Lecture 10: Simple Effects (cont. of Factorial ANOVA)

• 10. 1 Simple Effects

- Given an interaction one may want to examine whether an effect exists for all the levels of the other factor (and vice-versa)

- These tests are called ***simple effects*** (from Lecture 9's example):

    - Is there an "experience" effect for each level of method?

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- Table of Means

	L	*D*	*S*	*Row Means*
N	8	5	2	$\overline{X}_N = 5$
Y	6	9	6	$\overline{X}_Y = 7$
Column Means	$\overline{X}_L = 7$	$\overline{X}_D = 7$	$\overline{X}_S = 4$	$\overline{X}_{.} = 6$

- ANOVA Source Table

Source	*SS*	*df*	*MS*	*F*	*Sig.*
Experience	30	1	30	6	<.05
Method	60	2	30	6	<.01
Interaction	60	2	30	6	<.01
Within	120	24	5		
Total	270	29			

• 10. 2 Calculations

- Simple Effects for "Experience"

  - Row effects (variability due to the rows and variability due to the interaction)

$SS_R + SS_I = \Sigma_k SS_{R\ at\ k} = n\Sigma_k\Sigma_j (X_{jk} - X._k)^2 + \ldots +$

$= n\Sigma_j (X_{j1} - X._1)^2 + n\Sigma_j (X_{j2} - X._2)^2 + n\Sigma_j (X_{j3} - X._3)^2 + \ldots + n\Sigma_j (X_{jp} - X._p)^2$

SS_R(at lecture) = $5((8–7)^2 + (6–7)^2) = 10$; $df = j - 1 = 1$

SS_R(at discuss) = $5((5–7)^2 + (9–7)^2) = 40$; $df = j - 1 = 1$

SS_R(at study) = $5((2–4)^2 + (6–4)^2) = 40$; $df = j - 1 = 1$

$\therefore SS_{R\ at\ k} = 90$

  - Additivity of simple effects

$SS_R + SS_I = 30 + 60 = 90 = SS_{R\ at\ k}$

$df_R + df_I = 1 + 2 = 3 = df_{R\ at\ k}$

- Simple Effects F – Tests for "Experience"

  - Lecture

$$F = \frac{MS_{R(lecture)}}{MS_W} = \frac{10}{5} = 2, p > .05$$

  - Discussion

$$F = \frac{MS_{R(discussion)}}{MS_W} = \frac{40}{5} = 8, p < .01$$

  - Study

$$F = \frac{MS_{R(study)}}{MS_W} = \frac{40}{5} = 8, p < .01$$

- Simple Effects for "Method"

  - Column effects (variability due to the columns and variability due to the interaction)

  $SS_C + SS_I = \Sigma_j SS_{C\,at\,j} = n\Sigma_j\Sigma_k (X_{jk} - X._j)^2$

  $= n\Sigma_k (X_{k1} - X._1)^2 + n\Sigma_k (X_{k2} - X._2)^2$

  SS_C(no experience) $= 5((8–5)^2 + (5–5)^2 + (2–5)^2) = 5(9 + 0 + 9) = 90$

  $df_{c(\text{no exp})} = df = k - 1 = 2$

  SS_C(yes experience) $= 5((6–7)^2 + (9–7)^2 + (6–7)^2) = 5(1 + 4 + 1) = 30$

  $df_{c(\text{exp})} = df = k - 1 = 2$

  $\therefore SS_{\text{C at j}} = 120$

  - Additivity of simple effects

    $SS_C + SS_I = 60 + 60 = 120 = SS_{\text{C at j}}$

    $Df_C + df_I = 2 + 2 = 4 = df_{\text{C at k}}$

- Simple Effects F – Tests for "Method"

  - No Experience

  $$F = \frac{MS_{C(no\,\text{exp})}}{MS_W} = \frac{45}{5} = 9,\ p < .01$$

  - Yes Experience

  $$F = \frac{MS_{R(\text{exp})}}{MS_W} = \frac{15}{5} = 3,\ p > .05$$

- Simple Effects Table

Source	*SS*	*df*	*MS*	*F*	*Sig.*
Experience					
Exp at lecture	10	1	10	2	>.05
Exp at discuss	40	1	40	8	<.01
Exp at study	40	1	40	8	<.01
Method					
M at no exp	90	2	45	9	<.01
M at yes exp	30	2	15	3	>.05
Within	120	24	5		

• 10. 3 Follow up and Interpretation

- Interpretation:

- *Experience*: Differences in performance between both levels of experience occur for both discussion and study, but not for lecture (the sum of squares are ¼ than those for the other methods).

- Because there are only two groups in each test, we would simply look at the means to determine where the differences are.

- Individuals with experience perform better than those without experience when attending either discussion or study sessions. There are no differences between both groups for those individuals attending lectures.

- *Method*: Differences in performance with regard to the methods (i.e., effect due to the type of method) exist but only for those individuals with no experience (the sum of squares are three times larger than those for the "experience" group).

- Because this is an omnibus test, it is not clear which of the three groups are different.

- An inspection of the means suggest that the decrease in performance is linear from lecture to discussion to study. But this needs to be formally tested, for example, with orthogonal contrasts.

- Orthogonal contrasts (for the *method at no-experience* simple effects test)

	$\overline{X}_{lecture}$	$\overline{X}_{discussion}$	$\overline{X}_{study}$	Σ
c_1	1	-1	0	0
c_2	1/2	1/2	-1	0
$c_j c'_j$	1/2	-1/2	0	0

$\Psi_1 = (1 \text{ x } 8) + (-1 \text{ x } 5) + (0 \text{ x } 2) = 3$

$\Psi_2 = (.5 \text{ x } 8) + (.5 \text{ x } 5) + (-1 \text{ x } 2) = 4.5$

- 1st Contrast

$\Psi_1 = (1 \text{ x } 8) + (-1 \text{ x } 5) + (0 \text{ x } 2) = 3$

$$t = \frac{\Psi}{s_\Psi}$$

$$s_\Psi = \sqrt{MS_W\left(\frac{c_1^{\,2}}{n_1} + \frac{c_2^{\,2}}{n_2} + \ldots \frac{c_j^{\,2}}{n_j}\right)} = \sqrt{MS_W\left(\frac{\Sigma c_j^{\,2}}{n_j}\right)}$$

$$est.Var(\hat{\psi}) = 5\frac{1^2 + (-1^2)}{5} = 2$$

$$est.SE(\hat{\psi}) = \sqrt{2} = 1.414$$

$$t = \frac{\hat{\psi}}{\sqrt{est.Var(\hat{\psi})}} = \frac{3}{1.414} = 2.12$$

$df = N - J;\ 15 - 3 = 12$

$t_{(crit,\ \alpha = .05,\ \mathrm{df} = 27)} = 2.179$

$t_{(12)} = 2.12 < t_{crit}$, $p > .05$

95% CI = 3$\pm$ 2.179 (1.414) = 3$\pm$ 3.081 = (-.081, 6.081)

- 2nd Contrast

$\Psi_2 = (.5 \text{ x } 8) + (.5 \text{ x } 5) + (-1 \text{ x } 2) = 4.5$

$$est.Var(\hat{\psi}) = 5\frac{.5^2 + .5^2 + (-1^2)}{5} = 1.5$$

$$est.SE(\hat{\psi}) = \sqrt{2} = 1.225$$

$$t = \frac{\hat{\psi}}{\sqrt{est.Var(\hat{\psi})}} = \frac{4.5}{1.225} = 3.673$$

$t_{(27)} = 3.67 > t_{crit}$, $p < .05$

95% CI = 4.5 $\pm$ 2.179 (1.225) = 4.5 $\pm$ 2.669 = (1.83, 7.17)