

Lista sem gabarito

Karen dos Anjos Arcoverde

1 Questão 1

(a)

A matriz A é similar a matriz B se, e somente se, existe uma matriz inversível tal que:

$$A = P \cdot B \cdot P^{-1}$$

Pelo enunciado, sabemos que:

$$A = Q \cdot R \quad (1)$$

$$B = R \cdot Q \quad (2)$$

Utilizando a equação (2):

$$B = R \cdot Q$$

Como Q é ortogonal, então $Q^{-1} = Q^T$, assim Q^{-1} existe:

$$B \cdot Q^{-1} = R \cdot Q \cdot Q^{-1}$$

$$B \cdot Q^{-1} = R \quad (3)$$

Substituindo (3) em (1):

$$A = Q \cdot B \cdot Q^{-1}$$

Dessa forma, A e B são similares.

(b)

O método utilizado para a decomposição QR foi o de Gram-Schmidt.

Sabendo que a matriz A composta por $R \cdot Q$, onde A, Q e R são da forma:

$$A = \begin{pmatrix} | & | & | & | \\ a_1 & a_2 & a_3 & a_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$
$$Q = \begin{pmatrix} | & | & | & | \\ q_1 & q_2 & q_3 & q_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

$$R = \begin{pmatrix} q_1^T \cdot a_1 & q_1^T \cdot a_2 & q_1^T \cdot a_3 & q_1^T \cdot a_4 \\ 0 & q_2^T \cdot a_2 & q_2^T \cdot a_3 & q_2^T \cdot a_4 \\ 0 & 0 & q_3^T \cdot a_3 & q_3^T \cdot a_4 \\ 0 & 0 & 0 & q_4^T \cdot a_4 \end{pmatrix}$$

Considerando que cada coluna da matriz A é um vetor com nome a_1, a_2, a_3, a_4 e cada coluna da matriz Q serão os 4 vetores ortonormais encontrados em cada iteração (q_1, q_2, q_3, q_4) de modo que na primeira iteração teremos os dados fornecidos pelo professor:

$$a_1 = \begin{pmatrix} 0.7140299 \\ 0.6918437 \\ 0.6748934 \\ 0.7884872 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.6918437 \\ 1.2492256 \\ 1.0307285 \\ 0.6275049 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.6748934 \\ 1.0307285 \\ 1.0893471 \\ 0.5567456 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.7884872 \\ 0.6275049 \\ 0.5567456 \\ 0.9550583 \end{pmatrix}$$

Sabendo que a norma de um vetor $u \in R^n$ é o número real $\|u\|_2 = \sqrt{\langle u, u \rangle}$. Então, segue-se o seguinte algoritmo:

$$1. \ q_1 = \frac{a_1}{\|a_1\|_2}$$

$$2. \ q_2' = (I - q_1 \cdot q_1^T) \cdot a_2 = a_2 - q_1 \cdot q_1^T \cdot a_2$$

$$3. \ q_2 = \frac{q_2'}{\|q_2'\|_2}$$

$$4. \ q_3' = (I - q_1 \cdot q_1^T - q_2 \cdot q_2^T) \cdot a_3 = a_3 - q_1 \cdot q_1^T \cdot a_3 - q_2 \cdot q_2^T \cdot a_3$$

$$5. \ q_3 = \frac{q_3'}{\|q_3'\|_2}$$

$$6. \ q_4' = (I - q_1 \cdot q_1^T - q_2 \cdot q_2^T - q_3 \cdot q_3^T) \cdot a_4 = a_4 - q_1 \cdot q_1^T \cdot a_4 - q_2 \cdot q_2^T \cdot a_4 - q_3 \cdot q_3^T \cdot a_4$$

$$7. \ q_4 = \frac{q_4'}{\|q_4'\|_2}$$

$$8. \ R = \begin{pmatrix} q_1^T \cdot a_1 & q_1^T \cdot a_2 & q_1^T \cdot a_3 & q_1^T \cdot a_4 \\ 0 & q_2^T \cdot a_2 & q_2^T \cdot a_3 & q_2^T \cdot a_4 \\ 0 & 0 & q_3^T \cdot a_3 & q_3^T \cdot a_4 \\ 0 & 0 & 0 & q_4^T \cdot a_4 \end{pmatrix}$$

$$9. \ Q = \begin{pmatrix} | & | & | & | \\ q_1 & q_2 & q_3 & q_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

10. $A = R \cdot Q$

Na próxima iteração, as matrizes-colunas de A (a_1, a_2, a_3, a_4) mudarão, de forma que cada matriz-coluna de A será igual a cada matriz-coluna de $R \cdot Q$.

Fez-se várias vezes as iterações até que na quinta iteração, os valores na diagonal principal da matriz $R \cdot Q$ ficarão iguais aos da quarta iteração.

1º Iteração:

```

1  a1 =  [[0.7140299]
2      [0.6918437]
3      [0.6748934]
4      [0.7884872]]
5  a2 =  [[0.6918437]
6      [1.2492256]
7      [1.0307285]
8      [0.6275049]]
9  a3 =  [[0.6748934]
10     [1.0307285]
11     [1.0893471]
12     [0.5567456]]
13 a4 =  [[0.7884872]
14     [0.6275049]
15     [0.5567456]
16     [0.9550583]]
17 q1 =  [[0.49680381]
18     [0.48136722]
19     [0.46957363]
20     [0.54860931]]
21 q2' =  [[-0.18914089]
22     [ 0.3956148 ]
23     [ 0.19803132]
24     [-0.34534662]]
25 q2 =  [[-0.31935716]
26     [ 0.66798046]
27     [ 0.33436832]
28     [-0.58310456]]
29 q3' =  [[ 0.01965057]
30     [-0.10515473]
31     [ 0.14390639]
32     [-0.0487033 ]]
33 q3 =  [[ 0.10575722]
34     [-0.5659313 ]
35     [ 0.77448851]
36     [-0.26211586]]
37 q4' =  [[-1.71294927e-03]
38     [-9.10805482e-05]
39     [ 5.57797235e-04]
40     [ 1.15367355e-03]]
41 q4 =  [[-0.80000774]
42     [-0.04253783]
43     [ 0.26051099]
44     [ 0.53880625]]

```

```

45 R = [[ 1.43724722  1.7733048  1.64841302  1.47917066]
46      [ 0.          0.59225504  0.51257672 -0.20338878]
47      [ 0.          0.          0.1858083  -0.0908793 ]
48      [ 0.          0.          0.          0.00214117]]
49 Q = [[ 0.49680381 -0.31935716  0.10575722 -0.80000774]
50      [ 0.48136722  0.66798046 -0.5659313  -0.04253783]
51      [ 0.46957363  0.33436832  0.77448851  0.26051099]
52      [ 0.54860931 -0.58310456 -0.26211586  0.53880625]]
53 A = RQ = [[ 3.15317879e+00  4.14203696e-01  3.73934457e-02  1.17466352e-03]
54            [ 4.14203696e-01  6.85601138e-01  1.15120544e-01 -1.24852359e-03]
55            [ 3.73934457e-02  1.15120544e-01  1.67727299e-01 -5.61233532e-04]
56            [ 1.17466352e-03 -1.24852359e-03 -5.61233533e-04  1.15367355e-03]]

```

2º Iteração:

```

1  a1 = [[3.15317879e+00]
2      [4.14203696e-01]
3      [3.73934457e-02]
4      [1.17466352e-03]]
5  a2 = [[ 0.4142037 ]
6      [ 0.68560114]
7      [ 0.11512054]
8      [-0.00124852]]
9  a3 = [[ 0.03739345]
10     [ 0.11512054]
11     [ 0.1677273 ]
12     [-0.00056123]]
13 a4 = [[ 0.00117466]
14     [-0.00124852]
15     [-0.00056123]
16     [ 0.00115367]]
17 q1 = [[9.91413667e-01]
18     [1.30232769e-01]
19     [1.17571427e-02]
20     [3.69334423e-04]]
21 q2' = [[-0.08278005]
22     [ 0.62031702]
23     [ 0.10922683]
24     [-0.00143367]]
25 q2 = [[-0.1303052 ]
26     [ 0.97644952]
27     [ 0.17193545]
28     [-0.00225675]]
29 q3' = [[ 0.00159135]
30     [-0.02508144]
31     [ 0.14364405]
32     [-0.00027342]]
33 q3 = [[ 0.01091262]
34     [-0.17199536]
35     [ 0.98503553]
36     [-0.00187499]]
37 q4' = [[-7.35338576e-07]
38     [ 2.10683582e-06]

```

```

39 [ 2.56381985e-06]
40 [ 1.14937149e-03]]
41 q4 = [[-6.39771708e-04]
42 [ 1.83302494e-03]
43 [ 2.23061792e-03]
44 [ 9.99995628e-01]]
45 R = [[ 3.18048751e+00  5.01287967e-01  5.40366269e-02  9.95806374e-04]
46 [ 0.00000000e+00  6.35278124e-01  1.36376375e-01 -1.47128452e-03]
47 [ 0.00000000e+00  0.00000000e+00  1.45826262e-01 -3.27439177e-04]
48 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937651e-03]]
49 Q = [[ 9.91413667e-01 -1.30305205e-01  1.09126206e-02 -6.39771708e-04]
50 [ 1.30232769e-01  9.76449518e-01 -1.71995361e-01  1.83302494e-03]
51 [ 1.17571427e-02  1.71935450e-01  9.85035530e-01  2.23061792e-03]
52 [ 3.69334423e-04 -2.25675407e-03 -1.87499273e-03  9.99995628e-01]]
53 A = RQ = [[ 3.21909859e+00  8.43368823e-02  1.71437924e-03  4.24504242e-07]
54 [ 8.43368823e-02  6.43768272e-01  2.50734429e-02 -2.59386022e-06]
55 [ 1.71437924e-03  2.50734429e-02  1.43644663e-01 -2.15507257e-06]
56 [ 4.24504312e-07 -2.59386012e-06 -2.15507261e-06  1.14937149e-03]]

```

3º Iteração:

```

1  a1 = [[3.21909859e+00]
2  [8.43368823e-02]
3  [1.71437924e-03]
4  [4.24504312e-07]]
5  a2 = [[ 8.43368823e-02]
6  [ 6.43768272e-01]
7  [ 2.50734429e-02]
8  [-2.59386012e-06]]
9  a3 = [[ 1.71437924e-03]
10 [ 2.50734429e-02]
11 [ 1.43644663e-01]
12 [-2.15507261e-06]]
13 a4 = [[ 4.24504242e-07]
14 [-2.59386022e-06]
15 [-2.15507257e-06]
16 [ 1.14937149e-03]]
17 q1 = [[9.99656843e-01]
18 [2.61899221e-02]
19 [5.32382246e-04]
20 [1.31825301e-07]]
21 q2' = [[-1.68099277e-02]
22 [ 6.41118335e-01]
23 [ 2.50195756e-02]
24 [-2.60719841e-06]]
25 q2 = [[-2.61907655e-02]
26 [ 9.98896621e-01]
27 [ 3.89818356e-02]
28 [-4.06215442e-06]]
29 q3' = [[ 6.97307379e-05]
30 [-5.55728747e-03]
31 [ 1.42450500e-01]
32 [-2.03109159e-06]]

```

```

33 q3 = [[ 4.89136416e-04]
34      [-3.89824023e-02]
35      [ 9.99239777e-01]
36      [-1.42473878e-05]]
37 q4' = [[-2.65737072e-10]
38         [ 4.02144661e-09]
39         [ 1.65450142e-08]
40         [ 1.14937145e-03]]
41 q4 = [[-2.31202082e-07]
42        [ 3.49882244e-06]
43        [ 1.43948366e-05]
44        [ 1.00000000e+00]]
45 R = [[ 3.22020363e+00  1.01181531e-01  2.44693632e-03  3.55429767e-07]
46      [ 0.00000000e+00  6.41826513e-01  3.06004091e-02 -2.69079391e-06]
47      [ 0.00000000e+00  0.00000000e+00  1.42558877e-01 -2.06848723e-06]
48      [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
49 Q = [[ 9.99656843e-01 -2.61907655e-02  4.89136416e-04 -2.31202082e-07]
50      [ 2.61899221e-02  9.98896621e-01 -3.89824023e-02  3.49882244e-06]
51      [ 5.32382246e-04  3.89818356e-02  9.99239777e-01  1.43948366e-05]
52      [ 1.31825301e-07 -4.06215442e-06 -1.42473878e-05  1.00000000e+00]]
53 A = RQ = [[ 3.22174983e+00  1.68256775e-02  7.58958146e-05  1.51443706e-10]
54            [ 1.68256775e-02  6.42311196e-01  5.55720670e-03 -4.66901591e-09]
55            [ 7.58958146e-05  5.55720670e-03  1.42450500e-01 -1.63754932e-08]
56            [ 1.51516237e-10 -4.66892431e-09 -1.63755407e-08  1.14937145e-03]]

```

4° Iteração:

```

1  a1 = [[3.22174983e+00]
2        [1.68256775e-02]
3        [7.58958146e-05]
4        [1.51516237e-10]]
5  a2 = [[ 1.68256775e-02]
6        [ 6.42311196e-01]
7        [ 5.55720670e-03]
8        [-4.66892431e-09]]
9  a3 = [[ 7.58958146e-05]
10       [ 5.55720670e-03]
11       [ 1.42450500e-01]
12       [-1.63755407e-08]]
13 a4 = [[ 1.51443706e-10]
14       [-4.66901591e-09]
15       [-1.63754932e-08]
16       [ 1.14937145e-03]]
17 q1 = [[9.99986363e-01]
18       [5.22245641e-03]
19       [2.35570058e-05]
20       [4.70285337e-11]]
21 q2' = [[-3.35406847e-03]
22         [ 6.42205806e-01]
23         [ 5.55673132e-03]
24         [-4.66987335e-09]]
25 q2 = [[-5.22246474e-03]
26        [ 9.99948932e-01]

```

```

27 [ 8.65212910e-03]
28 [-7.27124362e-09]]
29 q3' = [[ 3.07998696e-06]
30 [-1.23203871e-03]
31 [ 1.42391758e-01]
32 [-1.63261811e-08]]
33 q3 = [[ 2.16295642e-05]
34 [-8.65213413e-03]
35 [ 9.99962569e-01]
36 [-1.14652493e-07]]
37 q4' = [[-9.48482264e-14]
38 [ 7.21648718e-12]
39 [ 1.31845677e-10]
40 [ 1.14937145e-03]]
41 q4 = [[-8.25218223e-11]
42 [ 6.27863794e-09]
43 [ 1.14711112e-07]
44 [ 1.00000000e+00]]
45 R = [[ 3.22179377e+00  2.01800212e-02  1.08272756e-04  1.26726204e-10]
46 [ 0.00000000e+00  6.42238604e-01  6.78902666e-03 -4.81960862e-09]
47 [ 0.00000000e+00  0.00000000e+00  1.42397088e-01 -1.64662583e-08]
48 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
49 Q = [[ 9.99986363e-01 -5.22246474e-03  2.16295642e-05 -8.25218223e-11]
50 [ 5.22245641e-03  9.99948932e-01 -8.65213413e-03  6.27863794e-09]
51 [ 2.35570058e-05  8.65212910e-03  9.99962569e-01  1.14711112e-07]
52 [ 4.70285337e-11 -7.27124362e-09 -1.14652493e-07  1.00000000e+00]]
53 A = RQ = [[ 3.22185522e+00  3.35422304e-03  3.35444903e-06 -1.89539327e-14]
54 [ 3.35422304e-03  6.42264546e-01  1.23203799e-03 -8.44816195e-12]
55 [ 3.35444903e-06  1.23203799e-03  1.42391758e-01 -1.31730023e-10]
56 [ 5.40532539e-14 -8.35735981e-12 -1.31778302e-10  1.14937145e-03]]

```

5° Iteração:

```

1 a1 = [[3.22185522e+00]
2 [3.35422304e-03]
3 [3.35444903e-06]
4 [5.40532539e-14]]
5 a2 = [[ 3.35422304e-03]
6 [ 6.42264546e-01]
7 [ 1.23203799e-03]
8 [-8.35735981e-12]]
9 a3 = [[ 3.35444903e-06]
10 [ 1.23203799e-03]
11 [ 1.42391758e-01]
12 [-1.31778302e-10]]
13 a4 = [[-1.89539327e-14]
14 [-8.44816195e-12]
15 [-1.31730023e-10]
16 [ 1.14937145e-03]]
17 q1 = [[9.99999458e-01]
18 [1.04108378e-03]
19 [1.04115392e-06]
20 [1.67770495e-14]]

```

```

21 q2' = [[-6.68648487e-04]
22      [ 6.42260358e-01]
23      [ 1.23203380e-03]
24      [-8.35742730e-12]]
25 q2 = [[-1.04108386e-03]
26      [ 9.99997618e-01]
27      [ 1.91827325e-03]
28      [-1.30124914e-11]]
29 q3' = [[ 1.36114290e-07]
30      [-2.73141271e-04]
31      [ 1.42388871e-01]
32      [-1.31758716e-10]]
33 q3 = [[ 9.55931730e-07]
34      [-1.91827330e-03]
35      [ 9.99998160e-01]
36      [-9.25342499e-10]]
37 q4' = [[-3.38370023e-17]
38      [ 1.29159273e-14]
39      [ 1.06358898e-12]
40      [ 1.14937145e-03]]
41 q4 = [[-2.94395710e-14]
42      [ 1.12373832e-11]
43      [ 9.25365758e-10]
44      [ 1.00000000e+00]]
45 R = [[ 3.22185697e+00  4.02287371e-03  4.78535372e-06 -2.78670350e-14]
46      [ 0.00000000e+00  6.42261887e-01  1.50517786e-03 -8.71577246e-12]
47      [ 0.00000000e+00  0.00000000e+00  1.42389133e-01 -1.32777137e-10]
48      [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
49 Q = [[ 9.99999458e-01 -1.04108386e-03  9.55931730e-07 -2.94395710e-14]
50      [ 1.04108378e-03  9.99997618e-01 -1.91827330e-03  1.12373832e-11]
51      [ 1.04115392e-06  1.91827325e-03  9.99998160e-01  9.25365758e-10]
52      [ 1.67770495e-14 -1.30124914e-11 -9.25342499e-10  1.00000000e+00]]
53 A = RQ = [[ 3.22185941e+00  6.68650002e-04  1.48249003e-07 -7.30823464e-14]
54      [ 6.68650002e-04  6.42263245e-01  2.73141265e-04 -1.05589498e-13]
55      [ 1.48249004e-07  2.73141265e-04  1.42388871e-01 -1.01510922e-12]
56      [ 1.92830617e-17 -1.49561861e-14 -1.06356225e-12  1.14937145e-03]]

```

Somente na 5ª Iteração os valores da diagonal principal de $R \cdot Q$ são iguais aos da 4ª Iteração com aproximação de 4 casas decimais e a matriz $R \cdot Q$ se tornou próxima a uma triangular superior, uma vez que todos os valores abaixo da diagonal principal são próximos de zero em 4 casas decimais. De forma que os autovalores da matriz A são revelados, arredondando em 4 casas decimais: **3.2219, 0.6423, 0.1424, 0.0011**.

Se fizermos mais uma iteração, todos os valores abaixo da diagonal principal realmente terão se tornado zeros, considerando as 4 primeiras casas decimais. Além disso, os valores da diagonal principal continuarão constantes iguais aos da 5ª Iteração na aproximação de 4 casas decimais.

6ª Iteração:

```

1 a1 = [[3.22185958e+00]
2      [1.33292376e-04]
3      [6.55180317e-09]
4      [6.87907112e-21]]
5 a2 = [[ 1.33292376e-04]

```



```

6  [ 6.42263220e-01]
7  [ 6.05549855e-05]
8  [-2.67650710e-17]]
9  a3 = [[ 6.55180154e-09]
10 [ 6.05549855e-05]
11 [ 1.42388729e-01]
12 [-8.58508289e-15]]
13 a4 = [[-7.31204284e-14]
14 [-9.06242897e-14]
15 [ 3.99064838e-14]
16 [ 1.14937145e-03]]
17 q1 = [[9.99999999e-01]
18 [4.13712556e-05]
19 [2.03354709e-09]
20 [2.13512444e-21]]
21 q2' = [[-2.65712357e-05]
22 [ 6.42263214e-01]
23 [ 6.05549851e-05]
24 [-2.67650713e-17]]
25 q2 = [[-4.13712556e-05]
26 [ 9.99999995e-01]
27 [ 9.42837508e-05]
28 [-4.16730564e-17]]
29 q3' = [[ 2.65852571e-10]
30 [-1.34249428e-05]
31 [ 1.42388722e-01]
32 [-8.58507981e-15]]
33 q3 = [[ 1.86709008e-09]
34 [-9.42837508e-05]
35 [ 9.99999996e-01]
36 [-6.02932568e-14]]
37 q4' = [[-4.30626773e-24]
38 [ 4.13640184e-20]
39 [ 6.92993521e-17]
40 [ 1.14937145e-03]]
41 q4 = [[-3.74662842e-21]
42 [ 3.59883817e-17]
43 [ 6.02932605e-14]
44 [ 1.00000000e+00]]
45 R = [[ 3.22185958e+00  1.59863612e-04  9.34659149e-09 -7.31241775e-14]
46 [ 0.00000000e+00  6.42263217e-01  7.39799283e-05 -9.06175495e-14]
47 [ 0.00000000e+00  0.00000000e+00  1.42388722e-01  3.98457285e-14]
48 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
49 Q = [[ 9.99999999e-01 -4.13712556e-05  1.86709008e-09 -3.74662842e-21]
50 [ 4.13712556e-05  9.99999995e-01 -9.42837508e-05  3.59883817e-17]
51 [ 2.03354709e-09  9.42837508e-05  9.99999996e-01  6.02932605e-14]
52 [ 2.13512444e-21 -4.16730564e-17 -6.02932568e-14  1.00000000e+00]]
53 A = RQ = [[ 3.22185959e+00  2.65712359e-05  2.89552537e-10 -7.31241832e-14]
54 [ 2.65712359e-05  6.42263221e-01  1.34249428e-05 -9.05899750e-14]
55 [ 2.89554172e-10  1.34249428e-05  1.42388722e-01  4.84308088e-14]
56 [ 2.45405106e-24 -4.78978212e-20 -6.92993479e-17  1.14937145e-03]]

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