**Theory of Computing**

**Tutorial 1**

**Exercise 1** (*Sets*)

**1.** Is each of the following a well-defined set? Give brief reasons for each of your answers.

1. The collection of all alphanumeric characters.
2. The collection of all tall people.
3. The collection of all integers *x* for which: 2*x* – 9 = 16.
4. The collection of all real numbers *x* for which: 2*x* – 9 = 16.
5. The collection of all good tennis players.

**2.** Prove the following identities:

1. A ∩ (A ∪ B) = A

**3.** Let U be the universe and *P*, *Q* and *R* three sets such that: **U** = {a, b, c, d, e, f, g, h}, *P* = {c, f}, *Q* = {a, c, d, e, f, h} and *R* = {c, d, h}

1. Draw a Venn diagram, showing these sets with all the elements entered into the appropriate regions.
2. Which of the sets *P*, *Q* and *R* are proper subsets of others?
3. Are *P* and *R* disjoint sets?

**Exercise 2** (*Graphs*)

1. Draw a picture of the graph with vertices {*v1*, *v2*, *v3, v4*} and edges {(*v1*, *v2*), (*v2*, *v3*), (*v3*, *v4*), (*v4*, *v1*), (*v1*, *v3*), (*v2*, *v4*)}.
2. Enumerate all cycles in this graph.
3. Let G = (V, E) be a simple undirected graph.
   1. What is the maximum degree a node can have?
   2. What is the maximum number of possible edges in this graph?

**Exercise 3** (*Relations and Functions*)

Determine if the given relation is a function:

1. {(2,4),(3,−7),(6,10)}
2. {(−1,8),(4,−7),(−1,6),(0,0)}
3. {(2,1),(9,10),(−4,10),(−8,1)}

**Exercise 4** (*Proof Techniques*)

1. Show that, , we have:

1. Prove that:
2. Prove that for any n > 0, if a2 is even, then a is even.
3. Prove that if a.b=n, then either a or b must be ≤, where a, b, and n are non-negative real numbers.