

C3-Feature Engineering

One Hot Encoding

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.90	0	yes	southwest	16884.9240
11	62	female	26.29	0	yes	southeast	27808.7251
14	27	male	42.13	0	yes	southeast	39611.7577
19	30	male	35.30	0	yes	southwest	36837.4670
23	34	female	31.92	1	yes	northeast	37701.8768

Sex Encoding { 0 : Female
1 : Male

Children Encoding

0	1	2	3	4	5	Number of child
0	0	0	0	0	0	0
0	1	0	0	0	0	1
0	0	1	0	0	0	2
0	0	0	1	0	0	3
0	0	0	0	1	0	4
0	0	0	0	0	1	5

age	sex	bmi	OHE-1	OHE-2	OHE-3	OHE-4	OHE-5
19	0	27.90	0	0	0	0	0
34	0	31.92	1	0	0	0	0

D) Split the data

1. First shuffle the rows of the data

	Bmi	Charges
1	27	10,000
...
9337		
9338	30	50,000
1338		

Train data

2. X_{train} is $0.7 \times 1338 = 9337$ rows
 Y_{train} is $\dots = 9337$ rows

Test data

3. X_{test} is $0.3 \times 1338 = 401$ rows
 Y_{test} is $0.3 \times \dots = 401$ rows

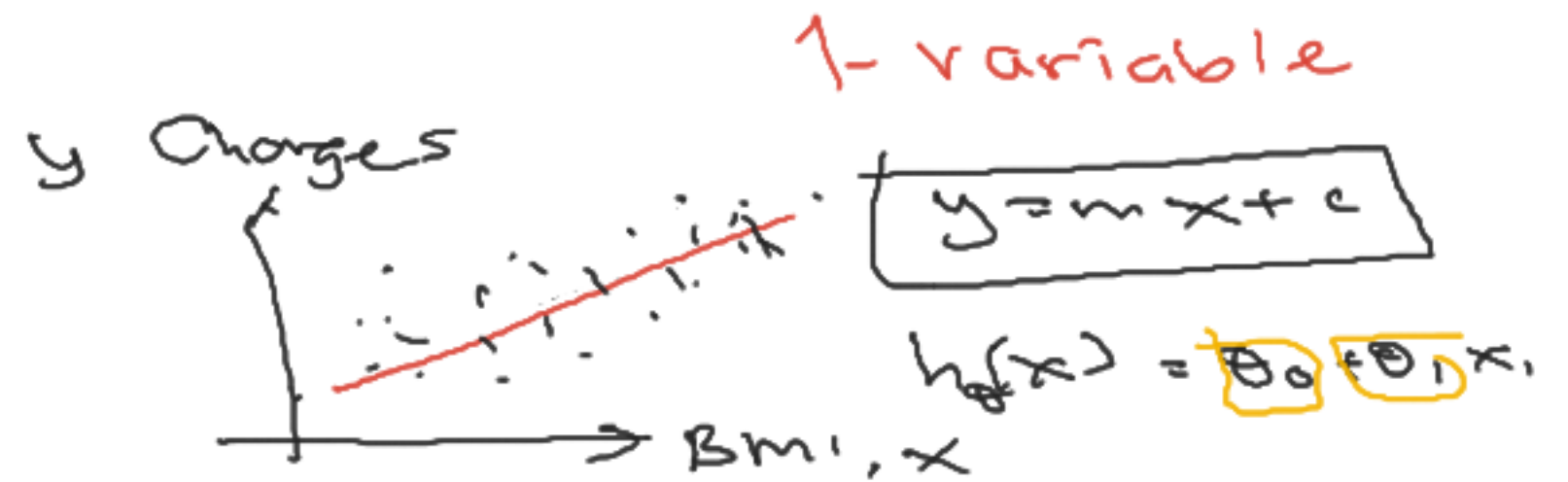
Smoker only has 274 rows

$$\underline{X_{\text{train}}} = \underline{0.7} \times \underline{274} = 191$$

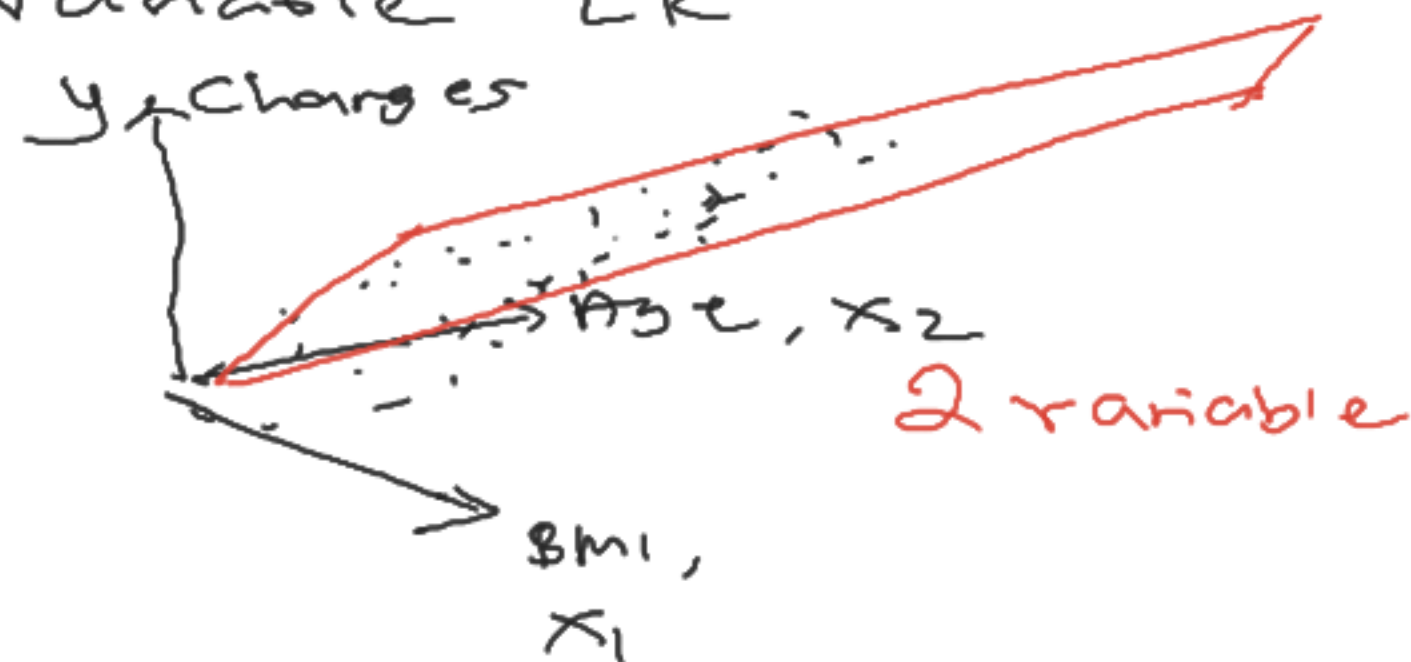
$$\underline{X_{\text{test}}} = \underline{0.3} \times \underline{274} = 82$$

Linear Regression (LR)

→ 1) One variable LR



2) Multi variable LR



$$y = m_1 x_1 + m_2 x_2 + c$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

Q How about 11 variables? What does the eqn look like?

$$A) h_\theta(x) = \theta_0 + \overset{m_1}{\theta_1} x_1 + \overset{m_2}{\theta_2} x_2 + \overset{m_3}{\theta_3} x_3 + \dots + \overset{m_{11}}{\theta_{11}} x_{11}$$

Linear Regression (LR)

- We are given this insurance.csv dataset.
- We want to do some LR on this dataset.
- For example. Can we predict how much a person has to pay for his/her insurance based on their BMI?

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

How to look at dataset

(input) Features, variable, dimension, attribute target output,

rows
= observations/
Samples

	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

i^{th} observation

Continuous variable

1. age
2. Bmi
3. Charges

Categorical variable

1. Sex
2. Children
3. Smoke
4. region

Revision of Linear Algebra

λ , $\lambda = 5$: scalar

$\mathbf{x}, \mathbf{x} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \rightarrow \mathbf{x}$ is a 1×3 row vector

$\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (column) vector

\mathbf{x} is a 3×1 column vector

$\mathbf{X} = \begin{bmatrix} 1 & 10 & 100 \\ 2 & 20 & 200 \\ 3 & 30 & 300 \end{bmatrix}$

\mathbf{X} is a 3×3 matrix

Continue

$$x_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 100 \\ 200 \\ 300 \end{bmatrix}$$

$$X = [x_1 \quad x_2 \quad x_3]$$

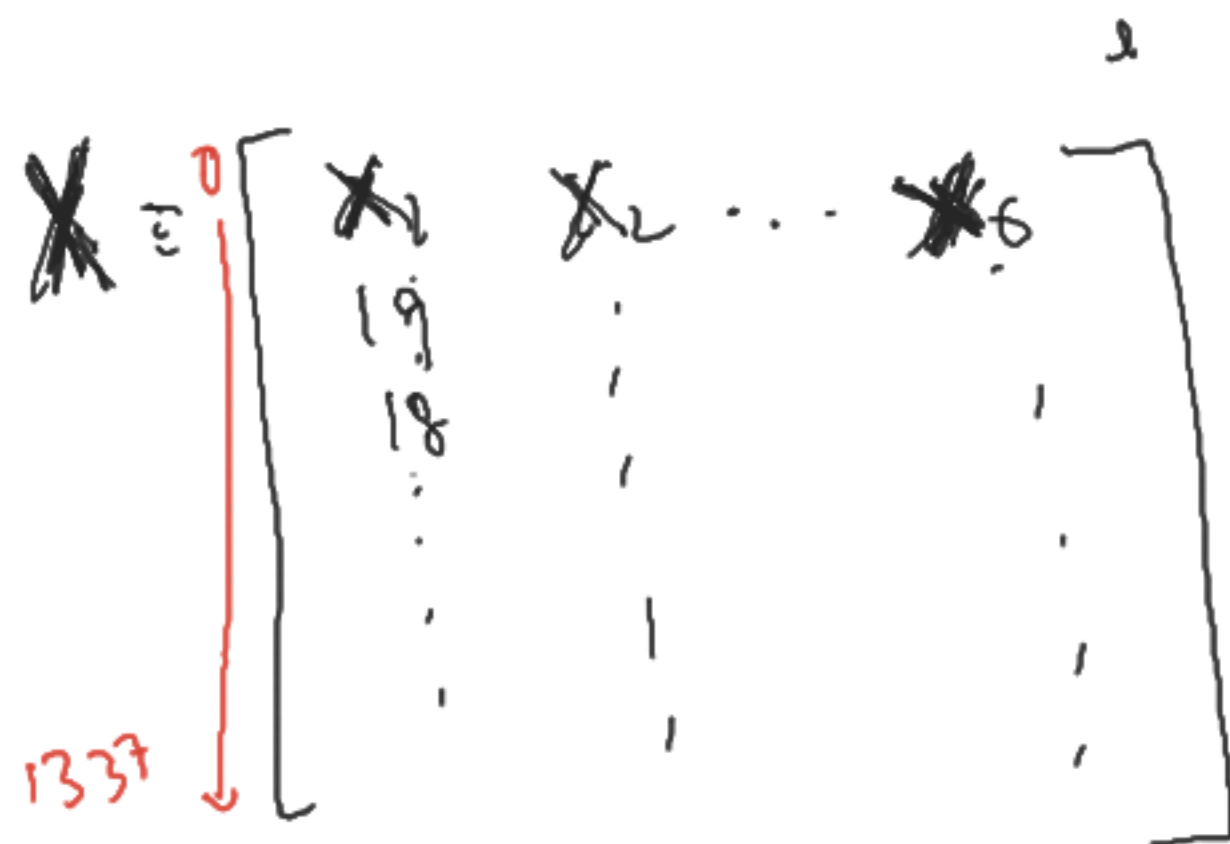
$$= \begin{bmatrix} 1 & 10 & 100 \\ 2 & 20 & 200 \\ 3 & 30 & 300 \end{bmatrix}$$

Now when we see a dataset, let's imagine it as vectors and matrices

	x_1	x_2	x_3	x_4	x_5	x_6	y
	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

x_i : the i th feature/attributes of the dataset. x_i is a col vector

y : a col vector of the target output



- we have 6 features.

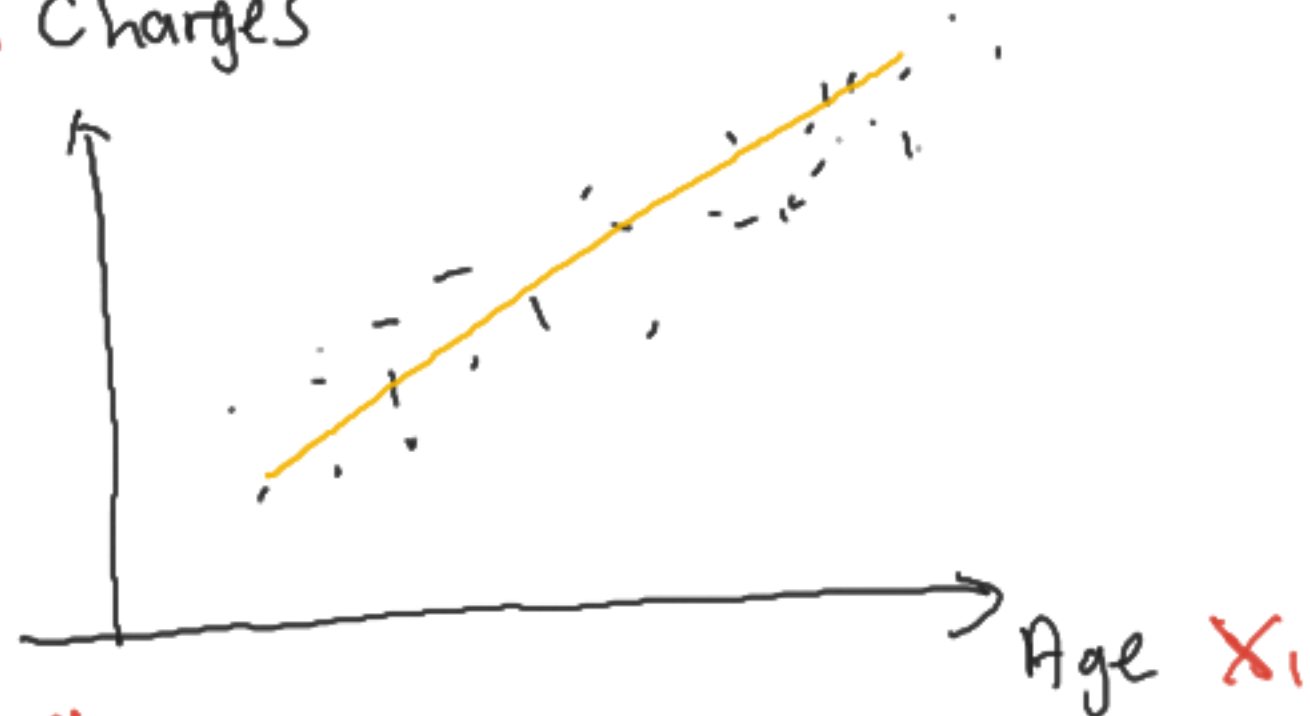
But how many rows of observation/samples do we have?

cols = 6
rows = 1338

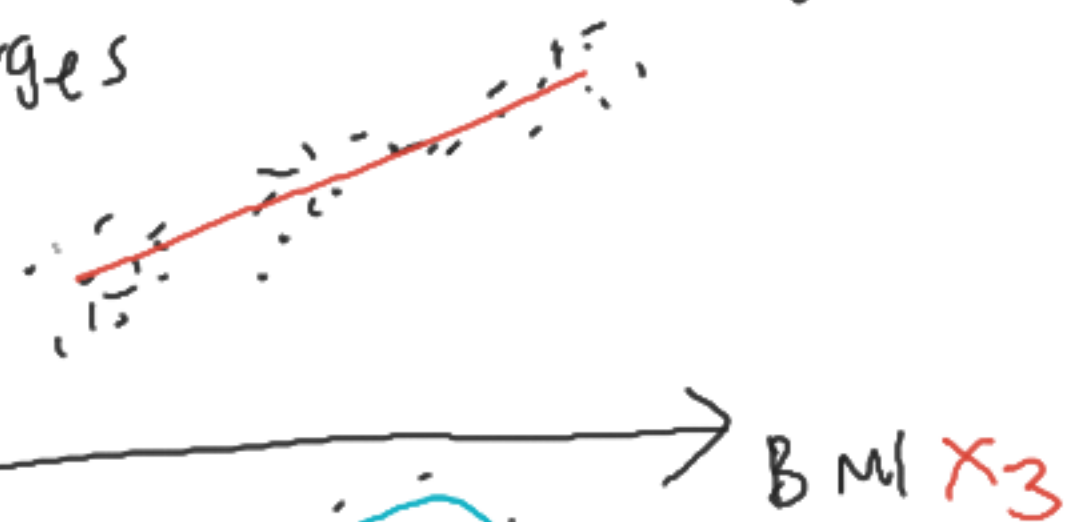
X is a 1338×6 matrix

Our 'intuition'

y Charges



y Charges



y Charges



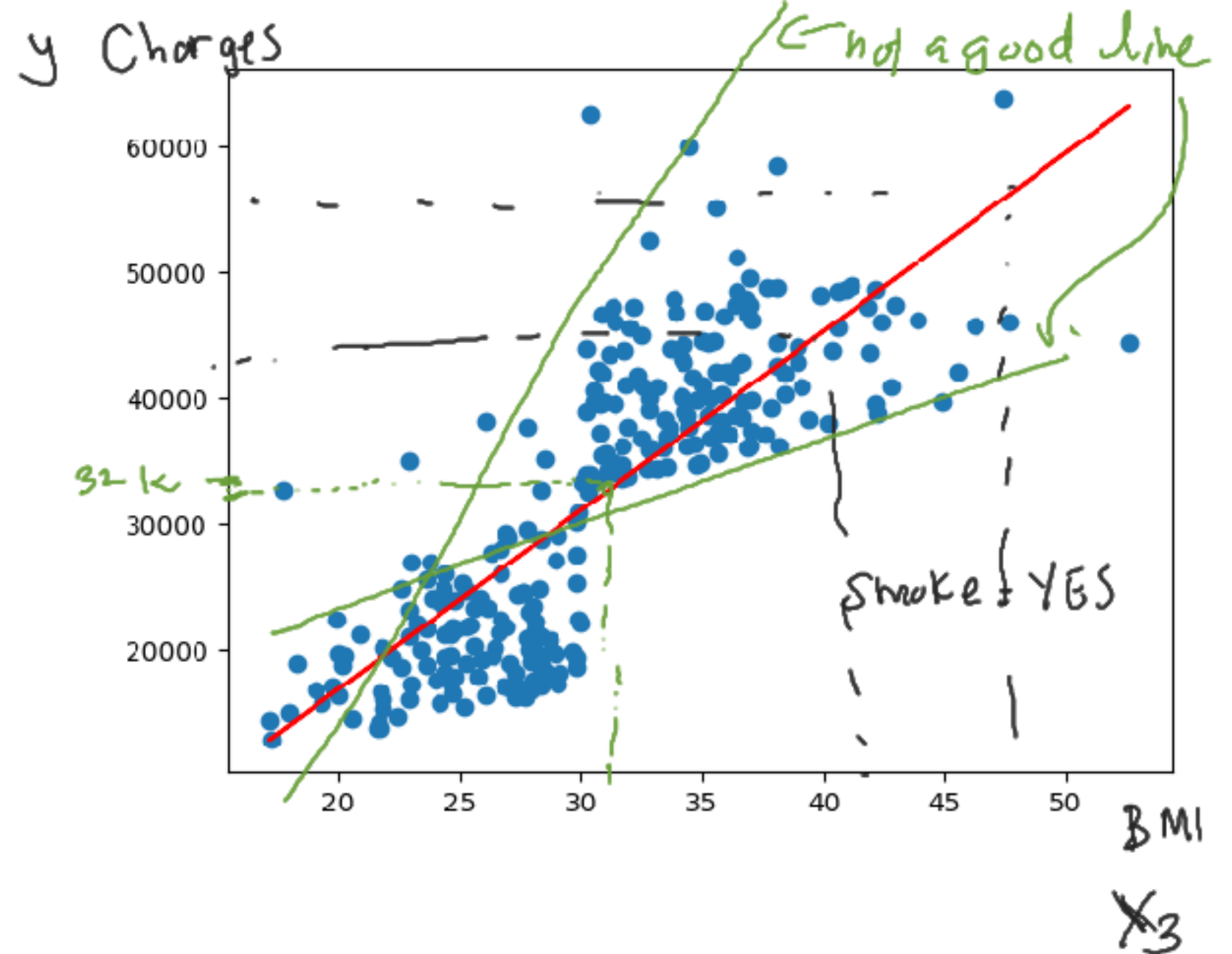
	X_1	X_2	X_3	X_4	X_5	X_6	y
	age	sex	bmi	children	smoker	region	charges
0	19	female	27.900	0	yes	southwest	16884.92400
1	18	male	33.770	1	no	southeast	1725.55230
2	28	male	33.000	3	no	southeast	4449.46200
3	33	male	22.705	0	no	northwest	21984.47061
4	32	male	28.880	0	no	northwest	3866.85520

X : features/attributes, independent variable

y: target output, dependant variable

Linear Regression with one variable

	X_3 BMI	y Charges
1	29	58000
2	30	60000
...		
1338	25	70000
1339	31	?



- = Linear regression is to find a linear line that fits the dataset.
- = We can make predictions for unseen observation. For example
What is the insurance charges for a person with BMI = 31?

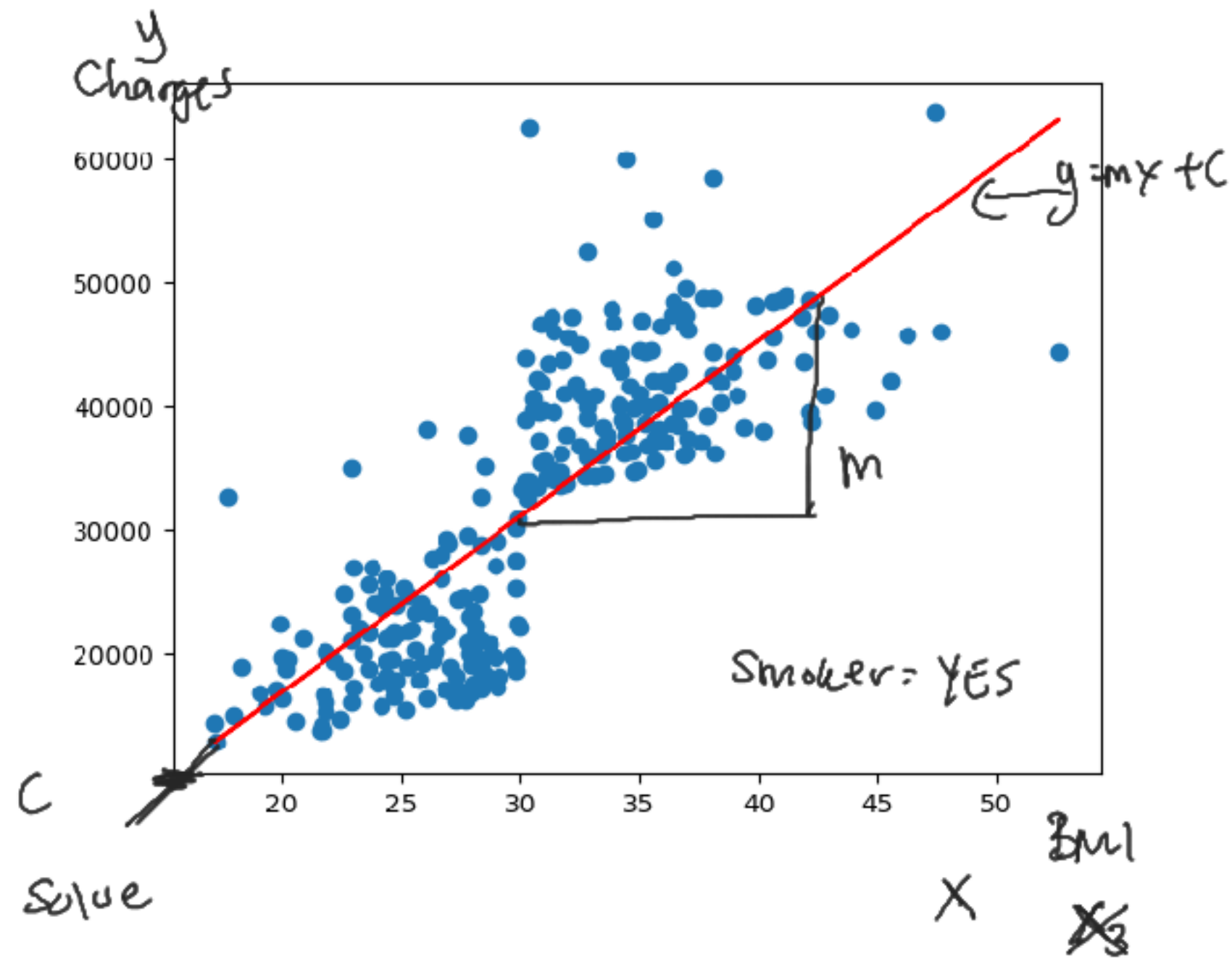
How do we find the best line (red) that fits the data?

$$y = \underline{m}x + \underline{c}$$

$h(x) = \theta_0 + \theta_1 x$

θ_0 : intercept θ_1 : slope

~ So our job (for one variable) is to solve for m and c .



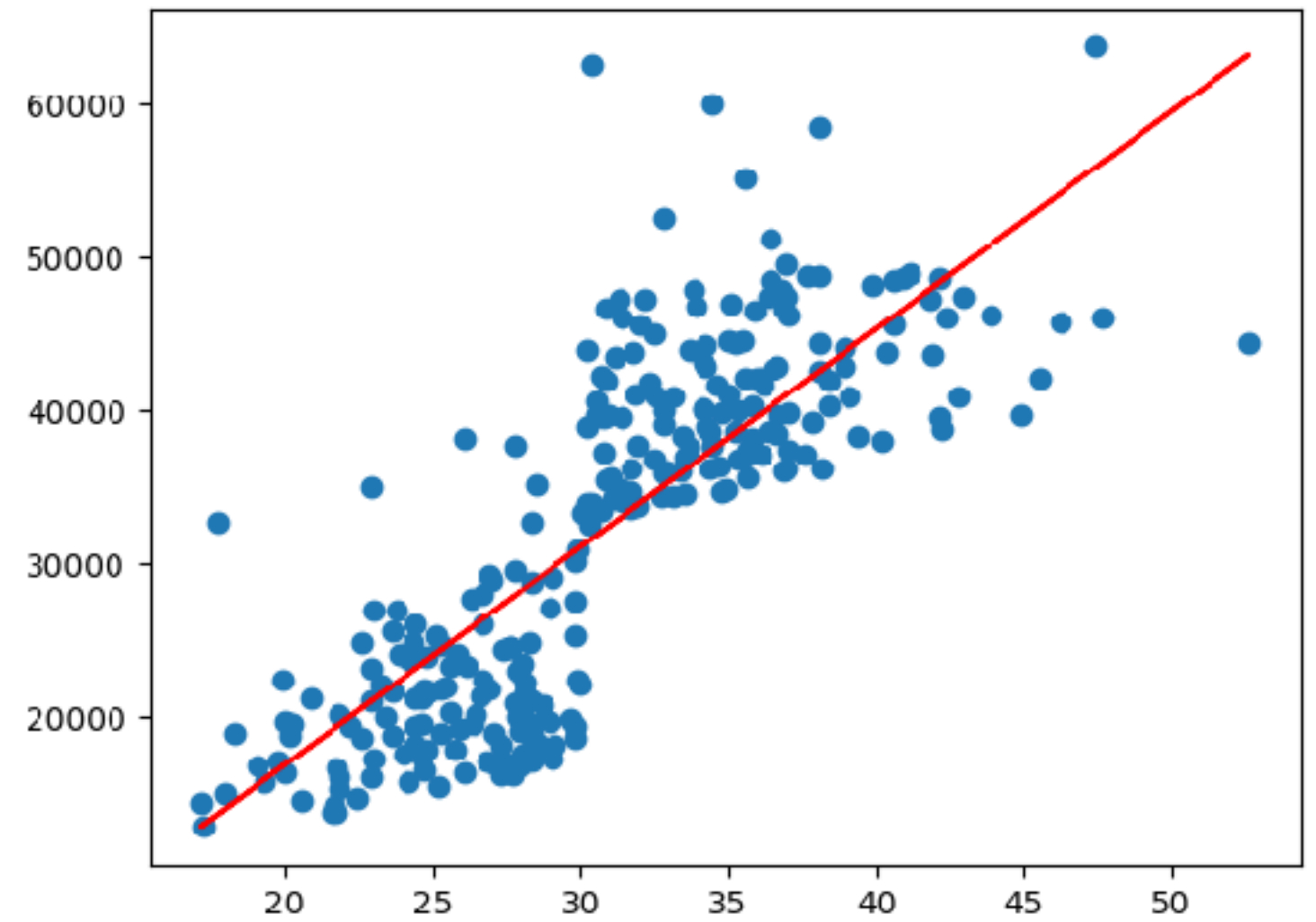
How do we find the 'best'
 m and c ?

$$y = mx + c$$

Two methods

A) Solve using equation

B) Use gradient descent



A) Solve using equation

Ordinary Least Square method (from Stats)

$$c, \hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x};$$

$$y = mx + c$$

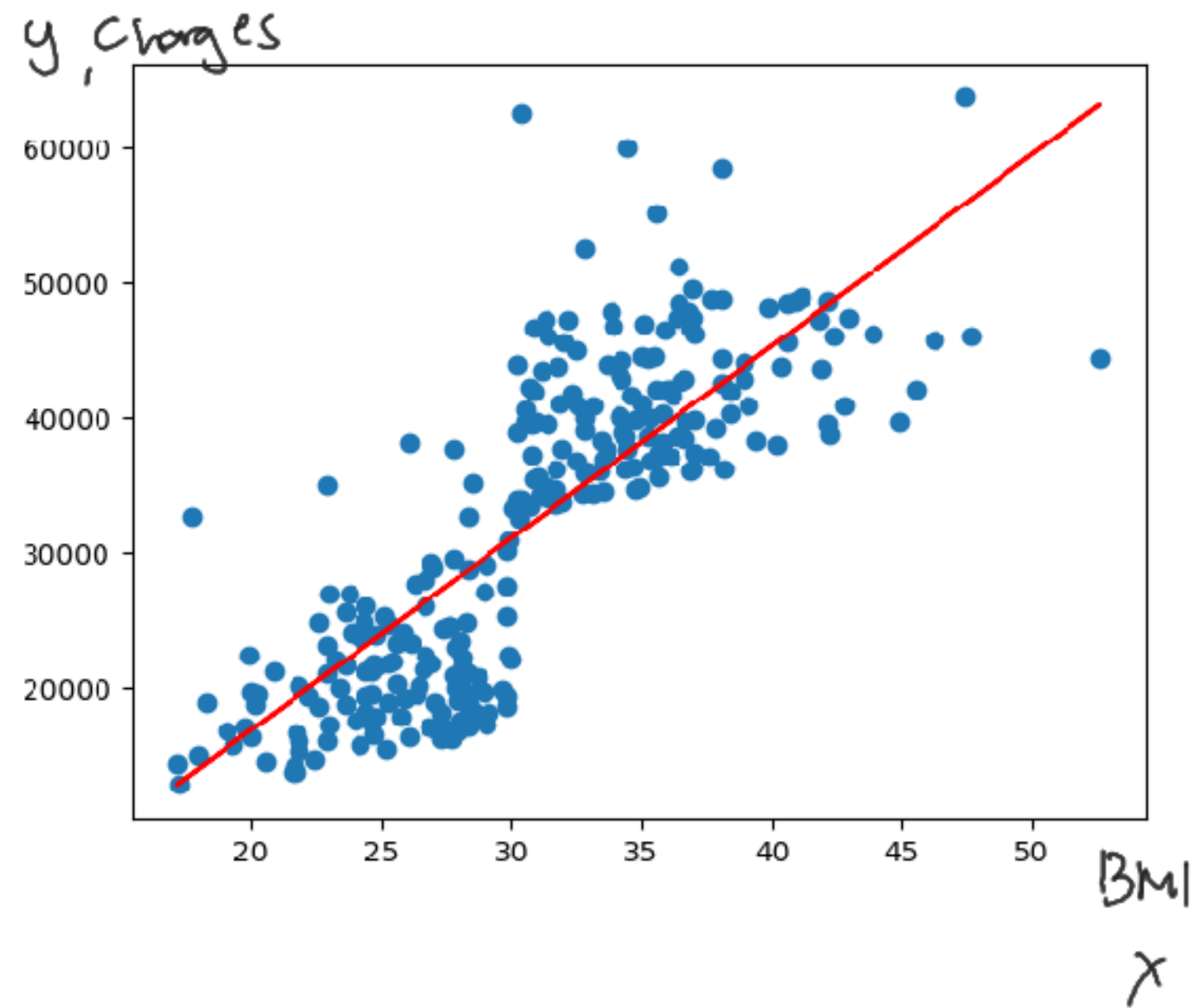
$$\underline{m}, \hat{\beta}_1 = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2},$$

$$y = \beta_0 + \beta_1 x$$

← what is this?

From our dataset

index	x_3 BMI	y Charges
1	19	
2	23	
...		
i	$x_i = 25$	$y_i = 20000$
$n = 1338$		



$$\bar{x} = \text{average of } x = \frac{1}{n} \sum_{i=1}^n x_i$$

$$\bar{y} = \text{" of } y = \frac{1}{n} \sum_{i=1}^n y_i$$

3.) Use gradient Descent

(gradient of the cost function)

We need to define a cost function

$$h(x) = mx + c \quad \leftarrow \text{our red line}$$

● : our data

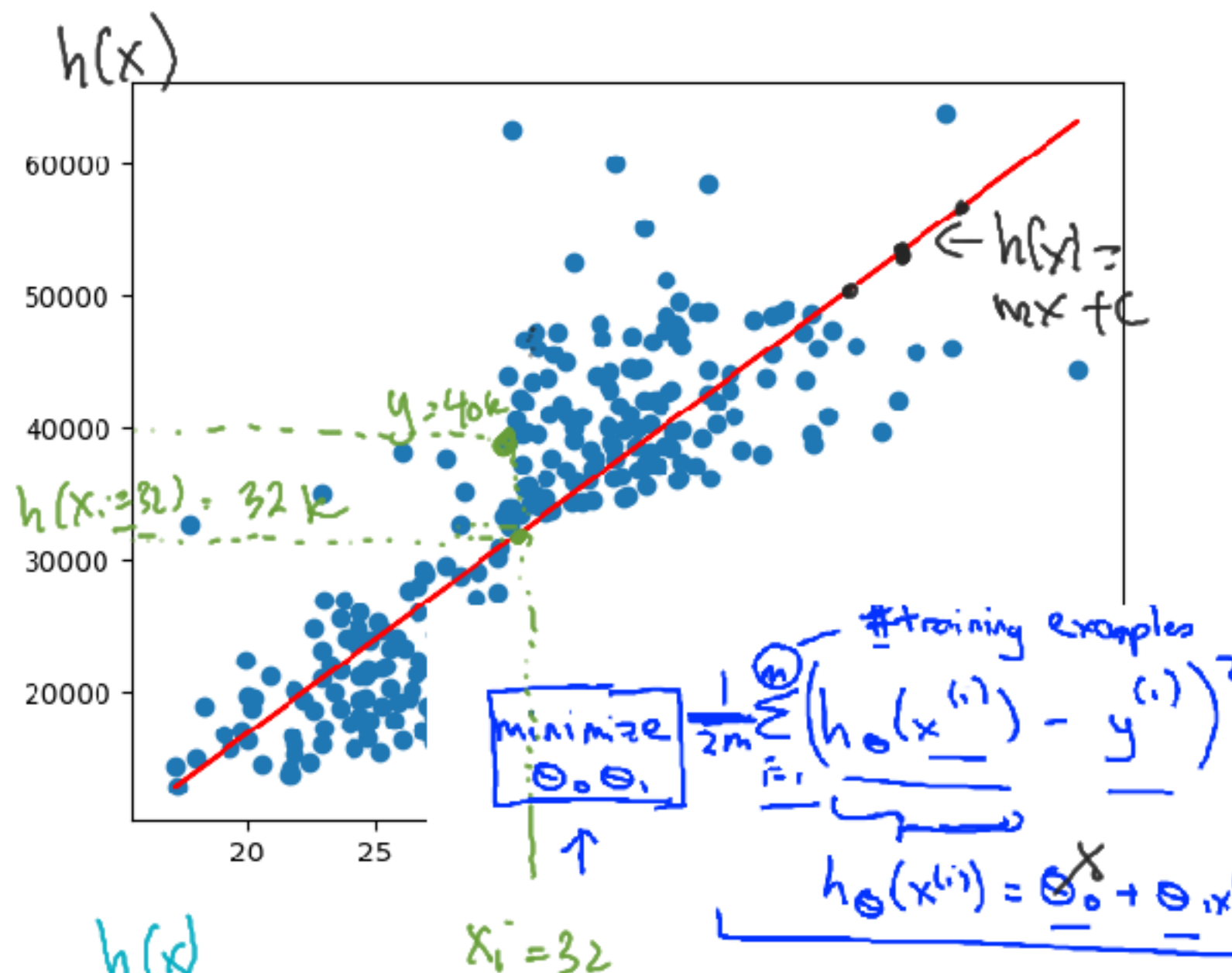
● : our prediction (line)

$$(h(x^i) - y^i)^2$$

∴ So our cost function is

$$J(c, m) = \frac{1}{2N} \sum_{i=1}^N (h(x^i) - y^i)^2$$

index	X BMI	y Charges	h(x)
1	27	49000	
2	31	45,000	
⋮			
j	$x^j = 32$	$y^j = 40,000$	32,000
⋮			
N = 1338	25	38 000	



ii)

at
or our
(y)

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Minimize $J(\theta_0, \theta_1)$
 θ_0, θ_1

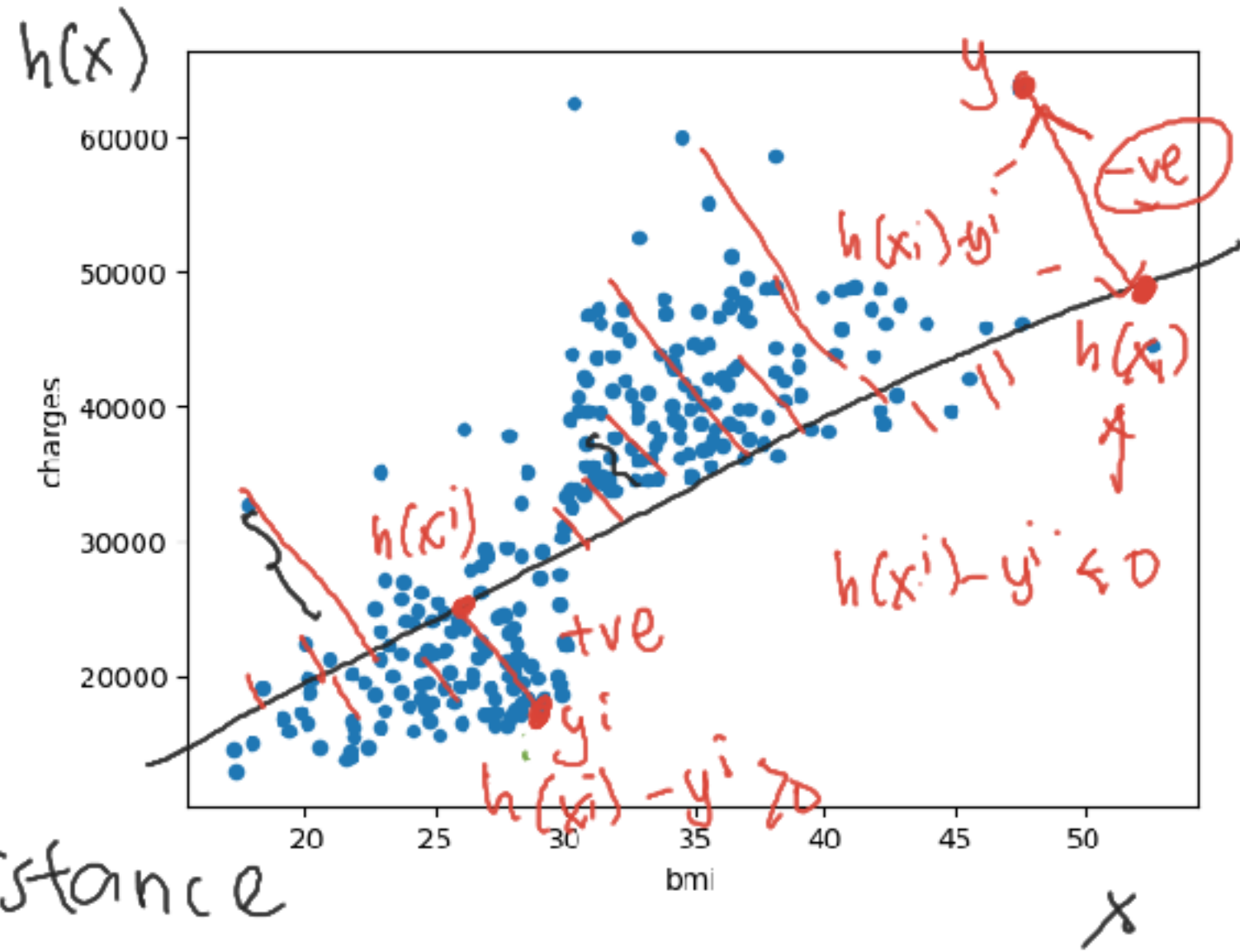
Squared error function

3.) Use gradient Descent
(gradient of the cost function)

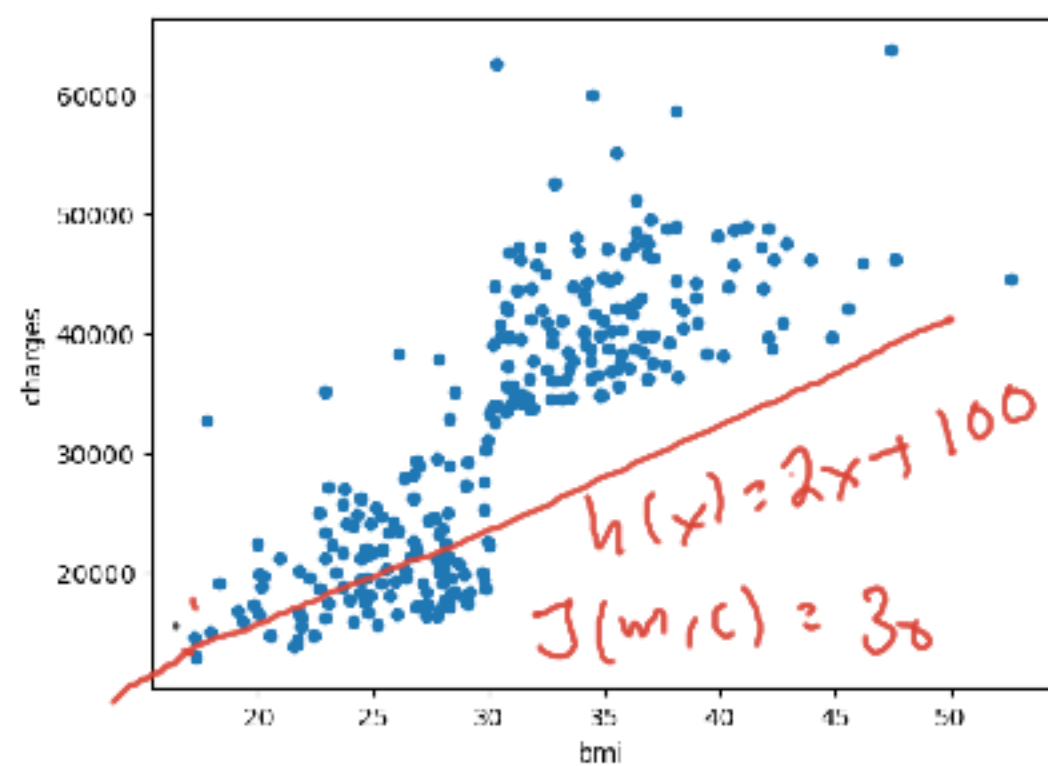
Cost function

$$h(x) = mx + c$$

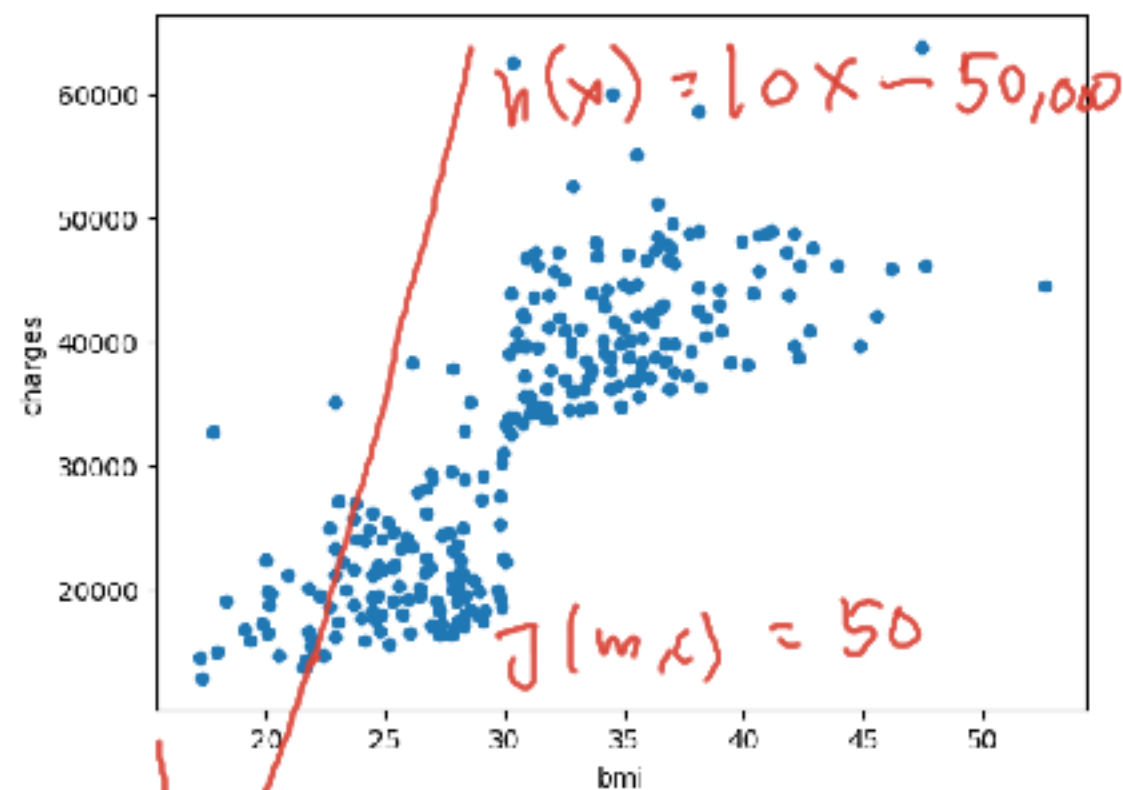
$$J(m, c) = \frac{1}{2N} \sum_{i=1}^{N=1338} (h(x^i) - y^i)^2$$



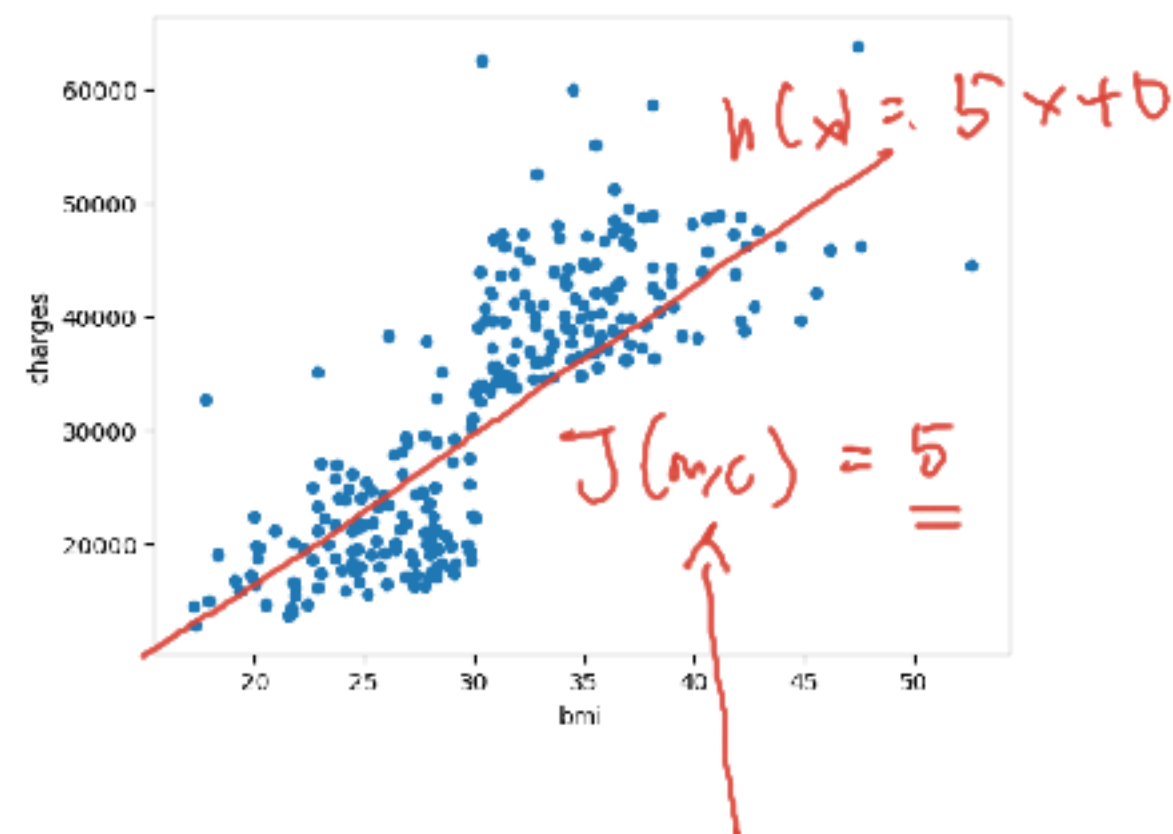
- The cost function calculates the distance between our predictions $h(x)$ and the original data.
- So then we need to find the values of m and c that minimizes the cost function J



$R^2 = 0.1$ bad



$R^2 = 0.01$ really bad



$m=5, c=0$
yields the lowest
error/cost function

$R^2 = 0.7$ Good ✓

How do we find the best m and c that minimizes the cost function?

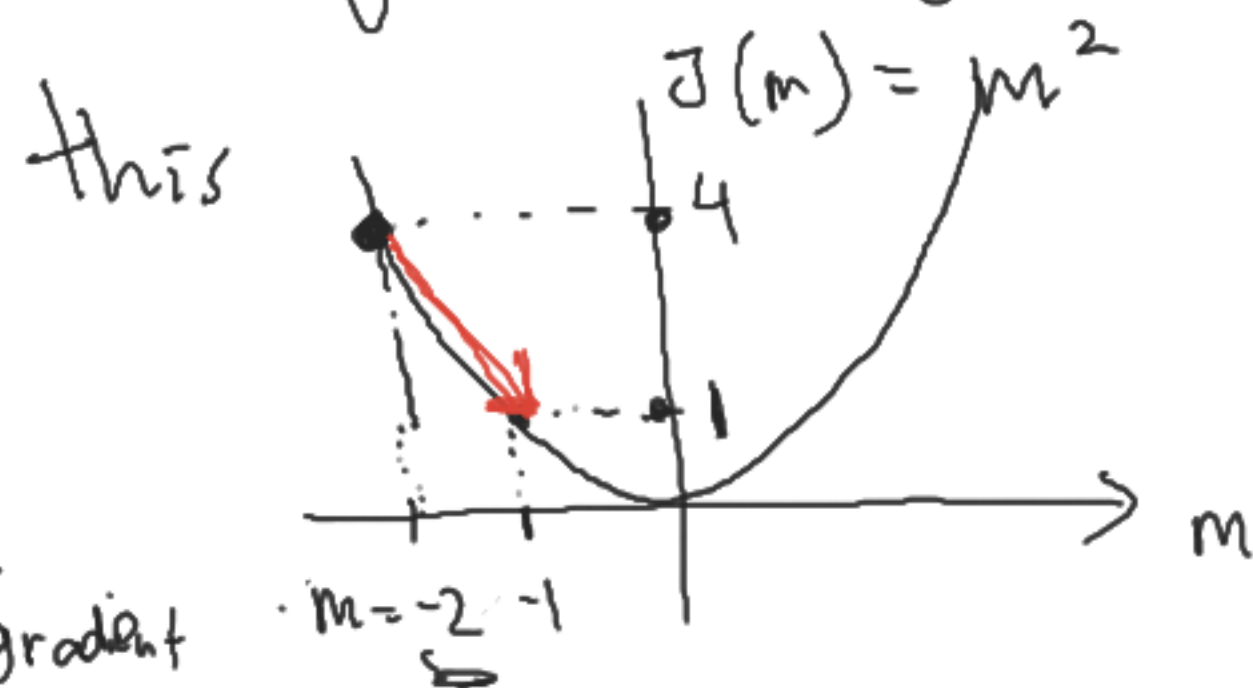
$$J(m, c) = \frac{1}{2N} \sum_{i=1}^{N=1338} (h(x^i) - y^i)^2$$

What does this cost function look like?

(I don't know)

How do we find the best m and c that minimizes the cost function?

Let's just imagine our cost function looks like



α : learning rate/step = $1/4$

Update using gradient descent

Derivative/gradient

$$J(m) = m^2$$
$$\frac{dJ(m)}{dm} = 2m$$

THIS IS HOW WE GO
DOWN THE GRADIENT OF
THE COST FUNCTION

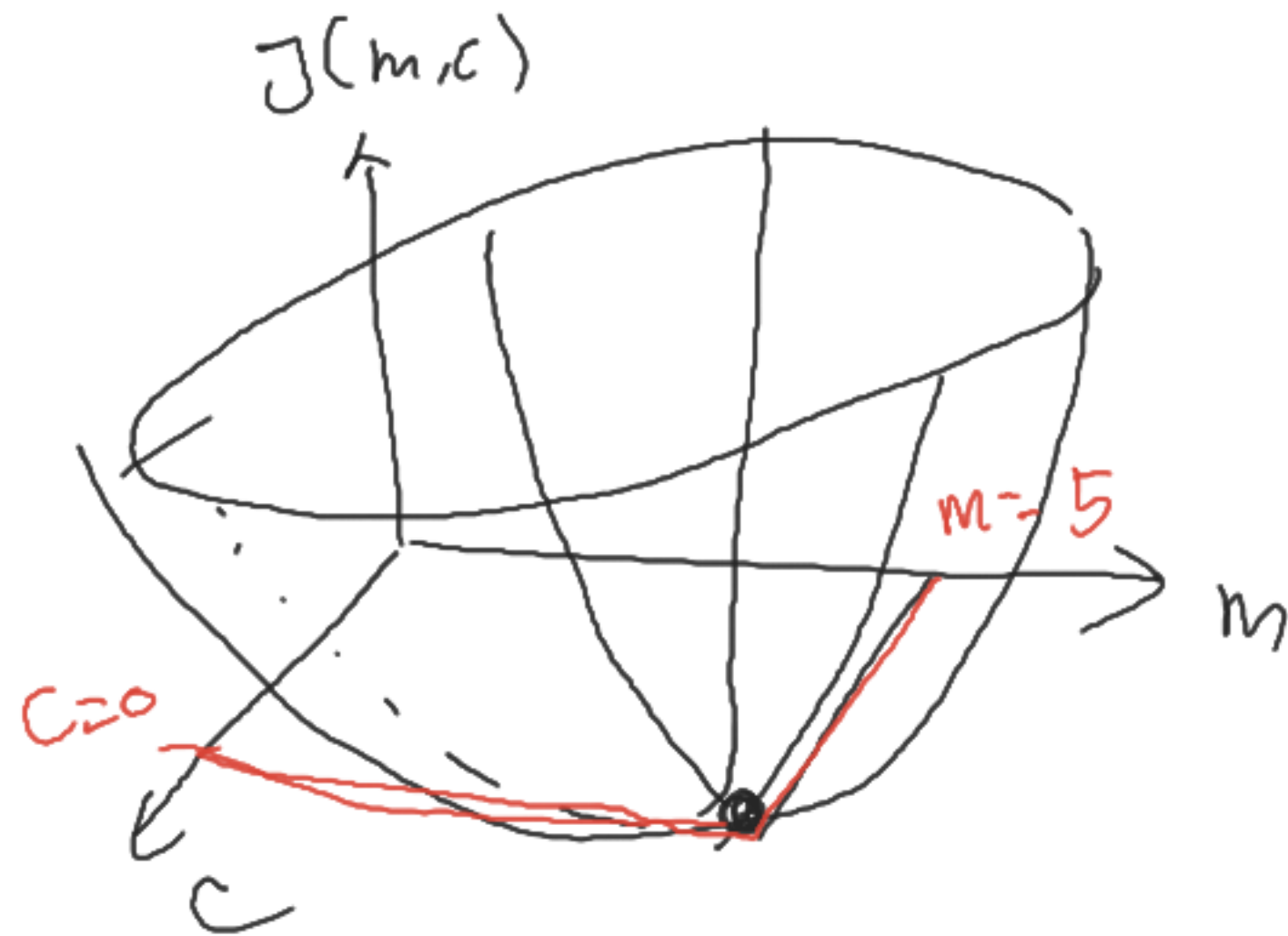
1st iter

$$m_{\text{new}} = m_{\text{old}} - \alpha \frac{dJ(m)}{dm}$$

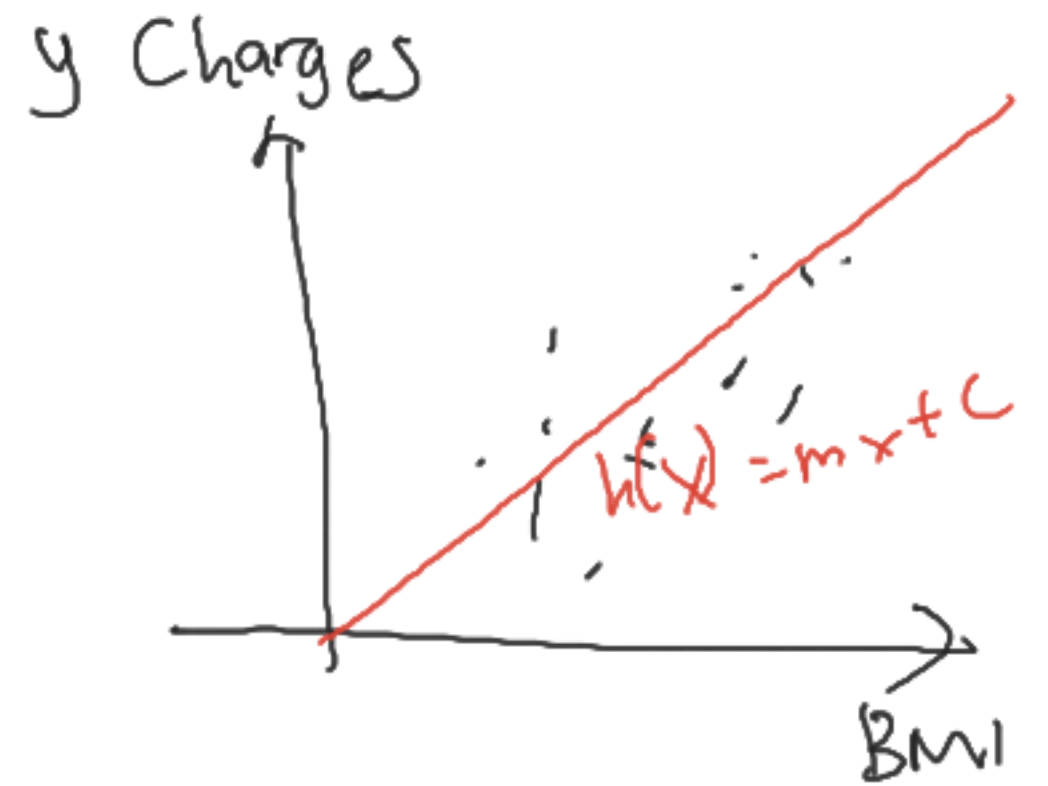
$$= -2 - \frac{1}{4} \cdot 2(-2)$$
$$= -1$$
$$y_{\text{new}} = m_{\text{new}}^2 = 1$$

We define the cost function $N=1338$

$$① J(\underline{m}, \underline{c}) = \frac{1}{2N} \sum_{i=1}^N (h(x^i) - y^i)^2$$



$C_{new} =$



② we find the best value of m and c which minimizes cost function J .

③ This is done by going down the ^{derivative} gradient of J

$$M_{new} = M_{old} - \alpha \frac{\partial J(m, c)}{\partial m}$$