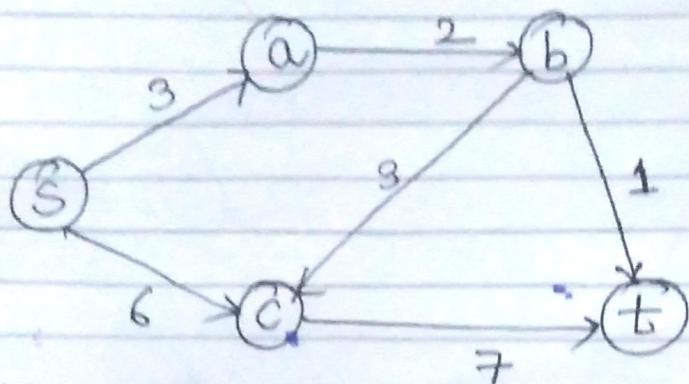


(2)



Find maxflow using push relabel algorithm.

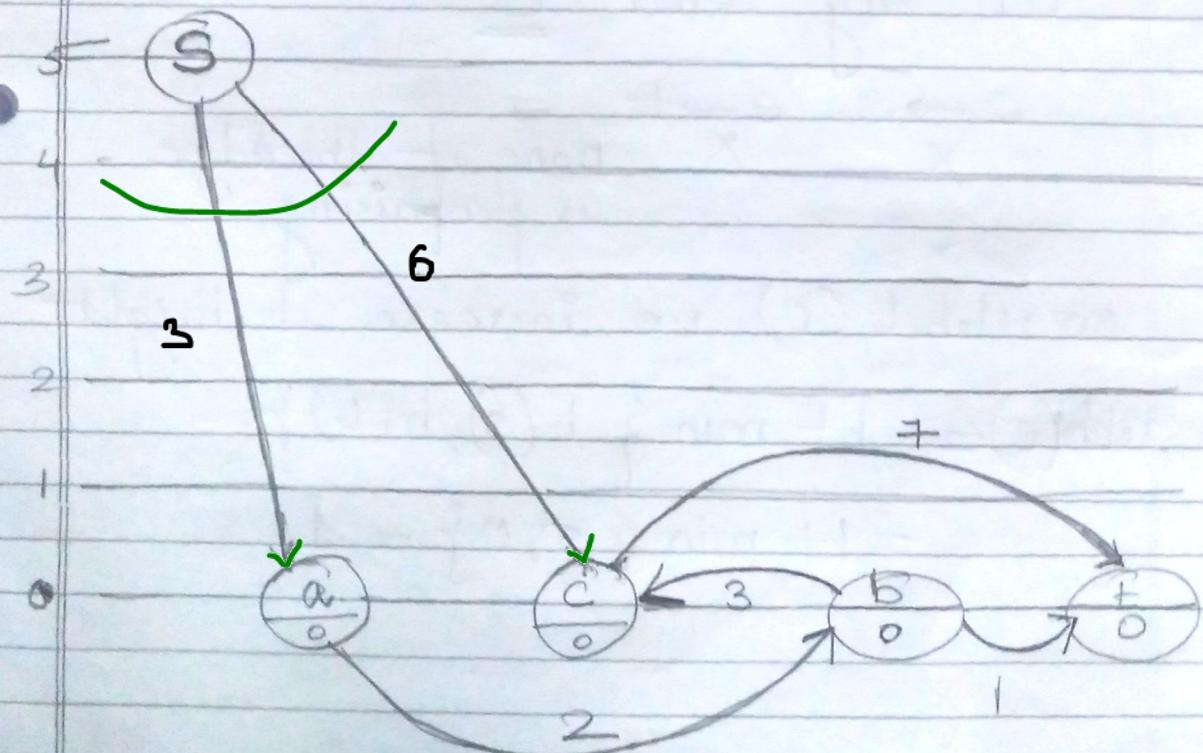
Soln

Initially flow is 0

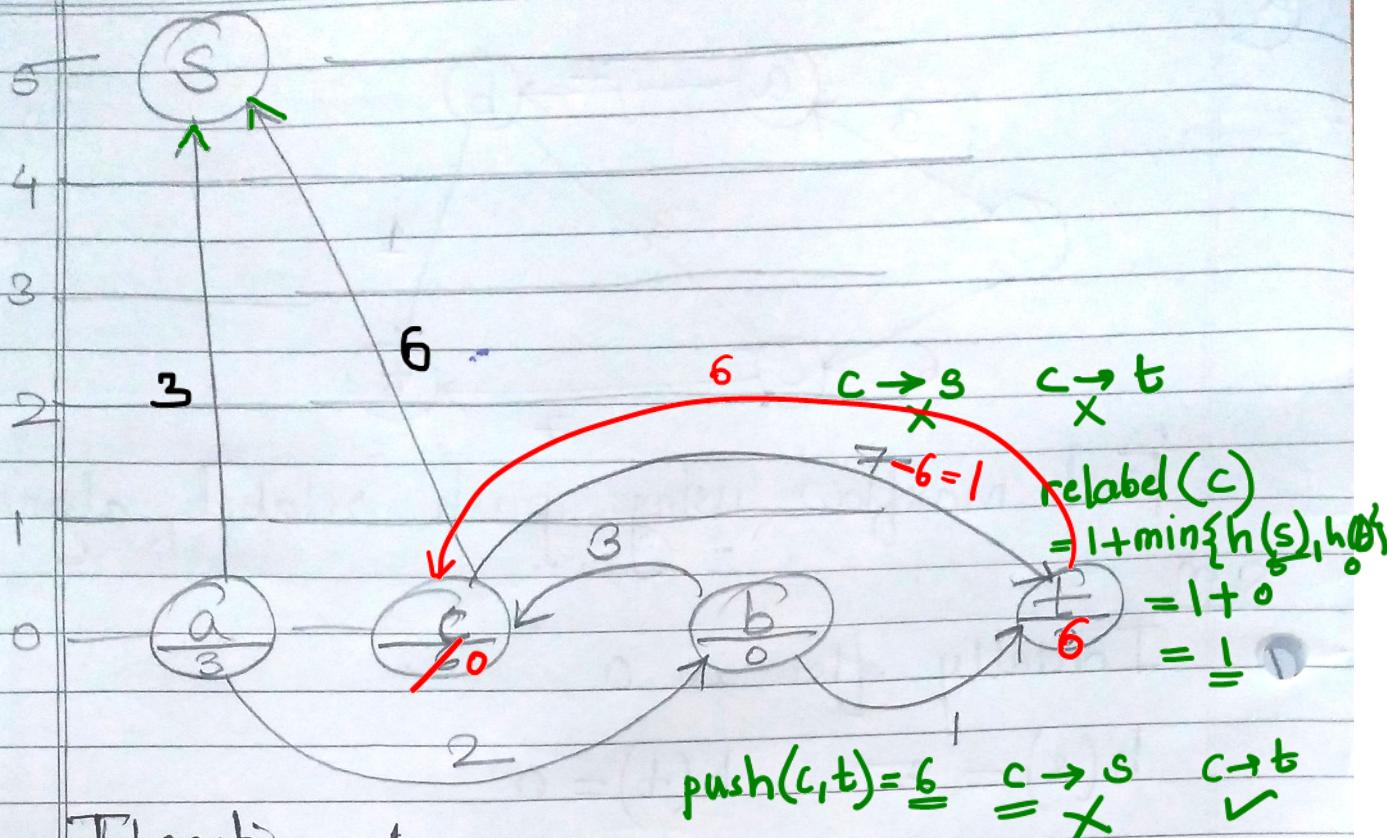
$$h(s) = 5 \quad h(t) = 0$$

$h(a) = h(b) = h(c) = 0$.. height of all internal nodes is 0.

$e(a) = e(b) = e(c) = 0$.. excess flow at internal node is 0



Saturate edges leaving from S.



Iteration 1

Select any active node.

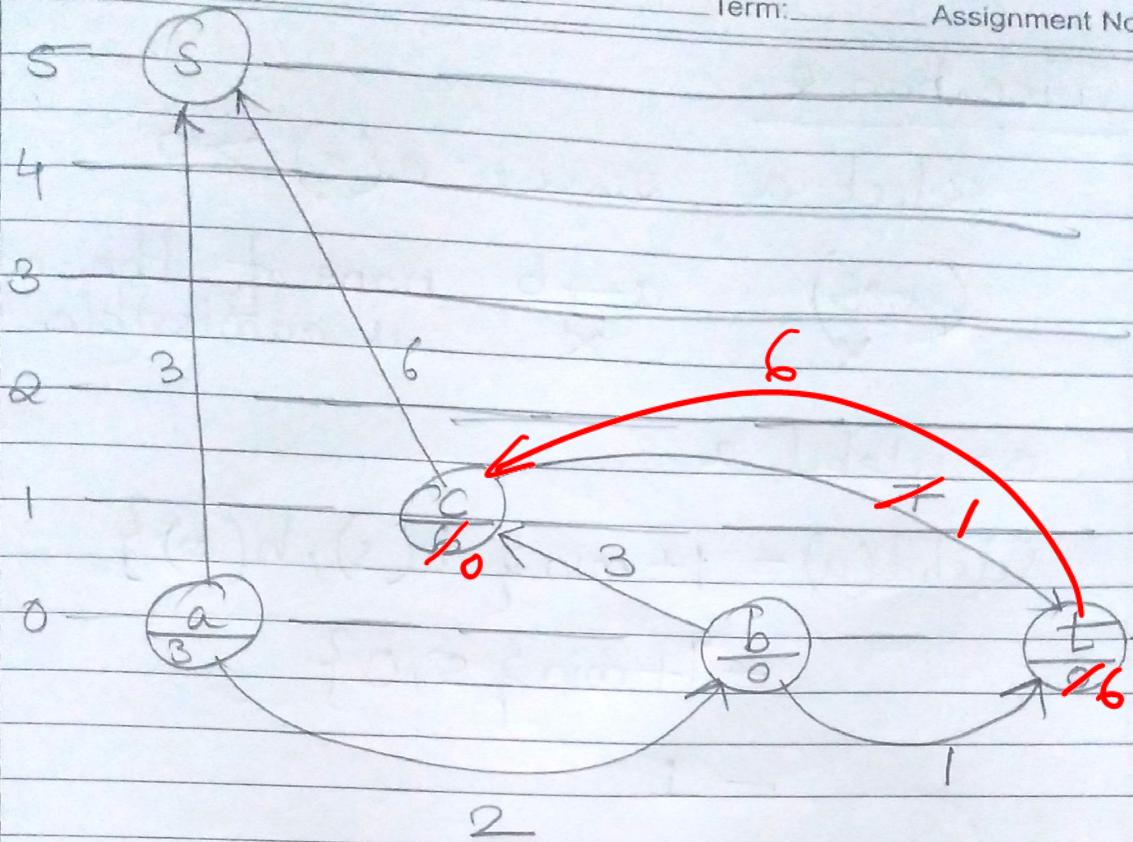
(Internal node which is overflaing
ie $e(\text{node}) > 0$)

Let say select c

$c \rightarrow s$ \cancel{x} $c \rightarrow t$ \cancel{x}
 none of the edge
 is promising

so relabel(c) i.e increase its height

$$\begin{aligned}\text{relabel}(c) &= 1 + \min\{h(s), h(t)\} \\ &= 1 + \min\{5, 0\} = 1\end{aligned}$$



Select c

$c \rightarrow t$ is admissible edge

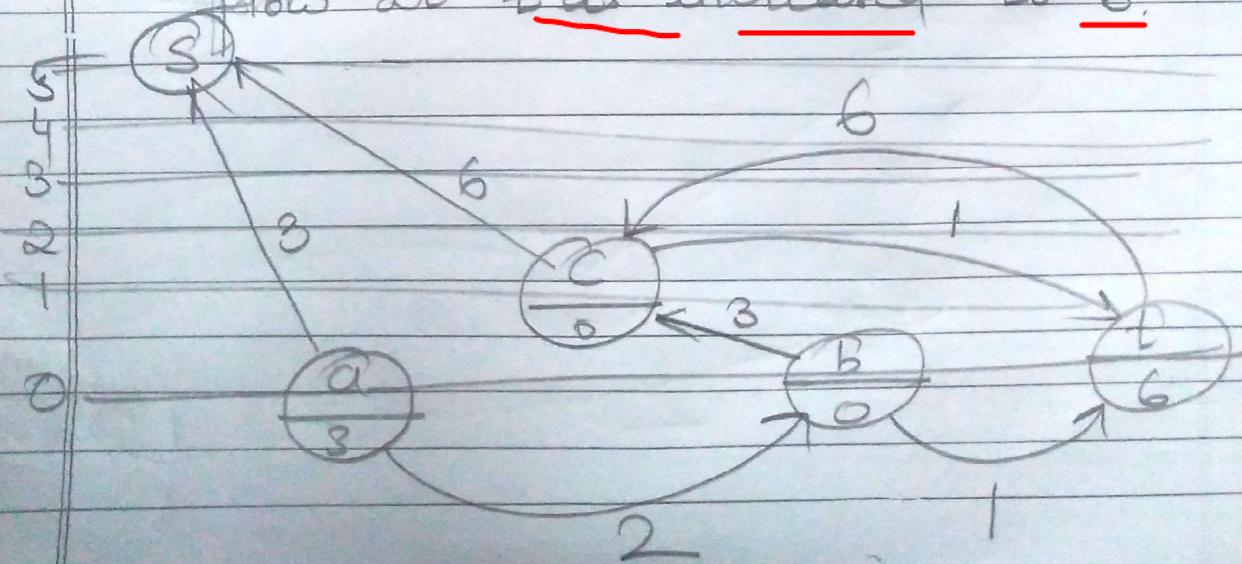
So

$$\text{push}(c, t) = df(c, t) = \min \{ e(c), cf(c, t) \}$$

$$= \min \{ 6, 7 \}$$

$$= 6$$

so $e(c) = e(c) - df(c, t) = 6 - 6 = 0$.
 Flows at tail increased to 6.



Iteration 2:

Select a since $e(a) > 0$.

$a \rightarrow s$
X

$a \rightarrow b$
X

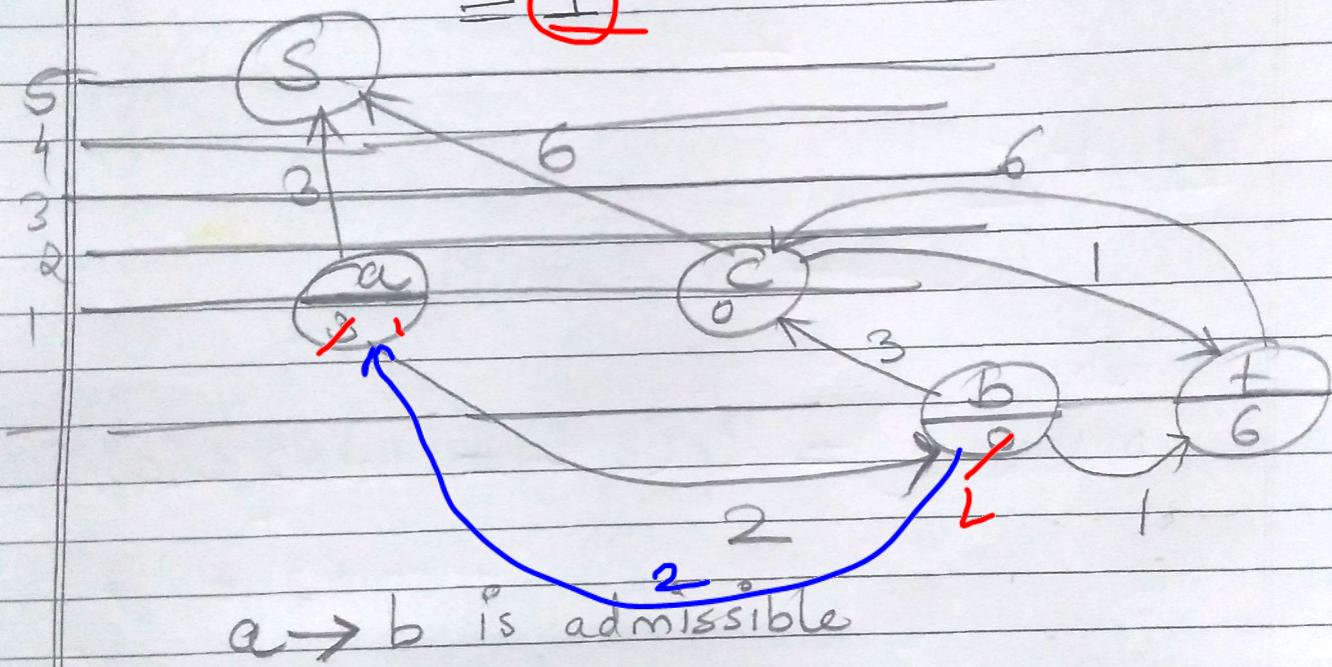
none of the edge
is admissible

so relabel a

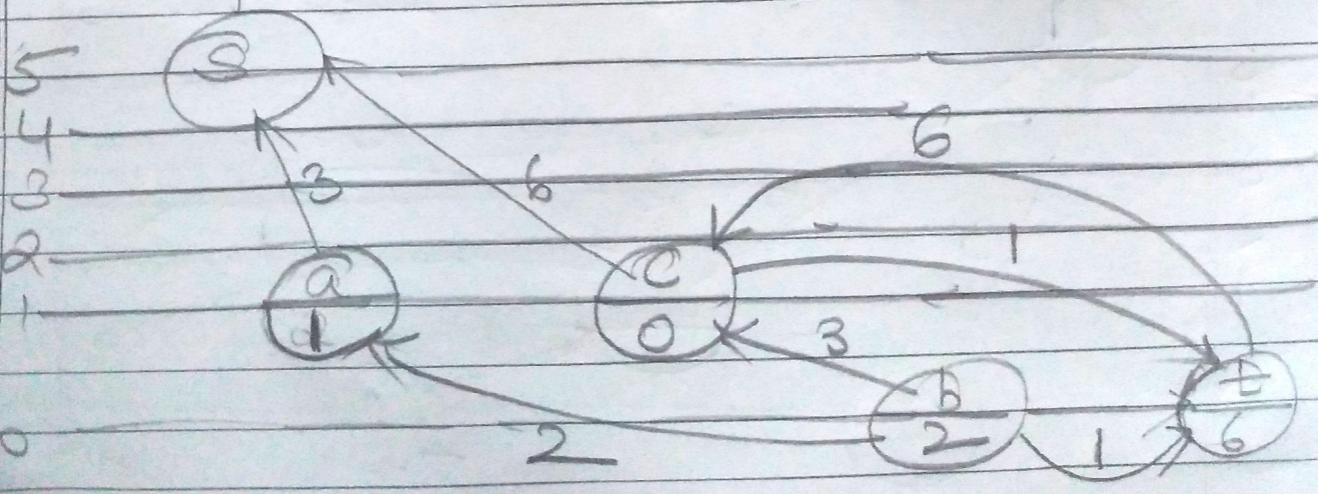
$$\text{relabel}(a) = 1 + \min \{ h(s), h(b) \}$$

$$= 1 + \min \{ s, 0 \}$$

$$= 1$$



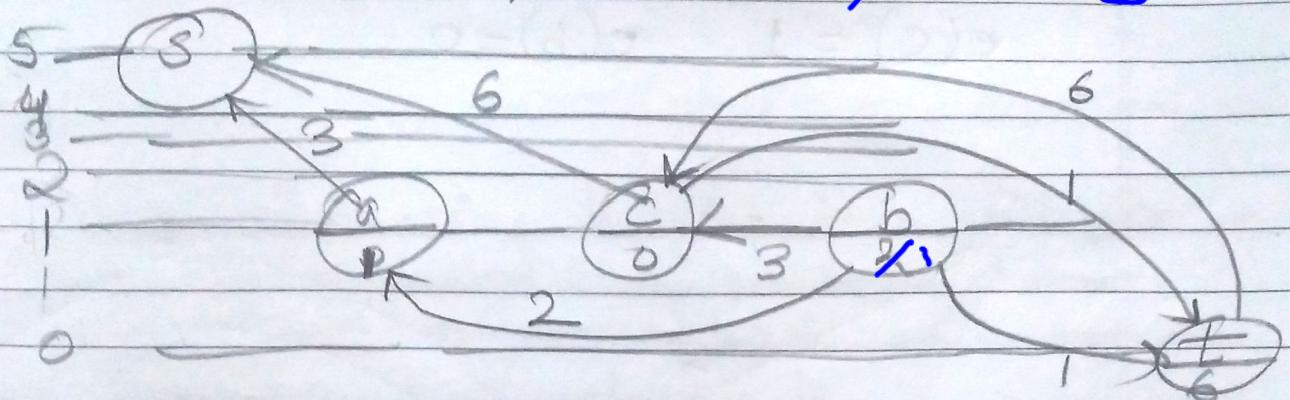
$$\text{so } \underline{\text{push}}(a, b) = 2 \quad \begin{cases} e(a) = 1 \\ e(b) = 2 \end{cases}$$



Iteration 3

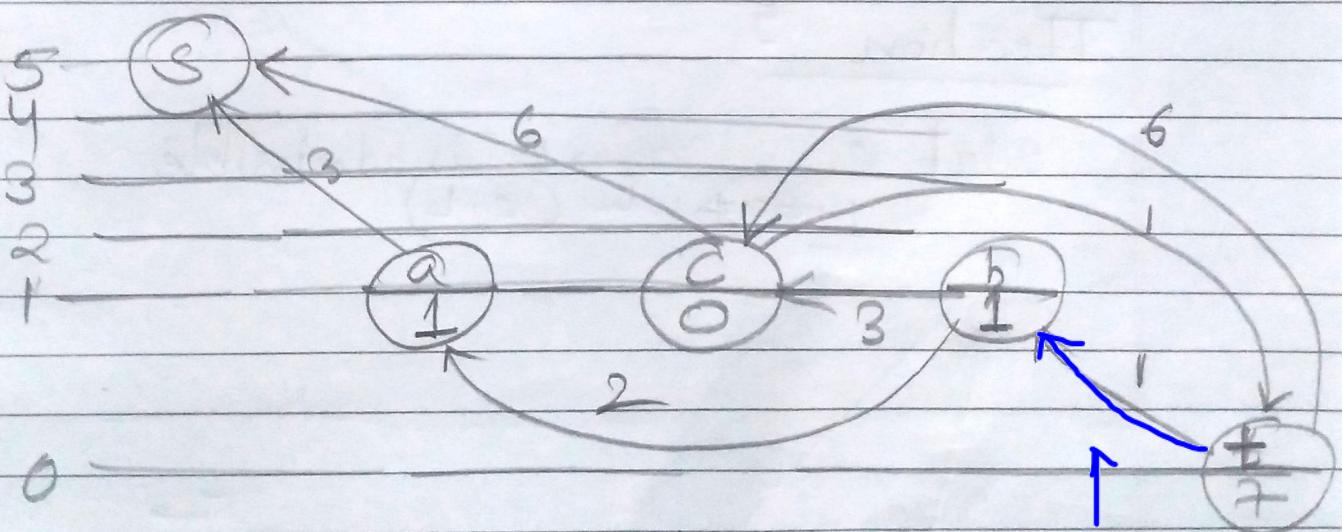
Select any internal node having $e(\text{node}) > 0$

so select b $b \rightarrow a$ $\cancel{b \rightarrow c}$, $b \rightarrow t$ relabel(b) $= 1$



so now $b \rightarrow t$ is admissible

push(b, t) = 1. so $e(b) = 1$
float at $t = 6 + 1 = 7$



Iteration 4.
select b .

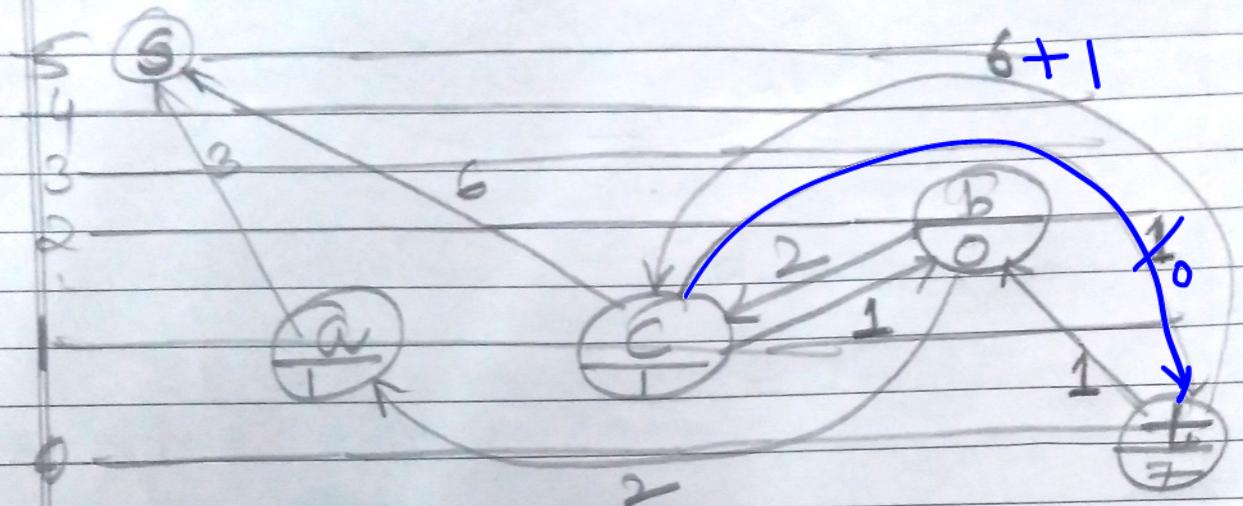
$b \rightarrow a \times$
 $b \rightarrow c \times$

$$\text{relabel}(b) = 1 + \min \{ h(a), h(c) \} \\ = 1 + \min \{ 1, 1 \} = 2$$

after relabelling b, $b \rightarrow c$ edge is
admissible

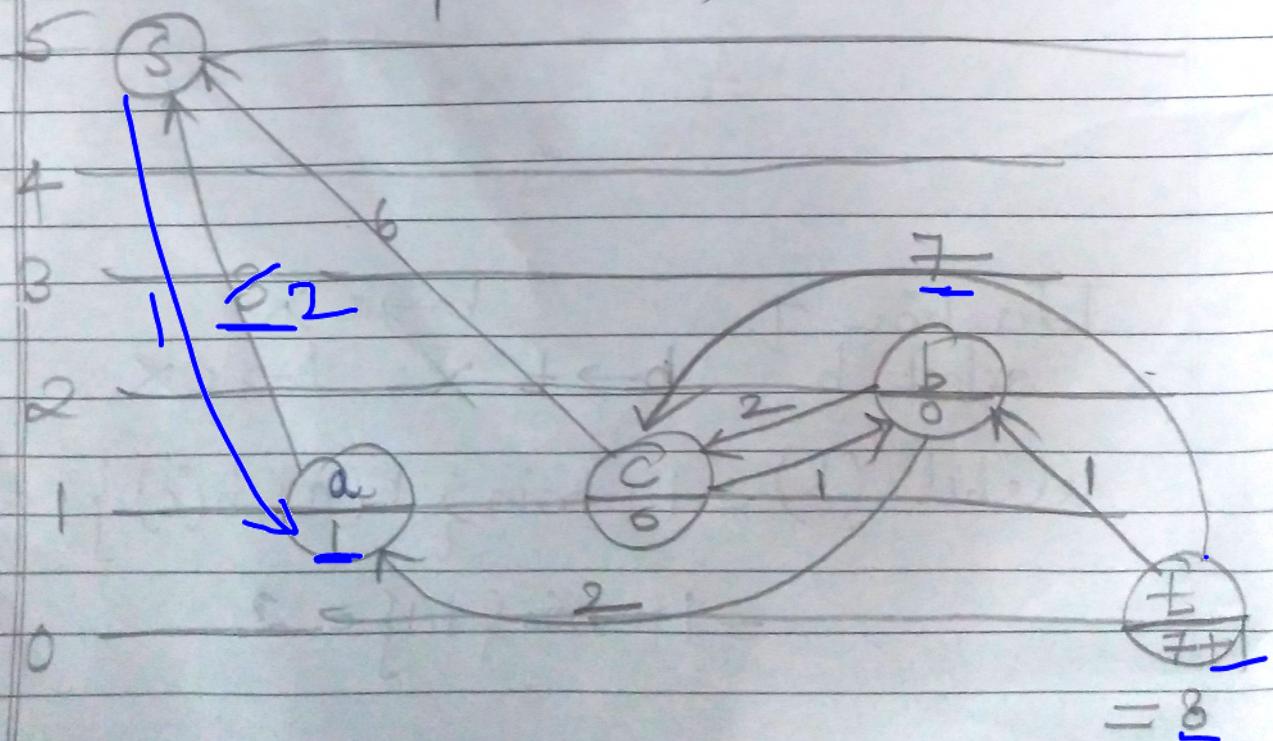
$$\text{so } \underline{\text{push}}(b, c) = \min \{ e(b), \underline{c_f}(bc) \} \\ = \min \left\{ \frac{1}{1}, \frac{3}{1} \right\} = \frac{1}{1}$$

$$\underline{e(c)} = 1 \quad \underline{e(b)} = 0$$



Iteration 5

Select C so $c \rightarrow t$ is admissible
so push (C, t)



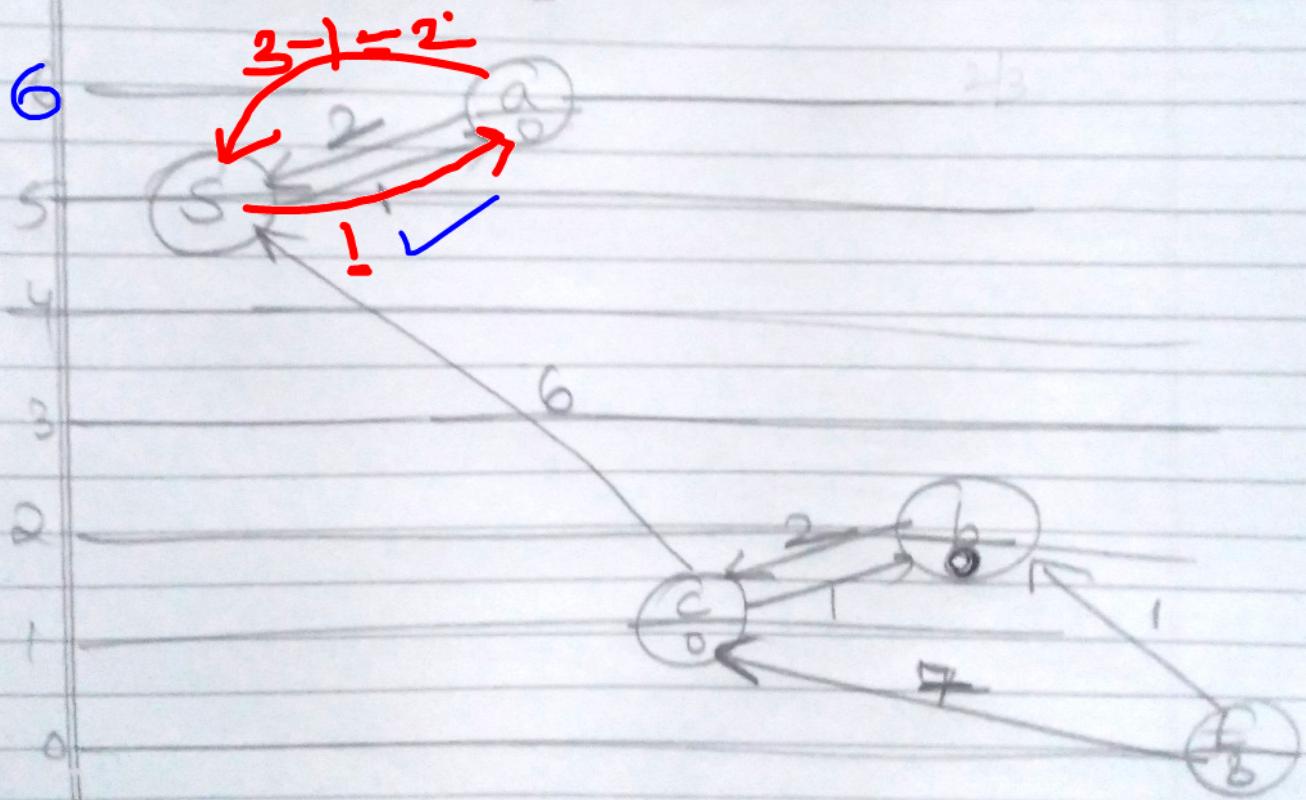
Iteration 6

Select a such $e(a) > 0$
 $\underline{a \rightarrow s} \times$

$$\text{So. } \underline{\text{selectable}}(a) = 1 + h(s)$$

$$= 1 + 5 = \underline{\underline{6}}$$

After increasing height of a , which is more than height of s node, the excess flow is given back to s , since it can't reach to b .



all internal node excess flow is 0,
so stop, & flow into t is maxflow.