Dear Student,

The journey to achieve success has begun. The CL Educate team brings to you an offering,
which incorporates **theme based learning** that revolves around different concepts with **diverse applications**. The outcome is an enriching learning experience.

Our integrated thematic methodology is driven by latest research, undertaken to enhance learning. Numerous practice exercises and tests have been incorporated to reinforce the conviction in one's ability. Our teaching experience coupled with extensive research has lent credence to our conviction that learning is at its best when concept based understanding and applications go hand in hand.

To enhance your learning and assimilation of relevant concepts, our attempt has been to identify the basic concepts (or themes) that are required to solve different questions in MBA entrance examinations. Our class exercises integrate the different types of questions requiring application of these concepts. Each set of concepts along with relevant question types therefore, forms a module. At the end of each module we expect the student to:

1) Clearly understand a concept through its repeated application in different question types.

2) Quickly and effectively apply the relevant concept to different question types in a time-bound examination scenario.

3) Develop long-lasting skills by imbibing each concept that is clearly covered through a module.

Armed with the latest tools for success, along with your diligence and positive attitude, you have begun your march towards success. Have faith in yourself!

The woods are lovely, dark and deep,

But I have promises to keep,

And **miles togo before I sleep**,

And **miles to go before I sleep**

(Robert Frost)

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How to use this book

1. Before you enter the class read the topics that are to be covered beforehand. This will help you immensely in understanding the concepts when they are taught in the class.

2. After each class, once again go through the relevant topics very carefully, in order to understand the concepts and relate them to what was taught in the class.

3. Do not directly jump to the practice problems but go through the solved examples first as they will enhance your problem solving skills and help in further clarifying concepts.

4. After you are through with the fundamentals and the solved examples, move on to the unsolved problems given at the end of the book and the practice exercises.

5. Start with the Level - I problems as they are easier. Move to the Level - II problems, if and only if you have completely understood the concept used in every problem in Level - I. Similarly, move to the Level - III problems after you have completed all the problems in Level - II.

Ratio and Proportion 1

Introduction

This is one of the most important topics in arithmetic from examination point of view. Ratio is an extension of the concept of fractions. These are fairly simple topics and will need a focussed approach to refresh these concepts that you learnt during your school days.

After Percentages, Ratio and Proportion is another topic that finds innumerable applications in our daily life. R&P can be used very effectively for building speed in problems from topics as varied as Simple and Compound Interest, Time Speed and Distance, Time and Work, Mixtures, etc. Even in Geometry, corresponding sides of similar figures are `proportional'!

A lot of problems that conventionally can be solved by forming equations, can very easily be solved verbally by using just ratios without the help of the equations. So, make sure that you do not discount this topic because it is a very simple, rather strive hard to identify the usage of ratios in diverse situations.

Learning Objectives

By the end of the chapter, you should be able to solve problems related to

• Ratio

• Direct and Inverse Proportion

• Partnership

• Alligation

• Mixtures and solutions

• Direct & Inverse Relation

• Partnership

• Problems on chain rule

Ratio

The concept of ratios is used for comparing two or more quantities of a similar kind. The definition of *ratio* is given below:

The ratio of the quantity "A" to the quantity "B" is a relation that tells us what multiple or fraction the quantity "A" is of the quantity "B".

The ratio of quantity A to quantity B is denoted by A: B and it is measured by the fraction $\frac{A}{B}$. The quantities

Page :

A and B are called the *terms* of the ratio. Also note that, as both the terms A and B of a ratio are of a similar kind or have the same units, the resulting ratio A: B has no unit and hence it is purely a number.

Consider the two quantities A and B such that A:B = 2:3. In what ways can we interpret this relation? Read the following.

(i) The ratio B: A is 3:2.

(ii) A is $\frac{2}{3}$rd part of B.

(iii) B is 1.5 times that of A.

(iv) B is 50% more than A.

(v) A is 33.33% less than B.

If both the terms of a ratio are multiplied or divided by the same (non-zero) quantity, then the value of the ratio remains the same. In terms of notations:

$\frac{a}{b} = \frac{ka}{kb}$ (Here, k is any non-zero real number.)

Example 1:

Solve these problems

(I) A and B got 175 and 225 marks respectively. What is the ratio of their marks?

(II) X scored 105 marks out of 150 and Y scored 175 marks out of 200. What is the ratio of the percentage marks scored by each?

Solution:

(I) $\frac{175}{225} = \frac{7}{9} = 7 : 9$

(II) $\frac{105}{150} : \frac{175}{200} = 4 : 5$

Example 2:

5 kg of wheat flour is mixed with 500 gm of sugar extract. What is the ratio of sugar extract to the rest of the mixture after adding 1.5 kg of water?

Solution:

We first need to express all quantities in a single unit.

Wheat flour = 5 kg

Water = 1.5 kg

Sugar extract = 500 gm = 0.5 kg

Total weight of the mixture = 7 kg

Ratio of sugar extract to the rest of mixture = $\frac{0.5}{6.5} = 1 : 13$

Example 3:

Divide Rs. 1000 between A and B in the ratio of 7 : 3.

Solution:

Let A gets Rs. 7x and B gets Rs. 3x as 7x + 3x = 1000 $\Rightarrow x = \frac{1000}{10} = 100$

$\Rightarrow$ A gets Rs. 700 and B gets Rs. 300

Example 4:

What must be subtracted from the numerator and the denominator of fraction $\frac{6}{7}$ to give a fraction equal to $\frac{16}{21}$?

Solution:

Let the number to be subtracted be x.

$$\frac{6-x}{7-x} = \frac{16}{21}$$

On solving, we get x = 2.8

Example 5:

Ram's father is thrice as old as Ram was, 2 years ago. Five years from now, his father's age will be 6 years more than twice the Ram's age. What is Ram's present age?

Solution:

Let Ram's present age be X and his father's present age be Y.

$Y = 3(X - 2)$ and $(Y + 5) - 6 = 2(X + 5)$.

Solving, we get, X = 17 years and Y = 45 years.

Example 6:

A's income is $\frac{2}{3}$rd of B's. B's income is 75% of C's. What is the ratio of C's income to A's income?

Solution:

B's income = $\frac{3}{2}$ of A's income.

C's income = $\frac{4}{3}$ of B's income $= \frac{4}{3} \times \frac{3}{2}$ of A's income

$\Rightarrow$ required ratio = 2 : 1.

Example 7:

Let the ratio A:B is measured by the fraction $\frac{X}{Y}$.

If the quantities A and B are fractions then can X and Y be integers?

Solution:

Let $A = \frac{a}{b}$ and $B = \frac{c}{d}$ Now $A : B = \frac{X}{Y} = \frac{ad}{bc}$.

As each of *a*, *b*, *c* and *d* are integers, *ad* and *bc* are integers as well. So, X and Y are integers.

Example 8:

Let one or both of the two quantities P and Q $(P \neq Q)$ are surds.

If $\frac{P}{Q} = \frac{p}{q}$ then can *p* and *q* simultaneously be integers?

Solution:

Yes, this is possible

e.g. if $P = 3\sqrt{2}$ and $Q = 2\sqrt{2}$ then $\frac{P}{Q} = \frac{3\sqrt{2}}{2\sqrt{2}} = \frac{3}{2} = \frac{p}{q}$

Here p and q are both integers.

Example 9:

Two numbers are in the ratio 2 : 3. The difference between their squares is 45. Find the numbers.

Solution:

Let the two numbers be 2x and 3x.

Then $(3x)^2 - (2x)^2 = 45$

$5x^2 = 45 \Rightarrow x = 3$ or -3.

Hence, the numbers are either (6, 9) or (-6, -9).

Example 10:

The ratio of two numbers is 3 : 4. If each number is increased by 6, the ratio becomes 4 : 5. Find the two numbers.

Solution:

Let the two numbers be 3x and 4x.

Then (3x + 6) : (4x + 6) = 4 : 5.

Solving for x, we get x = 6.

Hence, the numbers are 18 and 24.

Example 11:

The incomes of A and B are in the ratio 3 : 2 and the expenditures in the ratio 5 : 3. If each saves Rs. 2,000, what are their incomes?

Solution:

Let 3a, 2a be the incomes, and 5b and 3bbe the expenditures of A and B respectively.

We have 3a - 5b = 2a - 3b = 2000

Solving, a = 4000, b = 2000

$\therefore$ A's income = Rs. 12,000, B's income = Rs. 8,000.

Example 12:

The ratio of the prices of two houses was 16 : 23. Two years later when the price of the first had risen by 10% and that of the second by Rs. 477, the ratio of prices became 11 : 20. What were the original prices?

Solution:

Let the first and the second houses be priced 16x and 23x respectively.

The final values are 17.6x and 23x + 477.

The final ratio is 11 : 20.

Solving for x, we get x = 53.

So the values of the houses are

Rs. 848 and Rs. 1,219 respectively.

Example 13:

If x : y = 2 : 3, find the value of (3x + 2y) : (2x + 5y).

Solution:

Method 1: (3x + 2y) : (2x + 5y) = $\frac{3x+2y}{2x+5y}$

$$\frac{\left[3\left(\frac{x}{y}\right)+2\right]}{\left[2\left(\frac{x}{y}\right)+5\right]}=\frac{\left[3\left(\frac{2}{3}\right)+2\right]}{\left[2\left(\frac{2}{3}\right)+5\right]}=\frac{12}{19}$$

Method 2: Assume the values of x = 2 and y = 3 and work out the ratio.

Example 14:

If 2x + 3y : 3x + 5y = 18:19, find x : y.

Solution:

As in the previous problem, we obtain

$$\frac{2\left(\frac{x}{y}\right)+3}{3\left(\frac{x}{y}\right)+5}=\frac{2k+3}{3k+5}=\frac{18}{19}.$$

Solving for k, we get $k=\left(\frac{x}{y}\right)=-\frac{33}{16}$.

• *Ratios just give an idea of the relative sizes and not the actual magnitude*

• *If two variables are in the ratio a : b, the actual values of the variables can be assumed as ak and bk where k is a constant of proportionality.*

• *The ratio a : b is same as the ratio a × k : b × k where k is any constant*

• *When ratios are given, one can work with the values of the ratio and at end use unitary process to equate the value obtained using the values of ratio to the actual amount given in the question e.g. in example 12 on this page one could work as follows*

Price are 16 and 23

16 increases by 10% to become 17.6

New prices are 17.6 and 32 (ratio of 11 : 20)

Thus increase in price of second house = 9. But actual increase given is 477. Multiply the assumed prices (16 and 23) with 477/9 = 53

This process avoids equations and any use of variables

Comparison of Ratios

Question: Which of the following is/are correct?

(a) $\frac{113}{115}>\frac{13}{15}$ (b) $\frac{87}{85}<\frac{27}{25}$

(c) $-\frac{27}{17}>\frac{-13}{9}$ (d) $-\frac{15}{19}<-\frac{5}{9}$

To compare two ratios $\frac{a}{b}$ and $\frac{c}{d}$, first make their denominators of the same sign.

Now

(I) If $(ad-bc)>0$ the $\frac{a}{b}>\frac{c}{d}$

(II) If $(ad-bc)<0$ then $\frac{a}{b}<\frac{c}{d}$

(III) If $(ad-bc)=0$ then $\frac{a}{b}=\frac{c}{d}$

Apply the above concepts for each option. You will see that (a), (b) and (d) are correct and (c) is incorrect.

There are some other methods and shortcuts as well, to compare the ratios. Read the following concepts and understand the following examples carefully and make yourself comfortable with these methods as they are extremely helpful in Data Interpretation problems.

We will discuss some important properties of the ratios now. Let a ratio has value $\frac{A}{B}$. What happens when we add or subtract the same quantity from both the numerator and the denominator?

The result, in fact, depends on whether $\frac{A}{B} > 1$ or < 1? We have summarized the results for both the cases as under:

Case I: If $\frac{A}{B} > 1$.

1. $\frac{A+x}{B+x} < \frac{A}{B}$ $(x > 0)$

2. $\frac{A-x}{B-x} > \frac{A}{B}$ $(x > 0)$

Case II: If $\frac{A}{B} < 1$

3. $\frac{A+x}{B+x} > \frac{A}{B}$ $(x > 0)$

4. $\frac{A-x}{B-x} < \frac{A}{B}$ $(x > 0)$

You can verify the above results by choosing any arbitrary values for A, B and x. Try to memorize these results as these are extremely helpful in Data Interpretation problems.

Example 15:

Which one is the greatest of $\frac{13}{11}, \frac{15}{13}, \frac{11}{9}$ and $\frac{12}{10}$

Solution:

Difference in the numerator and denominator is 2 in each case. Also, the fractions are all more than 1. Therefore, greatest fraction is $\frac{11}{9}$.

[**Note:-** Here, Case I - (2) is used]

Example 16:

Which of the following is the smallest:

$\frac{14}{25}, \frac{57}{100}, \frac{49}{86}, \frac{3}{5}$

Solution:

One way of solving such a problem is to convert each into decimals and then compare.

Another way is to equalize either numerator or denominator of every two fractions that are being compared.

Suppose, we have $\frac{a}{b}$ and $\frac{c}{d}$ to be compared.

If a = c, then the fraction with the higher value of denominator is smaller.

If b = d, then the fraction with the lower value of numerator is smaller.

Comparing $\frac{14}{25}$ with $\frac{57}{100}$, $\frac{14}{25} = \frac{56}{100} < \frac{57}{100}$.

So, $\frac{57}{100}$ is eliminated.

Now comparing

$\frac{14}{25}$ with $\frac{49}{86}$, $\frac{14}{25} = \frac{98}{175}$ & $\frac{49}{86} = \frac{98}{172}$.

So, $\frac{49}{86}$ is eliminated

Comparing $\frac{14}{25}$ with , = $\frac{15}{25} > \frac{14}{25}$.

So $\frac{14}{25}$ is the smallest.

Example 17:

Pompo & Rompo are two countries engaged in a war. Pompo possesses 8 tanks and Rompo possesses 11 tanks. They get external support from neighbouring countries, of two tanks each. Which of the two countries is supposed to have a relatively greater increase in strength?

Solution:

For Pompo's = $\frac{\text{Final}}{\text{Initial}} = \frac{10}{8}$

For Rompos' $\frac{\text{Final}}{\text{Initial}} = \frac{13}{11}$

As $\frac{10}{8} > \frac{13}{11}$ Pompo had a relatively greater increase in strength.

Now, we will learn some very important properties of ratios by solving the following problems. These properties will be very useful in solving problems in algebra. Understand each step thoroughly.

Example 18:

If a: b = 2:5 then find the ratio $2a-3b:5a+7b$

Solution:

a: b is $\frac{2}{5}$

$$2a-3b:5a+7b = \frac{2a-3b}{5a+7b}$$

$$= \frac{2\frac{a}{b}-3\frac{b}{b}}{5\frac{a}{b}+7\frac{b}{b}} = \frac{2\times\frac{2}{5}-3}{5\times\frac{2}{5}+7} = -\frac{11}{45}$$

Proportions

When two ratios are equal, the four quantities composing them are said to be *proportionals*. In terms of the notations, if $\frac{A}{B} = \frac{C}{D}$ then A, B, C and D are proportionals. We use the symbol "::" to express it mathematically. So whenever we write A: B:: C: D, it is interpreted as A, B, C and D are proportionals. The terms A and D are called the *extremes* and the terms B and C are called the *means*. It is very easy to note that $A\times D = B\times C$. This result is more commonly expressed as:

"Product of the extremes = Product of the means".

Continued Proportion:

a, b, c, d, e, f...are said to be in continued proportion if

$$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f} = \ldots$$

When three quantities a, b and c are in continued proportion then b is called the *mean proportional* and c is called the *third proportional*.

$$\frac{a}{b} = \frac{b}{c} \Rightarrow b^2 = a\times c$$

When four quantities a, b, c and d are proportionals then d is called the *fourthproportional.*

Example 19:

Rs. 1,150 is to be divided between A, B and C such that the ratio of share of A to that of B is equal to 3 : 2 and share of B to share of C is equal to 3 : 4. Find their individual share.

Solution:

Here A : B = 3 : 2 = 9 : 6 and B : C = 3 : 4 = 6 : 8

Therefore, A : B : C = 9 : 6 : 8

1150 can be divided between them as follows:

A's share = $\frac{1150 \times 9}{23}$ = 450

B's share = $\frac{1150 \times 6}{23}$ = 300

C's share = $\frac{1150 \times 8}{23}$ = 400

Example 20:

In the year 1996, the monthly allowances given to A, B and C were in the ratio 5 : 3 : 1. If C's monthly allowance was Rs. 1000 then what was total allowances received by A, in that year?

Solution:

C's share = Rs. 1,000

A's share : C's share = 5 : 1

Therefore, A's monthly share = Rs. 5,000

A's share for the whole year = 5000 × 12 = Rs. 60,000

Example 21:

From a total of Rs. 159000, Rs. 5000 is to be divided between A and B in the ratio 2 : 3. The rest of the money is to be divided between A, B and C in the ratio 5 : 3 : 3. How much money did A and B get, respectively?

Solution:

Rs. 5,000 is to be divided between A and B in the ratio of 2 : 3.

A's share = Rs. 2,000

B's share = Rs. 3,000

Amount left is 1,54,000 and it is to be divided in the ratio of 5 : 3 : 3.

A's share = Rs. 70,000

B's share = Rs. 42,000

C's share = Rs. 42,000

A's total share = Rs. 72,000

B's total share = Rs. 45,000

Example 22:

Ratio of a : b is same as that of b : c, where a, b and c are positive numbers. If a = 10 and c = 40, find b.

Solution:

a : b = b : c

b × b = a × c

b × b = 400

$b = \sqrt{400} = \pm 20$

So, b = 20.

Example 23:

Divide 156 in 4 parts such that they are in continued proportion and the sum of the first and third parts is in a ratio of 1 : 5 to the sum of the second and the fourth part.

Solution:

$156 = a + b + c + d$

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = k \Rightarrow b^2 = ac \;\&\; c^2 = bd$

$\Rightarrow$ a, b, c, d are in a G..P.

Let the common ratio be r. We have:-

$\frac{a+c}{b+d} = \frac{1}{5}$ or $\frac{a+ar^2}{ar+ar^3} = \frac{1}{5}$

$\Rightarrow r = 5$

$\Rightarrow a + 5a + 25a + 125a = 156$

$\Rightarrow a = 1, b = 5, c = 25 \;\&\; d = 125$

Example 24:

Find the third proportional to 3, 5.

Solution:

Let the third proportional be "x" then 3:5 :: 5:x

$\frac{3}{5} = \frac{5}{x} \Rightarrow x = \frac{25}{3}$.

Example 25:

Find the fourth proportional to 1, 2 and 3.

Solution:

Let the fourth proportional be "x" then 1 : 2 :: 3 : x

$\frac{1}{2} = \frac{3}{x} \Rightarrow x = 6$.

Example 26:

Find the mean proportional to 27, 3.

Solution:

Let the mean proportional be "x".

27 : x :: x : 3

$\frac{27}{x} = \frac{x}{3} \Rightarrow x^2 = 81 \Rightarrow x = \pm 9$

As 27 and 3 are positive quantities, we will take x to be positive only. So x = 9.

Operations on Ratios:

Let $\frac{A}{B} = \frac{C}{D}$. Three very important results are derived from the following operations.

(1) Componendo operation

(2) Dividendo operation

(3) Componendo and Dividendo operation

Componendo Operation:

As $\frac{A}{B} = \frac{C}{D}$

$\frac{A}{B} + 1 = \frac{C}{D} + 1 \Rightarrow \frac{A+B}{B} = \frac{C+D}{D}$

This operation is called componendo. We will make use of this important result while solving problems in a variety of topics.

Dividendo Operation:

As $\frac{A}{B} = \frac{C}{D}$

$\frac{A}{B} - 1 = \frac{C}{D} - 1 \Rightarrow \frac{A-B}{B} = \frac{C-D}{D}$

This operation is called dividendo.

This is an equally important result.

Componendo and Dividendo Operation:

When we combine the results of componendo and the dividendo operations, we get following important result.

If $\frac{A}{B} = \frac{C}{D}$ then $\frac{A+B}{A-B} = \frac{C+D}{C-D}$

We will solve a few examples using the above results.

Example 27:

If $p : q :: r : s$ then prove that $2p+3q : 2p-3q :: 2r+3s : 2r-3s$

Solution:

We have $\frac{p}{q} = \frac{r}{s}$

Multiplying both the sides by $\frac{2}{3}$ we get

$\frac{2}{3} \times \frac{p}{q} = \frac{2}{3} \times \frac{r}{s}$ Or $\frac{2p}{3q} = \frac{2r}{3s}$

Using the componendo and dividendo property we get

$\frac{2p+3q}{2p-3q} = \frac{2r+3s}{2r-3s}$

Example 28:

Solve for x, $\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} = 2$

Solution:

$\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}} = 2$

Applying the Componendo and Dividendo:

$\frac{(\sqrt{1+x} + \sqrt{1-x}) + (\sqrt{1+x} - \sqrt{1-x})}{(\sqrt{1+x} + \sqrt{1-x}) - (\sqrt{1+x} - \sqrt{1-x})} = \frac{2+1}{2-1}$

Or $\frac{2\sqrt{1+x}}{2\sqrt{1-x}} = \frac{3}{1}$

Simplifying and then squaring on both sides, we get:

$$\frac{1+x}{1-x}=9$$

You can proceed to get $1+x=9\times(1-x)$ and solve for x. Another way of proceeding from here is to apply componendo and dividendo, one more time. So that we get:

$$\frac{(1+x)+(1-x)}{(1+x)-(1-x)}=\frac{9+1}{9-1}=\frac{5}{4}$$

This gives, $x=\frac{4}{5}$.

Example 29:

If $\frac{a}{b}=\frac{c}{d}$ then prove that

(a) $\frac{a\pm b}{b}=\frac{c\pm d}{d}$

(b) $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

(c) $\frac{a\pm c}{b\pm d}=\frac{a}{b}$

Solution:

(a) $\frac{a}{b}=\frac{c}{d}\Rightarrow\frac{a}{b}\pm1=\frac{c}{d}\pm1$ Or $\frac{a\pm b}{b}=\frac{c\pm d}{d}$

(b) $\frac{a}{b}=\frac{c}{d}$

$$\frac{a+b}{b}=\frac{c+d}{d}=k(\text{say})$$

$$\Rightarrow(a+b)=bk\text{ and }(c+d)=dk$$

$$\Rightarrow\frac{a+b}{c+d}=\frac{b}{d}\ \ldots(i)$$

Similarly,

$$\frac{a-b}{c-d}=\frac{b}{d}\ \ldots(ii)$$

From (i) and (ii)

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

(c) $\frac{a}{b}=\frac{c}{d}=k$, say

$$\Rightarrow a=bk\text{ and }c=dk$$

So that, $\frac{a\pm c}{b\pm d}=\frac{bk\pm dk}{b\pm d}=k=\frac{a}{b}$

Example 30:

If $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=K$, then prove that

(a) $\frac{a+c+e}{b+d+f}=K$

(b) $\frac{pa+qc+re}{pb+qd+rf}=K$

(p, q and r are not all zero)

(c) $\left(\frac{pa^n + qc^n + re^n}{pb^n + qd^n + rf^n}\right)^{\frac{1}{n}} = K$

(p,q and r are not all zero)

Solution:

$\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = K$, given

$\Rightarrow a = bK, c = dK$ and $e = fK$

Substituting, we get

(a) $\frac{a+c+e}{b+d+f} = \frac{bK+dK+fK}{b+d+f} = K$, hence proved.

(b) $\frac{pa+qc+re}{pb+qd+rf}$

$= \frac{pbK+qdK+rfK}{pb+qd+rf} = K$, hence proved

(c) $\left(\frac{p(a)^n + q(c)^n + r(e)^n}{pb^n + qd^n + rf^n}\right)^{\frac{1}{n}}$

$= \left(\frac{p(bK)^n + q(dK)^n + r(fK)^n}{pb^n + qd^n + rf^n}\right)^{\frac{1}{n}} = (K^n)^{\frac{1}{n}} = K$

Example 31:

If a, b, c, d, e are in continued proportion, prove that a . b . d . e = c^4.

Solution:

Since a, b, c, d and e are in continued proportion,

$\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = k$

where k is some constant.

$\therefore$ a = bk, b = ck, c = dk, d = ek

Expressing a, b, d, e in terms of c and k, we have

$a = ck^2$, $b = ck$, $d = \frac{c}{k}$ and $e = \frac{d}{k} = \frac{c}{k^2}$

$\therefore$ a . b . d . e = c^4

Example 32:

If A : B = 3 : 4; B : C = 8 : 9; C : D = 15 : 16,

(a) find the ratio A : D, and

(b) find A : B : C : D.

Solution:

(a) $\frac{A}{D} = \frac{A}{B} \times \frac{B}{C} \times \frac{C}{D} = \frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} = \frac{5}{8}$

So A : D = 5 : 8

(b) A : B = 3 : 4;

B : C = 8 : 9 = 4 : $\frac{9}{2}$;

$C : D = 15 : 16 = \frac{9}{2} : \frac{24}{5}$.

Hence, $A : B : C : D = 3 : 4 : \frac{9}{2} : \frac{24}{5}$

$= 30 : 40 : 45 : 48$.

Operations on proportions :

If we have $\frac{a}{b} = \frac{c}{d}$, we can also arrive at the following equalities :

- $\frac{a}{c} = \frac{b}{d}$

This process is called Alternendo.

- $\frac{b}{a} = \frac{d}{c}$

This process is called Invertendo.

- $\frac{a+b}{b} = \frac{c+d}{d}$

This process is called Componendo and is arrived at by adding 1 to each side

- $\frac{a-b}{b} = \frac{c-d}{d}$

This process is called Dividendo and is arrived at by subtracting 1 from both sides of the equation.

- $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ is Componendo and Dividendo.

Further, if $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots$ then each of this ratio is also equal to $\frac{(a \times k_1) + (c \times k_2) + (e \times k_3) \ldots}{(b \times k_1) + (d \times k_2) + (f \times k_3) \ldots}$

where k_1, k_2, k_3 are any real numbers except all of them being zero simultaneously.

A special relation of this is $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \ldots = \frac{a+c+e \ldots}{b+d+f \ldots}$

where $k_1 = k_2 = k_3 \ldots = 1$.

Example 33:

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f}$, prove that $\frac{(a+3c-5e)}{(b+3d-5f)} = \frac{a}{b} = \frac{c}{d} = \frac{e}{f}$.

Solution:

Let $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = k$

$a = bk, c = dk, e = fk$

So $\frac{(a+3c-5e)}{(b+3d-5f)} = \frac{k(b+3d-5f)}{(b+3d-5f)} = k$

Example 34:

If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b}$, what is the value of each of the fractions? $(a, b, c > 0)$

Solution:

If each of the ratio is equal to k, then

$a = (b + c)k, b = (c + a)k$ and $c = (a + b)k$.

Hence, a + b + c = (2a + 2b + 2c)k or
(a + b + c) - 2k(a + b + c) = 0

(a + b + c)(1 - 2k) = 0.

So $k = \frac{1}{2}$.

Example 35:

If $\frac{x}{b-c} = \frac{y}{c-a} = \frac{z}{a-b}$,

then prove ax + by + cz = 0.

Solution:

There are two approaches of doing this problem.

Method 1:

Assume that each of these ratio is equal to K and then follow the method as in the previous problem.

Method 2:

Assume any arbitrary values of a, b, c, x such that none of the denominators is 0. Find the corresponding values of y, z. Put it in the required equation and prove that the condition is true.

If, $\frac{a}{b} = \frac{c}{d} = x$ (say), then $\frac{a^2}{b^2} = \frac{c^2}{d^2} = ?$

$A : B : C = \frac{1}{3} : \frac{1}{4} : \frac{1}{5}$ implies A : B : C = 20 : 15 : 12.

But $\frac{A}{3} = \frac{B}{4} = \frac{C}{5}$ implies A : B : C = 3 : 4 : 5

Applications of Ratio

Example 36:

A dog pursues a cat and takes 5 leaps for every 6 leaps of the cat, but 4 leaps of the dog are equal to 5 leaps of the cat. Compare the speeds of the dog and the cat.

Solution:

Method 1:

4 leaps of the dog = 5 leaps of the cat = x say

1 leap of dog = $\frac{x}{4}$

1 leap of cat = $\frac{x}{5}$

In same time, dog takes 5 leaps and cat 6 leaps.

∴ In same time distance covered by dog and cat is $\frac{5}{4}x$ and $\frac{6}{5}x$.

∴ Ratio of speed $\frac{5}{4} : \frac{6}{5}$, i.e. 25 : 24.

Method 2:

Ratio of	Dog	Cat
Leap frequency	5	6
Leap length	4	5

Required ratio of speed is the ratio of cross product.

Speed of the dog : Speed of the cat = $5 \times 5 : 6 \times 4 = 25 : 24$.

The following one is difficult but intellectually stimulating. Consider income of two persons in ratio a : b and expenditure in ratio x : y.

If expenditure cannot exceed the income for either of them, find the condition under which we can ascertain who saves more?

E.g. Incomes of A and B are in the ratio 4 : 5 and expenditures are in the ratio 3 : 4.

If incomes are 400 and 500 and expenditures are 30 and 40 respectively, B saves more.

In the case incomes are 400 and 500 and expenditure are 330 and 440 respectively, A saves more.

However in case incomes of A and B are in the ratio 4 : 5 and expenditures in ratio 5 : 6, whatever be the absolute value of their incomes and expenditures, B will always be saving more.

Can you generalize what we have observed here?

Introduction to Mixtures:

One application of ratios that will be used extensively in problems on mixtures is to describe any mixture or solution. A solution of milk and water has milk and water in the ratio 2 : 3. Thus, if the solution has 2k litres of milk, there will be 3k litres of water and the total solution will be 5k litres. The fraction of the solution that is milk will be $\frac{2k}{5k} = \frac{2}{5}$.

Also as we have already seen the relation between fractions and percentages, we can also say that 40% of the solution is milk and consequently 60% of the solution is water. Thus here we have simply re-visited the fact that ratios are nothing but fractions and fractions can be converted to percentages and thus we can work with any of ratios, fractions or percentages, whichever is most suitable to us.

Further, two solutions can be mixed with each other to result in a third solution. The composition of resultant solution will depend on the ratio in which the two solutions are mixed. E.g. A 2 : 3 milk and water solution is mixed with a 4 : 1 milk and water solution in the ratio 1 : 2. In this, there are three ratios used and it would pay off well for one to understand these three ratios. 2 : 3 and 4 : 1 are ratio of milk and water in the two solutions that are being mixed whereas 1 : 2 is the ratio in which these two solutions are being mixed. Thus 2 : 3 and 4 : 1 are used to characterize the solutions as 40% and 80% milk solutions. 1 : 2 is used to denote that if 10 litres of 40% milk solution is taken, it has to be mixed with 20 litres of 80% solution.

Please note that the concentration of milk in the resultant solution depends not only on the concentration of the solutions being mixed (40% and 80% in this example) but also on the ratio in which these are being mixed. And this should be obvious since if we add just a spoonful of 40% solution to an entire jug of 80% solution, the resultant will still have a concentration marginally lower than 80% but if the two solutions are added in equal quantity (i.e. mixed in ratio 1 : 1), the resultant will be a 60% milk solution. More of this in chapter on Mixtures.

Races:

Problems on races are based on ratios though they appear as problems of Time Speed Distance. The statement "In a race of 1000 mts, A can give B 100 mts" means that when A runs 1000 mts, B runs 900 mts. Thus the ratio of the speeds of A and B is 10 : 9. Thus when A runs 600, B will run 540 mts.

Example 37:

In a game of 500 points A gives B 100 points and B gives C 100 points. In the same game, how many points can A give C?

Solution:

The answer is not 100 + 100 = 200. The statements just tell us the ratio of A : B is 5 : 4 and that of B : C is also 5 : 4. Using this we can get the ratio of A : C as 25 : 16. Thus when A scores 500 points, C will score 320 points and thus A can give C 180 points.

Direct and Inverse Proportion

Proportion plays a very major part in simplifying many tedious processes and methods. Proportions are based on how one particular parameter changes with respect to changes in other parameter/s.

Direct Proportion

When one variable x increases as another variable y increases and that too 'proportionally' (i.e. if y becomes two times, x also becomes twice and if y becomes 1.4 times, x also becomes 1.4 times), we say that x is Directly Proportional to y. This relation of direct proportion is denoted as $x \propto y$.

A few instances of direct proportion are

a. Distance covered is directly proportional to speed (time remaining constant)

b. Amount of work done is directly proportional to number of men working (rate of work and time for which worked remaining constant)

c. Expenditure in Rs is directly proportional to the price per unit (number of units bought remaining same)

If $x \propto y$ (x is directly proportional to y), we can convert the relation to an equation as $\frac{x}{y} = k$, where k is a constant.

Let's see, how is this so. Assume $x = x_1$ when $y = y_1$. As seen in the definition, if y becomes $1.4y_1$, then x will also become $1.4x_1$ and the ratio $\frac{1.4x_1}{1.4y_1}$ is same as $\frac{x_1}{y_1}$. The ratio of any values of x and y will always be the same.

Thus $\frac{x}{y}$ = constant and we will have $x = k \times y$, where k is called the constant of proportionality.

An example of the use of this follows.

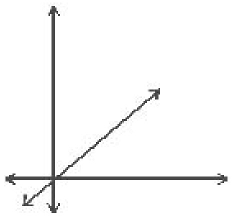
The cost of a picnic is directly proportional to the number of people going for the picnic. When 20 people go for the picnic, the cost of the picnic is Rs. 400. What would be the cost of the picnic when 50 people go for the picnic?

If C denotes the cost of the picnic and N denotes the number of people going for the picnic, we have the relation $C \propto N$. Thus we can say the ratio $\frac{C}{N}$ is a constant.

Thus we will have $\frac{400}{20} = \frac{?}{50}$.

Thus the cost when 50 people go on picnic is Rs. 1000.

The variable x can also be directly proportional to a power of y, say n. In this case the relation is $x \propto y^n$ and the ratio $\frac{x}{y^n}$ is constant.



Direct Proportion:

Note that the line passes through origin.

Direct Relation :

If x is Directly Related to or varies directly as y, then as y increases, x also increases but not proportionally.

The mathematical expression for this relationship is $x = k_1y + k_2$,

where k_1 and k_2 are any constants.

An instance of direct relation is the total cost of production being directly related to the number of items being produced.

The cost of a picnic is directly related to the number of people going on the picnic. The cost per person going on a picnic decreases from Rs. 100 to 90 when the number of people increases from 100 to 120. What will be the cost per person when the number of people going for the picnic is 150?

Please note that this is distinct from the example we saw in the case of Direct proportion. Over there the relation was "Directly Proportional", whereas here, the cost is "Directly Related". Another point to notice before proceeding to solve the problem is that the relation is given between the cost of the picnic whereas the data is given about cost per person.

Since, this is a relation of directly related, we have $C = k_1N + k_2$, where C is total cost of the picnic, N is the number of people going for the picnic and k_1 and k_2 are constants. Thus we have

$10000 = 100k_1 + k_2$ and $10800 = 120k_1 + k_2$

Solving these two equations simultaneously we get $k_1 = 40$ and $k_2 = 6000$ and the relation between cost of picnic and number of people is $C = 40N + 6000$.

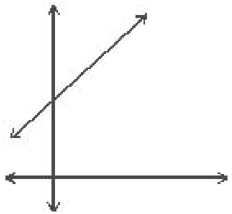
Substituting $N = 150$, we have $C = 12000$ and the cost per person is $\frac{12000}{150} = 80$.

The difference between direct proportion and direct relation can also be understood by the graphs given.

Note: In the graph, a linear relation is assumed i.e. y is directly proportional (or directly related) to x and not to any power of x. If the relation was to a power of x, the graph would be a curve and not a straight line.

In direct proportion the line passes through origin i.e. when the value of x is zero, y is also zero. But in the case of direct relation, even though x = 0, y is not zero but takes some value (theoretically can also be negative). Consider the example given of direct relation, that of cost of production. In real life, even if no unit is produced, yet there would be some cost incurred, usually called the fixed cost. In real life, total cost of production = fixed cost + (cost per unit × number of units produced)

Cost per unit is called as variable cost and will be incurred only if any unit is produced, e.g. raw material. Fixed cost is the cost incurred even if no unit is produced, e.g. rent of place, cost of machinery and even labour cost to an extent.



Direct Relation:

It is a upward sloping line but does not pass through origin.

Inverse Proportion

y is Inversely proportional to x if as x increases, y decreases proportionally.

The equation equivalent to x being inversely proportional to y i.e.

$x \propto \frac{1}{y}$, is xy = k, where k is a constant

For instance, the time taken to travel a constant distance decreases as speed increases (or vice versa) and that too proportionally i.e. if speed double, time taken halves and if speed becomes 1/3rd, time taken becomes thrice.

A few instances of Inverse Proportion are

1. The rate (efficiency) at which people work and the number of people needed to complete a given work.

2. The quantity that a person can purchase for a given amount and the price of the product

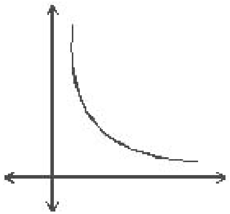
3. Concentration of a solution and the volume of the solution (given amount of solute remains constant). Understand this thoroughly as it will be used extensively in Mixtures. If water is mixed to 120 ml of spirit, as the volume of the solution increases (i.e. more water is added), the concentration of spirit in the solution decreases and that too proportionally. When volume of solution is 240 ml, concentration of spirit is 50% and when volume doubles to 480 ml, concentration halves to 25%.

In general, if y is inversely proportional to x^n, $y \times x^n$ = constant and the relation is expressed as $y \propto \frac{1}{x^n}$.

An illustration of Inverse Proportion follows.

The number of days required to build a wall is inversely proportional to the number of people building the wall. If 20 men can build the wall in 10 days, 25 men will build it in how many days?

As the number of days (n) required varies inversely to the number of people building the wall (m), we have the relation m × n = constant and thus 20 × 10 = 25 × ? and the number of days required will be 8 days.



Inverse Proportion

Inverse Relation

If y is inversely related to or varies inversely with x, then as x increases (or decreases), y decreases (or increase) but not proportionally.

The mathematical expression for Inverse Relation is $y = \frac{k_1}{x} + k_2$, where k_1 and k_2 are constants.

Example 38:

Speed of car `A' is twice that of truck `B'. Both started from a point X and reached a point Y. If `B' took 3 hours more than `A', what is the time taken by the truck `B'?

Solution:

Since distance is constant, speed & time are inversely proportional. Let

v_A = speed of A, v_B = speed of B, t_A = time taken by A; t_B = time taken by B

or $\frac{v_A}{v_B} = \frac{2}{1} \Rightarrow \frac{t_A}{t_B} = \frac{1}{2}$. If A takes x hrs. then $\frac{x}{x+3} = \frac{1}{2} \Rightarrow x = 3 \Rightarrow$ B takes 6 hrs.

Partnership

When two or more persons invest money in a common business, they are called *partners* and the business relation is called *partnership*.

In a partnership business, every partner invests two entities:

(i) Time

(ii) Money

It follows quite easily that different partners may invest different amounts of *money*. What may not follow that easily is that even the *time* for which the money is invested by the partners could be different. Now what is meant by that? It only means that partners join the business at different points of time. For example consider a business in which A, B and C are partners. Suppose A and B invested amounts a and b at the start of the business. Three months after that, C joined as a partner investing an amount c. At the end of the first year A and B have invested their respective amounts for one full year but C has invested his amount for 9 months only.

Hence, it follows that a partner's *Investment* is a product of the time and the money:

Investment = Time × Money

How to share a Profit (or a Loss)?

The total profit or loss is distributed among the different partners in direct proportion of their investments. So, if three partners A, B and C have invested amounts a, b and c for time periods t_A, t_B and t_C then the profit (or loss) p_A, p_B and p_C are such that:

$p_A : p_B : p_C :: t_A \times a : t_B \times b : t_C \times c$

It follows that if $t_A = t_B = t_C$ then $p_A : p_B : p_C :: a : b : c$

Example 39:

A and B enter into a partnership. A invested Rs. 2,000 and B invested Rs. 3,000. After 6 months, B withdrew from the business. At the end of the year, the profit was Rs. 4,200. How much would B get out of this profit?

Solution:

In partnership problems, the ratio in which the profit is shared is $I_A \times T_A : I_B \times T_B$, where I and T stand for investment and time respectively.

So the ratio in which A and B would share the profits is 2000 (12) : 3000 (6) = 4 : 3.

So B receives $\frac{3}{7} \times 4200$ = Rs. 1,800 from the total profits made in the investment.

Example 40:

Three people decided to start a partnership firm. The ratio of their investments at the outset of a project was $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. The amounts were invested for time periods that are in the ratio $1:\frac{1}{2}:\frac{1}{3}$. The profits were shared in the direct proportion to both the amount invested and the time for which it is invested. Find A's share, if B's share is Rs. 20,000.

Solution:

Ratio of investments is $I_A : I_B : I_C = \frac{1}{2}:\frac{1}{3}:\frac{1}{4} = 6:4:3$.

Ratio of the times for which it is invested is $T_A : T_B : T_C = 1:\frac{1}{2}:\frac{1}{3} = 6:3:2$.

Ratio of profits shared

$I_A\ T_A : I_B\ T_B : I_C\ T_C = 36 : 12 : 6 = 6 : 2 : 1.$

Hence, if B's share is Rs. 20,000,

A's share = 3 × Rs. 20,000 = Rs. 60,000.

Example 41:

A , B and C invested Rs. 1,000, Rs. 600 and Rs. 400 respectively to start a business. The profit is Rs. 200 which is to be divided among A, B and C in the ratio of their capital invested. What share does each of them get?

Solution:

As nothing specific is given about the time periods of their investments, we will assume that each of them invested for an equal period of time. Now, the ratio of the investments of A, B & C is the same as the ratio of the money that each of them invested or $p_A : p_B : p_C$ = 1000 : 600 : 400 = 5 : 3: 2

A's share of profit = $\frac{5}{10} \times 200$ = Rs. 100

B's share of profit = $\frac{3}{10} \times 200$ = Rs. 60

C's share of profit = $\frac{2}{10} \times 200$ = Rs. 40

Example 42:

Ram, Sham and Pran share profits in the ratio of 12 : 1 : 5. If Pran's share is Rs. 12,500, what was their total profit?

Solution:

Pran's share = 12500 = $\frac{5}{18}$ of total profit

Total profit = $\frac{12500 \times 18}{5}$ = Rs. 45,000

Example 43:

A, B and C enter into a partnership with an amount of Rs. 10,000 each. After 4 months, A invests an additional Rs. 2,000. Three months later, B invests Rs. 4,000, and C at the same time withdraws Rs. 2,000. Profit at the end of the year is Rs. 2,17,000. What are their respective shares if C is to be allowed Rs. 2,000 as monthly salary from profits at the end?

Solution:

A's capital investment in that year = 10000 × 4 + 12000 × 8 = 1,36,000

B's capital investment in that year. = $10000 \times 7 + 14000 \times 5 = 1{,}40{,}000$

C's capital investment in that year. = $10000 \times 7 + 8000 \times 5 = 1{,}10{,}000$

Ratio in which profits are to be shared = 68 : 70 : 55

Annual Salary of C = 2000×12 = Rs. 24,000

Profit to be shared = 217000 - 24000 = Rs. 193000

A's share = $\frac{193000 \times 68}{193}$ = Rs. 68,000

B's share = Rs. 70,000, C's share = Rs. 55,000.

Example 44:

A can do a piece of work in 12 days, B is 60% more efficient than A. Find the number of days that B takes to do the same piece of work.

Solution:

Method 1:

Ratio of the efficiencies is A : B = 100 : 160 = 5 : 8.

Since efficiency is inversely proportional to the number of days, the ratio of days taken to complete the job is 8 : 5.

So number of days taken by B = $\frac{5}{8} \times 12 = 7\frac{1}{2}$ days.

Method 2:

The time taken is inversely proportional to the efficiency of the person.

So if B was as efficient as A, he would have taken 12 days to finish the job. Since he is 1.6 times as efficient as A he would take $\frac{1}{1.6}$ times the days that A takes to finish the job.

Hence, B finishes the job in $\left(\frac{12}{1.6}\right)$ days $= 7\frac{1}{2}$ days

Example 45:

The height of Rajan is of the same proportion as the square root of his age (between 5 and 17 years). What will be the height of Rajan after 7 years, if he is 4 ft tall at the age of 9 years?

Solution:

Height is proportional to square root of age.

Or H = $k \cdot \sqrt{A}$; H = 4 ft at A = 9. Hence, k = $\frac{4}{3}$.

So when A = 16 years,
H = $\frac{4}{3}\sqrt{16} = 5\frac{1}{3}$ ft or 5 ft 4 inches.

Example 46:

A precious stone worth Rs. 6,800 is accidentally dropped and it breaks into three pieces. The weights of these three pieces are in the ratio 5 : 7 : 8. The value of the stone is proportional to the square of its weight. Calculate the loss in values if any incurred because of the breakage.

Solution:

Let the weights of three pieces be 5 g, 7 g and 8 g.

So the total weight of the unbroken stone = 5 + 7 + 8 = 20 g.

The price of the stone is directly proportional to the $(\text{weight})^2$.

So $6800 = k \times (20)^2$. Hence, $k = 17$.

So value of the broken pieces are $k \times (5)^2$, $k \times (7)^2$ and $k \times (8^2)$,

i.e. Rs. 425, Rs. 833, and Rs. 1,088 respectively.

Hence, the total value of the broken pieces = (425 + 833 + 1088) = Rs. 2,346.

So the total loss = 6800 - 2346 = Rs. 4,454.

Example 47:

The concentration of an acid solution is inversely proportional to the volume of the solution if the amount of acid is not changed. A 40% hydrochloric acid solution becomes 30% solution when 30 L of water is added to it. Find the original volume of the solution.

Solution:

The original concentration = 40%.

The final concentration = 30%.

$I_C \times I_V = F_C \times F_V$

where I_C and F_C indicate initial and final concentrations respectively.

I_V and F_V indicate initial and final volumes respectively.

$\therefore \frac{30}{40} = \frac{I_V}{F_V} \Rightarrow F_V = \frac{4}{3} I_V$

There is an increase of $\frac{1}{3} I_V$, which is 30 L.

Hence, initial volume = 30 × 3 = 90 L.

Example 48:

The cost of organizing a party varies directly as the number of invitees. If there are 40 invitees, the cost works out to Rs. 200 per head. If there are 50 invitees, the cost works out to Rs. 180 per head. Find

a. the variable cost and the fixed cost of conducting the party,

b. the total cost if 60 people attended the party.

Solution:

a. TC = VC (N) + FC

TC = Total cost

VC = Variable cost

FC = Fixed cost

200 × 40 = VC (40) + FC

180 × 50 = VC (50) + FC

VC (10) = 1000

$\therefore$ VC = Rs. 100;

Hence, fixed cost = Rs. 4,000.

b. The total cost when there are
60 participants is 60 × 100 + 4000 = Rs. 10,000.

Example 49:

The expenses for a picnic party are partly fixed and partly variable with the number of people going for the picnic. The charge comes to Rs. 50 per head, when there are 20 people, and comes to Rs. 40 per head when there are 35 people. Find the charge per head when there are 150 people.

Solution:

Total expense (E) = Fixed cost (f) + Variable cost (kv)

E = f + kv, where f and k are constants.

$50 \times 20 = f + 20k \Rightarrow 1000 = f + 20k$

and $40 \times 35 = f + 35k \Rightarrow 1400 = f + 35\,k$

$\Rightarrow 400 = 15\,k$

$\Rightarrow k = \frac{80}{3}$

So we have, $50 \times 20 = f + 20 \times \frac{80}{3}$

$f = 1000 - \frac{1600}{3} = \frac{1400}{3}$

Total expense when total number of people are 150 will be = f + 150 k

$= \frac{1400}{3} + 150 \times \frac{80}{3} = \frac{1400 + 12000}{3} = \frac{13400}{3}$

So the charge per head will be $= \frac{13400}{3 \times 150}$ = Rs. 29.78 approximately.

Example 50:

Three pipes of varying diameters can fill the vessels of 1, 2 and 3 L in 4, 18 and 48 min respectively. What is the ratio of their diameters? (Assume that speed of the flow is same in all the cases.)

Solution:

Volume per unit time = Area of cross section of the pipe × Speed of the flow

Hence, $\frac{v}{t} \propto d^2$ (Where d is the diameter, v is the volume and t is the time)

$d_1^2 : d_2^2 : d_3^2 = \frac{1}{4} : \frac{2}{18} : \frac{3}{48}$ or $d_1 : d_2 : d_3 = \frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 6 : 4 : 3$.

Example 51:

All the sides of a hexagon become three times the original length. Find the ratio of areas of the new and old hexagons.

Solution:

The ratio of the corresponding sides a : b = 1 : 3.

Hence, the ratio of the areas $a^2 : b^2 = 1^2 : 3^2 = 1 : 9$.

$\therefore$ The ratio of areas of the new and the old hexagons = 9 : 1.

A variable may be proportional to more than one variable at the same time. E.g. if a is directly proportional to b and inversely proportional to c, then we can arrive at the conclusion that the ratio $\frac{a \times c}{b}$ = constant.

Example 52:

There are three parameters: X, Y and Z. X is directly proportional to Y^2 and inversely proportional to Z^3. When X = 20, Y = 10 and Z = 5. What is the value of Y when X = 10 and Z = 10?

Solution:

$X \propto Y^2$

$X \propto \frac{1}{Z^3}$ $X \propto \frac{Y^2}{Z^3}$

$\Rightarrow X = k\frac{Y^2}{Z^3}$, where k is a constant

$k = \frac{XZ^3}{Y^2}$

So $\frac{20 \times 5^3}{10^2} = \frac{10 \times 10^3}{Y^2}$

$\Rightarrow Y^2 = \frac{10^5}{2 \times 5^3} \Rightarrow Y = \frac{100}{5} \Rightarrow Y = 20$

Chain Rule

An extension of the above concept is used in certain type of problems on work, popularly called chain rule. E.g. if 10 carpenters can make 10 chairs in 10 days working for 10 hours per day, how many chairs can 20 carpenters make in 20 days working 20 hours per day? And the answer is not 20 chairs!

In this problem there are 4 variables viz. number of carpenters (n), number of days worked (d), number of hours worked per day (h) and the amount of work (w) i.e. number of chairs made.

We have the following proportionality between the variables, $w \propto n, w \propto d \text{ and } w \propto h$. Thus we have $\frac{w}{n \times d \times h}$ = constant.

Substituting values, we have $\frac{?}{20 \times 20 \times 20} = \frac{10}{10 \times 10 \times 10}$.

Thus the number of chairs that will be made is 80.

While the above solution makes use of the proportionality concept we just learnt, all this is common sense and the problem can be solved simply using common sense without realizing that you are using the same concept of proportionality as follows :

We have to find number of chairs made and hence we start with 10, the given data for number of chairs that are made. Next we take each variable turn by turn and just use our common sense to see how will this variable affect the number of chairs made. While doing this, don't worry about how other variables will affect the number of chairs made.

Since the number of carpenters have increased from 10 to 20 i.e. become twice, the number of chairs made will also become twice. Thus number of chairs made $= 10 \times \frac{20}{10}$.

Next, since earlier the number of days worked were 10 and now they are working for 20 days, the number of chairs made will increase and that too proportionally. Hence the number of chairs made $= 10 \times \frac{20}{10} \times \frac{20}{10}$.

Finally, since each day, the carpenters are working for 20 hours instead of the earlier 10 hours, again the number of chairs made will be double. Thus the total number of chairs made will be $= 10 \times \frac{20}{10} \times \frac{20}{10} \times \frac{20}{10}$.

Let's solve one more problem using the funda just learnt...

If 10 pumps, each of 150 watt can raise 500 litres of water from a well in 6 hours, how many pumps of 250 watts are required to raise 1000 litres of water in 8 hours?

We have to find the number of pumps needed, so let's start with the knowledge that earlier 10 pumps were needed. Earlier the pumps were of 150 watts and now the pumps are of 250 watts. That means the pumps are more powerful now. Thus all other things remaining same, lesser number of pumps are needed i.e. $10 \times \frac{150}{250}$ pumps are needed. Taking the next variable, amount of water raised, earlier 500 litres of water was being raised and now we need to raise 1000 litres of water. Obviously more pumps will be needed as work involved is more. Thus number of pumps needed $= 10 \times \frac{150}{250} \times \frac{1000}{500}$. One more variable is still left to be considered. Earlier the pump raised the water in 6 hours, but now we have to do the work in 8 hours. Since more time is available for us, we can do with lesser pumps and thus the number of pumps needed $= 10 \times \frac{150}{250} \times \frac{1000}{500} \times \frac{6}{8} = 9$.

If 10 hens lay 10 eggs in 10 days, how many eggs will one hen lay in one day?

Example 53:

A garrison of 2,200 men is provisioned for 16 weeks at the rate of 45 g per day per man. How many men must leave the garrison so that the same provisions may last 24 weeks at 33 g per day per man.

Solution:

1. For more weeks, less men are needed. (Inverse)

2. For less grams, more men are needed. (Inverse)

Weeks	Grams	Men
16	45	2200
24	33	X

$X = \frac{2200 \times 16 \times 45}{24 \times 33} = 2000$

Hence, 2200 - 2000 = 200 men must leave the garrison.

Example 54:

If 10 men are able to complete 200 units of work in one day, how many units of work are done by 5 men in the same time? (Assume all the people are equally efficient.)

Solution:

Method 1:

The problem has two variables — the number of men and the number of units of work done. If the number of men is more, the number of units of work done is more. Hence, there is a direct proportionality between the two parameters.

Hence, $\frac{\text{Total number of units}}{\text{Number of men}}$ = Constant.

If the number of units of work done by 5 men in 1 day is x, then $\frac{x}{5} = \frac{200}{10} \Rightarrow x = 100$.

Method 2:

Ten men working for 1 day put in 10 man-days of work. In this time, they are producing 200 units of work. 5 men working for 1 day put in 5 man-days of work. Hence, they can produce 100 units of job only.

Example 55:

If 10 men can make 200 chairs working for 15 days at the rate of 8 hr per day, how many chairs can 5 men make working for 10 days at the rate of 6 hr a day? (Assume all people are equally efficient.)

Solution:

Method 1:

10 men working for 15 days at 8 hr per day put in 1,200 man-hours of work. They produce 200 chairs. If 5 men work for 10 days at 6 hr per day, they put in 300 man-hours. This is $\frac{1}{4}$ of the man-hours put in by the first set of men. Hence, the number of chairs that they can make $= \frac{1}{4} \times 200 = 50$.

Method 2: If the number of chairs produced by the second set of men is x, then

$$\frac{200}{10 \times 15 \times 8} = \frac{x}{5 \times 10 \times 6}$$

Solving for x, we get x = 50.

If you were to directly arrive at the expression, then it is $x = \left(\frac{5}{10}\right)\left(\frac{10}{15}\right)\left(\frac{6}{8}\right)(200) = 50$.

Let's bring in a new parameter into this problem-efficiency.

Example 56 :

If in example 55, the second set of people are half as efficient as the first set, what is the number of chairs that they can produce?

Solution:

The solution in example 55 presumes that the second set of people are as efficient as the first set. Hence, they are able to produce 50 chairs. If they were half as efficient, they would be able to produce only 25 chairs.

Example 57 :

In a camp of 100 students, there is ration which lasts for 8 days. After the first 2 days, 50 more students join them. How long will the ration last now? (Assume all the students have equal eating-capacity.)

Solution:

Suppose, no more students had joined, then the remaining ration would have lasted for 6 days. But since 50 more students are joining, it now lasts for fewer than 6 days. There are 2 ways of approaching this problem.

Method 1:

Let each student consume x kilograms per day. Hence, if the ration lasts for 8 days when there are 100 students, the amount of ration left at the end of 2 days = (100)(6)(x) = 600x kg. This has to be now consumed by 150 students.

Hence, the number of days this would last = $\frac{600x}{150x} = 4$ days.

Method 2:

This is the method where we use proportions. If there are more people, then the ration should last for lesser time. The number of students now is $\frac{3}{2}$ times the original number. Hence, the number of days it would last for would be $\frac{2}{3}$ times the original number of days,

i.e. $\left(\frac{2}{3}\right) \times 6 = 4$ days.

Example 58:

Two cogged wheels, one has 16 cogs and the other has 27, work into each other. If the second wheel turns 80 times in three-quarters of a minute, how often does the other wheel turn in 8 s?

Solution:

Cogs	Seconds	Turns
27	45	80
16	8	X

$X = 80 \times \frac{8}{45} \times \frac{27}{16}$ turns = 24 turns.

Therefore, fewer cogs would imply more turns.

If n men work for h hours per day at an efficiency of h and complete w amount of work in d days then the ratio

$\frac{n \times h \times d \times \eta}{w}$ = *constant*

Learning Outcomes

Fill in the blanks with an expression using the variables given in each question and any mathematical operator.

1. A, B and C invest Rs. a, Rs. b, Rs. c, respectively for t_a, t_b and t_c time periods respectively. The profit earned by them will be shared between them in the ratio

__ : _____ : _____

2. If $\frac{A}{x} = \frac{B}{y} = \frac{C}{z}$, then A : B : C is _____ : _____ : _____

3. If A : B = $\frac{1}{x} : \frac{1}{y}$, then A : B is _____ : _____

4. If A $\propto$ B; B $\propto \frac{1}{C}$; C $\propto \frac{1}{D}$, then A is directly / indirectly proportional to D.

5. The ratio of the number of Re. 1 coins, number of 50 ps. coins and the number of 25 ps. coins, each making up Rs. X, is

_____ : _____ : _____

6. The ratio of the amounts (in Rs.) made by 'a' Re. 1 coins, 'b' 50 ps. coins and 'c' 25 ps. coins is _____ : _____ : _____

7. In V_1 ltrs of salt solution with concentration of salt C, water is added till the solution is V_2 ltrs. Now the concentration of salt in the mixture is _____

8. Total cost is directly related to the number of units produced. If TC_1 and TC_2 is the total cost when n_1 and n_2 number of units are produced respectively, the cost incurred on each incremental unit produced is _____

Assignment:

Take any equation, say 3y = 2x + 12. Assume different values of x and compute the corresponding values of y and plot a table of the x and y values. Check that for every increase of 3 in the value of x, y increases by 2 (think why we started with values of 3 and 2 for increases in x and y); or for every increase of 1 in x, y increases by 2/3. Take different equations (with differing signs for coefficients of x and y, with constant term and also without constant term) and try to get a similar relation as found above for increases in x and corresponding increases/decreases in y. Extend this theory to understand what an equation really means.

Mixtures and Solutions 2

Introduction

Though an unusual topic for Maths (seems more like a chapter in Chemistry), rest assured we will not be studying reactions or alloys. For Maths, even a class consisting of boys and girls is a mixture of boys and girls! Or part of a loan taken at 4% and rest taken at 6% is also a mixture and so is part of a distance travelled at 40 kmph and remaining at 60 kmph. Once, one is clear about the basics of mixtures, which itself is just weighted average, proportion, fractions, one can apply these fundamentals to a range of problems. Thus make sure you develop a skill of identifying "mixtures" in the most unusual of situations.

Learning Objectives

• Weighted Average

• Alligation

• Mixing pure component with a solution

• Removal and Replacement

Weighted Average

Average :

We all know that the average is nothing but the sum of all observations divided by the number of observations. However, technically speaking this is the mean or the simple average. As we shall see further in this chapter, usually we will have to apply weighted average rather than simple average. But let us first start with simple problems on averages.

If in a group of 10 persons, the average amount that each one has is Rs. 16, we can easily understand that the sum total of the amount with the group is Rs. 160. Now if one more person having Rs. 20 joins this group, the new average amount that each one has can be calculated as $\frac{160+20}{10+1}=\frac{180}{11}=16.36$.

The same calculations could also be verbalized as : the new person has Rs. 4 more than the average of the group. Thus to calculate the average of the group after his inclusion, he will have to share this Rs. 4 equally among all the people including himself, such that each one gets Rs. 0.3636 more and the average of the group becomes 16.3636

Let us see another example with just this intellectually stimulating solution. A group has an average weight of 48 kgs. One person of the group weighing 60 kgs leaves the group and the average weight of the remaining group becomes 45 kgs. What was the original number of people in the group?

The person leaving the group carries away 12 kgs more than the average with him and each remaining person of the group contributes 3 kg. Thus the remaining number of people has to be 4 and the original number of people would have been 5.

As seen in the first example here, just because one person with Rs. 20 joins a group having on average Rs. 16 per person, the average of the group does not become the simple average of 16 and 20 i.e. 18. This is because of the concept of weighted average which we shall see next.

Weighted Average :

The problems on mixtures and solutions are basically an application of weighted average and hence let us start with an understanding of weighted average.

The Xth A class has 3 students who have scored 20, 30 and 40 marks respectively whereas Xth B has 5 students who have scored 60, 70, 80, 90 & 100 marks.

We all know how to find the average marks of the class. Consider Xth A and we know the average marks of a student of Xth A is $\frac{20+30+40}{3} = 30$.

Similarly the average marks of a student of class Xth B is $\frac{60+70+80+90+100}{5} = 80$.

We know the average marks of Xth A is 30 marks and that of Xth B is 80 marks. Thus, the average of both the classes together must be $\frac{30+80}{2} = 55$ marks, right?

Let us check, if we need to find average marks of all the 8 students, we need to total their marks and divide by 8 and thus the answer is $\frac{20+30+40+60+70+80+90+100}{8} = \frac{490}{80} = 61.25$!!!

Thus what we did earlier was wrong. Earlier we had 30 and 80 as the average marks of the two class and we simply found the average of the averages. This is wrong as we know that neither is 30 the total marks of class Xth A, nor is 80 the total marks of Xth B and nor is the number of students 2 (by which we had divided the sum of 30 and 80)

Thus whenever we are dealing with averages of two parts and we need to find the average of both the parts considered together, the average of the averages NEED NOT be the correct answer.

In fact, we see that the correct answer (61.25) is closer to 80 than to 30 (the middle value of 30 and 80 is 55). And this is because the class with average marks of 80 was larger in size than the class with average marks 30 and thus the larger class size pulled the average of both the classes taken together, closer to itself. i.e. the class with larger strength had more of an influence (or weight) on the average of both the classes considered together.

Infact we know that $\frac{(20+30+40)+(60+70+80+90+100)}{8}$

can be also written as $\frac{(30\times 3)+(80\times 5)}{3+5}$ where (30×3) is the total marks of the class Xth A (average $\times$ number of observations = sum of observations) and similarly (80×5) is the total marks of the class Xth B.

Thus, average marks of both the class taken together = $\frac{(30\times 3)+(80\times 5)}{3+5} = 61.25$.

This form is known as weighted average and we can say that 30 exerts a weight of 3 whereas 80 exerts a weight of 5.

The general form of weighted average is

$$A_{wt} = \frac{(A_1 \times wt_1)+(A_2 \times wt_2)+(A_3 \times wt_3)+\ldots\ldots+(A_n \times wt_n)}{wt_1+wt_2+wt_3+\ldots\ldots+wt_n},$$

where

A_{wt} is the weighted average of A_1, A_2, A_3, ..., A_n having weights wt_1, wt_2, wt_3, ..., wt_n respectively.

What happens if the weights are equal?

Let's say $wt_1 = wt_2 = wt_3 = \ldots = wt_n = y$, then

$$A_{wt} = \frac{(A_1\times y)+(A_2\times y)+(A_3\times y)+\ldots\ldots+(A_n\times y)}{y+y+y+\ldots\ldots+y}$$

$$A_{wt} = \frac{y\times(A_1+A_2+\ldots\ldots+A_n)}{y\times n}$$

$$A_{wt} = \frac{A_1 + A_2 + ... + A_n}{n}$$

i.e. the simple average as we all knew so far. Thus simple average is nothing but weighted average with weights being equal!!!

Please realise that in most of the problems we will encounter, it will be weighted average and please do not interpret it as simple average as long as you know that the weights are equal. E.g. if Abhishek scored 60% in Science and 80% in Maths, it is not entirely true that he scored an average of 70% in Science and Maths taken together. This will happen only when the maximum marks are for both Science and Maths are equal. If the Science paper was out of 300 and the Maths paper was out of 200 marks, Abhishek would have scored a total of 180 marks in Science and 160 marks in Maths and his average percentage in both Science and Maths together would be

$$\frac{(60\% \times 300) + (80\% \times 200)}{300 + 200} = \frac{180 + 160}{500} = \frac{340}{500} = 68\%$$

Two points worth noting in the above example : we again see that Science has drawn the average marks (68%) closer to itself (60%) than the exact middle value (70%) and we know the reason is since Science has a higher maximum marks and hence plays a heavier role in determining the average.

Second point worth noting is that, even if we did not know that the maximum marks of Science and Maths were 300 and 200 respectively and we just knew that the ratio of maximum marks of Science and Maths is 3 : 2, we would have arrived at the same average.

$$\frac{(60\% \times 300) + (80\% \times 200)}{300 + 200} = \frac{(60\% \times 3) + (80\% \times 2)}{3 + 2}$$

And by intuition also one could have gathered that the ratio of the weights is all that we need as we are just interested in how much relative importance does each average exert on the weighted average.

A mixture is any collection of parts with distinct characteristics. Thus a class is a mixture of boys with different (or same) age, height, marks scored and thus one can have weighted average age, height, marks scored of this mixture.

Instead of knowing or using the absolute value of the weights, we can solve or simplify even by using the ratio of the weights. Thus weighted average of 23 and 37 with weights 42 and 63 respectively, is just $\frac{23 \times 2 + 37 \times 3}{2 + 3}$

as 42 : 63 is same as 2 : 3.

3 kg of rice costing Rs. 5/kg is mixed with 4 kg of rice costing Rs. 6/kg. Then the weights are 3 and 4 and not 5 and 6. We are interested in the average cost and thus the costs 5 and 6 are the defining character of the two groups and the other values 3 and 4 are the weights.

Simple average is weighted average where all weights are equal.

Another faster or oral way of calculating the weighted average of two groups is as follows :

If we have to find the weighted average of A_1 and A_2($A_2 > A_1$) with weights w_1 and w_2, divide the difference (A_2 - A_1) in the ratio $w_1 : w_2$. Let say the result is x and y. The *weighted average will be $A_1 + y$ or $A_2 - x$.*

E.g. Weighted average of 40 and 72 with weights in ratio 1 : 3 will be $40 + \frac{3}{4} \times 32$ *or* $72 - \frac{1}{4} \times 32$ *i.e. 64*

Example 1:

The average age of the boys of class Xth is 18 years and the average age of the girls of the class is 16 years. What is the average age of the entire class, if the ratio of boys and girls is 3 : 4?

Solution:

$$\text{Average age} = \frac{\text{Total Age of all students}}{\text{Total number of students}}$$

$$= \frac{18 \times 3 + 16 \times 4}{3 + 4} = \frac{118}{7} = 16.86$$

Example 2:

In two alloys the ratios of copper to tin are 3 : 4 and 1 : 6 respectively. If 7 kg of the first alloy and 21 kg of the second alloy are mixed together to form a new alloy, then what will be the ratio of copper to tin in the new alloy?

Solution:

$$\frac{\text{Copper in new alloy}}{\text{Tin in new alloy}}$$

$$\frac{\text{Copper in 1st} + \text{Copper in 2nd}}{\text{Tin in 1st} + \text{Tin in 2nd}}$$

$$= \frac{\frac{3}{7} \times 7 + \frac{1}{7} \times 21}{\frac{4}{7} \times 7 + \frac{6}{7} \times 21} = \frac{3 + 3}{4 + 18} = \frac{6}{22} = \frac{3}{11}$$

Alternately: Let's just work with copper,

$$\text{Ratio of Copper in mixture} = \frac{\frac{3}{7} \times 7 + \frac{1}{7} \times 21}{7 + 21} = \frac{6}{28} = \frac{3}{14}$$

Thus Copper : Tin is 3 : (14 - 3) i.e. 3 : 11

Example 3:

In a zoo having just lions and lambs, each lion eats 5 kgs of food per day and each lamb eats 1000 gms of food each day. If there are 56 lions and 35 lambs, what is the average amount of food eaten by each animal per day?

Solution:

Instead of using 56 and 35, which will make the calculations tougher, let's just use the ratio of lions and lambs i.e. 8 : 5.

Average food eaten per day by each animal = $\frac{\text{Total food consumed}}{\text{Total number of animals}}$ =

$$\frac{5 \times 8 + 1 \times 5}{8 + 5} = \frac{45}{13} = 3.46 \text{ kg}$$

Example 4:

$\frac{3}{4}$ th of a loan is taken at interest of 8% p.a. and the balance is taken at 10% p.a. What is the average interest rate paid on the full loan?

Solution:

Please note here we need not assume any particular amount of loan taken and then calculate the interest paid on each parts and finally calculate the total interest paid as a percentage of total loan. The problem is simple application of weighted average. Go through the following solution and if not convinced, assume a loan of Rs. 1000 and do the above calculations and confirm that the answer is the same.

Average rate of Interest = $\frac{3\times 8+1\times 10}{3+1}=\frac{34}{4}=8.5\%$

Example 5:

Five litres of 30% alcohol solution is mixed with 10 L of 45% alcohol solution. What is the concentration of the resultant solution?

Solution:

It would be a long method trying to solve this problem with alligation rule. Apply the weighted average method in all such problems.

So the average concentration of the resultant solution = $\left(\frac{5}{15}\right)\times 30+\left(\frac{10}{15}\right)\times 45=40\%$.

Till now, we have restricted ourselves to ratios. Proportion, which is closely related to ratio is a very useful concept while solving a lot of problems.

Alligation

In what ratio must 40% milk solution be mixed with 70% milk solution to result in a 60% milk solution?

Let the 40% milk solution and the 70% milk solution be mixed in the ratio of m:n. Thus using the formula of weighted average we have, $\frac{(40\%\times m)+(70\%\times n)}{m+n}=60\%$.

Solving this, (40% × m) + (70% × n) = (60% × m) + (60% × n) i.e. 10% × n = 20% × m

and we have m : n = 1 : 2.

However, in the above solution, we have introduced variables and we needed to solve the equation. A procedure called "Alligation" simplifies this type of problems. Let us first see the procedure and then later we shall see the theory behind it

Write the two averages in a straight line and slightly apart and the weighted average in the center slightly below the straight line as follows:

40%   70%
   60%

Now, the ratio of mixing can be found by just subtracting the figures as follows :

40%   70%
   60%

70 – 60 = 10% : 60 – 40 = 20%
10 : 20
1 : 2

Please note that 70 - 60 i.e. 10 is written on the diagonally opposite end and similarly 60 - 40 = 20 is written on diagonally opposite end.

1 part corresponds to 40% as it is written below it and 2 parts corresponds to 70% as it is written below it.

We could also have done the whole process as

70%   40%
   60%

$$40-60=-20\% \quad : \quad 60-70=-10\%$$
$$2 \quad : \quad 1$$

This is also same as mixing 40% milk solution and 70% milk solution in ratio of 1 : 2 and not
2 : 1 as 1 part is written below 40% and 2 parts is below 70%

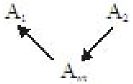
Thus visually we can solve this type of problems without the introduction of any variable.

Though, Alligation appears `magical', it is nothing but a re-arranged form of weighted average...

$$A_{wt}=\frac{(A_1\times w_1)+(A_2\times w_2)}{w_1+w_2}$$
$$\Rightarrow (A_{wt}\times w_1)+(A_{wt}\times w_2)=(A_1\times w_1)+(A_2\times w_2)$$
$$\Rightarrow (A_{wt}\times w_1)-(A_1\times w_1)=(A_2\times w_2)-(A_{wt}\times w_2)$$
$$\Rightarrow w_1\times(A_{wt}-A_1)=w_2\times(A_2-A_{wt})$$
$$\Rightarrow \frac{w_1}{w_2}=\frac{(A_2-A_{wt})}{(A_{wt}-A_1)}$$

This is the Alligation formula.

Understand that this is exactly what is done in Alligation visually...



A_2 - A_{wt} : A_{wt} - A_1

Thus please get it crystal clear that wherever Weighted Average can be used, Alligation can also be used and wherever Alligation can be used, Weighted Average can also be used as basically both are the same.

It is just that at places alligation can be slightly faster and at other instances, weighted average can be faster. A simple benchmark to follow is

If the weighted average is given and the ratio of weights is to be found out, Alligation can prove to be faster

and

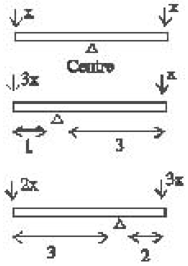
If the weights are given and the weighted average is to be found out, the weighted average formula can be faster.

Alligation,

A process or rule for the solution of problems concerning the compounding or mixing of ingredients differing in price or quality.

- Merriam-Webster Medical Dictionary

Alligation is common sense as evident in the balancing of the following ruler.



The position of the fulcrum is the weighted average of the two end points. See how the fulcrum moves to the heavier end (also the extent to which it moves) so that the ruler balances.

Alligation does not give the actual volumes to be mixed but only the ratio in which volumes are to be mixed.

Any problem that can be done using alligation can also be done using weighted average and vice-versa. Calculations are less if alligation approach is used when the weighted average is given and weighted average approach is used if weighted average is to be found out.

Example 6:

In what ratio must rice at Rs. 3.10 per kilogram be mixed with rice at Rs. 3.70 per kilogram, so that the mixture is worth Rs. 3.25 per kilogram?

Solution:

The conventional method is: let x kilograms of rice of Rs 3.10 per kilogram be mixed with y kilograms of rice of Rs. 3.70 per kilogram rice.

Hence, $x(3.1) + y(3.7) = (x + y)(3.25)$

or $x(3.25 - 3.1) = y(3.7 - 3.25)$

Hence, $\frac{x}{y} = \frac{3.7 - 3.25}{3.25 - 3.1} = \frac{3}{1}$.

So for every 3 kg of the first variety, 1 kg of the second is needed.

If you closely observe the last step, it is the Alligation procedure.

Example 7:

How many kilograms of salt costing Rs. 42 per kilogram must a man mix with 25 kg of salt costing Rs. 24 per kilogram, such that on selling the mixture at Rs. 40 per kilogram, there is a gain of 25% on the outlay?

Solution:

Average cost price of the mixture = $\frac{40 \times 100}{125}$ = Rs. 32 per kilogram

Applying the principle of alligation to find the ratio in which the constituents have to be mixed $\frac{42 - 32}{32 - 24} = \frac{10}{8} = \frac{5}{4}$.

So the ratio in which salt of Rs. 42 per kilogram is mixed with the salt of Rs. 25 per kilogram is 4 : 5.

Thus, the required quantity of salt of Rs. 42 per kilogram is $25 \times \frac{4}{5}$ = 20 kg.

Example 8:

A mixture of certain quantity of milk with 16 L of water is worth 90 paise per litre. If pure milk costs 108 paise per litre, what is the amount of milk in the mixture?

(Assume that water does not cost anything.)

Solution:

Price of milk per litre = 90 paise; price of water per litre = 0 paisa.

Applying the principle of alligation, we have

$$\frac{108 - 90}{90 - 0} = \frac{18}{90} = \frac{1}{5}$$

Ratio of milk to water in the mixture is 5 : 1.

Quantity of milk in the mixture = 5 × 16 = 80 L.

Example 9:

In what proportion must water be mixed with the spirit to gain $16\frac{2}{3}\%$ by selling it at the cost price?

Solution:

Let the selling price of 1 L of mixture be Re 1.

Gain = $16\frac{2}{3}\%$ = 16.66%.

So CP of 1 L of mixture = Rs. $\frac{1}{1.1666}$ = Rs. $\frac{6}{7}$.

CP of 1 L water = 0; CP of 1 L pure spirit = Re 1.

$$\frac{1-\frac{6}{7}}{\frac{6}{7}-0} = \frac{\frac{1}{7}}{\frac{6}{7}} = \frac{1}{6}.$$

$$= \frac{(\text{Quantity of water})}{(\text{Quantity of spirit})}$$

Example 10:

There are 65 students in a class. Among them, Rs. 39 is distributed so that each boy gets 80 paise and each girl gets 30 paise. Find the number of boys and girls in that class.

Solution:

Alligation is applicable for `money per boy or girl'.

Average money per student = $\frac{3900}{65}$ = 60 paise.

Applying the alligation rule, $\frac{80-60}{60-30} = \frac{2}{3}$.

Boys : Girls = 3 : 2

Number of boys = $\frac{65 \times 3}{3+2} = 39$.

Number of girls = 65 - 39 = 26

Example 11:

Anil buys two varieties of sugar costing Rs. 15 per kilogram and Rs. 20 per kilogram. He mixes these two varieties in a certain ratio that costs him Rs. 18 per kilogram. Find the ratio of the cheaper quantity to that of the dearer quantity.

Solution:

Using alligation Rs. 18 per kg. is the mean cost price.

15 20

18

(20–18) : (18–15)

Thus, ratio of cheaper quantity to dearer quantity is = (20 - 80) : (18 - 15) = 2 : 3

Example 12:

Akash buys a certain variety of rice and mixes it with another variety of rice costing Rs. 25 per kilogram in the ratio of 5 : 4. The mixture costs Rs. 20 per kg. What is the cost price of the cheaper variety?

Solution:

Using alligation

$$\begin{array}{ccc} x & & 25 \\ & 20 & \\ 5 & : & 4 \end{array}$$

$\frac{25-20}{20-x} = \frac{5}{20-x} = \frac{5}{4} \Rightarrow 20 - x = 4$ or, $x = 16$

∴ Price is Rs. 16 per kg.

Example 13:

A dealer buys 11 kg of wheat at Rs. 275 and mixes it with another quality of wheat in the ratio of 4 : 5. The price of the resulting mixture is Rs. 30 per kg. The price of this other quality of wheat is

Solution:

Price per kg of wheat is $= \frac{275}{11} = \text{Rs. } 25/\text{kg}$

using Alligation

$$\begin{array}{ccc} 25 & & x \\ & 30 & \\ 4 & & 5 \end{array}$$

$\therefore \frac{5}{x-30} = \frac{5}{4} \Rightarrow x = 34$

∴ Price is Rs. 34 per kg.

Example 14:

A trader mixes two varieties of apples, costing Rs. 24 per dozen and Rs. 36 per dozen and sells them at Rs. 33 per dozen, there by gaining 10% on the transaction. The ratio in which he mixes these varieties is:

Solution:

CP of the mixture is $= 33 \times \frac{10}{11} = 30$ per dozen

Using alligation,

$$\begin{array}{ccc} 24 & & 36 \\ & 30 & \\ 36-30 & : & 30-24 \end{array}$$

Ratio of the mixture is $\frac{36-30}{30-24} = 1:1$

When we use alligation method to solve problems from other areas, we change the terms involved in the above formula accordingly. Let us solve some other similar problems.

Carefully note how and on which quantities, we are applying alligation rule.

Example 15:

The ratio in which 30% alcohol solution should be mixed with 50% solution in order to get a 42% solution is:

Solution :

Using alligation

30 50

42

50–42 : 42–30

Thus, the required ratio is: = 8 : 12 = 2 : 3.

Example 16:

11 liter of water is mixed with a certain quantity of milk such that the mixture costs Rs. 1 per litre. If the cost of pure milk is 110 paise per litre, then the amount of milk in the mixture is:

Solution:

Price of water is zero. Thus

0 110

100

Water : Milk

(110–100) (100–0)

∴ water : Milk = (110 - 100) : (100 - 0)

11 : Milk = 10 : 100

∴ The mixture contains 110 litres of milk.

Example 17:

Rashmi has 40 cups. She sells some at a profit of 15% and the rest at a loss of 5%. She gains 10% on the entire transaction. Find the number of cups she sells at a profit of 15%.

Solution:

Using alligation

–5 15

10

5 : 15

Ratio of cups sold at a loss to that sold in profit is 5 : 15 = 1 : 3

∴ No. of cups sold at a profit of 15% = $\frac{3}{4} \times 40$ = 30 cups.

Example 18:

A man sells two cycles for Rs. 2100 and gains 5% in the transaction. He incurs a loss of 3% on one and a profit of 7% on the other. Find the cost of each cycle.

Solution:

Total CP of both the cycles is to be found out.

$$\text{Total CP} = \frac{2100 \times 100}{105} = \text{Rs. } 2000$$

Using alligation

–3 7

5

$C_1(=2)$: $C_2(=8)$

C_1 and C_2 are the cost prices of the cycles.

∴ $C_1 : C_2 = 1 : 4$

$$\Rightarrow C_1 = \frac{1}{5} \times 2000 = 400/- \text{ and } C_2 = \frac{4}{5} \times 2000 = 1600/-$$

Mixing Pure component with a solution

In most of the above problems, the two solutions being mixed themselves were a mixture. Let us see how the problem becomes much more simpler when one part being mixed is a pure component.

Example 20:

How many litres of water should be added to 30 litres of 50% milk solution to make it a 20% milk solution?

While the problem is very simple, we are attempting this just to understand the various methods by which it can be solved.

Method 1 : Use of Equation

Let us add x litres of water. Thus we will have,

$$\frac{\text{milk}}{\text{total solution}} = \frac{50\% \times 30}{30 + x} = \frac{20}{100}$$

Solving which we can find x = 45 litres.

Method 2 : Alligation

50%   0%

   20%

$0 - 20 = -20\% \quad : \quad 20 - 50 = -30\%$

$2 : 3$

Thus, the 50% milk solution and water should be added in ratio 2 : 3. Since we have 30 litres of 50 % milk solution, water to be added will be 45 litres.

Please note that in this problem, we are working with milk percentages and pure water has 0% milk. Thus we have taken 0%. We could also have worked with water percentages and we would arrive at the same answer.

50%   100%

   80%

$100 - 80 = 20\% \quad : \quad 80 - 50 = 30\%$

$2 : 3$

Just note that everything should be in terms of percentage of water. 50% milk solution will have 50% water and hence it remains at 50%, pure water will be 100% water and the resultant 20% milk solution will be 80% water solution.

Method 3 : Use of Inverse Proportion

Recollect that in the chapter on ratio and proportion, we studied that the concentration of solute is inversely proportional to the total volume, when amount of solute remains constant. In this case, the solute is milk and since water is being added, the amount of milk remains constant. Using the knowledge of inverse proportion we can say that 50% × 30 = 20% × v (If x and y are inversely proportional, x × y = constant). Thus v = 75 i.e. final volume is 75 litres and hence 45 litres of water must have been added.

Method 4 : Work on no change component

In problems like this, since we know that one solute (in this case milk) is remaining constant, the problem becomes very simple. The amount of milk will remain 50% × 30 = 15 litres even after adding water. This 15 litres is 20% and thus the total solution (i.e. 100%) will be 75 litres, meaning 45 litres of water is added.

Method 5 : Unitary method

	M	W	Total
50% milk solution:	1	1	2
20% milk solution:	1	4	5

Keep the value of milk the same as no milk is added and adjust the value of water so that the ratio of milk and water is that given in the question.

With these values, we see we have to add 3 litres of water for every 2 litres of original solution. Thus for 30 litres of original solution we need to add 45 litres of water.

Though the problem is very simple and any method would be fast enough, learning the various approaches would help you grasp the concepts very firmly and the knowledge of various methods may also be useful in further application-oriented problems.

Example 21:

In a mixture of 42 L, the ratio of milk to water is 6 : 1. Another 12 L of water is added to the mixture. Find the ratio of milk to water in the resultant mixture.

Solution:

Method 1:

Amount of milk in 42 L of mixture = $\frac{6}{7} \times 42 = 36$ L.

Amount of water = 6 L.

Since 12 L of water is being added, the new mixture now contains 36 L of milk and 18 L of water and hence, M : W = 36 : 18 = 2 : 1.

Method 2:

The average milk concentration originally is $\frac{6}{7}$. The volume increases to $\frac{9}{7}$ of the original volume.

Hence, since no milk is added and the concentration is inversely proportional to the volume of milk in the mixture, the new concentration would be $\frac{6}{7} \times \frac{7}{9} = \frac{2}{3}$.

The new ratio of milk and water is now 2 : 1.

Removal and Replacement

Next let us look at a standard type of problem based on removal and replacement. We will understand this with the help of an example. Before we get on to the actual problem, let us understand a simple enough fundamental.

When we remove x% of a solution of milk and water, we are removing x% of the amount of milk present and also x% of the amount of water present. E.g. from 50 litres of 2 : 3 milk and water solution, 20 litres are removed. In the 20 lts solution removed, milk and water will again be in the ratio of 2 : 3 as the solution is homogeneous. Also obviously the ratio of milk and water in the remaining solution will remain as 2 : 3 as we are just removing part of the solution and not changing its composition. All this should be very apparent and if it is not, have a look at the following simple calculations.

	Milk	Water	Total
Initially	20 lts	30 lts	50 lts
In the 20 litres solution removed	8 lts	12 lts	20 lts
In the left over solution	12 lts	18 lts	30 lts

On removing $\frac{20}{50} \times 100 = 40\%$ of the solution, we are removing $\frac{8}{20} \times 100 = 40\%$ of milk and also $\frac{12}{30} \times 100 = 40\%$ of the water present.

And again this should be obvious because if we remove different percentages of milk and water, we would be changing the composition of the mixture and the ratio of milk and water in the remaining solution will not remain the same as earlier, which it should.

Now on to the main problem, a container has 100 litres of milk and water solution in ratio 4:1. From this container 10 litres of solution is taken out and replaced by water. This process was repeated once more. How much amount of milk is in the container now and what is the ratio of milk and water of the solution now?

Initially amount of milk = $100 \times \frac{4}{5} = 80$ litres.

When 10 litres of the solution is removed, we are removing $\frac{1}{10}$th of the solution (10 litres out of 100 litres). In this process we will be removing $\frac{1}{10}$th of milk also (and also of water but that is not necessary as we will be working just on milk) and the amount of milk remaining will be $\frac{9}{10}$th.

Thus amount of milk remaining after removal of 10 lts of solution = $80 \times \frac{9}{10}$ lts

Next, the part removed is replaced but with only water. Thus the amount of milk will remain constant (now do you realise why we are working on milk and not on water...as milk is only removed and not added back).

Thus amount of milk remaining after removed part is replaced with water = $80 \times \frac{9}{10}$ lts

Also please note that the total volume of the solution is back to 100 lts.

When the process is repeated, it is just like a new beginning, from 100 litres of milk and water solution, containing $80 \times \frac{9}{10}$ litres of milk, 10 litres of solution is removed and replaced by water.

Again since $\frac{1}{10}$th of the solution is removed, $\frac{1}{10}$th of the milk will be removed and $\frac{9}{10}$th of milk will be left over. On replacing by water,

amount of milk will not change and thus final amount of milk = $80 \times \frac{9}{10} \times \frac{9}{10} = 64.8$ litres.

To find ratio of milk and water, let us find milk : total.

$\frac{\text{Milk}}{\text{Total}} = \frac{80}{100} \times \frac{9}{10} \times \frac{9}{10} = \frac{4}{5} \times \frac{9}{10} \times \frac{9}{10} = \frac{81}{125}$ and thus milk : water will be 81 : (125 - 81)

While the above solutions has taken a lot of written space, please note that we have just found out what fraction of the solution is removed ($\frac{1}{10}$th) and then found the fraction of solution left ($\frac{9}{10}$th) and multiplied the original volume

by this fraction as many times as the process is repeated.

Thus, in such standard problems of removal and replacement, we have,

Final amount of solute that is not replaced = Initial amount × (fraction left)^n

Final ratio of solute not replaced to total = Initial ratio × (fraction left)^n

Method 2:

The above problem could also be looked at using the inverse relation between concentration and amount of solution.

We all know very well that when part of solution is removed, percentage of milk does not change.

And when part removed is being replaced by water, amount of milk remains constant and thus concentration of milk is inversely proportional to amount i.e. $\boxed{C_2 = C_1 \times \frac{V_1}{V_2}}$.

In above problem, V_1 is the volume of the solution just before water is added (i.e. 90 litres) and V_2 is the volume of the solution after water is added (i.e. 100 lts). Since initial concentration of milk is 80%, concentration after first replacement will be $80\%\times\frac{9}{10}$ and after second replacement will be $80\%\times\frac{9}{10}\times\frac{9}{10}$.

Thus in such standard problems of removal and replacement, one could also remember the formula as (if one finds this easier than earlier one),

Final amount of solute that is not replaced = Initial amount $\times\left(\frac{\text{Vol. after removal}}{\text{Vol. after replacing}}\right)^n$

Final ratio of solute not replaced to total = Initial ratio $\times\left(\frac{\text{Vol. after removal}}{\text{Vol. after replacing}}\right)^n$

From a container filled with milk, 12 litres of solution is removed and replaced with water. After a total of two such operations the ratio of milk and water becomes 9 : 7. What is the capacity of the container?

We have final ratio of milk to total = initial ratio × (fraction of solution left after removal)^n

Thus, (fraction of solution left after removal)^2 = $\frac{\text{Final ratio of milk to total}}{\text{Initial ratio of milk to total}} = \frac{9/16}{1/1} = \frac{9}{16}$

Thus fraction of the solution left after removal = $\frac{3}{4}$th implying $\frac{1}{4}$th of the solution was removed

and we know that 12 litres was removed. Thus capacity of container = 48 litres.

Alternately if you prefer the formula, $\frac{9}{16} = 1\times\left(\frac{V-12}{V}\right)^2$

Avoid using alligation in problems involving replacement of part of a mixture as situations of these type have two operations

a) removal of part of a solution

b) adding second solution to remaining part of first.

Alligation is applicable only to second operation and thus you have to be very cautious to use the correct ratio of mixing if you are using alligation.

Replacing part of a mixture with a pure component

1. Work on the solute which is not replaced

2. If x% of solution is removed, x% of each solute is removed

3. Try to work in fractions rather than percentages.

Replacement of a part of solution with a pure component when done repetitively is exactly similar to compound interest with the rate of interest being negative. Rate of interest depends on the fraction of the solution being replaced.

If the method using the concept that concentration is inversely proportional to volume is crystal clear, get the formula if a pure component is first added and then an equal amount of solution is removed and this operation is done multiple times.

Example 22:

A container has 80 L of milk. From this container 8 L of milk was taken out and replaced by water. The process was further repeated twice. How much milk is in the container now?

Solution:

Method 1:

In this questions $\frac{8}{80} = \frac{1}{10}$ part of the mixture is replaced with water. So, after each operation $\frac{9}{10}$ of the milk is left. So, the amount of milk left at the end of three operations $= 80\left(1-\frac{1}{10}\right)^3$ kg = 58.32 L.

Method 2:

If 8 L of the mixture is taken out of the container, then we are left with $\frac{9}{10}$ of the original amount

of milk at the end of the first operation, i.e. 80 × $\frac{9}{10}$ = 72 L.

If in the second operation 8 L is again removed, we would be left with of the amount of milk that was left at the end of first operation, i.e. 72 × $\frac{9}{10}$ = 64.8 L.

Similarly, at the end of the third operation, we are finally left with 64.8 × $\frac{9}{10}$ = 58.32 L of milk in the container.

Example 23:

From a cask containing water, 9 L is taken out. It is replaced with an equal quantity of pure milk. This process is done twice. The ratio of water to milk in the cask now is 16 : 9. What is the volume of the cask?

Solution:

Let there be x litres in the cask.

After n number of process,

$$\frac{\text{Water left in the vessel after n time process}}{\text{Original quantity of water in vessel}} = \left(1-\frac{9}{x}\right)^n \Rightarrow \left(1-\frac{9}{x}\right)^2 = \frac{16}{25}$$

∴ x = 45 L.

Example 24:

A jar contains a mixture of two liquids A and B in the ratio 4 : 1. When 10 L of the mixture is replaced with liquid B, the ratio becomes 2 : 3. How many litres of liquid A was present in the jar earlier?

Solution:

Method 1:

If x is the volume of liquid B in the original mixture, then

$$\left\{4x - 10\left(\frac{4}{5}\right)\right\} : \left\{x - 10\left(\frac{1}{5}\right) + 10\right\} = 2 : 3$$

(This is the ratio of liquid A to liquid B.)

Solving for x, we get x = 4.

Hence, the volume of liquid A = 4x = 16 L.

Method 2:

Concentration of liquid B in the original mixture = $\frac{1 \times 100}{5} = 20\%$

Concentration of liquid B in the resultant mixture = $\frac{3 \times 100}{5} = 60\%$.

Concentration of B in second mixture which is added = 100% (since it is a pure mixture).

∴ The ratio in which the original mixture and second mixture are added = $\frac{100-60}{60-20} = \frac{40}{40} = 1:1$.

∴ Total mixture finally = 10 + 10 = 20 L.

And liquid A initially = $\frac{20 \times 4}{5}$ = 16 L.

Method 3:

The average composition of B in the first mixture is $\frac{1}{5}$.

The average composition of B in the second mixture = 1.

The average composition of B in the resultant mixture = $\frac{3}{5}$.

Hence, applying the rule of alligation, we have $\frac{\left[1-\left(\frac{3}{5}\right)\right]}{\left[\left(\frac{3}{5}\right)-\left(\frac{1}{5}\right)\right]} = \frac{\left(\frac{2}{5}\right)}{\left(\frac{2}{5}\right)} = 1$.

So the quantity of mixture in the jar finally = 10 + 10 = 20 L.

And quantity of A in the jar finally = $\frac{(20 \times 4)}{5}$ = 16 L.

The above could also be done with liquid A. Then, take the averages with respect to liquid A.

Example 25:

A container contained 100 kg of milk. From this container 10 kg of milk was taken out and replaced by water. This process was further repeated three times. How much milk does the container have now?

Solution:

Amount of liquid left after n operations, when the container originally contains `a' units of liquid from which `b' units are taken out each time and replaced is, $a \times \left\{\left(\frac{a-b}{a}\right)^n\right\}$ units.

$= 100\left(\frac{100-10}{100}\right)^4 \text{kg} = 100 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = 65.61 \text{ kg}$

Example 26:

A vessel contains 10 L of milk. First, 2L of the vessel's contents are removed and replaced by 2L of water. Subsequently, 4L of vessel's contents are removed and replaced by 4L of water. Finally, 6L of vessel's contents are removed and replaced by 6L of water. What is the amount of milk left in the vessel?

Solution:

Method 1:

The following table shows the quantities of milk and water:

	Milk (in liters)	Water (in liters)
Initial	10	0
After the first removal and replacement	$(10-2)=8$	2
After the second removal and replacement	$\left(8-\frac{4}{5}\times4\right)=\frac{24}{5}$	$\left(2-\frac{1}{5}\times4\right)+4=\frac{26}{5}$
After the third removal and replacement	$\left(\frac{24}{5}-\frac{12}{25}\times6\right)=\frac{48}{25}$	$\left(\frac{26}{5}-\frac{13}{25}\times6\right)+6=\frac{202}{25}$

As clear from the table, the amount of milk left in the vessel is $\frac{48}{25}=1.92$ litres

Method 2:

The amount of milk left in the vessel = $10\left(1-\frac{2}{10}\right)\left(1-\frac{4}{10}\right)\left(1-\frac{6}{10}\right)$ = 1.92 litres.

Example 27:

A vessel contained a 10L mixture of milk and water. First, 1L of the vessel's contents are removed and replaced by 1L of water. Subsequently, 2L of vessel's contents are removed and replaced by 2L of water. Finally, 3L of vessel's contents are removed and replaced by 3L of water. If the ratio of milk and water in the final solution is 1 : 1, then what was the ratio of milk and water in the original solution?

Solution:

Mathod 1:

Let the initial amount of milk and water be 10x and (10 - 10x) respectively.

The following table shows the quantities of milk and water:

	Milk (in liters)	Water (in liters)
Initial	$10x$	$10-10x$
After the first removal and replacement	$10x-1\times x=9x$	$10-9x$
After the second removal and replacement	$9x-2\times\frac{9}{10}x=\frac{36}{5}x$	$10-\frac{36}{5}x$
After the third removal and replacement	$\frac{36}{5}x-3\times\frac{18}{25}x=\frac{126}{25}x$	$10-\frac{126}{25}x$

As clear from the table, the ratio of milk and water in the final solution is $\frac{126x}{250-126x}=1.$

$\Rightarrow x=\frac{125}{126}.$

So, the ratio of milk and water = $\frac{125}{126}:\frac{1}{126}=125:1$

Mathod 2:

Let the initial amount of milk be `x' L.

We know that the final amount of milk is $\frac{1}{2}\times10=5$ L.

So, using the formula:

$$5=x\left(1-\frac{1}{10}\right)\left(1-\frac{2}{10}\right)\left(1-\frac{3}{10}\right)\Rightarrow x=\frac{625}{63}$$

So, amount of water initially: $10-\frac{625}{63}=\frac{5}{63}$

So, ratio of milk and water = $\frac{625}{63}:\frac{5}{63}=125:1$

Example 28:

A butler stole wine from a butt of cherry, which contained 40% of spirit solution. He replaced what he had stolen with wine containing only 16% spirit by volume. The resultant concentration of the mixture in the butt was then 24%. What fraction of the butt of cherry did he steal?

Solution:

Applying the rule of alligation,

$\frac{40\% - 24\%}{24\% - 16\%} = \frac{16\%}{8\%} = \frac{2}{1}$.

$= \frac{(\text{Quantity of } 16\% \text{ solution})}{(\text{Quantity of original strength})}$

$\frac{1}{3}$ of the butt of sherry was left and hence $\frac{2}{3}$ of the butt was removed.

Example 29:

Sheetal removed a certain quantity from a solution of 70% milk and replaced it by pure water. The concentration of the resulting mixture is now 60%. What fraction of milk did Sheetal remove?

Solution:

Pure water is 0% milk solution:

∴ Using alligation

0%   70%
   60%
10 : 60

Thus, the ratio of water to that of original solution is = 1 : 6

Thus, fraction removed is $= \frac{1}{7}$

Unusual Mixtures

The above concepts of mixtures can be used in a variety of situations. For this, one must just understand that a mixture need not be just a liquid mixed with another liquid or for that matter, a solid mixed with another solid to form an alloy. A class of girls and boys can also be considered to be a mixture of girls and boys!

Wherever there are two parts and we can consider the two parts taken together, the concepts of mixtures can be applied there. Consider the following few unusual mixtures.

Average Speed

Consider a person who travels from Mumbai to Pune (a distance of 180 kms) at a uniform speed 60 kmph. However, while coming back from Pune to Mumbai, he travels at a uniform speed of 90 kmph (thanks to the expressway). What is his average speed for the round journey?

And it is not $\frac{60+90}{2} = 75$ kmph!

Average Speed is defined as total distance divided by total time taken.

Thus in this case, average speed = $\frac{2\times180}{\frac{180}{60}+\frac{180}{90}} = \frac{2\times180}{3+2} = 72$ kmph

Average speed is also a special case of weighted average. However the weights are the time taken at each of the speed and not the distance travelled. In this case, the ratio of the time taken with the two different speed (and hence the ratio of the weights) is 3 : 2 and using these weights we come to the same answer :

$\frac{3\times60+2\times90}{3+2}=\frac{360}{5}$. In this process you would see that we do not really need to know the distance between Mumbai and Pune! Take the distance anything you like and see for yourself, the average speed is always going to be 72 with the individual speeds being 60 kmph and 90 kmph.

Compare the above problem with this one : On a round trip from Mumbai to Pune and back to Mumbai, Sachin travels for half the time at a speed of 60 kmph and other half the time at 90 kmph. What is the average speed for the round trip?

Let us solve the question by using our definition of average speed as the total distance divided by total time. If the total time taken is 2t, the average speed is $\frac{60t+90t}{2t}=\frac{60+90}{2}=75$ kmph.

Thus, in this case we see that the average speed is the simple average of the two speeds and rightly so as the weights i.e. the time taken is same for the two speeds.

Example 30:

A person travels 285 km in 6 hr. In the first part of the journey he travels at 40 km/hr by bus. In the second part, he travels at 55 km/hr by train. How much distance does he travel by train?

Solution:

Average speed for the entire journey $=\frac{285}{6}$ km/hr.

Average speed of the train in the second part of the journey = 55 km/hr.

Average speed in the first part of the journey = 40 km/hr.

Here the alligation principle is applied to find the ratio of time that he has travelled with speeds of 40 km/hr and 55 km/hr respectively (i.e. the right-hand side of the equation).

$$\frac{55-\frac{285}{6}}{\frac{285}{6}-40}=\frac{\frac{45}{6}}{\frac{45}{6}}=\frac{1}{1}$$

So, he has travelled for equal amounts of time with the two speeds, i.e. for 3 hr each.

Distance travelled by the train = 55 × 3 = 165 km.

Example 31:

What is the average speed of the entire journey if a person drives for 4 hr at a speed of 10 km/hr and for 6 hr at a speed of 20 km/hr?

Solution:

The average speed = $\left(\frac{4}{10}\right)\times10+\left(\frac{6}{10}\right)\times20=16$ km/hr.

The above problem could have been worded differently as "A person travels for 40% of the time at a speed of 10 km/hr and the rest of the time at a speed of 20 km/hr".

Example 32:

If I make a profit of 10% on one-fourth of the quantity sold and a loss of 20% on the rest, what is my average (net) profit or loss?

Solution:

The average profit/loss $=\left(\frac{1}{4}\right)(10)+\left(\frac{3}{4}\right)(-20)=-12.5\%$.

The same problem could have been worded as: A person makes a profit of 10% on 25% of the quantity and a loss of 20% on the rest, what is his percentage gain or loss on the whole?

Example 33:

A person takes a loan of Rs. 10000, partly from CICIC bank at 8% p.a. and remaining from CFDH bank at 10% p.a. He pays a total interest of Rs. 950 per annum. What amount of loan is taken from CICIC bank?

Solution:

Conventionally, the problem would be solved by assuming a loan of Rs. x from CICIC bank. Thus,

$0.08x + 0.1(10000-x) = 950$

$\therefore 1000 - 0.02x = 950$

$\therefore 0.02x = 50$ and $x = 2500$

However if we consider the total loan as a mixture of two loans one at 8% and other at 10% and the overall rate on both the loans taken together as 9.5%, we can easily apply alligation to get the ratio of the loan amounts as

$$\begin{array}{ccc} 8 & & 10 \\ \nwarrow & & \swarrow \\ & 9.5 & \end{array}$$

$$\begin{array}{ccc} 10 - 9.5 = 0.5 & : & 9.5 - 8 = 1.5 \\ 1 & : & 3 \end{array}$$

Thus amount of loan taken at 8% is ¼th of 10000 i.e. 2500

Example 34:

In a zoo, there are rabbits and pigeons. If heads are counted, there are 200 and if legs are counted, there are 580. How many pigeons are there?

Solution:

Here too the zoo can be considered as a mixture of rabbits and pigeons and the distinguishing criteria is rabbits have 4 legs per head and pigeons have 2 legs per head. The average number of legs per head for the entire mixture (rabbits & pigeons taken together) is $\frac{580}{200} = 2.9$. Applying alligation with this data,

$$\begin{array}{ccc} 2 & & 4 \\ \nwarrow & & \swarrow \\ & 2.9 & \\ 1.1 & : & 0.9 \end{array}$$

Thus number of pigeons = $\frac{11}{20} \times 200 = 110$ and balance 90 are rabbits.

Though this example can be solved by simultaneous equation, it is taken here to elucidate the application of the fundamentals of mixtures in such diverse examples as above. And the most efficient way to solve this problem would not be the simultaneous equation and nor this alligation. Simple common sense is enough to solve this problem. If all of the 200 animals were pigeons there should have been 400 legs. But there are 580. For each pigeon removed and replaced by a rabbit, the number of legs in the zoo increase by 2. We need an increase of 180. Thus 90 pigeons are removed and replaced by rabbits.

Practice Exercises 3

Introduction

There are 6 practice exercises out of which 2 are of level-1, 3 are of level 2 and 1 is of level 3 apart from non MCQ to strengthen you fundamental. While solving the exercises make sure that each and every concept is understood properly.

Problems for Practice (Non MCQ)

Level - 1

1. What number should be subtracted from each of the numbers 54, 71, 75 and 99, so that the first two terms bear the same ratio as last two terms in that order?

2. Divide 465 into three parts which are in the ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$.

3. Divide 170 into three parts such that the first part is 10 more than the second and its ratio with third part is 2 : 5.

4. The denominator of a number is three times its numerator. If the ratio of the numbers that are added to the numerator and denominator is 1 : 3, find the new fraction that is formed.

5. How many one-rupee, 50-paisa and 25-paisa coins are there if these are in the ratio 4 : 5 : 6 and together amount to Rs. 32?

6. An employer reduces the number of employees by $\frac{1}{9}$ and increases their wages by $\frac{1}{14}$. What is the ratio of the total wages paid to the employees before the hike and after that?

7. Father's age was five times his son's age 5 years ago and will be three times the son's age after 2 years. What is the ratio of their present ages?

8. Mohan's expenditure and savings are in the ratio 4 : 1. His income increases by 20%. If his savings also increases by 12%, by how much percentage would his expenditure increase?

9. A bag contains Rs.1,800 in the form of one-rupee, 50-paisa and 25-paisa coins in the ratio 3 : 4 : 12. Find the number of 50-paisa coins.

10. In ΔABC, ∠A : ∠B = 4 : 5 and ∠B is 30° more than ∠C. Find the three angles.

11. The ratio of volumes of 2 cylinders is x : y and the ratio of their heights is p : q. Find the ratio of the squares of their diameters.

12. p, q and r are three positive numbers and $Q = \frac{p+q+r}{2}$.

If (Q - p) : (Q - q) : (Q - r) = 2 : 5 : 7, then find the ratio of p, q and r?

13. The incomes of Ram and Shyam are in the ratio 8 : 11 and their expenditures are in the ratio 7 : 10. If each of them saved Rs.500, what are their incomes and expenditures?

14. A began a business with Rs.450 and was joined by B with Rs.300. When did B join if the profit at the end of the year was divided between the two in the ratio 2 : 1?

15. At a game, A can give B 20 points in 60, and can give C 15 points in 60. How many points can C give B in a game of 60?

16. 20 men take 10 days to complete a job working at the rate of 5 hr per day. How much time would 15 men, who work for 8 hr per day, take to complete the same job?

17. In question 16, if 15 men were 75% as efficient as the first set and the job is 25% more than the earlier job, how much time is needed to complete it?

18. The weights of 4 chairs are 10 kg, 8 kg, 12 kg and 6 kg. By how much would the average weight change if the heaviest chair was replaced by a 8 kg chair?

19. The average income of a household (all 4 earning) is Rs.15,000 per month. If the income of each person increases by Rs.1,000, by how much does the average increase?

20. In the above problem, if the income of each member increases by 20%, what will be the new average?

21. The average turnover of a company in January, February and March is Rs. 12 crore, while that of March, April, May and June is Rs. 14 crore. What is the turnover in March if the average for the first half of the year is Rs. 12 crore?

22. The average height of 4 friends is 168 cm. John, whose height is 170 cm, leaves the group. What is the new average?

23. Four friends have an average score of 72. If Tom is also included, the average becomes 70. What is Tom's score?

24. A vendor purchases 20 kg apples at Rs. 40 per kilogram and 10 kg apples at Rs. 50 per kilogram. What is the average cost?

25. A cyclist rides 20 km at 10 km/hr and another 20 km at 5 km/hr. What is his average speed?

26. A man travels 10 km on foot in 2 hr and 20 km by bus in 1 hr. What is his average speed?

27. A company sells 50% of the products at Rs. 20 per unit and another 30% of the products at Rs. 40 per unit. The rest remains as its inventory. What is the average price per unit of the items sold?

28. A group of 5 friends has an average weight of 6.5 kg. Two friends with average weight of 7 kg leave the group. What is the new average?

29. A class of 30 students with an average score of 70% is merged with another class of 20 students with average score of 60%. What is the average score of the combined class?

30. Two alloys of gold and copper were prepared by mixing metals in the ratio 7 : 3 and 7 : 13 respectively. Then equal quantities of the two alloys were melted together to form the third alloy. What is the proportion of gold and copper in the third alloy?

31. A mixture of 40 L of milk and water contains 10% water. How much water must be added to this mixture in order to make it a 20% water mixture?

32. A dishonest milkman professes to sell his milk at cost price. He mixes it with water and thereby gains 25%. What is the percentage of water in the mixture?

33. The ratio of milk and water in 66 L of adulterated milk is 5 : 1. Water is added to it to make the ratio 5 : 3. What is the quantity of water added?

34. A mixture of 729 ml contains milk and water in the ratio 7 : 2. How much more water is added to get a new mixture containing milk and water in the ratio 7 : 3?

35. A man purchases two shirts for Rs. 900. He sells one at profit of 15% and other at a loss of 10%. He neither gains nor losses anything. Find the cost price of each shirt.

Level - 2

36. Rs. 50 is divided among 6 men, 12 women and 17 boys so that 2 men get as much as 5 boys and 2 women as much as 3 boys. What is the share of each boy?

37. An army of 7,300 troops is formed into four battalions, so that $\frac{1}{2}$ of the first, $\frac{2}{3}$ of the second, $\frac{3}{4}$ of the third, $\frac{4}{5}$ of the fourth are all composed of the same number of men. How many men are there in each battalion?

38. A man divides his property among his three sons A, B and C in the ratio 2 : 3 : 5. B sells $\frac{1}{3}$ of his property to A and $\frac{1}{3}$ to C and C then sells $\frac{1}{4}$ of his entire property to A. A now has Rs. 18,000 worth of property. What was the total worth of property?

39. A, B and C invested their capitals in the ratio 5 : 6 : 8. At the end of the business term they received the profits in the ratio 5 : 3 : 1. Find the ratio of times for which they contributed their capitals.

40. Rs. 535 is divided among A, B and C so that if Rs. 15, Rs. 10, Rs. 30 be subtracted from their respective shares, the remainders would have been in the ratio 4 : 5 : 7. What was their initial shares (in Rs.)?

41. Three men rent a farm for Rs. 7,000 per annum. One man puts 110 cows for 6 months, the second puts 50 cows for 9 months and the third puts 440 cows for 3 months. What part of the rent should the second person pay?

42. A, B and C enter into a partnership. A contributes Rs. 3,200 for 4 months, B contributes Rs. 5,100 for 3 months and C contributes Rs. 2,700 for 5 months. If the total profits are Rs. 1,248 and if they divide the profits in the proportion of their investments, how much should B get?

43. A dog takes 6 leaps for every 4 leaps of a hare and 2 leaps of the dog are equal to 5 leaps of the hare. Compare their speeds.

44. The ratio of prices of two computers was 13 : 17. The price of the first computer went up, 10 years later, by 100% and that of the second went up by Rs. 2,500 and new ratio of their prices became 3 : 5. Find the original prices of the two computers.

45. A can beat B by 5 m in a 100 m race and B can beat C by 10 m in a 200 m race. Then in a race of 500 m, what is the lead that A can give C?

46. A group of 5 students has an average weight of 62 kg. One of the students needs to be replaced so that the average decreases by 2 kg. What should be the weight of the new student if the expelled student weighs 70 kg?

47. A crate of 10 different cases has an average weight of 25 kg. By how much will the average change if two cases of average weight 30 kg are replaced by two cases of average weight 32 kg?

48. Gold is 19 times as heavy as water and copper 9 times as heavy as water. In what ratio should these metals be mixed so that the mixture may be 15 times as heavy as water?

49. A shopkeeper buys two varieties of rice, the price of the first being twice the second. He sells the mixture at Rs. 36 per kilogram and makes a profit of 20%. If the ratio of quantities of the first and second variety in the mixture is 3 : 7, then what is the cost price of each variety of rice?

50. A mixture contains alcohol and water in the ratio 4 : 3. If 8 L of water is added to the mixture, the ratio of alcohol and water becomes 3 : 5. Find the quantity of alcohol in the mixture.

51. Two liquids are mixed in the ratio 3 : 5 and the mixture is sold at Rs. 120 per litre with a profit of 20%. If the first liquid is costlier than the second by Rs. 2 per litre, find the costs of the two liquids.

52. In three vessels, each of 10 L capacity, mixture of milk and water is filled. The ratios of milk and water are 2 : 1, 3 : 1 and 3 : 2 in the respective vessels. If all the three vessels are emptied into a single large vessel, find the ratio of milk and water in the resultant mixture.

53. Marmalade is made using two ingredients, sugar and orange peels in the proportion 2 : 5. The price of the sugar is three times the price of orange peels. The overall cost of production of a bottle of marmalade is $5.20 including $0.80 as labour charges. What is the value of sugar used in a jar of marmalade?

54. In two alloys, the ratio of copper and gold are in ratio 3 : 6 and 5 : 4. How many kilograms of the first alloy and the second alloy respectively should be melted together to get 64 kg of a new alloy with equal quantities of copper and gold?

55. Mother Dairy has two sources of milk supply. The first gives mixture of milk and water in ratio 5 : 8 and the second in the ratio 9 : 4. Mother Dairy supplies the mixture of the two types to Rama Milk Products, which require the mixture of milk and water to be in the ratio 7 : 6 for their products. In order to meet the requirements of Rama Milk Products, in what ratio should Mother Dairy mix the two mixtures?

56. A man has 60 pens. He sells some at a profit of 12% and the rest at a loss of 8%. On the whole, he makes a profit of 11%. How many pens did he sell at a profit of 12%?

57. A can contains a mixture of two liquids A and B in the ratio 7 : 5. When 9 L of mixture is taken out and the can is filled with B, the ratio of A and B is now 7 : 9. How many litres of liquid A was there in the can initially?

58. A man travelled a distance of 80 km in 7 hr, partly on foot at the rate of 8 km/hr and partly by a cycle at 16 km/hr. Find the distance travelled on foot.

59. 27 L of orange juice contains 312 calories and 9 L of mango juice contains 54 calories. If 18 L of mixture of both orange and mango juice contains 118 calories, find the proportion of orange juice in the mixture.

60. A man buys two horses for Rs.1,350. He sells one at a loss of 6% and the other at a gain of 7.5%. If on the whole he neither gains nor loses, what does each horse cost?

61. Fresh grapes contain 90% water by weight while dried grapes contain 20% water by weight. What will be the weight of dry grapes available from 20 kg of fresh grapes?

62. 3 L water is taken out from a vessel full of water and substituted by pure milk. This process is repeated two more times. Finally, the ratio of milk and water in the solution becomes 1701 : 27. Find the volume of the original solution.

63. A container contains 240 L of rum. 80 L is taken out of the container every day and an equal quantity of water is put into it. Find the quantity of rum that remains in the container at the end of the fourth day.

Level - 3

64. A vessel contains milk and water in the ratio 3 : 2. The volume of the contents is increased by 50% by adding water to this. From this resultant solution 30 L is withdrawn and then replaced with water. The resultant ratio of milk to water in the final solution is 3 : 7. Find the original volume of the solution.

Practice Exercise 1 - Level 1

1. A : B = 3 : 7 and the sum of A and B is 45. Find the value of B.

a. 28 b. 33.5 c. 31.5 d. 36 e. 13.5

2. A fraction bears the same ratio to $\frac{3}{7}$ as $\frac{1}{27}$ does to $\frac{1}{35}$. Find the fraction.

a. $\frac{4}{9}$ b. $\frac{1}{3}$ c. $\frac{3}{5}$ d. $\frac{5}{9}$ e. $\frac{9}{5}$

3. Mean proportional between 8 and 72 is

a. 24 b. 40 c. 16 d. 32 e. None of these

4. Fourth proportional to 3, 15 and 27 is

a. 39 b. 135 c. 81 d. 45 e. None of these

5. Third proportional to 20 and 30 is

a. 40 b. 55 c. 60 d. 50 e. 45

6. What must be added to each term of the fraction $\frac{4}{9}$, so that it becomes $\frac{2}{3}$?

a. 1 b. 6 c. 11 d. 16 e. $\frac{-6}{5}$

7. A, B and C join a partnership contributing Rs. 2,000, Rs. 1,500 and Rs. 1,250 respectively. What is A's share if total profit is Rs. 3,610?

a. Rs. 1,500 b. Rs. 2,290 c. Rs. 1,870 d. Rs. 1,520 e. Rs. 1,140

8. If X and Y shared in the ratio of 2 : 7, what is the ratio of X's share to the difference between Y's & X's shares?

a. 2 : 7 b. 4 : 10 c. 2 : 5 d. 5 : 7 e. None of these

9. What must be subtracted from each term of the ratio 68 : 49 so that it becomes 3 : 4?

a. - 125 b. 0 c. 5 d. 125 e. 105

10. Rs. 3,960 is divided among A, B and C such that half of A's part, one third of B's part and $\frac{1}{6}$th of C's part are equal. Then B's part is

a. Rs. 1,080 b. Rs. 960 c. Rs. 1,720 d. Rs. 1,540 e. Rs. 1,440

11. In question number 10 above, what is the ratio of A's share to the difference of B and C?

a. 2 : 9 b. 2 : 5 c. 4 : 5 d. 2 : 3 e. 1 : 3

12. Two numbers are in the ratio of 5 : 3. If 9 is subtracted from both of them, they become a ratio of 23 : 12. The first number is

a. 52 b. 53 c. 55 d. 54 e. 33

13. If 4x = 3y = 2z, then x : y : z is

a. 4 : 3 : 2 b. 2 : 3 : 4 c. 3 : 4 : 2 d. 3 : 4: 6 e. None of these

14. In a ratio equal to 4 : 9, the antecedent is 36, the consequent is

a. 79 b. 16 c. 72 d. 81 e. None of these

15. If $\frac{a}{3} = \frac{b}{4} = \frac{c}{7}$ then, $\frac{(a+b+c)}{c}$ is

a. 7 b. 2 c. $\frac{1}{2}$ d. $\frac{1}{7}$ e. Cannot be determined

16. Two whole numbers whose sum is 84 cannot be in the ratio of

a. 9 : 3 b. 3 : 5 c. 19 : 2 d. 2 : 19 e. All of these

17. It is given that 0.35 part of a number is equal to 0.07 part of another number. The ratio of the two numbers is

a. 1 : 2 b. 1 : 5 c. 2 : 1 d. 1 : 4 e. 5 : 1

18. Given that the ratio of A's money to that of B's money is 4 : 5 and B's money to C's is 2 : 3. If A has Rs. 800, then total amount of money among A, B and C is

a. Rs. 2,790 b. Rs. 3,300 c. Rs. 3,000 d. Rs. 3,620 e. None of these

19. The number 68 is divided into two parts such that one-seventh part of the first is equal to one-tenth part of the second. Find the first part.

a. 7 b. 22 c. 28 d. 32 e. 40

20. Rs. 9,700 has been divided among x, y and z such that if their shares are reduced respectively by Rs. 30, Rs. 20 and Rs. 50, the balances are in the ratio of 3 : 4 : 5. What is y's share?

a. Rs. 3,180 b. Rs. 3,220 c. Rs. 3,253.33 d. Rs. 3,200 e. Rs. 3,120

21. The sum of Rs. 530 is divided among A, B and C such that A gets Rs. 70 more than B and B gets Rs. 80 more than C. What is the ratio of the amount with A and C?

a. 25 : 18 b. 18 : 10 c. 9 : 5 d. 5 : 2 e. 7 : 2

22. An amount of money is distributed amongst A, B and C such that A gets half that of B and B gets twice that of C. What is the ratio of the share of B to that of the sum of the shares of A and B.

a. 2 : 5 b. 2 : 3 c. 3 : 2 d. 4 : 3 e. None of these

23. In a class of 500 students, the number of boys equals the number of girls. If $\frac{1}{5}$th of the girls left the class and 25 boys joined in, what is the ratio of the number of boys to the number of girls, now?

a. 3 : 2 b. 12 : 7 c. 11 : 8 d. 9 : 8 e. None of these

24. The ages of a man and his son bear a ratio of 7 : 2. After 15 years, their ages would be in the ratio of 2 : 1. What was father's age when the son was born?

a. 25 b. 30 c. 35 d. 42 e. Cannot be determined

25. Four years ago, a man's age was 6 times that of his son. 12 yrs from now, his age will be twice that of the son. What is the ratio of their present ages?

a. 6 : 1 b. 7 : 1 c. 8 : 2 d. 7 : 2 e. 7 : 4

26. Brass is an alloy of copper and zinc and has no other metal in it. In a sample of brass, copper and zinc are in the ratio of 13:7. How much copper will be there in a 500 kg sample of this alloy?

a. 300 kg b. 325 kg c. 175 kg d. 150 kg e. 350 kg

27. The numerator and the denominator of a rational number differ by 40 and its simplest form is $\frac{2}{7}$. What is the number?

a. $\frac{8}{48}$ b. $\frac{56}{16}$ c. $\frac{20}{60}$ d. $\frac{16}{56}$ e. $\frac{18}{63}$

28. In a mixture of 100 litres, the ratio of milk and water is 3 : 1. If 200 litres of water is added in the mixture, what will be the new ratio of milk and water?

shares of A and B.

a. 2 : 5 b. 2 : 3 c. 3 : 2 d. 4 : 3 e. None of these

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a. $\frac{8}{48}$ b. $\frac{56}{16}$ c. $\frac{20}{60}$ d. $\frac{16}{56}$ e. $\frac{18}{63}$

28. In a mixture of 100 litres, the ratio of milk and water is 3 : 1. If 200 litres of water is added in the mixture, what will be the new ratio of milk and water?

a. 1 : 3 b. 3 : 1 c. 2 : 5 d. 5 : 2 e. None of these

29. In 80 litres mixture of milk and water, milk and water are in the ratio of 5 : 3. If 16 litres of this mixture are replaced by 16 litres of milk then the ratio of milk and water in the resulting mixture becomes

a. 2 : 1 b. 6 : 3 c. 7 : 3 d. 8 : 3 e. 33 : 7

Practice Exercise 2 - Level 1

1. The ratio of boys and girls in a class of 72 is 7 : 5. How many more girls should be admitted to make the number of boys and girl equal?

a. 9 b. 12 c. 220 d. 240 e. 24

2. The incomes of A and B are in the ratio of 3 : 2 and their expenses are in the ratio of 5 : 3. If each of them saves Rs. 3000, then B's income is

a. Rs. 10,000 b. Rs. 6,000 c. Rs. 9,000 d. Rs. 12,000 e. Rs. 15,000

3. What must be added to the numerator and denominator of $\frac{7}{19}$ so that they bear the same ratio as 20 bears to 40?

a. 5 b. 7 c. 11 d. 12 e. 3

4. Three mixtures containing water and alcohol in the ratio of 5 : 2, 6 : 1 and 4:3 are mixed in equal quantities. The ratio of water to alcohol in the resulting mixture is

a. 5 : 2 b. 7 : 3 c. 6 : 4 d. 7 : 4 e. 2 : 1

5. If a carton containing dozen mirrors is dropped, which of the following cannot be the ratio of broken mirrors to unbroken mirrors?

a. 2 : 1 b. 3 : 1 c. 3 : 2 d. 1 : 1 e. 4 : 2

6. A certain ship floats with $\frac{3}{5}$ of its weight above the water. What is the ratio of the ship's submerged weight to its exposed weight?

a. 3 : 8 b. 2 : 5 c. 3 : 5 d. 2 : 3 e. None of these

7. In a contest 5% of the entrants won prizes. If 30 prizes were won, and no entrant won more than one prize, how many entrants did not win any prizes?

a. 300 b. 540 c. 570 d. 600 e. 575

8. Ratio of boys to girls in a class is 5 : 3. Which of these cannot be the number of students in the class?

a. 32 b. 40 c. 36 d. 56 e. None of these

9. An amount of money is to be distributed among A, B and C in the ratio 5 : 8 : 12 respectively. If the total share of B and C is four times that of A, what is A's share?

a. Rs. 3,000 b. Rs. 5,000 c. Rs. 2,000 d. Rs. 4,000 e. Data inadequate

10. An oculist charges $40 for an eye examination, frames and glass lenses, but $52 for an eye examination, frames and plastic lenses. If the plastic lenses cost four times as much as the glass lenses, how much do the glass lenses cost?

a. $3 b. $4 c. $5 d. $6 e. $12

11. The ratio of incomes of A and B is 3 : 5 and the ratio of their savings is 5 : 3. Whose saving is more?

a. A's b. B's c. They have equal savings

d. Depends upon the incomes of A and B.

e. Depends upon the expenditures of A and B.

12. The ratio of incomes of Pratima and Sujith is 3 : 5 and the ratio of their expenditures is 5 : 1. Who does save more?

a. Pratima b. Sujith c. Both save equally

d. Depends upon the incomes of Pratima and Sujit.

e. Depends upon the expenditures of Pratima and Sujit.

13. Equal quantities of a 1 : 5 and 3 : 5 of milk and water solution are mixed together. What will be the ratio of water and milk in the resultant solution?

a. 13 : 35 b. 4 : 10 c. 5 : 8 d. 35 : 13 e. 5 : 2

14. The mileage of a car is 16 km per litre, if the fuel is totally unadulterated. If due to adulteration the mileage reduces by 25%, by what percentage does the cost of maintaining the car per kilometre increase? (Assume that maintenance cost is equal to the fuel cost.)

a. 20% b. 25% c. 33.33% d. 36% e. 50%

15. A candidate was asked to find of a positive number. He found $\frac{5}{18}$ of the same by mistake. If his answer was 125 less than the correct one, then the original number was

a. 360 b. 240 c. 180 d. 270 e. None of these

16. The ratio of the ages of A and B is 5 : 7. If the difference between the present age of B and that of A 6 years hence is 2, then what is the total of present ages of A and B?

a. 24 years b. 52 years c. 56 years d. 45 years e. Cannot be determined

17. An amount of Rs. 680 was invested at 6% rate of interest and another sum of money was invested at 10% interest. If the average interest on the total at the end of the year was 7.5%, how much was invested at 10% interest?

a. Rs. 408 b. Rs. 412.50 c. Rs. 267.50 d. Rs. 340 e. Cannot be determined

Direction for questions 18 and 19: Answer the questions based on the following information.

A cubical container A of side 6 cm is filled with water. Another cubical container B of side 4 cm is filled with milk, and a third cubical container of side 10 cm is empty. The first container is tilted such that the water level coincides with the plane which is formed by joining the bottom side and the top side of the opposite faces of the container. The water that spills over is poured into the third container. The same is done with the second container and the milk that spills over is poured into the third container.

18. What is the ratio of water to milk in the third container?

a. 6 : 4 b. 36 : 16 c. 216 : 64 d. 3 : 1 e. None of these

19. What percentage of the third container is filled with liquid?

a. 100% b. 50% c. 36% d. 21% e. 14%

20. Father's age is four times the age of his elder son and five times that of his younger son. When the elder son has lived to three times his present age, then father's age will exceed twice that of his younger son's by 3 years. What is the father's age?

a. 30 years b. 32 years c. 35 years d. 40 years e. Data insufficient

21. A container has milk and water in the ratio 3 : 2. The volume of the container is 50 L.

Ten litres of the solution is removed and then replaced with water. What is the final amount of milk if this operation is done twice?

a. 19.2 L b. 32 L c. 40 L d. 24 e. None of these

22. Gopal wants to strengthen a 15% alcohol solution to the one containing 32% alcohol solution. Thus, the amount of pure alcohol to be added to 400 ml of the 15% solution is

a. 75 ml b. 60 ml c. 100 ml d. 25 ml e. 80 ml

Practice Exercise 3 - Level 2

1. A box containing a dozen ceramic mugs is dropped. Some of the mugs broke. Which of the following cannot be the ratio of broken and unbroken mugs?

a. 2 : 1 b. 5 : 7 c. 7 : 5 d. 3 : 2 e. None of these

2. A, B and C join a partnership. A invested Rs. 16,000 for 6 months, B invested Rs. 12,000 for $\frac{2}{3}$rd year and C invested Rs. 1,000 for 12 months. Calculate their profit sharing ratio.

a. 8 : 8 : 1 b. 10 : 8 : 7 c. 6 : 8 : 12 d. 8 : 7 : 10 e. 8 : 8 : 3

3. A starts a business with Rs. 4,000. B joins him after 3 months with Rs. 8,000. C puts a sum of Rs. 12,000 in the business for 2 months only in the same year. At the end of the year, the business generated a profit of Rs. 5,200. Find the share of B.

a. Rs. 1,500 b. Rs. 1, 800 c. Rs. 2,080 d. Rs. 4,000 e. Rs. 2,600

4. The manufacturing cost of a product involves only the expenses on Labour (L), the raw material costs (R), and the overheads cost (O). If L:R:O is 5:7:3 and the product is sold at 20% profit then what is the ratio of the raw material costs to the profit?

a. 5 : 2 b. 10: 3 c. 3 : 5 d. 7 : 3 e. None of these

5. In a partnership business, A, B and C invest money in the ratio 8:7:5. A withdraws half her money after 5 months. If the profit is Rs. 26,500 for the year, find B's share.

a. Rs. 9,800 b. Rs. 10,200 c. Rs. 10,500 d. Rs. 12,600 e. Rs. 8,500

6. Salim covers 910 km by boat, road and rail in the ratio 4 :3 : 6 respectively. Surprisingly, the speeds at which the journey was covered were also in the ratio 4 : 3 : 6 respectively. The total time taken for the journey was 89 hours. The ratio of the time taken by boat, road and rail was

a. 3 : 4 : 2 b. 1 : 1 : 1 c. 6 : 3 : 4 d. 4 : 3 : 6 e. Cannot be determined

7. A person has a total of Rs. 330 in Rs. 1.50 (assume), one-rupee and 25-paisa coins. They were in the ratio 2 : 5 : 1. How many one-rupee coins were there?

a. 400 b. 80 c. 40 d. 200 e. None of these

8. A servant is paid a total of Rs. 100 and a turban for a full year's service. If the servant works for only 9 months and received in return Rs. 65 and the turban, what is the value of the turban?

a. Rs. 9 b. Rs. 10 c. Rs. 40 d. Rs. 50 e. Rs. 25

9. In an examination, the ratio of pass candidates to failed was 3 : 1. If 8 more had appeared, and 6 less had passed, then the ratio would have changed to 1 : 1. The number of candidates who appeared for the examination is

a. 30 b. 60 c. 40 d. 80 e. None of these

10. A sporting goods store ordered an equal number of white and yellow tennis balls. The tennis ball company delivered 45 extra white balls, making the ratio of white balls to yellow balls $\frac{1}{5}:\frac{1}{6}$. How many white tennis balls did the store originally order for?

a. 450 b. 270 c. 225

d. 250 e. None of these

11. If $\frac{1}{2}$ of the number of white mice in a certain laboratory is $\frac{1}{8}$ of the total number of mice, and $\frac{1}{3}$ of the number of gray mice is $\frac{1}{9}$ of the total number of mice, then what is the ratio of white mice to gray mice in the laboratory?

a. 16 : 27 b. 2 : 3 c. 3 : 4 d. 4 : 5 e. 4 : 3

12. A dealer mixes three varieties of rice costing Rs. 20 per kg, Rs. 24 per kg and Rs. 30 per kg such that the mean price of the mixture Rs. 25 per kg. If the mixture contains 3 kg of the third variety and 2 kg of the first variety, how much quantity of the second variety does it contain?

a. 2 kg b. 5 kg c. 3 kg d. 10 kg e. 4 kg

13. A trader mixes three qualities of sugar costing Rs. 15 per kg, Rs 18 per kg and Rs 22 per kg and sells it at a price of Rs 22 per kg making a profit of 10%. If the mixture contains
4 kgs of the first quality of sugar, then find the difference between the amount of second quality of sugar and the third quality of sugar in the mixture.

a. 20 kg b.10 kg c. 3 kg d. 4 kg e. 12 kg

14. Ayan mixes three varieties of salt costing Rs. 15 per kg, Rs 18 per kg and Rs 20 per kg and sells the mixture at Rs 8 per kg and there by incurring a loss of 50%. If the mixture contains 6 kg of the first variety, then find the quantity of second and third variety of salt in the mixture, if both of them are integral multiple of a kg.

a. 1 kg, 1 kg b. 1 kg, 2 kg c. 2 kg, 1 kg d. 2 kg, 3 kg e. No integral solution

15. Ayan mixes three varieties of tea costing Rs. 20 per kg, Rs 30 per kg and Rs 24 per kg and sells it at Rs 30 per kg making a profit of 20%. If the mixture contains 1 kg of the first variety, then the quantity of the second and third variety can never be

a. 2 kg, 5 kg b. 4 kg, 15 kg c. 6 kg, 25 kg d. 7 kg, 30 kg e. None of these

16. Three mixtures containing milk and water in the ratio of 5 : 1, 2 : 1 and 3:1 are mixed in the ratio of 1 : 2 : 3. The ratio of milk to water in the resulting mixture is:

a. 3 : 1 b. 2 : 1 c. 4 : 3 d. 3 : 2 e. None of these

17. Two alloys containing copper and nickel in the ratio of 2 : 3 and 7 : 3 are melted and mixed in the ratio of 2 : 1 to get a new alloy. Find the ratio of copper to nickel in the new alloy.

a. 1 : 2 b. 1 : 3 c. 3 : 2 d. 2 : 5 e. 1 : 1

18. Krishna bought some wheat at Rs. 10 per kilogram. He buys 2 kg wheat from a ration shop at the rate of Rs. 8 per kilogram. He mixes the two and sells at Rs. 11 per kilogram. He makes 25% profit. What is the ratio of non-ration to ration wheat?

a. 1 : 4 b. 1 : 5 c. 5 : 1 d. 1: 3 e. None of these

19. A student's grade in a course is determined by 6 quizzes and one examination. If the examination counts thrice as much as each of the quizzes, what fraction of final grade is determined by the examination?

a. $\frac{1}{6}$ b. $\frac{1}{5}$ c. $\frac{1}{3}$ d. $\frac{1}{4}$ e. $\frac{2}{3}$

20. A student finishes the first half of an examination in two-thirds of the time it takes him to finish the second half. If the whole examination takes him an hour and half, how many minutes does he spend on the second half of the examination?

a. 36 min b. 54 min c. 60 min d. 44 min e. 30 min

21. Ram committed two mistakes in an examination where all the questions carried equal marks, and obtained 72%. If he had attempted 4 more questions and made 3 mistakes, he would have obtained 84%. If there was no negative marking for wrong answers, how many questions were asked?

a. 25 b. 30 c. 20 d. 35 e. None of these

22. In 8 days 50 men can do a job working 10 hr per day. If 20 boys and 15 girls work 15 hr per day, in how many days can the job be completed? Given that boys and girls work equally hard and each of them works half as hard as a man.

a. 15.24 days (approximately) b. 20 days c. 12 days

d. 14 days e. None of these

23. There are two vessels which are filled only with the pure components. Vessel I contains 20 L brandy and vessel II contains 20 L water. From vessel I, 5 L brandy is taken and placed in vessel II. Then 4 L mixture is transferred from vessel II to vessel I. Find the ratio of water in vessel I to brandy in vessel II.

a. 7 : 8 b. 8 : 7 c. 1 : 1 d. 2 : 1 e. None of these

24. A vendor buys milk at a certain price, adds water and sells the adulterated milk at the same rate as he bought it for. He makes a 30% profit. What is the percentage of water that he adds to milk?

a. 30% b. 15% c. 20% d. 60% e. 21%

25. Eight people enter into a partnership; 6 of them bring in Rs. 30 each. The seventh person brings in Rs. 10 more than the average of eight persons, and the eighth person brings in Rs. 55. What is the total sum brought in?

a. Rs. 40 b. Rs. 240 c. Rs. 280 d. Rs. 250 e. Rs. 270

26. Maddy is engaged in two transactions, where he earns same amount as profit in each case. He makes a 25% profit in the first transaction and a 20% gain in the second. If the same amount is earned as the profit, and the two transactions are combined, his profit becomes 22.22%. The sale price in the combined transaction is

a. 20 b. 22 c. 25 d. 21 e. Cannot be determined

27. A container having a liquid X is at a level of $\frac{4}{7}$ of its actual level. From this, if x litres is removed, the container is one-fourth full. Finally, 35 L is added to the container making it half full. The value of x and the capacity of the container are respectively

a. 45 L and 210 L b. 25 L and 140 L c. 45 L and 140 L

d. 40 L and 210 L e. None of these

Practice Exercise 4 - Level 2

1. A bottle of whisky contains of whisky and the rest is water. How much of the mixture must be taken away and substituted by equal quantity of water so as to have half whisky and half water?

a. 25% b. 45% c. $33\frac{1}{3}$% d. 50% e. 40%

2. A man covered a total distance of 1,000 km in 16 hr, partly in a taxi at 36 km/hr and partly in a bus at 80 km/hr. The distance covered by the bus is (approx.)

a. 770 km b. 640 km c. 720 km d. 680 km e. None of these

3. In what ratio must the 1 : 4 mixture of whisky and water be mixed with a 1 : 1 mixture to obtain a 2 : 3 mixture?

a. 2 : 3 b. 3 : 2 c. 2 : 1 d. 1 : 2 e. None of these

4. An alloy of copper and tin contains 77.78% of copper. After 18 kg of tin is added, the copper content gets reduced to 50%. How much copper and tin respectively does the new alloy have?

a. 40 kg and 15 kg b. 25.2 kg and 20 kg c. 20 kg and 23 kg

d. 17 kg and 23 kg e. None of these

5. A container contains 80 kg milk. From this, 8 kg milk was taken out and replaced by water. This process is repeated thrice. What is the ratio of the quantity of milk after the first draw to quantity of milk left after the third draw?

a. 10 : 9 b. 100 : 81 c. 100 : 64 d. 81 : 64 e. None of these

6. In a mixture of 35 L, the ratio of milk to water is 4 : 1. If 7 L water is added to the mixture, the ratio of milk to water changes to a new ratio. If we want the ratio of milk to water to change back to the original value, how much milk is to be added now?

a. 35 L b. 28 L c. 21 L d. 32 L e. None of these

7. A milk vendor sells 33 L milk containing milk and water in the ratio 12 : 1. He wants to make this a 11 : 2 solution of milk and water and to sell the entire quantity at the cost price. How much of water needs to be added?

a. 3 L b. $2\left(\frac{9}{11}\right)$ L c. 2.5 L d. 4 L e. None of these

8. Nupur has 73 L wine in a drum. She replaces 3.65 L of it with water and keeps doing so till the time the concentration of wine is less than 85%. The minimum number of operations that Nupur has to perform is

a. 3 b. 4 c. 2 d. once e. None of these

9. The ratio of incomes of Rashmi and Divya is 3 : 5 and the ratio of their expenditures is 2 : 3. Who does save more? (You have to assume that no one takes any loan from anywhere.)

a. Rashmi b. Divya c. Both save equally

d. Depends upon the incomes of Rashmi and Divya.

e. Depends upon the expenditures of Rashmi and Divya.

10. Ratio of incomes of Rohit and Sunil is 5 : 7 and the ratio of their expenditures is also 5 : 7. What is the ratio of their savings?

a. 2 : 3 b. 7 : 5 c. 5 : 7 d. 2 : 5 e. Cannot be determined

11. The ratio of sum of squares of first n natural numbers to square of sum of first n natural numbers is 17 : 325. The value of n is

a. 15 b. 25 c. 35 d. 30 e. None of these

12. IBM and SGI quote for a tender. On the tender opening day, IBM realizes that their quotations are in the ratio 7 : 4 and hence decreases its price during negotiations to make it Rs.1 lakh lower than SGI's quoted price. SGI then realizes that the final quotes of the two were in the ratio 3 : 4. What was the price at which IBM won the bid?

a. Rs. 7 lakh b. Rs. 4 lakh c. Rs. 3 lakh d. Rs. 1 lakh e. Rs. 1.5 lakh

13. A tea trader mixed two varieties of tea, one costing Rs. 3.50 per kilogram, and the other costing Rs. 4 per kilogram, and sells 40 kg of the mixture to a vendor at Rs. 4.50 per kilogram, and makes a profit of 20%. How much of each variety did the vendor mix?

a. 30 kg, 10 kg b. 20 kg, 20 kg c. 10 kg, 30 kg

d. 15 kg, 25 kg e. None of these

14. Each tree in a forest houses at least one bird. On half the number of trees, there are 2 sparrows each, on $\frac{1}{4}$ of the remaining there is one pigeon each, and the rest, that housed 4 birds each, are cut off for factory use. If there were in all 630 birds in the forest, how many trees were cut off?

a. 90 b. 80 c. 110 d. 180 e. Cannot be determined

15. The total surface area of a solid copper cube and a solid zinc cuboid are the same. The length, breadth and height of the cuboid are in the ratio 1 : 2 : 4. Both are melted together in a vessel. What is the ratio of copper and zinc in the resultant mixture?

a. $\left(\frac{14}{3}\right)^{\frac{3}{2}}:8$ b. $8:\left(\frac{14}{3}\right)^{\frac{3}{2}}$ c. $\left(\frac{3}{14}\right)^{\frac{3}{2}}:8$ d. $8:\left(\frac{3}{14}\right)^{\frac{3}{2}}$ e. None of these

16. In a cage, there are sparrows, parrots and doves in the ratio 3 : 7 : 5. If the number of parrots was more than the number of sparrows by a multiple of both 9 and 7, what is the minimum number of birds in the cage?

a. 945 b. 630 c. 252 d. 238 e. None of these

17. The price of a precious stone is proportional to its weight. What will be the loss incurred, if a stone weighing 28 g and costing Rs. 28,000 breaks into two pieces whose weights are in the ratio 15 : 13?

a. Rs. 12,000 b. Rs. 7,000 c. Rs. 21,000 d. Rs. 2,000 e. None of these

18. A housewife has 1 L of solution that contains milk and water in the ratio 3 : 1. She adds 250 ml of 3 : 2 solution of milk and water to it and then uses 250 ml of the combined mixture to make curd. How much of pure milk is she left with?

a. 1,000 ml b. 912.5 ml c. 750 ml d. 720 ml e. 680 ml

19. Monthly incomes of two persons are in the ratio 5 : 4 and their monthly expenditure are in the ratio 9 : 7. If each person saves Rs. 500 per month, then what are their monthly incomes?

a. Rs. 8,000 and Rs. 10,000 b. Rs. 3,750 and Rs. 3,000 c. Rs. 4,500 and Rs. 3,500

d. Rs. 5,000 and Rs. 4,000 e. Rs. 4,500 and Rs. 3,600

20. A, B and C started a business in which B and C were sleeping partners. They invested Rs. 4,000, Rs. 3,000 and Rs. 7,000 respectively for a period of one year. A is paid 10% of the profit as compensation for his work, and then the rest is shared in the ratio of their investments among all the three. If A gets Rs. 6,000, as his share of profit, find out the amount that B and C together receive.

a. Rs. 9,000 b. Rs. 7,500 c. Rs. 9,600 d. Rs. 10,800 e. Rs. 12,500

21. A 40 L solution contains spirit and water in the ratio 3 : 1. How much water should be added to bring the ratio to 5 : 2?

a. 2 L b. 3 L c. 4 L d. 3.8 L e. 5 L

22. How many litres of a 3% hydrogen peroxide solution should be mixed with a 6 L of a 30% hydrogen peroxide solution so as to get a 12% solution?

a. 3 L b. 6 L c. 9 L d. 12 L e. 15 L

23. The population of a town is 3,11,250. The ratio of the number of men and women is 1075 : 1000. There are 24% literates among men and 8% literates among women. What is the total number of literate people in the town? Approximately what percentage of people in the town are illiterate?

a. 48,900, 72.6% b. 4,8000, 70% c. 50,700, 83.75%

d. 77,812, 75% e. None of these

24. In a factory men, women and children were employed in the ratio 8 : 5 : 1 to finish a job and their individual wages were in the ratio 5 : 2 : 3. When 20 women were employed, total daily wages of all amounted to Rs. 318. Find the total daily wages paid to each category.

a. Rs. 280, Rs. 70 and Rs. 110 b. Rs. 240, Rs. 60, and Rs. 18 c. Rs. 380, Rs. 318, and Rs. 110

d. Rs. 160, Rs. 118, Rs. 40 e. Cannot be determined

25. A vessel contains 40 L milk. The milkman delivers 10 L to the first house, and adds an equal quantity of water. He does exactly the same at the second and third houses. What is the ratio of milk and water when he has finished delivering at the third house?

a. 27 : 37 b. 26 : 38 c. 1 : 4 d. 3 : 4 e. None of these

26. The average age of the members of a club increases to 27 when 3 more persons whose average age is 29 join them from 26. What is the number of members in the club now?

a. 6 b. 9 c. 12 d. 5 e. 10

27. The ratio of spirit and water in a mixture is 1 : 3. If the volume of the solution is increased by 25% by adding spirit only, what is the resultant ratio of spirit and water?

a. 2 : 3 b. 1 : 4 c. 1 : 2 d. 3 : 4 e. Cannot be determined

28. By mistake, in stead of dividing Rs. 117 among three persons A, B and C in the ratio $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$, it was divided in the ratio 2 : 3 : 4. Who gains the most and how much?

a. A, Rs. 28 b. B, Rs. 35 c. C, Rs. 25 d. C, Rs. 20 e. C, Rs. 27

29. How much water must be added to 28 L milk, costing Rs. 12 per litre, so that the value of the mixture will be Rs. 10.50 per litre?

a. 3.6 L b. 4.2 L c. 3 L d. 4 L e. 5 L

Practice Exercise 5 - Level 2

1. A purse contains one-rupee, ten-rupee and twenty-rupee notes only. The number of one-rupee notes is seven times the number of ten-rupee notes, and the number of twenty-rupee notes is four times the number of ten-rupee notes. What is the amount of money in the bag, if there were 12 twenty-rupee notes in this purse?

a. 291 b. 873 c. 582 d. 453 e. None of these

2. If there are Rs. 495 in a bag in denominations of one-rupee, 50-paisa and 25-paisa coins, which are in the ratio 1 : 8 : 16. How many 50-paisa coins are there in the bag?

a. 50 b. 220 c. 440 d. 480 e. None of these

3. In question 2, how much money is in the denomination of 25-paisa?

a. Rs. 220 b. Rs. 55 c. Rs. 110 d. Rs. 440 e. None of these

Direction for questions 4 and 5:

Answer the questions based on the following information.

The volumes of three vessels are in the ratio 1 : 2 : 4. The volume of the second vessel is 40 L. The first vessel is empty, the second is filled with milk and the third is filled with water. 50% of the volume from the second vessel is transferred to the first and that is replaced with the liquid from the third vessel. The contents of the first vessel is now emptied into the third vessel. This operation is done once more.

4. What is the ratio of milk to water in the first vessel at the end of the second operation, just before it is emptied into the third vessel?

a. 1 : 4 b. 1 : 3 c. 1 : 1 d. 3 : 1 e. 2 : 3

5. What fraction of the third container comprises milk finally?

a. 5 : 16 b. 5 : 11 c. 16 : 5 d. 11 : 5 e. 3 : 8

Direction for questions 6 to 8:

Answer the questions based on the following information.

Hansie invested some money with the bookies. He invested on the odds of South Africa winning which was 1 : 3 (for every Re 1 invested he gets Rs. 3 if the result is in his favour). He knew about fixing of the matches played by India and invested some amount at odds of 1 : 4 of India losing the match. Both India and South Africa won their respective matches. South Africa played Australia while India played Pakistan. The following questions are independent of each other.

6. If Hansie's profit on his investment was 150%, what is the ratio of investments in the two cases?

a. 5 : 1 b. 1 : 5 c. 1 : 4 d. 4 : 1 e. 5 : 2

7. If Hansie gained on the whole $2,00,000 and his investments were in the ratio 3 : 4, find the total amount invested.

a. $7,00,000 b. $2,80,000 c. $3,00,000 d. $4,00,000 e. $2,50,000

8. If the ratio of investments was 1 : 2 in the two cases, and India lost the match, what would the gain on investment be?

a. 466.66% b. 133.33% c. 266.66% d. 233.33% e. 166.66%

9. A, B and C had different amounts of money with them. C had an amount equal to the average of the total. A spent half of what he had, B two-thirds of what he had, and C spent all the money he had. What was remaining with them was half of B had originally. What was the percentage of the total amount spent by the three together?

a. 90% b. 80% c. 75% d. 60% e. 72%

10. Three jars contain alcohol to water solution in the ratios 3 : 5, 1 : 3 and 1 : 1. If all the three solutions are mixed, what will be the ratio of alcohol to water in the final solution?

a. 3 : 5 b. 3 : 4 c. 4 : 5 d. 5 : 9 e. Cannot be determined

11. A lump of two metals weighing 18 g is worth Rs. 87. If their weights be interchanged, it would be worth Rs. 78.60. If the price of one metal is Rs. 6.70 per gram, find the weight of the other metal in the mixture.

a. 10 g b. 8 g c. 9 g d. 11 g e. 12 g

12. Several litres of acid were drawn off from a 54 L vessel full of acid and an equal amount of water is added. Again the same volume of the mixture was drawn off and replaced by water. As a result, the vessel contained 24 L of pure acid. How much acid was drawn off initially?

a. 12 L b. 16 L c. 18 L d. 21 L e. 24 L

13. How many litres of a 90% solution of concentrated acid needs to be mixed with a 75% solution of concentrated acid to get a 30 L solution of 78% concentrated acid?

a. 24 L b. 22.5 L c. 6 L d. 17.5 L e. 25 L

14. Tina, Ishan, Abhishek and Fatima have a total of Rs. 80. If Tina's share increases by Rs. 3, Ishan's share increases by one-third of his share, Abhishek's share decreases by 20% and Fatima's share decreases by Rs. 4, all of them would have equal amounts of money. Fatima's original share is

a. Rs. 24 b. Rs. 24.75 c. Rs. 23.75 d. Rs. 20 e. Rs. 23.50

15. A tea merchant buys two varieties of tea - the price of the first being twice that of the second. He sells the mixture at Rs. 17.50 per kilogram thereby making a profit of 25%. If the ratio of the amounts of the first tea and the second tea in the mixture is 2 : 3, then the respective costs of each tea are

a. Rs. 20 and Rs. 10 b. Rs. 24 and Rs. 12 c. Rs. 16 and Rs. 8

d. Rs. 23 and Rs. 13 e. None of these

16. There is a certain number of cigarettes in a box. They are divided into such a way that the person who gets $\frac{1}{4}$ of the whole gets thrice of what the others get on an average. Find the number of people amongst whom the cigarettes are distributed.

a. 8 b. 9 c. 10 d. 12 e. 7

17. If p kg of sugar, worth Rs. 6 per kilogram, is mixed with q kg of sugar, worth Rs. 8 per kilogram, then the resulting mixture costs Rs. 7.50 per kilogram. What will a mixture cost if there are q kg of sugar worth Rs. 6 per kilogram and p kg of sugar worth Rs. 8 per kilogram?

a. Rs. 7.25 b. Rs. 7 c. Rs. 6.50 d. Rs. 6.25 e. Rs. 7.5

18. A 735 g sample of a 16% (by weight) solution of iodine in alcohol is kept for three days. Some of the alcohol gets evaporated and the concentration of the solution becomes 20% (by weight). what amount of alcohol gets evaporated ?

a. 140 g b. 147 g c. 135 g d. 215 g e. 150 g

19. Two vats contain 4 L and 6 L of wine solution respectively. If the contents of the two are added the result obtained is wine of 35% concentration. If 1 L from both is mixed, we get 35% concentration of wine. The amount of pure wine in the two vats is

a. 4.8 and 2.1 L b. 1.4 and 2.1 L c. 1.4 and 2.4 L d. 2 and 3 L e. 1.4 and 2.8 L

20. Out of 95 people present at the party, 60 were graduates and 35 were employed. Total number of men in the party was 55, and all of them were graduates. If 50% of the women at the party were employed, then the ratio of `employed graduate women' to 'unemployed men' is

a. Rs. 20 and Rs. 10 b. Rs. 24 and Rs. 12 c. Rs. 16 and Rs. 8

d. Rs. 23 and Rs. 13 e. None of these

16. There is a certain number of cigarettes in a box. They are divided into such a way that the person who gets $\frac{1}{4}$ of the whole gets thrice of what the others get on an average. Find the number of people amongst whom the cigarettes are distributed.

a. 8 b. 9 c. 10 d. 12 e. 7

17. If p kg of sugar, worth Rs. 6 per kilogram, is mixed with q kg of sugar, worth Rs. 8 per kilogram, then the resulting mixture costs Rs. 7.50 per kilogram. What will a mixture cost if there are q kg of sugar worth Rs. 6 per kilogram and p kg of sugar worth Rs. 8 per kilogram?

a. Rs. 7.25 b. Rs. 7 c. Rs. 6.50 d. Rs. 6.25 e. Rs. 7.5

18. A 735 g sample of a 16% (by weight) solution of iodine in alcohol is kept for three days. Some of the alcohol gets evaporated and the concentration of the solution becomes 20% (by weight). what amount of alcohol gets evaporated ?

a. 140 g b. 147 g c. 135 g d. 215 g e. 150 g

19. Two vats contain 4 L and 6 L of wine solution respectively. If the contents of the two are added the result obtained is wine of 35% concentration. If 1 L from both is mixed, we get 35% concentration of wine. The amount of pure wine in the two vats is

a. 4.8 and 2.1 L b. 1.4 and 2.1 L c. 1.4 and 2.4 L d. 2 and 3 L e. 1.4 and 2.8 L

20. Out of 95 people present at the party, 60 were graduates and 35 were employed. Total number of men in the party was 55, and all of them were graduates. If 50% of the women at the party were employed, then the ratio of `employed graduate women' to 'unemployed men' is

a. $\frac{2}{11}$ b. $\frac{1}{3}$ c. not greater than $\frac{1}{8}$

d. $\frac{1}{2}$ e. None of these

21. Three beakers have capacity of 250 ml, 650 ml, and 200 ml. 682 ml of water is poured into them so that the same fraction of each is filled. The volume filled in the largest beaker will be

a. 415 ml b. 403 ml c. 400 ml d. 424 ml e. 420 ml

22. An alloy contains tin and copper in the ratio 3 : 4, and 10 kg pure copper is melted and mixed with it. If the resultant alloy contains 38 kg of copper, what is the weight of the original alloy?

a. 15 kg b. 40 kg c. 28 kg d. 49 kg e. Cannot be determined

Practice Exercise 6 - Level 3

1. A vessel contains 12 L wine and another contains 4 L water. 3 L is removed from each of the vessels and transferred into one another. The same operation is repeated once more. What is the final ratio of wine to water in two vessels?

a. 1 : 3, 3 : 1 b. 3 : 1, 1 : 3 c. 3 : 1, 3 : 1 d. 2 : 3, 1 : 2 e. Cannot be determined

2. The ratio of incomes of Princy and Kunjumol is 1 : 2 and the ratio of their expenditures is 1 : 5. Who does save more? (You again have to assume that these girls do not take any loan from anywhere).

a. Princy b. Kunjumol c. Both save equally

d. Depends upon the incomes of Princy and Kunjumol

e. Depends upon the expenditures of Princy and Kunjumol

3. Five times A's income added to B's income is more than Rs. 51. Three times A's income is more than Rs. 21 from B's income. Find out the possible range of value of a and b if they represent A's and B's income respectively.

a. $a > 9, b < 6$ b. $a > 9, b = 6$ c. $a > 9, b > 6$

d. $a = 9, b > 6$ e. $a > 9$, but we can put no bounds on b

4. Average age of the students in a class of 50 is 13. The weights of each is directly proportional to the height and the height is also found to be in direct proportion to the age. A student of age 11 is 165 cm tall, with weight being 33 kg. Find the average weight of the class.

a. 33 kg b. 39 kg c. 36 kg d. 30 kg e. Data insufficient

Answer Key

Problems for Practice (Non MCQ)

1. 3 **2.** 225,150, 90 **3.** 40,30,100

4. $\frac{1}{3}$ **5.** 16, 20, 24 **6.** 21 : 20

7. 10 : 3 **8.** 22% **9.** 900

10. 60, 75, 45 **11.** xq : py **12.** 12 : 9 : 7

13. 4000,5500; 3500, 5000 **14.** after 3 months **15.** $6\frac{2}{3}$

16. 25/3 days **17.** 13.88 days **18.** -1 kg

19. Rs. 1000 **20.** Rs. 18000 **21.** Rs. 20 cr

22. 167.37 cm **23.** 62 **24.** Rs. 43.33

25. 6.67 kmph **26.** 10 kmph **27.** Rs. 27.5

28. 6.16 kg **29.** 66% **30.** 21 : 19

31. 5 L **32.** 20% **33.** 22 kg

34. 81 ml **35.** 360, 540 **36.** Re. 1

37. 2400, 1800, 1600, 1500 **38.** Rs. 40,000 **39.** 8 : 4 : 1

40. 135, 160, 240 **41.** $\frac{5}{27}$ **42.** Rs. 459

43. 15 : 4 **44.** Rs. 1234.08, Rs. 1613.81 **45.** 48.75 cm

46. 60 kg **47.** +0.4 kg **48.** 3 : 2

49. 46.14, 23.07 **50.** $\frac{96}{11}$ L **51.** 101.25, 99.25

52. 121 : 59 **53.** $2.4 **54.** 16 kg, 48 kg

55. 1 : 1 **56.** 57 **57.** 21 L

58. 32 km **59.** $\frac{1}{10}$ **60.** 750, 600

61. 2.5 kg **62.** 4 L **63.** 47.4 L

64. 80 L

Practice Exercise 1 - Level 1

1	c	**2**	d	**3**	a	**4**	b	**5**	e	**6**	b	**7**	d	**8**	c	**9**	d	**10**	a
11	d	**12**	c	**13**	d	**14**	d	**15**	b	**16**	b	**17**	b	**18**	b	**19**	c	**20**	b
21	d	**22**	b	**23**	c	**24**	a	**25**	d	**26**	b	**27**	d	**28**	a	**29**	c		

Practice Exercise 2 - Level 1

1	b	**2**	d	**3**	a	**4**	a	**5**	c	**6**	d	**7**	c	**8**	c	**9**	e	**10**	b
11	a	**12**	b	**13**	d	**14**	c	**15**	a	**16**	e	**17**	a	**18**	c	**19**	e	**20**	a
21	a	**22**	c																

Practice Exercise 3 - Level 2

1	d	**2**	a	**3**	e	**4**	d	**5**	c	**6**	b	**7**	d	**8**	c	**9**	c	**10**	c
11	c	**12**	b	**13**	b	**14**	a	**15**	e	**16**	e	**17**	e	**18**	e	**19**	c	**20**	b
21	a	**22**	a	**23**	e	**24**	a	**25**	c	**26**	e	**27**	c						

Practice Exercise 4 - Level 2

1	c	**2**	a	**3**	d	**4**	e	**5**	b	**6**	b	**7**	a	**8**	b	**9**	b	**10**	e
11	b	**12**	c	**13**	b	**14**	a	**15**	a	**16**	a	**17**	e	**18**	d	**19**	d	**20**	d
21	a	**22**	d	**23**	c	**24**	b	**25**	a	**26**	b	**27**	a	**28**	c	**29**	d		

Practice Exercise 5 - Level 2

1	a	**2**	c	**3**	a	**4**	c	**5**	a	**6**	a	**7**	a	**8**	c	**9**	c	**10**	e
11	b	**12**	c	**13**	c	**14**	c	**15**	a	**16**	c	**17**	c	**18**	b	**19**	b	**20**	c
21	b	**22**	d																

Practice Exercise 6 - Level 3

1	c	2	e	3	c	4	b

Explanations: Fundamentals of Ratio

Problems for Practice (Non MCQ)

Level - 1

1. (54 - x) : (71 - x) : : (75 - x) : (99 - x)

Solving, x = 3

2. Ratio = $\frac{1}{2} : \frac{1}{3} : \frac{1}{5}$ = 15 : 10 : 6

or first part = $\frac{15}{31} \times 465 = 225$

Second part = $\frac{10}{31} \times 465 = 150$

Third part = $\frac{6}{31} \times 465 = 90$

Hence, the three parts are 225, 150, 90.

3. Ratio of first, second and third is

2x : 2x - 10 : 5x

and 2x + 2x - 10 + 5x = 170

9x = 180

x = 20

So the three parts are 40, 30, 100 respectively.

4. If numerator = x and Denominator = 3x

∴ Fraction ⇒ $\frac{x+y}{3x+3y} = \frac{1}{3}$

5. Ratio of number of coins = 4 : 5 : 6

Suppose we have 4, 5 and 6 coins of Re. 1, 50-paisa and 25-paisa respectively. Then the total value = Rs. 8.

Since the aggregate sum is Rs. 32 (4 times the sum that we have arrived at), the number of coins of the denomination Re. 1, Re. 0.5 and Re. 0.25 must be 16, 20 and 24 respectively.

6. Let the initial number of employees = n.

New number of employees = $\frac{8}{9}n$

Let initial wages = x. New wages = $\frac{15}{14}x$

∴ Ratio of total wages before and after the change = (nx) : $\left(\frac{8}{9}n\right)\left(\frac{15}{14}x\right)$ = 21 : 20.

7. Let father's present age be F.

Son's present age be S.

∴ (F - 5) = 5[S - 5] and (F + 2) = 3(S + 2) or, F - 5S = -20 and F - 3S = 4

∴ 2S = 24; S = 12 and F = 40

∴ Ratio of ages of father and son = 10 : 3.

8. Mohan's expenditure = 4x

Mohan's savings = x

$\therefore$ Income = 5x

New income = (5x)(1.2) = 6x; new saving = 1.12x

New expenditure = 4.88x

Percentage increase in expenditure = $\left(\frac{4.88x-4x}{4x}\right)\times 100 = 22\%.$

9. Ratio of values of 1 rupee, 50 paisa and 25-paisa coin is $\frac{3}{1}:\frac{4}{2}:\frac{12}{4}$, i.e. 3 : 2 : 3

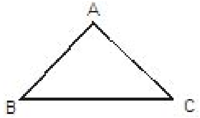
$\therefore$ 3x + 2x + 3x = 1800

8x = 1800 $\Rightarrow$ x = 225

Value of 50-paisa coin = 2 × 225 = 450

and number of 50-paisa coins = 450 × 2 = 900

10.



The angles of the triangle are

If ∠A = 4x; ∠B = 5x; ∠C = 5x - 30

∠A + ∠B + ∠C = 180 $\Rightarrow$ 4x + 5x + 5x - 30 = 180

$\Rightarrow$ 14x = 210 $\Rightarrow$ x = 15

So ∠A = 60°, ∠B = 75°, ∠C = 45°

11. The ratio of volumes of two cylinders is x : y and heights are p : q.

$$\frac{V_1}{V_2}=\frac{\pi r_1^2 p}{\pi r_2^2 q}=\frac{x}{y}=\frac{r_1^2}{r_2^2}=\frac{xq}{py}\Rightarrow\frac{d_1^2}{d_2^2}=\frac{xq}{py}$$

12. The given expression can be written as

$$\frac{Q-p}{2}=\frac{Q-q}{5}=\frac{Q-r}{7}=k$$

p = Q - 2k; q = Q - 5k; r = Q - 7k

So, $Q=\frac{Q-2k+Q-5k-Q-7k}{2}$

2Q = 3Q -14k

Q = 14k

p = 12k;

q = 9 k;

r = 7k;

$\therefore$ p : q : r = 12 : 9 : 7

13.

	Ram	Shyam
Incomes	8x	11x
Expenditures	7y	10y
Savings	500	500

$\therefore$ 8x - 7y = 500 $\Rightarrow$ 88x - 77y = 5500

11x - 10y = 500 $\Rightarrow$ 88x - 80y = 4000

$\therefore$ 3y = 1500, y = 500

$\Rightarrow$ x = 500

∴ Incomes of Ram and Shyam are Rs. 4,000

Rs. 5,500 their expenditures are Rs. 3,500,

Rs. 5,000 respectively.

14. If B joined for t months then profit ratio = (450 × 12) : 300t = 2 : 1

∴ $\frac{450 \times 12}{300t} = 2 \Rightarrow t = 9$ months

∴ B joined after 3 months.

15. When A gets 60 points, B gets 40 points

When A gets 60 points, C gets 45 points

∴ When C gets 45 points, B gets 40 points

When C gets 60 points, B gets $53\frac{1}{3}$ points

∴ C gives B $6\frac{2}{3}$ points in 60

16. Time $t = \frac{[20 \text{ men} \times 10 \text{ days} \times 5 \text{ hr}]}{[15 \text{ men} \times 8 \text{ hr}]} = \frac{25}{3}$ days

17. Time t = $\frac{20 \times 10 \times 5}{15 \times 0.75 \times 8} \times 1.25$ = 13.88 days (approximately).

Multiplication by 1.25 is because of 25% extra work load and 0.75 accounts for the efficiency.

18. Average weight of the chairs = $\frac{10+8+12+6}{4} = 9$ kg

Now a 12 kg chair is replaced by an 8 kg chair

∴ Average = $\frac{10+8+8+6}{4} = 8$ kg

i.e. a decrease of 1 kg in average.

Alternative method:

There is a loss of 4 kg on replacement of 12 kg chair by 8 kg. This loss will be equally compensated between all the four chairs. So, there is a decrease of 1 kg in average.

19. When the same number is added to all the quantities, the average increases by the number added.

Therefore, increase = Rs.1,000

20. New average will also increase by 20% of the present average.

Therefore, new average = 15,000 + 3,000 = Rs. 18,000

21. Turnover from January to March = 12 × 3 = 36

Turnover from March to June = 14 × 4 = 56

Turnover from January to June = 36 + 56 - Turnover of March.

Also average turnover from January to June is 12.

∴ 36 + 56 - Turnover in March = 12 × 6

So turnover in March = Rs. 20 crore.

22. If John leaves the group, number of students in the group will be 3.

So average height = $\frac{168 \times 4 - 170}{3} = 167.37$ cm (approximately)

23. Let Tom's score be x.

$\therefore \left[\frac{72 \times 4 + x}{5}\right] = 70 \Rightarrow x = 62$

24. Cost of 20 kg apples = 20 × 40 = Rs. 800

Cost of 10 kg apples = 10 × 50 = Rs. 500

Total cost of 30 kg = 800 + 500 = Rs. 1,300

∴ Average cost = $\frac{1300}{30}$ = Rs. 43.33 per kilogram

25. Time taken to ride 20 km at 10 km/hr = $\frac{20}{10}$ = 2 hr

Time taken for another 20 km at 5 km/hr = $\frac{20}{5}$ = 4 hr

∴ Total time taken = 6 hr

Total distance covered = 40 km

Average speed = $\frac{40}{6}$ = 6.67 km/hr

26. Average speed = $\frac{\text{Total distance}}{\text{Total time}} = \frac{10 + 20}{2 + 1} = 10 \text{ km/hr.}$

27. Let total number of units = 100

Revenue when 50 units are sold at Rs. 20 per unit = Rs. 1,000

Revenue when 30 units are sold at Rs. 40 per unit = Rs. 1,200

Total price = Rs. 2,200

Average price = $\frac{2200}{50 + 30}$ = Rs. 27.50

28. If 2 friends leave the group, then 3 are left.

New average = $\frac{(6.5 \times 5) - (7 \times 2)}{3} = 6.16 \text{ kg}$

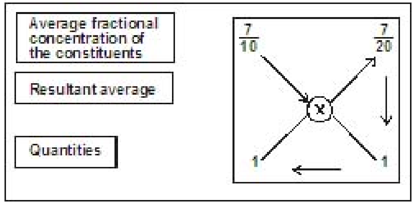
29. Average = $\frac{\text{Total score}}{\text{Number of students}} = \frac{30 \times 70 + 20 \times 60}{50} = 66\%$

30. **Method 1:**

Fraction of gold in first alloy = $\frac{7}{10}$.

Fraction of gold in second alloy = $\frac{7}{20}$.

Applying rule of alligation,



$\therefore x - \frac{7}{10} = \frac{7}{20} - x$

$\Rightarrow x = \frac{\frac{7}{20} + \frac{7}{10}}{2} = \frac{7}{2}\left[\frac{3}{20}\right] = \frac{21}{40}$

∴ Ratio of gold to copper = 21 : 19.

Method 2:

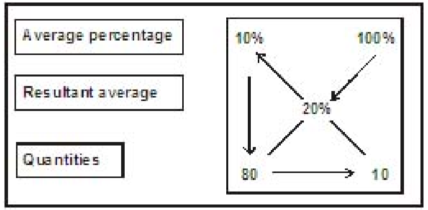
The first alloy is a 70% gold alloy and the second is a 35% gold alloy. If the two are mixed in equal quantities, the average concentration of gold in the resulting alloy is $\frac{70 + 35}{2} = 52.5\%$.

Therefore, the ratio of gold to copper is

52.5 :47.5 = 21 : 19.

31. To a mixture containing 10% water, pure water (100%) is mixed to get the resultant solution containing 20% water.

Applying the rule of alligation, average concentration of constituents



Ratio of the volumes = 8 : 1 or 40 : 5.

$\therefore$ To 40 L of solution, 5 L of water must be added.

Alternatively:

36 L of milk in the original mixture now becomes 80% of the mixture. Hence, the total volume of the new solution = 45 L. So the extra 5 L must be the water that was added.

32. **Method 1:**

Let CP of 1 L of milk = Rs. 100.

He mixed x litres of water.

SP of 1 L of mixture = 100.

$\therefore$ SP of (1 + x) L of mixture = 100(1 + x).

$P\% = \frac{SP - CP}{CP} \times 100 = \frac{100x}{100} \times 100 = 100x = 25$

$\therefore$ Volume of mixture = 1 + 0.25 = 1.25.

Percentage of water $= \frac{0.25}{1.25} \times 100 = 20\%$.

Method 2:

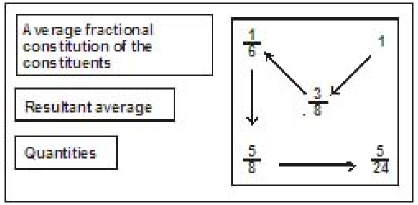
If the total cost of 100 L of milk is Rs. 100, the milkman is making a revenue of Rs. 125. Since he is selling at the same cost price, he must actually be selling 125 L. Hence, he is adding 25 L of water or 20% of the mixture is water.

33. Fraction of water in the adulterated milk $= \frac{1}{6}$th.

Fraction of water in pure water (added) = 1.

Fraction of water in resultant mixture $= \frac{3}{8}$th.

$\therefore$ Apply rule of alligation.



Ratio of the volumes of the solution to the water added = 3 : 1 = 66 : 22.

$\therefore$ 22 kg of water should be added.

Alternative method:

55 kg of milk is now $\frac{5}{8}$th of the new mixture.

Hence, new weight = $\frac{8}{5}\times 55$ = 88 kg.

Hence, the amount of water added = 88 - 66 = 22 kg.

34. The volume of the solution is $\frac{10}{9}$ times the original volume. Hence, if the original volume is 729 ml, the new volume = 810 ml. The increase is due to the addition of 81 ml of water.

(**Hint:** the factor $\frac{10}{9}$ is the ratio of $\frac{7}{9}$ and $\frac{7}{10}$, which are the average milk content in the initial and the final mixtures.)

35. Using alligation,

$$\frac{15-0}{0-(-10)}=\frac{CP_{II}}{CP_I}=\frac{3}{2}$$

$=\frac{CP_{II}}{CP_I}=\frac{\text{Cost price of second shirt}}{\text{Cost price of first shirt}}$ and $CP_I + CP_{II}$ = 900

$\therefore CP_I=\frac{2}{5}\times 900=360$

$CP_{II}=\frac{3}{5}\times 900=540$

Level - 2

36. 2 men = 5 boys and 3 boys = 2 women

$\therefore$ 1 boy gets $\frac{2}{5}$ of what man gets.

1 woman gets $\frac{1\times 3}{2}$ or 1.5 times of what a boy gets.

$\therefore$ A woman gets $\frac{3}{5}$ of what a man gets

$\therefore$ 6m + 12w + 17b = 6m + 12 × $\frac{3}{5}$m + 17$\left(\frac{2}{5}\right)$m = 20m

or m = Rs. 2.5. Similarly, b = Rs. 1, w = Rs. 1.5

m, b, w are shares of a man, a boy and a woman, respectively.

37. Assume that the persons in 4 battalions are a, b, c and d.

Then $\frac{1}{2}a=\frac{2}{3}b=\frac{3}{4}c=\frac{4}{5}d$

i.e. b = $\frac{3}{4}a$, c = $\frac{2}{3}a$ and d = $\frac{5}{8}a$

$\therefore a+\frac{3}{4}a+\frac{2}{3}a+\frac{5}{8}a=7300$

$\Rightarrow$ a = 2400, b = 1800, c = 1600 and d = 1500

38. Let the worth of the properties of A, B and C be 2x, 3x and 5x respectively. Total = 10x

B sells to A = $\frac{1}{3}$ (3x) = x and $\frac{1}{3}$ (3x) = x to C.

$\therefore$ A has 3x, B has x, C has 6x

C sells $\frac{1}{4}(6x)$ to A = $\frac{3}{2}x$

$\therefore$ A has now $3x+\frac{3x}{2}=\frac{9}{2}x$ = Rs. 18,000

Hence, x = Rs. 4,000

$\therefore$ Total worth of properties = 10x = Rs. 40,000.

39. Let A, B and C invested Rs. 5, Rs. 6 and

Rs. 8, respectively for a, b, and c months.

Then 5a : 6b : 8c is the ratio of profits which is given to be 5 : 3 : 1.

$\therefore$ 5a : 6b : 8c = 5 : 3 : 1.

$\therefore$ a : b : c = 1 : $\frac{1}{2}$: $\frac{1}{8}$ = 8 : 4 : 1.

40. Total amount deducted is 15 + 10 + 30 = 55.

Amount left = 535 - 55 = 480.

Now Rs. 480 is divided in the ratio 4 : 5 : 7.

So after deduction

share of A = $\frac{4}{16}\times 480 = 120$, share of B = $\frac{5}{16}\times 480 = 150$, share of C = $\frac{7}{16}\times 480 = 210$.

$\therefore$ Initial share of A = 120 + 15 = Rs. 135.

Initial share of B = 150 + 10 = Rs. 160.

Initial share of C = 210 + 30 = Rs. 240.

41. Ratio of contributions expected from the three men = (110 × 6) : (50 × 9) : (440 × 3) = 22 : 15 : 44.

$\therefore$ Second man must pay $\frac{15}{81}$th or $\frac{5}{27}$ th of the total.

42. B should get = $\left[\frac{5100\times 3}{(3200\times 4)+(5100\times 3)+(2700\times 5)}\right]1248 = \text{Rs. } 459$

43. 2 leaps of the dog = 5 leaps of the hare

or 1 leap of the dog = 2.5 leaps of the hare

$\therefore$ 6 leaps of the dog = 15 leaps of the hare

$\therefore$ Ratio of leaps of dog to hare = 15 : 4.

44. Let the original prices by 13x and 17x. So ratio for new prices = $\frac{13x+100\%\ \text{of}\ 13x}{17x+2500} = \frac{3}{5}$

$\frac{26x}{17x+2500} = \frac{3}{5}$

130x = 51x + 7500

79x = 7500

x = 94.93

So the original prices are 13 × 94.93= Rs. 1234.08 and 17 × 94.93 = Rs. 1613.81.

45. In a race of 100 m, A travels 100 m while B travels 95 m. This means in a race of 500 m, when A travels 500 m, B travels 95 × $\frac{500}{100}$ = 475 m.

Similarly, when B travels 200 m, C travels 190 m. So if B travels 500 m C travels 190 × $\frac{500}{200}$ = 475 m.

Hence, when B travels 475 m, C travels 475 × $\frac{475}{500}$ = 451.25.

Therefore, when A travels 500 m, C travels 451.25 m.

This means, A can give C a lead of 48.75 m.

46. Average = $\frac{\text{Total weight}}{\text{Number of students}}$

Average is decreased by 2 kg.

∴ New average = 60 kg

So $60 = \frac{62 \times 5 - 70 + x}{5} \Rightarrow x = 60$

Weight of new student x = 60 kg

47. Total weight = 25 × 10 kg

2 cases of weight 30 kg each are replaced by 2 cases of weight 32 kg each.

∴ Total weight = (25 × 10) - (2 × 30) + (2 × 32)

= 254 kg

Number of cases = 10

∴ Average = $\frac{254}{10}$ = 25.4 kg

∴ The average will change by 0.4 kg.

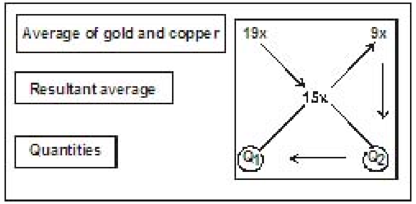
48. If water weights x kg/unit volume

weight of gold = 19x/unit volume

weight of copper = 9x/unit volume

∴ Weight of the mixture desired = 15x/unit volume

Apply rule of alligation:



$\frac{19x - 15x}{15x - 9x} = \frac{Q_2}{Q_1} \Rightarrow \frac{4}{6} = \frac{2}{3}$

49. Let the cost price of first variety be Rs. 2x per kilogram.

So the cost price of second variety becomes Rs. x per kilogram.

Selling price of mixture is Rs. 36 per kilogram after making profit of 20%.

So the cost price of mixture = $\frac{36}{1.2} = 30$ per kilogram

Applying the principle of alligation,

$\frac{2x - 30}{30 - x} = \frac{7}{3}$

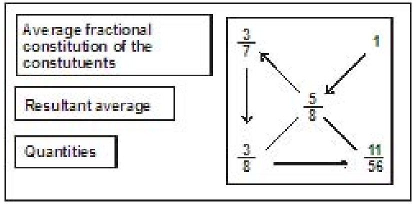
6x - 90 = 210 - 7x

13x = 300

x = 23.07

So the cost price of first variety = Rs. 46.14 and cost price of the second variety = Rs. 23.07.

50. Apply rule of alligation for fractional water content.



The ratio of the volumes of the alcohol mixture to water = 21 : 11. If only 8 L of water was added, then the original amount of the mixture = $\frac{21\times 8}{11}$.

Alcohol content in that = $\frac{4}{7}\times\frac{21\times 8}{11}=\frac{96}{11}$.

51. If the cost of second liquid is Rs. x per litre, then cost of first liquid is Rs. (x + 2) per litre.

Selling price of mixture is Rs. 120, after making a profit of 20%.

So the cost price of mixture = $\frac{120}{1.2}=100$ per litre.

Use Alligation $\frac{(x+2)-100}{100-x}=\frac{5}{3}$

$\frac{x-98}{100-x}=\frac{5}{3}$

8x = 794

x = Rs. 99.25 per litre

∴ Cost of first solution = 99.25 + 2 = Rs. 101.25 per litre and cost of second solution = Rs. 99.25 per litre.

52. $\frac{\text{Total fractional amount of milk}}{\text{Total fractional amount of water}}$

$=\frac{\frac{2}{3}+\frac{3}{4}+\frac{3}{5}}{\frac{1}{3}+\frac{1}{4}+\frac{2}{5}}=\frac{40+45+36}{20+15+24}=\frac{121}{59}$

53. Ratio of quantities = 2 : 5.

Ratio of prices = 3 : 1.

∴ Ratio of the values of sugar to orange peels = 6 : 5.

∴ $\frac{6}{11}[5.2-0.8]=\$2.4$ worth is sugar in marmalade

54. Ratio of copper in first alloy = $\frac{3}{9}$.

Ratio of copper in second alloy = $\frac{5}{9}$.

Ratio of copper in new alloy = $\frac{1}{2}$.

Applying the principle of alligation,

$\frac{\frac{5}{9}-\frac{1}{2}}{\frac{1}{2}-\frac{3}{9}}=\frac{Q_I}{Q_{II}}=\frac{\text{Quantity of first alloy}}{\text{Quantity of second alloy}}\Rightarrow\frac{Q_I}{Q_{II}}=\frac{1}{3}$

∴ Quantity of first alloy = $\frac{1}{4}\times 64=16$ kg and quantity of second alloy = $\frac{3}{4}\times 64=48$ kg.

55. Milk proportion in the total solution of first variety

$(V_I)=\frac{5}{13}$.

Milk proportion in the total solution of second variety $(V_{II})=\frac{9}{13}$.

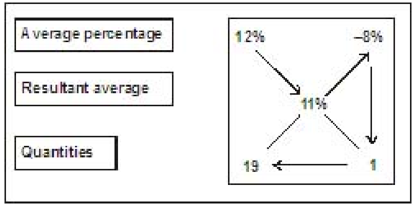
Milk proportion to total in the required mixture = $\frac{7}{13}$.

Applying the principle of alligation,

$$\frac{\frac{9}{13}-\frac{7}{13}}{\frac{7}{13}-\frac{5}{13}}=\frac{V_I}{V_{II}}$$

$$\frac{V_I}{V_{II}}=\frac{2}{2}=\frac{1}{1}$$

56. Applying the rule of alligation,



The ratio of the items that he is selling at a profit of

12% and at a loss of 8% is 19 : 1.

Hence, he sold $\frac{19}{20}\times 60$ = 57 pens at a 12% profit and $\frac{1}{20}\times 60$ = 3 pens at a 8% loss.

57. Let the volume of the mixture be x. Then

amount of A = $\frac{7}{12}x$, amount of B = $\frac{5}{12}x$.

When 9 L of mixture is taken out,

amount of A withdrawn = $\frac{7}{12}\times 9=\frac{21}{4}$ L;

amount of A left in the mixture = $\frac{7}{12}x-\frac{21}{4}$.

∴ Final amount of B in the mixture = $\frac{5}{12}x+\frac{21}{4}$.

(Since all the milk that was removed was substituted by water.)

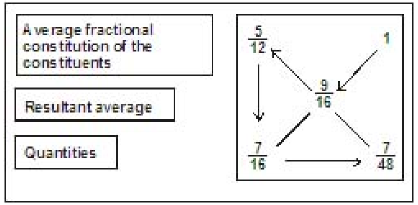
Now ratio of A and B = $\frac{\left(\frac{7}{12}x-\frac{21}{4}\right)}{\left(\frac{5}{12}x+\frac{21}{4}\right)}=\frac{7}{9}$, x = 36.

∴ Amount of A = $\frac{7}{12}\times 36=21$ L

Alternative method:

Use rule of alligation:

After 9 L is taken out, the ratio of A and B is 7 : 5. To this is added pure B to obtain a solution containing A and B in the ratio 7 : 9. Considering the rule of alligation for fractional B content,



∴ Ratio of the original solution left to that of the water added = 3 : 1 = 27 : 9.

Hence, the initial volume = 36 L.

∴ Amount of A initially = $\frac{7}{12}\times 36$ = 21 L.

58. Let t be the time for which he travelled on foot.

$\therefore$ $8t + 16(7 - t) = 80$ or $t = 4$ hr.

The distance travelled on foot = 32 km.

Alternatively:

The problem could also be done using the principle of alligation. The average speeds on foot and on a bicycle are given and the resulting average speed $\left(\text{i.e. } \frac{80}{7}\right)$ is also given. We can find the ratio of times and hence, the time taken for each part of the journey.

59. 9 L of orange juice contains = $\frac{312}{3} = 104$ calories.

9 L of mango juice contains = 54 calories and 9 L of mixture (orange and mango) contains

$= \frac{118}{2} = 59$ calories.

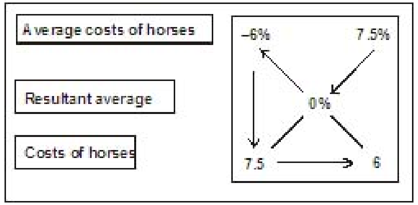
Use alligation.

$$\frac{104-59}{59-54} = \frac{V_m}{V_0} = \frac{\text{Volume of mango juice}}{\text{Volume of orange juice}}$$

$$\frac{45}{5} = \frac{V_m}{V_0} \text{ or } = \frac{V_m}{V_0} = \frac{9}{1}$$

So fraction of orange juice in 18 L mixture is $\frac{1}{10}$.

60. Apply rule of alligation:



$\therefore$ The ratio of the costs of the horses is 7.5 : 6 or 5 : 4.

$\therefore$ 5 : 4 is the ratio of cost of each horse.

$\therefore$ $\frac{5}{9} \times 1350 = 750$ is the cost of one which was sold at a loss of 6% and $\frac{4}{9} \times 1350 = 600$ is the cost of the other.

61. Fresh grapes contain 90% water. So 20 kg fresh grapes contain 18 kg water and 2 kg pulp. Now if 2 kg pulp contributes 80% of dried grapes, then the total amount of dried grapes = 2.5 kg.

62. Final ratio of water to total = Initial ratio of water to total x $\left[\frac{V-X}{V}\right]^n$

As the process is repeated two more times.

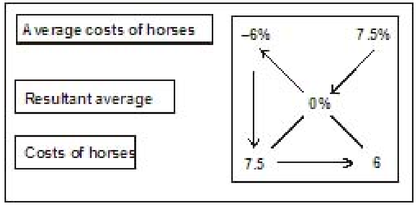
$\therefore$ The number of times we do the same operation will be 3.

$$\frac{27}{1728} = \frac{1}{1}\left[\frac{V-3}{V}\right]^3$$

$$\frac{3}{12} = \frac{V-3}{V} \Rightarrow V = 4 \text{ L}$$

63. Final ratio of rum to total = Initial ratio of rum to total $\times \left[\frac{V-X}{V}\right]^n = \frac{1}{1}\left[\frac{240-80}{240}\right]^4$

$$= \left[\frac{160}{240}\right]^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$



∴ The ratio of the costs of the horses is 7.5 : 6 or 5 : 4.

∴ 5 : 4 is the ratio of cost of each horse.

∴ $\frac{5}{9} \times 1350 = 750$ is the cost of one which was sold at a loss of 6% and $\frac{4}{9} \times 1350 = 600$ is the cost of the other.

61. Fresh grapes contain 90% water. So 20 kg fresh grapes contain 18 kg water and 2 kg pulp. Now if 2 kg pulp contributes 80% of dried grapes, then the total amount of dried grapes = 2.5 kg.

62. Final ratio of water to total = Initial ratio of water to total x $\left[\frac{V-X}{V}\right]^n$

As the process is repeated two more times.

∴ The number of times we do the same operation will be 3.

$$\frac{27}{1728} = \frac{1}{1}\left[\frac{V-3}{V}\right]^3$$

$$\frac{3}{12} = \frac{V-3}{V} \Rightarrow V = 4 \text{ L}$$

63. Final ratio of rum to total = Initial ratio of rum to total $\times \left[\frac{V-X}{V}\right]^n = \frac{1}{1}\left[\frac{240-80}{240}\right]^4$

$$= \left[\frac{160}{240}\right]^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

∴ Quantity of rum left = $\frac{16}{81} \times 240 = 47.40$ L

Level - 3

64. At the first stage, the concentration of milk is 60% $\left(= \frac{3}{5} \times 100\right)$. Since the volume increases by 50%, the volume becomes $\frac{3}{2}$ times of the original volume.

Hence, the concentration becomes $\left(\frac{2}{3} \times 60\%\right)$

= 40%. Now 30 L is withdrawn and replaced with water. Final concentration = 30% $\left(= \frac{3}{10} \times 100\right)$.

Hence, $\frac{1}{4}$th of milk has been removed from the mixture. $\left(\therefore \frac{40-30}{40} = \frac{10}{40} = \frac{1}{4}\right)$

Or $\frac{1}{4}$th of the mixture = 30 L

So the volume at the end of first state = 120 L.

This is times of the initial volume.

Hence, the initial volume = $\frac{2}{3} \times 120$ = 80 L.

Practice Exercise 1 - Level 1

1. c $\frac{A}{B}=\frac{3}{7}$. Let A = 3x, B = 7x

A + B = 45; 3x + 7x = 45, x = $\frac{45}{10}=4.5$

B = 7x = 31.5

2. d Let the fraction be $\frac{x}{y}$

$\frac{\frac{x}{y}}{\frac{3}{7}}=\frac{\frac{1}{27}}{\frac{1}{35}}$ or, $\frac{7x}{3y}=\frac{35}{27}$

$\frac{x}{y}=\frac{35}{27}\times\frac{3}{7}=\frac{5}{9}$

3. a If a, b and c are in continued proportion, the mean proportional is b.

Therefore, b^2 = ac, $b^2 = 8 \times 72$, b = $\sqrt{576}=24$

4. b If a, b, c and d are in proportion, $\frac{a}{b}=\frac{c}{d}$ or $\frac{3}{15}=\frac{27}{d}$,

or d = 135 is the fourth proportional.

5. e Third proportional, c = $\frac{b^2}{a}$ = $\frac{30\times30}{20}=45$

6. b Let the number added be x. Then,

$\frac{4+x}{9+x}=\frac{2}{3}$, 12 + 3x = 18 + 2x

x = 6

7. d A's share = $P\times\frac{x}{x+y+z}$ = 3610 × $\frac{2000}{4750}$

= Rs. 1,520

Where P = Profit and x, y and z are respective shares of A, B and C.

8. c $\frac{x}{y}=\frac{2}{7}$ Let x's share = 2a, y's share = 7a

$\frac{\text{x's share}}{\text{x's share} - \text{y's share}}=\frac{2a}{5a}=\frac{2}{5}$

9. d $\frac{68-x}{49-x}=\frac{3}{4}$, ⇒ 272 - 4x

= 147 - 3x ⇒ x = 125

10. a $\frac{A}{2}=\frac{B}{3}=\frac{C}{6}$ or 3A = 2B = C

$\text{B's part}=\frac{B}{\frac{2}{3}B+B+2B}\times3960$

= $\frac{3}{11}\times3960=3\times360=1080$

11. d C's share = 2 B's share

A's share = $\frac{2}{3}$ B' share

⇒ required ratio is $\frac{\frac{2}{3}\text{B's share}}{(2\text{B's share} - \text{B's share})}=\frac{2}{3}$

12. c Let the number be 5x and 3x, then $\frac{5x-9}{3x-9}=\frac{23}{12}$

60x - 108 = 69x - 207; 9x = 99, x = 11

The first number is $11 \times 5 = 55$

13. d $4x = 3y = 2z$

$\Rightarrow \frac{x}{y} = \frac{3}{4}$ & $\frac{y}{z} = \frac{2}{3} = \frac{4}{6}$

$\Rightarrow x : y : z = 3 : 4 : 6$

14. d In a ratio A:B, A is called the antecedent and B is called the consequent.

Let antecedent = 4x & consequent be 9x.

$4x = 36 \Rightarrow 9x = 81$.

15. b Let $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = k$

$a = 3k$, $b = 4k$, $c = 7k$

$\frac{a+b+c}{c} = \frac{3k+4k+7k}{7k} = \frac{14k}{7k} = 2$

16. b **Option a:**

Numbers = 9x, 3x

$12x = 84$

x = 7; possible.

Option b:

$8x = 84 \Rightarrow$ No whole number x is possible.

Option c and d:

$21x = 84$

x = 4; Possible.

17. b 0.35 of x = 0.07 of y

$\therefore \frac{x}{y} = \frac{0.07}{0.35} = \frac{1}{5}$

18. b B's money = $\frac{5}{4} \times 800 = 1000$

C's money = $\frac{3}{2} \times 1000 = 1500$

Therefore, total amount of money = Rs. 3,300

19. c Let the two parts be x, (68 - x)

$\frac{x}{7} = \frac{1}{10}(68 - x) = \frac{68}{10} - \frac{x}{10}$

$\frac{x}{7} + \frac{x}{10} = \frac{68}{10}$ or, $\frac{17x}{70} = \frac{68}{10}$, x = 28

20. b x's share $\Rightarrow 3x + 30$

y's share $\Rightarrow 4x + 20$

z's share $\Rightarrow 5x + 50$

Sum is 9700

$12x + 100 = 9700$, $12x = 9600$, $x = 800$

y's share = $4x + 20 = 3200 + 20 = 3220$

21. d Let B get's Rs. x

A get's Rs. x + 70

C get's Rs. x - 80

x + x + 70 + x - 80 = 530

3x - 10 = 530, 3x = 540

x = 180

$\frac{\text{A's share}}{\text{C's share}} = \frac{180+70}{180-80} = \frac{250}{100} = \frac{5}{2}$

22. b A = $\frac{1}{2}$B, B = 2C or $\frac{A}{B} = \frac{1}{2}, \frac{B}{C} = \frac{2}{1}$

A : B : C = 1 : 2 : 1

Shares of A, B, C are x, 2x, x

$\frac{B}{A+B} = \frac{2x}{x+2x} = \frac{2x}{3x} = \frac{2}{3}$

23. c Initially, Number of boys = 250

Number of girls = 250

New Batch: $\frac{1}{5} \times 250$ = 50 girls left & 25 boys joined the class

$\frac{\text{Boys}}{\text{Girls}} = \frac{275}{200} = \frac{11}{8}$

24. a Let Father's age be 7x, son's age = 2x

After 15 years,

$\frac{7x+15}{2x+15} = \frac{2}{1}$, 7x + 15 = 4x + 30, x = 5

Present age of father = 35

Present age of son = 10

Father's age when son was born = 35 - 10 = 25

25. d Let present age of son = x and

present age of father = y

(y - 4) = 6(x - 4) ... (i)

$(y+12) = 2(x+12)$ ... (ii)

from (i) and (ii),

y - 6x + 20 = 0

y - 2x - 12 = 0

4x = 32, x = 8 years, y = 28 years

Ratio of their present ages = $\frac{28}{8} = 7:2$

26. b Let copper = 13x

Zinc = 7x

In 500 kg, 13x + 7x = 500

x = $\frac{500}{20}$ = 25 kg

Copper = 13 × 25 = 325 kg

27. d Simplest form = $\frac{2}{7}$

Numerator = 2x

Denominator = 7x

7x - 2x = 40, x = 8

Number is $\frac{16}{56}$.

28. a $\frac{M}{W}=\frac{3}{1}$ in 100 L mixture

Milk = 75 L, Water = 25 L

After adding 200 L of water, water = 225 L and milk = 75 L

Ratio = $\frac{75}{225}$ = 1 : 3

29. c In 80 liters of mixture, milk = $\frac{5}{8}\times 80 = 50$ liters

Water = (80 - 50) = 30 liters.

When we take a 16 liter sample of this mixture.

Milk taken out $=\frac{5}{8}\times 16 = 10$ litres

Water taken out = (16 - 10) = 6 litres.

$\Rightarrow$ Milk available = 50 L - 10 L = 40 litres

Water available = 30 L - 6 L= 24 litres.

Now, 16 litres of milk are added,

$\Rightarrow$ Milk = 40*l* + 16 L = 56 L

Water = 24 litres

$\Rightarrow$ Milk : Water ratio is 56 : 24 = 7:3.

Practice Exercise 2 - Level 1

1. b $\frac{B}{G}=\frac{7}{5}$

Let number of boys = 7x

Let number of girls = 5x

7x + 5x = 72, x = 6

Number of boys = 42

Number of girls = 30

12 more girls should be admitted to make the ratio equal.

2. d Let A's income = Rs. 3x

B's income = Rs. 2x

A's expenses = Rs. 5y

B's expenses = Rs. 3y

A's savings = 3x - 5y = 3000 ... (i)

B's savings = 2x - 3y = 3000 ... (ii)

Solving (i) and (ii), x = 6000

B's income = 2x = Rs. 12,000

3. a Let the number to be added be, x.

Then, $\frac{7+x}{19+x}=\frac{1}{2}$, x = 5

4. a Take 7 L of the solution of each mixture,

(LCM of 5 + 2, 6 + 1, 4 + 3)

∴ net amount of water in the resulting mixture,

$\frac{5}{7}\times7+\frac{6}{7}\times7+\frac{4}{7}\times7$ = 15 L

The net amount of alcohol is,

$\frac{2}{7}\times7+\frac{1}{7}\times7+\frac{3}{7}\times7$ = 6 L

Ratio is, 15 : 6 = 5 : 2

5. c If the ratio is a : b, then a + b should divide 12 completely. Only 3 : 2 does not divide, the rest are all possible ratios.

6. d If the ship weighs 5 kg, weight above water = 3 kg.

Weight submerged = 2 kg.

Therefore, desired ratio = 2 : 3.

7. c 5% of x = 30. Therefore, x = 600. Therefore, number of entrants who lost = 570.

8. c If the ratio is a : b, then a + b should divide 8 completely. Only 36 does not fulfil this condition.

9. e Considering the ratio,

B's + C's shares = 20x and A's share = 5x.

Hence, the second sentence gives no additional information.

Since the total amount is not given, we cannot say anything about A's share.

10. b Plastic lenses are four times costlier than glass lenses.

∴ If cost of glass lenses = x, cost of plastic

lenses = 4x.

Cost of age examination and frames = y.

∴ y + x = 40 and y + 4x = 52

⇒ 3x = 12 ⇒ x = $4

Alternative method:

The price differential is due to plastic lenses which cost four times as much as the glass lenses.

∴ Price differential = Cost of plastic lenses over and above glass lenses = 3 (Cost of glass lenses).

∴ Cost of glass lenses = $\frac{1}{3}(52-40) = \$4$. (Compare with the above algebraic method.)

11. a Ratio of savings of A and B is 5 : 3. So it is obvious that A's savings is $66\frac{2}{3}\%$ more than that of B.

12. b Here Sujith earns more but spends less so definitely his savings will be more than that of Pratima.

13. d 1 : 5 and 3 : 5 solutions

Since equal quantities of both are mixed, we can take LCM of (1 + 5 = 6) and (3 + 5 = 8), i.e. 24.

Assuming that 24 L of both solutions are mixed together in the resultant solution.

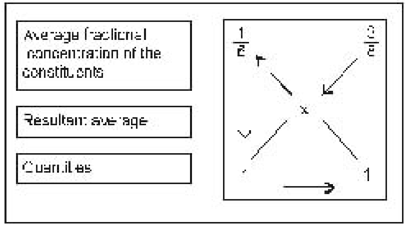
Milk = 4 + 9 = 13; Water = 20 + 15 = 35

Hence, the ratio of water and milk = 35 : 13.

Alternative method:

Proportion of milk in 1 : 5 solution = $\frac{1}{6}$.

Proportion of milk in 3 : 5 solution = $\frac{3}{8}$.



$$\frac{x-\frac{1}{6}}{\frac{3}{8}-x} = \frac{1}{1}$$

$$\Rightarrow 2x = \frac{1}{6} + \frac{3}{8} = \frac{4+9}{24} = \frac{13}{24}$$

$$\Rightarrow x = \frac{13}{48}$$

∴ Water : Milk = (48 - 13) : 13 = 35 : 13

14. c Mileage when petrol is adulterated

= $\frac{3}{4}$ of (16) = 12 km/L

Hence, the cost increases to a factor $\left(\frac{4}{3}\right)$ of the original cost of maintenance. So it increases by $\frac{1}{3} \times 100 = 33.33\%$.

Alternative method:

We can safely assume that the car runs for the same

number of kilometres.

Now mileage is reduced to $\frac{3}{4}$ of its original value, which is, say, x.

∴ Distance = Mileage × Number of litres

$\Rightarrow x \times y_1 = \frac{3}{4} x \times y_2 \Rightarrow \frac{y_2}{y_1} = \frac{4}{3}$ = 1.33

Therefore, we see that the consumption of fuel

increases by $33.\bar{3}\%$.

15. a If the number is x, $\frac{5}{8}x - \frac{5}{18}x = 125$ or x = 360.

16. e If A's age is 5x, B's age = 7x.

Case I: 7x - (5x + 6) = 2

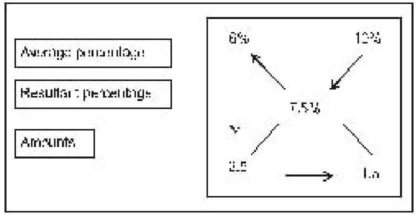
So x = 4.

Case II: (5x + 6) - 7x = 2

So x = 2

Hence, sum of their present ages = 12 × 4 = 48 years or 12 × 2 = 24 years.

17. a This can be solved using alligation.



So the amounts invested are in the ratio 2.5 : 1.5 = 5 : 3.

Since the amounts invested at the two rates are in the ratio 5 : 3, and Rs. 680 is invested at 6%,

amount invested at 10% = $\frac{680}{5} \times 3$ = Rs. 408.

__Questions 18 and 19:__ Half of the volumes of the containers A and B are emptied into the third container. Hence, the volumes emptied are 108 cm^3 and 32 cm^3 respectively.

18. c Hence, the ratio of water to milk is 108 : 32 or 216 : 64.

19. e $\frac{140}{1000} \times 100 = 14\%$

20. a

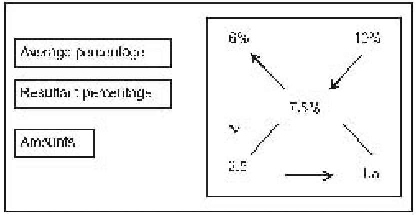
Father	Elder son	Younger son now
20x	5x	4x
After 10x years,		
30x	15x	14x

So 30x = 2 × 14x + 3

⇒ x = 1.5 \ 20x = 30 years.

21. a The final amount of milk after two operations = $\frac{4}{5} \times \frac{4}{5} \times \left(\frac{3}{5} \times 50\right) = 19.2\text{ L}$

22. c Let X ml be added to the solution.



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Father	Elder son	Younger son now
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So 30x = 2 × 14x + 3

⇒ x = 1.5 \ 20x = 30 years.

21. a The final amount of milk after two operations = $\frac{4}{5} \times \frac{4}{5} \times \left(\frac{3}{5} \times 50\right) = 19.2\text{ L}$

22. c Let X ml be added to the solution.

$$\Rightarrow \frac{[(15\% \text{ of } 400) + X]}{[400 + X]} = \frac{32}{100}$$

⇒ X = 100 ml

Alternative Method:

15% of 400 ml = 60 ml

Rest is water = 400 - 60 = 340 ml

Since, water is constant ⇒ 340 ml will be 68% (in the new mixture)

⇒ 340 is 68%

⇒ 100 is $\frac{340}{68} \times 100 = 500\text{ ml}$; Extra alcohol required 500 - 400 = 100 ml

Practice Exercise 3 - Level 2

1. d The question uses the same concept as in Q 17.

Number of mugs = 12

The sum of broken and unbroken mugs must be equal to 12.

Sum of ratio = 3, 12, 5, 6, (5 does not divide 12)

$\therefore$ 3 : 2 cannot be the ratio.

2. a A's effective investment = 16000 × 6 = Rs. 96,000

B's effective investment = 12000 × 8 = Rs. 96,000

C's effective investment = 1000 × 12 = Rs. 12,000

Profit sharing ratio = 96 : 96 : 12 or 8 : 8 : 1

3. e Ratio of their profits = Ratio of their investments

= 4000 × 12 : 8000 × 9 : 12000 × 2

48 : 72 : 24

2 : 3 : 1

Bs' share = $\frac{3}{6}$ × 5200 = 2600

4. d L : R : O = 5 : 7 : 3

Let Labour cost be 5x, Raw Material Cost

= 7x

Overheads = 3x, Total cost = 15x

Profits = 20% of 15x = 3x

$\frac{\text{Material cost}}{\text{Profit}} = \frac{7x}{3x} = \frac{7}{3}$

5. c Let A's, B's and C's investments be 8x, 7x, 5x

A's effective investment = (8x) × 5 + (4x) × 7

= 40x + 28x = 68 x

B's effective investment = 7x × 12 = 84x

C's effective investment = 5x × 12 = 60x

B's share = $\frac{84}{212} \times 26500$ = 10,500

6. b The ratio between the distance and the speed is the same. Since the ratios between the two are the same, the time has to be constant and hence the ratio needs to be 1 : 1 : 1 or mathematically,

$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

Hence, Ratio of time = $\frac{\text{Ratio of distance}}{\text{Ratio of speed}} = 1:1:1.$

7. d If there were 2, 5, 1 coins of the denominations

Rs. 1.50, one-rupee and 25-paisa respectively, then value of the coins would be Rs. 8.25. Hence, if the total value has to be Rs. 330, then the number of coins would have to be multiplied by a factor of 40. There would be in all 80 of Rs. 1.50 coins, 200 one-rupee coins and 40 25-paisa coins.

8. c Let the value of the turban be Rs. x.

Then $\frac{100+x}{65+x}=\frac{12}{9} \Rightarrow$ or x = Rs. 40.

9.c Suppose numbers of students who pass and fail are 3x and x respectively.

∴ Total number of students appearing = 3x + x = 4x.

In second case,

total number of students who appear = 4x + 8;

total number of students who pass = 3x - 6.

∴ Total number of students who fail

= (4x + 8) - (3x - 6) = x + 14.

Now we have $\frac{3x-6}{x+14}=1 \Rightarrow$ x = 10.

∴ Total number of students is 40.

10.c Ratio of white to yellow balls = 6 : 5.

Difference in the number of white and yellow balls = 6x - 5x = x = 45.

Therefore, number of white balls now available = 45 × 6.

Number of white balls ordered = (45 × 6) - 45 = 225.

11. c Ratio of white mice to total mice $=\frac{1}{\frac{1}{2}}:\frac{1}{\frac{1}{8}}=1:4=\frac{1}{4}:1$.

Ratio of gray mice to total mice $=\frac{1}{\frac{1}{3}}:\frac{1}{\frac{1}{9}}=3:9=1:3=\frac{1}{3}:1$.

∴ Ratio of white mice to gray mice = $\frac{1}{4}:\frac{1}{3}=3:4$.

12. b Let the required quantity be x kg.

∴ 20 × 2 + 24x + 30 × 3 = 25 (3 + x + 2)

⇒ x = 5

13. b Let the amount of second and third quality of sugar in the mixture be \`x' and \`y' kg respectively.

$\therefore 15\times4+18x+22y=22\times\frac{100}{110}[4+x+y] \Rightarrow y-x=10$

14. a Mean rice of the mixture is $8\times\frac{100}{50}=\text{Rs. }16/\text{kg}$

Let the quantity of second and third variety be \`a' and \`b' kg.

∴ 15 × 6 + 18a + 20b = 16[6 + a + b] $\Rightarrow a+2b=3$

a = 1 and b = 1 is the only integral solution.

15. e Let the quantity of second and third variety be x and y kg respectively.

$\therefore 20\times1+30x+24y=30\times\frac{100}{120}[1+x+y] \Rightarrow 5x-y=5$ ...(i)

All the four options (a), (b), (c) and (d) satisfy equation (i).

16. e Let's take

12 litre of 5 : 1 sample

24 litre of 2 : 1 sample and

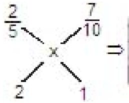
36 litre of 3 : 1 sample

Amount of milk in the final mixture $=\frac{5}{6}\times12+\frac{2}{3}\times24+\frac{3}{4}\times36=53\text{ L}$

Amount of water in the final mixture = 72 - 53 = 19 L

$\Rightarrow$ milk : water = 53 : 19.

17. e Using alligation we find the amount of copper (average amount of copper in the final mixture is x)



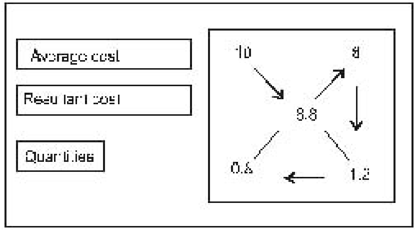
$$\Rightarrow \left(\frac{\frac{7}{10}-x}{x-\frac{2}{5}}\right)=\frac{2}{1} \Rightarrow x=\frac{1}{2}$$

Since, copper is $\frac{1}{2}$ of the new alloy the final ratio of copper to nickel will be 1 : 1.

18. e Selling at Rs. 11 per kilogram, profit percentage = 25%.

Therefore, CP = $\frac{11}{1.25}$ = Rs. 8.8

Applying the rule of alligation,



The required ratio is 2 : 3.

19. c Let the maximum marks in the quizzes be 1 and that in the examination is 3. Hence, the weightage of the examination = $\frac{3\times1}{6\times1+3\times1}=\frac{1}{3}$.

20. b If he spends `t' minutes on the second half, then he spends $\frac{2}{3}$t on the first half.

Therefore, $\frac{2}{3}t + t = 90$ or t = 54.

21. a Let us assume that out of 100, he obtained 72 marks. If he had attempted four more questions, he would have made one more mistake. Hence, three of them are correct. The three correct questions secure him 12 marks more. This means, each question carries $\frac{12}{3}=4$ marks and hence, number of questions is 25.

Alternative method:

We can clearly see that three correct answers increase the score by 12%. [Three correct questions as four new questions and one mistake]

100% score corresponds to $\frac{3}{12\%}\times100\%=25$ questions.

22. a 20 boys = 10 men. 15 girls = 7.5 men.

Therefore, total number of men = 17.5

Now 50 × 10 × 8 = (17.5 × 15)x, where x is the number of days.

Therefore, x = 15.24 (approximately).

Alternative method:

In these type of questions it is always easier to work

with man-hours (or man-days, etc). 50 men working 10 hr a day for 8 days is 50 × 10 × 8 = 4,000 man-hours.

Now 35 (boys and girls) working 15 hr a day for (say) x days is $\left(\frac{1}{2}\times35\times15\times x\right)$ man-hours

$\Rightarrow \frac{1}{2} \times 35 \times 15 \times x = 4{,}000 \Rightarrow x = \frac{8000}{35 \times 15} = 15.24$ days.

23. e When 5 L brandy is transferred, volume of brandy left in vessel I = 15 L.

Volume of brandy in vessel II = 5 L.

Volume of water in vessel II = 20 L.

Now, 4 L of mixture containing brandy and water in the ratio 1 : 4 is poured into pure brandy of volume 15 L.

4 L of mixture contains 3.2 L water and 0.8 L brandy.

∴ Volume of brandy left in vessel II = 5 - 0.8 = 4.2 L.

Ratio of water in vessel I to brandy in vessel II = 3.2 : 4.2 = 16 : 21.

24. a In 100 L pure milk, let him add `a' litres water.

Let CP of 100 L pure milk = Rs. 100.

CP of (100 + x) L adulterated milk = 100.

CP of 100 L adulterated milk = $\left(\frac{10000}{100+x}\right)$.

SP of 100 L adulterated milk = 100.

Profit percentage = $\frac{100\left(1-\frac{100}{100+x}\right)}{\left(\frac{10000}{100+x}\right)} \times 100$ or x = 30.

Therefore, 30 L is added or $\frac{30}{100} \times 100 = 30\%$ of water is added to pure milk.

25. c Six people contribute a total of Rs. 180.

Let the seventh person contributes Rs. x.

Eighth person contributes Rs. 55.

Total contributions of these eight persons = 235 + x.

Now $x = \frac{235+x}{8} + 10 \Rightarrow \frac{7}{8}x = \frac{1}{8}(235) + 10 \Rightarrow x = 45$

∴ Total collection = 235 + 45 = Rs. 280.

26. e No price values (either of profit amount or of cost price) is given. Obviously, SP cannot be determined.

27.c $\frac{4}{7}v - x = \frac{1}{4}v \Rightarrow 9v = 28x$

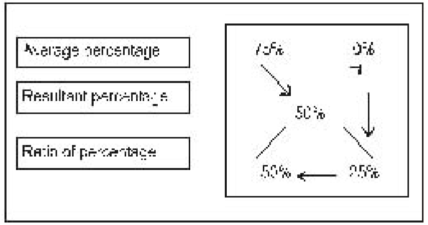
Since by adding 35 L the level rises from a quarter to a half, the volume of the vessel = 35 × 4 = 140 L.

Therefore, $28x = 9 \times 140 \Rightarrow x = 45\,L$.

Practice Exercise 4 - Level 2

1. c Percentage of whisky in mixture = 75%.

Percentage of whisky in water = 0%.

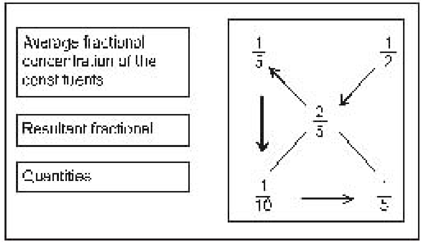


So mixture and water should be mixed in the ratio 2 : 1 or we need to replace $\frac{1}{3}$ of the mixture with water.

2. a Let x be the distance travelled by bus.

$\therefore \frac{x}{80}+\frac{1000-x}{36}=16 \Rightarrow$ x = 770 km (approximately)

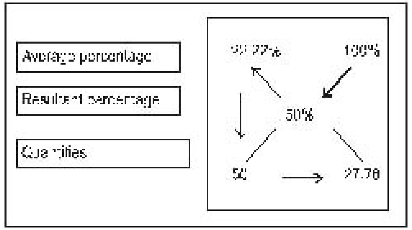
3. d Applying the rule of alligation:



$\therefore$ Ratio of volumes of two solutions = 1 : 2.

4. e Tin content in the alloy = 100 - 77.78% = 22.22%.

Using the principle of alligation,



Ratio of volumes = $\left(\frac{50}{27.78}\times 18\right):18=32.4:18$.

$\therefore$ Total tin content in the new alloy = (22.22% of 32.4) + 18 = 25.2 kg.

Total copper = (77.78% of 32.4) = 25.2 kg.

Hence, none of these is the correct option.

By the way, is there any need to do any calculations? In the question it is given that the new alloy has 50%, i.e. equal amount of copper and tin.

5.b Quantity of milk after first draw = $\frac{90}{100}\times 80=72\text{ kg}$.

Quantity of milk after second draw = $\frac{90}{100}\left(\frac{90}{100}\times 80\right)=64\text{ kg}$.

Quantity of milk after third draw = $\frac{9}{10}\times\frac{9}{10}\times\frac{9}{10}\times 80=58.32$

$\therefore$ Ratio = 100 : 81.

Alternative method:

We know that

$$\frac{\text{New amount of liquid X}}{\text{Original amount of liquid X}} = 1 - \left(\frac{\text{Amount of liquid taken out and replaced}}{\text{Total amount of liquid}}\right)^n$$

where n = Number of times the process is repeated $= 1 - \left(\frac{8}{80}\right)^2 = \left(\frac{9}{10}\right)^2$

$\therefore$ Ratio = $10^2 : 9^2$ = 100 : 81

6. b Initial respective amount of milk and water is 28 L and 7 L.

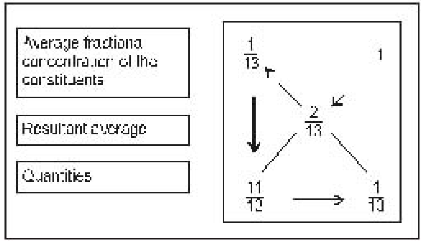
7 L of water added means now we have 14 L of water. To keep the ratio M : W same, amount of milk must be doubled, i.e. 28 L of milk must be added.

Alternative method:

In order to keep the solution of the same ratio, we need to add milk in the same ratio as already there in the solution.

$\therefore$ We need to add $\frac{4}{1} \times 7 = 28$ L of milk.

7. a Rule of alligation for fractional content of water:



or 11 : 1 or 33 : 3

$\therefore$ 3 L of water must be added to 33 L of solution of milk.

8.b In fact, she takes out 5% of the existing volume of wine in every operation.

After the first operation concentration of wine is 95%.

After the second operation concentration of wine is 95 × 0.95 = 90.25%.

After the third operation concentration of wine is 90.25 × (0.95) = 85.7375%.

Obviously, after the fourth operation the concentration of wine will be less than 85%.

9.b The ratio of incomes of Rashmi and Divya is 3 : 5. Ratio of their expenditures is 2 : 3, i.e. 3 : 4.5. Had the ratio of expenditures been 3 : 5, ratio of savings also would have been 3 : 5; but since ratio of their expenditures is 3 : 4.5 only, obviously savings of Divya will be something more than $\frac{5}{3}$ of savings of Rashmi and thus Divya will save more.

10.e

	Rohit	Sunil
Income	5x	7x
Expenditure	5y	7y
Saving	5x – 5y	7x – 7y

Ratio of savings is $\frac{5(x-y)}{7(x-y)} = 5 : 7$.

But when both of them would have spent their entire incomes then savings of both of them would be zero.

Hence, the ratio of their savings cannot be determined.

11.b Sum of squares of first n natural number is $\frac{n(n+1)(2n+1)}{6}$.

Square of sum of first n natural numbers is $\frac{n^2(n+1)^2}{4} = 17 : 325$

Now $\frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n^2(n+1)^2}{4}} = 17 : 325$.

On solving, we get n = 25.

12.c If IBM initially quoted Rs. 7x lakh, SGI quoted 4x lakh.

IBM's final quote = (4x - 1) lakh

$\frac{4x-1}{4x}=\frac{3}{4}$ or x = 1

IBM's bid winning price = Rs. 3 lakh.

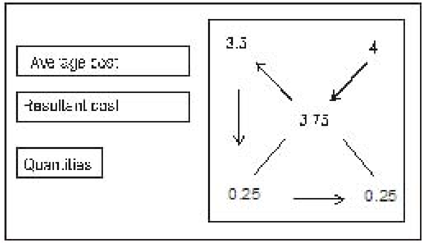
So IBM wins the bid at 4x - 1 = Rs. 3 lakh.

Note: Whoever quotes the minimum price, wins the bid.

13. b SP = 4.5 and profit = 20%.

Hence, CP = $\frac{450}{120}$ = 3.75

Using the alligation method,



i.e. 1 : 1

Hence, in a 40 kg mixture, there are 20 kg of the first variety and 20 kg of the second.

14. a Here we calculate the number of birds in terms of number of trees. We know that there are 630 birds.

Let the number of trees be x.

∴ Number of sparrows = $\frac{x}{2}\times 2$.

Also number of pigeons = $\left(\frac{x}{2}\times\frac{1}{4}\right)\times 1$.

Other birds = $\frac{x}{2}\times\frac{3}{4}\times 4$.

Thus, x + $\frac{x}{8}+\frac{3x}{2}=\frac{21}{8}x=630 \Rightarrow x=240$

Hence, other birds are there in $\frac{3x}{8}=90$ trees.

15.a $6a^2 = 2(lb + bh + lh)$

l : b : h = 1: 2 : 4. Therefore, b = 2l; h = 4l

Hence, $6a^2 = 2[2l^2 + 8l^2 + 4l^2]$

$6a^2 = 28l^2$

$a=\left(\frac{14}{3}\right)^{\frac{1}{2}} l \therefore a^3=\left(\frac{14}{3}\right)^{\frac{3}{2}} l^3$

Ratio of the volumes of the cube and cuboid = $a^3 : lbh = \left(\frac{14}{3}\right)^{\frac{3}{2}} : 8$

16. a If three kinds of birds are taken to be 3x, 7x and 5x respectively, then 7x - 3x = 63y (where y is any positive integer). As the number is a multiple of both 9 and 7, it has to be a multiple of 63.

$\Rightarrow x=\frac{63y}{4}$

Minimum value of y for which x is a natural number is 4.

$\Rightarrow x = 63$

Hence, the number of birds = 15x = 945.

17. e Price = k × Weight, where k is any constant.

Original weight of the stone = 28x.

Original price = 28kx.

New price = k(15x + 13x) = 28kx.

Hence, there is no profit, no loss.

Concept: Please note that since we have a linear function, there is neither profit nor loss.

18. d In a mixture of 1,000 ml, milk : water = 3 : 1.

Hence, milk = 750 ml, water 250 ml.

A 250 ml of 3 : 2 solution contains 150 ml milk and 100 ml water.

Total milk = 900 ml, total water = 350 ml.

After using 250 ml to make curd milk used = $\frac{250}{1250} \times 900$ = 180 ml.

Pure milk left = 900 - 180 = 720 ml.

19. d Let income and expenditure of the first person be 5x and 9y respectively.

Then income and expenditure of the second person are 4x and 7y respectively.

5x - 9y = 4x - 7y = 500 or x = 2y.

Therefore, y = 500, and x = 1000.

Incomes are Rs. 5,000 and Rs. 4,000 respectively.

Alternative method:

In the given choices, the choices (a) and (c) are not in the ratio 5 : 4. Now we can subtract Rs. 500 from incomes and check the ratio of expenditures in choice (b). Ratio of expenditures

= (3750 - 500) : (3000 - 500)

= 3250 : 2500 = 13 : 10

Similarly if we check for other options. (d) will satisfy.

Hence, the answer is (d).

20. d A, B and C are to share profit in the ratio 4 : 3 : 7.

But, since A is to receive 10% of profit, only 90% is to be shared.

A's total share = 10% of profit + $\frac{4}{14}$ × 90% of profit = 6000

or (10 + $\frac{180}{7}$)% of profit = 6000

or $\frac{250}{7}$% P = 6000 ⇒ P = Rs. 16,800

B and C's share combined = $\frac{10}{14}$ of 90% profit $= \frac{10}{14} \times \frac{90}{100} \times 16800$ = 10,800

21. a In the original mixture,

spirit = $\frac{3}{4}$ × 40 = 30 L,

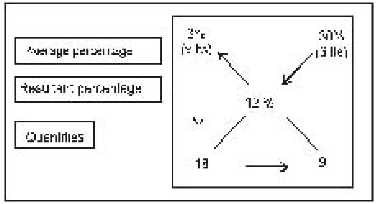
water = $\frac{1}{4}$ × 40 = 10 L.

To get the ratio 5 : 2, let's assume that we add x litres of water.

$\frac{30}{(10 + x)} = \frac{5}{2}$ or x = 2

Hence, 2 L of water has to be added.

22.d



i.e. 2 : 1

Hence x litres must be 12 L, i.e. (6 × 2).

23. c This is a classic case for approximations. We see that the ratio 1075 : 1000, when converted to percentage terms, is approximately 52% : 48%. Thus, we can calculate the literacy percentage as

$\frac{24}{100} \times 52 + \frac{8}{100} \times 48 \approx 16.3\%$

Now as 100 - 16.3 = 83.7%, only choices (c) and (e) are left. We calculate the number of literate people as

$\frac{16.3}{100} \times 311250 = 50{,}700$.

We could have approximated the ratio of men to women as 50% : 50% also.

24. b Let x be the number of children employed.

Then numbers of men and women are 8x and 5x respectively.

Let y be the common factor for wages.

Then wages of men, women and children are Rs. 5y, 2y and 3y respectively.

Hence, total wages of different categories will be

40xy, 10xy and 3xy respectively.

Total wages = (40 + 10 + 3)xy = 53xy = 318.

So xy = 6.

Thus, they will be getting 40 × 6, 10 × 6 and 3 × 6 respectively.

Alternative method:

Sum of all three wages have to be equal to Rs. 318. Only (b) displays this property.

25. a Milk in final/Milk in original = $[(a - b)/a]^n$,

where a = Quantity of mixture,

b = Amount taken out during each operation,

n = Number of times the operation is repeated.

Milk in final/Milk in original = $\left[\frac{40 - 10}{40}\right]^3 = \left(\frac{3}{4}\right)^3$.

Milk in final solution = 40 × $\frac{27}{64}$ = 16.875 L.

Water = 23.125 L.

Required ratio = 16.875 : 23.125 = 27 : 37.

26. b This question can either be done by substituting choices, or by using weighted averages.

If x be the number of members in club initially, then sum of ages of (x + 3) members now = 26x + 87 = 27(x + 3)

Hence, x = 6. So number of members now = 9.

Alternative method:

This method is called sum of deviations, as follows.

"The sum of all weighted deviations from the average is zero."

Thus, assuming we have x students initially.

$x(26 - 27) + 3(29 - 27) = 0$

$\Rightarrow -x + 6 = 0$

$x = 6$

Please note that always subtract the average from the number and not vice versa.

27. a Assuming that there is 1 L of spirit and 3 L of water,

the total volume of the mixture is 4 L.

After increasing its volume by 25%, volume of the mixture is 4 + 25% of 4 = 5 L.

So the ratio of volumes of spirit to water = 2 : 3.

28.c A ratio $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$ is equivalent to 6 : 4 : 3.

So in this case, A, B and C would have got Rs. 54, Rs. 36 and Rs. 27 respectively.

But actually the money was divided in the ratio

2 : 3 : 4 and shares of A, B and C in this case was 26, 39 and 52 respectively.

Hence, the answer is (c).

Alternative method:

$\frac{1}{2} : \frac{1}{3} : \frac{1}{4} = 0.5 : 0.\overline{33} : 0.25$

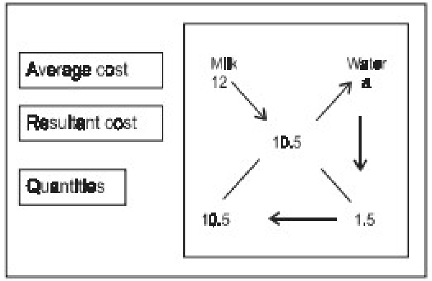
Thus, C, who was getting the least, finally got the maximum amount.

This eliminates (a) and (b).

Thus, gain of C = Rs.$117\left(\frac{4}{4+3+2} - \frac{0.25}{0.25+1/3+0.5}\right)$ = Rs. 25

Hence, the answer is (c).

29. d



i.e. 7 : 1

Hence, milk and water are in the ratio 7 : 1 since there is 28 L milk. Hence, water should be 4 L.

Practice Exercise 5 - Level 2

1. a If A, B and C represent the number of three denominations in increasing order of their values,

then A = 7B, C = 4B.

Since C = 12, B = 3 and A = 21, the total money in the purse = $21 \times 1 + 3 \times 10 + 12 \times 20$ = Rs. 291.

2.c If there are x number of one-rupee coins in the bag, then there are 8x and 16x, 50-paisa and 25-paise coins respectively.

Therefore, money in the bag = x + 4x + 4x = 9x = 495.

So x = 55.

Hence, there are $8 \times 55 = 440$ 50-paisa coins in the bag.

3. a There are 880 25-paisa coins.

Value of the money = 0.25×880 = Rs. 220.

4.c

	V1	V2	V3
Initial volume:	0 L	40 L milk	80 L W water
After operation 1:	0	20 L M + 20 L W	60 L w + 20 L milk
After operation 2:	0	15 L M + 25 L W	55 L w + 25 L milk

5.a Check from the above data.

6.a Suppose he invested Rs. x and Rs. y in the matches of South Africa and India respectively. From the first match he will get 3 x, but from the second he will get nothing.

Now $3x = (x + y) + (x + y)\frac{150}{100}$ or $\frac{x}{y} = \frac{5}{1}$.

7.a Suppose he invested 3X on the first match and 4X on the second match.

He will get back $3X \times 3 = 9X$.

So gain = 9X - 7X = 2X = 200000 or X = 100000

:. Total investment = $7X = 7 \times 100000 = 700000$

8.c Investment = x + 2x = 3x

He gets back 3x + 8x = 11x. So gain = 8x.

Gain percentage = $\frac{8x}{3x} \times 100$ = 266.66%.

9. c $C = \frac{A+B+C}{3} \Rightarrow$ Originally B had an amount = 2C - A

Now $A¢ = \frac{A}{2}$, $B¢ = \frac{1}{3}B$, $C¢ = 0$

$\therefore \frac{A}{2} + \frac{B}{3} = \frac{1}{2}(2C - A) = \frac{B}{2}$

$\therefore$ B = 3A, C = 2A

So amount spent = $(A + 3A + 2A) - \frac{3A}{2} = 4.5A$

Percentage of amount spent = 75%.

10.e We cannot determine the final ratio unless we know the volume in 3 jars.

11. b Mixing two lumps, we have 18 g from each metal.

So price of the second metal = $\frac{(87 + 78.60 - 6.7 \times 18)}{18}$ = Rs. 2.50 per gram

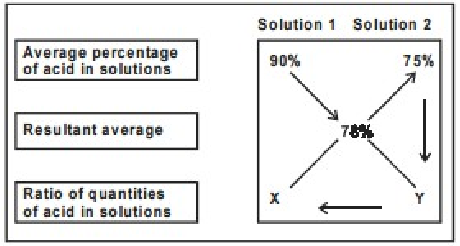
$\therefore 6.7x + 2.5(18 - x) = 87$

$\therefore x = 10$ g and $18 - x = 8$ g

12.c $54\left(1-\frac{x}{54}\right)^2 = 24 \Rightarrow \left(1-\frac{x}{54}\right)^2 = \frac{24}{54} = \frac{4}{9}$

$\Rightarrow 1-\frac{x}{54} = \frac{2}{3} \Rightarrow x = 18$

13 c Use alligations.



$\therefore \frac{90-78}{78-75} = \frac{Y}{X} \Rightarrow \frac{12}{3} = \frac{Y}{X} \Rightarrow \frac{X}{Y} = \frac{1}{4}$

So ratio of quantities of solutions = 1 : 4

Total quantity = 30 L

$\therefore$ Quantity of 90% solution of concentrated acid = $30 \times \frac{1}{5} = 6$ L

14. c Let Tina's share be T, Issan's be I, Abhishek's be A and Fatima's be F.

Given that $T + 3 = I + \frac{I}{3} = \frac{80A}{100} = F - 4$

You get

$T = F - 7$ ... (i)

$I = \frac{3}{4}(F - 4)$ ... (ii)

$A = \frac{5}{4}(F - 4)$ ... (iii)

Also given that $T + I + F + A = 80$ ... (iv)

Substituting the values from (i), (ii) and (iii) in (iv), we get

$\left[F + \frac{3F}{4} + \frac{5F}{4} + F\right] - 7 - 3 - 5 = 80$ or $F = 23.75$

15. a Let the price of one tea = x $\Rightarrow$ the price of other tea $= \frac{x}{2}$

Price of 1 kg $= \frac{2x}{5} + \frac{3}{5} \times \frac{x}{2} = \frac{7x}{10}$

But CP = $\frac{17.50 \times 100}{125} = 14$

(Only the first option satisfies the first condition.)

$\Rightarrow \frac{7x}{10} = 14 \Rightarrow x = 20$

So the price of the tea's are 20 and 10.

16. c If the person who gets $\frac{1}{4}$ of the whole gets thrice of what the others get on an average, each one will get

$= \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ of the whole.

Therefore, if there are x persons other than the person who gets one-fourth of the whole, then $\frac{1}{4} + \frac{x}{12} = 1$

So x = 9

Hence, total number of people = 10

17. c 6p + 8q = 7.5(p + q); 6q + 8p = n(p + q)

$14(p+q) = (7.5+n)(p+q) \Rightarrow n = 6.5$

18. b Amount of 16% iodine in the mixture = $735 \times \frac{16}{100}$ gm

Now the amount of iodine becomes 20% of the mixture.

Amount evaporated $= 735 - 735 \times \left(\frac{16}{100}\right) \times \left(\frac{100}{20}\right)$ = 147 g

19. b Whether you add them in the ratio 2 : 3 or 1 : 1, we get the same concentration.

Hence, both of them must have a 35% concentration. So (b) is the answer.

20. c Only 5 women were college graduates while 20 women were employed. Therefore, maximum number of `employed graduate women' can be 5. Since number of `unemployed men' was (55 - 15) = 40. Answer cannot be greater than $\frac{1}{8}$.

21. b Only possibility is if 650 is divided by a multiple of 13, because the other jars have a capacity that is not a multiple of 13. 403 is the only choice possible.

Alternative method:

As the beakers filled by same fractions,

$$\frac{a}{250} = \frac{b}{650} = \frac{c}{200} = \frac{a+b+c}{250+650+200}$$

$\Theta\ a + b + c = 682$

$$\therefore \frac{b}{650} = \frac{682}{1100}$$

b = 403 ml

22. d 10 kg copper is from alloy II.

Hence, 28 kg copper is from alloy I.

In alloy I, tin : copper = 3 : 4

Since, 4 units = 28 kg

Alloy = 7 units = 49 kg

Practice Exercise 6 - Level 3

1.c After first transfer,

amount of wine in first vessel = 9 L,

amount of water in first vessel = 3 L,

amount of wine in second vessel = 3 L,

amount of water in second vessel = 1 L.

3 L drawn from first vessel contains $\frac{3}{4}\times 3=\frac{9}{4}$ L of wine and $\frac{1}{4}\times 3=\frac{3}{4}$ L of water.

3 L drawn from second vessel contains $\frac{9}{4}$ L of wine and $\frac{3}{4}$ L of water.

∴ Amount of wine in first vessel after second transfer = $9-\frac{9}{4}+\frac{9}{4}=9$.

Amount of water in first vessel after second transfer = $3-\frac{3}{4}+\frac{3}{4}=3$.

Similarly, for second vessel, wine = $3-\frac{9}{4}+\frac{9}{4}=3$

Water = $1-\frac{3}{4}+\frac{3}{4}=1$

∴ Ratio is same in both the vessels.

Alternative method:

Ratio of wine to water in first vessel after first transfer

is (12 - 3) : 3 = 3 : 1.

Ratio of wine to water in second vessel after first transfer is 3 : (4 - 3) = 3 : 1.

Since the ratio of wine to water in both the vessels is equal. Any amount of such exchange any number of times will not alter the ratio of wine to water.

2.e Consider two cases:

i.	Princy	Kunjumol
Income	100	200
Expenditure	1	5
Saving	100 – 1 = 99	200 – 5 = 195

Thus, Kunjumol is saving more.

ii.	Princy	Kunjumol
Income	100	200
Expenditure	39	195
Saving	100 – 39 = 61	200 – 195 =5

Thus, Princy is saving more.

Hence, their savings depends upon their respective expenditures.

3. c 5a + b > 51 ... (i)

3a - b = 21 ... (ii)

8a > 72 or a > 9.

From (ii),

$a=\frac{21+b}{3}\Rightarrow\frac{21+b}{3}>9\Rightarrow b>6$

Combining a > 9, b > 6.

4. b Average age = 13. Number of students = 50

W µ H and H µ A ⇒ W µ A

∴ $W = K_1H$ and $H = K_2A \Rightarrow W = K_1K_2A$

Since the ratio of wine to water in both the vessels is equal. Any amount of such exchange any number of times will not alter the ratio of wine to water.

2.e Consider two cases:

i.	Princy	Kunjumol
Income	100	200
Expenditure	1	5
Saving	100 – 1 = 99	200 – 5 = 195

Thus, Kunjumol is saving more.

ii.	Princy	Kunjumol
Income	100	200
Expenditure	39	195
Saving	100 – 39 = 61	200 – 195 =5

Thus, Princy is saving more.

Hence, their savings depends upon their respective expenditures.

3. c $5a + b > 51$ ... (i)

$3a - b = 21$ ... (ii)

$8a > 72$ or $a > 9$.

From (ii),

$$a = \frac{21+b}{3} \Rightarrow \frac{21+b}{3} > 9 \Rightarrow b > 6$$

Combining $a > 9$, $b > 6$.

4. b Average age = 13. Number of students = 50

$W \mu H$ and $H \mu A \Rightarrow W \mu A$

$\therefore W = K_1H$ and $H = K_2A \Rightarrow W = K_1K_2A$

For $A = 11$, $H = 165$ and $W = 33$

$$K_1K_2 = \frac{33}{11} = 3$$

$\therefore W = 3A$

Since $W \mu A$, the averages should also be in direct proportion.

$\therefore W_{avg} = K_1K_2 A_{avg} = 3 \times 13 = 39$ kg