University of Pittsburgh

Algorithms and Data Structures 2
CS 1501

Fall 2022

Sherif Khattab

ksm73@pitt.edu

(Slides are adapted from Dr. Ramirez's and Dr. Farnan's CS1501 slides.)

Announcements

- Upcoming Deadlines
  - Lab 8: next Monday 11/14 @ 11:59 pm
  - Homework 8: next Monday 11/14 @ 11:59 pm

Previous lecture

- Minimum Spanning Tree (MST) problem
  - Prim's MST algorithm
    - running time analysis of the Best Edges implementation
    - an implementation that uses a heap
  - Kruskal's MST algorithm

This Lecture

- Weighted Shortest Paths problem
  - Dijkstra's single-source shortest paths algorithm
  - Bellman-Ford's shortest paths algorithm

Muddiest Points

- **Q: Please review an example of eager prims and kruskals again**
- Sure!

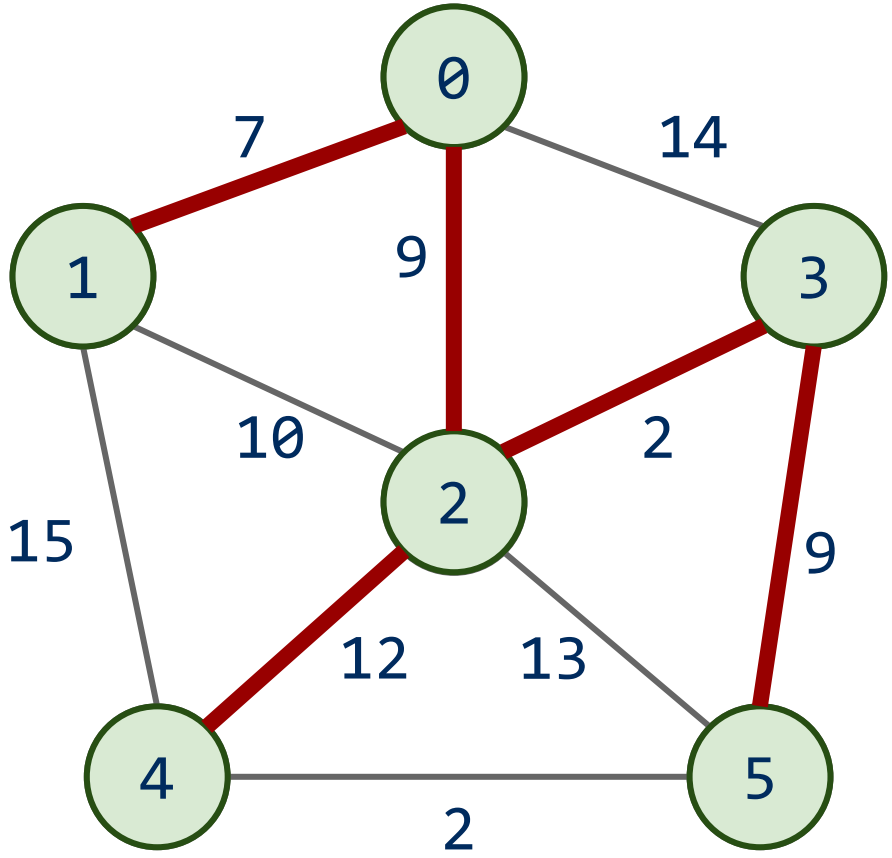
Problem of the Day: Weighted Shortest Paths

- Input:
  - A road network
    - Road segments and intersections
    - Road segments are labeled by travel time
      - From length and maximum speed
      - How do we get max speed?
  - Starting address and destination address
- Output:
  - A shortest path from source to destination

Dijkstra's algorithm

- Set a distance value of Double.POSITIVE_INFINITY for all vertices
- distance[start] = 0
- Set cur = start
- While destination is not visited:
  - For each unvisited neighbor x of cur:
    - Compute distance from start to x through cur
      - distance[cur] + weight of edge between cur and x
    - Update distance[x] if computed distance < distance[x]
  - Mark cur as visited
  - Let cur be the unvisited vertex with the smallest tentative distance from start

Dijkstra's example

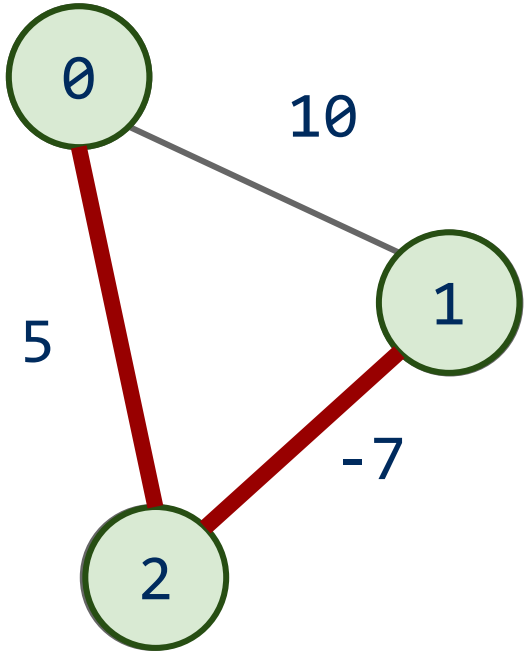


	Distance	Parent
0	0	--
1	7	0
2	9	0
3	11	2
4	21	2
5	20	3

Analysis of Dijkstra's algorithm

- How to implement?
  - Best path/parent array?
    - Runtime?
  - PQ?
    - Turns out to be very similar to Eager Prims
      - Storing paths instead of edges
    - Runtime?

Dijkstra's example with negative edge weights



	Distance	Parent
0	0	--
1	-2	1
2	5	0

Incorrect!

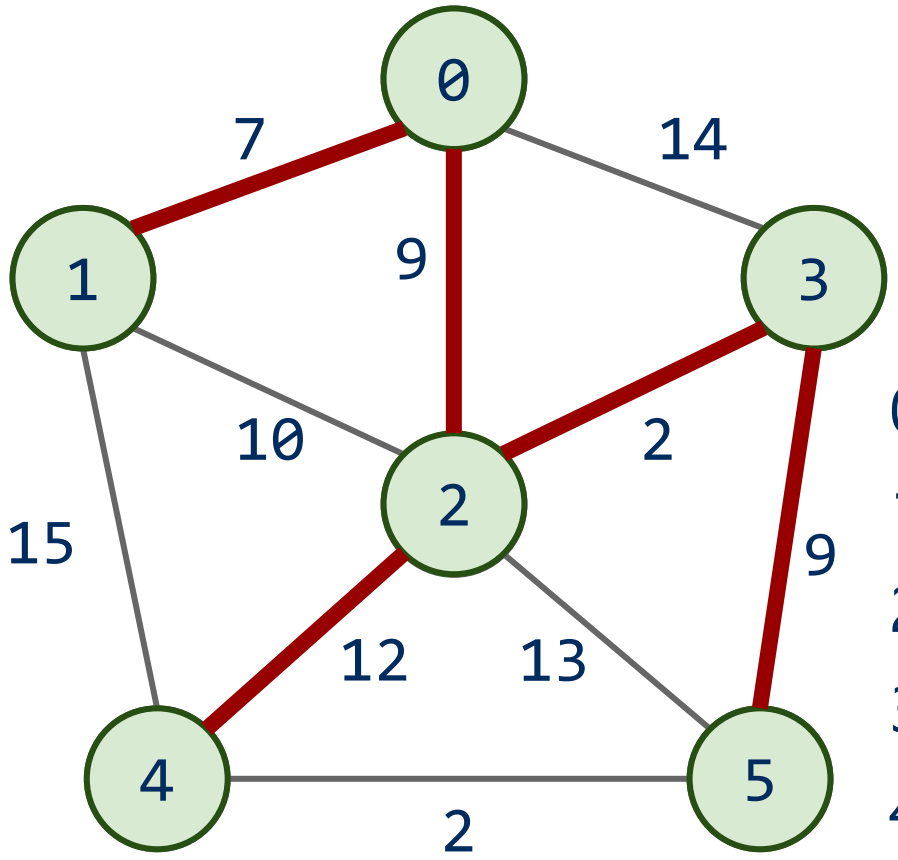
Analysis of Dijkstra's algorithm

Dijkstra's is correct only when all edge weights >= 0

Bellman-Ford's algorithm

- Set a distance value of Double.POSITIVE_INFINITY for all vertices
- Initialize a FIFO Q
- distance[start] = 0
- add start to Q
- While Q is not empty:
  - cur = pop a vertex from Q
  - For each non-parent neighbor x of cur:
    - Compute distance from start to x through cur
      - distance[cur] + weight of edge between cur and x
    - if computed distance < distance[x]
      - Update distance[x]
      - add x to Q if not already there

Bellman-Ford's example



	Distance	Parent
0	0	--
1	7	0
2	9	0
3	11	2
4	21	2
5	20	3

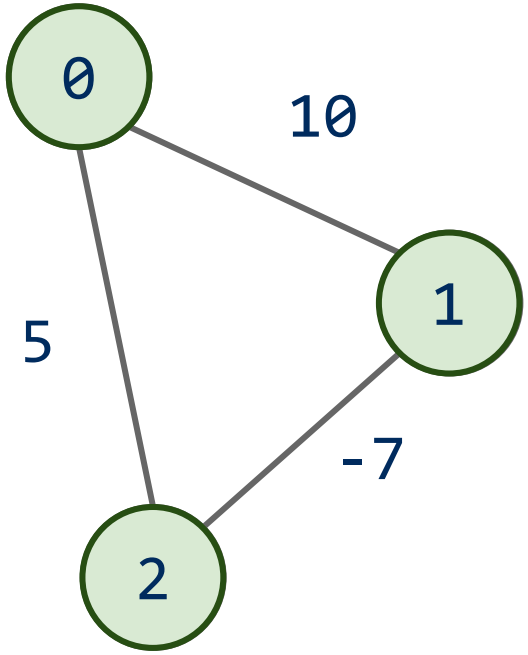
FIFO Q:

0
1
2
3
4
5

Analysis of Bellman-Ford's algorithm

- How to implement?
- Runtime?

Bellman-Ford's example with negative edge weights



	Distance	Parent
0	0	--
1	-4	2
2	3	1

FIFO Q:

0

1

2

Correct!