The QFM Operator System

Formal Foundations of Operator-Based
Quantum-Inspired Computation

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Abstract

This paper develops the operator-theoretic foundations of Quansistor Field Mathematics (QFM), a framework in which computation is expressed through multiplicative operator dynamics over the natural numbers. We introduce forward and backward prime-shift operators, their self-adjoint generators, diffusion dynamics, and the resulting Hamiltonian structure. The framework provides a rigorous substrate for quantum-inspired computation without physical qubits and forms the mathematical core of the Quantum Virtual Machine (QVM) architecture.

Contents

1 Introduction

Classical computation is traditionally formulated within an additive geometric paradigm. Information is stored in arrays, registers, or tensor grids, and computation proceeds by local transformations over linear or spatial indices. Even highly parallel architectures—such as GPUs or distributed clusters—remain fundamentally additive: neighboring states are defined by proximity in memory or geometry, and complexity is managed by spatial decomposition.

Quansistor Field Mathematics (QFM) departs from this paradigm at a foundational level. Instead of additive geometry, QFM adopts *multiplicative geometry*, in which the natural numbers $\mathbb{N}$ form the underlying index space and prime numbers generate elementary symmetries. Computational states are not arranged along a line or grid, but along the multiplicative structure induced by prime factorization.

This shift is not cosmetic. It replaces spatial locality with *arithmetic locality*, and linear translation with *multiplicative transport*. As a consequence, phenomena that appear non-local or global in additive models emerge naturally and locally in the multiplicative setting.

1.1 From Additive Motion to Multiplicative Evolution

In additive models, the elementary symmetry is translation: $f(x) \mapsto f(x+a)$. In QFM, the corresponding symmetry is generated by multiplication:

$$n \mapsto pn,$$

where p is prime. Each prime induces its own independent direction of motion, and the arithmetic structure of $\mathbb{N}$ replaces Euclidean space as the computational manifold.

Within this framework:

- computation becomes *spectral evolution* rather than sequential instruction execution,
- primes act as *generative symmetries* rather than constants,
- factorization emerges as a form of *diffusion*,
- and arithmetic complexity is encoded into operator spectra.

These principles define a computational model that is neither classical nor quantum in the physical sense, but which exhibits quantum-like behavior at the level of operator dynamics.

1.2 Operator-Centric Computation

QFM is fundamentally operator-based. States are not primary objects; operators are. A computation is specified not by a sequence of instructions but by the selection and composition of operators acting on an underlying Hilbert space.

For each prime p, QFM introduces a pair of elementary operators:

- a forward multiplicative shift operator A_p,
- and a backward multiplicative shift operator B_p.

These operators act on arithmetic amplitudes indexed by $\mathbb{N}$ and encode the elementary motions of the system. Their algebraic relationship gives rise to self-adjoint generators, diffusion processes, and ultimately to a Hamiltonian governing global evolution.

Crucially, this operator system is not an abstract curiosity. It forms the exact computational substrate of higher-level architectures, including:

- the Quantum Virtual Machine (QVM),
- the Quansistor Field Processor (QFP),
- quantum-inspired acceleration units (QPU),
- and low-level execution languages such as QWASM.

The present document focuses exclusively on the mathematical and operator-theoretic foundations. Architectural, runtime, and governance aspects are treated in separate whitepapers built on top of this core.

1.3 Why Multiplicative Geometry Matters

The multiplicative structure of $\mathbb{N}$ is exceptionally rich. A single multiplicative step can connect indices that differ by orders of magnitude. As a result:

- long-range interactions arise naturally,
- interference patterns emerge from operator superposition,
- and global correlations can be generated without explicit global control.

These features closely resemble properties usually attributed to quantum systems. However, in QFM they arise without physical qubits, wavefunction collapse, or probabilistic measurement. The dynamics are fully deterministic, reversible where required, and entirely classical in their implementation.

The term *quantum-inspired* is therefore used in a precise sense: the mathematics mirrors structures known from quantum theory, while the ontology remains purely operator-theoretic.

1.4 Scope and Structure of This Paper

This paper develops the QFM operator system in a step-by-step and self-contained manner.

- Section 2 introduces the Hilbert space of multiplicative states and discusses admissible weightings and inner products.
- Sections 3 and 4 define the forward and backward prime-shift operators and construct their self-adjoint combinations.
- Sections 5 and 6 develop diffusion dynamics and the full QFM Hamiltonian.
- Section 7 interprets the resulting spectra and connects them to arithmetic and statistical phenomena.
- Section 8 explains how these operators acquire computational meaning within QVM, QFP, and QPU architectures.
- Sections 9 and 10 situate the framework relative to the Smrk Hamiltonian and outline extensions.

The goal is to present a mathematically rigorous operator calculus that can serve simultaneously as:

- a foundation for quantum-inspired computation,
- a bridge between number theory and spectral dynamics,
- and a stable core for large-scale distributed execution systems.

1.5 What This Paper Does Not Claim

To avoid ambiguity, it is important to state explicitly what is *not* claimed here.

- No physical quantum behavior is asserted.
- No claim of computational supremacy is made.
- No physical interpretation of operators as particles or forces is required.

All results and constructions remain within classical mathematics and deterministic computation. Any quantum-like behavior arises strictly from the structure of operators and the geometry of the underlying arithmetic space.

With this foundation in place, we proceed to the formal construction of the multiplicative Hilbert space on which the entire operator system is built.

2 The Hilbert Space of Multiplicative States

The operator system of QFM acts on a Hilbert space whose geometry is induced not by spatial adjacency but by the multiplicative structure of the natural numbers. This section introduces the underlying state space, its admissible inner products, and the role of weighting in ensuring well-defined operator dynamics.

2.1 Arithmetic State Space

The fundamental index set of QFM is the set of natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$. A computational state is represented by a complex-valued function

$$\psi : \mathbb{N} \to \mathbb{C},$$

where $\psi(n)$ is interpreted as an *arithmetic amplitude* associated to the integer n.

Unlike classical register-based computation, these amplitudes are not addresses or memory cells; they form a distributed field over arithmetic space. The full collection of admissible states is obtained by imposing a Hilbert space structure.

2.2 Weighted ℓ^2 Spaces

We define the QFM state space as a weighted square-summable space

$$\mathcal{H} = \ell^2(\mathbb{N}, w),$$

where $w : \mathbb{N} \to \mathbb{R}_{>0}$ is a fixed weight function and

$$\|\psi\|^2 = \sum_{n=1}^{\infty} w(n)|\psi(n)|^2 < \infty.$$

The associated inner product is

$$\langle \psi, \phi \rangle = \sum_{n=1}^{\infty} w(n)\, \psi(n)\, \overline{\phi(n)}.$$

The choice of $w(n)$ determines the geometry of arithmetic space and strongly influences the analytic properties of QFM operators.

2.3 Canonical Choices of Weight

Two choices of weight appear naturally in QFM and will be used throughout this work.

Uniform Weight.

$$w(n) = 1.$$

This yields the standard Hilbert space $\ell^2(\mathbb{N})$ and represents a flat arithmetic geometry. It is convenient for conceptual exposition and for computational implementations where normalization is handled explicitly.

Conformal Arithmetic Weight.

$$w(n) = \frac{1}{n}.$$

This choice introduces scale invariance under multiplication and is particularly well adapted to analytic number-theoretic constructions, including Hamiltonians related to the Riemann zeta function. In this geometry, large integers contribute less weight, reflecting their increasing arithmetic complexity.

Both choices lead to separable Hilbert spaces and admit dense subspaces suitable for operator analysis.

2.4 Dense Domains and Test Functions

To define unbounded or partially bounded operators rigorously, we specify a canonical dense subspace

$$\mathcal{D} \subset \mathcal{H}$$

consisting of finitely supported functions on $\mathbb{N}$.

Elements of $\mathcal{D}$ vanish outside a finite set and are preserved under all elementary QFM operators. This space serves as the common core domain for forward shifts, backward shifts, and their algebraic combinations.

All adjoint relations stated later are understood initially on $\mathcal{D}$ and then extended by closure where appropriate.

2.5 Multiplicative Geometry of the State Space

The Hilbert space $\mathcal{H}$ inherits a geometry that differs fundamentally from additive or spatial models.

- Neighboring indices are defined multiplicatively, not additively.
- Prime numbers generate independent geometric directions.

- Distance is not metric in the usual sense, but spectral and operator-induced.

A single multiplication by a prime connects indices that may differ by several orders of magnitude. As a result, long-range correlations are intrinsic to the space rather than emergent from repeated local steps.

This multiplicative geometry underlies the non-local propagation effects observed later in diffusion and Hamiltonian evolution.

2.6 Hilbert Space Versus Physical State Space

It is important to distinguish the present construction from physical Hilbert spaces in quantum mechanics.

- The amplitudes $\psi(n)$ do not represent probabilities of physical measurement outcomes.
- No collapse postulate or measurement axiom is assumed.
- The Hilbert structure is a mathematical device encoding operator dynamics, not physical states.

The use of Hilbert space is therefore instrumental rather than ontological. It provides a precise language for spectral analysis, adjointness, and operator evolution, while remaining fully compatible with deterministic computation.

2.7 Preparation for Operator Dynamics

With the Hilbert space $\mathcal{H}$ fixed, we are prepared to define the elementary operators that generate QFM dynamics. These operators act naturally on $\mathcal{D}$, respect the multiplicative geometry of $\mathbb{N}$, and admit well-defined adjoints with respect to the chosen inner product.

The next section introduces the forward and backward prime-shift operators, which form the atomic building blocks of the entire QFM operator algebra.

3 Forward and Backward Prime-Shift Operators

The elementary dynamics of Quansistor Field Mathematics are generated by operators associated with prime numbers. Each prime p defines an independent multiplicative direction along which arithmetic amplitudes may propagate. In this section we introduce the forward and backward prime-shift operators, establish their basic properties, and analyze their adjoint relationship with respect to the QFM Hilbert space.

3.1 The Forward Prime-Shift Operator

For each prime number p, we define the *forward multiplicative shift operator* A_p by

$$(A_p\psi)(n) = \psi(pn),$$

for all $\psi \in \mathcal{D}$ and $n \in \mathbb{N}$.

Interpretation. The operator A_p implements a multiplicative translation along the p-direction. It moves amplitude from index n to index pn, thereby expanding arithmetic scale.

From a computational perspective, A_p corresponds to branching or expansion of the state space through the p-channel. A single application can increase the index by orders of magnitude, reflecting the intrinsic non-locality of multiplicative geometry.

Basic Properties.

- A_p is linear.
- A_p preserves finite support and thus maps $\mathcal{D}$ into itself.
- A_p is bounded on $\ell^2(\mathbb{N})$ but not self-adjoint.

The non-self-adjointness of A_p is expected: forward multiplicative motion alone is irreversible, corresponding to pure expansion without compensation.

3.2 The Backward Prime-Shift Operator

The *backward multiplicative shift operator* B_p is defined by

$$(B_p\psi)(n) = \mathbf{1}_{p|n}\,\psi\left(\frac{n}{p}\right),$$

where $\mathbf{1}_{p|n}$ is the indicator function of divisibility by p.

Interpretation. The operator B_p implements contraction along the multiplicative p-direction. Amplitude is transported from pn back to n, but only when the arithmetic structure allows it. Unlike A_p, this operator is selective: it annihilates components not divisible by p.

Basic Properties.

- B_p is linear.
- B_p preserves $\mathcal{D}$.
- B_p is not self-adjoint and not invertible.

The asymmetry between A_p and B_p reflects the arithmetic fact that multiplication by p is always possible, whereas division by p is conditional.

3.3 Adjoint Relationship

A central structural fact of QFM is that A_p and B_p form an adjoint pair up to a scalar factor determined by the chosen weight.

For the conformal arithmetic weight $w(n) = 1/n$, one verifies directly that

$$A_p^* = p\,B_p$$

on the dense domain $\mathcal{D}$.

Derivation. Let $\psi, \phi \in \mathcal{D}$. Then

$$\langle A_p \psi, \phi \rangle = \sum_{n=1}^{\infty} \frac{1}{n} \psi(pn) \, \overline{\phi(n)}.$$

Changing variables $m = pn$ yields

$$\langle A_p \psi, \phi \rangle = \sum_{m=1}^{\infty} \frac{p}{m} \psi(m) \, \overline{\phi\Big(\frac{m}{p}\Big)} = \langle \psi, p B_p \phi \rangle.$$

This identity is the cornerstone of all self-adjoint constructions in QFM.

3.4 Arithmetic Geometry and Non-Commutativity

The operators $\{A_p, B_p\}_{p \in \mathbb{P}}$ generate a non-trivial operator algebra. In particular:

- Operators associated with distinct primes generally do not commute.
- Forward and backward shifts along different prime directions interact through the arithmetic structure of $\mathbb{N}$.

For example, for distinct primes $p \neq q$,

$$A_p A_q = A_q A_p,$$

since multiplication is commutative, but

$$B_p B_q \neq B_q B_p$$

on general states, due to divisibility constraints.

This asymmetry encodes the partial order inherent in prime factorization and is responsible for the rich combinatorial structure of QFM dynamics.

3.5 Reversibility and Irreversibility

Individually, neither A_p nor B_p defines a reversible evolution. Pure forward motion leads to unbounded expansion, while pure backward motion collapses information.

Reversibility emerges only when forward and backward components are combined in a balanced manner. This observation motivates the construction of self-adjoint generators that incorporate both directions symmetrically.

3.6 Role as Primitive Computational Operations

In the broader QVM stack, the operators A_p and B_p play the role of primitive arithmetic instructions.

- A_p corresponds to expansion or branching in multiplicative space.
- B_p corresponds to contraction or filtering by arithmetic divisibility.

All higher-level dynamics—diffusion, Hamiltonian evolution, and spectral computation—are built from these two primitives.

The next section introduces their canonical self-adjoint combination, which serves as the elementary reversible generator of QFM dynamics.

4 Construction of Self-Adjoint Generators K_p

In both quantum mechanics and operator-based computation, reversible dynamics require self-adjoint generators. In the additive setting, this role is played by momentum and Hamiltonian operators. In QFM, the elementary generators must instead respect multiplicative geometry and the arithmetic structure of $\mathbb{N}$.

This section constructs, for each prime p, a canonical self-adjoint generator K_p by symmetrically combining the forward and backward prime-shift operators introduced earlier.

4.1 Motivation for Self-Adjointness

The forward operator A_p produces irreversible expansion, while the backward operator B_p produces irreversible contraction. Neither alone can generate stable evolution. Any long-term dynamics built solely from one of them either diverges or collapses.

Self-adjointness provides the necessary balance. A self-adjoint operator:

- generates reversible evolution,
- admits real spectral values,
- supports well-defined exponential dynamics,
- and preserves inner products under unitary or contractive flows.

The construction of K_p ensures that multiplicative motion along the p-direction becomes symmetric and energetically balanced.

4.2 Definition of the Generator

Let p be a prime number. We define the operator

$$K_p = \frac{1}{p}\,A_p + p\,B_p$$

on the dense domain $\mathcal{D} \subset \mathcal{H}$.

This particular weighting is not arbitrary. It is uniquely determined by the adjoint relation

$$A_p^* = p\,B_p$$

with respect to the conformal weight $w(n) = 1/n$.

4.3 Self-Adjointness

Proposition. The operator K_p is symmetric on $\mathcal{D}$ and admits a unique self-adjoint extension.

Proof (Formal). Using the adjoint relation, we compute

$$K_p^* = \left(\frac{1}{p}A_p + pB_p\right)^* = \frac{1}{p}A_p^* + pB_p^* = \frac{1}{p}(pB_p) + p\left(\frac{1}{p}A_p\right) = K_p.$$

Thus K_p is formally self-adjoint on $\mathcal{D}$. Since $\mathcal{D}$ is dense and invariant under K_p, standard results from operator theory ensure essential self-adjointness under mild growth conditions, which are satisfied in the present setting.

4.4 Geometric Interpretation

The operator K_p generates symmetric motion along the multiplicative p-axis. Whereas A_p and B_p individually favor expansion or contraction, their weighted sum produces oscillatory or diffusive behavior analogous to kinetic motion.

In this sense, K_p plays the role of a *multiplicative momentum operator*. It measures and propagates arithmetic variation along the p-direction while conserving total amplitude.

4.5 Comparison with Additive Quantum Mechanics

In standard quantum mechanics, the kinetic term arises from the Laplacian, which itself is built from forward and backward finite differences. The operator K_p is the multiplicative analogue of this construction.

- Additive space: $f(x+a) + f(x-a)$
- Multiplicative space: $\psi(pn) + \psi(n/p)$

The weights $1/p$ and p reflect the non-uniform geometry of arithmetic space and ensure invariance of the inner product.

4.6 Spectral Role of K_p

Each generator K_p contributes a distinct spectral component to the full QFM dynamics. Since primes are independent generators of multiplicative structure, the family $\{K_p\}_{p\in\mathbb{P}}$ forms a complete basis of elementary kinetic modes.

Key consequences include:

- arithmetic diffusion decomposes into independent prime channels,
- spectral interference arises from superposition across primes,
- global dynamics inherit fine structure from prime distribution.

This prime-resolved structure has no analogue in additive models and is a unique feature of QFM.

4.7 Computational Interpretation

From the perspective of execution systems:

- K_p is the smallest reversible computational primitive,
- it acts as an arithmetic ALU operation in QFP architectures,
- and it defines a frequency-like mode for QPU acceleration.

Higher-level operators, including diffusion generators and Hamiltonians, are constructed as weighted sums of the K_p.

4.8 Preparation for Global Dynamics

With the family $\{K_p\}$ defined, we are now equipped to assemble operators that govern global arithmetic evolution. The next section introduces multiplicative diffusion operators formed by summing K_p across primes with appropriate weights.

These operators describe how information propagates through arithmetic space as a collective phenomenon rather than a sequence of isolated prime transitions.

5 Multiplicative Diffusion and Global Dynamics

With the family of self-adjoint generators $\{K_p\}_{p\in\mathbb{P}}$ established, we now turn to collective dynamics. While each K_p describes reversible motion along a single multiplicative direction, the full behavior of the system emerges only when all prime directions act together.

This section introduces multiplicative diffusion operators and global dynamical generators formed as weighted sums over primes. These operators govern the large-scale propagation of arithmetic amplitudes and provide the first genuinely global notion of evolution in QFM.

5.1 From Local Generators to Global Flow

In additive geometries, diffusion arises from summing second-order finite differences across spatial directions. In QFM, the role of spatial directions is played by prime indices. Each prime contributes an independent channel of motion, and global propagation is obtained by superposing their effects.

Formally, this leads to operators of the form

$$\sum_{p\in\mathbb{P}} w_p K_p,$$

where $\{w_p\}$ are positive weights controlling the relative strength of each prime channel.

5.2 The Multiplicative Diffusion Operator

We define the *multiplicative diffusion operator* D by

$$(D\psi)(n) = \sum_{p\in\mathbb{P}} c_p\big(A_p\psi(n) + B_p\psi(n)\big),$$

or equivalently, in terms of self-adjoint generators,

$$D = \sum_{p\in\mathbb{P}} c'_p K_p,$$

where the coefficients $c_p, c'_p > 0$ are related by normalization constants.

Interpretation. The operator D describes arithmetic heat flow. Amplitude diffuses along all multiplicative directions simultaneously, with primes acting as local generators on a highly non-uniform arithmetic manifold.

5.3 Choice of Weights

The qualitative behavior of diffusion depends critically on the choice of weights.

Canonical Kinetic Weights.

$$c_p = \frac{1}{p}.$$

This choice treats primes as increasingly costly directions and ensures convergence of the prime sum. It is the natural analogue of uniform diffusion in logarithmic arithmetic space.

Analytic Weights.

$$c_p = p^{-s}, \qquad s > 1.$$

These weights appear naturally in analytic number theory and allow tuning of diffusion regimes through the complex parameter s.

Adaptive or Contextual Weights. In computational settings, weights may depend on runtime constraints, spectral feedback, or regulatory logic imposed by higher-level execution systems.

5.4 Self-Adjointness and Stability

Provided the weights satisfy mild summability conditions, the diffusion operator D is symmetric on $\mathcal{D}$ and admits a self-adjoint extension.

Self-adjointness ensures:

- real spectrum,
- stability of long-time evolution,
- well-defined exponential flows.

In contrast to purely forward or backward dynamics, diffusion balances expansion and contraction across the entire arithmetic structure.

5.5 Global Dynamics Operator

Beyond diffusion, we define the *global dynamics operator*

$$M = \sum_{p \in \mathbb{P}} w_p K_p, \qquad w_p > 0,$$

which serves as the fundamental generator of QFM time evolution.

Interpretation. The operator M is the multiplicative analogue of a Laplacian. It encodes the total arithmetic kinetic energy of the system and governs reversible global motion in arithmetic space.

Special regimes include:

- uniform weights: isotropic multiplicative geometry,
- logarithmic weights: Chebyshev-type dynamics,
- analytically continued weights: zeta-like operator geometry.

5.6 Evolution Semigroups

The global dynamics operator generates an evolution semigroup

$$\psi(t) = e^{-tM}\psi(0), \qquad t \geq 0.$$

This evolution exhibits:

- interference between prime channels,
- attenuation of high-energy modes,
- emergence of dominant spectral components.

Despite its formal similarity to quantum evolution, this flow remains entirely deterministic and does not rely on probabilistic interpretation.

5.7 Diffusion as Factorization Flow

A key insight of QFM is that multiplicative diffusion encodes arithmetic structure.

- Highly composite integers act as hubs of diffusion.
- Prime powers generate resonance-like structures.
- Factorization emerges as structured amplitude transport rather than discrete search.

In this sense, diffusion is not noise but structured exploration of arithmetic geometry.

5.8 Relation to Physical Analogies

While diffusion terminology is borrowed from physics, it is used here in a strictly operator-theoretic sense.

- No physical particles are assumed.
- No thermodynamic interpretation is required.
- Time is an abstract evolution parameter.

The analogy serves only to guide intuition; all results follow from the algebraic structure of operators on $\mathcal{H}$.

5.9 Preparation for Hamiltonian Construction

The operators introduced in this section form the kinetic backbone of QFM. To describe full computational energy landscapes, additional potential terms must be incorporated.

The next section introduces the complete QFM Hamiltonian, obtained by augmenting global dynamics with arithmetic potential functions that encode constraints, regulation, and analytic structure.

6 The Full QFM Hamiltonian

The operators introduced in the previous sections describe the kinetic aspect of arithmetic evolution: reversible motion and diffusion across multiplicative directions. To obtain a complete dynamical system, this kinetic structure must be augmented by potential terms that encode constraints, asymmetries, and arithmetic structure.

This section introduces the full QFM Hamiltonian, analyzes its components, and clarifies the role of arithmetic potentials in shaping spectral behavior.

6.1 General Structure of the Hamiltonian

The QFM Hamiltonian is defined as

$$H_{\mathrm{QFM}} = \sum_{p \in \mathbb{P}} K_p + V,$$

where:

- $\sum_p K_p$ represents the kinetic generator of multiplicative motion,
- V is a multiplication operator acting diagonally in arithmetic space.

The Hamiltonian acts initially on the dense domain $\mathcal{D}$ and admits self-adjoint extensions under standard summability and growth conditions.

6.2 The Potential Term

The potential operator V is defined by pointwise multiplication:

$$(V\psi)(n) = V(n)\,\psi(n),$$

where $V(n)$ is a real-valued arithmetic function.

The role of the potential is threefold:

- to introduce arithmetic inhomogeneity,
- to encode global constraints and regulation,
- to shape the spectral landscape of the operator.

6.3 Canonical Arithmetic Potentials

A general and flexible form of the potential is

$$V(n) = \alpha\,\Lambda(n) + \beta\,\log n + U(n),$$

where the individual components have distinct interpretations.

Von Mangoldt Term. The von Mangoldt function $\Lambda(n)$ singles out prime powers and introduces sharp arithmetic structure. Its inclusion creates localized energy contributions aligned with prime generation.

Logarithmic Term. The term $\log n$ introduces scale-dependent energy growth and ensures proper asymptotic control. It reflects the increasing arithmetic complexity of large integers.

Contextual Potential. The function $U(n)$ represents additional constraints. In abstract QFM, this term may be set to zero; in computational settings, it may encode regulatory logic, execution constraints, or contextual semantics imposed by higher-level systems.

6.4 Self-Adjointness of the Full Hamiltonian

Under mild conditions on the coefficients α, β, and the growth of $U(n)$, the operator H_{QFM} is symmetric on $\mathcal{D}$ and admits a self-adjoint extension.

The essential requirements are:

- real-valued potentials,
- relative boundedness of V with respect to the kinetic term,
- summability of the prime-indexed generators.

These conditions ensure that the Hamiltonian generates a well-defined spectral theory and stable evolution.

6.5 Hamiltonian Evolution

The Hamiltonian defines an evolution semigroup

$$\psi(t) = e^{-tH_{\mathrm{QFM}}}\psi(0),$$

which governs global arithmetic dynamics.

This evolution exhibits:

- interference between prime-induced modes,
- competition between kinetic diffusion and potential localization,
- attenuation or amplification of spectral components.

The flow is deterministic and reversible where the spectrum permits.

6.6 Interpretation as Computational Energy

In the computational interpretation of QFM:

- the kinetic term represents free arithmetic exploration,
- the potential encodes constraints, costs, or regulatory forces,
- the Hamiltonian defines the total computational energy landscape.

States evolve toward low-energy configurations, which correspond to arithmetically structured or spectrally stable patterns.

6.7 Riemann-Specific Hamiltonian Regime

A distinguished specialization of the QFM Hamiltonian is obtained by choosing

$$V(n) = \alpha\,\Lambda(n) + \beta\,\log n,$$

with fixed coefficients $\alpha, \beta \in \mathbb{R}$.

The resulting operator,

$$(H_{\mathrm{RH}}\psi)(n) = \sum_{p\in\mathbb{P}} \frac{1}{p}\big(\psi(pn) + \mathbf{1}_{p|n}\,\psi(n/p)\big) + (\alpha\Lambda(n) + \beta\log n)\,\psi(n),$$

coincides with the Smrk Hamiltonian formulation developed in subsequent work.

This operator is conjectured to be essentially self-adjoint and to exhibit spectral properties closely related to the non-trivial zeros of the Riemann zeta function.

6.8 Separation of Claims

It is important to emphasize that, at this stage:

- no spectral identification with zeta zeros is assumed,
- no proof of the Riemann Hypothesis is claimed,
- the Hamiltonian is presented as a mathematically well-defined operator model.

Any deeper number-theoretic interpretation depends on further analytic and spectral analysis.

6.9 Role Within the QFM Operator System

The Hamiltonian integrates all previously introduced structures:

- prime-shift operators define elementary motion,
- self-adjoint generators ensure reversibility,
- diffusion governs global propagation,
- potentials encode arithmetic and contextual structure.

It is therefore the central object of the QFM operator system and the primary interface between arithmetic geometry and computation.

6.10 Preparation for Spectral Analysis

With the Hamiltonian in place, we are now prepared to study its spectrum and the qualitative information it encodes about arithmetic structure.

The next section develops the spectral interpretation of QFM operators and explains how eigenvalues, resonances, and statistical features emerge from the Hamiltonian dynamics.

7 Spectral Interpretation and Arithmetic Meaning

The Hamiltonian structure developed in the previous section gives rise to a rich and highly nontrivial spectral theory. In QFM, spectral data do not merely describe abstract operator properties; they encode qualitative and quantitative information about arithmetic structure, multiplicative geometry, and global operator dynamics.

This section explains how the spectrum of QFM operators should be interpreted, what arithmetic meaning can be ascribed to eigenvalues and spectral distributions, and which conclusions are justified at the present level of rigor.

7.1 Spectrum as Global Arithmetic Invariant

Let H denote a self-adjoint QFM Hamiltonian. Its spectrum

$$\mathrm{Spec}(H) \subset \mathbb{R}$$

is invariant under unitary equivalence and therefore captures intrinsic features of the underlying operator system.

In the QFM setting, spectral values summarize how arithmetic amplitudes propagate across multiplicative geometry under the combined action of all prime directions and potential terms.

Unlike local operator coefficients, spectral data reflect global structure: they integrate information across infinitely many arithmetic scales.

7.2 Eigenstates and Arithmetic Structure

An eigenstate ψ satisfying

$$H\psi = \lambda\psi$$

represents a dynamically stable arithmetic configuration. Such states are invariant under Hamiltonian evolution up to a phase or scaling factor.

Qualitatively:

- low-energy eigenstates correspond to arithmetically coherent patterns,
- higher-energy modes exhibit increasing oscillation across multiplicative scales,
- localization phenomena may occur near highly structured integers such as prime powers.

Eigenstates are not interpreted as physical states but as invariant modes of operator dynamics.

7.3 Resonances and Prime Interference

The multiplicative geometry of $\mathbb{N}$ causes spectral interference between prime channels. Since global dynamics are assembled from sums over p, distinct prime-induced modes interact nontrivially.

This interaction gives rise to resonance-like phenomena:

- constructive interference amplifies specific arithmetic patterns,
- destructive interference suppresses others,
- resonance frequencies reflect correlations among prime directions.

Such effects have no direct analogue in additive lattices and are a distinctive feature of QFM.

7.4 Continuous Versus Discrete Spectrum

Depending on the choice of potential and weights, the QFM Hamiltonian may exhibit:

- purely continuous spectrum,
- mixed continuous and discrete components,
- or embedded eigenvalues.

The logarithmic potential term typically enforces spectral confinement, while pure kinetic operators tend toward continuous spectra.

Understanding the precise spectral type is essential for interpreting long-time dynamics and stability properties.

7.5 Statistical Properties of the Spectrum

In high-energy regimes, spectral statistics of QFM Hamiltonians often exhibit behavior reminiscent of random matrix ensembles.

Empirically and heuristically:

- level spacings may approach Wigner–Dyson distributions,
- correlations reflect arithmetic randomness of primes,
- deviations encode structured arithmetic input via potentials.

These observations motivate comparison with random matrix theory, but such comparisons are interpretative tools rather than formal identifications.

7.6 Arithmetic Meaning of Spectral Density

The density of states

$$\rho(\lambda) = \frac{d}{d\lambda} \#\{\text{eigenvalues} \leq \lambda\}$$

encodes how arithmetic complexity is distributed across energy scales.

In QFM:

- rapid growth of $\rho(\lambda)$ reflects increasing arithmetic branching,
- irregularities correspond to prime-induced structural features,
- smoothing effects arise from diffusion across multiplicative directions.

Thus spectral density provides a coarse but global measure of arithmetic complexity.

7.7 Relation to Analytic Number Theory

The inclusion of the von Mangoldt function and logarithmic potentials links QFM Hamiltonians to classical objects of analytic number theory.

While no direct equivalence is asserted, the operator framework naturally interfaces with:

- explicit formulae relating primes and spectral sums,
- trace-like expressions over arithmetic states,
- Hilbert–Pólya-inspired interpretations of zeta zeros.

The spectrum serves as an intermediary between discrete arithmetic data and continuous analytic behavior.

7.8 What the Spectrum Can and Cannot Tell Us

To maintain conceptual clarity, it is essential to delimit the scope of spectral interpretation.

What it can tell us:

- qualitative arithmetic structure,
- stability and coherence of operator dynamics,
- presence of interference and resonance phenomena.

What it does not claim:

- direct computation of prime distributions,
- automatic identification with zeta zeros,
- proof of deep number-theoretic conjectures.

Spectral analysis is a diagnostic and structural tool, not an oracle.

7.9 Spectral Interpretation as Computational Resource

From a computational viewpoint, spectral properties guide algorithmic design:

- low-energy modes define efficient computational pathways,
- spectral gaps inform convergence rates,
- resonance structures enable targeted amplification.

These features are exploited in higher-level QFM-based computation without requiring physical quantum resources.

7.10 Transition to Computational Semantics

Having established how arithmetic meaning emerges from spectral structure, we now turn to the interpretation of QFM operators as executable computational primitives.

The next section explains how the abstract operator system acquires concrete semantics within QVM, QFP, and QPU architectures.

8 Computational Semantics and Execution Interpretation

The QFM operator system is not merely a mathematical formalism. Its structure is explicitly designed to admit execution as a computational process. In this section we explain how abstract operators acquire computational meaning, how arithmetic amplitudes are interpreted as state, and how operator dynamics translate into executable semantics within quantum-inspired virtual architectures.

8.1 Operators as Computational Primitives

In QFM, operators are the primary computational objects. A computation is defined by the selection, composition, and parameterization of operators, rather than by imperative instruction sequences.

At the semantic level:

- states correspond to arithmetic amplitude distributions,
- operators correspond to transformations of global state,
- execution corresponds to operator application or evolution.

This contrasts with classical models, where state is primary and operators are transient. In QFM, the operator algebra defines the machine.

8.2 State Interpretation

A state $\psi \in \mathcal{H}$ represents a distributed configuration over arithmetic space. Individual amplitudes $\psi(n)$ are not registers or addresses; they encode participation of index n in the current computation.

Key semantic properties include:

- global accessibility: all indices participate simultaneously,
- implicit parallelism: updates propagate across multiplicative directions,
- non-local coupling: a single operator can affect distant indices.

State evolution is therefore inherently parallel and global, even when executed on classical hardware.

8.3 Primitive Execution Semantics

The elementary execution primitives are induced by the operators A_p, B_p, and K_p.

Forward Shift (A_p). Execution of A_p expands state along the p-direction, corresponding to multiplicative branching. This primitive underlies search, expansion, and exploration processes.

Backward Shift (B_p). Execution of B_p contracts state, filtering by divisibility. This primitive enforces arithmetic constraints and supports selective collapse of state components.

Self-Adjoint Generator (K_p). Execution of K_p produces balanced motion and serves as the minimal reversible computational step. It is the atomic unit of stable execution.

8.4 Composition and Operator Programs

Complex computations are expressed as compositions of operators:

$$\mathcal{O} = \sum_i \alpha_i O_i, \qquad \mathcal{O}_2 \circ \mathcal{O}_1, \qquad e^{-tH}.$$

Such compositions define operator programs rather than instruction lists. Execution order is determined by operator algebra and spectral properties, not by program counters.

This model supports:

- declarative specification of computation,
- automatic parallelization,
- algebraic optimization.

8.5 Evolution as Execution

Hamiltonian evolution

$$\psi(t) = e^{-tH}\psi(0)$$

is interpreted computationally as a continuous execution process.

Rather than executing discrete steps, the system evolves under global constraints toward energetically favored configurations.

Semantically:

- low-energy states represent computational attractors,
- convergence corresponds to solution emergence,
- interference encodes competition among candidate structures.

This form of execution replaces explicit control flow with spectral dynamics.

8.6 Determinism and Reproducibility

Despite its quantum-inspired structure, QFM execution is fully deterministic.

- Given the same operators and initial state, evolution is identical.
- No probabilistic measurement or collapse occurs.
- All intermediate states are in principle inspectable.

This property is essential for reproducible computation, auditability, and governed execution in large-scale systems.

8.7 Relation to Classical and Quantum Models

QFM execution semantics occupies an intermediate position between classical and quantum computation.

- Like classical systems, it is deterministic and inspectable.
- Like quantum systems, it exploits superposition and interference.
- Unlike both, it operates over arithmetic geometry rather than spatial or qubit-based structures.

This hybrid character enables quantum-like behavior without quantum hardware.

8.8 Abstraction from Physical Execution

The present semantics are independent of physical implementation.

- No assumption is made about processors, memory layout, or networks.
- Execution may be simulated, distributed, or accelerated.
- The same semantics apply across heterogeneous platforms.

This abstraction layer allows QFM to serve as a stable mathematical interface for diverse execution architectures.

8.9 Preparation for Architectural Integration

The computational semantics developed here define the contract between mathematics and execution. Subsequent documents instantiate this contract within concrete systems, including virtual machines, processors, and instruction languages.

The next section situates the QFM operator system relative to the Smrk Hamiltonian and clarifies how specialized Hamiltonians inherit these execution semantics while encoding additional analytic structure.

9 Relation to the Smrk Hamiltonian

The operator system developed in the preceding sections is intentionally general. It defines a broad class of multiplicative, self-adjoint Hamiltonians acting on arithmetic Hilbert spaces. Within this class, certain operators acquire special significance due to their analytic structure and their connection to classical problems in number theory.

This section clarifies the relationship between the general QFM Hamiltonian and the Smrk Hamiltonian, which arises as a distinguished specialization of the framework.

9.1 General Versus Specialized Hamiltonians

The QFM Hamiltonian

$$H_{\mathrm{QFM}} = \sum_{p \in \mathbb{P}} K_p + V$$

is defined abstractly, with minimal assumptions on the potential term $V(n)$. This generality allows QFM to be applied across a wide range of computational and arithmetic contexts.

The Smrk Hamiltonian is obtained by imposing a specific analytic form on the potential and kinetic weights, thereby selecting a narrow but highly structured regime within the space of admissible QFM Hamiltonians.

9.2 Definition of the Smrk Hamiltonian

The Smrk Hamiltonian is defined by the action

$$(H_{\mathrm{Smrk}}\psi)(n) = \sum_{p \in \mathbb{P}} \frac{1}{p}\big(\psi(pn) + \mathbf{1}_{p|n}\,\psi(n/p)\big) + \big(\alpha\,\Lambda(n) + \beta\,\log n\big)\,\psi(n),$$

where:

- the kinetic term uses canonical weights $1/p$,
- $\Lambda(n)$ is the von Mangoldt function,
- $\log n$ introduces logarithmic scaling,
- $\alpha, \beta \in \mathbb{R}$ are fixed parameters.

This operator fits precisely into the QFM Hamiltonian schema developed earlier.

9.3 Structural Properties

The Smrk Hamiltonian inherits the fundamental properties of QFM Hamiltonians:

- symmetry on a dense domain,
- formal self-adjointness,
- well-defined multiplicative dynamics,
- compatibility with spectral analysis.

What distinguishes it is not its algebraic form, but the arithmetic specificity of its coefficients and potential.

9.4 Analytic Motivation

The inclusion of the von Mangoldt function aligns the Hamiltonian with the explicit formulas of analytic number theory, where primes and prime powers play a central role.

The logarithmic term $\log n$ reflects asymptotic scaling behavior associated with arithmetic growth and ensures proper control of high-index modes.

Together, these terms embed classical arithmetic information directly into the operator framework.

9.5 Relation to the Hilbert–Pólya Program

The Smrk Hamiltonian is motivated by the Hilbert–Pólya philosophy, which suggests that non-trivial zeros of the Riemann zeta function may arise as spectral data of a suitable self-adjoint operator.

Within QFM:

- the operator is explicitly constructed,
- self-adjointness is structurally supported,
- spectral interpretation is mathematically meaningful.

However, the present framework does not assert nor require a direct identification of eigenvalues with zeta zeros. The Hamiltonian provides a structured arena in which such questions can be posed rigorously.

9.6 Separation of Mathematical Claims

It is essential to distinguish the levels of assertion involved.

Established within QFM:

- the Smrk Hamiltonian is a well-defined specialization of H_{QFM},
- it admits formal self-adjointness,
- its dynamics are well-defined on arithmetic Hilbert space.

Conjectural or Investigative:

- precise spectral correspondence with zeta zeros,
- completeness of eigenstates,
- equivalence to known trace formulas.

This separation ensures conceptual clarity and prevents overstatement.

9.7 Role Within the QFM Program

Within the broader QFM program, the Smrk Hamiltonian serves as:

- a benchmark analytic operator,

- a test case for spectral arithmetic hypotheses,
- a bridge between operator theory and number theory,
- a motivating example for computational experimentation.

It does not exhaust the possibilities of QFM, nor does QFM depend on its success.

9.8 Computational Perspective

From an execution standpoint, the Smrk Hamiltonian is particularly well suited for:

- numerical spectral experiments,
- distributed operator simulation,
- validation of QFM-based execution semantics,
- stress-testing of arithmetic diffusion algorithms.

Its structure balances analytic richness with computational tractability.

9.9 Conceptual Closure

The relationship between QFM and the Smrk Hamiltonian is thus one of inclusion, not dependence. QFM provides the language and machinery; the Smrk Hamiltonian selects a specific sentence written in that language.

With this distinction established, we can conclude the development of the QFM operator system proper and outline possible extensions and directions for future work.

10 Extensions and Outlook

The operator system developed in this paper defines a stable and flexible foundation for multiplicative, quantum-inspired computation. Its strength lies not in a single specialized construction, but in the breadth of extensions it admits while preserving mathematical rigor and computational interpretability.

This section outlines natural directions in which the QFM operator system may be extended, both mathematically and architecturally.

10.1 Alternative Hamiltonian Families

The QFM framework admits a wide class of Hamiltonians beyond the canonical forms discussed earlier. Variations may arise from:

- alternative weightings of prime generators,
- modified kinetic coefficients,
- non-linear or state-dependent potentials,
- restriction to selected subsets of primes.

Such Hamiltonians may encode domain-specific arithmetic structure or computational priorities while remaining compatible with the same underlying operator algebra.

10.2 Generalized Arithmetic Potentials

The potential term $V(n)$ may be enriched to encode additional arithmetic information. Examples include:

- divisor-counting functions,
- Möbius-type sign structures,
- arithmetic constraints derived from algebraic number fields,
- externally imposed regulatory or semantic conditions.

These generalizations allow the Hamiltonian to act as a programmable energy landscape, shaping computation through arithmetic structure rather than explicit control flow.

10.3 Non-Linear and Adaptive Dynamics

While the present work focuses on linear operators, QFM naturally admits controlled non-linear extensions.

Possible directions include:

- amplitude-dependent potentials,
- feedback-driven weight adaptation,
- normalization-preserving non-linear flows.

Such dynamics may be used to model learning, optimization, or constraint satisfaction processes within arithmetic space.

10.4 Distributed Hilbert Spaces

The multiplicative Hilbert space $\mathcal{H}$ may be decomposed into distributed subspaces, enabling parallel execution across computational nodes.

Key considerations include:

- shard-local operator evaluation,
- controlled boundary exchange of arithmetic amplitudes,
- global consistency of spectral evolution.

This perspective prepares the ground for execution within distributed virtual machines while preserving mathematical coherence.

10.5 Integration with Execution Architectures

The QFM operator system is designed to serve as the mathematical core of higher-level execution architectures.

In particular:

- QVM instantiates global operator evolution,
- QFP realizes controlled operator execution pipelines,
- QPU accelerates prime-parallel arithmetic dynamics,
- QWASM provides a low-level instruction language mapping directly to QFM operators.

Each of these systems implements a different layer of the same operator semantics, ensuring conceptual continuity across abstraction levels.

10.6 Numerical and Experimental Directions

The operator framework lends itself naturally to numerical exploration.

Potential avenues include:

- approximation of spectral densities,
- investigation of resonance statistics,
- empirical study of arithmetic diffusion patterns,
- validation of convergence and stability regimes.

Such experiments do not replace analytic work, but provide valuable intuition and falsifiability.

10.7 Relation to Broader Mathematical Structures

Beyond number theory, the QFM operator system interfaces with:

- non-commutative geometry,
- spectral graph theory on arithmetic graphs,
- operator algebras with arithmetic generators,
- dynamical systems on non-Euclidean index spaces.

These connections suggest that QFM may serve as a bridge between several established mathematical domains.

10.8 Conceptual Outlook

At a conceptual level, QFM reframes computation as operator evolution over arithmetic geometry. This shift enables quantum-like behavior without quantum ontology and replaces instruction-centric thinking with spectral dynamics.

The framework emphasizes:

- structure over control,
- evolution over sequencing,
- global coherence over local updates.

These principles are likely to remain relevant regardless of future implementation choices.

10.9 Preparation for Concluding Remarks

The developments outlined here indicate that the QFM operator system is not a closed construction but a generative foundation. It supports a wide range of mathematical investigation and computational realization while maintaining internal consistency.

We now conclude by summarizing the central contributions and clarifying the scope of claims established in this work.

11 Conclusion

This paper has developed the operator-theoretic foundations of Quansistor Field Mathematics as a coherent and self-contained framework for multiplicative, quantum-inspired computation.

Starting from the arithmetic Hilbert space, we introduced forward and backward prime-shift operators as the elementary generators of multiplicative motion. By combining these operators in a symmetric manner, we constructed self-adjoint generators that support reversible dynamics and admit rigorous spectral analysis.

Building on these primitives, we defined multiplicative diffusion and global dynamics operators that describe collective propagation across arithmetic space. The inclusion of arithmetic potential terms led to the full QFM Hamiltonian, which integrates kinetic motion, structural constraints, and analytic input into a single operator.

The resulting Hamiltonian framework provides:

- a mathematically well-defined operator calculus,
- a natural notion of global arithmetic evolution,
- a robust foundation for spectral interpretation,
- and a direct bridge to executable computational semantics.

A key theme throughout this work has been separation of structure and specialization. The QFM operator system was developed independently of any specific number-theoretic conjecture or computational architecture. Within this general framework, the Smrk Hamiltonian was identified as a distinguished specialization motivated by analytic number theory, but not as a prerequisite for the validity of the framework itself.

From a computational perspective, QFM reframes computation as operator evolution rather than instruction sequencing. Execution is interpreted as global transformation of arithmetic amplitudes, governed by spectral dynamics rather than explicit control flow. This viewpoint enables quantum-like behavior without reliance on physical quantum systems and preserves determinism, reproducibility, and inspectability.

The scope of claims established in this paper is deliberately limited:

- no physical quantum interpretation is asserted,
- no computational supremacy is claimed,
- no deep number-theoretic conjecture is resolved.

What is provided instead is a rigorous mathematical foundation and a clear conceptual interface for further work. The operator system defined here supports extension, specialization, numerical exploration, and architectural realization without modification of its core principles.

Future work will develop these foundations in several directions, including dedicated execution architectures, instruction-level semantics, distributed implementations, and targeted analytic investigations. Each of these efforts will build directly on the operator framework established in this paper.

In this sense, the QFM operator system is not presented as a final answer, but as a stable starting point: a precise language in which multiplicative computation, arithmetic geometry, and operator dynamics can be studied, implemented, and critically evaluated.