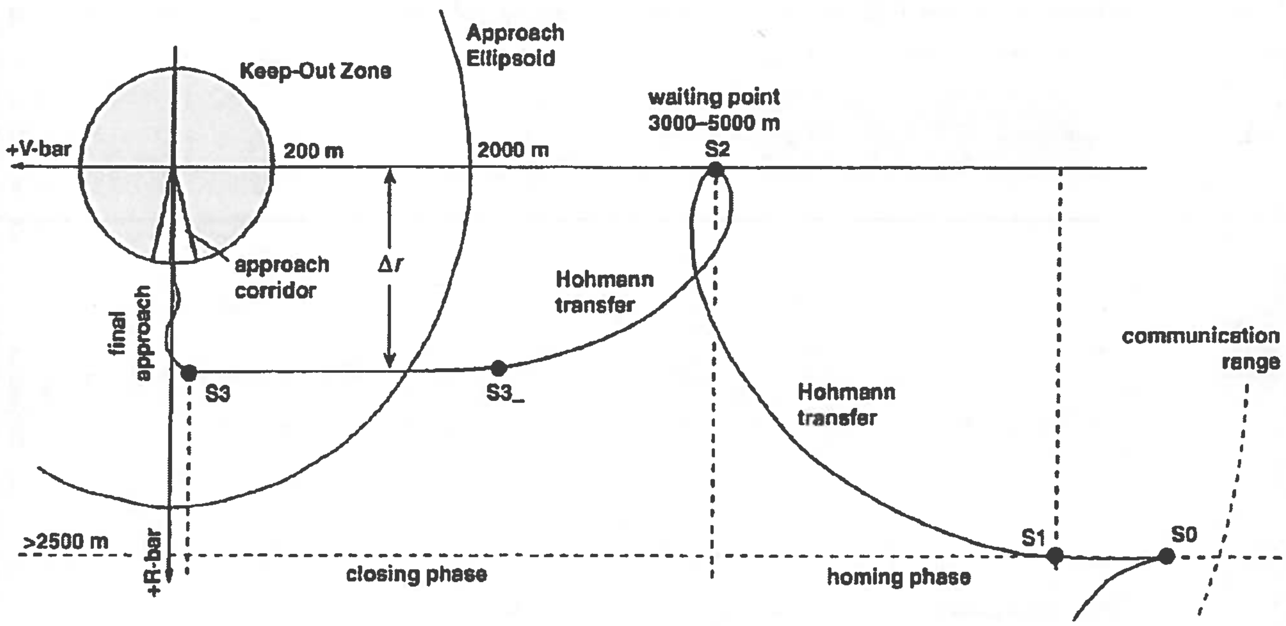


Figure 8.34 To +R-bar approach to ISS. A Hohmann transfer brings the interceptor first to the intermediate point S3 , where it crosses over into a circular orbit on which it drifts to the final point S3.