Fourier ~~&~~ & Domain transform Idea

if we multiply two signals the more the two near in freq the more its their integeration

$$\int_{-\infty}^{\infty} \underbrace{\sin(10t)}\,\underbrace{\sin(10t)}\,dt \uparrow\uparrow\uparrow$$ leading to signal energy and increasing harmonic ampllitude at this freq

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so if want to calculate each freq ampilitude in the freq domain we need to test each freq:

$$\int_{-\infty}^{\infty} x_1(t)\cos(2\pi f t)\,dt - j\int_{-\infty}^{\infty} x_1(t)\sin(2\pi f t)\,dt$$

$$= \int_{-\infty}^{\infty} x_1(t)\,\underbrace{\left[\cos(2\pi f t) - j\sin(2\pi f t)\right]}_{\Rightarrow \text{euler formula}}$$

$$= \int_{-\infty}^{\infty} x_1(t)\, e^{-j2\pi f t}\,dt \Rightarrow \text{FTFT}$$

From FTFT we can get different types of forriers:

For DTFT: $t = nT_s$

$$X(f) = \int_{-\infty}^{\infty} x_1(nT_s)\, e^{-j2\pi f \times nT_s} = \sum_{n=-\infty}^{\infty} x_1(n)\, e^{-j2\pi \frac{f}{f_s} n}$$

$2\pi \frac{f}{f_s} n = wn \Rightarrow \frac{w}{2\pi} = \frac{f}{f_s} \Rightarrow$ move to "w" domain with rep equal to 2π

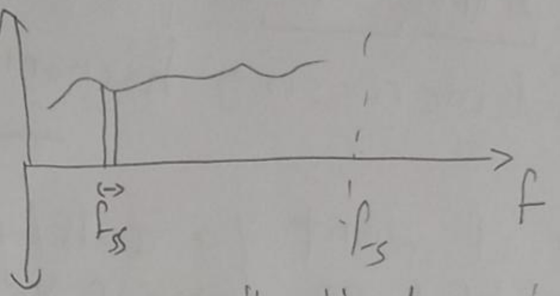
$$X(w) = \sum_{n=-\infty}^{\infty} x_1(n)\, e^{-jwn} \Rightarrow \text{DTFT}$$

DFT

"DTFT" isn't practical cause "∞" in time domain and in freq domain cause it's continous there
So we will sample freq domain with "number of samples equal to number of samples in time domain = N"

$f = k f_{ss}$

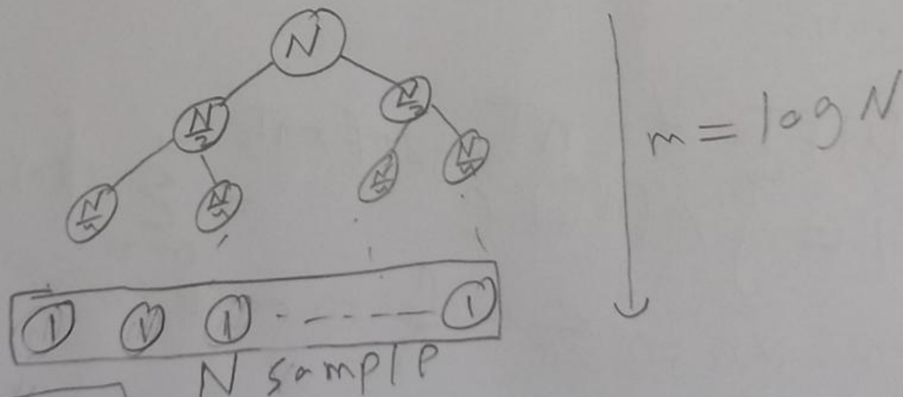
$N = \frac{f_s}{f_{ss}} = \frac{f_s k}{f}$



$\frac{N}{k} = \frac{f_s}{f}$ # we can transport to "k" domain

$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi \frac{k}{N} n} \Longrightarrow$ DFT $O(N^2)$

FFT: it's an algorithm try to make DFT more faster
instead of each freq sum all sample,
It use divide and conquar



$\boxed{O(N \log N)}$