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Answers

1 — $x[402] = 10\sin\left(314 \cdot 5.10^{-3} \cdot 402 + \frac{\pi}{3}\right)$

2 — $x[k] = \sum_{n=0}^{3} x[n] \cdot e^{-j\frac{2\pi}{5.10^{-3}} n.k}$

3 — $x(t) = t \cdot u(t+1) + u(t) - u(t-1) - t[u(t-1) - u(t-3)]$

4 — $y[n] = \{0, 2, 7, 5, 1\}$

5 — $y[n] = \{2, 6, 10, 10, 6, 2\}$

6 — $Y(s) = \frac{8z^2}{z-1} + \frac{(-10)z^2}{z-0.5} + \frac{(-1)z^2}{(z-0.5)^2}$

7 — Input X
A = X + B − 0.25C
Y = 2B
Output(Y)
C = B
B = A
Return

8 — $f_c = 200\ Hz$

9 — $Y(s) = \frac{10\ 400\pi}{s^2 + (400\pi)^2} \cdot \frac{1000}{5s + 1000}$

10 — $\frac{1}{j\pi}, \frac{1}{j\pi}$

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## Answers

11 – $-1 < a < 1$

12 – It is not periodic

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(1) $t = k.T_s \quad T_s = 5.10^{-3}s \quad t = 5.10^{-3}.k$

$x[k] = 10\sin(314.5.10^{-3}k + \pi/3) \quad 0 \le k \le 402$

$$\boxed{x[402] = 10\sin\left(314.5.10^{-3}.402 + \frac{\pi}{3}\right)}$$

(2)

$x[k] = \sum_{0}^{3} x[k].e^{-j\Omega_0.n} \qquad \Omega_0 = \frac{2\pi}{T_s} \qquad T_s = 5.10^{-3}$

$$\boxed{X[k] = \sum_{n=0}^{3} x[n].e^{-j\frac{2\pi}{5.10^{-3}}n.k}} = x[0] + x[1].e^{-j\frac{2\pi}{5.10^{-3}}k} + x[2]$$

(3) We can divide and write each part. Then we can sum them to get $x(t)$

1- $t.u(t+1)$
2- $u(t) - u(t-1)$
3- $-t[u(t-1) - u(t-3)]$

$$\boxed{x(t) = t.u(t+1) + u(t) - u(t-1) - t[u(t-1) - u(t-3)]}$$

(4) $y[n] = x[n] * h[n] \qquad x[n] = \{1,3,1\} \qquad h[n] = \{0,2,1\}$

$$\boxed{y[n] = \sum_{k=0}^{2} x[k].h[n-k]}$$

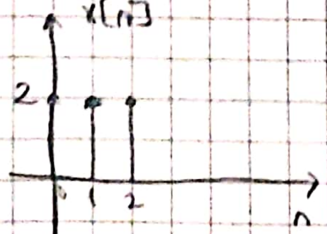
$y[n] = x[0].h[n] + x[1].h[n-1] + x[2]h[n-2]$

$y[0] = 0 \qquad y[2] = 1+6 \qquad y[4] = 1$

$y[1] = 2 \qquad y[3] = 3+2$

$$\boxed{y[n] = \{0,2,7,5,1\}}$$

(5) $x[n] = 2(u[n] - u[n-3])$




$y[n] = \sum_{k=0}^{2} x[k].h[n-k]$

$n = 6$

$y[n] = x[0].h[n] + x[1].h[n-1] + x[2].h[n-2]$

$y[1] = 2$

$y[2] = 4 + 2 = 6$

$y[3] = 4 + 4 + 2 = 10$

$y[4] = 2 + 4 + 4 = 10$

$y[5] = 2 + 4 = 6$

$y[6] = 2$

$$\boxed{y[n] = \{2,6,10,10,6,2\}}$$

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(6) $H(z) = \dfrac{2z^{-1}}{(1-0.5z^{-1})^2}$

$y[n] = H(z) . X(z)$

$X(z) = \dfrac{z}{z-1}$

$= \dfrac{2\cancel{z^{-1}}}{(1-0.5z^{-1})^2} \cdot \dfrac{\cancel{z}}{z-1}$

$= \dfrac{2}{1-z^{-1}+0.25z^{-2}} \cdot \dfrac{1}{z-1}$

$= \dfrac{2z^2}{(z^2-z+0.25)(z-1)} = \dfrac{A}{z-1} + \dfrac{B}{z-0.5} + \dfrac{C}{(z-0.5)^2}$

$(z-\frac{1}{2})^2(z-1)$

$8(z-0.5)^2 + B(z-1)(z-0.5) - 1(z-1)$

$A(z-0.5)^2 \Big|_{z=1} = 2z$

$A = 8$

$C(z-1)\Big|_{z=0.5} = 2z^2$

$C = -1$

$z=0 \quad 0 = 4 + \frac{B}{2} + 1 \qquad B = -10$

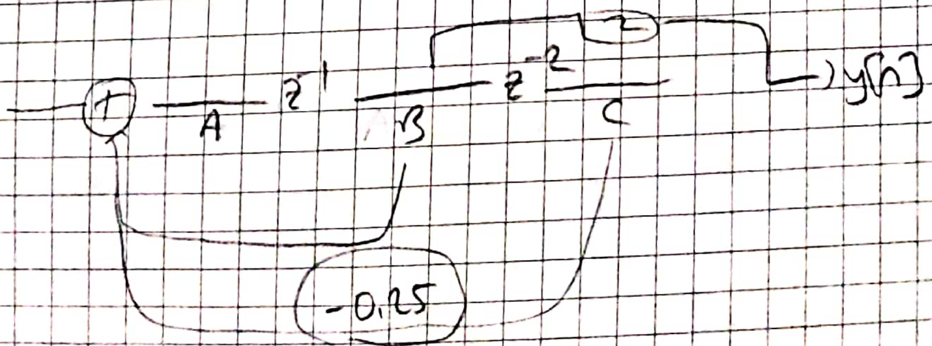
(7)

Input X
A = X + B − 0.25C
Y = 2B
Output(Y)
C = B
B = A
Return

$H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{2z^{-1}\,A(z)}{(1-z^{-1}+0.25z^{-2})A(z)}$

$Y(z) = 2z^{-1} . A(z)$

$A(z) = X(z) + z^{-1}A(z) - 0.25z^{-2}A(z)$



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(8) $H(s) = \frac{Y(s)}{X(s)} = \frac{1000}{5s+1000}$ $\quad H(j\omega) = \frac{1000}{5(j\omega)+1000}$

$$= \frac{1}{1 + \frac{5j\omega}{1000}}$$

$$\frac{j\omega}{200}$$

$$f_c = 200\ Hz$$

(9) Since we are given transfer function in s domain we can find output y(t) by

$x(t) = 10\sin(400\pi t)$

$y(s) = H(s) \cdot X(s)$

$$X(s) = 10 \cdot \frac{400\pi}{s^2 + (400\pi)^2}$$

$y(s) = \frac{10 \cdot 400\pi}{s^2+(400\pi)^2} \cdot \frac{1000}{5s+1000}$ then we need to come back t domain

$T = 0{,}2\ ms$ $\quad w = \frac{2\pi}{0.2} = 10\pi$

(10) $\int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t}\, dt$

$$= \int_0^{0.1} 5 \cdot e^{-j10\pi t}\, dt = \frac{5}{-j10\pi} \cdot e^{-j10\pi t} \Big|_0^{0.1} = \frac{1}{2j\pi} - \frac{e^{-j\pi}}{2j\pi}$$

$$+ \int_{0.1}^{0.2} -5 \cdot e^{-j10\pi t}\, dt = \frac{5}{j10\pi} \cdot e^{-j10\pi t} \Big|_{0.1}^{0.2} = \frac{e^{-2j\pi}}{2j\pi} - \frac{e^{-j\pi}}{2j\pi}$$

$$\boxed{= \frac{1}{j\pi}, \frac{1}{j\pi}}$$

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(11) $Y(z) = z^{-2}X(z) + 2az^{-1}Y(z) - a^2z^{-2}Y(z)$

$$Y(z)\left[1-2az^{-1}+a^2z^{-2}\right] = z^{-2}X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z^{-2}}{1-2az^{-1}+a^2z^{-2}} =$$ one zero at 1

$$= \frac{1}{z^2-2az+a^2} = \frac{1}{(z-a)(z-a)}$$

$|z| > |a|$

$|a| < 1 \rightarrow -1 < a < 1$

$z = a \rightarrow$ pole

(12) $\cos\left(\frac{14\pi}{3}t\right) \rightarrow T_1 = \frac{2\pi}{\frac{14\pi}{3}} = \frac{6}{14} = \frac{3}{7}$

$\sin\left(\frac{5\pi}{4}t\right) \rightarrow T_2 = \frac{2\pi}{\frac{5\pi}{4}} = \frac{8}{5}$

$x_1(t+T) + x_2(t+T) = x_1(t+mT_1) + x_2(t+kT_2)$

There must be positive integers

$mT_1 = kT_2 = T \quad \frac{T_1}{T_2} = \frac{k}{m} = \frac{5}{56}$

not rational

so it is not periodic.