



İTÜ  
Computer Engineering Department  
Assoc. Prof. Feza BUZLUCA  
Prof. Mustafa Ersel KAMAŞAK  
Asst. Prof. Sanem KABADAYI  
December 6, 2019

## BLG 231E DIGITAL CIRCUITS MIDTERM 2 SOLUTIONS

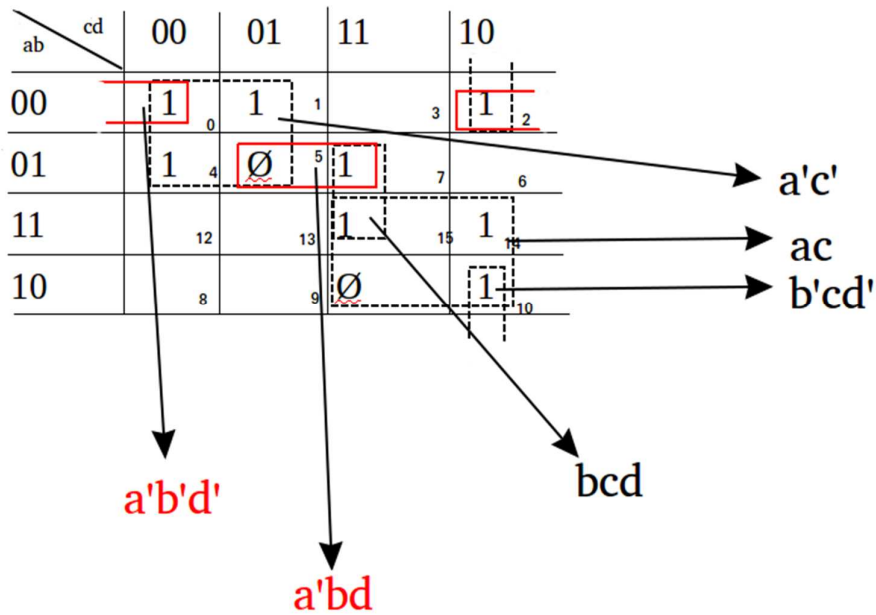
### QUESTION 1 (30 Points):

*Note that Parts (a) and (b) below are not related.*

a.

- i. Draw the Karnaugh map for the function, and write the missing 2 prime implicants in the box below.

(8 points)



- ii. Draw the prime implicant chart for this function. The cost of each variable is 3 units, and the cost of each complementation is 2 units. Do not simplify the chart. (7 points)

	0	1	2	4	7	10	14	15	Cost
$a'c'$	X	X		X					10
$ac$						X	X	X	6
$bcd$					X			X	9
$b'cd'$			X			X			13
$a'bd$					X				11
$a'b'd'$	X		X						15

**b. Find** the sufficient base (minimal covering sum) with the lowest cost for the following chart. Calculate the cost. Show your work in systematical steps (i.e., do not write results directly). (15 points)

**Step-1)** 4 is distinguished point → F is essential prime implicant  
Select F → delete 4, 7, 13, 14

**Step-2)** C covers D With less cost → Delete D

	0	2	5	8	10	11	Cost
A	X		X				8
B	X		X			X	10
C		X	X	X	X		6
E				X	X	X	10

**Step-3)** 2 is distinguished point → C is essential prime implicant  
Select C → delete 2, 5, 8, 10

	0	11	Cost
A	X		8
B	X	X	10
E		X	10

**Step-4)** B covers E with same cost → Delete E.

**Step-5)** 11 is distinguished point → B is essential prime implicant  
Select B.

**Final sufficient base = B+C+F**

**Cost is this sufficient base is  $6+10+8 = 24$  units.**

## QUESTION 2 (35 Points):

- a. F is a logic function with five inputs.  $Z = F(a,b,c,d,e)$

"1"-generating inputs (true points) are (01001, 01011, 01100, 01101, 01110, 01111, 10011, 10100, 11100, 11110).  
9 11 12 13 14 15 19

20 28 30  
 Find the set of all prime implicants of the function F using the **Quine-McCluskey** method. (You should clearly write out the expressions of the prime implicants.) **Important note:** that you should group according to the number of 1s. This means that the 1-generating inputs in the Quine-McCluskey table may not necessarily be in strictly increasing order.

15 pt

Num	a b c d e	Num	a b c d e	Num	a b c d e
9	0 1 0 0 1 ✓	9,11	0 1 0 - 1 ✓	9,11,13,15	0 1 - - 1
12	0 1 1 0 0 ✓	9,13	0 1 - 0 1 ✓	9,13,11,15	0 1 - - 1
20	1 0 1 0 0 ✓	12,13	0 1 1 0 - ✓	12,13,14,15	0 1 1 - -
11	0 1 0 1 1 ✓	12,14	0 1 1 - 0 ✓	12,14,13,15	0 1 1 - -
13	0 1 1 0 1 ✓	12,28	- 1 1 0 0 ✓	12,14,28,30	- 1 1 - 0
14	0 1 1 1 0 ✓	20,28	1 - 1 0 0	12,28,14,30	- 1 1 - 0
19	1 0 0 1 1	11,15	0 1 - 1 1 ✓		
28	1 1 1 0 0 ✓	13,15	0 1 1 - 1 ✓		
15	0 1 1 1 1 ✓	14,15	0 1 1 1 - ✓		
30	1 1 1 1 0 ✓	14,30	- 1 1 1 0 ✓		
		28,30	1 1 1 - 0 ✓		

No need to rewrite the same items

Set of prime implicants: (not marked):  $\bar{a}\bar{b}\bar{c}de$ ,  $ac\bar{d}\bar{e}$ ,  $\bar{a}be$ ,  $\bar{a}bc$ ,  $bc\bar{e}$

- b. List all the true points (1s) of the function F that the prime implicants you found in Part (a) cover. (Which prime implicant covers which points?)

10 pt

$\bar{a}\bar{b}\bar{c}de$  covers 10011 (19).

$ac\bar{d}\bar{e}$  covers 10100 (20) and 11100 (28).

$\bar{a}be$  covers 01001 (9), 01011 (11), 01101 (13), 01111 (15).

$\bar{a}bc$  covers 01100 (12), 01101 (13), 01110 (14), and 01111 (15).

$bc\bar{e}$  covers 01100 (12), 01110 (14), 11100 (28), and 11110 (30).

- c. Determine which of the prime implicants you found in Part (a) are **essential** prime implicants.

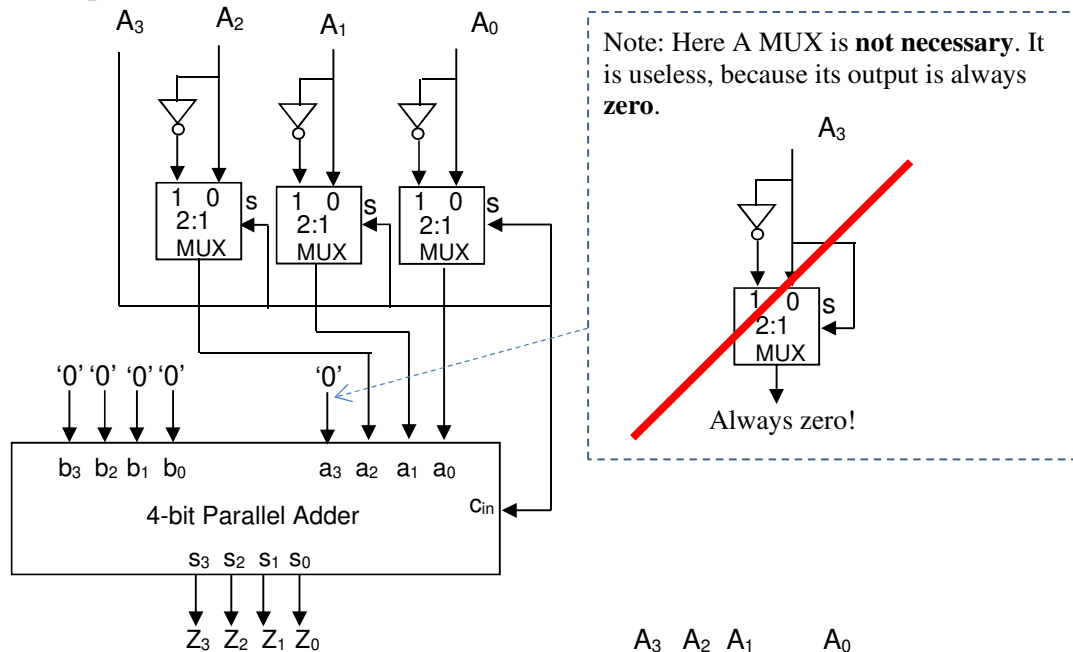
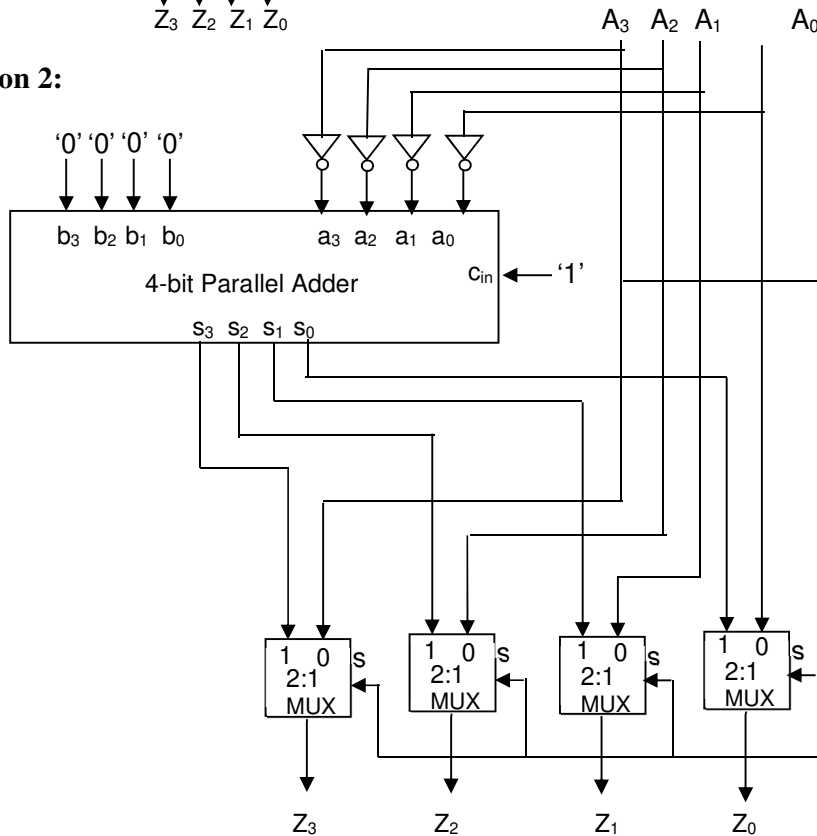
A prime implicant is essential if it covers a minterm that cannot be covered by any other prime implicant. So, the essential prime implicants are

10 pt

$\bar{a}\bar{b}\bar{c}de$  (covers 19),  $ac\bar{d}\bar{e}$  (covers 20),  $\bar{a}be$  (covers 9),  $bc\bar{e}$  (covers 30)

**QUESTION 3 (35 Points):****a. A** ( $A_3, A_2, A_1, A_0$ ) $A_3$  is the most significant bit and the sign of the number.If  $A_3 = 0$ ,  $Z = A$ If  $A_3 = 1$ ,  $Z = \bar{A} + 1$  (2's complement of  $A$ )

There are different possible proper solutions. Two of them are given below. (15 points)

**Solution 1 (cheaper):****Solution 2:**

**b.** The X-Y latch given on the right is a 1-bit memory unit.

**i.** Truth table: (10 points)

X	Y	Q(t)	Q(t+1)
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

**ii.** The characteristic equation: (5 points)

$$Q(t + 1) = \bar{X}Q(t) + XY$$

**iii.** (5 points)

Operation	X	Y
Set	1	1
Reset	1	0
Don't Change	0	$\Phi$