Optimal Packet Length for Throughput Maximization in Correlated Multi-User Channels

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Abstract—**In this paper, a multi-user uplink channel with correlated Rayleigh fading coefficients is considered. Optimal data packet length is derived for throughput maximization of the system with stop and wait automatic repeat request (SW-ARQ). For the throughput formulation of the SW-ARQ system, the packet error rate (PER) is analytically derived using a two-state Markov chain modelling of the signal-to-interference ratio (SIR) of the time-varying channel. For the PER formulation, second-order channel statistics including average level crossing rate (LCR) and average outage duration (AOD) are derived. Numerical results indicate the accuracy of the obtained theoretical results. The effect of increasing the number of interferers on the throughput is investigated w.r.t given SIR threshold.**

I. Introduction

For adaptive transmission, packet size can play an important role in wireless networks. Therefore, adapting data packet size along with automatic repeat request (ARQ) is widely deployed in wireless networks to improve the system throughput. Transmissions in wireless networks are usually assumed to have independent and identically distributed (i.i.d.) characteristics, which may not hold in many scenarios. One motive for such assumption can be the perfect interleaving. Perfect interleaving is difficult to accomplish in practical systems, and also increases the complexity and delay. Oppositely, correlated channel structures can be exploited to improve the system performance and decrease the complexity. For these reasons, in this paper, optimal data packet length is derived for a multi-user system with ARQ under flat Rayleigh fading assumption taking into consideration channel time correlation properties.

In the literature, outage probability is commonly linked to the system performance. However in practice, it is the average outage duration (AOD) that provides highly accurate results for system design and analysis. Similarly, average level crossing rate (LCR) is another important second-order channel statistics, which is critical for real-time applications. By using these channel statistics, which capture the channel time correlation, the probabilities of correlated error structures can be evaluated. Such probabilities provide important measures for the selection of transmission modes such as transmission rates, modulation schemes and data packet lengths, which are the point of interest in this paper.

Multi-user communications based on second-order channel statistics is less studied in literature. In [1], LCR and AOD are derived for a point-to-point system subject to frequency-selective fading where the receiver deploy maximal-ratio combining (MRC). In [2], again only LCR and AOD are derived for a receiver with a single antenna, however, the system is a multi-user system subject to flat Rayleigh fading. In our work, we consider multi-user system with unequal mobility speeds for every transmitter with unequal transmitting powers having single antenna each, and multiple antennas at the receiver which deploys MRC. We derive an approximate LCR expression for the signal-to-interference ratio (SIR) of the system under consideration. Then we model the system using a two-state Markov chain model utilizing the derived LCR expression to derive an expression for the packet error rate (PER).

Many works have been done in literature on ARQ-based systems due to their practical importance. Different ARQ schemes, mainly stop and wait (SW) ARQ, go-back-N (GBN) ARQ and selective-repeat (SR) ARQ, have been analyzed for different AWGN and fading channels. In [3], the authors consider a point-to-point system and evaluate the throughput without data packet length optimization. In [4], again a point-to-point system is studied under slow Rayleigh flat fading and an expression for the optimum data packet length maximizing the throughput was derived. In our work, we derive the optimum data packet length for the *multi-user* system described above when deploying SW-ARQ scheme, via utilizing the Finite State Markov Chain (FSMC) Model developed throughout the paper. We express the optimum data packet length in terms of the system's parameters and discuss their effects on it.

II. System Model

We consider a mobile radio system with $N+1$ transmitter nodes and a single receiver node. Each transmitter and the receiver has a single antenna and L antennas, respectively. N transmitter nodes are interfering users, thus there is a single desired transmitter. All paths in the system are subject to flat Rayleigh fading with unit average power. The desired transmitter power and the interfering transmitter power is p_{D} and p_n, $n \in \mathcal{N} \triangleq \{1, \ldots, N\}$, respectively. This model can be regarded as each transmitter has a unity power, while the fading paths have different average powers, i.e. non-i.i.d fading paths. The receiver performs maximal-ratio combining (MRC) over the received signal. We consider interference-limited scenario, hence the received noise, being negligible w.r.t. the interference, is not considered. This model can be generalized to any system with L i.i.d. diversity paths, where again the receiver performs MRC over the L diversity paths. The diversity can be achieved in time or frequency, not necessarily in space.

Let $\mathbf{h}_{\mathrm{D}}(t)$ and $\mathbf{h}_n(t)$ denote the $L \times 1$ channel vectors between the receiver and the desired user and n^{th} interferer, respectively. The received signal before applying MRC, $\mathbf{y}(t)$, is given by

$$\mathbf{y}(t) = \mathbf{h}_{\mathrm{D}}(t)s_{\mathrm{D}}(t) + \sum_{n=1}^{N} \mathbf{h}_n(t)s_n(t) \tag{1}$$

where $s_{\mathrm{D}}(t)$ and $s_n(t)$ are the transmitted symbols at time instant t from the desired user and the n^{th} interferer, respectively, with average powers $E\{|s_{\mathrm{D}}|^2\} = p_{\mathrm{D}}$ and $E\{|s_n|^2\} = p_n,\ \forall n \in \mathcal{N}$. Let $\alpha_{\mathrm{D},l}(t),\ \forall l \in \mathcal{L} \triangleq \{1,\ldots,L\}$, denote the Rayleigh distributed random envelope of the received signal of the desired user over the l^{th} diversity path. The output signal-to-interference ratio (SIR) $\gamma(t)$ of the system is given by

$$\gamma(t) = \frac{p_{\mathrm{D}}\|\mathbf{h}_{\mathrm{D}}(t)\|^2}{\sum\limits_{n=1}^{N} p_n|\mathbf{w}^H\mathbf{h}_n(t)|^2} = \frac{p_{\mathrm{D}}\sum\limits_{l=1}^{L}\alpha_{\mathrm{D},l}^2(t)}{\sum\limits_{n=1}^{N} p_n\ \alpha_{\mathrm{I},n}^2(t)} = \frac{r_{\mathrm{D}}^2}{r_{\mathrm{I}}^2}, \tag{2}$$

where r_{D}^2 and r_{I}^2 denote the numerator and denominator of the SIR, respectively, $\mathbf{w} = \mathbf{h}_{\mathrm{D}}/\|\mathbf{h}_{\mathrm{D}}\|$ is the receive filter, and $\alpha_{\mathrm{I},n} = |\mathbf{w}^H\mathbf{h}_n|$. Then, $\alpha_{\mathrm{D},l}^2(t)$ and $\alpha_{\mathrm{I},n}^2(t)$ are standard exponential random processes with unit means [5].

Given an SIR threshold $\gamma = \gamma_{\mathrm{th}}$ for the desired user, the level crossing rate (LCR) is defined as [6]

$$\mathrm{LCR}(\gamma_{\mathrm{th}}) = \int\limits_{x=0}^{\infty} x f_{\dot{\gamma}(t),\gamma(t)}(x,\gamma_{\mathrm{th}})dx, \tag{3}$$

where $\dot{\gamma}(t)$ is the time derivative of the SIR $\gamma(t)$, and $f_{\dot{\gamma}(t),\gamma(t)}(.,.)$ is the joint probability density function (PDF) of $\dot{\gamma}(t)$ and $\gamma(t)$.

The average duration for which the SIR remains below a threshold, namely the average outage duration (AOD), is defined as [6]

$$\mathrm{AOD}\left(\gamma_{\mathrm{th}}\right) = \frac{F_{\gamma(t)}\left(\gamma_{\mathrm{th}}\right)}{\mathrm{LCR}\left(\gamma_{\mathrm{th}}\right)}, \tag{4}$$

where $F_{\gamma(t)}\left(\gamma_{\mathrm{th}}\right) = \mathrm{Prob}(\gamma(t) \leq \gamma_{\mathrm{th}})$ is the cumulative distribution function (CDF) of $\gamma(t)$. In the next section, the LCR and AOD of the system are derived.

III. Channel Statistics and Modeling

A. Level Crossing Rate (LCR)

The received desired signal power, the numerator of (2), is comprised of equal power components since the paths from the intended transmitter to the receiver are assumed i.i.d. with $E\{\alpha_{\mathrm{D},l}^2\} = 1,\ \forall l$, and the transmitting power p_{D} is the same at each received diversity path. On the other hand, although the paths from the interferers to the receiver are i.i.d., each interferer has a different transmitting power p_n, thus the components of the total interference signal at the receiver are of unequal average powers. This is obvious from the denominator of (2) where $\alpha_{\mathrm{I},n}^2,\ \forall n$, are of equal unit mean while p_n are of different values.

In [1], Fukawa et al. derived an LCR expression for a lower bound of the SIR of a point-to-point system subject to multipath fading with MRC for two different cases: (1) *Equal Average Power (EAP)* where $p_{\mathrm{D},l} = p_D, \forall\ l \in \mathcal{L}$ and $p_n = p_{\mathrm{I}}, \forall\ n \in \mathcal{N}$, and (2) *Unequal Average Power (UAP)* where $p_{\mathrm{D},l} \neq p_{\mathrm{D},j}$ for $l \neq j$ and $p_n \neq p_m$ for $n \neq m$. The LCR expression is exact for the EAP case while it is an approximation for the UAP case. We utilize the approach and findings of [1] to find the LCR of our system. For space considerations, we list the main equations only, and refer the interested reader to follow the derivations in [1]. The LCR in (3) can be re-written as

$$\mathrm{LCR}(\gamma_{\mathrm{th}}) = \int\limits_{\dot{r}_{\mathrm{D}}=-\infty}^{\infty} \int\limits_{\dot{r}_{\mathrm{I}}=-\infty}^{\dot{r}_{\mathrm{D}}/\sqrt{\gamma_{\mathrm{th}}}} \int\limits_{r_{\mathrm{I}}=0}^{\infty} \left(\dot{r}_{\mathrm{D}} - \sqrt{\gamma_{\mathrm{th}}}\dot{r}_{\mathrm{I}}\right) \times f_{r_{\mathrm{D}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}}}\left(\sqrt{\gamma_{\mathrm{th}}}r_{\mathrm{I}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}}\right)\ dr_{\mathrm{I}}\ d\dot{r}_{\mathrm{I}}\ d\dot{r}_{\mathrm{D}}, \tag{5}$$

where $f_{r_{\mathrm{D}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}}}(r_{\mathrm{D}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}})$ is the joint PDF of the envelopes of the total desired and total interference signals and their time derivatives. It is given by

$$f_{r_{\mathrm{D}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}}}(r_{\mathrm{D}},\dot{r}_{\mathrm{D}},r_{\mathrm{I}},\dot{r}_{\mathrm{I}}) = f_{r_{\mathrm{D}},\dot{r}_{\mathrm{D}}}(r_{\mathrm{D}},\dot{r}_{\mathrm{D}})f_{r_{\mathrm{I}},\dot{r}_{\mathrm{I}}}(r_{\mathrm{I}},\dot{r}_{\mathrm{I}}) \tag{6}$$

since the desired and interfering signals are independent. The joint PDFs in the above equation can be obtained as

$$f_{r_{\mathrm{D}},\dot{r}_{\mathrm{D}}}(r_{\mathrm{D}},\dot{r}_{\mathrm{D}}) = \frac{\sqrt{2}\ r_{\mathrm{D}}^{2L-1}\ e^{-\frac{r_{\mathrm{D}}^2}{p_{\mathrm{D}}}}}{\pi^{3/2}\ (L-1)!\ p_{\mathrm{D}}^{L+1/2}f_{\mathrm{D}}}\ e^{-\frac{\dot{r}_{\mathrm{D}}^2}{2\pi^2 p_{\mathrm{D}} f_{\mathrm{D}}^2}} \tag{7}$$

$$f_{r_{\mathrm{I}},\dot{r}_{\mathrm{I}}}(r_{\mathrm{I}},\dot{r}_{\mathrm{I}}) \approx \frac{\sqrt{2}\ e^{-\frac{\dot{r}_{\mathrm{I}}^2}{2\dot{\sigma}_{\mathrm{I}}^2}}}{\sqrt{\pi\dot{\sigma}_{\mathrm{I}}^2}} \sum_{n=1}^{N} \frac{\delta_n r_{\mathrm{I}}}{p_n} e^{-\frac{r_{\mathrm{I}}^2}{p_n}}, \tag{8}$$

where

$$\delta_n = \prod_{\substack{k=1\\k\neq n}}^{N} \frac{p_n}{p_n - p_k}, \qquad \dot{\sigma}_{\mathrm{I}}^2 = \frac{\pi^2 f_{\mathrm{I,max}}^2 \sum\limits_{n=1}^{N} p_n^2}{\sum\limits_{n=1}^{N} p_n} \tag{9}$$

and $f_{\mathrm{I,max}} = \max_n(f_{\mathrm{I},n})$. $f_{\mathrm{D},l}$, and $f_{\mathrm{I},n}, \forall l \in \mathcal{L}$ and $\forall n \in \mathcal{N}$, denote the Doppler frequency of the l^{th} and n^{th} component of the desired and interference signals. While $f_{\mathrm{I},n}$ are different for every n, $f_{\mathrm{D},l} = f_{\mathrm{D}},\ \forall l \in \mathcal{L}$. This is because the interfering signals belong to different users moving with different speeds, while the desired signal is transmitted by a single user.

Note that (8) follows from approximating the variance $E\{\dot{r}_{\mathrm{I}}^2\}$ by $\dot{\sigma}_{\mathrm{I}}^2$, ignoring the fact that $E\{\dot{r}_{\mathrm{I}}^2\}$ is dependent on r_{D} and its components. This approximation was found to be very close to the exact case in [1], and we found the same applies for our system but the figure is omitted due to space limitation. Note that one main difference between our system and the system in [1] is that ours have multiple interferers possessing multiple maximum Doppler frequencies due to different speeds, while in [1] there is one maximum Doppler frequency only since all components belong to the same interfering user. The term maximum frequency refers to the Doppler frequency associated to the path of zero angle of arrival, and $f_{\text{I,max}}$ is the maximum of all such frequencies.

By substituting (6) in (5), we find $\text{LCR}(\gamma_{\text{th}}) = AB$, where

$$A = \int_{\dot{r}_{\text{D}}=-\infty}^{\infty} f(\dot{r}_{\text{D}}) \int_{\dot{r}_{\text{I}}=-\infty}^{\dot{r}_{\text{D}}/\sqrt{\gamma_{\text{th}}}} (\dot{r}_{\text{D}} - \sqrt{\gamma_{\text{th}}}\dot{r}_{\text{I}})\, f(\dot{r}_{\text{I}})\; d\dot{r}_{\text{I}}\; d\dot{r}_{\text{D}}$$

$$B = \int_{r_{\text{I}}=0}^{\infty} f_{r_{\text{D}}}(\sqrt{\gamma_{\text{th}}} r_{\text{I}}) f(r_{\text{I}})\; dr_{\text{I}}. \tag{10}$$

Here the subscripts of the PDFs are dropped when not needed to simplify the notations. By following similar steps as in Appendix B in [1], A can be given by

$$A = \sqrt{\frac{\pi^2 f_{\text{D}}^2 p_{\text{D}} + \gamma_{\text{th}} \dot{\sigma}_{\text{I}}^2}{2\pi}} \tag{11}$$

However, for the derivation of B, a different approach from [1] is needed as shown below

$$B = \frac{2\, \gamma_{\text{th}}^{L-0.5}}{(L-1)!\; p_{\text{D}}^L} \sum_{n=1}^{N} \frac{2\delta_n}{p_n} \int_{r_{\text{I}}=0}^{\infty} r_{\text{I}}^{2L}\; e^{-\frac{\gamma_{\text{th}} p_n + p_{\text{D}}}{p_{\text{D}} p_n} r_{\text{I}}^2}\; dr_{\text{I}} \tag{12}$$

$$= \frac{2\sqrt{p_{\text{D}}}\gamma_{\text{th}}^{L-\frac{1}{2}}\Gamma(L+\frac{1}{2})}{(L-1)!} \sum_{n=1}^{N} \frac{\delta_n p_n^{L-\frac{1}{2}}}{(\gamma_{\text{th}} p_n + p_{\text{D}})^{L+\frac{1}{2}}}, \tag{13}$$

where [7, Sec. 3.326, Eq. 2] is used to solve the integral in (12), and $\Gamma(L+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^L}(2L-1)!!$ is the Gamma function as given in [7, Sec. 8.339, Eq. 2].

Hence an approximate closed-form expression for the LCR of the SIR given in (2) is obtained by the multiplication of (11) and (13).

B. Average Outage Duration (AOD)

The AOD can now be easily found by substituting in (4) for the LCR expression derived above and the CDF $F(\gamma_{\text{th}})$ which is given by [8] as

$$F_\gamma(\gamma_{\text{th}}) = \sum_{n=1}^{N} \delta_n \frac{(\gamma_{\text{th}} p_n / p_{\text{D}})^L}{(1 + \gamma_{\text{th}} p_n / p_{\text{D}})^L} \tag{14}$$

C. Two-State Markov Chain Model

To find the throughput, the packet error rate (PER) of the system needs to be derived first. In this section, PER is derived for uncoded transmission. The PER calculation requires a long computational time in a conventional computer simulation since it requires the demodulation of all bits in the packet to determine if all bits in the packet are received correctly, else the packet is considered erroneous. To reduce the computational time, it is considered that the data packet is received correctly only if the received SIR is above a certain threshold γ_{th} during the whole data packet duration T_{p} [1], as shown in Fig. 1. If at any instant, the SIR drops below the threshold, the received data packet is considered to be erroneous. This process is modeled by using the two-state Markov model described in [1]. The system is in good state G if the SIR is greater than or equal to the threshold γ_{th}, which can identify a target PER. The system is in bad state B otherwise. Hence, PER P_e is the probability of being in state B. In other words, $P_e = P_b = 1 - P_g$, where P_b and P_g are the probabilities of being in states B and G, respectively, also known as steady state probabilities. P_g is given as follows [1],

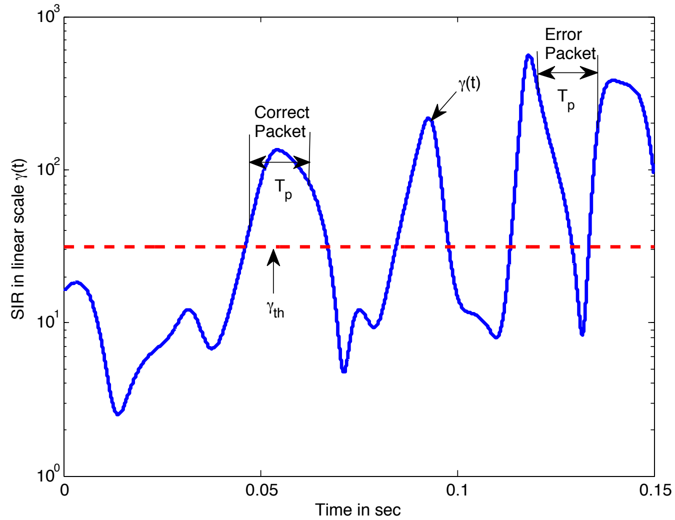

Fig. 1. The received SIR over a time interval where $L = 2, N = 2, \gamma_{\text{avg}} = \gamma_{\text{th}} = 15$ dB, $f_{\text{D}} = f_{\text{I,max}} = 35$ Hz.

$$P_g = P_{\text{CF}}\; e^{-N_c T_{\text{p}}/P_{\text{CF}}} \tag{15}$$

where $P_{\text{CF}} = 1 - F_\gamma(\gamma_{\text{th}})$ is the complimentary CDF and to simplify the notation, define $N_c = \text{LCR}(\gamma_{\text{th}})$. Substituting the LCR and the CDF derived in previous subsections, the PER of the uncoded system is obtained. It is worth mentioning that the PER used in this paper considers the channel time variations since it is derived using the FSMC model which captures the channel time variations and correlation, second-order channel statistics, through LCR.

IV. ARQ-Based System

A. ARQ Basics

Many practical wireless packet systems deploy ARQ schemes to improve the transmission reliability. In this section, the stop and wait ARQ (SW-ARQ) scheme with unlimited number of retransmissions is deployed in the system described in section II. In such a scheme, during a single ARQ round, the transmitter transmits a packet and waits for an acknowledgement (ACK) from the receiver. If the transmitter receives an ACK, then the transmitter transmits the next packet, otherwise it retransmits the last packet. The duration of a single ARQ round is given by $T_{\text{ARQ}} = T_s(m_{\text{t}} + m_{\text{ov}})$, where T_s is the transmitted symbol duration, and m_{t} is the number of symbols per data packet, i.e., data packet length. m_{ov} is the equivalent number of symbols induced by the ARQ protocol overhead, e.g., ACK transmission duration and the guard interval for processing.

For a SW-ARQ scheme with unlimited number of retransmissions, assuming a PER P_e and that the receiver detects any data packet error that occurs, then the average number of retransmissions $\bar{M}$ is given as [3]

$$\bar{M} = (1 - P_e) \sum_{k=1}^{\infty} k(P_e)^{k-1} = \frac{1}{1 - P_e}. \tag{16}$$

Then the throughput R of such a system is given by [4]

$$R = \frac{m_{\rm r}}{T_{\rm tot}} = \frac{m_{\rm t}\,(1 - P_e)}{(m_{\rm t} + m_{\rm ov})\,T_s}, \tag{17}$$

where $m_{\rm r}$ is the number of user's information symbols, and $T_{\rm tot}$ is the total transmission time which is given by

$$T_{\rm tot} = \frac{m_{\rm r}}{m_{\rm t}}\ \bar{M}\ T_{\rm ARQ}. \tag{18}$$

B. Optimal Packet Length

Since $P_e = 1 - P_g$, then from (15) and (17), the throughput can be re-written as

$$R(m_{\rm t}) = \frac{P_{\rm CF}\ m_{\rm t}\ \exp\left(-\frac{N_c T_s}{P_{\rm CF}} m_{\rm t}\right)}{(m_{\rm t} + m_{\rm ov})\,T_s}, \tag{19}$$

where the data packet duration is $T_{\rm p} = m_{\rm t} T_s$. Thus we can see that the throughput of the ARQ-based uncoded multiuser system under correlated fading channel is a function of the data packet length $m_{\rm t}$. In the following, we find the optimal data packet length that maximizes the throughput of such a system as function of the system parameters. We take the derivative of $R(m_{\rm t})$ w.r.t. $m_{\rm t}$ and equate it to zero as follows

$$\frac{\partial R(m_{\rm t})}{\partial m_{\rm t}} = \frac{e^{-N_c T_s / P_{\rm CF}}}{(m_{\rm t} + m_{\rm ov})^2} \left(\frac{m_{\rm ov} P_{\rm CF}}{T_s} - N_c m_{\rm t}^2 - N_c m_{\rm ov} m_{\rm t} \right) = 0 \tag{20}$$

Then, by solving the above quadratic equation, the optimal data packet length $m_{\rm opt}$ is given as

$$m_{\rm opt} = \frac{m_{\rm ov}}{2} \left(\sqrt{1 + \frac{4P_{\rm CF}}{m_{\rm ov} N_c T_s}} - 1 \right), \tag{21}$$

where $P_{\rm CF}$ and N_c are functions of $\gamma_{\rm th}$ and the systems characteristics: powers of the desired and interfering users, $p_{\rm D}$ and p_n, respectively, the maximum Doppler frequencies of the desired and interfering users, $f_{\rm D}$ and $f_{\rm I,max}$, respectively, the number of interferers N, and the number of diversity paths L.

C. Discussion

From (19), the data packet length $m_{\rm t}$ contributes in two functions that have opposite behaviors. To elaborate, we re-write the throughput as $R(m_{\rm t}) = \frac{P_{\rm CF}}{T_s} f_1(m_{\rm t}) f_2(m_{\rm t})$, where

$$f_1(m_{\rm t}) = \frac{m_{\rm t}}{m_{\rm t} + m_{\rm ov}}, \qquad f_2(m_{\rm t}) = e^{-\frac{N_c T_s}{P_{\rm CF}} m_{\rm t}} \tag{22}$$

It can be seen that $f_1(m_{\rm t})$ is monotonically increasing while $f_2(m_{\rm t})$ is monotonically decreasing with $m_{\rm t}$. However, their steepness, i.e. how fast the curve increases or decreases, depends on the values of $m_{\rm ov}$ and $N_c T_s / P_{\rm CF}$, respectively. It can be seen that as $m_{\rm ov}$ increases, the slope (steepness) of $f_1(m_{\rm t})$ decreases, thus taking longer time to reach the asymptotic value 1. This is confirmed by (21), where for a fixed $N_c T_s / P_{\rm CF}$, as $m_{\rm ov}$ increases, the term inside the brackets decreases, while that outside it increases. However, the effect of the latter is greater, thus resulting in increasing $m_{\rm opt}$ as $m_{\rm ov}$ increases. This agrees with intuition, as it makes sense to increase the data packet length as the overhead increases. Note that this does not contradict with the fact that as we increase the data packet length, we suffer from more channel changes within the packet leading to increasing the probability of error. In fact, this is true and here comes the role of finding an optimum data packet length that is long enough to overcome the large overhead and small enough to reduce the PER.

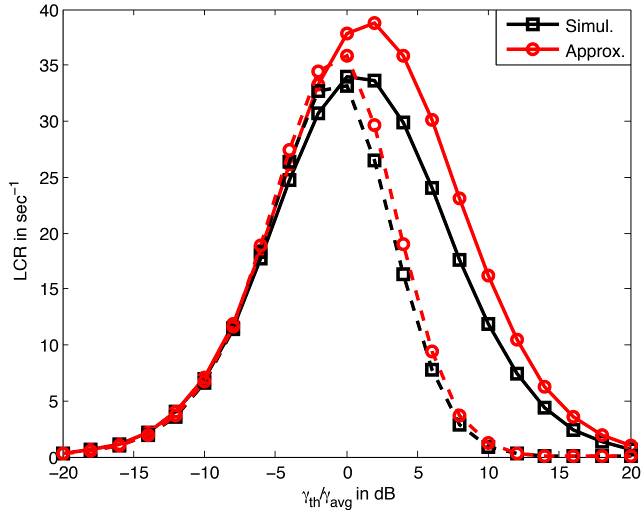

Fig. 2. LCR comparison and validation. Solid lines: $L = 2,\ N = 2$. Dashed lines: $L = 2,\ N = 4$. For both systems: $f_s = 1$ Msymbols/sec, $f_{\rm D} = f_{\rm I,max} = 35$ Hz, and $\gamma_{\rm avg} = 10$ dB.

On the other hand, as $m_{\rm ov}$ decreases, $m_{\rm opt}$ decreases as well, till the point where the throughput is monotonically decreasing, depending on the relative values of $m_{\rm ov}$ and $N_c T_s / P_{\rm CF}$. In this case, the optimum data packet length would be the minimum data packet length allowed by the system.

From (21), for a fixed $m_{\rm ov}, N_c$, and $P_{\rm CF}$, as T_s decreases, i.e. the data rate increases, $m_{\rm opt}$ increases. This is because fixed N_c and $P_{\rm CF}$ correspond to fixed Doppler frequencies in which case as the symbol time decreases, we can have more symbols within one data packet such that the data packet *time* duration, $T_{\rm p}$, is still the same and the normalized data packet length, $T_{\rm p} f_{\rm d}$, is the same as well, thus experiencing the same type of fading as with larger T_s and less number of symbols, where $f_{\rm d} = \max(f_{\rm D}, f_{\rm I,max})$.

With all other parameters fixed, as the SIR threshold $\gamma_{\rm th}$ increases, the complimentary CDF $P_{\rm CF}$ increases monotonically while N_c increases till around the average SIR value of the received signal then starts decreasing as the threshold $\gamma_{\rm th}$ further increases as shown in Fig. 2. It is not straightforward to deduce the relation between $m_{\rm opt}$ and $\gamma_{\rm th}$ and it needs further investigation through (21).

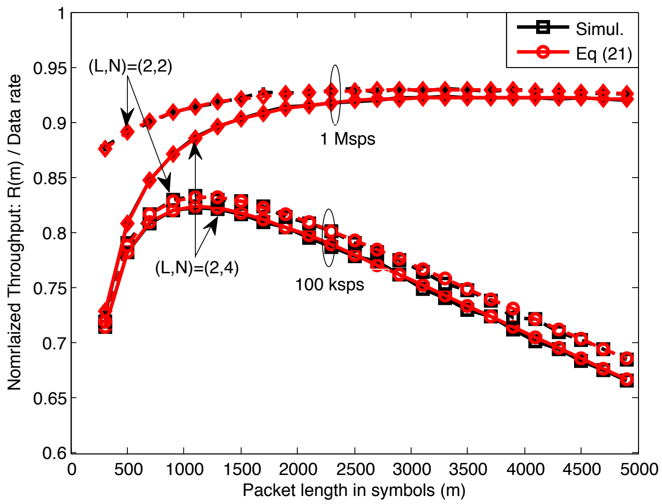

Fig. 3. Throughput comparison and validation. For both systems: $\gamma_{\text{th}} = 0$ dB, $\gamma_{\text{avg}} = 10$ dB, and $f_{\text{D}} = f_{\text{I,max}} = 35$ Hz.

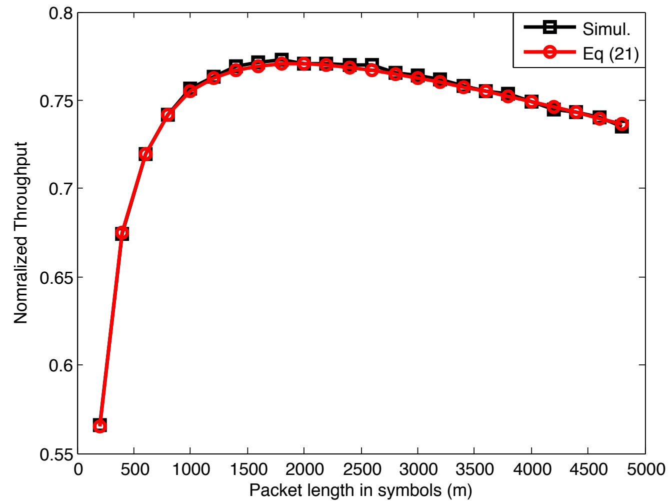

Fig. 4. Throughput validation, where $f_s = 1$ Msymbols/sec, $L = 2$, $N = 2$, $\gamma_{\text{th}} = 10$ dB, $\gamma_{\text{avg}} = 15$ dB, and $f_{\text{D}} = f_{\text{I,max}} = 35$ Hz.

V. Numerical Results

To validate the expressions obtained for the LCR, and the optimum data packet length maximizing the throughput in an ARQ-based system, we evaluate these expressions and compare them with the simulation results of such a system. Simulations are conducted, using Matlab R2014a. Each channel process, i.e. $\alpha_{\text{D},l}(t)$ and $\alpha_{\text{I},n}(t)$, $\forall l, n$, is simulated using modified Jake's model [9] to ensure that each path is independent of the others. 500 packets with different lengths are generated, for at least 500 Monte Carlo simulations and the overhead length m_{ov} is set to 100 symbols.

Fig. 2 compares the approximated LCR expression obtained in section III-A and the LCR obtained from simulations for two different systems with $L = 2$ and $N = 2, 4$ where the data rate f_s is set to 1 Msymbols/sec (Msps). The LCR is plotted versus the SIR threshold, γ_{th}, normalized to the average SIR of the system which is defined as [1] $\gamma_{\text{avg}} = Lp_{\text{D}}/\sum_{n=1}^{N} p_n$. In both systems the desired user exhibits 35 Hz maximum Doppler frequency, corresponding to a vehicle moving at a speed of about 20 km/hr in a system with carrier frequency 1.9 GHz. The interferers exhibit maximum Doppler frequencies of values 30 and 35 Hz, and $3, 30, 15$ and 35 Hz for the two- and four-interferer systems, respectively. The interferers have unequal average powers, however, the two-interferer system's powers are of far values (1 and 0.1), while the four-interferer system's powers are of close values ($1.2, 1.59, 1.98$, and 2.31). The power of the desired user is chosen such that the average SIR is 10 dB. The figure shows that the approximation's accuracy is dependent on the system's parameters. In particular, the approximate LCR of the 2-interferer and 4-interferer systems have a maximum deviation from the simulation LCR of about 6.14 sec^{-1} and 3.13 sec^{-1} at normalized threshold SIR of 6 dB and 2 dB, respectively. However, it is noticed that for low values of normalized threshold SIR (around -5 dB), the approximation is highly accurate, then accuracy decreases as the threshold approaches the average SIR, then increases once more at high SIR thresholds.

In Fig. 3, the throughput normalized to the data rate is plotted for both systems described above for normalized SIR threshold of -10 dB, for two data rates, namely 1 Msps and 100 ksps. It can be shown that (19) matches the simulation results very well although it depends on the LCR approximation. This is true because at the chosen normalized SIR threshold (-10 dB), both the approximate LCR and the simulated one agree to a great extent as shown in Fig. 2. The figure also shows that there is indeed an optimum data packet length maximizing the throughput, and that the value obtained from (21) matches that obtained from simulation. In particular, the theoretical and simulation m_{opt} were found to be 1121 and 1133 symbols, respectively, for the two-interferer system, while for the four-interferer system they are 1159 and 1171 symbols, respectively. Although the throughput is normalized in the figure, it is shown that as the data rate increases, the normalized throughput increases. This is because (19) shows that the dependence of the normalized throughput on the data rate ($1/T_s$) still exists within the PER expression and that as the data rate increases, there is less variation in the signal level for a fixed data packet length and Doppler frequencies. Hence, the probability of maintaining the whole packet above the threshold level increases and PER decreases increasing the (normalized) throughput. From the previous argument, it seems that reducing the data packet length and increasing the data rate is the optimum choice, however, the existence of the ARQ overhead reduces the throughput as the data packet length is reduced, thus finding the optimum data packet length through (19) is essential. Indeed, Fig. 3 shows that as the data rate increases, the optimum data packet length increases.

Fig. 4 plots the normalized throughput of the two-interferer system with data rate 1 Msps, but the average SIR is 15 dB and the threshold is set to 10 dB, i.e. the normalized threshold is -5 dB. Comparing it with Fig. 3, we see that as the normalized SIR threshold increases, the maximum throughput decreases and the rate by which the throughput decreases w.r.t. data packet length increases.

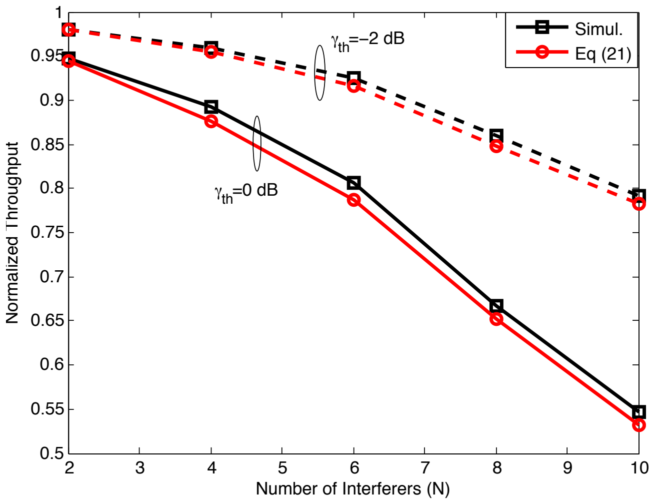

Fig. 5. Effect of increasing number of interferers on the normalized throughput, where $L = 10$, $f_s = 100$ ksymbols/sec, $f_{\text{D}} = f_{\text{I,max}} = 35$ Hz with $\gamma_{\text{avg}} \approx 8, 5, 3, 2$ and 1 dB for $N = 2, 4, 6, 8$ and 10, respectively. Solid lines: $\gamma_{\text{th}} = 0$ dB. Dashed lines: $\gamma_{\text{th}} = -2$ dB

To investigate the effect of increasing the number of interferers, the normalized throughput of a system with a fixed number of receiver antennas $L = 10$ vs. varying number interfering users N is plotted in Fig. 5 for two threshold values, namely $\gamma_{\text{th}} = -2$, and 0 dB. The data packet length for every simulation point is set to the optimum length m_{opt} obtained from (21). The interferers have unequal powers and travel with unequal speeds where the maximum Doppler frequency of all interferers is 35 Hz, which is the same as the Doppler frequency of the desired transmitter. The simulation settings suggest that as the number of interferers increases, the average SIR decreases, which is set to $\gamma_{\text{avg}} \approx 8, 5, 3, 2$ and 1 dB for $N = 2, 4, 6, 8$ and 10, respectively. The figure shows that for $\gamma_{\text{th}} = 0$ dB, the throughput deteriorates massively as number of interferers increases, where it falls from almost 0.95 when only 2 interferers exist to 0.55 when $N = 10$, thus amounting to only half of the data rate. On the other hand, for $\gamma_{\text{th}} = -2$ dB, the slope of decrease of the decrease in the throughput is much less pronounce than that in case of $\gamma_{\text{th}} = 0$ dB, where at $N = 10$ it loses only almost one fifth of the data rate as opposed to half of the data rate in the previous case.

From the above discussions, clearly the SIR threshold is a critical parameter. A joint optimization problem can be formulated to find the optimum threshold value and data packet length. Finally, to get accurate simulation results, intensive Monte Carlo simulations with high number of symbols are needed over a long time. Thus it is imperative to have accurate theoretical expressions of LCR, PER and throughput for design purposes. The proposed approximation in the LCR expression, and the simple two-state Markov model provide good accuracy at low normalized threshold values which are usually used in practical systems. For higher values, a tighter or an exact expression can be required.

VI. Conclusion

In this paper, we derived the optimum data packet length that maximizes the throughput of a SW-ARQ based multi-user system subject to correlated Rayleigh fading. To do so, we modeled the system using a two-state Markov chain model which needs an expression of the CDF and the LCR which were obtained through using Fukawa's results [1] and adapting them to suit the system under consideration, making some approximations through the derivation. Simulations showed the validity of the model proposed, and revealed the effect of increasing the number of interferers and the associated role of choosing an appropriate SIR threshold value in the performance of the system. An exact LCR expression and a tighter approximation that accounts for the thermal noise as well are addressed in the journal version [10].

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