**Theory of Computing**

**Solution tutorial 1**

**Exercise 1** (*Sets*)

**1.** Is each of the following a well-defined set? Give brief reasons for each of your answers.

1. The collection of all alphanumeric characters.

Well-defined set. It does clearly describe the elements of the set. The set could be described as {x| x ϵ [a..z] or x ϵ [A..Z] or x ϵ [0..9] }

1. The collection of all tall people.

Not well-defined set. ‘Tall’ is not a well-defined property.

1. The collection of all integers *x* for which: 2*x* – 9 = 16.

Well-defined set but empty set. Because the solution will be a real number.

1. The collection of all real numbers *x* for which: 2*x* – 9 = 16.

Well-defined set that contains {x| x=(19+9)/2, x ϵ R}

1. The collection of all good tennis players.

Not well-defined set. ‘Good’ is not a well-defined property.

**2.** Prove the following identities:

=

= B

1. A ∩ (A ∪ B) = A

A ∩ (A ∪ B)

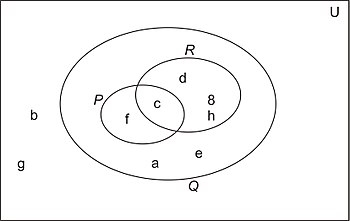
= (A ∪ ø) ∩ (A ∪ B)

= A ∪ (ø ∩ B)

= A ∪ ø = A

**3.** Let U be the universe and *P*, *Q* and *R* three sets such that: **U** = {a, b, c, d, e, f, g, h}, *P* = {c, f}, *Q* = {a, c, d, e, f, h} and *R* = {c, d, h}

1. Draw a Venn diagram, showing these sets with all the elements entered into the appropriate regions.



1. Which of the sets *P*, *Q* and *R* are proper subsets of others?

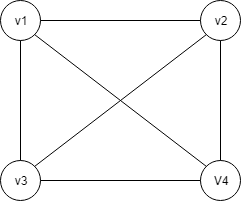
*P* ⊂ *Q*; *R* ⊂ *Q*

1. Are *P* and *R* disjoint sets?

No.

**Exercise 2** (*Graphs*)

1. Draw a picture of the graph with vertices {*v1*, *v2*, *v3, v4*} and edges {(*v1*, *v2*), (*v2*, *v3*), (*v3*, *v4*), (*v4*, *v1*), (*v1*, *v3*), (*v2*, *v4*)}.



1. Enumerate all cycles in this graph.

{(*v1*, *v2), (v2, v4), (v4, v3), (v3, v1)*}

{(*v1*, *v2), (v2, v3), (v3, v1)*}

{(*v1*, *v4), (v4, v3), (v3, v1)*}

{(*v1*, *v2), (v2, v4), (v4, v1)*}

{(*v2*, *v3), (v3, v4), (v4, v2)*}

Let G = (V, E) be a simple undirected graph.

* 1. What is the maximum degree a node can have?

n - 1 (with n the number of vertices).

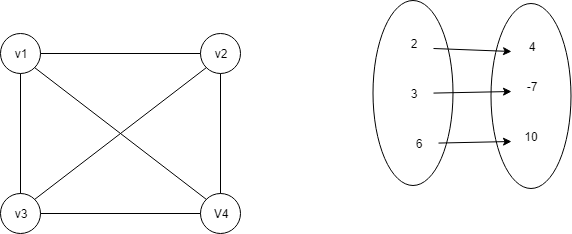
* 1. What is the maximum number of possible edges in this graph?

n(n-1)/2

**Exercise 3** (*Relations and Functions*)

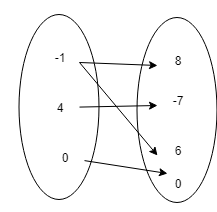
Determine if the given relation is a function:

1. {(2, 4), (3, −7), (6, 10)}



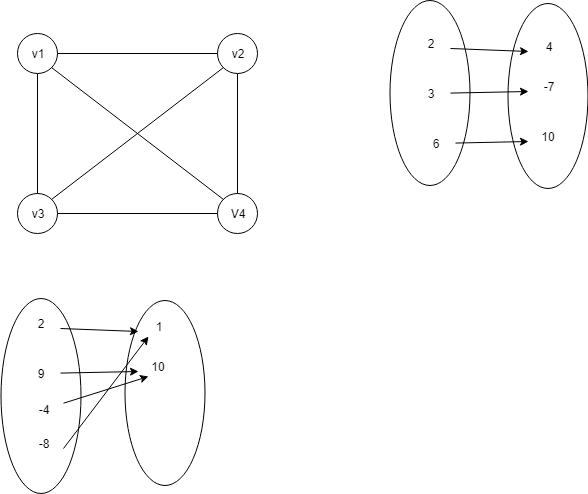
In this case, no matter which number you pick from the 1st set there is exactly one number in the second set which is associated to it. Therefore, this relation is a function.

1. {(−1,8), (4, −7), (−1, 6), (0, 0)}



We have found a number (-1) from the 1st list that has two numbers in the 2nd list associated with it. Therefore, this relation is NOT a function.

1. {(2, 1), (9, 10), (−4, 10), (−8, 1)}



In this case, no matter which number you pick from the 1st list there is exactly one number in the second list which is associated to it. Therefore, this relation is a function.

**Exercise 4** (*Proof Techniques*)

1. Show that, , we have:

Basis

For n=0, we have:

And

So, the statement holds for n=0.

Induction assumption

Let us suppose that we have, for n ϵ N:

We want to prove now that:

1. Prove that:

2n+1 -1=

Basis

For n=0

20+1 -1 = = 20 =1

Induction assumption:

Let n ϵ N, we suppose that we have:

2n+1 -1=

We want to prove that:

2(n+1)+1 -1=

= +2(n+1)

= 2n+1 -1+2(n+1)

= 2n+2-1

=2(n+1)+1-1

1. Prove that for any a , if a2 is even, then a is even.

Let a2 be even. Suppose that a is odd, then there exists k such as: with.

Then:

+ 1

Even number +1 : gives an odd number

Which gives: which is a contradiction.

1. Prove that if a.b = n, then either a or b must be ≤, where a, b, and n are non-negative real numbers.

Suppose that:

→ Impossible (contradiction)