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BLG 231E DIGITAL CIRCUITS MIDTERM 1 SOLUTIONS

QUESTION 1 (35 Points):

Note that Parts (a), (b), and (c) below are not related.

a. (*10 points*)

Since the carry is "0" after A-B, there is barrow; therefore, A<B.
The largest possible 8-bit unsigned integer is 1111 1111. Since the carry is "0" after A+B; and the smallest possible A is 0000 0001, t**he largest possible binary B is 1111 1110**.

b. (*10 points*)

Since B is positive and A>B, A is also positive.
After A+B, the result is negative. **There is overflow**, because positive + positive → negative.
For overflow the smallest 8-bit result must be decimal 128.
Because A>B and result must be at least 128, **the smallest possible decimal A is 65** (B is 63).

c. (*7 points*)

A=$9 = 1001; B = $5A = 0101 1010

Remember, to convert a hexadecimal number to a binary number, it is not necessary to convert it into decimal. Hexadecimal numbers are just used to represent binary numbers easily. They can be converted to binary directly.

i. (Unsigned extension)

A=0000 1001	0000 1001	A = 9
B=0101 1010 (2' comp.)	+1010 0110	B = 90
	1010 1111 (Barrow)	Z cannot be represented as an 8-bit unsigned integer.

ii. (*8 points*)

(Signed extension)

A=1111 1001	1111 1001	A = - 7
B=0101 1010 (2' comp.)	+1010 0110	B = 90
	1 1001 1111	Z = -97

BLG 231E DIGITAL CIRCUITS MIDTERM 1 (Question 2 of 3)

QUESTION 2 (35 Points):
<u>Note</u> that Parts (a), (b), and (c) below are <u>not</u> related.

a.

i. (*5 points*)

Dual of XOR: $(\mathbf{X}+\overline{\mathbf{Y}})(\overline{\mathbf{X}}+\ \mathbf{Y}\ )$

$$= \mathbf{X}\,\overline{\mathbf{X}} + \mathbf{X}\,\mathbf{Y} + \ \overline{\mathbf{X}}\,\overline{\mathbf{Y}} + \ \mathbf{Y}\,\overline{\mathbf{Y}}$$
$$= \mathbf{X}\,\mathbf{Y} + \ \overline{\mathbf{X}}\,\overline{\mathbf{Y}}$$

ii. (*10 points*)

Complement of XOR: $\overline{(X \oplus \ Y)}$

$$= \overline{(\mathbf{X}\,\overline{\mathbf{Y}}\ + \overline{\mathbf{X}}\,\mathbf{Y})} \quad \text{De Morgan}$$
$$= (\overline{\mathbf{X}} + \mathbf{Y})(\ \mathbf{X} + \overline{\mathbf{Y}}) \ \text{Distributuve}$$
$$= \mathbf{X}\,\overline{\mathbf{X}} + \ \overline{\mathbf{X}}\,\overline{\mathbf{Y}} + \mathbf{X}\,\mathbf{Y} + \mathbf{0} + \ \mathbf{Y}\,\overline{\mathbf{Y}} \ \text{(Absorptions)}$$
$$= \overline{\mathbf{X}}\,\overline{\mathbf{Y}} + \mathbf{X}\,\mathbf{Y}$$

b. (*10 points*)

Expression:

$$\boldsymbol{Z} = \ \overline{\overline{\overline{\boldsymbol{X}} \cdot \overline{\boldsymbol{Y}}}}$$

Boolean operator (with explanation): NOR
A NAND gate with inverted inputs is equivalent of the OR gate (DeMorgan).
Because of the last NOT gate, this is the NOR operator.

c. (*10 points*)

$$\boldsymbol{E} = \boldsymbol{a\,b\,c\,d} + \boldsymbol{a\,b\,c\,\overline{d}}\ + \boldsymbol{\overline{a}\,\overline{b}\,c\,d} + \ \boldsymbol{\overline{a}\,\overline{b}\,d} + \ \boldsymbol{\overline{a}\,b\,d} + \ \boldsymbol{b\,c\,d} = ?$$

$$= \boldsymbol{a\,b\,c\,(d+\ \overline{d}\,)} + \boldsymbol{a\,b\,c\,\overline{d}}\ + \boldsymbol{\overline{a}\,\overline{b}\,c\,d} + \ \boldsymbol{\overline{a}\,\overline{b}\,d} + \ \boldsymbol{\overline{a}\,b\,d} + \ \boldsymbol{b\,c\,d}$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,\overline{b}\,c\,d} + \ \boldsymbol{\overline{a}\,\overline{b}\,d} + \ \boldsymbol{\overline{a}\,b\,d} + \ \boldsymbol{b\,c\,d}$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,\overline{b}\,d}\ \ (\boldsymbol{c}+\boldsymbol{1}) + \ \boldsymbol{\overline{a}\,b\,d} + \ \boldsymbol{b\,c\,d}$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,\overline{b}\,d}\ \ + \ \boldsymbol{\overline{a}\,b\,d} + \ \boldsymbol{b\,c\,d}$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,\overline{b}\,d}\ \ + \ \boldsymbol{\overline{a}\,b\,d}$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,d}\ \ (\boldsymbol{\overline{b}}\ + \boldsymbol{b}\,)$$
$$= \boldsymbol{a\,b\,c}\ + \boldsymbol{\overline{a}\,d}$$

QUESTION 3 (30 Points):
Note that Parts (a) and (b) below are not related.

a. Consider the completely specified function $h(X,Y,Z) = \mathrm{U}_1(1,2,4,6,7)$.

i. (*6 points*)

$$\overline{h(Y,Z,X)} = \mathrm{U}_0(1,2,4,5,7)$$

ii. (*6 points*)

$$h(X,Y,Z) == (X+Y+Z)(X+\bar{Y}+\bar{Z})(\bar{X}+Y+\bar{Z})$$

b. (*9 points*)

$$g(X,Y,Z) = X\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + XY\bar{Z}$$

c. (*9 points*)