Presentado por: Miguel Fernando Becerra **Código:** 2201888

Karen Sarat Anaya Verdugo 2200813

Taller tensores

Sección 3.3.5

Punto 2

- **Inciso a.**

Sección 3,3,5

2. Encuentre:

a. La parte simétrica S^i_j y antisimétrica A^i_j del tensor R^i_j.

$$R^i_j = \begin{pmatrix} \frac{1}{2} & 1 & \frac{3}{2} \\ 2 & \frac{5}{2} & 3 \\ \frac{7}{2} & 4 & \frac{9}{2} \end{pmatrix}$$

► Parte simétrica

$$S^i_j = \frac{1}{2}\left(R^i_j + R^j_i\right)$$

$$S^i_j = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 5/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$S^i_j = \frac{1}{2}\left[\begin{pmatrix} 1 & 3 & 5 \\ 3 & 5 & 7 \\ 5 & 7 & 9 \end{pmatrix}\right]$$

$$S^i_j = \begin{pmatrix} 1/2 & 3/2 & 5/2 \\ 3/2 & 5/2 & 7/2 \\ 5/2 & 7/2 & 9/2 \end{pmatrix}$$

► Parte antisimétrica

$$A^i_j = \frac{1}{2}\left(R^i_j - R^j_i\right)$$

$$A^i_j = \frac{1}{2}\left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} - \begin{pmatrix} 1/2 & 2 & 7/2 \\ 1 & 5/2 & 4 \\ 3/2 & 3 & 9/2 \end{pmatrix}\right]$$

$$A^i_j = \frac{1}{2}\left[\begin{pmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{pmatrix}\right]$$

$$A^i_j = \begin{pmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{pmatrix}$$

Podemos notar que ambas partes cumplen con la propiedad

$$R^i_j = S^i_j + A^i_j$$

por lo tanto el resultado es acertado.

- **Inciso b**

b. $R_{kj} = g_{ik} R^i_j$, $R^{ki} = g^{ik} R^i_j$, $T_j = g_{ij} T^i$ ¿Qué se concluye con estos datos?

$$T_i = \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} \qquad g^{ij} = g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $R_{kj} = g_{ik} R^i_j = g_{ki} R^i_j$

g_{ik} al ser una matriz métrica sus índices se pueden intercambiar y no cambiará.

$$R_{kj} = \left[\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \right]$$

$$R_{kj} = \begin{pmatrix} 1/2 & 1 & 3/2 \\ -2 & -5/2 & -3 \\ 7/2 & 4 & 9/2 \end{pmatrix}$$

- $R^{ki} = g^{jk} R^i_j$

Al expandirlo.

$R^{11} = g^{j1} R^1_j = g^{11} R^1_1 = R^1_1$
$R^{12} = g^{j1} R^2_j = g^{11} R^2_1 = R^2_1$
$R^{13} = g^{j1} R^3_j = g^{11} R^3_1 = R^3_1$
$R^{21} = g^{j2} R^1_j = g^{22} R^1_2 = -R^1_2$
$R^{22} = g^{j2} R^2_j = g^{22} R^2_2 = -R^2_2$
$R^{23} = g^{j2} R^3_j = g^{22} R^3_2 = -R^3_2$
$R^{31} = g^{j3} R^1_j = g^{33} R^1_3 = R^1_3$
$R^{32} = g^{j3} R^2_j = g^{33} R^2_3 = R^2_3$
$R^{33} = g^{33} R^3_3 = R^3_3$

Por lo tanto:

$$R^{ki} = \begin{pmatrix} 1/2 & 2 & 1/2 \\ -1 & -3/2 & -4 \\ 3/2 & 3 & 9/2 \end{pmatrix}$$

- $T_j = g_{ij} T^i$

$R^{11} = g_{11} T^1 = T^1$
$R^{12} = g_{22} T^2 = -T^2$
$R^{13} = g_{33} T^3 = T^3$

$(1/3, -2/3, 1)$

- **Inciso c & d**

c.

$$R^i_j T_i \underset{\substack{\uparrow \\ \text{Conmutamos}}}{=} T_i R^i_j \qquad\qquad T_i = g_{ij} T^j$$

$$= \left[\left(1/3, -2/3, 1 \right) \begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \right] \cancel{\times}$$

$$= \left(\frac{7}{3}, \frac{8}{3}, 3 \right)$$

$$R^i_j T^j$$

$$= \left[\begin{pmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{pmatrix} \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} \right]$$

$$= \begin{pmatrix} 7/3 \\ 16/3 \\ 25/3 \end{pmatrix}$$

$$\underbrace{R^i_j T_i} T^j$$

$$= \left[\left(7/3, 8/3, 3 \right) \begin{pmatrix} 1/3 \\ 2/3 \\ 1 \end{pmatrix} \right]$$

$$= \frac{50}{9}$$

d. $R^i_j S^j_i = R^1_1 S^1_1 + R^1_2 S^2_1 + R^1_3 S^3_1 + R^2_1 S^1_2 + R^2_2 S^2_2 + R^2_3 S^3_2 + R^3_1 S^1_3 +$

$R^3_2 S^2_3 + R^3_3 S^3_3$

$$= \overset{1/4}{(1/2(1/2))} + \overset{3/2}{(1(3/2))} + \overset{15/2}{(3/2(5/2))} + \overset{3}{(2(3/2))} + \overset{25/4}{(5/2(5/2))}$$

$$+ \underset{21/2}{(3(7/2))} + \underset{35/2}{(5/2)(7/2)} + \underset{14}{(4(7/2))} + \underset{81/4}{(9/2(9/2))}$$

$$= \frac{273}{4}$$

- $R^i_j A^j_i = R^1_1 A^1_1 + R^1_2 A^2_1 + R^1_3 A^3_1 + R^2_1 A^1_2 + R^2_2 A^2_2 + R^2_3 A^3_2$

$+ R^3_1 A^1_3 + R^3_2 A^2_3 + R^3_3 S^3_3$

$$= (1/2(0)) + (1(-1/2)) + (3/2(-1)) + (2(1/2)) + (5/2(0))$$

$$+ (3(-1/2)) + (1(7/2)) + (4(1/2)) + (9/2(0))$$

$$= 3$$

CS Escaneado con CamScanner

c) $\underset{j\times i}{A_i^j}\ \underset{i\times 1}{T^i} = \begin{bmatrix} 0 & -1/2 & -1 \\ 1/2 & 0 & -1/2 \\ 1 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$

$A_i^j T^i = \begin{bmatrix} -4/3 \\ -1/3 \\ 2/3 \end{bmatrix} = M^j$

d) $A_i^j T^i T_j = \underset{j\times 1}{M^j}\ \underset{1\times j}{T_j}$

$= M^1 T_1 + M^2 T_2 + M^3 T_3$

$= \left(-\frac{4}{3}\right)\left(1/3\right) + \left(-\frac{1}{3}\right)\left(-\frac{2}{3}\right) + \left(\frac{2}{3}\right)\left(1\right)$

$= \frac{4}{9}$

- **Inciso e**

e) • $R^i_j - 2S^i_j R^M_M = R^i_j - 2\delta^i_j \frac{15}{2}$

$$= R^i_j - 15\delta^i_j$$

$$= \begin{bmatrix} 1/2 & 1 & 3/2 \\ 2 & 5/2 & 3 \\ 7/2 & 4 & 9/2 \end{bmatrix} - \begin{bmatrix} 15 & 0 & 0 \\ 0 & 15 & 0 \\ 0 & 0 & 15 \end{bmatrix}$$

$$= \begin{bmatrix} -29/2 & 1 & 3/2 \\ 2 & -25/2 & 3 \\ 7/2 & 4 & -21/2 \end{bmatrix} = S^i_j$$

• $(R^i_j - 2S^i_j R^M_M)T_i = S^i_j T_i = T_i S^i_j$

$T_i = [1/3, -2/3, 1]$

$$\rightarrow = [1/3, -2/3, 1] \begin{bmatrix} -29/2 & 1 & 3/2 \\ 2 & -25/2 & 3 \\ 7/2 & 4 & -21/2 \end{bmatrix}$$

$$= [-8/3, 38/3, -12] = M_j$$

• $(R^i_j - 2\delta^i_j R^M_M)T_i T^j = S^i_j T_i T^j = M_j T^j$

$$= [-8/3, 38/3, -12] \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$$

$$= -\frac{40}{9}$$

Punto 8

- **Inciso a & b**

8.

$q^1 = x+y$; $q^2 = x-y$; $q^3 = 2z$

a. $\vec{q}_1 = (1,1,0)$ $\vec{q}_2 = (1,-1,0)$ $\vec{q}_3 = (0,0,2)$

Compruebe que el sistema es ortogonal.

$$\vec{q}_1 \cdot \vec{q}_2 = (1\times 1) + (1\times -1) + (0\times 0)$$
$$= 1 - 1 + 0 = 0$$
$$\vec{q}_1 \cdot \vec{q}_3 = (1\times 0) + (1\times 0) + (0\times 2)$$
$$= 0 + 0 + 0 = 0$$
$$\vec{q}_2 \cdot \vec{q}_3 = (1\times 0) + (-1\times 0) + (0\times 2)$$
$$= 0 + 0 + 0 = 0$$

Como los producto punto entre los 3 vectores es 0, conforman un sistema de coordenadas ortogonales.

b. Encuentre los vectores base para este sistema de coordenadas.

$$X = \frac{\partial x}{\partial x} x^j$$

$$\frac{\partial x}{\partial x} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \frac{\partial x^1}{\partial x} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \frac{\partial x}{\partial x} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} = \frac{\partial x}{\partial x} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

CS Escaneado con CamScanner

- **Inciso c**

c) $|W^1\rangle = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $|W^2\rangle = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$, $|W^3\rangle = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$

$$\eta_{\mu\nu} = \begin{bmatrix} \langle W_1|W^1\rangle & \langle W_1|W^2\rangle & \langle W_1|W^3\rangle \\ \langle W_2|W^1\rangle & \langle W_2|W^2\rangle & \langle W_2|W^3\rangle \\ \langle W_3|W^1\rangle & \langle W_3|W^2\rangle & \langle W_3|W^3\rangle \end{bmatrix}$$

$$\eta_{\mu\nu} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\rightarrow d\tilde{x}^i = \frac{\partial \tilde{x}^i}{\partial x^j} dx^j$$

$$= \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} dx \\ dy \\ dz \end{bmatrix}$$

$$d\tilde{x}^i = \begin{bmatrix} dx + dy \\ dx - dy \\ 2dz \end{bmatrix}$$

- **Inciso d & e.**

d. Encuentre las expresiones en el sistema (q_1, q_2, q_3) para los vectores.

$A = 2\hat{j}$, $B = \hat{i} + 2\hat{j}$, $C = \hat{i} + 7\hat{j} + 3\hat{k}$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 \\ 2 & 2 & 7 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 2 & 3 & 8 \\ -2 & -1 & -6 \\ 0 & 0 & 6 \end{pmatrix}$$

↑
Matriz
De transformación

e. Encuentre en el sistema (q^1, q^2, q^3) las expresiones para la siguientes relaciones vectoriales.

$$A \times B = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix}$$

$$A \cdot C = \begin{pmatrix} 2 \\ -2 \\ 0 \end{pmatrix} \begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix} = (16) + (12) + (0) = 28$$

$$(A \times B) \cdot C = \begin{pmatrix} 0 \\ 0 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ -6 \\ 6 \end{pmatrix} = 0 + 0 + 24 = 24$$

¿Qué se puede decir si se compara esas expresiones en ambos sistemas de coordenadas?

Si se compara con coordenadas cartesianas el resultado será diferente debido a las bases usadas para el cálculo.

Escaneado con CamScanner

- **Inciso f**

$$T^{i'} = \frac{\partial \bar{x}^{i'}}{\partial x^{j}} T^{j} \qquad T^{j} = \begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{i' \times j} \underbrace{\begin{bmatrix} 1/3 \\ 2/3 \\ 1 \end{bmatrix}}_{j \times 1}$$

$$T^{i'} = \begin{bmatrix} 1 \\ -1/3 \\ 2 \end{bmatrix}$$

$$g^{i'j'} = \frac{\partial \bar{x}^{i'}}{\partial x^{\alpha}} \frac{\partial \bar{x}^{j'}}{\partial x^{\beta}} g^{\alpha\beta} \qquad \text{simetrica} \qquad g^{ij} = g_{ij} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{i \times \alpha} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{j' \times \beta \,\to\, \beta \times j'} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\alpha \times \beta}$$

$$= \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{i \times \alpha} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\alpha \times \beta} \underbrace{\begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}}_{\beta \times j'}$$

$$g^{i'j'} = \begin{bmatrix} 0 & -1/2 & 0 \\ -1/2 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Sección 3.4.3

Punto 6

- **Inciso a & b**

6. a) los campos que mide el observador es

$$F_{\mu\alpha} = \begin{bmatrix} 0 & E_x & 0 & 0 \\ -E_x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

un observador con velocidad $\beta = v\hat{z}$

El Tensor transforma

$$\tilde{F}_{\beta'\sigma'} = \Lambda^{\mu}_{\beta'}\Lambda^{\alpha}_{\sigma'}F_{\mu\alpha} \longleftarrow$$ Solo tiene valores F_{10} y F_{01}, de resto todo es 0

$$\tilde{F}_{\beta'\sigma'} = \Lambda^{0}_{\beta'}\Lambda^{1}_{\sigma'}F_{01} + \Lambda^{1}_{\beta'}\Lambda^{0}_{\sigma'}F_{10}$$ $E_x = F_{01} = -F_{10}$

$$\tilde{F}_{\beta'\sigma'} = \left(\Lambda^{0}_{\beta'}\Lambda^{1}_{\sigma'} - \Lambda^{1}_{\beta'}\Lambda^{0}_{\sigma'}\right)F_{01}$$

Recordemos que:

- $$\partial_\mu F^{\mu 1} = \partial_0 F^{01} + \partial_1 F^{11} + \partial_2 F^{21} + \partial_3 F^{31}$$
$$= -\frac{\partial E_x}{\partial t} + 0 + \frac{\partial cB_z}{\partial y} - \frac{\partial cB_y}{\partial z} = -\frac{\partial E_x}{\partial t} + c(\vec{\nabla}\times\vec{B})_x$$

- $$\partial_\mu F^{\mu 2} = \partial_0 F^{02} + \partial_1 F^{12} + \partial_2 F^{22} + \partial_3 F^{32}$$
$$= -\frac{\partial E_y}{\partial t} + -\frac{\partial cB_z}{\partial x} + 0 + \frac{\partial cB_x}{\partial z} = -\frac{\partial E_y}{\partial t} + c(\vec{\nabla}\times\vec{B})_y$$

- $$\partial_\mu F^{\mu 3} = \partial_0 F^{03} + \partial_1 F^{13} + \partial_2 F^{23} + \partial_3 F^{33}$$
$$= -\frac{\partial E_z}{\partial t} + \frac{\partial cB_y}{\partial x} - \frac{\partial cB_x}{\partial y} + 0 = -\frac{\partial E_z}{\partial t} + c(\vec{\nabla}\times\vec{B})_z$$

Si tomamos $c=1$ y recordando $J^i = (c\rho, J^1, J^2, J^3)$

$$\partial_\mu F^{\mu 0} = 4\pi c\rho = 4\pi J^0$$

$$\partial_\mu F^{\mu 1} = 4\pi J^1$$

$$\partial_\mu F^{\mu 2} = 4\pi J^2$$

$$\partial_\mu F^{\mu 3} = 4\pi J^3$$

$$\rightarrow \underbrace{\partial_\mu F^{\mu\nu} = 4\pi J^\nu}$$

$$\tilde{F}_{\beta'\sigma'} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot E_x \gamma [V^2 + 1]$$

B) $F^{\mu\nu} = \eta^{\mu\alpha}\eta^{\nu\beta} F_{\alpha\beta}$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{bmatrix}$$

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

$$\vec{\nabla}\times\vec{B} - \frac{\partial \vec{E}}{\partial t} = 4\pi\vec{J}$$

$\partial_\mu F^{\mu\nu}$

- $\partial_\mu F^{\mu 0} = \partial_0 F^{00} + \partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30}$

$$= 0 + \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \vec{\nabla}\cdot\vec{E} = 4\pi\rho$$

- $\partial_\mu F^{\mu 1} = \partial_0 F^{01} + \partial_1 F^{11} + \partial_2 F^{21} + \partial_3 F^{31}$

$$= \frac{\partial E_x}{\partial t} + 0 + \frac{\partial c B_z}{\partial y} - \frac{\partial c B_y}{\partial z} = -\frac{\partial E_x}{\partial t} + c(\vec{\nabla}\times\vec{B})_x$$

Recordamos que:

$$\Lambda^0_{0'} = \gamma \quad , \quad \Lambda^i_{0'} = \gamma V^i \quad , \quad \Lambda^i_{j'} = \delta^i_{j'} + V^i V_j \frac{(\gamma - 1)}{\vec{V}^2}$$

observamos que $\quad \Lambda^{j'}_i = \eta^{\beta' j'} \eta_{\alpha i} \Lambda^\alpha_{\beta'}$ cuando $j' = 0$

$$\Lambda^0_i = \eta^{\beta' 0} \eta_{\alpha i} \Lambda^\alpha_{\beta'} \longrightarrow \eta^{\beta' 0}$$ será diferente 0 cuando $\beta' = 0$

$$= \eta^{00} \eta_{\alpha i} \Lambda^\alpha_0 \longrightarrow$$ lo mismo con $\eta_{\alpha i}$, será $\neq 0$ cuando $\alpha = i$

$$= -\eta_{ii} \Lambda^i_0 \longrightarrow i = 1,2,3$$

$$\boxed{\Lambda^0_i = -\Lambda^i_0} = -\gamma V^i$$

$$\rightarrow \tilde{F}_{\beta'\sigma'} = (\underbrace{\Lambda^0_{\beta'} \Lambda^1_{\sigma'} - \Lambda^1_{\beta'} \Lambda^0_{\sigma'}}) F_{01}$$

Solo pueden ser 1, porque el observador solo tiene vel en x, recuerde $\Lambda^0_i = -\gamma V^i$

- $\tilde{F}_{01} = (\Lambda^0_0 \Lambda^1_1 - \Lambda^1_0 \Lambda^0_1) E_x$

$$= [\gamma \cdot \gamma - \gamma \cdot V^x \cdot (-\gamma V^x)] E_x$$

$$\tilde{F}_{01} = E_x \gamma^2 [(V^x)^2 + 1]$$

Como tiene que ser antisimetrico

$$\rightarrow \tilde{F}_{10} = -E_x \gamma^2 [(V^x)^2 + 1]$$

- **Inciso c**

$\nabla \cdot B = 0$

$\nabla \times E = \frac{\partial B}{\partial E}$

c) $\partial_\gamma F_{\mu\nu} + \partial_\mu F_{\nu\gamma} + \partial_\nu F_{\gamma\mu} = 0$

$\rightarrow \varepsilon^{\mu\nu\alpha\beta} \partial_\nu F_{\alpha\beta} = 0$

cuando $\mu = 0$ $\qquad \varepsilon^{0\nu\alpha\beta} = \varepsilon^{\nu\alpha\beta}$

$$0 = \varepsilon^{0\nu\alpha\beta} \partial_\nu F_{\alpha\beta} = \varepsilon^{\nu\alpha\beta} \partial_\nu F_{\alpha\beta}$$

$\rightarrow \underline{F_{ij} = \varepsilon_{ijk} B^k}$

$\rightarrow$

$$= \varepsilon^{\nu\alpha\beta} \partial_\nu \varepsilon_{\alpha\beta k} B^k$$

$$= \varepsilon_{k\alpha\beta} \varepsilon^{\nu\alpha\beta} \partial_\nu B^k$$

$$= 2 \delta^\nu_k \partial_\nu B^k$$

$$= 2 \partial_k B^k$$

$$= 2(\vec{\nabla} \cdot \vec{B}) = 0$$