Probability

Career Launcher

QA - 32

CEX-Q-0233/18

Contents

- Probability

Number of Questions : 20

1. A box contains 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball selected is the smallest red ball is:

(CAT 1993)

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$

(3) $\frac{1}{6}$ (4) $\frac{2}{3}$

2. 30 Students of a class were asked to prepare one project each. Each project was to be assessed by 2 professors namely Don and Ron. Don usually rejects 40% of the projects that he evaluates while Ron accepts only 40% of the projects that he evaluates. When a student gets his project evaluated from both the professors, what is the probability of getting a rejection from both?

(1) 0.5 (2) 0.8
(3) 0.24 (4) 0.6

3. Bag I contains 3 red and 4 black balls while bag II contains 5 red and 6 black balls. One ball is drawn at random from one of the bags and it was found to be red. Find the probability that it was drawn from bag II.

(1) $\frac{2}{3}$ (2) $\frac{32}{41}$

(3) $\frac{35}{68}$ (4) $\frac{1}{5}$

4. 7 people were sitting around a circular table. What is the probability of 2 of them always being next to each other?

(1) 0.33
(2) 0.1
(3) 0.167
(4) None of these

5. Out of 2n consecutive integers, if (n + 1) integers are selected at random, what is the probability that the HCF of selected numbers is 1?

(1) 1 (2) $\frac{1}{(n+1)}$

(3) $\frac{1}{2}$ (4) $\frac{1}{(n+2)}$

6. Ramesh has a special set of cards for new game he invented. He has two packs of 6 cards each with their value ranging from 1 to 6. In that game, Ramesh shuffles the cards of both packs (i.e. 12 cards) and draws 4 cards. He tries to find any pair with the same value. What is probability that at least one pair will have the same value?

(1) $\frac{18}{35}$ (2) $\frac{17}{33}$

(3) $\frac{14}{37}$ (4) $\frac{16}{33}$

7. You buy 20 eggs from the market of which 2 are rotten. You randomly pick 4 eggs to prepare scrambled eggs for breakfast (if any one of the 4 eggs are rotten, then it will make the breakfast unetable). What is the probability that the preparation turns out to be uneatable?

(1) 0.368 (2) 0.5
(3) 0.116 (4) None of these

8. If a 4 digit number is formed with digits 1, 2, 3 and 5. What is the probability that the number is divisible by 25, if repetition of digits is not allowed?

(CAT 1995)

(1) $\frac{1}{12}$ (2) $\frac{1}{24}$

(3) $\frac{1}{6}$ (4) $\frac{1}{18}$

9. A chord AB of a circle subtends an angle 60° on the centre of the circle. If any point C is taken randomly on circumference of the circle, then what is the probability that centre of the circle lies inside the triangle ABC?

(1) $\frac{1}{4}$

(2) $\frac{1}{6}$

(3) $\frac{1}{9}$

(4) Cannot be determined

10. A bag contains 6 red marbles and 4 blue marbles. If a marble is drawn with replacement, what is the minimum number of draws required so that probability of drawing at least one blue ball is more than 0.7?

(1) 6 (2) 4
(3) 3 (4) 2

11. A group of friends have decided to take a road trip. Whenever they come to a fork in the road, they will toss a fair coin to decide whether to head right or left. If the coin shows heads, they will head right. If the coin shows tails, they will head left. If the friends have made seven decisions, what is the probability that they took exactly four lefts?

(a) $\frac{1}{16}$ (2) $\frac{1}{32}$

(3) $\frac{21}{25}$ (4) $\frac{35}{128}$

12. In a defective 6 faced die with numbers 1 to 6 inscribed, the probability of getting an odd number is twice the probability of getting an even number. Find the probability of getting a 2 digit prime number on adding 2 successive throws of the die, if among the even numbers all are equally likely and same is true for odd numbers.

(1) $\frac{2}{81}$ (2) $\frac{4}{81}$

(3) $\frac{1}{3}$ (4) $\frac{1}{9}$

13. Ramesh plans to order a birthday gift for his friend from an online retailer. However, the birthday coincides with the festival season during which there is a huge demand for buying online goods and hence deliveries are often delayed. He estimates that the probability of receiving the gift, in time, from the retailers A, B, C and D are 0.6, 0.8, 0.9 and 0.5 respectively. He orders from all four retailers simultaneously. What would be the probability that his friend would receive the gift in time?

(XAT 2015)

(1) 0.004 (2) 0.006
(3) 0.216 (4) 0.994
(5) 0.996

14. Aditya has a total of 18 red and blue marbles in two bags (each bag has marbles of both colors). A marble is randomly drawn from the first bag followed by another randomly drawn from the second bag, the probability of both being red is 5/16. What is the probability of both marbles being blue?

(XAT 2014)

(1) $\frac{1}{16}$ (2) $\frac{2}{16}$

(3) $\frac{3}{16}$ (4) $\frac{4}{16}$

(5) None of the above

15. A coin of radius 3 cm is randomly dropped on a square floor full of square shaped tiles of side 10 cm each. What is the probability that the coin will land completely within a tile?

(1) 0.91 (2) 0.5
(3) 0.49 (4) 0.36
(5) 0.16

(XAT 2018)

16. Find the probability that in 10 throws of a fair die a score which is a multiple of 3 will be obtained in at least 8 of the throws.

(1) $\frac{201}{3^{10}}$ (2) $\frac{2}{3}$

(3) $\frac{1}{100}$ (4) $\frac{1}{8\times9\times10}$

17. The scheduling officer for a local police department is trying to schedule additional patrol units in each of two neighbourhoods – southern and northern. She knows that on any given day, the probabilities of major crimes and minor crimes being committed in the northern neighbourhood were 0.418 and 0.612, respectively, and that the corresponding probabilities in the southern neighbourhood were 0.355 and 0.520. Assuming that all crime occur independent of each other and likewise that crime in the two neighbourhoods are independent of each other, what is the probability that no crime of either type is committed in either neighbourhood on any given day?

(XAT 2011)

(1) 0.069 (2) 0.225
(3) 0.690 (4) 0.775
(5) None of the above

18. The supervisor of a packaging unit of a milk plant is being pressurised to finish the job closer to the distribution time, thus giving the production staff more leeway to cater to last minute demand. He has the option of running the unit at normal speed or at 110% of normal - "fast speed". He produces 60% of packets at the fast speed and the rest at the normal speed. A packet produced at the fast speed is twice as likely to be damaged as compared to a packet produced at the normal speed. If the probability of randomly selected packet from the unit being damaged is 0.112, what is the probability that the packet will not be damaged at normal speed?

(XAT 2010)

(1) 0.81 (2) 0.93
(3) 0.75 (4) 0.60
(5) None of the above

19. There are four machines in a factory. At exactly 8 pm, when the mechanic is about to leave the factory, he is informed that two of the four machines are not working properly. The mechanic is in a hurry, and decides that he will identify the two faulty machines before going home, and repair them next morning. It takes him twenty minutes to walk to the bus stop. The last bus leaves at 8 : 32 pm. If it takes six minutes to identify whether a machine is defective or not, and if he decides to check the machines at random, what is the probability that the mechanic will be able to catch the last bus?

(XAT 2011)

(1) 0 (2) $\frac{1}{6}$

(3) $\frac{1}{4}$ (4) $\frac{1}{3}$

(5) 1

20. A rectangle is drawn such that none of its sides has length greater than ‘a’. All the lengths greater than ‘a’ are equally likely. The probability that the rectangle has its diagonal greater than ‘a’ is

(1) 1

(2) $\frac{\pi}{4}$

(3) $1 - \frac{\pi}{4}$

(4) Cannot be determined

QA - 32 : Probability
Answers and Explanations

1	3	**2**	3	**3**	3	**4**	1	**5**	1	**6**	2	**7**	1	**8**	1	**9**	2	**10**	3
11	4	**12**	2	**13**	5	**14**	3	**15**	5	**16**	1	**17**	1	**18**	2	**19**	4	**20**	3

1. 3 Let R denotes the event where a red ball is selected. Let S be the event wherein the smallest red ball is selected.
Thus, probability of selecting the smallest ball given that it is red is given by,

$$P\left(\frac{S}{R}\right)=\frac{P(S\cap R)}{P(R)}=\frac{\left(\frac{1}{18}\right)}{\left(\frac{6}{18}\right)}=\frac{1}{6}.$$

2. 3 In this question, the number of students i.e. 30 is a redundant data and thus needs to be ignored.
The probability of a project getting rejected by Don is 0.4 and that of by Ron is 0.6.
Hence, the probability of a project being rejected by both Don and Ron is 0.4 × 0.6 = 0.24.

3. 3 Probability of selection of bag I and bag II each is $\frac{1}{2}$.

Probability of picking up a red ball from bag I $=\frac{3}{7}$

$\Rightarrow$ Probability of a red ball from bag I $=\frac{3}{7}\times\frac{1}{2}$

Probability of picking up a red ball from bag II $=\frac{5}{11}$

$\Rightarrow$ Probability of a red ball from bag II $=\frac{5}{11}\times\frac{1}{2}$

Now, the required probability $=\frac{\frac{5}{11}\times\frac{1}{2}}{\frac{3}{7}\times\frac{1}{2}+\frac{5}{11}\times\frac{1}{2}}=\frac{35}{68}$.

4. 1 Let us consider the 2 people are sitting next to each other.
Thus, considering them to be as one individual we can say that there are 6 people to be arranged around a circular table and this can be done in 5! ways.
And the 2 people can be arranged between themselves in 2 ways.
Thus, the total ways in which the two people are always together = 5! × 2 = 240 ways.
And the number of ways in which all the 7 people can be arranged around the circular table is 6! = 720.
Thus, the probability of having the two people always sitting together is $\frac{240}{720}$ = 0.33.

5. 1 Out of 2n numbers, if we select (n + 1) numbers, there must be a pair of consecutive numbers. So, in all the cases the probability of getting 1 as their HCF is 100% i.e. 1.

6. 2 Let us first find the probability of having no pairs in the four drawn cards.
The first card is picked at random.
The next card can be picked from the remaining 11 cards with the exception of 1 card which is of the same value as the first card.
The third card can be picked from the remaining 10 cards with the exception of the cards having the same value as the first 2 picked cards.
The fourth card can be picked from the remaining 9 cards with the exception of the cards having the same value as the first 3 picked cards.
So, the probability of the above event

$$=\frac{10}{11}\times\frac{8}{10}\times\frac{6}{9}=\frac{16}{33}$$

So, the probability of picking at least one pair

$$=\frac{1+16}{33}=\frac{17}{33}.$$

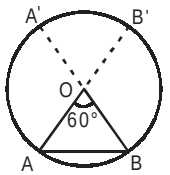
7. 1 For the preparation to be uneatable it should have at least 1 rotten egg.
The probability of having all good eggs in the selected 4 eggs is $\frac{^{18}C_4}{^{20}C_4}=\frac{3060}{4845}$ = 0.632

Thus, the probability of having a spoiled preparation = the probability of having at least one rotten egg in the 4 eggs picked for making the preparation = 1 – P(Having all good eggs for the preparation) = 1 – 0.632 = 0.368.

8. 1 For the number to be divisible by 25, the last 2 digits of the number should be divisible by 25, i.e. the last 2 digits should be 00, 25, 50 or 75.
Thus, in the given case, we are supposed to consider numbers ending with 25.
The total number of ways in which a four digit number can be formed using the given four digits
$= 4 \times 3 \times 2 \times 1 = 24$ ways.
And the number of ways in which the four digit number can be formed with ending 25 = 2
[i.e. the thousands and hundreds place can be filled in 2 ways, 13 or 31.]

Hence, the required probability = $\frac{2}{24} = \frac{1}{12}$.

9. 2



Centre will lie inside ΔABC only when C lies on minor Arc A'B'.

$\therefore$ Required probability = $\frac{60}{360} = \frac{1}{6}$.

10. 3 In this case, it is easy to calculate the probability of not drawing a blue marble. The probability of an event occurring plus the probability of the event not occurring is equal to 1. If the probability of drawing a blue marble is at least 70%, the probability of not drawing a blue marble can be at most 30%.
There is a probability of 6/10 on each draw such that the marble drawn is not blue. The probability of event A and event B occurring is the probability of event A times the probability of event B, given that the event A has already occurred. Since the probability of not drawing a blue marble on each draw is 6/10, we must determine how many times one has to multiply 6/10 by itself in order to reduce the probability to below 30%.
$(0.6)^n < 0.30$
$(0.6)^1 = 0.6$;
$(0.6)^2 = 0.36$;
$(0.6)^3 = 0.216$
Thus, 3 draws are required.

11. 4 The number of possible outcomes can be determined by the multiplication principle. The multiplication principle tells us that the number of ways in which an independent events can occur together can be determined by multiplying together the number of possible outcomes for each event. There are two outcomes possible while tossing a coin i.e., heads or tails.
Thus, the number of possible outcomes are $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$.
The number of ways in which 4 tails can occur out of 7 tosses is the number of combinations of 7 objects taken 4 at a time.
The number of combinations of n objects taken r at a time is given by $C(n, r) = \frac{n!}{(r!(n-r)!)}$.

$C(7, 4) = \frac{7!}{(4!(3!))} = \frac{5040}{144} = 35$

Hence, the probability of occuring of 4 tails is $\frac{35}{128}$.

12. 2 Let probability of getting an even number be x.
$\Rightarrow$ Probability of getting an odd number = 2x
In a particular throw either an odd number, out of three odd numbers, or an even number, out of three even numbers, will appear.
So, $3(x) + 3(2x) = 1$

$\Rightarrow x = \frac{1}{3}$

Now a two-digit prime number possible in two successive throws is 11 only i.e. sum of 5 and 6.

So, required probability $= \frac{1}{9} \times \frac{2}{9} + \frac{2}{9} \times \frac{1}{9} = \frac{4}{81}$

13. 5 The probabilities of not receiving the gift from retail A, B, C and D are 0.4, 0.2, 0.1 and 0.5 respectively.
Then probability of receiving the gift from at least one retailer = $\{1 - (0.4 \times 0.2 \times 0.1 \times 0.5)\} = 0.996$.

14. 3 Let the total number of balls in the bag I be 'x'; the total number of balls in bag II be $(18 - x)$; the number of red balls in bag I and bag II be 'a' and 'c' respectively; and the number of blue balls in bag I and bag II be 'b' and 'd' respectively.
$\therefore a + b = x$ and $c + d = 18 - x$

$\therefore \frac{a}{x} \times \frac{c}{18-x} = \frac{5}{16}$

Let $a \times c = 5k$ and $x \times (18 - x) = 16k$, where k is a natural number.
Each bag has marbles of both colors.

$\therefore 2 \le x \le 16$ and $2 \le (18 - x) \le 16$

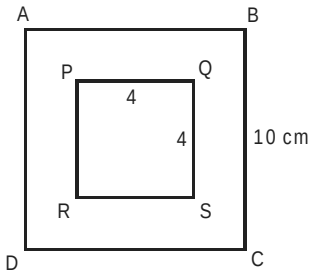
As, $x + (18 - x) = 18 \Rightarrow$ Max $\{x \times (18 - x)\} = 81$

$\therefore k \le 5$

By hit and trial, the only possible value of k comes out to be 5.
$\therefore x = 10$, $18 - x = 8$, $a = 5$, $b = 5$, $c = 5$ and $d = 3$

Hence, the required probability = $\frac{5}{10} \times \frac{3}{8} = \frac{3}{16}$.

15. 5



Here the centre of the circle must be within the smaller square drawn as shown in the figure above.

$\Rightarrow$ Probability $= \frac{4\times 4}{10\times 10} = 0.16$

16. 1 There are two multiples of 3 i.e. 3 and 6.

So, probability of getting 3 or 6 = $\frac{2}{6} = \frac{1}{3}$

$\Rightarrow$ Probability of not getting 3 or 6 = $\frac{2}{3}$

The probability of getting 3 or 6 in 'n' throws is given by ${}^{10}C_n \times \left(\frac{1}{3}\right)^n \times \left(\frac{2}{3}\right)^{10-n}$.

For at least 8 throws, n will be 8, 9 or 10
$\Rightarrow$ Required probability

$= {}^{10}C_8\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)^2 + {}^{10}C_9\left(\frac{1}{3}\right)^9\left(\frac{2}{3}\right)^1 + {}^{10}C_{10}\left(\frac{1}{3}\right)^{10}\left(\frac{2}{3}\right)^0$

$= \frac{1}{3^{10}}[180 + 20 + 1] = \frac{201}{3^{10}}.$

17. 1 Required probability = (1 – 0.418) (1 – 0.612) (1 – 0.355) (1 – 0.520) $\approx$ 0.069

18. 2 Probability of damaged packets in all packaging runs = 0.6 × 2x + 0.4 × x, where 'x' is the probability of packaging at normal speed.

$\Rightarrow$ 0.112 = 0.6 × 2x + 0.4 × x

$\Rightarrow$ 0.112 = 1.2x + 0.4x

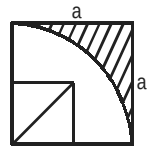
$\Rightarrow$ x = 0.07

$\therefore$ Probability of non damaged packets at normal speed = 1 – 0.07 = 0.93.

19. 4 In order to catch the bus, the mechanic has to leave the factory in 12 minutes. As inspecting one machine takes 6 minutes, he will identify the two faulty machines in 12 minutes if the first two machines he inspects are either both faulty or both working properly.

Required probability $= \frac{1}{2}\times\frac{1}{3} + \frac{1}{2}\times\frac{1}{3} = \frac{1}{3}$

20. 3 We will draw a square of side 'a' and an arc of radius 'a'. All rectangles with diagonal less than or equal to a will lie within or on the quadrant of the circle as shown below.



Hence, required probability = $\frac{a^2\left(1-\frac{\pi}{4}\right)}{a^2} = 1 - \frac{\pi}{4}$.