

# Simplicial Homology

An introduction to simplicial homology theory

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## Abstract

This paper examines the fundamental ideas of simplicial structures that lead to simplicial homology theory and introduces singular homology to demonstrate the equivalence of homology groups of homomorphic topological spaces. It concludes with a proof of the equivalence of simplicial and singular homology groups.

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