Chinese Journal of Aeronautics, (2014),27(1): 1–11

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Chinese Journal of Aeronautics

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Survey of orbital dynamics and control of space rendezvous

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Received 15 December 2012; revised 20 March 2013; accepted 5 May 2013
Available online 1 August 2013

KEYWORDS

Automated control;
Relative dynamics equation;
Relative navigation;
Space rendezvous;
Survey;
Trajectory optimization

Abstract Rendezvous orbital dynamics and control (RODC) is a key technology for operating space rendezvous and docking missions. This paper surveys the studies on RODC. Firstly, the basic relative dynamics equation set is introduced and its improved versions are evaluated. Secondly, studies on rendezvous trajectory optimization are commented from three aspects: the linear rendezvous, the nonlinear two-body rendezvous, and the perturbed and constrained rendezvous. Thirdly, studies on relative navigation are briefly reviewed, and then close-range control methods including automated control, manual control, and telecontrol are analyzed. Fourthly, advances in rendezvous trajectory safety and robust analysis are surveyed, and their applications in trajectory optimization are discussed. Finally, conclusions are drawn and prospects of studies on RODC are presented.

1. Introduction

Shenzhou-8 and Shenzhou-9 spacecraft successfully docked with the Tiangong-1 space lab on November 3rd 2011 and June 18th 2012, respectively, which marked the Chinese breakthrough in the rendezvous and docking (RVD) technology.[1] RVD denotes the technology that two spacecraft meet in space with the same velocity and then join into a complex. It is a key operational technology for complicated space missions such as assembling a space station and repairing a satellite in space.

Rendezvous orbital dynamics and control (RODC) is a key technology of RVD, and also an active research field of spacecraft dynamics and control. RODC has a research history of more than 50 years, and many new research ideas and results on this topic are still coming out. Researchers have published many papers and several monographs on RODC.[2–6]

This paper surveys the research status of RODC and discusses its prospects. The contents are organized as follows. Section 2 makes an introduction of RVD missions, and Section 3 describes the basic relative dynamics equations and surveys its improved versions. Next, studies on rendezvous trajectory optimization are commented in Section 4, and studies on relative navigation are reviewed in Section 5. Furthermore, Section 6 summarizes close-range control methods, and Section 7 surveys research on rendezvous trajectory safety and robustness. Finally, research prospects on RODC are discussed in Section 8.

2. RVD missions

In the past half-century, major space countries performed hundreds of RVD missions.[3] The chaser spacecraft can be divided

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Peer review under responsibility of Editorial Committee of CJA.

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http://dx.doi.org/10.1016/j.cja.2013.07.042

into more than ten series, including Gemini,[7] Apollo,[8] space shuttle,[9] Experimental Satellite System-11 (XSS-11),[10] Demonstration of Autonomous Rendezvous Technology (DART),[11] and Autonomous Space Transport Robotic Operations (ASTRO)[12] from USA, the Soyuz and Progress spacecraft from Russia or the former Soviet Union,[13] the automated transfer vehicle (ATV) from the European Space Agency (ESA),[14] the Engineering Test Satellite VII (ETS-VII)[15] and H-II Transfer Vehicle (HTV)[16] from Japan, and the Shenzhou spaceship from China.[1]

As shown in Fig. 1, a typical RVD process can be divided into several phases, including phasing, close-range rendezvous, final approaching, and docking. A complex-vehicle flight phase and a departure phase could also be included in a general concept. The close-range rendezvous phase can be further divided into the homing and closing phases. The close-range rendezvous phase and the final approaching phase can be called by a uniform name: an autonomous control phase.

In the phasing phase, the chaser executes several maneuvers under the guidance of the ground telemetry tracking and command (TT&C) network, so that the navigation sensors of the chaser can catch the target. The major objectives of this phase include adjusting the phase angle between the two spacecraft, reducing the orbital plane differences, increasing the orbital height, and initiating the relative navigation.

In the homing phase, the chaser controls itself autonomously, and the final position of this phase is P_2, a station-keeping point located a couple of kilometers away from the target. The major objectives of this phase are to acquire the target orbit and to reduce the relative velocity. In the closing phase, the chaser reduces the relative distance further, and its position is transferred to P_3, a station-keeping point located hundreds of meters away from the target.

The final approaching phase starts from P_3 and ends when the chaser touches the target. In this phase, the chaser approaches the target along a straight line as much as possible, in order to satisfy the strict requirements of docking for the relative position, velocity, attitude, and angular rate.

Absolute navigation and control are mainly used in the phasing phase, relative navigation and control are mainly used in the close-range rendezvous phase, and orbit and attitude combined six-degree of freedom (6-DOF) control is used in the final approaching phase.

3. Relative dynamics equations

When the two spacecraft have a long relative distance, their movements are usually described in an Earth-centered coordinate system. When the distance between the two spacecraft is short enough, the relative movement is usually described in a target-centered orbital coordinate system. This orbital coordinate system is given as: the center is located in the target's center of mass; the z axis, also called R-bar, is along the position vector from the target to the Earth; the y axis, also called H-bar, is in the opposite direction to the orbit normal; the x axis, also called V-bar, is in the direction of the velocity and completes the right-handed system.

A linear relative dynamics equation set in a rectangular coordinate is widely used, and multiple improved versions have been developed from it.

3.1. Basic relative dynamics equations

When the two spacecraft run on neighboring near-circular orbits, the relative distance is much shorter than the spacecraft's geocentric distance, and the orbital perturbations are ignored, so the relative movement can be effectively described by a linear dynamics equation set as given below:

$$\begin{cases} \ddot{x} - 2\omega\dot{z} = a_x \\ \ddot{y} + \omega^2 y = a_y \\ \ddot{z} + 2\omega\dot{x} - 3\omega^2 z = a_z \end{cases} \tag{1}$$

where ω is the orbital angular rate of the target, and a_x, a_y, and a_z are the thrust acceleration components.

This equation set is referred to as the Clohessy–Whiltshire (C–W) or Hill equations,[17] and has a closed-form analytical solution. Based on the C–W equations, relative motion can be divided into two independent parts: in-plane motion (the x–z plane) and out-of-plane motion (the y direction), where only the motion in the directions of x and z are coupled with each other. When the control forces disappear, the out-of-plane trajectory waves as a trigonometric function, and the in-plane trajectory relates closely to its initial state.

3.2. Improved relative dynamics equations

The C–W equation set is derived on the assumptions that the two spacecraft run on neighboring two-body circular orbits and the relative distance between the two spacecraft is much shorter than their geocentric distance. Moreover, first-order

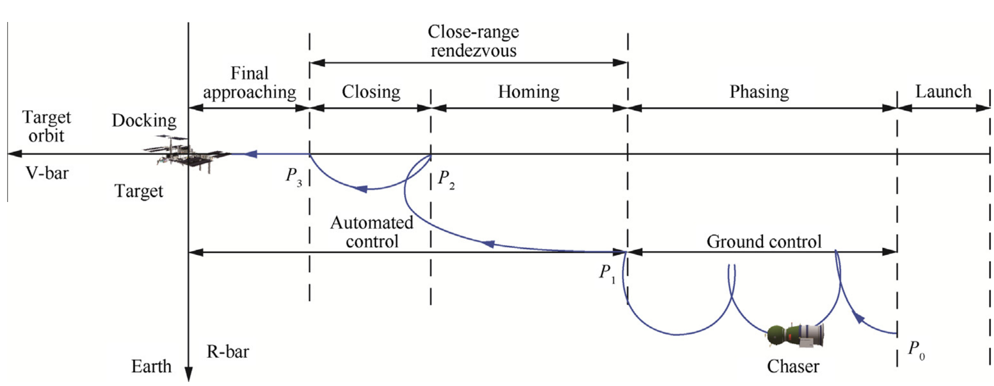

Fig. 1 Spacecraft rendezvous and docking process.

approximations are used so that second- and higher-order terms of relative positions and velocities are ignored. The C–W equation set is only effective for relative trajectories with close distance and short time. It needs improvements in order to describe relative trajectories not satisfying these assumptions.

(1) Relative dynamics equations for large angle gaps

In the phasing phase, the difference between the two spacecraft's arguments of latitude is not a small value, and the relative distance could be the same order as the spacecraft's geocentric distance. Then, the relative position and velocity using a rectangular coordinate are not able to describe this relative trajectory effectively. Baranov[18] used the difference in argument of latitude, the difference in orbital radii, and their first-order derivatives to describe relative trajectories with a large angle gap. This dynamics equation set can be expressed below[18]:

$$\begin{cases} \Delta\dot{r} = \Delta v_r \\ \Delta\dot{\theta} = -\omega_0 \dfrac{\Delta r}{r_0} + \omega_0 \dfrac{\Delta v_t}{v_0} \\ \Delta\dot{z} = \Delta v_z \\ \Delta\dot{v}_r = \omega_0^2 \Delta r + 2\omega_0 \Delta v_t + \Delta a_r \\ \Delta\dot{v}_t = -\omega_0 \Delta v_r + \Delta a_t \\ \Delta\dot{v}_z = -\omega_0^2 \Delta z + \Delta a_z \end{cases} \tag{2}$$

where r_0, ω_0, and v_0 are the mean radius, orbital angular rate, and orbital velocity of the target, respectively; Δr, $\Delta\theta$, and Δz the relative position components; Δv_r, Δv_t, and Δv_z the relative velocity components; Δa_r, Δa_t, and Δa_z the acceleration components in an Earth-centered cylindrical coordinate system.

The essentiality of Baranov's idea is to build relative dynamics equations in an Earth-centered cylindrical coordinate system. This equation set was used in the phasing phase of Russian (or the former Soviet Union) RVD missions, as well as by Tang et al.[5] for Shenzhou phasing mission design.

(2) Relative dynamics equations for elliptic orbits

The linear relative dynamics equation set for elliptical orbits was derived by Tschauner and Hempel,[19] and therefore was named after them as the T–H equation set. Although the T–H equation set is linear, researchers are still not able to give a simple closed-form analytical solution for it.

Melton[20] derived the state transition matrix (STM) of the T–H equation for orbits with small eccentricity, based on the solution of the C–W equation and using a perturbation theory. Carter[21] derived a complicated STM using the true anomaly as the independent variable. Yamanaka and Ankersen[22] derived a simpler STM than Carter's, also using the true anomaly as the independent variable. Some researchers already studied the targeting algorithms for elliptic rendezvous missions using the STM given by Yamanaka and Ankersen, which was based on the following equation set[22]:

$$\begin{cases} \ddot{x} + k\omega^{3/2} x - 2\omega\dot{z} - \dot{\omega} z - \omega^2 x = a_x \\ \ddot{y} + k\omega^{3/2} y = a_y \\ \ddot{z} - 2k\omega^{3/2} z + 2\omega\dot{x} + \dot{\omega} x - \omega^2 z = a_z \end{cases} \tag{3}$$

where μ is the gravitational constant of Earth, and $k\omega^{3/2} = \mu/r^3$, with r being the distance from the center of the Earth to the target spacecraft.

However, when the relative state is propagated as time, the Kepler equation needs to be solved in every time step, and the effectiveness of this STM for practical applications still needs validation.

(3) Relative dynamics equations including second-order terms

The precision of the C–W equation decreases as the relative distance increases, so the second-order terms of relative positions and velocities should be taken into account to improve the precision of relative dynamics equations.

London[23] derived the equation set of the quadratic terms, and obtained its approximate solution based on the solution of the C–W equation. Karlgaard and Lutze[24] presented an approximate solution of the second-order relative motion equation set in spherical coordinates. Kechichian[25] applied the second-order equation set to a rendezvous trajectory with an initial relative distance of 2000 km. Zhu and Li[26] extended the second-order equation set to include constant thrusts, and considered the small eccentricity. The widely used second-order relative dynamics equation set is given below[23]:

$$\begin{cases} \ddot{x} - 2\omega_0\dot{z} + 3\omega_0^2 \dfrac{xz}{r_0} = a_x \\ \ddot{y} + \omega_0^2 y + 3\omega_0^2 \frac{yz}{r_0} = a_y \\ \ddot{z} + 2\omega_0\dot{x} - 3\omega_0^2 z + \dfrac{3}{2}\omega_0^2 \dfrac{x^2}{r_0} + \dfrac{3}{2}\omega_0^2 \dfrac{y^2}{r_0} - 3\omega_0^2 \dfrac{z^2}{r_0} = a_z \end{cases} \tag{4}$$

(4) Relative dynamics equations considering orbital perturbations

Studies on satellite formation presented some nonlinear relative dynamics equation sets considering orbital perturbations, in order to design relative formation orbits with robustness or special configurations.[27] For instance, Gim and Alfriend[28] derived a complicated STM for elliptic orbits under perturbations, and Ross[29] derived a time-varying coefficient relative dynamics equation set considering the J_2 perturbation. However, these nonlinear equations are complicated and not convenient to calculate maneuvers. To conquer this problem, Schweighart and Sedwick[30] derived a constant coefficient relative equation set based on the C–W equation by adding the long-term effect of the J_2 perturbation. This equation set has a closed-form analytical solution and is convenient to calculate maneuvers. Pollock et al.[31] presented an analytical solution for relative motion subject to Lorentz-force perturbations.

Kechichian[25] combined the second-order terms and the J_2 perturbation, Yamada et al.[32] added the J_2 perturbation to the elliptical STM, and Zhang and Zhou[33] presented a second-order elliptical STM with the J_2 perturbation.

However, in close-range rendezvous missions with the duration of one or two orbital revolutions, corrections from orbital perturbations cannot improve the precision a lot, and therefore relative dynamics equations considering orbital perturbations have not been widely used in rendezvous missions as in satellite formation missions.

(5) Relative dynamics equations based on orbital element differences

For rendezvous missions with large initial orbital plane differences, the rectangular coordinate is not effective any more, and orbital element differences are employed. Labourdette and Baranov[34] derived a relative dynamics equation set based on orbital element differences with the J_2 perturbation, and used it to calculate maneuvers of a long-time rendezvous mission on a Mars orbit. Zhang et al.[35] corrected and extended this equation set and applied it to a long-time multi-spacecraft rendezvous mission on a low Earth orbit. On the other hand, orbital element differences are also used in satellite formation orbital design and analysis, but the equations used are not convenient to calculate maneuvers.[27]

4. Rendezvous trajectory optimization

Design variables of rendezvous trajectory design are usually burn time and maneuver impulse (or burn thrust vector), while optimization objectives could be propellant cost (or total velocity increment), total time of flight, trajectory safety index, or trajectory robustness.[5] The constraints considered for operational missions include rendezvous time windows based on sun illumination and TT&C conditions, terminal position and velocity, minimum burn time interval, passive trajectory safety requirements, and working requirements of navigation sensors. The optimal rendezvous problem has been extensively studied for a long time. A good survey of this work was provided by Jezewski et al.[36] in an early paper. This paper surveys the studies on rendezvous trajectory optimization according to the following three categories: the linear rendezvous, the nonlinear two-body rendezvous, and the perturbed and constrained rendezvous.

4.1. Optimization of linear rendezvous trajectories

Early studies mainly focused on the propellant-optimal linear impulse rendezvous problems. Based on the Pontryagin maximal principle, Lawden[37] proposed the widely used primer-vector theory of impulse maneuvers, and provided the first-order necessary conditions for propellant-optimal trajectories. Subsequent researchers, such as Prussing,[38] Carter,[39] and Jezewski,[40] improved the primer-vector theory. They proved that necessary conditions given by Lawden were also sufficient conditions for coplanar linear rendezvous problems, and burn time, impulse magnitude and directions could be calculated analytically based on boundary conditions by the primer-vector theory. In addition, the optimality of solutions could be judged by observing the primer-vector locus, and the optimal number of maneuvers could also be obtained. Lion and Handelsman[41] improved the primer-vector theory to the non-optimal primer-vector theory, based on which a trajectory not satisfying Lawden's necessary conditions could be improved to an optimal one by adding maneuvers or coasting arcs. Recently, Wang et al.[42] extended the primer theory to help optimize a four-impulse elliptical rendezvous trajectory, while Baranov and Roldugin[43] employed it to search for a non-degenerate six-impulse analytical solution to the rendezvous problem in close near-circular noncoplanar orbits.

When the design objective is not the propellant or the maneuver direction is limited, the primer-vector theory cannot be applied any more, and numerical optimization algorithms should be employed. For linear rendezvous problems, Li and Xiao[44] employed genetic algorithms to optimize an objective combining velocity increment and time of flight, and Luo et al.[45] used a hybrid genetic algorithm to obtain the minimum time of flight with consideration of impulse constraints.

If the ratio of the working time of thrusters to the orbital period is not a small number, the finite-thrust maneuver model is needed. Carter[21,46–48] obtained fruitful achievements in finite-thrust optimal rendezvous, and in his series work, the indirect method based on Pontryagin's maximum principle was employed to solve the finite-thrust rendezvous problem with different considerations including thrust bound constraints, propellant max loss, thrust saturation, etc.

Also using the indirect method, Lembeck and Prussing[49] investigated the optimal linear rendezvous trajectory combining impulse and low-thrust maneuver, while Guelman and Aleshin[50] proposed a two-stage solution for the bounded, low-thrust, fixed-time, fuel-optimal constrained terminal approach direction rendezvous problem.

The pseudospectral method is a recently widely applied method for spacecraft trajectory optimization, which was employed by Boyarko et al.[51] to solve the problem of minimum-time and minimum-energy optimal trajectories of rendezvous of a powered chaser and a passive tumbling target with considerations of both translational and rotational dynamics, specified ending conditions, and collision-avoidance constraints, as well as by Ma et al.[52] to obtain a suboptimal solution for rendezvous with power-limited propulsion systems, fixed docking direction, and collision avoidance. Apart from the general indirect and direct methods, there are some other new methods, for example, Bevilacqua et al.[53] proposed a rapid algorithm for the generation of feasible quasi-optimal spacecraft rendezvous trajectory, which was based on the direct method of calculus of variations and exploring the advantages of the inverse dynamics approach, and Ulybyshev[54] formulated the finite-thrust rendezvous problem as a linear programming problem by introducing the idea of pseudo-impulses and solved it using the interior point algorithms.

4.2. Optimization of nonlinear two-body rendezvous trajectories

The optimal first-order necessary conditions can also be applied to two-body orbits, but they are not the sufficient conditions any more. For two-body coplanar circular-to-circular orbital transfer problems, Hohmann provided the analytical equations for optimal maneuver impulses, and Prussing proved the optimality of these equations and provided the conditions that the Hohmann transfer needed to satisfy.[55] Hohmann transfer equations can only be applied to coplanar circular-to-circular transfers with specified time of flight, and they are usually used to help design the nominal trajectories of practical missions. On the other hand, the more general two-body rendezvous problems are usually solved by Lambert's method, which has been studied for more than 200 years. Early studies mainly focused on improving the algorithm's universalization and convergence rate, while recent studies focused on how to solve the algorithm's singularity points.[56]

The solution to a multi-revolution Lambert's problem needs an additional search for the best revolution number based on the solution to a single revolution problem. There are in total $2N + 1$ trajectories for a two-impulse rendezvous problem of coasting N revolutions most, and studies on multi-revolution Lambert's problems mainly focused on how to choose the trajectory consuming minimum propellant as fast as possible.[57] Recently, He et al.[58] solved the multiple-revolution Lambert problem using the transverse eccentricity-vector-based algorithm. Zhang et al.[59] gave a solution to the fixed-time multiple-revolution Lambert problem with constraints on the perigee and apogee altitudes.

Nonlinear two-body rendezvous problems could also be solved directly by nonlinear programming, and the primer-vector theory and Lambert's method could help improve the convergence rate. Gross and Prussing[60] provided the analytical formulation of the first-order derivative of the total velocity increment, based on the primer-vector theory and Lambert's method. Hughes et al.[61] and Luo et al.[62] calculated the last two impulses of a multiple-impulse rendezvous problem using Lambert's method, so that the optimization search is limited in the feasible field. Luo et al.[63] further proposed an interactive optimization approach by combining the primer vector theory, the Lambert algorithm, and a parallel simulated annealing algorithm. Chen et al.[64] applied the interval branch and bound optimization algorithm to a time-open constrained Lambert rendezvous problem.

For a finite-thrust nonlinear rendezvous problem, a fast and reliable method is to use an impulse solution as the initial reference and then optimize it with nonlinear programming.[65] However, if the thrust is very small, the impulse solution is not a good initial reference any more. Researchers usually transferred a finite-thrust nonlinear rendezvous problem into a two-point boundary value problem using the Pontryagin maximal principle[66] or into a nonlinear programming problem using the direct methods including direct shooting, the collocation method, and the pseudospectral method,[67,68] and then solved the transferred problem using a nonlinear programming algorithm. Park et al.[66] proposed a novel method of evaluating an optimal continuous rendezvous trajectory as well as an optimal feedback control via generating functions. Recently, with the background of deep exploration missions using electric propulsion, the low-thrust rendezvous problem has been studied extensively,[68–70] and a good survey on this topic has been made by Gao.[69]

4.3. Optimization of perturbed and constrained rendezvous trajectories

Because the linear and nonlinear two-body rendezvous studies do not consider orbital perturbations, they cannot be directly applied to a practical phasing mission. Many operational constraints still need to be taken into account during the trajectory optimization.

The time of flight for a phasing phase is long, so its trajectory should be calculated by high-precision numerical integration. The results of linear and nonlinear two-body models can be used as initial references, and can also help the convergence of the iteration with trajectory perturbations. Weeks and D'Souza[71] improved the multi-level shooting iteration method of the space shuttle phasing maneuvers calculation into the on-board calculation level, and the effects of maneuvers on orbital height, phase-angle, and orbital plane were considered separately. For the combined maneuver strategy, Baranov[18] calculated impulses by linear equations and improved the results to a perturbed trajectory by iterations. Luo et al.[72] solved a special-point maneuver phasing strategy by combing a genetic algorithm and the Newton iteration method, and optimized a combined maneuver phasing strategy by a hybrid approach[62] in which a two-body solution was firstly obtained using the Lambert algorithm and a parallel simulated annealing algorithm, and then the perturbed solution was obtained by the sequential quadratic programming (SQP) algorithm. Zhang et al.[73] proposed a fast and hybrid approach for the optimization of long-duration phasing maneuvers.

The optimization of a close-range rendezvous phase could directly use the linear dynamics model. The sun illumination and TT&C conditions could be satisfied by choosing rendezvous windows and adjusting rendezvous-phasing time. Path constraints, such as the field of view angle, trajectory safety, and effect of thruster plumes, need to be considered directly during trajectory optimization. Richards et al.[74] considered the effects of collision and thruster plumes in the close-range flyby planning. Luo et al.[75,76] considered TT&C conditions and the field of view angle constraints in the close-range rendezvous trajectory planning. Zhang et al.[77] proposed a hybrid optimization approach for a multi-segment rendezvous trajectory, with consideration of the requirements of sun illumination, station keeping, and sensor transition. Recently, Epenoy[78] optimized an elliptical continuous-thrust rendezvous trajectory subject to collision avoidance constraints.

Generally, perturbed and constrained rendezvous problems are very complicated and cannot be solved analytically just by orbital dynamics knowledge but need powerful optimization algorithms. The progress in numerical computation improved studies in this field. Early studies mainly used gradient-based optimization algorithms. Gross and Prussing[60] employed a variable-metric method to solve a multiple-impulse rendezvous problem. Jezewski and Rozendaal[79] employed the SQP algorithm to optimize the space shuttle rendezvous trajectory. Hughes et al.[61] tested and compared the performances of SQP, the simplex method, and the quasi-Newton method on solving multiple impulse rendezvous problems. Recent studies in this field mainly used intelligent optimization algorithms, such as the genetic algorithm, simulated annealing, the particle swarm algorithm, and hybrid algorithms.[45,62,63,72,73,80]

5. Relative navigation

Navigation sensors on the chaser can be divided into two categories: relative ones and absolute ones.[81] Relative navigation sensors are mainly used to determine the chaser's position, velocity, attitude, and angular rate with respect to the target, including satellite relative navigation equipment, microwave radars, lidars, optical imaging sensors, and television cameras. Absolute navigation sensors are mainly used to determine position and attitude in the inertial frame, such as satellite navigation equipment, inertial measurement units, and optical attitude sensors.

The determination of relative states is mainly related to two aspects: the methods to determine instantaneously measured relative states based on sensors' output and the filters to reduce errors based on historical information. For the former, if the output of sensors includes geometric information such as distance, velocity, and attitude, the instantaneously measured relative states can be obtained directly by coordinate transformation; if the measurement output is image information, image recognition technologies should be employed. For instance, Zhang et al.[82] studied a charge-coupled device binocular vision measurement strategy to improve noise resistance ability during image recognition, and Zhang et al.[83] studied a monocular vision measurement approach to estimate relative position and attitude based on feature points.

For the latter, most studies used the C–W equation as the filter equation of states, some studies for elliptical rendezvous filters used the T–H equation, and the second-order equations were also used in some studies. The extended Kalman filter is the major filter algorithm used for rendezvous navigation, and the particle filter was also employed in a few studies. Recently, a method automatically tuning the Kalman filter by estimating measurement and processing noise covariance was proposed to improve the robustness of the filter for an elliptical rendezvous navigation problem.[84]

For rendezvous missions between small spacecraft or with non-cooperative targets, small and lightweight devices are preferred in order to reduce energy requirements and the total mass. The maximum measuring distance in these missions is much less than that in cooperative missions, and even only relative angle information could be obtained at a relative distance of a few tens of kilometers. In missions with angle-only measurement or a maneuverable target, the distance information has large uncertainties, which brings new requirements for relative navigation. Chari[85] analyzed the observability of angle-only relative navigation. Wodffinden and Geller[86,87] presented the conditions needed to satisfy for observable angle-only relative navigation, and proposed a method using active maneuvers to improve navigation precision. Recently, Li et al.[88,89] proposed a multi-objective closed-loop trajectory design method under angle-only relative navigation. For missions with maneuverable target spacecraft, Liu and Xu[90] proposed a two-step multi-model estimation method.

6. Close-range control

The chaser in a close-range or final approaching rendezvous phase is usually controlled automatically by on-board computers. It can also be manually controlled by astronauts on the chaser, and even telecontrolled by operators on the target or in the mission control center.

6.1. Automated control

In the phases with relative distances from a few tens of kilometers to hundreds of meters, the trajectory and the attitude are controlled separately, and the maneuver impulses are mainly calculated based on the initial and final relative states using the C–W equation, which is referred to as a C–W targeting method.[4] Most spacecraft, such as Soyuz, Progress, ATV, HTV, and Shenzhou, employed this method, while the space shuttle used a simple iterative targeting method based on the Lambert algorithm and the trajectory numerical integration with low-order gravity perturbations.[9] Another targeting method based on the C–W targeting is called the glide slope targeting, in which some path points are deployed along the approaching line. Maneuvers between each two path points are calculated based on the C–W targeting, so that the maneuver points are directed to the target and the relative velocity could decrease as the relative distance decreases. Pearson,[91] Hablani et al.,[92] Wang and Cao[93] designed different path-point relations, and obtained different glide slope targeting properties.

From the relative distance of hundreds of meters to docking, in order to keep the trajectory exactly in the approaching gate and to reduce approaching time, a trajectory-attitude combined control strategy of 6-DOF is usually used, in which the thruster commands are not calculated in advance as the C–W targeting, but are sent by triggering conditions of targeting laws.

On the other hand, for improving the robustness and adaptivity of rendezvous controls, modern control methods are employed. Karr and Freeman,[94] Chen and Xu[95] proposed different rendezvous fuzzy control methods. Youmans and Lutze[96] proposed a neural network based rendezvous control method. Shibata and Ichikawa[97] proposed a feedback controller for circular and elliptic rendezvous by using the property of null controllability with vanishing energy for the linear C–W and T–H equations. Luo and Tang[98] designed a rendezvous linear quadratic controller using a simulated annealing algorithm. Sharma et al.[99] presented a near-optimal feedback control methodology for minimum-fuel rendezvous near elliptic orbits accounting for nonlinear differential gravity.

Cairano et al.[100] proposed a model predictive control approach for spacecraft rendezvous and proximity maneuvering which could effectively handle the constraints on thrust magnitude, line-of-sight, and approach velocity. Yang et al.[101] considered the impulse controlled rendezvous process as a switching system and proposed a novel feedback control approach based on linear matrix inequality and genetic algorithm. Gao et al.[102] designed an H-infinity state-feedback controller for spacecraft rendezvous systems subject to parameter uncertainties, external perturbation, control input constraints, and poles constraint via a Lyapunov approach, which could guarantee the closed-loop systems to meet the multi-objective design requirements. Gao et al.[103] proposed a robust H-infinity controller for spacecraft rendezvous missions on elliptical orbits. Zhou et al.[104] proposed a controller based on Lyapunov differential equations for an elliptical rendezvous problem by considering both magnitude and energy constraints. Zhou et al.[105] designed a circular orbital rendezvous controller with actuator saturation and time delay based on a parametric Lyapunov equation approach.

6.2. Manual control and telecontrol

The manual control, as an important control method, was usually used by the space shuttle, while the Russian spacecraft used it as a backup of the automated control. On the other hand, the telecontrol is an important backup of the automated control for unmanned spacecraft, and it can also be applied to missions with non-cooperative targets.[106]

The manual control needs good optical observation devices and drones on the target. The professional training of operators could benefit the success of operations, because it can make the operators be familiar with the relative motion and improve their skills.

The navigation sensors of a telecontrol rendezvous mission are similar to a manual control rendezvous mission, and the Progress spacecraft are already installed with telecontrol RVD systems.[106] Under a telecontrol model, measurement and control information is transferred between the chaser and the target or the mission control center. The time delay of the signal transfer could affect the control a lot, and may lead to a low mission success rate. Therefore, the telecontrol rendezvous is a little worse than the direct manual control rendezvous in the stableness from the point view of controlling.

Zhou et al.[107] used the virtual reality technology to display the rendezvous trajectory predicted by the C–W equation, which could help reduce the effect of time delay.

7. Rendezvous trajectory safety and robustness

RVD is a planned collision between two spacecraft, and only allows very small trajectory errors. If the chaser deviates from the planned trajectory due to orbital errors or failures, the two spacecraft, which are planned to approach each other, may collide with each other out of the docking point with a high relative velocity, and therefore lead to a serious accident. In consequence, trajectory safety and robustness are very important for rendezvous missions.

7.1. Rendezvous trajectory safety

There are two major categories of rendezvous trajectory safety problems: the collision between a spacecraft and space debris, and the collision between two spacecraft. Rendezvous trajectory safety here mainly relates to the latter.

7.1.1. Trajectory safety analysis

For avoiding undesired collisions, a target-centered cuboid or spheroid is usually used as a safety control zone. The chaser is only allowed to approach the target from an approach corridor with a very small velocity; otherwise it is treated as a non-safe approaching operation. The International Space Station (ISS) has two safety control zones[3]: the external control zone, called the approach ellipsoid (AE), is a target-centered ellipsoid with a size of 4 km × 2 km × 2 km, and the inside control zone, called the keep out sphere (KOS), is a target-centered sphere with a radius of 200 m.

Before entering the approach corridor, a passive trajectory safety model is required so that when the thrust control ceases at any point of the trajectory, the resulting free trajectory will never go into the control zone for a certain period of time (usually an orbital revolution). Studies on passive trajectory safety mainly focused on safety zone analysis and safe velocity calculation. Fehse[3] provided the definition and rules of rendezvous control zones, Yamanake[108] derived the safety velocity equations for a V-bar approach with a cuboid control zone, and Zhu et al.[109] analyzed the safety velocities of the cuboid, spheroid, and cone zones.

An active trajectory safety model should be involved after the chaser enters the approach corridor. The safety boundary is set outside of the approach corridor. If the relative states have gone beyond the safety boundary, a collision avoidance maneuver (CAM) would be triggered to change the trajectory into a non-collision safety trajectory. The approach trajectory with an active safety model allows a higher approach velocity than that with a passive model, and therefore it can reduce the time of flight required. Fehse[3] used the impulse in the opposite direction to the velocity as the CAM, and Zhu et al.[109] presented a method to calculate CAMs based on the safety boundary.

The rendezvous trajectory safety considering errors is a probability problem. Patera[110] proposed a method to calculate the collision probability between two spacecraft. Based on Patera's work, Wang et al.[111] proposed a safety analysis method using the collision probability. Luo et al.[112] proposed a safety performance index based on the collision probability and the distance between the trajectory error ellipsoid and the safety control zone.

7.1.2. Trajectory safety optimization

Jacobsen et al.[113] used the reciprocal of the time of flight from the out-of-control point to the collision point as the flyby safety index, and employed the genetic algorithm to optimize the mixed index combining the safety index and the propellant consummation. Roger and McInnes[114] used the Laplace potential function to express the safety control zone during the safety trajectory optimization. Richards et al.[74] added 0–1 variables to express the constraints of collision avoidance and plumes' pollution avoidance for a space station flyby problem, and then solved this problem using a mixed integer programming (MIP) approach. Breger and How[115] accumulated the probabilities of collisions caused by different failures in different trajectory points, formulated the passive and active safety indexes, and then proposed an on-line method for obtaining optimal safety trajectory. Luo et al.[76,116,117] defined the minimum distance between the chaser and the target in the chaser's free-flying path as the trajectory safety performance index, and then completed the multi-objective optimization design of an impulsive rendezvous that included the minimum characteristic velocity, minimum flight duration, and maximum safety performance index. The Pareto solution set was obtained by the multi-objective nondominated sorting genetic algorithm for linear rendezvous[116] and two-body rendezvous, while a tradeoff solution for nonlinear perturbed rendezvous.[117]

7.2. Rendezvous trajectory robustness

By deploying station-keeping points, a rendezvous mission could be suspended under emergency conditions, so that the rendezvous mission can be continued after isolating and solving the abnormal factors. On the other hand, the practical rendezvous trajectories under errors always deviate from the nominal ones. In-flight retargeting strategies are usually adopted in engineering applications to reduce the effect of errors, and therefore the robustness of the rendezvous trajectory could be improved. Tang el al.[5] proposed an in-flight retargeting strategy for phasing rendezvous maneuvers.

Analysis for the effect of errors is an important aspect of trajectory robustness studies. The Monte Carlo simulations are usually used in engineering applications to obtain the trajectory scattering field. Another important method is to propagate errors analytically or semi-analytically based on relative dynamics equations.

For the phasing rendezvous problem, Zhang et al.[118-] expanded the first-order terms of the orbital propagation process of the quasi-mean-element method, and obtained the first-order transition matrix of errors. Zhang et al.[119]employed an experimental design method to analyze the effects of major errors on terminal conditions and the interaction effects between errors. Liang et al.[120]proposed a semi-analytical method using the covariance analysis description equation technique to calculate the terminal state dispersions induced by dynamics model errors, navigation errors, and actuation errors.

When the relative distance is short, the state transition matrix based on linear dynamics equations can be directly used to propagate the error covariance matrix and maintain good precision. This method is referred to as the linear covariance

(LinCov). Gossner[121] used the LinCov to analyze the effect of maneuver errors, Geller[122] extended the LinCov to the closed-loop process of guidance, navigation, and control, while Wodffinden and Geller[86] further extended the LinCov to missions with angle-only measurement.

Recently, trajectory robustness performance has been considered in rendezvous trajectory optimization. Tang et al.[123] defined a robustness objective based on the LinCov, and then employed a multi-objective genetic algorithm to optimize trajectories considering robustness. Li et al.[89] extended this method to rendezvous problems under closed-loop control, Luo et al.[124] extended this method from a linearized model to a nonlinear model, and Zhang et al.[77] employed this robustness index to the optimization of a multi-phase rendezvous mission.

8. Conclusions and prospects

As mentioned above, studies on RODC have achieved great successes and many valuable results. As the development of the space industry, RVD has a wide range of demands in assembly and operation of large spacecraft, on-orbit service, and deep-space exploration. The control manner of RVD has been moving from manual or automated control towards autonomous control, and mission background is progressively varying from low Earth orbits to high Earth orbits such as geostationary orbits,[125,126] and even to deep space.[127] Moreover, the docking orbit needs to allow a highly elliptical one. Under the new background of RVD missions, RODC still has many theoretical and technological problems for further studies.

8.1. Relative dynamics equations

For Earth-orbit missions with high eccentricities or long durations or interplanetary missions with three-body gravity effect, existing relative dynamics equations have considerable limitations in the precision and the convenience of maneuver calculation.

For instance, the STM given by Yamanaka and Ankersen[22] has been widely used by theoretical studies on the guidance and control of elliptical rendezvous missions. However, during practical applications, the relative state should be propagated as time, and then the Kepler's equation needs to be solved in every time step. This would bring several difficulties. Firstly, the computation cost of the guidance law based on the STM is much higher than that based on the C–W equations. Secondly, more absolute state information of the target spacecraft is required during the guidance based on the STM, rather than only the mean orbital angular rate during the guidance based on the C–W equations, and therefore absolute navigation is needed for an elliptical rendezvous mission from the point view of relative dynamics equations.

It will play a fundamental role in these RVD missions to derive new relative dynamics equations or new solution formulations which can overcome these limitations.

8.2. Rendezvous trajectory optimization

For the optimization of rendezvous trajectories, some studies have started to employ multi-objective optimization algorithms to optimize propellant, time of flight, trajectory safety index, and trajectory robustness index at the same time.[76,89,116,117,123,124]

New propulsion manners, such as the solar electric propulsion and the nuclear electric propulsion, bring new requirements for low-thrust rendezvous trajectory optimization and make it more difficult to solve.[68–70]

Multi-spacecraft service and multi-asteroid exploration have been the focus of future space missions, so approaches to optimize the rendezvous sequence and the trajectory connecting each mission at the same time would be very valuable.[35,128,129] When the rendezvous sequence (permutation integer number) and the transfer time or burn impulse (continuous number) are both used as design variables, the rendezvous trajectory optimization problem becomes a mixed integer nonlinear programming problem which is typically much more difficult to solve than both the nonlinear programming and mixed integer linear programming problems.

On the other hand, for practical engineering applications, studies on simple approaches and fast algorithms for rendezvous trajectory optimization would help improve the autonomy and reliability of spacecraft.

8.3. Relative navigation

During RVD missions, optical imaging measurement and satellite navigation are becoming the major relative measuring manners. Methods to improve measuring efficiency and reliability, to extend maximum measuring distance, and to solve relative states from measuring data reliably are important to practical missions. Navigation approaches with strong anti-disturbance are also of great significance. On the other hand, topics for non-cooperative rendezvous navigation, such as information fusion, active maneuvers to improve observability, and relative measurement for tumbling or maneuvering target spacecraft, deserve further studies.[87,88]

8.4. Close-range control

Most of the control methods proposed in existing studies are much more theoretical, and cannot be directly applied to engineering missions, thus further studies should take much more practical constraints into considerations. Furthermore, current studies mainly focus on low Earth circular orbits, but future new RVD missions, for example, a RVD mission in a geostationary orbit, an asteroid rendezvous mission using low-thrust propulsion, and an autonomous RVD mission around Mars orbit, will bring lots of new requirements on proximity operations control which need further studies.

Acknowledgements

This study was co-supported by the National Natural Science Foundation of China (Nos. 10902121 and 11222215), National Basic Research Program of China (No. 2013CB733100), and the Foundation for the Author of National Excellent Doctoral Dissertation of China (No.201171).

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