Problem 1.   Trapped Particle Radiation

Assume two spacecraft orbit Earth, one at an altitude of 3000km, the other at 20,000km.

Part (a)

Remind yourself — are these orbits above or below geostationary?

> A geostationary orbit (GEO) is a circular geosynchronous orbit in the plane of the Earth's equator with a radius of approximately 42,164 km (measured from the center of the Earth). A satellite in such an orbit is at an altitude of approximately 35,786 km above mean sea level[1].

As such, both of these satellites are below geostationary.

Part (b)

Assuming they are both shielded with 4mm aluminum and have the same electronics, which spacecraft is more likely to suffer Single-Event Upsets? Why?

The most likely place for a satellite to experience a Single-Event Upset is within the Van Allen radiation belts, and one of the safest regions for satellites to be is within the inner and out belt. The inner belt extends from 1,000 to 6,000 kilometers above the Earth, and the outer belt extends from 13,000 to 60,000 kilometers above the Earth's surface.

The inner belt contains high concentrations of electrons in the range of hundreds of keV and energetic protons with energies exceeding 100 MeV, trapped by the relatively strong magnetic fields in the region[2]. Due to this large flux, the satellite in the 3,000km altitude would be more likely to suffer Single-Event Upsets.

Problem 2.   Upper Atmosphere Physics

Part (a)

What is the change in density of the exosphere at Hubble Space Telescope (HST) altitude between the date of its fourth Shuttle re-boost (Servicing Mission 3B) and the date of your new spacecrafts re-boost?

The HST is currently orbiting with a Perigee 551.4 km and an Apogee of 555.6 km[3]. Servicing Mission 3B took place in March 2002[4], which was during a solar maximum (see Figure 1). The date of our proposed mission is January 2020, and the next solar cycle is expected to be "one of the weakest in centures"[5], and should peak around 2022. While it's

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[1] https://en.wikipedia.org/wiki/Geosynchronous_orbit

[2] Gusev, A. A.; Pugacheva, G. I.; Jayanthi, U. B.; Schuch, N. (2003). "Modeling of Low-altitude Quasi-trapped Proton Fluxes at the Equatorial Inner Magnetosphere". Brazilian Journal of Physics 33 (4): 775781. Bibcode:2003BrJPh..33..775G.

[3] "HST Satellite details 1990-037B NORAD 20580". N2YO. January 27, 2015. Retrieved January 27, 2015.

[4] https://en.wikipedia.org/wiki/STS-109

[5] http://science.nasa.gov/science-news/science-at-nasa/2006/10may_longrange/
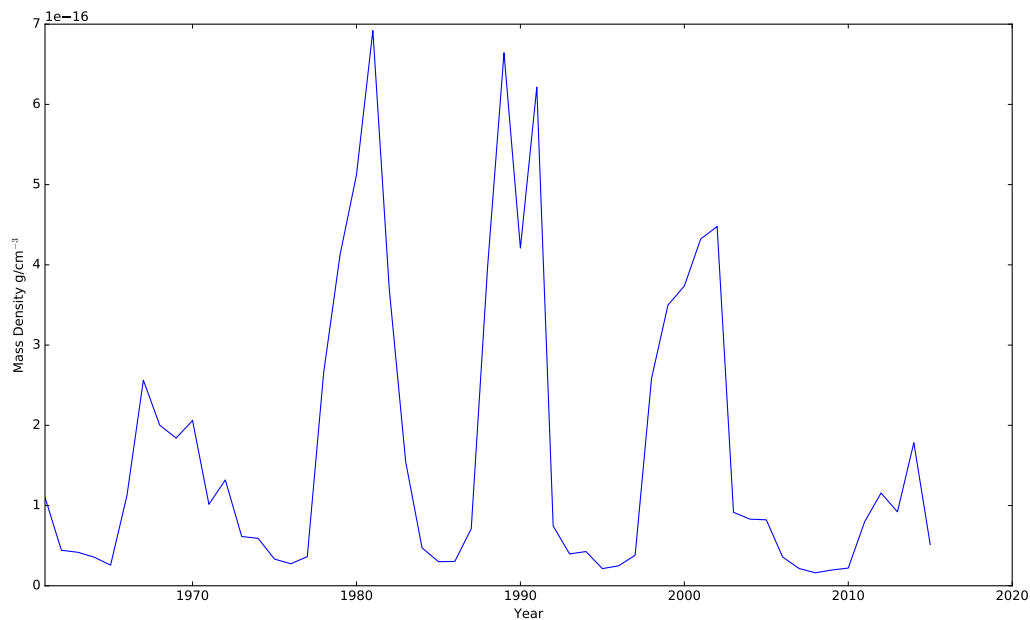
Figure 1: Mass density of exosphere from 1961-2015.

impossible to know what the density of the exosphere will be in 2020, we should expect the density to be 50% to 100% of what is was during Servicing Mission 3B.

Part (b)

Using the NASA NRLMSISE-100 atmosphere model, list the following parameters for todays date and the HST altitude:

  (a) Atomic oxygen species density

  (b) Atmosphere mass density

  (c) Exosphere temperature

The most recent data available is from September 2015. The information for an altitude of 553km is:

  (a) Atomic oxygen species density, $1.743\text{E}+06 \ \text{cm}^{-3}$

  (b) Atmosphere mass density, $5.419\text{E-}17 \ \text{g/cm}^{-3}$

  (c) Exosphere temperature, 810 K

Part (c)

What role do each of these environmental parameters play in the mission design of your spacecraft?

Problem 3.   Orbital Debris and Ballistic Limit Equations (BLE)

Part (a)

Using eqn 2-3 of the NASA MMOD handbook and Fig 2-4 of the NASA Orbital Debris Engineering Model (both in MMOD folder on SmartSite), determine the critical particle diameter for a Probability of No Penetration (PNP) of 0.877. Assume $1m^2$ exposed area and a two-week mission.

$$PNP = exp\left[-\sum_{i=1}^{n}(FAt)_i\right]$$

over n spacecraft regions, where F (number/m²-year), of meteoroid and debris impacts that exceed the failure limits (or ballistic limits), the exposed area, A (m²), and duration or time exposed to the MMOD flux, t (year). Plugging in the given values, we have

$$PNP = exp\left[-\sum_{i=1}^{n}(FAt)_i\right]$$
$$PNP = exp\left[-(FAt)\right]$$
$$FAt = ln(\frac{1}{PNP})$$
$$F = ln(\frac{1}{PNP})\frac{1}{At}$$
$$F = ln(\frac{1}{0.877})\frac{1}{(1)(2*7*24*60*60)}$$
$$F = 1.09e-07\ [1/m^2/s]$$

Looking at Figure 2-4, we find a diameter of $\approx 0.1mm$.

Part (b)

What would the critical particle diameter be for the same PNP if your spacecraft stayed attached to HST for a second re-boost in 2 years?

$$F = ln(\frac{1}{PNP})\frac{1}{At}$$
$$F = ln(\frac{1}{0.877})\frac{1}{(1)(2*365*24*60*60)}$$
$$F = 2.08e-09\ [1/m^2/s]$$

Looking at Figure 2-4, we find a diameter of $\approx 0.2mm$.

Part (c)

Consider a perpendicular impact of a particle of the size found in part a and another particle with 1mm diameter. Use the Design Equations 4-1 and 4-2 to compute the penetration depth for:

(a) Silicate particle at 0.5g/cm^3 and 23 km/s (micro-meteroid)

(b) Steel particle at 7.8 g/cm^3 and 7 km/s (orbital debris)

With target materials of:

(a) 6061-T6 Alum

　　(i) BHN: 95

　　(ii) C_t: 5.433 km/s

　　(iii) Density: 2.7 g/cm^3

(b) 7075-T6 Alum

　　(i) BHN: 150

　　(ii) C_t: 6.350 km/s

　　(iii) Density: 2.81 g/cm^3

For $\dfrac{\rho_p}{\rho_t} < 1.5$,

$$P_\infty = 5.24 d^{(19/18)} BHN^{-0.25} \left(\frac{\rho_p}{\rho_t}\right)^{0.5} \left(\frac{V\cos\theta}{C_t}\right)^{2/3}$$

For $\dfrac{\rho_p}{\rho_t} \geq 1.5$,

$$P_\infty = 5.24 d^{(19/18)} BHN^{-0.25} \left(\frac{\rho_p}{\rho_t}\right)^{2/3} \left(\frac{V\cos\theta}{C_t}\right)^{2/3}$$

	Micro-meteroid	Orbital Debris
6061	0.166	0.355
7075	0.131	0.278

Table 1: P_∞ (cm) for combinations of particles and target materials

Part (d)

No protection case: Use Performance Equation 4-6 to estimate the required spacecraft wall thickness to avoid detached spall due to impact of the 1mm particle for the four material cases in part c.

$$d_c = \left[\frac{t}{k}\frac{BHN^{0.25}\left(\rho_t/\rho_p\right)^{0.5}}{5.24\left(V\cos\theta/C_t\right)^{2/3}}\right]^{18/19} \implies t = \left[\frac{d_c^{19/18}}{k}\frac{BHN^{0.25}\left(\rho_t/\rho_p\right)^{0.5}}{5.24\left(V\cos\theta/C_t\right)^{2/3}}\right]^{-1}$$

To avoid detached spall with an incidence angle of $\theta = 0$, this simplifies to:

$$t = \left[\frac{d_c^{19/18}}{11.528}\frac{\left(\rho_t/\rho_p\right)^{0.5}}{\left(V/C_t\right)^{2/3}}BHN^{0.25}\right]^{-1}$$

	Micro-meteroid	Orbital Debris
6061	255.192	29.234
7075	209.310	23.978

Table 2: t (cm) for combinations of particles and target materials

Part (e)

Compare your answers in part d to equation 4-4.

The shielding thickness will prevent detached spall, given that:

$$t \leq 2.2P_\infty$$

After investigating our four cases, we find

	Micro-meteroid	Orbital Debris
6061	False	False
7075	False	False

Table 3: Results for combinations of particles and target materials

Part (f)

Whipple Shield design: For the two 1mm particle compositions/velocities in part c, use equations 4-21 and 4-22 to estimate the bumper and rear-wall thickness required to defeat the threat particles. Assume

(a) Particles are spherical

(b) Bumper standoff $= 10.2$cm (Fig 4-1)

(c) Rear wall: 2219-T87 Alum, 0.5cm thickness ($\sigma = 57000$ psi $= 57$ksi)

(d) Perpendicular velocity impact ($\theta = 0$)

$$t_b = c_b d \rho_p / \rho_b$$
$$t_w = c_w d^{0.5} \left(\rho_p \rho_b\right)^{1/6} \left(M_p\right)^{1/3} V_n / S^{0.5} \left(70/\sigma\right)^{0.5}$$

where c_b depends on the ratio of $S/d = 10.2cm/0.1cm = 102 \implies c_b = 0.2$. Since the particles are spherical, $M_p = \rho_p V_p = \frac{4}{3}\pi \rho_p r_p^3$.

	Micro-meteoroid	Orbital Debris
6061	0.004	0.058
7075	0.004	0.056

Table 4: t_b (cm) for combinations of particles and target materials

	Micro-meteoroid	Orbital Debris
6061	0.027	0.033
7075	0.027	0.033

Table 5: t_w (cm) for combinations of particles and target materials

Part (g)

Estimate the protection capability limits for your Whipple Shields: Compute the critical projectile performance diameter using equations 4-23, 4-24, and 4-25 for various relative velocities, for two types of spherical particle, one silicate, the other steel. Assume the same parameters as for part f.

Problem 4.   Acoustic Shielding

Part (a)

Download and install a sound-level app on your smartphone.

Part (b)

Measure the sound pressure level (dB) in some noisy environment (home stereo? EFL?).

Part (c)

Remind yourself: what is the definition of the Decibel, and how is the reference soundpressure level defined?

Part (d)

Use a Styrofoam cup (or similar) as a payload shroud for your phone. Plug the open end of the shroud with isolation (tissues?). Measure the change in dB sensed by your phone.

Part (e)

Add some kind of insulation to the inside walls of your payload shroud and repeat part d.

Part (f)

Discuss results.

Problem 5.   Numerical Integration Review

Consider the first-order initial-value problem: $\dfrac{dy}{dt} = t + y$, $y(0) = 0$ with exact solution $y(t) = e^t - t - 1$

Part (a)

Program Euler and Runge-Kutta solvers (write your own) and plot the results over a range t=0 to 1.0, with step sizes h=0.01, 0.1, and 0.5. Plot results (y vs t) and compare to the exact solution.

Part (b)

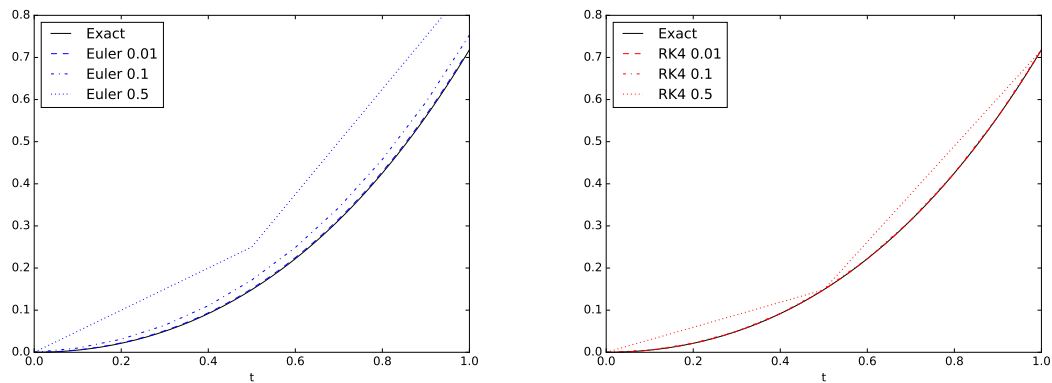Repeat step a with library functions for Euler and RK4
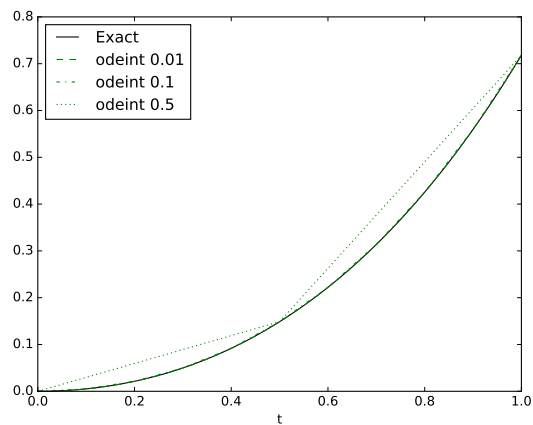
Figure 2: Results from Euler (left) and RK4 (right) solvers.

Figure 3: Results from scipy's RK4 solver.