Zewail City University of Science and Technology

Probability and Stochastic Processes - CIE 327

Project Report

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Introduction

A communication system is a model that represents the exchanges of information between two stations, the transmitter and the receiver. Signals or information transfer from source to destination via what is termed a channel. A channel is a representation of the path that signals take from source to destination. To transmit signals in a communication system, they must first be processed via a series of phases, starting with signal representation and progressing through signal shaping, encoding, and modulation. After preparing the transmitted signal, it is routed through the channel's transmission line, where it encounters several impairments such as noise, attenuation, and distortion.

A communication system's purpose is to transport data from one location to another. As illustrated in figure (1), a typical communication system is composed of three major components.

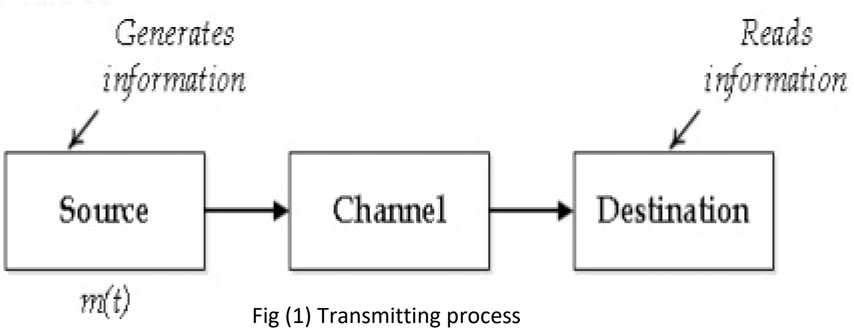


Fig (1) Transmitting process

Theoretical base

M-pulse amplitude modulation (M-PAM)

Modulation Definition

Modulation is changing the characteristic of the carrier signal (sinusoidal wave) such as frequency, amplitude and so on according to the modulation signal (signal needed to be sent).

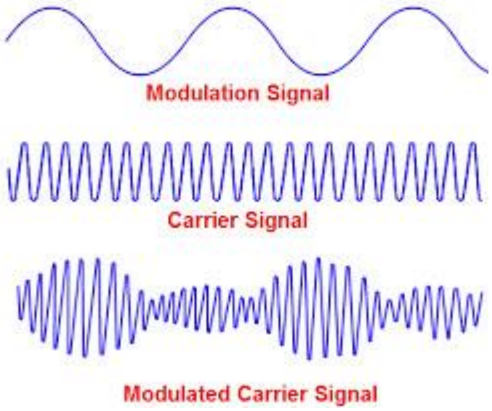


Fig (2)

Types of modulation

There are two types of modulating signal:

1- Continuous-wave Modulation
2- Pulse Modulation

Underneath these two types there are different ways as shown in the figure (3).
Our focus is on how pulse amplitude modulation works.

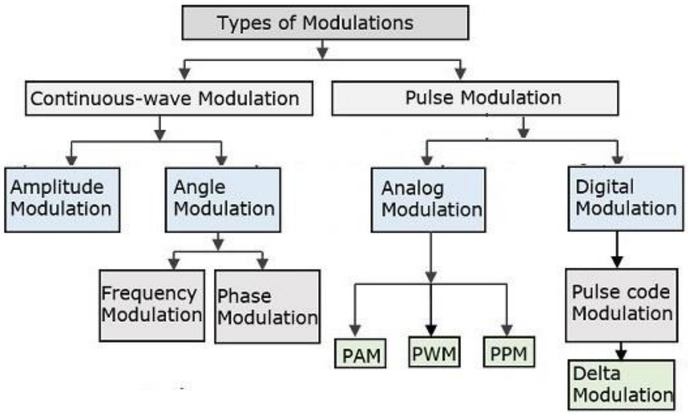


Fig (3) Types of modulation

In pulse amplitude modulation the amplitude of each pulse is specified by the instantaneous amplitude of the modulating signal sampled at equal intervals as shown below. Each sample is made directly proportional to the amplitude of the signal at the instant of sampling.

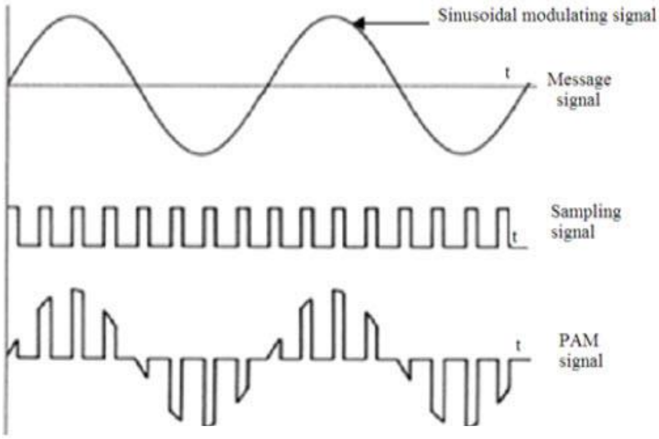


Fig (4) Pulse amplitude modulation

Additive white gaussian Noise (AWGN)

The basic idea behind adding a noise signal to the transmitted signal is to simulate the existence of random signal added by nature characteristics. As the noise signal is defined as random signal, it must follow the gaussian distribution in time domain with time average zero.

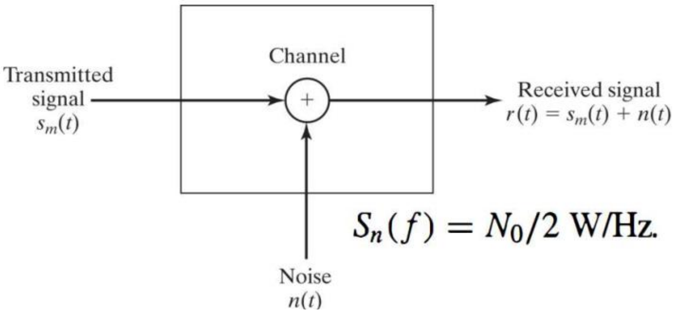


Fig (5) Additive white noise

Matched Filters

Matched filter is a filter used in communications to “match” a particular transit waveform. It passes all the signal frequency components while suppressing any frequency components where there is only noise and allows to pass the maximum amount of signal power.

The purpose of the matched filter is to maximize the signal to noise ratio at the sampling point of a bit stream and to minimize the probability of undetected errors received from a signal.

To achieve the maximum SNR, we want to allow through all the signal frequency components, but to emphasize more on signal frequency components that are large and so contribute more to improving the overall SNR.

A basic problem that often arises in the study of communication systems is that of detecting a pulse transmitted over a channel that is corrupted by channel noise (i.e., AWGN).

Let us consider a received model, involving a linear time-invariant (LTI) filter of impulse response h(t), and the filter input x(t) consists of a pulse signal g(t) corrupted by additive channel noise w(t) of

$$x(t) = g(t) + w(t), \qquad 0 \le t \le T$$
$$y(t) = g_o(t) + n(t)$$

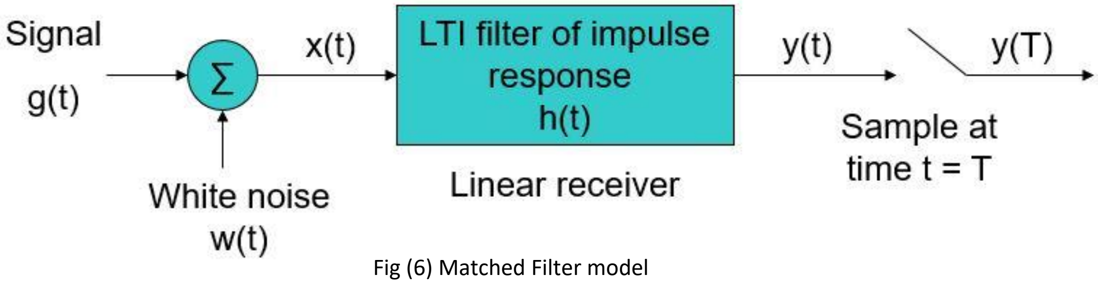


Fig (6) Matched Filter model

zero mean and power spectral density No/2; Then the resulting output y(t) is composed of go(t) and n(t), the signal and noise components of the input x(t), respectively.

The linear receiver optimizes the design of the filter so as to minimize the effects of noise at the filter output and improve the detection of the pulse signal.

Signal to noise ratio is:

$$SNR = \frac{|g_o(T)|^2}{\sigma_n^{\ 2}} = \frac{|g_o(T)|^2}{E\left[n^2(t)\right]}$$

Where |go(T)|2 is the instantaneous power of the filtered signal, g(t) at point t = T, and σn2 is the variance of the white Gaussian zero mean filtered noise.

Deriving the matched filter

We sampled at t = T because that gives you the max power of the filtered signal.

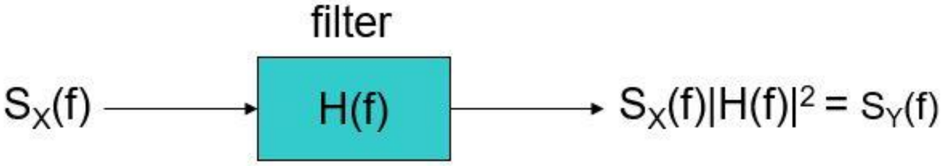
We get Fourier transform to go(t):

$$G_o(f) = G(f)H(f)$$
$$so g_o(t) = \int G(f)H(f)e^{j2\pi ft}df$$

$$then|g_o(t)|^2 = |\int G(f)H(f)e^{j2\pi ft}df|^2$$

$$\sigma_n{}^2 = E[n(t)^2] - E[n(t)]$$
$$\sigma_n{}^2 = E[n(t)^2]$$
$$E[n(t)^2] = var[n(t)] = R_n(0)$$
$$R_n(\tau) = \int S_n(f)e^{j2\pi f\tau}df$$
$$R_n(0) = \int S_n(f) \cdot 1df$$

Autocorrelation is inverse Fourier transform of power spectral density



SX(f) is PSD of white Gaussian noise

$$S_X(f) = \frac{N_o}{2}$$

$$S_n(f) = \frac{N_o}{2}|H(f)|^2$$
$$\sigma_n{}^2 = E[n(t)^2] = R_n(0) = \int \frac{N_o}{2}|H(f)|^2df = \frac{N_o}{2}\int |H(f)|^2df\ so$$
$$SNR = \frac{|\int H(f)G(f)e^{j2\pi fT}\,df|}{\frac{N_o}{2}\int |H(f)|^2df}$$

To maximize, use Schwartz Inequality:

$$\int |\Phi_1(x)|^2\, dx < \infty$$
$$\int |\Phi_2(x)|^2\, dx < \infty$$

$$\longrightarrow \quad |\int \Phi_1(x)\Phi_2(x)dx|^2 \leq \int_{-\infty}^{\infty} |\Phi_1(x)|^2\, dx \int_{-\infty}^{\infty} |\Phi_2(x)|^2\, dx$$

We pick φ1(x)=H(f) and φ2(x)=G(f)ej2πfT and want to make the numerator of SNR to be large as possible

$$|\int H(f)G(f)e^{2\pi fT}df| \leq \int |H(f)|^2 df \cdot \int |G(f)e^{j2\pi fT}|^2 df$$
$$\frac{|\int H(f)G(f)e^{2\pi fT}df|}{\frac{N_o}{2}\int |H(f)|^2 df} \leq \frac{\int |H(f)\,|^2 df \int |G(f)|^2 df}{\frac{N_o}{2}\int |H(f)|^2 df}$$
$$SNR \leq \frac{\int |G(f)|^2 df}{\frac{N_o}{2}} = \frac{2\int |G(f)|^2 df}{N_o}$$

Inverse transform:

Assume g(t) is real. This means g(t)=g*(t)

if

$$F\{g(t)\} = G(f)$$
$$F\{g^*(t)\} = G^*(-f)$$

Then

$$G(f) = G^*(-f)$$
$$G^*(f) = G(-f)$$

Find h(t) (inverse transform of H(f))

$$h(t) = k\int G(-f)e^{-j2\pi fT}e^{j2\pi ft}\,df$$
$$= k\int G(-f)e^{-j2\pi f(T-t)}df$$
$$= k\int G(f)e^{j2\pi f(T-t)}df$$
$$h(t) = kg(T-t)$$

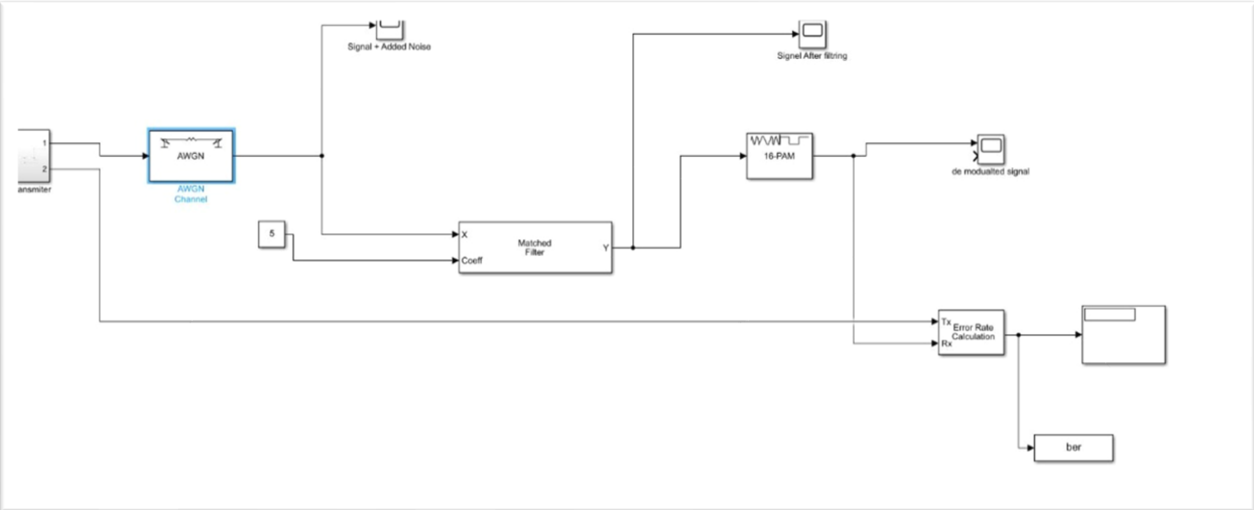
Simulation

Code

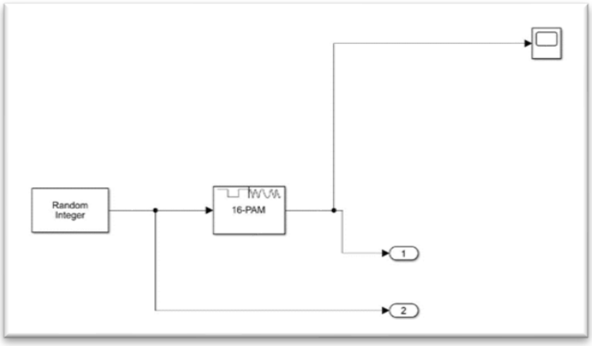
```
m = 16;
n = 1 ;
x = 50000;
T = [0: 0.001 : x*0.001-0.001];
SNR = 2;
OrignalSignal = randi([0 1], n,x);
PAMSignal = pammod(OrignalSignal,m);
Signal_Noise = awgn(PAMSignal,SNR);
FilterWave = phased.LinearFMWaveform('PulseWidth',1e-4,'PRF',5e3);
wave = getMatchedFilter(FilterWave);
Filter = phased.MatchedFilter('Coefficients',wave);
FilteredSignal = Filter(Signal_Noise);
DeModulatedSignl = pamdemod (FilteredSignal ,m );
[NumErrors,BER] = biterr(OrignalSignal,DeModulatedSignl);
figure(1)
subplot(3,2,1), plot(T,OrignalSignal); title ('Orignal Signal'); xlabel('Time(S)'); ylabel('Signal');
ylim([-1 2]);xlim([0 0.5])
subplot(3,2,2), plot(T,PAMSignal); title ('M-PAM Modlated Signal');xlabel('Time(S)'); ylabel('Signal')
ylim([-16 -12]);xlim([0 0.5])
subplot(3,2,3), plot(T,Signal_Noise); title ('Signal added WGN');xlabel('Time(S)'); ylabel('Signal')
xlim([0 50])
subplot(3,2,4), plot(T,FilteredSignal); title ('Filtered Signal');xlabel('Time(S)'); ylabel('Signal')
xlim([0 50])
subplot(3,2,5), plot(T,DeModulatedSignl); title ('M-PAM De Modulated Signal');xlabel('Time(S)');
ylabel('Signal')

ylim([-1 4]);xlim([0 0.5])
```

Simulink Model



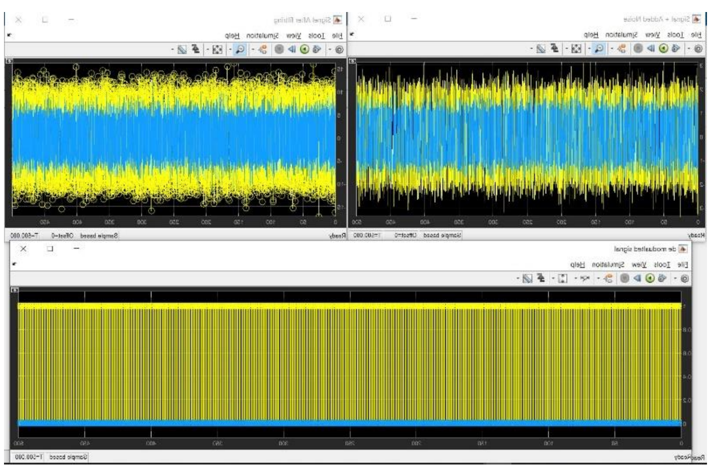
Transmitting model



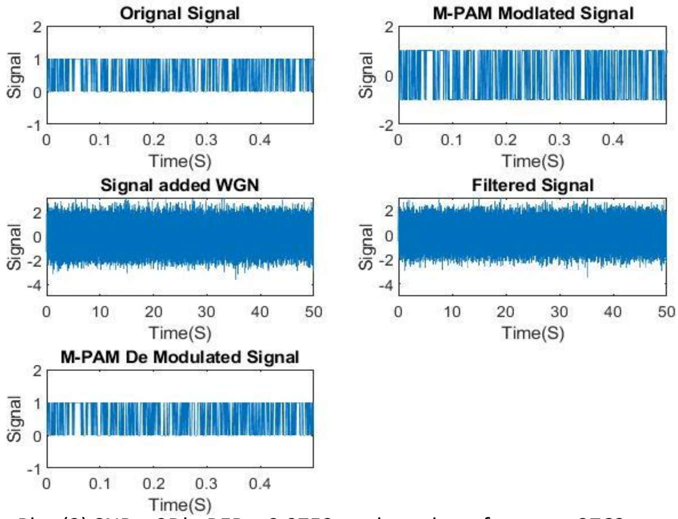
Results and Comments

All test were done under 50000 bits, (Simulink | Code).

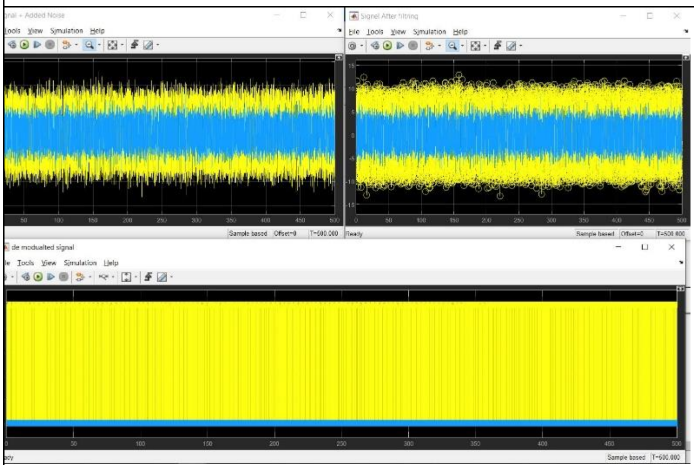
2-PAM



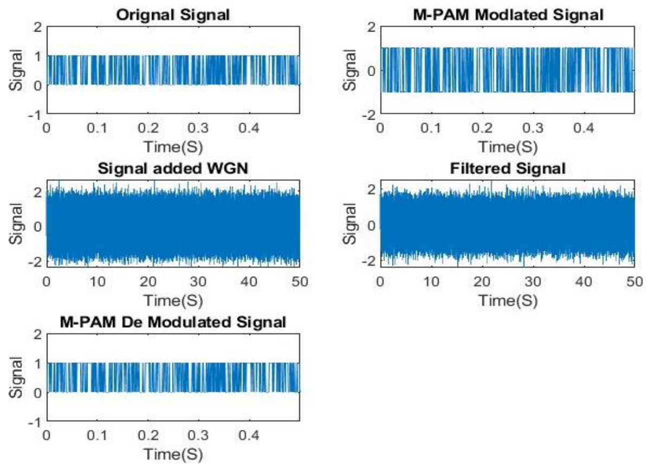
Plot (1) SNR = 2Db, BER = 0.03784, and number of errors = 1892



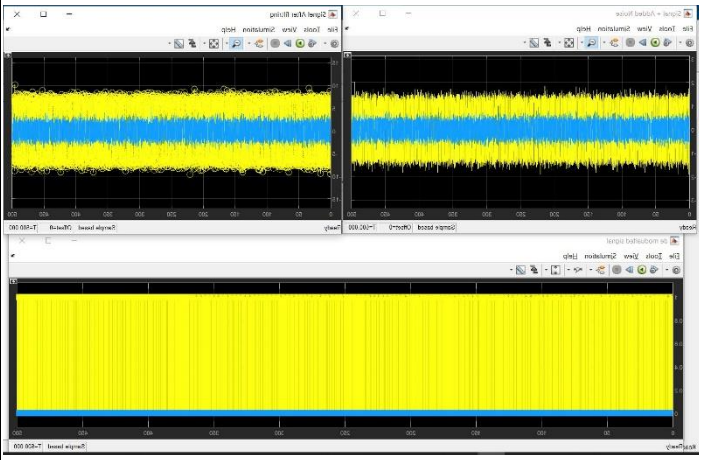
Plot (2) SNR = 2Db, BER = 0.0753, and number of errors = 3763



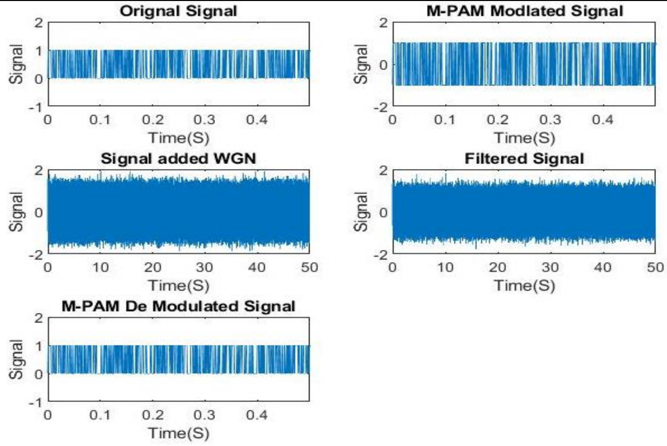
Plot (3) SNR = 5Db, BER = 0.00612, and number of errors = 306



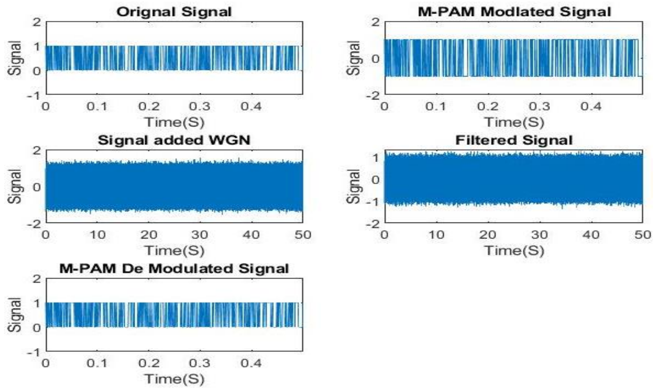
Plot (4) SNR = 5Db, BER = 0.0206, and number of errors = 1032



Plot (5) SNR = 10Db, BER = 0, and number of errors = 0

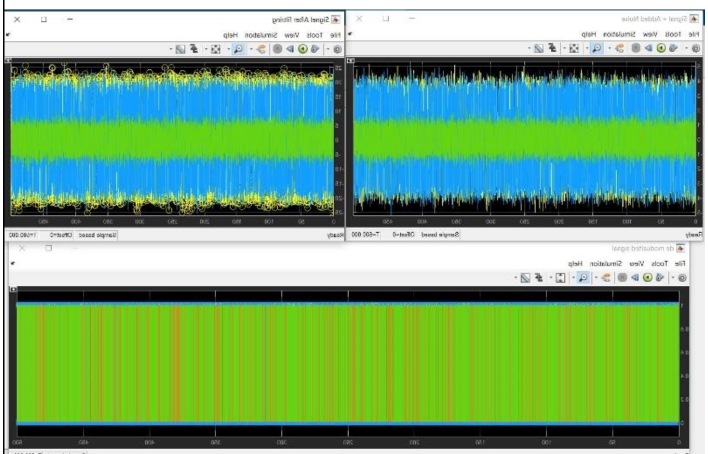


Plot (6) SNR = 10Db, BER = 0.00026, and number of errors = 13

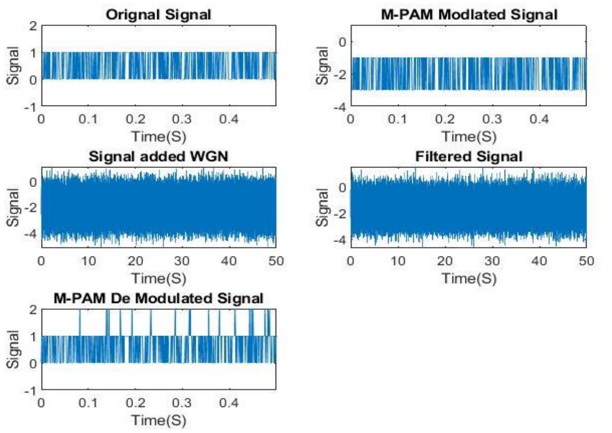


Plot (7) SNR = 15Db, BER = 0, and number of errors = 0

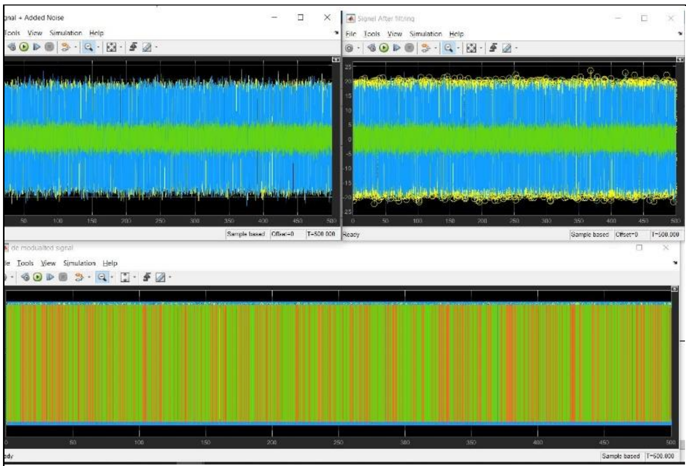
4-PAM



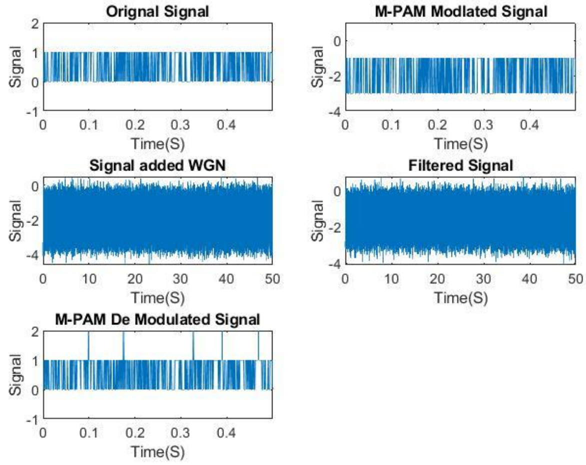
Plot (8) SNR = 2Db, BER = 2245, and number of errors = 11230



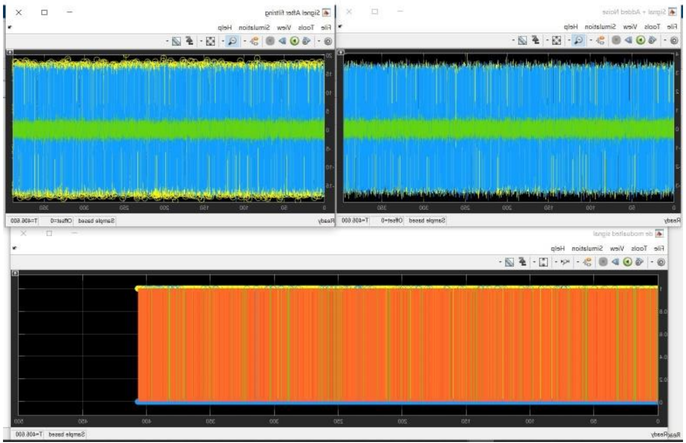
Plot (9) SNR = 2Db, BER = 0.0965, and number of errors = 9645



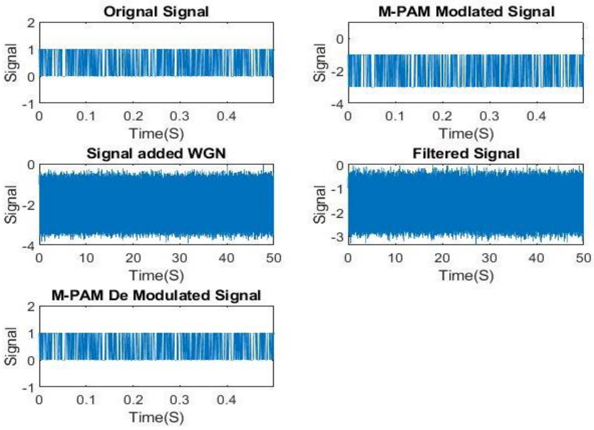
Plot (10) SNR = 5Db, BER = 0.2342, and number of errors =11710



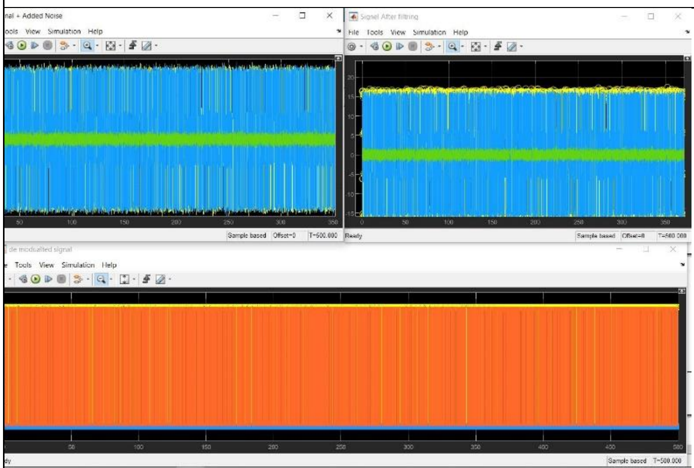
Plot (11) SNR = 5Db, BER = 0.0457, and number of errors = 4567



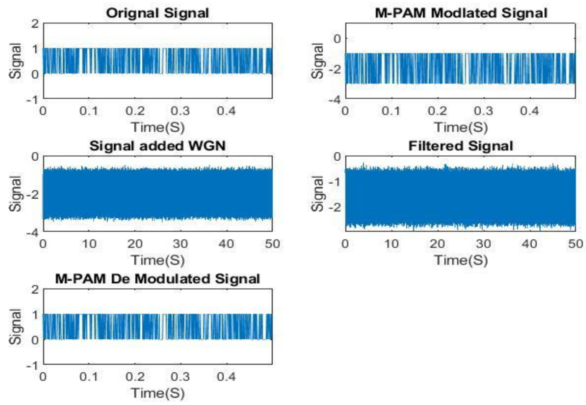
Plot (12) SNR = 10Db, BER = 0.2459, and number of errors = 10000



Plot (13) SNR = 10Db, BER = 0.0063, and number of errors = 628

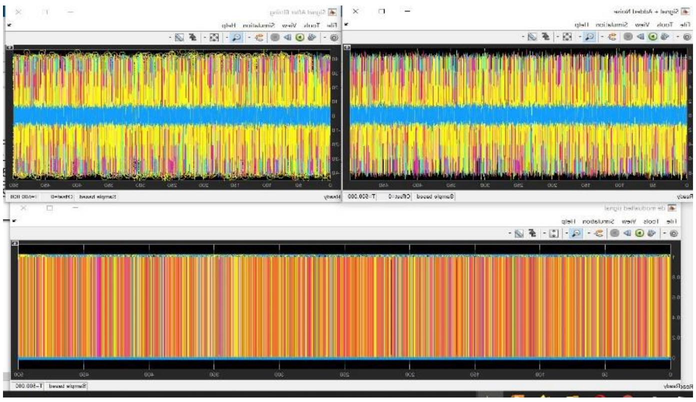


Plot (14) SNR = 15Db, BER = 0.2511 and number of errors = 12560

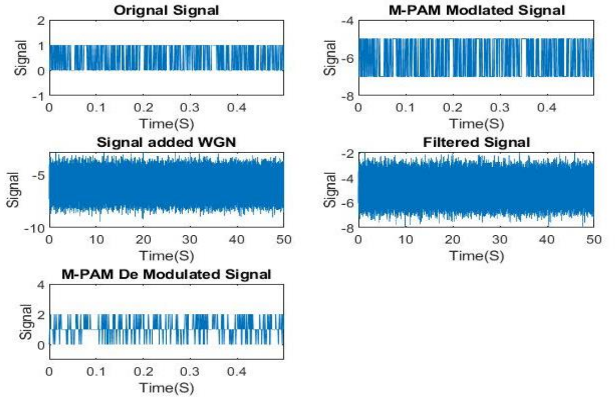


Plot (15) SNR = 15Db, BER = 0.00014, and number of errors = 7

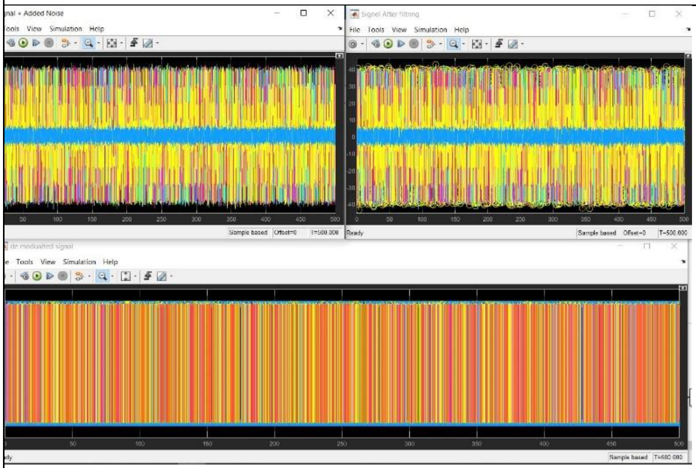
8-PAM



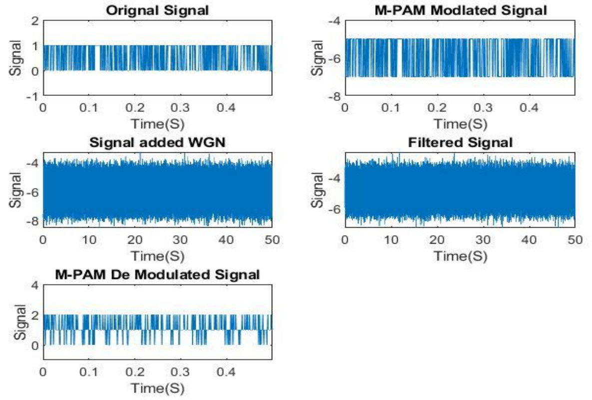
Plot (16) SNR = 2Db, BER = 0.3464, and number of errors = 17330



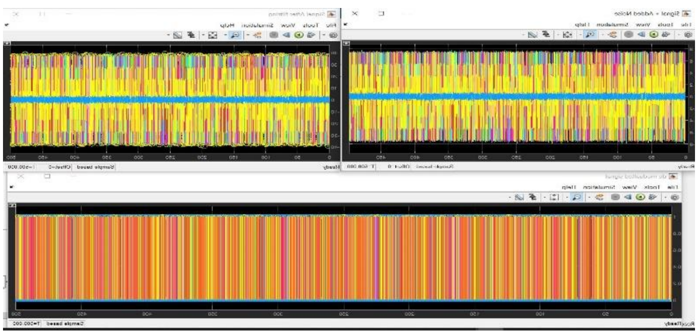
Plot (17) SNR = 2Db, BER = 0.4131, and number of errors = 41311



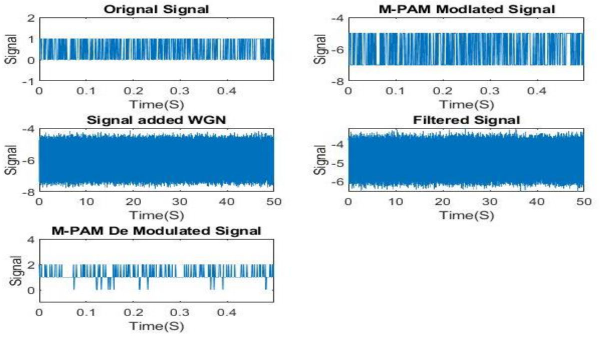
Plot (18) SNR = 5Db, BER = 0.3601, and number of errors = 18010



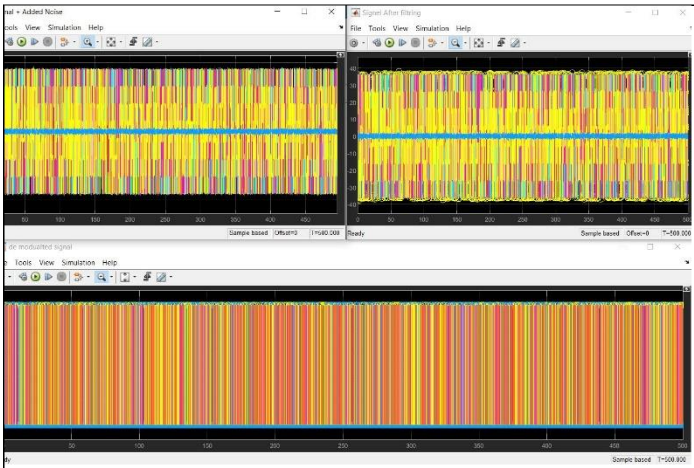
Plot (19) SNR = 5Db, BER = 0.4215, and number of errors = 42147



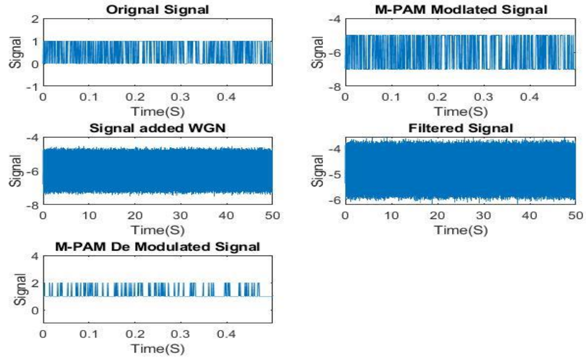
Plot (20) SNR = 10Db, BER = 0.3882, and number of errors = 19420



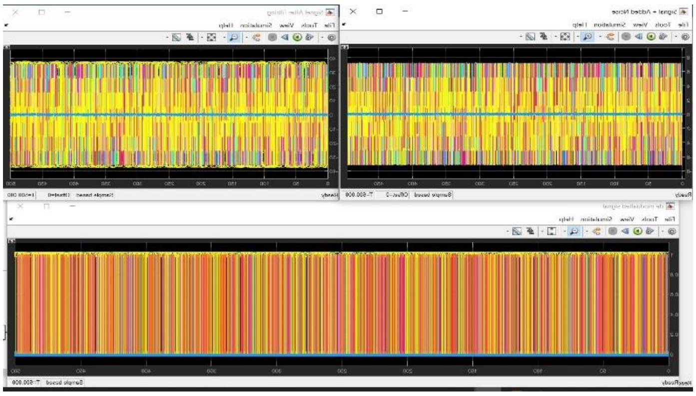
Plot (21) SNR = 10Db, BER = 0.4347, and number of errors = 43467



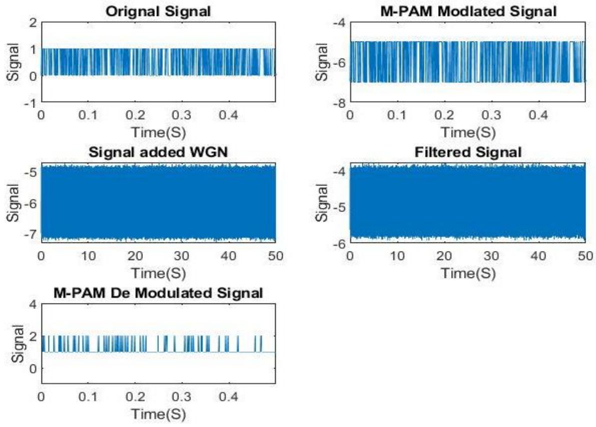
Plot (22) SNR = 15Db, BER = 0.4066, and number of errors = 20340



Plot (23) SNR = 15Db, BER = 0.4161, and number of errors = 41613

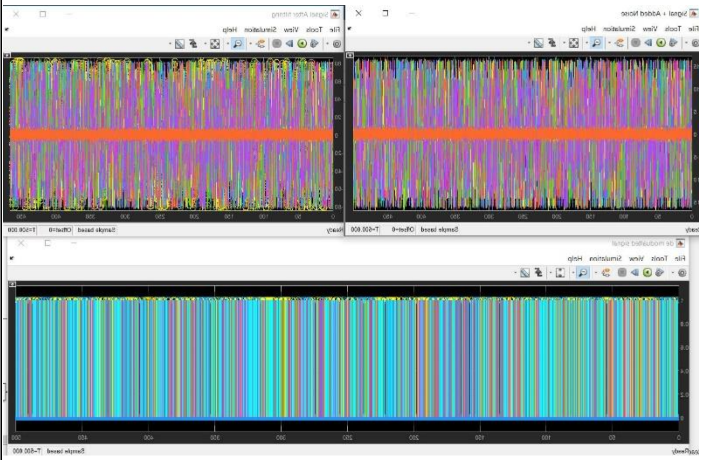


Plot (24) SNR = 20Db, BER = 0.415, and number of errors = 20760

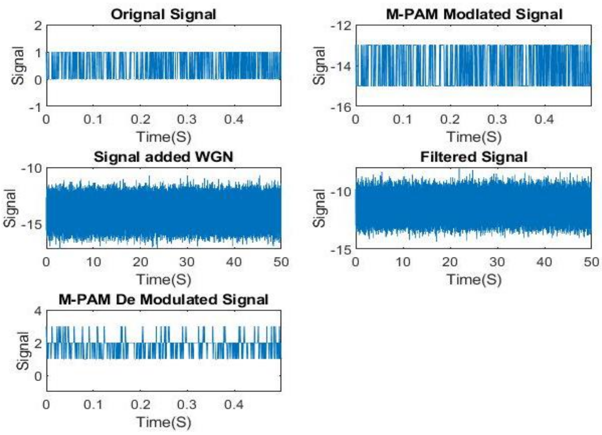


Plot (25) SNR = 20Db, BER = 0.3612, and number of errors = 36117

16-PAM

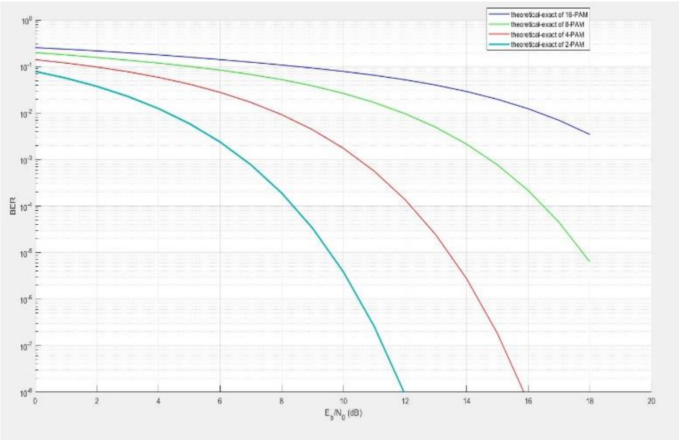


Plot (26) SNR = 2Db, BER = 0.3806, and number of errors = 19040

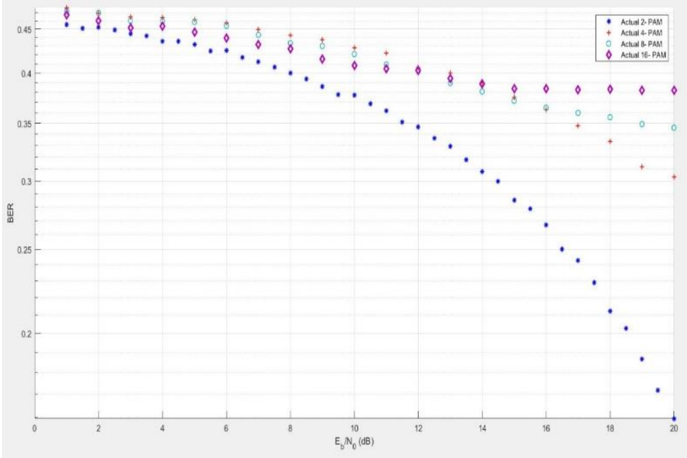


Plot (27) SNR = 2Db, BER = 0.4696, and number of errors = 70343

Theoretical vs Actual



Plot (27) Theoretical results



Plot (28) Actual results

In conclusion, bit error rate inversely proportional with SNR as when signal to noise ratio increase, the BER approaches zero and also noticed that while using M-PAM, the BER reaches zero on higher SNRs. Also noticed on the comparison between the actual and theoretical graph, the Ber reaches zero on higher SNRs values due to the efficiency of the real matched filter.

Reference

[1] *M-ary Pulse Amplitude Modulation*. M-ary pulse amplitude modulation. (n.d.). https://people.eecs.ku.edu/~perrins/class/F08_700/lab/MPAM/.

[2] says:, M., & *, N. (2021, March 4). *Pulse amplitude modulation (PAM) : Working, types & its applications*. ElProCus. https://www.elprocus.com/pulse-amplitude-modulation/.

[3] Jaynes, E. T. (2003). "14.6.1 The classical matched filter". Probability theory: The logic of science. Cambridge: Cambridge University Press.