CATapult Courseware

Module 4

Practice Exercise Solutions

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PRACTICE EXERCISE-1

1. a) 4. (8^{4n+2} will give a units digit of 4)
   b) 9. ($9^{\text{odd power}}$ will give a units digit of 9)
   c) 3. (3^{4n+1} will give a units digit of 3)
   d) 9. ($9^{\text{odd power}}$ will give a units digit of 9)

2. Find the equivalent in base 10 of
   a) The first 4-digit number in base 3 $= (1000)_3 = 0 \times 3^0 + 0 \times 3^1 + 0 \times 3^2 + 1 \times 3^3 = (27)_{10}$
   b) The last 3-digit number in base 8 $= (777)_8 = 7 \times 8^0 + 7 \times 8^1 + 7 \times 8^2 = (511)_{10}$
   c) the number 121 in base 7 $= (121)_7 = 1 \times 7^0 + 2 \times 7^1 + 1 \times 7^2 = (64)_{10}$
   d) the largest 3-digit number with distinct digits in base 5 $= (432)_5 = 2 \times 5^0 + 3 \times 5^1 + 4 \times 5^2 = (117)_{10}$
   e) the smallest 4-digit number with distinct digits in base 6 $= (1023)_6 = 3 \times 6^0 + 2 \times 6^1 + 0 \times 6^2 + 1 \times 6^3 = (231)_{10}$

3. $(4654)_7 = 4(7)^3 + 6(7)^2 + 5(7)^1 + 4(7)^0$
   $= 1372 + 294 + 35 + 4$
   $= (1705)_{10}$
   1705 divided by 14 gives 11 as remainder.
   Hence, (2).
   Alternatively,
   $4(7)^3$ and $6(7)^2$ is divisible by 14. Therefore the remainder when $(4654)_7$ is divided by 14, is equal to remainder of $\frac{5(7)^1 + 4(7)^0}{14}$ i.e., 11

4. The two numbers $(3565)_{10}$ and $(1773)_{10}$ are relatively prime.
   $\therefore$ HCF = 1, and LCM = 3565 × 1773 = 6320745, HCF + LCM = 6320746
   In base 5, the solution is written as:

5	6320746	
5	1264149	1
5	252829	4
5	50565	4
5	10113	0
5	2022	3
5	404	2
5	80	4
5	16	0
	3	1
		3

   $\therefore (6320746)_{10} = (3104230441)_5$
   Hence, (1).

   Alternatively,
   As HCF + LCM has unit digit 6, and hence in base 5, number will end in 1. Only option (1) has last digit as 1.

5. $\log_3 x = a \Rightarrow x = 3^a$
   $\log_{12} y = a \Rightarrow y = 12^a$
   $G = \sqrt{xy}$
   $G = \sqrt{3^a \times 12^a} = \sqrt{36^a} = \sqrt{6^a \times 6^a} = 6a$
   $\log_6 G = \log_6 6^a = a \log_6 6 = a \times 1 = a$
   Hence, (4).

6. $\log_{0.008} \sqrt{5}$
   $\Rightarrow \frac{\log 5^{\frac{1}{2}}}{\log \frac{8}{1000}} \Rightarrow \frac{\frac{1}{2}\log 5}{\log \frac{1}{125}}$
   $\Rightarrow \frac{\frac{1}{2}\log 5}{\log 5^{-3}} \Rightarrow \frac{\frac{1}{2}\log 5}{-3\log 5} \Rightarrow \frac{1}{2} \div -3 = \frac{1}{2} \times \frac{1}{-3}$
   $\Rightarrow -\frac{1}{6}$
   $\log_{\sqrt{3}} 81 = \log_{\sqrt{3}} \left(\sqrt{3}\right)^8 = 8 \log_{\sqrt{3}} \left(\sqrt{3}\right) = 8$
   $\log_{0.008} \sqrt{5} + \log_{\sqrt{3}} 81 - 7$
   $\Rightarrow -\frac{1}{6} + 8 - 7 = -\frac{1}{6} + 1 = \frac{5}{6}$.
   Hence, (3).

7. Given $\log_3 5 = \log_5 (x + 2)$
   Now
   $\log_3 3 < \log_3 5 < \log_3 9$
   $1 < \log_3 5 < 2$
   $5 < 5 \log_3 5 < 5^2$
   $5 < 5\log_5 (2 + x) < 25$
   $5 < 2 + x < 25$
   $3 < x < 23$
   Hence, (4).

8. $2\log_{a_2} a_1 \times 2\log_{a_3} a_2 \times 2\log_{a_4} a_3 \ldots \times 2\log_{a_n} a_{n-1} \times \log_{10} a_n$
   $= 2^{n-1}\left[\frac{\log a_1}{\log a_2} \times \frac{\log a_2}{\log a_3} \times \frac{\log a_3}{\log a_4} \times \ldots \times \frac{\log a_{n-1}}{\log a_n}\right] \times \log_{10} a_n$
   $= 2^{n-1} \frac{\log a_1}{\log a_n} \times \frac{\log a_n}{\log 10} = 2^{n-1} \frac{\log a_1}{\log 10}$
   $= 2^{n-1} \log_{10} a_1$. Hence, (3).

9. $\log_{81}32 = \frac{5}{4}\log_{11}2 \times \log_a b$

$\Rightarrow \log_{3^4}2^5 = \frac{5}{4}\log_{11}2 \times \log_a b$

$\Rightarrow \frac{5}{4}\log_3 2 = \frac{5}{4}\log_{11}2 \times \log_a b$

$\Rightarrow \frac{\log 2}{\log 3} = \frac{\log 2}{\log 11} \times \log_a b$

$\Rightarrow \log_3 11 = \log_a b$

As $3 \le a \le 11$

When a = 3, b = 11 ...(i)

When a = 11, then $\log_3 11 = \log_{11} b$

but $\log_3 11 = k \Rightarrow 3^k = 11$

As $3^2 = 9$ and $3^3 = 27 \Rightarrow 2 < k < 3$

and $11^k = b \Rightarrow 121 < b < 1331$ ... (ii)

From (i) and (ii),

$11 \le b < 1331$

Hence, (2).

10. $2\log\left(\frac{b}{a}\right) = \log\left(\frac{a}{b}\right) + \log\left(\frac{bc}{a^2}\right)$

$\log\frac{b^2}{a^2} = \log\left(\frac{a}{b} \times \frac{bc}{a^2}\right)$

$\log\frac{b^2}{a^2} = \log\left(\frac{c}{a}\right)$

$\log\frac{b^2}{a^2} - \log\frac{c}{a} = 0$

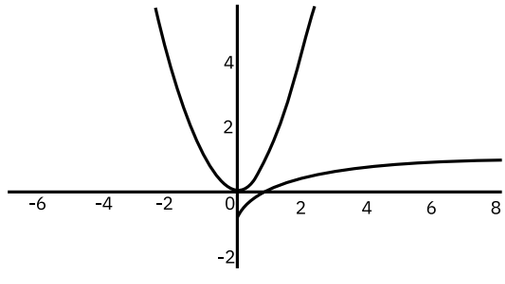
$\log\left[\frac{b^2}{a^2} \times \frac{a}{c}\right] = 0$

$\log\frac{b^2}{ac} = 0$

$\Rightarrow \frac{b^2}{ac} = 1$

$\Rightarrow b^2 = ac$. Hence, (3).

11. The function $y = x^2$ is positive for all values of x except x = 0. The function $\log_{10}x$ is negative for all values of x such that x < 1. At x = 1, $x^2 = 1$, while $\log_{10}x = 0$. As the value of x increases, the function $y = x^2$ increases more rapidly than the function $y = \log_{10}x$, as shown below:



Thus it can be seen that the two curves do not intersect. Hence (1).

12. $\log_y x = a\log_z y = b\log_x z = ab$

$\frac{a\log y}{\log z} \times \frac{b\log z}{\log x} = ab \times ab$

$ab \times \frac{\log y}{\log x} = (ab)^2 \Rightarrow \frac{1}{\log_y x} = ab \Rightarrow 1 = ab\log_y x$

Also, $\log_y x = ab \Rightarrow 1 = (ab)^2$

Thus, for (a, b) = (2, 2)

$1 \neq (ab)^2$. Hence, (5).

13. Let the base of the numbers 44, 11 and 1034 be x

$\therefore (44)_x \times (11)_x = (1034)_x$

$\Rightarrow (4 \times x^0 + 4 \times x^1)(1 \times x^0 + 1 \times x^1)$

$= (4 \times x^0 + 3 \times x^1 + 0 \times x^2 + 1 \times x^3)$

$\Rightarrow (4x + 4)(x + 1) = (x^3 + 0 + 3x + 4)$

$\Rightarrow 4(x + 1)(x + 1) = (x^3 + 3x + 4)$

$\Rightarrow 4(x^2 + 2x + 1) = x^3 + 3x + 4$

$\Rightarrow 4x^2 + 8x + 4 = x^3 + 3x + 4$

$\Rightarrow x^3 - 4x^2 - 5x = 0$

$\Rightarrow x(x^2 - 4x - 5) = 0$

$\Rightarrow x(x^2 - 5x + x - 5) = 0$

$\Rightarrow x(x - 5)(x + 1) = 0 \quad \Rightarrow x = -1, 0$ or 5

But as the base of the number system has to be a natural number, x = 5

Now $(3111)_5$ when expressed in decimal number system = $1 \times 5^0 + 1 \times 5^1 + 1 \times 5^2 + 1 \times 5^3 = 1 + 5 + 25 + 375 = 406$. Hence, (1).

14. $(297)_B$ in decimal form $\Rightarrow 7 + 9B + 2B^2$

$\Rightarrow 2B^2 + 7B + 2B + 7 \Rightarrow B(2B + 7) + 1(2B + 7)$

$\Rightarrow (2B + 7)(B + 1)$ ..... (I)

$(792)_B$ in decimal form $\Rightarrow 2 + 9B + 7B^2$

$\Rightarrow 7B^2 + 7B + 2B + 2 \Rightarrow 7B(B + 1) + 2(B + 1)$

$\Rightarrow (B + 1)(7B + 2)$ ..... (II)

Now dividing (II) by (I) we get $\Rightarrow \frac{7B + 2}{2B + 7}$

Now as $(297)_B$ is a factor of $(792)_B$ (2B + 7) has to be a factor of 7B + 2 or $\frac{7B + 2}{2B + 7}$ has to be a natural number.

Only for B = 19, $\frac{7B + 2}{2B + 7} = \left(\frac{7 \times 19 + 2}{2 \times 19 + 7} = \frac{135}{45}\right) = 3$ and is a natural number. Hence, (5).

15. $\log_4 a^2 = 2.\log_4 a = 2.\frac{\log_2 a}{\log_2 4} = 2.\frac{\log_2 a}{2} = \log_2 a$

$\log_8 a^3 = 3.\log_8 a = 3.\frac{\log_2 a}{\log_2 8} = 3.\frac{\log_2 a}{3} = \log_2 a$

Therefore each of the three numbers is $\log_2 a$. Their average is $\log_2 a$. Hence $a = 2^3 = 8$

16. $$\frac{\log_4 7 + \log_3 7}{\log_4 7 \times \log_3 7} = \frac{\frac{1}{\log_7 4} + \frac{1}{\log_7 3}}{\frac{1}{\log_7 4} \times \frac{1}{\log_7 3}}$$

$= \log_7 4 + \log_7 3 = \log_7 12$. Hence (4).

17. $$\frac{1}{\log_2 x} + \frac{1}{\log_3 x} + \ldots + \frac{1}{\log_9 x} = 2$$

$\therefore \log_x 2 + \log_x 3 + \ldots + \log_x 9 = 2$

$\therefore \log_x (2 \times 3 \times \ldots \times 9) = 2$

$\therefore x^2 = 9!$

$$\frac{1}{\log_2 y} + \frac{1}{\log_3 y} + \ldots + \frac{1}{\log_8 y} = 2$$

$\therefore \log_y 2 + \log_y 3 + \ldots + \log_y 8 = 2$

$\therefore \log_y (2 \times 3 \times \ldots \times 8) = 2$

$\therefore y^2 = 8!$

$$\therefore \frac{x^2}{y^2} = \frac{9!}{8!} = 9$$

$$\therefore \frac{x}{y} = 3$$

Hence, (1).

18. $\text{Log } 5^{30} = 30 \log 5 = 30 \log 10 - 30 \log 2 = 30(1) - 30 * 0.3010 = 30 - 9.03 = 20.97$
Hence no of digits in $5^{30} = 21$
Hence, (3).

19. $\log_{(x-a)}(8x^3 - 36x^2 + 54x - 27) = 3 + \log_{(x-a)}8$
$\therefore \log_{(x-a)}(8x^3 - 36x^2 + 54x - 27) - \log_{(x-a)}8 = 3$
$\therefore \log_{(x-a)}[(8x^3 - 36x^2 + 54x - 27) \div 8] = 3$

$$\therefore \log_{(x-a)}\left(x^3 - \frac{9}{2}x^2 + \frac{27}{4} - \frac{27}{8}\right) = 3$$

$$\therefore \left(x - \frac{3}{2}\right)^3 = (x - a)^3$$

$\therefore a = \frac{3}{2}$. Hence, (3).

20. $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$
$= \log_x x - \log_x y + \log_y y - \log_y x$
$= 1 - \log_x y + 1 - \log_y x$
$= 2 - (\log_x y + \log_y x)$
For $x \geq y$, $y > 1$
$\log_y x \geq 1$
$\therefore \log_y x + \log_x y > 1$
$\therefore 2 - (\log_x y + \log_y x) < 2 - 1 = 1$
Hence, $\log_x\left(\frac{x}{y}\right) + \log y\left(\frac{y}{x}\right)$ cannot be 1.
Hence, (4).

21. $x^u = 256$
taking log on both sides we get,
$u \log_2 x = \log_2 256$
it can also be written in the following ways.
(1) $u\log_2 x = 8\log_2 2$
(2) $u\log_2 x = 4\log_2 4$
(3) $u\log_2 x = 2\log_2 16$
(4) $u\log_2 x = 1\log_2 256$
Consider, $u = (\log_2 x)^2 - 6 \log_2 x + 12$ ... (i)
(1) $u \log_2 x = 8 \log_2 2$
$\therefore x = 2, u = 8$
Substituting in (i), we get
$u = \log_2 2 - 6 \log_2 2 + 12$
$= 1 - 6 + 12 = 7$
$\therefore$ LHS $\neq$ RHS
(2) $u \log_2 x = 4 \log_2 4$
$\therefore u = 4, x = 4$
$\therefore u = (\log_2 4)^2 - 6 \log_2 4 + 12$
$\therefore 4 - 12 + 12 = 4$
$\therefore$ LHS = RHS
(3) $\therefore u = 2, x = 16$
$\therefore u = (\log_2 16)^2 - 6 \log_2 16 + 12$
$\therefore 16 - 24 + 12$
$= 4$
$\therefore$ LHS $\neq$ RHS
(4) $u = \log_2 16$
$\therefore u = 1, x = 256$
$\therefore u = (\log_2 256)^2 - 6 \log_2 256 + 12$
$= 64 - 48 + 12 = -4$
$\therefore$ LHS $\neq$ RHS
$\therefore$ there is only one solution for x.
Hence, (2).

22. $\log x - \log \sqrt{x} + \log \sqrt[3]{x} - \log \sqrt[4]{x} = \frac{7}{24}$

$$\therefore \log x - \log x^{\frac{1}{2}} + \log x^{\frac{1}{3}} - \log x^{\frac{1}{4}} = \frac{7}{24}$$

$$\therefore \log x - \frac{1}{2}\log x + \frac{1}{3}\log x - \frac{1}{4}\log x = \frac{7}{24}$$

$$\therefore (\log x) \times \left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right) = \frac{7}{24}$$

$$\therefore (\log x) \times \left(\frac{7}{12}\right) = \frac{7}{24}$$

$$\therefore \log x = \frac{1}{2}$$

$$\therefore x = 10^{\frac{1}{2}} = \sqrt{10}$$

$$\text{Now, } \log\left(\frac{x^3 - x^2 + x - 1}{x - 1}\right) = \log(x^2 + 1) = \log 11.$$

The value of log 11 will be between 1 and 1.5.
Hence (3).

PRACTICE EXERCISE-2

1. To find rightmost non-zero digit of 20^{2520}, we use cyclicity of 2 which is 4.
2520 is divisible by 4 and $2^4 = 16$.
Thus, rightmost non-zero digit is 6.
Hence, (3).

2. $90^{468} = (9 \times 10)^{468} = 9^{468} \times 10^{468}$
10^{468} is 1 followed by 468 zeroes. Thus $9^{468} \times 10^{468}$ has 468 zeroes at the end.
Hence 469th digit, from the right of 90^{468} will be the units' digit of 9^{468}.
Units' digit of 9^{468} = units' digit of $(81)^{234} = 1$.
Hence, (1).

3.

12	4760	
	396	8
	33	0
	2	9
	0	2

$\therefore (4760)_{10} = (2908)_{12}$.
Hence, (1).

4. $(16)_8 = (14)_{10}$
$(5)_{10} = (5)_{10}$
$(3)_4 = (3)_{10}$
$(101)_2 = (5)_{10}$
$(3)_8 = (3)_{10}$

$$A_{10} = \frac{(3)_8 \times (16)_8 \times (5)_{10}}{(3)_4 \times (101)_2}$$

$$= \frac{(3)_{10} \times (14)_{10} \times (5)_{10}}{(3)_{10} \times (5)_{10}} = (14)_{10}.$$

Hence, (1).

5. Powers of 7 follows a cyclicity of 4 for successive powers (beginning with 7^1) with last two digits ending with 07, 49, 43, 01, 07.... and so on.
Thus, 7^{2008} ends with 01. Hence, (3).

6. Since 2 has a cyclicity of 4,
i.e. $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$ ..., the last digits (2, 4, 8, 6) occur in a cycle of 4.
Since cyclicity of the power of 2 is 4, so 2^{51} can be written in $2^{4(12)+3}$ or unit digit will be $2^3 = 8$.
Hence, (2).

7. $\log_{12} 18$

$$= \frac{\log_{10} 18}{\log_{10} 12} = \frac{\log_{10}(2 \times 3)^2}{\log_{10}(2^2 \times 3)} = \frac{\log_{10} 2 + 2\log_{10} 3}{2\log_{10} 2 + \log_{10} 3}$$

$$= \frac{x + 2y}{2x + y}$$

$\therefore 2x.\log_{10}2 + 4x.\log_{10}3 + y.\log_{10}2 + 2y.\log_{10}3$
$= 2x.\log_{10}2 + x.\log_{10}3 + 4y.\log_{10}2 + 2y.\log_{10}3$
$\therefore 3x.\log_{10}3 = 3y\log_{10}2$

$$\therefore \frac{x}{y} = \frac{\log_{10} 2}{\log_{10} 3} = \log_3 2.$$

Hence, (1).

8. $\log_{12}81 = p \Rightarrow 81 = 12^p$
$\Rightarrow 3^{(4-p)} = 2^{2p}$
Taking log on both the sides,
$(4 - p)(\log 3) = (2p)(\log 2)$

$$\therefore \frac{\log 3}{\log 2} = \frac{2p}{(4-p)}$$

$$\therefore \frac{\log 3 + \log 2}{\log 2} = \frac{2p + (4-p)}{(4-p)} \quad \text{...(by componendo)}$$

$$\therefore \frac{\log 3 + \log 2}{\log 2} = \frac{(4+p)}{(4-p)}$$

$$\therefore \frac{\log 6}{\log 2} = \frac{(4+p)}{(4-p)}$$

$$\therefore \frac{(4-p)}{(4+p)} = \frac{\log 2}{\log 6}$$

$$\therefore 3\frac{(4-p)}{(4+p)} = 3 \times \frac{\log 2}{\log 6} = \frac{\log 2^3}{\log 6} = \frac{\log 8}{\log 6} = \log_6 8$$

Hence, (3).

9. $2^x = 3^{\log_5 2}$
Taking log on both the sides,
$x \log 2 = (\log_5 2)(\log 3)$

$$\therefore x = \frac{(\log_5 2)(\log 3)}{(\log 2)} = \frac{(\log 2)(\log 3)}{(\log 5)(\log 2)} = \frac{(\log 3)}{(\log 5)} = \log_5 3$$

(1): $1 + \log_3 \frac{5}{3} = 1 + \frac{\log\left(\frac{5}{3}\right)}{\log 3} = 1 + \frac{\log 5 - \log 3}{\log 3}$

$$= \frac{\log 5}{\log 3} = \log_3 5$$

(2): $\log_5 9 = 2(\log_5 3)$

(3): $\log_5 8 = 3(\log_5 2)$

(4): $1 + \log_5 \frac{3}{5} = 1 + \frac{\log\left(\frac{3}{5}\right)}{\log 5} = 1 + \frac{\log 3 - \log 5}{\log 5}$

$$= \frac{\log 3}{\log 5} = \log_5 3$$

Hence, (4).

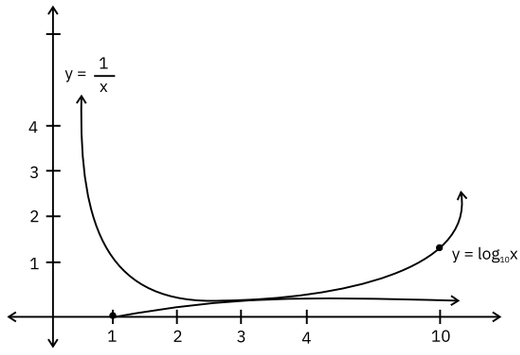
10. $\log_2(5 + \log_3 a) = 3$
$\therefore (5 + \log_3 a) = 2^3 = 8$
$\therefore \log_3 a = 3$
$\therefore a = 3^3 = 27$

$4a + 12 + \log_2 b = (4 \times 27) + 12 + \log_2 b = 120 + \log_2 b$
$\log_5(4a + 12 + \log_2 b) = \log_5(120 + \log_2 b) = 3$
$\therefore (120 + \log_2 b) = 5^3 = 125$
$\therefore \log_2 b = 5$
$\therefore b = 2^5 = 32$
Therefore, $a + b = 27 + 32 = 59$
Hence, (3).

11. $\log_s (pqr) = \log_s p + \log_s q + \log_s r$
Given: $p^3 = s^6$ or $p = s^2$. Therefore, $\log_s p = 2$.
On similar lines, $\log_s q = \frac{3}{2}$ and $\log_s r = \frac{6}{5}$
Therefore, $\log_s(pqr) = \log_s p + \log_s q + \log_s r = \frac{47}{10}$
Hence, (1).

12. $\frac{1}{\log_2 100} - \frac{1}{\log_4 100} + \frac{1}{\log_5 100} - \frac{1}{\log_{10} 100}$
$+ \frac{1}{\log_{20} 100} - \frac{1}{\log_{25} 100} + \frac{1}{\log_{50} 100}$
$= \log_{100} 2 - \log_{100} 4 + \log_{100} 5 - \log_{100} 10 + \log_{100} 20$
$- \log_{100} 25 + \log_{100} 50$
$= \log_{100}\left(\frac{2 \times 5 \times 20 \times 50}{4 \times 10 \times 25}\right) = \log_{100}(10) = \frac{1}{2}$.
Hence, (2).

13. $y = \log_{10} x$, $y = \frac{1}{x}$
They meet at $\frac{1}{x} = \log_{10} x$



Here, we see the graph has only 1 intersection point.
Hence, (2).

14. Since, $\log_3(2)$, $\log_3(2x - 5)$ and $\log_3\left(2^x - \frac{7}{2}\right)$ are in A.P., we can write.
$2 \times \log_3(2x - 5) = \log_3(2) + \log_3\left(2^x - \frac{7}{2}\right)$
$\log_3(2^x - 5)^2 = \log_3(2^{x+1} - 7)$
$(2^x - 5)^2 = 2^{x+1} - 7$
$x = 2$ and $x = 3$ both satisfy the equation. But $x = 2$ cannot be the answer as
$\log_3(2^x - 5) = \log_3(2^2 - 5) = \log_3(4 - 5) = \log_3(-1)$
But logarithm of negative number is not defined. Hence, $x = 3$ is the answer.
Hence, (4).

15. If the digit 5 is skipped, the base system changes from 10 to 9 and also the actual value of any digit greater than 5 will be reduced by 1.
$\therefore$ In base 9, actual value of 3016 will be $3 \times 9^3 + 0 \times 9^2 + 1 \times 9^1 + 5 \times 9^0 = 2201$. Hence, (1).

16. Since log A, log B and log C are in arithmetic progression, therefore log B – log A = log C – log B.
$\therefore \log\left(\frac{B}{A}\right) = \log\left(\frac{C}{B}\right)$
$\therefore \frac{B}{A} = \frac{C}{B}$
$\therefore B^2 = AC$
$\therefore$ B, AC, and ABC is same as B, B^2, and B^3, which form a GP. Hence (2).

17. $\log(324)^5 = 5 \log(324) = 5 \log(81 \times 4)$
$= 4 [\log(81) + \log(4)] = 5 [\log(34) + \log(2^2)] = 5 [4 \log(3) + 2\log(2)]$
$= 5 [4 \times 0.4771 + 2 \times 0.3010] = 5(1.9084 + 0.6020)$
$= 5 \times (2.5104)$
ie, $\log(324)^5 \approx 12.552$
Hence, number of digits in $(324)^5 = 12+1 = 13$
Hence, (3).

18. We have,
$(abc)_8 = (64a + 8b + c)_{10}$ and
$(abc)_6 = (36a + 6b + c)_{10}$
$\therefore (abc)_8 - (abc)_6 = (28a + 2b)_{10}$
$\therefore 28a + 2b = 90$ or $14a + b = 45$
The maximum value that a, b and c can take is 5.
Therefore, a = 3 and b = 3 is the only solution that satisfies this equation. The variable 'c' is not present in the equation, so it can take any value between 0, 1, 2, 3, 4 and 5 (total 6 values), since in base 6 highest value c can take is 5.
Therefore, the requirement is satisfied for 330, 331, 332, 333, 334 and 335 (total 6 numbers). Hence, (2).

19. Let the base 4 number be ab [a can take values 1, 2 and 3 and b can take values 0, 1, 2 and 3].
The decimal value of the number = $4^1 \times a + 4^0 \times b = 4a + b$
If the number is in base 5 then the decimal value of the number = $5^1 \times a + 5^0 \times b = 5a + b$
$\therefore$ The difference (say D) between the squares in base 10 is given by,
$D = (5a + b)^2 - (4a + b)^2 = 9a^2 + 2ab$
When a = 1, b = 0, 1, 2, 3, the corresponding values of

D = 9, 11, 13, 15
When a = 2, b = 0, 1, 2, 3, the corresponding values of D = 36, 40, 44, 48
When a = 3, b = 0, 1, 2, 3, the corresponding values of D = 81, 87, 93, 99
$\therefore$ D can take all the values in the options except 79.
Hence, (4).

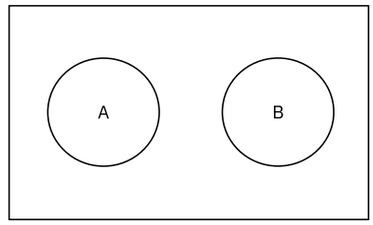
20. Of the four given options 63 and 75 are multiples of 3, thus, their remainder cannot be 1.
Now,
$31 = (1111)_2 = (1011)_3 = (111)_5$
and $91 = (1011011)_2 = (10101)_3 = (331)_5$
Hence, (4).

21. Suppose $\log_2 x = z$.
Thus, $(z + 1)(z - 6) = y$
Thus, y will be positive when $z > 6$ or when $z < -1$.
Hence, $\log_2 x < -1$ or $\log_2 x > 6$
But if $\log_2 x < -1$, then x won't be an integer and hence it is not possible.
Also, as x is 2-digit integer, $6 < \log_2 x < 7$
But if $\log_2 x$ is less than 7, then y will never be a 2-digit integer. Hence, the equation has no solutions. Therefore, the required answer is 0.

QA-4.2 | SET THEORY

PRACTICE EXERCISE-1

1.



$A \subset A \cup B$ is true. Hence, (3).

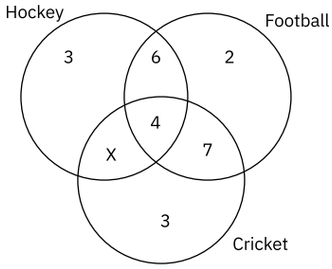
2. The shaded region is represented by $(A \cup B) \cap (A \cup C)$. Hence (3).

3. Here
$A \equiv \{... -7, -5, -3, -2, 2, 3...\}$
$B \equiv \{... -3, -2, -1, 1, 2,...\}$
$C \equiv \{... \sqrt{2}, \sqrt{3}, 2, \sqrt{5} ...\}$. Hence, (4).

4. $A \equiv \{0\}$
$B \equiv \{\}$ {there is no prime number which is cube of a natural number}
$C \equiv \{-1\}$
$D \equiv \{2\}$
Thus we see that only B is an empty set. Hence, (2).

5. I. As all parallelograms are quadrilaterals $B \subset A$ and all squares are rectangles $C \subset D$. Thus, I is true.
II. $A \cup B = A$ = Set of all quadrilaterals as $B \subset A$.
$C \cup D = D$ = Set of all rectangles as $C \subset D$. Thus, II is true.
III. $A \cap C = C$. $\because C \subset A$
$B \cap D = D$ $\because D \subset B$. Thus, III is true.
Hence, (4).

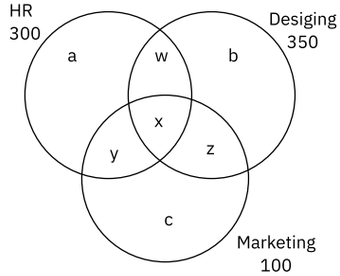
For answers to questions 6 and 7:



6. $3 + 3 + 2 + x + 6 + 7 + 4 = 30$.
$\therefore x = 5$. Hence, (4).

7. From the figure, number of boys who can play at least 2 games = 6 + 5 + 4 + 7 = 22. Hence, (4).

8.



Now total strength = 300 + 350 + 100 – (w + x) – (y + x) – (z + x) + x
$\therefore$ 600 = 750 – (w + y + z) – 2x
But w + y + z = 120
$\therefore$ 600 = 750 – 120 – 2x
$\therefore$ 2x = 30
$\therefore$ x = 15. Hence, (3).

9. Now 600 = (a + b + c) + (w + y + z) + x
= a + b + c + 120 + 15
$\therefore$ a + b + c = 600 – 135 = 465. Hence, (1).

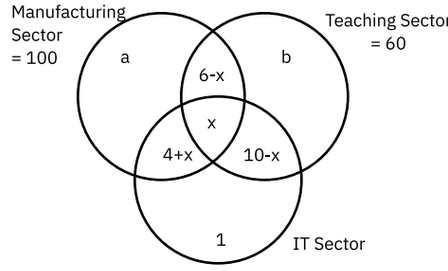
10. The required number is (w + y + z) + x
= 120 + 15 = 135. Hence, (2).

11. Given that a + c = 250
Also a + b + c = 465 (using answer 9).
$\therefore$ b = 215. Hence, (3).

12. As we know a + b + c = 465 and a + c = 250 (Using question 11).But we cannot find out a. Hence, (4).

13. The required value is x + z; while we know only x, we can't calculate z. Hence, (4).

14. Let the number of people employed in all the three sectors be x.
The given data can be represented as

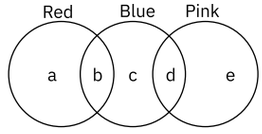


100 = a + 6 – x + x + 4 + x
100 = a + 10 + x
$\therefore$ a = 90 – x
Also, 60 = b + 6 – x + x + 10 – x
$\therefore$ b = 44 + x

Total population = a + b + 1 + 20
=90–x+44+x+1+20
= 134 + 20 + 1
= 155

Hence, (3).

Answers to questions 15 and 16:

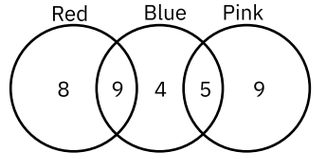


$a + b + c + d + e = 35$ ... (i)
$a + c + e = 21$ ... (ii)
$\therefore b + d = 14$ ... (iii)
$b = a + 1$ ... (iv)
$d = e - 4$ ... (v)
$a \geq d$
$c < 5$ ... (vi)
$a > 4$
$e > 8$

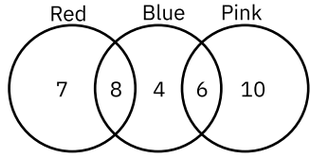
Using (ii) and (vi), probable values of a, c, e, d are:

a	c	e	d
7	4	10	6
8	4	9	5
7	3	11	7
8	3	10	6

Using (iii), set of values where c = 3 cannot be used as values of b will not follow.
Case (i):



Case (ii):



As number of diaries > number of employees, we are sure about number of employees having only Red covered, only Blue covered or only Pink covered, but the number of employees having 2 diaries might have more than 1 diary of the same coloured cover.

15. Using case (ii), the minimum possible number of red covered diaries is 7 + 8 = 15. Hence, (1).

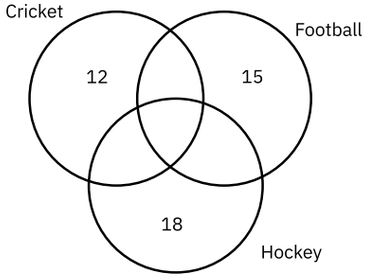
16. Minimum requirement of pink covered diaries = 5 + 9 = 14. Hence, (1).

17. Suppose 'a' is the number of students who study only one language, 'b' is the number of students who study exactly two languages and 'c' is the number of students who study all the three languages.

Therefore we have, a + b + c = 60
a + 2b + 3c = 30 + 20 + 40 = 90
From the two equations, b + 2c = 30
Maximum number of students who study three languages will be obtained by equating b to zero.
$\therefore$ Maximum number of students who study three languages = $\frac{30}{2}$ = 15. Hence (3).

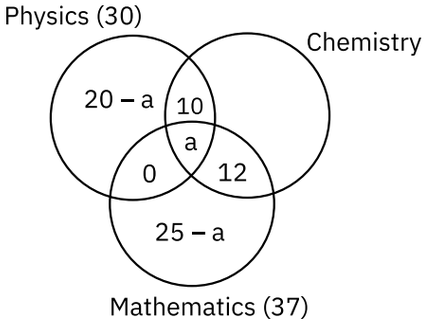
18. If we observe the set T, we get that the sum of the first and last term is 470, the sum of the second and the second last term is 470 and so on.
Hence, only one of those two terms will be in 5. (eg. only one of 3 and 467)
i.e., we can have maximum half of the consecutive terms of T in S, starting.
Also, the terms in T are in A.P., where a = 3, d = 8
$T_n = a + (n - 1)d \Rightarrow 467 = 3 + (n - 1)8 \Rightarrow n = 59$
$\therefore$ The maximum number of terms in S = $\frac{59 + 1}{2}$ = 30. Hence, (4).

19. We can draw a Venn diagram as follows:



To maximize the number of students who play exactly two sports, the number of students who play all the three sports should be minimum, that is, 0.
$\therefore$ Required answer = 60 – 12 – 15 – 18 = 15.

20. The given situation can be represented through Venn diagram as follows:



Now, (20 – a) + (25 – a) = 45 $\therefore$ a = 0.
$\therefore$ The number of students who like only Chemistry = 73 – 20 – 10 – 12 – 25 = 6
Therefore, the required answer is 6.

PRACTICE EXERCISE-2

1. B, C, and D are empty sets. A = {0} and therefore is not an empty set. Hence, (1).

2. The given shaded region can be represented as (B∩A) U[(BUA) – B], A as well as (B∩A) + (B'∩A). Hence, (4).

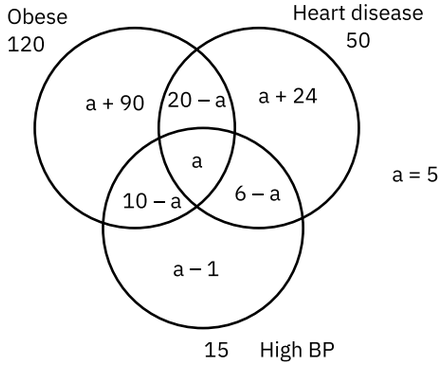
3. A ≡ {....–4,–2,2,4,.....}
B ≡ {2,3,5,7....}
C ≡ {1,3,5.....}
Now A∩B = {2}
B∩C = {3,5,....}
A∩C = { }
Hence, (1).

4. I. I is a true as it is the criteria which element can be included in a set and not the number of elements in the given set which defines the validity of a set.
∴ Set of natural numbers is a well-defined set.
II. II is false as a null set is an empty set and not even '0' is included.
III. III is true and S2 is an improper subset of S1.
IV. IV is false as S2 is not even a subset because all the elements of S2 are not included in S1.
Hence, (2).

5. $n(C \cup E \cup T)$
$= n(C) + n(E) + n(T) – n(C \cap T) –$
$n(E \cap T) – n(C \cap E) + n(C \cap T \cap E)$
= 72 + 39 + 75 – 53 – 26 – 32 + 21 = 96
$n(C \cup E \cup T)' = 100 – 96 = 4$
4 people had neither cereal, toast nor egg.
Hence, (2).

Answers to questions 6 and 7:

Using the information the following diagram can be drawn:

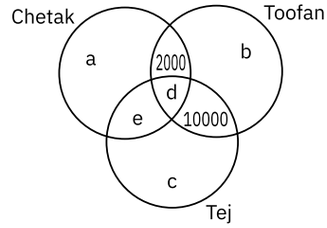


6. There are 4 persons suffering only from high BP. Hence, (4).

7. From the Venn diagram, the number of obese persons having neither heart disease nor high BP is 95.
Hence, (2).

8. Given information can be filled as follows:
Difference in literate and illiterate population in city x = |x – y| = 40% of total population
And total population x + y = 100% of total population.
∴ x = 30% and y = 70% of total population
OR
x = 70% and y = 30% of total population
But, 0.6y > 0.8x
$\therefore y > \frac{4}{3}x$
∴ x = 30% and y = 70% of total population
∴ Number of people who had at least one lucid dream in their live = 0.8x + 0.6y
= 0.8 × 30 + 0.6 × 70 = 66%
Hence, (3).

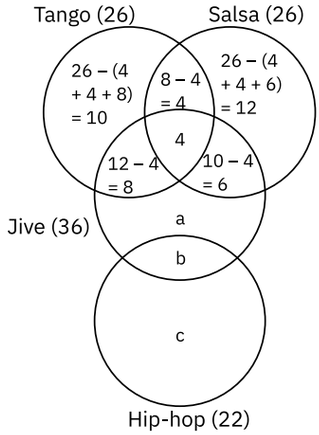
9.



$2000 + d = \frac{10000 + d}{2}$
∴ d = 6000
$2000 + d = \frac{e + d}{3}$
∴ e = 18000 (∵ d = 6000)
c = 0.2a ... (i)
a + 2000 + 6000 + 18000 = 18000 + 6000 + 10000 + c
a = 10000
∴ c = 2000
The number of people who bet on Tej
= e + d + 10000 + c
= 18000 + 6000 + 10000 + 2000
= 36000. Hence, (3).

Answers to questions 10 to 12:

There is no intersection of Hip-hop with Tango and Salsa.

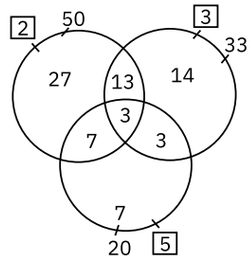


10. Total students admitted
= 10 + 4 + 12 + 4 + 8 + 6 + a + b + c
Students doing Jive and Hip-hop = b = 8
∴ a = 36 – (8 + 4 + 6 + 8)
a = 10
and c = 22 – b = 22 – 8 = 14
∴ Total students admitted = 44 + 10 + 8 + 14 = 76
Hence, (2).

11. Required Ratio = 10 : 12 = 5 : 6. Hence, (1).

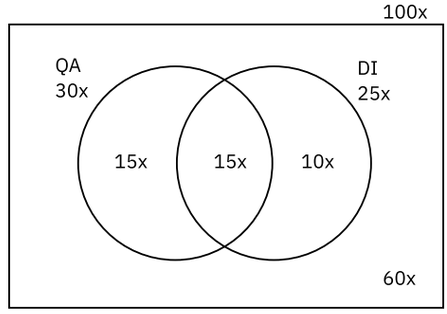
12. a + b = 36 – 8 – 4 – 6 = 18
Hence, (1).

13.



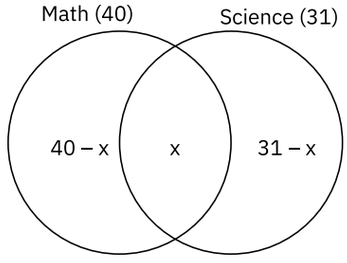
The number of integers that are divisible by 2 is 50.
The number of integers that are divisible by 3 is 33.
The number of integers that are divisible by 5 is 20.
The number of integers that are divisible by 2, 3 and 5 is 3.
The number of integers that are divisible by only 2 and 3 is 16 – 3 = 13.
The number of integers that are divisible by only 3 and 5 is 6 – 3 = 3.
The number of integers that are divisible by only 2 and 5 is 10 – 3 = 7.
So, the number of integers that are divisible only by 2 = 50 – (13 + 3 + 7) = 27
Similarly, the number of integers that are divisible only by 3 = 33 – (13 + 3 + 3) = 14
and, the number of integers that are divisible only by 5 = 20 – (7 + 3 + 3) = 7
So, the total number of numbers that are divisible by 2 or 3 or 5 is = 27 + 14 + 7 + 13 + 3 + 7 + 3 = 74
The numbers that are not divisible by 2 or 3 or 5 is 100 – 74 = 26.
Hence, (1).

14. Let the total number of students be 100x.
∴ Number of students who like QA = 30x, number of students who like DI = 25x and the number of students who don't like either of the sections = 60x.
∴ Number of students who like at least one section = 100x – 60x = 40x.
∴ Number of students who like both the sections = 30x + 25x – 40x = 15x
We can create the Venn diagram as follows.



Looking at the options, only option 4 is correct.
Hence (4).

15. Parvez's survey can be represented as follows:



$\therefore \frac{71 - 2x}{x} = \frac{35}{18}$
∴ x = 18
∴ The actual number of students who like both the subjects is 31 – 18 = 13
∴ The actual number of students who like exactly one subject
= 40 – 13 + 31 – 13 = 45
∴ Required ratio = 45 : 13
Hence (4).

16. $S = \{2, 3, 4, \ldots 2n + 1\}$
Total number of elements in $S = 2n$

$$X = \frac{3 + 5 + \ldots + (2n + 1)}{n}$$

$$Y = \frac{2 + 4 + 6 \ldots + 2n}{n}$$

$$\therefore X - Y = \frac{(3 - 2)(5 - 4) \ldots + (2n + 1 - 2n)}{n}$$

$$= \frac{1 + 1 + \ldots + 1 \text{ (n times)}}{n} = \frac{n}{n} = 1$$

Hence, (4).

17. When union of the sets is equal to the sum of their individual elements that means, there is absolutely no intersection between the sets.
$\therefore P \cap Q = \phi$ .... a null set. Thus, [1] is true.
$R \cap Q = \phi$ and $P \cap R = \phi$
$\therefore (R \cap Q) \cup (P \cap R) = \phi$. Thus, [2] is true.
$(P \cap Q \cap R) = \phi$ ... Thus, [3] is true.
Hence, (4).

18. $6x^2 + x - 15 = 0$; $(2x - 3)(3x + 5) = 0$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{5}{3}; A \equiv \left\{\frac{3}{2}, -\frac{5}{3}\right\}$$

$2x^2 - 5x + 3 = 0$; $(2x - 3)(x - 1) = 0$

$$\therefore x = \frac{3}{2} \text{ or } 1; \; B \equiv \left\{\frac{3}{2}, 1\right\}$$

$2x^2 + x - 3 = 0$; $(2x + 3)(x - 1) = 0$

$$\therefore x = -\frac{3}{2} \text{ or } 1; \; C \equiv \left\{-\frac{3}{2}, 1\right\}$$

$A \cap B \cap C = \phi$. Hence, (4).

19. Let the number of students who like apple be x.
Then, the number of students who like only apple will be 0.625x and those who like both apple and mango will be 0.375x.
$\therefore$ Number of students who don't like any fruit = $0.3 \times (x + 18) = 0.5 \times (0.375x + 18)$
$\therefore 0.3x + 5.4 = 0.1875x + 9$
$\therefore 0.1125x = 3.6$
$\therefore x = 32$
$\therefore$ Total number of students = $(32 + 18)1.3 = 65$

20. Suppose 'x' is the number of students who study only one language, 'y' is the number of students who study two languages and 'z' is the number of students who study all the three languages.
Therefore we have,
$x + 2y + 3z = 30 + 40 + 50 = 120$ ..... (I)
Also, $x + y + z = 100$ ..... (II)
From equation (I) and (II),
$y + 2z = 20$
The maximum value of 'z' is therefore $\frac{20}{2} = 10$
Therefore, the required answer is 10.

QA-4.3 | BASIC P&C - I

PRACTICE EXERCISE

1. One can hoist 1 or 2 or 3 or 4 or 5.
So number of ways = ${}^5P_1 + {}^5P_2 + {}^5P_3 + {}^5P_4 + {}^5P_5 = 5 + 20 + 60 + 120 + 120 = 325$. Hence, (4)

2. 21 persons can be arranged in (21 – 1)! = 20! ways.
Treating the host and the two persons as one, we can arrange the 19 persons in (19 – 1)! = 18! ways.
But the two can be arranged on either side of the host in 2 ways.
So total number of arrangements = 2 × 18!. Hence, (1).

3. At least 1 green can be selected in $2^5 – 1 = 31$ ways;
At least 1 blue in $2^4 – 1 = 15$ ways;
At least 1 red or no red can be selected in $2^3 = 8$ ways.
Hence the total number of selections
= 31 × 15 × 8 = 3720. Hence, (3).

4. The required answer is: ${}^7C_3 \times \frac{{}^4C_2}{2!} = \frac{7!}{4!3!} \times \frac{4!}{2!2!2!}$
= 105.
Hence (3).

5. The first card can be selected in ${}^{52}C_1 = 52$ ways.
The second card can be selected in ${}^{39}C_1 = 39$ ways.
The third card can be selected in ${}^{26}C_1 = 26$ ways.
$\therefore$ Required number of ways = 52 × 39 × 26 = 4 × 13 × 3 × 13 × 2 × 13 = 24 × 13^3. Hence (3).

6. Case I: There is only one black ball.
Only one black ball can be filled in 6 ways (1, 2, 3, 4, 5 and 6)
Case II: There are two black balls.
Two black balls can be filled in 5 ways (12, 23, 34, 45 and 56)
Case III: There are three black balls.
Three black balls can be filled in 4 ways (123, 234, 345 and 456)
Case IV: There are four black balls.
Four black balls can be filled in 3 ways (1234, 2345 and 3456)
Case V: There are five black balls.
Five black balls can be filled in 2 ways (12345 and 23456)
Case VI: There are six black balls.
Six black balls can be filled in only 1way (123456)
The total number of ways = 1 + 2 + 3 + 4 + 5 + 6 = 21
Hence, (2).

7. The number of possible routes are 12.
Assuming that one starts from the city A,
They required routes are:
ABDA, ABDCA, ABCA, ABCDA, ACDA, ACBDA, ADBA, ACDBA, ACBA, ADCBA, ADCA and ADBCA.
Hence, (2).

8. This is the problem of distributing dissimilar objects into dissimilar groups.
Each judge can vote in favour of any of the three candidates. Therefore, each judge can vote in 3 different ways. Therefore, the total number of ways in which the judges can vote is $3^4 = 81$. Hence (4).

9. There a total of 4 boys and 5 girls, out of which exactly 3 girls have to be invited. But the number of boys is not given; hence it could be 0, 1, 2, 3 or 4.
Thus, the total number of required selections = ${}^5C_3 \times ({}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4) = 160$. Hence, (2).

10. 3 points out of 12 can be selected in ${}^{12}C_3$ ways,
out of which ${}^7C_3$ triangles are impossible.
$\therefore$ Number of triangles formed = ${}^{12}C_3 – {}^7C_3$
= 220 – 35 = 185. Hence, (4).

11. All 8 directors can shift, hence number of ways for those 8 directors = 8!
Also, the chairman and director can sit in 2 different ways.
Thus, the total number of ways = 8! × 2
Hence, (2).

12. Each digit can be choosen in ${}^5C_1$ ways.
${}^5C_1 \times {}^5C_1 \times {}^5C_1 = 5 \times 5 \times 5 = 125$.
(*Note*: The number can repeat in a code)
One of these 125 numbers is the number 111 that definitely will not be the code.
But he will have to open the mechanical lock to check it.
$\therefore$ There are 125 combinations.
$\therefore$ In 124 attempts the thief will definitely know the lock code. Hence, (4).

13. Total number of ways
$= {}^6C_0 \times {}^6C_5 + {}^6C_1 \times {}^5C_4 + {}^6C_2 \times {}^4C_3 + {}^6C_3 \times {}^3C_2 + {}^6C_4 \times {}^2C_1 + {}^6C_5 \times {}^1C_0$
(representing cases where no males are selected; 1 male is selected while his partner is not; two males are selected while their partners are not, and so on, through to: no female is selected).
= 192. Hence, (4).

Alternatively,
the number of ways the committee can be formed = ${}^6C_5 \times 2^5 = 6 \times 32 = 192$. Hence, (4).

14. The student can select 1 Maths book out of 6 in ${}^6C_1 = 6$ ways.
Similarly, he can select 1 Chemistry book and 1 Science book in 3 and 4 ways respectively.
$\therefore$ Total number of ways = $6 \times 3 \times 4 = 72$
Hence, (3).

15. Number of possible systems = $4 \times 2 \times 4 \times 3 = 96$. Hence, (1).

16. The number of triangles that can be formed using the points on the 2 given straight lines
= (Combinations of 2 points on line 1 and 1 point on line 2) + (Combinations of 1 point on line 1 and 2 points on line 2)
= ${}^{10}C_2 \times 11 + {}^{11}C_2 \times 10 = 1045$. Hence, (3).

17. Consider a maximum of 2 candidates being selected, then the total number of candidates = $(2 \times 2) + 1 = 5$
The total number of ways in which atleast 1 candidate can be selected out of the total number of 5 candidates among which at most 2 can be selected = ${}^5C_1 + {}^5C_2 = 5 + 10 = 15$ ways.
Next, consider a maximum of 3 candidates being selected, then the total number of candidates = $(3 \times 2) + 1 = 7$
The total number of ways in which atleast 1 candidate can be selected out of the total number of 7 candidates among which at most 3 can be selected = ${}^7C_1 + {}^7C_2 + {}^7C_3 = 7 + 21 + 35 = 63$ ways.
Thus, the maximum number of candidates that can be selected for scholarship = 3. Hence, (1).

18. Number of ways in which mangoes can be selected = 2^3. But this also includes, the case where all three mangoes are not selected Hence, number of ways in which at least one mango is selected = $(2^3 - 1)$
Similarly, Number of ways in which apples and oranges are selected is $(2^4 - 1)$ and $(2^2 - 1)$
$\therefore$ Total number of selections = $(2^3 - 1)(2^4 - 1)(2^2 - 1) = 315$. Hence, (3).

19. The number of ways in which 5 doctors' coats can be distributed among 5 lawyers = 5! = 120.
The number of ways in which 5 lawyers' coats can be distributed among 5 doctors = 5! = 120.
$\therefore$ Required answer = $120 \times 120 = 14400$ ways
Therefore, the required answer is 14400.

20. From each point in a set, 6 lines can be drawn that pass through a point in the other set. One line each passes through all the points in a set.
$\therefore$ Required answer = $6 \times 6 + 2 = 38$.

QA-4.4 BASIC P&C - II

PRACTICE EXERCISE

1. We can use the digits 0, 4, 4, 5, 5 and 6. Clearly a 7-digit (or greater) number cannot be made with these. Thus we have to fit these 6 objects in 6 places.
This can be done in $\frac{6!}{2!2!} = 180$ ways.
But these 180 ways include $\frac{5!}{2!2!}$ numbers begining with 0. $\therefore$ The required answer is $180 - \frac{5!}{2!2!} = 150$.
Hence, (2).

2. We have 8 letters (assuming R & O to be one) of which A and E appear twice.
The letters can be arranged in $\frac{8!}{2!2!}$ ways.
R & O can arrange themselves in 2! ways.
Thus required number of ways
$= \frac{8!}{2!2!} \times 2! = \frac{8!}{2!}$. Hence, (4).

3. If the group consisted of n persons then possible number of ways when Sush does not sit with one particular friend
$= n! - 2 \times (n-1)!$
Now, $n! - 2 \times (n-1)! = 480$
$\Rightarrow (n-1)!\,(n-2) = 480$
$= 2 \times 3 \times 4 \times 5 \times 4$
$\Rightarrow (n-1)!\,(n-2) = 5! \times 4$
Hence there were 6 persons in the group including Sush, i.e., she went with five friends. Hence, (1).

4. The Committee may contain 1 lady + 4 gents or 2 ladies + 3 gents or 3 ladies + 2 gents, or 4 ladies + 1 gent or 5 ladies + 0 gents.
This can be done
$({}^4C_1 \times {}^6C_4) + ({}^4C_2 \times {}^6C_3) + ({}^4C_3 \times {}^6C_2) + ({}^4C_4 \times {}^6C_1)$
$= 60 + 120 + 60 + 6 = 246$. Hence, (1).
Alternatively,
6 gentlemen + 4 ladies = 10
$\therefore$ Choose 5 from 10 in ${}^{10}C_5 = 252$
If there is no lady in the committee, then the number of ways of choosing 5 from 6 = ${}^6C_5 = 6$.
$\therefore$ Required number of ways = 252 – 6 = 246.

5. The first place can be taken by 4 or 7 in 2 ways
The remaining three places can be taken by the four numbers in 4 × 4 × 4 ways
$\therefore$ Total number of ways = 2 × 4 × 4 × 4 = 128.
Hence, (2).

6. 6 A's can be put in 9 boxes in ${}^9C_6$ ways.
$= {}^9C_6 = \frac{9!}{6! \times 3!} = \frac{9 \times 8 \times 7}{3 \times 2} = 84$.
There are six ways in which one row or one column is completely empty.
$\therefore$ Required number of ways = 84 – 6 = 78.
Hence, (3).

7. The 3 identical Physics books can be arranged in one way.
__ P __ P __ P __
If all the 3 Chemistry books are together, there are 4 places where they can be arranged.
Therefore required number of ways = 4. Hence (2).

8. The numbers can be of the following forms:
9 6 _ _ _ (No. of possibilities = 8 × 7 × 6 = 336)
9 _ 6 _ _ (No. of possibilities = 8 × 7 × 6 = 336)
9 _ _ 6 _ (No. of possibilities = 8 × 7 × 6 = 336)
9 _ _ _ 6 (No. of possibilities = 8 × 7 × 6 = 336)
_ 9 6 _ _ (No. of possibilities = 3 × 7 × 6 = 126)
_ 9 _ 6 _ (No. of possibilities = 3 × 7 × 6 = 126)
_ 9 _ _ 6 (No. of possibilities = 3 × 7 × 6 = 126)
_ _ 9 6 _ (No. of possibilities = 3 × 7 × 6 = 126)
_ _ 9 _ 6 (No. of possibilities = 3 × 7 × 6 = 126)
_ _ _ 9 6 (No. of possibilities = 3 × 7 × 6 = 126)
$\therefore$ Total possibilities = 2100. Hence, (1).

9. F and G are to be selected
Therefore, we have to select 6 out of the remaining 13.
Case (1) E is selected
If E is selected, A and I will also be selected but B, H and K cannot be selected. Therefore, we have select 3 out of remaining 7, which can be done in ${}^7C_3 = \frac{7!}{4!3!}$ = 35 ways.
Case (2) E is not selected
If E is not selected, A and I will also not be selected. Therefore, we have to select 6 from the remaining 10.
This can be done in ${}^{10}C_6 = \frac{10!}{6!4!} = 210$ ways
$\therefore$ Total number of ways = 35 + 210 = 245. Hence, (1).

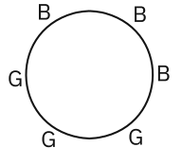
10. This is a problem of arranging 11 out of 14 players in 11 positions where 1 player is sure to be selected and one is sure not to be selected. Thus effectively the coach has to select only 10 (Sachin being already selected) out of 12 players (excluding Sachin and Sourav) and further arrange 11 players in 11 batting slots. The number of ways
$= {}^{12}C_{10} \times 11! = \frac{12 \times 11}{2} \times 11! = 66 \times 11!$.
Hence, (1).
Note : The direct formula for the total number of arrangements of n things taken r at a time, in which a particular thing always occurs
$= r({}^{n-1}P_{r-1})$
$= 11({}^{13-1}P_{11-1})$ [13, as Sourav is not a part of the selection]
$= 11({}^{12}P_{10})$

$= 11 \times \frac{12!}{2!}$

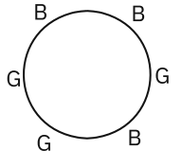
$= 66 \times 11!$

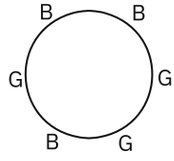
11. The four persons who wish to sit facing forward can be seated in ${}^5P_4$ ways and the 3 who wish to sit facing towards the rear can be seated in ${}^5P_3$ ways and the remaining 3 can be seated in the remaining 3 seats in ${}^3P_3$ ways
∴ Total number of ways = ${}^5P_4 \times {}^5P_3 \times {}^3P_3 = 43200$.
Hence, (2).
Alternatively,
In the row facing forward there is one spare seat which can be filled by any one of the three indifferent person in three ways. Then in each row the 5 passengers can be reseated in 5! ways.
∴ Total number of arrangements = $3 \times 5! \times 5! = 43200$

12. First 6 red beads are arranged. This can be done in only one way. The 6 red beads create 6 empty places, in which 3 blue and 3 green beads are to be arranged. Therefore now the question is the number of ways of arranging 3 blue and 3 green beads along a circle.
Following are the cases of arranging 3 blue and 3 green beads in a necklace
1) All 3 blue beads are together:
This can be done in only one way, as shown below.

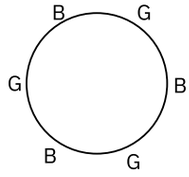


2) When 2 blue beads are together:
This can be done in the following ways –





In the case of a necklace, these two are in reality identical. When first arrangement is flipped, we get the second.
3) When no blue beads are together
This can be done in the following way



Therefore, the required number of ways = 3
Hence, (3).

13. After distributing one chocolate of type A to the each of the 4 students, 3 chocolates remain. These can be distributed either as 3-0-0-0 (4 ways), 2-1-0-0 (12 ways) or 1-1-1-0 (4 ways). Therefore, total 20 ways.
After distributing one chocolate of type B to the each of the 4 students, 2 chocolates remain. These can be distributed either as 2-0-0-0 (4 ways) or 1-1-0-0 (6 ways). Therefore, total 10 ways.
After distributing one chocolate of type C to the each of the 4 students, 1 chocolate remains. This can be distributed in 4 ways.
∴ Total number of ways = $20 \times 10 \times 4 = 800$ ways.

14. Let the number be represented as 2a-2b-2c, where 2a, 2b and 2c are its digits. If a is 0, then the number is a 2-digit number. If both a and b are 0, then the number is a 1-digit number.
Now, 2a + 2b + 2c = 10. Therefore, a + b + c = 5.
We need to find the total whole number solutions of a + b + c = 5 which is $\frac{7!}{2! \times 5!} = 21$.
Out of these 21, we need to subtract 3 cases (5-0-0, 0-5-0 and 0-0-5) because a digit cannot be 10.
Therefore, the required answer is 18.

15. The one letter that is enclosed in the right envelope can be selected out of the 5 letters in ${}^5C_1 = 5$ different ways.
The number of ways in which the 4 envelopes can be enclosed in the wrong envelopes is equal to the derangement of 4

$= 4!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!}\right)$

$= 24 \times \left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24}\right) = 24 \times \left(\frac{9}{24}\right) = 9$

Therefore, the number of ways = $5 \times 9 = 45$.
Therefore, the required answer is 45.

16. Out of the 5 digits, the first digit has to be 1. The remaining 4 digits can be selected in $6 \times 5 \times 4 \times 3 = 360$ ways.

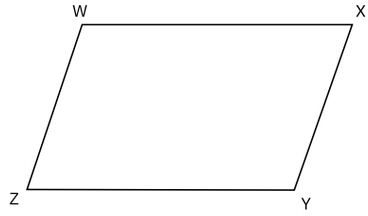
17. If B4 is connected to S5, then B1 can be connected to any of S3, S4 or S6, and the remaining 4 bulbs can be connected to any of the remaining 4 switches. Therefore, number of possibilities = $3 \times 4!$
Similarly, if B4 is connected to S6, then B1 can be connected to any of S3, S4 or S5, and the remaining 4 bulbs can be connected to any of the remaining 4 switches. Therefore, number of possibilities = $3 \times 4!$
∴ Required answer = $3 \times 4! + 3 \times 4! = 144$

18. Numbers having at least one 5 in different ranges are calculated as follows:

Range	Numbers
96-199	19
200-299	19
300-399	19
400-499	19
500-599	100
600-699	19
700-749	5
Total	200

Therefore, the required answer is 200.

19.



Since W is one of the vertices, the other two vertices of the triangle can be selected in $^{21}C_2$ ways. Out of these, we need to remove the cases where all the three points are collinear (i.e., when all the three points are along WX or WZ).
$\therefore$ The required number of total ways = $^{21}C_2 - {}^5C_2 - {}^6C_2 = 210 - 10 - 15 = 185$ ways.
Therefore, the required answer is 185.

20. We need to find the number of 3-digit and 4-digit numbers that can be formed using the digits of 1269. The last two digits as a 2-digit number should be divisible by 4.

	Form	No. of possibilities
3-digit number	_ 1 2	2
	_ 1 6	2
	_ 9 2	2
	_ 9 6	2
4-digit number	_ _ 1 2	2
	_ _ 1 6	2
	_ _ 9 2	2
	_ _ 9 6	2

$\therefore$ Total number of ways = 16

QA-4.5 | FACTORIALS, BINOMIAL THEOREM, REMAINDERS

PRACTICE EXERCISE - 1

1. In the expansion of $(a + 3)^{100}$:
a) 100
b) All terms containing 3^4, 3^5, ... 3^{100} i.e., 5^{th} to 101^{st} terms are divisible by 81

3rd term = $^{100}C_2\ a^{98}\ 3^2$

$= \frac{100\times99}{2\times1} \times a^{98} \times 9$

$= 50 \times 99 \times 9 \times a^{98}$

$= 50 \times 11 \times a^{98} \times 81$

So 3rd term is divisble by 81

4th term = $^{100}C_3\ a^{97}\ 3^3$

$= \frac{100\times99\times98}{3\times2\times1} \times a^{97} \times 27$

$= 50 \times 33 \times 98 \times a^{97} \times 27$

$= 50 \times 98 \times 11 \times a^{27} \times 81$

So, 4^{th} term is also divisible by 81.
In all 99 terms are divisible by 81.

c) $^{100}C_{50}\ (3)^{50}\ a^{50}$

2.

a. 10	f. 5	k. 1
b. 1	g. 1	l. 10
c. 6	h. 1	m. 1
d. 1	i. 1	
e. 3	j. 10	

Hint for (f): $9^{99} = 3^{198} = 3^3 \times (3^5)k = 27 \times (11k+1)$

3. The divisor, $66 = 11 \times 2 \times 3$.
So the highest factor = 11.
Hence, (1).

4. $a \equiv b \pmod n$ means,
b is the remainder when a is divided by n.
$2^1 \equiv 2 \pmod{15}$
$2^2 \equiv 4 \pmod{15}$
$2^3 \equiv 8 \pmod{15}$
$2^4 \equiv 1 \pmod{15}$
$2^{144} = (2^4)^{36} \equiv 2^{144} \equiv 1 \pmod{15}$
Hence, (1).

5. We have to find the number of zeros that 125! will end in.
The deciding factor to calculate the zeros will be the maximum power of 5 dividing 125!.
In 125! the number of multiples of 5
$= \frac{125}{5} = 25,$

Number of multiples of 25 = $\frac{125}{25} = 5,$

Number of multiples of 125 = $\frac{125}{125} = 1.$

$\therefore$ Maximum power of 5
= 25 + 5 + 1 = 31
$\therefore$ 125! will end in 31 zeros.
Thus 10^{31} will completely divide 125!.
Hence, (4).

6. (a, n)! = product of n consecutive natural numbers starting from 'a' which is at least divisible by n!.
(n)! = product of n consecutive natural numbers.
For n = 2, (a, n)! = a(a + 1) and n! = 2
a(a + 1) is divisible by 2!.
For n = 3, (a, n)! = a(a + 1)(a + 2) and n! = 6.
One of the factors of a(a + 1)(a + 2) is divisible by 3 and other by 2.
Thus, proceeding in this manner, (a, n)! and n! have HCF = n!.
$\therefore$ H = n!. Hence, (2).

7. The product of the first 25 odd terms is given as
$1 \times 3 \times 5 \times 7 \times ..... \times 47 \times 49$

$= \frac{1 \times 2 \times 3 \times 4 \times 5..... \times 47 \times 48 \times 49 \times 50}{2 \times 4 \times 6 \times ..... \times 48 \times 50}$

$= \frac{50!}{2^{25}(1 \times 2 \times 3 \times .... \times 25)}$

$= \frac{50!}{2^{25}(25!)}$. Hence, (2).

8. Terms beyond 4! will have 5 as a factor and hence are divisible by 5.
$\therefore$ Remainder will be zero for numbers beyond 4!
$\therefore$ The remainder will be the same when,
$(1!)^3 + (2!)^3 + (3!)^3 + (4!)^3$ is divided by 5
The remainders when these numbers are divided by 5 are as follows:

$\frac{(1!)^3}{5} \equiv 1, \frac{(2!)^3}{5} \equiv 3, \frac{(3!)^3}{5} \equiv 1, \frac{(4!)^3}{5}$

$= \frac{(24)^3}{5} \equiv 4$

$\therefore$ The remainder of $\frac{(1+3+1+4)}{5}$ is 4.

Hence, (4).

9. $n(n^2 - 1) = n(n - 1)(n + 1) = (n - 1)(n)(n + 1)$
n is odd.
(n – 1) is divisible by 2, (n + 1) is divisible by 4
Also, one of these numbers is definitely divisible by 3.
Therefore, the number is definitely divisible by $2 \times 3 \times 4 = 24$. Hence, (2).

10. $n(n^4 - 1) = n(n^2 - 1)(n^2 + 1)$
$= n(n - 1)(n + 1)(n^2 + 1)$
n – 1, n and n + 1 are consecutive natural numbers.

As every 2nd natural number is even and every 3rd natural number is divisible by 3, the product of any three consecutive natural numbers is divisible by 6.
Now, if n is of the form 5k or $5k \pm 1$ then one of the n, n – 1 or n + 1 will be divisible by 5.
If n is of the form $5k \pm 2$, then $n^2 + 1 = (5k \pm 2)^2 + 1 = 5m + 5$ which is divisible by 5.
In any case, $n(n-1)(n+1)(n^2+1)$ is divisible by both 6 and 5, hence it is divisible by 30. Hence, (4).
Alternatively,
Consider option (4), if (4) is not true, then none of (1), (2), (3) is true. Hence, (4) has to be true.

11. $2500! \times 2500! = (1 \times 2 \times 3 \times .. \times 2500)2500!$
$5000! = (5000 \times 4999 \times 4998 \times ... \times 2501) \times 2500!$
Clearly, $5000! > 2500! \times 2500!$
$3600! \times 1400! = (1 \times 2 \times 3 \times ... \times 1400) \times 3600!$
$5000! = (5000 \times 4999 \times 4998 \times ... \times 3601) \times 3600!$
Clearly, $5000! > 3600! \times 1400!$
$4000! \times 1000! = (1 \times 2 \times 3 \times ... \times 1000) \times 4000!$
$5000! = (5000 \times 4999 \times 4998 \times ... \times 4001) \times 4000!$
Clearly, $5000! > 4000! \times 1000!$
Hence (2).

12. Using binomial theorem, the expansion is given by:
$$\left(x + \frac{1}{2x}\right)^{10} = x^{10} + {}^{10}C_1 \times x^9 \times \frac{1}{2x} + {}^{10}C_2 \times x^8 \times \frac{1}{(2x)^2} + ...$$
Therefore, the constant term in the expansion is given by:
$$\therefore k = {}^{10}C_5 \times \left(\frac{1}{2}\right)^5 = \frac{10!}{5!\,5!} \times \frac{1}{32}$$
$$\therefore 40k = \frac{40}{32} \times \frac{10!}{5!\,5!} = 315$$

13. To find the last three digits of the product, consider the remainder when it is divided by 1000
$$\frac{1011 \times 2012 \times 3013 \times 4014 \times 5015 \times 6016}{1000}$$
$$= \frac{1011 \times 1006 \times 3013 \times 2007 \times 1003 \times 3008}{25}$$
Remainder = 11 × 6 × 13 × 7 × 3 × 8 = 144144
Since we divided the numerator and denominator by 40, so 144144 × 40 = 5765760
∴ The last three digits will be 760, which is the required answer.

14. Remainder when 100^{25} is divided by 101 is –1 or 100.
Remainder when 100^{25} is divided by 99 is 1.
∴ Required answer is 100 – 1 = 99.

15. $6^4 = 1296$
$6^{66} = (6^4)^{16} \times 6^2$
∴ Required remainder is $(-1)^{16} \times 36 = 36$.

16. 2016 is divisible by 7. So 2016^{2017} gives a remainder 0. Also, 2017 leaves a remainder 1 when divided by 7. So 2017^{2016} will leave a remainder of 1. So the overall remainder will be 0 + 1 = 1.

17. $2592 = 2^5 \times 3^4$
From the series, all numbers including $(4!)^4$ and above will be divisible by 2592.
So we need to find the remainder when $(1!)^4 + (2!)^4 + (3!)^4$ is divided by 2592.
$(1!)^4 + (2!)^4 + (3!)^4 = 1 + 16 + 1296 = 1313$.
Therefore, the required answer is 1313.

18. The general term T_r in the expansion of $\left(\sqrt[6]{x} + \sqrt[9]{y}\right)^{54}$ is given as:
$$T_r = {}^{54}C_r \left(\sqrt[6]{x}\right)^r \left(\sqrt[9]{y}\right)^{54-r} = {}^{54}C_r (x)^{\frac{1}{6}}$$
$$= {}^{54}C_r (x)^{\frac{r}{6}} (y)^{6-\frac{r}{9}}$$
For the term to be free of radicals, both $\frac{r}{6}$ and $6 - \frac{r}{9}$ should be whole numbers.
'r' can be 0, 18, 36 and 54.
There are 4 such terms. Hence (2).

19.
$$\frac{7^{84}}{342} = \frac{(7^3)^{28}}{342} = \frac{(342+1)^{28}}{342}$$
Thus, the remainder when 7^{84} is divided by 342 is 1. Hence, (2).

20. $(4)^{4^2} . (5)^{5^2} = 4^{16}.5^{25} = 2^{32}.5^{25} = 2^7.10^{25} = 128 \times 10^{25}$.
Thus, we have is 128 followed by 25 zero's. Thus, 28 digits are present in the number. Hence, (3).

21. The number of zeroes depends on the number of fives and the number of twos. Here close scrutiny shows that the number of twos is the constraint. The expression can be written as:
5 × (5 × 2) × (5 × 3) × (5 × 2 × 2) × (5 × 5) × (5 × 2 × 3) × (5 × 7) × (5 × 2 × 2 × 2) × (5 × 3 × 3) × (5 × 5 × 2)
Number of 5's = 12
Number of 2's = 8. Hence, (4).

22. Let us find the highest power of 72 in 200!.
$72 = 2^3 \times 3^2$
The highest power of 2 in 200! is calculated as follows:
$$\left[\frac{200}{2}\right] + \left[\frac{200}{4}\right] + \left[\frac{200}{8}\right] + \left[\frac{200}{16}\right] + \left[\frac{200}{32}\right] + \left[\frac{200}{64}\right] + \left[\frac{200}{128}\right] + ...$$
= 100 + 50 + 25 + 12 + 6 + 3 + 1 = 197
∴ The highest power of $2^3 = 65$
The highest power of 3 in 200! is calculated as follows:

$\left[\frac{200}{3}\right] + \left[\frac{200}{9}\right] + \left[\frac{200}{27}\right] + \left[\frac{200}{81}\right] + ...$
$= 66 + 22 + 7 + 2 = 97$
$\therefore$ The highest power of $3^2 = 48$

$\therefore$ The highest power of 72 in 200! is 48. So, 200! will not be completely divided by 72^{50}. Hence (4).

PRACTICE EXERCISE-2

1.

a) $(99)^{999} = (100 - 1)^{999}$
$\Rightarrow 100^{999} - {}^{999}C_1\, 100^{998} + {}^{999}C_2\, 100^{997}\, 1^2 .... + {}^{999}C_{998}\, 100(1)^{998} - 1^{999}$
Except the last term, all the other terms end in atleast 2 zeros
Last term is equal to 1
$\therefore (99)^{999}$ ends in 00–01 = 99

b) $\Rightarrow (101)^{202} = (100 + 1)^{202}$
$\Rightarrow 101^{202} + {}^{202}C_1\, 100 + 1 + {}^{202}C_2\, 100^{200}\, 1^2 + ... + {}^{202}C_{201}\, 100 . 1^{201} + 1^{202}$
Except the last term all terms end in an atleast 2 zeros
Last term is equal to 1
$\therefore (101)^{202}$ ends in 00 + 01 = 01

c) 43^{4343}
$\Rightarrow (1849)^{2171} \times 43^1$
$(1849)^{2171} = (1850 - 1)^{2171}$
$\Rightarrow (1850)^{2171} - {}^{2171}C_1 (1850)^{2170}\, 1 + {}^{2171}C_2\, (1850)^{2169}\, 1^2 .... + {}^{2171}C_{2170}\, (1850)\, 1^{2170} - 1^{2171}$
Except last 2 terms all other terms end in at least 2 zeros
${}^{2171}C_{2170}\, (1850) . 1^{2170} = 2171 \times 1850$ ends in 50
1^{2171} ends in 01
last 2 digits of $(1849)^{2171} \Rightarrow 50 - 01 = 49$
So last 2 digits of $(1849)^{2171} \times 43^1$
$= 49 \times 43 = 07$
So last 2 digits of $43^{4343} = 07$

d) 75^{2020}
$\Rightarrow (70 + 5)^{2020} = (70)^{2020} + {}^{2020}C_1 (70)^{2019} 5^1 + ... + {}^{2020}C_{2019}\, (70) 5^{2019} + 5^{2020}$
Except the last term all other terms end with at least 2 zeros. 5 raised to any power ends with 25.
So, last digit of 75^{2020} ends with 00 + 25 = 25

e) $(37)^{486} = (1369)^{243}$
$(1369)^{243} = (1370 - 1)^{243}$
$\Rightarrow (1370)^{243} - {}^{243}C_1 (1370)^{242}\, (1) + {}^{243}C_2 (1370)^{241} 1^2 .... + {}^{243}C_{242} (1370)^1 1^{242} - 1^{243}$
Except last 2 terms all will end with at least 2 0's.
${}^{243}C_{242} (1370)^1 . 1^{242} = 243 \times 1370$ will end in 10
Also $(1)^{243}$ ends in 01
So ${}^{243}C_{242} (1370) . 1^{242} - 1^{243}$ will end in 09

2.

a) $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$
$= 2^8 \times 3^4 \times 5^2 \times 7$
$\Rightarrow (2^2 \times 5^2) \times 2^6 \times 3^4 \times 7$
$\Rightarrow (2^2 \times 5^2) \times 64 \times 81 \times 7$
$\Rightarrow$ will end in 2 zeros with a right most non zero digit of 8 ($\because 4 \times 1 \times 7 = 28$)

b) $16! = 2^{15} \times 3^6 \times 5^3 \times 7^2 \times 11^1 \times 13^1$
$\Rightarrow (2^3 \times 5^3) \times 2^{12} \times 3^6 \times 7^2 \times 11^1 \times 13^1$
$\Rightarrow (2^3 \times 5^3) \times 729 \times 49 \times 11 \times 13 \times 4096$
will ending with 3 zeros end a rightmost non zero digit of 8($\because 6 \times 9 \times 9 \times 1 \times 3$ ends in 8)

c) $24! = 2^{22} \times 3^{10} \times 5^4 \times 7^3 \times 11^2 \times 13$
$\Rightarrow (2^4 \times 5^4) \times 2^{18} \times 3^{10} \times 7^3 \times 11^2 \times 13$
will end with 4 zeros and a rightmost non zero digit of 4 ($\because 4 \times 9 \times 3 \times 1 \times 3$ ends in 4)

3.

a) Number of 2's in 20!
$\therefore \left[\frac{20}{2}\right] + \left[\frac{20}{4}\right] + \left[\frac{20}{8}\right] + \left[\frac{20}{16}\right] = 18$
Number of 3's in 20! $\Rightarrow \left[\frac{20}{3}\right] + \left[\frac{20}{9}\right] = 8$
$\therefore$ Number of 0's in 20! in base 6 = 8

b) Number of 2's in 20! = 18
Number of 0's in 20! in base 8($= 2^3$) $= \left[\frac{18}{3}\right] = 6$

c) Number of 3's in 20! = 8
$\therefore$ Number of 0's in 20! in base 9 ($= 3^2$)
$\Rightarrow \left[\frac{8}{2}\right] = 4$

4. 20![1 + 21 + 21 × 22 + 21 × 22 × 23 + 21 × 22 × 23 × 24]
20![266134]
So no. of zeros will be same as no. of zeros in 20! $\Rightarrow \left[\frac{20}{5}\right] = 4$

5.

a) 49^{25}
$\Rightarrow (32 + 17)^{25}$
$\Rightarrow 32^{25} + {}^{25}C_1\, 32^{24}\, 17^1 + {}^{25}C_2\, 32^{23}\, 17^2 + ... + {}^{25}C_{24}\, 32\, 17^{24} + 17^{25}$
Now each of the terms except the last term 17^{25} is divisible by 32.
$17^{25} = 17^{24} \times 17 = (289)^{12} \times 17$
$\frac{(289)^{12}}{32}$ = will give a remainder of $(1)^{12} = 1$
$\frac{17}{32}$ will give a remainder of 17
$\therefore 17^{25}$ will give a remainder of 17 when divided by 32.

b) $(125)^{38}$
$\Rightarrow (128 - 3)^{38} = 128^{38} - {}^{38}C_1\ 128^1\ 3^{37} + ... - {}^{38}C_{37}(128)3^{37} + 3^{38}$
Each of the terms except last term 3^{38} is divisible by 32
$(3^{38}) = (3^4)^9 \times 3^2$
$\Rightarrow (3^4)^9 \times 3^2 = (81)^9 \times 9$
$\left(\frac{81}{32}\right)$ will give a remainder of 17
$(17)^9 = 17^8 \times 17 = (17^2)^4 \times 17$
17^2 will give a remainder of 1 when divided by 32
$\therefore 17^9$ will give a remainder of 17 when divided by 32
Also 9 divided by 32 gives a remainder of 9.
Now 17 × 9 will give a remainder of 25 when divided by 32.

6. If on division of a natural number, a, by another, b, the remainder is t, we write $a \equiv t \pmod b$
Thus $3 \equiv 3 \pmod 6$; also $3^2 = 9$
and $3^2 \equiv 3 \pmod 6$; further $3^3 = 27$
and $3^3 \equiv 3 \pmod 6$
In general, $3^n \equiv 3 \pmod 6$ for any natural number n.
Thus, in particular, $3^{58} \equiv 3 \pmod 6$.
Hence, (4).

7. Power of 2 in
$10! \Rightarrow \left[\frac{10}{2}\right] + \left[\frac{10}{4}\right] + \left[\frac{10}{8}\right] = 8$
$11! \Rightarrow \left[\frac{11}{2}\right] + \left[\frac{11}{4}\right] + \left[\frac{11}{8}\right] = 8$
$12! \Rightarrow \left[\frac{12}{2}\right] + \left[\frac{12}{4}\right] + \left[\frac{12}{8}\right] = 10$
Now, 2^{10} will divide the factorial of any number greater than 12. So, k cannot be 9 for any n. Hence, (4).

8. Each of 5!, 6!, 7! ... 100! is divisible by 15.
3! + 4! = 3!(1 + 4) is also divisible by 15.
$\therefore$ 1! + 2! is the only term that is not divisible by 15.
Hence, x = 1! + 2! = 3. Hence,(2).

9. $\frac{3^{41} + 7^{41}}{13}$
$= \frac{3^{39} \times 3^2 + 7^{40} \times 7}{13}$
$= \frac{(3^3)^{13} \times 3^2 + (7^2)^{20} \times 7}{13}$
$= \frac{(27)^{13} \times 3^2 + (49)^{20} \times 7}{13}$
$= \frac{(26 + 1)^{13} \times 3^2 + (52 - 3)^{20} \times 7}{13}$
$= \frac{(x + 1)^9 + (y + 3^{20})7}{13} = z + \frac{9 + 3^{20} \times 7}{13}$
Where x is a multiple of 26 and y is a multiple of 52.
Also, $z = \frac{9 + 3^{20} \times 7}{13}$ which is a natural number.

Now, $\frac{9 + (3)^{20} \times 7}{13}$
$= \frac{9}{13} + \frac{(3^3)^6 \times 3^2 \times 7}{13}$
$= \frac{9}{13} + \frac{(26 + 1)^6 \times 9 \times 7}{13}$
$= \frac{9}{13} + \frac{P}{13} + \frac{63}{13}$, P is a multiple of 26
$\therefore \frac{P}{13}$ is a natural number.
$= \frac{9}{13} + \frac{63}{13} = \frac{72}{13} = 5\frac{7}{13}$
Thus, the remainder is 7. Hence, (1).

10. $n \times n! = (n + 1 - 1)n!$
$= (n + 1)n! - n!$
$= (n + 1)! - n!$
$\therefore p = (2! - 1!) + (3! - 2!) + (4! - 3!) + ---- + (13! - 12!)$
$\therefore p = 13! - 1$
$\therefore p + 3 = 13! + 2$
$\therefore$ p + 3 when divided by 13! will have remainder 2.
Hence, (2).

11. Sum of digits of 12570 is 15.
Since 15 is divisible by 3, 12570 is divisible by 3.
$(243)^4 = (3^5)^4 = 3^{20}$
Thus, $(12570)^{20}$ is divisible by 3^{20}. Hence, (1).

12.

n	1	2	3	4	5	6	7	8
7^n	07	49	343	2401	16807	117649	823543	5764801

It can be observed that the tens digits follows the pattern {0, 4, 4, 0, 0, 4, 4, 0, ...}. Thus, tens digit of $(45407)^{356}$ is zero.
Hence, (2).
Note: One need not calculate n^{th} power of 45407; it is sufficient to multiply the last two digits to get the answer as the second last digit is zero. The last two digits of higher power of 45407 will depend only on the last digit, that is 7.
Alternatively,
$7^4 = 2401$ and tens digit of $(2401)^{89}$ is zero.

13. Consider $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
The coefficient of a^2 or $b^2 = \frac{2!}{2!} = 1$ and the coefficient of ab or bc or ca = $\frac{2!}{1!1!} = 2$.
Consider $(a + b + c)^3 = (a^2 + b^2 + c^2 + 2ab + 2bc + 2ca) \times (a + b + c)$
$= a^3 + b^3 + c^3 + 3a^2b + 3ab^2 + 3b^2c + 3bc^2 + 3a^2c + 3ac^2 + 6abc$
The coefficient of a^3 or b^3 or $c^3 = \frac{3!}{3!} = 1$, the coefficient of a^2b, ab^2, b^2c, bc^2, a^2c or $ac^2 = \frac{3!}{1!2!} = 3$, and

the coefficient of abc = $\frac{3!}{1!1!1!} = 6$.

In the same way, the coefficient of $a^2b^2c^2$ in $(a + b + c)^6$ will be $\frac{6!}{2!2!2!} = 90$.

But in our case, c = 2.

$\therefore$ The coefficient of a^2b^2 in $(a + b + 2)^6$ will be $4 \times 90 = 360$. Hence, (3).

14. The product of n consecutive numbers is divisible by n!

Thus, the product of numbers from 110 to 126 is perfectly divisible by 17!

18! = 18! × 17!. 18 is a factor of 2's and 3's.

Number of 2's in 18! = $\frac{18}{2} + \frac{18}{4} + \frac{18}{8} + \frac{18}{16} = 16$ ..... (1)

Number of 3's in 18! = $\frac{18}{3} + \frac{18}{9} = 8$ ....(2)

Let X represent the product ! 110 × 111 ×.... 126

Total number of 2's in X = (no. of even terms in X) + (no. of terms in X divisible by 4) + (no. of terms in X divisible by 8) + (no. of terms in X divisible by 16) = 9 + 4 + 2 + 1 = 16 ...(3)

Total number of 3's in X = (no. of terms in X divisible by 3) + (no. of terms in X divisible by 9) = 6 + 2 = 8 ..... (4)

From (1), (2), (3) and (4), we say that X is divisible by 18!

The term 114 in X is divisible by 19.

Thus, the product is divisible by 19 × 18! = 19!

Now, the number of 2's in 20! = 18 exceeds the number of 2's in X.

Hence, the product is not divisible by 20! and hence not even by 21!. Hence, (1).

15. $784 = 7^2 \times 2^4$

For n! to be divisible by 784, it should have at least two 7s.

$\therefore$ The minimum value of n is 14.

So we need to find the highest power of 12 that will divide 14!.

$12 = 2^2 \times 3^1$

Number of 2s in 14! = $\frac{14}{2} + \frac{14}{4} + \frac{14}{8} = 7 + 3 + 1 = 11$

Highest power of 2^2 in 14! = $\left[\frac{11}{2}\right] = 5$

Number of 3s in 14! = $\frac{14}{3} + \frac{14}{9} = 4 + 1 = 5$

We need two 2s and one 3.

$\therefore$ The highest power of 12 that divides 14! = 5, which is the required answer.

16. $(((16)^{17})^{18})^{19} = (14 + 2)^{18k} = 14m + 2^{18k}$

14m when divided by 7 gives remainder 0. So, we need to find the remainder when 2^{18k} is divided by 7.

$2^{18k} = (2^3)^{6k} = (7 + 1)^{6k} = 7n + 1$

7n + 1, when divided by 7, gives a remainder 1.

$\therefore$ The required answer is 1.

17. The number of zeroes at the end of 120! will be equal to the number of powers of 10 in 120!

The number of powers of 10 in 120! is same as the number of powers of 5 in 120! (because the number of 2s in 120! will be greater than the number of 5s).

Number of 5s is 120! = $\left[\frac{120}{5}\right] + \left[\frac{120}{25}\right] = 24 + 4 = 28$

Therefore, the required answer is 28.

18. Consider the terms of the numerator individually.

$\frac{7! \times 6!}{9!} = \frac{6!}{9 \times 8}$

The remainder obtained in this case is 0.

$\frac{8! \times 7!}{9!} = \frac{7!}{9!}$

7! is completely divisible by 9, so the remainder in this case is 0.

Therefore, the required answer is 0.

19. Last 2 digits of 25^{63} will be 25 as 5 raised to any power always ends in 25

$63^{25} = (63)^{24} \times 63$

$= (3969)^{12} \times 63$

$(3969)^{12} = (3970 - 1)^{12}$

$\Rightarrow (3970)^{12} - {}^{12}C_1\, 3970^{11}\,(1) + ... - {}^{12}C_{11}\, 3970(1)^{11} + 1^{12}$

Except last 2 terms all terms will end with at least 2 zeros

${}^{12}C_{11}\, 3970(1)^{11} = 12 \times 3970$ will end in 40

1^{12} will end in 01

$(3969)^{12}$ will end in 00 – 40 + 01 = 61

Also 63^{25} will end in last 2 digits of 61 × 63 or 43

$63^{25} \times 25^{63}$ will end in last 2 digits of 25 × 43 or 75.

20. $101^{76} = (100 + 1)^{76} = 100^{76} + 76 \times 100^{75} + ... + 76 \times 100^1 + 1$

$= [100^{76} + 76 \times 100^{75} + ... + {}^{76}C_{74} \times 100^2] + 7600 + 1$

$M \times 100^2 + 7601$; M is some natural number.

$\therefore$ Third digit from the right in 101^{76} is 6.

21. We will first find the remainder when 7^{84} is divided by 11.

$7^{84} = 7^{3\times28} = (343)^{28} = (341 + 2)^{28}$

The remainder when 7^{84} is divided by 11.

= remainder when 2^{28} is divided by 11.

$2^{28} = 2^{7\times4} = 128^4 = (132 - 4)^4$

$\therefore 4^4 = 16^2 = (11 + 5)^2$

$\therefore$ The remainder when $5^2 = 25$ is divided by 11 = 3

The required least number = 11 – 3 = 8

22. We arrive at the numbers by trial and error: a = 48, b = 49, c = 50.

So, $\left[(48)^{49}\right]^{50} \equiv \left[(-2)^{49}\right]^{50} \mod 5 \equiv \left[(-2)^{50}\right]^{49}$

$\left[(4)^{25}\right]^{49} = (-1)^{odd} = -1$ and hence will give a remainder of 4.

QA-4.6 | PROBABILITY

PRACTICE EXERCISE-1

Answers to questions 1 to 6:

The following tabular representation of the 50 students can be made:

	Boys	Girls
Coffee	20	5
Not Coffee	10	15

1. 15 out of 50 satisfy the condition. Hence 3/10 or 0.3

2. Out of 49 students (excluding Y), 24 drink coffee. Therefore the required answer is 24/49.

3. 5 out of 20 girls drink coffee. Therefore the required answer is 1/4 or 0.25

4. 5 out of 25 coffee-drinkers are girls. Therefore the required answer is 1/5 or 0.2

5. ${}^{25}C_2 / {}^{50}C_2 = 12/49$

6. Either X = (boy, coffee) and Y = (girl, coffee) or vice versa. Therefore the required answer is $2 \times 20/50 \times 5/49 = 4/49$

7. A throw amounting to 18 must be made up of 6, 6, 6 and this can occur in 1 way, 17 can be made up of 6, 6, 5 which can occur in 3 ways, 16 may be made up of 6, 6, 4 and 6, 5, 5 each of which arrangements can occur in 3 ways.
Therefore the number of favourable cases is = 1 + 3 + 3 + 3 = 10 and the total number of cases i s $6^3 = 216$.
$\therefore$ Required probability = $\frac{10}{216} = \frac{5}{108}$. Hence, (3).

8. Three items of clothing can be selected from 15 items in ${}^{15}C_3$ ways i.e. $\frac{15!}{12! \times 3!} = 5 \times 7 \times 13 = 35 \times 13 = 455$.
$\therefore$ Probability that all three are selected is
$\frac{{}^4C_1 \times {}^5C_1 \times {}^6C_1}{455} = \frac{120}{455} = \frac{24}{91}$. Hence, (3).

9. Probability that Sudhir chooses the correct lane is $\frac{1}{4}$.
After choosing the correct lane the probability of choosing the correct house is $\frac{1}{4} \times \frac{1}{5} = \frac{1}{20}$
$\therefore$ Probability that the first house that Sudhir checks is not Jitu's = $1 - \frac{1}{20} = \frac{19}{20}$. Hence, (2).

10. The four boys for the part can be selected in either of the following ways.
(1) 2 groups of 2 boys of same name are selected in ${}^3C_2 = 3$.
(2) 1 group of 2 boys of same names and other 2 boys having different names are selected in ${}^3C_1 \times {}^6C_4 = 45$ ways.
(3) 4 boys having different names are selected in ${}^6C_4 = 15$ ways.
$\therefore$ Total number of ways of selecting four boys for the party = 3 + 45 + 15 = 63
If two boys named Suresh go for the party, then other two boys going for the party with them will either be boys with same name or boys with different names.
$\therefore$ The total number of ways = ${}^2C_1 + {}^6C_2 = 2 + 15 = 17$.
$\therefore$ The required probability = $\frac{17}{63}$. Hence, (3).

11. If 100 persons are surveyed, female = 45, male = 55.
Female smokers = $\frac{20}{100} \times 45 = 9$
Male smokers = $\frac{40}{100} \times 55 = 22$
P(selected person smokes) = $\frac{(9 + 22)}{100} = \frac{31}{100}$.
$P\left(\frac{\text{male}}{\text{selected person smokes}}\right) = \frac{p\,(\text{male and smoker})}{p\,(\text{smoker})}$
$= \frac{\frac{22}{100}}{\frac{31}{100}} = \frac{22}{31}$. Hence, (1).

12. Let p(1) be the probability of clearing 1st section and p'(1) = 1 – p(1) the probability of not clearing the first section and so on.
$\therefore$ $p(1) = \frac{1}{5}$, $p'(1) = \frac{4}{5}$
$p(2) = \frac{2}{3}$, $p'(2) = \frac{1}{3}$
$p(3) = \frac{3}{4}$, $p'(3) = \frac{1}{4}$
$\therefore$ Probability of clearing exactly one section
= p(1)p'(2)p'(3) + p'(1)p(2)p'(3) + p'(1)p'(2)p(3)
$= \frac{1}{5} \times \frac{1}{3} \times \frac{1}{4} + \frac{2}{3} \times \frac{4}{5} \times \frac{1}{4} + \frac{3}{4} \times \frac{4}{5} \times \frac{1}{3}$
$= \frac{1}{60} + \frac{8}{60} + \frac{12}{60}$
$= \frac{21}{60} = \frac{7}{20}$. Hence, (1).

13. Let p(A) = probability of selecting a red bag.
p(B) = probability of selecting a black bag.
p(C) = probability of selecting non yellow ball from red bag.
p(D) = probability of selecting non yellow ball from black bag.

= p(A).p(C) + p(B).p(D)

$= \left(\frac{4}{7}\right)\left(\frac{5}{9}\right) + \left(\frac{3}{7}\right)\left(\frac{8}{17}\right)$

$= \frac{1}{7}\left[\frac{20}{9} + \frac{24}{17}\right]$

$= \frac{1}{7}\left[\frac{340 + 216}{153}\right] = \frac{556}{1071}$. Hence, (4).

14. P (B wins the game) = 1 – P(A wins the game) – P(The game is a draw)

$\therefore$ P(B wins the game) = $1 - \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$

Required probability = P(ABA) + P(BAB)

$\therefore$ Required probability = $\frac{1}{2} \times \frac{1}{6} \times \frac{1}{2} + \frac{1}{6} \times \frac{1}{2} \times \frac{1}{6} = \frac{1}{24} + \frac{1}{72} = \frac{1}{18}$. Hence, (1).

15. There are 21 two-digit prime numbers. For the sum of the digits to be odd, the tens digit must be even.
We have the following cases: 23, 29, 41, 43, 47, 61, 67, 83, 89 (total = 9)

$\therefore$ Required probability = $\frac{9}{21} = \frac{3}{7}$. Hence, (1).

16. Let A, B and C be the events that I clear the stages CAT, GD and PI respectively, then

P(A) = $\frac{3}{5}$, P(B) = $\frac{2}{5}$ and P(C) = $\frac{4}{5}$.

$\therefore$ The required probability

= P(A) × P(B) × P(C) + P(A) × P(B) × P(C')

= P(A) × P(B)

$= \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$. Hence, (3).

17. No one wins if no one hits the bulls eye.

Now, the probability of dead heat = $\frac{1}{27}$

$\therefore \frac{1}{27}$ = P(A wins) × P(B wins) × ...

Let the probabilities of others be $\frac{1}{x}$ each

Hence, $\frac{2}{9} \times \frac{2}{3} \times \left(\frac{1}{x}\right)^6 = \frac{1}{27} \Rightarrow x = (4)^{\frac{1}{6}} = \sqrt[6]{4}$

Hence, P[no one wins] = P(A loses) × P(B loses) × P(one of the other loses) × ...

$= \frac{7}{9} \times \frac{1}{3} \times \left(1 - \frac{1}{\sqrt[6]{4}}\right) \times \left(1 - \frac{1}{\sqrt[6]{4}}\right) \times$ (6 times)

$= \frac{7}{27} \times \left(\frac{\sqrt[6]{4} - 1}{\sqrt[6]{4}}\right) = \frac{7}{27 \times 4}\left(\sqrt[6]{4} - 1\right)^6$

$= \frac{7}{108}\left(\sqrt[6]{4} - 1\right)^6 = \frac{7}{108}\left(\sqrt[3]{2} - 1\right)^6$ .Hence, (2).

18. Total number of employees = 100.
The number of employees having at least two loans = Grade A + Grade B = Y (say)
Probability of an employee having at least two loan options

$= \frac{Y}{\text{Total numer of employees}} = \frac{Y}{100} = \frac{1}{2}$

Thus, Y = Grade A + Grade B = 50.
Also, the probability of obtaining at least one loan = $\frac{\text{Grade A + Grade B + Grade C}}{\text{Total number of employees}} = \frac{7}{10}$

Thus, Grade A + Grade B + Grade C = 70
From the equations, we have,
Grade C = 20
Therefore, probability of obtaining exactly one loan = $\frac{20}{100}$ = 0.2. Hence, (2).

19. Total unique number of scores of a team batting first = 200 (200, 201...399). This situation can be looked upon as if we have a giant 'die' with 200 faces, which is being rolled twice. We will get any one number between 200 and 399 on the first throw. The score for the second match will be the same as the first if the same number is obtained on the second throw of the die as well.

$\therefore$ Required probability = $\frac{1}{200}$ = 0.5%. Hence (2).

20. The number of ways of choosing two squares is = $^{64}C_2$

$= \frac{64!}{62!2!} = 63 \times 32 = 2016$

If the first square happens to be any of the four corner ones, the second square can be chosen in only 1 way. If the first square happens to be any of the 24 squares on the edge of the chess board, the second square can be chosen in 2 ways. If the first square happens to be any of the 36 remaining squares, the second square can be chosen in 4 ways. The two squares between themselves can be arranged in 2! = 2 ways.

$\therefore$ The number of ways of selecting two squares = $\frac{4 \times 1 + 24 \times 2 + 36 \times 4}{2} = 98$

$\therefore$ Required probability = $\frac{98}{2016} = \frac{7}{144}$. Hence, (2).

21. The sum can be 4, 9 or 16. The various combinations of numbers that sum up to 4, 9 and 16 are as follows.

Sum	Possible numbers	Total possibilities
4	1, 1, 2	3
9	1, 2, 6	6
	1, 3, 5	6
	1, 4, 4	3
	2, 2, 5	3
	2, 3, 4	6
	3, 3, 3	1
16	4, 6, 6	3
	5, 5, 6	3

$\therefore$ Total favourable outcomes = 3 + 6 + 6 + 3 + 3 + 6 + 1 + 3 + 3 = 34

Total outcomes = 6 × 6 × 6 = 216

$\therefore$ Required probability = $\frac{34}{216} = \frac{17}{108}$. Hence, (1).

PRACTICE EXERCISE-2

1. Note that in the first case their results are independent, while in the second they are dependent

Event	Probability in Case I	Probability in Case II
a. Ricky loses	3/5	3/5
b. Both Ricky & Shane win	4/35	0
Reason	2/5x2/7 (independent events)	Not possible
c. Exactly one of them wins	16/35	24/35
Reason	P(R wins & S loses) + P(S wins & R loses) = 2/5x5/7 + 3/5x2/7	2/5 + 2/7 (events "R wins" & "S wins" are mutually exclusive)
d. At least one of them wins	4/7	24/35
Reason	P(both lose) = 3/5 x 5/7 = 3/7	Both cannot win simultaneously
e. Ricky wins given that Shane loses	2/5	14/25
Reason	R's winning is independent of S's winning / losing	P(R wins & S loses) / P(S loses) = P(R wins)/P(S loses) = (2/5)/(5/7)
f. At least one of them loses	31/35	1
Reason	1 – (2/5x2/7)	Both cannot win

Answer to questions from 2 to 7:

Each die can show values from 1 to 6. ∴ Sample Space = 36 cases (both for sum and product).

2. The sum can be 4 (3 ways – (1,3), (2,2), (3,1)) or 9 (4 ways – (6,3), (5,4), (4,5), (3,6)). Hence 7/36

3. The product can be 1 (1 way – (1,1)) or 4 (3 ways – (1,4), (2,2), (4,1)) or 9 (1 way - (3,3)) or 16 (1 way - (4,4)) or 25 (1 way - (5,5)) or 36 (1 way - (6,6)). Hence totally 8 ways → 8/36 = 2/9

4. The sum can be 2 (1 way) or 3 (2 ways) or 5 (4 ways) or 7 (6 ways) or 11 (2 ways). Hence 15/36 = 5/12

5. There are 6 cases ((1, 2), (1, 3), (1, 5), (2, 1), (3,1) (5, 1)). Hence 6/36 = 1/6

6. There are 3 cases((5, 6), (6, 5), (6, 6)). Hence 3/36 = 1/12

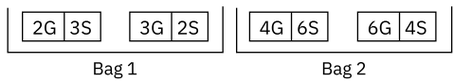
7. This is not true in 6 cases ((4, 6), (6, 4), (5,5), (5, 6), (6, 5), (6,6)). Hence 1 – 6/36 = 5/6

8. Probability that A wins (p1) = $\frac{1}{6}$, that B wins (p2) = $\frac{1}{10}$ and that C wins (p3) = $\frac{1}{8}$. As a dead heat is impossible, these are mutually exclusive events, so the chance that one of them will win the race is p1 + p2 + p3 i.e., $\frac{1}{6} + \frac{1}{10} + \frac{1}{8} = \frac{47}{120}$. Hence, (1).

9. The number of ways in which a number can be chosen between 1 to 30 is 30, so that the total number of ways of choosing numbers is 30 × 30 = 900.
Again, there are 30 ways in which both Rajeev and Kapil can draw the same number.
∴ The probability that they will draw the same number in the first draw is $\frac{30}{900} = \frac{1}{30}$. Hence, (4).
Alternatively,
The problem can also be seen as follows: Whichever number Rajeev draws, Kapil has 30 choices out of which only one is correct. Hence, the required probability is $\frac{1}{30}$.

10. Probability that Sachin scores a goal is 3P.
∴ The probability that Saurav scores the goal and Sachin misses it = P(1–3P). Hence, (1).

11. Since all the rooms have same combination of bags, so it does not matter if the person enters any of the rooms.
In one such room



Probability of picking a gold coin from either of the pockets of bag 1 = $\frac{1}{2}\left[\frac{2}{5} + \frac{3}{5}\right] = \frac{1}{2}$.
Similarly, probability of picking a gold coin from either of pockets of bag 2
$= \frac{1}{2}\left[\frac{4}{10} + \frac{6}{10}\right] = \frac{1}{2}$.
∴ Required probability = $\frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{1}{2}$.
Hence, (3).

12. (i) $P(A \cap B) = P(A).P(B/A) = 0.2 \times 0.5 = 0.1$
(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.2 + 0.5 - 0.1 = 0.6$
(iii) $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.5} = 0.2$
∵ 0.6 > 0.2 > 0.1 we get
$P(A \cup B) > P(A/B) > P(A \cap B)$. Hence, (3).

13. 5 members can be selected out of 10 in ${}^{10}C_5$ ways
$\therefore n(S) = {}^{10}C_5 = \frac{10!}{5! \times 5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2}$
$= 252$
Let A be the event that the team includes exactly 2 Indians and 3 Englishman.
$\therefore n(A) = {}^{6}C_2 \times {}^{4}C_3 = \frac{6!}{4! \times 2!} \times 4$
$= 15 \times 4 = 60$
$\therefore p(A) = \frac{n(A)}{n(S)} = \frac{60}{252} = \frac{15}{63} = \frac{5}{21}$. Hence, (4).

14. The situation that exactly two of them hit bullseye involves three cases: A & B hit bullseye but not C, B & C hit bullseye but not A, and A & C hit bullseye but not B.
∴ Required probability
$= \frac{1}{3} \times \frac{1}{4} \times \frac{1}{2} + \frac{1}{4} \times \frac{2}{3} + \frac{1}{3} \times \frac{1}{2} \times \frac{3}{4} = \frac{1+2+3}{24} = \frac{1}{4}$. Hence, (1).

15. Probability of getting plumbing contract = $P(A) = \frac{2}{3}$
Probability of not getting an electric contract = $P(B') = \frac{5}{9}$
Hence, probability of getting an electric contract = $P(B) = 1 - \frac{5}{9} = \frac{4}{9}$
Probability of getting atleast one contract = $P(A \cup B) = \frac{4}{5}$
$P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$\Rightarrow P(A \cap B) = \frac{2}{3} + \frac{4}{9} - \frac{4}{5} = \frac{14}{45}$. Hence, (2).

16. Let A be the event that a cube is transferred from the first box to the second and let B be the event that a cube is selected from the second box.
$P(B) = P(A \cap B) \cup (A' \cap B)$ where $A \cap B$ and $A' \cap B$ are mutually exclusive.
$\therefore P(B) = P(A \cap B) + P(A' \cap B)$
$P(A) = \frac{8}{14} = \frac{4}{7}$
$P(B/A) = \frac{7}{19}$
$\therefore P(A \cap B) = P(A).P(B/A) = \frac{4}{7} \times \frac{7}{19} = \frac{4}{19}$
Similarly $P(A' \cap B) = P(A') \times P(B/A')$
$P(A') = \frac{6}{14} = \frac{3}{7}$ $\quad P(B/A') = \frac{6}{19}$
$\therefore P(A' \cap B) = \frac{3}{7} \times \frac{6}{19} = \frac{18}{133}$
$\therefore$ Required probability = $\frac{4}{19} + \frac{18}{133} = \frac{46}{133}$.
Hence, (4).

17. E_1 : atleast 1 head = (H, T), (T, H), (H, H)
E_2 : getting a picture card.
E_3 : sum < 9
Sample space $(S_1 \cup S_2 \cup S_3)$: S
$= 4 \times {}^{52}C_1 \times 36$
$n(E_1) = 3$
$n(E_2)$ = {4 kings + 4 queen + 4 jack} = 12
E'_3 (i.e., sum ≥ 9) = {(6, 6), (6, 5), (5, 6), (5, 5), (5, 4), (4, 5), (6, 4), (4, 6), (6, 3), (3, 6)}
$n(E'_3) = 10 \Rightarrow n(E_3) = 36 - 10 = 26$
$\therefore n(E) = n(E_1 \cap E_2 \cap E_3) = 3 \times 12 \times 26$

$\therefore$ Required probability = $\frac{3 \times 26 \times 12}{4 \times 52 \times 36} = \frac{1}{8}$.
Hence, (4).

18. Let the number of black balls in bag A be x.
Since the probability is 0.25, therefore the number of white balls = 3x
Let the number of white balls in bag B be y.
Since the probability is 0.25, therefore the number of black balls = 3y
Total number of black balls in bag C = x + 3y and total number of balls in bag C = 4x + 4y

$\therefore \frac{x + 3y}{4x + 4y} = \frac{7}{15}$

$15x + 45y = 28x + 28y$
$13x = 17y$

$4x + 4y = 4(x + y) = 4 \times \left(\frac{17}{13}y + y\right) = \frac{120}{13}y$

For this number to be an integer, y should take values like 13, 26, 39, etc. such that the required number of total balls in bag C becomes 120, 240, 360, etc. Therefore, 4x + 4y = 300 is not possible. Hence (3).

19. The favourable outcomes are (1, 1, 2), (1, 2, 3), (1, 3, 4), (1, 4, 5), (1, 5, 6), (2, 1, 1), (2, 1, 3), (2, 2, 4), (2, 3, 5), (2, 4, 6), (3, 2, 1), (3, 1, 2), (3, 1, 4), (3, 2, 5), (3, 3, 6), (4, 3, 1), (4, 2, 2), (4, 1, 3), (4, 1, 5), (4, 2, 6), (5, 4, 1), (5, 3, 2), (5, 2, 3), (5, 1, 4), (5, 1, 6), (6, 5, 1), (6, 4, 2), (6, 3, 3), (6, 2, 4), (6, 1, 5) = Total 30 favourable outcomes.
Total outcomes = 6 × 6 × 6 = 216
$\therefore$ Required probability = $\frac{30}{216} = \frac{5}{36}$. Hence (1).

20. Let's denote 5 paise, 10 paise and 20 paise coins by O, E_1 and E_2 respectively.
Initially,
Rajesh O E_1 E_2
Ramesh O E_1 E_2
There are two possibilities:
(i) Rajesh wins odd coin first.
It's probability = $\frac{2}{9}$
(Out of 9 cases, 2 cases are E_1 + O, E_2 + O)
Then before the second game,
Rajesh O O E_1 E_2
Ramesh E_1 E_2
$\therefore$ Probability of Rajesh winning even coin in the second game = $\frac{1}{2}$
(ii) Rajesh wins even coin in the 1st game
It's probability = $\frac{2}{9}$ (Out of 9 cases 2 cases are O + E_1, O + E_2)
Rajesh O E_1 E_2 E_1/E_2
Ramesh O E_2/E_1
$\therefore$ Probability of Rajesh winning odd coins in 2nd game = $\frac{3}{8}$
$\therefore$ Total probability = $\frac{2}{9} \times \frac{1}{2} + \frac{2}{9} \times \frac{3}{8} = \frac{7}{36}$
Hence, (1).

21. **Case I:**
Probability of Ramesh winning odd coin in first game = $\frac{1}{9}$ (Out of 9 cases only 1 case is O + O)
Before 2nd game,
Rajesh E_1 E_2
Ramesh O O E_1 E_2
Probability of winning even coin in 2nd game = $\frac{1}{2}$
Before 3rd game,
Rajesh E_2/E_1
Ramesh O O E_1 E_2 E_1/E_2
Probability of winning even coin in 3rd game = $\frac{3}{5}$
Probability of winning (O, E_1/E_2, E_2/E_1)
$= \frac{1}{9} \times \frac{1}{2} \times \frac{3}{5} = \frac{1}{30}$
Case II:
Probability of Ramesh winning even coin in first game = $\frac{4}{9}$
(Out of 9 cases 4 cases are $E_1 + E_2$, $E_1 + E_1$, $E_2 + E_1$, $E_2 + E_2$)
Before 2nd game,
Rajesh E_1/E_2 O
Ramesh E_1 E_2 E_2/E_1 O
Probability of winning odd coin in 2nd game = $\frac{1}{8}$
Before 3rd game,
Rajesh E_1/E_2
Ramesh E_1 E_2 E_2/E_1 O O
Probability of winning even coin in 3rd game = $\frac{3}{5}$
$\therefore$ Probability of winning (E_1/E_2, O, E_2/E_1) = $\frac{4}{9} \times \frac{1}{8} \times \frac{3}{5} = \frac{1}{30}$
Case III:
Probability of Ramesh winning even coin in first game = $\frac{4}{9}$
Before 2nd game,
Rajesh E_1/E_2 O
Ramesh E_1 E_2 E_2/E_1 O
Probability of winning even coin in 2nd game = $\frac{3}{8}$
Before 3rd game,
Rajesh O
Ramesh E_1 E_2 E_1 E_2 O
Probability of winning odd coin in 3rd game = $\frac{1}{5}$
$\therefore$ Probability of winning (E_1/E_2, E_2/E_1, O) = $\frac{4}{9} \times \frac{3}{8} \times \frac{1}{5} = \frac{1}{30}$
$\therefore$ Total probability = $\frac{1}{30} + \frac{1}{30} + \frac{1}{30} = 0.1$
Hence, (4).

QA-4.7 | APPLICATIONS OF P&C, PROBABILITY

PRACTICE EXERCISE-1

1. We want solution set of the equation $x + y + z = 14$ for $1 \le x < y < z$
i.e., we want solution set of the equation $x_1 + y_1 + z_1 = 11$ for $0 \le x_1 < y_1 < z_1$, where $x_1 = x - 1$, $y_1 = y - 1$ and $z_1 = z - 1$.
$\therefore$ Total number of solutions to $(x_1 + y_1 + z_1 = 11) = {}^{11+3-1}C_{3-1} = \frac{13!}{11!2!} = 78$
These solutions include cases
(i) $x_1 = y_1 \neq z_1$
(ii) $x_1 = z_1 \neq y_1$
(iii) $y_1 = z_1 \neq x_1$

(i) For $x_1 = y_1 \neq z_1$.
$(x_1, y_1, z_1) = (0, 0, 11), (1, 1, 9), (2, 2, 7), (3, 3, 5), (4, 4, 3), (5, 5, 1)$
Similarly we have 6 solutions for each of remaining two cases.
$\therefore$ These $6 \times 3 = 18$ solutions to be excluded.
$\therefore$ $78 - 18 = 60$ solutions are such that $x_1 \neq y_1 \neq z_1$.
There are 6 permutations of x_1, y_1 and z_1. As $x_1 < y_1 < z_1$, we need to consider only one solution among these 6.
$\therefore$ Among 60 solutions only $\frac{60}{6} = 10$ valid solutions. Hence, (1).

2. The two colours of the pentagonal faces can be chosen in ${}^7C_2$ ways = 21 ways.
Other five faces can be considered as objects arranged in circular manner.
These faces can be painted in $(5 - 1)!$
$= 4! = 24$ ways.
Total number of ways in which a pentagonal prism can be painted $= 21 \times 24 = 504$. Hence, (4).

3. The number of ways in which none of the wires are connected with their respective telephones according to their extension number
$$= 6!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!} + \frac{1}{6!}\right)$$
$$= 6!\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \frac{1}{720}\right)$$
$$= 6!\left(\frac{53}{144}\right)$$
Total number of ways of connecting six wires = 6!
$\therefore$ The required probability
$$= \frac{6!\left(\frac{53}{144}\right)}{6!} = \frac{53}{144}.$$ Hence, (4).

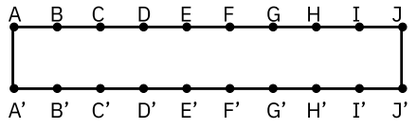
4. Let R be the event that the student gives at least two right answers and R^c be the event that he gives at the most one right answer.
$\therefore P(R) = 1 - P(R^C)$
Here R^c is the event that the student gives no right answer or he gives exactly one right answer to the question.
Let A = event that no right answer is given by the student.
B = Event that exactly one right answer is given by the student.
$\therefore P(R^c) = P(A) + P(B)$
Then $P(A) = \left(\frac{2}{3}\right)^8$
In the event B the student can give right answer for the 1^{st} or 2^{nd} or .... 8^{th} question and for remaining seven questions he does not give right answers.
$$\therefore P(B) = 8\left[\frac{1}{3} \times \left(\frac{2}{3}\right)^7\right] = 4\left(\frac{2}{3}\right)^8$$
$$\therefore P(R) = 1 - \left[\left(\frac{2}{3}\right)^8 + 4\left(\frac{2}{3}\right)^8\right]$$
$$\therefore P(R) = 1 - 5\left(\frac{2}{3}\right)^8.$$ Hence, (4).

5. Anand has to win at least three out of the remaining 4 matches. The sequence could be either WWWW, WWWL, WWLW, WLWW or LWWW. Now the probability of one win is 0.75 i.e. $\frac{3}{4}$. Hence the probability of WWWW is $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{81}{256}$. Also the probability for any of the other sequences (3 wins and a loss) is $\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{27}{256}$. Hence the required probability will be $\frac{81}{256} + 4 \times \frac{27}{256} = \frac{189}{256}$
Hence, (3).

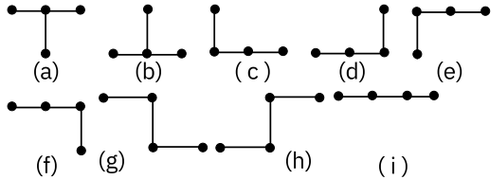
6. Out of 52 cards, 26 are red and 26 are black.
$\therefore P(A) = P(\text{card is red}) = \frac{26}{52} = \frac{1}{2}$.
Out of 52 cards, 13 are diamond and 13 are spade.
$\therefore P(B) = P(\text{diamond or spade}) = \frac{26}{52} = \frac{1}{2}$.
Out of 52 cards 13 are heart, and there are 4 kings of which 1 king is king of heart.
$\therefore$ Total favourable outcomes are $13 + 4 - 1 = 16$.
$\therefore P(C) = \frac{16}{52} = \frac{4}{13}$. Thus, $P(A) = P(B) > P(C)$.
Hence, (3).

7. For the condition, one of the two cards at the end of row must be removed every time. ∴ The probability that all the spades can be removed without a break (hole) occurring in the row of spades is as follows:
$\frac{2}{8} \times \frac{2}{7} \times \frac{2}{6} \times \frac{2}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{2}{2} = \frac{1}{315}$.
Hence, (1).

8. The sitting arrangement will be as follows where the lines joining the point shows that the persons (points) are knowing each other.



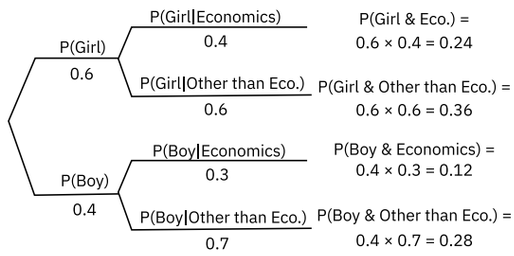
Number of different group of 4 persons with 4 pairs i.e., number of squares formed in above figure i.e., there are 9 ways of choosing 4 persons form 20 so that it contains 4 pairs with required property.
The chosen 4 persons have 3 required pairs if the 3 persons sit like:



There are 8 ways of choosing 4 persons for each sitting arrangement of 4 persons for cases (a) to (h) and 14 ways for case (i).
∴ Probability of choosing four persons with exactly 4 required pairs
$= \frac{9}{9 + (8 \times 8) + 14} = \frac{9}{87}$. Hence, (4).

9. If answer is marked randomly, probability of getting it right is $\frac{1}{4}$.
∴ Required probability: ${}^{10}C_2(0.25)^2(0.75)^8$. Hence, (1).

10. We can construct the tree diagram as follows—



∴ Required probability: $\frac{0.24}{0.24 + 0.12} = \frac{24}{36} = \frac{2}{3}$
Hence, (2).

11. Probability that Kamlesh will be selected = P(K) = 0.4
Probability that both Kamlesh and Vimlesh will be selected = $P(K \cap V) \leq 0.2$
Probability that neither of them will be selected = 0.1
∴ Probability that either of them will be selected = $P(K \cup V) = 1 - 0.1 = 0.9$
$\therefore P(K \cup V) = P(K) + P(V) - P(K \cap V)$
$\therefore P(V) = 0.9 - 0.4 + P(K \cap V) = 0.5 + P(K \cap V)$
Since $P(K \cap V) \leq 0.2$, $P(V) \leq 0.7$.
Also, $P(V) \geq (0.9 - 0.4)$ i.e. $P(V) \geq 0.5$. Hence, (2).

12. Only numbers that have exactly three factors are the squares of prime numbers.
Squares of 122 and 123 are 14884 and 15129 respectively. Therefore, we need to calculate the number of prime numbers between 1 and 122.
Between 1 and 122, there are 30 prime numbers. (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113).
Therefore required probability: $\frac{30}{15000} = \frac{1}{500}$
Hence (2).

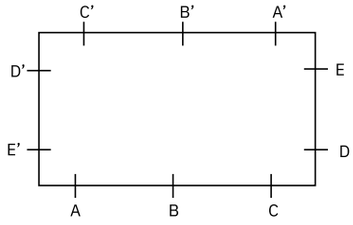
13. A, C, E and G among themselves can be arranged in 4! ways. Among these, only one combination satisfies the condition required in the question. The position of B, D, F and H do not matter.
∴ Required probability = $\frac{1}{24}$. Hence, (4).

14. The given problem can be viewed as, there are 20 equidistant points on a circle. We have to find the number of triangles which are not right-angled triangles formed by joining any three points on the circle.
The total number of triangles formed by these 20 points = ${}^{20}C_3 = 20 \times 57$
These points will form right-angled triangles if and only if one of the sides of the triangle is a diameter.
The total number of diameters = $\frac{20}{2} = 10$.
10 diameters form $10 \times 18 = 180$ right angled triangles.
∴ The required number of sets of three light poles = $(20 \times 57) - 180 = 960$. Hence, (2).

15. The five wooden blocks can be arranged among themselves in 5! = 120 ways. Out of these, there are two ways in which they can be arranged either in increasing or decreasing order of their sizes. Therefore, the required probability = $\frac{118}{120} = \frac{59}{60}$. Hence (2).

16. Three numbers can be selected from the given set of 8 numbers in ${}^8C_3$ ways = 56 ways.
The possible combinations when the three numbers selected are in GP are (1,2,4), (1,3,9), (1,4,16), (2,4,8), (3,9,27) and (4,8,16).
∴ Required probability = $1 - \frac{6}{56} = \frac{25}{28}$. Hence, (3).

17. We have,



The seats denoted by A and A', B and B' etc are identical. Therefore, there are five distinct chairs along the table. The first person can be seated in 5 different ways. Once the first person is seated, there are now 9 distinct chairs so the second person can be seated in 9 different ways. Therefore, the total number of ways in which the two persons can be seated around the table = 5 × 9 = 45.
There are three distinct seats on which the first person can be seated along the longer side of the table. If the first person sits in the middle seat, there are two places on which the second person can be seated so that they are next to each other. On the other hand, if the first person sits on the chairs at the end of the side, there is only one place where the second person can be seated so that the two are next to each other. Therefore, the total number of ways in which the two persons are next to each other = 2 + 2 = 4.

Therefore, the required probability = $\frac{4}{45}$. Hence, (4).

18. Consider a group of 3 consecutive students as one block. There will be 3 blocks, say B_1, B_2 and B_3. The remaining 3 students can be put in four positions as shown below:
$\_ B_1 \_ B_2 \_ B_3 \_$
The number of ways of choosing those 3 remaining students is same as the number of whole number solutions to a + b + c + d = 3, which is $\frac{6!}{3! \times 3!}$ = 20 ways.

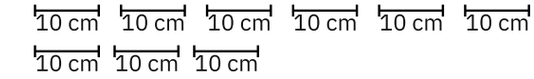
The number of ways in which the three types of roses can be distributed to the three blocks = 3! = 6 ways.
Therefore, the required number of ways = 20 × 6 = 120 ways

19. Let the number of individuals who were invited to the party be 'x'.
Therefore, the number of persons who remained while the power went off = x – 6
Number of roses exchanged between these "x – 6" individuals = (x – 6)(x – 7)
Number of roses given by the 6 individuals who left early = 6 × 6 = 36
$\therefore$ (x – 6)(x – 7) + 36 = 192
$\therefore$ (x – 6)(x – 7) = 156 = 13 × 12
$\therefore$ x = 19
Therefore, the required answer is 19.

20. If there are 'n' players in the tournament, the number of matches possible = ${}^nC_3$.
The number of matches involving only Russian players was 84. If R is the number of Russian players, ${}^RC_3$ = 84. The number of matches involving only American players was 165. If A is the number of American players, ${}^AC_3$ = 165.
We have, ${}^4C_3$ = 4, ${}^5C_3$ = 10, ${}^6C_3$ = 20, ${}^7C_3$ = 35, ${}^8C_3$ = 56, ${}^9C_3$ = 84, ${}^{10}C_3$ = 120 and ${}^{11}C_3$ = 165.
Therefore, the number of Russian players = 9 and the number of American players = 11.
The number of matches involving players from both America and Russia = Total number of possible matches – Number of matches in which only Russian players participate – Number of matches in which only American players participate
= ${}^{20}C_3 - 84 - 165 = \frac{20!}{17!3!} - 84 - 165 = 1140 - 84 - 165 = 891$
Therefore, the required answer is 891.

PRACTICE EXERCISE-2

1. For achieving a total length of 108 cm we need to use nine sticks of length 10 cm and three sticks of length 6 cm (9 × 10 + 3 × 6 = 108).
Nine longer sticks can be arranged in one way (as they are identical).



There are 10 spaces for the 3 shorter sticks. These places can be occupied by shorter sticks in $^{10}C_3$ ways.
i.e., $\frac{10!}{3! \times 7!}$ = 120 ways. Hence, (2).

2. From chosen four triangles Jenny will not be able to make a rhombus if no two of the four triangles are congruent.
∴ Number of ways of choosing four triangles from five non-congruent triangles = $^5C_4$. Hence, (3).

3. The sum can be divisible by 5 when the sum is 5, 10 or 15. The various possibilities and their count are given below.

Sum	Possible numbers	Total possibilities
5	1, 1, 3	3
	1, 2, 2	3
10	1, 3, 6	6
	2, 2, 6	3
	1, 4, 5	6
	2, 3, 5	6
	2, 4, 4	3
	3, 3, 4	3
15	5, 5, 5	1
	6, 5, 4	6
	6, 6, 3	3

∴ Favourable outcomes = 3 + 3 + 6 + 3 + 6 + 6 + 3 + 3 + 1 + 6 + 3 = 43
Total outcomes = 6^3 = 216
∴ Required probability = $\frac{43}{216}$. Hence (1).

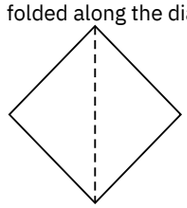
4. Four cousins have already taken their chocolates such that equal number of chocolates are left in each box for the fifth cousin to choose his chocolates.
So, the number of chocolates left in each box = 25 – 14 = 11.
∴ The fifth cousin can choose his chocolates in $^{11}C_5 \times {}^{11}C_2 + {}^{11}C_2 \times {}^{11}C_5 + {}^{11}C_4 \times {}^{11}C_3 + {}^{11}C_3 \times {}^{11}C_4$ ways.
i.e., in $2[{}^{11}C_2 \times {}^{11}C_5 + {}^{11}C_4 \times {}^{11}C_3]$ ways.
Hence, (2).

5. Mohit has the digits 2, 2, 2, 3, 3 and 4. Using these digits he has to make a three-digit number.
i] All three digits are different.
This can be done in 3! = 6 ways.
ii] All the three digits are 2. This can be done in one way.
iii] Two digits are 2 and one digit can be selected from 3 and 4 in 2 ways.
Then the three digits can be arranged in $\frac{3!}{2!}$ = 3 ways.
∴ Total number of ways when two digits are 2 and one digit is either 3 or 4
= 3 × 2 = 6 ways.
iv] Two digits are 3 and one digit is either 2 or 4.
This also can be done in 6 ways.
∴ Total number of ways of forming a three-digit number = 6 + 1 + 6 + 6 = 19 ways. Hence, (3).

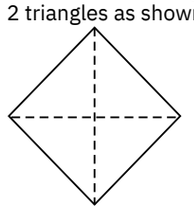
6. One can select three numbers in $^{17}C_1 \times {}^{17}C_1 \times {}^{17}C_1$ ways.
= 17 × 17 × 17 = 4913 are the possible combinations.
If one tries all of these combinations he has to spend 4913 × 5 = Rs.24,565.
And definitely one of the combination will give him Rs.50,000.
Similarly, for 2 numbers to match one has to spend minimum 17 × 17 × 5 = 1445.
In this case also, it is possible to make a profit.
If 2 numbers are selected then the numbers of tickets to be bought are 289 and for the 3 numbers the number of tickets to be bought will be 4913.
∴ To get the profit, number of minimum tickets to be bought = 289. Hence, (4).

7. Since E is definitely selected, the remaining 4 friends can be selected from among 7 friends in $^7C_4$ = 35 ways.
The required condition is satisfied in two cases.
Case 1: __ > __ > __ > E > __
The first 3 positions can be filled by any among 5 people (A, C, D, F and H). This can be done in $^5C_3$ = 10 ways. The 5th position can be filled by either B or G, so 2 ways. Therefore, total number of ways = 10 × 2 = 20 ways.
Case 2: __ > __ > __ > __ > E
The 4 positions can be filled by any among 5 people (A, C, D, F and H).
This can be done in $^5C_4$ = 5 ways.
∴ Required probability = $\frac{20 + 5}{35} = \frac{25}{35} = \frac{5}{7}$
Hence (4).

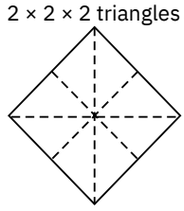
8. The maximum number of triangles which we get after four foldings is 16.
In the 1st fold we will get only two triangles when folded along the diagonal as shown.



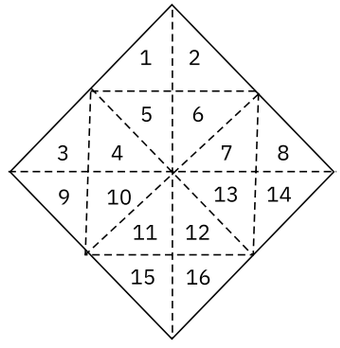
In the 2nd fold along with the first fold we will get 2 × 2 triangles as shown.



In the 3rd fold along with 1st and 2nd folds we will get 2 × 2 × 2 triangles as shown.



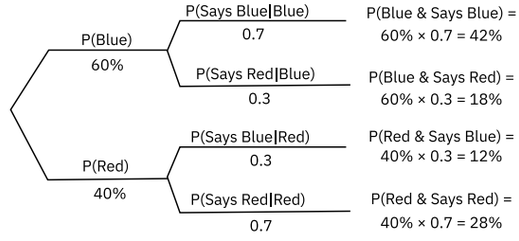
In the 4th fold along with 1st, 2nd and 3rd we will get 2 × 2 × 2 × 2 triangles.



Thus, there are 16 triangles which do not contain any other triangles.
Number of ways of putting 19 identical balls in 16 triangles such that each triangle contains at least one ball = ${}^{19-1}C_{16-1} = {}^{18}C_{15}$
Number of ways of putting 19 balls in 16 triangles such that no triangle is empty and there is a triangle containing exactly 3 balls = 16 × 15
∴ The required number of ways = ${}^{18}C_{15} - 16 \times 15$
$= \frac{18 \times 17 \times 16}{3!} - 16 \times 15 = 576$. Hence, (2).

9. P(student passes) = 1 – Getting zero questions correct) – P(Getting one question correct) – P(Getting two questions correct)
P(Getting zero questions correct)
$= {}^{10}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{10} = \left(\frac{3}{4}\right)^{10}$
P(Getting one question correct)
$= {}^{10}C_1 \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^9 = 10 \times \frac{1}{4}\left(\frac{3}{4}\right)^9 = \frac{5}{2}\left(\frac{3}{4}\right)^9$
= P(Getting two questions correct)
$= {}^{10}C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 = 45 \times \frac{1}{16}\left(\frac{3}{4}\right)^8 = \frac{45}{16}\left(\frac{3}{4}\right)^8$
$\therefore$ P(Student fails) $= \left(\frac{3}{4}\right)^{10} + \frac{5}{2}\left(\frac{3}{4}\right)^9 + \frac{45}{16}\left(\frac{3}{4}\right)^8$
$= \left(\frac{3}{4}\right)^8 \left[\frac{9}{16} + \frac{5}{2} \times \frac{3}{4} + \frac{45}{16}\right]$
$= \left(\frac{3}{4}\right)^8 \left[\frac{9}{16} + \frac{30}{16} + \frac{45}{16}\right]$
$= \left(\frac{3}{4}\right)^8 \left[\frac{21}{4}\right]$
$\therefore$ P(Student passes) $= 1 - \left(\frac{3}{4}\right)^8 \left[\frac{21}{4}\right]$
Hence, (4).

10. We can construct the tree diagram as follows —



$\therefore$ Required probability : $\frac{0.42}{0.42 + 0.12} = \frac{7}{9}$
Hence, (2).

11. Tinku removed the letters A, B, C, D, E, F and G. From these, he took 5 letters. The probability that those were F, A, C, E and D = $\frac{1}{{}^7C_5} = \frac{1}{21}$.
After selecting the 5 letters, the probability that they are in the order F-A-C-E-D = $\frac{1}{5!} = \frac{1}{120}$
Required probability = $\frac{1}{21} \times \frac{1}{120} = \frac{1}{2520}$
Hence (2).

12. $x^2 - y^2 = (x + y)(x - y)$, so this product must be divisible by 3. Therefore, either of $(x + y)$ or $(x - y)$ or both of them must be divisible by 3.
We can formulate a table regarding the various possibilities of x and y.

Value of y	Corresponding values of x	Total pairs
1	2, 4, 5, 7, 8, 10	6
2	4, 5, 7, 8, 10	5
3	6, 9	2
4	5, 7, 8, 10	4
5	7, 8, 10	3
6	9	1
7	8, 10	2
8	10	1

Therefore, required favourable outcomes = 6 + 5 + 2 + 4 + 3 + 1 + 2 + 1 = 24
Total outcomes = 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45
$\therefore$ Required probability = $\frac{24}{45} = \frac{8}{15}$. Hence, (4).

13. Total number of ways
$= 2 \times \left[\frac{10!}{6!4!} + \frac{10!}{5!5!}\right] = 924$. Hence, (4).

14. There are 4 suits in a pack of 52 cards. 4 cards from each suit can be selected in $^{13}C_4$ ways. A particular suit can be selected in 4 ways. The 5^{th} card can be any of the remaining 39 cards.
$\therefore$ Required probability = $\frac{4 \times {}^{13}C_4 \times 39}{{}^{52}C_5}$
$= \frac{4 \times 13 \times 12 \times 11 \times 10 \times 39 \times 5 \times 4 \times 3 \times 2}{52 \times 51 \times 50 \times 49 \times 48 \times 4 \times 3 \times 2}$
$= \frac{143}{3332}$
Hence (3).

15. In all, there are 12 switches of which one is for the two red lamps.
Case (i):
The hall is illuminated with 2 red lamps and other two lamps from remaining 11 lamps. These two lamps can be chosen in $^{11}C_2$ ways.
Case (ii):
All the four lamps from remaining 11 lamps can be chosen in $^{11}C_4$ ways
$\therefore$ Total number of ways = $^{11}C_2 + {}^{11}C_4$
$= 55 + 330$
$= 385$
Hence, (4).

16. The number of whole number solutions to equation $x + y + z = 30$ without any restriction = $\frac{32!}{30!2!} = 496$
There is only one solution when $x = y = z$ and that is $x = y = z = 10$.
Now let us calculate the number of solutions to the equation such that $x = y \neq z$. The solutions are (0,0,30), (1,1,28), (2,2,26)...(15,15,0) or total 16 solutions. However, the solution (10,10,10), which has already been counted under $x = y = z$, is getting double counted here. That needs to be taken out. Therefore, the number of solutions such that $x = y \neq z$ is $16 - 1 = 15$.
Similarly, there are 15 solutions such that $y = z \neq x$ and $x = z \neq y$.
Therefore, the number of solutions such that $x \neq y \neq z$ is $496 - 1 - 15 \times 3 = 450$
The three numbers x, y and z can be arranged in 3! = 6 different ways. Out of these, only one way has $x < y < z$.
Therefore, the required number of solutions = $\frac{450}{6} = 75$
Therefore, the required answer is 75.

17. We have, $20 = 2^2 \times 5^1$. Therefore, we need at least one 5 and two 2s in the product. Therefore, at least one of 10 and 15 must be selected.

Case 1: Only 10 is selected but 15 is not selected
We need at least one even number in the remaining two. The remaining two numbers can be selected out of the following: 11, 12, 13, 14, 16, 17, 18, 19 (total 8, out of which 4 are even and 4 are odd).
The number of ways of selecting two numbers out of these 8 such that at least one is even is $^8C_2 - {}^4C_2 = 28 - 6 = 22$.

Case 2:Only 15 is selected but 10 is not selected
We need two 2s in the remaining two numbers. The remaining two numbers can be selected out of 11, 12, 13, 14, 16, 17, 18, 19.
If 12 is selected, the remaining number can be any one of 11, 13, 14, 16, 17, 18, 19 (total 7 ways)
If 16 is selected, the remaining number can be any one of 11, 13, 14, 17, 18, 19 (total 6 ways). Note we don't count 12 here because the case (15,12,16) has already been counted earlier.
If neither 12 nor 16 is selected, the remaining two numbers have to be 14 and 18. (total 1 way).

Case 3: Both 10 and 15 are selected
The third number will have to be an even number. The remaining even numbers are 12, 14, 16 and 18. (total 4 ways)

Therefore, the required answer is: 22 + 7 + 6 + 1+ 4 = 40

18. Vowels in KARAKORAM – A, A, O, A
Consonants in KARAKORAM – K, R, K, R, M
The vowels as one unit and the consonants can be re-arranged in $\frac{6!}{2!2!}$ = 180 ways.
The vowels among themselves can be arranged in $\frac{4!}{3!}$ = 4 ways.
∴ Required answer = 180 × 4 = 720 ways.

19. Case 1:
1G and 5B can be selected in $^5C_1 \times {}^6C_5$ ways.
Selecting 3 (There is only one case such as 1G and 2B)
$1 \times {}^5C_2$
Selecting 1 (The monitor is a boy)
∴ Number of ways = 2
∴ Total = $^5C_1 \times {}^6C_5 \times {}^5C_2 \times 2 = 5 \times 6 \times 10 \times 2 = 600$ ways.

Case 2:
2G and 4B can be selected in $^5C_2 \times {}^6C_4$ ways.
Selecting 3
There are two cases (a) and (b)
a] 2G and 1B ∴ $^2C_2 \times {}^4C_1$
b] 1G and 2B = $^2C_1 \times {}^4C_2$
Selecting a monitor
Case (a) A girl will be a monitor in 2 ways.
Case (b) A boy will be a monitor in 2 ways.
∴ Total = $^5C_2 \times {}^6C_4 [{}^2C_2 \times {}^4C_1 \times 2 + {}^2C_1 \times {}^4C_2 \times 2]$
$= 10 \times 15 [1 \times 4 \times 2 + 2 \times 6 \times 2] = 4800$
Case 3:
3B and 3G can be selected in $^5C_3 \times {}^6C_3$ ways.
Selecting 3
$^3C_1 \times {}^3C_2 + {}^3C_2 \times {}^3C_1 = 2 \times {}^3C_1 \times {}^3C_2$
Selecting a monitor in each case = 2.
∴ Total = $^5C_3 \times {}^6C_3 [{}^3C_1 \times {}^3C_2 \times 2 + {}^3C_1 \times {}^3C_2 \times 2]$
$= 2 \times {}^5C_3 \times {}^6C_3 \times {}^3C_1 \times {}^3C_2 \times 2$
$= 4 \times 10 \times 20 \times 3 \times 3 = 7200$
Case 4:
4G and 2B in $^5C_4 \times {}^6C_2 = 5 \times 15 = 75$ ways.
Selecting 3 and a monitor
$^4C_2 \times {}^2C_1 \times 2 + {}^4C_1 \times {}^2C_2 \times 2 = 24 + 8 = 32$
∴ Total = 75 × 32 = 2400
Case 5:
5G and 1B in $^5C_5 \times {}^6C_1 = 1 \times 6 = 6$
To choose 3 and 1 monitor $^5C_2 \times {}^1C_1 \times 2$
$= 10 \times 2 = 20$
Total = 6 × 20 = 120
∴ The required number = 600 + 4800 + 7200 + 2400 + 120 = 15120.
Hence, (3).
Alternatively,
Two prefects can be chosen in $^5C_1 \times {}^6C_1 = 30$ ways of the remaining 9, one monitor can be chosen in 9 ways. Of the remaining 8, 3 coordinators can be chosen $^8C_3 = 56$ ways
∴ Total number of ways = 30 × 9 × 56 = 15120.

20. As seen in case 3 and case 4 in the above question, (1) and (2) can definitely be solved.
(3) Since there are greater number of boys than girls in the class hence boys have a greater chance to make it to as a monitor. Hence, (3) is true and can be answered.
(4) Equal number of boys and girls will be left when the number of boys getting selected in the group of 6 coordinates is greater than number of girls by one. But this not possible as the number of students wanted here is 6, an even number. Thus, (4) can never be solved. Hence, (4).

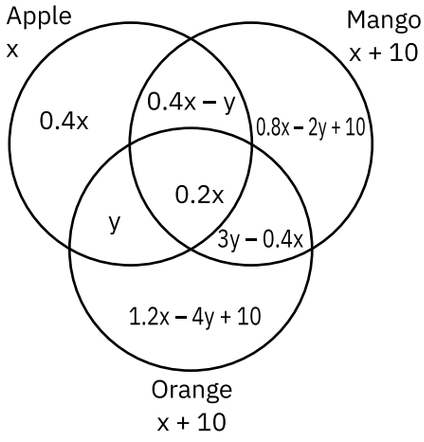
Data Interpretation
&
Logical Reasoning

DI-4.1 VENN DIAGRAMS

PRACTICE EXERCISE-1

Answers to questions 1 to 4:

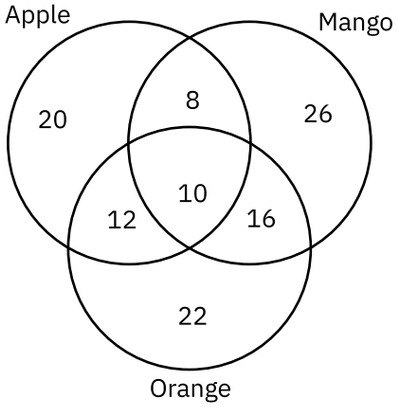
From statement 1, the number of students who like at least one fruit = 0.95 × 120 = 114
Let the number of students who like Apple be x. Therefore, from statements 2 and 6, the number of students who like Mango is x + 10, which is also equal to the number of students who like Orange.
Now, using statements 3, 4 and 5 and using variables, we can create the following Venn diagram.



Using statement 1, we get: x + 0.8x – 2y + 10 + 3y – 0.4x + 1.2x – 4y + 10 = 114
∴ 2.6x – 3y = 94

Using statement 7, we get: y + 0.4x – y + 0.2x + 3y – 0.4x = 0.4x + 0.8x – 2y + 10
∴ –x + 5y = 10

Solving the two equations simultaneously, we get x = 50 and y = 12. Therefore, the Venn diagram can be represented as follows:



Using this, all the questions can be answered.

1. Required answer = $\frac{8+16}{60} \times 100 = 40\%$. Hence, (3).

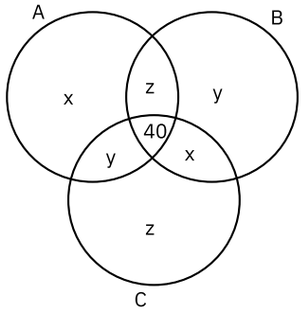
2. Required answer = $\frac{12}{50} \times 100 = 24\%$. Hence, (2).

3. Required answer = $\frac{10}{60} \times 100 = 16.67\%$. Hence, (1).

4. Required answer = $\frac{20+26+22}{114} \times 100 = 59.65\%$. Hence, (4).

Answers to questions 5 to 8:

Suppose x is the number of students who have watched only A, y is the number of students who have watched only B and z is the number of students who have watched only C.
Therefore we have the following



We have,
The number of students who have watched only B is $\frac{1}{8}$th the total number of students. Therefore,

$y = \frac{2x+2y+2z+40}{8}$

$\therefore 8y = 2x + 2y + 2z + 40$

$\therefore 6y = 2x + 2z + 40$

$\therefore y = \frac{2x+2z+40}{6}$ ... (I)

The number of students who have watched only A is $\frac{1}{2}$ times the number of students who have watched only B. Therefore

$x = \frac{y}{2}$ or $x = \frac{x+z+20}{6}$

$\therefore 6x = x + z + 20$

$\therefore 5x = z + 20$

$\therefore x = \frac{z+20}{5}$

The number of students who have watched only A is $\frac{1}{3}$ times the number of students who have watched only C. Therefore

$z = 3x$ or $z = \frac{3z+60}{5}$

$\therefore$ 5z = 3z + 60

$\therefore$ z = 30; x = $\frac{z + 20}{5}$ = $\frac{50}{5}$ = 10; y = 2x = 20

Now all the questions can be answered.

5. The number of students who watched only one movie = 10 + 20 + 30 = 60. Hence, (2).

6. The number of students who watched only two movies = 10 + 20 + 30 = 60. Hence, (2).

7. The number of students who watched both B and C = 10 + 40 = 50. Hence, (3).

8. Total number of students = 10 + 30 + 20 + 20 + 40 + 10 + 30 = 160. Hence, (1).

Answers to questions 9 to 12:

From point 2, the number of students specializing in Finance and Systems is two times the number of students specializing in Marketing and Systems, but is four less than the number of students specializing in Operations and HR.

The only values corresponding to these are:
Finance & Systems: 20 (A and C)
Marketing & Systems: 10 (C and F)
Operations & HR: 24 (B and D)

Therefore it can be seen that C stands for Systems, A stands for Finance and F stands for Marketing. B and D stand for Operations and HR. Therefore E stands for General Management.

From point 1, The number of students specializing in Finance and Operations both (A and B/D) is two times the number of students who are specializing in General Management and HR both (E and D/B), but was one-third the number of students specializing in Operations and HR both (B and D). The number of students specializing in Finance and Operations both is 8. Therefore the number of students specializing in General Management and HR both is 4 (which is D and E both). From the table, it can be seen that since E stands for General Management, D stands for HR.

Just to confirm, the number of students specializing in Finance and Operations both (24) is three times the number of students specializing in Finance and Operations both, as required.

Therefore we have the following:

	Finance	Operations	Systems	HR	General Management	Marketing
Finance	24					
Operations	8	40				
Systems	20	0	14			
HR	0	24	0	12		
General Management	0	0	0	4	4	
Marketing	0	0	10	0	36	2

Now all the questions can be answered.

9. Total number of students in the institute is just the addition of all the numbers given in the table. Therefore the total number of students = 24 + 8 + 20 + 40 + 24 + 14 + 10 + 12 + 4 + 4 + 36 + 2 = 198. Therefore the required answer is 198.

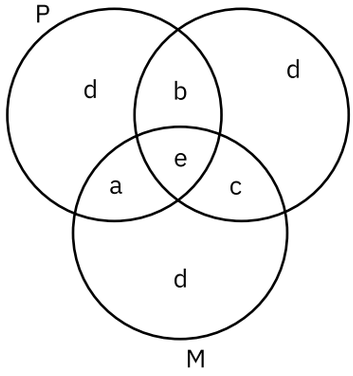
10. The number of students who are specializing in Finance = Number of students specializing only in Finance + Number of students specializing in Finance and one other subject. The required number of students is 24 + 8 + 20 = 52. Therefore the required answer is 52.

11. The required difference = 0 – 0 = 0. Therefore the required answer is 0.

12. The number of students who are specializing only in Marketing = 2. The number of students specializing only in Operations = 40. Therefore the required difference is 38.

Answers to questions 13 to 16:

13. Using the information given in the question let us represent it in the Venn diagram shown below. The diagram depicts the number of candidates getting 80 percentile and above in at least one or more of the subjects amongst students getting 90 percentile overall.

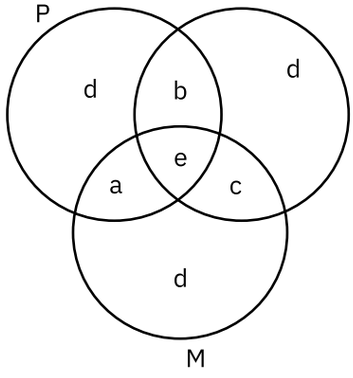


The number of candidates scoring 80 percentile and above in only Physics, only Chemistry and only Math is the same. Let this be 'd'

Let 'a' – number of candidates scoring 80 percentile and above only in Physics and Math.
Let 'b' – number of candidates scoring 80 percentile above only in Physics and Chemistry.
Let 'c' – number of candidates scoring 80 percentile and above in Chemistry and Math.
Let 'e' – number of candidates scoring 80 percentile and above in all 3 subjects.
a + b + c = 150
Also a + b + c + 3d + e = 200
$\Rightarrow$ 3d + e = 50
Given that (2d + c) : (2d + a) : (2d + b) = 4 : 2 : 1
This implies 6d + a + b + c is a multiple of 7. We already know that a + b + c = 150. So 6d + 150 is a multiple of 7. This implies that 6d + 3 will also be a multiple of 7. So d will be 3, 10, 17. But as 3d + e = 50, it implies that d < 17. So d will be either 3 or 10.

The number of students who scored 90 percentile and above and scored at least 80 percentile in Physics (but not in Chemistry and Math) will be eligible for the BIE entrance test. This is equal to d which is either 3 or 10. Hence, (1).

14. Using the information given in the question let us represent it in the Venn diagram shown below. The diagram depicts the number of candidates getting 80 percentile and above in at least one or more of the subjects amongst students getting 90 percentile overall.



The number of candidates scoring 80 percentile and above in exactly each of Physics, chemistry and Math is the same. Let this be 'd'

Let 'a' – number of candidates scoring 80 percentile and above only in Physics and Math.

Let 'b' – number of candidates scoring 80 percentile above only in Physics and Chemistry.

Let 'c' – number of candidates scoring 80 percentile and above in Chemistry and Math.

Let 'e' – number of candidates scoring 80 percentile and above in all 3 subjects.

$a + b + c = 150$

Also $a + b + c + 3d + e = 200$

$\Rightarrow 3d + e = 50$

Given that $(2d + c) : (2d + a) : (2d + b) = 4 : 2 : 1$

This implies $6d + a + b + c$ is a multiple of 7. We already know that $a + b + c = 150$. So $6d + 150$ is a multiple of 7. This implies that $6d + 3$ will also be a multiple of 7. So d will be 3, 10, 17. But as $3d + e = 50$, it implies that $d < 17$. So d will be either 3 or 10.

Now $3d + e = 50$

Also, $d = 3$ or 10

So, $e = 41$ or 20

But it is given that e is a multiple of 5, so $e = 20$

Now $\frac{20 + c}{20 + a} = \frac{2}{1}$, $\frac{20 + c}{20 + b} = \frac{4}{1}$ and $\frac{20 + a}{20 + b} = \frac{2}{1}$

Solving the above expression we get the following equations:

$c - 2a = 20$ .... (i)

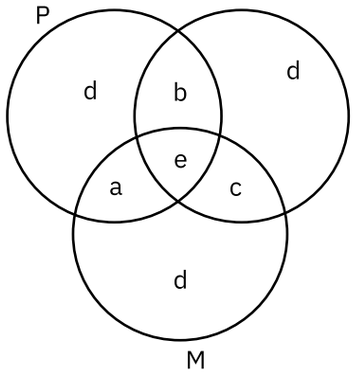
$c - 4b = 60$ .... (ii)

$a - 2b = 20$ .....(iii)

Adding (i), (ii) and (iii) we get

-a - 6b + 2c = 100 ..... (iv)
Also a + b + c = 150 .... (v)
Now adding (iv) and (v) we get
-5b + 3c = 250 ....(vi)
Solving (ii) and (vi) we get
b = 10 and c = 100
$\therefore$ a = 150 - 10 - 100 = 40
Now, the number of candidates who scored 90 percentile overall and above who score 80 percentile and above in P and M is a + e = 40 + 20 = 60
The required answer is 60.

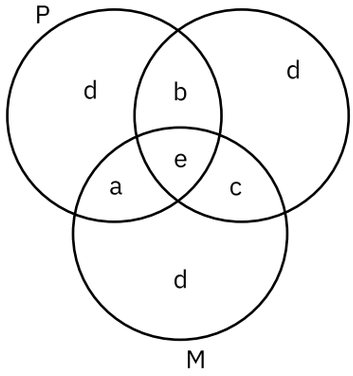
15. Using the information given in the question let us represent it in the Venn diagram shown below. The diagram depicts the number of candidates getting 80 percentile and above in at least one or more of the subjects amongst students getting 90 percentile overall.



The number of candidates scoring 80 percentile and above in exactly each of Physics, chemistry and Math is the same. Let this be 'd'
Let 'a' – number of candidates scoring 80 percentile and above only in Physics and Math.
Let 'b' – number of candidates scoring 80 percentile above only in Physics and Chemistry.
Let 'c' – number of candidates scoring 80 percentile and above in Chemistry and Math.
Let 'e' – number of candidates scoring 80 percentile and above in all 3 subjects.
a + b + c = 150
Also a + b + c + 3d + e = 200
$\Rightarrow$ 3d + e = 50
Given that (2d + c) : (2d + a) : (2d + b) = 4 : 2 : 1
This implies 6d + a + b + c is a multiple of 7. We already know that a + b + c = 150. So 6d + 150 is a multiple of 7. This implies that 6d + 3 will also be a multiple of 7. So d will be 3, 10, 17. But as 3d + e = 50, it implies that d < 17. So d will be either 3 or 10.
Since the number of candidates who are at 90 percentile and above and also at or 80th percentile in all 3 sections is a multiple of 5 (which in the Venn Diagram is 'e'), the value of e(as explained in the previous question) will have to be 20. Now the number of candidates who were shortlisted for the AIE test will be those who get 90 percentile overall and number of candidates who get 80 percentile in atleast 2 out of P, C and M which in the Venn Diagram will be the sum of a, b, c and e.

We already know a + b + c = 150 and e = 20
$\therefore$ a + b + c + e = 150 + 20 = 70
The required answer is 170.

16. Using the information given in the question let us represent it in the Venn diagram shown below. The diagram depicts the number of candidates getting 80 percentile and above in at least one or more of the subjects amongst students getting 90 percentile overall.



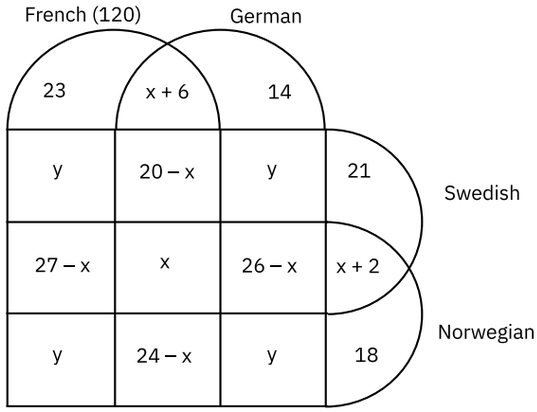
The number of candidates scoring 80 percentile and above in exactly each of Physics, chemistry and Math is the same. Let this be 'd'
Let 'a' – number of candidates scoring 80 percentile and above only in Physics and Math.
Let 'b' – number of candidates scoring 80 percentile above only in Physics and Chemistry.
Let 'c' – number of candidates scoring 80 percentile and above in Chemistry and Math.
Let 'e' – number of candidates scoring 80 percentile and above in all 3 subjects.
a + b + c = 150
Also a + b + c + 3d + e = 200
$\Rightarrow$ 3d + e = 50
Given that (2d + c) : (2d + a) : (2d + b) = 4 : 2 : 1
This implies 6d + a + b + c is a multiple of 7. We already know that a + b + c = 150. So 6d + 150 is a multiple of 7. This implies that 6d + 3 will also be a multiple of 7. So d will be 3, 10, 17. But as 3d + e = 50, it implies that d < 17. So d will be either 3 or 10.
The number of candidates who are at or above 90th percentile overall and also at or above 80th percentile in P (as indicated in the Venn Diagram) = a + b + d + e. As indicated in the answers to the previous question a = 3 or 10. Now we have already seen that fi d = 10, a = 40, e = 20 and b = 10
So, then a + b + d + e = 10 + 40 + 20 + 10 = 80
This does not satisfy the condition given in the question.
So, then d = 3
If d = 3, 9 + e = 50 e = 41
Also, $\frac{6+c}{6+a} = \frac{2}{1}$, $\frac{6+c}{6+b} = \frac{4}{1}$ and $\frac{6+a}{6+b} = \frac{2}{1}$
Solving the above expressions we get
c - 2a = 6 .... (i)
c = 4b = 18 ..... (ii)

a - 2b = 6 .... (iii)
Adding (i), (ii) and (iii) we get
-a - 6b + 2c = 30 .... (iv)
We alrady know a + b + c = 150 .... (v)
Adding (iv) and (v) we get -5b + 3c = 180 ... (vi)
Solving (ii) and (vi) we get
b = 18 and c = 90
Now a = 150 - 18 - 90 = 42
So a + b + d + e = 42 + 18 + 3 + 41 = 104, which satisfies the condition given in the question.
Now the number of candidates who appear separately for the BIE test will be those who got 90 percentile and above overall and got 80 percentile and above only in P plus there who got 80 percentile only in P but got less than 90th percentile overall.
Candidates who got 80th percentile only in P and got less than 90 in percentile overall = 400 - 104 = 296
Number of candidates who get 80 percentile and above only in P and got 90 in percentile and above overall = d = 3
So, the number of candidates who sit for the separate test for BIE = 296 + 3 = 299.
Hence, (1).

PRACTICE EXERCISE-2

Answers to questions 1 to 4:

Using the information given, we can generate the following



Now all the questions can be answered.

1. Total number of students who study French = 120.

   $\therefore$ 2y – x = 20 or 2y = x + 20

   Therefore, the minimum and maximum values that x can take are 0 and 20.

   The number of students who study French and Norwegian (but no other language) is y. Therefore, the required answer is 0.5 × (20 + 20) = 20. Hence, (1).

2. From the figure, the number of students who study only Norwegian and Swedish (but no other language) = x + 2

   From the previous answer, the maximum value of x is 20.

   Therefore, the required answer is 20 + 2 = 22. Hence, (2).

3. From the figure, the number of students who study only German and Norwegian (but no other language) = y = 14 (given).

   From the figure, the number of students who study French, Swedish and Norwegian but not German = 27 – x = 47 – 2y = 47 – 28 = 19. Hence, (3).

4. The total number of students in the academy can be calculated by adding all the values from the figure, which comes out to be 201 + 2y.

   The maximum value of y is 20. Therefore, the required answer is 201 + 40 = 241. Hence, (4).

Answers to questions 5 to 8:

Let the number of students specializing in only one, only two and all 3 disciplines be '**a**', '**b**' & '**c**' respectively.
Therefore, from condition (1),(2) and (3), the number of students specializing in 2016-17 in only one, only two and all three disciplines is '**a – 20'**, '**b + 5'** and '**2c**' respectively.

For 2015-16, a + b + c = 200 .....(i)

& for 2016-17 a – 20 + b + 5 + 2c = 200.....(ii)

From equations (i) & (ii) we get,

c = 15 & a + b = 185 .....(iii)

Now from condition (4) we have,

(a – 20) – (b + 5) = 30 .....(iv)

Solving (iii) & (iv), a = 120; b = 65.

From condition (5) & (6), the number of students specializing in Finance in 2015-16 is 115, and that in Marketing is 80. Let the number of students specializing in Operations be 'z',

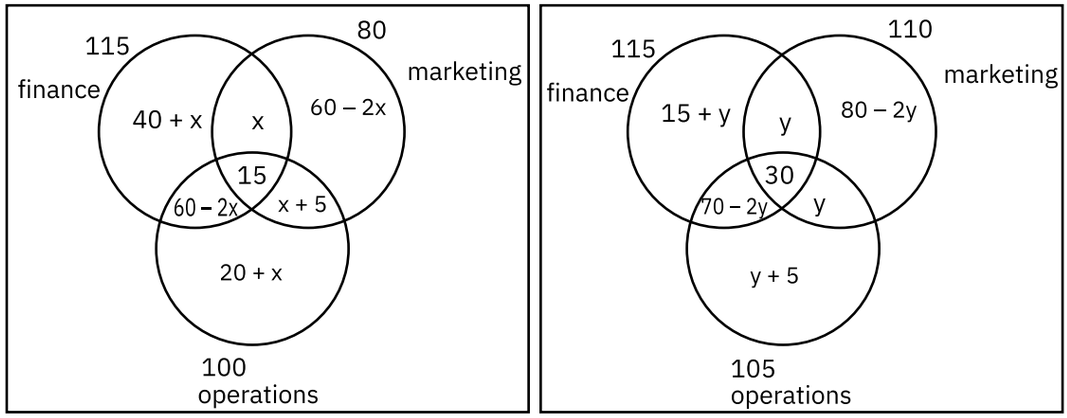
Therefore, 115 + 80 + z = a(1) + b(2) + c(3)

Therefore, z = 100, i.e the number of students specializing in Operations in 2015-16 is 100.

Similarly, the number of students specializing in Operations in 2016-17 is 105.

From condition (7), if the number of students specializing in only Finance and Marketing in 2015-16 is 'x', then the number of students specializing in only Marketing and Operations is 'x + 5'.

From condition (8), if the number of students specializing in only Finance and Marketing in 2016-17 is 'y', then the number of students specializing in only Marketing and Operations is 'y'. Rest is shown in the Venn diagram.



From this, we can solve all the questions.

5. 40 + x is definitely greater than 20 + x.
For 40 + x > 60 – 2x, x > 6.6 and maximum x can be 30.
Therefore, 11.66 < B**oth Marketing & Operations (but not Finance)** $\leq$ 35.
The only value among the options that satisfies this condition is 12. Hence, (2).

6. Hence, (3).

7. Hence, (2).

8. Hence, (1).

Answers to questions 9 to 12:

9. As every student has to choose at least one subject, and the students who choose Electronics or Comp Science have to choose Mathematics, students who choose only one subject are the ones who choose Mathematics.
Furthermore, the number of students who choose Electronics + number of students who choose Computer Science have to be a multiple of 9. (9, 18, 27, 36, 45, 54....). For number of students who choose one subject to be minimum, the addition has to be maximum. At the same time, it should be less than half of students choosing only Mathematics.
Hence, number of students who choose Electronics + Number of students who choose Computer Science = 45.
Hence minimum value of students who choose only one subject = 150 – 45 = 105.
Hence (3).

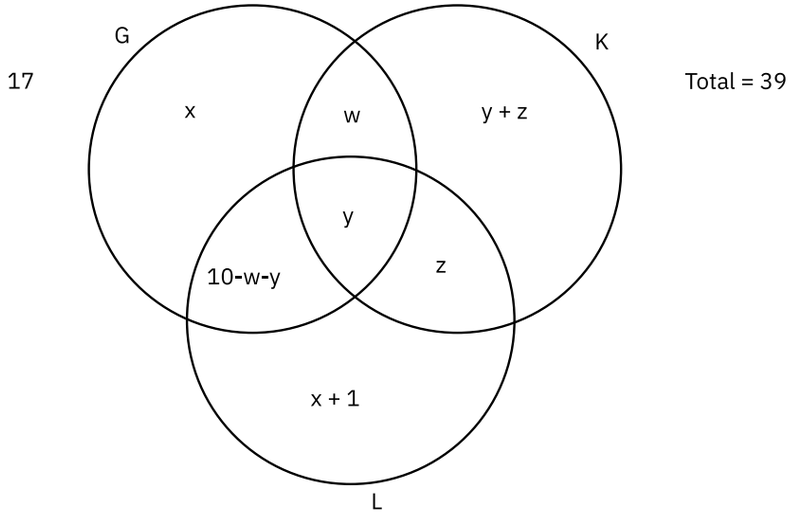
10. We have, 150 = 9 × 16 + 6. Also the number of students who choose Electronics and Computer Science is a multiple of 9, the number of students choosing only one subject (Mathematics) will always be in the form of (9k + 6); where k is an integer. Since it is also a multiple of 13, the only possible value is 78.
The sum of Computer Science & Electronics students will hence be 150 – 78 = 72.
Since the Electronics students can't be zero, their minimum value has to be 8 for it to be a multiple of 8. The maximum value of Electronics students will be 72.
Hence, no of students choosing Comp Science = 0 (minimum) and 64 (maximum).
Hence (4).

11. Total students who opt for Computer Science is 72 – 24 =48.
Percentage of students who opt for Computer Science is hence $\frac{48}{150} \times 100 = 32\%$
Hence (2).

12. In this case, the total number of students will be 240. Of these, the number of students who opt only for Mathematics is either 78 or 195. Consequently, the number of students who choose Electronics and Computer Science is 162 or 45. For the first case, the minimum and maximum values will be 2 and 154, whereas for the second case, the minimum and maximum values will be 5 and 37. Hence (3).

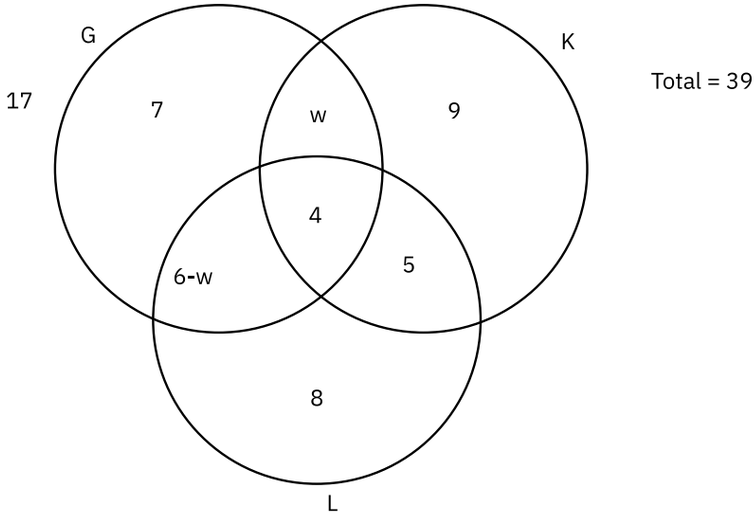
Answers to questions 13 to 16:

Using the information given in the set, we can construct the following venn diagram:



Since 10 students who play G enrolled in at least one other sport, the number of students who enrolled only in G = 17 – 10 = 7. Therefore x = 7. Also from statement 1, the number of students who enrolled in all the three sports = $\frac{7+1}{2}$ = 4.

As 17 students enrolled in G, the number of students who did not enrol in G = 39 – 17 = 22. Therefore, x + 1 + z + y + z = 22. We know that x = 7 and y = 4. Therefore we have the following: 8 + 4 + 2z = 22 or z = 5.



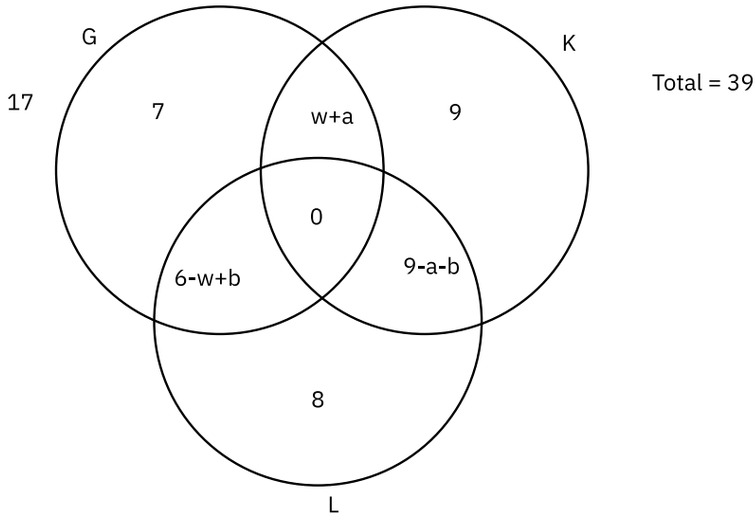
From statement 5, 6 – w > w or w = 0 or 1 or 2.

Now first two questions can be answered.

13. The required answer is 6 – 2 = 4.

14. If the ratio of the numbers of students enrolled in K and L are in the ratio 19:22, 18 + w23 – w = 1922. Therefore w = 1. Therefore total enrollment in L = 23 – 1 = 22. Hence, (2).

Additional information to questions 15 and 16
Out of 4 students who are enrolled in all the three, suppose 'a' students dropped out of L and 'b' students dropped out of K. Therefore the number of students who dropped out of G = 4 - a - b. Therefore we have the following:



If the number of students enrolled in K reduced by 1 that means out of the 4 students who had enrolled in all the three, one student dropped out of K i.e. b = 1.
Now, if the number of students enrolled in G was 6 less than the number of students enrolled in L, we have the following:
(7 + w + a + 6 – w + b + 6 = 6 – w + b + 9 – a – b + 8)
$\therefore$ 19 + a + b = 23 – w – a
$\therefore$ 2a + b + w = 4
Since b = 1, the only solution for the equation 2a + b + w = 4 is a = 1, b = 1 and w = 1.
Now both the questions can be answered.

15. The required number of students = w + a = 2. Therefore the required answer is 2.

16. The required number of students = 6 – w + b = 6 – 1 + 1 = 6. Hence (2).

DI-4.2 | DATA INTERPRETATION MISC SETS-II

PRACTICE EXERCISE-1

Answers to questions 1 to 4:

Suppose the number of employees in R&D, Marketing, Finance and Engineering are 100a, 100b, 100c and 100d respectively.
From table 1,
Number of employees in R & D who got A grade = 33.33a ...(I)
Number of employees in Finance who got A grade = 40c ...(II)
Number of employees in Engineering who got A grade = 40d ...(III)
From table 2, (I) = (II)=(III)

$\therefore \frac{100}{3}a = 40c$

$\therefore a = 1.2c$
Also, $40c = 40d$
$\therefore c = d$
From table 1,
Number of employees in Marketing who got B grade = 15b ...(IV)
Number of employees in Engineering who got B grade = 6d ...(V)
From table 2, (IV) = 2 (III)
$\therefore 15b = 12d$
$\therefore b = 0.8d$
Therefore the number of employees in R&D, Marketing, Finance and Engineering are in the ratio 12:8:10:10. Suppose the number of employees in R&D, Marketing, Finance and Engineering are 120x, 80x, 100x and 100x respectively. Therefore total number of employees working for the company is 400x.

1. Number of employees who got 'A' grade = 120x × 33.33% + 80x × 10% + 100x × 40% + 100x × 40% = 128x

   $\therefore$ Required percentage = $\frac{128x}{400x} \times 100 = 32\%$. Hence, (2).

2. Total number of employees in Finance department = 100x
   Total number of employees in departments other than Finance who got 'C' grade = 120x × 20% + 80x × 45% + 100x × 20% = 80x

   $\therefore$ Required ratio = $\frac{100x}{80x} = 4.5x$
   Hence, (3).

3. 1. Number of employees in R&D department who got 'D' grade = $120x \times 31\frac{2}{3}\% = 38x$
   2. Number of employees in Marketing department who got 'C' grade = 80x × 45% = 36x
   3. Number of employees in Engineering department who got 'D' grade = 100x × 34% = 34x
   4. Number of employees in Finance department who got 'A' grade = 100x × 40% = 40x
   Hence, (4).

4. Total number of employees who got 'C' grade = 120x × 20% + 80x × 45% + 100x × 18% + 100x × 20% = 98x

Consider option (1)

If 45 employees in Finance department got 'C' grade, 18x = 45 or x = 2.5. If x = 2.5, number of employees in R&D department = 2.5 × 120 = 300. Similarly number of employees in Marketing department = 2.5 × 80 = 200, number of employees in Finance = 2.5 × 100 = 250 and the number of employees in Engineering = 2.5 × 100 = 250.

If we substitute these numbers, we get the following Table (1)—

	R&D	Marketing	Finance	Engineering
A	100	20	100	100
B	45	30	70	15
C	60	90	45	50
D	95	60	35	85

All the numbers are natural numbers and therefore this option is possible.

It can be seen that we get the same table if we consider options (2) and (3).

Now consider option (4). If 75 employees in Engineering department got 'C' grade, every number in the table mentioned above will have to be multiplied by 1.5. In that case, some numbers in the table will not remain natural numbers anymore. Therefore option (4) cannot be correct.

Hence, (4).

Answers to questions 5 to 8:

Using the information on the number of days required to complete the different modules and the prerequisites for various modules, we can generate the following table:

Module	Cycle 1	Cycle 2	Cycle 3	Cycle 4
A	Days 1-3	Days 37-39	Days 74-76	Days 110-112
B	Days 4-8	Days 40-44	Days 77-81	Days 113-117
C	Days 4-7	Days 40-43	Days 77-80	Days 113-116
D	Days 9-14	Days 45-50	Days 82-87	Days 118-123
E	Days 15-18	Days 51-54	Days 88-91	Days 124-127
F	Days 19-25	Days 55-61	Days 92-98	Days 128-134
G	Days 19-28	Days 55-64	Days 92-101	Days 128-137
H	Days 29-36	Days 65-72	Days 102-109	Days 138-145
I	Days 29-31	Days 65-67	Days 102-104	Days 138-140
J	Days 29-33	Days 65-69	Days 102-106	Days 138-142

The institute will be closed on days 73 and 146.
In the first cycle, we have the following timeline:

Days	1-3	4-7	8	9-14	15-18	19-25	26-28	29-31	32-33	34-36
Modules	A	B, C	B	D	E	F, G	G	H, I, J	H, J	H
Number of modules	1	2	1	1	1	2	1	3	2	1

5. On 20 days, only one module is offered (1-3, 8, 9-14, 15-18, 26-28, 34-36). On 13 days, two modules are offered (4-7, 19-25, 32-33). On 3 days, three modules are offered (29-31).

Therefore the number of module days = 20 × 1 + 13 × 2 + 3 × 3 = 55. Hence, (2).

6. The institute will be closed on day 73 and 146. In 2016 (a leap year), number of days till 30th April = 31 + 29 + 31 + 30 = 121. Therefore 146th day will be on 25th May. Hence, (1).

7. From the table, it can be seen that Ajay will have to choose two modules out of H, I and J which are being offered simultaneously. Ajay can finish the course earliest if he takes all modules except module I in the first cycle and takes the remaining module I in the second cycle. He will be able to finish module I on day 67.

   In 2016 (a leap year), the number of days up-to 29th February is: 31 + 29 = 60. Therefore he will be able to finish the course on 7th March, which is 67th day of 2016. Hence, (2).

8. 10th March is 70th day of the year 2016.

   Modules B and C are being offered simultaneously. Shweta can take only one of them in the first cycle. Therefore she can take only A and B or A, B and D in the first cycle. Similarly modules F and G are being offered simultaneously. She can take only one of them. Therefore we have the following:

If she takes only A and B in the first cycle			If she takes A, B and D in the first cycle		
Module	Days	Cycle	Module	Days	Cycle
A	1 to 3	1	A	1 to 3	1
B	4 to 8	1	B	4 to 8	1
C	40 to 43	2	D	9 to 14	1
D	45 to 50	2	C	40 to 43	2
E	51 to 54	2	E	51 to 54	2

   Thus upto 54th day, she would have completed 5 modules – A, B, C, D and E. Modules F and G are being offered simultaneously. She can take only one of them.

   If she takes module F, that will be completed between 55th and 61st day. She will be unable to take any more module in the second cycle. So the maximum number of modules she can take is 6.

   If she takes module G, that will be completed between 55th and 64th day. After completing module G, she can take module H or J. Module H will not be completed by 70th day. However module J can be completed between 65th and 69th day. Therefore the maximum number of modules that can be completed by 70th day is 7. Hence, (2).

Answers to questions 9 to 12:

We have the profit made on the different products in the year 2014 as follows:

Product	A	B	C	D
Profit	20	30	40	50

Since the profit on all products follows a peculiar pattern, we get the profit on the different products over the four years as follows:

	A	B	C	D	Average
2012	20.8333	31.25	41.6667	52.0833	36.4583
2013	25	37.5	50	62.5	43.75
2014	20	30	40	50	35
2015	24	36	48	60	42

The cost price of A and D over the years is as given below:

	A	D	Average	Sum of CPs of all products	Sum of CPs of B and C
2012	40	50	45	180	90
2013	70	80	60	240	90
2014	40	40	52.5	210	130
2015	50	40	50	200	110

Selling Price = Cost Price + Profit. Therefore, the Selling Prices over the years are as follows:

	A	D	Average	Sum of SPs of all products	Sum of SPs of B and C
2012	60.8333	102.8333	81.4583	325.8333	162.1667
2013	95	142.5	103.75	415	177.5
2014	60	90	87.5	350	200
2015	74	100	92	368	194

Now all the questions can be answered.

9. Average sales of product D = $\frac{102.8333 + 142.5 + 90 + 100}{4}$ = 108.83 Lakhs. Hence, (3).

10. Required percent change = $\frac{74 - 60}{60} \times 100$ = 23.33%. Hence, (2).

11. Required percent change = $\frac{110 - 90}{90} \times 100$ = 22.22%. Hence, (4).

12. The ratios of total sales of D to total sales of A over the years:

2012: $\frac{102.0833}{60.8333}$, which is clearly greater than 1.6

2013: $\frac{142.5}{95}$ = 1.5

2014: $\frac{90}{60}$ = 1.5

2015: $\frac{100}{74}$, which is slightly greater than 1.33. Hence (1).

Answers to questions 13 to 16:

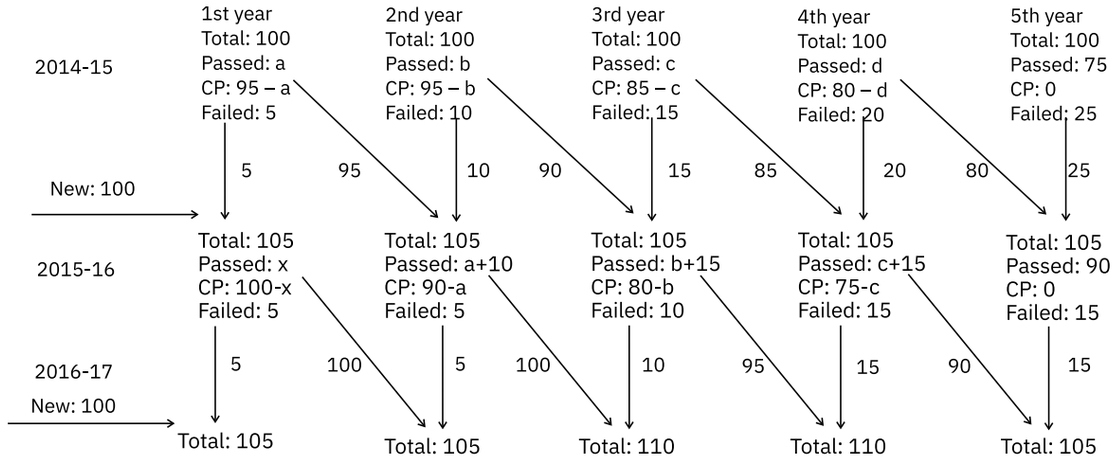
Consider 1st year students in 2015-16:
Total number of students = 105, out of which new admissions = 100.
Therefore 5 students in the 1st year in 2015-16 were the ones who failed in the 1st year in 2014-15 and repeated the 1st year in 2015-16. .....(I)

Consider 1st year students in 2014-15:
Total number of students = 100. From statement (I), 5 students failed.
Therefore, total 95 students either unconditionally passed or were conditionally promoted with 1 or 2 backlogs.
Suppose the number of students who unconditionally passed = a
$\therefore$ The number of students who were conditionally promoted with 1 ot 2 backlogs = 95 – a

Consider 2nd year students in 2015-16:
Total number of students = 105. Out of these, 95 were promoted from 1st year in 2014-15. Therefore 10 students in the 2nd year in 2015-16 were the ones who failed in the 2nd year in 2014-15 and repeated the 2nd year in 2015-16 .... (II)

Consider 2nd year students in 2014-15:
Total number of students = 100. From statement (II). 10 students failed.
Therefore total 90 students either unconditonally passed or were conditionally promoted with 1 ot 2 backllogs.
Suppose the number of students who unconditionally passed = b
∴ The number of students who were conditionally promoted with 1 or 2 backlogs = 90 – b

From statement (4), the number of students who got up-to 2 backlogs in the 2nd year in 2015-16 = (95 – a) – 5 = 90 – a. This way, we can complete the following using the information given (Note that a student can either pass or fail in the 5th year and he/she cannot be conditionally promoted to the next year in the 5th year).



Now all the questions can be answered.
CP stands for conditionally promoted.

13. Therefore, the required answer is 75. Hence, (2).

14. The total number of students who failed in the 5 years combined in 2015-16 = 5 + 5 + 10 + 15 + 15 = 50. Hence, (3).

15. It can be seen that in 2015-16, the number of students who were conditionally promoted from the 4th year to the 5th year with upto 2 backlogs = 75 – c and the number of students who were conditionally promoted from the 3rd year to the 4th year with upto 2 backlogs in 2014-15 = 85 – c. Therefore, the maximum value of c is 75. The number of students who passed the 3rd year exam without any backlog in 2014-15 = c. Therefore, the maximum number of students who passed the 3rd year exam in 2014-15 can be 75. Hence, (1).

16. The number of students who passed the 3rd year examination in 2015-16 without any backlog = b + 15. The number of students who got 1 or 2 backlogs in the 3rd year (and were conditionally promoted to the 4th year) in 2015-16 = 80 – b. $\therefore \frac{b+15}{80-b} = 18$

$\therefore$ b + 15 = 1440 – 18b $\therefore$ 19b = 1425 $\therefore$ b = 75
The number of students who passed the 2nd year examination in 2014-15 = b.
Therefore, the required answer is 75. Hence, (2).

PRACTICE EXERCISE-2

Answers to questions 1 to 4:

Using the information given, we can generate the following table:

	Male	Female	Total
Beijing	20.13% of 154 = 31	23.76% of 101 = 24	31 + 24 = 55
London	38.96% of 154 =60	22.77% of 101 = 23	60 + 23 = 83
Rio	154 – 31 – 60 = 63	101 – 24 – 23 = 54	63 + 54 = 117
Total	60.39% of 255 = 154	39.61% of 255 = 101	255

Now all the questions can be answered.

1. The number of male players who participated in Athletics in London Olympics = 13.33% of 60 = 8
The number of female players who participated in Athletics in Rio Olympics = 31.48% of 54 = 17
Therefore, the required difference is 17 – 8 = 9.
Therefore, the required answer is 9. Hence, (3).

2. We have,

	Beijing Olympics		London Olympics		Rio Olympics	
Event	Male	Female	Male	Female	Male	Female
Athletics	9.68%	54.17%	13.33%	26.09%	26.98%	31.48%
Shooting	22.58%	8.33%	11.67%	17.39%	14.29%	5.56%
Boxing	16.13%	0.00%	11.67%	4.35%	4.76%	0.00%
Swimming	12.90%	0.00%	1.67%	0.00%	1.59%	1.85%
Tennis	6.45%	8.33%	8.33%	8.70%	3.17%	3.70%
Archery	3.23%	12.50%	5.00%	13.04%	1.59%	5.56%
Other events	29.03%	16.67%	48.33%	30.43%	47.62%	51.85%
Number of players	“29.03% of 31 = 9”	“16.67% of 24 = 4”	“48.33% of 60 = 29”	“30.43% of 23 = 7”	“47.62% of 63 = 30”	“51.85% of 54 = 28”

Therefore, the required sum is 9 + 4 + 29 + 7+ 30 + 28 = 107.
Therefore, the required answer is 107. Hence, (4).

3. The required difference is 63 – 54 = 9.
Therefore, the required answer is 9. Hence, (3).

4. We have,

Beijing Olympics		London Olympics		Rio Olympics	
Male	Female	Male	Female	Male	Female
31	24	60	23	63	54

From visual inspection, it can be seen that among men, the number of players who participated in Athletics in Rio Olympics (26.98% of 63 = 17) is the highest. Similarly, among women, the number of players who participated in Athletics in Rio Olympics (31.48% of 54 = 17) is the highest.
Therefore, the required answer is 17 + 17 = 34. Hence, (2).

Answers to questions 5 to 8:

5. Students selecting 1 elective could be (maximum) 8. Students selecting 2 electives could be (maximum) $^8C_2$ = 28. So overall there could be a maximum of 36 students in the batch. Therefore, the required answer is 36.

6. There could be 6 students who have chosen C and an elective other than G, and one who has selected only C. Similarly, there could be 6 students who have chosen G and an elective other than C, and one who has selected only G. Hence there could be at most 14 people who would attend classes with Joffrey.
Therefore, the required answer is 14.
Alternatively, there could be a maximum of 36 students. People choosing 2 electives other than C or G would be $^6C_2$ = 15. People choosing only one elective other than C or G would be 6. Hence 36 – 15 (2 electives) – 6 (1 elective) – 1 (Joffrey himself) would give us 14 people.

7. There could be 7 other students who have chosen only one elective other than F. Also the number of students who selected 2 out of the other 7 electives would be at most $^7C_2$ = 21. So overall, there could be at most 21 + 7 = 28 students who do not attend any lecture with Gregor. Therefore, the required answer is 28.
Alternatively, out of at most 36 students there would be 7 who are attending F along with some other elective. So 36 – 7 – 1 (Gregor himself) = 28 others would not attend any lecture with Gregor.

8. Suppose Petyr has selected A and D, while Varys has selected B and D. There would be 5 others who have selected D along with one of C, E, F, G or H. There would be 1 who has selected D alone. And there would be 1 who has selected A and B. Thus both Petyr and Varys would encounter these 7 people.
Therefore, the required answer is 7.

Answers to questions 9 to 13:

C got 3 different perfect square scores in 3 different subjects. These scores must be 49, 64 and 81. His marks in Mathematics and Statistics were equal. Therefore, his marks in Mathematics and Statistics were $\frac{318 - 49 - 64 - 81}{2}$ = 62. Also, C's highest score was in Physics. Therefore, C scored 81 in Physics and 49/64 in Chemistry and Biology.

From statement (5), D scored 97 in Biology. Therefore, so far we have

	Physics	Chemistry	Mathematics	Biology	Statistics	Total
A		87	72		58	340
B		64		96		354
C	81	64/49	62	49/64	62	318
D				97		402
E	67	44				388
Total	338	332	383	398	351	

Therefore, D's marks in Chemistry could have been 332 – (87 + 64 + 64 + 44) = 73 or 332 – (87 + 64 + 49 + 44) = 88. But D's score in Chemistry was a prime number. Therefore, D scored 73 and C scored 64 in Chemistry. From statement (3), D's score in Physics = 71, in Mathematics = 82 and in Statistics = 79.
Therefore so far we have,

	Physics	Chemistry	Mathematics	Biology	Statistics	Total
A		87	72		58	340
B		64		96		354
C	81	64	62	49	62	318
D	71	73	82	97	79	402
E	67	44				388
Total	338	332	383	398	351	

From statement (7), B scored equal marks in Mathematics and Statistics. Suppose that is equal to x. Then we can fill the table as follows

	Physics	Chemistry	Mathematics	Biology	Statistics	Total
A	2x – 75	87	72	198 – 2x	58	340
B	194 – 2x	64	x	96	x	354
C	81	64	62	49	62	318
D	71	73	82	97	79	402
E	67	44	167 – x	2x – 42	152 – x	388
Total	338	332	383	398	351	

The marks scored by students are between 44 and 99 (both inclusive).

Consider student A:
$44 \le 2x - 75 \le 99$ or $59.5 \le x \le 88$ or $60 \le x \le 88$
$44 \le 198 - 2x \le 99$ or $49.5 \le x \le 77$ or $50 \le x \le 77$
Consider student B:
$44 \le 194 - 2x \le 99$ or $47.5 \le x \le 75$ or $48 \le x \le 75$
Consider student E:
$44 \le 167 - x \le 99$ or $68 \le x \le 123$
$44 \le 2x - 42 \le 99$ or $43 \le x \le 70.5$ or $43 \le x \le 70$
$44 \le 152 - x \le 99$ or $53 \le x \le 108$
Combining all these statements, we get $68 \le x \le 70$
We need at least one student who scored 99 in at least one subject. This is possible if x = 68, which will ensure that E scored 99 in Mathematics. Therefore, we have

	Physics	Chemistry	Mathematics	Biology	Statistics	Total
A	61	87	72	62	58	340
B	58	64	68	96	68	354
C	81	64	62	49	62	318
D	71	73	82	97	79	402
E	67	44	99	94	84	388
Total	338	332	383	398	351	

Now all the questions can be answered.

9. Hence, (2).

10. Hence, (1).

11. Hence, (1).

12. Hence, (3).

13. Hence, (2).

Answers to questions 14 to 17:

Using the information given in the table, we can generate the following:

	SY		FY			
	Only Physics	Only Chemistry	Both Physics & Chemistry	Only Physics	Only Chemistry	Both Physics & Chemistry
2012	60	40	20	50	60	10
2013	70	50	0	30	70	20
2014	60	40	30	100	60	30
2015	70	40	10	70	30	40

Now all the questions can be answered.

14. The number of students who got a first class in at least one subject in SY in 2014 = 60 + 40 + 30 = 130. Therefore the number of students who did not get a first class in SY in 2014 = 180 – 130 = 50.

    Therefore the required answer is 50.

15. Therefore the required answer is 40.

16. From the table it can be seen that the number of students who got a first class in FY and SY in different years is as follows:

    2012: 60 + 40 + 20 + 50 + 60+ 10 = 240

    2013: 70 + 50 + 0+ 30+ 70+ 20 = 240

    2014: 60 + 40+ 30+ 100+ 60+ 30 = 320

    2015: 70 + 40+ 10+ 70+ 30+ 40 = 260

    Therefore the required answer is 2014.

17. From the explanatory answer to the previous question, the required sum = 240 + 240 + 320 + 260 = 1060

    Therefore the required answer is 1060.

LR-4.1 | LOGICAL REASONING MISC SETS

PRACTICE EXERCISE-1

Answers to questions 1 to 4:

Using the table given in the question, we can develop the following table.

	Hometown					Destination				
	Abhijeet	Ajit	Arijit	Ashutosh	Avijit	Abhijeet	Ajit	Arijit	Ashutosh	Avijit
Agra			No			No				
Ahmedabad				No						No
Ahmednagar	No						No			
Allahabad					No				No	
Amritsar		No						No		

Using conditions (1) to (5), we can generate the following:

	Hometown					Destination				
	Abhijeet	Ajit	Arijit	Ashutosh	Avijit	Abhijeet	Ajit	Arijit	Ashutosh	Avijit
Agra			No		No	No	No			
Ahmedabad				No						No
Ahmednagar	No		No				No		No	
Allahabad	No				No	No			No	
Amritsar		No						No		

Using conditions (6) and (7), we can generate the following:

	Hometown					Destination				
	Abhijeet	Ajit	Arijit	Ashutosh	Avijit	Abhijeet	Ajit	Arijit	Ashutosh	Avijit
Agra			No		No	No	No			
Ahmedabad	No			No					No	No
Ahmednagar	No		No	No	No	No	No		No	No
Allahabad	No			No	No	No			No	No
Amritsar		No						No		

From this table, we can see that Ahmednagar is Ajit's hometown and Arjit's destination. We can further see that Allahabad is Arjit's hometown and Ajit's destination. Now, we can further fill the table as shown below:

	Hometown					Destination				
	Abhijeet	Ajit	Arijit	Ashutosh	Avijit	Abhijeet	Ajit	Arijit	Ashutosh	Avijit
Agra		No	No		No	No	No	No		
Ahmedabad	No	No	No	No	Yes	Yes	No	No	No	No
Ahmednagar	No	Yes	No	No	No	No	No	Yes	No	No
Allahabad	No	No	Yes	No	No	No	Yes	No	No	No
Amritsar		No	No		No	No	No	No		

Now all the questions can be answered.

1. Hence, (1).

2. Hence, (3).

3. Hence, (2).

4. We don't have information about Ashutosh's hometown or his destination. Therefore options (1) and (4) are incorrect.
   Using statement I, if Ashutosh is travelling to Agra, we get
   We have,

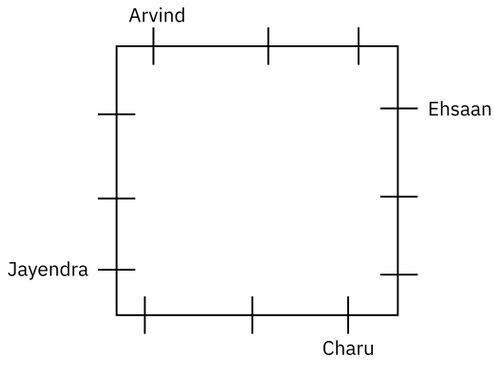
	Hometown				Destination					
	Abhijeet	Ajit	Arijit	Ashutosh	Avijit	Abhijeet	Ajit	Arijit	Ashutosh	Avijit
Agra	Yes	No	No	No	No	No	No	No	Yes	No
Ahmedabad	No	No	No	No	Yes	Yes	No	No	No	No
Ahmednagar	No	Yes	No	No	No	No	No	Yes	No	No
Allahabad	No	No	Yes	No	No	No	Yes	No	No	No
Amritsar	No	No	No	Yes	No	No	No	No	No	Yes

   Which means that statement (II) is correct.
   Therefore Statement (II) is correct only if Statement (I) is correct. Hence, (2).

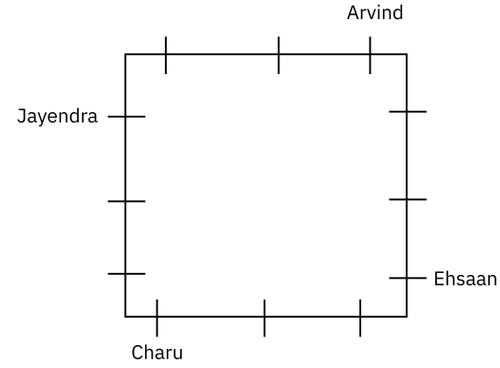
Answers to questions 5 to 8:

Using condition (1), we get the following possibilities:

Possibility 1

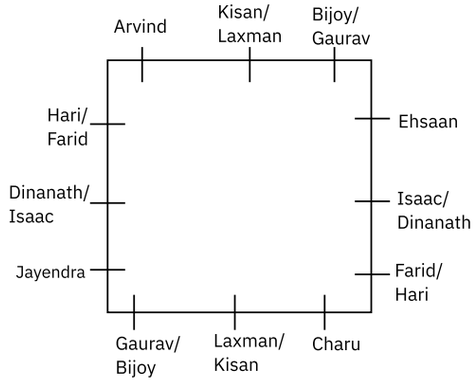


Possibility 2

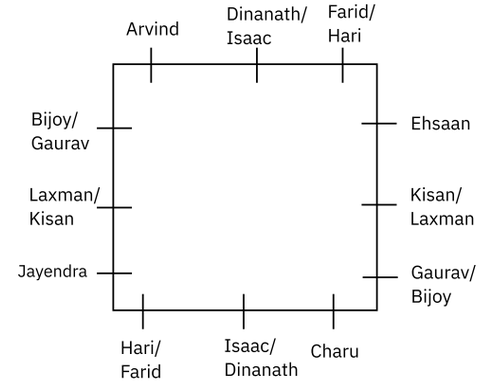


Using condition (2), we can see that exactly five persons sit between Farid and Hari in both clockwise and anti-clockwise directions. Similarly exactly five persons sit between Bijoy and Gaurav in both clockwise and anti-clockwise directions. Therefore we get the following possibilities:

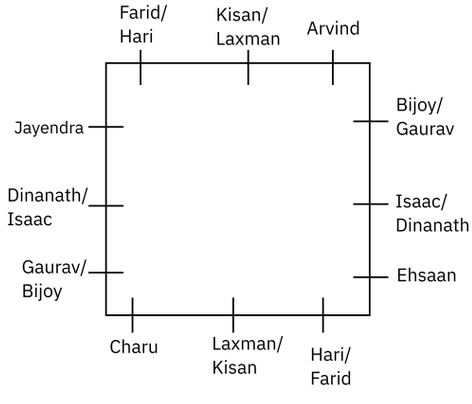
Possibility 1a



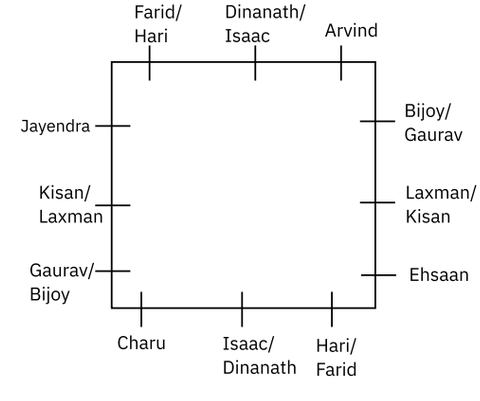
Possibility 1b



Possibility 2a



Possibility 2b



Now all the questions can be answered.

5. It can be seen that Dinanath (possibility 1b), Farid and Hari (possibility 1a) can be seated next to Arvind. Hence, (4).

6. It can be seen that Bijoy & Charu and Gaurav & Charu and Ehsaan & Hari can be seated facing each other. But Ehsaan and Dinanath cannot be seated facing each other. Hence, (3).

7. If Kisan is sitting adjacent and to the left of Jayendra (Possibility 1b), either Kisan or Laxman can sit to the right of Gaurav. Hence, (4).

8. If Charu, Laxman and Hari are sitting along the same side of the table (Possibilty 2a), then Arvind, Kisan and Farid definitely sit on the side opposite to Charu, Laxman and Hari. On the remaining two sides, Ehsaan, Isaac/Dinanath & Bijoy/Gaurav and Jayendra, Dinanath/Isaac & Gaurav/Bijoy sit. Therefore options (2) and (3) are not definitely correct. Hence, (1).

Answers to questions 9 to 12:

Based on conditions 5 and 7, we get the following:

Airline	Destination	Slot
Cathay Pacific		
Lufthansa		10
KLM	Tehran	10
Etihad		
Scandinavian		
Qantas		
Delta		

From condition 8, there will be only 1 slot with 1 flight. All other slots will have 2 flights each.
From conditions 2 & 7, Etihad will be in the same slot as Cathay Pacific and Qantas will be in the same slot as Delta or Scandinavian.
From the archives information and using conditions 4 & 8, Delta has only 1 slot available, i.e. at 1100 hrs.
Using conditions 2 and 6 we get that Scandinavian flies in the 1200 hrs slot.
Thus we get -

Airline	Destination	Slot
Cathay Pacific		9
Lufthansa		10
KLM	Tehran	10
Etihad		9
Scandinavian		12
Qantas		
Delta		11

Now Qantas may fly in the same slot as either Scandinavian or Delta. Hence, one of them will always fly to Panama City while the other will fly to Osaka.

Thus, out of archives table, destinations in rows 3 and 4 are wrong and those in rows 1 and 2 are correct.
Therefore, we get:

Airline	Destination	Slot
Cathay Pacific	Hanoi	900
Lufthansa	Skopje	1000
KLM	Tehran	1000
Etihad	Yalta	900
Scandinavian	Osaka / Panama City	1200
Qantas	Nairobi	1100 or 1200
Delta	Osaka / Panama City	1100

Now all the questions can be answered.

9. The required order and ranks are as follows:

Destination	Rank
Yalta	1
Tehran	2
Skopje	3
Panama City	4
Osaka	5
Nairobi	6
Hanoi	7

Qantas goes to Nairobi. Therefore, the required answer is 6.

10. The required answer is 6.

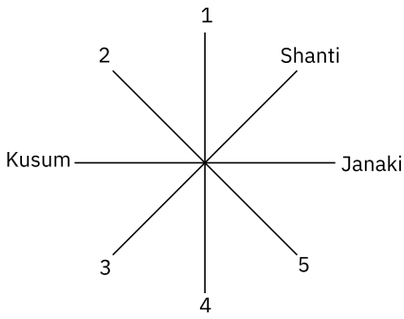
11. The flight to Hanoi can leave London the earliest at 0900 hrs. It takes (5 + 5) hours to travel to and fro, and 4 hours is the halt. Thus, it will reach after 14 hours, that is, at 2300 hrs.

12. As the flight can leave the earliest at 0900 hrs, I should reach Yalta at 1600 hrs London time. I have 3 hours transit so that makes it 1900 hrs. Thus, it will be 1900 hrs London time when I board the flight for Singapore from Yalta. Hence, local time in Yalta at the time of boarding will be 2100 hrs.

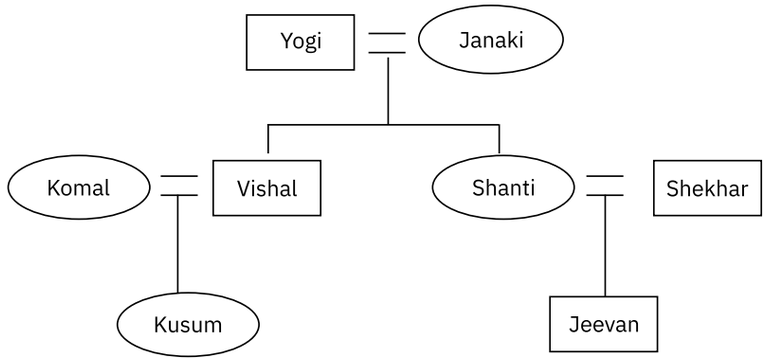
Answers to questions 13 to 16:

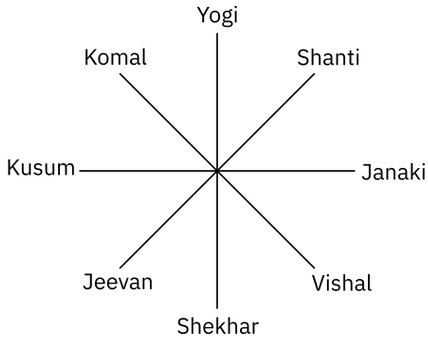
From the given data, we can conclude that Shanti and Shekhar belong to the 2nd generation and Jeevan and Kusum belong to the 3rd generation.
Therefore, from statement 2, Kusum is Shanti's niece. Therefore, Jeevan is Shanti's son.
From statement 3, since Janaki has a son, she must be from the 1st generation. Since her son-in-law is Shekhar, her husband and son are Yogi and Vishal (in no particular order).
Thus, the 2nd generation has one son and one daughter who are both married.

Now, based on the results inferred so far, the circular arrangement will be as follows:



From statement 5, the only possible positions for the grandfather are positions 1 and 2. But position 2 will contradict with statements 3 and 6. Also, the grandfather cannot be Vishal because that will contradict with statement 4. Thus, Yogi is the grandfather whose wife is Janaki. Their son is Vishal whose wife has to be Komal (from statement 8). Thus, we get the final family tree and seating arrangement as follows:





Using this, all questions can be answered.

13. Hence, (4).

14. Hence, (1).

15. Hence, (2).

16. Hence, (3).

PRACTICE EXERCISE-2

Answers to questions 1 to 4:

From conditions 1 to 7, we get the following table:

Name	Nationality	Occupation	Place of Residence
Anothny	A/M/D/T	M/E	A/D
Margeaux	A/M/D/T	M/E	A/D
Samuel	Singapore	Archeologist	M/T
Dorothy	A/M/T	S/T	M/T
Ernstmayer	A/M/D/T	S/T	Sao Paulo
Theodore	Estonia	Dermatologist	Edinburgh

(The initials are used in table for simplicity).
If we assign numbers to each entity from 1 to 4 (Name is assigned as 1 and place of residence as 4), and if we use condition 1, each person has exactly 2 entities matching, viz: 1-2, 1-3, 1-4, 2-3, 2-4 & 3-4.
Thus, we get a new table:

Name	Nationality	Occupation	Place of Residence	Possible Matching Entities
Anothny	A/M/D/T	M/E	A/D	1 & 4
Margeaux	A/M/D/T	M/E	A/D	1 & 3
Samuel	Singapore	Archeologist	M/T	1& 2
Dorothy	A/M/T	S/T	M/T	2 & 3
Ernstmayer	A/M/D	S/T	Sao Paulo	3 & 4
Theodore	Estonia	Dermatologist	Edinburgh	2 & 4

Using conditions 8, 9 & 10, we get the following table:

Name	Nationality	Occupation	Place of Residence	Matching Entities
Anothny	M/D	Engineer	Amsterdam	1 & 4
Margeaux	American	Mining Expert	Dresden	1 & 3
Samuel	Singaporean	Archeologist	Tijuana	1& 2
Dorothy	Turkish	Tarot Reader	Milwaukee	2 & 3
Ernstmayer	M/D	Sportsman	Sao Paulo	3 & 4
Theodore	Estonian	Dermatologist	Edinburgh	2 & 4

Now all the questions can be answered.

1. Hence, (4).

2. Hence, (1).

3. Hence, (3).

4. Hence, (1).

Answers to questions 5 to 8:

The time taken by Anil, Bharat, Chandru, Deepak and Eckert for finishing any assignment (in minutes) is 120, 120, 180, 360 and 180 respectively. Therefore, the number of minutes taken by any two of them working together to finish any assignment is as follows:

Combination	Number of minutes
Anil + Bharat	60
Anil + Chandru	72
Anil + Deepak	90
Anil + Eckert	72
Bharat + Chandru	72
Bharat + Deepak	90
Bharat + Eckert	72
Chandru + Deepak	120
Chandru + Eckert	90
Deepak + Eckert	120

Therefore, depending on the time requirements of different assignments, the following combinations can be assigned to work on the same:

Time requirement	Possible combinations
60	Anil + Bharat
72	Anil + Chandru; Anil + Eckert; Bharat + Chandru or Bharat + Eckert
90	Anil + Deepak; Bharat + Deepak or Chandru + Eckert
120	Chandru + Deepak or Deepak + Eckert

The following table shows the possible total time required to finish the two assignments of the day:

Total time required	Possible combination of minutes	Maximum number of days on which this combination is possible
132	60 + 72	1
150	60 + 90	1
162	72 + 90	3
180	60 + 120	1
192	72 + 120	2
210	90 + 120	2

From this table and points 2 and 3, we have the following:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Total time requirement	192	162	180	162	162
Time for the two assignments	72 + 120	72 + 90	60 + 120	72 + 90	72 + 90

Only combination [Amit + Bharat] can complete the assignment in 60 minutes. Therefore, they work on the first assignment on Wednesday.
Bharat + Chandru do not work together on any assignment on Thursday. Also, Eckert does not work on Thursday. Therefore Anil + Chandru is the only possible combination that can work on the first assignment on Thursday. Eckert does not work on Tuesday as well. Therefore Bharat + Chandru is the only possible combination that can work on the first assignment on Tuesday.
Out of four pairs that can work on the first assignment on Monday (72 minutes), we have already assigned two – Amit + Chandru and Bharat + Chandru to work on other assignments that take 72 minutes. Since Anil does not work on Monday, the only possible pair that can work on the first assignment on Monday is Bharat + Eckert. Therefore Anil + Eckert work on the first assignment on Friday.
So far we have –

Day	Times for assignments	First assignment	Second assignment
Monday	72 + 120	Bharat + Eckert	
Tuesday	72 + 90	Bharat + Chandru	
Wednesday	60 + 120	Anil + Bharat	
Thursday	72 + 90	Anil + Chandru	
Friday	72 + 90	Anil + Eckart	

Same person does not work on both assignments on Tuesday. Therefore the only possible pair that can work on the second assignment on Tuesday (90 minutes) is Anil + Deepak.
The second assignment on Thursday (90 minutes) can be completed either by Bharat + Deepak or Chandru + Eckert. Since Eckert does not work on Thursday, Bharat + Deepak work on the second assignment on Thursday. This leaves Chandru + Eckert to work on the second assignment on Friday. That means Eckert is the only employee who works on both assignments of the day and that day is Friday.
The second assignment on Monday (120 minutes) can be completed either by Chandru + Deepak or Deepak + Eckert. But Eckert does not work on Monday. Therefore, Chandru + Deepak work on the second assignment on Monday while Deepak + Eckert work on the second assignment on Wednesday.

Therefore we have –

Day	Times for assignments	First assignment	Second assignment
Monday	72 + 120	Bharat + Eckert	Chandru + Deepak
Tuesday	72 + 90	Bharat + Chandru	Anil + Deepak
Wednesday	60 + 120	Anil + Bharat	Deepak + Eckert
Thursday	72 + 90	Anil + Chandru	Bharat + Deepak
Friday	72 + 90	Anil + Eckart	Chandru + Eckert

Now all the questions can be answered.

5. Hence, (4).

6. Hence, (1).

7. Hence, (4).

8. Hence, (2).

Answers to questions 9 to 13:

From statements 1 and 7, Lewis ran the first lap for Japan, followed by Bailey then another person and finally Isner.
From statement 4, Kohler ran the 2nd lap and Maxim ran the 3rd lap.
From statement 2, Dalton, Isner and Nolan ran the 4th lap with Dalton belonging to Uzbekistan (from statement 6). Also from statement 6, Charis ran the 3rd lap for Uzbekistan.
From statement 3, Gordon and Festus ran the 3rd lap.

So far, we have the following:

	Country			Others in this lap
Lap	Japan	Uzbekistan		
1	Lewis			
2	Bailey		Kohlberg	
3		Charis	Maxim	Gordon, Festus
4	Isner	Dalton		Nolan

From statement 5, we can conclude that Oberlin, Johannes and Planck ran the first lap.
From statement 8, we can conclude that Austin and Heller ran the 2nd lap whereas Erwin ran the 4th lap.

Thus, we have the following:

	Country			Others in this lap
Lap	Japan	Uzbekistan		
1	Lewis			Oberlin, Johannes, Planck
2	Bailey		Kohlberg	Austin, Heller
3		Charis	Maxim	Gordon, Festus
4	Isner	Dalton		Nolan, Erwin

But Austin and Erwin have to be in the same team. Thus, we get the final table as follows:

	Country				Others in this lap
Lap	Japan	Uzbekistan			
1	Lewis				Oberlin, Johannes, Planck
2	Bailey	Heller	Kohlberg	Austin	
3		Charis	Maxim		Gordon, Festus
4	Isner	Dalton	Nolan	Erwin	

9. Hence, (2).

10. Hence, (4).

11. We get the table as follows:

	Country				Others in this lap
Lap	Japan	Uzbekistan	USA	Jamaica	
1	Lewis				Oberlin, Johannes, Planck
2	Bailey	Heller	Kohlberg	Austin	
3	Festus	Charis	Maxim	Gordon	
4	Isner	Dalton	Nolan	Erwin	

Thus, Festus passed the baton to Isner. Hence, (3).

12. We get the table as follows:

	Country			
Lap	Japan	Uzbekistan	USA	Jamaica
1	Lewis	Planck/	Oberlin/	Johannes
		Oberlin	Planck	
2	Bailey	Heller	Kohlberg	Austin
3	Festus	Charis	Maxim	Gordon
4	Isner	Dalton	Nolan	Erwin

Thus, Johannes passed the baton to Austin. Hence, (4).

13. We get the table as follows:

	Country			
Lap	Japan	Uzbekistan	USA	Jamaica
1	Lewis	Planck	Oberlin	Johannes
2	Bailey	Heller	Kohlberg	Austin
3	Festus	Charis	Maxim	Gordon
4	Isner	Dalton	Nolan	Erwin

Hence, (2).

Answers to questions 14 to 17:

Government Employees can stay for either 3 or 4 hours. That means the latest time when Government Employees can enter is 12 noon. Therefore, the number of Government Employees who enter at 1 PM, 2 PM and 3 PM is 0. Similarly, the earliest time when the Government Employees can leave is 11 AM. Therefore, the number of Government Employees who leave at 8 AM, 9 AM and 10 AM is 0. Similarly, the number of History Students who enter at 2 PM and 3 PM is 0 and the number of History Students who leave at 8 AM and 9 AM is 0. Similarly, the number of Journalists who enter at 3 PM is 0 and the number of Journalists who leave at 8 AM is 0.
Therefore, we have

	Entry			Exit		
	History Students	Government Employees	Journalists	History Students	Government employees	Journalists
8:00 AM			1	0	0	0
9:00 AM	4			0	0	
10:00 AM	0			3	0	0
11:00 AM		0		0		0
12 noon	1	2				8
1:00 PM		0	0	2		
2:00 PM	0	0	2	0	1	0
3:00 PM	0	0	0		2	
Total	10	8	20	10	8	20

It can now be seen that the Journalists who entered at 8 AM must have left at 9 AM (after 1 hour). Since no Journalist left at 10 AM and 11 AM, the number of Journalists who entered at 9 AM must have been 0. Similarly, the only Government Employee who left at 2 PM must have entered at 10 AM. Similarly, the 3 History Students who left at 10 AM must have entered at 8 AM. The only time when the 4 History Students who entered at 9 AM can leave is 11 AM or 12 noon. Since no History student left at 11 AM, the number of History students who left at 12 noon is 4.

2 History students left at 1 PM. They must have entered at 11 AM. Therefore, the number of History Students who entered at 1 PM = 0 and the number of History students who left at 3 PM = 1.

So far, we have one Government Employee who entered at 10 AM and left at 2 PM. Additionally, there can be a few other Government Employees who entered at 10 AM, who left at 1 PM. Suppose the number of Government Employees who entered at 10 AM = 1 + x, out of which 1 left at 2 PM and the remaining x Government Employees left at 1 PM.
∴ The number of Government Employees who entered at 8 AM and 9 AM = 8 – 2 – (1 + x) = 5 – x.
Suppose the number of Government Employees who entered at 8 AM = y
∴ The number of Government Employees who entered at 9 AM = 5 – x – y.

The Government Employees who entered at 8 AM could have left at 11 AM or 12 noon. Out of y Government Employees who entered at 8 AM, suppose z Government Employees left at 12 noon and remaining y – z Government Employees left at 11 AM. The Government Employees who entered at 9 AM could have left at 12 noon or 1 PM. Suppose w Government Employees out of 5 – x – y Government Employees who entered at 9 AM leave at 12 noon and remaining 5 – x – y – w leave at 1 PM.
Consider the Government Employees who left at 1 PM. x Government Employees who left at 1 PM had entered at 10 AM. 5 – x – y – w Government Employees who left at 1 PM had entered at 9 AM.
∴ Total number of Government Employees who left at 1 PM = x + 5 – x – y – w = 5 – y – w.

Therefore, we have

	Entry			Exit		
	History Students	Government Employees	Journalists	History Students	Governement employees	Journalists
8 AM	3	y	1	0	0	0
9 AM	4	5 – x – y	0	0	0	1
10 AM	0	1 + x		3	0	0
11 AM	2	0		0	y – z	0
12 noon	1	2		4	z + w	8
1 PM	0	0	0	2	5 – y – w	
2 PM	0	0	2	0	1	0
3 PM	0	0	0	1	2	
Total	10	8	20	10	8	20

The two Journalists who entered at 2 PM must have left at 3 PM.
Therefore, the number of Journalists who left at 1 PM = 20 – 2 – 8 – 1 = 9.
The 8 Journalists who left at 12 noon must have entered at 10 AM or 11 AM. Out of these 8 Journalists, suppose ‘a’ entered at 11 AM. Therefore, the number of Journalists who entered at 10 AM = ‘8 – a’.

The 9 Journalists who left at 1 PM must have entered at 11 AM or 12 noon. Out of these 9 Journalists, suppose 'b' entered at 11 AM. Therefore, the number of Journalists who entered at 12 noon = '9 – b'
Therefore, we have the following —

	Entry			Exit		
	History Students	Government Employees	Journalists	History Students	Governement employees	Journalists
8 AM	3	y	1	0	0	0
9 AM	4	5 – x – y	0	0	0	1
10 AM	0	1 + x		3	0	0
11 AM	2	0		0	y – z	0
12 noon	1	2		4	z + w	8
1 PM	0	0	0	2	5 – y – w	
2 PM	0	0	2	0	1	0
3 PM	0	0	0	1	2	
Total	10	8	20	10	8	20

Now all the questions can be answered.

14. The total number of visitors who entered at 8 AM or 9 AM = 3 + 4 + 1 + 4 + (5 – x – y) = 13 – x.
But we don't know the value of x.
Hence, (4).

15. We have:
a + b = 9 – b and we are looking for the value of 8 – a
a = 9 – 2b
8 – a = 8 – (9 – 2b)
8 – a = 2b – 1
Therefore this number has to be an odd number.
Hence, (2).

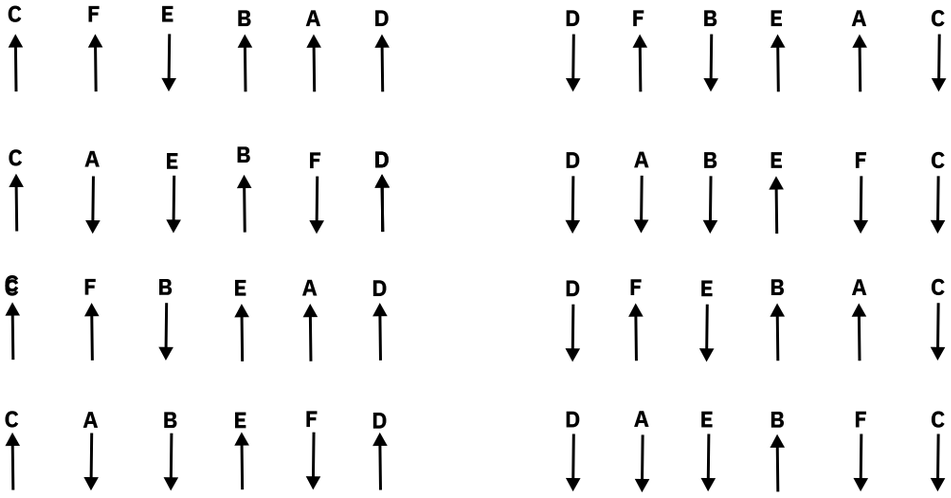
16. Hence, (3).

17. Hence, (3).

PRACTICE EXERCISE-3

Answers to questions 1 to 3:

Using the information given, we can generate the following 8 possibilities:



Now all the questions can be answered.

1. Hence, (1).

2. Hence, (3).

3. Hence, (2).

Answers to questions 4 to 8:

Out of the four women, one is supporting the government stand and three are opposing the government stand. Therefore, four men are supporting the government stand and two are opposing the government stand. From statement (6), we can identify that E and J are the two men who are opposing the government stand. From statement (4), we can see that the only woman who is supporting the government stand is the Doctor.
Therefore so far we have,

Supporting government stand				Opposing government stand			
Name	Gender	Institute	Profession	Name	Gender	Institute	Profession
	F	I-6	Doctor	E	M	I-1	
	M			J	M	I-9	
	M				F		
	M				F		
	M				F		

From statement (1), the only woman supporting the government stand can be B or C and she is the Doctor. Therefore, one of B and C is the Doctor and the other is the Social Worker. From statement (7), the two men who are supporting the government stand are Banker and Writer. Therefore, so far we have

Supporting government stand				Opposing government stand			
Name	Gender	Institute	Profession	Name	Gender	Institute	Profession
B/C	F	I-6	Doctor	E	M	I-1	
	M	I-3	Banker	J	M	I-9	
	M	I-7	Writer	C/B	F		Social Worker
	M				F		
	M				F		

From statement (2), the Marketing Professional and the Writer are taking the same sides. So are the Politician and the Bureaucrat. From statement (5), one of the Professor and the Journalist is supporting the government stand while the other is opposing the government stand. Using statements (2), (3) and (5), we have,

Supporting government stand				Opposing government stand			
Name	Gender	Institute	Profession	Name	Gender	Institute	Profession
B/C	F	I-6	Doctor	E	M	I-1	
	M	I-3	Banker	J	M	I-9	
	M	I-7	Writer	C/B	F		Social Worker
	M	I-8	Marketing Professional		F		
	M		Professor/ Journalist		F		
				Politican,Bureaucrat, Journalist/Professor			

Using statement (9), we have,

Supporting government stand				Opposing government stand			
Name	Gender	Institute	Profession	Name	Gender	Institute	Profession
B/C	F	I-6	Doctor	E	M	I-1	Politician
	M	I-3	Banker	J	M	I-9	Bureaucrat
	M	I-7	Writer	C/B	F		Social Worker
	M	I-8	Marketing Professional	H	F		Consultant
	M	I-2/I-5	Professor	F	F		Journalist
A, D, G, I				I-4 & I-10; Either I-2 or I-5			

Now all the questions can be answered.

4. Therefore, the required answer is 10.

5. Therefore, the required answer is 2.

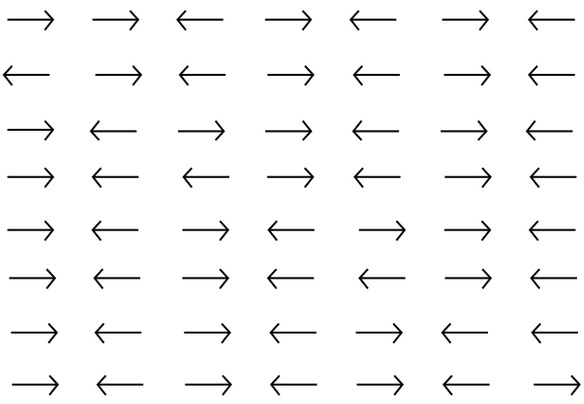
6. Therefore, the required answer is 4.

7. Therefore, the required answer is 5.

8. Therefore, the required answer is 2.

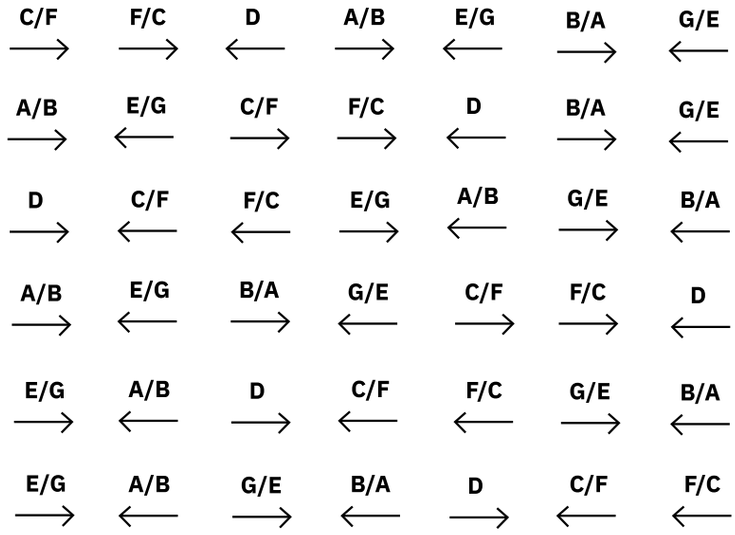
Answers to questions 9 to 12:

The following 8 arrangements are possible.

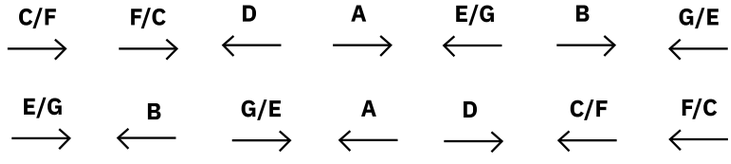


The 7th person A will be facing either the direction same as that of B, C and F or the direction same as that of D, E and G.
Consider statement 4. Among C, D and F, C and D are facing opposite directions and C and F are facing the same direction. Therefore, C/F is the person who is not facing the person next to him. The rest are facing the person sitting next to them. Thus, from the above 8 arrangements, arrangement numbers 2 and 8 can be eliminated. Also, note that A will be facing the direction

same as that of B, C and F. Thus, we get the following cases:



From statement 5, we get the following possibilities:



Using this, all the questions can be answered.

9. Hence, (2).

10. Hence, (3).

11. Hence, (1).

12. Hence, (1).

Answers to questions 13 to 16:

We have the following list of professors who can express their views on the courses offered by different departments:

Department	Professors
Finance	Awasthi, Bhalla
Marketing	Panwar, Gokhale
Operations	Chandrachud, Awasthi
Systems	Chandrachud, Subramaniam
HR	Mukherjee, Gowda
Economics	Mukherjee, Gowda, Shah, Nayar

From point 4, Prof. Panwar attends only morning session about Marketing courses while Prof. Awasthi attends morning session about Finance. Therefore Prof. Awasthi attends afternoon session about Operations. Prof. Gokhale is the only other professor who can attend the afternoon session on Marketing while Prof. Bhalla is the only other professor who can attend the afternoon session on Finance. Prof. Chandrachud can offer his views on Operations and Systems. Since Prof. Awasthi attends afternoon session on Operations, Prof. Chandrachud attends morning session on Operations and the afternoon session on Systems. Prof. Subramaniam is the only other professor who can express his views on Systems courses. Therefore Prof. Subramaniam attends morning session on Systems. Therefore so far we have

Department	Morning	Afternoon
Finance	Awasthi	Bhalla
Marketing	Panwar	Gokhale
Operations	Chandrachud	Awasthi
Systems	Subramaniam	Chandrachud
HR	Mukherjee/Gowda	Gowda/Mukherjee
Economics	Gowda/Mukherjee	Mukherjee/Gowda

Prof. Shah and Prof. Nayar can attend either morning or afternoon session on Economics.

Now all the questions can be answered.

13. It can be seen that Profs. Awasthi, Panwar, Chandrachud, Subramaniam, Mukherjee, Gowda, Shah and Nayar can attend morning session. Hence, (3).

14. Out of the four professors who can express their views on Economics courses, Prof. Shah and Prof. Nayar can attend only one session while Prof. Mukherjee and Prof. Gowda attend both sessions. However since Prof. Mukherjee and Prof. Gowda have to express their views on HR courses as well, one of them talks on Economics and the other talks on HR. Therefore we have,

Department	Morning	Afternoon
HR	Mukherjee/Gowda	Gowda/Mukherjee
Economics	Gowda/Mukherjee + Shah/Nayar	Mukherjee/Gowda+ Nayar/Shah

Thus it can be seen that all the three options – 1, 2, and 3 are not possible. Hence, (4).

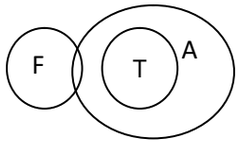
15. Hence, (3).

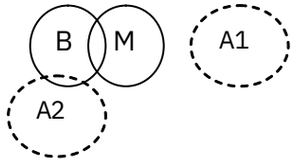
16. It can be seen that options I and II are possible but option III is not possible since Prof. Subramaniam cannot attend the meeting in the afternoon session. Hence, (3).

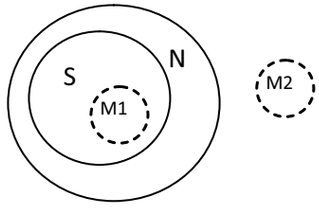
Verbal Ability

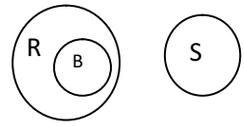
VA-4.1 | SYLLOGISMS

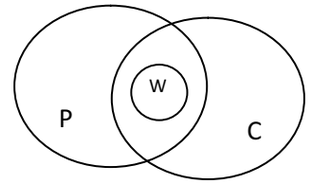
PRACTICE EXERCISE 1

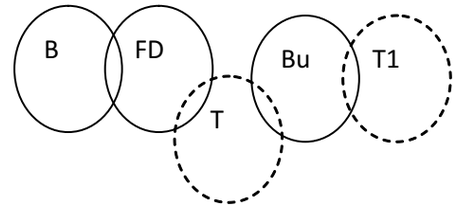
1. The conclusion is of the form 'Some X is not Y', which is a form in which Y (juries) is distributed. So 'juries' has to be distributed in one of the premises too. It is present in the major premise, which is of the form 'Some X is Y', in which both X and Y are not distributed. Thus, Rule 2 is being violated and this makes the conclusion invalid.
2. The middle term 'unethical' is not distributed in either of the premises, thus violating Rule 1 and making the conclusion invalid.
3. No rule is broken. Hence the conclusion is valid.
4. No rule is broken. Hence the conclusion is valid.
5. No rule is broken. Hence the conclusion is valid.
6. 

From the Venn diagram, it is evident that some frogs may be amphibians. So, the conclusion is invalid.
7. 

From the Venn diagram, it is evident that some books may be adventure. So, the conclusion is invalid.
8. 

From the Venn diagram, it is evident that all media may be social. So, the conclusion is invalid.
9. 

From the Venn diagram, it is evident that no scarves can be bands. So, the conclusion is invalid.
10. 

From the Venn diagram, it is evident that some plastic is copper. So, the conclusion is valid.
11. 

From the above diagram, two conclusions can be definitely derived: (1) Some traders (the ones who are businessmen) are definitely not fashion designers, i.e., B and (2) Some barbers (the ones who are fashion designers) are definitely not businessmen, i.e., D. A & C are not necessarily true. As there is no option with only B and D, none of the conclusions follow. Hence, (4).
12. Taking (1), we see that E is not a logical conclusion from B and D because we don't know whether Reena and Meena are sisters. (2) is also invalid because A contradicts F. In (3), E contradicts what is stated in C; hence the set is logically inconsistent. Taking (4), we see that if Reena and Meena are sisters and sisters are often known to quarrel, we can definitely conclude that Reena and Meena quarrel often. So, sequence ABE is valid. Hence, (4).
13. If all crows are birds and all crows lay eggs, we cannot definitely say that all birds lay eggs because there may be other birds, which are not crows, that don't lay eggs; so (1) is invalid. In case of

(2), D does not follow from B and E as B says that 'all birds are not crows' (which is the same as 'no birds are crows'). (3) is also invalid because A and B contradict each other. Taking (4), we see that if all crows are birds and all birds are warm blooded, we can definitely conclude that crows are warm-blooded. Thus, the sequence ACF is valid. Hence, (4).

14. Ts are sub-set of squares, which is the sub-set of rectangles. Hence, Ts are sub-set of rectangles. Hence, the conclusion of group A is correct.

So, the answer has to be either option 3 or 4. In both the options 3 and 4, the group C is mentioned. So, let us check group B to eliminate one out of these two options.

In group B, both the premises start with "some...". Hence, no definite conclusion can be drawn for the premises in group B.

Hence, (3).

15. When we have two premises starting with 'some...', we cannot have a definite conclusion. Hence, the conclusion of group A is incorret.

Since group B is not among the answer options, we can check for group C.

In group C, there is nothing common between 'faith' and 'strong' and 'biceps'. We cannot arrive at a particular conclusion.

Now, consider group D. Since the word 'weak' is repeated in all the sentences including the conclusion, the conclusion is incorrect.

Hence, (4).

16. Consider group A. Cycles are sub-set of men and some cycles are icicles. Hence, some men are icicles. Hence, the conclusion of group A is correct.

So, the answer cannot be option (3).

Since group B is in options 1, 2 and 4, let us check group C.

In group C, since one of the premise is negative, the conclusion has to be negative. But the given conclusion is not negative. So, the conclusion is incorrect.

Hence, (2).

17. No sun is not white means all sun is white. As all moon is sun, moon will always be white. So, group A has the correct conclusion.

Each of the groups B and C have more than 3 terms. The deduction can have only 3 terms. Hence, the conclusions of groups B and C are incorrect.

Hence, (1).

18. [A] – Those who laugh are cowboys. Some sphinxes laugh. So, some sphinxes are not cowboys. Hence, it is correct.

[B] – It is only mentioned that all ghosts are fluroscent. Nothing is said about the the colour of singers. Hence, this is incorrect.

[C] – Since we do not know the set of people who swear, we cannot conclude that whether the people who are hanged are cricketers or not.

[D] – Crazy people are sub-set of whistlers. It is given that some crazy people are pianists, which mean some whistlers are pianists. Hence, this option is correct.

Hence, (3).

19. [A] – Warriors are sub-set of good people. Good people are sub-set of knights. Hence it is incorrect.

[B] – Since the first premise is negative, the conclusion has to be negative. But the conclusion is not negative. Hence, it is incorrect.

[C] – Some meals are pizzas. Since all pizzas are snacks, some meals are snacks. Hence, it is correct.

[D] – Some barkers are musk-deer. Since all barkers are sloth bears, some musk-deer are sloth bears and vice versa.

Hence, (1).

20. (1) – C cannot be the conclusion, because apart from a software company some other companies could also employ knowledge workers. Hence, we cannot conclude Tara Tech is a software company.

(2) – Tara Tech is a software company. Since all software companies employ knowledge workers, Tara Tech also employs knowledge workers. Hence, B is the perfect conclusion for the premises A and C.

(3) – B cannot be the conclusion for C and D as only some software companies employ knowledge workers.

(4) – E cannot be the conclusion for the premises

A and C as it stresses that Tara Tech employs ONLY knowledge workers.

Hence, (2).

21. (1) – From C and B, all we know that 'traffic congestion' and 'increase in carbon monoxide' are injurious to health. This is not the view of statement A. Hence, this is incorrect.

(2) – We cannot assume that, it is not only the increase of carbon monoxide that is hazardous to health. Hence, E cannot be the perfect conclusion for the premises B and D.

(3) – The phrase 'traffic congestion' is repeated in all the three statements. Hence, this option is incorrect.

(4) – Increase in carbon monoxide is hazardous to health. Since traffic congestion increases carbon monoxide, it is hazardous to health. Hence, C is the perfect conclusion for B and A.

Hence, (4).

22. (1) – All sweets are tasty, no apple is tasty, hence it can be said that no apple is a sweet. Hence, A is the perfect conclusion for C and E.

(2) – Some apples are sweets and some are not tasty. From these, it cannot be concluded that all sweets are tasty. Hence, this option is incorrect.

(3) – All sweets are tasty and some apples are sweets, so it can be concluded that some apples are definitely tasty but nothing can be said about some apples not being tasty. Hence, this option is incorrect.

(4) – No apple is tasty or sweet. From these, nothing can be said about all sweets being tasty. Hence, this option is incorrect.

Hence, (1).

23. (1) – If all polluted towns should be destroyed and Town Meghana is polluted, nothing can be said about some towns in India. Hence, this option is incorrect.

(2) – Some towns in India are polluted. Since all polluted towns should be destroyed, some towns in India should be destroyed. Hence, E is the perfect conclusion for B and A.

(3) – Some towns in India are polluted along with town Meghana. From these, nothing can be concluded. Hence, this option is incorrect.

(4) – From C and D, nothing can be concluded about all polluted towns. Hence, this option is incorrect.

Hence, (2).

24. (1) – Bundledas is a patriot. Since no patriot is a criminal, he cannot be a criminal. Hence, B is the perfect conclusion for A and C.

(2) – From A, we see that no patriot is a criminal but nothing is said about a patriot being a criminal or not. Hence, nothing can be concluded from A and B. Hence, this option is incorrect.

(3) – From A, we see that no patriot is a criminal but nothing is said about a patriot being a criminal or not. Hence, nothing can be concluded from A and D. Hence, this option is incorrect.

(4) – From A, we see that no patriot is a criminal but nothing is said about a patriot being a criminal or not. Hence, nothing can be concluded from A and B. Hence, this option is incorrect.

Hence, (1).

25. (1) – From D and C, we can see that Balaram likes ants and is an ant eater, these do not say anything about ant eaters liking ants as such. Hence, this option is incorrect.

(2) – From A and D, we cannot conclude that Balaram is an ant eater since 'ant eaters' and 'Balaram' could still be two disjoint sets. Hence, this option is incorrect.

(3) – From A and B, nothing is said about Balaram and hence nothing can be concluded. Hence, this option is incorrect.

(4) – Balaram is an ant eater. Since all ant eaters likes ants, he must like ants. Hence, D is the perfect conclusion for A and C.

Hence, (4).

26. (1) – From A and C, we cannot conclude that Ram is a popular actor since 'actors' and 'Ram' could still be two disjoint sets. Hence, this option is incorrect.

(2) – Since all actors are handsome and some actors are popular, some popular people (actors) should be handsome. Hence, E is the perfect conclusion for A and B.

(3) – From D and C, it cannot be said that what applies to Ram applies to all actors. Hence, this option is incorrect.

(4) – Since only some popular people are handsome, Ram could be one of them or not, hence he need not be handsome. Hence, this option is incorrect.

Hence, (2).

27. (1) – BTI is a modern industry. Since any modern industry is technology driven, BTI must be technology driven. Hence, C is the perfect conclusion for A and B.

(2) – From A and B, C can be definitely concluded as shown above. But D uses the word 'may be'. Hence, this option is incorrect.

(3) – From B and C, it can be seen that BTI is technology driven and is a modern industry. From this, nothing can be concluded about all modern industries being industry driven. Hence, this option is incorrect.

(4) – From E and B, C cannot be concluded as 'BTI' and 'technology driven industry' could still be disjoint sets. Hence, this option is incorrect.

Hence, (1).

28. (1) – From B and C, it can be said some smart people are not babies. But D cannot be concluded. Hence, this option is incorrect.

(2) – All Golmal islanders are blue coloured people and some smart people are not blue coloured, thus they are not Golmal islanders as well. Hence, E is the perfect conclusion for A and B.

(3) – From C and D, nothing can be concluded. Hence, this option is incorrect.

(4) – Option 2 is shown to be valid. Hence this option is incorrect.

Hence, (2).

29. (1) – From A and B, E cannot be concluded since 'MBAs' and 'Ram and Sita' could still be disjoint sets. Hence, this option is incorrect.

(2) – From E and C, nothing can be said about Sita. Hence, this option is incorrect.

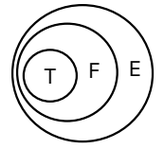
(3) – Ram and Sita are MBAs. Since all MBAs are in great demand, both have to be in great demand. Hence, B is the perfect conclusion for A and E.

(4) – From E and B, it cannot be concluded that they are in great demand because they are MBAs, there could be other reasons as well. Hence, this option is incorrect.

Hence, (3).

PRACTICE EXERCISE 2

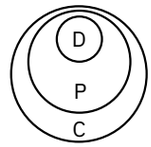
1.



T = Trout; F = Fish; E = Things that lay eggs

If Trout are fish and all fish lay eggs, then Trout will lay eggs as well. Hence, (4).

2.

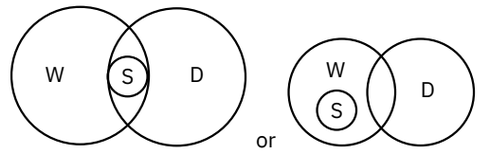


D = Dictator P = Absolute Power

C = Absolute corruption

Though option (1) might confuse you, you will see that statement (F) does not flow logically from the first two statements. Options (2) and (3) do not make sense. Hence, (4).

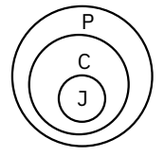
3.



W = Women; D = Dancers; S = Sheela

All that this question needs is a little close examination to realize that statements (A) and (B) are irrelevant, and have no third statement to connect them – therefore you have to make the set of the remaining four statements, and the correct one is option (2) – 'DFC'. Hence, (2).

4.

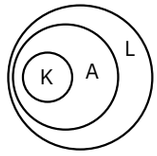


J = Jumbo; C = Cricketers

P = Those who play ball

On seeing the statements, we can immediately eliminate (C) and (F), as the terms in them are not repeated in any other statement ('bat' and 'teeth'). Statement (E) cannot be connected to any other statement either. So this leaves us with (A), (B) and (D), which are featured only in option (4), 'b a d'. Hence, (4).

5.



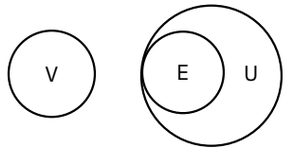
K = What Keto likes to do

A = Adults

L = Those who wear Long Pants

Again statements (B) and (C) can be discarded - they do not have a third statement to complete the sequence. With the other four sentences, you can make the sequence 'AFE'. Please note that the same three statements in sequences 'AEF' or 'FEA' are not correct! Hence, (2).

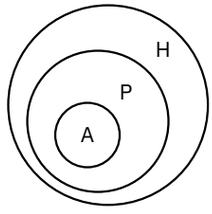
6.



V = Vagrants; E = Exits; U = Uncouth

If no vagrants are uncouth and all exits are uncouth, then no exits are vagrants. Hence, (2).

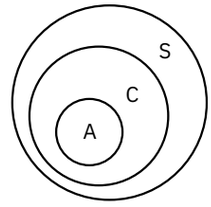
7.



A = Apes; P = Primates; H = Things that have hair

Only 'DEA' forms a logical sequence. Hence, (4).

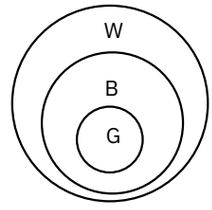
8.



A = Ashet; C = Ship's crew; S = Submarine crew

The first clue is that statement (C) has to be a part of the logical sequence, as no logical sequence can exist without it. But that eliminates only one option. The second clue is that statement (C) has to be either the first or the second statement. That leaves us with options (1) and (2). Then we can see that option (1) - 'BCE' - is the answer. Hence, (1).

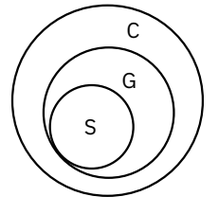
9.



G = Green; B = Black; W = White

This is a question where at least five logical sets can be formed – CDA, ABC, DEF, ECB and FAD. The one that is present in the options is 'CDA', option (1). Hence, (1).

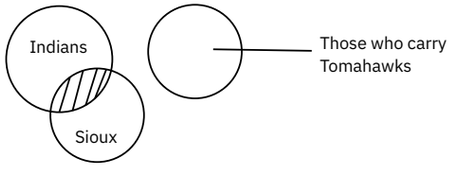
10.



S = Susheel; G = Golfers; C = Cricket Players

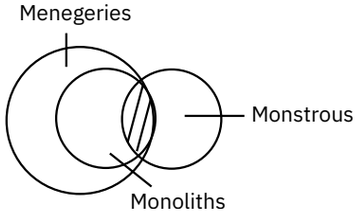
This is of the type 'All this ... are that'. Statement (B) must be a premise. The answer is 'EBC'. Hence, (4).

11.



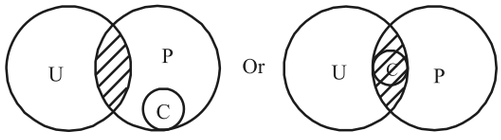
As some Sioux are Indians, none of whom carry Tomahawks, some Sioux do not carry Tomahawks. Hence,(3).

12.

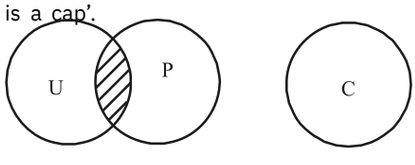


If all monoliths are menageries and some monoliths are monstrous, then some menageries (which are monoliths) are obviously monstrous. Hence, (1).

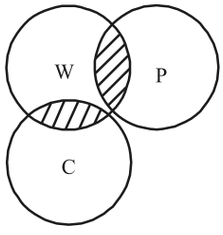
13. Statement A is not valid because 'some chairs are desks' is drawn only on the basis of the first premise. If 'some umbrellas are pencils' and 'all chocolates are pencils', we can conclude that 'some chocolates may be umbrellas', but we cannot say for sure that 'some chocolates are umbrellas'.



Again, if 'some watches are pins', and 'no cap is a pin', then 'some watches may or may not be caps'. We cannot conclude for sure that 'no watch is a cap'.

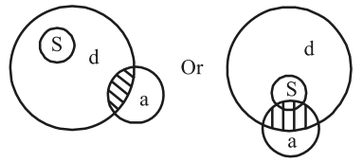


or

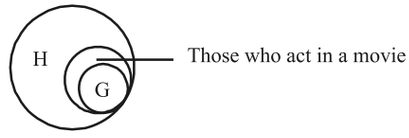


Thus, none of these statements follow. Hence, (4).

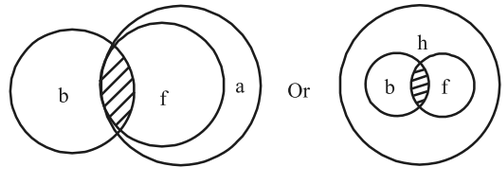
14. Statement A is not valid: if 'all singers can dance', and 'some animals can dance', we cannot conclude that 'some animals are singers'. We can only conclude that 'some animals may be singers'.



Statement B is valid: 'if only heroes act in a movie' and 'Gautam acts in a movie', we can conclude that 'Gautam is a hero'.

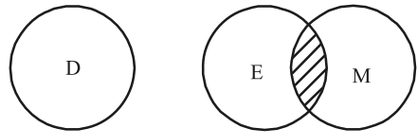


Statement C is also valid: if some brave are famous and all famous are handsome then we can conclude that some brave are handsome.

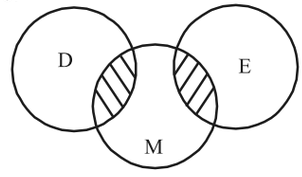


Thus, B and C are valid. Hence, (1).

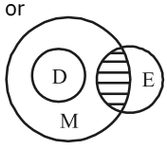
15. Statement A is valid: if 'no doctor is an engineer 'and 'some managers are engineers', then it can be concluded that 'some managers – those that are engineers – are not doctors':



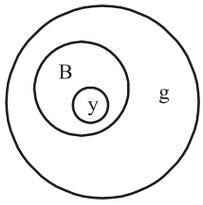
or



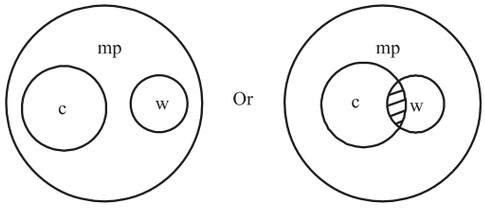
or



Statement B is not valid: the conclusion should be 'all yellow are green' to make it valid.



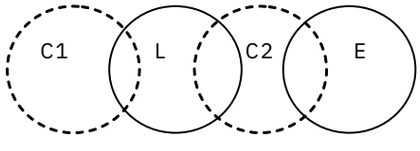
Statement C is also not valid: If 'all companies are making profits' and 'Werner is making profits' then 'Werner may be a company', but we cannot conclude that 'Werner is a company'.



Hence, (3).

16. None of the options hold. You may be tempted to mark (2), but just because 'Ravi plays cricket 'and 'all players play cricket' does not imply that 'Ravi is a player'. Hence, (4).

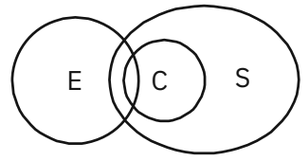
17. Both A and C violate rule 1, as their middle terms, 'juices' and 'ice-creams' respectively, are not distributed (note that words like 'many' mean essentially the same thing as 'some' for the purpose of syllogisms). Only B is correct, as can be seen in this diagram:



Those cappuccinos that are not lattes could be espressos. Though both the premises are negative, this syllogism is valid since the conclusion is uncertain. Hence, (2).

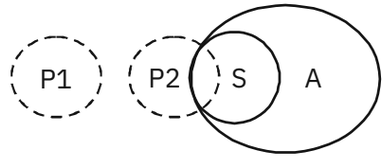
18. Statement A violates rule 4: a positive conclusion cannot be drawn from a negative premise. Statement B violates rule 5, since both its premises are particular. Statement C violates rule 3, as both its premises are negative. Thus, none of these are valid syllogisms. Hence, (4).

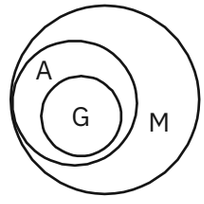
19. Statements A and B violate rule 1: the middle terms, 'eggs' and 'omelettes' respectively, are not distributed. Only C is correct as can be seen from the following diagram:



Hence, (3).

20. Statement B violates rules 2 and 3: the term 'zoology' is distributed in the conclusion, but not in either premise; and both premises are negative. However, both A and C are correct (note that 'not all' in a can mean either 'none' or 'some'). Refer to the following diagrams:





Hence, (2).

21. In statement A, the conclusion is derived only from the second premise, not both of them. Statements B and C violate rule 2: the term 'stealthy' in B and 'want revenge' in C are distributed in the conclusions, but not in the premises. Thus, none of these statements are valid syllogisms. Hence, (4).

22. The use of 'only' reverses the usual direction of implication in 'if' sentences. So the statement means that if you borrowed a second book from the library, you must have returned the first one, and if you didn't return the first one, you cannot

have borrowed a second one – i.e. AB and DC. Hence, (4).

23. The formula for answering such questions is 'If/when X then Y' and 'If/when not Y then not X'. So the correct combinations in this case are CA and BD. Hence, (1).

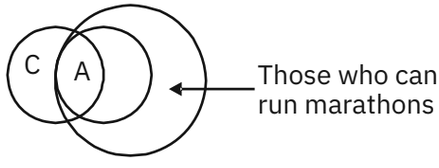
24. In this question, the 'if' portion consists of two parts ('Riya goes to the party' and 'Ritika goes to the party'). Both of these need to be affirmed or negated together. Since none of the statements include these two parts together, we cannot form a valid pair from any of them. Hence, (4).

25. In such questions, we can say for sure that if the voucher was not used at one location, it must have been used at the other (as it must be used at least one place). So, DC and BA become logical necessities. Many logicians advocate that DC & CD and BA & AB would be equally correct, as, according to them, the occurrence of both the events (i.e. using the voucher at the mall and online) and the occurrence of none of the events are logical impossibilities. However, some schools of thought believe that in either/or cases, the occurrence of both events is a logically acceptable possibility. This logic accepts only pairs such as DC and BA as logical necessities. According to this logic, CD, AB, AC and CA are all logical possibilities. As it is not possible to predict the logic followed by an examiner, it is best to use the options to your advantage. Hence, (3).

26. Based on the formula 'If X then Y' and 'If not Y then not X', the correct combinations are CB and DA. Hence, (1).

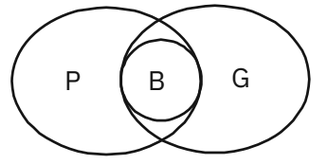
27. Both the premises are particular, which violates rule 5. So no certain conclusion can be drawn from them. Hence, (4).

28. B only rephrases the first premise. A is a valid conclusion, as can be seen from the following diagram:



Hence, (1).

29. B can be inferred on combining the two premises. A is also valid, as can be seen in the following diagram:

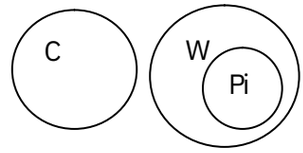


Hence, (3).

30. We cannot derive a certain conclusion from an uncertain premise, so A is incorrect. B is wrong - we already know that all rocks are stones, so there is no need for the uncertainty. Also, it is inferred only from one premise. Hence, (4).

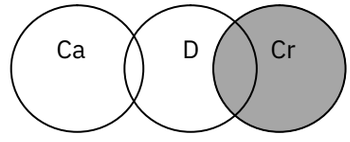
PRACTICE EXERCISE 3

1. 'Only X is Y' means that 'all Y is X'. So, in such statements, Y is distributed. In option (1), the term 'puddings', which is distributed in the conclusion, is not distributed in either premise. Similarly, 'puddings' is distributed in the conclusion of (2), but not in either premise. So both these options violate rule 2. In (3), the only possible conclusion is that 'No cheesecakes are puddings', so the given conclusion (E), which implies that we know for certain about only some cheesecakes, is incorrect. Refer to the following diagram:



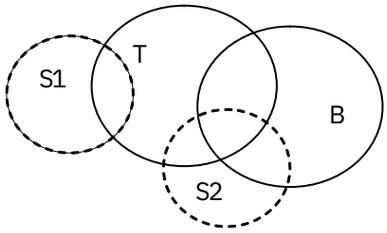
Only (4) is a valid syllogism, as can be seen in the above diagram. Hence, (4).

2. Option (2) is invalid, as the term 'credit' in the conclusion is not present in the premises. Option (3) violates rule 4, as it has a negative premise (A) and a positive conclusion (B). Option (4) has two particular premises, so it violates rule 5, and is thus also invalid. Only option (1) is valid, as can be seen in the following diagram:



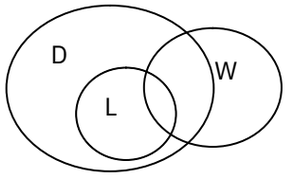
That part of debit which is credit is definitely not cash. Hence, (1).

3. The conclusion in option (1) has a term that does not appear in its premises, viz. 'Targaryens', so it is clearly invalid. The conclusion of option (3) contradicts one of its premises, so it too is invalid. Neither premise of option (4) has distributed the middle term, so no certain conclusion can be drawn from the premises. Option (2) can be represented by the following diagram:



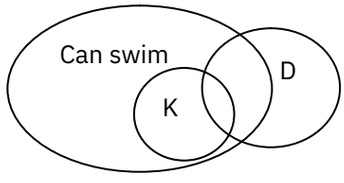
We cannot draw any certain conclusion from C and B, but we can conclude that some Starks may be Baratheons. Hence, (2).

4. In options (1) and (3), the term 'deer' is distributed in the conclusion but not in the premises, so they violate rule 2. Since option (2) has two particular premises ('some ...' and 'some ...') and a certain conclusion, it violates rule 5. Option (4) can be represented by the following diagram:



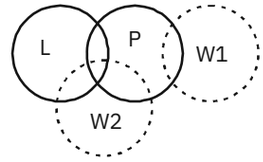
As can be seen from the diagram, those wolves that are lions must also be deer. Hence, (4).

5. Based on C and A, we can infer that no krakens are dragons . But D implies that we know for sure only about some krakens, and we don't know about the rest. Since we know about all krakens, D is not universal enough. So option (1) is wrong. Based on rule 4, we can eliminate option (2), as it has a negative conclusion, though both its premises are positive. Similarly, option (4) can be eliminated on the basis of rule 3, i.e. it has two negative premises. Only option (3) is correct:



As can be seen from the diagram, those dragons that cannot swim cannot be krakens. Hence, (3).

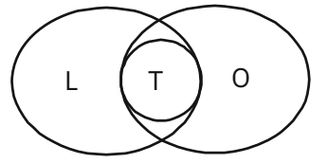
6. Option (1) violates rule 4, so it is incorrect. This also eliminates option (4). One cannot draw a certain conclusion from an uncertain premise, so (2) is wrong as well. Only option (3) is correct, as can be seen from the following diagram:



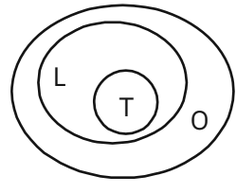
Hence, (3).

7. Option (1) violates rule 1: the middle term 'carbon' is not distributed. In (2), the term 'carbon' is distributed in the conclusion, but not in the premises, so rule 2 is violated. (3) has two negative premises, so it violates rule 3. Thus none of these are valid. (Note that 'X are never Y' means that 'No X are Y', and 'X are always Y' means that 'All X are Y'.) Hence, (4).

8. Option (3) is invalid, as (d) is drawn only from one premise; also, it says that there is only a possibility that some Lokis are Odins, while (c) states that some definitely are. Option (1) is valid, as can be seen in the following diagram:

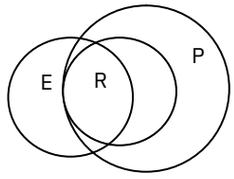


Those Odins that are Thors must also be Lokis. Option (2) is also valid - refer to the following diagram:



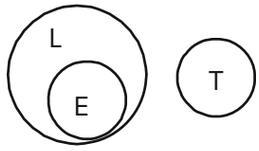
Hence, (4).

9. Option (2) is invalid, as the conclusion includes a term, 'evil', that is not in the premises. (3) violates rule 2, as the term 'rings' is distributed in the conclusion, but not the premises. Only (1) is valid, as can be seen from the following diagram:



Hence, (1).

10. Option (1) is incorrect, as based on the premises, some lemon are definitely earl grey, not 'may be'. (2) is incorrect, as it is not possible to derive a certain conclusion from an uncertain premise. (4) has a negative premise but positive conclusion, so it violates rule 4. Only (3) is correct - refer to the following diagram:



Hence, (3).

11. The statements in option (1) contradict each other, so it can be eliminated immediately. The students may leave the classroom or stay in it for any number of reasons. So CA and BD are not necessarily valid. But DB is valid: the statement implies that whenever the bell rings, the students always leave the classroom, so if they didn't leave the classroom, we can infer that the bell did not ring. Hence, (3).

12. The statement clearly states that Harry could choose only one of the two subjects, Transfiguration or Potions. So if he chose one, he would not be able to take up the other. Thus, both AD and CB are valid combinations. Hence, (4).

13. In this statement, the use of 'only' inverts the usual cause-effect direction: so the statement implies that Kajal does not pick up her mobile under any other conditions, except when it buzzes. So if she picks up her mobile, that implies that it is buzzing, i.e. BA. But there is no guarantee she will pick it up whenever it is buzzing; nor can anything be inferred if she does not pick it up. D says 'ringing' instead of 'buzzing', which is not the term used in the statement. Hence, (2).

14. The formula for answering such questions is 'If X then Y' and 'If not Y then not X'. So the correct combinations in this case are C (X) and A (Y), and D (not Y) and B (not X – note that since X is a negative statement, it becomes positive when negated). Since CA is not included among the options, the only correct possibility is DB. Hence, (4).

15. D does not include either of the alternatives mentioned in the statement, so the option that includes it - option (2) - can be eliminated at once. Both the alternatives in the statement could be true, but are not necessarily implied from the main statement. So BA is not necessarily true, while CA is; it correctly suggests that we ordered from one place (Domino's in this case), when not from the other. Hence, (1).

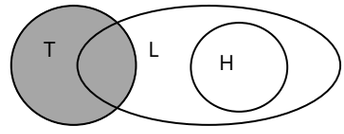
16. Ram never drives when he drinks does not imply that for all the times Ram doesn't drink he drives, thus (1) is out. (2) does not make sense. (3) cannot be assumed for the same reason as (1). However (4) is true. Hence, (4).

17. If only intelligent investors manage money well then the fact that Harsh is not an intelligent investor implies that he does not manage money well. Hence, (1).

18. The computer may move for other reasons as well, therefore (1) is not necessarily true. However, (2) is true. Hence, (2).

19. The main statement can be read as: 'All good girls go to heaven'. So, if Sheela is a good girl, she will go to heaven, and if she does not go to heaven, she is definitely not a good girl. But, if she is not a good girl, she may or may not go to heaven. Hence, (3).

20. Refer to the following diagram:



As can be seen, conclusion A follows: that leather which is tanned is definitely not hide. But B does not follow. Hence, (1).

21. The conclusion, which is of the form 'Some X is not Y', is a particular negative. A negative conclusion must have a negative premise. Since both premises are affirmative, Rule 4 is violated. Hence the conclusion is invalid.

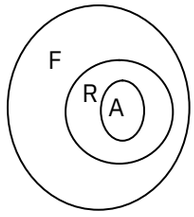
22. Rule 3, two negative premises not being allowed, is violated. Hence the conclusion is invalid.

23. No rule is broken. Hence the conclusion is valid.

24. The middle term, 'foods high in calcium', is not distributed in either premise, violating Rule 1. Hence the conclusion is invalid.

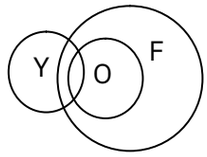
25. No rule is broken. Hence the conclusion is valid.

26.



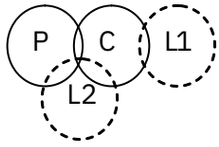
From the Venn diagram, it is evident that some apples are flowers. So, the conclusion is valid.

27.



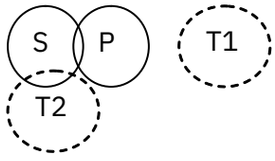
From the Venn diagram, it is evident that not all fruit is orange. So, the conclusion is invalid.

28.



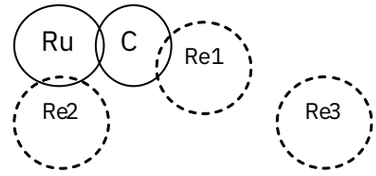
From the Venn diagram, it is evident that some panes may be lucid. So, the conclusion is valid.

29.

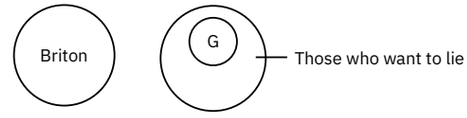


From the Venn diagram, it is evident that some trousers may be shirts. So, the conclusion is invalid.

30. As the second premise is uncertain (some carpets may be red), nothing can be said for sure about rugs being red. So, the conclusion is invalid.

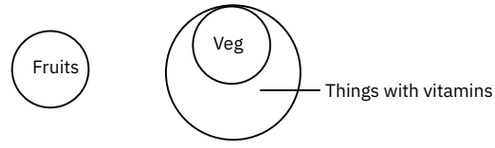


31.



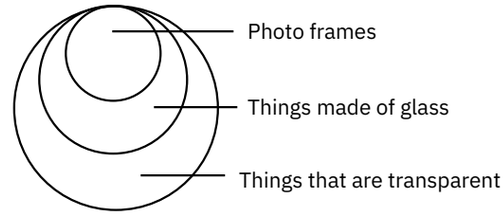
If 'no Briton wants to lie' and 'Grant wants to lie' then 'Grant cannot be a Briton'. The rest of the statements are invalid. Hence, (3).

32.

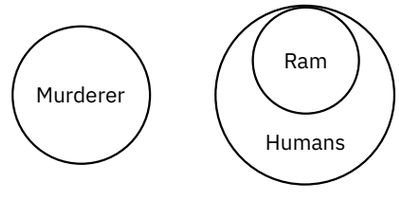


If 'no fruits have vitamins' and 'all vegetables have vitamins', then 'vegetables cannot be fruits'. The rest of the statements are invalid. Hence, (1).

33. 'All thunderstorms are accompanied by hail' does not imply that the reverse is also true. Thus A does not hold. Only 'some' not 'all who talk do not smile', so B is invalid.

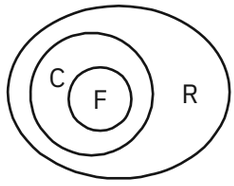


If 'all things made of glass are transparent' and 'photoframes are made of glass' then they have to be transparent as well.



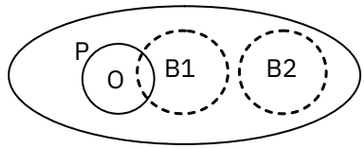
'If no murderer is human' and 'Ram is human' then 'Ram cannot be a murderer'. Hence, (3).

34. In statement A, the term 'freerunning' is distributed in the conclusion but not in the premises, so it violates rule 2 (note that 'not all freerunning is dangerous' is equivalent to 'some freerunning is not dangerous'). In statement C, the middle term 'sands' is not distributed, so it violates rule 1. Only B is correct, as can be seen from the following diagram:

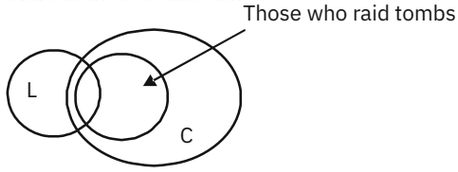


Hence, (2).

35. The first two segments of B have nothing in common, so it can be dismissed at once. Statement D violates rule 4, so it can be eliminated as well. Statement A is valid, as can be seen from the following diagram:

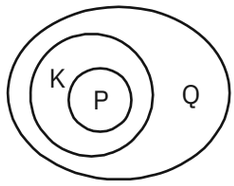


Statement C is also valid:



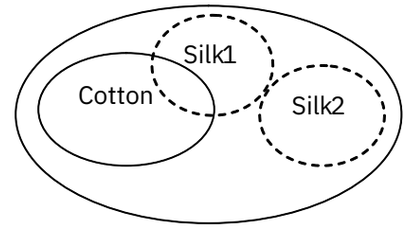
Thus, both A and C are valid. Hence, (2).

36. The statement 'Only queens are kings' means that 'All kings are queens'. Thus we get the following diagram:



Thus, conclusion B is valid, but A is not. Hence, (2).

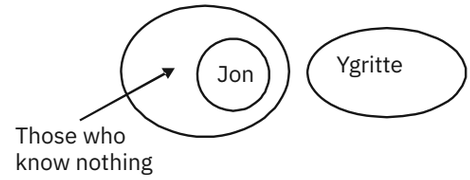
37. Since the middle term 'satin' is not distributed, A is definitely wrong. However, using the following representation, it is possible to draw an uncertain conclusion such as B from the two statements.



Thus, only B is correct. Hence, (2).

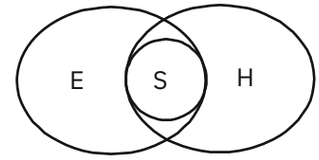
38. Since both the premises are negative, no certain conclusion can be drawn from them (this is rule 3 as discussed in the chapter). Thus both A and B are incorrect. Hence, (4).

39. Based on the statements, we can draw the following diagram:



As can be seen, there is no overlap between the categories 'Jon' and 'Ygritte', so A is correct and B is wrong. Hence, (1).

40. Refer to the following diagram:



Putting both the premises together, we get A. Also, at least some of those things that have hilts - viz. swords - have edges. Hence, (3).

PRACTICE EXERCISE 4

1. ABE – Since, we do not know whether Brand X is a cigar or not, E cannot be the conclusion for A and B. Hence, option (1) is incorrect.

   BDF - All cigars are hazardous to health. As cham-cham is a cigar, it is hazardous to health. Hence, option (2) is correct.

   ABD - D mentions that cham-cham is a brand of cigar. As the premises A and B do not mention about cham-cham, D cannot be the right conclusion for A and B.

   ABC- As C contradicts A, C cannot be the conclusion for A and B.

   Hence, (2).

2. The group ADF is logically related.

   A: Laxman is a man

   B: No man is island

   F: Laxman is not an island.

   Hence, (3).

3. ABC – The term 'pious' is distributed in the conclusion but not in the premises. This violets rule 2. Hence, option (1) is incorrect.

   BCD – The term 'air' is distributed in the conclusion but not in the premises. This violates rule 2. Hence, option (2) is incorrect.

   CDE – Since all bosses are pious and all pious need air, it can be concluded that all bosses need air. Hence, option (3) is correct.

   CEF – The term 'water is distributed in the conclusion but not in the premises. This violates rule 2. Hence, option (4) is incorrect. Hence, (3).

4. ABF – The premise A is a negative premise. So, the conclusion F should be a negative one, but it is not. Hence, option (1) is incorrect.

   CDF – The premise C is a negative premise. So, the conclusion F should be negative one, but it is not. Hence, option (2) is incorrect.

   BDF – All nurses are attendants. As some nurses are qualified, some attendants are qualified.

   BDE – The premise B is a particular premise. So, the conclusion E should be a particular one, but it is not. So, option (4) is incorrect.

   Hence, (3).

5. It is very evident that the group ADF is logically related.

   A: Mary is John's wife

   D: Husbands and wives danced the last waltz.

   F: John danced last with Mary.

   Hence, (1).

6. BEF – All well paid persons are professional. Since no college athlete is professional. So we can conclude that 'no college athlete is well paid'. Hence, option (1) is correct.

   ABF – Since the premise A is a particular premise, the conclusion F has to be particular statement, but it is not. Hence, option (2) is incorrect.

   BDF – No college athlete is professional. Since, all professionals are well-paid; we cannot conclude that there is nothing in common between college athlete and well paid persons. Hence, option (3) is incorrect.

   ACF – Since, both the premises A and C are particular premises, there is no absolute conclusion that can be drawn. Hence, option (4) is incorrect.

   Hence, (1).

7. CDE – All people with high ideals are intolerant. Since, no poor thinker is intolerant; no poor thinker has high ideals. Hence, option (1) is correct.

   CDF – All people with high ideals are intolerant. Since, no poor thinker is intolerant; no poor thinker has high ideals. So, option (2) is incorrect.

   ABD – Both the premises A and B are particular premises. So, there is no absolute conclusion that can be drawn. Hence, option (3) is incorrect.

   BCF – All people with high ideals are intolerant. Since, some poor thinkers are intolerant; some poor-thinkers may or may not have high ideals. So, option (4) is incorrect.

   Hence, (1).

8. ACE – Engineers are a subset to singers, who are a subset to music lovers. So, all engineers are music lovers. Hence, option (1) is incorrect.

   ACF – Engineers are a subset to singers, who are a subset to music lovers. So, all engineers are music lovers. Hence, option (2) is correct.

   ABF – Since B is a negative premise the conclusion has to be negative, but it is not. Hence, option (3) is incorrect.

   ACD – Engineers are a subset to singers, who are a subset to music lovers. So, all engineers are

music lovers. So, option (4) is incorrect.

Hence, (2).

9. ACE – Since, both the premises A and C are particular premises; there is no absolute conclusion that can be drawn. Hence, option (1) is incorrect.

BCE – Sociable people are a subset to well-dressed people. Since, some well-dressed people are dull; we cannot conclude that some sociable people are dull. Hence, option (2) is incorrect.

ADE – Since, the premise D is a negative premise the conclusion has to be a negative statement, but it is not. Hence, option (3) is incorrect.

BEF – Sociable people are a subset to well-dressed people. Since, some sociable people are dull; we can conclude that some dull ones are well dressed. Hence, option (4) is correct.

Hence, (4).

10. ABE – Iran and Iraq are members of UN. Since Iran and Iraq are not friends, not all members of UN are friends. Hence, option (1) is correct.

ABD – Iran and Iraq are members of UN. Since Iran and Iraq are not friends, we can only conclude that not all members of UN are friends. Hence, option (2) is incorrect.

CDF – Since, we do not know whether Iran and Iraq are members of UN or not, the given conclusion in option (3) is incorrect.

AEF – Since, we do not know whether Iran and Iraq are neighbours or not, the given conclusion in option (4) is incorrect.

Hence, (1).

11. BCE – There are more than 3 terms in this set of deduction. They are 'women, good manager, manager and intuitive'. Every deduction should have only 3 terms. Hence option (1) is incorrect.

ABD – In the premises, A and B, the name Supriya is not mentioned at all. But the conclusion D mentions about her. Hence, it is incorrect.

ACF – Good managers are intuitive. Supriya is a good manager. So she is intuitive. Hence, option (3) is correct.

ADF – Good managers are intuitive. Since we do not know whether Supriya is good manager or not, the given conclusion is incorrect. Hence, option (4) is incorrect.

Hence, (3).

12. BEF – Both the premises B and E are particular premises, hence, there is no absolute conclusion that can be drawn. So, option (1) is incorrect.

ADF – Sincere people are a subset to graduates who are a subset to dull people. So, the conclusion has to be 'all sincere people are dull', but the given conclusion is incorrect. Hence, option (2) is incorrect.

ABF – Since premise B is a particular premise, the conclusion F has to be a particular statement, but it is not. Hence, option (3) is incorrect.

ADC – Sincere people are a subset to graduates who are subset to dull people. So, the conclusion has to be 'All sincere people are dull'. Hence, option (4) is correct.

Hence, (4).

13. BEF – The premise B mentions about Sham loosing the game, premise E mentions the condition to win the game but the conclusion F is irrelevant. Hence, option (1) is incorrect.

ACD – Sham won a lottery, but he is not intelligent. So, to win a lottery, it is not necessary to be intelligent. Hence, option (2) is correct.

BDE – Premise B mentions about chess, premise D mentions about lottery, but the conclusion is drawn for only chess. Hence, option (3) is incorrect.

BDF – The premise B mentions about Sham loosing the game, premise D mentions the condition to win the lottery but the conclusion F is irrelevant. Hence, option (4) is incorrect.

Hence, (2).

14. BCF – There are four terms 'girls, bad, good people, and educated'. Every deduction should contain only 3 terms. Hence, option (1) is incorrect.

ACD – Boys are a subset to good people which are a subset to educated people. So boys are educated. Hence, option (2) is correct.

DEF – Premises D and E mention about the boy, since the conclusion F mentions about a girl, option (3) is incorrect.

ADF – Premises A and D mention about the boy, since the conclusion F mentions about a girl, option (4) is incorrect.

Hence, (2).

15. BCE – The phrase 'those who eat meat' is repeated in all the three statements including the conclusion. Hence, option (1) is incorrect.

ABE – Statements A and B contradict each other. Hence, option (2) is incorrect.

ACD – There are four terms'who eat meat, herbivorous, vegetarians and carnivorous'. Every deduction should contain only 3 terms. Hence, option (3) is incorrect.

ACF – Vegetarians are those who eat meat who are called herbivorous. So vegetarians are herbivorous. Hence, option (4) is correct.

Hence, (4).

16. BEF – There are four terms 'roses, nectars, shrubs and thorns'. Every deduction should contain only 3 terms. Hence, option (1) is incorrect.

FEB – There are four terms 'roses, nectars, shrubs and thorns'. Every deduction should contain only 3 terms. Hence, option (2) is incorrect.

BDE – Shrubs is a subset to roses which is a subset to nectar. So, all shrubs have nectars. Hence, option (3) is correct.

ACF – The word 'thorns'- is repeated in all the three statements including the conclusion. Hence, option (4) is incorrect.

Hence, (3).

17. AEF – Both the premises A and E are particular premises, hence, there is no absolute conclusion that can be drawn. So, option (1) is incorrect.

BCF – Dabra is a subset to Abra, which is a subset to cabra. So some cabra are dabra. Hence, option (2) is correct.

ABD – The premise A is a particular premise so, the conclusion has to be particular but it is not. Hence, option (3) is incorrect.

BCE – The word 'Abra'- is repeated in all the three statements including the conclusion. Hence, option (4) is incorrect.

Hence, (2).

18. DFA – No seasons are autumns. Since all springs are autumns, no seasons are springs. Hence, option (1) is correct.

BEF – Both the premises B and E are particular premises, hence, there is no absolute conclusion that can be drawn. So, option (2) is incorrect.

CEB – Both the premises C and E are particular premises, hence, there is no absolute conclusion that can be drawn. So, option (3) is incorrect.

DEB – The premise D is a negative premise and the premise E is a particular premise, so, the conclusion has to be particular negative but it is not. Hence, option (4) is incorrect.

Hence, (1).

19. ABC – There are four terms 'falcons, fly-high, blind and birds'. Every deduction should contain only 3 terms. Hence, option (1) is incorrect.

CDF – Falcons is a subset to birds which is a subset to yellow. So falcons are a subset to yellow. Hence, option (2) is correct.

DEF – There are four terms 'falcons, birds, yellow and thirsty'. Every deduction should contain only 3 terms. Hence, option (3) is incorrect.

BCA – The word 'falcons'- is repeated in all the three statements including the conclusion. Hence, option (4) is incorrect.

Hence, (2).

20. AED – Both the premises A and E are negative premises, hence, there is no absolute conclusion that can be drawn. So, option (1) is incorrect.

BCF – The premise B is a particular premise, so the conclusion has to be a particular premise, but it is not. Hence, option (2) is incorrect.

BEF – The premise E is a negative premise and the premise B is a particular premise, so, the conclusion has to be particular negative but it is not. Hence, option (3) is incorrect.

ACE – No wires are hooks. Since all springs are wires, no springs are hooks. Hence, option (4) is correct.

Hence, (4).

21. ACD – Both the premises A and C are negative premises, hence, there is no absolute conclusion that can be drawn. So, option (1) is incorrect.

ADF – Both the premises A and D are negative premises, hence, there is no absolute conclusion that can be drawn. So, option (2) is incorrect.

ABC – No plane is chain. Since all manes are chains, no plane is mane. Hence, option (3) is correct.

CDF – Both the premises C and D are negative premises, hence, there is no absolute conclusion that can be drawn. So, option (4) is incorrect.

Hence, (3).

22. CDE – Toys is a subset to dolls. Since some toys are nice, some nice things are dolls. Hence, option (1) is correct.

CEF – The premise E is a particular premise so, the conclusion has to be particular but it is not. Hence, option (2) is incorrect.

ACD – There are four terms 'rolls, toys, dolls and nice'. Every deduction should contain only 3 terms. Hence, option (3) is incorrect.

CEF – The premise E is a particular premise so, the conclusion has to be particular but it is not. Hence, option (4) is incorrect.

Hence, (1).

23. ACE – The premise C is a negative premise and the premise A is a particular premise, so, the conclusion has to be particular negative but it is not. Hence, option (1) is incorrect.

BDF – Sky-scrapers is a subset to structures. Since some sky-scrapers are not buildings, some structures are not buildings. Hence, option (2) is correct.

CDE – The premise C is a negative premise so, the conclusion has to be negative but it is not. Hence, option (3) is incorrect.

ACF – No structure is a sky-scraper. Since some buildings are not sky-scrapers, we cannot say some buildings are not structures. Hence, option (4) is incorrect.

Hence, the correct answer is option (2).

24. BDE – The premise B is a negative premise and the premise D is a particular premise, so, the conclusion has to be particular negative but it is not. Hence, option (1) is incorrect.

ACB – No bin is a basket, but bins are subset to buckets. There may or may not be something in common between baskets and buckets. So, option (2) is incorrect.

CDF – The premise C is a negative premise and the premise D is a particular premise, so, the conclusion has to be particular negative but it is not. Hence, option (3) is incorrect.

ABF – Bins are subset to buckets. Since no bucket is a basket, no basket is bin. Hence, option (4) is correct.

Hence, (4).

25. AFE – Both premises A and F are particular premises. Hence, there is no absolute conclusion that can be drawn.

BCF – Bad things are subset of men, which is a subset of sad. So, the bad things are subset of sad. In other words, some sad are bad things. Hence, option (2) is correct.

BDA – Both men and bad things are subset to sad. So we cannot bring out an absolute relationship between 'men' and 'bad things'. Hence, option (3) is incorrect.

BCE – The word 'men' is repeated in all the premises including the conclusion. So, option (4) is incorrect.

Hence, (2).

26. ADF – Dubbles is subset of rubbles. Some bubbles are not dubbles. So, the conclusion that can be drawn is– 'Some bubbles may or may not be rubbles'. Hence, option (1) is incorrect.

DEF – Dubbles is subset of rubbles. Some dubbles are bubbles. So, the conclusion that can be drawn is– 'Some rubbles are bubbles'. Hence, option (2) is incorrect.

ABC – Both premises A and B are particular premises. Hence, there is no absolute conclusion that can be drawn.

BDF – Some dubbles are not bubbles. All dubbles are rubbles. So, the conclusion is that– 'Some rubbles are not bubbles'. Hence, option (4) is correct.

Hence, (4).

27. ABC – The premise B is a negative premise. So the conclusion C should be negative, but it is not. Hence, option (1) is incorrect.

BEF – Both the premises B and E are negative premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (2) is incorrect.

ABF – There is nothing in common between bright Toms and Dicks. Since all the Toms are bright, there is nothing in common between Tom and Dicks. Hence, option (3) is correct.

CDA – Both the premises C and E are particular

premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (4) is incorrect.

Hence, (3).

28. BCD – The premise B is a particular premise. So the conclusion D should be particular, but it is not. Hence, option (1) is incorrect.

CDA – Witches are subset to devils who are subset to nasty. So, witches are subset to nasty. Hence, option (2) is correct.

DEC – The premise E is a particular premise. So the conclusion C should be particular, but it is not. Hence, option (3) is incorrect.

FEC – The premise E is a particular premise. So the conclusion C should be particular, but it is not. Hence, option (4) is incorrect.

Hence, (2).

29. ABC – Jingoes are subset to bingoes. Since there is nothing in common between bingoes and tingoes, there is nothing in common between jingoes and tingoes. Hence, option (1) is correct.

ACB – Both the premises A and C are negative premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (2) is incorrect.

DFA – Both the premises D and F are particular premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (3) is incorrect.

BDA – The premise D is a particular premise. So, the conclusion A should be particular, but it is not. Hence, option (4) is incorrect

Hence, (1).

30. ABC – Tin is a subset of copper. There is something common between tins and pins. So, we cannot conclude that copper is subset of pins. Hence, option (1) is incorrect.

CEF – Copper is subset of pins. There is something common between pins and tins. There may or may not be something in common between copper and tins. Hence, option (2) is incorrect.

CDA – Copper is subset of pins. There is something in common between copper and tins. So, there is also something common in between pins and tins. Hence, option (3) is correct.

ABE – The word 'tins' is repeated in the statements A, B and E. Hence, E cannot be the right conclusion for A and B. Hence, option (4) is incorrect.

Hence, (3).

31. BEA – All birds lay eggs. An ostrich is a bird, it lays eggs. Hence, option (1) is correct.

ABE – There are four terms in this group. They are eggs, ostrich, birds and fly. Deductions must have only three terms. Hence, option (2) is incorrect.

DEC – Just because an ostrich is a bird that cannot fly, we cannot conclude that all birds will fly or will not fly. Hence, option (3) is incorrect.

ECB – There are four terms in this group. They are eggs, ostrich, birds and fly. Deductions must have only three terms. Hence, option (4) is incorrect.

Hence, (1).

32. BED – Because, all papers and woods are good, we cannot conclude all papers are woods. Hence, option (1) is incorrect

BDF – Wood is subset of good and wood is also subset of paper, so 'some good are paper' or 'some paper is good'. Hence, option (2) is correct.

FAB – Both F and A are 'particular premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (3) is incorrect.

FBA – Wood is subset of good. There is something in common between paper and good. Hence, there may or may not something in common between wood and paper. Hence, option (4) is incorrect.

Hence, (2).

33. EAC – Bricks are subset of tricks, which is subset of shrieks. So, all bricks are shrieks. Hence, some bricks are shrieks. Hence, option (1) is correct.

BCD – Both B and C are 'particular premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (2) is incorrect.

ABC – Bricks are subset of tricks. Since some tricks are shrieks we cannot conclude that some bricks are shrieks. Hence, option (3) is incorrect.

EDC – D is a negative premise. So the conclusion C has to be negative. But here it is not. Hence, option (4) is incorrect.

Hence, (1).

34. BEF – The word 'strife' is repeated in the statements B, E and F. So, F cannot be the right conclusion for B and E. Hence, option (1) is incorrect.

FCB – The premise F is a negative premise. So the conclusion B has to be negative, but it is not. Hence, option (2) is incorrect.

ABF – All life is strife. Since there is nothing common between life and wife, we cannot conclude that 'no wife is strife'. Hence, option (3) is incorrect.

BDE – Life is subset of strife. Wife is subset of life. So some wife is strife.

Hence, (4).

35. ABC – The word 'mosquitoes, is repeated in statements C, E and F. Hence, F cannot be the right conclusion for C and E. Hence, option (2) is incorrect.

CEF – Since the conclusion is negative at least one of the premises has to be negative. However, since both the premises are positive, the conclusion cannot be negative. Hence, option (2) is incorrect.

ADE – Both A and D are 'particular premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (3) is incorrect.

EDB – Owls is a subset of mosquitoes. Since, some owls are flies, some flies are mosquitoes. This option is correct.

Hence, (4).

36. ADE – Six is a subset of five. Since some six is twelve, some twelve is five. Hence, this option is correct.

ABC – There are four terms in this group. They are six, five, four and ten. Deductions must have only three terms. Hence, option (2) is incorrect.

AEF – There are five terms in this group. They are six, five, twelve, four and ten. Deductions must have only three terms. Hence, option (3) is incorrect.

EDC – Both E and D are 'particular premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (4) is incorrect.

Hence, (1).

37. ADE – 'Poor girls' is subset of 'rich boys', which is subset of 'rich girls'. So 'poor girls' is a subset of 'rich girls'. Hence, option (1) is correct.

ABC – 'Poor girls' is a subset of 'rich boys'. 'Rich girls' is a subset of 'rich boys'. But the conclusion says that 'poor girls' is not a subset of 'rich girls'. 'Poor girls' may or may not be subset of 'rich girls'. Hence, option (2) is incorrect.

BCD – The word 'rich girls' is repeated in the statements B, C and D. Hence, D cannot be the right conclusion for B and C. Hence, option (3) is incorrect.

DEF – 'Rich boys' is a subset of 'rich girls'. 'Poor girls' is a subset of 'rich girls'. We cannot definitely conclude saying 'rich boys' is a subset of 'poor girls'. Hence, option (4) is incorrect.

Hence, (1).

38. BCA – Sandal is subset of band, which is a subset of sand. So, some sand is sandal. Hence, option (1) is correct.

AFE – Both A and F are 'particular premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (2) is incorrect.

DEC – Both D and E are 'negative premises'. Hence, there is no absolute conclusion that can be drawn. Hence, option (3) is incorrect.

CED – Statements C and E are contradicting each other. Hence, option (4) is incorrect.

Hence, (1).

39. ADC – D is a negative statement, so, C also has to be also negative, but it is not. Hence, option (1) is incorrect.

ABE – There is nothing common between birds and viviparous. Since all mammals are viviparous, there is nothing common between birds and mammals. Hence, option (2) is correct.

FBA – There are four terms in this group. They are bats, mammals, viviparous and birds. Deductions must have only three terms. Hence, option (3) is incorrect.

AFC – There are four terms in this group. They are bats, mammals, viviparous, bird. Deductions must have only three terms. Hence, option (4) is incorrect.

Hence, (2).

40. ABE – A is a negative statement. So, E also has to be negative, but it is not. Hence, option (1) is incorrect.

CED – C is a negative statement, so, D also has to be also negative, but it is not. Hence, option (2) is incorrect.

FEB – All women like to work. Since, some nurses are women, some nurses like to work. Hence, option (3) is correct.

BEF – The premises B and E are particular premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (4) is incorrect.

Hence, (3).

41. DAC – All oranges are sweet. Since, some oranges are apples, some sweet things are apples. Hence, option (1) is correct.

CDA – The premises C and D are particular premises. Hence, there is no absolute conclusion that can be drawn. Hence, option (2) is incorrect.

BCA – All oranges are apples. It is given that some sweet things are apples. So, oranges may or may not be sweet. Hence, option (3) is incorrect.

FEC – All sweet are sour. It is given that some apples are sour. So, we cannot conclude that some sweet things are apples. So, option (4) is incorrect.

Hence, (1).

42. ACB – Zens are a subset of Marutis. Marutis are a subset of fragile. Hence, Zens are a subset of fragile. Hence, option (1) is correct.

EFD – There are four terms in this group. They are Marutis, stable, Opels and weak. Deductions must have only three terms. Hence, option (2) is incorrect.

CEA – There are four terms in this group. They are Marutis, fragile, Opels and Zen. Deductions must have only three terms. Hence, option (3) is incorrect.

ABC – Zens are a subset of Marutis. Zens are a subset of fragile. So, we cannot conclude that Marutis are subset of fragile. Hence, option (4) is incorrect.

Hence, (1).

43. DEF – There are four terms in this group. They are dogs, sheep, indoors and open. Deductions must have only three terms. Hence, option (1) is correct.

DCA – There are four terms in this group. They are dogs, sheep, indoors and open. Deductions must have only three terms. Hence, option (2) is incorrect.

ABE – Sheep sleep indoors. Dogs sleep in the open. So, all dogs are not sheep. Hence, some dogs are not sheep. Hence, option (3) is correct.

FBD – The word 'sheep' is repeated in the statements F, B and D. Hence, D cannot be the right conclusion for F and B. Hence, option (4) is incorrect.

Hence, (3).

44. [A] – The water buffaloes may or may not be pre-historic creatures. This eliminates options (3).

[B] – All politicians are frank and no frank people are crocodiles. Hence, no politician is a crocodile. This is correct. Retain it.

[C] – Since there are two negative premises, there cannot be an absolute conclusion. This eliminates options (1) and (4).

Hence, (2).

45. [A] – Since not all actors are pretty, MD may or may not be pretty. Hence, this is incorrect.

[B] – The word 'cops' is repeated in all the three statements. Hence it is incorrect.

[C] – All cops are brave. Since some men are cops, some men are brave. Hence, this is correct.

[D] – Since we do not know whether MD is an actor or not, MD may or may not be pretty. So, this is incorrect.

Hence, (2).

46. [A] – The perfect conclusion should be– 'All young people of citizens of Yes Islands speak only the truth'.

Hence, the given conclusion is incorrect.

[B] – Since some Yes Islands are in the Atlantic, it does not mean that some citizens of Yes Islands are in the Atlantic. Hence, the given conclusion is incorrect.

[C] – Some young people are citizens of Yes Islands. Since all citizens of Yes Islands speak the truth, some young people speak the truth. This is correct. This eliminates option (4).

Hence, (3).

47. [A] – Since not all viviparous are mammals, some fishes may be or may not be mammals. Hence, the given conclusion is incorrect.

[B] – Since one of the premises is negative, the conclusion has to be negative. Hence, the given conclusion is incorrect.

[C] – Since no mammal is oviparous, and some

creatures are oviparous, some creatures are not mammals. Hence, the given conclusion is correct. This eliminates option (4).

Hence, (3).

48. [A] – Since we do not know the relationship between the group of people that sing, the given conclusion is incorrect. This eliminates option (1).

[B] – All giants climb beanstalks. Since, some chicken do not climb beanstalks, some chicken are not giants. The given conclusion is correct.

[C] – Since we do not know the relationship of the group of people that live in snowdrifts, the given conclusion is incorrect.

[D] – From the given statements, we cannot infer if Akbar is shorter than Anthony. So the given conclusion is incorrect.

Hence, (2).

49. [A] – Since the relationship between the farmers and snake catchers may or may not be there, the given conclusion is incorrect.

[B] – Since only a few kangaroos are made of teak, we cannot say whether Poonam is made of teak or not. Hence, the conclusion is incorrect.

[C] – Since all matadors eat grass and no bulls eat grass, it can be concluded that no matador is bull. Hence, the given conclusion is correct.

[D] – Skunks are sub-set of polar bears. Since some skunks drive Cadillacs, some polar bears drive cadillacs.

Hence, (4).

50. Let us analyze all options separately.

Option (1) – It is not valid as the middle term does not concur with the first term.

Option (2) – It is not the correct option as the middle term is the conclusion if first and second terms were the arguments.

Option (3) – Contradiction of first and second term rules out this option as well.

Option (4) – This is a valid option as all the conditions are satisfied.

Hence, (4).

51. We will consider and analyze all options separately.

Option (1) – It is not a valid option as all pens, erasers and pencils will form the same set. Conclusions cannot be drawn.

Option (2) – Scales and pens do not figure in the first two terms and no conclusion can be drawn. Thus it is not a valid answer.

Option (3) – III is a logical derivative from the other two statements. And hence, this is the correct option.

Option (4) – All scales are not pens but 'all pens are scales' is the conclusion that can be drawn. Hence, not the correct option.

Hence, (3).

VA-4.2 | SUMMARY AND PARAGRAPH COMPLETION

PRACTICE EXERCISE 1

1. The paragraph uses examples to support the argument that art need not depict what is real any more. Option (3) correctly interprets this point, which doesn't mean that films or art have an 'unreal appearance', as options (1) and (4) claim. Option (2) is too extreme as it claims that the appearance of art has changed 'forever'. Hence, (3).

2. Option (1) incorrectly states that the Greeks knew only two ways of 'producing' art; they knew only two ways of 'mechanically reproducing' it. Option (2) incorrectly implies that the Greeks invented printing. Option (3) wrongly mentions that mechanical reproduction of art increased 'steadily' with time. Thus, only option (4) correctly summarizes the entire paragraph, the two most important points of which are the advent of mechanical reproduction and the significance of printing. Hence, (4).

3. According to the paragraph, the untranslatability of poetry is a myth. Thus, options (1) and (4) are negated as they wrongly state that poetry is untranslatable. Between options (2) and (3), the latter is a better option as it mentions why poetry is believed to be untranslatable. Hence, (3).

4. Option (1) correctly summarizes the points mentioned in the paragraph. Option (2) wrongly states that Confucius was a historian while the paragraph only talks about Confucius's love for history. Option (3) is negated as it suggests that Confucius preserved the Chou dynasty's civilization, which is not implied in the paragraph (the paragraph mentions that the cultural values and norms worked well for the Chou dynasty). Option (4) wrongly relates the cultural values and social norms with the outmoded rituals. Hence, (1).

5. Only option (1) correctly summarizes the given paragraph by incorporating all the important points in it. Option (2) wrongly mentions that the deep spiritual connections between *dhvani* and *artha* developed simultaneously. According to the passage, the deep spiritual connection developed between the communicable word and the thought that inspired it. Option (3) is wrong as it mentions that there is a spiritual connection between the spoken and the communicable word. Option (4) is incorrect as both the *Vakyapadia* and Bhartrhari talk about the spoken word but distinguish it differently. Hence, (1).

6. Option (1) wrongly mentions that the aesthetics of social realism were associated with European art movements. The paragraph does not talk about the freedom of painters which negates option (2). Option (3) correctly summarizes the main points of the paragraph. The paragraph mentions that Abstract expressionism rejected the aesthetics of social realism. But option (4) wrongly implies that Abstract expressionism used social realism but was not restricted by it. Hence, (3).

7. The first sentence mentions that Stipple has set out to provide a solution to stop the loss of metadata. Nowhere in the paragraph is it mentioned that Stipple has succeeded in doing that. Thus, options (1), (3) and (4) wrongly assume that Stipple was successful in doing so. Hence, (2).

8. Albert Camus was known for his philosophy of the absurd and not existentialism. Thus, option (1) is negated. Option (2) correctly summarizes the paragraph. Option (3) wrongly mentions that the novels are a gateway to the philosophy of existentialism. The novels actually explore Camus' ideas of philosophy of the absurd. Option (4) wrongly mentions that Camus was a trendsetter of the philosophy of the absurd which is not mentioned in the passage. Hence, (2).

9. The paragraph mentions that the Large Hadron Collider (LHC) is a 'kind of time machine', not an actual one, so option (1) wrongly calls it a time machine. Option (2) correctly summarizes the important points in the paragraph. Option (3) is also wrong as it is not certain from the paragraph whether the goal of the scientists would be fulfilled or not. It is not mentioned whether the LHC will certainly provide the answers that the scientists are looking for, so option (4) is incorrect as well. Hence, (2).

10. The paragraph talks only about American grocery market. options (1), (3) and (4) talk about grocery markets in general and do not restrict themselves to the American grocery market. Only option (2) incorporates all the points mentioned in the passage correctly. Hence, (2).

11. Option (1) which says that the Sumerian civilization put an end to the nomadic way of life is an extreme statement and thus negates it. (2) wrongly implies that the development of writing enabled it to progress which is not mentioned in the passage. (3) assumes that the Sumerian civilization is definitely the first one to do all these things, whereas the passage mentions that it is 'arguably' so. Thus, only (4) is a correct summary. Hence, (4).

12. The passage does not mention anything about 'latent demand'. Hence, option (2) is out. Option (3) is beyond the purview of the passage. Option (4) presents an idea that somehow contradicts the last sentence of the passage. Only option (1) logically follows from the passage because it carries further the idea of competition in the marketplace. Hence, (1).

13. Option (1) is in no way connected to the passage. Option (2) goes against the pervading tone of the passage, which is gloomy. The letter is in the form of an earnest request, so it will try to project a desperate picture and not draw a conclusion. So option (3) is more logical as a concluding sentence than option (4). Hence, (3).

14. The passage talks of a situation where Arabs have been deprived. Option (1) does not logically follow. Option (3) is not correct because here, we are not talking of any mixing of two cultures. Option (4) is a positive scenario that does not go with the negative tone of the passage. Option (2) continues the line of thought and the word 'also' connects it well with the preceeding information. Hence, (2).

15. The passage talks of allocating personal space to each employee in the form of a separate locker, drawer or desk. Option (1) talks of company policies that have no bearing in the passage. Option (2) is also irrelevant here, as there is no mention of a manual in the passage. Option (3) does not specify which area it wants to mean by the expression 'in this area'. Only option (4) is directly and logically connected to the subject of discussion. Hence, (4).

16. Option (1) does not follow because there is no mention of any particular year in the passage. The comparison to a torture device in the passage helps us to negate option (2), which is a very positive statement. Option (3) is beyond the purview of the passage. Only option (4) logically follows from the passage because it dwells on the negative effects of the two things that are being compared in the passage. Hence, (4).

17. The paragraph puts forth the paradox that people fight over material gains whereas a precious substance like water is not fought for. Therefore, the next sentence should be option (2) which clarifies the paradox by explaining why water is not an expensive commodity and why it is not fought over. Therefore, option (2) is the correct answer. That countries do not fight over water is not the cause for water being found all over the world. Therefore, option (1) is incorrect. Options (3) and (4) imply that the central idea of the paragraph is that countries should not go to war over oil because water is more important for sustenance. This is however not the central idea. Therefore, options (3) and (4) are incorrect. Hence, (2).

18. Option (2) elaborates on the characteristics of the phobia that the earlier sentences have described. Therefore, it is the most natural continuation from the preceding sentence. Option (1) just repeats the characteristics of agoraphobia which have been stated earlier in the passage. Therefore, this cannot be the concluding line of the paragraph. The paragraph is in an informative tone whereas option (3) provides a very stark opinion-based view. Hence, it can be eliminated. Option (4) is counterintuitive as it suddenly reduces the severity of the issue that the paragraph presents. Therefore, it can be eliminated. Hence, (2).

19. The passage talks about people who are dissatisfied with their belief systems. Therefore, when they come in contact with people of different religious beliefs it seems more likely that they will try and seek an understanding of those alternative belief systems rather than criticize or think of such people as misguided. Therefore, option (4) is correct. This is also why option (3) is incorrect. The passage has an inquisitive tone. It does not pass any value judgments. Option (1) is highly critical and is hence wrong. 'Agnostic' means a person who doubts or denies that ultimate knowledge in an area of study (esp. religion) is possible. There is no indication of the people seeking to understand the world being agnostic. Therefore option (2) is incorrect. Hence, (4).

20. The text describes the growing trend of use of the mobile phones for making payments. Therefore, the appropriate sentence to conclude the paragraph would be a sentence which can create an impact by stating the effect of the trends stated earlier.

Option (1) shows the result of the increased use of mobile phones in terms of a large increase in payments. Hence, it is the most appropriate answer. Options (2) and (3) describe supporting facts with respect to the use of mobile phones for payments. However, they do not represent the concluding sentiment of the paragraph. Option (4) mentions how a credit card-free life is still far from reality. However, the tone of the passage suggests a very rapid growth of mobile based payments. Thus, the two have different outlooks. Therefore, option (4) cannot be the answer. Hence, (1).

21. The entire paragraph has a tone which suggests a positive outlook towards the earth's environment and an expectation of enormous change – both in terms of environmental impact as well as the way the world would look. Option (4) comes closest in reflecting this outlook. Therefore, the most appropriate answer is option (4). Options (1) and (3) assume that the prediction is a target to be reached and suggest ways or actions to reach it. However, the paragraph is only trying to enable the reader to visualize a green future and is not suggesting any actions. Therefore, options (1) or (3) cannot be the answer. From the text, it cannot be inferred that the author believes that the goal specified is difficult to achieve. Hence, option (2) cannot be the answer. Hence, (4).

22. The description calls the coverage an exposé – thereby hinting that the author trusts the veracity of the film's contents. We can therefore expect that in the last statement the author will conclude in favour of the activists. Options (1) and (3) are both in support of the activists' point of view. However, option (3) is logically incorrect. On the basis of media coverage, the implementation of laws may be doubted but not their existence of laws. Also, option (1) is in line with what effect an exposé is expected to bring about. Therefore, option (1) is correct. Option (2) is logically inconsistent as activists campaigning against such experiments will not make an appeal for higher funds for those very experiments. Option (4) makes the film look like a commercial production which it was not. Therefore, this is an unlikely concluding sentence for this paragraph. Hence, (1).

23. The sentence before the blank mentions the number of people living in slavery around the world, while the sentence after the blank gives the number of Indians living in slavery. Therefore, the missing sentence must address both, i.e. number of people living in slavery both in India and the world. Only option (3) does this. (Another clue can be obtained from the numbers: 14 million is approximately half of 29.8 million.) Option (1) only addresses the number of people living around the world in slavery. Although options (2) and (4) are relevant to the topic, they do not provide a link between the relevant sentences. Hence, (3).

24. If the performers lost their royal patronage (as mentioned in the second sentence) and had to retain their identities (as mentioned in the sentence after the blank), they would have likely moved to other locations where people did not recognise them. Only option (1) mentions this. Options (2) and (4) talk about the possible effects of the loss of royal patronage but do not explain why the performers had to retain their identities. Option (3) is irrelevant to the subject of the paragraph. Hence, (1).

25. The first sentence talks about the postcards received by Ruma from her father and the sentence after the blank talks about her not having replied to his letters. The remaining sentences explain why she could not write back. Therefore, the missing sentence must refer to this one-way correspondence. All options, except (3), fail to provide a logical link between the opening sentence and the remaining sentences. Hence, (3).

PRACTICE EXERCISE 2

1. Option (1) wrongly mentions that there is nothing in common between philosophy and music. Option (2) wrongly implies that only the common ground between the two disciplines have been studied by both philosophers as well as musicians. Philosophers have been using music in their composition, although in a simplistic way. Thus, option (3) is also negated. Only option (4) summarizes the passage correctly. Hence, (4).

2. Dvorak's musical compositions were opposed as they were not considered the new American voice. Option (1) wrongly states that he was opposed by the Native Americans. According to the passage, many people who considered themselves to be American opposed Dvorak, not the Native American themselves. Option (2) wrongly claims that Dvorak plagiarized (used another's work) American music. Nothing about plagiarism has been mentioned in the paragraph. Option (3) incorrectly states that the songs of the Native Americans and African-Americans were considered to be the voice of the new America. Only option (4) correctly summarizes the paragraph. Hence, (4).

3. Option (1) wrongly implies that new methods of agriculture led to further developments, thus, it is negated. Option (2) wrongly mentions that the rise of the educated middle class was an upheaval in the eighteenth century. Option (3) mentions that the taste of the middle class in music was different as compared to the earlier connoisseurs, but this information is not mentioned in the paragraph. Option (4) covers all the points in the paragraph correctly. Hence, (4).

4. According to the last sentence of the paragraph, Smetacek suspects that plankton could be used to change the Earth's environment. Options (1), (2) and (4) wrongly imply that he has definitely come to this conclusion. Also, option (1) does not mention plankton's abundance or that it is at the base of the global food chain - points that led to Smetacek's conjectures. Option (2) also misses out on the former point. Only option (3) correctly summarizes the paragraph. Hence, (3).

5. Option (2) wrongly implies that human rights advocates consider all the three conditions needed for democracy impossible. Option (3) is an extreme statement as it mentions that human rights advocates think that one cannot obtain freedom from the state. Human rights do not protect freedom but seek the state's protection and security. Thus, option (4) is also negated. Only option (1) correctly summarizes what natural rights and human rights are and the difference between the two. Hence, (1).

6. Only option (1) correctly summarizes the entire paragraph. Option (2) incorrectly mentions that the temperance activists increased excise revenue. However, according to the paragraph, it was the presidency which increased its revenue. Option (3) is also wrong as it mentions that the temperance activists passed the 1878 Act while the 1878 Act was actually passed by the presidency. Option (4) mentions that the presidency 'got around this act' (which means 'to outwit') which is incorrect. In the passage it mentions that the Act provided a way to increase the revenue. Hence, (1).

7. Option (1) changes the meaning of being counterproductive according to the Taoists. Option (3), which does not mention the comparison to the Confucian philosophy, is incomplete. The Taoists did not think that all work was counterproductive, only that work that is against the natural order of things. Thus, option (4) is also negated. Only option (2) is a complete summary. Hence, (2).

8. Option (1) unnecessarily focuses on the comparison of the special genetic code with the qwerty keyboard, though it is only a minor example in the paragraph. The genetic code has not been 'frozen in a special manner'; rather, it is called a 'frozen accident'. Thus, option (2) is negated. Option (4) wrongly mentions that alterations are done by the protein molecules. According to the paragraph, the mutation would affect the codon table, which would in turn alter the protein molecules. Only option (3) summarizes the given paragraph correctly. Hence, (3).

9. X-rays are said to be used as diagnostic tools today, but there is no mention in the passage that they were not used as such in their early days, in addition to their more 'miraculous' uses. So option (1) is not quite a correct reading of the passage. In this passage, it is only stated that some people made claims that X-rays had miraculous abilities (such as showing ghosts or projecting images into the brain). These claims were not necessarily true. So option (4), which suggests that they were true, is incorrect. The passage focuses on the claims made by some people regarding X-rays, rather than what their audience believed, so option (2), which focuses on the former, is a better summary than option (3), which focuses on the latter. Hence, (2).

10. This passage is about odd-looking buildings in general, of which the city hall in Philadelphia is just one example. So option (1), which focuses only on this building, is not a correct summary. The point of the passage is that such buildings may be significant despite not looking very good; this aspect of their looks is not mentioned in option (2) at all, so it can be ruled out as well. Option (3) is not quite correct, as it wrongly suggests that odd-looking buildings are always significant, as opposed to the passage, which merely suggests that they can become significant. Only option (4) correctly summarizes the passage. Hence, (4).

11. The main point of the passage is that human nature supposedly changed because people started carrying smartphones at all times, which meant that they could be reached at any time and any place, and thus had no privacy left. Option (1), which fails to mention the result of carrying smartphones, is therefore not a complete summary. Similarly, option (4), which does not mention the point about human nature changing, is also incomplete. In the passage, the idea that people could be reached at all times and everywhere is framed as a bad thing (see negative words such

as 'intruded'), indicating a lack of privacy. Option (3), which makes this point clear, is therefore a better summary than option (2), which frames this issue in a neutral way. Hence, (3).

12. Option (1) fails to mention mercury, which is one of the liquids discussed in the passage, so it can be ruled out at once. Option (4) incorrectly states that liquids with low surface tension roll off a surface – rather, that is the case with liquids with high surface tension. The classification scheme in option (2) does not have a basis in the passage (which talks of a 'spectrum' rather than 'types'); also, it does not explain the reason for the difference in surface tension. Only option (3), which correctly explains this and summarizes the main points of the passage, is the correct option. Hence, (3).

13. Here the paragraph presents a contrast in the administration of George W. Bush and Obama. Thus, the last sentence should present the contrast and the reason for Obama's strategy. Therefore, option (4) is most appropriate as it captures both the points needed for a statement to be an apt concluding statement. Option (1) is incorrect as it is a very extreme conclusion of the passage and cannot be the true motive behind the Obama administration's measures. Option (2) cannot be correct as a reversal in the American approach cannot lead to the restoration of peace in Iran. In the given text, the US government has taken steps to engage Iran diplomatically. However, option (3) is giving a contradictory idea when it says that the government is funding some anti-Iranian institutions. Therefore, option (3) is incorrect. Hence, (4).

14. The passage talks of the drastic decrease in the population of Europe and America. The population is low among the ethnic groups. The next sentence logically should bring in the immigrants into the picture because these are the two sources for increasing the population. Hence, (2).

15. Option (3) is negated because it is beyond the purview of the passage. Option (4) is negated because the volcano is supposed to erupt at intervals of 20 to 200 years according to the passage. Therefore the passage does not give a definite reason for the occurences but indicates a probability. Option (1) provides additional information which may or may not be true. Since the last sentence of the paragraph talks about the huge rock being unstable, the next sentence would most likely be option (2). Hence, (2).

16. Option (1) cannot be the answer as it merely restates what is spoken of in the first sentence of the passage. Option (2) talks of switching from one commodity to a better one but there is no contradiction here. Option (3) is not correct because, in the option at least, there seems to be no unpleasant situation. However, option (4) is correct, as it sums up the point of the passage by stating that such matters are always subjective. Hence, (4).

17. The third sentence of the paragraph refers to the uniqueness of the story i.e. the antagonist and the protagonist being the same. Therefore, logically, what follows should elaborate on this. Option (2) clearly elaborates on this aspect and is hence the correct answer. Option (1) conveys that the antagonist is not known or evident. This is not logical if the audience can see that the protagonist and antagonist is the same person. The reference to Nash in option (3) assumes that the reader already knows that the protagonist and antagonist are personas of Nash himself. This is however not correct as none of the earlier sentences suggest this. Therefore, option (3) is inappropriate. Option (4) is contradictory to the preceding sentence stating that the story has a 'commonplace approach'. Therefore, option (4) can be eliminated. Hence, (2).

18. Option (2) clearly states the trend in the video game industry as a whole and therefore concludes the paragraph. Hence, option (2) is correct. People complain because they think that video games encourage violence, not because their market is stagnant. Thus answer option (1) is incorrect. Option (3) is contradicted by the passage. The passage says that gamers are devoted. Thus option (3) is incorrect. The passage says nothing about the ballet market. Ballet is only mentioned as an example to show that non-gamers often complain of gaming. Thus option (4) is incorrect. Hence, (2).

19. Option (2) links to the sentence in the paragraph which states the various things that new generation electronic storage devices are expected to do. Also, it gives a good conclusion to the paragraph. Hence, option (2) is correct. Option (1) states information that cannot be deduced from the passage. Hence, option (1) is incorrect. Option (3) is not a point in favour of the new devices, which does not fit into the flow of the paragraph, which is very positive about the new devices. Thus option (3) is incorrect. The introduction of CDs in option (4) does not fit as a suitable conclusion to the paragraph. Thus, option (4) is incorrect. Hence, (2).

20. The last sentence talks about the desire of humans to bring things closer and to accept reproductions of unique objects. Thus, option (1) is the best option as it combines both wishes of humans and mentions that the urge to fulfil them is getting stronger. None of the other options connect meaningfully to the last two sentences of the paragraph. Hence, (1).

21. Option (2) is negated because there is a confusion here and Ekman could not be sure. Option (3) is negated because it does not show any confusion. Option (4) is negated because the passage is about Ekman's thoughts. Hence, (1).

22. Option (1) is negated because there is no connection between the Caribbean Islands and the Bars of Eastern Europe. Option (3) is negated because the topic is about drinks and not about architecture. Option (2) is negated because bars serve alcoholic drinks and not fruit juices. The contrast between the last sentence and option (4) – 'they are not unusual' but instead 'unfamiliar' gives a proper link to the sentences. The word 'tipples', meaning alcoholic drink also confirms the link. Hence, (4).

23. The paragraph talks about diesel engines and their reputation for being smoky and smelly. Also, it mentions that diesel engines used in luxury cars are quite clean. So, option (3), which compares the diesel engines of larger vehicles with the diesel engines used in luxury cars, is an appropriate completion of the paragraph. Option (1) is incorrect, as it compares the cost of petrol and diesel and is therefore not related to the paragraph. Option (2) starts with the word 'however', which suggests that it should contradict the parapgraph, but, a preference of diesel cars over petrol cars supports the paragraph, not contradicts it. Hence, option (2) is incorrect. Option (4) makes no sense, as the consumers' problem with diesel vehicles is not that they lack spark plugs, but that they are supposedly smoky and smelly. Hence, (3).

24. The given paragraph discusses the improving market conditions. It also states that some companies are still hesitant to spend cash. Therefore, the last sentence should delve into the reason as to why these companies doubt the strength of the recovery. Thus, option (3) is the most appropriate answer. Option (1) is a mere repetition of the idea stated in the paragraph. Options (2) and (4) show that the companies are confident about the market, which goes against the idea of the companies being cautious in their expenditure. Hence, (3).

PRACTICE EXERCISE 3

1. The idea that there is no risk of going out of business in the world of consumable products and services, as stated in option (4), finds no mention in the passage. So option (4) can be ruled out at once. Option (3) does not state why the sale of consumable products and services is self-sustaining (and why the sale of durable goods is not), so it is not a complete summary; additionally, it claims that the sellers of durable goods may go out of business, while the passage states it as a certainty. The passage focuses on the sale of goods and services, not their production, so option (1) is incorrect, and only option (2) is a correct summary. Hence, (2).

2. Option (3) mentions 'casting the opposite as weaker than it really is' which is not a 'very effective strategy'. This is borne out by the rest of the paragraph. Therefore, option (3) is logically the correct answer option. The word 'fallacy' in sentence 3 of the paragraph indicates that 'casting the opposite in feeble light' is not correct or effective thus ruling out options (1) and (2). Option (4) is logically inconsistent for this blank. It is ideal for an ending to the paragraph. Hence, (3).

3. The section of the paragraph before the blank defines 'deliberation'. After the blank, it states that 'in the light of the other argument', thus indicating that the correct answer option should be about arguments being presented. Among the given options, only option (1) mentions how the arguments are presented- i.e. most sincerely. Options (2), (3) and (4) do not mention the quality of arguments required for the blank. Therefore, they can be negated. Hence, (1).

4. The section of the paragraph before the blank mentions animal rights being annoying as millions of animals are killed for human use, the benefits accruing to medical research are also mentioned. After the blank, the author states that 'super intelligent beings could make similar use of humans' thus drawing a comparison. Hence, the blank should initiate and lead to this conclusion. Among the given options, only option (2) inquires about the argument being fair to animals vis-à-vis animals-humans and humans-super intelligent beings. Hence, (2).

5. The paragraph mentions the 'eye of the mind' and a 'mechanical world' based on fact and reason. After

the blank, it states that 'so today more and more people are opening the other eye' thus indicating that the correct answer option should be about the disadvantage of the 'eye of the mind' which is making people open the other eye. Among the given options, only option (2) highlights a disadvantage of the 'eye of the mind' by mentioning that the mind-made world is dangerous. Hence, (2).

6. The paragraph in the end says "he was seeking honour where honour was not to be found, and looking for pleasure....." According to the style of writing of the author, Briggs' friend would go to places to find pleasure where none lay in store. Hence, (4).

7. The paragraph talks about how Byki helps foreign language learners in mastering new words. The last sentence of the paragraph talks about tracking words that need the most work. Only option (1) completes the paragraph meaningfully by mentioning that learners get more exposure to difficult words. Option (2) talks about the association between a word and its meaning and emphasizes how the meaning must be conveyed. These ideas are not discussed in the paragraph. Option (3) and (4) talk about forming sentences, which is too premature as the paragraph focuses on mastering vocabulary. Hence, (1).

8. The paragraph talks about how we learn language as a child and how it becomes second nature. Option (1) talks about second nature 'reactions', which are out of context of the paragraph. Option (3) talks about 'precise and vague' language which is also out of context. Between options (2) and (4), (2) continues the same train of thought as the penultimate sentence, while option (4) talks about 'communication', that too the 'difficult' kind, which the paragraph does not discuss. Hence, (2).

9. The last sentence of the paragraph asserts what the best level of self-efficacy is. So, a sentence that supports this assertion is now required. From that perspective, (3) is a better option than (1), as the former provides some important benefits and especially since the penultimate sentence talks about growth and development (and option (3) talks about gaining experience). Option (2) unnecessarily makes a comparison with people with low self-efficacy. Option (4) cannot really be inferred from the last sentence of the paragraph. Hence, (3).

10. The paragraph describes the physiological changes in the author when he is hunting. The last sentence talks about the state of his mind. So option (4) is the best option. Option (1) mentions a realization that the author has already had long back. Option (2) talks about 'meditation', which has not been talked about so far. Option (3) draws an unnecessary comparison, and is not about the author's state of mind. Hence, (4).

11. The paragraph is about how a poet's function is different from a historian's. The last sentence describes what a historian should do, specifically with respect to his study of events, so the next sentence should describe what a poet should do. Only options (3) and (4) talk about the poet. Option (3) talks about the poet's analysis of the historian's materials, not how the poet goes about his own work. Option (4) is appropriate as it describes what a poet must do about the events he studies, and it uses a suitable connector ('on the other hand'). Hence, (4).

12. The paragraph is about trying to answer the question of whether or not poetry gives us propositional knowledge. Options (1) and (2) in no way help us to know more about propositional knowledge; rather, they talk about developing new skills like writing poetry or learning to interpret a poem. Option (4) talks about 'subjective experience' which is not mentioned in the paragraph at all. Only option (3) completes the paragraph by addressing 'propositional knowledge'. Hence, (3).

13. In the paragraph, the author is trying to see whether poetry can be used for writing philosophy. The last sentence poses a question that only option (2) answers. Hence, (2).

14. The last sentence talks about values changing with the changes in social groups. Option (1) rightly concludes that 'meanings' depend on domination and power, as the social group that has the power to re-evaluate and change values ends up determining 'meanings'. Hence, (1).

15. The paragraph talks about the popular perception of war in Europe before 1914: it was similar to a duel. Britain stood apart from other European nations in that it substituted duelling with cricket. Option (2) appropriately states the qualities of men that are evident in the last sentence of the paragraph and that ultimately led to war. Option (1) talks about the male code of conduct a century later, which may start a new paragraph but should

not conclude the given paragraph. The information contained in option (3) doesn't contrast with the last sentence, so 'yet' is unwarranted. Though option (4) talks about the war, the reasons provided for the war are not related to the paragraph at all. Hence, (2).

16. The paragraph talks about how Socratic culture replaced the tragic Greek culture with one filled with reason. The last sentence of the paragraph carries this same line of thought. As all options begin with 'thus', we need a conclusion for the paragraph. Only option (1) correctly concludes the paragraph by talking about the change in culture and giving due importance to 'reason'. Option (2) talks about Socratic culture being the formative force of the modern period. But this is beyond the purview of the paragraph. Option (3) mentions 'modern rationalism' and 'Enlightenment optimism', which too are beyond the purview of the paragraph. Option (4) undermines the acceptance of Socratic culture by making it seem to be just an alternative rather than a replacement. The description of tragic Greek culture is also a bit over-the-top. Hence, (1).

17. The passage is about the romantics' focus on nature and their contribution. Thus, the concluding sentence must follow the same line of thought. Option (1) is not ironic based on the information given in the passage. Option (2) abruptly moves on to a completely new point and 'similar means' has no antecedent in the last sentence of the paragraph. Option (3) abruptly talks about the end of the romantic movement. Option (4) continues the paragraph by talking more about the romantics' contribution. Hence, (4).

18. The paragraph is about poetry predating literacy. The last sentence mentions priestly incantations. Thus option (3), which also mentions 'scriptures', is the best option. Option (3) supports the information given in the last sentence of the paragraph. Option (1) is repetitive. Option (2) could be imagined as continuing the paragraph if the author has no more to say about liturgy; but in the presence of options (3), (2) is unsuitable. Option (4) talks about 'these forms of poetry', which has no antecedent in the paragraph. Hence, (3).

19. The last sentence of the paragraph talks about how the shape of atoms affects the texture of substances. Thus, only option (4), which talks about the relative density and fragility of materials also getting affected ('by the same means'), concludes the paragraph properly. Option (1) talks about 'division', which is not discussed in the paragraph. Option (2) states a very specific case (when atoms are separated by a void and cannot fuse), which is different from the case discussed in the paragraph (where atoms do bond). Option (3) is too philosophical a take for this technical paragraph. Hence, (4).

20. According to the last few sentences of the paragraph, when we do not expect swear words in some settings, they stand out. Thus, option (3), which mentions that a change in the context in which taboo words occur can change the way we experience them, completes the paragraph suitably. Option (1) is too extreme as the paragraph has just established the emotional impact of taboo words. Option (2) talks about the varied meanings and impact of taboo words, which the paragraph doesn't talk about. Option (4) talks about why we use taboo words, which is not covered by the paragraph. Hence, (3).

21. The clue is the word 'affair' in the last sentence. It likens the author's relationship with a book to a relationship between two people. Only option (3) continues on the same lines and provides a conclusion to the events that take place in the last three sentences. This makes it a better choice than option (1), which is also in a different tense. Option (2) repeats what has already been said in the paragraph. Option (4) blames the Internet for reducing the attention span of the author, which is out of context of the paragraph. Hence, (3).

22. The penultimate sentence talks about writing differently to different people. Option (2) continues in the same vein as the paragraph and talks about having fun while sending a postcard home. Options (1) and (3) don't relate to writing a message on a postcard. Option (4) does, but it is not a conclusion that follows from the paragraph. Hence,(2).

23. The last sentence talks about the beginning of the Vietnam War but specifically relates it to America. Option (1) is a twist in the tale and abruptly talks about the general Vietnamese belief. option (2) also is from the Vietnamese point of view, whereas the entire paragraph is from America's. Option (3) presupposes opposition to the war, whereas it has been introduced in option (4). Option (4) is from the American point of view and talks about events that took place at the same time as the beginning of America's Vietnam war. American's opposition to the war can be expected after reading the penultimate sentence of the paragraph. Hence, (4).

24. Only option (2), which brings out the symbiotic relationship between life and death, meaningfully completes the paragraph, as the last sentence talks about death enabling life. Option (1) changes the topic from frogs to *Homo sapiens*. It should be the opener of a subsequent paragraph. Option (3) merely repeats the point made in the paragraph. Option (4) is too critical to follow the paragraph. Hence, (2).

25. Option (1) talks about social functions in general while the paragraph talks about the social functions of artwork. The last two sentences of the paragraph highlight the importance of import, so, option (2) is an appropriate option. Option (3) can be negated as it claims that social functions are antithetical (opposed) to society, which is self-contradictory. Option (4) just rephrases the third sentence of the paragraph. Hence, (2).

26. The entire paragraph is about readers becoming writers. Thus, option (1) (which mentions that the distinction between an author and the public is blurring) is correct. Options (2) and (3) are about concepts that have not been discussed in the paragraph i.e. polytechnic/specialized training and progressive/reactionary manner. As the paragraph doesn't talk about readers' reactions, option (4) is also inappropriate. Hence, (1).

VA-4.3 | CRITICAL REASONING 1

PRACTICE EXERCISE

1. The author has compared the salmon and mackerel to the average people in society while the shark and whale are the powerful and moneyed members. He talks about the ease with which the whales and sharks get away by breaking through the net i.e. by bypassing the rules. Therefore option (4) is right. Options (1) and (2) are too extreme, and (3) is out of context. Hence, (4).

2. Both options (2) and (3) are true and follow from the passage. Hence, (4).

3. The author says that we need to develop our creativity more and concludes that as a country, we are not as innovative as we could be. From this we can say that the author assumes that creativity is necessary for being more innovative. Hence, (3).

4. Neither option (2) nor (4) has any basis in the paragraph. Option (1) is the author's premise. The last sentence of the paragraph '...shouldn't they know little more?' suggests that option (3) is the conclusion. Hence, (3).

5. The clue lies in the last sentence. To address corruption, it is important to target the various departments of municipal corporations. This means that corruption is prevalent at these levels. And that the common man stands to benefit from the same as he interacts with individuals at these levels rather than politicians. Hence, (2).

6. If the quantity of pesticides used in Coke and Pepsi is considered safe for human consumption, then their use is justified. This may not be so in case of the use of human bones in Ramdev's ayurvedic medicines. Hence, (3).

7. A journalist records day-to-day life, while a historian focuses on broader currents of change in society and the individuals involved. According to the passage, Herodotus recorded both. Hence, (1).

8. The statistics show only an increase in rate of the diseases, but we cannot ascertain the fact that more women suffer from such diseases. Hence, (1).

9. Options (2), (3) and (4) all explain how the amount of food production per worker per hour has increased over the years. Only option (1) cannot account for this as the total number of workers has no relation to the amount produced by each worker. Hence, (1).

10. From the last sentence of the passage, we can infer that the same situation may not affect all people in the same way: 'it depends on the person ...' So we cannot infer a universal statement such as option (4). Hence, (4).

11. We can first eliminate alternatives options (2) and (4). Both of them at best suggest difficulties in the proposed method of generating energy. But they do accept the conclusions that energy can be generated and put to use and electricity bills can be reduced by such a method. In case of option (3), although the reduction in bills is marginal it is still better than nothing. But if the power so generated would be expensive to transmit, then the author's assertion that the electricity bill would be reduced will not be necessarily true. Hence, (1).

12. Option (2) talks about students finding computer courses uninteresting but there is no reason to believe that it would affect the quality of teaching. Options (1) and (4) give reasons for low quality teaching in general. A technical subject like computers requires proper infrastructure. If that is missing it could be a major reason behind the underachievement in teaching computing in schools. Hence, (3).

13. Since the passage talks of a win-win situation for all, look for an option which talks about a gain for everybody involved – the designers involved, the targeted customers and the big companies. Options (1) and (4) are therefore clearly ruled out. Option (2), which mentions a win for the customers as well as the companies, is a better fit than option (3), which mentions a win for only the customers. Hence, (2).

14. The opponents of the law claim that as long as people do not harm others, they should be allowed to take risks with their own lives. So, any reason that would induce them to accept the law would have to show that people who risk their lives - in this case, heart patients who participate in adventure sports - do indeed cause harm to others. Both options (1) and (3) describe only instances of harm to the participants themselves. Option (2) is not an instance of harm - the insurance companies would simply be doing their job. Hence, (4).

15. There is no suggestion that either option (1) or (4) is true. The ecology of the dam site is not mentioned either, so option (2) is incorrect. Only option (3) can be inferred from the paragraph. Hence, (3).

16. Option (1) acknowledges the risk involved and the delay in normal motor development and therefore does not question the decision effectively. Option (3) prescribes what would be ideal but does not question the decision either. Option (2) points out a benefit gained from walkers and therefore questions the banning. Hence, (2).

17. Options (2) and (3) presume what is not detailed in the passage. Option (4) is much too conclusive. But option (1) is clearly the basis for why Singapore is seeking a change. Hence, (1).

18. Option (1) answers only a part of the question. Option (2) does not explain why the two broods looks different. Option (4) does not explain the fact that the second brood eat leaves but look like twigs. Option (3) is correct as it explains the phenomenon in terms of season-specific camouflage. Hence, (3).

19. The author compares pearls to apples, in order to show that artificially grown pearls – i.e. cultured pearls – are not fake pearls, the same way that artificially grown apples – i.e. ones grown in an orchard – are real apples. So option (3) best sums up what the argument is trying to prove. Options (2) and (4) are the general viewpoint that the author is arguing against, not for. Option (1) has no basis in the passage – the author simply claims that both artificially and naturally grown things can be equally authentic, not that one is 'more authentic' than the other. Hence, (3).

20. The argument in the given passage is that all life has a common origin. The similarity of the genetic code is taken as a major piece of evidence supporting this conclusion. If a form of life were discovered that had a very different sort of genetic code than the rest, this evidence would not support that conclusion, and would in fact cast doubt on it. Hence, (1).

21. According to the passage, conifers do grow very well in India (on plantations), so options (1) and (2), which suggest that they don't grow well at all, are not very likely explanations. The issue in the passage is the lack of native – i.e. naturally growing – conifers in India, whereas option (4) deals with trees that are deliberately planted, so it is irrelevant to the question. The most likely answer is option (3): if conifers require a regular water supply to grow well, they would not grow naturally in a country where it rains only some months of the year – but they would grow quite well on plantations, where they would presumably be watered regularly. Hence, (3).

VA-4.4 | CRITICAL REASONING 2

PRACTICE EXERCISE 1

1. Options (1), (2) and (3) are outside the scope of the passage. The passage suggests that there are similarities in the way computer and biological viruses spread. Computer viruses get restricted to a small group since the individuals in the group have the same people as neighbours. Hence, we can logically infer that even in the case of diseases, if the contact is mainly local, they will spread slowly. Hence, (4).

2. Options (1), (2) and (4) all show how new forms of technology have changed the nature of media and its dissemination, thus leading to conflict between the two. In spite of the word 'fight' in option (3), it does not illustrate media and technology being in conflict with each other. Hence, (3).

3. The argument of the passage is that the notions of group ethics were the foundation for a stable society. Options (1), (2) and (4) do not shake the validity of this notion per se. Even if the ideas were founded on notions of hierarchy and served the rich, they do not as such affect a stable society. It is only with option (3) that the effects of this as seen in social instability weaken the notion of group norms being conducive to stability. Hence, (3).

4. Option (1) is not the correct answer as most of the instances picked out by the author in the passage benefit one sector while depriving another. However, option (1) does not deprive the sector for which the reforms are intended. For the same reason, option (3) is also ruled out. However, option (2) is like the situation already discussed in the passage. Hence, (2).

5. Option (1) is a serious problem, since the target consumers are people who do not have access to opthalmologists. The phenomenon mentioned in option (2) would seriously affect the spectacle wearer's vision. Option (3) mentions yet another source of unnecessary trouble for the wearer. Therefore all these can affect the market potential of Adapative Spectacles. Hence, (4).

6. Option (1) can be negated immediately, as it actually supports the argument that current weather forecasting tools are unreliable and cannot predict weather accurately. Option (2) is irrelevant, as the novelty of a technology doesn't necessarily imply that it is accurate. Option (4) talks about only a decade, which is a very small fraction of the time period under consideration, viz., 100 years. However, (3) highlights the fact that such a forecast is made using all available data with all possibilities factored into the calculation. Thus, it is quite likely that the forecast would be accurate – which serves to weaken the author's argument that the technique is unreliable. Hence, (3).

7. The key is to look for the option that reverses the attitude expressed in the passage – that farmers have failed at their job. Options like (1) and (3) are peripheral to the issue. Option (4) merely states that farmers are not the only culprits, but that doesn't mean that they are not at fault, nor does it change the fact that growing food is not economical. Option (2), however, gives a comparative analysis of how much more profitable farming is than tourism, which suggests that farming is not going into a loss and is not a burden to the economy. It can be concluded that the land is better used for farming than tourism. Hence, (2).

8. The argument of the passage is that followers, for various reasons, opt to follow the leaders all the time and do what others want them to do. Option (2) talks about specific examples, which may not be true of the general follower-leader

relationship. Option (3) is incorrect as the passage is not about past vs. present followers, but about the general nature of followers - to follow leaders all the time. Option (4) does not undermine the argument, as it only changes the definition of who is a follower - 'lines dividing...is blurring'. Only option (1) weakens the argument, as it contradicts the idea that subordinates only do what leaders want them to do. Hence, (1).

9. The passage states that companies focus on minor innovations instead of major innovations; however, these fail to achieve revenue goals that companies seek. Options (1) and (2) are a rephrasing of the last line and first line of the passage, respectively. The first half of option (4) cannot be concluded from the passage. Only option (3) can be concluded from the passage. Hence, (3).

10. 'The dollar firmed but pared earlier gains against the Euro.' This implies that a Republican sweep would raise the dollar to an advantage against the Euro. Hence, (1).

11. Read the last sentence carefully. Options (1) and (3) do not ensure that the individual will not have to file returns. Only option (2) does so. Hence, (2).

12. People would opt for adversarial machinery in spite of the drawbacks because it is a readymade solution – they do not have to hunt for solutions. Hence, (3).

13. An assumption is stated based on the passage which states that the volume of air travel between Delhi and Mumbai would increase owing to reduced air fares on the three airlines. The passage mentions the increase in volume of air travel by reducing air fare. Option (1) does not add credence to the general belief presented in the passage. Option (2) might appear as the correct one, but the passage is based on an assumption for a particular air route. Had the passage been based on concrete numbers, this option would have been the correct one. Option (3) connects with the belief presented in the passage. As stated, if the air travellers in India are price-conscious, a reduction in air fare would result in increased volume of air travel as the hindrance – the cost of air travel – would be eliminated. Option (4) nullifies the assumption stated in the passage. If 80% of the passengers have their travel sponsored by their companies, the volume of air travel would not be affected much, as the main gainers of the fare cut would be the 20% individual travellers. Hence, (3).

14. It is mentioned in the passage what was expected of a Brahmin priest. The passage also mentions the fact that the Vedas were memorised by Brahmins and that there was no written reference for them for quite some time. Option (1) is incorrect as it is mentioned in the passage that the same was 'expected' of a Brahmin priest and hence was not an obligation. Option (2) surmises the main thought presented in the passage as the Brahmins passed on the Vedas verbally to future generations for a considerable time. The Vedic priest can thus be referred as a recorded audio cassette. Option (3) might come across as the correct option, but it is too vague. McNeill did study specific pattern of Brahmin priests, but not their behaviour in general. Option (4) has been stated in the passage. The question asks us to find an inference. Thus, this is an incorrect option. Hence, (2).

15. The gist of the passage is to ensure that the people have enough savings when they grow old as the state doesn't have any support mechanism for the elderly, owing to financial crunch, and the elderly have lesser avenues to seek help due to the crumbling joint family system. Option (1) is not related to what has been stated in the passage. The passage mentions the relation between the insurance sector, the collapse of the joint family and personal savings. It does not mention the growth or development of the insurance sector. Thus, option (2) is incorrect. Option (3) weakens the argument presented in the passage. If,

according to the option, the country moves to a developed status, then there will be support from the government through insurance companies to help the elderly. The premise presented in the passage gets defeated in that case. Option (4) will strengthen, rather than weaken the case. Hence, (3).

16. The passage mentions the relation of biodiversity with educational performance and poverty. It states that educational performance is directly related to the level of biodiversity in the region and that poverty can be ignored while establishing this relationship. Higher the biodiversity, lower is the education performance and lower the bio-diversity, higher is the education performance (ignoring poverty levels). Options (1), (2) and (3) negate this relationship. Option (4) correctly identified the relation between the level of biodiversity and educational performance. It goes out to state that despite levels of poverty, educational performance is better in regions where there is low biodiversity. Hence, (4).

17. Option (1) is an assumption and is not the central idea of the passage. Option (2) has no relation to the passage. Option (3) is a exaggeration - the free-rider problem is only one of the problems associated with giving employees stock options. Option (4) is the main point the author is trying to make, as he mentions several flaws in the practice of offering employees stock options. Hence, (4).

18. According to the passage, reductionists believe that everything about nature and human beings can be explained in material terms. So option (1) is a mistake they are likely, not unlikely, to make. Option (2) can be inferred from the last sentence of the passage: if genes predetermine culture, then human culture can be called a 'natural fallout of man's innate tendencies'. Reductionists, who believe that everything has a material basis, would be unlikely to believe in something as spiritual as the soul, so option (4) is also wrong. But on the same grounds, option (3) is unlikely - spirituality is not something associated with reductionists. Hence, (3).

19. The legislation is meant to protect the rights of independent businessmen. This would not become invalid on account of its misuse by a few as suggested by option (1). Laws are frequently misused, but this does not invalidate the law itself. Again, the law is not meant for the peasant class, so option (2) is irrelevant. Option (4) does not necessarily invalidate the legislation, as we cannot tell what the police will do on their own initiative. Only option (3) is a situation in which the amendment may become meaningless - if the Communist Party can unilaterally decide whether any particular piece of property is 'legally obtained' or not, then the entrepreneurs have not gained much from this legislation. Hence, (3).

20. Even if organic farming has no specific benefits, that does not mean it is bad, so option (1) is wrong. Just because an alternative to organic farming exists (as in option (2)), it does not cast organic farming in a negative light. Option (3) merely states that it is difficult to say what, if any, negative effects organic farming will have. Only option (4) suggests a serious ill-effect. Hence, (4).

PRACTICE EXERCISE 2

1. The theme of the passage is that democracy cannot exist in a two party system since politics is not a sport. We need to find the option that strengthens this argument. Option (1) will weaken rather than strengthen the argument. If a two party system functions well, it will be at variance to what is mentioned in the passage. Therefore, it will not strengthen the argument. Option (2) will not strengthen the argument. If politics is a dirty game we do not know whether this will strengthen the argument as we do not have sufficient data from the passage to be able to arrive at a conclusion. Option (3) will strengthen the argument. The passage mentions that a two party system is not good for democracy and option (3) corroborates that. Hence, (3).

2. The theme of the passage is that democracy cannot exist in a two party system since politics is not a sport. Option (1) will weaken the argument. If politics is like sport, then it is at a variance to what is mentioned in the passage. Option (2) will strengthen the argument rather than weakening it since it is in consonance with the passage. Option (3) is not related to the passage in any way. We do not have sufficient data to be able to validate this statement. Hence, (1).

3. Options (1) and (2) are incorrect as they are in consonance with the theme of the passage. If music is the life of man and life equals civilization, then it follows that music is necessary for civilization – as stated in the passage. Hence, they do not weaken the given argument. Option (3) does weaken the given argument. If there is no relation between art and civilization – as mentioned in the option – then it is in variance with what is mentioned in the passage. Hence, (3).

4. Option (1) does not strengthen the argument. In order for civilization to exist, people must be vibrantly alive through music, dance and art. If music, dance and art are merely human activities and the vibrancy of life is missing – then according to the passage it will not lead to civilization – as vibrant life is essential for civilization. Option (2) strengthens the argument by linking the contribution of vibrant people to civilization. Option (3) does not strengthen the argument. If music injects new life in man it does not necessarily follow that it will lead to civilization as for civilization to occur, people must be vibrantly alive. Option (3) is silent on the 'vibrantly alive' part. Hence, (2).

5. Option (1) is incorrect as it is logically in line with the main idea of the passage. Hence, it does not weaken the given argument. Option (2) is incorrect as it is nearly in line with the main idea of the passage. According to the passage social justice and economic prosperity go hand in hand which is what is mentioned in this option. Hence, it does not weaken the given argument. Option (3) is weakens the given argument by negating the point that 'development can be planned' – as mentioned in the passage. Option (4) merely adds more data and does not weaken the main data. Hence, (3).

6. The passage mentions raising the standard of living of millions and that economic development is inspired by social justice. Option (1) strengthens the argument as it directly links the social justice to raising the standard of living (or economic development). Option (2) is incorrect as 'planning for every state' is not connected to the theme of the passage. It will not strengthen the argument in any way. Option (3) is incorrect as 'increase in production' is not connected to the theme of the passage. We do not know whether this measure will lead to a raise in the standard of living of our teeming millions as the passage has not mentioned production at all. Hence, (1).

7. Option (1) is correct since 'if generation of hydroelectric power will be costlier than oil', then it would not be a logical alternative to oil. Note that an alternative source of energy is being sought due to the high price of oil. This source should be cheaper than oil. Hence, option (1) weakens the argument. Option (2) will strengthen rather than weaken the argument. If oil prices rise further it would be more imperative to find alternative sources of energy that are cheaper. Option (3) is incorrect as it not a relevant statement in the given context. It does not weaken the argument in any way. Hence, (1).

8. The passage mentions dedicating one's entire life to understanding and perfecting the language of another country. In option (4) it states that you need to first understand your own countrymen, language and literature. Then only can you appreciate another country's language and culture which undermines the argument present in the passage. Therefore, option (4) is the correct answer. Options (1) and (2) do not relate directly to the argument while option (3) mentions only language and ignores people and literature. Hence, (4).

9. The passage mentions the writer being productive and writing well when he has abundant experience and constantly seeks new experiences. Options (2) and (3) mention 'meeting people' and going to 'new places' which strictly speaking do not constitute 'experience'. Option (1) supports the passage by mentioning that by fresh experience, a writer will be able to get ideas. Hence, (1).

10. The passage mentions that private property if permitted within limits can be good for society. Options (2) and (3) outright condemn working towards the common good of humanity, which is not expressed in the paragraph. The paragraph is about private property. Option (4) cites an example by mentioning the USA, which is not related. Only option (1) mentions that private property does bring about a tremendous disparity which weakens the argument in the paragraph. Hence, (1).

11. Both statements 1 and 2 are mentioned in the paragraph and grade O mental capacity would mean a mark of 15 out of 26, where 26 is the lowest and 15 is lesser than the average 13 score. Hence, (4).

12. The author mentions new traits in a person whom you knew very well and which can be surprising. Option (1) mentions men being inconsistent which is not what the author projects. Option (2) mentions men being unpredictable, again this is not the idea that the author wants to convey. Option (3) mentions about the unknown facets of a person, which when manifested can be surprising even to those who know him well. This option summarises the paragraph smoothly and in a succinct manner. Hence, (3).

13. The passage mentions the stalling of various infrastructure projects owing to disconnect between groups of planners. Option (1) cannot be inferred from the passage. Stalling of the monorail project was a result of other options that came up. Option (2) might appear as the correct answer, but there is no debate or discussion between the two groups of planners that is mentioned in the passage. On the contrary, the projects were stalled as the two groups never consulted each other. Option (3) is the correct option as the two groups of planners mooted their own ideas which were stalled due to the other group. Hence, there are projects mentioned in the passage that would be stalled indefinitely. Hence, (3).

14. The passage mentions abolition of the concept of hierarchy and structural superiority in the organisation. It mentions the idea of fostering an open environment in the organization, wherein even a junior employee can question the CEO. Option (1) is incorrect. We cannot infer that the company encouraged this step because it was concerned about its reputation among its employees. Option (2) is the correct option as it follows directly from the passage. "Designations are forgotten during these meetings..."Option (3) is incorrect as the idea

mentioned in this option (inter-personnel problem) has not been mentioned in the passage. Hence, (2).

15. The passage mentions antique dealers who source antiques from remote areas in India and sell them abroad for a hefty profit. The locals from whom the antiques are sourced have no idea of the importance of their antiques. This activity is thriving due to activist disunity as well as local indifference. Option (1) follows directly from the passage as it clearly mentions the blend of activist disunity and local indifference and how this is enabling the dealers to thrive. Option (2) mentions a different perspective (only Indians) which has not been supported in the passage. Option (3) is a fact stated in the passage and is not an inference from the passage. We are looking for an inference from the passage. Option (4) is incorrect as the market mentioned in the passage is not in India, but in Western countries. Hence, (1).

16. The passage mentions the contention of the country's self-appointed moral police who believe that the moral fabric of India is being ruined by certain forms of cinema. Option (1) might appear to be the correct answer but it is a fact presented partially in the passage and is not an inference from it. The word 'vulnerable' is too strong a word to use in this context. Option (2) follows from the passage as it is mentioned that "...moral fabric must be protected and defended against..." This implies that the Indian audience is impressionable and therefore must be protected. Option (3) might appear to be the correct answer but it is not an inference based on the facts presented in the passage. We cannot infer from the passage that the moral police thinks itself to be the 'sole authority' as mentioned in this option. Option (4) is incorrect as there is a correct option available among the other three options. Hence, (2).

17. The premise that the passage states for universal peace is that everyone should be equally rich so as to counter any misgivings that arise from difference in levels of wealth. The passage also identifies that the rich are aggressive against the poor as they feel insecure and are afraid of the poor. Option (1) highlights what is mentioned in the passage about the reason for aggressiveness of the rich against the poor as it is mentioned '... their aggressiveness stemmed from fear...' Option (2) is a contradiction of the thought presented in the passage. Hence, (1).

18. The author discusses two types of travellers, The first traveller has a desire to see as many objects of interest as possible when visiting a place. The second type of traveller believes in visiting only a few objects of interest, but appreciating the beauty and the architecture of the places he visits. Option (1) is incorrect as the author does not side with any one of the two types of travellers. Option (2) is incorrect as the worth of a place for the first traveller is in the number of sites that he visits. The option contradicts this reason. Secondly, we cannot infer that this type of traveller does not understand the worth of any place he travels to. Option (3) follows from the passage as it is mentioned in the last sentence of the passage that the second type of traveller "... they allow the spirit of the place to sink in to their minds, and only visit such monuments as the time they have at their disposal allows them to contemplate without haste". Hence, (3).

19. The author mentions the importance of using English in a particular country as it would help the natives of this country. It is possible that he wants introduction of English to another country so that people from his country have no problem conversing with the masses. Option (1) is incorrect as the thought is not reflected anywhere in the passage. Although option (2) might appear as the correct answer, the author refers only to the native subjects. Secondly, we don't know who the word "useful" used in the passage is referring to and

in relation to what. Thus, this option cannot be considered as a direct inference from the passage. Option (3) is incorrect as it is an assumption and not an inference. Hence, (4).

20. The passage mentions the marketing strategies and economic policies of the country that address the aspirations of the middle-class. Option (1) is correct as it follows directly from the passage. It is mentioned in context of the middle class that "Since the mid-80s, that has been the focus of the economic policy..." Option (2) is incorrect as there is no mention of a 'deprived' or 'pampered' middle class in the passage. Hence, (1).