2014 American Control Conference (ACC)
June 4-6, 2014. Portland, Oregon, USA

Improving SOC Accuracy Using Collective Estimation for Lithium Ion Battery Cells in Series

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***Abstract*—This paper presents new methods for improving state of charge (SOC) estimation accuracy for Lithium Ion battery cells connected in series. The methods benefit from the fact that the cells share a common current trajectory. These methods extend previously studied techniques for SOC estimation, like the Extended Kalman Filter. While the existing literature focuses on estimating SOC for individual cells separately, we consider the cells in a series string collectively. We show that estimation accuracy is increased for cells in series both when they are 1) balanced and 2) un-balanced. We validate these methods against a control case using Monte Carlo simulation.**

I. Introduction

Battery state of charge (SOC) is an important metric for users of devices powered by Lithium Ion (Li-Ion) batteries because it dictates the amount of time they can use these devices without interruption [1]. One example is determination of the distance remaining before an electric vehicle would need to stop. In addition to user convenience, accurate knowledge of SOC is critical to health-conscious battery control [2]. SOC is related to the amount of charge stored within a battery cell and realistically cannot be measured in a direct fashion [3]. As a result, battery SOC estimation is a difficult but well-studied problem.

This problem has been approached in many different ways. Voltage-based open-loop methods that relate open circuit voltage (OCV) of a battery cell to SOC, and current-based methods that count the charge carriers as they enter or leave the battery (Coulomb counting) provide varying degrees of accuracy [4]. More sophisticated closed-loop methods that use both voltage and current-based techniques, like the Kalman filter are both more accurate and robust [5]. Simple and computationally efficient models that represent the battery with equivalent electrical elements are perhaps most commonly used for this purpose [6], [7], [8]. Electrochemistry-based models capture the battery's dynamics in more detail and can be used if higher accuracy is desired and more computational resources are available [8], [9], [10], [11].

In practice, all of these techniques are applied separately to individual cells within a battery pack, as illustrated in Figure 1. This approach ignores additional information available due to the fact that all cells in series share the same current. In vehicle applications cells are often connected in series to reach a necessary voltage [2]. Moreover, the voltage of each cell is monitored to avoid over-charge or over-discharge [12]. As such, it is reasonable to study an approach that capitalizes on these series connections for collective SOC estimation.

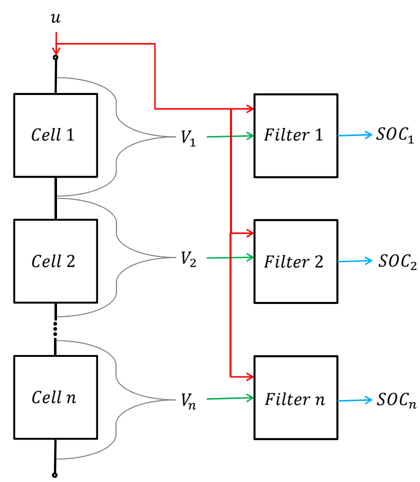

Fig. 1. An illustration of the conventional approach to SOC estimation; the use of a single filter per cell.

Our first hypothesis is that if SOC for a balanced set of cells in series is estimated collectively (i.e. the additional information is used) as opposed to individually, the SOC estimation accuracy can be improved for all involved cells. Hereafter, we refer to this idea as collective estimation. Because of the flatness of the curve relating OCV to SOC for many Li-Ion cells, as seen in Figure 2, estimation of SOC can be difficult in the middle operating region (approximately 20% to 80% SOC) [13]. Estimation accuracy in this flat region can be improved through collective estimation. We test the first hypothesis by considering the variation in estimation error for a cell as a function of the string size considered in the filter.

The results of the first hypothesis would be applicable in a typical battery mangement system (BMS). In our second hypothesis, we take a more unconventional approach and assert that if heterogeneity exists among the considered cells in such a way that the SOC for some is in the non-flat regions, these cells can help improve SOC estimation accuracy for the entire string[1]. We test this hypothesis by considering only a two-cell string, keeping one cell in the middle SOC range, and varying the SOC for the other cell. We expect the

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978-1-4799-3274-0/$31.00 ©2014 AACC

estimation error for the first cell to decrease with an increase in the slope of the OCV-SOC curve for the second cell.

This paper is organized as follows: In the next section, we present the key ideas of our hypotheses in detail and then present a discussion of the battery model used. We also draw an analytical insight which leads into a discussion of two algorithms for collective estimation. In section III, we discuss the simulation setup and the results of the simulation experiments for both implementations, and then conclude the section with a brief discussion of the effect of sensor noise on the effectiveness of the first implementation. Section IV contains concluding remarks for the paper.

II. Key Ideas

The fundamental idea behind collective SOC estimation is that additional information available due to knowledge that cells in series experience the same current trajectory can help improve estimation accuracy. This notion has previously been studied in the field of mobile robot localization as presented in [14], which reported higher estimation accuracy when considering a group of robots as a collective and using the additional information provided by the relative position measurements of the robots. This is very similar to our first hypothesis. The second hypothesis is suggested by the nonlinearities inherent in the OCV-SOC curve, as seen in Figure 2. The high slopes in the non-flat region of this curve can lead to better estimation accuracy for cells within that region which, according to the second hypothesis, can improve SOC estimation for other cells in series.

In the remainder of this section, we will present details of the battery model used in this paper, and then discuss two implementations of the collective estimation method.

A. The Battery Model

For the purposes of this paper, we used a first-order equivalent circuit battery model, details for which can be found in [7]. The model assumes that the battery is a nonlinear capacitor, and uses the battery SOC as the only state where the SOC is defined in [4] as normalized useful charge remaining. This model is simple yet it captures the integrator nature of a battery as well as the nonlinearities in the OCV-SOC relationship. It has also been used quite effectively for algorithm design and to draw conclusions about error dynamics of SOC estimators as seen in [5], [15].

The output of this model is cell terminal voltage, given by

$$y_i\left(t\right) = g_i\left(x_i\right) - u\left(t\right)R_i, \;\; i = 1, 2, ..., n \tag{1}$$

for n cells in a series string. In equation 1, x_i is the state of charge for the i^{th} cell, $u\left(t\right)$ represents the current input the string is subjected to (positive in discharge), R_i represents the internal resistance for the i^{th} cell, and $g\left(x\right)$ is a relationship between battery SOC and OCV. A plot for $g_i\left(x_i\right)$ for the 1 Ah battery we use in this paper is shown in Figure 2.

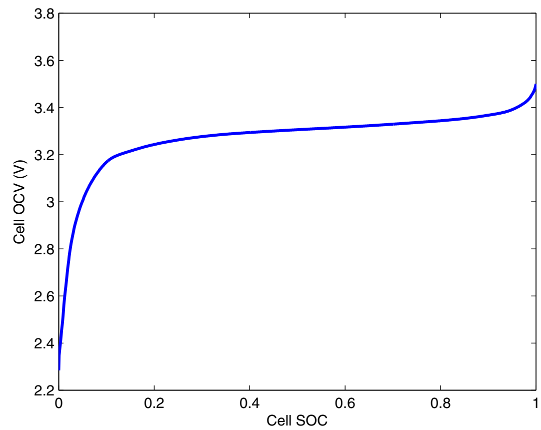

Fig. 2. The OCV-SOC curve for a 1 A-h $LiFePO_4$ battery cell is shown here. Note the extremely small slope in the middle of the curve. The data for this plot were measured in-house by two of this paper's co-authors.

This measurement model can be linearized around a suitable operating point to yield an output equation of the form

$$\delta y_i\left(t\right) = \alpha_i x_i - u\left(t\right)R_i \tag{2}$$

where α_i represents the slope of the OCV-SOC curve at x_i and δy_i is a perturbation from a nominal voltage value. When considering a single cell, the slope represents the output matrix which is also the observability matrix when n equals 1. The lower the slope gets, the more ill-conditioned this matrix becomes, causing increased difficulty in state estimation.

The model's state dynamics are given by

$$\dot{x}_i = -\frac{u\left(t\right)}{Q_i} \tag{3}$$

where Q_i represents the maximum usable charge that can be stored in the battery. Note that while these state dynamics can depend on the value of SOC for some battery types because of self-discharge, Li-Ion batteries exhibit very little self-discharge, which can be ignored in this simple model [10]. From equation 3, we can see that batteries behave as current integrators. Therefore, if α_i is zero, and one is forced to use open-loop Coulomb counting, white noise in the current measurement would cause the estimator error variance to diverge. Moreover, added bias to the measurement would cause the error mean to diverge as well. This fact makes it important for us to account for the current bias term in the model. We can make the current measurement bias a time-invariant state in our model similar to [16]. Doing so changes the model's input for estimation purposes to

$$u\left(t\right) = u_m\left(t\right) - b, \quad \dot{b} = 0 \tag{4}$$

where $u_m\left(t\right)$ is a measured current (known) and b is the measurement bias (unknown), assumed to be constant.

[1] While a typical BMS attempts to minimize SOC heterogeneity in a series string of cells. We conjecture that such heterogeneity could lead to better SOC estimates. As such we hope that our results provide a first order reason to study the tradeoff between battery pack balancing and SOC estimation accuracy.

B. Collective Estimation Using a Collective Kalman Filter (CKF)

When considering the linearized model for n cells in series with the current bias term collectively (equations 2, 3, and 4), the observability matrix takes on the following form

$$\mathcal{O}=\begin{bmatrix} \alpha_1 & \cdots & 0 & R_1 \\ \vdots & \ddots & 0 & \vdots \\ 0 & \cdots & \alpha_n & R_n \\ 0 & \cdots & 0 & \frac{\alpha_1}{Q_1} \\ \vdots & & \vdots & \vdots \\ 0 & \cdots & 0 & \frac{\alpha_n}{Q_n} \\ 0 & \cdots & 0 & 0 \\ \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{5}$$

For the system to be locally observable about an operating point, this matrix needs to have full column rank [17]. If any of the α_i values are particularly small compared to the others (i.e. the cells are in the flat region), this matrix will run the risk of being ill-conditioned. Considering the cells as separate entities, this will cause estimation difficulties for the cells in the flat OCV-SOC region. Notice however that the last column of the observability matrix suggests that a redundancy exists for the bias estimate in this case. This means that as long as at least one cell is not in the flat OCV-SOC region, the collective bias estimate can keep the mean SOC error from drifting for all cells. This insight motivates our first implementation of collective estimation: the construction of a single Kalman filter for the collective $n+1$ state system. This approach is similar to that presented in [14]. Since the output equation for the model used is non-linear, an extended Kalman filter like the one presented in [5] is used. Figure 3 illustrates this approach graphically.

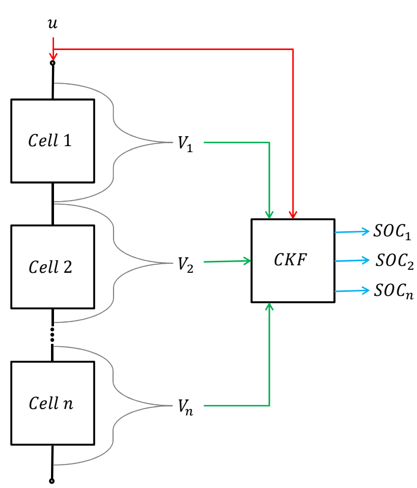

Fig. 3. Instead of using multiple Kalman filters, one for each cell, we suggest that a single Kalman filter be used. This way, the estimator has access to information from other cells in the string.

C. Collective Estimation Using ΔSOC Innovations

All cells in a series string experience the same current, which is also the time rate of change of stored charge. Therefore, if all cells in the string have the same parameters (a fair assumption when considering pristine Li-Ion battery packs for vehicles), they should all experience the same $\dot{x}_i$ at a given instant in time. This means that if individual Kalman filters for the different cells in series run in parallel, they should all report the same estimated $\dot{x}_i$ ($\hat{\dot{x}}_i$). The second implementation of collective estimation is based on enforcing conservation of charge among the estimators, by attempting to reduce the differences between the $\hat{\dot{x}}_i$ terms. We call this the ΔSOC innovations method. This method involves adding a new innovations term in the equation for state estimators as shown below

$$\hat{\dot{x}}_i = -\frac{1}{Q_i}u + k_i\left(y_i - \hat{y}_i\right) + k_{\Delta,i}\left(\Delta\dot{x}\right)_i \tag{6}$$

where k_i is the observer gain for the i^{th} cell, $\hat{y}_i$ is a predicted output based on the best state estimate, $\left(\Delta\dot{x}\right)_i$ is the ΔSOC innovations term, and $k_{\Delta.i}$ is a gain similar to k_i . Figure 4 illustrates this idea graphically.

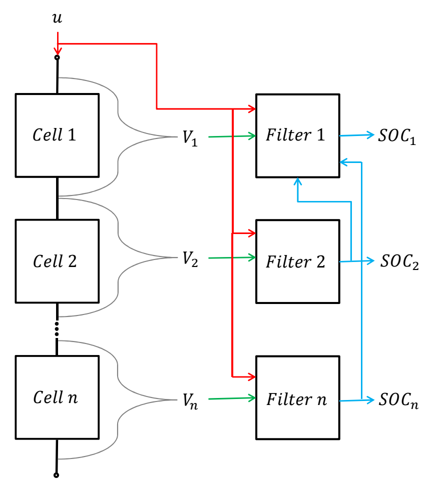

Fig. 4. A single filter per cell can can give better estimates if charge is conserved among individual estimators This is the approach used by the ΔSOC method.

The $\Delta\dot{x}$ term can be constructed in a variety of ways. We consider two cases that correspond well with the previous section and our hypotheses. If all cells in the series are in the flat OCV-SOC region, it can be defined as

$$\left(\Delta\dot{x}\right)_i = \hat{\dot{x}}_i - \frac{1}{n-1}\sum_{i=1,\, i\neq j}^{n}\hat{\dot{x}}_j \tag{7}$$

Alternatively, if one cell in the series string (say the j^{th} cell) is in the non-flat region, its $\hat{\dot{x}}$ value can be taken as truth and the other cells can correct their SOC estimates against it. Then the $\Delta\dot{x}$ term can be defined as

$$\left(\Delta\dot{x}\right)_i = \hat{\dot{x}}_i - \hat{\dot{x}}_j \tag{8}$$

From the structure of equation 6 we can see that the absolute value of the k_Δ term should not be greater than unity to avoid overcorrection. We use this insight during simulation to determine a good value for k_Δ.

III. Simulation Results

In this section, we present the results of some simulation experiments designed to test the collective estimation hypotheses using the implementations suggested in the last section. Each implementation is tested twice: once to test for the effect of considering additional cells in the estimation process for a balanced string (first hypothesis), and a second time to test the effect of a single additional cell in the non-flat region (second hypothesis). First we briefly discuss the simulation setup, and then we present the results of the simulations for each implementation.

A. Simulation Setup

The simulation is set up using a discrete-time version of the equivalent circuit model of a 1 Ah Li-Ion battery cell. The fixed time step used (Δt) is one second, and each test is run for one hour. For each experiment, 1000 tests are run, and the estimation error at the last time step is saved. The ensemble statistics of these errors are used to compare estimation accuracy. Additive Gaussian noise is assumed for the voltage and current sensors with an additional current bias. For each test, the same current profile is applied to the series string, shown in Figure 5.

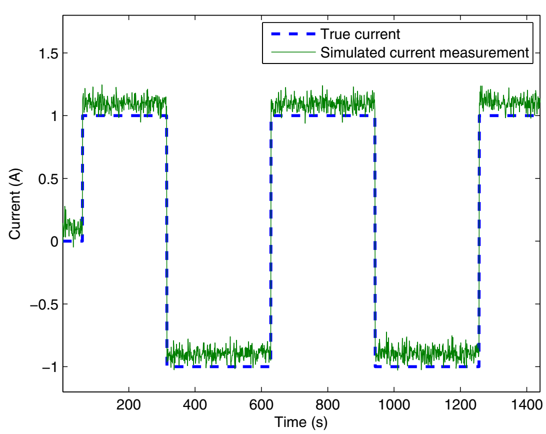

Fig. 5. Current trajectory used in the experiments, alongside the simulated measurement.

The profile is designed to periodically charge and then discharge the cells by the same amount. We do this to keep the cells' SOC hovering around the initial value. Table 1 lists the parameters used during these simulations.

B. Results: CKF

For the CKF method, we run two experiments. The first experiment is run with a varying number of cells being considered in the collective Kalman filter. Each cell in the string has the same initial SOC of 50%, which corresponds to the flat region of the OCV-SOC curve. Figure 6 shows the effect on both standard deviation and mean of the estimation error.

TABLE I
Simulation Parameters

Parameter	Value
Cell capacity	0.9871 Ah
Cell internal resistance	0.1067 Ω
Current sensor bias	0.1 A
Current sensor Gaussian noise variance	1×10^{-4} A^2
Voltage sensor Gaussian noise variance	1×10^{-6} V^2

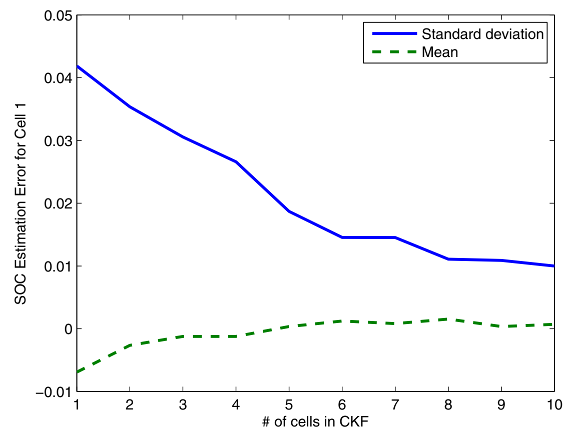

Fig. 6. The effect on estimation accuracy of adding cells at the same initial SOC to the Kalman filter.

Note that the mean of the errors hovers about zero and is within the reported standard deviation. The one cell case is the control that represents convention. As more cells are considered in the CKF, the standard deviation of errors is always lower than the control and decreases to about 25% of the control value for 9 additional cells, consistent with the first hypothesis.

We run the second experiment for only two cells in the CKF, but the initial SOC for the second cell is varied between 20% and 95%. The first cell's initial SOC is kept at 50%. Figure 7 shows the relevant results.

Based on the second hypothesis, we expect estimation accuracy to improve over the control in all cases, but improve more when the second cell is in the non-flat region (i.e. maximum heterogeneity), and this is confirmed by the results. What is remarkable to note is that estimation accuracy benefits gained from considering only one additional cell at 95% SOC exceed those gained from considering nine additional cells in the flat region.

C. Results: ΔSOC Method

For the ΔSOC method, we run an additional two experiments. The first experiment is designed to find a best value for the k_Δ parameter. The simulation is run with two cells,

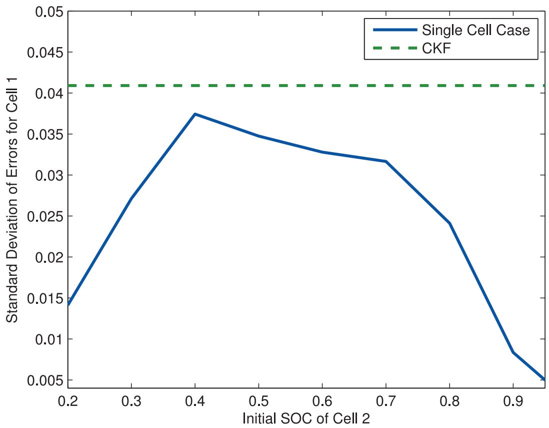

Fig. 7. The effect on estimation accuracy of adding only one additional cell at various initial SOC values, compared to the single cell control case.

each subject to its individual two state Kalman filter. The first cell's initial SOC is set at 50%, and the second cell's at 95% SOC. The discrete time equivalent of $\Delta \dot{x}_i$ term is taken at the k^{th} time step as

$$(\Delta \dot{x})_d = \left[\left(\hat{x}^-_{1,k} - \hat{x}^+_{1.k-1} \right) - \left(\hat{x}^+_{2,k} - \hat{x}^+_{2,k-1} \right) \right] / \Delta t \quad (9)$$

where $\hat{x}^-_{1,k}$ is the a priori estimate of the first cell's SOC at the k^{th} time step, and $\hat{x}^+_{1.k-1}$ is the a posteriori estimate of the first cell's SOC at the $k-1^{th}$ time step (the same convention applies to cell 2). Note that the ΔSOC innovations term is added to only the estimator for cell 1, because cell 2 is in the non-flat region and thus converges to an accurate SOC estimate quickly. Figure 8 shows a plot that compares the value of k_Δ to the mean and standard deviations of estimation error.

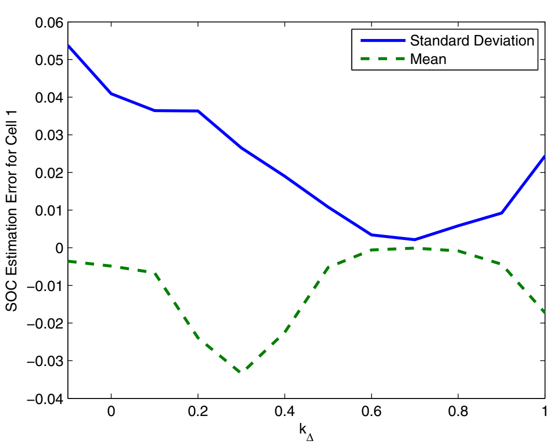

Fig. 8. Estimation accuracy for cell 1 (at 50% SOC) as a function of the k_Δ value, while cell 2 is kept at 95% SOC.

For this experiment, the best value for k_Δ appears to be ≈ 0.7 as it minimizes the error standard deviation. Consistent with the second hypothesis, the ΔSOC method performs better than the control (seen in Figure 8 when $k_\Delta = 0$) for this value of the gain.

Using the k_Δ value found in the last experiment, we run the next experiment in which the ΔSOC method is applied to a string of multiple cells all at the same SOC of 50%. The ΔSOC term used here is the difference between the change reported by cell 1, and an average of the changes reported by all the cells in the series string. Figure 9 shows a plot of the error standard deviation and mean as a function of the number of cells being considered for estimation.

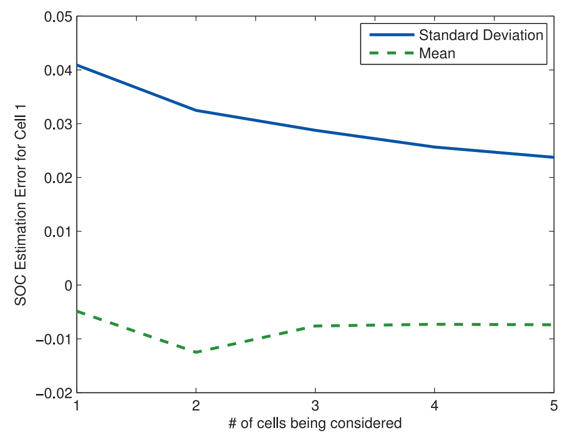

Fig. 9. Estimation accuracy for cell 1 (at 50% SOC) as a function of the number of cells contributing to the ΔSOC innovations term.

Improvements in estimation accuracy are smaller for this study than for the CKF method. This likely has to do with the choice of k_Δ which may be dependent on the configuration of the estimator.

D. Discussion

The hypotheses put forth in this paper appear to be valid in light of the simulation results presented in the last two sections. Both the CKF method and the ΔSOC methods perform well. Analytical means for determination of a good k_Δ value for the latter is necessary as it may be a function of the string configuration. Considering some worst case scenarios, we find that it is possible for the CKF method to fail if the additive noise in the voltage sensor exceeds the voltage drop associated with the current sensor's bias across the internal resistor. This is tested by running the second CKF experiment again after increasing the voltage sensor noise standard deviation to 100 mV. Figure 10 shows the results of this experiment.

The CKF has the opposite effect on the error standard deviation in this case compared to that of section III-B. The accuracy improvements obtained by the CKF are tied to its enhanced bias estimation ability but a high enough voltage measurement noise can reduce the effect of the bias term on the estimator, making it less important. In practice, this limitation would be of little concern given the typical values

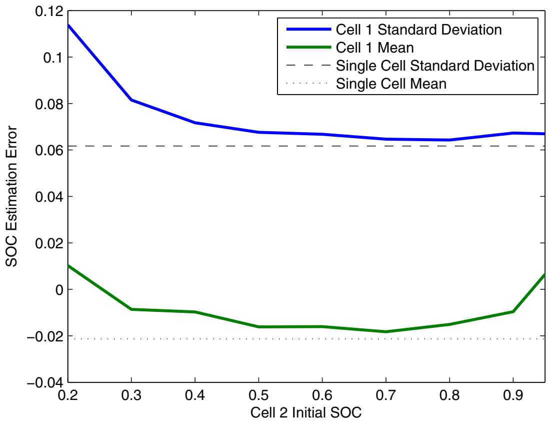

Fig. 10. The effect on estimation accuracy of adding only one additional cell at various initial SOC values. Note that the standard deviation of the additive gaussian noise in the voltage sensor is higher in this case than the voltage drop across the internal resistor associated with the measurement bias.

of voltage and current measurement errors in commercial battery packs.

IV. Conclusions

This paper presents new insights in the area of Li-Ion battery state of charge estimation. We hypothesize that collective estimation of SOC for Li-Ion cells connected in series leads to improved accuracy and that estimation accuracy can be improved if the series string is unbalanced. We suggest two method to test these hypotheses, the CKF method and the ΔSOC method. The results seen in simulation provide evidence that these hypotheses are valid. However, the CKF method can have opposite the desired effect if the standard deviation of the noise in the voltage sensor is higher than the voltage drop across the cell's internal resistor associated with the bias term in the current sensor.

These results are preliminary, and were obtained through simulation of a simplified model that does not capture higher-order dynamics of a battery cell. We conjecture that the same estimation accuracy improvements will be gained when these algorithms are tested against an electro-chemistry based model, or against an actual cell. This conjecture stems from the facts that our methods are concerned with the integrator nature of a battery and with the series string structure found within battery packs. These facts do not change when a more sophisticated battery model or an actual cell is considered. Using an electro-chemistry based model, or an actual Li-ion cell connected in the loop, might decrease estimation accuracy (assuming a simple model is used in the state estimator), but the relative estimation accuracy should improve through the use of our methods.

There is also some concern about how cell heterogeneity can be reached in a real world application. A typical battery management system will attempt to always keep a series string balanced [3]. For the work presented, we are concerned not with how cell to cell heterogeneity is reached, but only with the resulting gains in SOC estimation accuracy.

Acknowledgment

This work was funded by the Advanced Research Projects Agency - Energy (ARPA-E) and National Science Foundation (NSF). The authors gratefully acknowledge this support.

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