DeepBayes Summer School - Theoretical Assignment

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Problem 1

The random variable η is defined as a number of successes of k Bernoulli trials with the probability of success p, where k is drawn from Poison distribution with the parameter λ. Thus:

$$p(\eta|k, p, \lambda) = p(\eta|k, p)p(k|\lambda).$$

We can marginalise out k in order to obtain $p(\eta|p, \lambda)$:

$$\begin{aligned}
p(\eta|k, p, \lambda) &= \sum_{k>l}^{\infty} \binom{k}{l} p^l (1-p)^{k-l} e^{-\lambda} \frac{\lambda^k}{k!} \\
&= e^{-\lambda}\lambda^l \sum_{k>l}^{\infty} \frac{\lambda^{k-l}}{k!} \frac{k!}{l!(k-l)!} p^l (1-p)^{k-l} \\
&= e^{-\lambda}\lambda^l \frac{p^l}{l!} \sum_{k>l}^{\infty} \lambda^{k-l} \frac{1}{(k-l)!} (1-p)^{k-l} \\
&= e^{-\lambda}\lambda^l \frac{p^l}{l!} \sum_{k>l}^{\infty} \frac{(\lambda(1-p))^{k-l}}{(k-l)!} \\
&= e^{-\lambda}\lambda^l \frac{p^l}{l!} e^{\lambda(1-p)} \\
&= e^{-p\lambda} \frac{(p\lambda)^l}{l!},
\end{aligned} \tag{1}$$

which indeed shows that η has Poisson distribution with the parameter $p\lambda$

Problem 2

The time needed by a strict reviewer is normally distributed:

$$t_1 \sim \mathcal{N}(\mu_1 = 30,\, \sigma_1 = 10), \tag{2}$$

analogically, the time needed by a kind reviewer:

$$t_2 \sim \mathcal{N}(\mu_2 = 20,\, \sigma_2 = 5). \tag{3}$$

Each submission is randomly assigned to a reviewer r with the probability $p(r = \text{strict}) = p(r = \text{kind}) = 0.5$. Given the review time $t = 10$, the probability that the application was checked by the kind reviewer can be calculated using Bayes theorem and marginalising to find the denominator:

$$\begin{aligned} p(r = \text{kind}|t) &= \frac{p(t|r = \text{kind})p(r = \text{kind})}{p(t)} \\ &= \frac{p(t|r = \text{kind})p(r = \text{kind})}{p(t|r = \text{kind})p(r = \text{kind}) + p(t|r = \text{strict})p(r = \text{strict})} \\ &= \frac{p(t|r = \text{kind})}{p(t|r = \text{kind}) + p(t|r = \text{strict})} \end{aligned} \tag{4}$$

Since the prior probability is equal for both reviewers, the conditional probability $p(r = \text{kind}|t)$ depends only on the likelihoods of observing the time t under the two revision models (kind, strict). The respective likelihoods can be computed by evaluating the Normal distributions from equations 2 and 3 at time t.

$$p(r = \text{kind}|t) = \frac{\frac{1}{\sqrt{2\pi\sigma_2^2}} exp(\frac{-(t-\mu_2)^2}{2\sigma_2^2})}{\frac{1}{\sqrt{2\pi\sigma_1^2}} exp(\frac{-(t-\mu_1)^2}{2\sigma_1^2}) + \frac{1}{\sqrt{2\pi\sigma_2^2}} exp(\frac{-(t-\mu_2)^2}{2\sigma_2^2})} \tag{5}$$

Finally, the numerical calculation yields:

$$p(r = \text{kind}|t = 10) \approx \frac{0.005}{0.01 + 0.005} = 0.6(6). \tag{6}$$