STA461: Randomized Complete Block Designs

Lynn Lin

November 9, 2018

Introduction

- In a randomized complete block (RCB) design, experimental units are grouped into "blocks" that are thought to be similar
- The random assignment of units to treatments is done separately within each block. The rationale for doing this is that, in the resulting dataset, the proportion of units receiving each treatment is identical across blocks
- If the blocking factor is related to the outcome, then blocking can substantially increase the precision of treatment comparisons over a completely-randomized (CR) design

RCB

- The RCB assumes that a population of experimental units can be divided into a number of relatively homogeneous subpopulations or blocks
- The treatments are then randomly assigned to experimental units such that each treatment occurs equally often (usually once) in each block (i.e. each block contains all treatments)
- Blocks usually represent levels of naturally-occurring differences or sources of variation that are unrelated to the treatments, and the characterization of these differences is not of interest to the researcher
    - Nuisance factor: A factor that probably has an effect on the response, but is not a factor that we are interested in

Nuisance Factor

- When present in experiment, nuisance factor has effect on response but its effect is not of interest
- Nuisance factor can come in 3 types
    - Noise factors
    - Blocking factors
    - Covariates

Nuisance Factor

- Noise factors are factors that randomly affect the response, without any systematic pattern to the effect, and are not measured
    - Example: Corn yield may be influenced by small insects, which may affect some corn plots more than others
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    - Example: Some plots may get more rain than other plots. We can't control the rain, but we can measure it with a rain gauge placed in each experimental corn plot
    - Analysis of Covariance

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- Blocking factors are factors for which we can assign levels, but we do not want to interpret the results
    - Example

Randomized Complete Block Design

- b blocks each consisting of (partitioned into) a experimental units
- a treatments are randomly assigned to the experimental units within each block
- Typically after the runs in one block have been conducted, then move to another block
- Results in restriction on randomization because randomization is only within blocks
- Data within a block are dependent on each other (sharing the same block effect)

Statistical Model

Checking Assumptions (Diagnostics)

- Assumptions
  - Model is additive (no interaction between treatment effects and block effects, i.e., additivity assumption)
  - Errors are independent and normally distributed
  - Constant variance
- Checking normality: Histogram, QQ plot of residuals
- Checking constant variance
  - Residual Plot: Residuals vs. $\hat{y}_{ij}$
  - Residuals vs blocks
  - Residuals vs treatments

Checking Assumptions

- Additivity
  - Interaction Plot: $\hat{y}_{ij}$ vs. treatment levels for each block
    - Parallel lines: block effects are same for different treatments, so NO interaction
    - Lines are not parallel: block effects can be different for different treatments, so possible interaction between block and treatment
- If interaction exists, usually try transformation to eliminate interaction

Treatments Comparison

- Multiple Comparisons/Contrasts
  - Procedures (methods) are similar to those for Completely Randomized Design (CRD)
    - r is replaced by b in all formulas
    - Degrees of freedom error is $(b-1)(a-1)$