

A* Algorithm:

```
using System.Diagnostics;
```

```
namespace N_PUZZLE
```

```
{  
    class NPuzzleSolution  
    {  
        public static Stopwatch CalculateTime = new Stopwatch(); // $\theta(1)$   
        public static HashSet<int> HashMatrix; // $\theta(1)$   
        public static PriorityQueue<Node, int> priorityQueue; // $\theta(1)$   
        public static int z_index_x; // $\theta(1)$   
        public static int z_index_y; // $\theta(1)$   
        public static Node n1; // $\theta(1)$   
        public static int AStarAlgorithm(int[,] Array2D, int MatrixSize, int[] Array1D)  
        //  $E \log(V)$        $E$ : Number of steps  
  
        //       $V$ : priorityQueue.Count  
        {  
            priorityQueue = new PriorityQueue<Node, int>(); // $\theta(1)$   
            HashMatrix = new HashSet<int>(); // $\theta(1)$   
            Node n = new Node(Array2D, MatrixSize, null); // $\theta(n^2)$        $n$ : matrix_size  
  
            while (priorityQueue.Count > 0) // $E$ : number of steps  
            {  
                CalculateTime.Start();  
                n1 = priorityQueue.Dequeue(); // $\theta(\log(V))$        $V$ : priorityQueue.Count  
                HashMatrix.Add(n1.Hash_Matrix); // $\theta(1)$   
                Node tmp = n1; // $\theta(1)$   
                if (n1.Cost - n1.level == 0) // $\theta(1)$   
                {  
                    CalculateTime.Stop();  
                    return tmp.level; // $\theta(1)$   
                }  
                n1.Get_adjts(n1); // $\theta(n^2)$        $n$ : matrix_size  
            }  
            return 0;  
        }  
        /// Hamming Function  
        public static int HammingFun(int[,] Mat, int MatrixSize) // $\theta(\text{MatrixSize}^2)$   
        {  
            int Hamming_value = 0; // $\theta(1)$   
            for (int i = 0; i < MatrixSize; i++) // $\theta(\text{MatrixSize}^2)$   
            {  
                for (int j = 0; j < MatrixSize; j++) // $\theta(\text{MatrixSize})$   
                {  
                    if (Mat[i, j] == 0) // $\theta(1)$   
                        continue; // $\theta(1)$   
                    else if ((i * MatrixSize + j + 1) != Mat[i, j]) // $\theta(1)$   
                        Hamming_value += 1; // $\theta(1)$   
                }  
            }  
            return Hamming_value;  
        }  
        /// Manhattan Function  
        public static int ManhattanFun(int[,] Matrix2D, int MatrixSize) // $\theta(n^2)$        $n$ :  
        MatrixSize  
        {  
            int Col = 0; // $\theta(1)$ 
```

```

int Row = 0; //θ(1)
int Manhattan_value = 0; //θ(1)
for (int i = 0; i < MatrixSize; i++) //θ(n^2)      n: MatrixSize
{
    for (int j = 0; j < MatrixSize; j++) //θ(n)      n: MatrixSize
    {
        if (Matrix2D[i, j] == 0) //θ(1)
            continue; //θ(1)
        else if (Matrix2D[i, j] != (i * MatrixSize + j) + 1) //θ(1)
        {
            Col = ((Matrix2D[i, j] - 1) % MatrixSize); //θ(1)
            Row = ((Matrix2D[i, j] - 1) / MatrixSize); //θ(1)
            Manhattan_value += (Math.Abs(Row - i) + Math.Abs(Col - j));
        }
    }
}
return Manhattan_value;
}
/// Check Solvable or not
public static bool CheckSolvability(int[] Matrix1D, int MatrixSize) //θ(n^4) =
θ(s^2)      n: MatrixSize, s: Matrix Length
{
    int inv = 0; //θ(1)
    //compare from first cell to the pre last
    for (int i = 0; i < (MatrixSize * MatrixSize); i++)
    {
        if (Matrix1D[i] == 0) //θ(1)
        {
            // Get Zero Index
            z_index_x = i / MatrixSize; //θ(1)
            z_index_y = i % MatrixSize; //θ(1)
            continue;
        }
        //compare with the cell after i cell till the last cell
        for (int j = (i + 1); j < (MatrixSize * MatrixSize); j++) //θ(n^2)
        {
            if (Matrix1D[j] == 0) //θ(1)
                continue; //θ(1)
            else if (Matrix1D[i] > Matrix1D[j]) //θ(1)
                inv++; //θ(1)
        }
    }
    if (MatrixSize % 2 != 0 && inv % 2 == 0) //θ(1)
        return true; //θ(1)
    else if (MatrixSize % 2 == 0 && inv % 2 != 0 && z_index_x % 2 == 0) //θ(1)
        return true; //θ(1)
    else if (MatrixSize % 2 == 0 && inv % 2 == 0 && z_index_x % 2 != 0) //θ(1)
        return true; //θ(1)
    return false; //θ(1)
}
public static void PrintPath(Node n) //θ(n^2 * c)      n: MatrixSize      c:
number of nodes in path
{
    List<Node> list = new List<Node>(); //θ(1)
    while (n.parent != null) //Best: θ(1),      Worst: θ(v)
    {
        list.Add(n); //θ(1)
        n = n.parent; //θ(1)
    }
    int number_of_movements = 1; //θ(1)

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        for (int i = list.Count - 1; i >= 0; i--) //O(n^2 * c)      n: MatrixSize
        {
            Console.WriteLine("-----> #" + number_of_movements); //O(1)
            for (int j = 0; j < list[i].matrix_size; j++) //O(n^2)      n:
            {
                for (int k = 0; k < list[i].matrix_size; k++) //O(n)      n:
                {
                    Console.Write(list[i].matrix[j, k] + " "); //O(1)
                    Console.WriteLine(); //O(1)
                }
                number_of_movements++; //O(1)
                Console.WriteLine(); //O(1)
            }
        }
    }
}

```

Node Class:

```

using System;
using System.Collections.Generic;
using System.Linq;
using System.Text;
using System.Threading.Tasks;
using static N_PUZZLE.NPuzzleSolution;
namespace N_PUZZLE
{
    public class Node
    {
        public static int ManOrHam = 0; //O(1)
        public int[,] matrix; //O(1)
        public int matrix_size; //O(1)
        public Node parent; //O(1)
        public int z_x; //O(1)
        public int z_y; //O(1)
        public int Cost; //O(1)
        public int level; //O(1)
        public int Hash_Matrix; //O(1)

        public Node(int[,] Matrix2D, int Size, Node par) //O(n^2)      n: matrix_size
        {
            matrix = Matrix2D; //O(1)
            matrix_size = Size; //O(1)
            Hash_Matrix = GetMatrixHash(this.matrix); //O(n^2)      n: matrix_size
            if (par == null) //O(n^2)      n: matrix_size
            {
                level = 0; //O(1)
                parent = null; //O(1)
                z_x = z_index_x; //O(1)
                z_y = z_index_y; //O(1)
                Console.WriteLine("-----\n[1] Hamming Distance\n[2]
                Manhattan Distance"); //O(1)
                Console.Write("\nEnter your choice [1-2]: "); //O(1)
                char Ch = (char)Console.ReadLine()[0]; //O(1)
                if (Ch == '1') //O(n^2)      n: MatrixSize
                {

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        Cost = HammingFun(matrix, matrix_size); //θ(n^2)      n: matrix_size
        ManOrHam = 1; //θ(1)
    }
    else //θ(n^2)      n: MatrixSize
    {
        Cost = ManhattanFun(matrix, matrix_size); //θ(n^2)      n: matrix_size
    }
    priorityQueue.Enqueue(this, Cost); //θ(log(n))      n:
priorityQueue.Count
}
    else //θ(n^2)      n: matrix_size
    {
        parent = par; //θ(1)
        level = parent.level + 1; //θ(1)
        if (ManOrHam == 1) //θ(n^2)      n: MatrixSize
        {
            Cost = HammingFun(matrix, matrix_size) + level; //θ(n^2)      n:
matrix_size
        }
        else //θ(n^2)      n: matrix_size
        {
            Cost = ManhattanFun(matrix, matrix_size) + level; //θ(n^2)      n:
matrix_size
        }
    }
}
public void Get_adjts(Node x) //θ(n^2)      n: matrix_size
{
    if (x.z_x + 1 < x.matrix_size) //θ(n^2)      n: matrix_size
    {
        int[,] mat = new int[x.matrix_size, x.matrix_size]; //θ(1)
        Array.Copy(x.matrix, mat, x.matrix_size * x.matrix_size); //θ(n^2)
n: matrix_size
        mat[x.z_x, x.z_y] = mat[x.z_x + 1, x.z_y]; //θ(1)
        mat[x.z_x + 1, x.z_y] = 0; //θ(1)
        Node n1 = new Node(mat, matrix_size, x); //θ(n^2)      n: matrix_size
        n1.z_x = x.z_x + 1; //θ(1)
        n1.z_y = x.z_y; //θ(1)
        bool visited = HashMatrix.Contains(n1.Hash_Matrix); //θ(n)      n:
HashMatrix.Count
        if (!visited) //θ(log(n))      n: priorityQueue.Count
        {
            priorityQueue.Enqueue(n1, n1.Cost); //θ(log(n))      n:
priorityQueue.Count
        }
    }
    if (x.z_x - 1 >= 0) //θ(n^2)      n: matrix_size
    {
        int[,] mat = new int[x.matrix_size, x.matrix_size]; //θ(1)
        Array.Copy(x.matrix, mat, x.matrix_size * x.matrix_size); //θ(n^2)
n: matrix_size
        mat[x.z_x, x.z_y] = mat[x.z_x - 1, x.z_y]; //θ(1)
        mat[x.z_x - 1, x.z_y] = 0; //θ(1)
        Node n2 = new Node(mat, matrix_size, x); //θ(n^2)      n: matrix_size
        n2.z_x = x.z_x - 1; //θ(1)
        n2.z_y = x.z_y; //θ(1)
        bool visited = HashMatrix.Contains(n2.Hash_Matrix); //θ(n)      n:
HashMatrix.Count
        if (!visited) //θ(log(n))      n: priorityQueue.Count
        {
            priorityQueue.Enqueue(n2, n2.Cost); //θ(log(n))      n:
priorityQueue.Count
        }
    }
}

```

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    }
    if (x.z_y + 1 < x.matrix_size) //θ(n^2)      n: matrix_size
    {
        int[,] mat = new int[x.matrix_size, x.matrix_size]; //θ(1)
        Array.Copy(x.matrix, mat, x.matrix_size * x.matrix_size); //θ(n^2)
n: matrix_size
        mat[x.z_x, x.z_y] = mat[x.z_x, x.z_y + 1]; //θ(1)
        mat[x.z_x, x.z_y + 1] = 0; //θ(1)
        Node n3 = new Node(mat, matrix_size, x); //θ(n^2)      n: matrix_size
        n3.z_x = x.z_x; //θ(1)
        n3.z_y = x.z_y + 1; //θ(1)
        bool visited = HashMatrix.Contains(n3.Hash_Matrix); //θ(n)      n:
HashMatrix.Count
        if (!visited) //θ(log(n))      n: priorityQueue.Count
        {
            priorityQueue.Enqueue(n3, n3.Cost); //θ(log(n))      n:
priorityQueue.Count
        }
    }
    if (x.z_y - 1 >= 0) //θ(n^2)      n: matrix_size
    {
        int[,] mat = new int[x.matrix_size, x.matrix_size]; //θ(1)
        Array.Copy(x.matrix, mat, x.matrix_size * x.matrix_size); //θ(n^2)
n: matrix_size
        mat[x.z_x, x.z_y] = mat[x.z_x, x.z_y - 1]; //θ(1)
        mat[x.z_x, x.z_y - 1] = 0; //θ(1)
        Node n4 = new Node(mat, matrix_size, x); //θ(n^2)      n: matrix_size
        n4.z_x = x.z_x; //θ(1)
        n4.z_y = x.z_y - 1; //θ(1)
        bool visited = HashMatrix.Contains(n4.Hash_Matrix); //θ(n)      n:
HashMatrix.Count
        if (!visited) //θ(log(n))      n: priorityQueue.Count
        {
            priorityQueue.Enqueue(n4, n4.Cost); //θ(log(n))      n:
priorityQueue.Count
        }
    }
}

public int GetMatrixHash(int[,] matrix_hash) //θ(n^2)      n: matrix_size
{
    int RandomHash = 193; //θ(1)
    for (int row = 0; row < matrix_size; row++) //θ(n^2)      n: matrix_size
    {
        for (int col = 0; col < matrix_size; col++) //θ(n)      n: matrix_size
        {
            RandomHash = RandomHash * 59 + (matrix_hash[row, col]); //θ(1)
        }
    }
    return RandomHash; //θ(1)
}
}
}

```

Program.Cs:

```
using System;
using System.Collections.Generic;
using System.Linq;
using System.Text;
using System.IO;
using System.Diagnostics;
using static N_PUZZLE.NPuzzleSolution;

namespace N_Puzz
{
    class Program
    {
        static void Main(string[] args) //0(n^2)      n: MatrixSize
        {
            Console.WriteLine("N Puzzle Problem:\n[1] Sample test cases\n[2] Complete
testing"); //0(1)
            Console.Write("\nEnter your choice [1-2]: "); //0(1)
            char choice = (char)Console.ReadLine()[0]; //0(1)
            bool succeed = false; //0(1)
            switch (choice)
            {
                case '1':
                    Console.WriteLine("-----\n[1] Solvable\n[2]
UnSolvable"); //0(1)
                    Console.Write("\nEnter your choice [1-2]: "); //0(1)
                    char ch = (char)Console.ReadLine()[0]; //0(1)
                    if (ch == '1')
                    {
                        Console.WriteLine("-----\n[1] 8 Puzzle (1)\n[2]
8 Puzzle (2)"); //0(1)
                        Console.WriteLine("[3] 8 Puzzle (3)\n[4] 15 Puzzle - 1");
                        //0(1)

                        Console.WriteLine("[5] 24 Puzzle 1\n[6] 24 Puzzle 2"); //0(1)
                        Console.Write("\nEnter your choice [1-2-3-4-5-6]: "); //0(1)
                        char ch1 = (char)Console.ReadLine()[0]; //0(1)
                        if (ch1 == '1') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//8Puzzle.txt"); //0(n^2)      n: MatrixSize
                        else if (ch1 == '2') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//20Puzzle.txt"); //0(n^2)      n: MatrixSize
                        else if (ch1 == '3') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//14Puzzle.txt"); //0(n^2)      n: MatrixSize
                        else if (ch1 == '4') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//15Puzzle.txt"); //0(n^2)      n: MatrixSize
                        else if (ch1 == '5') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//11Puzzle.txt"); //0(n^2)      n: MatrixSize
                        else if (ch1 == '6') //0(n^2)      n: MatrixSize
                            succeed =
                                Check("../net6.0//Sample//Solvable//24Puzzle.txt"); //0(n^2)      n: MatrixSize
                    }
                    else if (ch == '2')
                    {
```

```

        Console.WriteLine("-----\n[1] 8 Puzzle - Case 1
(1)\n[2] 8 Puzzle(2) - Case 1"); //0(1)
        Console.WriteLine("[3] 8 Puzzle(3) - Case 1\n[4] 15 Puzzle -
Case 2\n[5] 15 Puzzle - Case 3"); //0(1)
        Console.Write("\nEnter your choice [1-2-3-4-5]: "); //0(1)
        char ch1 = (char)Console.ReadLine()[0]; //0(1)
        if (ch1 == '1') //0(n^2) n: MatrixSize
            succeed = Check("../net6.0//Sample//Unsolvable//8 Puzzle -
Case 1.txt");
        else if (ch1 == '2') //0(n^2) n: MatrixSize
            succeed = Check("../net6.0//Sample//Unsolvable//8
Puzzle(2) - Case 1.txt");
        else if (ch1 == '3') //0(n^2) n: MatrixSize
            succeed = Check("../net6.0//Sample//Unsolvable//8
Puzzle(3) - Case 1.txt");
        else if (ch1 == '4') //0(n^2) n: MatrixSize
            succeed = Check("../net6.0//Sample//Unsolvable//15 Puzzle
- Case 2.txt");
        else if (ch1 == '5') //0(n^2) n: MatrixSize
            succeed = Check("../net6.0//Sample//Unsolvable//15 Puzzle
- Case 3.txt");
    }
    break;
case '2':
    Console.WriteLine("-----\n[1] Solvable\n[2]
UnSolvable\n[3] Very Larg"); //0(1)
    Console.Write("\nEnter your choice [1-2-3]: "); //0(1)
    char ch2 = (char)Console.ReadLine()[0]; //0(1)
    if (ch2 == '1')
    {
        Console.WriteLine("-----\n[1] Manhattan &
Hamming\n[2] Manhattan Only"); //0(1)
        Console.Write("\nEnter your choice [1-2]: "); //0(1)
        char ch3 = (char)Console.ReadLine()[0]; //0(1)
        if (ch3 == '1')
        {
            Console.WriteLine("-----\n[1] 50 Puzzle
(1)\n[2] 99 Puzzle - 1"); //0(1)
            Console.WriteLine("[3] 99 Puzzle - 2\n[4] 9999 Puzzle");
            //0(1)
            Console.Write("\nEnter your choice [1-2-3-4]: "); //0(1)
            char ch4 = (char)Console.ReadLine()[0]; //0(1)
            if (ch4 == '1') //0(n^2) n: MatrixSize
                succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan & Hamming//18Puzzle.txt");
            else if (ch4 == '2') //0(n^2) n: MatrixSize
                succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan & Hamming//18Puzzle2.txt");
            else if (ch4 == '3') //0(n^2) n: MatrixSize
                succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan & Hamming//38Puzzle.txt");
            else if (ch4 == '4') //0(n^2) n: MatrixSize
                succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan & Hamming//4Puzzle.txt");
        }
        if (ch3 == '2')
        {
            Console.WriteLine("-----\n[1] 15 Puzzle 1
(1)\n[2] 15 Puzzle 3"); //0(1)
            Console.WriteLine("[3] 15 Puzzle 4\n[4] 15 Puzzle 5");
            //0(1)
            Console.Write("\nEnter your choice [1-2-3-4]: "); //0(1)
            char ch4 = (char)Console.ReadLine()[0]; //0(1)

```

```

        if (ch4 == '1') //0(n^2)      n: MatrixSize
            succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan Only//46Puzzle.txt");
        else if (ch4 == '2') //0(n^2)      n: MatrixSize
            succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan Only//38Puzzle.txt");
        else if (ch4 == '3') //0(n^2)      n: MatrixSize
            succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan Only//44Puzzle.txt");
        else if (ch4 == '4') //0(n^2)      n: MatrixSize
            succeed = Check("../net6.0//Complete//Solvable
puzzles//Manhattan Only//45Puzzle.txt");
    }
}
else if(ch2 == '2')
{
    Console.WriteLine("-----\n[1] 15 Puzzle 1 -
Unsolvable\n[2] 99 Puzzle - Unsolvable Case 1"); //0(1)
    Console.WriteLine("[3] 99 Puzzle - Unsolvable Case 2\n[4] 9999
Puzzle"); //0(1)

    Console.WriteLine("\nEnter your choice [1-2-3-4]: "); //0(1)
    char ch4 = (char)Console.ReadLine()[0]; //0(1)
    if (ch4 == '1') //0(n^2)      n: MatrixSize
        succeed = Check("../net6.0//Complete//Unsolvable
puzzles//15 Puzzle 1 - Unsolvable.txt");
    else if (ch4 == '2') //0(n^2)      n: MatrixSize
        succeed = Check("../net6.0//Complete//Unsolvable
puzzles//99 Puzzle - Unsolvable Case 1.txt");
    else if (ch4 == '3') //0(n^2)      n: MatrixSize
        succeed = Check("../net6.0//Complete//Unsolvable
puzzles//99 Puzzle - Unsolvable Case 2.txt");
    else if (ch4 == '4') //0(n^2)      n: MatrixSize
        succeed = Check("../net6.0//Complete//Unsolvable
puzzles//9999 Puzzle.txt");
    }
    else if(ch2 == '3') //0(n^2)      n: MatrixSize
        succeed = Check("../net6.0//Complete//V. Large test
case//TEST.txt");
    if (succeed)
        Console.WriteLine("\nCongratulations... your program runs
successfully"); //0(1)
    break;
}
}
///  terun 1Darry to 2Darray.....
private static int[,] Conver_To_2D_Array(int[] Matrix1D, int MatrixSize)
//0(n^2)      n: MatrixSize
{
    int[,] Array2D = new int[MatrixSize, MatrixSize]; //0(1)
    for (int i = 0; i < MatrixSize; i++) //0(n^2)      n: MatrixSize
    {
        for (int j = 0; j < MatrixSize; j++) //0(n)      n: MatrixSize
            Array2D[i, j] = Matrix1D[i * MatrixSize + j]; //0(1)
    }
    return Array2D; //0(1)
}
public static bool Check(string Name_of_file) //0(n^2)      n: MatrixSize
{
    FileStream File = new FileStream(Name_of_file, FileMode.Open,
FileAccess.Read); //0(n^2)      n: MatrixSize
    StreamReader Stream_Reader = new StreamReader(File);
    int wrongAnswer = 0; //0(1)
    int MatrixSize = int.Parse(Stream_Reader.ReadLine()); //0(1)

```



```

int[] Matrix1D = new int[MatrixSize * MatrixSize]; //0(1)
int k = 0; //0(1)
for (int i = 0; i < MatrixSize; i++) //0(n^2)      n: MatrixSize
{
    string line = Stream_Reader.ReadLine(); //0(1)
    string[] ss = line.Split(' '); //0(1)

    for (int j = 0; j < MatrixSize; j++) //0(n)      n: MatrixSize
    {
        int x = Int32.Parse(ss[j]); //0(1)
        Matrix1D[k++] = x; //0(1)
    }
}

int RightAnswer = int.Parse(Stream_Reader.ReadLine()); //0(1)
int receivedResult;
if (CheckSolvability(Matrix1D, MatrixSize)) //0(n^4) = 0(s^2)      n:
MatrixSize, s: Matrix Length
{
    receivedResult =
N_PUZZLE.NPuzzleSolution.AStarAlgorithm(Conver_To_2D_Array(Matrix1D, MatrixSize),
MatrixSize, Matrix1D);
}
else
{
    Console.WriteLine("\n====="); //0(1)
    Console.WriteLine("      Not Solvable      "); //0(1)
    Console.WriteLine("====="); //0(1)
    return false; //0(1)
}
if (receivedResult != RightAnswer) //0(1)
{
    Console.WriteLine("\nwrong answer at number of movements --> expected =
" + RightAnswer + " received = " + receivedResult + "\n"); //0(1)
    return false; //0(1)
}
else //0(1)
{
    if (MatrixSize == 3) //0(n^2 * c)      n: MatrixSize      c: number of
nodes in path
    {
        PrintPath(n1); //0(n^2 * c)      n: MatrixSize      c: number of
nodes in path
    }
    Console.WriteLine("\n# of movements = " + n1.level); //0(1)
    Console.WriteLine("\nime : " + CalculateTime.Elapsed); //0(1)
    Console.WriteLine("\nCongratulations... :)\n"); //0(1)
    return true; //0(1)
}
Stream_Reader.Close(); //0(1)
File.Close(); //0(1)
}
}
}

```

Comparing Table:

File Name	Manhattan Time	Hamming Time	Num. Of Moves
50 Puzzle(1).txt	00:00:00.0072954	00:00:00.0874090	18
99 Puzzle - 1.txt	00:00:00.0018032	00:00:00.0018385	18
99 Puzzle - 2.txt	00:00:00.0019596	00:00:00.0025287	38
9999 Puzle.txt	00:00:00.0024273	00:00:00.0022700	4
