Generalized Stereo:

Epipolar Geometry

Generalized Stereo

Key idea: Any two images showing an overlapping view of the world can be treated as a stereo pair...

... we just have to figure out how the two views are related.

Some of the most "beautiful" math in vision concerns describing how multiple views are related, geometrically.

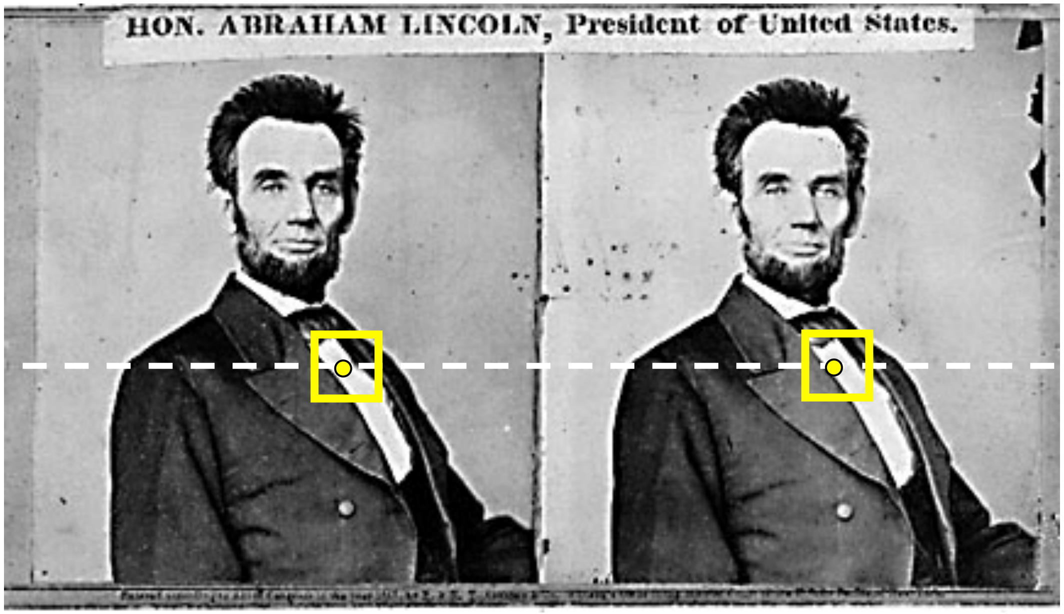
Recall: Epipolar Constraint

Important Stereo Vision Concept:

Given a point in the left image, we don’t have to search the whole right image for a corresponding point.

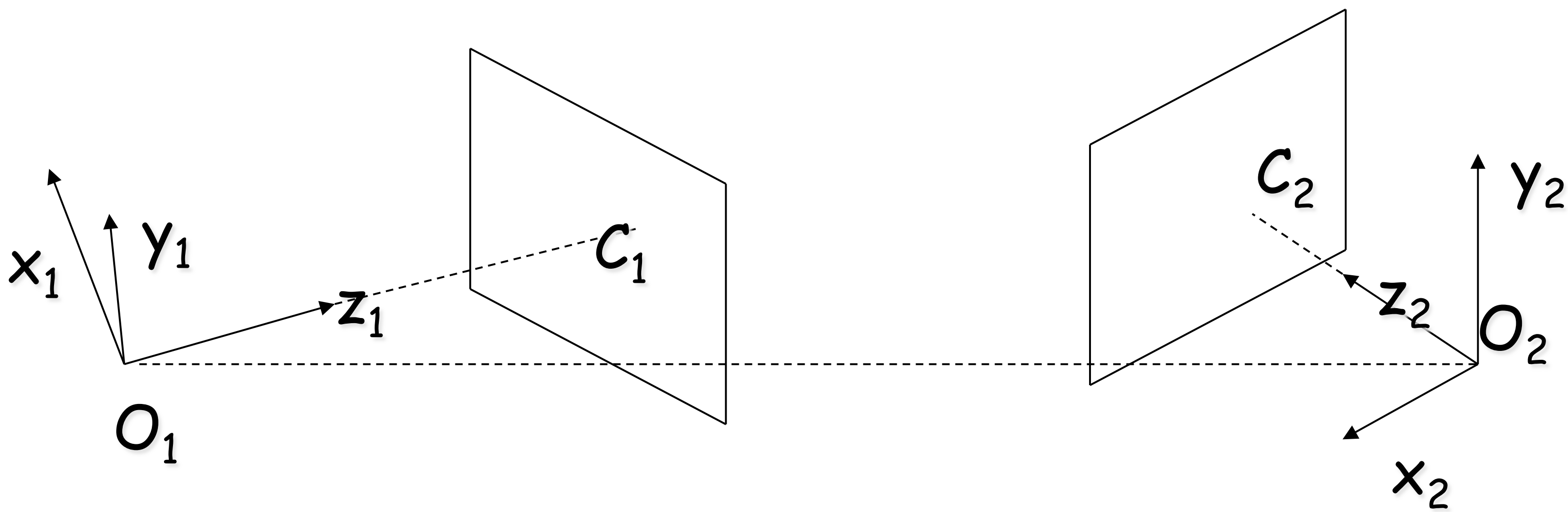
The “epipolar constraint” reduces the search space to a one-dimensional line.

Review: Epipolar Constraint



Corresponding features are constrained to lie along conjugate epipolar lines (on the same row in the case of our simple setup).

General Stereo



In general, the cameras may be related by an arbitrary transformation (R,T)

Epipolar Matrix

In general, intrinsic camera parameters may be different, and even unknown

Fundamental Matrix

EPIPOLAR GEOMETRY

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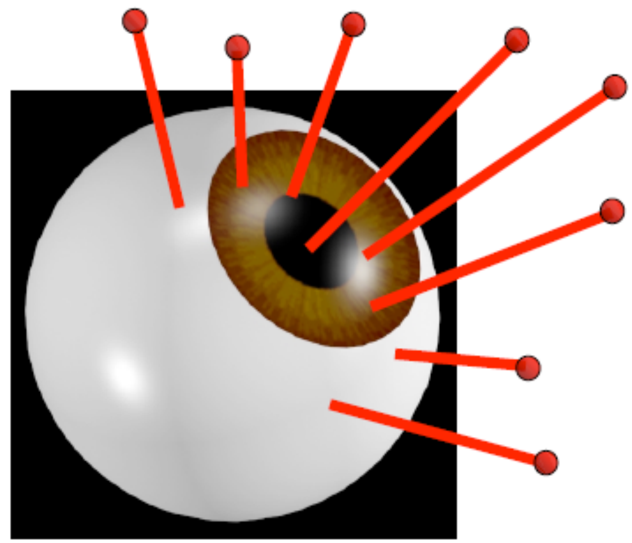
Epipolar Geometry

A Visualization

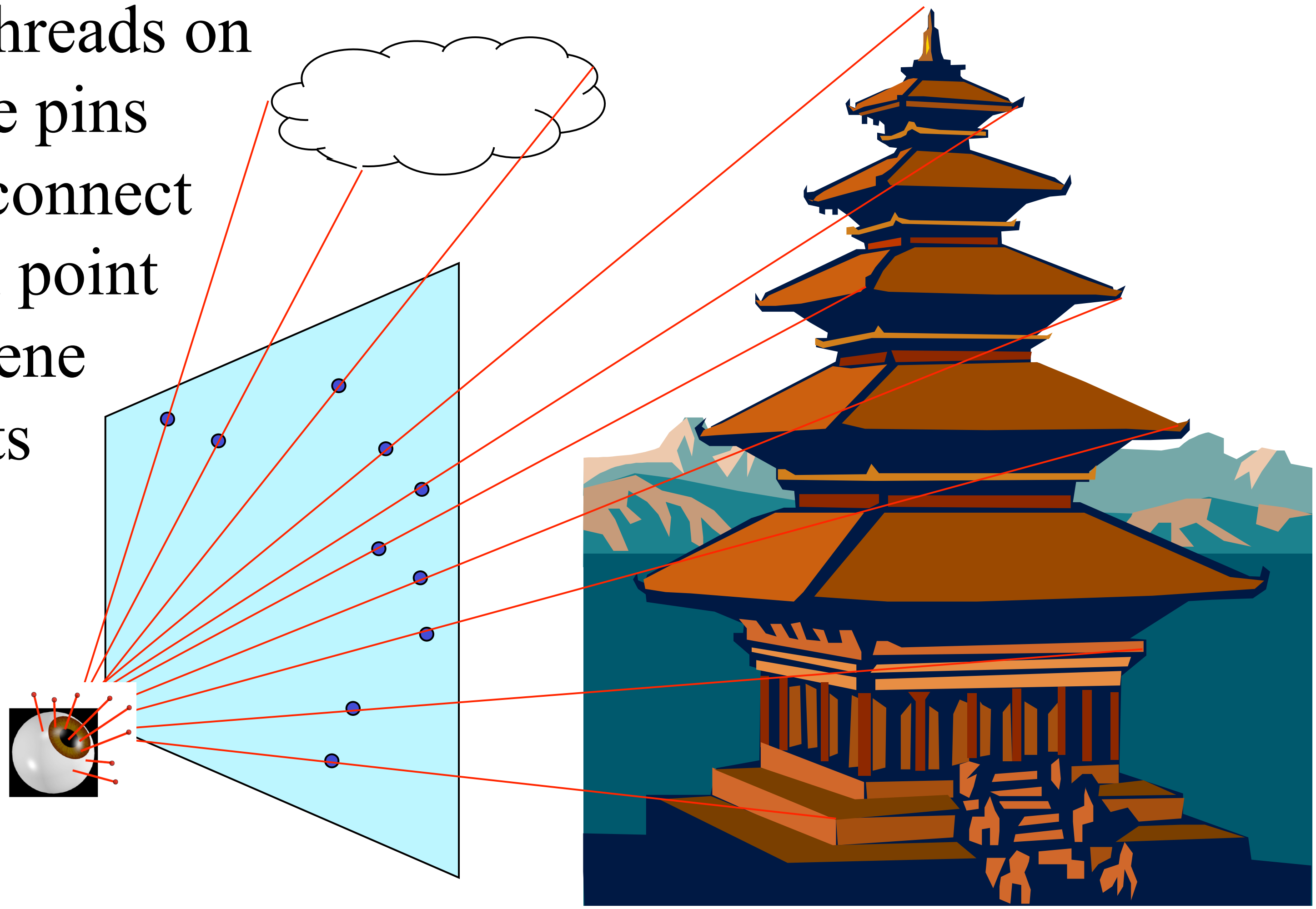
Would would Pinhead's eye look like close up?



answer

Rays to Points in Scene

Tie threads on to the pins and connect focal point to scene points



Now what would this look like to a second observer?

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Rays Seen from Second Observer

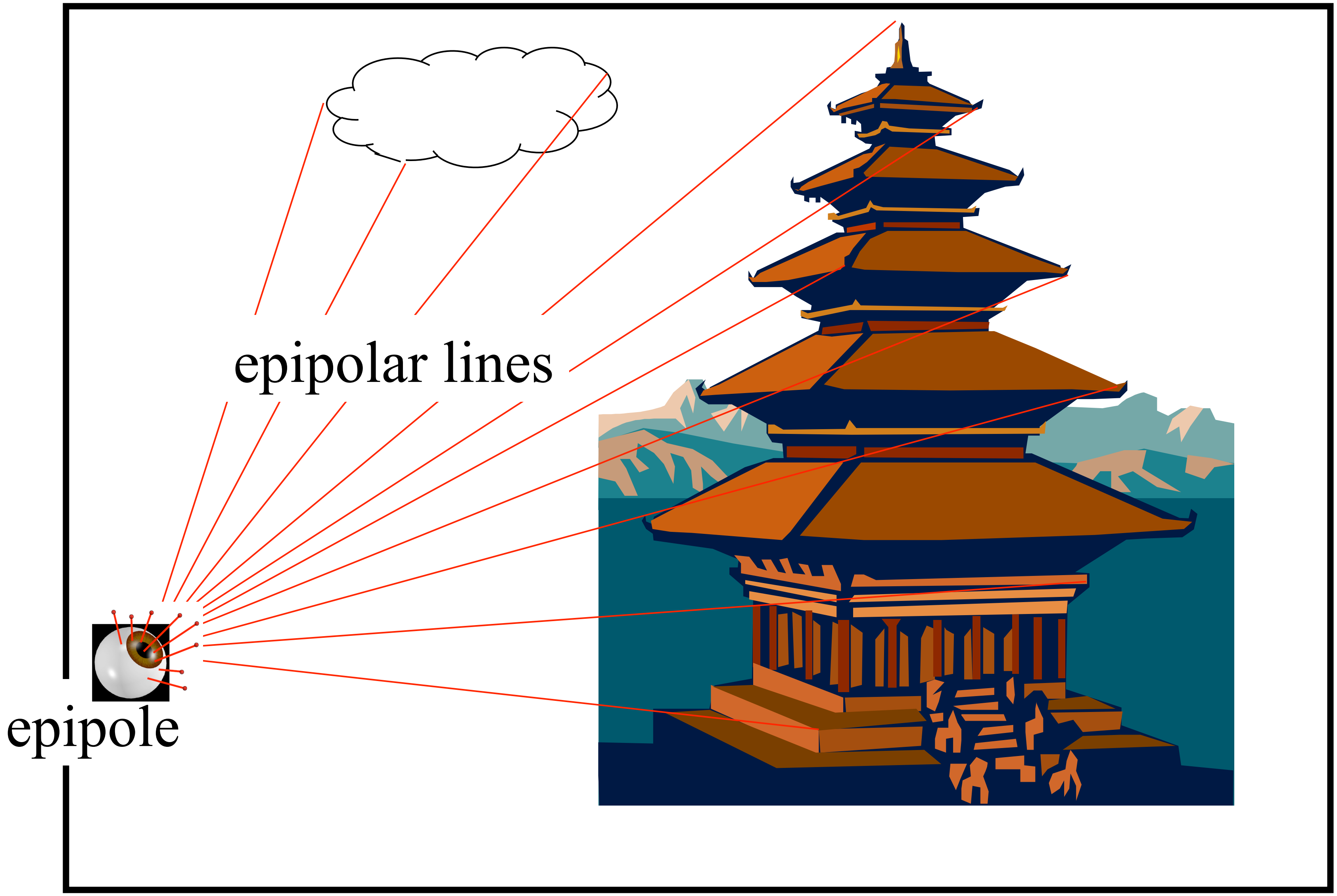

Image 2

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Rays Seen by the First Viewer

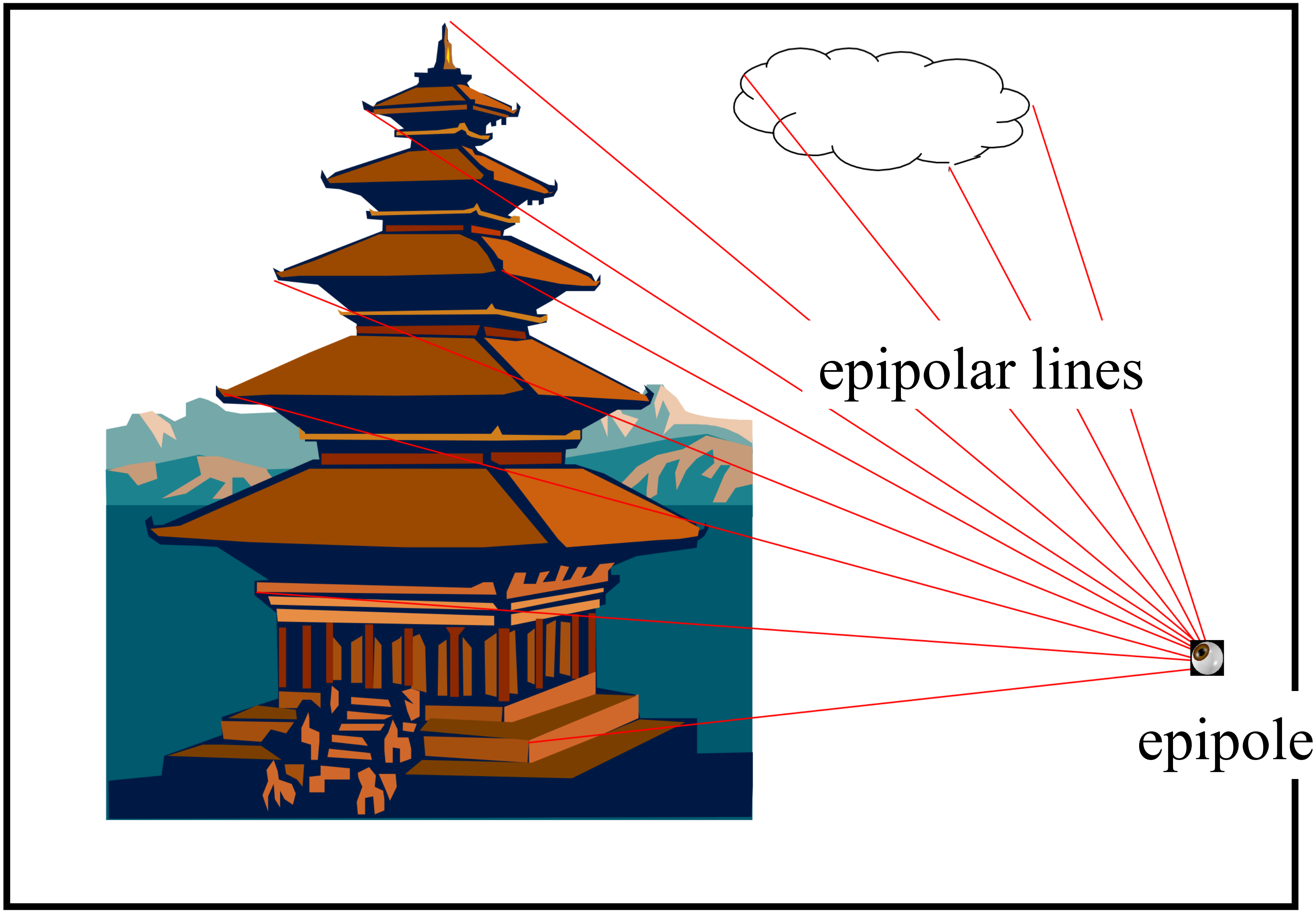

Image 1

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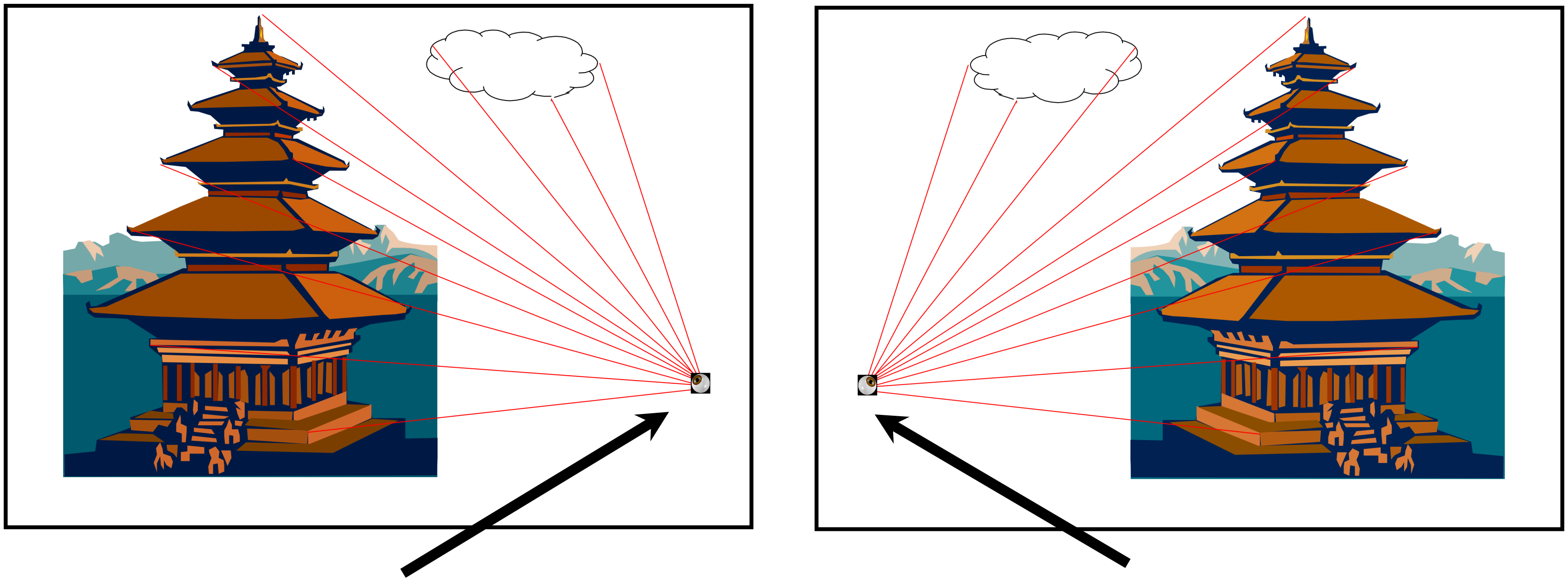
Epipolar Geometry

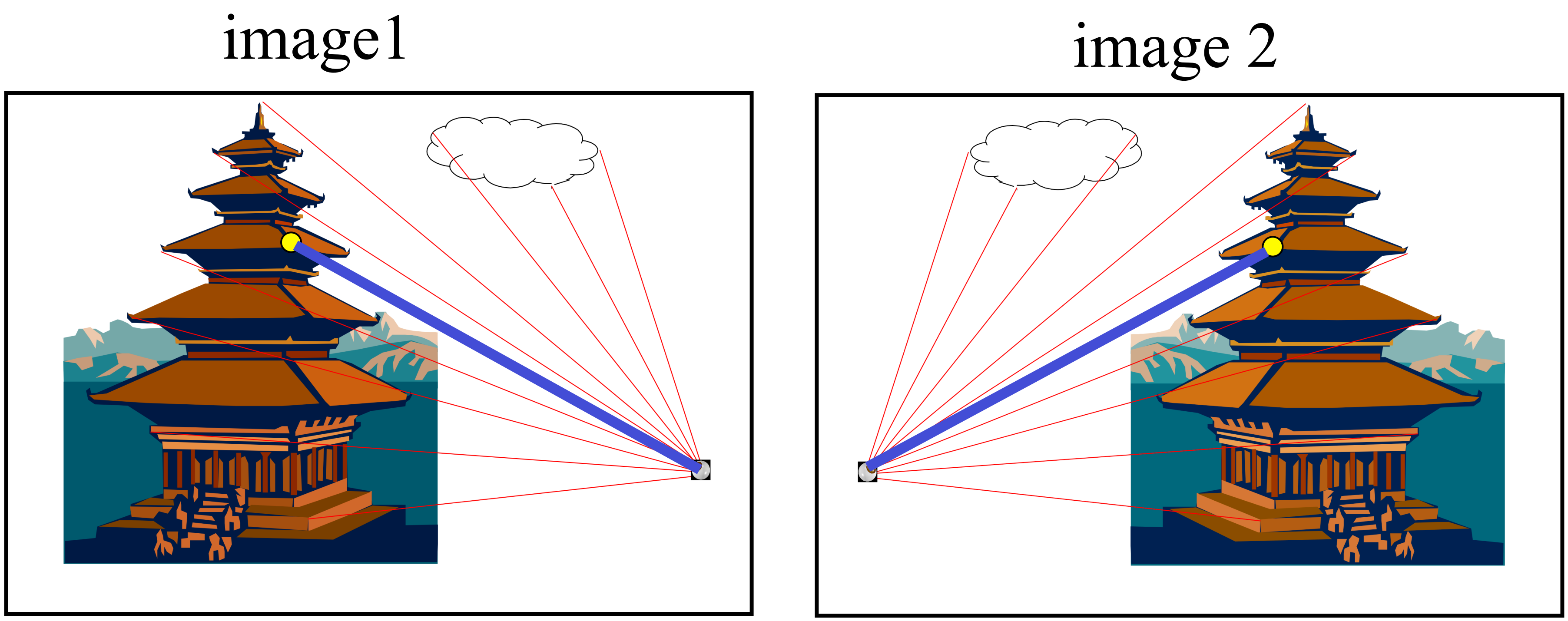
image1 image 2



Epipole : location of cam2 as seen by cam1.

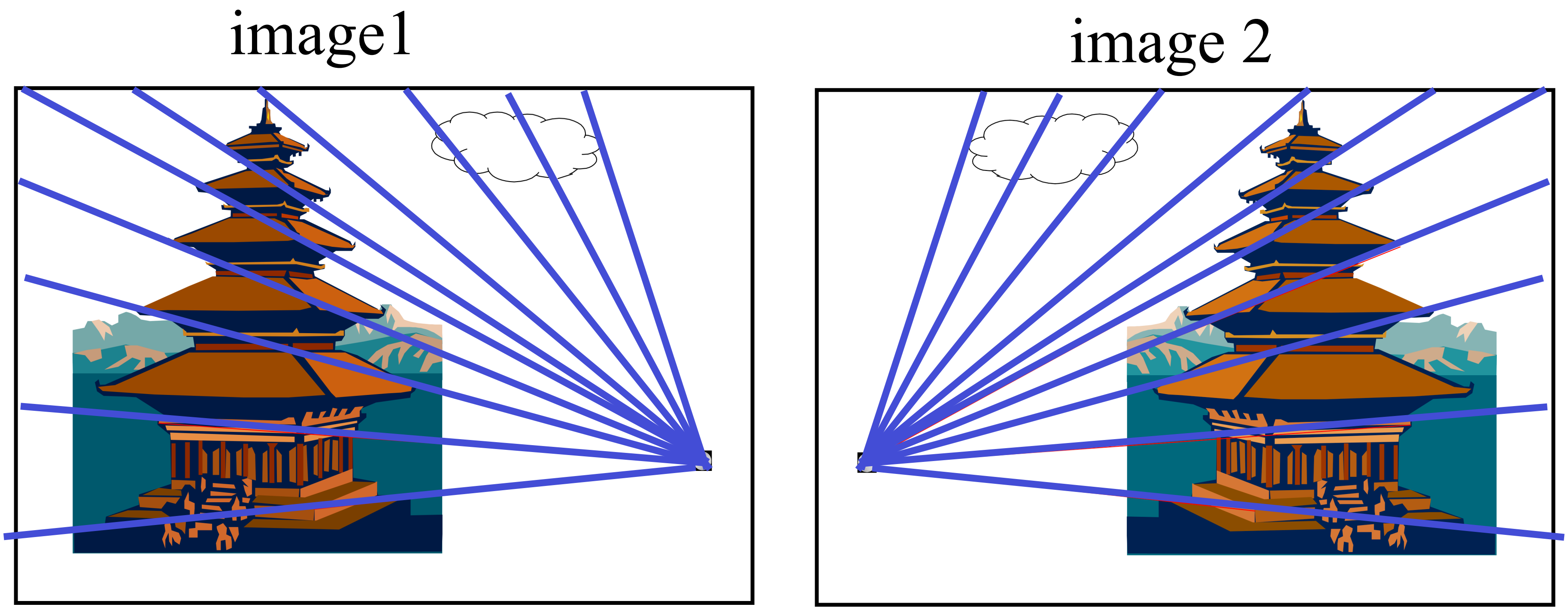
Epipole : location of cam1 as seen by cam2.

Epipolar Geometry



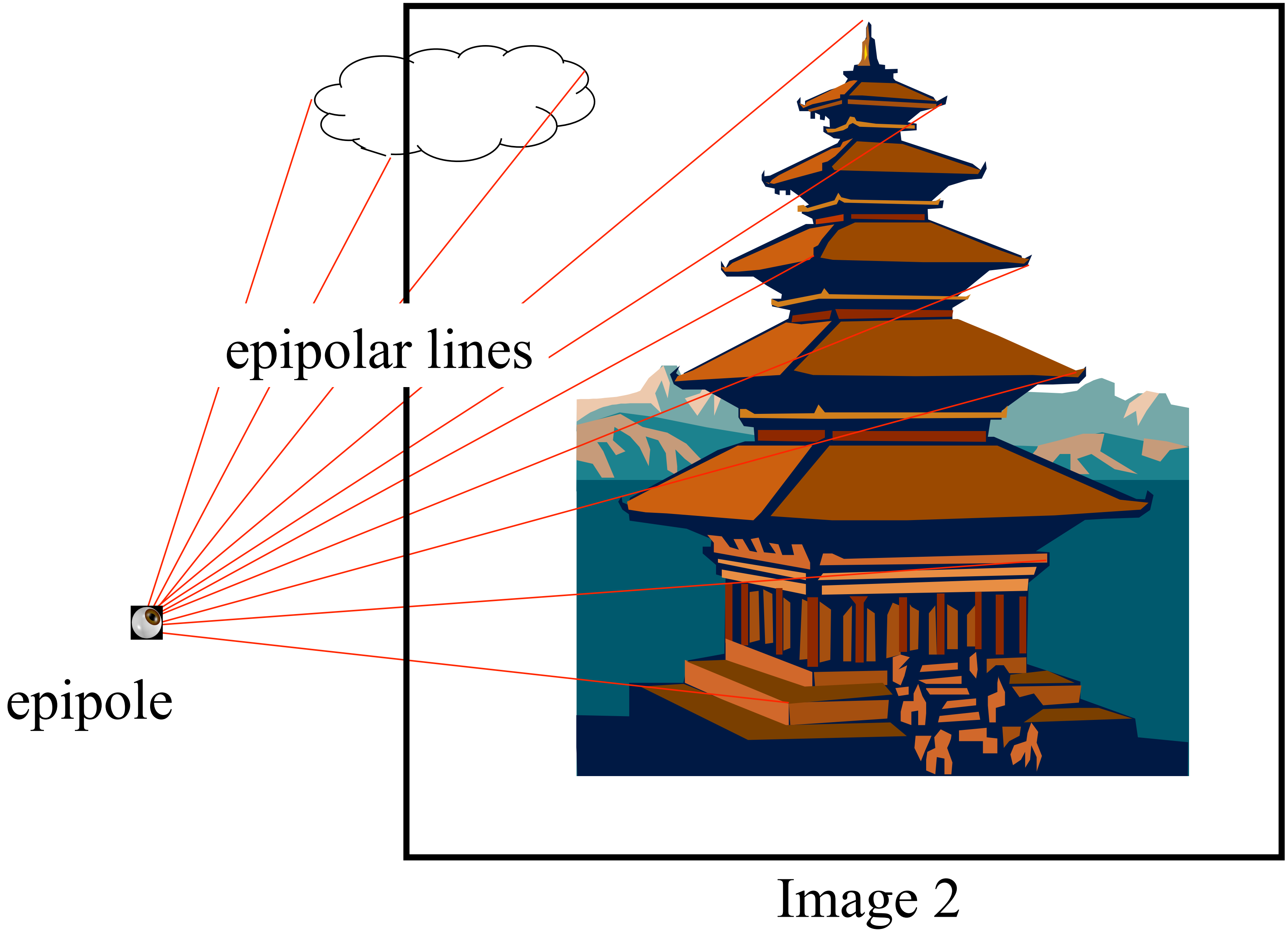
Corresponding points
lie on conjugate epipolar lines

Epipolar Geometry



Conjugate epipolar lines induce a generalized 1D “scan-line” ordering on the images (analogous to traditional scan line ordering of rows in an image)

Epipole not Necessarily in Image



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In fact it may be infinitely far off

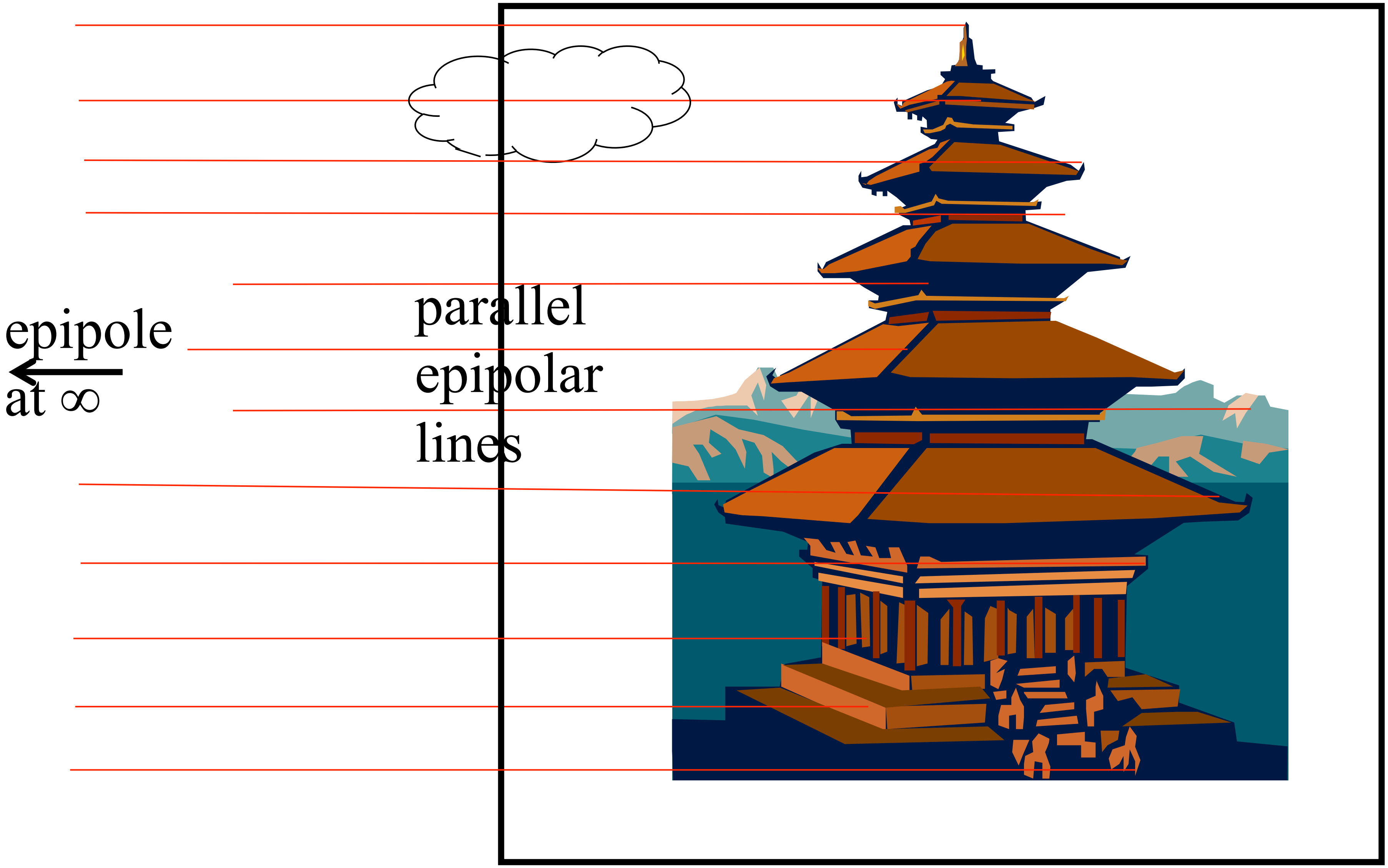
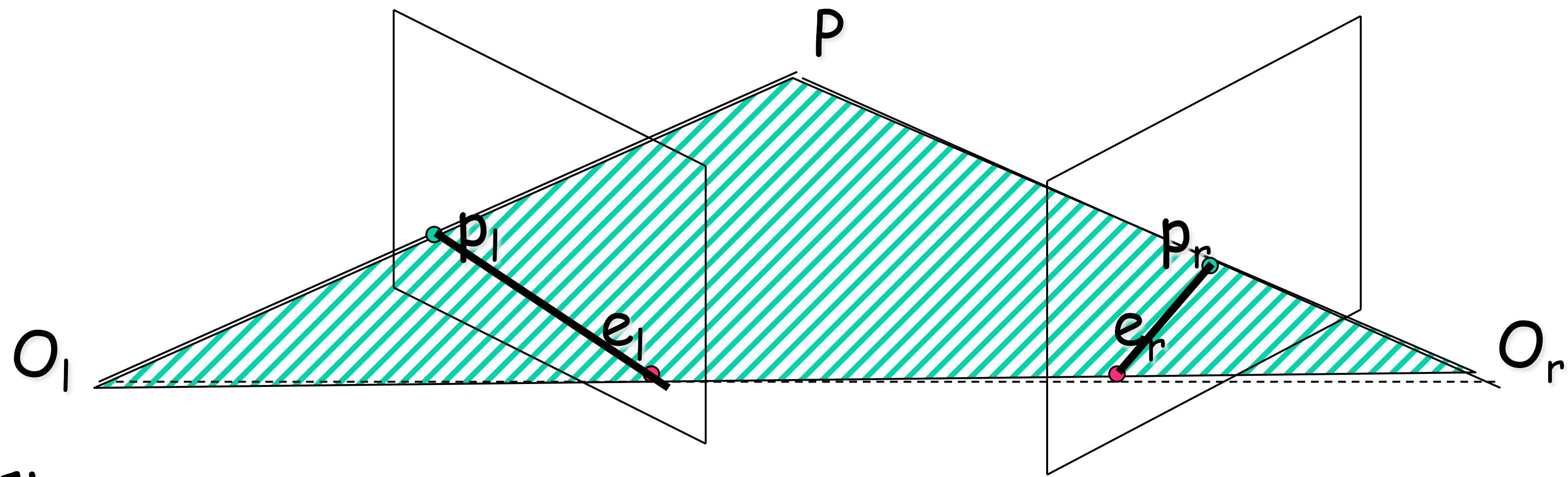


Image 2

Epipolar Geometry

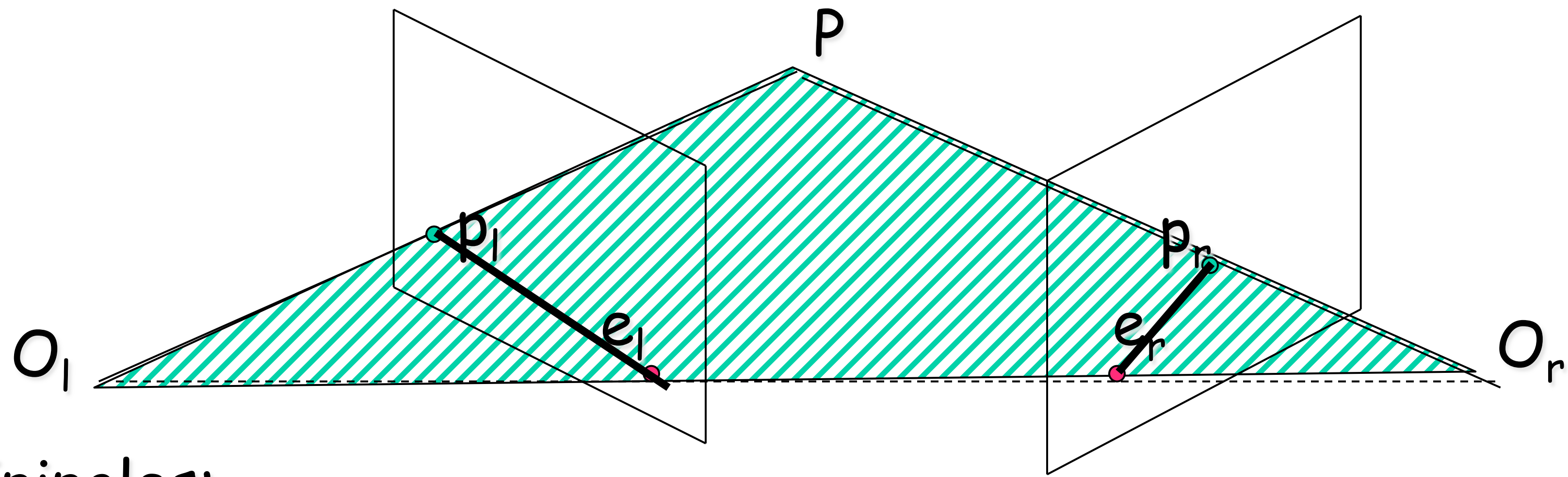


Epipoles:
- e_l: left image of O_r
- e_r: right image of O_l

Epipolar plane:
- Three points: O_l,O_r, and P define an epipolar plane

Epipolar lines and epipolar constraint:
- Intersections of epipolar plane with the image planes
- Corresponding points are on “conjugate” epipolar lines

Epipolar Constraint:



Given Epipoles:
- e_l: left image of O_r
- e_r: right image of O_l

Given p_l:
- consider its epipolar line: p_l e_l
- find epipolar plane: O_l,p_l,e_l
- intersect the epipolar plane with the right image plane
- search for p_r on the right epipolar line

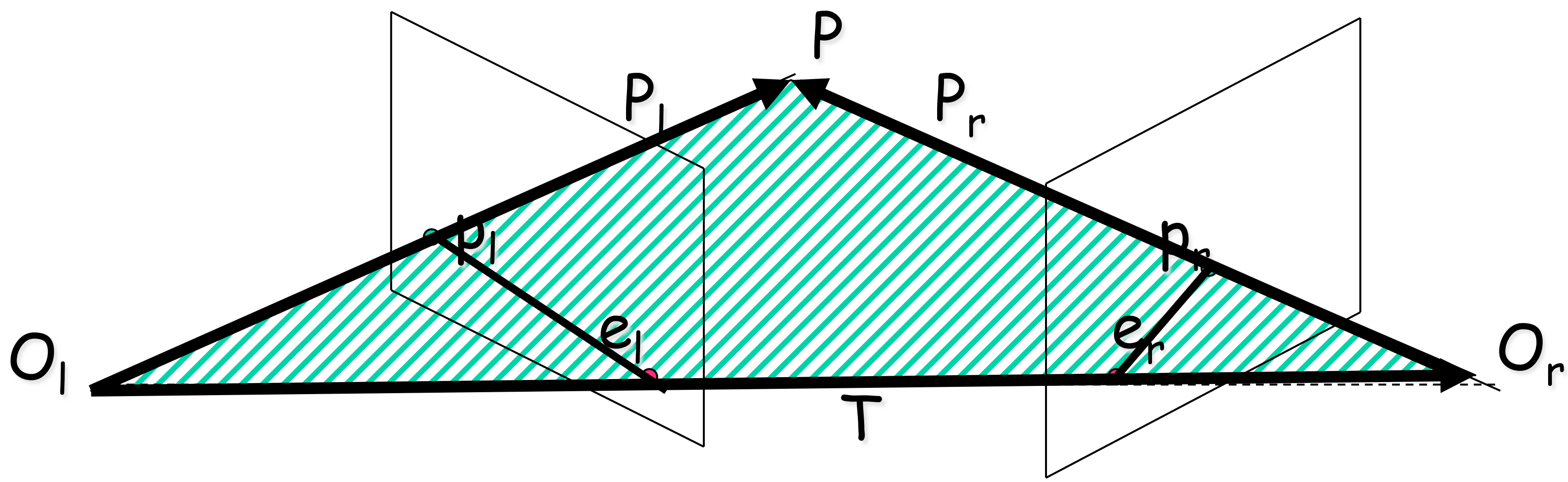
Essential Matrix

We now are going to derive the "essential matrix" for stereo, which encodes the geometry of a pair of cameras as a 3x3 matrix along with a set of useful algebraic constraint equations.

This is one of the most beautiful ideas in vision.

I hope you will follow along as we do it step-by-step.

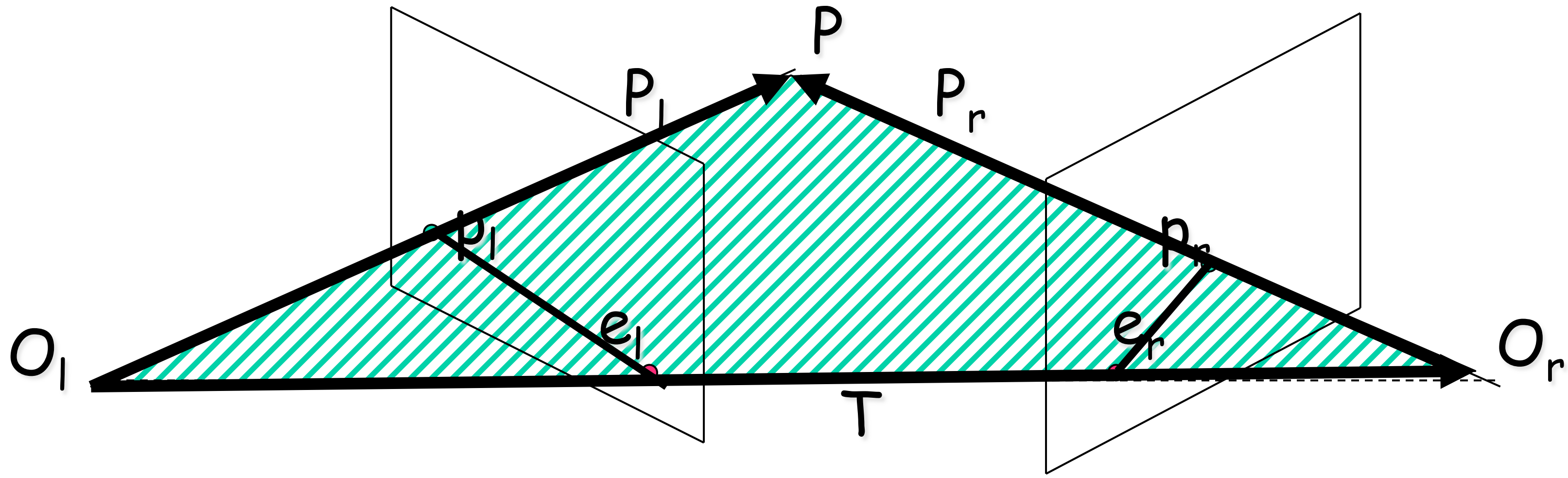
Essential Matrix



$$P_r = R(P_l - T)$$

Does this look familiar? Recall world to camera transformation by (R,T). Here, we are transforming from camera to camera.

Essential Matrix

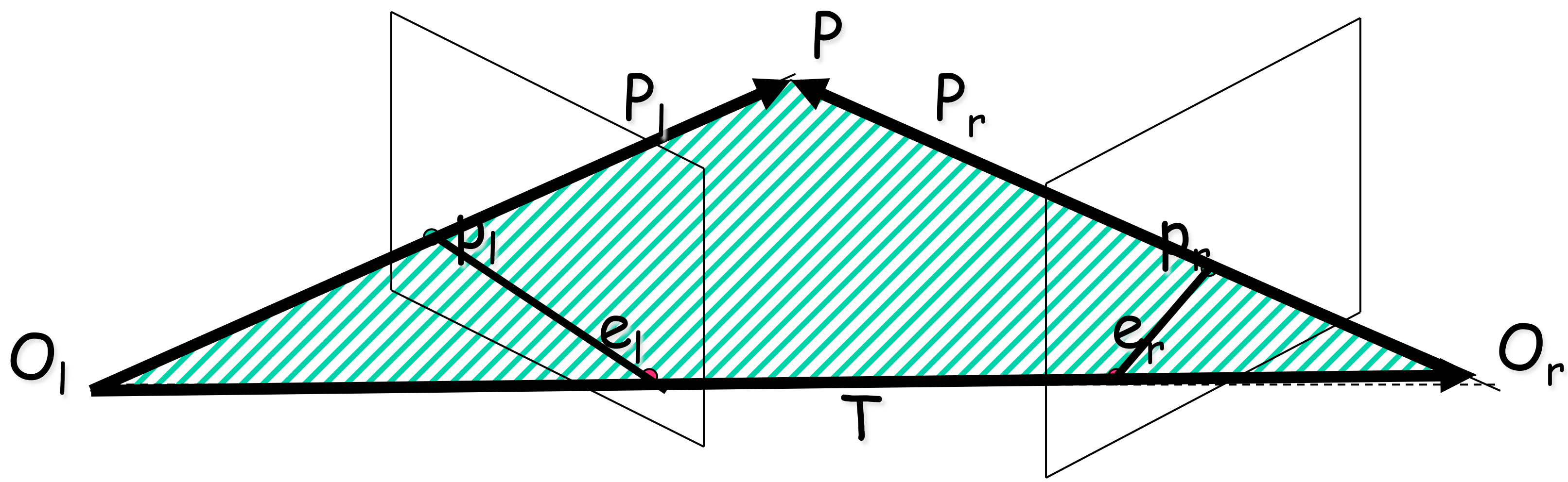


$$P_r = R(P_l - T)$$

$$P_l - T = R^{-1}P_r = R^T P_r$$

Essential Matrix

Epipolar constraint: P_l, T and P_l - T are coplanar:

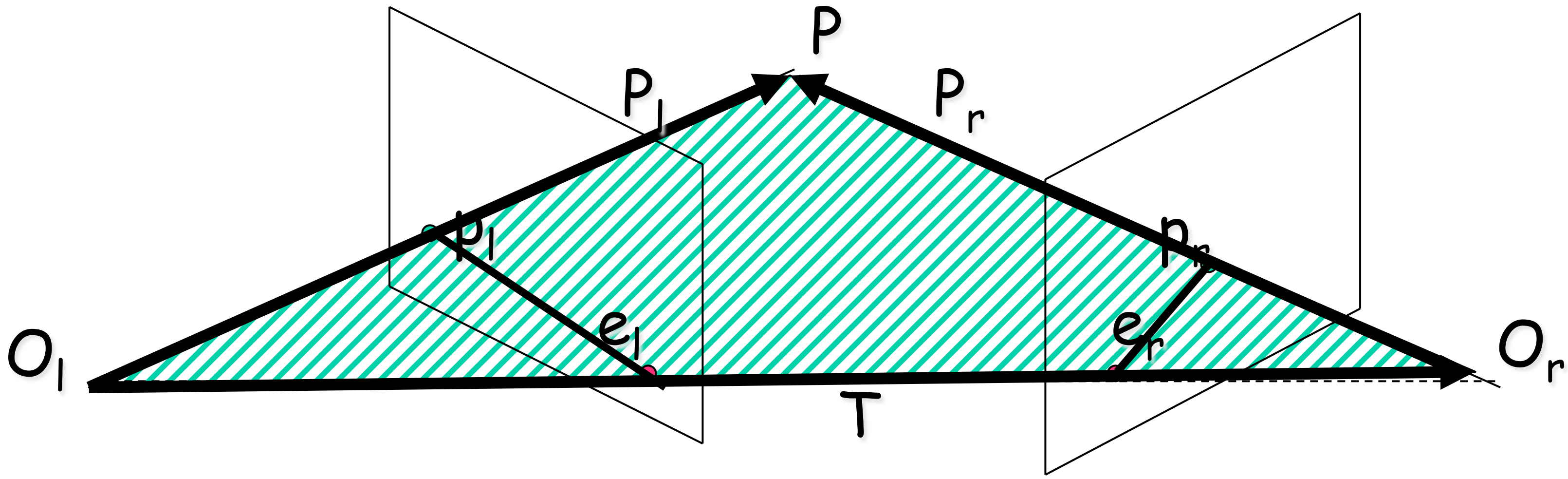


$$(P_l - T)^T \cdot T \times P_l = 0$$

$$P_l - T = R^T P_r \quad \Rightarrow (R^T P_r)^T \cdot T \times P_l = 0$$

Essential Matrix

Epipolar constraint: P_l, T and P_l - T are coplanar:



$$(R^T P_r)^T \cdot T \times P_l = 0$$

$$(P_r^T R) \cdot (T \times P_l) = 0$$

Vector Product as a Matrix Multiplication

$$T \times P_l = \begin{vmatrix} i & j & k \\ T_x & T_y & T_z \\ P_{l_x} & P_{l_y} & P_{l_z} \end{vmatrix}$$

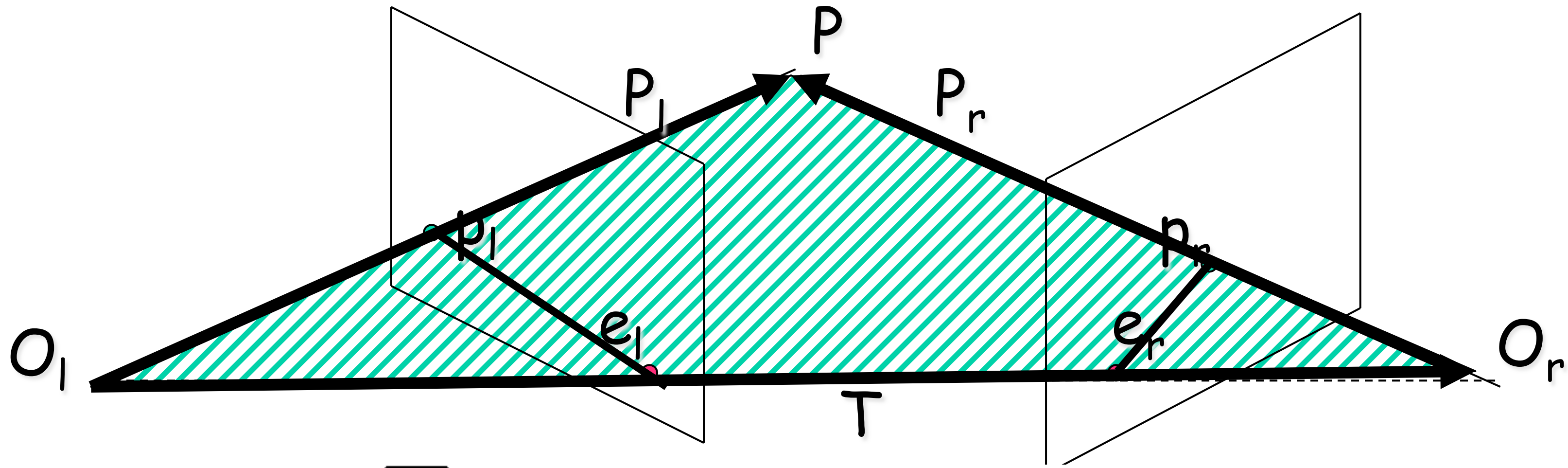
$$T \times P_l = (T_y P_{l_z} - T_z P_{l_y})i + (T_z P_{l_x} - T_x P_{l_z})j + (T_x P_{l_y} - T_y P_{l_x})k$$

$$T \times P_l = S P_l = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \begin{bmatrix} P_{l_x} \\ P_{l_y} \\ P_{l_z} \end{bmatrix} = \begin{bmatrix} T_y P_{l_z} - T_z P_{l_y} \\ T_z P_{l_x} - T_x P_{l_z} \\ T_x P_{l_y} - T_y P_{l_x} \end{bmatrix}$$

S has rank 2 ; it depends only on T

Essential Matrix

Epipolar constraint: P_l, T and P_l - T are coplanar:

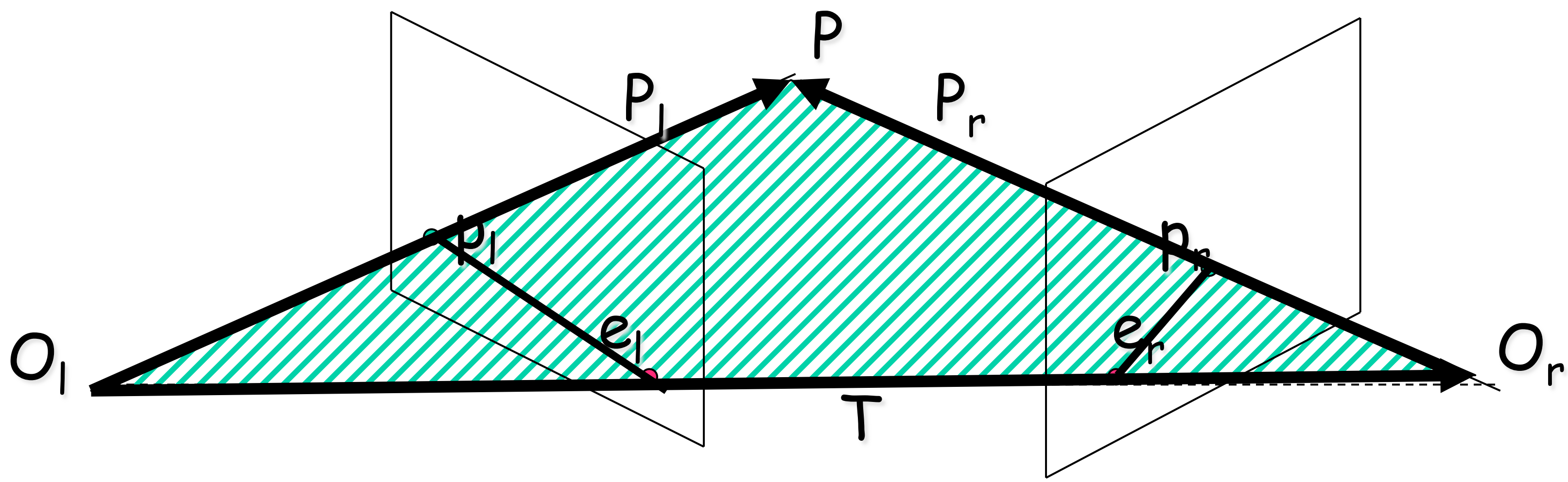


$$(P_r^T R) \cdot (T \times P_l) = 0$$

$$P_r^T R S P_l = 0$$

Essential Matrix

Epipolar constraint: P_l, T and P_l - T are coplanar:



$$P_r^T R S P_l = 0$$

Essential Matrix:

$$E = RS \qquad P_r^T E P_l = 0$$

Essential Matrix Properties

$$E = RS$$

- has rank 2
- depends only on the EXTRINSIC Parameters (R & T)

Longuet-Higgins equation

$$P_r^T E P_l = 0$$

$$p_l = \frac{f_l}{Z_l} P_l \qquad p_r = \frac{f_r}{Z_r} P_r$$

$$\left(\cancel{\frac{Z_r}{f_r}} p_r\right)^T E \left(\cancel{\frac{Z_l}{f_l}} p_l\right) = 0$$

$$\boxed{p_r^T E p_l = 0}$$

Longuet-Higgins equation

$$P_r^T E P_l = 0$$

This relates viewing rays

Importance of Longuet-Higgens ...

This relates 2D film points

$$\boxed{p_r^T E p_l = 0}$$

Epipolar Lines

- Let l be a line in the image:

$$au + bv + c = 0$$

- Using homogeneous coordinates:

$$\tilde{p} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \quad \tilde{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad \boxed{\tilde{p}^T \tilde{l} = \tilde{l}^T \tilde{p} = 0}$$

Epipolar Lines

- Remember:

$$p_r^T E p_l = 0$$

$$\tilde{l}_r = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

p_r belongs to epipolar line in the right image defined by

$$\tilde{l}_r = E p_l$$

Epipolar Lines

- Remember:

$$p_r^T E p_l = 0$$

$$\tilde{l}_l^T = \begin{bmatrix} a \\ b \\ c \end{bmatrix}^T$$

p_l belongs to epipolar line in the left image defined by

$$\tilde{l}_l = E^T p_r$$

Epipoles

- Remember: epipoles belong to the epipolar lines

$$e_r^T E p_l = 0 \qquad p_r^T E e_l = 0$$

- And they belong to all the epipolar lines

$$e_r^T E = 0 \qquad E e_l = 0$$

Essential Matrix Summary

Longuet-Higgins equation $$p_r^T E p_l = 0$$

Epipolar lines:

$$\tilde{p_r}^T \tilde{l_r} = 0 \qquad \tilde{p_l}^T \tilde{l_l} = 0$$

$$\tilde{l_r} = E p_l \qquad \tilde{l_l} = E^T p_r$$

Epipoles:

$$e_r^T E = 0 \qquad E e_l = 0$$