### **Number System**

#### **HCF and LCM**

Highest Common Factor (HCF) – HCF of two or more numbers is the greatest number that perfectly divides each of them

Lowest Common Multiple (LCM) – The LCM of two or more numbers is the least number which is perfectly divisible by each of them

If two numbers 'a' and 'b' are given, and their LCM and HCF are provided then:

LCM 
$$(a, b) \times HCF (a, b) = a \times b$$

$$LCM of Fractions = \frac{LCM of Numerator}{HCF of Denominator}$$

$$HCF of Fractions = \frac{HCF of Numerator}{LCM of Denominator}$$

For two co-prime numbers, product of two numbers = LCM of the numbers

Sum of first n natural numbers:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

Sum of squares of first n natural numbers:

$$1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers:

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

- The product of 'n' consecutive natural numbers is always divisible by n!
- Also, square of any natural number can be written in the form of 4n or 4n+1.
- Square of a natural number can only end in 0, 1, 4, 5, 6 or 9. Second last digit of a square of a natural number is always even except when last digit is 6. If the last digit is 5, second last digit has to be 2.
- Any prime number greater than 3 can be written as  $6k \pm 1$ .

If n is even, (n)(n+1)(n+2) is divisible by 24

(m + n)! is always divisible by  $m! \times n!$ 

 $x^n + y^n$  is divisible by (x + y) when n is odd

 $x^n - y^n$  is always divisible by (x - y)

 $x^n - y^n$  is always divisible by (x + y) when n is even

 $x^n - x$  is divisible by x (if n is prime)

Wilson's theorem: (p-1)! + 1 is divisible by p when p is prime

**Euler's number / Euler's totient function** for a number  $N = a^m * b^n * c^p$  ... (where a, b, c are the prime factors of N) is given by:  $E(N) = N \left(1 - \frac{1}{a}\right) \left(1 - \frac{1}{b}\right) \left(1 - \frac{1}{c}\right)$  ... (remember that the power is not important)

**Euler's theorem** states that:  $a^{xE(N)}$  mod b where a and b are coprime is always 1. If a and b are not coprime, you can cancel out the common factor from one of the terms and then proceed with the division. Just remember to multiply the cancelled off part to your final answer.

**Fermat's little theorem** states that:  $a^{x(b-1)} \mod b$  where b is a prime number is always 1.

If  $N = a^p \times b^q$  then, sets of factors of N which are co-prime to each other can be calculated as [(p+1)(q+1)-1]+pq

If  $N=a^p\times b^q\times c^r$  then, sets of factors of N which are co-prime to each other can be calculated as [(p+1)(q+1)(r+1)-1]+pq+qr+pr+3pqr

# Cyclicity of powers

| Digit | Power |   |   |   | Cyclicity |
|-------|-------|---|---|---|-----------|
|       | 1     | 2 | 3 | 4 | Cyclicity |
| 0     | 0     | 0 | 0 | 0 | 1         |
| 1     | 1     | 1 | 1 | 1 | 1         |
| 2     | 2     | 4 | 8 | 6 | 4         |
| 3     | 3     | 9 | 7 | 1 | 4         |
| 4     | 4     | 6 | 4 | 6 | 2         |
| 5     | 5     | 5 | 5 | 5 | 1         |
| 6     | 6     | 6 | 6 | 6 | 1         |
| 7     | 7     | 9 | 3 | 1 | 4         |
| 8     | 8     | 4 | 2 | 6 | 4         |
| 9     | 9     | 1 | 9 | 1 | 2         |

Note: The fifth power of any number has the same units place digit as the number itself.

#### **Properties of Surds:**

$$\left[\sqrt[n]{a}\right]^n = a$$

$$\sqrt[n]{a}\sqrt[n]{b} = \sqrt[n]{ab}$$

$$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$$

## Laws of Logarithms:

$$log_b 1 = 0$$

$$log_a a = 1$$

$$\log_a b \times \log_b a = 1$$

$$log_b(m \times n) = log_b m + log_b n$$

$$\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$$

$$\log_b m^n = n \log_b m$$

$$\log_b m = \frac{\log_a m}{\log_a b} = \log_a m \times \log_b a$$

$$b^{log_bn} = n$$

### **Percentages**

$$Percentage change = \frac{Final \, Value - Initial \, Value}{Initial \, Value} \times 100$$

# Successive percentage change/Successive discounts

If a quantity changes by a% and then by b%, successive percentage change is given by

Percentage Change =  $a + b + \frac{ab}{100}$  (For increase, keep sign positive. For decrease, use negative sign)

# **Special Question Type**

If one factor increases by 
$$\frac{x}{y}$$
 the other factor will reduce by  $\frac{x}{x+y}$ 

| Number | Reciprocal (1/Number) | Number | Reciprocal (1/Number) |
|--------|-----------------------|--------|-----------------------|
| 1      | 100%                  | 9      | 11.11%                |
| 2      | 50%                   | 10     | 10%                   |
| 3      | 33.33%                | 11     | 9.09%                 |
| 4      | 25%                   | 12     | 8.33%                 |
| 5      | 20%                   | 13     | 7.69%                 |
| 6      | 16.66%                | 14     | 7.14%                 |
| 7      | 14.28%                | 15     | 6.66%                 |
| 8      | 12.50%                | 16     | 6.25%                 |

### **Averages**

Simple Average or 
$$AM = \frac{Sum \text{ of all observations}}{Total number of observations}$$

In case of simple average, every observation or element has the same weight. But in case of different weights, calculate weighted average:

Weighted Average = 
$$\frac{w_1x_1 + w_2x_2 + w_3x_3 + \dots + w_nx_n}{w_1 + w_2 + w_3 + \dots + w_n}$$

If a < b, then for a positive quantity x

$$\frac{a+x}{b+x} > \frac{a}{b}$$
 and  $\frac{a-x}{b-x} < \frac{a}{b}$ 

If a > b, then for a positive quantity x

$$\frac{a+x}{b+x} < \frac{a}{b}$$
 and  $\frac{a-x}{b-x} > \frac{a}{b}$ 

If a: b: : c: d or 
$$\frac{a}{b} = \frac{c}{d}$$
, then

Using Componendo 
$$\rightarrow \frac{a+b}{b} = \frac{c+d}{d}$$

Using Dividendo 
$$\rightarrow \frac{a-b}{b} = \frac{c-d}{d}$$

Using Componendo and Dividendo 
$$\rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

# Profit, Loss and Discount

The most important thing is SP/CP ratio

If SP/CP is greater than 1, SP>CP -> Profit. Profit percentage = Profit/CP\*100 = (SP-CP)/CP\*100

If SP/CP is less than 1, SP<CP -> Loss. Loss percentage = Loss/CP\*100 = (CP-SP)/CP\*100

If SP/CP is 1. SP = CP. No profit No Loss.

In case of faulty weights,

Percentage Profit = error / actual quantity sold \* 100

Percentage Profit = [(claimed weight of item / actual weight of item) - 1]\*100

Discount = Marked Price - Selling Price

Discount Percentage = Discount / Marked Price \*100

Buy x get y free: Percentage Discount = y / (x+y) \*100

#### Simple Interest and Compound Interest

Simple Interest = 
$$\frac{P \times N \times R}{100}$$

P = Principal

N = Number of years

R = Rate of interest

Principal + Interest over x period = Amount after x period

Compound interest

$$A = P \left( 1 + \frac{r}{100} \right)^n$$

Interest = Amount - Principal

Simple Annual Growth Rate (SAGR) = Growth rate / number of years \* 100

Here, Growth rate is nothing but the percentage change over the period

Compounded Annual Growth Rate (CAGR) = (Final value / Initial value)(1/number of years) - 1

The difference between 2 years' simple interest and compound interest is given by  $P \times \left[\frac{R}{100}\right]^2$ 

#### **Mixtures and Solutions**

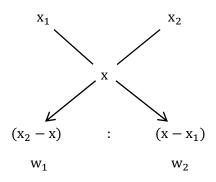
**Successive Replacement:** 

$$\frac{\text{Quantity of milk remaining after } n^{th} \text{ replacement}}{\text{Quantity of total mixture}} = \frac{(x-y)^n}{x}$$

Where x is the original quantity, y is the quantity that is replaced and n is the number of times the replacement process is carried out.

Alligation Rule: The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributed of these two items from the average attribute of the resultant mixture.

$$\frac{w_1}{w_2} = \frac{(x_2 - x)}{(x - x_1)}$$



Wine and Water formula

Let Q - volume of a vessel, q - quantity of a mixture of water and wine be removed each time from a mixture, n - number of times this operation is done and A - final quantity of wine in the mixture, then,

$$\frac{A}{Q} = (\frac{1-q}{Q})^n$$

## Time, Speed and Distance

$$Speed = \frac{Distance}{Time}$$

$$1 \text{ km/hr} = \frac{5}{18} \text{m/s}$$
 And  $1 \text{ m/s} = \frac{18}{5} \text{km/hr}$ 

If the ratio of speeds is a:b:c, then the ratio of times taken is  $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$  (Provided distance is same)

Speed is a relation between time and distance.

 $S \propto D$  and  $S \propto \frac{1}{T}$ ; i.e. if speed is doubled, distance covered in a given time also doubles and if speed is doubled, time taken to cover a distance would be half.

Average Speed:

Average Speed = 
$$\frac{Total\ distance\ travelled}{Total\ time\ taken} = \frac{d_1 + d_2 + d_3 + \cdots}{t_1 + t_2 + t_3 + \cdots}$$

If the distance is constant, then average speed is given by harmonic mean of two speeds:

$$S_{avg} = \frac{2 S_1 S_2}{S_1 + S_2}$$

If the time is constant, then average speed is given by arithmetic mean of two speeds:

$$S_{avg} = \frac{S_1 + S_2}{2}$$

Relative Speed: For Trains,

Time = 
$$\frac{\text{Sum of lengths}}{\text{Relative speed}} = \frac{L_1 + L_2}{S_1 \pm S_2}$$

For Boats and Streams:

 $S_{downstream} = S_{boat} + S_{stream}$ 

 $S_{upstream} = S_{boat} - S_{stream}$ 

#### Time and Work

Number of days to complete the work  $=\frac{1}{\text{Work done in one day}}$ 

- If 'A' can finish a work in 'X' time and 'B' can finish the same work in 'Y' time, then both of them together can finish that work in  $\frac{X*Y}{(X+Y)}$  time
- If 'A' can finish a work in 'X' time and 'A' & 'B' together can finish the same work in 'S' time then 'B' can finish that work in  $\frac{X*S}{X-S}$  time.
- If A can finish a work in X time and B in Y time and C in Z time then all of them working together will finish the work in  $\frac{X*Y*Z}{XY+YZ+XZ}$  time

#### **Pipes and Cisterns**

If a pipe can fill a tank in 'x' hour and another pipe can fill it in 'y' hour, then the fraction of tank filled by both the pipes together in 1 hour is given by,

$$\frac{1}{x} + \frac{1}{y} = \frac{x + y}{xy}$$

Or, the number of hours required to fill the tank by both pipes is given by

$$\frac{xy}{x+y}$$

# Algebra

For a quadratic equation  $ax^2 + bx + c = 0$ 

Sum of roots = 
$$-\frac{b}{a}$$
; Product of roots =  $\frac{c}{a}$ 

For a cubic equation  $ax^3 + bx^2 + cx + d = 0$ 

Sum of roots  $=-\frac{b}{a}$ ; Sum of the product of roots taken two at a time  $=\frac{c}{a}$ ; Product of roots  $=-\frac{d}{a}$ 

#### **Binomial Theorem**

If n is a natural number that is greater than or equal to 2 then according to the binomial theorem:

$$(x + a)^n = {}_0^n Cx^n a^0 + {}_1^n Cx^{n-1} a^1 + {}_2^n Cx^{n-2} a^2 + \dots + {}_n^n Cx^0 a^n$$

For an Arithmetic progression (A.P.) whose first term is a and the common difference is d

i) 
$$n^{th}$$
 term =  $t_n = a + (n - 1)d$ 

ii) Sum of the first n terms = 
$$S_n = \frac{n}{2}(a+L) = \frac{n}{2}(2a+(n-1)d)$$
 where L = last term = a+(n-1)d

For an A.P. with 4 terms, assume numbers as (a-3d), (a-d), (a+d), (a+3d) with difference as 2d. Makes calculation easier

For a Geometric progression (G.P.) whose first term is a and the common difference is r

i) 
$$n^{th}$$
 term =  $t_n = ar^{n-1}$ 

ii) Sum of the first n terms = 
$$S_n = \frac{a(1-r^n)}{1-r}$$
 if  $r < 1$ ;  $S_n = \frac{a(r^n-1)}{r-1}$  if  $r > 1$ 

If a, b, c are in G.P., then  $\log_a n$ ,  $\log_b n$ ,  $\log_c n$  are in H.P.

1. Arithmetic Mean (AM) = 
$$\frac{x_1 + x_2 + x_3 \dots + x_n}{n}$$

2. Geometric Mean (GM) = 
$$\sqrt[n]{x_1x_2x_3...x_n}$$

3. Harmonic Mean (HM) = 
$$\frac{n}{(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n})}$$

4. For two numbers:

$$AM = \frac{a+b}{2}$$
  $GM = \sqrt{ab}$   $HM = \frac{2ab}{(a+b)}$ ; and, in all the cases,  $AM \ge GM \ge HM$ 

5. 
$$(a+b)^2=a^2+2ab+b^2$$
;  $a^2+b^2=(a+b)^2-2ab$ 

6. 
$$(a-b)^2 = a^2 - 2ab + b^2$$
;  $a^2 + b^2 = (a-b)^2 + 2ab$ 

7. 
$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

8. 
$$(a + b)^3 = a^3 + b^3 + 3ab(a + b)$$

9. 
$$(a-b)^3 = a^3 - b^3 - 3ab(a-b)$$

10. 
$$a^2 - b^2 = (a + b)(a - b)$$

11. 
$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

12. 
$$a^3 + b^3 = (a + b)(a^2 + ab + b^2)$$

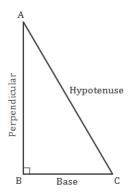
13. 
$$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

14. 
$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \dots + b^{n-1})$$

15. The roots of the quadratic equation  $ax^2 + bx + c = 0$  and  $a \ne 0$  are  $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 

# Geometry

## Pythagoras Theorem

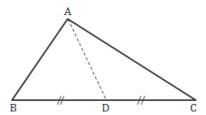


 $a^2 + b^2 = c^2$ ; Where a and b are sides making a right angle, and c is the hypotenuse.

Common Pythagorean Triplets

$$(3, 4, 5)$$
  $(5, 12, 13)$   $(7, 24, 25)$   $(8, 15, 17)$   $(9, 40, 41)$   $(11, 60, 61)$   $(12, 35, 37)$   $(16, 63, 65)$   $(20, 21, 29)$ 

## **Apollonius theorem**

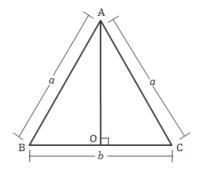


The sum of the squares of any two sides of any triangle equals twice the square on half the third side, together with twice the square on the median bisecting the third side"

Specifically, in any triangle ABC, if AD is a median, then

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$

In an isosceles triangle, the perpendicular from the vertex to the base or the angular bisector from vertex to base bisects base.



In any triangle the angular bisector of an angle bisects the base in the ratio of the other two sides.

If X is a point inside a rectangle ABCD, then

$$AX^2 + CX^2 = BX^2 + DX^2$$

A quadrilateral formed by joining the angle bisectors of another quadrilateral is a rectangle.

# AREA (A) of a:

Square  $A = s^2$ ; where s = any side of square

Rectangle A = lb; where l = length and b = breadth

Parallelogram A = bh; where b = base and h = height

Triangle  $A = \frac{1}{2}$  bh; where b = base and h = height

Triangle  $A = \sqrt{s(s-a)(s-b)(s-c)} \quad \text{where } s = (a+b+c)/2 \qquad \text{*Heron's formula}$ 

Circle  $A = \pi r^2$ ; where  $\pi = 3.14$  and r = radius

Trapezium  $A = \frac{1}{2} (b_1 + b_2) h;$  where h is height and  $b_1$  and  $b_2$  are parallel sides

Sphere  $S = 4\pi r^2$ ; where S = Surface area

# SURFACE AREA (SA) of a:

Cube  $SA = 6s^2$ ; where s = any side

Cylinder (lateral)  $SA = 2\pi rh$ ; where  $\pi = 3.14$  and r = radius and h = height

# PERIMETER (P) of a:

Square P = 4s; where s = any side

Rectangle P = 2(l + b); where l = length and w = width

Triangle  $P = (S_1 + S_2 + S_3);$  where s = side

Any shape P = sum of the length of all sides

Circle (Circumference)  $C = 2\pi r = \pi d$ ; where  $\pi = 3.14$  and d = diameter, r = radius

# VOLUME (V) of a:

| Cube                    | $V = S^3$ :                 | where $S = any side$                               |
|-------------------------|-----------------------------|--|
| dube                    | v — 3 ,                     | where 5 – any side                                 |
| Rectangular Container   | V = lwh;                    | where $l = length$ , $w = width$ and $h = height$  |
| Square Pyramid          | $V = \frac{1}{3} b^2 h;$    | where $b = base length and h = height$             |
| Cylinder                | $V = \pi r^2 h;$            | where $\pi = 3.14$ , $r = radius$ and $h = height$ |
| Cone                    | $V = \frac{1}{3}\pi r^2 h;$ | where $\pi = 3.14$ , $r = radius$ and $h = height$ |
| Sphere                  | $V = \frac{4}{3}\pi r^2;$   | where $\pi = 3.14$ , $r = radius$ and $V = volume$ |
| Right Circular Cylinder | $V = \pi r^2 h;$            | where $r = radius$ , $V = volume$ and $h = height$ |

# Coordinate geometry

| Distance between two points        | $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ | where $(x_1, y_1)$ and $(x_2, y_2)$ are two     |
|------------------------------------|--|---|
|                                    |  | Points of a coordinate plane                    |
| Slope of a Line                    | $m = \frac{y_2 - y}{x_2 - x_1}$            | where $(x_1, y_1)$ and $(x_2, y_2)$ are two     |
|                                    |  | Points of a Co-ordinate plane                   |
| Standard Equation of a Circle      | $(x - h)^2 + (y - k)^2 = r^2$              | where r is radius and (h, k) is the center      |
| Standard equation of a Circle      | $(x - h)^2 + (y - k)^2 = r^2$              | where r is radius and (h, k) is the center      |
| Point-Slope equation of a line     | $y - y_1 = m (x - x_1)$                    | where m is the slope and the point $(x_1, y_1)$ |
| Slope-Intercept equation of a line | y = mx + b                                 | where m is the slope and b is the y-intercept   |

# Special geometry properties:

- 1. Sum of the 2 interior opposite angles of a triangle is always equal to: exterior angle
- 2. In a triangle, the sum of the 2 angles is equal to the third angle, considering interior angles only, then the triangle is : right angled
- 3. Sum of the interior angles of a polygon having 'n' sides is equal to: (n-2)180°
- 4. The angle made by the altitude of a triangle with the side on which it is drawn is equal to : 90 degrees
- 5. When the bisector of any angle is perpendicular to the opposite side, then the triangle is: equilateral
- 6. Number of pairs of vertical angles formed when 2 lines intersect are : 2
- 7. The bisectors of the angle at the vertex of an isosceles triangle: bisects the base and is perpendicular to it
- 8. If 2 angles of a triangle are congruent, the sides opposite of these angles are: congruent
- 9. The straight line joining the midpoints of any 2 sides of a triangle is : parallel to the third side
- 10. The point of intersection of the medians of the triangle is called : centroid
- 11. The point of intersection of the altitudes of the triangle is called: orthocenter

- 12. The bisector of the exterior angle at the vertex of an isosceles triangle is: parallel to the base
- 13. In an isosceles triangle ABC, if D, E, F are the midpoints of the base BC and the equal sides AB, AC respectively, then: DF=DE
- 14. Medians of a triangle pass thru the same point which divides each median in the ratio: 2:1
- 15. A median divides a triangle into 2 triangles of equal areas
- 16. If the diagonal of a quadrilateral bisect each other and are perpendicular, the quadrilateral is: rhombus
- 17. If diagonal AC = diagonal BD and AC is perpendicular to BD in a parallelogram ABCD then it is: rhombus
- 18. If the midpoints of the sides of a quadrilateral are joined, then the figure formed is : a parallelogram if the diagonals of a parallelogram are equal then it's a : rectangle
- 19. If a line is drawn parallel to 1 side of a triangle, the other 2 sides are divided: in the same ratio
- 20. The ratios of areas of similar triangles is equal to the ratio of: squares on the corresponding sides
- 21. In triangle ABC, AD is perpendicular to BC. If  $AD^2 = BD \times DC$ , the triangle is : right angled
- 22. In a parallelogram ABCD, E is a point on AD. AC and BE intersect each other at F. then: BF x FA=EF x FC
- 23. Equal chords of a circle subtend equal angles at the: center
- 24. Angles in the same segment of a circle are: equal
- 25. P is the center of a circle of radius r and distance between the center of the circle and any point r on a given line PR. the line doesn't intersect the circle when: PR>r
- 26. Chord PQ of a circle is produced to 0. T is a point such that OT becomes a tangent to the circle. Then:  $OT^2=OP \times OQ$
- 27. P is the midpoint of an arc APB of a circle. The tangent at P is: parallel to the chord AB
- 28. For any regular polygon, the sum of the exterior angles is equal to  $360^{\circ}$ , hence measure of any external angle is equal to  $\frac{360}{n}$  (where 'n' is the number of sides)
- 29. For any regular polygon, the sum of interior angles = $(n 2) \times 180^{\circ}$ 
  - So measure of one angle is  $\frac{n-2}{n}$  x 180.
- 30. If any parallelogram can be inscribed in a circle, it must be a rectangle.
- 31. If a trapezium can be inscribed in a circle it must be an isosceles trapezium (i.e. oblique sides equal).
- 32. For an isosceles trapezium, sum of a pair of opposite sides is equal in length to the sum of the other pair of opposite sides (i.e. AB + CD = AD + BC, taken in order)

33. For any quadrilateral whose diagonals intersect at right angles, the area of the quadrilateral is

 $0.5 \times d_1 \times d_2$ ; where  $d_1$  and  $d_2$  are the length of the diagonals

34. For a cyclic quadrilateral, Area =  $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ ; where  $s=\frac{a+b+c+d}{2}$ 

Further, for a cyclic quadrilateral, the measure of an external angle is equal to the measure of the interior opposite angle.

- 35. Area of a Rhombus = Product of Diagonals/2
- 36. Given the coordinates (a, b); (c, d); (e, f); (g, h) of a parallelogram , the coordinates of the meeting point of the diagonals can be found out by solving for

$$\left[\left(\frac{a+e}{2}\right),\left(\frac{b+f}{2}\right)\right] = \left[\left(\frac{c+g}{2}\right),\left(\frac{d+h}{2}\right)\right]$$

- 37. Area of a triangle
  - ½ x base x altitude
  - ½ ab sinC or ½ bc sinA or ½ca sin B

- 
$$\sqrt{s(s-a)(s-b)(s-c)}$$
; where  $s = \frac{a+b+c}{2}$ 

- $\frac{a \times b \times c}{4R}$ ; where R is the circumradius of the triangle
- r x s ,where r is the inradius of the triangle
- 38. In any triangle

- 
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$
 {where R is circumradius}

- 
$$a^2 = b^2 + c^2 - 2bc \cos A$$

- 
$$\sin 2A = 2 \sin A \times \cos A$$

$$- \cos 2A = \cos^2(A) - \sin^2(A)$$

- 39. The ratio of the radii of the circumcircle and incircle of an equilateral triangle is 2:1
- 40. In any triangle, the angular bisector of an angle bisects the base in the ratio of the other two sides.
- 41. Distance between a point  $(x_1, y_1)$  and a line represented by the equation ax + by + c = 0 is given by  $\frac{|ax_1 + by_1 + c|}{(a^2 + b^2)^2}$
- 42. Where a rectangle is inscribed in an isosceles right angled triangle; then, the length of the rectangle is twice its breadth and the ratio of area of rectangle to area of triangle is 1:2

## Permutations, Combinations and Probability

If a function can be done in x ways and for each of these functions, the other function can be done in y ways. Together, they can be done in  $(x \times y)$  ways

If a function can be done in x ways and the other function can be done in y ways, then either of these functions can be done in (x + y) ways

Permutations:  $_{r}^{n}P = \frac{n!}{(n-r)!}$ 

Combinations:  ${}_{r}^{n}C = \frac{n!}{r!(n-r)!}$ 

Number of ways of distributing n identical things among r persons when each person may get any number of things =  ${n+r-1 \atop r-1}$ C

## Odds

Number of favourable outcomes Odds in favor =Number of unfavourable outcomes

Number of unfavourable outcomes Odds against = Number of favourable outcomes

|                       | Selection             | Arrangement         |
|-----------------------|-----------------------|---------------------|
| n similar things      | (n+1)                 | 1                   |
| n distinct things     | 2 <sup>n</sup>        | n!                  |
| n similar, m distinct | (n + 1)2 <sup>m</sup> | $\frac{(n+m)!}{n!}$ |

#### Miscellaneous

- Calendar repeats after every 400 years.
- Leap year is always divisible by 4, but century years are not leap years unless they are divisible by 400.
- In a normal year 1st January and 2nd July and 1st October fall on the same day. In a leap year 1st January, 1st July and 30<sup>th</sup> September fall on the same day.
- The speed of an hour hand in a clock is 30 degrees/hour and that of the minute hand is 360 degrees/hour; relative speed between the two is  $\frac{11}{2}$  degrees/minute Number of squares in a square of side  $n*n = 1^2 + 2^2 + 3^2 + \cdots + n^2$
- Number of rectangles in square of side  $n*n = 1^3 + 2^3 + 3^3 + \cdots + n^3$