counting

(1) unusual

(a) 5 unique letters:

Case 1: 1 fixed u → u_ _ _ _ $= \binom{4}{4} = 1$

Case 2: 2 fixed u → uu_ _ _ $= \binom{4}{3} = 4$

Case 3: 3 fixed u → uuu_ _ $= \binom{4}{2} = 6$

$\Big\}\ 1+4+6 = 11$

(b) Case 1: 1 u → $5! = 120$

Case 2: 2u → $5!/2! \times 4$ subsets → 240

Case 3: 3u → $5!/3! \times 6$ subsets → 120

$\Big\}\ 120+240+120 = 480$

(2) *unordered*

- 1^{st} card of each pair: $\binom{13}{2}$ ← 13 values in a deck, ← 2 cards in a pair
- Picking 2 cards from the 4 suits (to have a pair): $\binom{4}{2}$ ← for the 1^{st} value
- A same as above: $\binom{4}{2}$ ← for the second value
- final, lone card: $\binom{11}{1}$ ← only 11 values left
- choosing from the 4 suits: $\binom{4}{1}$

$$\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot \binom{11}{1} \cdot \binom{4}{1}$$

$$\frac{13!}{2!\,11!} \cdot \frac{4!}{2!\,2!} \cdot \frac{4!}{2!\,2!} \cdot \frac{11!}{10!\,1!} \cdot \frac{4!}{3!\,1!} = 78 \times 6 \times 6 \times 11 \times 4$$

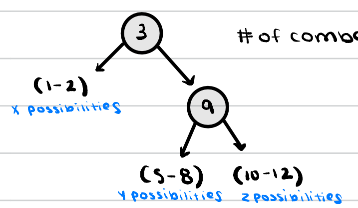
$$= \boxed{2{,}598{,}960}$$

(3) couple listens to no songs: stars = songs = 16, bars = # of couples − 1 → 5 $\Big\{\binom{16+6-1}{6-1} = \binom{21}{5}$

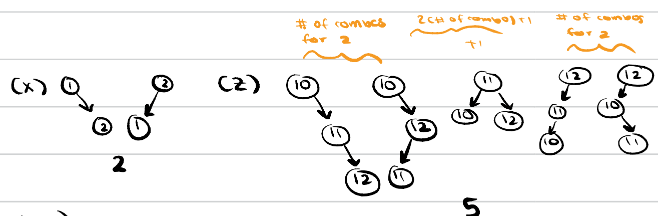
couple listens to one song: stars = songs = 16 − 1 (since couple listens once), bars = # of couples − 1 → 5 $\Big\{\binom{15+6-1}{6-1}$

$\Big\{\binom{21}{5} + \binom{20}{5} = \boxed{35853}$

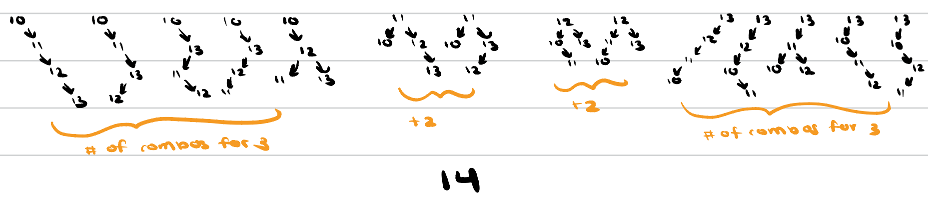
(4)



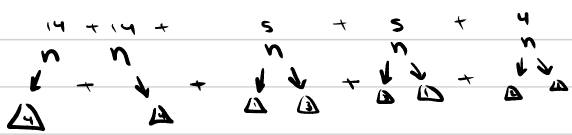
# of combos = x · y · z



(4 nodes)



(5 nodes)



→ 42

∴ $\boxed{\text{total} = 2 \times 42 \times 5 = 420}$

(5) Case 1: 4 nurses serving

(7,1,1,1), (6,2,1,1), (5,3,1,1), (5,2,2,1), (4,4,1,1), (4,3,2,1), (4,2,2,2), (3,2,2,3), (3,3,3,1)

Case 2: 3 nurses serving

(8,1,1), (7,2,1), (6,3,1), (6,2,2), (5,4,1), (5,3,2), (4,4,2), (3,3,4)

$\boxed{17}$