Fall 2016 Astro 250: Stellar Populations

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Problem Set 2 – Intro to Probability and Stats

Assigned: 9/12/16
Due: 9/26/16

Please complete the problems using your github repository.

Problem 1.

There is a minor mistake on the Initial Mass Function page of Wikipedia. What is it?

Problem 2.

Consider a single power-law IMF of the form:

$$P(M|\theta) = c\, M^{-\alpha} \tag{1}$$

where

$$c = \frac{1}{\int_{M_{min}}^{M_{max}} M^{-\alpha} dM} \tag{2}$$

and $\theta = (M_{min}, M_{max}, \alpha)$.

For simplicity, assume perfect knowledge of the masses and that observational effects are negligible.

(a) Write code that generates a list of N stellar masses between a given M_{min} and M_{max} from a power-law distribution with an index of α.

(b) Write code that will perform inference on the set of fake data you generated in part (a) using `emcee`.

(c) Generate a fake dataset assuming $M_{min} = 3\, M_{\odot}$, $M_{max} = 15\, M_{\odot}$, $N = 1000$ and $\alpha = 1.35$. In your inference code, let α and M_{max} be free parameters (but fix $M_{min} = 3M_{\odot}$). Given this fake dataset, to what precision can you constrain α and M_{max}?

(d) Show how the precision to which α and M_{max} can be recovered depends on N, from $N \sim 10$ to $N \sim 10,000$. Summarize your results in plots. It is OK to discretely and uniformly select values of N in $\log_{10}$ space (e.g., $\log_{10}(N) = 1, 2, 3, 4$). *Hint: In the limit that N is a small number, you may want to generate multiple datasets to verify the fidelity of your confidence intervals, as stochastic effects can be important.*

Problem 3.

In this problem, we will attempt to re-create Salpeter's original IMF measurement.

(a) Using the data for mass and number density in Table 2 and/or Figure 2 in Salpeter (1955), fit a power-law using an optimizer or least squares fitter (e.g., `scipy.optimize`).

(b) Same as part (a) only using your own inference code and `emcee`. Compare the two results: How close are they to one another? How close are they to the value reported in Salpeter (1955)?

Problem 4.

There are claims in the literature that the low-mass IMF slope may systematically deviate from the Galactic value in low-mass dwarf galaxies (e.g., Wyse et al. 2002; Geha et al. 2013). However, these measurements are made over a fairly limited mass range (usually $\sim 0.5 - 0.8\, M_{\odot}$) and done so assuming that a power-law is a reasonable approximation for the low-mass IMF.

Suppose the true IMF for stars with $M < 1M_{\odot}$ in all galaxies is actually a Chabrier IMF, i.e., a log-normal at low-masses. Ignoring corrections for stellar multiplicity, this IMF has the functional form:

$$\xi(m)\Delta m = \frac{0.15}{m} \exp \frac{-(log(m) - log(0.08))^2}{(2 \times 0.69)^2} \tag{3}$$

(a) Using the Chabrier IMF from above, generate a list of N=10,000 (perfectly known) stellar masses between 0.5 and 0.8 $M_{\odot}$.

(b) Now, assuming a single-slope power-law IMF model (as done in the literature), infer the value of the spectral index α. How does this compare with the canonical Kroupa IMF found in the Milky Way?

Problem 5.

Using python-FSPS:

(1) Generate the spectrum for a 10 Myr simple stellar population (assume no dust, fixed metallicity, etc – the only variable of interest is age). Plot how the spectrum from $1500 - 10000$Å changes for three different high-mass IMF ($> 1\, M_{\odot}$) values: $\alpha = 0.8, 1.3, 1.8$, holding the lower portions of the IMF fixed.

(2) Generate the spectrum of a 10 Gyr simple stellar population. Plot how the spectrum from $5000-20000$Å changes for three different IMF forms: Salpeter IMF, a Kroupa IMF, and the van Dokkum IMF.