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Taller operadores lineales

Sección 4.1.4

Punto 4 & 5.

4. Suponga que $AB = BA$, demuestre:

$(A+B)^2 = A^2 + 2AB + B^2$

$(A+B)(A+B)$

$A^2 + AB + BA + B^2$

Como $AB = BA$ entonces,

$A^2 + 2AB + B^2$,,

$(A+B)^3 = A^3 + 3A^2B + 3AB^2 + B^3$

$(A+B)(A+B)(A+B)$

$A^2 + AB + BA + B^2 (A+B)$

$= A^3 + A^2B + BA^2 + B^2A + A^2B + AB^2 + B^3 + B^3$

Como $AB = BA$, $A = A$ y $B = B$, $A^2B = BA^2$ y $B^2A = AB^2$.

$= A^3 + 3A^2B + 3AB^2 + B^3$.

5. $[L_-, L_+] = I$ $L : L_- L_+$

$L|x\rangle = \lambda|x\rangle$ $|y\rangle = L_+|x\rangle$ Demostrar: $L|y\rangle = (\lambda+1)|y\rangle$

① $I = L_- L_+ - L_+ L_-$

$L_- L_+ = I + L_+ L_-$

De:

$|y\rangle = L_+|x\rangle$

$L(|y\rangle) = L L_+|x\rangle$ ②

Reemplazamos ① en ②

De:

$L_+ L|x\rangle = L_+\lambda|x\rangle \longrightarrow \lambda$ puede conmutar

$L_+ L|x\rangle = \lambda L_+|x\rangle$

Reemplazamos: $|y\rangle = L_+|x\rangle$

$L_+ L|x\rangle = \lambda|y\rangle$ ③

$L|y\rangle = (I + L_+ L_-) L_+|x\rangle$

$L|y\rangle = L_+|x\rangle + L_+ L_- L_+|x\rangle$

$L|y\rangle = L_+|x\rangle + L_+ L|x\rangle$

$L|y\rangle = L_+|x\rangle + \lambda|y\rangle$

$L|y\rangle = |y\rangle + \lambda|y\rangle$

$[L_-, L_+] = I$ $L = L_- L_+$

$L|x\rangle = \lambda|x\rangle$ $|z\rangle = L_-|x\rangle$ $L|z\rangle = (\lambda - 1)|z\rangle$

$L|z\rangle = \lambda|z\rangle - |z\rangle$

$L_- L_+ - L_+ L_- = I$

$L_+ L_- = L_- L_+ - I$ ①

$L|z\rangle = L L_-|x\rangle$ ①

$L|z\rangle = (L_- \boxed{L_+) L_-}|x\rangle$

$L|z\rangle = L_-(L_- L_+ - I)|x\rangle$

$= L_-(L - I)|x\rangle$

$= (L_- L - L_-)|x\rangle$

De: $L|x\rangle = \lambda|x\rangle$

$L_-(L|x\rangle) = L_-(\lambda|x\rangle)$

$L_- L|x\rangle = \lambda L_-|x\rangle$

$L_- L|x\rangle = \lambda|z\rangle$

$\rightarrow L|z\rangle = L_- L|x\rangle - L_-|x\rangle$.

$L|z\rangle = \lambda|z\rangle - |z\rangle$

Sección 4.2.9

Punto 1

1

II $(PQ)^{-1} = Q^{-1}P^{-1}$

$\rightarrow PQ(PQ)^{-1} = P\underbrace{QQ^{-1}}_{I}P^{-1} = PP^{-1} = I$ ✓

$\rightarrow (PQ)^{-1}PQ = Q^{-1}\underbrace{P^{-1}P}_{I}Q = Q^{-1}Q = I$ ✓

III $PQ^{-1} = Q^{-1}P$ supongamos es cierto

$PQ^{-1}(PQ^{-1})^{-1} = I$

$PQ^{-1}(Q^{-1}P)^{-1} = PQ^{-1}P^{-1}Q = P(PQ)^{-1}Q$, si $[P,Q]=0$

$= P(QP)^{-1}Q$

$= PP^{-1}Q^{-1}Q = I$

IV $(e^A)^\lambda = e^{A\lambda}$

$(e^A)^\lambda = \left(\sum_{n=0}^{\infty} \frac{A^n}{n!}\right)^\lambda = \left[I + A + \frac{A^2}{2!} + \dots\right]^\lambda$

$= \left[I^\lambda + A^\lambda + \frac{(A^\lambda)^2}{2!} + \dots\right]$

$= \sum_{n=0}^{\infty} \frac{(A^\lambda)^n}{n!}$

$= e^{A^\lambda}$

b A hermitico, $A = A^\dagger$. $\tilde{A} = U^{-1} A U$ U unitario $U^{-1} = U^\dagger$

$$\tilde{A}^\dagger = (U^{-1} A U)^\dagger = U^\dagger A^\dagger U^{-1\dagger}$$
$$= U^\dagger A^\dagger U^{\dagger -1}$$
$$= U^{-1} A^\dagger U^{-1 -1} = U^{-1} A U = \tilde{A}$$

$$\rightarrow \underbrace{\tilde{A}^\dagger = \tilde{A}}$$

c) A hermitico, $A = A^\dagger$, sea $T = e^{iA}$

$$T^\dagger = (e^{iA})^\dagger = e^{(iA)^\dagger} = e^{-iA^\dagger} = e^{-iA}$$
$$(\lambda A)^\dagger = \lambda^* A^\dagger$$

$$T T^\dagger = e^{iA} \cdot e^{-iA} = I \longrightarrow \underbrace{T^\dagger = T^{-1}}_{\text{unitario}}$$

d) K antihermitico $K = -K^\dagger$, $\tilde{K} = U^{-1} K U$ U unitario

$$\tilde{K}^\dagger = (U^{-1} K U)^\dagger = U^\dagger K^\dagger U^{-1\dagger}$$
$$= U^\dagger K^\dagger U^{\dagger -1}$$
$$= U^{-1} K^\dagger U^{-1 -1} = -U^{-1} K U = -\tilde{K}$$

$$\rightarrow \underbrace{\tilde{K}^\dagger = -\tilde{K}}$$

I $(P^{+})^{-1} = (P^{-1})^{+}$

$\langle x|y\rangle = \langle x|I|y\rangle$

$= \langle x|P P^{-1}|y\rangle$

$= \langle y|(P^{-1})^{+} P^{+}|x\rangle^{*}$

$= \langle y|x\rangle^{*}$

$(P^{-1})^{+} P^{+} = I$

$(P^{-1})^{+} P^{+} (P^{+})^{-1} = I (P^{+})^{-1}$

$(P^{-1})^{+} = (P^{+})^{-1}$

V $Pe^{Q}P^{-1} = e^{PQP^{-1}}$

$P\left[I + Q + \frac{Q^2}{2} + \dots\right]P^{-1} = \left[I + PQP^{-1} + \frac{PQ^2}{2!}P^{-1} + \dots\right]$

$e^{PQP^{-1}} = \left[I + PQP^{-1} + \frac{PQP^{-1}PQP^{-1}}{2!} + \dots\right]$

Y así sucesivamente

PQ^2P

Entonces ya que la ec ① = ②

$Pe^{Q}P^{-1} = e^{PQP^{-1}}$

e A y B hermiticos

AB hermitico si y solo si $[A,B]=0$

$$(AB)^\dagger = B^\dagger A^\dagger = BA \quad \text{si } [A,B]=0$$
$$= AB$$

$$\longrightarrow \underbrace{(AB)^\dagger = AB}$$

f S real y antisimetrico $S^T = -S$

I $[I-S, I+S] = (I-S)(I+S) - (I+S)(I-S)$
$$= I^2 + S - S - S^2 - [I^2 - S + S - S^2]$$
$$= I - S^2 - I^2 + S^2$$
$$= 0 \quad , \quad \text{conmutan}$$

II $(1-S)(1+S)$ simetrico

$$((1-S)(1+S))^T = (1+S)^T(1-S)^T \qquad (A+B)^T = A^T + B^T$$
$$= (1+S^T)(1-S^T)$$
$$= (1-S)(1+S) \longrightarrow \underbrace{((1-S)(1+S))^T = (1-S)(1+S)}_{\text{simetrico}}$$

$(1-S)(1+S)^{-1}$ ortogonal, $M^{-1} = M^T$

$$((1-S)(1+S)^{-1})^T = ((1+S)^{-1})^T(1-S)^T = ((1+S)^T)^{-1}(1-S)^T$$
$$= (1-S)^{-1}(1+S) \qquad \text{tienen que conmutar}$$
$$= ((1+S)(1-S)^{-1})$$
$$= \underbrace{((1-S)(1+S)^{-1})^{-1}}$$

9. Sea $\mathbb{R} = \begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix}$

$$\mathbb{R} = (I - S)(I + S)^{-1}$$

$$\mathbb{R}(I + S) = (I - S)$$

$$\begin{pmatrix} \cos\theta & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1+S_{11} & S_{12} \\ S_{21} & 1+S_{22} \end{pmatrix} = \begin{pmatrix} 1-S_{11} & -S_{12} \\ -S_{21} & 1-S_{22} \end{pmatrix}$$

$$\begin{pmatrix} \cos\theta + \cos\theta S_{11} + \operatorname{sen}\theta S_{21} & \cos\theta S_{12} + \operatorname{sen}\theta + \operatorname{sen}\theta S_{12} \\ -\operatorname{sen}\theta - \operatorname{sen}\theta S_{11} + \cos\theta S_{21} & -\operatorname{sen}\theta S_{12} + \cos\theta + \cos\theta S_{12} \end{pmatrix}$$

$$\cos\theta + \cos\theta S_{11} + S_{11} - 1 + \operatorname{sen}\theta S_{21} = 0$$

① $[S_{11}[\cos\theta + 1] + S_{21}[\operatorname{sen}\theta] = 1 - \cos\theta$

② $S_{12}[1 + \cos\theta] + S_{22}\operatorname{sen}\theta = -\operatorname{sen}\theta$

③ $-\operatorname{sen}\theta S_{11} + S_{21}[1 + \cos\theta] = \operatorname{sen}\theta$

④ $-\operatorname{sen} S_{12} + S_{22}[1 + \cos\theta] = 1 - \cos\theta$

$$\begin{pmatrix} \cos\theta + 1 & 0 & \operatorname{sen}\theta & 0 \\ 0 & 1+\cos\theta & 0 & \operatorname{sen}\theta \\ -\operatorname{sen}\theta & 0 & 1+\cos\theta & 0 \\ 0 & -\operatorname{sen}\theta & 0 & 1+\cos\theta \end{pmatrix} \begin{pmatrix} S_{11} \\ S_{12} \\ S_{21} \\ S_{22} \end{pmatrix} = \begin{pmatrix} 1-\cos\theta \\ -\operatorname{sen}\theta \\ \operatorname{sen}\theta \\ 1-\cos\theta \end{pmatrix}$$

entonces: $S_{11} = 0 \quad S_{22} = 0$

Con $S_{12} = -S_{21}$

$$S_{12} = \frac{-2\sin\theta}{\sin^2\theta + \cos^2\theta + 2\cos\theta + 1} = \frac{-2\sin\theta}{2(1+\cos\theta)}$$

- $S_{12} = \dfrac{-\sin\theta}{(1+\cos\theta)}$

$$S_{21} = \frac{2\sin\theta}{\sin^2\theta + \cos^2\theta + \cos\theta + 1} = \frac{2\sin\theta}{2[1+\cos\theta]}$$

- $S_{21} = \dfrac{\sin\theta}{(1+\cos\theta)}$

Sección 4.3.8

Punto 2

2. Espacio Vectorial $P(t)_4$.

Dos bases: $\{1, t, t^2, t^3, t^4\}$

$\{P_0, P_1, P_2, P_3, P_4\}$ Polinomios Legendre

Producto interno:
$$\langle f|g\rangle = \int_{-1}^{1} f(t)g(t)\,dt$$

a) $|f\rangle_t \longleftrightarrow f(t) = 5t + 3t^2 + 4t^3$

Primera base: $f(t) = 0\cdot 1 + 5\cdot t + 3t^2 + 4\cdot t^3 + 0\cdot t^4$

$$f(t) = 5t + 3t^2 + 4t^3$$

Segunda base:

$$5t + 3t^2 + 4t^3 = a_0 1 + a_1 t + a_2 \frac{1}{2}(3t^2 - 1) + a_3 \frac{1}{2}(5t^3 - 3t) + a_4 \frac{1}{8}(35t^4 - 30t^2 + 3)$$

$$= a_0 + a_1 t + a_2 \frac{3t^2}{2} - \frac{a_2}{2} + a_3 \frac{5t^3}{2} - a_3 \frac{3t}{2} + a_4 \frac{35t^4}{8} - a_4 \frac{15t^2}{4} + a_4 \frac{3}{8}$$

$$(1)\quad 5t + 3t^2 + 4t^3 = \left(a_0 - \frac{a_2}{2} + \frac{3}{8}a_4\right) + \left(a_1 - \frac{3}{2}a_3\right)t + \left(\frac{3}{2}a_2 - \frac{15}{4}a_4\right)t^2 + \left(\frac{5}{2}a_3\right)t^3 + \left(\frac{35}{8}a_4\right)t^4$$

- $a_0 - \frac{a_2}{2} + \frac{3}{8}a_4 = 0$
- $a_1 - \frac{3}{2}a_3 = 5$
- $\frac{3}{2}a_2 - \frac{15}{4}a_4 = 3$
- $\frac{5}{2}a_3 = 4$
- $\frac{35}{8}a_4 = 0$

Solucionando obtenemos: $a_0 = 1$, $a_1 = 37/5$, $a_2 = 2$, $a_3 = 8/5$, $a_4 = 0$

$$|f\rangle_t = 5t + 3t^2 + 4t^3 = 1P_0 + \frac{37}{5}P_1 + 2P_2 + \frac{8}{5}P_3 + 0P_4$$

Para hallar la matriz de transformación:

sea $|g\rangle = a_0 1 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 = a_0' P_0 + a_1' P_1 + a_2' P_2 + a_3' P_3 + a_4' P_4$

La relación entre las coordenadas la hallamos en ① (página anterior)

$$a_0 = a_0' - \frac{a_2'}{2} + \frac{3}{8} a_4' \qquad a_3 = \frac{5}{2} a_3'$$

$$a_1 = a_1' - \frac{3}{2} a_3'$$

$$a_4 = \frac{35}{8} a_4'$$

$$a_2 = \frac{3}{2} a_2 - \frac{15}{4} a_4$$

$$\longrightarrow a^i = \frac{\partial x^i}{\partial \tilde{x}^j} \tilde{a}^j \qquad \frac{\partial x^i}{\partial \tilde{x}^j} = \begin{bmatrix} 1 & 0 & -1/2 & 0 & 3/8 \\ 0 & 1 & 0 & -3/2 & 0 \\ 0 & 0 & 3/2 & 0 & -15/4 \\ 0 & 0 & 0 & 5/2 & 0 \\ 0 & 0 & 0 & 0 & 35/8 \end{bmatrix}$$

Para pasar de polinomios en base Legendre a la otra base.

La inversa nos permite pasar de la base "Cartesiana" a base Legendre.

b) $\mathbb{P}_2 = |e_i\rangle\langle e^i|_2$ $\qquad\qquad$ $\tilde{\mathbb{P}}_2 = |\tilde{e}_i\rangle\langle\tilde{e}^i|_2$

para la base "cartesiana" $\qquad\qquad$ Para la base P. Legendre

$\rightarrow |f\rangle_t = 5t + 3t^2 + 4t^3 = 5|e_1\rangle + 3|e_2\rangle + 4|e_3\rangle$ cartesianas

$$\mathbb{P}_2|f\rangle_t = |e_i\rangle\langle e^i|\,|f\rangle_t \quad , i=0,1,2$$

$$= |e_i\rangle\langle e^i|5|e_1\rangle + |e_i\rangle\langle e^i|3|e_2\rangle + |e_i\rangle\langle e^i|4|e_3\rangle$$

$$= 5|e_1\rangle\underbrace{\langle e^1|e_1\rangle}_{2/3} + 3|e_2\rangle\underbrace{\langle e^2|e_2\rangle}_{2/5} + 0$$ (cuando que $\langle e^i|e_j\rangle \neq \delta^i_j$)

$$\mathbb{P}_2|f\rangle_t = \frac{10}{3}|e_1\rangle + \frac{6}{5}|e_2\rangle = \underbrace{\frac{10}{3}t + \frac{6}{2}t^2}$$

$\rightarrow |f\rangle_t = 1|\tilde{e}_0\rangle + \frac{37}{2}|\tilde{e}_1\rangle + 2|\tilde{e}_2\rangle + \frac{8}{5}|\tilde{e}_3\rangle$

$$\tilde{\mathbb{P}}_2|f\rangle_t = |\tilde{e}_i\rangle\langle\tilde{e}^i|1|\tilde{e}_0\rangle + |\tilde{e}_i\rangle\langle\tilde{e}^i|\frac{37}{2}|\tilde{e}_1\rangle + |\tilde{e}_i\rangle\langle\tilde{e}^i|2|\tilde{e}_2\rangle + |\tilde{e}_i\rangle\langle\tilde{e}^i|\frac{8}{5}|\tilde{e}_3\rangle$$

$$= 1|\tilde{e}_0\rangle\underbrace{\langle\tilde{e}^0|\tilde{e}_0\rangle}_{2} + \frac{37}{2}|\tilde{e}_1\rangle\underbrace{\langle\tilde{e}^1|\tilde{e}_1\rangle}_{2/3} + 2|\tilde{e}_2\rangle\underbrace{\langle\tilde{e}^2|\tilde{e}_2\rangle}_{2/5} + 0$$

$$= 2|\tilde{e}_0\rangle + \frac{37}{3}|\tilde{e}_1\rangle + \frac{4}{5}|\tilde{e}_2\rangle$$

$$\tilde{\mathbb{P}}_2|f\rangle_t = \underbrace{2P_0 + \frac{37}{3}P_1 + \frac{4}{5}P_2}$$

$\rightarrow \mathbb{P}_2$ sobre $|f\rangle$ pero expresado en base Legendre

$$\mathbb{P}_2|f\rangle = |e_i\rangle\langle e^i|1|\tilde{e}_0\rangle + |e_i\rangle\langle e^i|\frac{37}{2}|\tilde{e}_1\rangle + |e_i\rangle\langle e^i|2|\tilde{e}_2\rangle + |e_i\rangle\langle e^i|\frac{8}{5}|\tilde{e}_3\rangle$$

$$= \left[2|e_0\rangle + \frac{2}{3}|e_2\rangle + 0\right] + \left[\frac{37}{3}|e_1\rangle\right] + \left[\frac{8}{15}|e_2\rangle + 0\right] + 0$$

$$\mathbb{P}_2|f\rangle = \underbrace{2|e_0\rangle + \frac{37}{3}|e_1\rangle + \frac{6}{5}|e_2\rangle}$$

d) Sean las bases $B = \{|y>, |t^3>, |t>\} = \{|E_\alpha>\}$
y $P = \{|1>\ |t>, |l$

En base de monomios

$$\mathbb{D} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\frac{d}{dt}(1) = 0$

$\frac{d}{dt}|t> = |1>$

$\frac{d}{dt}|t^2> = 2|t>$

$\frac{d}{dt}|t^3> = 3|t^2>$

$\frac{d}{dt}|t^4> = 4|t^3>$

Imagenes de los monomios

$$\mathbb{D}|3t^4> = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 0 \end{pmatrix} = |12t^3>$$

$$e^{\mathbb{D}} = \sum_{n=0}^{\infty} \frac{\mathbb{D}^n}{n!} = \mathbb{I} + \mathbb{D} + \frac{\mathbb{D}^2}{2} + \frac{\mathbb{D}^3}{6} + \frac{\mathbb{D}^4}{24} + \frac{\mathbb{D}^5}{120} + \dots$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$\mathbb{D}$ es nilponte para potencias mayores que 4.
Entonces, es correcto afirmar:

$$\mathbb{T} = e^{\mathbb{D}} = \sum_{n=0}^{4} \frac{D^m}{n!} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

En la base de polinomios de legendre.

$\frac{d}{dt}|1> = 0 = |0>$

$\frac{d}{dt}|\frac{1}{2}(3t^2-1)> = 3t = 3|t>$

$\frac{d}{dt}|t> = 1 = |1>$

$\frac{d}{dt}|\frac{1}{2}(5t^2-3t)> = \frac{15}{2}t^2 - \frac{3}{2} =$

$$= 5\left(\frac{3}{2}t^2 - \frac{3}{10} - \frac{2}{10} + \frac{2}{10}\right)$$

$$= 5\left(\frac{3}{2}t^2 - \frac{5}{10}\right) + \frac{10}{10} = 5\left(1/2\,(3t^2-1)\right) + 1$$

$$= 5\,|\tfrac{1}{2}(3t^2-1)\rangle + |1\rangle$$

$$\frac{d}{dt}\,|\tfrac{1}{8}(35t^4 - 30t^2 + 3)\rangle = \frac{35}{2}t^3 - \frac{15}{2}t$$

$$= 7\left(\frac{5}{2}t^3 - \frac{15}{14}t - \frac{6}{14}t + \frac{6}{14}t\right)$$

$$= 7\left(\frac{5}{2}t^3 - \frac{21}{14}t\right) + 3t = 7\,|\tfrac{1}{2}(5t^3 - 3t)\rangle + 3|t\rangle$$

$$\mathbb{D} = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\mathbb{T} = e^{\mathbb{D}} = \sum_{n=0}^{4} \frac{\mathbb{D}^n}{n!} = \begin{pmatrix} 1 & 1 & 3/2 & 7/2 & 75/8 \\ 0 & 1 & 3 & 15/2 & 41/2 \\ 0 & 0 & 1 & 5 & 35/2 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}_p = T_{(p)}{}^{\alpha}{}_{\beta}$$

¿como transforman las representaciones matriciales de $\mathbb{T}$?

$$T_{(p)}{}^{\alpha}{}_{\beta} = \frac{\partial p^\alpha}{\partial e^\mu}\frac{\partial e^\nu}{\partial p^\beta} T_{(B)}{}^{\mu}{}_{\nu} = \frac{\partial p^\alpha}{\partial e^\mu} T_{(B)}{}^{\mu}{}_{\nu}\frac{\partial e^\nu}{\partial p^\beta}$$

$$= \begin{pmatrix} 1 & 0 & 1/3 & 0 & 1/5 \\ 0 & 1 & 0 & 3/5 & 0 \\ 0 & 0 & 2/3 & 0 & 4/7 \\ 0 & 0 & 0 & 2/5 & 0 \\ 0 & 0 & 0 & 0 & 8/35 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 1 & 3 & 6 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 3/2 & 7/2 & 75/8 \\ 0 & 1 & 3 & 15/2 & 41/2 \\ 0 & 0 & 1 & 5 & 35/2 \\ 0 & 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} = T_{(B)}{}^{\alpha}{}_{\beta}$$

$$\Rightarrow T_{(p)}{}^{\alpha}{}_{\beta} = \frac{\partial p^\alpha}{\partial e^\mu}\frac{\partial e^\nu}{\partial p^\beta} T_{(B)}{}^{\mu}{}_{\nu}$$

Igualmente y analogamente: $T(B)^{\mu}{}_{\nu} = \dfrac{\partial e^\mu}{\partial p^\alpha}\dfrac{\partial p^\beta}{\partial e^\nu} T_{(p)}{}^{\alpha}{}_{\beta}$