Lecture 8:
Harris Corner Detection

Reading: T&V Section 4.3
or Szeliski Section 4.1.1

Today we will see an example of a feature that is not a linear operator (not computable by convolution with a filter).

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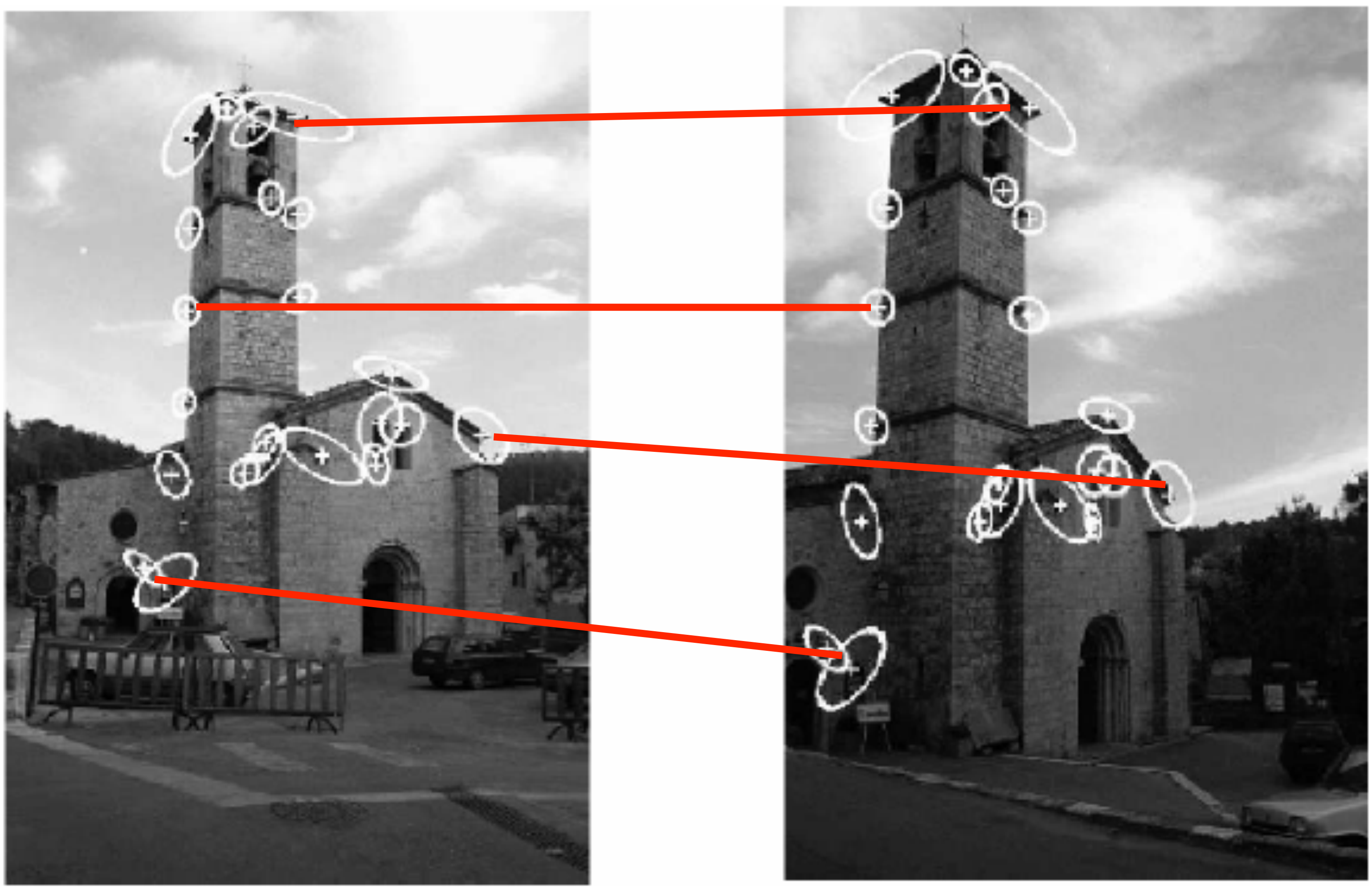
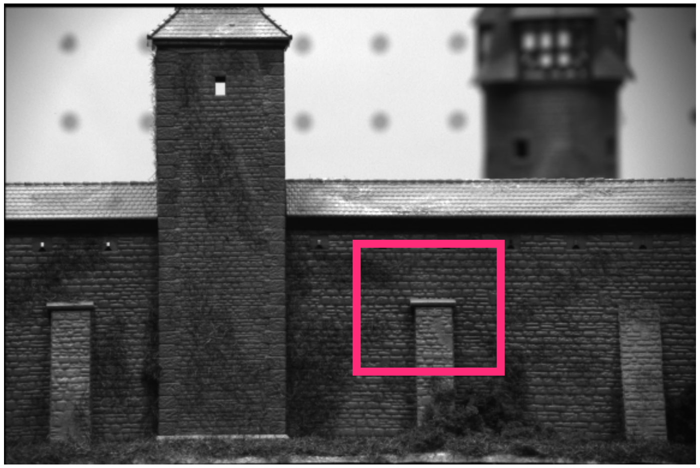
Recall: Matching Problem

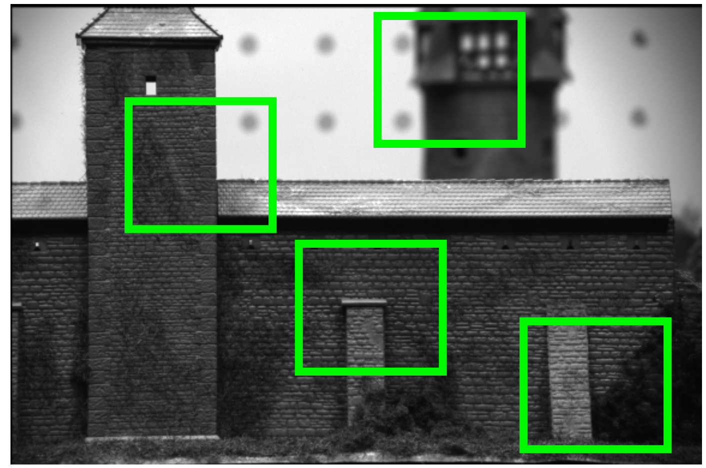
Vision tasks such as stereo and motion estimation require finding corresponding features across two or more views.



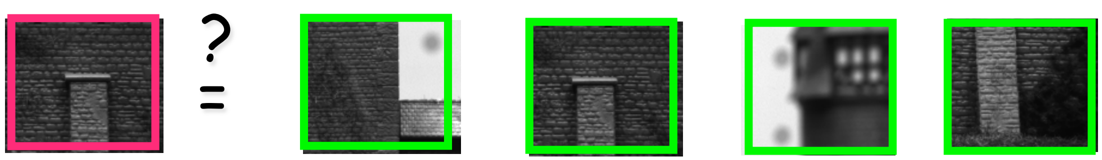
Recall: Patch Matching

Elements to be matched are image patches of fixed size

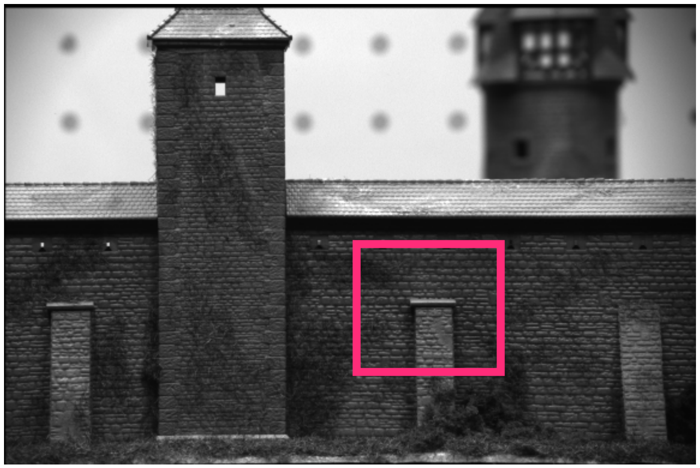


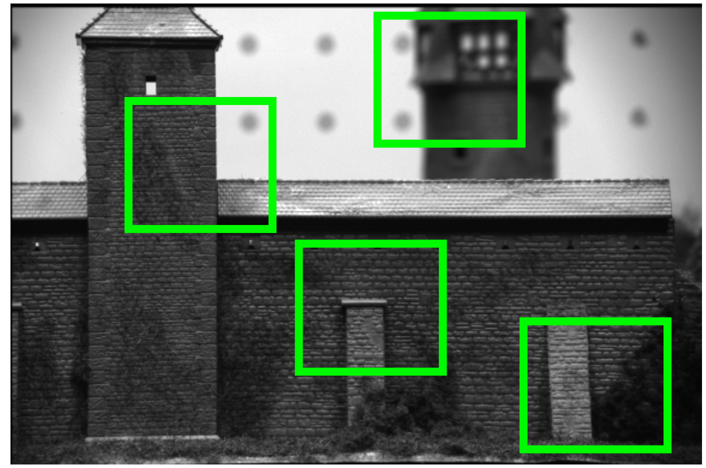


Task: find the best (most similar) patch in a second image

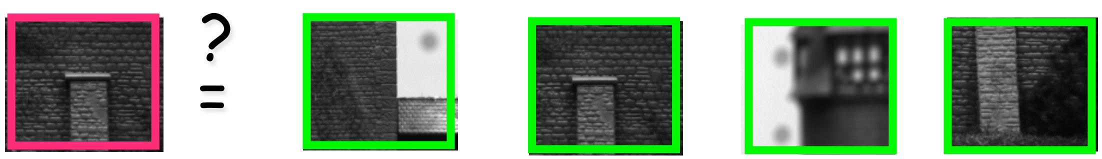


Not all Patches are Created Equal!

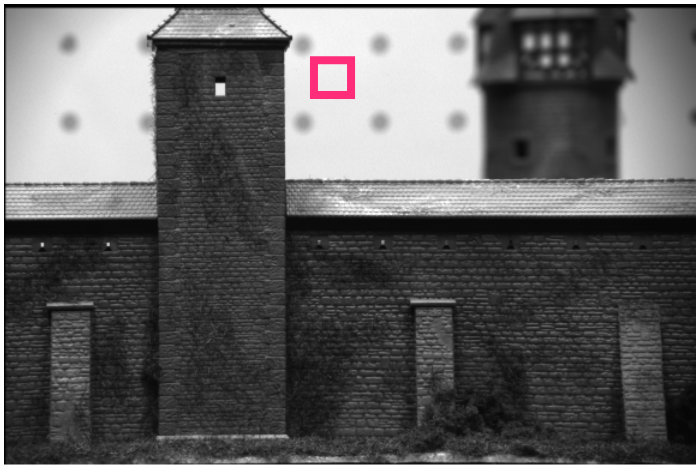




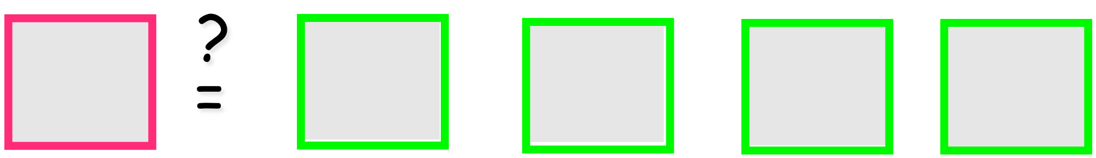
Initiution: this would be a good patch for matching, since it is very distinctive (there is only one patch in the second frame that looks similar).



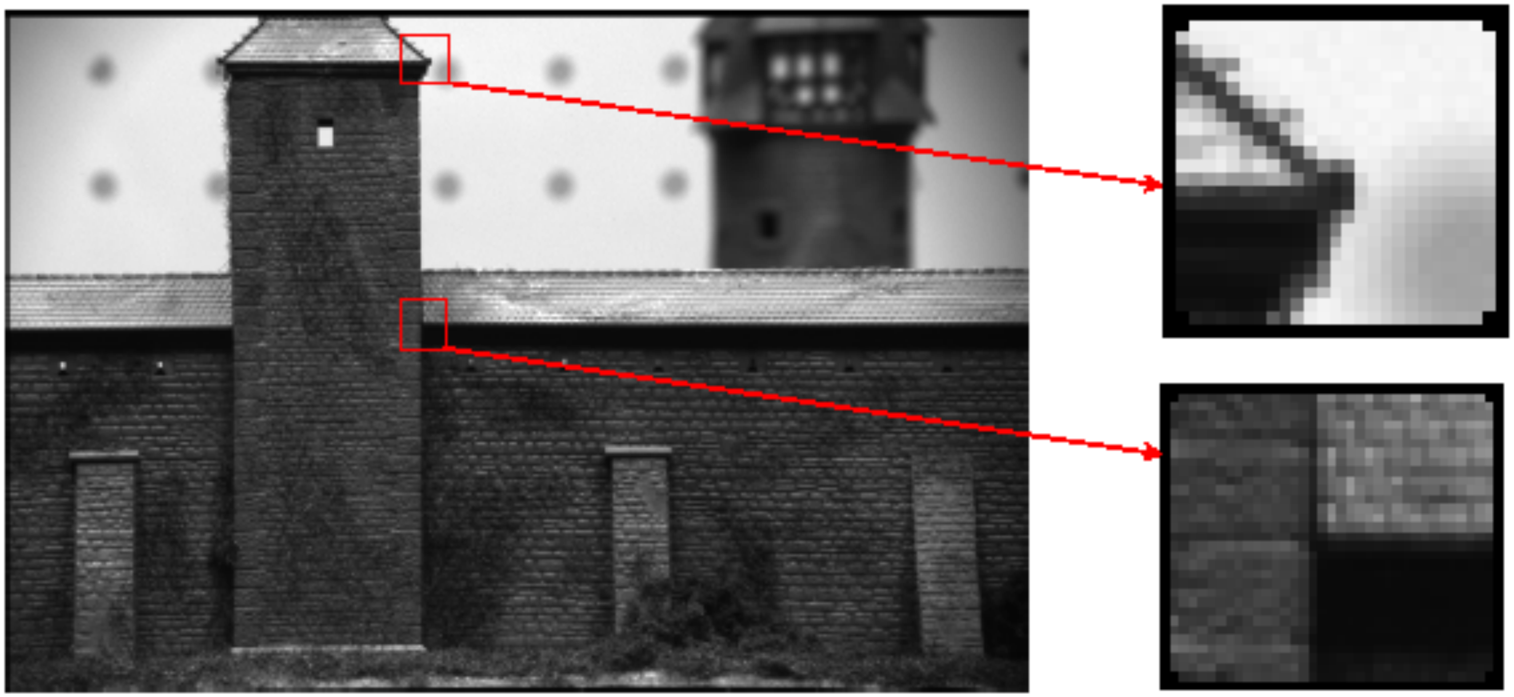
Not all Patches are Created Equal!



Initiuition: this would be a BAD patch for matching, since it is not very distinctive (there are many similar patches in the second frame)



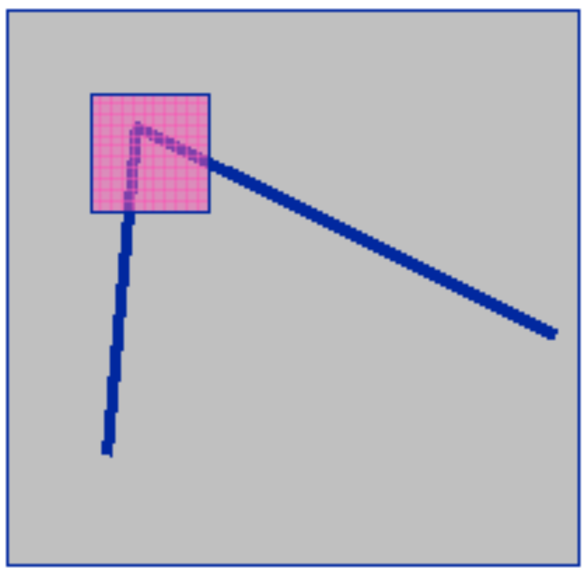
What are Corners?



- Intuitively, junctions of contours.
- Generally more stable features over changes of viewpoint
- Intuitively, large variations in the neighborhood of the point in all directions
- They are good features to match!

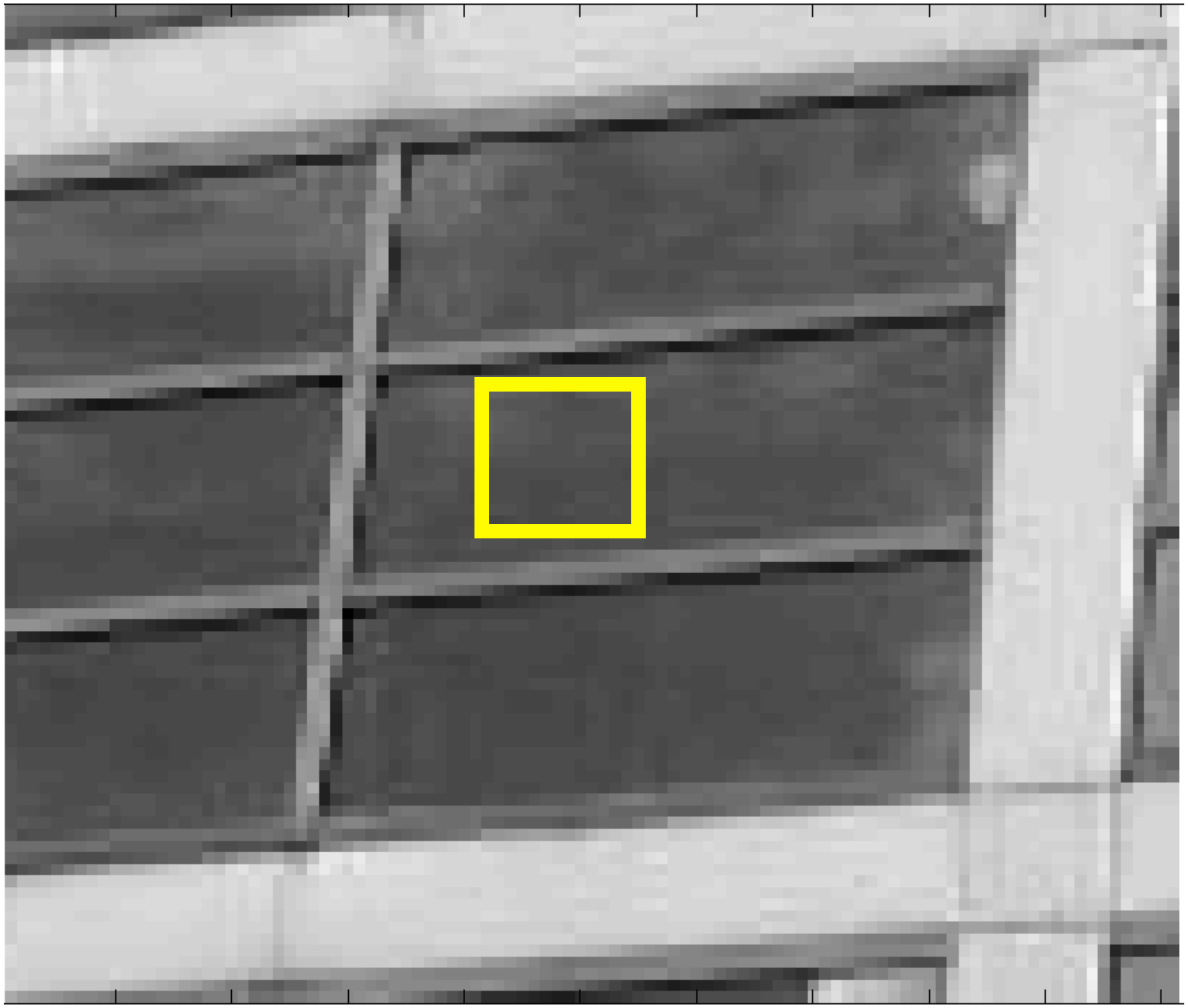
Corner Points: Basic Idea

- We should easily recognize the point by looking at intensity values within a small window
- Shifting the window in *any direction* should yield a *large change* in appearance.

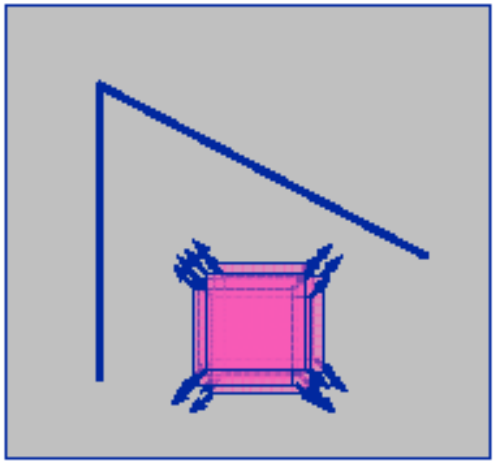


Appearance Change in Neighborhood of a Patch

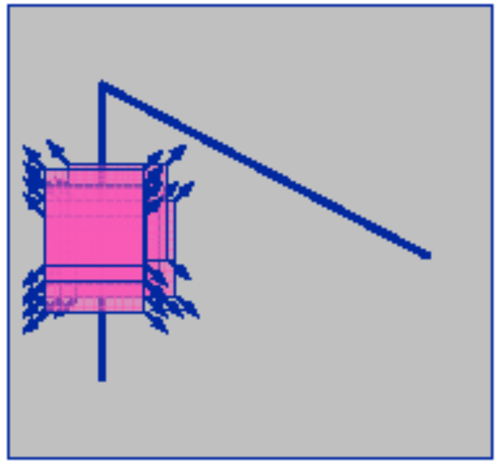
Interactive "demo"



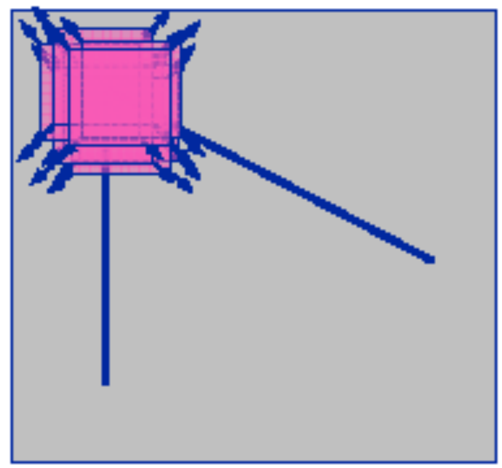
Harris Corner Detector: Basic Idea



"flat" region:
no change in
all directions



"edge":
no change along
the edge direction



"corner":
significant change
in all directions

Harris corner detector gives a mathematical approach for determining which case holds.

Harris Detector: Mathematics

Change of intensity for the shift $[u,v]$:

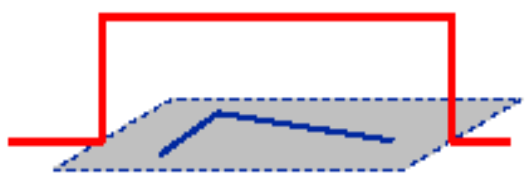
$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

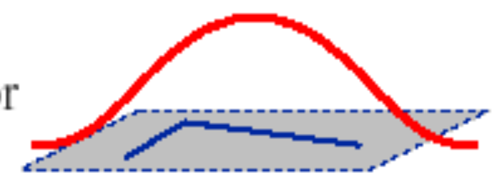
Intensity

Window function $w(x,y) =$



1 in window, 0 outside

or



Gaussian

Harris Detector: Intuition

Change of intensity for the shift $[u,v]$:

SSD! Sum of squared differences

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u,y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

For nearly constant patches, this will be near 0.
For very distinctive patches, this will be larger.
Hence... we want patches where E(u,v) is LARGE for any small u and v.

Taylor Series for 2D Functions

$$f(x+u,y+v) = f(x,y) + uf_x(x,y) + vf_y(x,y) +$$

First partial derivatives

$$\frac{1}{2!}\left[u^2 f_{xx}(x,y) + uvf_{xy}x,y + v^2 f_{yy}(x,y)\right] +$$

Second partial derivatives

$$\frac{1}{3!}\left[u^3 f_{xxx}(x,y) + u^2 v f_{xxy}(x,y) + uv^2 f_{xyy}(x,y) + v^3 f_{yyy}(x,y)\right]$$

Third partial derivatives

$+ \ \ldots$ (Higher order terms)

First order approx

$$f(x+u,y+v) \approx f(x,y) + uf_x(x,y) + vf_y(x,y)$$

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Harris Corner Derivation

$$\sum [I(x+u, y+v) - I(x,y)]^2$$

$$\approx \sum [I(x,y) + uI_x + vI_y - I(x,y)]^2$$ First order approx

$$= \sum u^2 I_x^2 + 2uvI_xI_y + v^2 I_y^2$$

$$= \sum \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$ Rewrite as matrix equation

$$= \begin{bmatrix} u & v \end{bmatrix} \left(\sum \begin{bmatrix} I_x^2 & I_xI_y \\ I_xI_y & I_y^2 \end{bmatrix} \right) \begin{bmatrix} u \\ v \end{bmatrix}$$

Harris Corner Matrix

$$\left(\sum \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} \right)$$

A real(-number), symmetric matrix

=> computed from gradients in a patch

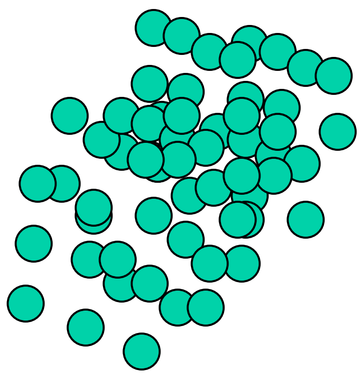
=> has real eigenvalues

=> can be decomposed as R D R^t

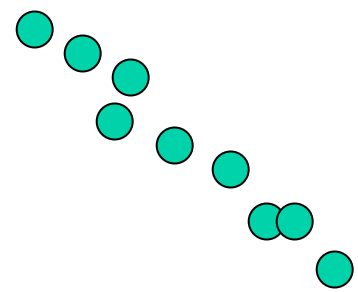
=> describes the shape of an ellipse!

Aside: Elliptical Blob Fitting

Problem: describe location and coarse shape of a set of points.



vs

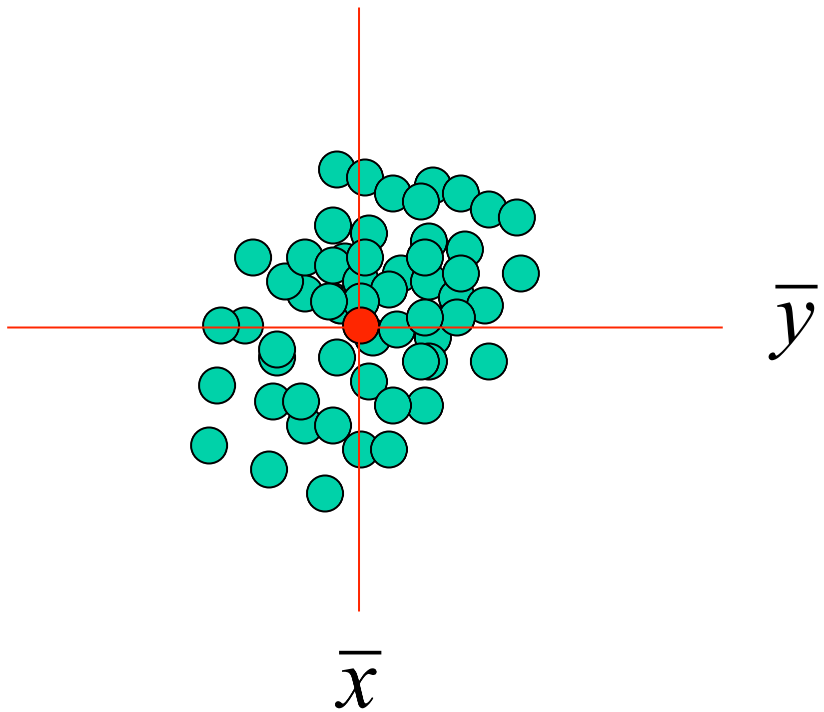


Elliptical Blob Fitting (cont)

Location: first moments

$$\bar{x} = \frac{1}{n}\sum_{1}^{n} x_i$$

$$\bar{y} = \frac{1}{n}\sum_{1}^{n} y_i$$



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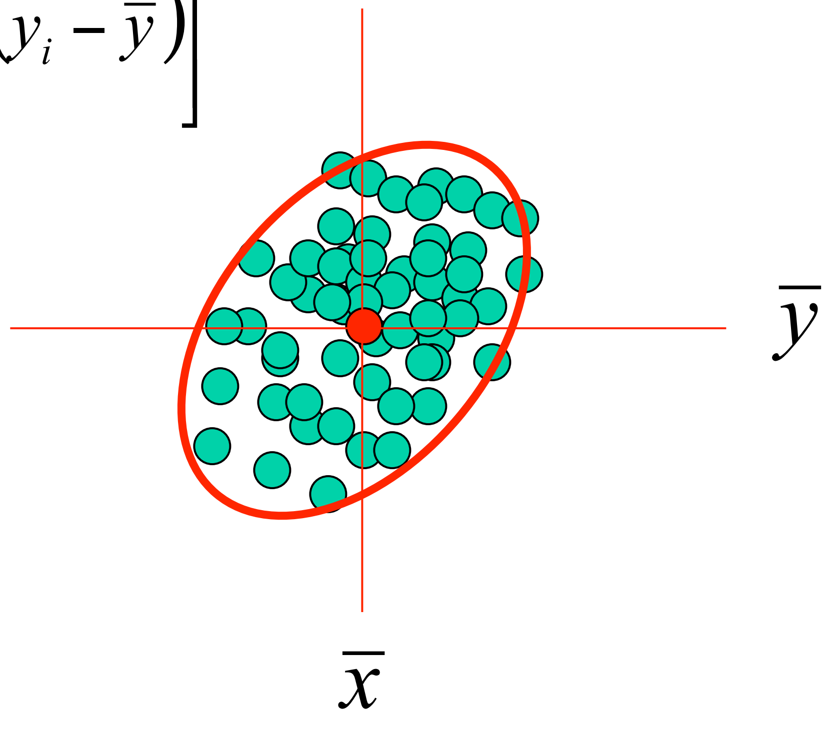
Elliptical Blob Fitting (cont)

Coarse Shape: second <u>central</u> moments

$$\mathbf{S} = \frac{1}{n}\begin{bmatrix} \sum_{1}^{n}(x_i - \bar{x})(x_i - \bar{x}) & \sum_{1}^{n}(x_i - \bar{x})(y_i - \bar{y}) \\ \sum_{1}^{n}(x_i - \bar{x})(y_i - \bar{y}) & \sum_{1}^{n}(y_i - \bar{y})(y_i - \bar{y}) \end{bmatrix}$$

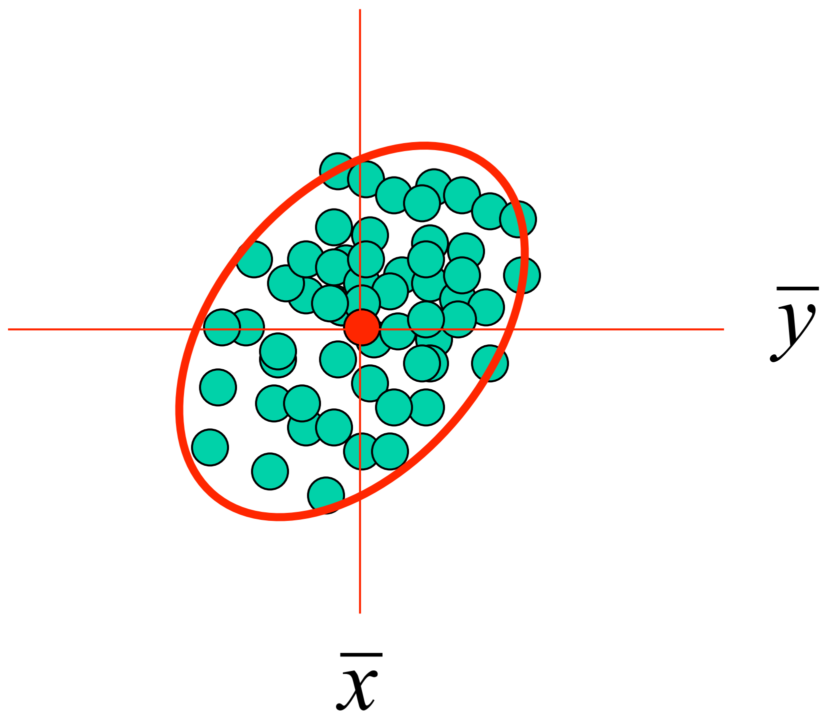
Aka "scatter matrix"
(or "covariance matrix")



Elliptical Blob Fitting (cont)

How to determine the shape and orientation of the ellipse?

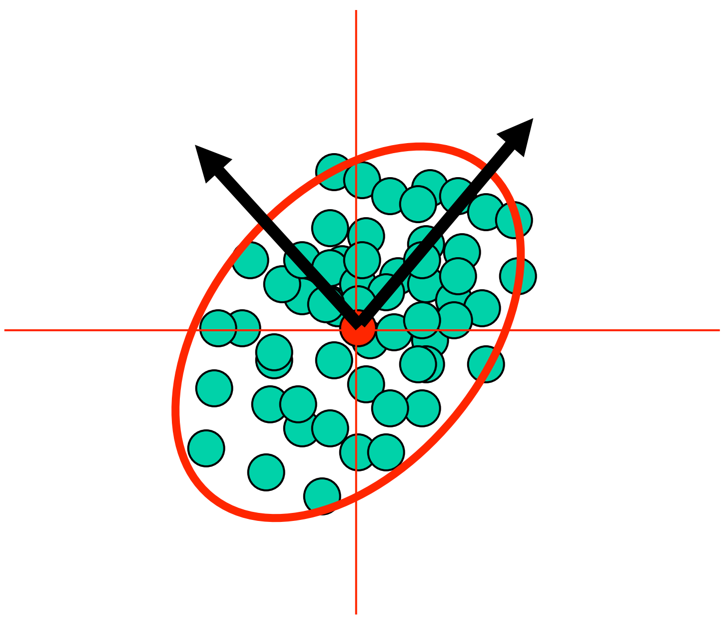
Computing eigenvalues and eigenvectors of the scatter matrix S.

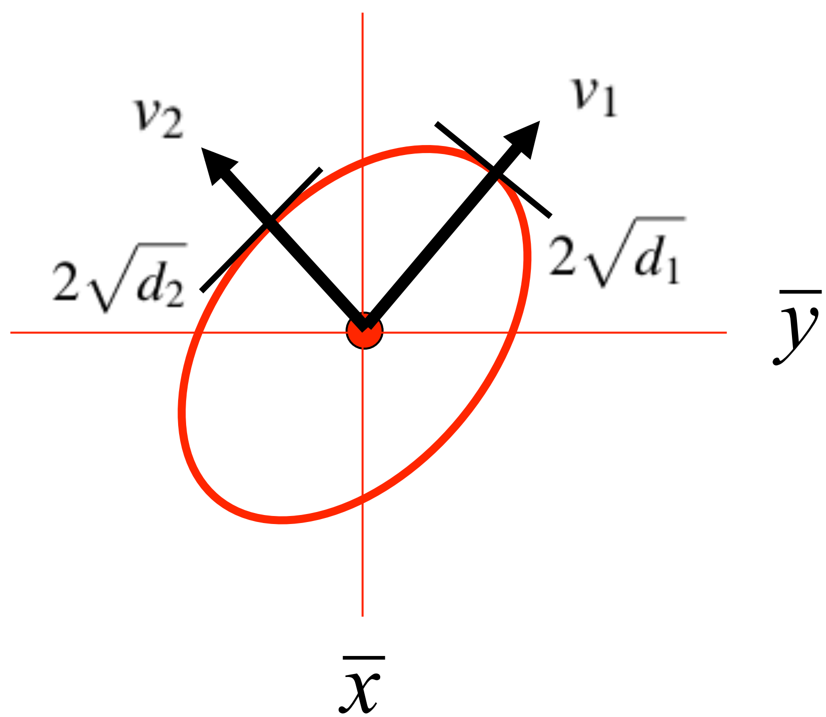


Back to Elliptical Blob Fitting

Decompose scatter matrix $S = PDP^T$
where

$$P = \begin{bmatrix} v_{1x} & v_{2x} \\ v_{1y} & v_{2y} \end{bmatrix} \qquad D = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix} \qquad d_1 \geq d_2$$





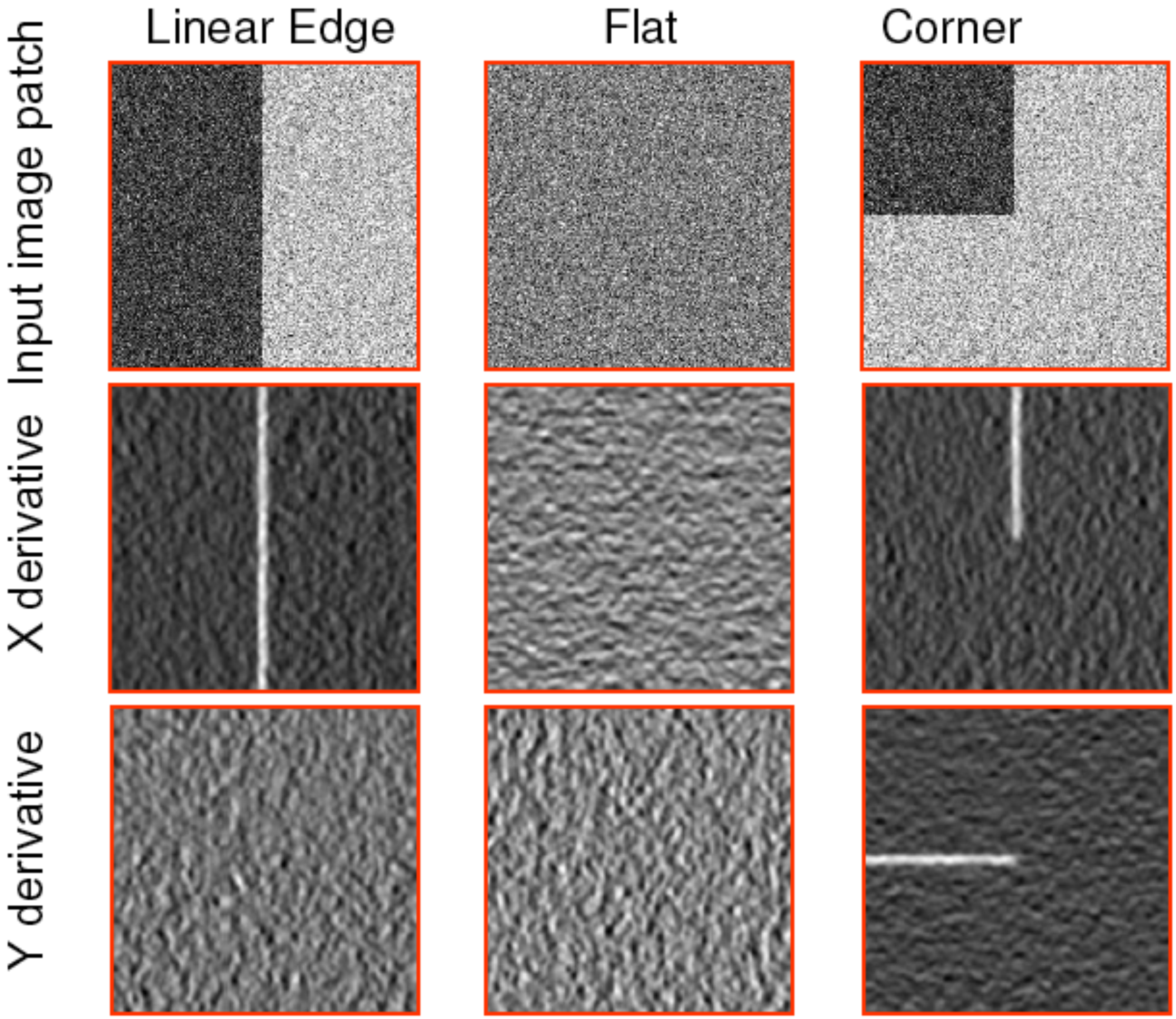
Intuitive Way to Understand Harris

Treat gradient vectors as a set of (dx,dy) points with a center of mass defined as being at (0,0).

Fit an ellipse to that set of points via scatter matrix

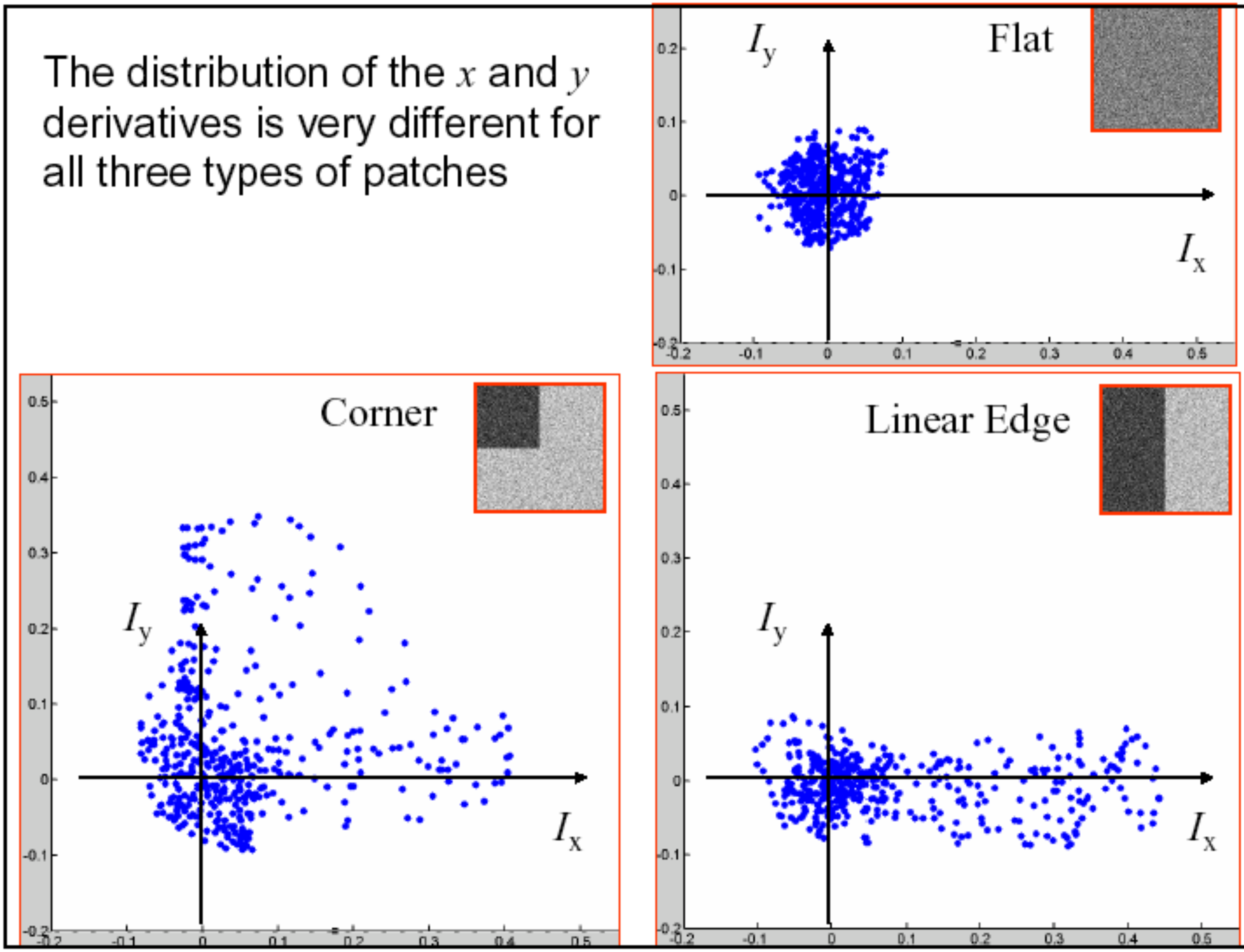
Analyze ellipse parameters for varying cases…

Example: Cases and 2D Derivatives



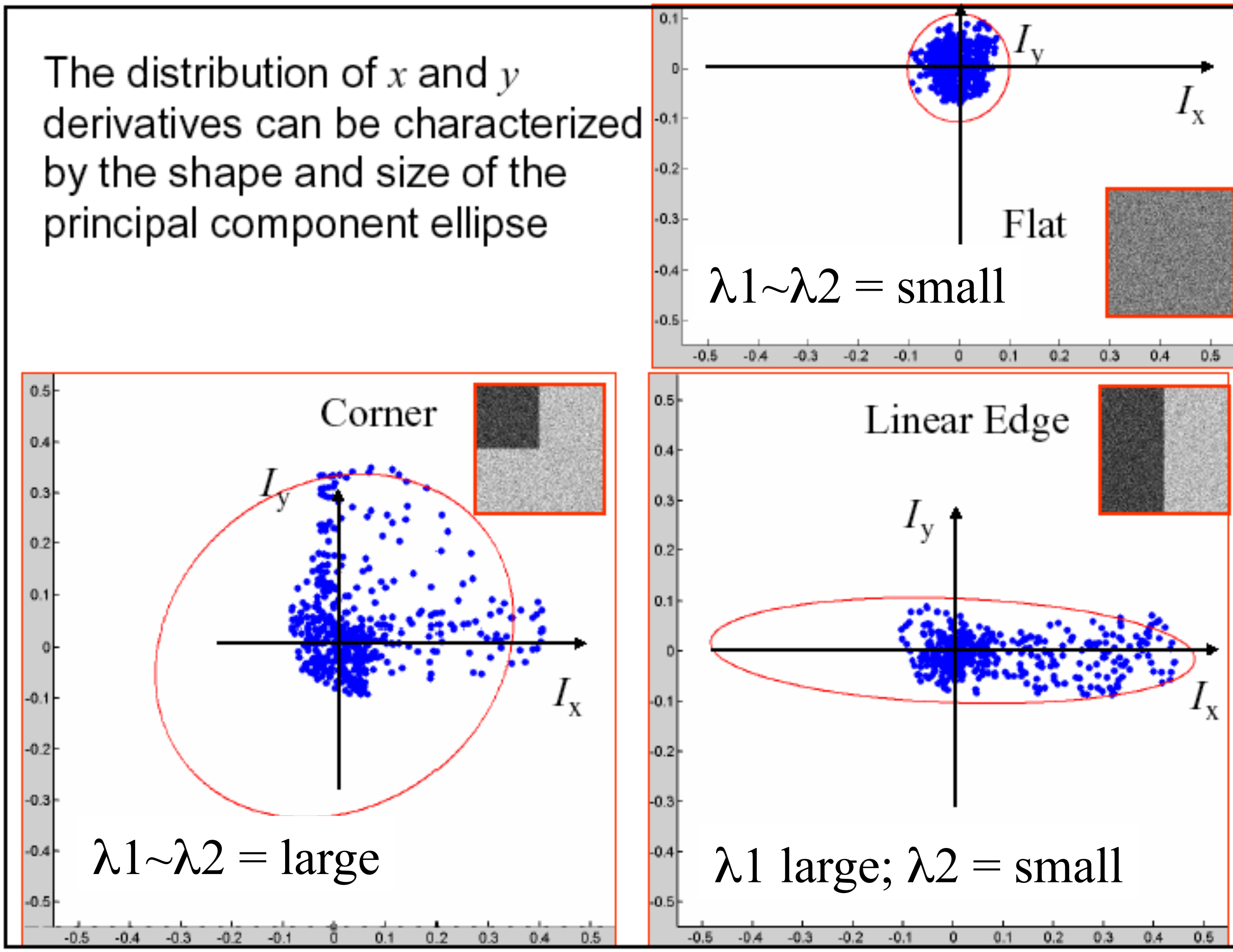
Plotting Derivatives as 2D Points

The distribution of the *x* and *y* derivatives is very different for all three types of patches



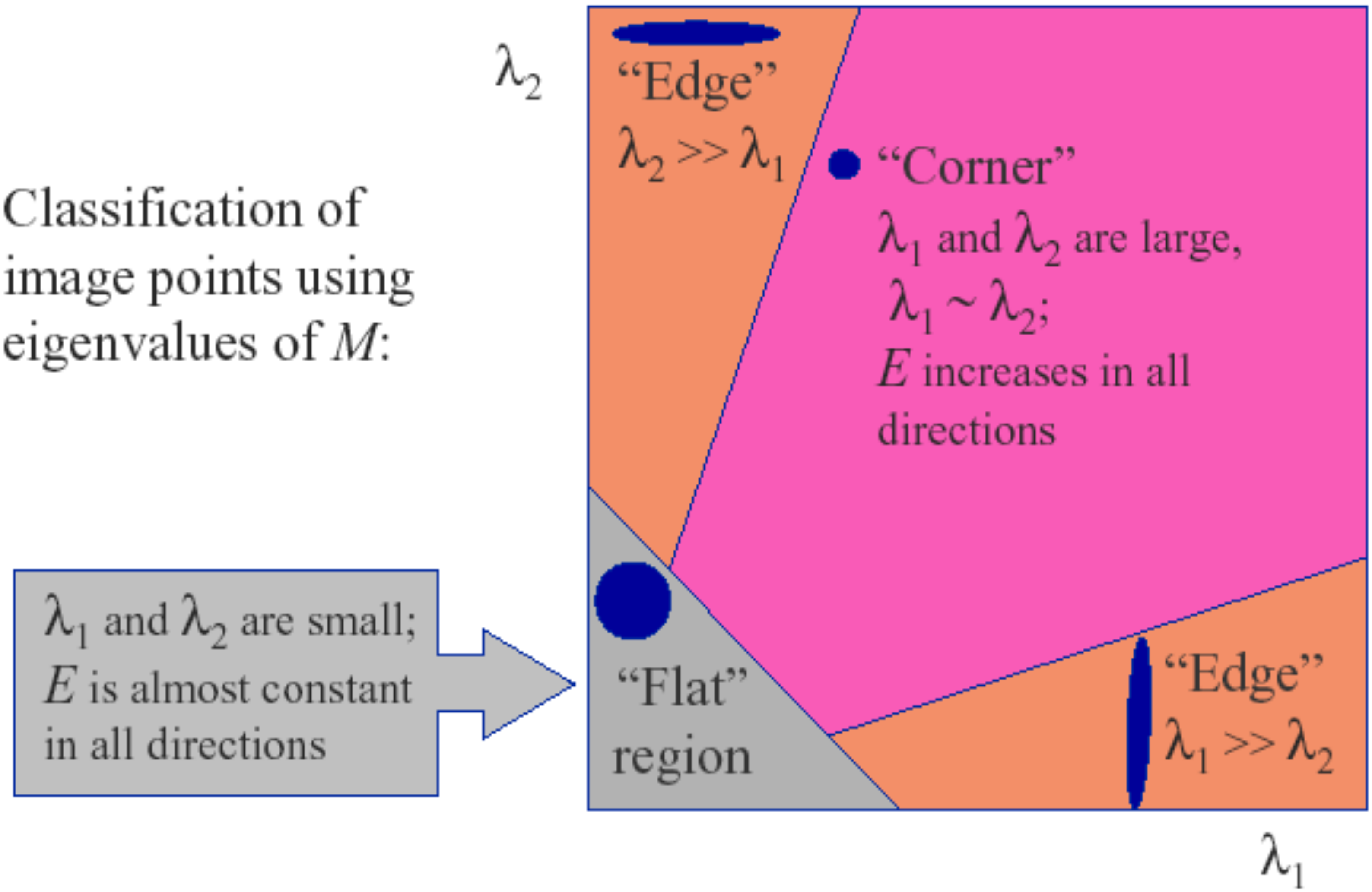
Fitting Ellipse to each Set of Points

The distribution of x and y derivatives can be characterized by the shape and size of the principal component ellipse



Classification via Eigenvalues

Classification of image points using eigenvalues of M:



Corner Response Measure

Measure of corner response:

$$R = \det M - k\left(\operatorname{trace} M\right)^2$$

$$\det M = \lambda_1 \lambda_2$$
$$\operatorname{trace} M = \lambda_1 + \lambda_2$$

(k is an empirically determined constant; $k = 0.04 - 0.06$)

Important Point: Harris R score is a function of eigenvalues, but we never have to explicitly compute them! Instead, we just compute determinant and trace of a 2x2 matrix, which is easy.

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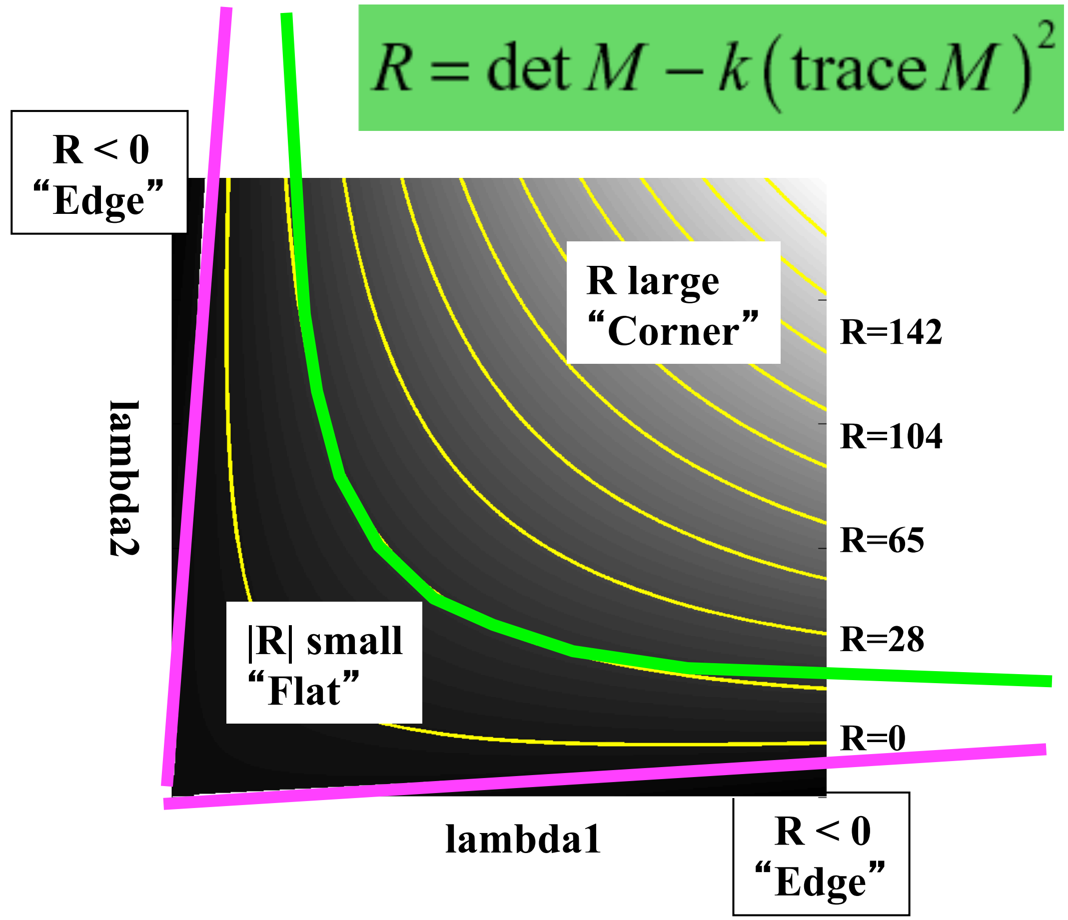
Classification via R Value

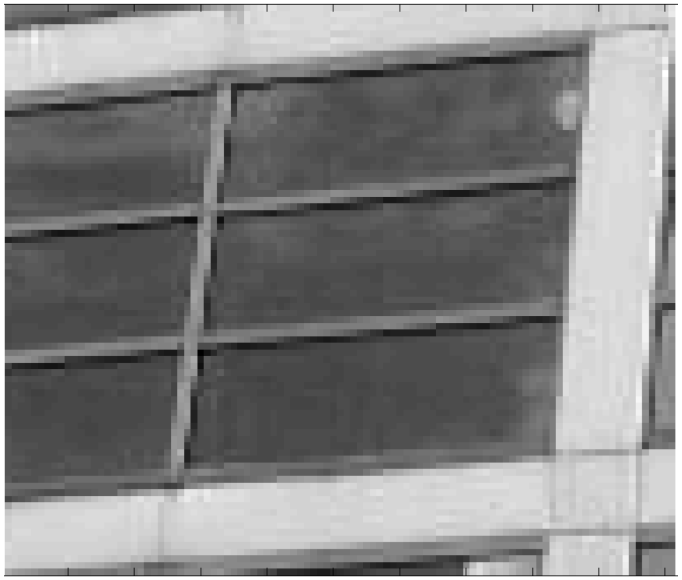
- R depends only on eigenvalues of M
- R is large for a corner
- R is negative with large magnitude for an edge
- $|R|$ is small for a flat region

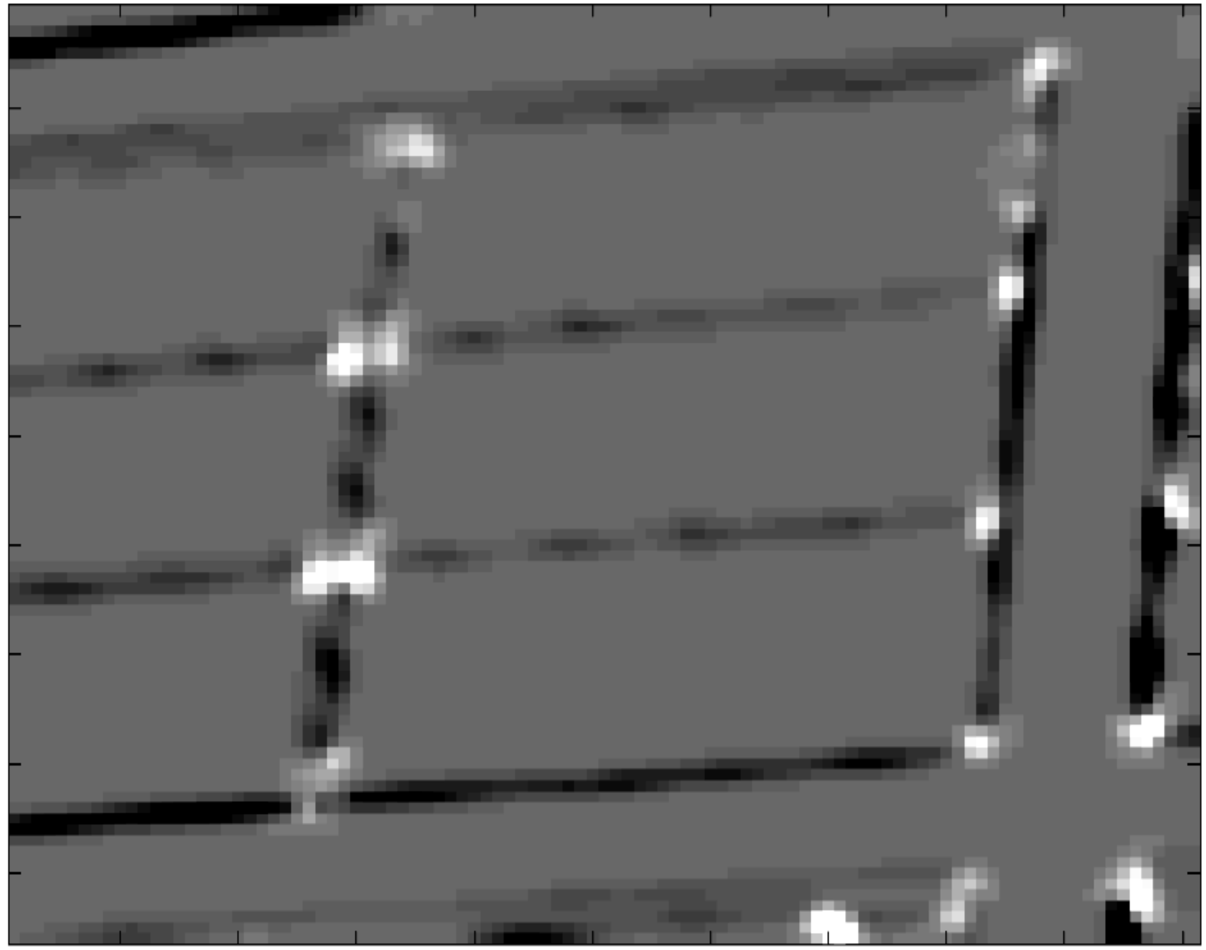
$$R = \det M - k\left(\operatorname{trace} M\right)^2$$



Compare with earlier slide "Classification via Eigenvalues"

Corner Response Example

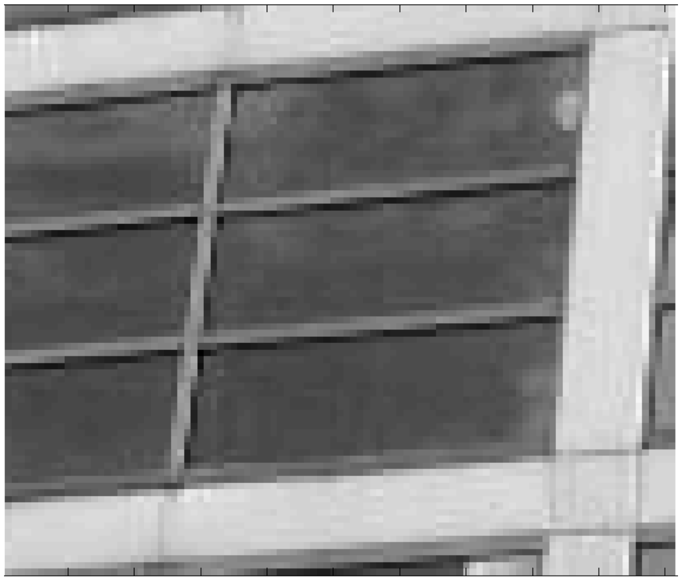


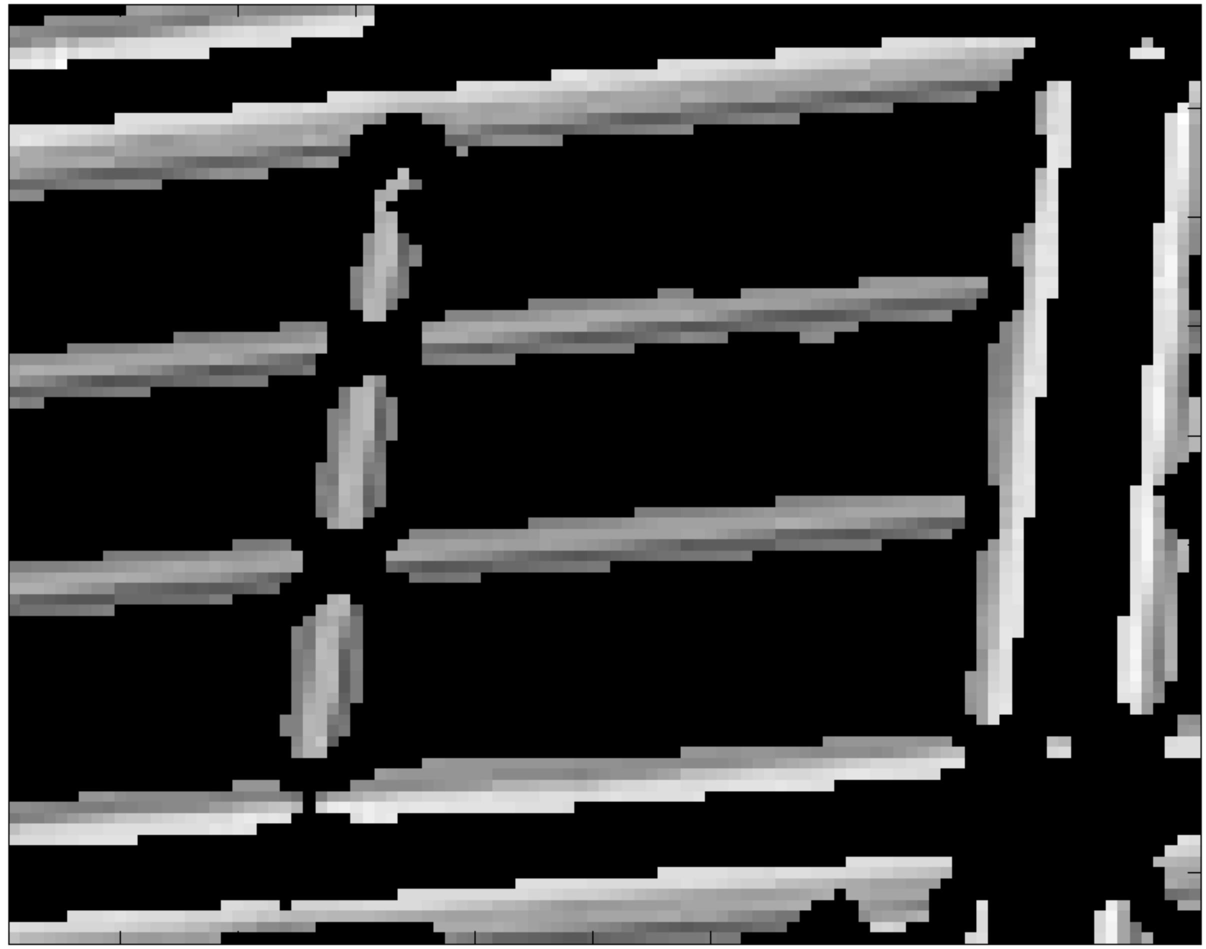


Harris R score.

Ix, Iy computed using Sobel operator
Windowing function w = Gaussian, sigma=1

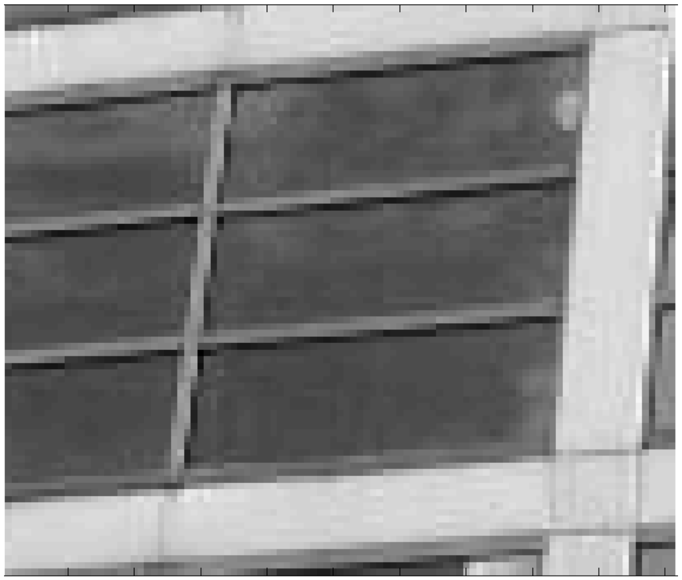
Corner Response Example

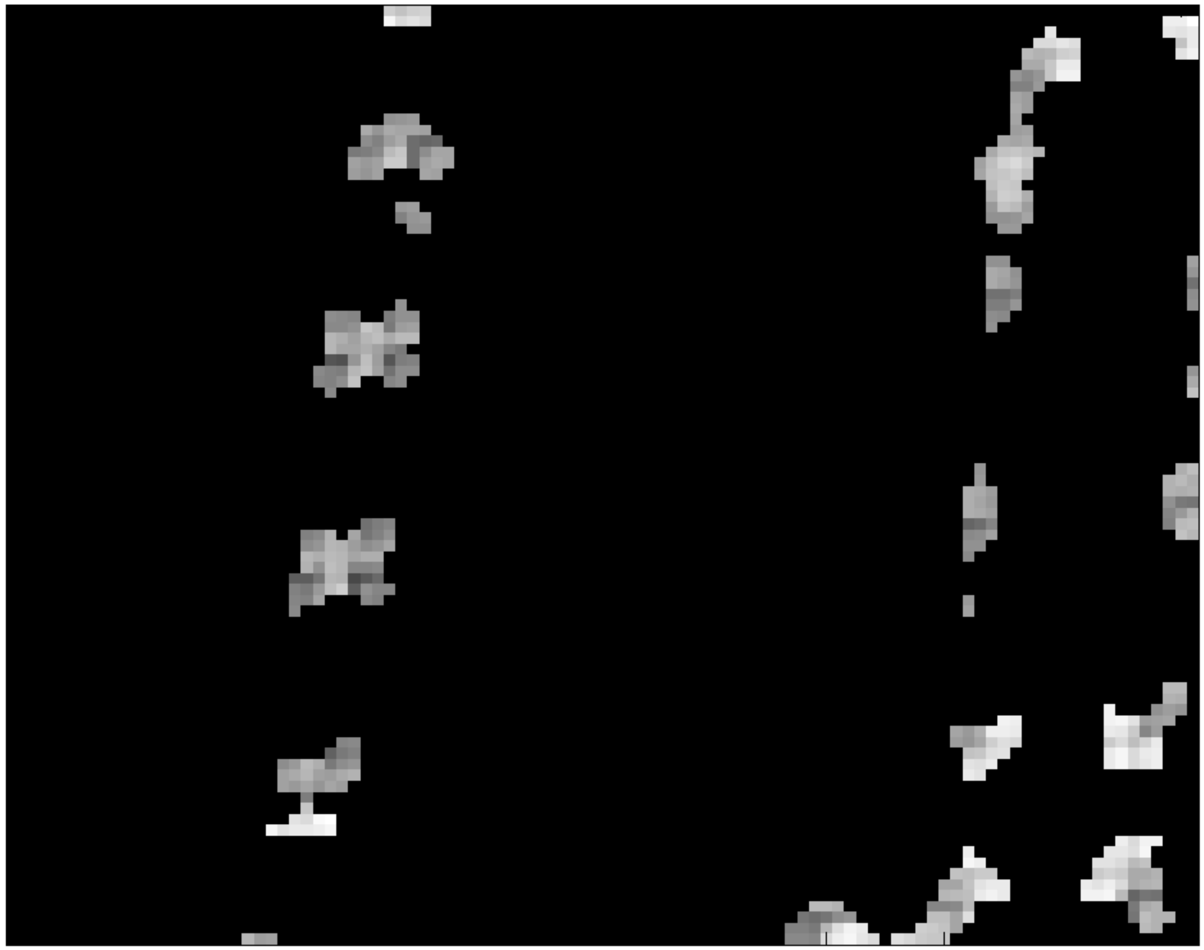




Threshold: R < -10000
(edges)

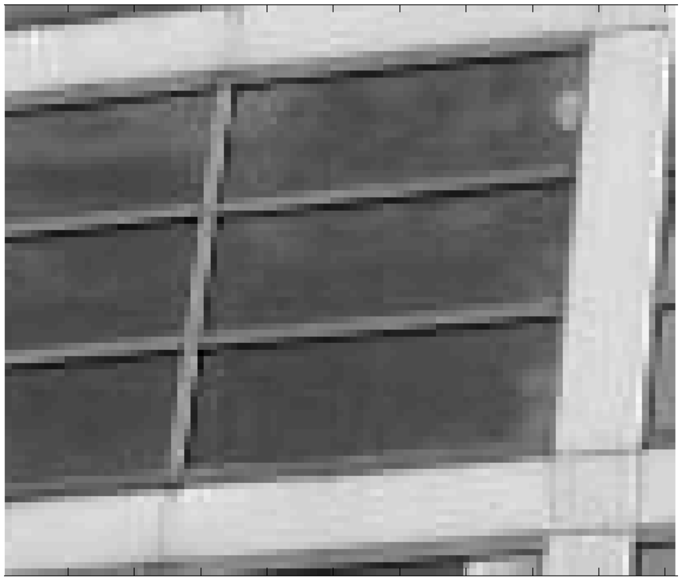
Corner Response Example

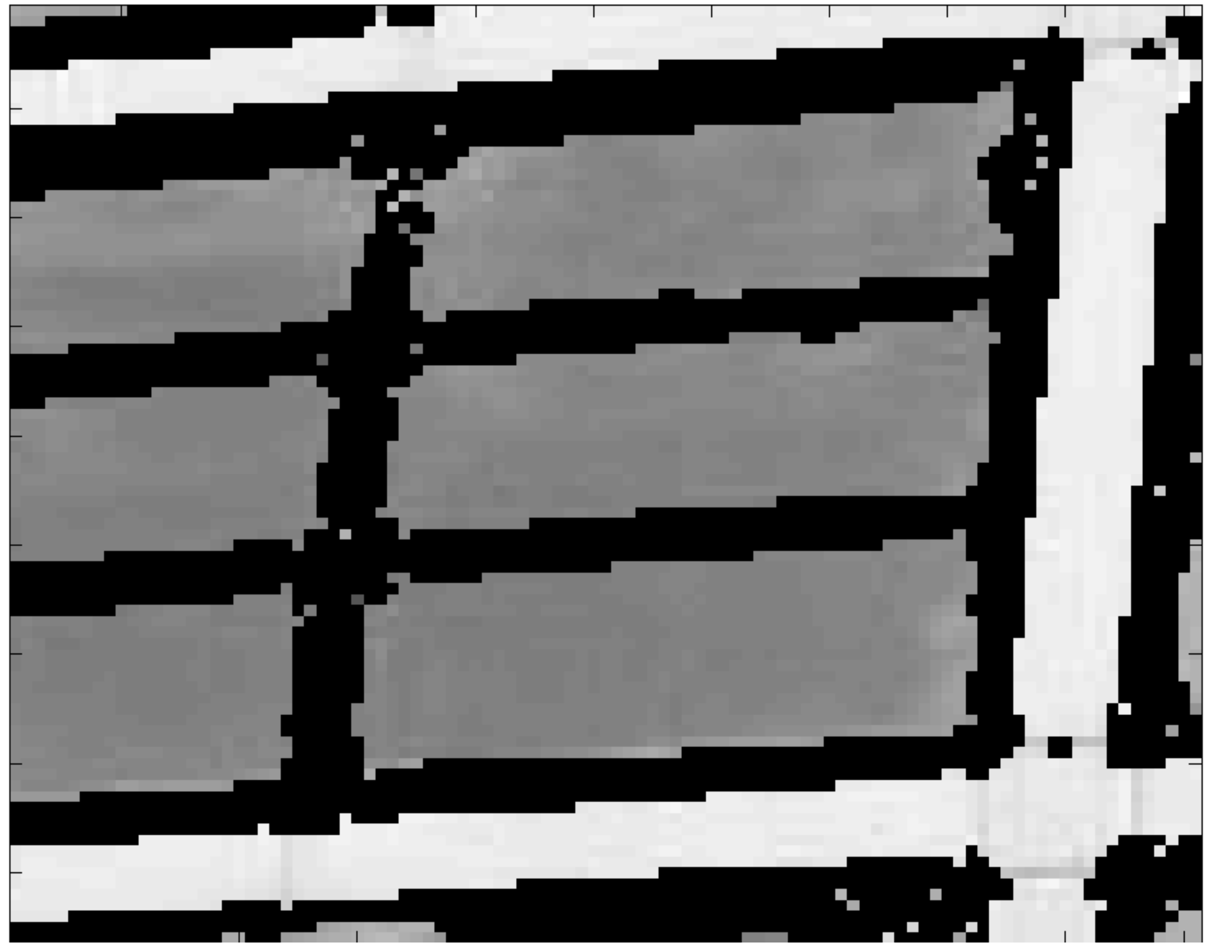




Threshold: > 10000
(corners)

Corner Response Example





Threshold: -10000 < R < 10000
(neither edges nor corners)

Harris Corner Detection Algorithm

1. Compute x and y derivatives of image

$$I_x = G_\sigma^x * I \quad I_y = G_\sigma^y * I$$

2. Compute products of derivatives at every pixel

$$I_{x2} = I_x . I_x \quad I_{y2} = I_y . I_y \quad I_{xy} = I_x . I_y$$

3. Compute the sums of the products of derivatives at each pixel

$$S_{x2} = G_{\sigma'} * I_{x2} \quad S_{y2} = G_{\sigma'} * I_{y2} \quad S_{xy} = G_{\sigma'} * I_{xy}$$

4. Define at each pixel (x, y) the matrix

$$H(x,y) = \begin{bmatrix} S_{x2}(x,y) & S_{xy}(x,y) \\ S_{xy}(x,y) & S_{y2}(x,y) \end{bmatrix}$$

5. Compute the response of the detector at each pixel

$$R = Det(H) - k(Trace(H))^2$$

6. Threshold on value of R.