Stat 461 Spring 2015 Homework #8

Due Monday, April 6

You must show all of your work in order to receive full and/or partial credit. No work = No Credit.
Post any questions you may have on Piazza.

1. **60 points A consumer organization studied the effect of age and sex of automobile owner on size of cash offer for a used car by utilizing 12 persons in each of three age groups (young, middle, elderly) who acted as the owner of a used car. Six male and six female volunteers were used in each age group.**

   The observations (in hundred dollars), classified by age (factor A) and sex of owner (factor B), was used to obtain the following cell means and ANOVA table.

$$\bar{Y}_{11.} = 21.67, \bar{Y}_{12.} = 21.33, \bar{Y}_{21.} = 27.83,$$
$$\bar{Y}_{22.} = 27.67, \bar{Y}_{31.} = 22.33, \bar{Y}_{32.} = 20.50$$

Source	SS	df	MS
Treatment	327.22	5	65.444
A (age)	316.722	2	158.361
B (sex)	5.444	1	5.444
AB interactions	5.056	2	2.528
Error	71.667	30	2.389
Total	398.889	35	

   (a) **Estimate μ_{11} with a 95 percent confidence interval. Interpret your interval estimate.**

   $\bar{Y}_{11.} \pm t_{0.025,30}\sqrt{\frac{MSE}{n}} = 21.67 \pm 2.042\sqrt{\frac{2.389}{6}} = (20.38, 22.96)$

   We are 95% confident that the true mean of group 11 (young and male) is between 20.38 and 22.96.

(b) **Estimate $D = \mu_{.1} - \mu_{.2}$ by means of a 95 percent confidence interval.**
$(\bar{Y}_{.1} - \bar{Y}_{.2}) \pm t_{0.025,30}\sqrt{\frac{MSE}{3n}(1+1)} = (23.94 - 23.17) \pm 2.042\sqrt{\frac{2.389\times 2}{3\times 6}}$
$= (-0.28, 1.82)$

(c) **Obtain all pairwise comparisons among the factor A level means; use the Tukey procedure with a 90 percent family confidence coefficient.**
$(\bar{Y}_{1.} - \bar{Y}_{2.}) \pm \frac{1}{\sqrt{2}}q[0.9,3,30]\sqrt{\frac{2MSE}{2n}} = (21.5 - 27.75) \pm \frac{1}{\sqrt{2}}3.017\sqrt{\frac{2.389}{6}}$
$= (-7.60, -4.90)$
$(\bar{Y}_{2.} - \bar{Y}_{3.}) \pm \frac{1}{\sqrt{2}}q[0.9,3,30]\sqrt{\frac{2MSE}{2n}} = (27.75 - 21.415) \pm \frac{1}{\sqrt{2}}3.017\sqrt{\frac{2.389}{6}}$
$= (4.99, 7.68)$
$(\bar{Y}_{3.} - \bar{Y}_{1.}) \pm \frac{1}{\sqrt{2}}q[0.9,3,30]\sqrt{\frac{2MSE}{2n}} = (21.415 - 21.5) \pm \frac{1}{\sqrt{2}}3.017\sqrt{\frac{2.389}{6}}$
$= (-1.43, 1.26)$
Based on 90% confidence interval, we can conclude that $\mu_{1.}$ and $\mu_{3.}$ are equal, while $\mu_{2.}$ is different from the other two.

(d) **Estimate the contrast:**

$$L = \frac{\mu_{1.} + \mu_{3.}}{2} - \mu_{2.}$$

with a 95 percent confidence interval. Interpret your interval estimate.
$(\frac{\bar{Y}_{1.}+\bar{Y}_{3.}}{2} - \bar{Y}_{2.}) \pm t_{0.025,30}\sqrt{\frac{MSE}{2n}(\frac{1}{4}+\frac{1}{4}+1)} = (\frac{21.5+21.415}{2} - 27.75) \pm 2.042\sqrt{\frac{2.389\times 3}{2\times 6\times 2}}$
$= (-7.41, -5.18)$
Since the interval does not include 0, we can conclude that the true average of young and elderly groups($\frac{\mu_{1.}+\mu_{3.}}{2}$) is significantly different from the true mean of middle group($\mu_{2.}$).

(e) **Suppose that in the population of female owners, 30 percent are young, 60 percent are middle-aged, and 10 percent are elderly. Obtain a 95 percent confidence interval for the mean cash offer in the population of female owners.**
$L = \frac{3}{10}\mu_{12} + \frac{6}{10}\mu_{22} + \frac{1}{10}\mu_{32}$
$(\frac{3}{10}\bar{Y}_{12.} + \frac{6}{10}\bar{Y}_{22.} + \frac{1}{10}\bar{Y}_{32.}) \pm t_{0.025,30}\sqrt{\frac{MSE}{n}(\frac{9}{100} + \frac{36}{100} + \frac{1}{100})}$
$= 25.051 \pm 2.042\sqrt{\frac{2.389\times 46}{6\times 100}}$
$= (24.18, 25.92)$

2. 40 points **A research laboratory was developing a new compound for the relief of severe cases of hay fever. In an experiment with 36 volunteers, the amounts of the two active ingredients (factors A and B) in the compound were varied at three levels each. Randomization was used in assigning four volunteers to each of the nine treatments. The summarized data on hours relief follow.**

Factor A (ingredient 1)	Factor B (ingredient 2)	Sample mean	Sample variance
Low $(i = 1)$	Low $(j = 1)$	2.475	7.588
Low $(i = 1)$	Medium $(j = 2)$	4.600	7.519
Low $(i = 1)$	High $(j = 3)$	4.575	7.106
Medium $(i = 2)$	Low $(j = 1)$	5.450	5.232
Medium $(i = 2)$	Medium $(j = 2)$	8.925	6.360
Medium $(i = 2)$	High $(j = 3)$	9.125	5.464
High $(i = 3)$	Low $(j = 1)$	5.975	5.446
High $(i = 3)$	Medium $(j = 2)$	10.275	7.708
High $(i = 3)$	High $(j = 3)$	13.250	8.950

In this design, we have a=3, b=3, and n=4.
Then, MSE=SSE/(ab(n-1))=3*sum(sample variance)/27=6.819

(a) **Estimate μ_{23} with a 95 percent confidence interval. Interpret your interval estimate.**

$\bar{Y}_{23.} \pm t_{0.025,27}\sqrt{\frac{MSE}{n}} = 9.125 \pm 2.052\sqrt{\frac{6.819}{4}} = (6.45, 11.81)$

We are 95% confident that the true mean of group 23(medium A and high B) is between 6.45 and 11.81.

(b) **Estimate $D = \mu_{12} - \mu_{11}$ with a 95 percent confidence interval. Interpret your interval estimate.**

$(\bar{Y}_{12} - \bar{Y}_{11}) \pm t_{0.025,27}\sqrt{\frac{MSE}{n}(1+1)} = (4.600 - 2.475) \pm 2.052\sqrt{\frac{6.819\times 2}{4}}$
$= (-1.66, 5.91)$

Based on 95% confidence interval, we can conclude that there is no significant difference between μ_{12} and μ_{11}, since the interval includes 0.

(c) The analyst is interested in the following contrasts:

$$L_1 = \frac{\mu_{1.} + \mu_{3.}}{2} - \mu_{2.}$$
$$L_2 = \frac{\mu_{1.} + \mu_{2.}}{2} - \mu_{3.}$$
$$L_3 = \frac{\mu_{2.} + \mu_{3.}}{2} - \mu_{1.}$$

Obtain confidence intervals for these contrasts; use the Scheffe multiple comparison procedure with a 90 percent family confidence coefficient. Interpret your findings.

$$\left(\frac{\bar{Y}_{1.}+\bar{Y}_{3.}}{2} - \bar{Y}_{2.}\right) \pm \sqrt{(3-1)F_{0.9,2,27}}\sqrt{\frac{MSE}{3n}\left(\frac{1}{4}+\frac{1}{4}+1\right)}$$
$$= \left(\frac{3.883+9.833}{2} - 7.833\right) \pm \sqrt{2 \times 2.51}\sqrt{\frac{6.819\times 3}{3\times 4\times 2}}$$
$$= (-3.07, 1.07)$$

$$\left(\frac{\bar{Y}_{1.}+\bar{Y}_{2.}}{2} - \bar{Y}_{3.}\right) \pm \sqrt{(3-1)F_{0.9,2,27}}\sqrt{\frac{MSE}{3n}\left(\frac{1}{4}+\frac{1}{4}+1\right)}$$
$$= \left(\frac{3.883+7.833}{2} - 9.833\right) \pm \sqrt{2 \times 2.51}\sqrt{\frac{6.819\times 3}{3\times 4\times 2}}$$
$$= (-6.07, -1.93)$$

$$\left(\frac{\bar{Y}_{2.}+\bar{Y}_{3.}}{2} - \bar{Y}_{1.}\right) \pm \sqrt{(3-1)F_{0.9,2,27}}\sqrt{\frac{MSE}{3n}\left(\frac{1}{4}+\frac{1}{4}+1\right)}$$
$$= \left(\frac{7.833+9.833}{2} - 3.883\right) \pm \sqrt{2 \times 2.51}\sqrt{\frac{6.819\times 3}{3\times 4\times 2}}$$
$$= (2.93, 7.07)$$

Based on the three Scheffe intervals, we can conclude that we cannot reject the first one, $H0: L_1 = 0$, since the interval includes 0. However, we can reject the rest, $H0: L_2 = 0$ and $H0: L_3 = 0$, since the intervals do not include 0.