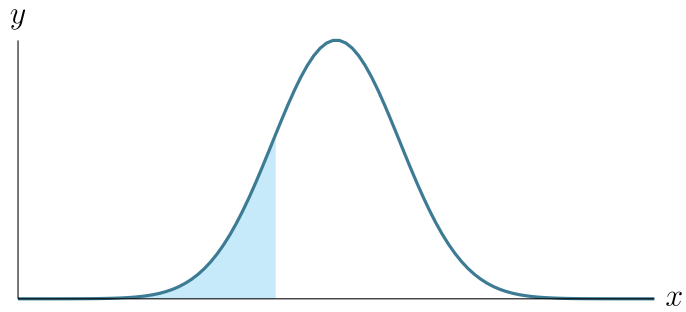
Problem 1.

Part (a)

What is the probability of having a z-value less than -0.95?

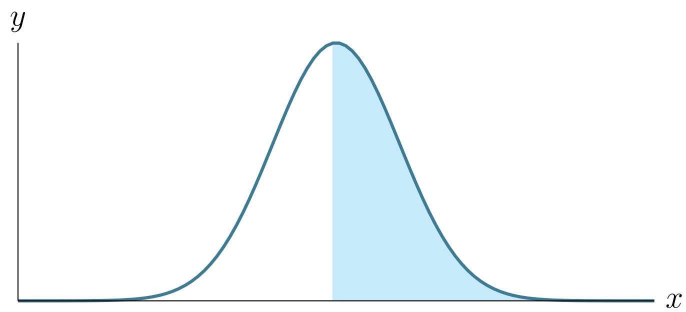
$$p(z < -0.95) = 0.1711$$



Part (b)

What is the probability of having a z-value greater than -0.06?

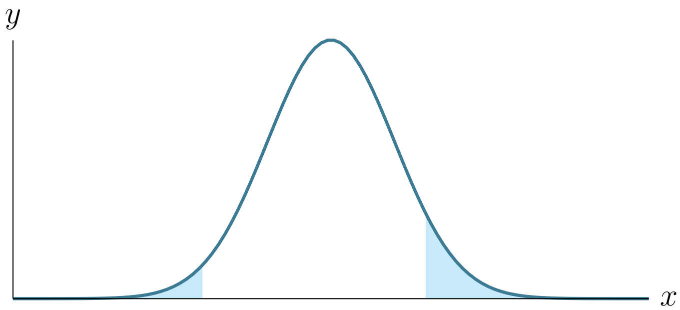
$$\begin{aligned} p(z > -0.06) &= 1 - p(z < -0.06) \\ &= 1 - 0.4761 = 0.5239 \end{aligned}$$



Part (c)

What is the probability of having a z-value less than -3.02 or greater than 1.49?

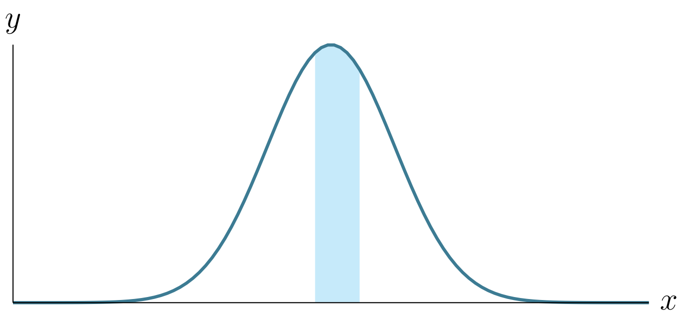
$$\begin{aligned} p(z < -3.02) + p(z > 1.49) &= p(z < -3.02) + [1 - p(z < 1.49)] \\ &= 0.0013 + [1 - 0.9319] \\ &= 0.0694 \end{aligned}$$



Part (d)

What is the probability of having a z-value between -0.25 and 0.45?

$$\begin{aligned} p(-0.25 < z < 0.45) &= p(z > -0.25) - p(z > 0.45) \\ &= [1 - p(z < -0.25)] - [1 - p(z < 0.45)] \\ &= [1 - 0.4013] - [1 - 0.6736] \\ &= 0.2723 \end{aligned}$$



Problem 2.

Part (a)

What are the critical t-values for a two-tailed t-test given an alpha of .05 and df of 1?

12.71

Part (b)

What is the critical t-value for a right-tailed t-test given an alpha of .05 and df of 10?

1.812

Part (c)

What is the critical t-value for a left-tailed t-test given an alpha of .01 and df of 100?

$$2.626 - 2.364 = 0.262$$

Part (d)

What are the critical t-values for a two-tailed t-test given an alpha of .05 and df of 4, 9, 14, 29, 99, and 1000? What is the absolute difference between each of these values and the critical z-values for a corresponding two-tailed z-test given an alpha of .05?

df	t-value	diff
4	2.776	0.816
9	2.262	0.302
14	2.145	0.185
29	2.045	0.085
99	1.984	0.024
1000	1.962	0.002
z	1.960	-

Problem 3.

Part (a)

Given an alpha of .05 what is the minimum sample size required to reject the null hypothesis using a two-tailed t-test given a t-value of 1.995?

80

Part (b)

Given an alpha of .05 what is the minimum sample size required to reject the null hypothesis using a one-tailed t-test given a t-value of -1.70?

29

Part (c)

Given an alpha of .01, what is the minimum sample size required to reject the null hypothesis using a two-tailed t-test given a t-value of -2.98?

13

For intermediary calculations (e.g., mean and sd) round to four decimal places. Round your final answer to two decimal places.

Problem 4.

Imagine that the common house fly lives for an average (mu) of 21 days. After some selective breeding, you have a small sample (N = 10) of specially bred flies that you think had longer than average lives compared to the common fly. Use the data below to test this. Report your conclusion and explain what this means.

Lifespan of Mutant flies (days) = {27, 25, 20, 25, 23, 21, 27, 25, 25, 22}

To solve this problem, we define two hypotheses: $H_0 : \mu = 21$, $H_1 : \mu \neq 21$. Alpha level is chosen as $\alpha = .05$. Results will be significant if the sample mean falls in either extreme .025 of all possible results. $\nu = N - 1 = 9$; $Q = .025$; $t = 2.262$. If obtained $t > |2.262|$, reject H_0.

$$
\begin{aligned}
\overline{x} &= \frac{27 + 25 + 20 + 25 + 23 + 21 + 27 + 25 + 25 + 22}{10} \\
&= 24.0000 \\
s^2 &= \frac{1}{N-1} \sum_{i=1}^{N} (x_i - \overline{x})^2 \\
&= \frac{1}{9}[(27-24)^2 + (25-24)^2 + (20-24)^2 + (25-24)^2 + (23-24)^2 \\
&+ (21-24)^2 + (27-24)^2 + (25-24)^2 + (25-24)^2 + (22-24)^2] \\
&= 5.7778 \\
s &= \sqrt{5.7778} = 2.4037 \\
t &= \frac{[\overline{x} - \mu]}{s/\sqrt{N}} \\
&= \frac{24 - 21}{2.4037/\sqrt{10}} \\
&= 12.4808 \\
SE &= \frac{s}{\sqrt{N}} = \frac{2.4037}{\sqrt{10}} = 0.7601 \\
\text{Upper } 95\% &= \bar{x} + (SE \times 1.96) = 25.4898 \\
\text{Lower } 95\% &= \bar{x} - (SE \times 1.96) = 22.5102
\end{aligned}
$$

The flies in our sample live an average of 24.0000 ± 0.7601 days. Since $t = 12.4808 > 2.365$, we reject the null hypothesis H_0. We conclude that the specially bred flies have longer than average lives compared to the common fly.

Problem 5.

Now imagine that, in addition to your immortal fly project, you have another selective breeding program where you are trying to breed flies that have much shorter life spans. You want to see if they are dying faster than your flies bred to have a longer life span. Use the data below to test this. Report your conclusion and explain what this means.

Lifespan of Mutant flies (days) = {27, 25, 20, 25, 23, 21, 27, 25, 25, 22}

Lifespan of Short-lived flies (days) = {24, 23, 19, 21, 22, 20, 25, 27, 21, 22}

To solve this problem, we define two hypotheses: $H_0 : \mu_m - \mu_{sl} = 0$, $H_1 : \mu_m - \mu_{sl} > 0$. Alpha level is chosen as $\alpha = .05$. Results will be significant if the sample mean falls in positive extreme .025 of all possible results. $\nu = N_1 + N_2 - 2 = 18$; $Q = .025$; $t = 2.101$. If obtained $t > |2.101|$, reject H_0.

From above, we know that $\overline{x}_m = 24.0000$ and $s_m = 2.4037$.

$$
\begin{aligned}
\overline{x}_{sl} &= \frac{24 + 23 + 19 + 21 + 22 + 20 + 25 + 27 + 21 + 22}{10} \\
&= 22.4000 \\
s_{sl}^2 &= \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2 \\
&= \frac{1}{9}[(24-22.4)^2 + (23-22.4)^2 + (19-22.4)^2 + (21-22.4)^2 + (22-22.4)^2 \\
&+ (20-22.4)^2 + (25-22.4)^2 + (27-22.4)^2 + (21-22.4)^2 + (22-22.4)^2] \\
&= 5.8222 \\
s_{sl} &= \sqrt{5.8222} = 2.4129 \\
\text{est.}\sigma_{diff} &= \sqrt{\frac{(n_m - 1)s_m^2 + (n_{sl} - 1)s_{sl}^2}{n_m + n_{sl} - 2}\left(\frac{n_m + n_{sl}}{n_m n_{sl}}\right)} \\
&= \sqrt{\frac{(10-1)5.7778 + (10-1)5.8222}{10 + 10 - 2}\left(\frac{10+10}{10 * 10}\right)} \\
&= 1.0770 \\
t &= \frac{(\overline{x}_m - \overline{x}_{sl})}{\text{est.}\sigma_{diff}} \\
&= \frac{24 - 22.4}{1.0770} \\
&= 1.4856 \\
95\% \text{ CI} &= (\overline{x}_m - \overline{x}_{sl}) \pm t_{(\alpha/2;\nu)}(\text{est.}\sigma_{diff}) \\
&= 1.6 \pm (2.101)(1.0770) = (-0.6628, 3.8628)
\end{aligned}
$$

While $\overline{x}_{sl} < \overline{x}_m$, this is not significant as $t = 1.4856 < 2.101$, given $\alpha = .05$, H_0 cannot be rejected. The p is .95 that the true difference between $\mu_m - \mu_{sl}$ is contained in this 95% CI interval.

Problem 6.

It just so happens that you recall an important detail about these fly experiments. It turns out that, to control for environment, the flies were assigned to live in the same cages for the duration of their lives. That is Fly 1 of the Mutant Flies and Short-lived flies lived in the same enclosure, as did Fly 2 of the Mutant Flies and Short-lived Flies, and so on. This suggests that each pair of flies had some common factors and that the samples are not actually independent. Given this information, conduct a second test of differences between the two samples. Report your conclusion and explain what this means.

Lifespan of Mutant flies (days) = 27, 25, 20, 25, 23, 21, 27, 25, 25, 22

Lifespan of Short-lived flies (days) = 24, 23, 19, 21, 22, 20, 25, 27, 21, 22

n	x_m	x_{sl}	x_d	$(x_d - \overline{x}_d)^2$
1	27	24	3	1.9600
2	25	23	2	0.1600
3	20	19	1	0.3600
4	25	21	4	5.7600
5	23	22	1	0.3600
6	21	20	1	0.3600
7	27	25	2	0.1600
8	25	27	-2	12.9600
9	25	21	4	5.7600
10	22	22	0	2.5600
	$\overline{x}_m = 24.0000$	$\overline{x}_{sl} = 22.4000$	$\sum x_d = 16.0000$	$\sum(x_d - \overline{x}_d)^2 = 3.0400$
	$(s_m = 2.4037)$	$(s_{sl} = 2.4129)$	$\overline{x}_d = 1.6000$	

To solve this problem, we define two hypotheses: $H_0 : \mu_m - \mu_{sl} = \mu_d = 0$, $H_1 : \mu_m - \mu_{sl} > 0$. Alpha level is chosen as $\alpha = .05$. $\nu = N - 1 = 9$; $Q = .025$; $t = 2.262$. If obtained $t >|2.262|$, reject H_0.

$$
\begin{aligned}
s_d^2 &= \frac{\sum(x_d - \overline{x}_d)^2}{n-1} = \frac{3.0400}{9} = 0.3378 \\
s_d &= 0.5812 \\
s_{\overline{d}} &= \frac{s_d}{\sqrt{n}} = \frac{0.5812}{\sqrt{10}} = 0.1838 \\
t &= \frac{\overline{x}_d - \mu_d}{s_{\overline{d}}} = \frac{\overline{x}_d}{s_{\overline{d}}} = \frac{1.6000}{0.1838} = 8.7051 \\
95\%\ \text{CI} &= (\overline{x}_m - \overline{x}_{sl}) \pm t_{(\alpha/2;\nu)} s_{\overline{d}} \\
&= 1.6000 \pm (2.262)(0.1838) = (1.1842, 2.0158)
\end{aligned}
$$

Since $t = 8.7051 > 2.365$, we reject the null hypothesis H_0. We conclude that, $\mu_{sl} < \mu_m$, the short-lived flies have shorter than average lives compared to the mutant flies.