Assignment 9

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1.Brainstorming.

A researcher investigated whether brainstorming is more effective for larger groups than for smaller ones by setting up four groups of agribusiness executives, the group sizes being two, three, four, and five, respectively. He also set up four groups of agribusiness scientists, the group sizes being the same as for the agribusiness executives. The researcher gave each group the same problem: "How can Canada increase the value of its agricultural exports?" Each group was allowed 30 minutes to generate ideas. The variable of interest was the number of different ideas proposed by the group. The results, classified by type of group (factor A) and size of group (factor B), were:

Table 1:

	B_1 (Two)	B_2 (Three)	B_3 (Four)	B_4 (Five)
A_1 (Agribusiness executives)	18	22	31	32
A_2 (Agribusiness scientists)	15	23	29	33

Assume that no-interaction ANOVA model is appropriate.

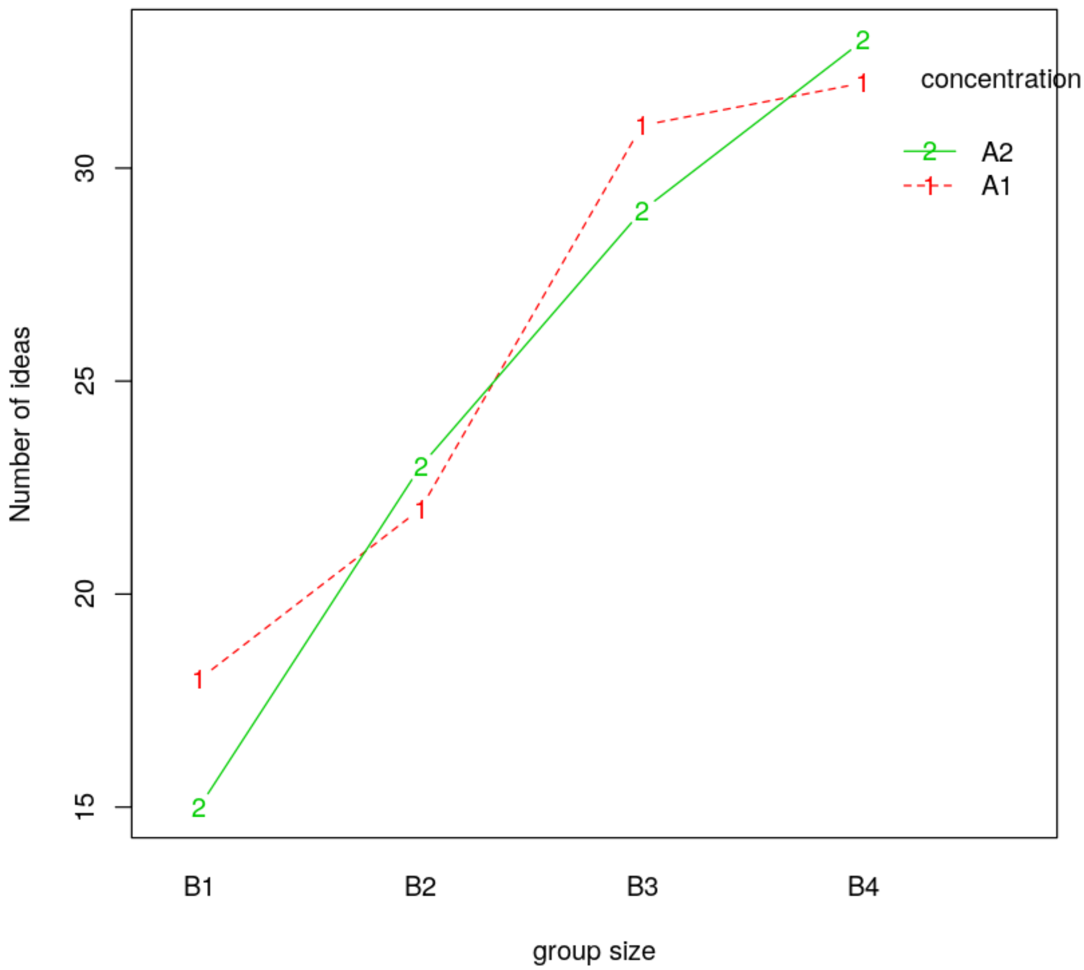
Q1

1. 20 points Plot the data in an interaction plot. Does it appear that interaction effects are present? Does it appear that factor A and factor B main effects are present? Discuss

```
In [6]: A = c(rep("A1", 4), rep("A2", 4))
        B = rep(c('B1', 'B2', 'B3', 'B4'), 2)
        num_ideas = c(18, 22, 31, 32, 15, 23, 29, 33)
        df = data.frame(A = A, B=B, num_ideas=num_ideas)
        df
```

A	B	num_ideas
A1	B1	18
A1	B2	22
A1	B3	31
A1	B4	32
A2	B1	15
A2	B2	23
A2	B3	29
A2	B4	33

```
In [11]: interaction.plot(x.factor = df$B, trace.factor = df$A,
                          response = df$num_ideas, type ="b",col = 2:3,
                          xlab ="group size", ylab ="Number of ideas",
                          trace.label ="concentration")
```



The interaction plot shows a weak interaction, with the lines for concentration=A1 and concentration=A2 being nearly parallel.

Factor A main effects are not present because with with factor A on the Y axis, the the lines for concentration A1 and A2 are almost overlapping each other, indicating that for each one of factor B, there's no significant difference between A1 and A2.

Factor B main effects are present because with factor B on the X axis, the lines for concentration A1 and A2 are not horizontal, the absolute value of slope is relatively large, indicating that for each one of factor A, there's significant difference among B1 to B4, thus we could conclude that factor A is present.

Q2

2. 40 points (By hand) Conduct separate tests for type of group and size of group main effects. In each test, use level of significance α = 0.01 and state the alternatives, decision rule, and conclusion. What is the P-value for each test?

$$\mu_{..} = \frac{18+15+22+23+31+29+32+33}{8} = 25.375$$

$$\mu_{A1.} = \frac{18+22+31+32}{4} = 25.75$$

$$\mu_{A2.} = \frac{15+23+29+33}{4} = 25$$

$$\mu_{B1.} = \frac{18+15}{2} = 16.5$$

$$\mu_{B2.} = \frac{22+23}{2} = 22.5$$

$$\mu_{B3.} = \frac{31+29}{2} = 30$$

$$\mu_{B4.} = \frac{32+33}{2} = 32.5$$

$$SS_A = 4 \cdot (25.75 - 25.375)^2 + 4 \cdot (25 - 25.375)^2 = 1.125$$

$$SS_B = 2 \cdot (16.5 - 25.375)^2 + 2 \cdot (22.5 - 25.375)^2 + 2 \cdot (30 - 25.375)^2 + 2 \cdot (32.5 - 25.375)^2 = 318.375$$

$$MS_A = \frac{SS_A}{Df} = 1.125$$

$$MS_B = \frac{SS_B}{Df} = \frac{318.375}{3} = 106.125$$

$$SS_{total} = (18 - 25.375)^2 + (22 - 25.375)^2 + (31 - 25.375)^2 + (32 - 25.375)^2 + (15 - 25.375)^2$$

$$+(23 - 25.375)^2 + (29 - 25.375)^2 + (33 - 25.375)^2 = 325.875$$

$$SSE = SS_{total} - SS_A - SS_B = 325.875 - 1.125 - 106.125 = 6.375$$

$$MSE = \frac{SSE}{Df} = \frac{6.375}{3} = 2.125$$

$$F_A = \frac{MS_A}{MSE} = \frac{1.125}{2.125} = 0.529$$

$$F_B = \frac{MS_B}{MSE} = \frac{106.125}{2.125} = 49.941$$

Two-way ANOVA table :

	Df	SS	MS	F value
A	1	1.125	1.125	0.529
B	3	318.375	106.125	49.941
Residuals	3	6.375	2.125	/

In [12]:
```
# check my answer:
modelAB<-aov(num_ideas~A+B, data=df)
anova(modelAB)
```

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
A	1	1.125	1.125	0.5294118	0.519497962
B	3	318.375	106.125	49.9411765	0.004641637
Residuals	3	6.375	2.125	NA	NA

For factor A (type of group):
Null hypothesis:

$$H_0 : \alpha_1 = \alpha_2 = 0$$

Alternative:

$$H_1 : \text{ there exists at least one in } \alpha_1, \alpha_2 \text{ that's not 0}$$

since the p value $p = 0.519 > 0.01$, so we are not confident enough to reject the null hypothesis, thus we could make the conclusion that there are no significant difference between factor α_1 and α_2. It is consistent with the result we got from the interaction plot above that the factor A main effects are not present.

For factor B (size of group):
Null hypothesis:

$$H_0 : \beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$$

Alternative:

$$H_1 : \text{ there exists at least one in } \beta_1, \beta_2, \beta_3, \beta_4 \text{ that's not 0}$$

since the p value $p = 0.0046 < 0.01$, so we are confident enough to reject the null hypothesis, thus we could make the conclusion that there are significant difference among factor β_1 to β_4. It is consistent with the result we got from the interaction plot above that the factor B main effects are present.

Q3

3. 40 points (By hand) Obtain confidence intervals for $D_1 = \mu_2 - \mu_1, D_2 = \mu_3 - \mu_2, D_3 = \mu_4 - \mu_3$; use the Bonferroni procedure with a 95 percent family confidence coefficient. State your findings.

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In [42]: library(multcomp)
         summary(glht(aov(num_ideas~A+B, data = df), linfct = mcp(B = 'Tukey')))
```

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 Simultaneous Tests for General Linear Hypotheses

Multiple Comparisons of Means: Tukey Contrasts


Fit: aov(formula = num_ideas ~ A + B, data = df)

Linear Hypotheses:
             Estimate Std. Error t value Pr(>|t|)
B2 - B1 == 0    6.000      1.458   4.116  0.07553 .
B3 - B1 == 0   13.500      1.458   9.261  0.00802 **
B4 - B1 == 0   16.000      1.458  10.976  0.00494 **
B3 - B2 == 0    7.500      1.458   5.145  0.04217 *
B4 - B2 == 0   10.000      1.458   6.860  0.01905 *
B4 - B3 == 0    2.500      1.458   1.715  0.44723
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
(Adjusted p values reported -- single-step method)
```

$$\hat{D}_1 = \mu_2 - \mu_1 = \frac{22+23}{2} - \frac{18+15}{2} = 6$$

$$\hat{D}_2 = \mu_3 - \mu_2 = \frac{31+29}{2} - \frac{22+23}{2} = 7.5$$

$$\hat{D}_3 = \mu_4 - \mu_3 = \frac{32+33}{2} - \frac{31+29}{2} = 2.5$$

$$S(\hat{D}_1) = S(\hat{D}_2) = S(\hat{D}_3) = \sqrt{MSE \cdot (\frac{1}{2} + \frac{1}{2})} = 1.458$$

$$B = t(1 - \alpha/(2g);\, df_A \cdot df_B) = t(1 - 0.05/6; 3) = t(0.99167; 3)$$

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In [45]: qt(0.99167, 3)
```

4.8573697522534

$$\therefore B = 4.857$$

Thus, using the Bonferroni procedure, the 95% family-wise CI for D_1, D_2 and D_3 is:

$$D_1 : 6 \pm 4.857 \cdot 1.458 = [-1.08, 13.08]$$

$$D_2 : 7.5 \pm 4.857 \cdot 1.458 = [0.42, 14.58]$$

$$D_3 : 2.5 \pm 4.857 \cdot 1.458 = [-4.58, 9.58]$$

From the CI above, since 0 is included in the CI of D_1, D_3, 0 not included in the CI of D_2, thus we can conclude that there's significant difference between μ_3 and μ_2, with μ_3 significantly larger than μ_2; there not enough confidence to guarantee that there exists significant difference between μ_2 and μ_1, μ_4 and μ_3.