

A GENERATIVE MODEL FOR *GAIA* ASTROMETRIC ORBIT CATALOGS: SELECTION FUNCTIONS FOR BINARY STARS, GIANT PLANETS, AND COMPACT OBJECT COMPANIONS

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Version August 22, 2024

ABSTRACT

Astrometry from *Gaia* DR3 has produced a sample of $\sim 170,000$ Keplerian orbital solutions, with many more anticipated in the next few years. These data have enormous potential to constrain the population of binary stars, giant planets, and compact objects in the Solar neighborhood. But in order to use the published orbit catalogs for statistical inference, it is necessary to understand their selection function: what is the probability that a binary with a given set of properties ends up in a catalog? We show that such a selection function for the *Gaia* DR3 astrometric binary catalog can be forward-modeled from the *Gaia* scanning law, including individual 1D astrometric measurements, the fitting of a cascade of astrometric models, and quality cuts applied in post-processing. We populate a synthetic Milky Way model with binary stars and generate a mock catalog of astrometric orbits. The mock catalog is quite similar to the DR3 astrometric binary sample, suggesting that our selection function is a sensible approximation of reality. Our fitting also produces a sample of spurious astrometric orbits similar to those found in DR3; these are mainly the result of scan angle-dependent astrometric biases in marginally resolved wide binaries. We show that *Gaia*'s sensitivity to astrometric binaries falls off rapidly at high eccentricities, but only weakly at high inclinations. We predict that DR4 will yield ~ 1 million astrometric orbits, most for systems with periods of > 1000 d and $G < 15$. We provide code to simulate and fit realistic epoch astrometry for *Gaia* DR3 or DR4 and determine whether any hypothetical binary would receive a cataloged orbital solution.

Subject headings: astrometry – catalogues – methods: statistical – binaries: general

1. INTRODUCTION

The third data release of the *Gaia* mission (DR3; [Gaia Collaboration et al. 2016, 2023a](#)) included astrometric orbital solutions for $\sim 168,000$ binary systems, representing a vast improvement in sample size and completeness over all previous work ([Gaia Collaboration et al. 2023b](#)). These data have already enabled discovery of several astrophysically interesting objects, including a sample of compact objects in au-scale orbits with stellar companions (e.g. [Shahaf et al. 2023](#)) and a sample of massive planets orbiting nearby stars ([Holl et al. 2023a](#)). For a review of the *Gaia* binary star sample, see [El-Badry \(2024\)](#).

Epoch-level astrometric data was not published in DR3, and several quality cuts were imposed on the published orbital solutions. These cuts depend on a variety of quantities, such as the signal-to-noise ratio of the photo-center semi-major axis, the orbital period, and the parallax and eccentricity uncertainties. These cuts – combined

with the fact that *Gaia*'s sensitivity to astrometric orbits depends on quantities such as the eccentricity and orientation in ways that are difficult to predict analytically – make it a nontrivial problem to use the *Gaia* astrometric binary sample for population inference, and few attempts at modeling the sample have been made so far.

In this paper, we use a model of the solar neighborhood's binary population to build a forward-model for the *Gaia* DR3 astrometric binary catalog. This entails modeling the *Gaia* observations at the level of individual epochs, predicting the observation times, scan angles, and simulated one-dimensional astrometry for each simulated source from the *Gaia* scanning law. We include common false positives and reproduce the cascade of single-star, accelerating source, and full orbital astrometric models employed in producing the observed catalog. Because our modeling results in a mock catalog of orbital solutions that resembles the one actually published in DR3, we believe that it captures the most important selection effects and false positives of the real catalog. Therefore it can be used both to interpret the DR3 binary sample and forecast what will be discover-

able in future data releases.

The remainder of this paper is organized as follows. In Section 2, we summarize the basics of *Gaia* observations and the astrometric signal caused by a binary. Section 3 describes the Galactic model, simulated binary population, and the assumptions we make to predict 1D epoch astrometry. We discuss the astrometric model cascade in Section 4 and present the resulting mock catalog in Section 5. Finally, Section 6 presents a Monte Carlo selection function that can be used to model and fit epoch astrometry for arbitrary binaries. Section 7 summarizes our main results.

2. HOW GAIA OBSERVES

How *Gaia* observes has been summarized extensively in other work, and we refer to [Gaia Collaboration et al. \(2016\)](#) for a detailed description. We summarize the most important aspects here.

Gaia is equipped with two telescopes whose fields of view are separated by 106.5 degrees. The satellite rotates with a period of 6 hours, such that the two telescopes sweep out an annulus with a width of about 0.7 deg on the sky. The rotation axis precesses with a period of 63 days and a fixed tilt angle of 45 degrees with respect to the Sun direction, causing this annulus to rotate on the sky. At the same time, the spacecraft orbits the Sun with a period of a year. The combination of these three rotations results in full-sky coverage, though with the observing cadence and distribution of scan angles varying significantly across the sky (for details, see [Holl et al. 2023b](#)). For stars brighter than $G = 15$, the median number of visibility periods used in DR3 is 20, with a (16-84)% range of 16-27. Here a visibility period refers to a group of observations separated by other groups of observations by at least 4 days.

As a source moves across the astrometric field of view (FOV), it is independently observed by 8-9 different CCDs. Each CCD observation results in an independent measurement of a source’s position in the along-scan (AL) direction relative to a reference position assigned to the source at a reference time. Crucially, *only* a 1D measurement in the AL direction is made with high precision and used in the astrometric solution. At the bright end – which is the regime relevant to most astrometric binaries published so far – the per-CCD observations currently reach an AL precision of order 0.12 mas. All the CCD transits in a given FOV transit are spread over a period of less than a minute, which is much shorter than the orbital periods of astrophysically plausible binaries to which *Gaia* is sensitive. The 8-9 CCD transits can thus be regarded as essentially independent measurements of the same quantity, and can be averaged to obtain smaller uncertainties in the CCD-averaged measurements. Our modeling in this work suggest that these measurements are indeed nearly independent and not systematics-dominated, such that the per-FOV transit uncertainties are ~ 3 times smaller than the per-CCD uncertainties (Appendix C).

Gaia astrometry is built upon a global astrometric solution, whose calculation requires the simultaneous optimization of millions of attitude and calibration parameters, and astrometric parameters for ~ 100 million stars ([Lindegren et al. 2012](#)). Most of these parameters are of little interest for the analysis of one source. It is pos-

sible to transform the astrometric measurements for a single source to “local plane coordinates” in a tangent plane to the unit sphere in the vicinity of each source, as described by [Lindegren & Bastian \(2022\)](#). The astrometric measurements required to describe the motion of a source can then be represented with just a small file containing the AL displacements, their uncertainties, and associated metadata (scan angles, parallax factors, transit times, etc.) for each source. Thus far, such epoch astrometry has been published for only one source: the binary Gaia BH3 ([Gaia Collaboration et al. 2024b](#)).

3. GALACTIC MODEL AND BINARY POPULATION

We now describe the Galactic model and binary population from which we forward-model *Gaia* observations and construct a mock astrometric orbit catalog. This will allow us to validate our selection function by comparing the mock catalog to the DR3 data.

Our modeling uses **Galaxia** ([Sharma et al. 2011](#)), a code to generate synthetic resolved-star surveys. **Galaxia** implements the Besançon model of the Milky Way ([Robin et al. 2003](#)). We use a modified version of the code described by [Lam et al. \(2020\)](#). We only attempt to model sources within 2 kpc of the Sun, because $\approx 98.6\%$ of all astrometric orbits published in DR3 are found within 2 kpc.

Galaxia predicts a total of 1.1 billion sources (all representing single stars) within 2 kpc of the Sun. Comparing to the *Gaia* DR3 `gaia_source` catalog, we find that **Galaxia** predicts 40% more sources than are observed (with `parallax_over_error > 10`) within both 50 and 100 pc. Since *Gaia* is expected to be nearly complete within these volumes for sources above the hydrogen burning limit ([Gaia Collaboration et al. 2021](#)), we conclude that **Galaxia** predicts 40% too many sources in the Solar neighborhood. We thus discard a randomly selected 40% of the predicted sources, selected independently of distance and apparent magnitude, before proceeding.

Galaxia does not include binary stars. We thus use **COSMIC** ([Breivik et al. 2020](#)) to generate a zero-age binary population according to the population demographics constraints from [Moe & Di Stefano \(2017\)](#). We assume a [Kroupa \(2001\)](#) primary mass function¹ and draw mass ratios, periods, and eccentricities from the covariant, mass-dependent distributions inferred by [Moe & Di Stefano \(2017\)](#). The assumed binary fraction is $\approx 41\%$ for solar-type primaries, increasing to 57% for $M_1 = 3 M_\odot$ and 80% at $M_1 = 6 M_\odot$. Below $0.8 M_\odot$, the binary fraction is assumed to decrease linearly with $\log M_1$, from 40% at $0.8 M_\odot$ to 0 at $0.08 M_\odot$. Almost all of the binaries predicted to be detectable in DR3 have primary masses between 0.7 and $1.3 M_\odot$, so it is the binary population in this mass range – which is constrained primarily by the [Raghavan et al. \(2010\)](#) survey of nearby G stars – that is most important for our modeling.

COSMIC predicts populations of single and binary stars. We match both binaries and singles to the **Galaxia**-predicted sources, continuing until the total number of stellar systems (singles plus binaries, with each binary

¹ Note that this is different from the more top-heavy mass function assumed in **COSMIC** by default.

representing a single system) in the zero-age COSMIC-predicted population exceeds the number of *Galaxia* sources within 2 kpc by 10%. This accounts for the fact that the *Galaxia* catalog only contains stars that are still alive; for a Kroupa IMF and constant star formation history over 12 Gyr, about $\approx 10\%$ of all stars that have been born will have already terminated their evolution.

Massive stars are preferentially found in the Galactic disk and thus are subject to higher extinction than typical lower-mass stars. To account for this, we place each simulated binary at the position of the *Galaxia* star whose mass is most similar to that of the primary. We assign inclinations by drawing from a $\sin(i)$ distribution. We draw longitudes of the ascending node, Ω , and arguments of periastron, ω , from $\mathcal{U}(0, 2\pi)$. We define the reference phase as $\phi_0 = \frac{2\pi T_p}{P}$, where T_p is the epoch of periastron, and draw ϕ_0 from $\mathcal{U}(0, 1)$.

We assign G -band absolute magnitudes to the primary and secondary components using the *isochrones* package (Morton 2015), using MIST models (Choi et al. 2016) and assuming a uniform age distribution between 0 and 12 Gyr. We remove binaries with initial masses $> 8 M_\odot$ whose primaries have died, assuming that most would-be black hole or neutron star binaries are destroyed or dramatically tightened by common envelope evolution and supernova kicks. We similarly assume that most binaries containing white dwarfs (WDs; i.e., primaries with initial masses $< 8 M_\odot$ that have terminated their evolution) shrink to short periods and are not detectable astrometrically. However, motivated by the results of Shahaf et al. (2024) and Yamaguchi et al. (2024), we assume that 10% of white dwarf + luminous star binaries with initial separations of $(2 - 6)$ au form wide post-common envelope binaries with separations that are 50% of their initial separations. This prescription is quite simplified and is not expected to capture all aspects of the observed population. The 10% fraction is chosen to approximately match the number of binaries with large mass functions in the observed catalog (Section 5). We assign white dwarf masses using the initial-final mass relation of Weidemann (2000). We remove binaries in which either star currently fills its Roche lobe or would have filled it previously (e.g., red clump stars that would have filled their Roche lobes at the tip of the first giant branch). Because most binaries that are astrometrically detectable contain main-sequence stars in au-scale orbits, the treatment of evolutionary effects has minor effects on the overall properties of the observable population.

We calculate extinctions to each binary using the combined19 dust map in the *mwdust* package (Bovy et al. 2016), which combines the maps from Drimmel et al. (2003), Marshall et al. (2006), and Green et al. (2019). We assume $A_G = 2.8E(B - V)$, where $E(B - V)$ is on the Schlegel et al. (1998) scale. We then calculate each binary's total G -band apparent magnitude, angular separation, ρ , and magnitude difference, $\Delta G = G_2 - G_1$. Binaries that are resolved or marginally resolved are removed from the sample. As we show in Appendix B, the transition between resolved and unresolved binaries depends on both ρ and ΔG , and in DR3 can be approximated as $\Delta G [\text{mag}] = \frac{1}{25} \left(\frac{\rho}{\text{mas}} - 200 \right)$, where binaries with ΔG larger than this value are unresolved. We remove resolved binaries from the sample, as well as un-

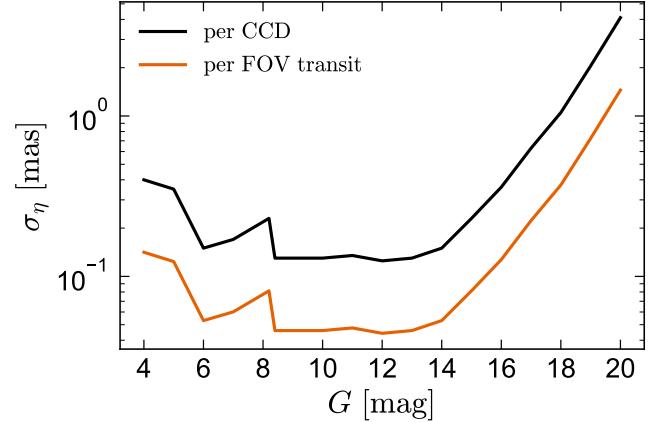


FIG. 1.— Assumed AL displacement uncertainties. Black line shows the uncertainty per CCD transit, adopted from Holl et al. (2023a). Tan line shows our assumed uncertainty per FOV transit and is smaller than the black line by a factor of $\sqrt{8}$, representing the result of averaging measurements from an average of 8 uncorrelated CCD measurements per FOV transit. The uncertainties are smallest at $G = 8 - 14$ and are dominated by calibration systematics in this magnitude range, but we find (Appendix C) that measurements from individual CCDs can still be usefully confined to yield FOV transit-averaged uncertainties a factor of $\approx \sqrt{8}$ smaller. The uncertainties at $G > 14$ are photon-limited and thus rise rapidly toward faint magnitudes. Calibration problems worsen at $G \lesssim 8$.

resolved binaries with $G > 19$, which were not fit with binary solutions in DR3. We only generate mock astrometry for binaries, discarding single stars and neglecting both higher-order multiples and chance alignments of physically unassociated stars. We are left with 46 million unresolved binaries with $G < 19$ within 2 kpc.

As a consistency check, we calculated the total number of predicted binaries with $M_G = 3 - 6$ (since solar-type binaries dominate the astrometric binary sample in DR3), finding 9.2×10^4 and 1.03×10^6 within 200 pc and 500 pc, respectively. These fractions represent 44% and 45%, respectively, of the total number of *Gaia* sources with $M_G = 3 - 6$ within the same volumes. The good agreement between these fractions and the 41% assumed binary fraction for solar-type stars suggests that the *Galaxia* model is reasonably accurate.

We mock-observe each binary using the scan times and scan angles predicted by the *Gaia* Observation Forecast Tool (*GOST*)². Because querying *GOST* involves a relatively slow web query, we first pre-computed the scanning law at 49152 uniformly spaced points on the sky (healpix level 6), and then used the saved value at the point closest to each simulated source. This results in an effective resolution of about 0.9 degrees, which is comparable to the 0.7 degree across-scan FOV.

We add Gaussian noise to the predicted AL displacements according to the empirical median CCD AL abscissa uncertainty in DR3 (Holl et al. 2023a, their Figure 3, “EDR3 adjusted” model). Each FOV transit contains 8 or 9 individual CCD transits, each separated by a few seconds. We combine these into a single CCD-averaged value per transit, with uncertainty $\sqrt{8}$ times smaller than the predicted per-CCD uncertainty. We choose this factor (rather than $\sqrt{9}$) to account for the fact that some

² <https://gaia.esac.esa.int/gost/>

CCD measurements flagged as outliers are discarded. We show the assumed per-CCD and per-FOV transit uncertainties in Figure 1. We verified that applying this method of single sources fit with five-parameter astrometric solutions results in astrometric uncertainties that are in good agreement with those published in *Gaia* DR3 (see Appendix C).

For sources brighter than $G = 13$, we add unmodeled Gaussian noise to account for underestimated uncertainties due to various systematics, which have been shown to increase at $G < 13$ (e.g. Lindegren et al. 2021b,a; El-Badry et al. 2021, 2023c). We draw the per-FOV transit σ of this noise from $\mathcal{U}(0, 0.04)$ mas. The 0.04 mas value is motivated by our fit to the epoch astrometry for Gaia BH3 (Gaia Collaboration et al. 2024b), the only *Gaia* source with currently published epoch astrometry.

When modeling astrometry for *Gaia* DR3, we only include observations between JD 2456891 and JD 2457902 (Halbwachs et al. 2023). For DR4 observations, we include scans taken between JD 2456891 and JD 2458868. Due to a variety of issues (see Lindegren et al. 2021b), about 10% of FOV transits do not result in usable data. To account for this, we reject FOV transits for each source with 10% probability.

3.1. Generating mock observations

We now consider how an unresolved or marginally resolved binary affects the epoch astrometry. Our modeling follows Lindegren (2022). In a given scan, a source is observed with scan angle ψ , defined as degrees east of north. We define ρ as the sky-projected, instantaneous angular separation of the two stars, and θ as the binary's position angle. Then $\Delta\eta = \rho \cos(\psi - \theta)$ is the angular separation projected onto the scan angle. Finally, we define $\xi = \Delta\eta/u$, where u quantifies the resolution of the *Gaia* line spread function. In detail, u will depend on the flux ratio, but Lindegren (2022) and Holl et al. (2023b) find that $u = 90$ mas provides a reasonable approximation for modeling astrometry of binaries.

We assume that the two stars each contribute flux along the AL direction as 1D Gaussians with unit variance separated by ξ , and with amplitudes scaled according to the flux ratio, $f = 10^{(G_1 - G_2)/2.5}$, where G_1 and G_2 are the magnitudes of the primary and secondary. We further assume that the measured AL displacement will be the *peak* of the resulting flux profile. At sufficiently large angular separations, the two peaks will be resolved, and in this case we assume the image parameter determination (IPD) pipeline will correctly centroid the peak of the primary. We predict the AL displacement relative to the binary's center of mass in these two cases as

$$\delta\eta = \begin{cases} uB(f, \xi) - \frac{q}{1+q}\Delta\eta & \text{if } 0 < |\xi| \leq 3 - f, \\ -\frac{q}{1+q}\Delta\eta & \text{if } 3 - f < |\xi|. \end{cases} \quad (1)$$

Here $B(f, \xi)$ is obtained by iteratively solving the equation

$$B = \frac{f\xi}{f + \exp(\xi^2/2 - \xi B)}. \quad (2)$$

for B , beginning at $B = 0$. This yields the AL coordinate of the peak of the total flux profile (see Lindegren 2022, for details). Inspection of Equation 1 reveals that

in the limit of small ξ (i.e., close separations), it reduces to $\delta\eta = \left(\frac{f}{f+1} - \frac{q}{1+q}\right)\Delta\eta$. In this case, the observations exactly trace the photocenter. As ξ increases, the peak of the flux profile no longer exactly traces the photocenter, but is displaced toward the primary. For typical flux ratios, the difference between the first case in Equation 13 and the photocenter displacement becomes significant ($\gtrsim 0.05$ mas) only for separations $\Delta\eta \gtrsim 30$ mas. This means that *Gaia* effectively observes the photocenter for the vast majority of binaries with periods short enough to be astrometrically constrained. On the other hand, for marginally resolved wide binaries, *Gaia* does *not* observe the photocenter. The difference between the observed AL displacements and the photocenter will vary with scan angle, and this can lead to spurious orbits with a range of periods related to the scanning law, as we explore in Section 5.2 and Appendix E.

The 2nd case of Equation 1 corresponds to cases where two peaks are detected in the AL flux profile and the IPD processing correctly identifies the centroid of the primary. It is rarely relevant for binaries published in DR3, because resolvable pairs are removed by the cut on `ipd_frac_multi_peak` used in selecting sources for processing with orbital solutions (Appendix B).

4. FITTING THE ASTROMETRIC DATA

We now describe how we fit the epoch astrometry generated above. We attempt to reproduce the procedure used to generate the binary solutions published in DR3 whenever possible, as described by Halbwachs et al. (2023). The astrometric model cascade is illustrated in Figure 2. We emphasize that our main goal is to reproduce the DR3 catalog of orbital solutions (i.e., `nss_solution_type = Orbital` or `AstroSpectroSB1`), so the cascade shown in Figure 2 is somewhat simplified compared to the one actually used in DR3.

It is not known *a priori* which sources are single stars and which are detectable binaries or higher-order multiples. In *Gaia* DR3, the guiding principle was to publish the simplest astrometric solution that could satisfactorily explain the data, beginning with a single-star model.

4.1. Five-parameter single-star solution

All sources are first fit with a 5-parameter, single-star astrometric model. It predicts AL displacements given by

$$\eta = [\Delta\alpha^* + \mu_\alpha^* t] \sin\psi + [\Delta\delta + \mu_\delta t] \cos\psi + \Pi_\eta \varpi. \quad (3)$$

Here $\Delta\alpha^*$ and $\Delta\delta$ represent the source position at a reference time t_{ref} , relative to the reference point (α_0, δ_0) assigned to each source.³ μ_α^* and μ_δ are proper motions, and ϖ is the source parallax. The scan angles, ψ , and parallax factors, Π_η , are precomputed for each source by GOST; the latter are defined such that multiplying the true parallax by Π_η gives the source's parallactic motion in the AL direction at the time of the relevant scan. Note that ψ and Π_η are functions of time.

Given a list of observation times, t_i , measured AL displacements, η_i , their uncertainties, $\sigma_{\eta,i}$, the scan angles

³ For *Gaia* DR3, $t_{\text{ref}} = \text{J2016.0} = \text{JD } 2457389.0$. For DR4, we anticipate $t_{\text{ref}} = \text{J2017.5} = \text{JD } 2457936.875$.

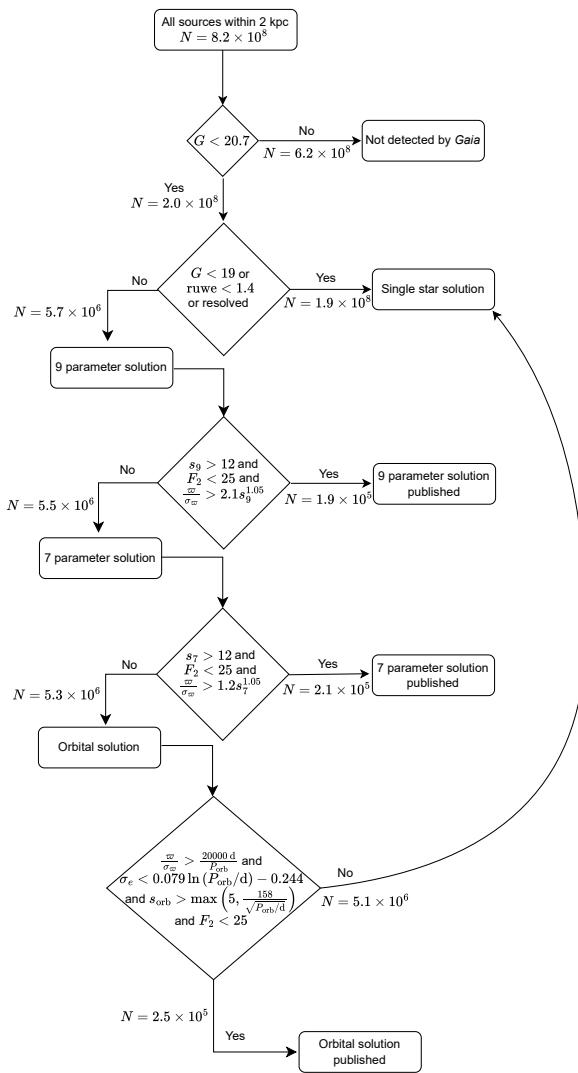


FIG. 2.— Astrometric model cascade used in constructing the mock catalog of orbital solutions. The cascade approximates the one used in generating the DR3 binary catalog [Halbwachs et al. \(2023\)](#), their Figure 1). The guiding principle is to only fit more complex models (e.g. orbits) in cases where simpler models (single-star and acceleration solutions) produce a manifestly poor fit. The number of sources retained and excluded at each branch is indicated. Only a small fraction (< 1%) of all binaries detected by *Gaia* within 2 kpc ultimately receive astrometric orbital solutions in DR3.

ψ_i , and parallax factors, $\Pi_{\eta,i}$, the maximum-likelihood values of the astrometric parameters $\{\Delta_\alpha^*, \mu_\alpha^*, \Delta\delta, \mu_\delta, \varpi\}$ and their uncertainties can be obtained via linear regression. These parameters can then be used to calculate the predicted AL displacements, $\eta_{\text{pred},i}$, and a corresponding χ^2 statistic:

$$\chi^2 = \sum_i \frac{(\eta_{\text{pred},i} - \eta_i)^2}{\sigma_{\eta,i}^2}. \quad (4)$$

From this we calculate a “unit weight error”,

$$\text{UWE} = \sqrt{\frac{\chi^2}{n_{\text{obs}} - 5}}, \quad (5)$$

where n_{obs} is the number of observations and 5 represents the number of free parameters. UWE is analogous to the RUWE statistic published in *Gaia* DR3 ([Lindegren 2018; Lindegren et al. 2021b](#)), where the latter is renormalized to correct for empirical trends in the astrometric uncertainties with color and apparent magnitude.

Various authors have shown that many sources with RUWE > 1.4 are binaries (e.g. [Belokurov et al. 2020; Penoyre et al. 2022](#)). However, not all binaries are expected to have RUWE > 1.4, since in some cases the orbital period is too long or the astrometric uncertainties are too large for the companion to produce detectable deviations from single-object motion. In *Gaia* DR3, only sources with RUWE > 1.4 were processed with binary models. We thus fit all 55 million simulated unresolved binaries satisfying $G < 19$ with single-star solutions, discarding those with UWE < 1.4. This leaves us with 7.3 million sources to fit with more complex models. Following the DR3 procedure, we also discarded 5×10^4 sources that were observed in fewer than 12 visibility periods.

4.2. Acceleration solutions

The simplest extension of the single-star solution is a 7-parameter acceleration solution, which adds two free parameters, $\dot{\mu}_\alpha^*$ and $\dot{\mu}_\delta$, for acceleration in the right ascension and declination directions:

$$\eta = \left[\Delta\alpha^* + \mu_\alpha^* t + \frac{1}{2} \dot{\mu}_\alpha^* t^2 \right] \sin \psi + \left[\Delta\delta + \mu_\delta t + \frac{1}{2} \dot{\mu}_\delta t^2 \right] \cos \psi + \Pi_\eta \varpi. \quad (6)$$

This model is appropriate if the deviation from single-object motion can be satisfactorily described by a constant accelerations, as is expected for binaries with orbital periods much longer than the observing baseline.

A similar 9-parameter model can be defined for sources with variable acceleration, as might be expected for binaries with periods only a few times longer than the observing baseline:

$$\eta = \left[\Delta\alpha^* + \mu_\alpha^* t + \frac{1}{2} \dot{\mu}_\alpha^* t^2 + \frac{1}{6} \ddot{\mu}_\alpha^* t^3 \right] \sin \psi + \left[\Delta\delta + \mu_\delta t + \frac{1}{2} \dot{\mu}_\delta t^2 + \frac{1}{6} \ddot{\mu}_\delta t^3 \right] \cos \psi + \Pi_\eta \varpi. \quad (7)$$

Here $\ddot{\mu}_\alpha^*$ and $\ddot{\mu}_\delta$ represent the acceleration derivatives.⁴

Adding additional free parameters will in general always lead to a better fit, so it is useful to quantify whether the data significantly constrain the additional free parameters. For acceleration solutions,

⁴ Equations 6 and 7 are not identical to the models fit by [Halbwachs et al. \(2023\)](#), their Equations 6 and 7) because they include an additional time offset to make the source positions at the reference epoch similar between accelerating and single-star solutions. This does not affect the magnitude of the inferred accelerations, the significance, or the goodness-of-fit.

“significance” is quantified as the modulus of the two-dimensional vector of additional free parameters (relative to the next-simplest model) divided by its uncertainty:

$$s = \frac{1}{\sigma_1 \sigma_2} \sqrt{\frac{p_1^2 \sigma_2^2 + p_2^2 \sigma_1^2 - 2p_1 p_2 \rho_{12} \sigma_1 \sigma_2}{1 - \rho_{12}^2}}. \quad (8)$$

Here p_1 and p_2 represent the additional parameters of the model, σ_1 and σ_2 their uncertainties, and ρ_{12} the correlation coefficient between them. For the 7-parameter model (Equation 6), p_1 and p_2 represent the acceleration terms, $\dot{\mu}_\alpha^*$ and $\dot{\mu}_\delta$; for the 9-parameter model, they represent the acceleration derivatives, $\ddot{\mu}_\alpha^*$ and $\ddot{\mu}_\delta$.

The goodness-of-fit of acceleration and orbital solution is quantified by the F_2 statistic (Wilson & Hilferty 1931), which is a transformation of χ^2 :

$$F_2 = \sqrt{\frac{9\nu}{2}} \left[\left(\frac{\chi^2}{\nu} \right)^{1/3} + \frac{2}{9\nu} - 1 \right]. \quad (9)$$

Here ν is the number of degrees of freedom; i.e., the number of astrometric measurements minus the number of free parameters in the model. For well-behaved solutions with reliably-estimated uncertainties, F_2 is expected to follow a normal distribution, $\mathcal{N}(0, 1)$. Values of F_2 significantly above 1 suggest a poor solution or underestimated uncertainties. To correct for underestimated uncertainties, the uncertainties of the astrometric parameters are re-scaled by a constant,

$$c = \sqrt{\frac{\chi^2}{\nu \left(1 - \frac{2}{9\nu}\right)^3}}, \quad (10)$$

such that sources with poor fits have inflated uncertainties at fixed apparent magnitude.

In *Gaia* DR3, the 9-parameter variable acceleration solution (Equation 7; *not* the simpler 7-parameter model) was the first model tried for sources with $\text{RUWE} > 1.4$. The acceptance criteria applied before testing the orbital model were:

$$\begin{cases} s_9 > 12 \\ F_2 < 25 \\ \frac{s_9}{\sigma_\varpi} > 2.1 s_9^{1.05} \end{cases} \quad (11)$$

where s_9 is the significance of the 9-parameter solution (Equation 8) and ϖ and σ_ϖ are the parallax and parallax uncertainty of the same solution.

Solutions satisfying these cuts were accepted with 9-parameter solutions (i.e., other models were not tried). Sources not accepted at this stage were fit with 7-parameter acceleration solutions and accepted or rejected using similar criteria:

$$\begin{cases} s_7 > 12 \\ F_2 < 25 \\ \frac{s_7}{\sigma_\varpi} > 1.2 s_7^{1.05} \end{cases} \quad (12)$$

As described by Halbwachs et al. (2023), the acceleration solutions selected at this stage do not constitute the full sample published in the `nss_acceleration_astro` catalog, as some lower-significance “alternative” solutions

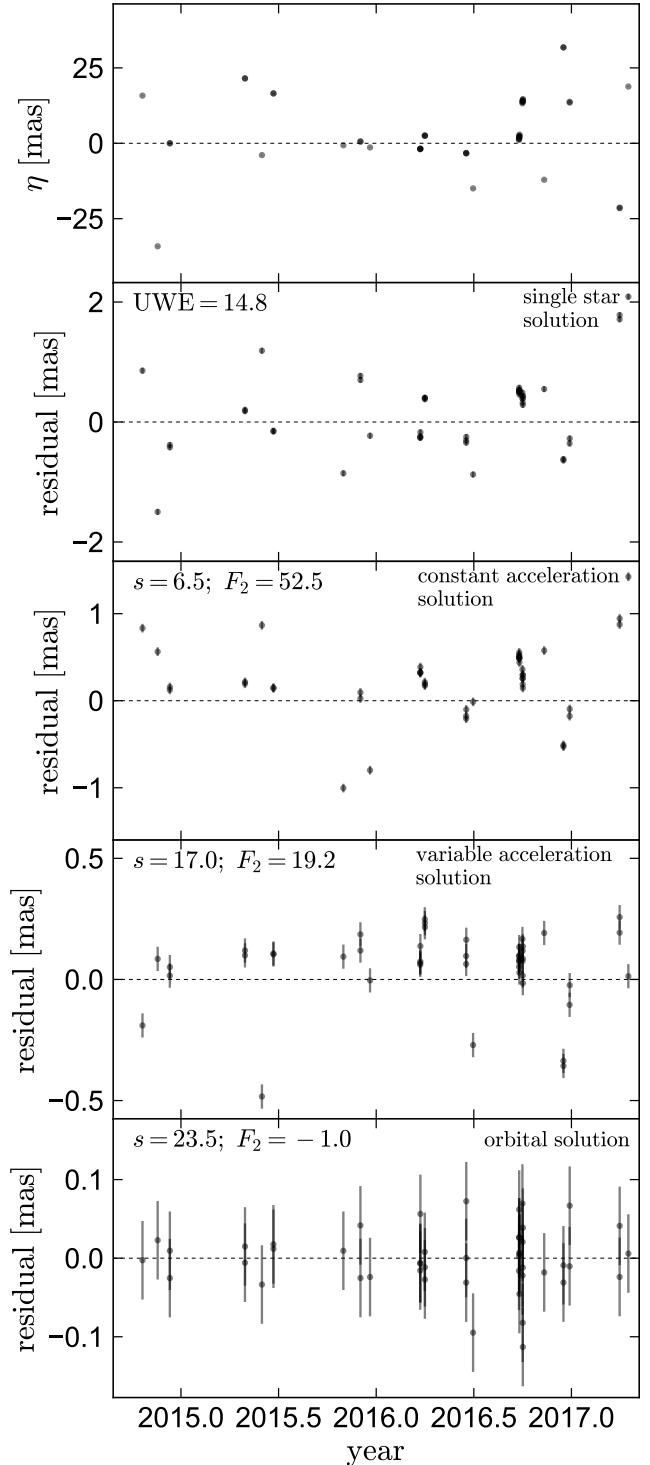


FIG. 3.— Example epoch astrometry for a simulated binary with $P_{\text{orb}} = 902$ d, $e = 0.5$, and $\varpi = 2.5$ mas. Top panel shows the raw AL displacements. Second panel shows residuals with respect to the best-fit single star model, which provides a manifestly bad fit, as quantified by $\text{UWE} = 14.8$. Third and 4th panels show residuals for 7- and 9-parameter acceleration solutions. Although neither of these yields a very good solution, the 9-parameter (variable acceleration) solution provides a good enough fit ($F_2 < 25$ and $s > 12$) that it would have been provisionally accepted in the processing of DR3, and thus no orbital solution would have been attempted for this source in DR3. Bottom panel shows the residuals with respect to the best-fit Keplerian orbital solution, which yields a good solution that would have been accepted *if* the source reached that stage in the astrometric solution cascade.

were added at later stages and more stringent cuts on significance and F_2 were applied later in filtering. But importantly for our purposes, none of the initially accepted acceleration solutions were fit with orbital solutions. We thus remove from further consideration sources whose 7- or 9-parameter acceleration solutions satisfied Equations 11 or 12. A total of 4.1×10^5 sources were thus removed; their properties are examined in Section 5.2.1.

Sources with $\text{RUWE} > 1.4$ that failed the acceptance criteria for both 7- and 9-parameter acceleration solutions were next fit with orbital solutions.

4.3. Orbital solutions

The final astrometric model we consider is a full orbital solution. This model presumes that the 1D astrometry traces the photocenter of two bodies moving in a Keplerian orbit. The displacement in the AL direction is given by

$$\begin{aligned}\eta = & [\Delta\alpha^* + \mu_\alpha^* t + BX + GY] \sin\psi \\ & + [\Delta\delta + \mu_\delta t + AX + FY] \cos\psi + \Pi_\eta \varpi.\end{aligned}\quad (13)$$

Here A , B , F , and G are the Thiele-Innes elements, which for our purposes are free parameters. The nonlinear orbital parameters P_{orb} , e , ϕ_0 enter via the functions

$$X = \cos E - e, \quad Y = \sqrt{1 - e^2} \sin E, \quad (14)$$

where the eccentric anomaly E is the solution to Kepler's equation,

$$E - e \sin E = \frac{2\pi t}{P_{\text{orb}}} - \phi_0. \quad (15)$$

The Thiele-Innes elements are transformations of the angles Ω , ω , i , and a_0 that can also parameterize the photocenter ellipse (representing the longitude of the ascending node, argument of periastron, inclination, and photocenter semi-major axis; e.g. Binnendijk 1960). As described in Appendix A, Equation 13 is linear in 9 parameters and can be solved via matrix inversion once the three nonlinear parameters P_{orb} , e , and ϕ_0 are specified. Unlike with the single-star and accelerating solutions, nonlinear optimization in at least 3 parameters is required to find the best-fit parameters, and the likelihood surface is often bumpy due to period aliases. The problem is qualitatively similar to the problem of fitting a Keplerian model to a radial velocity timeseries, and the same optimizations algorithms that have been employed for that problem are also useful here.

The fitting approach we use is described in detail in Appendix A. In brief, we use adaptive simulated annealing in 3 dimensions to find the maximum-likelihood values of P_{orb} , e , and ϕ_0 . We then calculate uncertainties on all 12 parameters from the Hessian matrix evaluated at the maximum-likelihood solution. Finally, we calculate the photocenter semi-major-axis a_0 , and its uncertainty, σ_{a_0} , from the Thiele-Innes elements. The significance of orbital solutions is quantified as the ratio of the photocenter semi-major-axis to its uncertainty:

$$s_{\text{orb}} = \frac{a_0}{\sigma_{a_0}}. \quad (16)$$

Following *Gaia* DR3, the initial selection criteria for or-

bital solutions were:

$$\begin{cases} s_{\text{orb}} > 5 \\ F_2 < 25 \end{cases} \quad (17)$$

We fit all sources within 2 kpc that satisfy $\text{UWE} > 1.4$ and were not assigned acceleration solutions with an orbital model. Almost 1 million solutions passing the cuts in Equation 17 were saved for further analysis. A majority of these sources were ultimately rejected in post-processing, as described in Section 4.4.

Figure 3 shows simulated epoch astrometry of an example binary. We simulate a binary with $P_{\text{orb}} = 902$ d, $e = 0.5$, and $d = 400$ pc, located at coordinates $(\alpha, \delta) = (153, 34)$ deg. We assume $M_1 = 1.0 M_\odot$ and $M_2 = 0.7 M_\odot$, with both components on the main sequence and a flux ratio $f = 0.1$. This leads to an apparent magnitude $G = 12.5$, corresponding to AL uncertainty of 0.128 mas per CCD transit, or 0.045 mas per FOV transit. The top panel shows the transit-averaged AL displacements, while the next 4 panels show residuals of these displacements with respect to the best-fit single-star model, constant and variable-acceleration models, and Keplerian orbital model. The figure illustrates a perhaps unexpected feature of the astrometric model cascade used in DR3: there are cases where a high-quality orbital solution could have been derived, but an acceleration solution is selected instead. This fact is also discussed by Halbwachs et al. (2023); we investigate accelerations solutions further in Section 5.2.1.

We ran the full cascade of astrometric models on all 46 million unresolved binaries brighter than $G = 19$ within 2 kpc. Running on a single 64-core node, this required about 80 hours of wallclock time, or a total of 5k CPU hours. This suggest that it will be feasible to fit all epoch astrometry published in DR4 with a variety of models at modest computational expense.

4.4. Post-processing

Figure 4 shows properties of the provisionally-accepted orbital solutions satisfying Equation 17. The figure is inspired by Figure 3 of Halbwachs et al. (2023) and shows the same quantities for our simulated solutions as that figure shows for actual solutions calculated with DR3 data. Only 10% of the data are shown in each panel, and the color scale is logarithmic.

For each solution, we calculate the astrometric mass function,

$$f_{m, \text{ast}} = \left(\frac{a_0}{\varpi} \right)^3 \left(\frac{P_{\text{orb}}}{1 \text{ yr}} \right)^{-2}. \quad (18)$$

This quantity is related to the companion's mass and is also useful for distinguishing between real and spurious solutions. For a binary containing a dark companion, $f_{m, \text{ast}}$ represents the mass in solar units required to explain the observed orbit if the luminous star is a massless test particle. Because the photocenter orbit in a luminous binary is significantly smaller than the true semi-major axis – and because the luminous stars are *not* massless – $f_{m, \text{ast}}$ is usually significantly lower than the true companion mass (e.g. Shahaf et al. 2019). The largest mass function expected for binaries containing two main sequence stars is about $0.1 M_\odot$. Binaries in which the primary is a giant or the secondary is itself a

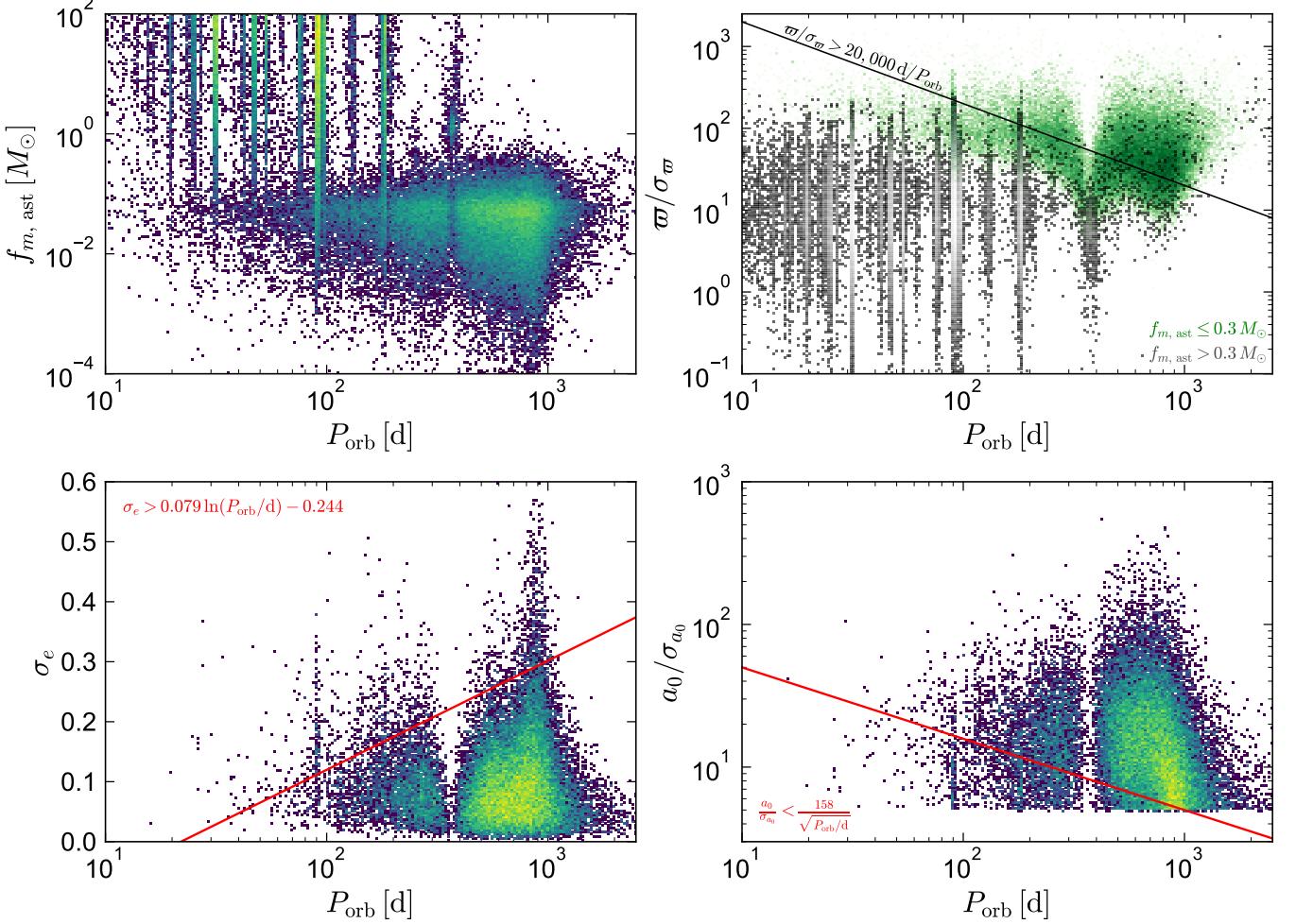


FIG. 4.— Selection of the astrometric solutions for our mock astrometric binary catalog. This figure is inspired by Figure 3 of [Halbwachs et al. \(2023\)](#), who present the same quantities for the astrometric solutions of real sources published in *Gaia* DR3. Upper left panel shows all solutions with $a_0/\sigma_{a_0} > 5$ and $F_2 < 25$. The cloud of points at long P_{orb} and low $f_{m, \text{ast}}$ consists mostly of good solutions, while the vertical stripes of solutions with high $f_{m, \text{ast}}$ and short periods are all spurious. In the upper right, green and gray show the distributions of solutions with $f_{m, \text{ast}} < 0.3 M_{\odot}$ (presumed mostly reliable) and $f_{m, \text{ast}} > 0.3 M_{\odot}$ (presumed mostly spurious). Black line shows the empirical cut (Equation 19) below which solutions are discarded. Bottom panels show only solutions that pass this cut. Red lines in these panels show additional cuts applied to the final solutions: only solutions below (left panel) and above (right panel) the red lines were retained.

close binary can reach $f_{m, \text{ast}} \sim 0.3 M_{\odot}$. Systems containing WDs can approach $f_{m, \text{ast}} \sim 1 M_{\odot}$ in the most extreme cases, but this requires a low-mass primary and a massive WD, so most WD binaries are also confined to $f_{m, \text{ast}} \lesssim 0.3 M_{\odot}$. In summary, a large majority of astrophysical binaries are expected to have $f_{m, \text{ast}} \lesssim 0.3 M_{\odot}$. Orbits with significantly larger $f_{m, \text{ast}}$ are thus either spurious or contain black holes and neutron stars.

The upper left panel of Figure 4 shows the orbital periods and mass functions of binaries initially accepted with orbital solutions. A cloud of solutions with $f_{m, \text{ast}} \lesssim 0.3 M_{\odot}$ and $100 \lesssim P_{\text{orb}}/\text{d} \lesssim 1000 \text{ d}$ is evident, consisting mostly of binaries with well-measured orbital solutions. In addition, there is a secondary population of solutions with P_{orb} mostly falling in several narrow vertical stripes and mass functions extending to much larger values, including well beyond the limits of the plot. As we explore further in Section 5.2, these are almost all wide binaries – with typical periods of 100–1000 years – and spurious apparent astrometric motion on periods related to the

observing cadence. A wide range of periods are present: the most common values are 91, 31.5, and 182 days, but some spurious solutions exist at almost all periods.

[Holl et al. \(2023b\)](#) discuss the origin of these periods in detail. The goodness-of-fit (Equation 9) of the spurious solutions in our simulations is generally unremarkable compared to the good solutions, and there is no simple way to reliably distinguish all of them from solutions of genuine binaries that happen to have the same periods. However, [Halbwachs et al. \(2023\)](#) found that most of the spurious solutions can be removed from the sample with a period-dependent cut on the fractional parallax uncertainty, ϖ/σ_{ϖ} . The cut they used is illustrated in the upper right panel of Figure 4:

$$\frac{\varpi}{\sigma_{\varpi}} > \frac{20,000 \text{ d}}{P_{\text{orb}}} \quad (19)$$

Following [Halbwachs et al. \(2023\)](#), we color sources with $f_{m, \text{ast}} < 0.3 M_{\odot}$ (mostly good solutions) in green in Figure 4, and sources with $f_{m, \text{ast}} > 0.3 M_{\odot}$ (mostly spurious

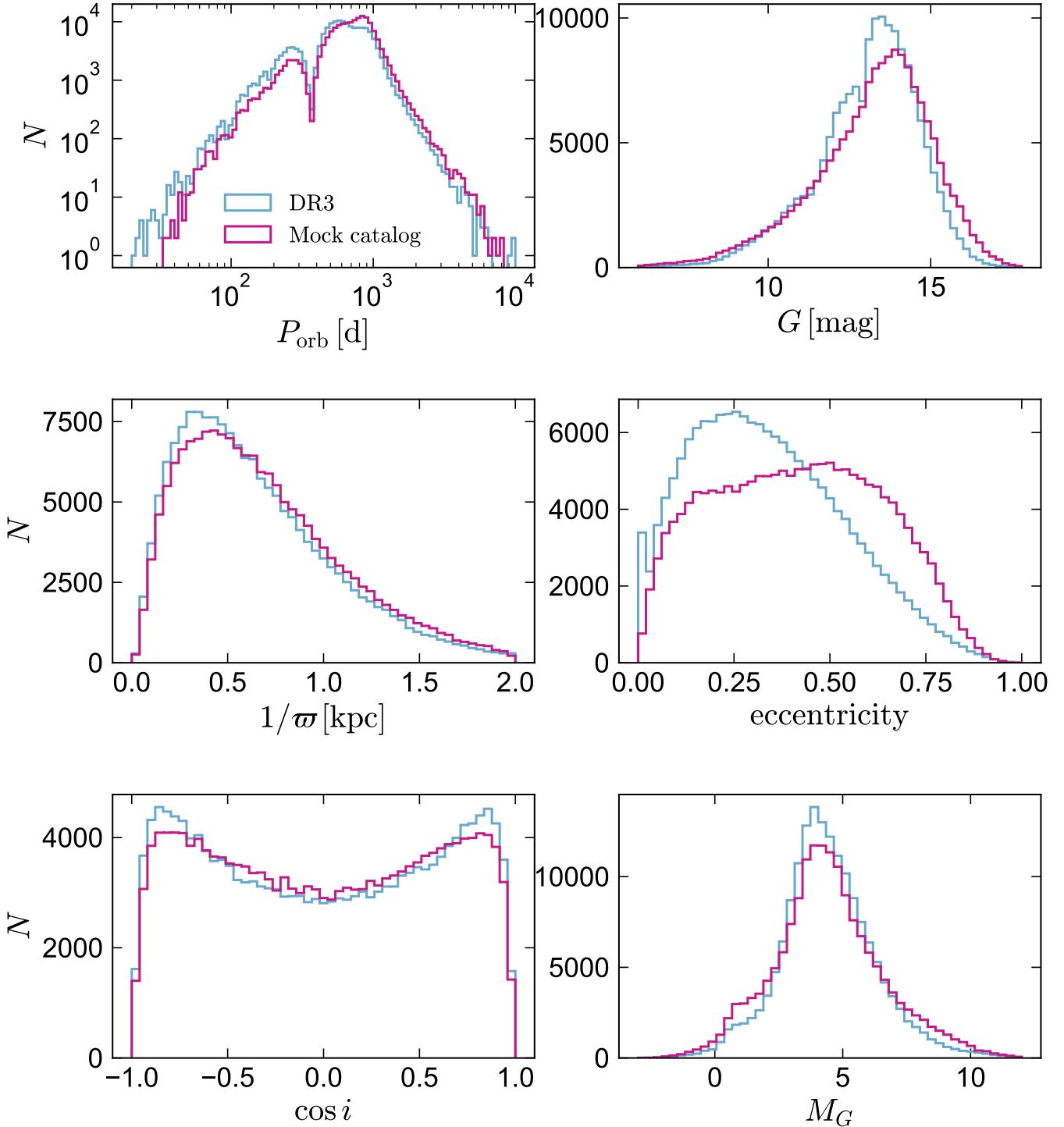


FIG. 5.— Comparison of the astrometric orbital solutions published in DR3 (blue) and the mock catalog after cuts (violet). Each panel shows the distribution of one parameter, marginalized over all other parameters. The mock catalog’s distributions of orbital period, apparent magnitude, distance, inclination, and absolute magnitude are all in reasonably good agreement with the DR3 data. The mock catalog predicts more eccentric orbits than are observed, suggesting the input eccentricity distribution has too many high-eccentricity orbits.

solutions) in gray. A majority of the spurious solutions fall below the black line and are thus filtered out by Equation 19, as was also the case with the real DR3 data.

Filtering of the spurious solutions comes at the expense of a significant number of good orbital solutions for real binaries. In total, about 50% of all the green points in

the right panel of Figure 4 fall below the dashed line and are excluded. At periods shorter than 1 year, the fraction is almost 75%. As expected, binaries with periods near one year have large parallax uncertainties – because their orbital motion is degenerate with parallactic motion – and are essentially all eliminated by Equation 19. This is

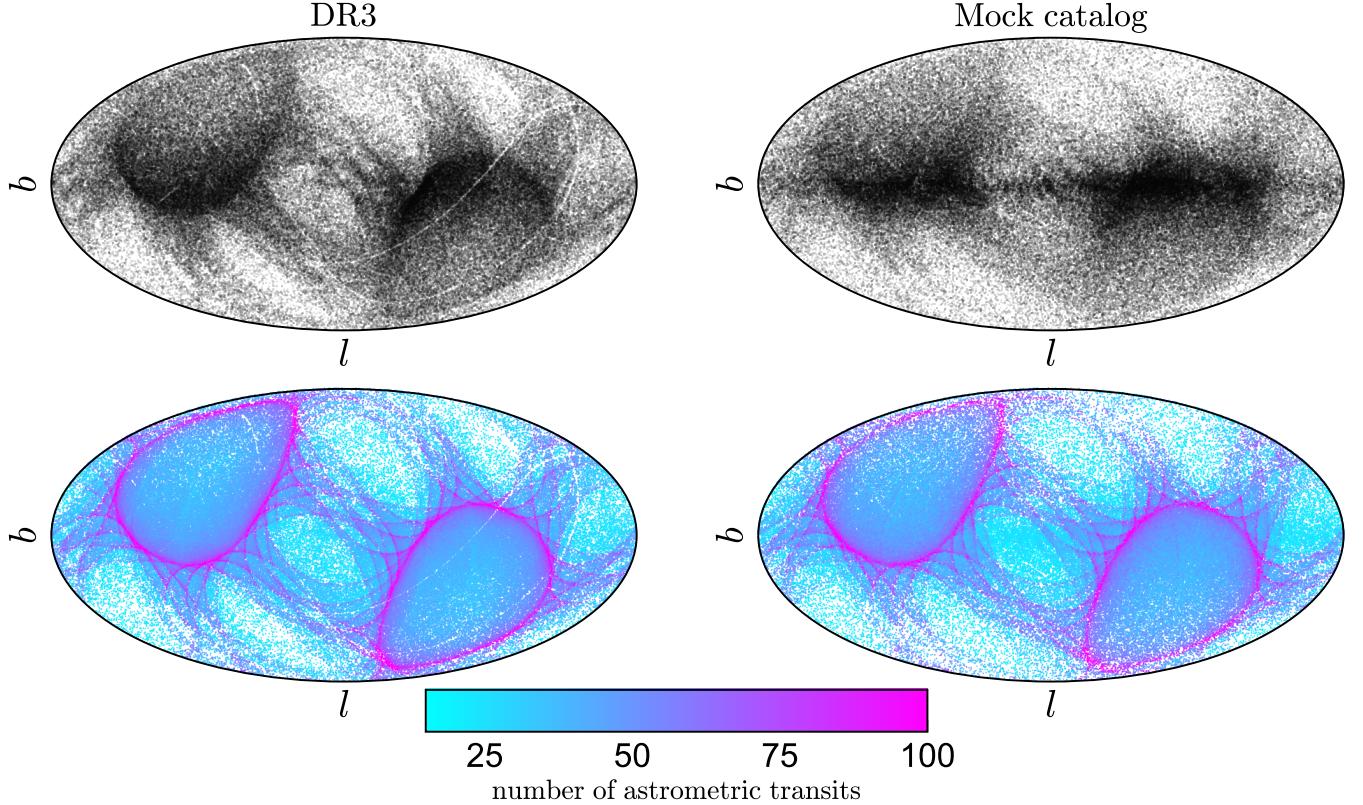


FIG. 6.— Galactic sky distribution of binaries in the DR3 astrometric binary catalog (left) and our mock catalog (right). The Galactic center is in the middle of the projection. In the bottom panels, sources are colored by the number of astrometric transits during the DR3 observing window. Imprints of the *Gaia* scanning law are clearly visible in both catalogs but are sharper in the observed catalog. A few narrow ribbons in which there are no sources in the DR3 catalog – whose origin is uncertain – are not present in the mock catalog.

desirable, since the left panel of Figure 4 shows that most such binaries have vastly overestimated mass functions.

The bottom panels of Figure 4 show the solutions that remain after removal of sources not satisfying Equation 19. The majority of the spurious solutions at short periods have been removed, but some vertical stripes associated with scan angle-dependent periods are still apparent. The panels show the eccentricity uncertainty (lower left) and the photocenter semi-major axis divided by uncertainty (i.e., the significance; lower right). Red lines show two additional cuts imposed by Halbwachs et al. (2023) in post-processing:

$$\sigma_e < 0.079 \ln(P_{\text{orb}}/\text{d}) - 0.244 \quad (20)$$

$$\frac{a_0}{a} > \frac{158}{\sqrt{P_{\text{orb}}/\text{d}}}. \quad (21)$$

These filters were chosen empirically because most of the manifestly spurious solutions fall above and below them, respectively. We retain only the simulated solutions that pass both cuts. After Equation 19 has been applied, only a few percent of the surviving solutions are removed by these cuts. However, Figure 4 shows that the filters are successful in removing the visible band of spurious solutions at $P_{\text{orb}} \approx 91$ d that survived Equation 19.

The cuts described above leave us with a catalog of 249,000 astrometric orbital solutions, which can be compared to the 168,000 solutions published in *Gaia* DR3.⁵ The mock catalog is larger by a factor of 1.5.

Several factors may account for the extra binaries in the mock catalog. Some real binaries were likely removed from the DR3 sample through cuts we did not attempt to model, such as a cut on BP/RP excess factor (Riello et al. 2021). A cut on this quantity was intended to remove marginally resolved binaries from the sample but likely also removed some unproblematic binaries in moderately crowded regions. Our modeling also did not attempt to account for the fact that many binaries are part of triples. Triples are generically less likely to receive an astrometric solution, both because the presence of a marginally-resolved tertiary will disturb the astrometry of the inner binary, and because in some cases the photocenter’s motion will be more complicated than a single Keplerian ellipse. The astrometry of some binaries will also be perturbed by the presence of an unassociated background star, which is not accounted for in our modeling. And finally, the intrinsic binary population

⁵ We note that 33467 of the astrometric orbital solutions published in DR3 are AstroSpectroSB1, meaning that they were computed from a combination of astrometric and RV data. We have not attempted to forward-model the *Gaia* RV data. However, having an accepted Orbital solution was a necessary prerequisite for being processed with an AstroSpectroSB1 solution in DR3, so we expect our modeling to still be largely applicable to the sources with these solutions.

assumed in constructing the mock catalog is unlikely to be an exact match to reality.

We compare the properties of the final simulated and observed catalogs in the following sections. To facilitate clear comparison of the distributions of binary properties between the observed and simulated catalogs, we discard a random 33% of the simulated binaries in the following comparison, such that the simulated comparison sample has the same number of binaries as the observed catalog.

5.1. Comparison to the DR3 catalog

Figure 5 compares the properties of the observed and simulated catalogues of orbital solutions. From upper left to lower right, panels show distributions of orbital period, apparent G -band magnitude, distance, eccentricity, inclination, and absolute magnitude. Most of these parameters have similar distributions in the observed and mock catalogs. The simulated catalog has somewhat fewer short-period orbits relative to long period ones: binaries with periods less than 1 year make up 13% of the mock catalog, but 21% of the full DR3 catalog and 18% of the DR3 solutions with `Orbital` (as opposed to `AstroSpectroSB1`) solutions.

Since short-period binaries are disfavored by the ϖ/σ_{ϖ} cut (Equation 19), a natural question is whether the simulated binaries have larger σ_{ϖ} on average. However, we show in Figure 7 that this does not appear to be the case. Another possibility is that the period distribution assumed in our mock catalog simply has more long-period orbits than reality.

The apparent magnitude distributions of the two catalogs are quite similar, though the discontinuity at $G = 13$ is somewhat more pronounced in the observed sample. The observed sample also peaks at a slightly closer distance and – likely as a corollary – has slightly fewer luminous ($M_G \lesssim 2$) stars. Both samples are biased against edge-on orbits, but the bias is somewhat stronger in the observed sample. All these differences are rather modest.

The most significant difference between the two catalogs is in their eccentricity distributions, where the mock catalog has significantly higher typical eccentricities. It seems most plausible that this simply reflects an imperfect input binary population assumed in our simulations. For solar-type primaries, which dominate the sample, the eccentricity distribution is taken from Moe & Di Stefano (2017) and assumes $p(e) \propto e^{0.2}$ at periods of 100-1000 d. That is, the input eccentricity distribution is more eccentric than uniform. This distribution was inferred by Moe & Di Stefano (2017) by fitting the eccentricities of 15 solar-type binaries with $P_{\text{orb}} = 100 - 1000$ d from the Raghavan et al. (2010) sample. However, only three of these binaries have $e > 0.6$, and we consider it likely that the true eccentricity distribution has less support at high eccentricities than $p(e) \propto e^{0.2}$. Refining the inferred population statistics of binaries in this period range using the *Gaia* sample is a promising avenue for future work.

5.1.1. Sky distribution

Figure 6 compares the sky distribution of binaries in the observed and mock catalog. Like the observed catalog, the mock catalog has an overdensity of binaries in the Galactic plane, but an underdensity toward the Galactic center. This is mainly a consequence of the *Gaia*

scanning law: the bottom panels of the figure show the number of individual FOV transits included in the astrometric solution for each source during the DR3 observing window. The number of FOV transits per source in the mock catalog matches the observed catalog with high fidelity, since the GOST-predicted scanning law is used in simulating our mock observations. Curiously, imprints of the scanning law in the observed catalog are somewhat more pronounced than in the mock catalog, perhaps suggesting that the quality of the *Gaia* observations and/or treatment of systematics varies with position on the sky.

Several narrow ribbons on the sky with essentially no good solutions are also evident in the observed catalog. No such features are present in the mock catalog, indicating that their cause is not included in our modeling. We found that these features all correspond to regions of the sky observed just before or after several “decontamination” events in which the *Gaia* mirrors and CCDs were heated to remove accumulated ice (e.g. Lindegren et al. 2021b). Astrometric measurements obtained close to decontamination events are expected to have poor quality and were discarded when calculating the single-star astrometric solutions reported in the `gaia_source` table. We suspect that these measurements were unintentionally included when calculating orbital solutions, and that the presence of a single spurious astrometric measurement prevented the calculation of a good solution in regions observed close to decontamination events.

5.1.2. Astrometric uncertainties

Figure 7 compares the astrometric uncertainties of the observed and mock catalogs. Recall that these quantities are calculated from the Hessian matrix; i.e., from the curvature of the likelihood function in the vicinity of the best-fit solution. They thus depend mainly on the assumed epoch-level astrometric uncertainties and number of scans per source. The left panels show the photocenter semi-major axis, a_0 , and its uncertainty, σ_{a_0} . As been pointed out in other work (e.g. Shahaf et al. 2023; El-Badry et al. 2023a), these two quantities are correlated in the observed catalog. Such a correlation may not be naively expected, since σ_{a_0} for a given set of orbital parameters and scan cadence is expected to depend only on the epoch-level astrometric uncertainties in the limit where a whole orbit is observed. The same correlation is present in the mock catalog. The correlation arises because of the cut of $s_{\text{orb}} > \max\left(5, \frac{158}{\sqrt{P_{\text{orb}}/\text{d}}}\right)$ employed in constructing the catalog, which removes sources in the upper left of the σ_{a_0} vs. a_0 plane.

5.2. Spurious solutions

Figure 8 investigates the spurious solutions found in the initial mock catalog, most of which are ultimately removed by the post-processing cuts. We define spurious solutions as those whose inferred periods are inconsistent with their true periods by at least 5σ , and good solutions as all others that satisfy $s_{\text{orb}} > 5$ and $F_2 < 25$. We plot a random subset of 10^4 solutions in each class, but note that spurious solutions outnumber good solutions by a factor of ~ 3 in the full sample before additional cuts are applied.

Most of the spurious solutions have true periods of decades to millenia and angular separations of 50-200

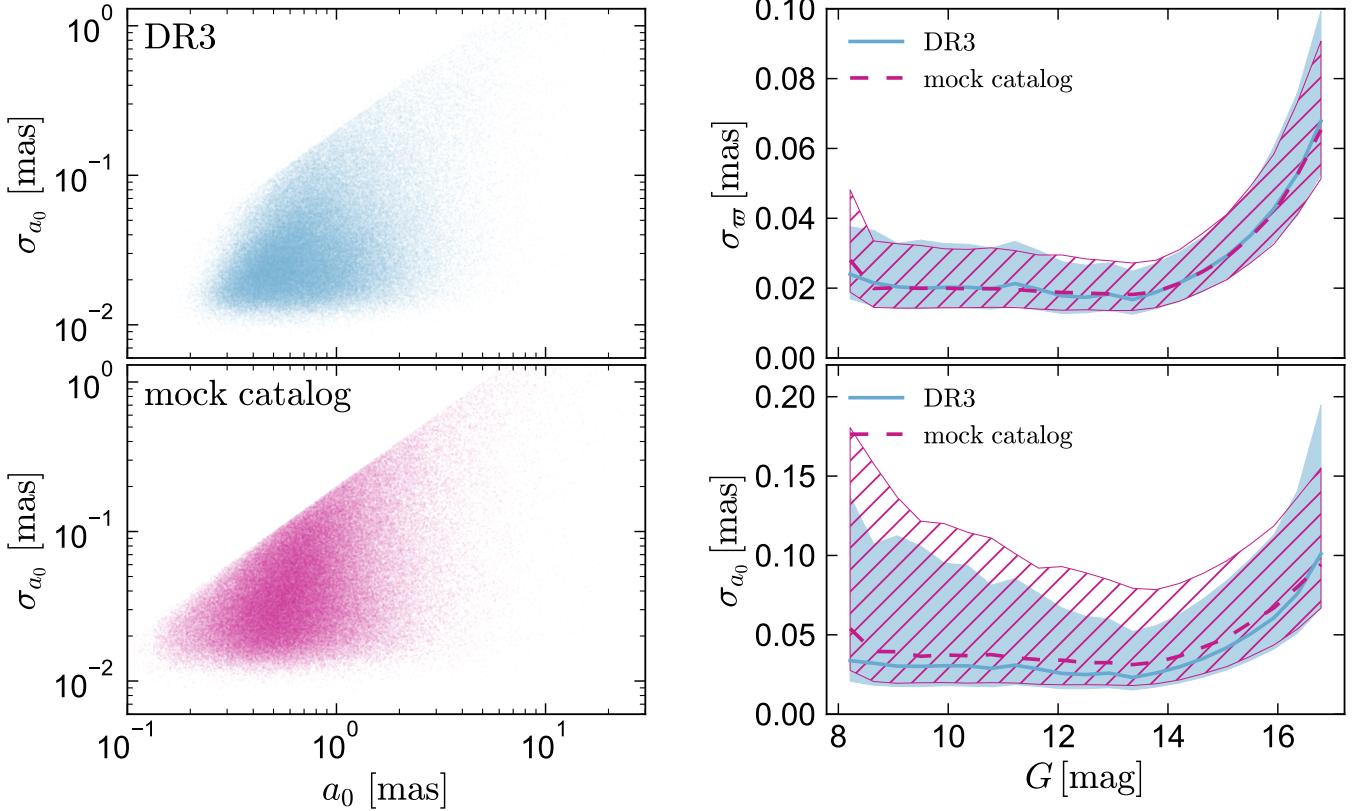


FIG. 7.— Left panels: Photocenter semi-major axis and uncertainty of the *Gaia* DR3 astrometric orbital solutions (top) and solutions in our mock catalog (bottom). In both sets of solutions, σ_{a_0} is correlated with a_0 , but this is primarily a selection effect owing to the cut on $s = a_0/\sigma_{a_0}$. Right panels: uncertainty in parallax (top) and photocenter semi-major axis (bottom) as a function of G -band apparent magnitude. We show the median and middle 68% uncertainty range for both the real and synthetic catalogs. The agreement is generally quite good, though σ_{a_0} is on average slightly larger in the mock catalog.

mas. These binaries experience little actual orbital motion during the DR3 observing baseline, but their marginally resolved separations combined with the one-dimensional nature of *Gaia* astrometry gives rise to spurious scan-angle dependent signals (Appendix E) with a variety of periods related to the scanning law (e.g. Holl et al. 2023b).

In addition to the large population of spurious solutions related to marginally resolved sources, there are also some spurious solutions with shorter true periods. These include (1) a significant population of sources with inferred $P_{\text{orb}} \approx 1000$ d and true $P_{\text{orb}} = 10^{3-4}$ d – for which the uncertainties inferred from linearized fitting are likely unreliable – and (2) a small number of sources with shorter periods for which fitting simply yielded the wrong period. It is possible that the number of spurious solutions with true periods < 1000 d could be reduced through improvements to the optimization algorithm used in orbit fitting (Appendix A), but since these solutions are few in number and are almost all removed in post-processing, we do not concern ourselves with them further here.

Given their short periods, most of the spurious solutions have very large mass functions, and could be eliminated with a cut of $f_{m,\text{ast}} < 1 M_\odot$. This approach is, however, not desirable, because the small number of good solutions with high mass functions are of significant astrophysical interest. Similarly, since spurious solutions

are primarily clustered around particular periods, most of them could be eliminated by filtering out solutions with these periods. A related approach, which is advocated by Holl et al. (2023b), would be to filter out solutions for which the AL displacements can be well-fit by a sinusoidal function of the scan angle. However, this would come at the expense of some true orbits that happen to have periods close to the problematic periods associated with the scanning law.

The bottom panels of Figure 8 shows the distribution of good and bad solutions on the plane of the sky. Although spurious solutions are found across most of the sky, their distribution is different from that of good solutions, and spurious solutions of different periods are concentrated at different positions on the sky (see also Holl et al. 2023b, their Figures C.6 and C.7).

Figure 8 suggests that – since true and spurious solutions have different distributions of many parameters, but do not completely separate in any parameter – a classifier could be devised to distinguish between good and bad solutions with higher purity and completeness than the cuts used in DR3. Indeed, Rybizki et al. (2022) designed such a classifier for single-star solutions, which achieved significantly better performance than cuts on any simple combination of astrometric quality flags. It would be straightforward to train a similar classifier for binary solutions, and astrometry from a mock catalog may form a useful component of the training set.

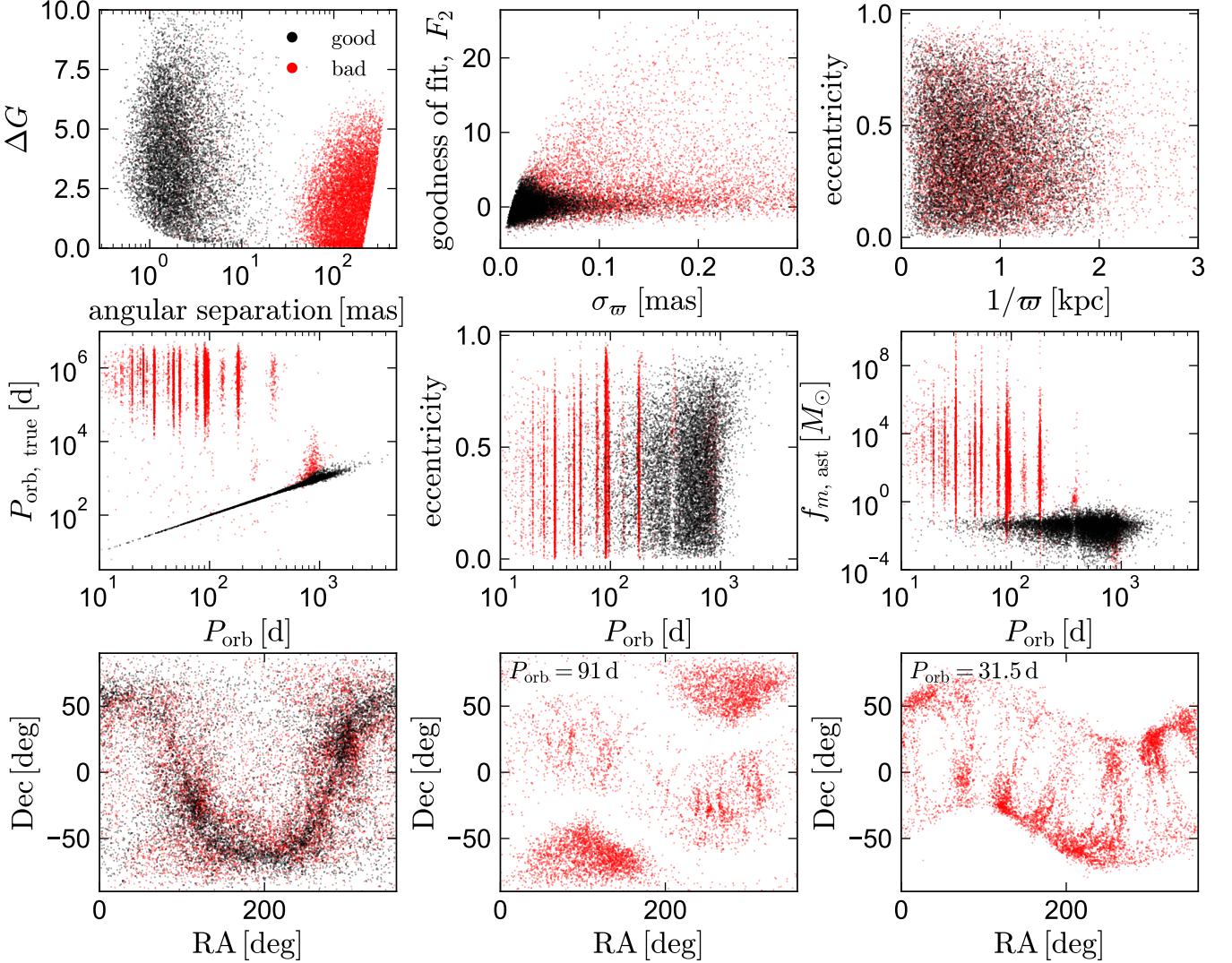


FIG. 8.— Comparison of good (black) and spurious (red) astrometric solutions, before post-processing. We show a random subset of 10^4 solutions in each category. We define spurious solutions as those for which the inferred period differs from the true period by at least 5σ . Upper left shows true angular separation and magnitude difference between the two stars. A large majority of the bad solutions are marginally resolved pairs with separations of order 100 mas. The inferred eccentricities, parallaxes, parallax uncertainties, and goodness of fit values of these solutions largely overlap with the good solutions. Almost all of the spurious solutions have inferred periods of a year or less – mostly related to the scanning law – while 85% of the true solutions have longer inferred periods. Most, but not all, of the bad solutions have unphysically large mass functions. Bottom panels shows the sky distributions. Spurious solutions are found across most of the sky, but spurious solutions of different periods are found in different locations.

5.2.1. Accelerating solutions

Figure 9 shows properties of a random subset of the sources in the mock catalog that received 7- and 9-parameter acceleration solutions. Recall that sources for which an accelerating solution provided a “good” fit (Equations 11 and 12) were removed from the astrometric model cascade and never fit with orbital solutions in DR3.

As one might expect, a majority of the sources with acceleration solutions have orbital periods longer than the DR3 observing baseline, and sources with 7-parameter solutions have longer periods on average than those with 9-parameter solutions. However, the period distributions are quite broad, and both sets of solutions have a short-period tail extending to $P_{\text{orb}} < 1000$ d, with the shortest-period 9-parameter solutions having periods of order 500

d. The presence of this short-period tail will complicate any attempts to infer companion masses for sources with acceleration solutions. A majority of these short-period solutions would have passed the post-processing cuts with full orbital solutions. In future data releases, it will be possible to fit their epoch astrometry with orbital solutions.

5.3. Predictions for DR4 and DR5

To make predictions for the *Gaia* DR4 binary sample, we include observations predicted by **GOST** before 20 January, 2020, corresponding to 1977 days of observations. For DR5, we include all predicted observations until January 2025. We assume that the epoch-level astrometric uncertainties and unmodeled additional noise will be the same as in DR3 – likely making these predictions conservative, since treatment of systematics is expected to

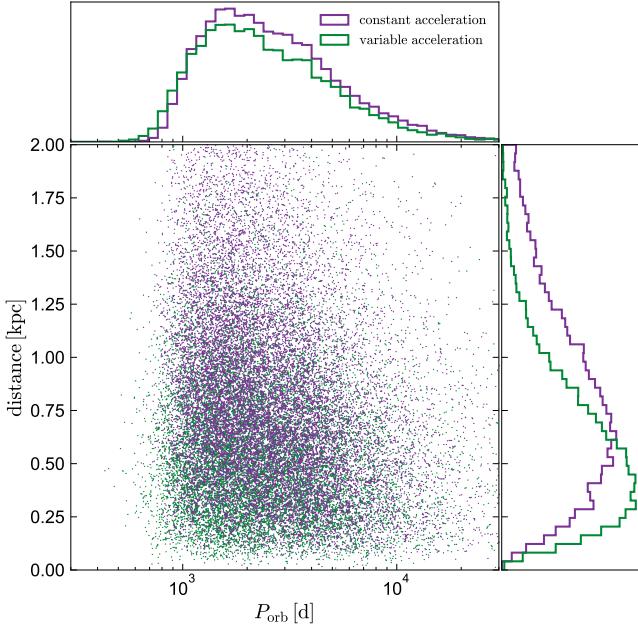


FIG. 9.— Orbital periods and distances of the accelerating solutions in the mock catalog. We show 10% of all solutions. The period distributions of sources with variable-acceleration and constant-acceleration solutions peak at 2300 and 2400 d, respectively, but both extend to periods of $> 10^4$ d. The fraction of solutions with true periods below 1000 d is 7% and 4%, respectively.

improve in future releases. We also employ the same selection criteria and post-processing cuts (Equations 19–21), though these may be relaxed in future releases.

Figure 10 shows results of simulations including observations with the DR4 (top) and DR5 (bottom) time baselines. Comparing to Figure 4, it is clear that (a) most good solutions will be available at longer orbital periods in future releases, and (b) there will be significantly fewer spurious solutions. The smaller number of spurious periods likely owes in part to a reversal in *Gaia*'s precession during 2019–2020 (including the last 6 months of data included in DR4; Lindegren et al. 2021b), which eliminates some spurious periods. The “hole” near $P_{\text{orb}} \approx 1$ yr is also predicted to become progressively narrower in DR4 and DR5.

In the lower left panel of Figure 10, a ridge of higher-mass function binaries can be seen at $P_{\text{orb}} \gtrsim 2000$ d. This is made up primarily of binaries containing a WD, which produce large photocenter orbits and are predicted to be common at long periods, where they can escape a common envelope event during the late-stage evolution of the WD progenitor. To avoid flagging these orbits as spurious solutions, we increase the threshold above which solutions in the right panels of Figure 10 are colored gray to $f_{m, \text{ast}} = 1 M_{\odot}$, compared to $0.3 M_{\odot}$ in Figure 4.

Figure 11 compares the predicted yields of the final astrometric catalog in DR3 (black), DR4 (gold), and DR5 (magenta). Here we employ exactly the same quality cuts used in DR3. Our modeling predicts 1.2 million astrometric binaries in DR4, a factor of 5 increase compared to its predictions for DR3. This increase owes both to the longer observing baseline – leading to a detected period distribution that peaks at ~ 1600 days – and to the increased number of observations per object, which results

in smaller parallax uncertainties and better constraints on a_0 . Because longer-period binaries have larger orbits, they are astrometrically detectable to larger distances, causing the volume within which binaries are detectable to grow rapidly as the observing baseline increases.

Typical binaries in the DR4 sample are predicted to be somewhat fainter and more distant than in the DR3 sample, with a median distance and apparent magnitude of $d = 0.85$ kpc and $G = 14.3$, compared to $d = 0.63$ kpc and $G = 13.6$ in DR3. An additional improvement anticipated in DR4 is the publication of epoch astrometry for all sources, including those not satisfying Equations 19, 20, and 21. If we ignore those filters and only require $a_0/\sigma_{a_0} > 5$, our modeling predicts 1.5 million binaries in DR4. The improvement compared to the case with no cuts is smaller than in DR3, because the filters primarily remove sources with short periods, for which the search volume is smaller.

Another significant improvement can be expected in DR5: the same modeling and cuts used in DR3 leads to a predicted 3.9 million orbital solutions in DR5. The true number can be expected to be even larger, because our `Galaxia` simulation only extends to 2 kpc, and Figure 11 suggests that a significant number of binaries detectable in DR5 will be beyond this limit. The period distribution in DR5 is predicted to peak at $P_{\text{orb}} \approx 3000$ d, with a median apparent magnitude of $G = 15.0$ and a median distance of $d = 1.01$ kpc. In the likely event that treatment of systematics improves in future data releases, the expected sample of orbits will be even larger. It could likely also be enlarged by reducing the threshold on a_0/σ_{a_0} below 5.

6. A MONTE CARLO SELECTION FUNCTION

Our modeling makes it possible to ask, for any hypothetical binary with particular properties (sky position, apparent magnitude, period, flux ratio, mass, distance, etc.), whether it would likely have been detectable in a given *Gaia* data release: we can simply simulate many realizations of the binary with different orientations, noise draws, etc., fit the mock astrometry for each realization, and calculate what fraction of the mock fits result in a solution that passes all of the quality cuts imposed on the published solutions. The main downside of this approach is that it is slow, and like any Monte Carlo method, it is noisy when the number of simulated realizations is small. For a complementary approach based on analytic approximations of the selection function, see Lam et al. (in preparation).

6.1. A main-sequence binary

Figure 12 illustrates this approach for a luminous binary. We consider a system containing two MS stars with a $1.0 M_{\odot}$ primary and mass ratio $q = M_2/M_1 = 0.7$. In the fiducial case, we consider a distance of 0.7 kpc, a period of 500 d, an eccentricity of 0.3. We simulate 500 realizations of this binary, assigning each one a different sky position and corresponding extinction and scan pattern, and a different random orientation and phase, as quantified by Ω , ω , i , and T_p . In assigning random positions at a given distance, we assume an underlying stellar population distributed as an exponential disk with scale height $h_z = 0.3$ kpc.

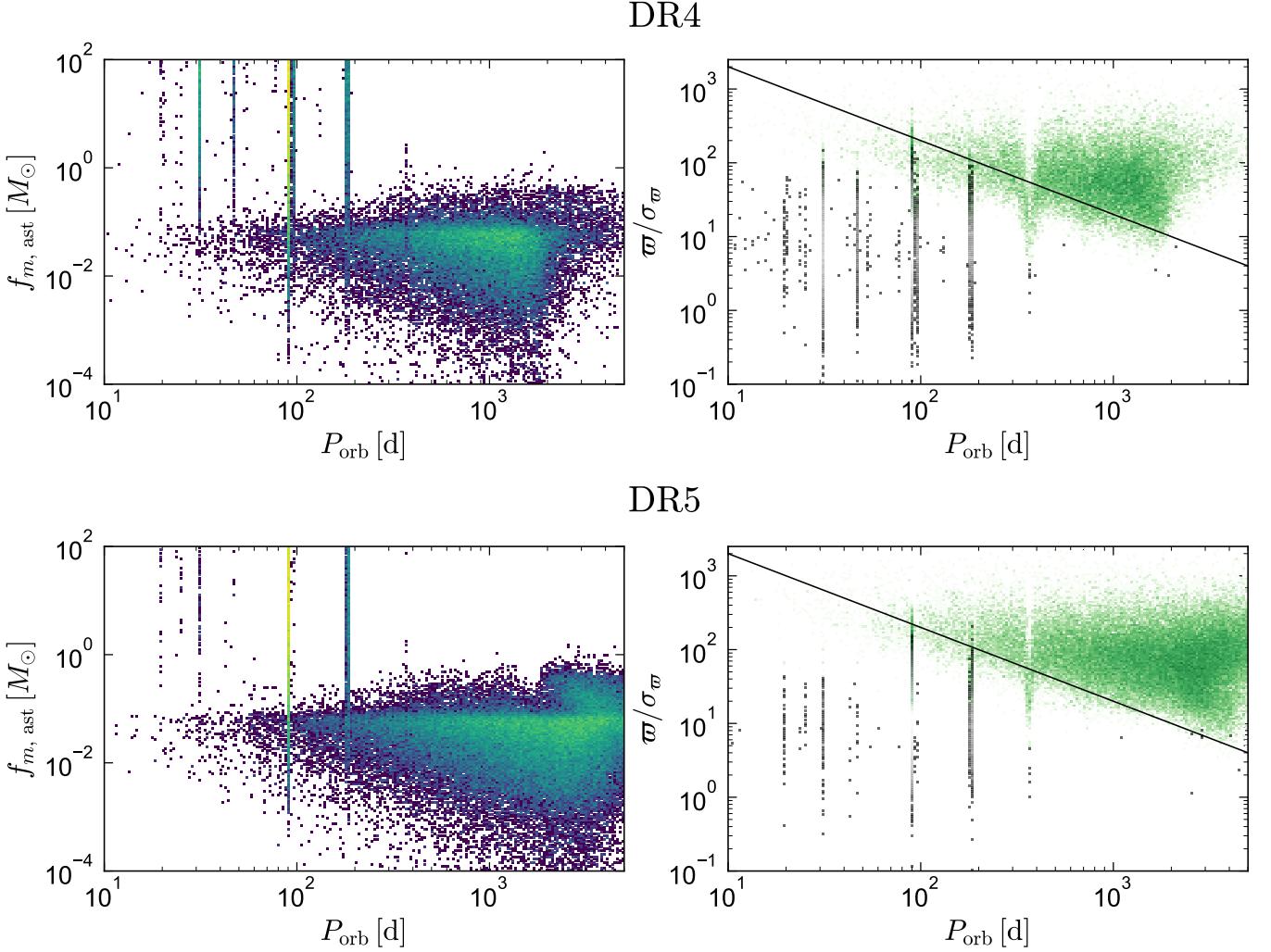


FIG. 10.— Analogous to the top panels of Figure 4, but for *Gaia* DR4 (top) and DR5 (bottom). Note that the x-axis extends to longer periods than in Figure 4. We show 2% of all solutions. The number of spurious periods associated with the scanning law is predicted to decrease compared to DR3, while the parallaxes uncertainties will decrease and sensitivity to long orbital periods will increase. Most solutions satisfying a_0/σ_{a_0} in DR5 also fall above the w/σ_w cut employed in DR3 (Equation 19).

We fit the simulated epoch astrometry for each binary with the astrometric cascade described in Section 4, and finally calculate what fraction of the simulated binaries lead to accepted orbital solutions that survive all the quality cuts imposed in post-processing. Figure 12 shows the results of such experiments for a range of distances, eccentricities, orbital periods, and mass ratios. In each panel, we show how the detection probability changes when one property is varied and the rest are held fixed. Black and gold lines show detection probabilities in DR3 and DR4, respectively.

The detection probability in DR3 falls with distance as expected, but the fall-off is relatively gradual, dropping from 85% at 0.5 kpc to 63% at 0.7 kpc, 25% at 0.9 kpc, and 1% and 1.1 kpc. The detection probability decreases with increasing eccentricity, but the drop-off only becomes steep at $e > 0.6$. The DR3 detection probability is largest near periods of 600 days, decreasing at shorter periods due to smaller orbits and at longer periods due to less complete phase coverage. Sensitivity to periods of 300-400 days is also severely reduced. Sensitivity varies weakly with q between 0.2 and 0.85 due to the competing

effects of a larger orbit at large q and less light blending at small q . Not surprisingly, all detection probabilities increase in DR4. The increase is most significant at periods longer than 1000 d, which are inaccessible in DR3.

6.2. Compact object companions

Calculations similar to those shown in Figure 12 can be carried out for any hypothetical class of binary. As another test case, we consider wide compact object binaries with properties similar to *Gaia* BH1 (El-Badry et al. 2023b; Nagarajan et al. 2024), BH2 (El-Badry et al. 2023c), BH3 (Gaia Collaboration et al. 2024b), and *Gaia* NS1 (El-Badry et al. 2024). As in the previous case, we generate 500 realizations of each system spread across the sky. The masses, orbital periods, and absolute magnitudes we assume for each system are listed in Table 1. Figure 13 shows detection probabilities as a function of distance. Note that the x-axis varies between panels.

Both *Gaia* BH1 and BH2 had a lower than 50% probability of being detected and passing all quality cuts to be published in DR3. For BH1, this reflects the fact that the observed parallax significance was only barely high

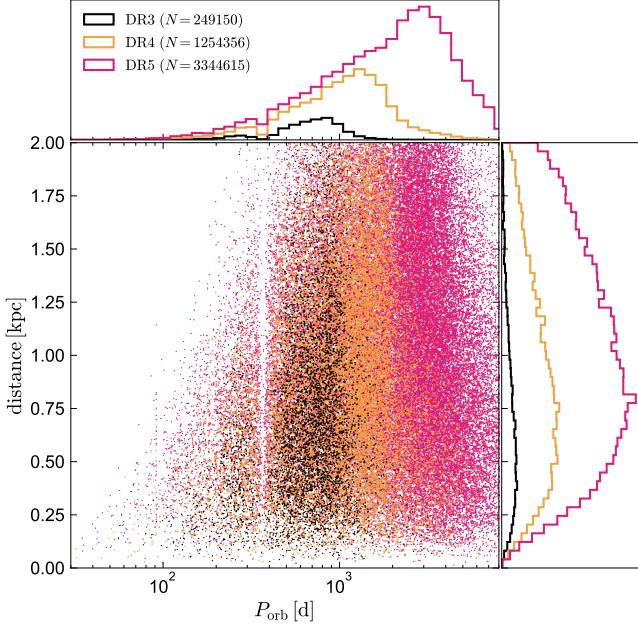


FIG. 11.— Comparison of predicted binary populations in DR3 (black), DR4 (gold), and DR5 (magenta), assuming exactly the same cuts are employed in future releases as in DR3. 2% of the full predicted sample is shown. The DR4 and DR5 samples are predicted to be ~ 5 and ~ 12 times larger than the DR3 sample. This is primarily because these data will be sensitive to binaries with longer periods, and longer-period binaries have larger orbits that can be detected to larger distances.

enough to satisfy Equation 19; for BH2, it is a consequence of the fact that the orbital period is longer than the DR3 observing baseline. In DR4, the observing baseline will be longer and the parallax uncertainties smaller, so the detection probabilities will rise to 82 and 99.6%. Gaia BH3 has an orbital period much longer than the DR3 observing baseline and thus would have been detectable in DR3 only for the most fortuitous orbital orientations and phases. In DR4, the system’s detection probability at its observed distance is 47%, reflecting the fact that the orbital period still exceeds the observing baseline by more than a factor of 2. Our modeling predicts that Gaia NS1-like binaries would have received an orbital solution for 74% of orientations and sky positions in DR3, and for 95% in DR4.

TABLE 1
Adopted properties of four BH/NS binaries and detection probabilities in *Gaia* DR3 and DR4. Detection probabilities as a function of distance are shown in Figure 12.

System	$M_{G,0}$ [mag]	P_{orb} [d]	M_* [M_\odot]	M_2 [M_\odot]	e	d [kpc]	P_{DR3}	P_{DR4}
Gaia BH1	4.50	185.4	0.93	9.3	0.43	0.48	0.31	0.82
Gaia BH2	1.38	1277	1.0	9.0	0.52	1.16	0.43	0.99
Gaia BH3	1.72	4250	0.78	33.0	0.73	0.59	0.12	0.47
Gaia NS1	3.75	731	0.78	1.90	0.12	0.73	0.74	0.95

6.3. Giant planets

Figure 14 shows the results of similar calculations for giant planets, which are ultimately expected to be detected by *Gaia* in large numbers (e.g. Casertano et al. 2008; Sozzetti et al. 2014; Perryman et al. 2014). For

the fiducial case, we consider a $5 M_{\text{Jup}}$ planet orbiting a $0.2 M_\odot$ star at a distance of 40 pc, with a period of 500 d and eccentricity of 0.2. We then show the effects of varying distance, stellar mass, orbital period, and planet mass in each panel, and again compare the DR3 and DR4 completeness. We note that the Figure assumes the same conservative quality cuts used in producing the general catalog of astrometric orbits; in reality, looser quality cuts may be adopted for planet candidates (e.g. Holl et al. 2023a).

Figure 14 suggests that *Gaia* DR3 is nearly complete for the fiducial case within 25 pc, while DR4 will be nearly complete within 50 pc. The sensitivity varies non-monotonically with M_* because higher-mass stars wobble less due to a fixed-mass planet but are brighter and thus have more precise astrometry at fixed distance, except at the closest distances where calibration systematics become important. The sharp drop-off visible at $\approx 0.9 M_\odot$ reflects the fact that at $d = 40$ pc, stars above this mass are brighter than $G \approx 8$, where *Gaia* astrometry becomes significantly less precise (Figure 1).

A key takeaway from Figure 14 is that *Gaia* is *much* more sensitive to massive planets with $M \approx 10 M_{\text{Jup}}$ than to Jupiter-mass or lower-mass planets. This is explored further in Figure 15, which shows detection probabilities estimated from simulated DR4 observations for planets with $P_{\text{orb}} = 1500$ d (near optimal for DR4) and a range of stellar and planet masses. For $1 M_{\text{Jup}}$ planets (left panel), we find that DR4 will be $> 50\%$ complete within ≈ 40 pc for $M_* = 0.2 M_\odot$ and $0.5 M_\odot$. The predicted sensitivity to $1 M_{\text{Jup}}$ planets is significantly lower for $1 M_\odot$ stars, which wobble less and are in the systematics-dominated bright star regime at the relevant distances.

On the other hand, at $M_{\text{planet}} = 10 M_{\text{Jup}}$, the 50% completeness limit ranges from 120 to 240 pc and increases with M_* . At distances $d < 300$ pc, the total number of stars within a distance d scales nearly as d^3 , and so a 3–10 times farther detection limit at $M_{\text{planet}} = 10 M_{\text{Jup}}$ compared to $M_{\text{planet}} = 1 M_{\text{Jup}}$ translates to a 27-1000 times larger search volume.

The predictions in Figure 15 can be compared to those of Holl et al. (2022), who forecasted the number of brown dwarfs and giant planets detectable with a 5-year *Gaia* mission (comparable to the DR4 observing baseline). They found that a $10 M_{\text{Jup}}$ planet orbiting a solar-type star could be detected with $> 50\%$ probability out to a distance limit of 300 pc, which is comparable to but somewhat farther than our prediction of 240 pc. Similarly, they predict a 50% completeness limit of 250 pc for a $10 M_{\text{Jup}}$ planet orbiting a $0.65 M_\odot$ MS star, whereas we find a somewhat closer limit of 195 pc. We compare to their predictions further in Appendix D, concluding that their slightly more optimistic predictions are mainly the result of (a) adopting a lower detectability threshold than the limits that were used for *Gaia* orbits published in DR3, and (b) assuming somewhat more optimistic epoch-level astrometric uncertainties than have been achieved thus far.

The occurrence rate of giant planets in wide orbits is still somewhat uncertain, and we do not attempt to predict the number that will eventually be discovered by *Gaia* here. The method we developed can be straightforwardly applied to a simulated planet population using

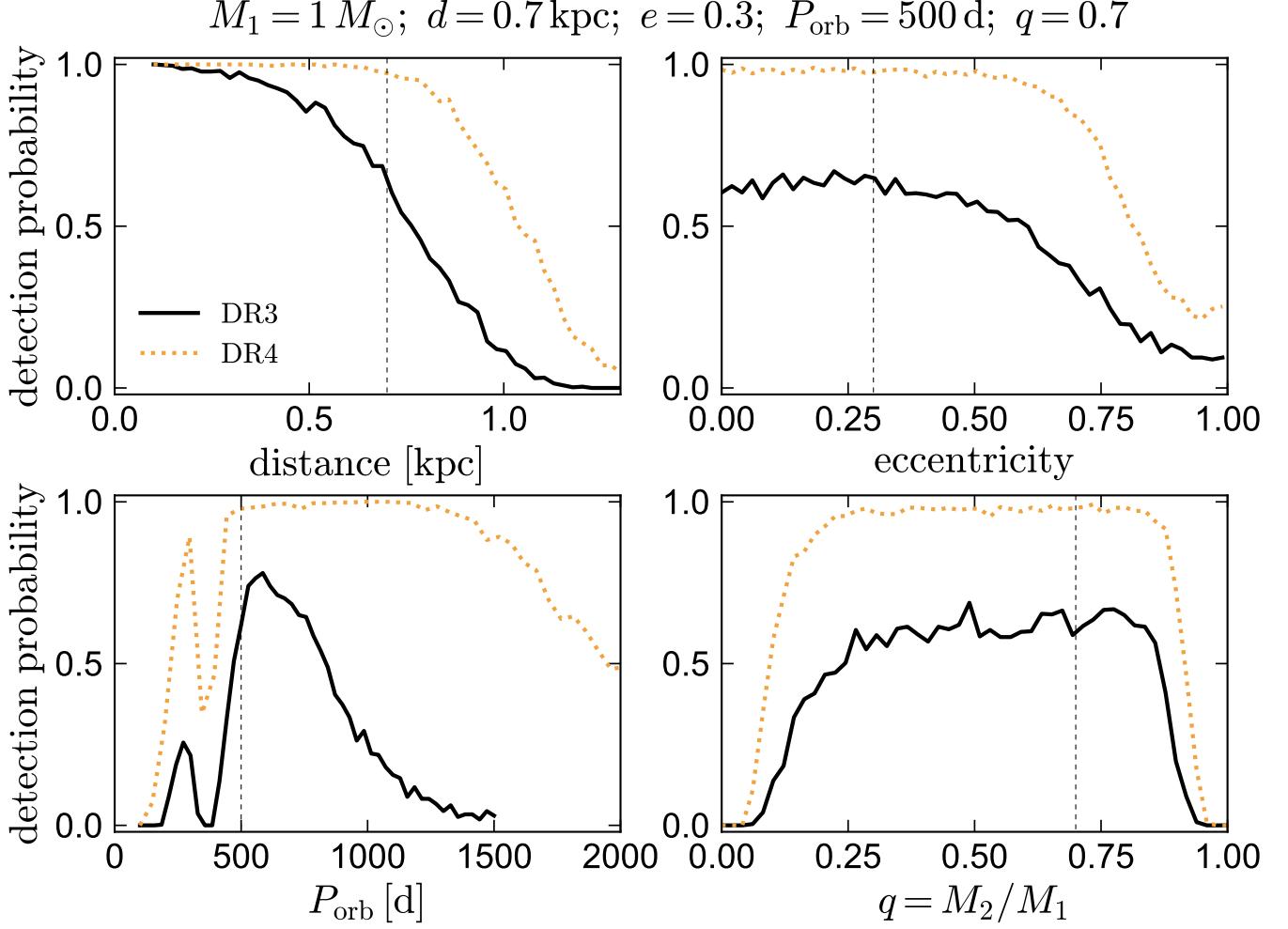


FIG. 12.— Probability of a binary containing two MS stars being published with an astrometric orbital solution in DR3 (solid black) or DR4 (dotted yellow). For the fiducial case, we take $M_1 = 1 M_\odot$, $d = 0.7 \text{ kpc}$, $e = 0.3$, $P_{\text{orb}} = 500 \text{ d}$, and $q = 0.7$. From top left to lower right, panels show the effects of varying distance, eccentricity, orbital period, and mass ratio. Other parameters are fixed at their fiducial values, which are marked with dashed vertical lines. For each combination of parameter values, we simulate 500 binaries distributed across the sky, drawn from an exponential disk with scale height $h_z = 0.3 \text{ kpc}$. We account for position-dependent extinction and *Gaia* scanning frequency.

the same strategy we used for binary stars in Section 5 (see also Holl et al. 2022). We note that most papers making predictions for *Gaia* astrometric exoplanet discoveries in the past assumed a 5-year mission duration, but the actual duration will foreseeable be 11 years. As we showed in Figure 11, the expected asymmetric yield from an 11-year mission (DR5) is at least a factor of 3 larger than that of a 5-year mission (DR4), because the search volume increases rapidly with increased sensitivity to long-period orbits.

7. SUMMARY

We have produced an approximate generative model for the sample of astrometric binaries with orbital solutions published in DR3, which appears to capture the most important selection effects of the real catalog. The method can also straightforwardly be applied to future *Gaia* data releases. As we will explore in other work, this approach is quite useful for interpreting the contents of observed binary catalogs, performing population inference, and forecasting the contents of future data re-

leases.

Our key results are as follows:

1. *A public code for generating and fitting epoch astrometry:* We provide Python code to simulate and fit *Gaia* epoch astrometry [here](#). The code is suitable for fitting large numbers of sources with a variety of astrometric solutions, as will be possible after *Gaia* DR4.
2. *Mock catalog:* We use the binary population demographics model of Moe & Di Stefano (2017) to populate a synthetic Galaxy with binaries and forward-model their *Gaia* observations and astrometric model fitting (Figure 3). We apply the same quality cuts used to construct the *Gaia* DR3 astrometric orbit sample to the mock data (Figure 4). This results in a mock catalog whose properties are similar but not identical to the catalog of astrometric binaries published in DR3 (Figures 5, 6, and 7). The most significant tension is in the eccentricity distribution, where the mock catalog contains more

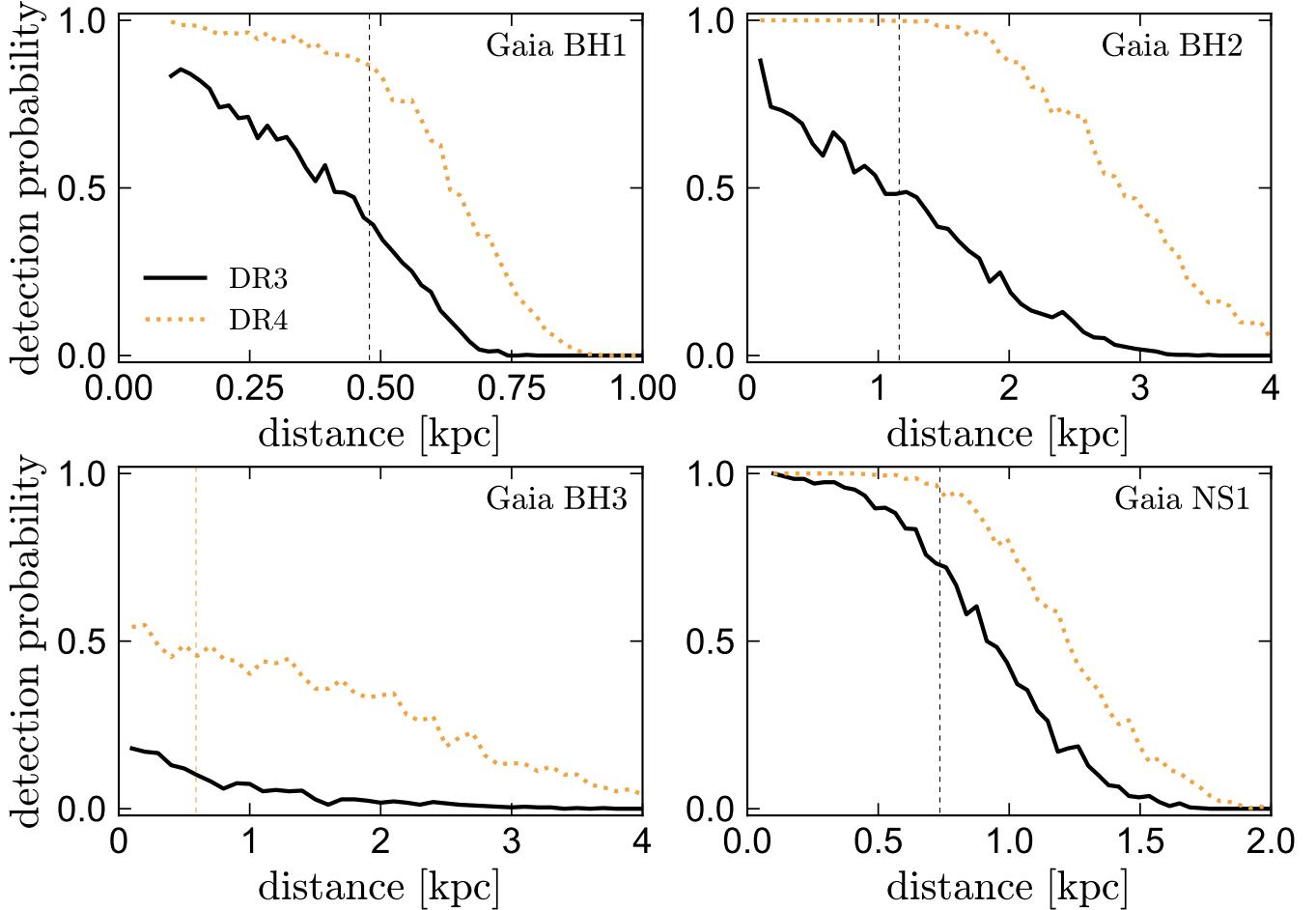


FIG. 13.— Probability of detecting known BH/NS + luminous star binaries in Gaia DR3 (solid black) and DR4 (dotted gold). At each distance, we simulated 500 realizations of each system, spread across the sky and with different orientations. The properties assumed for each system are listed in Table 1. Dashed vertical lines show the actual distances to the four systems.

high-eccentricity orbits. Varying the properties of the assumed binary population and comparing to the observed catalog is a promising avenue to refine the underlying binary model. Such efforts are left to future work.

3. *Spurious solutions:* Our forward-modeling predicts a population of spurious orbital solutions with properties similar to the spurious solutions initially selected in DR3, most of which were ultimately filtered out by stringent quality cuts (Figure 4). A large majority of the spurious solutions have one of a few discrete periods related to the *Gaia* scanning law and imply implausibly large astrometric mass functions (Figure 8). Most of the spurious solutions arise from marginally-resolved wide binaries with orbital periods much longer than the *Gaia* observational baseline (Appendix E). Many but not all of these systems can be resolved with ground-based high-resolution imaging. This modeling suggests that *Gaia* will be significantly less sensitive to astrometric binaries that are in triples, where a distant tertiary will often disturb the astrometry of the inner binary.

4. *Acceleration solutions:* About twice as many acceler-

erating solutions as orbital solutions are accepted into the mock catalog. The most common true orbital periods for 9- and 7- parameter accelerating solutions in the DR3 mock catalog are ~ 2300 and ~ 2400 days, respectively, about twice the DR3 observing baseline (Figure 9). A significant number of sources for which 9-parameter solutions are accepted could have yielded high-significance orbital solutions but were not fit with orbital solutions because the significance of the acceleration solution was high enough for the source to be removed from further processing in DR3. Acceleration solutions are of limited astrophysical utility – without a known orbital period or angular separation, only coarse limits can be set on the nature of the companion. Epoch astrometry in DR4 and beyond will allow orbital solutions to be constrained for many of the sources with accelerating solutions in DR3.

5. *Predictions for DR4 and DR5:* Our modeling predicts that *Gaia* DR4 will yield at least 5 times as many orbital solutions as DR3, even if the same conservative quality cuts are adopted as in DR3 and the treatment of systematics does not improve. Another factor of ~ 3 improvement is expected in DR5 (Figure 11). Most of the new solu-

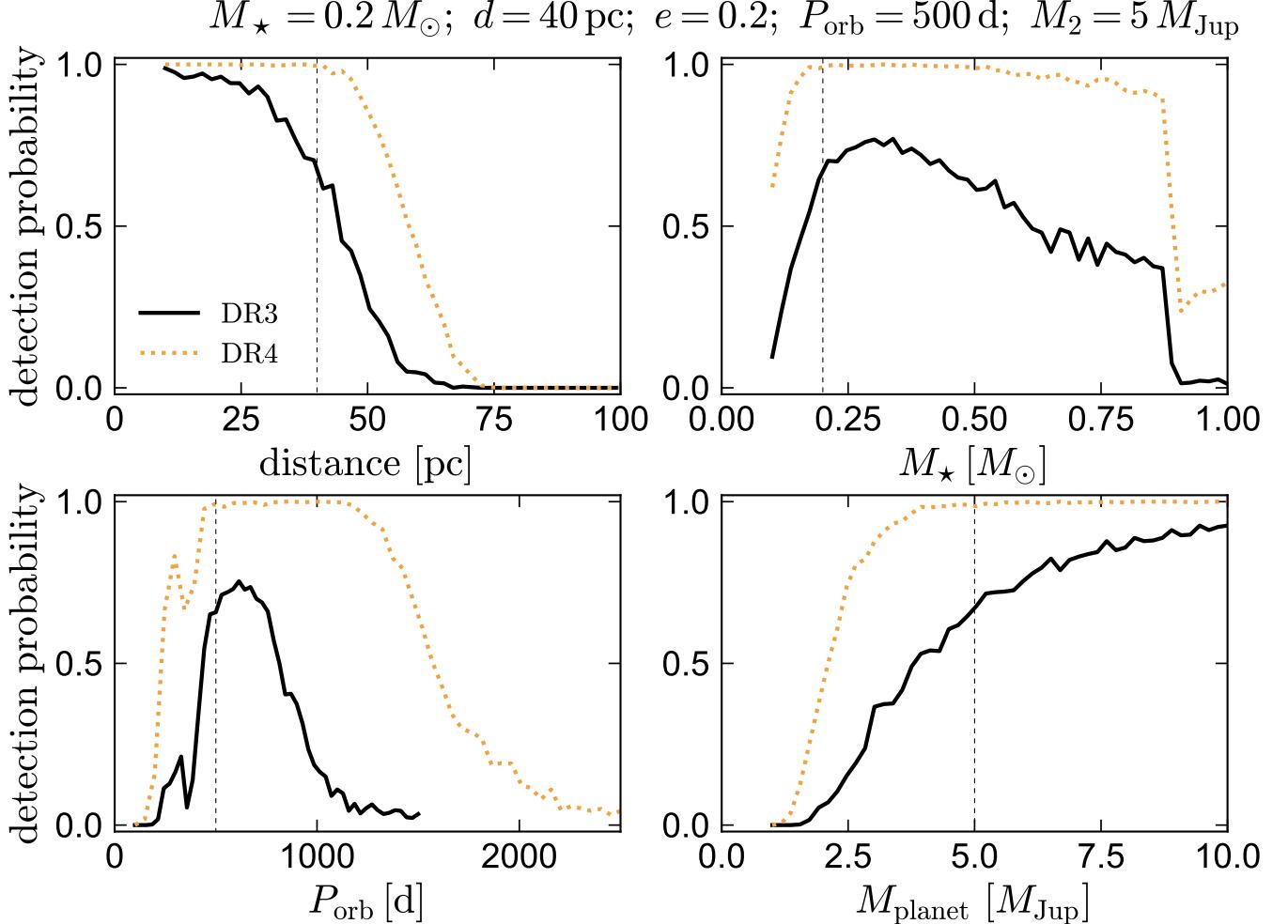


FIG. 14.— Probability of a giant planet companion published with an astrometric orbital solution in DR3 (solid black) or DR4 (dotted yellow). For the fiducial case, we take $M_\star = 0.2 M_\odot$, $d = 40$ pc, $e = 0.2$, $P_{\text{orb}} = 500$ d, and $M_2 = 5 M_{\text{Jup}}$. From top left to lower right, panels show the effects of varying distance, host star mass, orbital period, and planet mass. Other parameters are fixed at their fiducial values, which are marked with dashed vertical lines. The detection probability is largest for close distances, relatively low-mass stars, periods comparable to the observing baseline, and high-mass planets.

tions will come from binaries with periods longer than 1000 d, which could not be solved in DR3. Such binaries have larger orbits at fixed mass than shorter-period binaries and can thus be astrometrically constrained to larger distance. As a result, the effective search volume for astrometric orbits increases rapidly with the observational baseline. The number of spurious solutions is also expected to decrease significantly in future data releases (Figure 10).

6. *Empirical selection function:* Our modeling allows us to construct a Monte Carlo model of the *Gaia* selection function: for any hypothetical binary with a given set of observable properties, we can simulate many realizations of the epoch astrometry, varying the binary’s orientation, sky position, and noise. We then forward-model the *Gaia* orbit fitting and quality cuts and finally calculate what fraction of the simulated realizations would have passed all cuts to be included in the DR3 astrometric binary sample, or the cuts used in future

data releases. We provide publically available code to use this selection function.

Application of this method are shown in Figures 12, 13, 14, and 15 which consider luminous binaries, compact object companions, and giant planets. One intuitive but previously uncharacterized prediction of this modeling is that *Gaia* is significantly less sensitive to high-eccentricity orbits than to low-eccentricity orbits. The bias becomes severe at eccentricities $e \gtrsim 0.6$ (Figure 12).

Our simulations suggest that binaries similar to *Gaia* BH1, BH2, and BH3 will be detectable to distances of ≈ 0.7 , 3, and 2 kpc, respectively in *Gaia* DR4 (Figure 13), suggesting that significant numbers of such binaries will emerge from DR4. Quantitative predictions will be made in future work. The search volume within which most giant planets will be detected will also increase significantly (Figure 14). Massive planets ($M = 10 M_{\text{Jup}}$) in few-year periods will be detectable around low-mass stars out to ~ 110 pc, and around solar-type stars

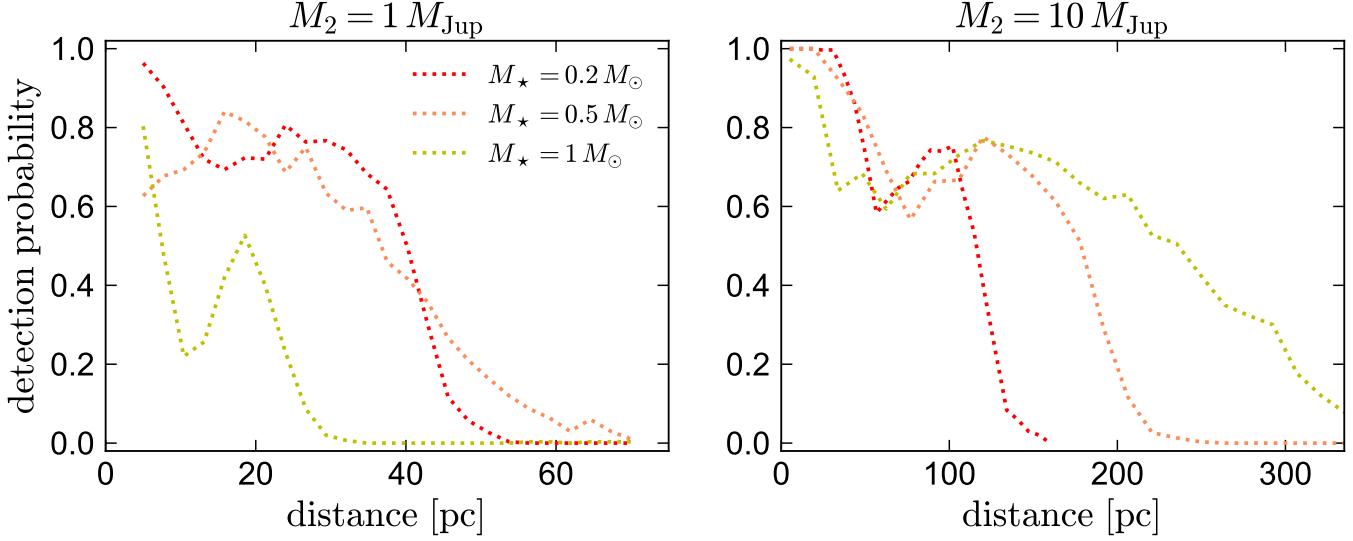


FIG. 15.— Detection probability for giant planets with $P_{\text{orb}} = 1500$ d in DR4. We assume $e = 0.2$ and consider two planet masses (left and right panels) and three stellar masses (red, orange, and yellow lines). The non-monotonic decrease in detection probability with increasing distance is a consequence of the nearest stars being very bright and thus having larger astrometric uncertainties (Figure 1). Trends with M_* are similarly nontrivial: low-mass stars have larger orbits at fixed M_{planet} , but they become faint at closer distances. *Gaia* is much more sensitive to high-mass planets than to low-mass planets: for M_* = 0.2, 0.5, and $1.0 M_{\odot}$, the discovery volume is respectively ≈ 30 , 100 , and 1000 times larger for $M_{\text{planet}} = 10 M_{\text{Jup}}$ than for $M_{\text{planet}} = 1 M_{\text{Jup}}$.

out to ~ 250 pc. Jupiter-mass planets will be detectable within $\lesssim 40$ pc (Figure 15).

ACKNOWLEDGMENTS

This research was supported by NSF grant AST-2307232. HWR acknowledges support from the European Research Council for the ERC Advanced Grant [101054731].

This work has made use of data from the European Space Agency (ESA) mission *Gaia* (<https://www.cosmos.esa.int/gaia>), processed by the *Gaia* Data Processing and Analysis Consortium (DPAC, <https://www.cosmos.esa.int/web/gaia/dpac/consortium>). Funding for the DPAC has been provided by national institutions, in particular the institutions participating in the *Gaia* Multilateral Agreement.

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APPENDIX

A. FITTING ASTROMETRIC ORBITAL SOLUTIONS

We suppose that a binary is observed at times $t_i = \{t_1, t_2, \dots, t_n\}$. Each observation comes with a known scan angle ψ_i and parallax factor $\Pi_{\eta,i}$. The data are the observed along-scan displacements η_i , and their uncertainties, $\sigma_{\eta,i}$. For a given guess of $\vec{\mu}_{\text{nonlinear}} = \{P, \phi_0, e\}$, we can numerically solve Equation 15 for the eccentric anomalies E_i corresponding to each t_i , and thus calculate an array of X_i and Y_i from equation 14. We can then rewrite Equation 13 as

$$\vec{\eta} = \mathbf{A}\vec{\mu}, \quad (\text{A1})$$

where

$$\mathbf{A} = \begin{bmatrix} \sin \psi_1 & t_1 \sin \psi_1 & \cos \psi_1 & t_1 \cos \psi_1 & \Pi_{\eta,1} & X_1 \sin \psi_1 & Y_1 \sin \psi_1 & X_1 \cos \psi_1 & Y_1 \cos \psi_1 \\ \sin \psi_2 & t_2 \sin \psi_2 & \cos \psi_2 & t_2 \cos \psi_2 & \Pi_{\eta,2} & X_2 \sin \psi_2 & Y_2 \sin \psi_2 & X_2 \cos \psi_2 & Y_2 \cos \psi_2 \\ \dots & \dots \\ \sin \psi_n & t_n \sin \psi_n & \cos \psi_n & t_n \cos \psi_n & \Pi_{\eta,n} & X_n \sin \psi_n & Y_n \sin \psi_n & X_n \cos \psi_n & Y_n \cos \psi_n \end{bmatrix} \quad (\text{A2})$$

is the design matrix, and

$$\vec{\eta} = \begin{bmatrix} \eta_1 \\ \eta_2 \\ \dots \\ \eta_n \end{bmatrix}; \quad \vec{\mu} = \begin{bmatrix} \Delta\alpha^* \\ \mu_\alpha^* \\ B \\ G \\ \Delta\delta \\ \mu_\delta \\ A \\ F \\ \varpi \end{bmatrix} \quad (\text{A3})$$

The maximum-likelihood solution to Equation A1 is

$$\vec{\mu} = [\mathbf{A}^\top \mathbf{C}^{-1} \mathbf{A}]^{-1} [\mathbf{A}^\top \mathbf{C}^{-1} \vec{\eta}], \quad (\text{A4})$$

where \mathbf{C} is the covariance matrix,

$$\mathbf{C} = \begin{bmatrix} \sigma_{\eta,1}^2 & 0 & 0 & 0 \\ 0 & \sigma_{\eta,2}^2 & \dots & 0 \\ 0 & \dots & \dots & 0 \\ 0 & 0 & 0 & \sigma_{\eta,n}^2 \end{bmatrix}, \quad (\text{A5})$$

and we assume the individual astrometric measurements are not covariant.

Solving the orbit is thus reduced to a nonlinear optimization problem in three dimensions. For each guess of $\mu_{\text{nonlinear}}$, we can solve Equation A4 for μ , plug the result in Equation A1, and finally evaluate the likelihood, which compares the observed and predicted η_i assuming Gaussian uncertainties. The main challenge is in finding the optimal $\mu_{\text{nonlinear}}$, because period aliases can make the posterior multimodal.

We use adaptive simulated annealing (Ingber 1989), a global optimization method designed to perform well on bumpy likelihoods. This method has been used previously to fit astrometry (Pourbaix 1998); our implementation borrows from the work of Iglesias-Marzoa et al. (2015), who used the method to fit RVs of binaries. We verified using simulations that for a wide variety of orbital periods and configurations the optimizer achieves very similar performance to the `kepmode1` package (Delisle & Ségransan 2022), which is used in the supplemental code released by Gaia Collaboration et al. (2024a) to fit the epoch astrometry for Gaia BH3, but our fitting is ~ 3 times faster. This most likely does not reflect any superiority of the optimization algorithm, but is simply a consequence of the fact that the core functions in our implementation are written in C.

Limited information is available about the details of the fitting algorithm used for *Gaia* DR3. As described by Halbwachs et al. (2023), the problem was linearized, and the maximum-likelihood solution in 3D was searched for with a nonlinear optimizer. The optimizer was initialized at $e = 0$ on a period grid ranging from 10 to ≈ 1600 d. How

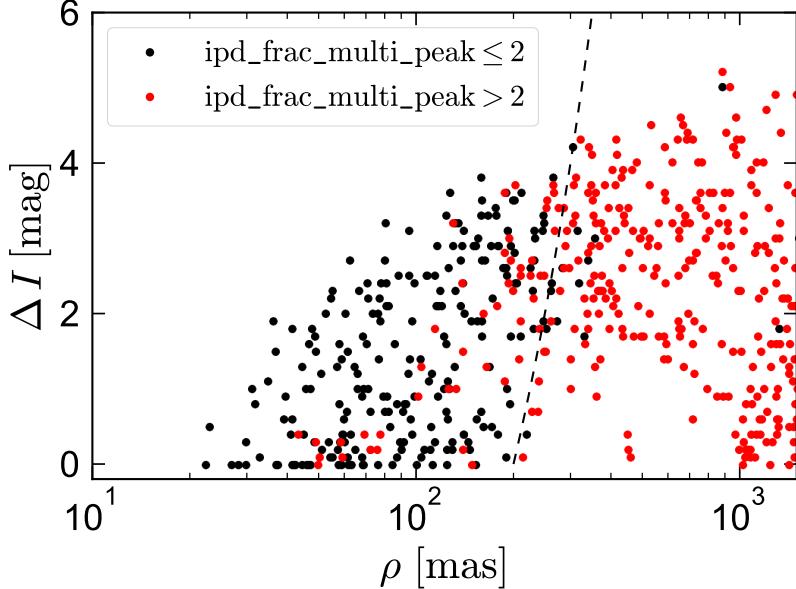


FIG. 16.— Magnitude difference vs. angular separation for wide pairs resolved by [Tokovinin \(2023\)](#) with speckle imaging. Red points show pairs in which the primary has $\text{ipd_frac_multi_peak} > 2$, and thus would not have been eligible for an astrometric binary solution. Black points show pairs with $\text{ipd_frac_multi_peak} \leq 2$, which would have been eligible for a solution. Dashed black line shows ΔI [mag] = $\frac{1}{25}(\frac{\rho}{\text{mas}} - 200)$.

dense a grid was searched and the periastron time at which the optimizers were initialized is not specified. Here we proceed under the assumption that the optimization was near-optimal.

Once the maximum likelihood solution has been identified, the task remains to estimate parameter uncertainties. Properly accounting for uncertainties in both the linear and nonlinear parameters requires returning to twelve dimensions (without analytic marginalization over the linear parameters). We approximate the covariance matrix of the astrometric parameters as

$$\mathbf{C}_{\text{ast}} = (\mathbf{J}^\top \mathbf{J})^{-1}, \quad (\text{A6})$$

where \mathbf{J} is the Jacobian matrix, defined as

$$\mathbf{J}_{ij} = \frac{\partial r_i}{\partial \theta_j}. \quad (\text{A7})$$

Here $r_i = (\eta_{\text{pred},i} - \eta_i)/\sigma_{\eta,i}$ is the uncertainty-scaled residual of the i th datapoint, and θ_j is the j th astrometric parameter; we evaluate these derivatives numerically using a step size of 10^{-8} in all parameters. Finally, we take the uncertainties of the astrometric parameters as the square root of the diagonal elements of the covariance matrix, and we transform Thiele-Innes elements into Campbell elements following [Halbwachs et al. \(2023\)](#).

We also experimented with using a Markov Chain Monte Carlo approach. We found that for well-constrained solutions that pass all the cuts imposed in DR3, this yields uncertainties in good agreement with those inferred from the linear analysis. We opted to use the linear approach because it more closely matches the method used by [Halbwachs et al. \(2023\)](#) and because it is faster, particularly for slow-to-converge solutions with periods longer than the observing baseline.

B. EFFECTS OF IPD RESOLVED BINARY CUTS

As described by [Halbwachs et al. \(2023\)](#), one of the first steps in selecting astrometric binary candidates in *Gaia* DR3 was the removal of sources with $\text{ipd_frac_multi_peak} > 2$ of $\text{ipd_gof_harmonic_amplitude} > 0.1$. This cut was designed to remove marginally resolved sources, whose astrometry is likely to be problematic if not subject to special processing.

In order to assess the likely effects of these cuts on our simulated binary sample, we investigated the $\text{ipd_frac_multi_peak}$ and $\text{ipd_gof_harmonic_amplitude}$ values of a sample of close pairs resolved by [Tokovinin \(2023\)](#) with speckle imaging in the *I*-band. As shown in Figure 16, we found that most pairs with $\rho \gtrsim 200$ mas have $\text{ipd_frac_multi_peak} > 2$, meaning that two separate peaks were detected in at least 2% of CCD observations. Most pairs closer than 200 mas have $\text{ipd_frac_multi_peak} = 0$, and pairs with larger flux ratios have $\text{ipd_frac_multi_peak} < 2$ out to slightly larger separations. We found that few sources satisfy $\text{ipd_gof_harmonic_amplitude} > 0.1$ at any separation, so this cut has little additional effect on the selection.

The dashed black line in Figure 16 shows an empirical boundary,

$$\Delta I \text{ [mag]} = \frac{1}{25} \left(\frac{\rho}{\text{mas}} - 200 \right). \quad (\text{B1})$$

Most sources below this boundary will be discarded and will not be fit with astrometric binary models. In our simulations, we apply the same cut in the G -band. Although the G - and I -bands are not identical, the dependence on ΔI in Figure 16 is relatively weak, so we expect this to be a servicable approximation.

C. PARALLAX UNCERTAINTIES OF 5-PARAMETER SOLUTIONS

To validate the per-FOV transit uncertainties σ_η used in our modeling (Figure 1), we generated epoch astrometry for a population of simulated single stars, again using `Galaxia` to model the underlying stellar population. We simulate 1% of the sources with $G < 19$ within 2 kpc, again fitting the cascade of astrometric models used in DR3. All sources ultimately receive 5-parameter solutions. In Figure 17, we compare the σ_ϖ distributions for the simulated 5-parameter solutions of all sources with $\text{UWE} < 1.4$ to those in DR3 with $\text{ruwe} < 1.4$. Each panel shows a 0.5-mag wide apparent magnitude bin (e.g., the $G = 9$ bin includes sources with $G = 8.75 - 9.25$). Overall, the simulated sources reproduce the observed σ_ϖ distributions quite well. This implies that our assumed uncertainties are realistic, at least on average, since for a fixed scanning law, the astrometric parameter uncertainties scale linearly with the epoch-level astrometric uncertainties.

D. COMPARISON TO PREVIOUS WORK

In forecasting exoplanet discoveries with *Gaia*, Perryman et al. (2014) quantified detectable via $\Delta\chi^2$, the difference between the χ^2 of the best-fit 5 parameter solution and that of the correct 12-parameter solution. An advantage of quantifying detectability in terms of $\Delta\chi^2$ is that it is inexpensive to calculate, since it does not require blind fitting of the orbital solution. Modeling a five-year *Gaia* mission, they proposed a threshold of $\Delta\chi^2 \gtrsim 50$ as representing a robust detection, and $\Delta\chi^2 \gtrsim 100$ as yielding orbital parameter constraints with uncertainties of 10% or better. These same thresholds were used by Holl et al. (2022) in forecasting brown dwarf discoveries.

To compare to these works, we calculated $\Delta\chi^2$ values for a random subset of 10^6 simulated binaries used in constructing the mock catalog. Figure 18 shows the distribution of $\Delta\chi^2$ and $s_{\text{orb}} = a_0/\sigma_{a_0}$ values for systems that are fit with orbital solutions (i.e., those with $\text{UWE} > 1.4$ that are not accepted with acceleration solutions). At $s_{\text{orb}} \gtrsim 5$, there is a clear correlation between the two quantities, but there is also significant scatter. Orbits with low eccentricity typically have the highest s_{orb} at fixed $\Delta\chi^2$. The median $\Delta\chi^2$ at $s_{\text{orb}} = 5$ is 160, somewhat higher than the thresholds used by Perryman et al. (2014) and Holl et al. (2022). There is also a significant population of systems with $\Delta\chi^2$ up to 10^3 and $s_{\text{orb}} \ll 5$. These systems all have best-fit eccentricities near unity, and most have true eccentricities $\gtrsim 0.8$. A handful of low-eccentricity binaries with spurious orbital solutions also enter the sample. The astrometric uncertainties blow up in the limit of $e \rightarrow 1$, and as a result, many high-eccentricity binaries will not sufficiently well-constrained orbits to enter the DR3 binary sample.

E. ASTROMETRY FOR MARGINALLY RESOLVED SOURCES

Figure 19 illustrates the effects of marginally resolved binaries on *Gaia* astrometry. We consider a MS+MS binary with a mass ratio $q = 0.7$, a G -band flux ratio $f = 0.2$, and a projected separation $\rho = 250$ mas. The top two rows show the positions of the two stars relative to the center of mass, and the expected total 1D flux profile as a function of AL coordinate, all for three different scan angles. The peak of the total flux profile shifts by ~ 100 mas across observations. The third row shows $\delta\eta$ predicted by Equation 1 as a function of scan angle. The total variation has an amplitude of about 100 mas, but most of this simply reflects position bias: i.e., the fact that the photocenter and barycenter are not in the same place. Position bias will be absorbed into the best fit solution, so we fit the black line in the 3rd panel with a function $\delta\eta = c_1 \sin \psi + c_2 \cos \psi$, where c_1 and c_2 are free parameters that correspond to the position bias in $\Delta\alpha^*$ and $\Delta\delta$ (Equation 3). The best-fit position bias is shown with a dashed cyan line, and the residuals are shown in the 4th panel. It is these residuals that lead to spurious astrometric wobble. More examples of such residuals can be found in Figures 20 and 21 of Holl et al. (2023b).

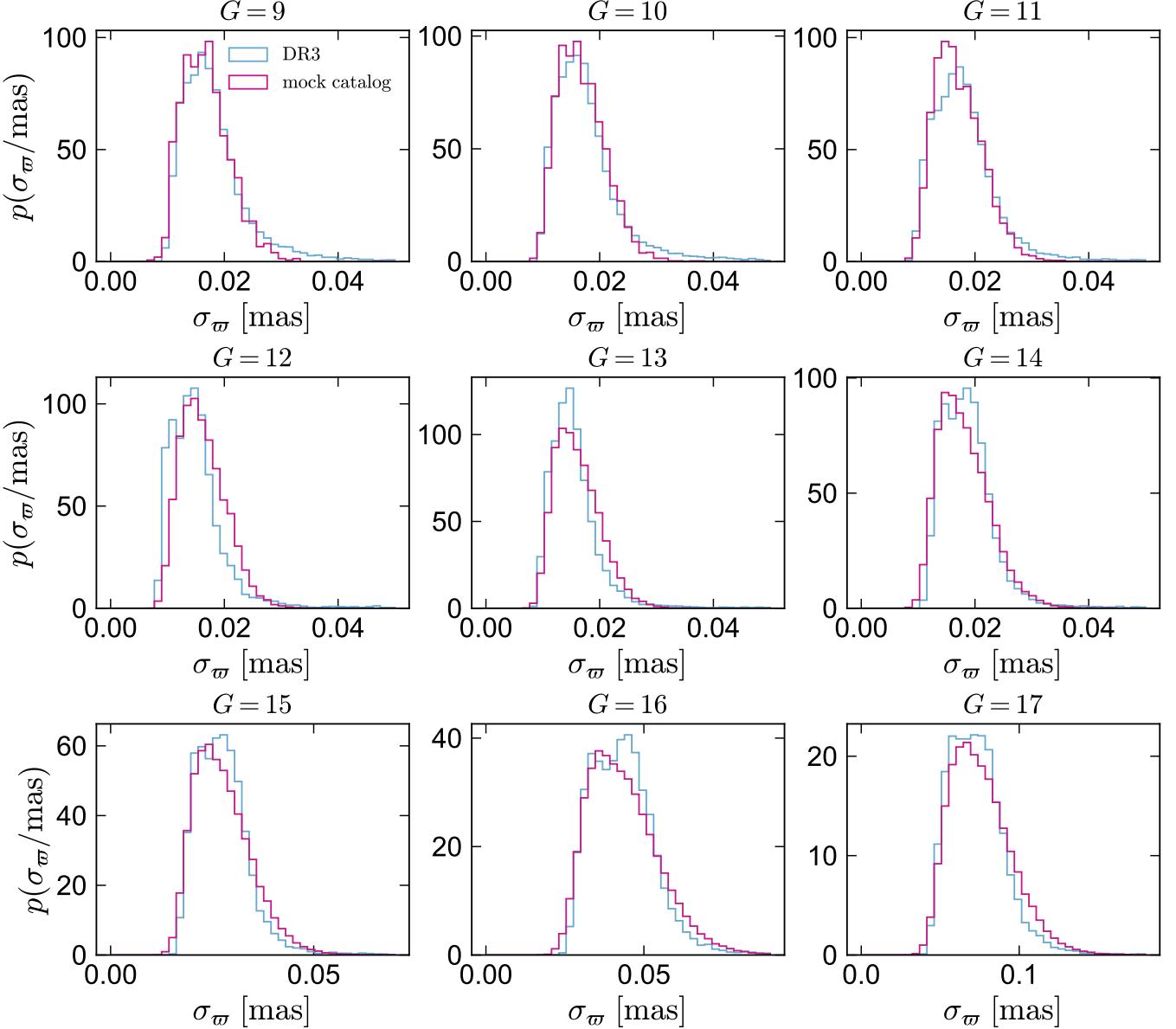


FIG. 17.— Parallax uncertainty distributions for single-star solutions with $\text{ruwe} < 1.4$ in DR3 (blue) and in the mock catalog (violet). Each panel shows a 0.5 mag-wide bin apparent magnitude bin centered on the value listed in the title. The generally good agreement between the observed and simulated distributions implies that our simulated epoch-level astrometric uncertainties are reasonably accurate, and in particular that measurements from 9 CCDs with a single FOV transit are nearly independent.

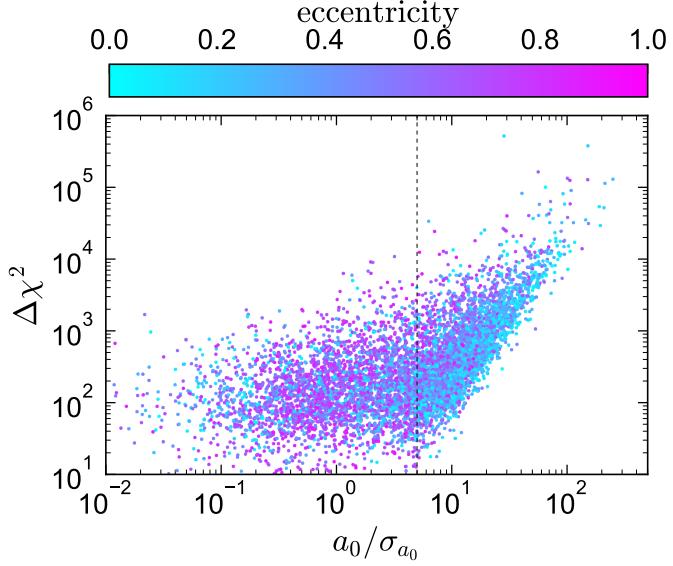


FIG. 18.— χ^2 difference between 5-parameter single-star and 12-parameter orbital solutions for simulated binaries, vs. signal-to-noise ratio of the photocenter semi-major axis. Dashed vertical line shows $a_0/\sigma_{a_0} = 5$, the minimum threshold adopted in DR3. This threshold corresponds roughly to a cut of $\Delta\chi^2 \gtrsim 160$, but there is considerable dispersion in $\Delta\chi^2$ at fixed a_0/σ_{a_0} . Points are colored by their true eccentricity. Some binaries with $\Delta\chi^2$ up to 1000 – mainly systems with high eccentricity – have poorly constrained a_0 despite having relatively large $\Delta\chi^2$. Systems with lower eccentricities generally have higher a_0/σ_{a_0} at fixed $\Delta\chi^2$.

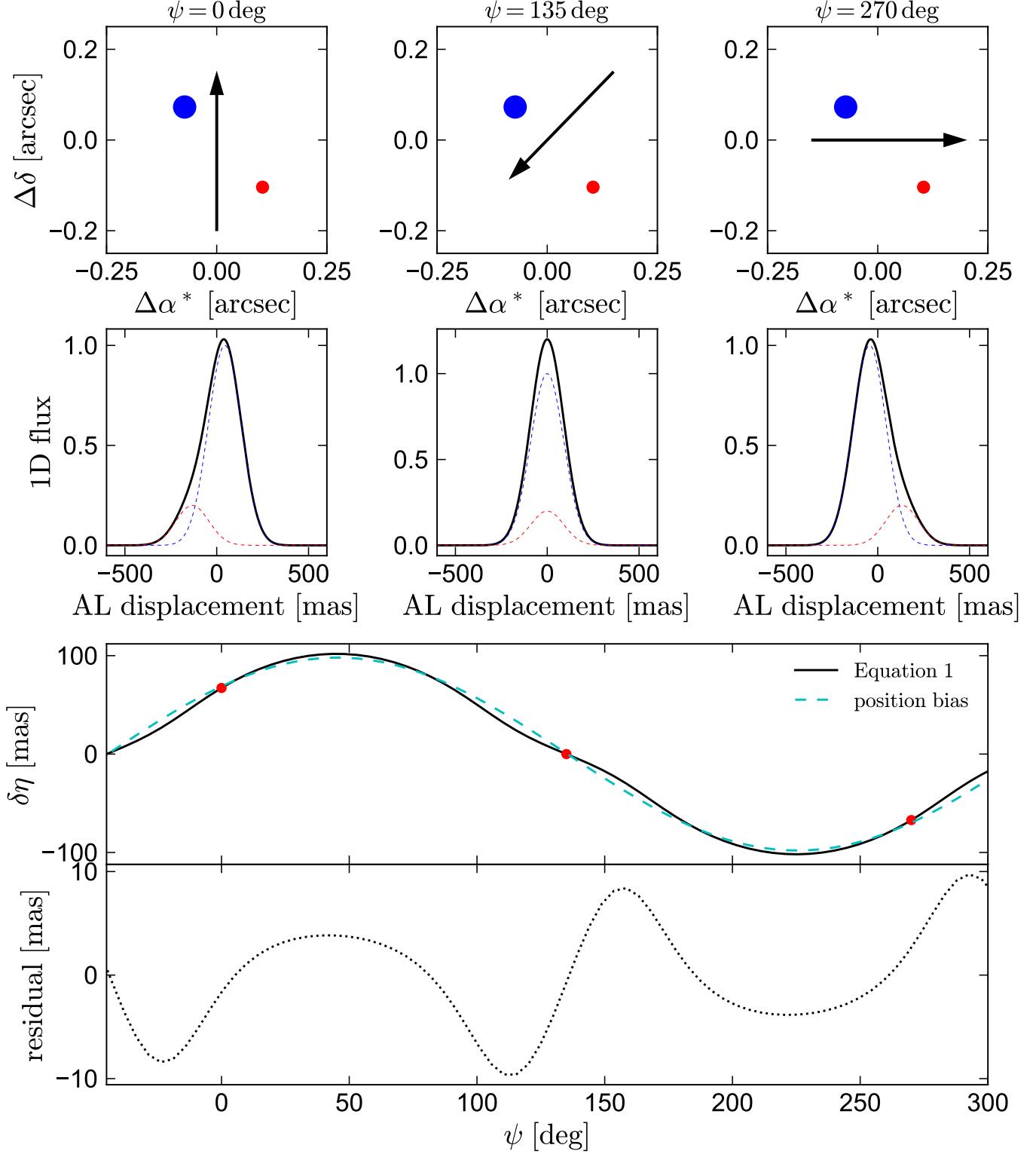


FIG. 19.— Illustration of how marginally resolved wide binaries lead to spurious astrometric orbits. Top panels show a pair separated by 0.25 arcsec, with the scan angle indicated by a black arrow. 2nd row shows the total flux integrated over the across-scan direction. Displacement is relative to the pair's center of mass, which is at the origin. 3rd panel shows $\delta\eta$ as predicted by Equation 1, with red points corresponding to the panels above. Dashed cyan line shows the best-fit sinusoid, which is absorbed as a positional bias. The residual with respect to this sinusoid (bottom panel) manifests as a spurious astrometric wobble.