Lecture 11: Repeated Measures ANOVA

• 11. 1 Purpose

- A special case of ANOVA when assessments are repeated over time

- Also called *Profile Analysis* because it analyzes a profile of scores across the levels of a repeated-measures factor

- DVs are same variables over time (using same scaling)

• 11. 2 Research Questions

- Groups Hypothesis
    - do groups score similarly on the collected set of measures?
    - main effect for the (between-subjects) grouping factor

- Flatness Hypothesis
    - is the DV similar across assessments?
    - main effect for the repeated-measures factor
    - the optimal linear combination of orthogonal contrasts that capture the profile of scores

- Parallelism Hypothesis (If grouping variable)
    - do different groups have parallel profiles?
    - interaction between the grouping and the repeated measures factor

• 11. 3 Practical Issues

- Robust to departures of multivariate normality

- Very sensitive to outliers

- Homogeneity of variance-covariance matrices is not a problem unless sample sizes are highly unequal

- Linearity of relationships among DVs is assumed

- **11. 4 Data**

timepoint

group	*subject*	1	2	…	*p*
1	1	y_{111}	y_{121}	…	$y_{1\,p1}$
1	2	y_{211}	y_{221}	…	$y_{2\,p1}$
1	.	.	.	…	.
1	n_1	$y_{n1\,11}$	$y_{n1\,21}$	…	$y_{n1\,p1}$
g	1	y_{11g}	y_{12g}	…	y_{1pg}
g	2	y_{21g}	y_{22g}	…	y_{2pg}
g	.	.	.	…	.
g	n_g	y_{ng1g}	y_{ng2g}	…	y_{ngpg}

$i = 1,\ldots n_k$ subjects in group k (with $n = \Sigma_k n_k$)
$j = 1, \ldots p$ time points
$k = 1,\ldots g$ groups

- **11. 5 Model**

$Y_{ijk} = \mu + \alpha_k + \beta_j + \alpha\beta_{kj} + \varepsilon_{ijk}$

μ = grand mean

α_k = effect of group k ($\Sigma k\ \alpha_k = 0$)

β_j = effect of time j ($\Sigma j\ \beta_j = 0$)

$(\alpha\beta)_{kj}$ = interaction effect of time j and group k $[\Sigma_j \Sigma_k (\alpha\beta)_{kj} = 0]$

ε_{ijk} = error for subject i in group k at time j

$\varepsilon_{ijk} \sim N(0, \sigma_e^2)$

- ## 11. 6 Equations: Test of Levels

- Main effect for the (between-subjects) grouping factor

$$\Sigma_i \Sigma_k (Y_{ik} - \overline{Y}.)^2 = np\Sigma_k (\overline{Y}_k - \overline{Y}.)^2 + p\Sigma_i\Sigma_k (Y_{ik} - \overline{Y}_k)^2$$

$$df_b = k - 1$$

$$df_w = N - k$$

- ## 11. 7 Equations: Test of Flatness

- Hypothesis of GM of segments different from zero

$$SS_{bj} = \Sigma_j n \cdot (\overline{Y}_j - \overline{Y}.)^2$$

$$SS_{\mathrm{w}} = \Sigma_i\Sigma_j (Y_{ij} - \overline{Y}_j)^2$$

- Useful to introduce transformations (contrasts) to examine specific hypotheses (e.g., linear, quadratic, cubic)

- ## 11. 8 Equations: Test of Parallelism

- Hypotheses are segments (adjacent different scores)
- Equivalent to one-way ANOVA on the segments

$$SS_b = \Sigma_j n \cdot (\overline{Y}_j - \overline{Y}.)^2$$

$$SS_w = \Sigma_i\Sigma_j (Y_{ij} - \overline{Y}_j)^2$$

- if H_0 is rejected:
    - group curves across time are not parallel
    - group differences are not the same across time

- **11. 9 Example**

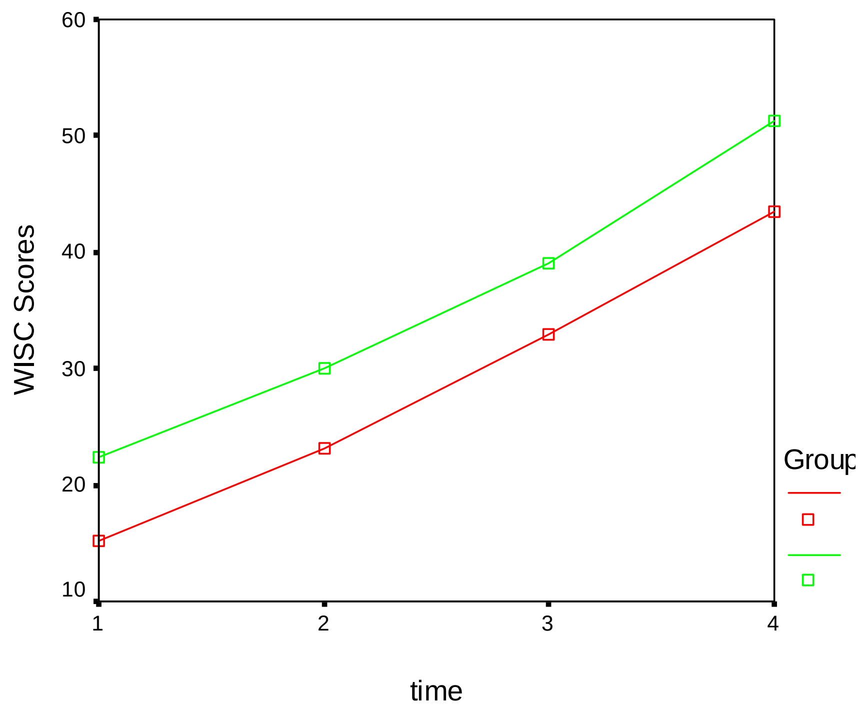
- Data are from 204 children on the WISC measured at four time points.

- Question 1. Are there differences in the WISC scores across measurement occasions? If so, what is the form of the trend?

- Question 2. Are there group differences in the trend?

SPSS Input File

```
MANOVA wisc1 to wisc4 BY group(1,2)
        /WSFACT = time(4)
        /PRINT = cellinfo(means) TRANSFORM homog (bart boxm) sign(univ) signif (hf gg hypoth eigen
        dimenr) error(sscp)
        /DISCRIM STAN COR
        /CONTR (time) = poly
        /RENAME = const lin quad cubic .
```



```
 Cell Means and Standard Deviations
 Variable .. TOTAL1
      FACTOR           CODE                  Mean  Std. Dev.          N

  GROUP                  1                 15.516      5.642        104
  GROUP                  2                 22.177      5.235        100
 For entire sample                         18.781      6.376        204

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 Variable .. TOTAL2
      FACTOR           CODE                  Mean  Std. Dev.          N

  GROUP                  1                 23.448      6.699        104
  GROUP                  2                 29.781      6.372        100
 For entire sample                         26.552      7.255        204

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 Variable .. TOTAL3
      FACTOR           CODE                  Mean  Std. Dev.          N

  GROUP                  1                 33.111      7.486        104
  GROUP                  2                 38.969      6.899        100
 For entire sample                         35.982      7.763        204

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 Variable .. TOTAL4
      FACTOR           CODE                  Mean  Std. Dev.          N

  GROUP                  1                 44.020     10.295        104
  GROUP                  2                 50.795      9.358        100
 For entire sample                         47.341     10.393        204
```

These are the means by measurement occasion and group level. They indicate an increase in WISC scores over time and differences between the two groups across all times. Such differences seem to be constant across the occasions (i.e., lack of interaction). But this visual inspection needs to be formally tested.

```
Univariate Homogeneity of Variance Tests

 Variable .. TOTAL1
       Bartlett-Box F(1,122284) =                  .56213, P =  .453

 Variable .. TOTAL2
       Bartlett-Box F(1,122284) =                  .25146, P =  .616

 Variable .. TOTAL3
       Bartlett-Box F(1,122284) =                  .66898, P =  .413

 Variable .. TOTAL4
       Bartlett-Box F(1,122284) =                  .91168, P =  .340
```

These are the Box's M tests of homogeneity of the var-cov matrices of the groups, indicating we are not violating the assumption of symmetry.

```
 Cell Number .. 1
 Determinant of Covariance matrix of dependent variables =     163388.00003
 LOG(Determinant) =                                                12.00388

Cell Number .. 2
 Determinant of Covariance matrix of dependent variables =     163273.39285
 LOG(Determinant) =                                                12.00318

Determinant of pooled Covariance matrix of dependent vars. =    169812.02490
 LOG(Determinant) =                                                 12.04245

 Multivariate test for Homogeneity of Dispersion matrices
 Boxs M =                          7.85947
 F WITH (10,194389) DF =            .76917, P =   .659 (Approx.)
 Chi-Square with 10 DF =           7.69207, P =   .659 (Approx.)
```

This test calculates the determinant of the var-cov matrix for each group and then compares them.

```
 Orthonormalized Transformation Matrix (Transposed)

                CONST        LIN       QUAD      CUBIC

 TOTAL1          .500      -.671       .500      -.224
 TOTAL2          .500      -.224      -.500       .671
 TOTAL3          .500       .224      -.500      -.671
 TOTAL4          .500       .671       .500       .224

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```

This is the transformation matrix. It contains orthogonal contrasts (as many as WS levels – 1) that serve as the dependent variables in the WS analysis (time in this example). The first one (CONST) is the contrast that SPSS uses for testing the BS effect. The others are linear, quadratic, and cubic contrasts because we specified to use POLYNOMIAL contrasts. But we can specify other SPSS contrasts, or write our own ones that represent specific hypotheses.

This part is the Between-Subjects test

```
 Order of Variables for Analysis

   Variates     Covariates
    CONST

    1 Dependent Variable
    0 Covariates

Tests of Between-Subjects Effects.

 Tests of Significance for CONST using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN CELLS             36277.62     202    179.59
 GROUP                     8370.78       1   8370.78     46.61      .000
```

If we average across trials, there are reliable differences in WISC scores between groups. Because there are only two groups, the interpretation is easy.

```
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 Estimates for CONST
 --- Individual univariate .9500 confidence intervals

 GROUP
  Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

        2   -6.4069509     .93845   -6.82715     .00000   -8.25737   -4.55653
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```

And this one is the Within-Subjects test.

```
 Order of Variables for Analysis
   Variates     Covariates

    LIN
    QUAD
    CUBIC

    3 Dependent Variables
    0 Covariates
```

These three DVs are really orthogonal contrasts (i.e., the weights are not correlated) of the original four levels of the TIME variable. This is a multivariate procedure that considers the 3 variables simultaneously.

```
Tests involving 'TIME' Within-Subject Effect.

 Mauchly sphericity test, W =      .63406
 Chi-square approx. =            91.45077 with 5 D. F.
 Significance =                      .000

 Greenhouse-Geisser Epsilon =      .74873
 Huynh-Feldt Epsilon =             .76131
 Lower-bound Epsilon =             .33333

AVERAGED Tests of Significance that follow multivariate tests are equivalent
to
univariate or split-plot or mixed-model approach to repeated measures.
Epsilons may be used to adjust d.f. for the AVERAGED results.
```

Mauchly test is significant, indicating sphericity (i.e., lack of homogeneity) in of the var-cov matrix of the transformed variables. This matrix is now decomposed into the **E** matrix and the **H** matrix (for the group by time and time tests). Then, we generate the eigenstructure of the $\mathbf{HE}^{-1}$ matrix, and this is evaluated for multivariate “significance.”

This is the **E** matrix.

```
 - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
 WITHIN CELLS Sum-of-Squares and Cross-Products

                  LIN       QUAD      CUBIC

 LIN         4779.577
 QUAD         858.060   1905.500
 CUBIC        541.908    146.697   1741.202

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```

And this is the **H** matrix for the TIME effect (flatness in T & F’s book).

```
EFFECT .. TIME
 Adjusted Hypothesis Sum-of-Squares and Cross-Products

                  LIN       QUAD      CUBIC

 LIN        92228.440
 QUAD        7804.877    660.492
 CUBIC        275.958     23.353       .826

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 Multivariate Tests of Significance (S = 1, M = 1/2, N = 99 )

 Test Name         Value    Exact F Hypoth. DF   Error DF  Sig. of F

 Pillais          .95329 1360.54208       3.00     200.00       .000
 Hotellings     20.40813 1360.54208       3.00     200.00       .000
 Wilks            .04671 1360.54208       3.00     200.00       .000
 Roys             .95329
 Note.. F statistics are exact.

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```

```
Eigenvalues and Canonical Correlations

Root No.    Eigenvalue        Pct.   Cum. Pct.  Canon Cor.

       1        20.408     100.000     100.000        .976

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EFFECT .. TIME (Cont.)
Univariate F-tests with (1,202) D. F.

Variable   Hypoth. SS   Error SS Hypoth. MS   Error MS          F  Sig. of F

LIN        92228.4398 4779.57675 92228.4398   23.66127 3897.86498       .000
QUAD        660.49161 1905.50001  660.49161    9.43317   70.01800       .000
CUBIC          .82570 1741.20241     .82570    8.61981     .09579       .757

EFFECT .. TIME (Cont.)
Standardized discriminant function coefficients
          Function No.

Variable           1
LIN            1.050
QUAD           -.154
CUBIC          -.180

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Correlations between DEPENDENT and canonical variables
          Canonical Variable

Variable           1
LIN             .972
QUAD            .130
CUBIC           .005

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```

And this is the **H** matrix for the interaction effect (parallelism in T & F's book).

```
EFFECT .. GROUP BY TIME
 Adjusted Hypothesis Sum-of-Squares and Cross-Products

                   LIN       QUAD      CUBIC

 LIN              .045
 QUAD            -.942     19.740
 CUBIC           -.520     10.900      6.019


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 Multivariate Tests of Significance (S = 1, M = 1/2, N = 99 )

 Test Name         Value    Exact F Hypoth. DF   Error DF  Sig. of F

 Pillais          .01455     .98448       3.00     200.00       .401
 Hotellings       .01477     .98448       3.00     200.00       .401
 Wilks            .98545     .98448       3.00     200.00       .401
 Roys             .01455
 Note.. F statistics are exact.


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 Eigenvalues and Canonical Correlations

 Root No.    Eigenvalue        Pct.   Cum. Pct.  Canon Cor.

        1          .015     100.000     100.000        .121

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 EFFECT .. GROUP BY TIME  (Cont.)
 Univariate F-tests with (1,202) D. F.

 Variable   Hypoth. SS   Error SS Hypoth. MS   Error MS          F  Sig. of F

 LIN           .04494 4779.57675     .04494   23.66127     .00190       .965
 QUAD        19.73998 1905.50001   19.73998    9.43317    2.09261       .150
 CUBIC        6.01907 1741.20241    6.01907    8.61981     .69828       .404

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 EFFECT .. GROUP BY TIME  (Cont.)

>Note # 12188
>Because there are no functions significant at level alpha, MANOVA will not
>report any canonical discriminant or correlation analysis for this effect.
```

It seems that the increase of WISC scores over time is mostly linear (as we inferred from the plot).

```
* * * * * * A n a l y s i s   o f   V a r i a n c e -- design   1 * * * * * *

Tests involving 'TIME' Within-Subject Effect.

 AVERAGED Tests of Significance for TOTAL using UNIQUE sums of squares
 Source of Variation          SS      DF        MS         F  Sig of F

 WITHIN CELLS            8426.28     606     13.90
    (Greenhouse-Geisser)          453.73
    (Huynh-Feldt)                 461.35
    (Lower bound)                 202.00
 TIME                   92889.76       3  30963.25   2226.81      .000
    (Greenhouse-Geisser)            2.25             2226.81      .000
    (Huynh-Feldt)                   2.28             2226.81      .000
    (Lower bound)                   1.00             2226.81      .000
 GROUP BY TIME             25.80       3      8.60       .62      .603
    (Greenhouse-Geisser)            2.25                 .62      .557
    (Huynh-Feldt)                   2.28                 .62      .560
    (Lower bound)                   1.00                 .62      .432
```

Although Mauchly’s test showed violation of (non) sphericity, the results do not differ.

These are contrasts of the transformed variables (linear, quadratic, cubic) examined one at a time. In case one cares, the p-values reported are not corrected for multiple testing. In any case, they indicate the same results than before: a clear linear effect (with some quadratic component) without a time by group interaction.

```
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Estimates for LIN
--- Individual univariate .9500 confidence intervals

TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       1   21.2667365     .34063   62.43288     .00000   20.59508   21.93839

GROUP BY TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       2   .014845647     .34063     .04358     .96528    -.65681     .68650

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Estimates for QUAD
--- Individual univariate .9500 confidence intervals

TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       1   1.79970810     .21508    8.36768     .00000    1.37562    2.22380

GROUP BY TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       2   -.31112996     .21508   -1.44659     .14956    -.73522     .11296

Estimates for CUBIC
--- Individual univariate .9500 confidence intervals

TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       1   .063632435     .20560     .30950     .75726    -.34176     .46902

GROUP BY TIME

 Parameter     Coeff.  Std. Err.    t-Value     Sig. t Lower -95%  CL- Upper

       2   -.17180400     .20560    -.83563     .40435    -.57720     .23359

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```
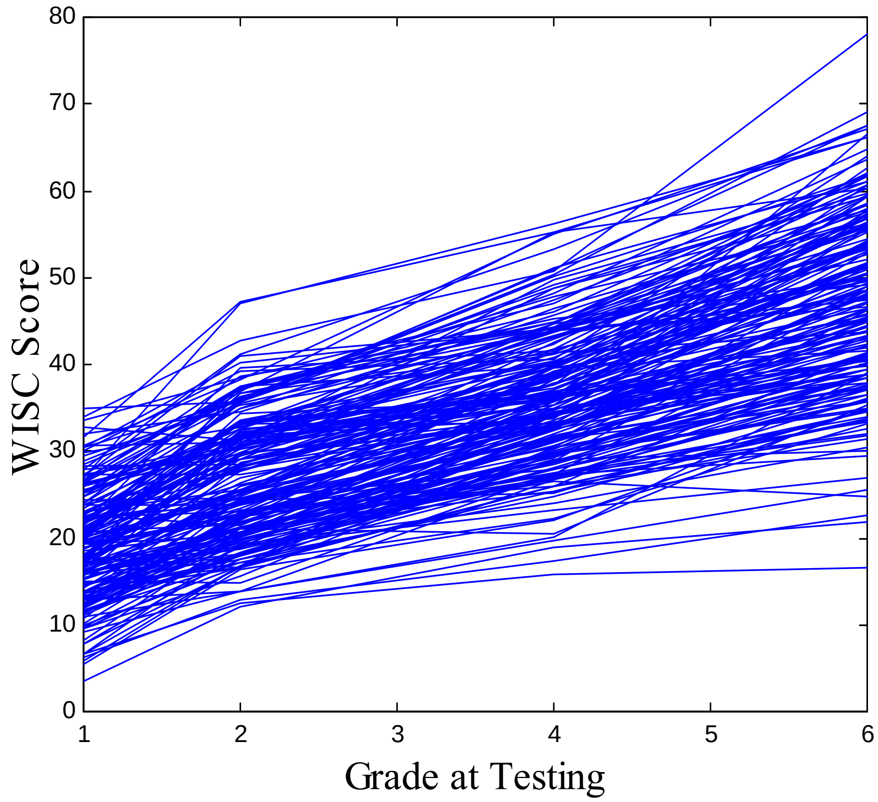

- **11. 10 Individual Data**



- **11. 11 Limitations of RM ANOVA**

- Group effects vs. individual change or growth

- Needs balanced designs
  - same number of observation per subject
  - same interval across assessments (and across subjects)

- ANOVA assumes sphericity

- ANOVA can't handle missing data

- Time is treated as a categorical variable

- Limited handling of covariates

- Other approaches are now available (e.g., mixed models)

- **11. 13 Comparing ANOVA vs. RM ANOVA**

Obs	id	wisc1	wisc2	wisc3	wisc4	group
1	1	9.17	12.91	18.10	26.10	1
2	2	6.67	13.86	19.73	30.93	1
3	3	7.89	14.19	19.94	31.36	1
4	4	22.13	24.97	41.76	49.91	2
5	5	30.03	39.27	40.65	63.95	2
6	6	27.93	41.12	53.27	67.59	2

------------------------------ group=1 ------------------------------

Variable	N	Mean	Std Dev	Minimum	Maximum
wisc1	3	7.9100000	1.2501200	6.6700000	9.1700000
wisc2	3	13.6533333	0.6645550	12.9100000	14.1900000
wisc3	3	19.2566667	1.0071908	18.1000000	19.9400000
wisc4	3	29.4633333	2.9206563	26.1000000	31.3600000

------------------------------ group=2 ------------------------------

Variable	N	Mean	Std Dev	Minimum	Maximum
wisc1	3	26.6966667	4.0918618	22.1300000	30.0300000
wisc2	3	35.1200000	8.8386933	24.9700000	41.1200000
wisc3	3	45.2266667	6.9878060	40.6500000	53.2700000
wisc4	3	60.4833333	9.3358949	49.9100000	67.5900000

------------------------------ total --------------------------------

Variable	N	Mean	Std Dev	Minimum	Maximum
wisc1	6	17.3033333	10.6397419	6.6700000	30.0300000
wisc2	6	24.3866667	13.0257816	12.9100000	41.1200000
wisc3	6	32.2416667	14.9087161	18.1000000	53.2700000
wisc4	6	44.9733333	18.0816976	26.1000000	67.5900000
wisc	24	29.7262500	17.0552921	6.6700000	67.5900000

- **Time only (no groups)**

ANOVA Model

The GLM Procedure

Dependent Variable: wisc

Source	DF	Sum of Squares	Mean Square	F Value
Model	3	2529.845279	843.281760	4.05
Error	20	4160.463483	208.023174	
Corrected Total	23	6690.308762		

Source	Pr > F
Model	0.0211
Error	
Corrected Total	

R-Square	Coeff Var	Root MSE	wisc Mean
0.378136	48.51943	14.42301	29.72625

Source	DF	Type I SS	Mean Square	F Value
time	3	2529.845279	843.281760	4.05

Source	Pr > F
time	0.0211

RM ANOVA Model

Repeated Measures Analysis of Variance
Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value
wisc	3	2529.845279	843.281760	47.62
Error(wisc)	15	265.609546	17.707303	

		Adj Pr > F	
Source	Pr > F	G - G	H - F
wisc	<.0001	<.0001	<.0001
Error(wisc)			

Greenhouse-Geisser Epsilon	0.6475
Huynh-Feldt Epsilon	1.0525

- **Time and Group**

ANOVA Model

The GLM Procedure

Dependent Variable: wisc

Source	DF	Sum of Squares	Mean Square	F Value
Model	7	6205.502163	886.500309	29.26
Error	16	484.806600	30.300412	
Corrected Total	23	6690.308762		

Source	Pr > F
Model	<.0001
Error	
Corrected Total	

R-Square	Coeff Var	Root MSE	wisc Mean
0.927536	18.51758	5.504581	29.72625

Source	DF	Type I SS	Mean Square	F Value
time	3	2529.845279	843.281760	27.83
group	1	3546.099704	3546.099704	117.03
group*time	3	129.557179	43.185726	1.43

Source	Pr > F
time	<.0001
group	<.0001
group*time	0.2722

Source	DF	Type III SS	Mean Square	F Value
time	3	27.079027	9.026342	0.30
group	1	3546.099704	3546.099704	117.03
group*time	3	129.557179	43.185726	1.43

Source	Pr > F
time	0.8264
group	<.0001
group*time	0.2722

RM ANOVA Model

The GLM Procedure
Repeated Measures Analysis of Variance
Tests of Hypotheses for Between Subjects Effects

Source	DF	Type III SS	Mean Square	F Value
group	1	3546.099704	3546.099704	40.67
Error	4	348.754233	87.188558	

Source	Pr > F
group	0.0031
Error	

Univariate Tests of Hypotheses for Within Subject Effects

Source	DF	Type III SS	Mean Square	F Value
wisc	3	2529.845279	843.281760	74.38
wisc*group	3	129.557179	43.185726	3.81
Error(wisc)	12	136.052367	11.337697	

		Adj Pr > F	
Source	Pr > F	G - G	H-F-L
wisc	<.0001	<.0001	<.0001
wisc*group	0.0396	0.0694	0.0396
Error(wisc)			

Greenhouse-Geisser Epsilon	0.6615
Huynh-Feldt-Lecoutre Epsilon	1.3101

- **11. 13 Repeated Measures as Mixed Models**

```
TITLE 'Linear Grade';
PROC MIXED NOCLPRINT COVTEST; CLASS id;
MODEL wisc = grade / SOLUTION DDFM=BW CHISQ;
RANDOM INTERCEPT grade / SUBJECT=id TYPE=UN GCORR; RUN;
```

Subjects	204
Max Obs Per Subject	4
Number of Observations Read	816
Number of Observations Used	816
Number of Observations Not Used	0

Estimated G Correlation Matrix

Row	Effect	id	Col1	Col2
1	Intercept	1	1.0000	0.6459
2	grade	1	0.6459	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	id	31.7680	3.7716	8.42	<.0001
UN(2,1)	id	3.5368	0.5781	6.12	<.0001
UN(2,2)	id	0.9438	0.1686	5.60	<.0001
Residual		10.1043	0.7074	14.28	<.0001

Fit Statistics

-2 Res Log Likelihood	4918.8
AIC (smaller is better)	4926.8
AICC (smaller is better)	4926.8
BIC (smaller is better)	4940.0

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	19.6977	0.4302	203	45.78	<.0001
grade	5.5407	0.08935	611	62.01	<.0001

```
TITLE 'Quadratic Grade';
PROC MIXED NOCLPRINT COVTEST; CLASS id;
MODEL wisc = time time*time/ SOLUTION DDFM=BW ;
RANDOM INTERCEPT time time*time /SUBJECT=id TYPE=UN GCORR; RUN;
```

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	id	33.4131	4.1269	8.10	<.0001
UN(2,1)	id	1.6747	2.0101	0.83	0.4048
UN(2,2)	id	0	.	.	.
UN(3,1)	id	1.4516	0.7152	2.03	0.0424
UN(3,2)	id	0.07148	0.2284	0.31	0.7543
UN(3,3)	id	0.2882	0.1478	1.95	0.0256
Residual		8.4434	0.5919	14.27	<.0001

Fit Statistics

-2 Res Log Likelihood	4850.6
AIC (smaller is better)	4862.6
AICC (smaller is better)	4862.7
BIC (smaller is better)	4882.6

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	18.7947	0.4507	203	41.70	<.0001
time	6.8208	0.3184	610	21.42	<.0001
time*time	0.8967	0.1084	610	8.27	<.0001

```
TITLE 'Linear Grade Plus Covariate on Level and Slope';
PROC MIXED NOCLPRINT COVTEST; CLASS id;
MODEL wisc = time groupc time*groupc / SOLUTION DDFM=BW SOLUTION;
RANDOM INTERCEPT time / SUBJECT=id TYPE=UN GCORR; RUN;
```

Estimated G Correlation Matrix

Row	Effect	id	Col1	Col2
1	Intercept	1	1.0000	0.7783
2	grade	1	0.7783	1.0000

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	id	21.6545	2.7830	7.78	<.0001
UN(2,1)	id	3.5334	0.5053	6.99	<.0001
UN(2,2)	id	0.9518	0.1698	5.61	<.0001
Residual		10.1043	0.7074	14.28	<.0001

Fit Statistics

-2 Res Log Likelihood	4852.7
AIC (smaller is better)	4860.7
AICC (smaller is better)	4860.7
BIC (smaller is better)	4874.0

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	19.7604	0.3682	202	53.66	<.0001
grade	5.5408	0.08959	610	61.84	<.0001
groupc	3.1943	0.3682	202	8.67	<.0001
grade*groupc	0.004116	0.08959	610	0.05	0.9634

```
TITLE4 'Quadratic Grade Plus Covariate on Level and Slopes';
PROC MIXED NOCLPRINT COVTEST; CLASS id;
MODEL wisc = grade grade*grade groupc grade*groupc
             grade*grade*groupc/ SOLUTION DDFM=BW ;
RANDOM INTERCEPT grade grade*grade / SUBJECT=id TYPE=UN; RUN;
```

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr Z
UN(1,1)	id	22.5314	3.1001	7.27	<.0001
UN(2,1)	id	3.1864	1.3051	2.44	0.0146
UN(2,2)	id	0	.	.	.
UN(3,1)	id	0.1266	0.2594	0.49	0.6254
UN(3,2)	id	0.06584	0.05850	1.13	0.2604
UN(3,3)	id	0.01319	0.02195	0.60	0.2739
Residual		9.6349	0.6762	14.25	<.0001

Fit Statistics

-2 Res Log Likelihood	4843.3
AIC (smaller is better)	4855.3
AICC (smaller is better)	4855.4
BIC (smaller is better)	4875.3

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	19.4551	0.3875	202	50.21	<.0001
grade	6.1374	0.2181	608	28.14	<.0001
grade*grade	-0.1185	0.04261	608	-2.78	0.0056
groupc	3.3653	0.3875	202	8.69	<.0001
grade*groupc	-0.3301	0.2181	608	-1.51	0.1307
grade*grade*groupc	0.06640	0.04261	608	1.56	0.1197