Probability

(1) $1 \times \frac{14}{15} \times \frac{13}{15} \times \frac{12}{15} \times \frac{11}{15} \times \frac{10}{15} \times \frac{9}{15} \times \frac{8}{15} = \boxed{10.124\%}$

P(no student has to answer more than 1 Q)

(2) $\underline{5}\ \underline{4}\ \underline{7}\ \underline{6}\ \underline{5} \rightarrow$ # of ints that fit $= 5 \cdot 5 \cdot 4 \cdot 7 \cdot 6 = 4200$
total #s $= 100{,}000$

1,3,5,7,9   0,2,4,6,8

indiv prob. of # fitting $= 4200/100{,}000 = 0.042$

Bernoulli's: $n=8,\ p=0.042$, $k=5,\ q=0.958$ $\Rightarrow \binom{8}{5} 0.042^5 \cdot 0.958^3 = \boxed{0.000643\%}$

(3) *if $P(A|B) = P(A)$, then independent

— $P(A) = 1 - [P(\text{only 1 die shows } 4+) + P(\text{no die show } 4+)]$

$= 1 - 3\left(\overset{0\text{-}3}{\frac{3}{6}} \cdot \overset{0\text{-}3}{\frac{3}{6}} \cdot \overset{4\text{-}6}{\frac{3}{6}}\right) - 1\left(\overset{0\text{-}3}{\frac{3}{6}} \cdot \overset{0\text{-}3}{\frac{3}{6}} \cdot \overset{0\text{-}3}{\frac{3}{6}}\right) = 1 - \frac{3}{8} - \frac{1}{8} = 1 - \frac{1}{2} = \frac{3}{4}$

— $P(B) = \left(\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}\right) \times 6 = \frac{1}{36}$

6 diff cases

$P(A|B)$: when B occurs, all die are the same value; there are 6 ways for this to occur
— 3 of the 6 result in A $\therefore P(A) = \frac{1}{2}$ $\therefore P(A|B) = \frac{1}{2}$
$\therefore P(A|B) = P(A) \therefore$ $\boxed{\text{independent}}$

(4) Flush $= E \rightarrow \binom{4}{1} \cdot \binom{13}{5}$

1 suit   5 cards of the 13 diff. values

All cards $= S \rightarrow \binom{52}{5}$ ← the # of combos of choosing 5 cards

$P(E) = \frac{|E|}{|S|} \rightarrow \dfrac{\frac{4!}{1!3!} \cdot \frac{13!}{5!8!}}{\frac{52!}{5!47!}} \rightarrow \dfrac{4 \cdot 1287}{2598960} = \boxed{0.198079\%}$

$E[x] = \sum_k k\,P(k) \rightarrow = \frac{1}{p} = \frac{1}{0.198079}$

$\boxed{= \sim 505}$

(5) A = superstar played → 75%

$\bar{A}$ = superstar doesn't play → 50%

$P(B|A)$: if the superstar played, what is the probability they win → 70%

$P(B|\bar{A})$: if the superstar doesn't play, what is the prob. they win → 30%

$P(A|B)$: *what we want* if we won 4/5 games, what is the probability the superstar plays

$P(A|B) = \dfrac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})} \rightarrow \dfrac{0.75 \times \binom{5}{4}(0.7)^4(0.3)}{0.75 \times \binom{5}{4}(0.7)^4(0.3) + 0.25 \times \binom{5}{4}(0.3)^{3}}$

$= \boxed{0.874}$