Lista sem gabarito

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1 Questão 1

(a)

A matriz A é similar a matriz B se, e somente se, existe uma matriz inversível tal que:

$$A = P \cdot B \cdot P^{-1}$$

Pelo enunciado, sabemos que:

$$A = Q \cdot R \tag{1}$$

$$B = R \cdot Q \tag{2}$$

Utilizando a equação (2):

$$B = R \cdot Q$$

Como Q é ortogonal, então $Q^{-1} = Q^T$, assim Q^{-1} existe:

$$B \cdot Q^{-1} = R \cdot Q \cdot Q^{-1}$$

$$B \cdot Q^{-1} = R \tag{3}$$

Substituindo (3) em (1):

$$A = Q \cdot B \cdot Q^{-1}$$

Dessa forma, A e B são similares.

(b)

O método utilizado para a decomposição QR foi o de Gram-Schmidt.
Sabendo que a matriz A composta por $R \cdot Q$, onde A, Q e R são da forma:

$$A = \begin{pmatrix} | & | & | & | \\ a_1 & a_2 & a_3 & a_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

$$Q = \begin{pmatrix} | & | & | & | \\ q_1 & q_2 & q_3 & q_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$$

$$R = \begin{pmatrix} q_1^T \cdot a_1 & q_1^T \cdot a_2 & q_1^T \cdot a_3 & q_1^T \cdot a_4 \\ 0 & q_2^T \cdot a_2 & q_2^T \cdot a_3 & q_2^T \cdot a_4 \\ 0 & 0 & q_3^T \cdot a_3 & q_3^T \cdot a_4 \\ 0 & 0 & 0 & q_4^T \cdot a_4 \end{pmatrix}$$

Considerando que cada coluna da matriz A é um vetor com nome a_1, a_2, a_3, a_4 e cada coluna da matriz Q serão os 4 vetores ortonormais encontrados em cada iteração (q_1, q_2, q_3, q_4) de modo que na primeira iteração teremos os dados fornecidos pelo professor:

$$a_1 = \begin{pmatrix} 0.7140299 \\ 0.6918437 \\ 0.6748934 \\ 0.7884872 \end{pmatrix}$$

$$a_2 = \begin{pmatrix} 0.6918437 \\ 1.2492256 \\ 1.0307285 \\ 0.6275049 \end{pmatrix}$$

$$a_3 = \begin{pmatrix} 0.6748934 \\ 1.0307285 \\ 1.0893471 \\ 0.5567456 \end{pmatrix}$$

$$a_4 = \begin{pmatrix} 0.7884872 \\ 0.6275049 \\ 0.5567456 \\ 0.9550583 \end{pmatrix}$$

Sabendo que a norma de um vetor $u \in R^n$ é o número real $\|u\|_2 = \sqrt{< u, u >}$. Então, segue-se o seguinte algoritmo:

1. $q_1 = \frac{a_1}{\|a_1\|_2}$
2. $q_2^{'} = \left(I - q_1 \cdot q_1^T\right) \cdot a_2 = a_2 - q_1 \cdot q_1^T \cdot a_2$
3. $q_2 = \frac{q_2^{'}}{\left\|q_2^{'}\right\|_2}$
4. $q_3^{'} = \left(I - q_1 \cdot q_1^T - q_2 \cdot q_2^T\right) \cdot a_3 = a_3 - q_1 \cdot q_1^T \cdot a_3 - q_2 \cdot q_2^T \cdot a_3$
5. $q_3 = \frac{q_3^{'}}{\left\|q_3^{'}\right\|_2}$
6. $q_4^{'} = \left(I - q_1 \cdot q_1^T - q_2 \cdot q_2^T - q_3 \cdot q_3^T\right) \cdot a_4 = a_4 - q_1 \cdot q_1^T \cdot a_4 - q_2 \cdot q_2^T \cdot a_4 - q_3 \cdot q_3^T \cdot a_4$
7. $q_4 = \frac{q_4^{'}}{\left\|q_4^{'}\right\|_2}$
8. $R = \begin{pmatrix} q_1^T \cdot a_1 & q_1^T \cdot a_2 & q_1^T \cdot a_3 & q_1^T \cdot a_4 \\ 0 & q_2^T \cdot a_2 & q_2^T \cdot a_3 & q_2^T \cdot a_4 \\ 0 & 0 & q_3^T \cdot a_3 & q_3^T \cdot a_4 \\ 0 & 0 & 0 & q_4^T \cdot a_4 \end{pmatrix}$
9. $Q = \begin{pmatrix} | & | & | & | \\ q_1 & q_2 & q_3 & q_4 \\ | & | & | & | \\ | & | & | & | \end{pmatrix}$

10. $A = R \cdot Q$

Na próxima iteração, as matrizes-colunas de A (a_1, a_2, a_3, a_4) mudarão, de forma que cada matriz-coluna de A será igual a cada matriz-coluna de $R \cdot Q$.

Fez-se várias vezes as iterações até que na quinta iteração, os valores na diagonal principal da matriz $R \cdot Q$ ficarão iguais aos da quarta iteração.

1° Iteração:

```
a1 =  [[0.7140299]
 [0.6918437]
 [0.6748934]
 [0.7884872]]
a2 =  [[0.6918437]
 [1.2492256]
 [1.0307285]
 [0.6275049]]
a3 =  [[0.6748934]
 [1.0307285]
 [1.0893471]
 [0.5567456]]
a4 =  [[0.7884872]
 [0.6275049]
 [0.5567456]
 [0.9550583]]
q1 =  [[0.49680381]
 [0.48136722]
 [0.46957363]
 [0.54860931]]
q2' =  [[-0.18914089]
 [ 0.3956148 ]
 [ 0.19803132]
 [-0.34534662]]
q2 =  [[-0.31935716]
 [ 0.66798046]
 [ 0.33436832]
 [-0.58310456]]
q3' =  [[ 0.01965057]
 [-0.10515473]
 [ 0.14390639]
 [-0.0487033 ]]
q3 =  [[ 0.10575722]
 [-0.5659313 ]
 [ 0.77448851]
 [-0.26211586]]
q4' =  [[-1.71294927e-03]
 [-9.10805482e-05]
 [ 5.57797235e-04]
 [ 1.15367355e-03]]
q4 =  [[-0.80000774]
 [-0.04253783]
 [ 0.26051099]
 [ 0.53880625]]
```

```
R =  [[ 1.43724722  1.7733048   1.64841302  1.47917066]
 [ 0.          0.59225504  0.51257672 -0.20338878]
 [ 0.          0.          0.1858083  -0.0908793 ]
 [ 0.          0.          0.          0.00214117]]
Q =  [[ 0.49680381 -0.31935716  0.10575722 -0.80000774]
 [ 0.48136722  0.66798046 -0.5659313  -0.04253783]
 [ 0.46957363  0.33436832  0.77448851  0.26051099]
 [ 0.54860931 -0.58310456 -0.26211586  0.53880625]]
A = RQ =  [[ 3.15317879e+00  4.14203696e-01  3.73934457e-02  1.17466352e-03]
 [ 4.14203696e-01  6.85601138e-01  1.15120544e-01 -1.24852359e-03]
 [ 3.73934457e-02  1.15120544e-01  1.67727299e-01 -5.61233532e-04]
 [ 1.17466352e-03 -1.24852359e-03 -5.61233533e-04  1.15367355e-03]]
```

2° Iteração:

```
a1 =  [[3.15317879e+00]
 [4.14203696e-01]
 [3.73934457e-02]
 [1.17466352e-03]]
a2 =  [[ 0.4142037 ]
 [ 0.68560114]
 [ 0.11512054]
 [-0.00124852]]
a3 =  [[ 0.03739345]
 [ 0.11512054]
 [ 0.1677273 ]
 [-0.00056123]]
a4 =  [[ 0.00117466]
 [-0.00124852]
 [-0.00056123]
 [ 0.00115367]]
q1 =  [[9.91413667e-01]
 [1.30232769e-01]
 [1.17571427e-02]
 [3.69334423e-04]]
q2' =  [[-0.08278005]
 [ 0.62031702]
 [ 0.10922683]
 [-0.00143367]]
q2 =  [[-0.1303052 ]
 [ 0.97644952]
 [ 0.17193545]
 [-0.00225675]]
q3' =  [[ 0.00159135]
 [-0.02508144]
 [ 0.14364405]
 [-0.00027342]]
q3 =  [[ 0.01091262]
 [-0.17199536]
 [ 0.98503553]
 [-0.00187499]]
q4' =  [[-7.35338576e-07]
 [ 2.10683582e-06]
```

```
 [ 2.56381985e-06]
 [ 1.14937149e-03]]
q4 =  [[-6.39771708e-04]
 [ 1.83302494e-03]
 [ 2.23061792e-03]
 [ 9.99995628e-01]]
R =  [[ 3.18048751e+00  5.01287967e-01  5.40366269e-02  9.95806374e-04]
 [ 0.00000000e+00  6.35278124e-01  1.36376375e-01 -1.47128452e-03]
 [ 0.00000000e+00  0.00000000e+00  1.45826262e-01 -3.27439177e-04]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937651e-03]]
Q =  [[ 9.91413667e-01 -1.30305205e-01  1.09126206e-02 -6.39771708e-04]
 [ 1.30232769e-01  9.76449518e-01 -1.71995361e-01  1.83302494e-03]
 [ 1.17571427e-02  1.71935450e-01  9.85035530e-01  2.23061792e-03]
 [ 3.69334423e-04 -2.25675407e-03 -1.87499273e-03  9.99995628e-01]]
A = RQ =  [[ 3.21909859e+00  8.43368823e-02  1.71437924e-03  4.24504242e-07]
 [ 8.43368823e-02  6.43768272e-01  2.50734429e-02 -2.59386022e-06]
 [ 1.71437924e-03  2.50734429e-02  1.43644663e-01 -2.15507257e-06]
 [ 4.24504312e-07 -2.59386012e-06 -2.15507261e-06  1.14937149e-03]]
```

3° Iteração:

```
a1 =  [[3.21909859e+00]
 [8.43368823e-02]
 [1.71437924e-03]
 [4.24504312e-07]]
a2 =  [[ 8.43368823e-02]
 [ 6.43768272e-01]
 [ 2.50734429e-02]
 [-2.59386012e-06]]
a3 =  [[ 1.71437924e-03]
 [ 2.50734429e-02]
 [ 1.43644663e-01]
 [-2.15507261e-06]]
a4 =  [[ 4.24504242e-07]
 [-2.59386022e-06]
 [-2.15507257e-06]
 [ 1.14937149e-03]]
q1 =  [[9.99656843e-01]
 [2.61899221e-02]
 [5.32382246e-04]
 [1.31825301e-07]]
q2' =  [[-1.68099277e-02]
 [ 6.41118335e-01]
 [ 2.50195756e-02]
 [-2.60719841e-06]]
q2 =  [[-2.61907655e-02]
 [ 9.98896621e-01]
 [ 3.89818356e-02]
 [-4.06215442e-06]]
q3' =  [[ 6.97307379e-05]
 [-5.55728747e-03]
 [ 1.42450500e-01]
 [-2.03109159e-06]]
```

```
q3 =  [[ 4.89136416e-04]
 [-3.89824023e-02]
 [ 9.99239777e-01]
 [-1.42473878e-05]]
q4' =  [[-2.65737072e-10]
 [ 4.02144661e-09]
 [ 1.65450142e-08]
 [ 1.14937145e-03]]
q4 =  [[-2.31202082e-07]
 [ 3.49882244e-06]
 [ 1.43948366e-05]
 [ 1.00000000e+00]]
R =  [[ 3.22020363e+00  1.01181531e-01  2.44693632e-03  3.55429767e-07]
 [ 0.00000000e+00  6.41826513e-01  3.06004091e-02 -2.69079391e-06]
 [ 0.00000000e+00  0.00000000e+00  1.42558877e-01 -2.06848723e-06]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
Q =  [[ 9.99656843e-01 -2.61907655e-02  4.89136416e-04 -2.31202082e-07]
 [ 2.61899221e-02  9.98896621e-01 -3.89824023e-02  3.49882244e-06]
 [ 5.32382246e-04  3.89818356e-02  9.99239777e-01  1.43948366e-05]
 [ 1.31825301e-07 -4.06215442e-06 -1.42473878e-05  1.00000000e+00]]
A = RQ =  [[ 3.22174983e+00  1.68256775e-02  7.58958146e-05  1.51443706e-10]
 [ 1.68256775e-02  6.42311196e-01  5.55720670e-03 -4.66901591e-09]
 [ 7.58958146e-05  5.55720670e-03  1.42450500e-01 -1.63754932e-08]
 [ 1.51516237e-10 -4.66892431e-09 -1.63755407e-08  1.14937145e-03]]
```

4° Iteração:

```
a1 =  [[3.22174983e+00]
 [1.68256775e-02]
 [7.58958146e-05]
 [1.51516237e-10]]
a2 =  [[ 1.68256775e-02]
 [ 6.42311196e-01]
 [ 5.55720670e-03]
 [-4.66892431e-09]]
a3 =  [[ 7.58958146e-05]
 [ 5.55720670e-03]
 [ 1.42450500e-01]
 [-1.63755407e-08]]
a4 =  [[ 1.51443706e-10]
 [-4.66901591e-09]
 [-1.63754932e-08]
 [ 1.14937145e-03]]
q1 =  [[9.99986363e-01]
 [5.22245641e-03]
 [2.35570058e-05]
 [4.70285337e-11]]
q2' =  [[-3.35406847e-03]
 [ 6.42205806e-01]
 [ 5.55673132e-03]
 [-4.66987335e-09]]
q2 =  [[-5.22246474e-03]
 [ 9.99948932e-01]
```

```
 [ 8.65212910e-03]
 [-7.27124362e-09]]
q3' =  [[ 3.07998696e-06]
 [-1.23203871e-03]
 [ 1.42391758e-01]
 [-1.63261811e-08]]
q3 =  [[ 2.16295642e-05]
 [-8.65213413e-03]
 [ 9.99962569e-01]
 [-1.14652493e-07]]
q4' =  [[-9.48482264e-14]
 [ 7.21648718e-12]
 [ 1.31845677e-10]
 [ 1.14937145e-03]]
q4 =  [[-8.25218223e-11]
 [ 6.27863794e-09]
 [ 1.14711112e-07]
 [ 1.00000000e+00]]
R =  [[ 3.22179377e+00  2.01800212e-02  1.08272756e-04  1.26726204e-10]
 [ 0.00000000e+00  6.42238604e-01  6.78902666e-03 -4.81960862e-09]
 [ 0.00000000e+00  0.00000000e+00  1.42397088e-01 -1.64662583e-08]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
Q =  [[ 9.99986363e-01 -5.22246474e-03  2.16295642e-05 -8.25218223e-11]
 [ 5.22245641e-03  9.99948932e-01 -8.65213413e-03  6.27863794e-09]
 [ 2.35570058e-05  8.65212910e-03  9.99962569e-01  1.14711112e-07]
 [ 4.70285337e-11 -7.27124362e-09 -1.14652493e-07  1.00000000e+00]]
A = RQ =  [[ 3.22185522e+00  3.35422304e-03  3.35444903e-06 -1.89539327e-14]
 [ 3.35422304e-03  6.42264546e-01  1.23203799e-03 -8.44816195e-12]
 [ 3.35444903e-06  1.23203799e-03  1.42391758e-01 -1.31730023e-10]
 [ 5.40532539e-14 -8.35735981e-12 -1.31778302e-10  1.14937145e-03]]
```

5° Iteração:

```
a1 =  [[3.22185522e+00]
 [3.35422304e-03]
 [3.35444903e-06]
 [5.40532539e-14]]
a2 =  [[ 3.35422304e-03]
 [ 6.42264546e-01]
 [ 1.23203799e-03]
 [-8.35735981e-12]]
a3 =  [[ 3.35444903e-06]
 [ 1.23203799e-03]
 [ 1.42391758e-01]
 [-1.31778302e-10]]
a4 =  [[-1.89539327e-14]
 [-8.44816195e-12]
 [-1.31730023e-10]
 [ 1.14937145e-03]]
q1 =  [[9.99999458e-01]
 [1.04108378e-03]
 [1.04115392e-06]
 [1.67770495e-14]]
```

```
q2' =  [[-6.68648487e-04]
 [ 6.42260358e-01]
 [ 1.23203380e-03]
 [-8.35742730e-12]]
q2 =  [[-1.04108386e-03]
 [ 9.99997618e-01]
 [ 1.91827325e-03]
 [-1.30124914e-11]]
q3' =  [[ 1.36114290e-07]
 [-2.73141271e-04]
 [ 1.42388871e-01]
 [-1.31758716e-10]]
q3 =  [[ 9.55931730e-07]
 [-1.91827330e-03]
 [ 9.99998160e-01]
 [-9.25342499e-10]]
q4' =  [[-3.38370023e-17]
 [ 1.29159273e-14]
 [ 1.06358898e-12]
 [ 1.14937145e-03]]
q4 =  [[-2.94395710e-14]
 [ 1.12373832e-11]
 [ 9.25365758e-10]
 [ 1.00000000e+00]]
R =  [[ 3.22185697e+00  4.02287371e-03  4.78535372e-06 -2.78670350e-14]
 [ 0.00000000e+00  6.42261887e-01  1.50517786e-03 -8.71577246e-12]
 [ 0.00000000e+00  0.00000000e+00  1.42389133e-01 -1.32777137e-10]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
Q =  [[ 9.99999458e-01 -1.04108386e-03  9.55931730e-07 -2.94395710e-14]
 [ 1.04108378e-03  9.99997618e-01 -1.91827330e-03  1.12373832e-11]
 [ 1.04115392e-06  1.91827325e-03  9.99998160e-01  9.25365758e-10]
 [ 1.67770495e-14 -1.30124914e-11 -9.25342499e-10  1.00000000e+00]]
A = RQ =  [[ 3.22185941e+00  6.68650002e-04  1.48249003e-07 -7.30823464e-14]
 [ 6.68650002e-04  6.42263245e-01  2.73141265e-04 -1.05589498e-13]
 [ 1.48249004e-07  2.73141265e-04  1.42388871e-01 -1.01510922e-12]
 [ 1.92830617e-17 -1.49561861e-14 -1.06356225e-12  1.14937145e-03]]
```

Somente na 5° Iteração os valores da diagonal principal de $R \cdot Q$ são iguais aos da 4° Iteração com aproximação de 4 casas decimais e a matriz $R \cdot Q$ se tornou próxima a uma matriz diagonal, uma vez que todos os valores abaixo e acima da diagonal principal são próximos de zero em 4 casas decimais. Como esse procedimento triangulariza a matriz. Se ela é simétrica, acaba por diagonalizá-la. De forma que os autovalores da matriz A são revelados, arredondando em 4 casas decimais: **3.2219, 0.6423, 0.1424, 0.0011**.

Se fizermos mais duas iterações, todos os valores abaixo e acima da diagonal principal realmente terão se tornado zeros, considerando as 4 primeiras casas decimais. Além disso, os valores da diagonal principal continuarão constantes iguais aos da 5° Iteração na aproximação de 4 casas decimais.

6° Iteração:

```
a1 =  [[3.22185941e+00]
 [6.68650002e-04]
 [1.48249004e-07]
 [1.92830617e-17]]
```

```
a2 =  [[ 6.68650002e-04]
 [ 6.42263245e-01]
 [ 2.73141265e-04]
 [-1.49561861e-14]]
a3 =  [[ 1.48249003e-07]
 [ 2.73141265e-04]
 [ 1.42388871e-01]
 [-1.06356225e-12]]
a4 =  [[-7.30823464e-14]
 [-1.05589498e-13]
 [-1.01510922e-12]
 [ 1.14937145e-03]]
q1 =  [[9.99999978e-01]
 [2.07535433e-04]
 [4.60134916e-08]
 [5.98507222e-18]]
q2' =  [[-1.33292361e-04]
 [ 6.42263079e-01]
 [ 2.73141228e-04]
 [-1.49561909e-14]]
q2 =  [[-2.07535434e-04]
 [ 9.99999888e-01]
 [ 4.25279308e-04]
 [-2.32867025e-14]]
q3' =  [[ 6.01550268e-09]
 [-6.05549855e-05]
 [ 1.42388729e-01]
 [-1.06355448e-12]]
q3 =  [[ 4.22470387e-08]
 [-4.25279308e-04]
 [ 9.99999910e-01]
 [-7.46937198e-12]]
q4' =  [[-1.20710772e-20]
 [ 2.31140084e-17]
 [ 8.58509350e-15]
 [ 1.14937145e-03]]
q4 =  [[-1.05023291e-17]
 [ 2.01101293e-14]
 [ 7.46938121e-12]
 [ 1.00000000e+00]]
R =  [[ 3.22185948e+00  8.01942381e-04  2.11487299e-07 -7.31042982e-14]
 [ 0.00000000e+00  6.42263150e-01  3.33696244e-04 -1.06032789e-13]
 [ 0.00000000e+00  0.00000000e+00  1.42388742e-01 -1.02364931e-12]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
Q =  [[ 9.99999978e-01 -2.07535434e-04  4.22470387e-08 -1.05023291e-17]
 [ 2.07535433e-04  9.99999888e-01 -4.25279308e-04  2.01101293e-14]
 [ 4.60134916e-08  4.25279308e-04  9.99999910e-01  7.46938121e-12]
 [ 5.98507222e-18 -2.32867025e-14 -7.46937198e-12  1.00000000e+00]]
A = RQ =  [[ 3.22185958e+00  1.33292376e-04  6.55180154e-09 -7.31204284e-14]
 [ 1.33292376e-04  6.42263220e-01  6.05549855e-05 -9.06242897e-14]
 [ 6.55180317e-09  6.05549855e-05  1.42388729e-01  3.99064838e-14]
 [ 6.87907112e-21 -2.67650710e-17 -8.58508289e-15  1.14937145e-03]]
```

7° Iteração:

```
a1 =  [[3.22185958e+00]
 [1.33292376e-04]
 [6.55180317e-09]
 [6.87907112e-21]]
a2 =  [[ 1.33292376e-04]
 [ 6.42263220e-01]
 [ 6.05549855e-05]
 [-2.67650710e-17]]
a3 =  [[ 6.55180154e-09]
 [ 6.05549855e-05]
 [ 1.42388729e-01]
 [-8.58508289e-15]]
a4 =  [[-7.31204284e-14]
 [-9.06242897e-14]
 [ 3.99064838e-14]
 [ 1.14937145e-03]]
q1 =  [[9.99999999e-01]
 [4.13712556e-05]
 [2.03354709e-09]
 [2.13512444e-21]]
q2' =  [[-2.65712357e-05]
 [ 6.42263214e-01]
 [ 6.05549851e-05]
 [-2.67650713e-17]]
q2 =  [[-4.13712556e-05]
 [ 9.99999995e-01]
 [ 9.42837508e-05]
 [-4.16730564e-17]]
q3' =  [[ 2.65852571e-10]
 [-1.34249428e-05]
 [ 1.42388722e-01]
 [-8.58507981e-15]]
q3 =  [[ 1.86709008e-09]
 [-9.42837508e-05]
 [ 9.99999996e-01]
 [-6.02932568e-14]]
q4' =  [[-4.30626773e-24]
 [ 4.13640184e-20]
 [ 6.92993521e-17]
 [ 1.14937145e-03]]
q4 =  [[-3.74662842e-21]
 [ 3.59883817e-17]
 [ 6.02932605e-14]
 [ 1.00000000e+00]]
R =  [[ 3.22185958e+00  1.59863612e-04  9.34659149e-09 -7.31241775e-14]
 [ 0.00000000e+00  6.42263217e-01  7.39799283e-05 -9.06175495e-14]
 [ 0.00000000e+00  0.00000000e+00  1.42388722e-01  3.98457285e-14]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.14937145e-03]]
Q =  [[ 9.99999999e-01 -4.13712556e-05  1.86709008e-09 -3.74662842e-21]
 [ 4.13712556e-05  9.99999995e-01 -9.42837508e-05  3.59883817e-17]
```

```
 [ 2.03354709e-09  9.42837508e-05  9.99999996e-01  6.02932605e-14]
 [ 2.13512444e-21 -4.16730564e-17 -6.02932568e-14  1.00000000e+00]]
A = RQ =  [[ 3.22185959e+00  2.65712359e-05  2.89552537e-10 -7.31241832e-14]
 [ 2.65712359e-05  6.42263221e-01  1.34249428e-05 -9.05899750e-14]
 [ 2.89554172e-10  1.34249428e-05  1.42388722e-01  4.84308088e-14]
 [ 2.45405106e-24 -4.78978212e-20 -6.92993479e-17  1.14937145e-03]]
```