Theory of Computing

Tutorial 2

Part 1 (*Sets and Languages*)

1. Let $\Sigma = \{a, m, n, t\}$ be an alphabet. Which of the following strings belong to Σ^*(Give the equivalent of each string).

a. a^4
b. $(mn)^2t^3$
c. m^3y^5

2. Give the equivalent of each of the following strings.

a. 0^21^2
b. $(01)^2$
c. $((00)^21^3)^2$

3. Consider the following languages:

a. $L_1 = \{a^mba^n: m \in N \wedge n \in N\}$ over $\Sigma = \{a,b\}$

Which of the following strings belong to L_1?

a) aaabaa b) aaab c) aabbaaa

b. $L_2 = \{ a^{n^2} : n \in N^+\}$ over $\Sigma = \{a\}$

Which of the following strings belong to L_2?

a) a b) aaaaaa c) aaaaa

c. $L_3 = \{s^jts^j: j \in N^+ \wedge (s=0 \vee s=1) \wedge (t=+ \vee t=-)\}$ over $\Sigma = \{0, 1, +, -\}$

Which of the following strings belong to L_3?

a) 0++0 b)00+00 c) 100+1256 d) 1111-1111

4. Give a formal expression that describes a language where each string.

- contains two a: an a at the beginning of the string and an a at the end of the string.
- contains an even number of b between the two a.

5. Consider the following language L defined over $\Sigma = \{t,o\}$ such as: $L = \{(to)^{2n}: n \in N \}$

a. Describe in a sentence the set of strings of this language.
b. Which of the following string belong to L?

a) ttoo b) tototo c) ttoto d) totototo

6. Give a formal expression that describes the set of strings over the alphabet $\Sigma = \{0\}$ where each string contains a prime number of 0.

7. Determine if each of the given expression is true or false. Justify your answer.

a. The empty set $\varnothing$ is a language.
b. $\varnothing \not\subseteq \{\varepsilon\}$
c. $\varepsilon \in \varnothing$

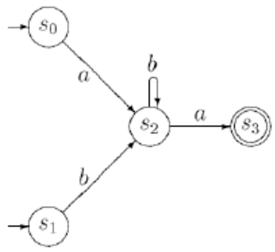
d. ε ⊆ ε
e. ∅ = {ε}
f. The string εaεεa is an element of the language {a, aa, aaa}
g. If a language is presented as the set {ε, εε, εεε}, this set contain three strings.
h. Σ* is a language for all alphabet Σ.
i. Let L be a language. ε ϵ L.
j. Σ*- Σ* is a language.

Part 2 *(Finite Auatomata)*

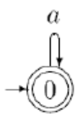
Exercise 1

Classify the following automata, first determine if the description is correct or not, then determine wither the automaton is deterministic or non-deterministic. Explain why.

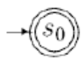
a) Σ= {a, b}



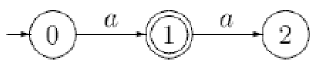
b) Σ= {a}



c) Σ= {a}

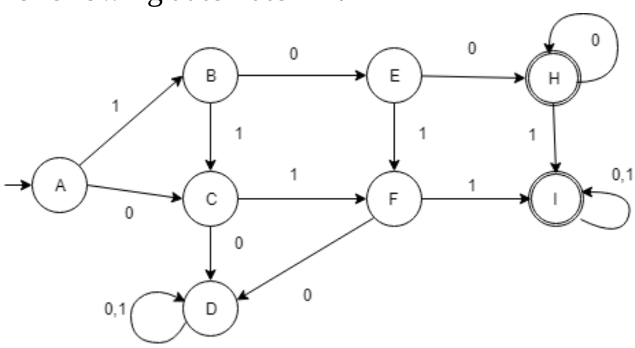


d) Σ= {a}



Exercise 2 (*Deterministic Finite Automata*)

1. Let us consider the following automaton A:



Describes how the automaton processes each of the following strings:

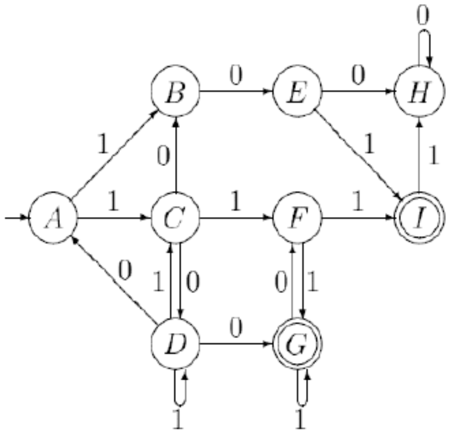
a. 10000

b. 10011
c. 1101

2. Construct a DFA that accepts the set of strings of length 2 over Σ= {0, 1}.
3. Construct a DFA that recognizes a language L defined over Σ with: Σ = {0, 1} and L = { s | |s| is even}.
4. Construct a DFA that recognizes a language L defined over Σ with: Σ = {a , b, c} and L= { s | |s| is odd}
5. Construct a DFA that recognizes a language L defined over Σ with: Σ = {0 , 1} and L= { s | |s| mod 4=0}
6. Construct a DFA that recognizes a language L defined over Σ with: Σ = {0,1} and L= { s | |s|=1 or |s|⩾3}.
7. Construct a DFA for the following language L = {ω| ω contains an equal number of occurrences of 01 and 10} defined over the alphabet Σ = {0, 1}.

Exercise 3 (*Nondeterministic Finite Automata*)

1. Let us consider the following NFA:



Describes how the automaton processes each of the following strings:

a. 10101101
b. 1110
c. 10001

2. Construct the NFAs that can recognize the following languages:

a) L1 = {ω| ω is a string in which at least one ai occurs even number of times (not necessarily consecutively), where 1 ⩽ i ⩽ 3 over Σ = {a1, a2, a3}}.
b) L2 = {ω| ω contains two 0s separated by a substring whose length is a multiple of 3}, Σ = {0, 1}.