Algebra - 5

Career Launcher

QA - 21

CEX-Q-0222/18

Contents

- Modulus
- Logarithm

Number of questions : 30

Modulus

1. How many of the following statement(s) is(are) always true?
   A. $|x+y| \geq |x-y|$
   B. $|x| \pm |y| \geq |x \pm y|$
   C. $|a \times x| = a|x|$ (where, a is a constant)
   D. $|2-3x| = 3\left|x-\frac{2}{3}\right|$
   (1) One (2) Two
   (3) Three (4) Four

2. If $7 + 3|x| = 13$, find the value of x.
   (1) –2.5 (2) ± 6
   (3) ± 2 (4) $\pm\frac{7}{2}$

3. Find the solution set (x, y, z), where x, y and z are real numbers, for the system of equations $x + y + z = 5$, $2|y| + z = 7$ and $|x| + 2x = -4$.
   (1) (–4, 3, 6) (2) $\left(\frac{-4}{3}, 2, 3\right)$
   (3) (–4, 3, 1) (4) No solution

4. How many integral values of x satisfy the inequality $|[x + 3]| < 5$?
   {Here, [x] denotes the greatest integer less than or equal to x}
   (1) 6 (2) 7
   (3) 8 (4) 9

5. If $|a| < |b|$, and $|b| < |c|$ and $b < a < c$, then which of the following are necessarily true?
   I. a is positive.
   II. b is negative.
   III. c is positive.
   (1) Only I (2) I and II
   (3) II and III (4) I, II and III

6. If a and b are positive integers such that $|a - 2| \leq 3$ and $|b + 3| \leq 4$. What is the minimum value of $\frac{a^2}{b^2}$?
   (1) $\frac{1}{49}$ (2) $\frac{25}{49}$
   (3) 1 (4) $\frac{4}{49}$

7. Find the range of values of real x that satisfy the equation $|2x+7| \geq |3x+9|$
   (1) $\left[-\frac{7}{2}, -2\right]$ (2) $(-\infty, -2]$
   (3) $\left(-\infty, -\frac{16}{5}\right)$ (4) $(-\infty, -1]$

8. If $|3x| + 7|x| \leq 60$. Find the value of x.
(1) $-7 < x < 7$ (2) $x < 6$
(3) $-6 < x < 6$ (4) $-6 \leq x \leq 6$

Directions for questions 9 and 10: Answer the question based on the following information.

$|x| + |y| = 7$, $|x|^2 + |y|^2 = 25$ $(x, y \in R)$

9. How many values of $x^3 + y^3$ are possible?
(1) 1 (2) 2
(3) 3 (4) 4

10. What is the value of $|x - y|^2$?
(1) 1
(2) 49
(3) 4
(4) Cannot be determined

11. For a real number x, let [x] denote the greatest integer less than or equal to x. Then the number of real solutions of $|2x - [x]| = 4$ is
(1) 1 (2) 2
(3) 3 (4) 4

12. What is the minimum value of the function?
(1) $F(x) = |x - 1| + |x - 2| + |x - 4|$
(2) $F(x) = |x - 1| + |x - 2| + |x - 4| + |x - 8|$

13. What is the area bounded by the graphs
(1) $|x| + |y| = 10$
(2) $|x + 2| + |y - 3| = 10$
(3) $|x - y| + |x + y| = 10$

14. Find the number of integral values of x that satisfy the inequality $||2x - 19| - 7| < 5$.
(1) 13 (2) 12
(3) 11 (4) 10

Logarithm

15. If a, b, c, d are positive quantities such that $a^2 = b^3 = c^5 = d^6$ then $\log_d (abc)$ equals
(1) 5.8 (2) 6.0
(3) 6.5 (4) 6.2

16. The value of $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \cdots + \frac{1}{\log_{43} n}$ is
(1) $\frac{1}{\log_{43!} n}$ (2) $\frac{1}{\log_{43} n}$
(3) $\frac{1}{\log_{43} n!}$ (4) $\frac{1}{\log_{42} n}$

17. If $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$, then $x + y$ is equal to
(1) 2 (2) $\frac{65}{8}$
(3) $\frac{10}{3}$ (4) None of the above

18. Find the sum of the series

$$\frac{1}{\log_3 9} + \frac{1}{\log_9 9} + \frac{1}{\log_{27} 9} + \ldots\ldots + \frac{1}{\log_{3^n} 9}.$$

(1) $\frac{n(n+1)}{2}$ (2) $\frac{2}{n(n+1)}$
(3) $\frac{n(n+1)(2n+1)}{12}$ (4) $\frac{n(n+1)}{4}$

19. If $\log_7 \log_5 \left(\sqrt{x+5} + \sqrt{x}\right) = 0$, what is the value of x?
(1) 2 (2) 3
(3) 4 (4) 5

20. If $x \geq y$ and $y > 1$, then the value of the expression $\log_x\left(\frac{x}{y}\right)+\log_y\left(\frac{y}{x}\right)$ can never be

(1) –1.3 (2) –0.5
(3) 0 (4) 1

21. What is the number of
(1) Digits in 2^{50} ($\log_{10}2 = 0.301$)
(2) Zeroes between the decimal point and the first significant digit after the decimal point in 2^{-50} ? ($\log_{10}2 = 0.301$)

22. If $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$, then the possible value of x is given by

(1) 10 (2) $\frac{1}{100}$
(3) $\frac{1}{1000}$ (4) None of these

23. If logyx = (a.logzy) = (b.logxz) = ab, then which of the following pairs of values for (a, b) is not possible?

(1) $\left(-2, \frac{1}{2}\right)$ (2) (1, 1)
(3) (0.4, 2.5) (4) (2, 2)

Challenging

24. If $\frac{\log x}{b-c} = \frac{\log y}{c-a} = \frac{\log z}{a-b}$, then which of the following is/are correct?
I. $xyz = 1$
II. $x^a y^b z^c = 1$
III. $x^{b+c} y^{c+a} z^{a+b} = 1$
IV. $x^{b+c} y^{c+a} z^{a+b} = 0$

(1) Only I and II (2) Only I, II and III
(3) Only II and IV (4) Only I, II and IV

25. If $\frac{\log x}{a^2+b^2+ab} = \frac{\log y}{b^2+c^2+bc} = \frac{\log z}{c^2+a^2+ac}$ then, the value of $x^{(a-b)}.y^{(b-c)}.z^{(c-a)}$ is

(1) 0 (2) 1
(3) abc (4) $(xyz)^{abc}$

26. How many positive integral values of x are possible, if $\log_{0.5}\log_7\left(\frac{x^3-1}{x^3+1}\right) > 0$?

(1) 0 (2) 1
(3) 2 (4) 3

27. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at x = 0, f(x) is

(CAT 2004)

(1) maximized whenever a > 0, b > 0
(2) maximized whenever a > 0, b < 0
(3) minimized whenever a > 0, b > 0
(4) minimized whenever a > 0, b < 0

28. Find the number of real values of x, where, x is a real number, that satisfy the equation $|2x-7|+|x-5| = 14$.

(1) 0 (2) 1
(3) 2 (4) 3

29. The set of all real numbers in (–2, 2) satisfying $2^{|x|}-\left|2^{x-1}-1\right| = 2^{x-1}+1$ is

(1) {–1, 1} (2) {–1} ∪ [1, 2)
(3) (–2, –1) ∪ [1, 2) (4) [–2, –1] ∪ {1}

30. Let $\log_{12} 18 = a$. Then $\log_{24} 16$ is equal to

(1) $\frac{8-4a}{5-a}$ (2) $\frac{1}{3+a}$
(3) $\frac{4a-1}{2+3a}$ (4) $\frac{8-4a}{5+a}$

QA - 21 : Algebra - 5
Answers and Explanations

CEX-Q-0222/18

1	1	**2**	3	**3**	4	**4**	4	**5**	3	**6**	3	**7**	1	**8**	4	**9**	4	**10**	4
11	2	**12**	–	**13**	–	**14**	4	**15**	4	**16**	1	**17**	4	**18**	4	**19**	3	**20**	4
21	–	**22**	2	**23**	4	**24**	2	**25**	2	**26**	1	**27**	4	**28**	3	**29**	1	**30**	1

1. 1 A. $|x + y| \geq |x - y|$ is not always true.
Let x = 4 and y = 2, it follows the given condition but for x = 4 and y = –2, it doesn't follow the condition .
B. This condition is not followed
C. $|a \times x| = a|x|$ this holds true only for non-negative values of a, so it is not always true.
D. This is always true.
Hence, only one statement is true.

2. 3 $7 + 3|x| = 13 \Rightarrow 3|x| = 6$
$\Rightarrow |x| = 2$
So, $x = \pm 2$

Alternative method:
Use the options and try.

3. 4 $|x| + 2x = -4$
If $x > 0$, then $3x = -4$ (not possible)
If x < 0, then x = – 4
Now, x + y + z = 5
Therefore, y + z = 9 and $2|y| + z = 7$
If y > 0, then y + z = 9 and 2y + z = 7, but this gives a negative value of y, so not possible.
If y < 0, then y + z = 9 and z – 2y = 7, but this gives a positive value of y, which is not possible
Hence, there is no solution set for the given system of equations.

Alternative method:
x + y + z = (given)
But options (2) and (3) do not satisfy this condition.
Now, the check option (1), which does not satisfy 2|y| + z = 7
Hence, option (4) is correct

4. 4 $|[x + 3]| < 5$
$-5 < [x + 3] < 5$
If we take x = 2, [x + 3] = 5 and if we take x = –8 [x + 3] = –5.
Therefore all integer value of x which are greater than –8 and less than 2 satisfy the given inequality.
Therefore, 9 integer values of x satisfy the inequality.

5. 3 Let as solve the question taking the opposite of whatever is stated
I. If a is negative, then b has to be negative, since b < a. but |b| can be greater than |a|.
Hence, (I) is not necessarily true.
II. If b is positive, then a also has to be positive since a > b. Now, when a > b, and both are positive |a| has to be greater than |b|, which contradicts the given condition. Hence, b cannot be positive.
So (II) is necessarily true.
III. If c is negative, then b has to be negative since b < c, but |b| will be greater than |c|.
Thus, c cannot be negative and hence (III) is also true.

6. 3 To make $\frac{a^2}{b^2}$ minimum, a must be minimum and b must be maximum. Also, a and b are +ve integers, so from the given inequations
a = 1, 2, 3, 4, 5 and b = 1
So, minimum value of $\frac{a^2}{b^2} = 1$.

7. 1 The intervals given in option (2), (3) and (4) contains large negative numbers for which the given inequation does not hold true.
So, there must be a lower limit, which is –7/2.
Hence, option (1) is correct.

8. 4 $|3x| + 7|x| \leq 60$
For $x \geq 0$
$3x + 7x \leq 60$
$x \leq \frac{60}{10} \Rightarrow x \leq 6$
For x < 0
$-3x + 7(-x) \leq 60$
$\Rightarrow -10x \leq 60 \Rightarrow x \geq -6$
$\Rightarrow$ value of x is $-6 \leq x \leq 6$

Alternative method:

The inequation given in the question contains ≤ but only option (4) contains equals to sign. So, option (4) is correct.

For questions 9 and 10:

Let |x| = m and |y| = n

m + n = 7 and $m^2 + n^2 = 25$

or $m^2 + (7 - m)^2 = 25$

Solving above equation we get m = 3 or 4 and hence n = 4 or 3 or $x = \pm 3$ or ± 4 and $y = \pm 4$ or ± 3

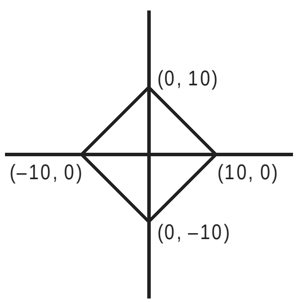
9. 4 It doesn't matter what value is assigned to x and y among ± 3 and ± 4 because the expression is $x^3 + y^3$
Cleary a total of four values are possible for $x^3 + y^3$. This will happen as (x, y) is retrieved from the following sets.
(– 3, – 4) or (+ 3, + 4) or (– 3, + 4) or (3, – 4)

10. 4 Here, x and y can be +ve and –ve both.
So, value of $|x - y|^2$ cannot be determined uniquely.

11. 2 |2x – [x]| = |x + (x – [x])| = 4
Here, x – [x] = fractional part of x = {x}
i.e. |x + {x}| = 4
This will only be satisfied for x = 3.5 and x = 4
Hence, only two solutions are possible.

12. (1) The minimum value will occur at x = 2, which is 3.
(2) The minimum value will occur at 2 ≤ x ≤ 4, which is 9.

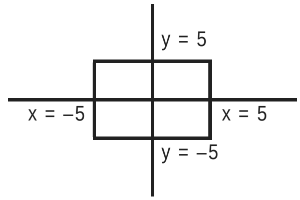
13. (1) The graph of the function |x| + |y| = 10 is given below



So, area = $4 \times \frac{1}{2} \times 10 \times 10 = 200$ sq. unit

(2) The given region is same as previous one, but the origin is shifted. So, area remains unchanged, which is 200 sq. unit

(3) The graph of |x – y| + |x + y| = 10 is given as



So, required area = 10 × 10 = 100 sq. unit.

14. 4 $||2x - 19| - 7| < 5$

$-5 < |2x - 19| - 7 < 5$

$\Rightarrow 2 < |2x - 19| < 12$

$\Rightarrow -12 < 2x - 19 < -2$ or $2 < 2x - 19 < 12$

$\Rightarrow 7 < 2x < 17$ or $21 < 2x < 31$

$\Rightarrow \frac{7}{2} < x < \frac{17}{2}$ or $\frac{21}{2} < x < \frac{31}{2}$

Therefore, there are 10 integer values of x that satisfy the inequality i.e. x = 4, 5, 6, 7, 8, 11, 12, 13, 14 and 15.

15. 4 Let $a^2 = b^3 = c^5 = d^6$
Therefore, $a = d^3$, $b = d^2$ and $c = d^{1.2}$
$\text{Log}_d(abc) = \text{Log}_d(d^{6.2}) = 6.2$

16. 1 Since $\frac{1}{\log_a b} = \log_b a$

So, the given expression becomes

$\log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 43$

Since $\log_n 1 = 0$ and $\log_a p + \log_a q = \log_a pq$,

So, $\log_n 43! = \frac{1}{\log_{43!} n}$

17. 4 Since $\log_2 x + \log_x 2 = \frac{10}{3} = \log_2 y + \log_y 2$ and $x \neq y$,

which means if we convert $\log_2 x + \log_x 2 = \frac{10}{3}$ into a quadratic equation, then x will take two values, same would be for y. i.e. we have to find the sum of the roots.

So, $\log_2 x + \log_x 2 = \frac{10}{3}$

$\log_2 x + \frac{1}{\log_2 x} = \frac{10}{3}$

$3(\log_2 x)^2 - 10(\log_2 x) + 3 = 0$

So, $\log_2 x + \log_2 y = \frac{-(-10)}{3} = \frac{10}{3}$.

18. 4 $\because \log_{a^m} b^n = \frac{n}{m}\log_a b$

$$\Rightarrow \frac{1}{\log_3 9}+\frac{1}{\log_9 9}+\frac{1}{\log_{27} 9}+........+\frac{1}{\log_{3^n} 9}$$

$$=\frac{1}{2}+\frac{1}{1}+\frac{3}{2}+......+\frac{n}{2}$$

$$\Rightarrow \frac{1}{2}+\frac{2}{2}+\frac{3}{2}+.....+\frac{n}{2} = \frac{n(n+1)}{4}$$

Alternative method:
Take n = 1

Then the answer should be $\frac{1}{\log_3 9}=\frac{1}{2}$, put in the options. 1st and 2nd option get eliminated. Now put n = 2 and 3rd option gets eliminated. So, answer is option (4).

19. 3 $\log_7 \log_5\left(\sqrt{x+5}+\sqrt{x}\right)=0$

For the equation to be 0, $\sqrt{x+5}+\sqrt{x}$ must be equal to 5. i.e. $\log_5\left(\sqrt{x+5}+\sqrt{x}\right)$ = 1.

Putting x = 4, satisfies the equation.

20. 4 $P=\log_x\left(\frac{x}{y}\right)+\log_y\left(\frac{y}{x}\right)$

$= \log_x x - \log_x y + \log_y y - \log_y x$
$= 2 - \log_x y - \log_y x$
Let, $t = \log_x y$

$$\Rightarrow P = 2 - t - \frac{1}{t} = -\left[\sqrt{t}-\frac{1}{\sqrt{t}}\right]^2$$

Which can never be positive, out of given option it can't assume a value of +1. So (4) is ans.

21. (1) Let $y = 2^{50}$
$\log_{10} y = = 50 \times \log_{10} 2 = 50 \times 0.301 = 15.05$
$y = 10^{15.05} = 10^{0.05+15} = 10^{15} \times 10^{0.05}$
Since $(10)^{0.05}$ is any number of the form a.bc…
So, number of digits in y = 15 + 1 = 16.

(2) let $y = 2^{-50}$
$\log_{10} y = -50 \log_{10} 2 = -50 \times 0.301$
$\Rightarrow y = 10^{-15.05} = 10^{-16+0.95} = 10^{-16} \times 10^{0.95}$
Here, $10^{0.95} \approx 8.88…$

So, $\frac{10^{0.95}}{10^{16}} = 0.\underbrace{0……}_{15\text{ zeroes}}8…$

i.e. 15 zeroes.

22. 2 $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$

$$\log_{10}\left[\frac{x}{\sqrt{x}}\right]=\log_x 100$$

$$\therefore \log_{10}\sqrt{x}=\frac{\log_{10}100}{\log_{10}x}$$

$$\therefore \frac{1}{2}\log_{10}x=\frac{2}{\log_{10}x}$$

$$\therefore (\log_{10}x)^2=4$$

$$\therefore \log_{10}x=\pm 2$$

$$\therefore \log_{10}x=2 \quad\text{or}\quad \log_{10}x=-2$$

$$\therefore 10^2=x \quad\text{or}\quad 10^{-2}=x$$

$$\therefore x=100 \quad\text{or}\quad x=\frac{1}{100}$$

23. 4 $\log_y^x = a.\log_z^y = b.\log_x^z = a\times b$

$$a=\frac{\log_y^x}{\log_z^y}\text{ and } b=\frac{\log_y^x}{\log_x^z}$$

$$\Rightarrow a\times b=\frac{\log_y^x}{\log_z^y}\times\left(\frac{\log_y^x}{\log_x^z}\right)$$

$$=\frac{\left(\frac{\log_k^x}{\log_k^y}\right)}{\left(\frac{\log_k^y}{\log_k^z}\right)}\times\frac{\left(\frac{\log_k^x}{\log_k^y}\right)}{\left(\frac{\log_k^z}{\log_k^x}\right)}\ \{\text{For some base k}\}$$

$$=\left(\frac{\log_k^x}{\log_k^y}\right)^3=\left(\log_y^x\right)^3=(ab)^3$$

So, $ab - a^3b^3 = 0$
or, $a \times b(1 - a^2b^2) = 0$
$\Rightarrow ab = \pm 1$
Only option (4) does not satisfy.

24. 2 $\frac{\log x}{b-c}=\frac{\log y}{c-a}=\frac{\log z}{a-b}=K$, say. Let B is base.

Then $\log_B x = K(b-c) \Rightarrow x = B^{K(b-c)}$ and $y = B^{K(c-a)}$ and $z = B^{K(a-b)}$.

Adding, we get

$$\log_B x+\log_B y+\log_B z$$
$$=\{K(b-c)+K(c-a)+K(a-b)\}=0$$

or $\log_B(xyz) = 0 \Rightarrow (xyz) = B^{(0)} = 1$

$\Rightarrow$ xyz = 1

Option (1): xyz = 1, is correct.

Option (2): $x^a.y^b.z^c$

$$= \left[B^{K(b-c)}\right]^a \times \left[B^{K(c-a)}\right]^b \times \left[B^{K(a-b)}\right]^c$$

$$= B^{K[a(b-c)+b(c-a)+c(a-b)]}$$

$$= B^{K(0)} = B^0 = 1$$

Option (2) is correct.

Option (3): $x^{b+c}.y^{c+a}.z^{a+b}$

$$\left[B^{k(b+c)(b-c)}\right] \times \left[B^{k(c+a)(c-a)}\right] \times \left[B^{k(a+b)(a-b)}\right]$$

$$= B^{K\left[(b^2-c^2)+(c^2-a^2)+(a^2-b^2)\right]}$$

$$= B^0 = 1$$

Option (3) is correct.

Option (4): is wrong as the expression evaluates to 1 as in (3) and not zero.

In all, options (1), (2) and (3) are correct.

25. 2 Let $\dfrac{\log x}{a^2+b^2+ab} = \dfrac{\log y}{b^2+c^2+bc} = \dfrac{\log z}{c^2+a^2+ac} = k_1$

and $x^{(a-b)}.y^{(b-c)}.z^{(c-a)} = k_2$

$$(a-b)\log x + (b-c)\log y + (c-a)\log z = \log k_2$$

$$\Rightarrow k_1\Big\{(a-b)\left(a^2+b^2+ab\right) + (b-c)\left(b^2+c^2+bc\right)$$

$$+(c-a)\left(c^2+a^2+ac\right)\Big\} = \log k_2$$

$$\Rightarrow \log k_2 = k_1\left(a^3-b^3+b^3-c^3+c^3-a^3\right)$$

$$\Rightarrow \log k_2 = 0 \quad \Rightarrow k_2 = 1$$

Hence, (2) is the answer.

26. 1 If 0 < a < 1 and $\log_a x > 0$, then 0 < x < 1.

So, $0 < \log_7 \dfrac{(x^3-1)}{(x^3+1)} < 1$

$$\Rightarrow 1 < \frac{x^3-1}{x^3+1} < 7$$

But $\dfrac{x^3-1}{x^3+1}$ is always less than 1, for all positive values of x.

So, no solution.

27. 4 When a > 0, b < 0,

ax^2 and $-b\,|x|$ are non negative for all x,

i.e. $ax^2 - b|x| \geq 0$

$\therefore$ $ax^2 - b\,|x|$ is minimum at x = 0 when a > 0, b < 0.

28. 3 $|2x-7| + |x-5| = 14$

If $x \geq 5, \Rightarrow 2x-7+x-5 = 14$

$$\Rightarrow 3x = 26$$

$$\therefore x = \frac{26}{3}$$

If $\dfrac{7}{2} \leq x < 5, \Rightarrow 2x-7+5-x = 14$

$$\Rightarrow x = 16$$

But x lies between 3.5 and 5, hence x cannot be equal to 16

If $x < \dfrac{7}{2}, \Rightarrow 7-2x+5-x = 14$

$$\Rightarrow x = \frac{-2}{3}$$

Therefore, there are two real values of x that satisfy the equation.

29. 1 We will go by options.

We put x = 1.5, which doesn't satisfy the given equation. So option (2) and (3) are rejected.

Again, we take x = –2, which again do not satisfy the given equation.

Hence, the only solution set is {–1, 1}, which is option (1).

30. 1 $\log_{12} 18 = a \Rightarrow \dfrac{\log 18}{\log 12} = a \Rightarrow \dfrac{\log 2 + 2\log 3}{2\log 2 + \log 3} = a$

Let, log 2 = x and log 3 = y

So, $\dfrac{x+2y}{2x+y} = a \Rightarrow \dfrac{y}{x} = \dfrac{1-2a}{a-2}$

Now, $\log_{24} 16 = \dfrac{4\log 2}{\log 3 + 3\log 2} = \dfrac{4}{\dfrac{\log 3}{\log 2}+3}$

$$= \frac{4}{3+\left(\dfrac{1-2a}{a-2}\right)} = \frac{4(a-2)}{(a-5)} = \frac{4a-8}{a-5}$$