CATapult Courseware

SPEED MATH

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Registered Office: 6th Floor, NCL Building, 'E' Block, Near Bandra Family Court,
Bandra Kurla Complex (BKC), Bandra (E), Mumbai - 400051
Tel.: +91 22 66170000
Toll Free: 1800-1234-467
CIN : U80220MH1999PTC121823
E-mail : support@imsindia.com Website: www.imsindia.com

PREFACE

This book discusses different speed mathematics tricks. Although you can carry out all the calculations using the traditional methods, in an exam like CAT, time is of essence and knowledge of speed mathematics tricks will help you save precious time that you can use to solve other quesions. In CAT, typically no direct questions involving such calculations are asked. However, a problem (typically a data interpretation set) may involve multiple calculations of this nature. You can effectively employ these techniques for solving such problems.

This book contains 3 chapters with number of solved and unsolved examples. Additionally there are 20 tests at the end. You should practise all the solved as well as unsolved examples in the chapters so that you develop required level of familiarity with the methods. Whenever you come across any problem where you can apply any of the methods described in the booklet, make sure that you solve the problem using those methods.

Take the tests after you have solved all the solved and unsolved examples in the three chapters. We suggest you not to take all the tests at once but spread them out during your preparation so that you will always be in touch with the methods described in the book.

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1 BASIC OPERATIONS — ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION

In this chapter, you will learn different ways of performing four basic operations, namely addition, subtraction, multiplication and division, using speed mathematics techniques. Although you cannot expect any direct questions on the four basic operations, knowledge of these speed mathematics techniques will immensely help you in solving calculation intensive Data Interpretation questions.

1.1 ADDITION TECHNIQUE

Example: 4218 + 3267

Break down the second number into thousands, hundreds, tens and units.

Therefore 3267 = 3000 + 200 + 60 + 7.

$\therefore$ 4218 + 3267 = 4218 + (3000 + 200 + 60 + 7)

$$= \underbrace{4218 + 3000} + 200 + 60 + 7$$
$$= \underbrace{7218 + 200} + 60 + 7$$
$$= \underbrace{7418 + 60} + 7$$
$$= 7478 + 7$$
$$= 7485$$

It is easier and faster to perform addition when you break down the second number this way. With sufficient practice, you will be able to perform addition orally in matter of seconds.

Example: 28492 + 15361

15361 = 10000 + 5000 + 300 + 60 + 1

$\therefore$ 28492 + 15361 = 28492 + (10000 + 5000 + 300 + 60 + 1)

$$= \underbrace{28492 + 10000} + 5000 + 300 + 60 + 1$$
$$= \underbrace{38492 + 5000} + 300 + 60 + 1$$
$$= \underbrace{43492 + 300} + 60 + 1$$
$$= \underbrace{43792 + 60} + 1$$
$$= 43852 + 1$$
$$= 43853$$

Example: 172829 + 38498

38498 = 30000 + 8000 + 400 + 90 + 8

172829 + 38498 = 172829 + (30000 + 8000 + 400 + 90 + 8)

$$= \underbrace{172829 + 30000} + 8000 + 400 + 90 + 8$$

$$= \underbrace{202829 + 8000} + 400 + 90 + 8$$

$$= \underbrace{210829 + 400} + 90 + 8$$

$$= \underbrace{211229 + 90} + 8$$

$$= 211319 + 8$$

$$= 211327$$

Example: 14329 + 8187 + 12081

When you have more than two numbers, perform additions of two numbers at a time.

14329 + 8187 + 12081 = (14329 + 8187) + 12081

$$= 14329 + (8000 + 100 + 80 + 7) + 12081$$

$$= \underbrace{14329 + 8000} + 100 + 80 + 7 + 12081$$

$$= \underbrace{22329 + 100} + 80 + 7 + 12081$$

$$= \underbrace{22429 + 80} + 7 + 12081$$

$$= \underbrace{22509 + 7} + 12081$$

$$= 22516 + 12081$$

$$= 22516 + 12081$$

$$= 22516 + 10000 + 2000 + 80 + 1$$

$$= \underbrace{22516 + 10000} + 2000 + 80 + 1$$

$$= \underbrace{32516 + 2000} + 80 + 1$$

$$= \underbrace{34516 + 80} + 1$$

$$= 34596 + 1$$

$$= 34597$$

EXERCISE 1.1

DIRECTIONS for questions: Perform the following additions using the technique outlined above—

1.	395 + 298 =		2.	803 + 753 =	
3.	129 + 918 =		4.	2049 + 4663 =	
5.	7627 + 1107 =		6.	9602 + 7323 =	
7.	82499 + 71633 =		8.	98475 + 40467 =	
9.	37824 + 89142 =		10.	18090 + 4037 =	
11.	5234 + 76585 =		12.	92135 + 6899 =	
13.	278 + 429 + 671 =		14.	1428 + 3893 + 5248 =	
15.	38478 + 1883 + 14913 =				

1.2 SUBTRACTION TECHNIQUE

Similar to addition, subtraction technique works faster if you break down the second number.

Example: 447 – 289

$289 = 200 + 80 + 9$

$$
\begin{aligned}
\therefore\ 447 - 289 &= 447 - (200 + 80 + 9) \\
&= \underbrace{(447 - 200)} - (80 + 9) \\
&= 247 - (80 + 9) \\
&= \underbrace{(247 - 80)} - 9 \\
&= 167 - 9 \\
&= 158
\end{aligned}
$$

Example: 4824 – 2193

$$
\begin{aligned}
&= 4824 - (2000 + 100 + 90 + 3) \\
&= \underbrace{(4824 - 2000)} - (100 + 90 + 3) \\
&= 2824 - 100 - 90 - 3 \\
&= \underbrace{(2824 - 100)} - (90 + 3) \\
&= \underbrace{(2724 - 90)} - 3 \\
&= 2634 - 3 \\
&= 2631
\end{aligned}
$$

Example: 38212 – 14778

$14778 = 10000 + 4000 + 700 + 70 + 8$

$$
\begin{aligned}
\therefore\ 38212 - 14778 &= 38212 - (10000 + 4000 + 700 + 70 + 8) \\
&= \underbrace{(38212 - 10000)} - (4000 + 700 + 70 + 8) \\
&= 28212 - (4000 + 700 + 70 + 8) \\
&= \underbrace{(28212 - 4000)} - (700 + 70 + 8) \\
&= 24212 - 24212 - (700 + 70 + 8) \\
&= 24212 - \underbrace{(24212 - 700)} - (70 + 8) \\
&= (23512 - (70 + 8) \\
&= \underbrace{(23512 - 70)} + 8 \\
&= 23442 - 8 \\
&= 23434
\end{aligned}
$$

Example: 47218 – 8324

$$
\begin{aligned}
&= 47218 - (8000 + 300 + 20 + 4)\\
&= \underbrace{(47218 - 8000)} - (300 + 20 + 4)\\
&= 39218 - (300 + 20 + 4)\\
&= \underbrace{(39218 - 300)} - 20 - 4\\
&= 38918 - (20 + 4)\\
&= \underbrace{(38918 - 20)} - 4\\
&= 38898 - 4\\
&= 38894
\end{aligned}
$$

EXERCISE 1.2

DIRECTIONS for questions: Perform the following subtractions using the technique outlined above—

1.	986 – 591 =		2.	804 – 611 =	
3.	492 – 175 =		4.	7314 – 2817 =	
5.	8516 – 4209 =		6.	6044 – 5106 =	
7.	85036 – 61441 =		8.	67841 – 19082 =	
9.	46578 – 21034 =		10.	12145 – 4289 =	
11.	30753 – 7959 =		12.	21246 – 9495 =	
13.	43327 – 18841 =		14.	39837 – 8148 =	
15.	52175 – 38226 =				

1.3 MULTIPLICATION TECHNIQUES

A) Multiplication of two numbers both of which are close to same power of 10

Suppose we want to multiply 98 and 93. Following is the process for speedy calculations

Step I: Identify the power of 10 closest to the two numbers (in this case 100). Treat that as base for calculations.

Step II: Calculate the difference of the two numbers from the base and write it in the following fashion

98/–02
× 93/–07

Important: Make sure that the difference written has as many digits as the number of zeroes in the base. Precede the difference with zeroes if required.

Step III: Take the algebraic sum of the numbers along ether diagonal and write the following fashion.

98 /–02
× 93 /–07

91

(98 – 07 = 93 – 02 = 91)

Note that algebraic sum of the numbers along either diagonal will give the same result.

Step IV: Take product of the differences and write in the following fashion

98 /–02
× 93 /–07

91 /14

(–2) × (–7) = 14

Make sure that there are as many digits in the product of differences as number of zeroes in the base. Precede the product with zero if needed.

Therefore the product of 98 and 93 is 91/14 or 9114.

Similarly we can carry out other multiplications as follows

1)	2)	3)
96 /–04	98 /–02	98 /–02
× 91 /–09	× 88 /–12	× 99 /–01
87 / 36	86 /24	97 /02
or 8736	or 8624	or 9702

Same approach can be extended when both numbers are greater than the base, as shown below

1)	2)	3)
103 /+03	106 /+06	102 /+02
× 108 /+08	× 109 /+09	× 108 /+08
111 /24	115 /54	110 /16
or 11124	or 11554	or 11016

If one number is greater than the base and the base and the other number is smaller than the base, following change is incorporated

1)	2)	3)
105 /+05	103 /+03	109 /+09
× 98 /–02	× 96 /–04	× 92 /–08
103 /–10	99 /–04	101 /–72
= 10300 – 10 = 10290	= 9900 – 12 = 9888	= 10100 – 72 = 10028

Same approach is used when the base is higher power of 10, as shown below

1)	2)	3)
989 /–011	1008 /+008	1018 /+08
× 993 /–007	× 1012 /+012	× 993 /–007
982 /077	1020 /096	1011 /–126
or 982077	or 1020096	or 1011000 – 126 = 1010874
(base:1000)	(base:1000)	(base:1000)

4)	5)	6)
9992 /–0008	10008 /+0008	10012 /+0012
× 9983 /–0017	× 9993 /–0007	× 10007 /+0007
9975 /0136	10001 /–0056	10019 /0084
or 99750136	= 100010000 – 56	10019 /0084
	= 100,009,944	or 100,190,084
(base:10000)	(base:10000)	(base:10000)

EXERCISE 1.3A

DIRECTIONS for questions: Perform the following multiplications using the technique outlined above—

1. 97 × 92 =
2. 105 × 112 =
3. 105 × 97 =
4. 994 × 991 =
5. 994 × 1007 =
6. 1003 × 1008 =
7. 86 × 93 =
8. 88 × 104 =
9. 111 × 117 =
10. 988 × 996 =
11. 993 × 1008 =
12. 1014 × 1019 =
13. 989 × 994 =
14. 993 × 1008 =
15. 1006 × 1018 =

B) Multiplication of two numbers both of which are close to the same multiple of a power of 10

If we want to multiply two numbers that are close to a multiple of a power of 10, say 50, we take 10 as the base and 50 as the working base and proceed as follows

Suppose we want to multiply 43 and 48

Step I: Use a multiple of power of 10 as working base for calculations. Here we use 50 as the working base. 10 is still base of the calculations.

Step II: Take the differences of the two numbers from the working base as shown below

$$\begin{array}{ll} & 43\ /{-7} \\ \times & 48\ /{-2} \\ \hline & 41 \end{array}$$

Important: When your write the difference, make sure that there are as many digits in the differences as the number of zeroes in the base(10).

Step III: Take algebraic sum of numbers along either diagonal, as shown below

$$\begin{array}{ll} & 43\ /{-7} \\ \times & 48\ /{-2} \\ \hline & 41 \end{array}$$

Step IV: Take product of the differences as shown below

$$\begin{array}{ll} & 43\ /{-7} \\ \times & 48\ /{-2} \\ \hline & 41\ /14 \end{array}$$

Step V: The working base used is 5 times 10. Therefore multiply the left part by 5.

$$\begin{array}{ll} & 43\ /{-7} \\ \times & 48\ /{-2} \\ \hline & 41\ /14 \\ \times & 5 \\ \hline & 205\ /14 \end{array}$$

Answer is 205 /14. There is only one zero in the base (10). Therefore we can have only one digit in the part to the right of '/'. Therefore we can have only '4' to the right of '/' and carry 1 forward

Therefore required product is (205 + 1) /4 or 206 /4 or 2064.

<u>Note that we could have used working base of 40 to solve the same problem as shown below. Now the working base (40) is 4 times the base (10). So you have to multiply by 4 in step 5.</u>

$$\begin{array}{ll} & 43\ /{+3} \\ \times & 48\ /{+8} \\ \hline & 51\ /24 \\ \times & 4 \\ \hline & 204\ /24 \end{array}$$

Therefore required product is 204/24 or (204 + 2)/4 or 206/4 or 2064

The same problem could have been solved with working base of 50 (100 ÷ 2) as follows – note that now the base (100) has two zeroes so the differences will require two digits

$$\begin{array}{ll} & 43\ /{-07} \\ \times & 48\ /{-02} \\ \hline & 41\ /14 \\ \div & 2 \\ \hline & 20.5\ /14 \end{array}$$

= 20.5 + 14
= 2050 + 14 = 2064

Similarly we can carry out other calculations as follows

1) 84 × 78 (Base = 10, working base = 10 × 8 = 80)

$$\begin{array}{ll} & 84\ /{+4} \\ \times & 78\ /{-2} \\ \hline & 82\ /{-8} \\ \times & 8 \\ \hline & 656\ /{-8} \end{array}$$

= 6560 – 8 = 6552

2) 63 × 72 (Base = 10, working base = 10 × 7 = 70)

$$\begin{array}{ll} & 63\ /{-7} \\ \times & 72\ /{+2} \\ \hline & 65\ /{-14} \\ \times & 7 \\ \hline & 455\ /{-14} \end{array}$$

= 4550 – 14 = 4536

3) 54 × 58 (Base = 10, working base = 10 × 5 = 50)

$$\begin{array}{ll} & 54\ /{+4} \\ \times & 58\ /{+8} \\ \hline & 62\ /32 \\ \times & 5 \\ \hline & 310\ /32 \end{array}$$

= 3100 + 32 = 3132

4) 54 × 58 (Base = 10, working base = 10 × 6 = 60)

$$\begin{array}{ll} & 54\ /{-6} \\ \times & 58\ /{-2} \\ \hline & 52\ /12 \\ \times & 6 \\ \hline & 312\ /12 \\ = & 3120 + 12 = 3132 \end{array}$$

5) 54 × 58 (Base = 100, working base = $\frac{100}{2}$ = 50)

$$\begin{array}{ll} & 54\ /{+04} \\ \times & 58\ /{+08} \\ \hline & 62\ /32 \\ \div & 2 \\ \hline & 31\ /32 \\ = & 3132 \end{array}$$

From examples (3), (4) and (5), you can see that you can take any convenient base to carry out calculations.

EXERCISE 1.3B

DIRECTIONS for questions: Perform the following multiplications using the technique outlined above—

1.	38 × 43 =		2.	53 × 56 =	
3.	81 × 76 =		4.	63 × 59 =	
5.	71 × 74 =		6.	59 × 64 =	
7.	392 × 403 =		8.	291 × 296 =	
9.	506 × 491 =		10.	804 × 811 =	
11.	589 × 586 =		12.	687 × 693 =	
13.	293 × 306 =		14.	409 × 398 =	
15.	492 × 509 =				

C) General method of multiplication of numbers with two digits

Suppose you have to calculate 37×82

<u>Step I:</u> Multiply the units digits in the two numbers. If the product is greater than 10, retain the units place and carry forward the ten's place of the product to the left as shown below –

$$\begin{array}{r} 37 \\ \times \ 82 \\ \hline 4 \\ 1\ \ \end{array}$$

Here product of units digits of the two numbers (7×2) is 14. In the product, units' digit (4) is retained and 1 is carried forward to the left

<u>Step II:</u> Take sum of products of numbers along the two diagonals.

Here there are two pairs numbers along diagonals i.e. (7 and 8, 3 and 2). $7 \times 8 + 3 \times 2 = 62$. In the sum, units place is retained and ten's place is carried forward to the left as shown.

$$\begin{array}{rrr} & 3 & 7 \\ \times & 8 & 2 \\ \hline & 2 & 4 \\ 6 & 1 & \end{array}$$

<u>Step III:</u> Take product of the two digits in the ten's place of the two numbers. In this example, product is $3 \times 8 = 24$, out of which, 4 is retained and 2 is carried forward to the left as shown

$$\begin{array}{rrrr} & & 3 & 7 \\ & \times & 8 & 4 \\ \hline & 4 & 2 & 4 \\ 2 & 6 & 1 & \end{array}$$

<u>Step IV:</u> Take the sum of two rows. That is the product of 37 and 82

$$\begin{array}{rrrr} & & 3 & 7 \\ & \times & 8 & 2 \\ \hline & 4 & 2 & 4 \\ 2 & 6 & 1 & \\ \hline 3 & 0 & 3 & 4 \end{array}$$

Examples

1) 28×93

<u>Step I:</u> $8 \times 3 = 24$

$$\begin{array}{rrr} & 2 & 8 \\ \times & 9 & 3 \\ \hline & & 4 \\ & 2 & \end{array}$$

<u>Step II:</u> $3 \times 2 + 8 \times 9 = 78$

$$\begin{array}{rrrr}
 & & 2 & 8 \\
\times & & 9 & 3 \\
\hline
 & & 8 & 4 \\
 & 7 & 2 &
\end{array}$$

<u>Step III:</u> $2 \times 9 = 18$

$$\begin{array}{rrrr}
 & & 2 & 8 \\
 & \times & 9 & 3 \\
\hline
 & 8 & 8 & 4 \\
1 & 7 & 2 & \\
\hline
2 & 6 & 0 & 4
\end{array}$$

or $28 \times 93 = 2604$

2) 37×43

$$\begin{array}{rrrr}
 & & 3 & 7 \\
\times & & 4 & 3 \\
\hline
 & 2 & 7 & 1 \\
1 & 3 & 2 & \\
\hline
1 & 5 & 9 & 1
\end{array}$$

3) 63×87

$$\begin{array}{rrrr}
 & & 6 & 3 \\
\times & & 8 & 7 \\
\hline
 & 8 & 6 & 1 \\
4 & 6 & 2 & \\
\hline
5 & 4 & 8 & 1
\end{array}$$

4) 53×78

$$\begin{array}{rrrr}
 & & 5 & 3 \\
\times & & 7 & 8 \\
\hline
 & 5 & 1 & 4 \\
3 & 6 & 2 & \\
\hline
4 & 1 & 3 & 4
\end{array}$$

Note: if the sum of products of digits along the diagonals ends in a three digit number, the digit in the hundreds place of the sum is written in the third line to the left of the ten's place digit in the second line as shown

5) 98 × 89

<u>Step I:</u> 8 × 9 = 72

```
    9 8
  × 8 9
  -----
      2
    7
```

<u>Step II:</u> 8 × 8 + 9 × 9 = 64 + 81 = 145

```
    9   8

  × 8   9
  -------
    5   2
  4 7
1
```

<u>Step III:</u> 9 × 8 = 72

```
    9 8

  × 8 9
  -----
  2 5 2
7 4 7
1
-------
8 7 2 2
```

or 98 × 89 = 8722

EXERCISE 1.3C

DIRECTIONS for questions: Perform the following multiplications using the technique outlined above—

1.	48 × 89 =		2.	67 × 98 =	
3.	32 × 74 =		4.	52 × 79 =	
5.	84 × 57 =		6.	56 × 83 =	
7.	22 × 72 =		8.	34 × 76 =	
9.	83 × 37 =		10.	59 × 23 =	
11.	72 × 91 =		12.	26 × 63 =	
13.	22 × 83 =		14.	63 × 78 =	
15.	26 × 82 =				

D) General method for product of two 3 digits numbers

Suppose you have to multiply 342 × 567

Step I: Calculate product of units' digits of the two numbers. If product is greater than 10, carry forward the ten's digit to the left

Here 2 × 7 = 14

```
     3  4  2

  ×  5  6  7
  ----------
           4
        1
```

Step II: Take sum of products of units place in the first number and ten's place in the second number and the unit's place in the second number and ten's place in the first number.

Here, 4 × 7 + 6 × 2 = 40

```
     3  4  2

  ×  5  6  7
  ----------
        0  4
     4  1
```

Step III: Take the sum of the following three numbers

i) Unit's place in first number and hundred's place in second number

ii) Unit's place in second numebr and hundred's place in first number.

iii) Ten's places in the two numbers

Here, 7 × 3 + 5 × 2 + 4 × 6 = 55.

```
     3  4  2

  ×  5  6  7
  ----------
     5  0  4
   5 4  1
```

Step IV: Take the sum of ten's place of the first number and hundred's plae of second number and ten's place of the second number and hundred's place of the first number.

Here 6 × 3 + 5 × 4 = 38

```
     3  4  2

  ×  5  6  7
  ----------
   8 5  0  4
 3 5 4  1
```

Step V: Take product of the hundred's places of the two numbers.

Here $3 \times 5 = 15$

```
     3  4  2

  ×  5  6  7
  ---------
   5 8 5  0  4
 1 3 5 4  1
 -----------
 1 9 3 9  1 4
```

or $342 \times 567 = 193914$

Examples

1) **313 × 483**

Step I: $3 \times 3 = 9$

```
     3  1  3

  ×  4  8  3
  ----------
           9
```

Step II: $3 \times 1 + 8 \times 3 = 27$

```
     3  1  3

  ×  4  8  3
  ----------
        7  9
     2
```

Step III: $3 \times 3 + 4 \times 3 + 8 \times 1 = 29$

```
     3  1  3

  ×  4  8  3
  ----------
     9  7  9
  2  2
```

Step IV: $3 \times 8 + 4 \times 1 = 28$

```
     3  1  3

  ×  4  8  3
  ----------
   8 9  7  9
 2 2 2
```

Step V: $3 \times 4 = 12$

```
     3  1  3

  ×  4  8  3
  ----------
   2 8 9 7  9
 1  2 2 2
 -----------
 1  5 1 1 7 9
```

or $313 \times 483 = 151179$

2) 747 × 939

<u>Step I</u>: 7 × 9 = 63

$$\begin{array}{ccccc} & & 7 & 4 & 7 \\ & \times & 9 & 3 & 9 \\ \hline & & & & 3 \\ & & & 6 & \end{array}$$

<u>Step II</u>: 4 × 9 + 3 × 7 = 57

$$\begin{array}{ccccc} & & 7 & 4 & 7 \\ & \times & 9 & 3 & 9 \\ \hline & & & 7 & 3 \\ & & 5 & 6 & \end{array}$$

<u>Step III</u>: 7 × 9 + 9 × 7 + 4 × 3 = 138

$$\begin{array}{ccccc} & & 7 & 4 & 7 \\ & \times & 9 & 3 & 9 \\ \hline & & 8 & 7 & 3 \\ & & 3\,5 & 6 & \\ & 1 & & & \end{array}$$

<u>Step IV</u>: 7 × 3 + 9 × 4 = 57

$$\begin{array}{ccccc} & & 7 & 4 & 7 \\ & \times & 9 & 3 & 9 \\ \hline & 7 & 8 & 7 & 3 \\ & 5\,3 & 5 & 6 & \\ & 1 & & & \end{array}$$

<u>Step V</u>: 7 × 9 = 63

$$\begin{array}{ccccc} & & 7 & 4 & 7 \\ & \times & 9 & 3 & 9 \\ \hline & 3\,7 & 8 & 7 & 3 \\ 6 & 5\,3 & 5 & 6 & \\ & 1 & & & \\ \hline 7 & 0\,1 & 4 & 3 & 3 \end{array}$$

or 747 × 939 = 701433

3) 192 × 341

<u>Step I</u>: 2 × 1 = 2

$$\begin{array}{ccccc} & & 1 & 9 & 2 \\ & \times & 3 & 4 & 1 \\ \hline & & & & 2 \end{array}$$

Step II: $1 \times 9 + 4 \times 2 = 17$

$$\begin{array}{rrrrr} & & 1 & 9 & 2 \\ & \times & 3 & 4 & 1 \\ \hline & & & 7 & 2 \\ & & 1 & & \end{array}$$

Step III: $1 \times 1 + 3 \times 2 + 9 \times 4 = 43$

$$\begin{array}{rrrrr} & & 1 & 9 & 2 \\ & \times & 3 & 4 & 1 \\ \hline & & 3 & 7 & 2 \\ & 4 & 1 & & \end{array}$$

Step IV: $1 \times 4 + 9 \times 3 = 31$

$$\begin{array}{rrrrr} & & 1 & 9 & 2 \\ & \times & 3 & 4 & 1 \\ \hline & 1 & 3 & 7 & 2 \\ 3 & 4 & 1 & & \end{array}$$

Step V: $1 \times 3 = 3$

$$\begin{array}{rrrrr} & & 1 & 9 & 2 \\ & \times & 3 & 4 & 1 \\ \hline 3 & 1 & 3 & 7 & 2 \\ 3 & 4 & 1 & & \\ \hline 6 & 5 & 4 & 7 & 2 \end{array}$$

or $192 \times 341 = 65472$

4) 237×83. We can write 83 as 083 to make it a 3-digit number

Step I: $7 \times 3 = 21$

$$\begin{array}{rrrrr} & & 2 & 3 & 7 \\ & \times & 0 & 8 & 3 \\ \hline & & & & 1 \\ & & & 2 & \end{array}$$

Step II: $3 \times 3 + 8 \times 7 = 65$

$$\begin{array}{rrrrr} & & 2 & 3 & 7 \\ & \times & 0 & 8 & 3 \\ \hline & & & 5 & 1 \\ & & 6 & 2 & \end{array}$$

<u>Step III</u>: $2 \times 3 + 0 \times 7 + 3 \times 8 = 30$

$$\begin{array}{rrrrr} & & 2 & 3 & 7 \\ & \times & 0 & 8 & 3 \\ \hline & & & 5 & 1 \\ & 3 & 6 & 2 & \end{array}$$

<u>Step IV</u>: $2 \times 8 + 0 \times 3 = 16$

$$\begin{array}{rrrrr} & & 2 & 3 & 7 \\ & \times & 0 & 8 & 3 \\ \hline & 6 & 0 & 5 & 1 \\ 1 & 3 & 6 & 2 & \end{array}$$

<u>Step V</u>: $0 \times 2 = 2$

$$\begin{array}{rrrrr} & & 2 & 3 & 7 \\ & \times & 0 & 8 & 3 \\ \hline 0 & 6 & 0 & 5 & 1 \\ 1 & 3 & 6 & 2 & \\ \hline 1 & 9 & 6 & 7 & 1 \end{array}$$

or $237 \times 83 = 19671$

EXERCISE 1.3D

DIRECTIONS for questions: Perform the following multiplications using the technique outlined above—

1. $876 \times 143 =$
2. $139 \times 775 =$
3. $224 \times 446 =$
4. $332 \times 624 =$
5. $863 \times 732 =$
6. $153 \times 391 =$
7. $828 \times 283 =$
8. $734 \times 418 =$
9. $452 \times 704 =$
10. $945 \times 232 =$
11. $928 \times 473 =$
12. $367 \times 283 =$
13. $572 \times 947 =$
14. $328 \times 843 =$
15. $803 \times 631 =$

E) Miscellaneous Methods

To multiply (or divide) by 5: This is equivalent to multiplying (or dividing) by $\frac{10}{2}$. For example:

218 × 5 = 109 × 10 = 1090
1387 × 5 = 693.5 × 10 = 6935

428/5 = 856/10 = 85.6
1257/5 = 2514/10 = 251.4

This can be extended to multiplying or dividing by 25 as well (for example 347 × 225 = $\frac{347}{4}$ × 100 = 86.75 × 100 = 8675, while 724/25 = 724 × $\frac{4}{100}$ = $\frac{2896}{100}$ = 28.96)

To multiply by 11: This is equivalent to multiplying by ten and adding the original number. Thus we can do it in a single step as shown in the adjacent box.

Suppose we want to calculate 578 × 11

	5	7	8
5	12	15	8
5	13	5	8

Step I: keep the last digit as it is i.e. 8
Write 8 as the last digit of the answer

Step II: Add the last two digits
8 + 7 = 15 so write 5, carry 1

Step III: Add the first two digits
7 + 5 = 12 + 1 (carried) = 13; write 3, carry 1

Step IV: Write the first digit
5 + 1 (carried) = 6

So 578 × 11 = 6358
Similarly we can write 459 × 11 = 5049
This technique can be extended to bigger numbers as well (for example, 25473 × 11 = 280203). If you like, you can use the same basic logic to create a similar method for multiplying by 12 or for multiplying by 111, 1111 etc. For example:
427 × 111 = 47397
232 × 12 = 2784

To multiply numbers whose difference is even:
We know that $(a - b)(a + b) = a^2 - b^2$. So if we have to multiply two numbers which have an even difference, we can let the difference be 2b and call the numbers as (a − b) and (a + b). Let us look at some examples of this approach:
23 × 37 = (30 + 7)(30 − 7) = 900 − 49 = 851
26 × 18 = (22 + 4)(22 − 4) = 484 − 16 = 468
294 × 306 = (300 − 6)(300 + 6) = 90000 − 36 = 89964

EXERCISE 1.3E

I. Multiply the following pairs of numbers:

1) 11 × 16	2) 353 × 11	3) 67 × 101	4) 26 × 34
5) 50 × 112	6) 11 × 334	7) 76 × 55	8) 87 × 29
9) 61 × 69	10) 59 × 44	11) 333 × 999	12) 236 × 64
13) 593 × 607	14) 131 × 476	15) 345 × 111	16) 131 × 313
17) 285 × 607	18) 697 × 703	19) 34 × 10101	20) 228 × 269

F) Approximation

We don't always need an exact or accurate answer. A reasonable approximation is good enough, in many DI-style questions. So let's try to understand how to approach this, through some examples:

a) 7436 × 2507

Let's approximate this as 7436 × 2500. (Note that the error introduced here is 7/2500 H" 0.28%)
Now 25 = 100/4 so let's do it as 7436 × 10000/4 = **18590000**
The exact answer is **18642052**, so we have an error of just around 5/1800

b) 5872 × 2176

Neither number is very close to an "easy" round number. Let's analyse this then. First of all, if I want to find a × b and a is increased, what should happen to b to keep the value of a × b same? It should logically decrease. So here if we change, say, the second number from 2176 to 2000, we will need to increase the first number. But by how much?
One tempting thing would be to increase by the same value (i.e. 176) but a little thinking will show us that this doesn't make sense. Let me demonstrate using a smaller example: if we want to do 51 × 11, reducing 11 to 10 and increasing 51 to 52 will give us 520 – the correct answer is 561.
The thing to realise is that instead of changing by the same **quantity** in opposite directions, we should change by the same **percentage**. So since 51 is roughly 5 times 11, the change also should be 5 times. Thus if we were to change 11 (by subtracting 1) to 10, we must change 51 (by adding 5) to 56, and with this we would get 560, a pretty accurate answer.

Coming back to our original problem then, 5872 is very roughly 3 times 2176 and so if we change 2176 by –176 $\approx$ –180, we should change 5872 by ~ +540 to 6412. Hence our approximation become 6412 × 2000 = **12824000**
The exact answer is **12777472**, so we have an error of around 5/1200.

EXERCISE 1.3F

I. Pick the closest answer:

1) The value of **7.1 × 4.49** is closest to
(a) 32.7 (b) 30.1 (c) 31.9 (d) 33.5

2) The value of **67.62 × 4.47** is closest to
(a) 302.3 (b) 289.1 (c) 274.7 (d) 265.3

3) The value of **11.45 × 60.73** is closest to
(a) 717.3 (b) 645.8 (c) 672.4 (d) 695.4

4) The value of **39.95 × 89.08** is closest to
(a) 3234.5 (b) 3456.7 (c) 3558.7 (d) 3682.3

5) The value of **87.15 × 22.16** is closest to
(a) 1931.2 (b) 2021.2 (c) 1757.4 (d) 1291.8

6) The value of **97.51 × 92.65** is closest to
(a) 8887.7 b) 9034.3 (c) 9293.2 (d) 8907.9

7) The value of **28.52 × 49.7** is closest to
(a) 1714.7 (b) 1474.1 (c) 1741.7 (d) 1417.4

8) The value of **24.09 × 2.26** is closest to
(a) 49.7 (b) 54.4 (c) 52.9 (d) 56.3

9) The value of **50.06 × 33.36** is closest to
(a) 1645.2 (b) 1627.1 (c) 1670.0 (d) 1608.1

10) The value of **23.78 × 42.46** is closest to
(a) 899.8 (b) 1009.7 (c) 1112.3 (d) 1078.8

II. Pick the closest answer:

1) The value of **78.51 × 95.84** is closest to
(a) 7524.4 (b) 7245.6 (c) 6958.7 (d) 7879.2

2) The value of **33.47 × 44.42** is closest to
(a) 1486.7 (b) 1554.6 (c) 1599.1 (d) 1610.1

3) The value of **85.73 × 25.74** is closest to
(a) 2025.1 (b) 2109.9 (c) 2370.2 (d) 2206.7

4) The value of **94.98 × 2.09** is closest to
(a) 189.8 (b) 187.2 (c) 198.5 (d) 207.3

5) The value of **67.65 × 70.94** is closest to
(a) 4585.2 (b) 4799 (c) 4882.3 (d) 4648.3

6) The value of **17.06 × 63.13** is closest to
(a) 998.9 b) 1077.0 (c) 1147.6 (d) 1008.3

7) The value of **30.47 × 2.87** is closest to
(a) 87.4 (b) 81.6 (c) 83.8 (d) 91.1

8) The value of **65.91 × 23.37** is closest to
(a) 1435.1 (b) 1328.9 (c) 1717.2 (d) 1540.3

9) The value of **25.55 × 14.39** is closest to
(a) 312.2 (b) 347.4 (c) 397.9 (d) 367.7

10) The value of **76.77 × 35.59** is closest to
(a) 2610.7 (b) 2992 (c) 2732.2 (d) 2813.4

1.4 DIVISION TECHNIQUES

Several times data interpretation questions require calculation of ratios.
Suppose the following table shows the sales of four products A, B, C and D of a company.

Product	Sales (in Rs.)
A	17,312,283
B	32,021,917
C	8,399,120
D	12,019,776
Total	69,753,096

If you have to calculate percent contribution of product A to the total sale, you will have to perform complex calculation $\frac{17,312,283}{69,753,096}$. At times you don't have to calculate the answer to two decimal places accuracy and only an approximate answer suffices. In this chapter, we will study a very powerful approximation technique.

In order to use this method, we need to know that 10% of a number is obtained by moving the decimal by one position to the left. For example 10% of 483 = 48.3.

Similarly 1% of a number is obtained by moving the decimal by two positions to the left. For example 1% of 483 = 4.83

5% of a number can be obtained by multiplying 1% by 5 or by dividing 10% by 2. For example, 5% of 483 = 1% of 483 × 5 = 4.83 × 5 = 24.15 = $\frac{10\% \text{ of } 483}{2} = \frac{48.2}{2}$ = 24.15.

Similarly 0.5% of 483 = 5 × 0.1% of 483 = 5 × 0.483 = 2.415

Once we know 5% of a number, other percentages such as 6%, 7% etc can be calculated by adding 1%, 2% etc. to 5%. Alternately 6% can be calculated by multiplying 1% by 6.

For example, calculate 7% of 1296

$$\begin{array}{rcr} 1\% \text{ of } 1296 & = & 12.96 \\ \underline{\times \qquad 7} & = & \underline{\times \quad 7} \\ 7\% \text{ of } 1296 & & 90.72 \end{array}$$

or

$$\begin{array}{rrcr} & 10\% \text{ of } 1296 & = & 129.6 \\ \therefore & 5\% \text{ of } 1296 & = & 64.8 \\ + & \underline{1\% \text{ of } 1296} & = & \underline{12.96} \\ & 6\% \text{ of } 1296 & = & 77.76 \\ + & \underline{1\% \text{ of } 1296} & = & \underline{12.96} \\ & 7\% \text{ of } 1296 & = & 90.72 \end{array}$$

or

$$\begin{array}{rrcr} & 10\% \text{ of } 1296 & = & 129.6 \\ - & \underline{1\% \text{ of } 1296} & = & \underline{12.96} \\ & 9\% \text{ of } 1296 & = & 116.64 \\ - & \underline{1\% \text{ of } 1296} & = & \underline{-12.96} \\ & 8\% \text{ of } 1296 & = & 103.68 \\ - & \underline{1\% \text{ of } 1296} & = & \underline{-12.96} \\ & 7\% \text{ of } 1296 & = & 90.72 \end{array}$$

Solved Examples

1) **Calculate** $\frac{1874}{7126}$

You can see that 10% of 7126 = 712.6. Therefore 20% of 7126 = 2 × 712.6 = 1425.2 and 30% of 7126 = 3 × 712.6 = 2137.8.

Thus you can see that $\frac{1874}{7126}$ lies between 20% and 30%.

1425.2   1874   2137.8

20%   30%

From visual observation you can see that 1874 is more than 400 away from 20% (1425.2) and about 250 away from 30% (2137.8).

$$\begin{array}{rrcr} & 10\% \text{ of } 7126 & = & 712.6 \\ \therefore & 5\% \text{ of } 7126 & = & 356.3 \\ + & \underline{20\% \text{ of } 7126} & = & \underline{1425.2} \\ & 25\% \text{ of } 7126 & = & 1781.5 \end{array}$$

1781.5   1874   2137.8

25%   30%

Now,

$$\begin{array}{rrcr} & 1\% \text{ of } 7126 & = & 712.6 \\ + & \underline{25\% \text{ of } 7126} & = & \underline{1781.50} \\ & 26\% \text{ of } 7126 & = & 1852.76 \\ + & \underline{1\% \text{ of } 7126} & = & \underline{71.26} \\ & 27\% \text{ of } 7126 & = & 1923.98 \end{array}$$

1852.76   1874   1923.98

26%   27%

Therefore $\frac{1874}{7126}$ lies between 26% and 27%.

Now,

$$\begin{array}{rrcr} & 1\% \text{ of } 7126 & = & 71.26 \\ \therefore & 0.5\% \text{ of } 7126 & = & 35.63 \\ + & \underline{26\% \text{ of } 7126} & = & \underline{1852.76} \\ & 26.5\% \text{ of } 7126 & = & 1888.39 \end{array}$$

1852.76   1874   1888.39

26%   26.50%

Thus you can see that $\frac{1874}{7126}$ lies between 26% and 26.5%.

Now,

$$\begin{array}{rrcr} & 0.5\% \text{ of } 7126 & = & 35.63 \\ \therefore & 0.25\% \text{ of } 7126 & = & 17.815 \\ + & 26\% \text{ of } 7126 & = & 1852.76 \\ \hline & 26.25\% \text{ of } 7126 & = & 1870.575 \end{array}$$

1870.575 1874 1888.39

26.25% 26.50%

Now,

$$\begin{array}{rrcr} & 0.5\% \text{ of } 7126 & = & 35.63 \\ \therefore & 0.05\% \text{ of } 7126 & = & 3.563 \\ + & 26.25\% \text{ of } 7126 & = & 1870.575 \\ \hline & 26.30\% \text{ of } 7126 & = & 1874.138 \end{array}$$

$$\therefore \frac{1874}{7126} \approx 26.30\%.$$

Note that depending on requirements you can decide what level of approximation is suitable i.e., if the answer between 20% and 30% is good enough or between 25% and 30% is good enough or between 26% and 27% is good enough or if you have to go for level of accuracy upto two decimal places (26.30%)

2) $\frac{27132}{59340}$

$$\begin{array}{rrcrl} & 10\% \text{ of } 59340 & = & 5934 & \\ \therefore & 40\% \text{ of } 59340 & = & 23736 & \text{and} \\ & 50\% \text{ of } 59340 & = & 29670 & \end{array}$$

Therefore $\frac{27132}{59340}$ is between 40% and 50%.

Now,

$$\begin{array}{rrcr} & 10\% \text{ of } 59340 & = & 5934 \\ \therefore & 5\% \text{ of } 59340 & = & 2967 \\ & 40\% \text{ of } 59340 & = & 23736 \\ \hline & 45\% \text{ of } 59340 & = & 26703 \end{array}$$

Therefore $\frac{27132}{59340}$ is between 45% and 50%.

$$\begin{array}{rrcr} & 1\% \text{ of } 59340 & = & 593.40 \\ + & 45\% \text{ of } 59340 & = & 26703 \\ \hline & 46\% \text{ of } 59340 & = & 27296.40 \end{array}$$

Therefore $\frac{27132}{59340}$ is between 45% and 46%.

Now,

$$\begin{array}{rlcr}
 & 46\% \text{ of } 59340 & = & 27296.40 \\
- & \underline{0.1\% \text{ of } 59340} & = & \underline{-\ 59.34} \\
 & 45.9\% \text{ of } 59340 & = & 27237.06 \\
- & \underline{0.1\% \text{ of } 59340} & = & \underline{-\ 59.34} \\
 & 45.8\% \text{ of } 59340 & = & 27177.72 \\
- & \underline{0.1\% \text{ of } 59340} & = & \underline{-\ 59.34} \\
 & 45.7\% \text{ of } 59340 & = & 27118.38
\end{array}$$

Therefore $\frac{27132}{59340}$ is between 45.7% and 45.8%.

Once you practise this technique, you will not have to write all the intermediate steps and carry out calculations as follows—

3) $\frac{13562}{42398}$

$$\begin{array}{rlcr}
 & 10\% \text{ of } 42398 & \approx & 4240 \\
\therefore & 30\% \text{ of } 42398 & \approx & 12720 \\
+ & \underline{1\% \text{ of } 42398} & & \underline{+\ 424} \\
 & 31\% \text{ of } 42398 & \approx & 13144 \\
+ & \underline{1\% \text{ of } 42398} & & \underline{+\ 424} \\
 & 32\% \text{ of } 42398 & \approx & 13568 \\
- & \underline{0.01\% \text{ of } 42398} & & \underline{-\ 4.24} \\
 & 31.99\% \text{ of } 42398 & & 13563.78
\end{array}$$

$\therefore \frac{13562}{42398} \approx 31.99\%.$

4) $\frac{73132}{118243}$

$$\begin{array}{rlcr}
 & 10\% \text{ of } 118243 & \approx & 11824 \\
\therefore & 60\% \text{ of } 118243 & \approx & 70944 \\
 & \underline{1\% \text{ of } 118243} & \approx & \underline{+\ 1182.4} \\
 & 61\% \text{ of } 118243 & \approx & 72126.4 \\
 & \underline{1\% \text{ of } 118243} & \approx & \underline{+\ 1182.4} \\
 & 62\% \text{ of } 118243 & \approx & 73308.8 \\
- & \underline{0.1\% \text{ of } 118243} & \approx & \underline{-\ 118.4} \\
 & 61.9\% \text{ of } 118243 & \approx & 73190.56 \\
- & \underline{0.05\% \text{ of } 118243} & \approx & \underline{-\ 59.12} \\
 & 61.85\% \text{ of } 118243 & \approx & 73131.44
\end{array}$$

$\therefore \frac{73132}{118243} \approx 61.85\%.$

5) $\dfrac{18412}{8367}$

$$
\begin{array}{rlcr}
 & 10\% \text{ of } 83670 & = & 8367 \\
\therefore & 20\% \text{ of } 83670 & = & 16734 \\
+ & 1\% \text{ of } 83670 & \approx & 836.70 \\
\hline
 & 21\% \text{ of } 83670 & = & 17570.70 \\
+ & 1\% \text{ of } 83670 & \approx & 836.70 \\
\hline
 & 22\% \text{ of } 83670 & = & 18407.40 \\
+ & 0.005\% \text{ of } 83670 & = & 4.1835 \\
\hline
 & 22.005\% \text{ of } 83670 & = & 18411.5835
\end{array}
$$

$$\therefore \frac{18412}{83670} \approx 22.005\% \approx 0.22005$$

$$\therefore \frac{18412}{8367} = \frac{18412}{83670} \times 10 \approx 0.22005 = 2.2005$$

Alternatively

$$\frac{18412}{8367} = \frac{16734 + 1678}{8367} = 2 + \frac{1678}{8367}$$

$$
\begin{array}{rlcr}
 & 10\% \text{ of } 8367 & = & 836.7 \\
 & 20\% \text{ of } 8367 & = & 1673.4 \\
+ & 0.05\% \text{ of } 8367 & = & 4.18 \\
\hline
 & 20.05\% \text{ of } 8367 & = & 1677.58
\end{array}
$$

$$\therefore \frac{1678}{8367} \approx 20.05\%$$

$$\therefore \frac{18412}{8367} \approx 2 + 20.05\% \approx 2.2005$$

6) $\dfrac{68385}{231418}$

$$
\begin{array}{rlcr}
 & 10\% \text{ of } 231418 & \approx & 23142 \\
\therefore & 30\% \text{ of } 231418 & \approx & 69426 \\
- & 1\% \text{ of } 231418 & - & 2314 \\
\hline
 & 29\% \text{ of } 231418 & \approx & 67112 \\
+ & 0.5\% \text{ of } 231418 & & 1157 \\
\hline
 & 29.5\% \text{ of } 231418 & \approx & 68269 \\
+ & 0.05\% \text{ of } 231418 & & 115.7 \\
\hline
 & 29.55\% \text{ of } 231418 & \approx & 68384.7
\end{array}
$$

$$\therefore \frac{68385}{231418} \approx 29.55\%$$

EXERCISE 1.4

DIRECTIONS for questions: Find the range between which the following ratios lie to 0.1% accuracy (For example, ratio $\frac{1873}{6147} \approx$ 30.47% i.e. between 30.4% and 30.5%)

1. 2888 ÷ 8353 is between _______ and _______
2. 4556 ÷ 5043 is between _______ and _______
3. 1244 ÷ 8291 is between _______ and _______
4. 1985 ÷ 5713 is between _______ and _______
5. 3637 ÷ 6981 is between _______ and _______
6. 5760 ÷ 8849 is between _______ and _______
7. 9048 ÷ 67636 is between _______ and _______
8. 5933 ÷ 44458 is between _______ and _______
9. 8242 ÷ 47539 is between _______ and _______
10. 75413 ÷ 87162 is between _______ and _______
11. 19483 ÷ 59558 is between _______ and _______
12. 57934 ÷ 73831 is between _______ and _______
13. 17892 ÷ 38413 is between _______ and _______
14. 82313 ÷ 417168 is between _______ and _______
15. 52918 ÷ 63667 is between _______ and _______

1.5 Division: Another Approximation Method

We can also use a method similar to that used for approximate multiplication. Again, let's try to understand how to approach this, through some examples:

1) Suppose we need to approximate $\frac{802}{489}$

In the case of multiplication, we could adjust either of the numbers. But here, it makes sense to make the denominator a nice round number. Making the numerator a nice friendly number won't help if the denominator is difficult! So let us first round the denominator off, to the nearest number whose table we know (in this case 500)

Now we need to change the numerator. In division, if we increase one of the numbers, we need to increase the other as well to keep the value the same, while if we decrease one, we need to decrease the other. (Note the contrast to multiplication, where we needed to change the numbers in opposite directions!)

So since we have increased the denominator from 489 to 500, we also need to increase the numerator. By how much? Again, in the same ratio. We can either use percentages (as we did in multiplication) or approximate the ratio of the original numbers by using the first two digits as follows: $\frac{802}{489}$ ~ $\frac{80}{48}$ ~ $\frac{5}{3}$.

We have changed the denominator by +11. So we'll change the numerator by around $\frac{5}{3}$ × (+11) = +18. Now the number becomes $\frac{802+18}{489+11}$ = $\frac{820}{500}$. Now this is an easy calculation which yields **1.64** (the exact value, for comparison, is **1.6401**)

2) Suppose we need to approximate $\frac{983}{724}$

First let us make the denominator 700.

The rough ratio is $\frac{983}{724}$ ~ $\frac{98}{72}$ ~ $\frac{4}{3}$.

We have changed the denominator by –24. So we'll change the numerator by around $\frac{4}{3}$ × (–24) = –32.

Now the number becomes $\frac{983-32}{724-24}$ = $\frac{951}{700}$ ~ **1.358** (the exact value, for comparison, is **1.3577**)

We can also use this for 4 digit numbers (and with a little practice, larger ones as well:

3) Suppose we need to approximate $\frac{3748}{5711}$

First let us make the denominator 6000.

The rough ratio is $\frac{3748}{5711} \sim \frac{37}{57} \sim \frac{2}{3}$.

Change in denominator is around +300. So change the numerator by around $\frac{2}{3} \times (+300) = +200$.

Now the number becomes $\frac{3748+200}{5711+300} = \frac{3948}{6000}$ ~ **0.658** (the exact value, for comparison, is **0.6563**)

4) Suppose we need to approximate $\frac{1475}{8403}$

First let us make the denominator 8000.

The rough ratio is $\frac{1475}{8403} \sim \frac{14}{84} \sim \frac{1}{6}$.

Change in denominator is around –400. So change the numerator by around $\frac{1}{6} \times (-400) = -66$.

Now the number becomes $\frac{1475-66}{8403-400} = \frac{1409}{8000}$ ~ **0.176** (the exact value, for comparison, is **0.1755**)

5) Suppose we need to approximate $\frac{5349}{2570}$

First let us make the denominator 2500.

The rough ratio is $\frac{5349}{2570} \sim \frac{53}{25} \sim \frac{2}{1}$.

Change in denominator is around –70. So change the numerator by around $\frac{2}{1} \times (-70) = -140$.

Now the number becomes $\frac{5349-140}{2570-70} = \frac{5209}{2500}$ ~ **2.08** (the exact value, for comparison, is **2.0813**)

EXERCISE 1.5A

1) The value of $\frac{730}{131}$ is closest to

(a) 5.689 (b) 5.721 (c) 5.979 (d) 5.572

2) The value of $\frac{164}{745}$ is closest to

(a) 0.2112 (b) 0.2079 (c) 0.2201 (d) 0.2345

3) The value of $\frac{752}{343}$ is closest to

(a) 1.978 (b) 2.411 (c) 2.321 (d) 2.192

4) The value of $\frac{651}{378}$ is closest to

(a) 1.695 (b) 1.722 (c) 1.763 (d) 1.795

5) The value of $\frac{795}{399}$ is closest to

(a) 1.992 (b) 2.036 (c) 2.133 (d) 1.778

6) The value of $\frac{875}{105}$ is closest to

(a) 8.333 b) 8.566 (c) 8.679 (d) 8.207

7) The value of $\frac{561}{294}$ is closest to

(a) 1.867 (b) 1.843 (c) 1.822 (d) 1.908

8) The value of $\frac{194}{882}$ is closest to

(a) 2.199 (b) 0.235 (c) 0.219 (d) 2.356

9) The value of $\frac{755}{758}$ is closest to

(a) 0.966 (b) 1.013 (c) 0.919 (d) 0.996

10) The value of $\frac{878}{304}$ is closest to

(a) 2.752 (b) 2.888 (c) 2.626 (d) 2.473

EXERCISE 1.5B

1) The value of $\frac{7444}{4820}$ is closest to

(a) 1.376 (b) 1.544 (c) 1.423 (d) 1.616

2) The value of $\frac{3874}{4575}$ is closest to

(a) 0.8467 (b) 0.8892 (c) 0.9124 (d) 0.9357

3) The value of $\frac{4415}{9479}$ is closest to

(a) 0.4143 (b) 0.4344 (c) 0.4657 (d) 0.4949

4) The value of $\frac{6446}{1494}$ is closest to

(a) 0.4324 (b) 4.835 (c) 4.314 (d) 0.4835

5) The value of $\frac{8486}{3467}$ is closest to

(a) 2.123 (b) 2.756 (c) 2.637 (d) 2.448

6) The value of $\frac{9726}{1620}$ is closest to

(a) 6.004 b) 5.893 (c) 6.121 (d) 6.237

7) The value of $\frac{8111}{9724}$ is closest to

(a) 0.8114 (b) 0.8341 (c) 0.8463 (d) 0.8528

8) The value of $\frac{2312}{1144}$ is closest to

(a) 1.892 (b) 2.021 (c) 1.933 (d) 1.981

9) The value of $\frac{6022}{2768}$ is closest to

(a) 2.176 (b) 2.344 (c) 2.531 (d) 2.292

10) The value of $\frac{5519}{1935}$ is closest to

(a) 2.852 (b) 2.992 (c) 2.732 (d) 3.003

ANSWERS

EXERCISE 1.1

1. 693	2. 1556	3. 1047	4. 6712
5. 8734	6. 16925	7. 154132	8. 138942
9. 136966	10. 22127	11. 81819	12. 99034
13. 1378	14. 10569	15. 55274	

EXERCISE 1.2

1. 395	2. 193	3. 317	4. 4497
5. 4307	6. 938	7. 23595	8. 48759
9. 25544	10. 7856	11. 22794	12. 11751
13. 24486	14. 31689	15. 13949	

EXERCISE 1.3A

1. 8924	2. 11760	3. 10185	4. 985054
5. 1000958	6. 1011024	7. 7998	8. 9152
9. 12987	10. 984048	11. 1000944	12. 1033266
13. 983066	14. 1000944	15. 1024108	

EXERCISE 1.3B

1. 1634	2. 2968	3. 6156	4. 3717
5. 5254	6. 3776	7. 157976	8. 86136
9. 248446	10. 652044	11. 345154	12. 476091
13. 89658	14. 162782	15. 250428	

EXERCISE 1.3C

1. 4272	2. 6566	3. 2368	4. 4108
5. 4788	6. 4648	7. 1584	8. 2584
9. 3071	10. 1357	11. 6552	12. 1638
13. 1826	14. 4914	15. 2132	

EXERCISE 1.3D

1. 125268	2. 107725	3. 99904	4. 207168
5. 631716	6. 59823	7. 234324	8. 306812
9. 318208	10. 219240	11. 438944	12. 103861
13. 541684	14. 276504	15. 506693	

Exercise 1.3.E:

1. 176	2. 3883	3. 6767	4. 884	5. 5600
6. 3674	7. 4180	8. 2523	9. 4209	10. 2596
11. 332667	12. 15104	13. 359951	14. 62356	15. 38295
16. 41003	17. 172995	18. 489991	19. 343434	20. 61332

Exercise 1.3.F.I:

1. [c].	2. [a].	3. [d].	4. [c].	5. [a].
6. [b].	7. [d].	8. [b].	9. [c].	10. [b].

Exercise 1.3.F.II:

1. [a].	2. [a].	3. [d].	4. [c].	5. [b].
6. [b].	7. [a].	8. [d].	9. [d].	10. [c].

EXERCISE 1.4

1. 34.5% and 34.6%	2. 90.3% and 90.4%
3. 15.2% and 15.3%	4. 34.7% and 34.8%
5. 52.0% and 52.1%	6. 65.0% and 65.1%
7. 13.3% and 13.4%	8. 13.3% and 13.4%
9. 17.3% and 17.4%	10. 86.5% and 86.6%
11. 32.7% and 32.8%	12. 78.4% and 78.5%
13. 46.5% and 46.6%	14. 19.7% and 19.8%
15. 83.1% and 83.2%	

Exercise 1.5.A:

1. [d]	2. [c]	3. [d]	4. [b]	5. [a]
6. [a]	7. [d]	8. [c]	9. [d]	
10. [b]				

Exercise 1.5.B:

1. [b]	2. [a]	3. [c]	4. [c]	5. [d]
6. [a]	7. [b]	8. [b]	9. [a]	
10. [a]				

2 | ADVANCED OPERATIONS

2.1 Square numbers

If a number is multiplied by itself, the product so obtained is called the square of that number.

Example

Square of 7 is $7 \times 7 = 49$ or $7^2 = 49$

The square of a natural number is called ***Perfect square.***

Students are expected to know all the squares of numbers up-to 30. They are given in the table below:

$1^2 = 1$	$6^2 = 36$	$11^2 = 121$	$16^2 = 256$	$21^2 = 441$	$26^2 = 676$
$2^2 = 4$	$7^2 = 49$	$12^2 = 144$	$17^2 = 289$	$22^2 = 484$	$27^2 = 729$
$3^2 = 9$	$8^2 = 64$	$13^2 = 169$	$18^2 = 324$	$23^2 = 529$	$28^2 = 784$
$4^2 = 16$	$9^2 = 81$	$14^2 = 196$	$19^2 = 361$	$24^2 = 576$	$29^2 = 841$
$5^2 = 25$	$10^2 = 100$	$15^2 = 225$	$20^2 = 400$	$25^2 = 625$	$30^2 = 900$

Shortcut method of calculation of squares of numbers

A) Square of a number ending in 5

Multiply the number of tens by the next higher integer and annex 25 to the right of the product.

Example

55^2 :Number of tens : 5. $5 \times 6 = 30$. Hence, squre is 3025.

Exercise 2.1.A

Calculate squares of the following numbers

1) 35 2) 65 3) 85 4) 115 5) 125

B) Square of a number ending in 1

Following is the process of calculation of its square –

i) Units place in the square is one.

ii) Ten's place in the square is equal to two multiplied by the tens' place in the number whose square we wish to calculate.

iii) Hundreds' place in the square is equal to the square of the tens' place of the number whose square we wish to calculate.

1) Given number : 41.
Step 1: Units place in the square is 1.
Step 2: Tens' place in the square $= 2 \times 4 = 8$

Step 3: Hundreds' place in the square = 4^2 = 16 or 1 is carried forward to thousands place.

∴ Square of 41 is 1681

2) Given number : 71.

Step 1: Units place in the square is 1.

Step 2: Tens place in the square = 2 × 7 = 14 or tens' place is 4 and 1 is carried forward to hundreds' place.

Step 3: Hundred's place in the square = 7^2 = 49. Including 1 carried forward from tens' place, hundred's place = 49 + 1 = 50 or hundreds' place is 0 and 5 is carried forward to thousand's place.

∴ Square of 71 is 5041.

Exercise 2.1.B

Calculate squares of the following numbers

1) 31 2) 51 3) 61 4) 91 5) 121

C) Square of a number close to 100 but less than 100

Following is the process

i) Calculate the difference between given number and 100.

ii) Deduct the difference between given number and 100 from the given number.

iii) Annex square of the difference to the number calculated in step (ii). If the square is a three digit number carry forward the hundreds' place to the number calculated in step (ii). If it is a one digit number. precede it with zero.

1) Given number : 96.

Step 1: Difference between 100 and 96 is 4.

Step 2: 96 – 4 = 92

Step 3: 4^2 = 16

∴ Required square is 9216.

2) Given number: 89.

Step 1: Difference between 100 and 89 is 11.

Step 2: 89 – 11 = 78

Step 3: 11^2 is 121.

∴ 1 is carried forward.

∴ Required square is (78 + 1)21 or 7921

Exercise 2.1.C

Calculate squares of the following numbers

1) 98 2) 97 3) 93 4) 87 5) 84

D) Square of a number close to 100 but greater than 100

Following is the process

Step 1: Calculate the difference between given number and 100.

Step 2: Add the difference between given number and 100 to the given number.

Step 3: Annex square of the difference to the number calculated in step (ii). If the square is a three digit number, carry forward the hundreds' place to the number calculated in step (ii). If it is a one digit number, precede it with zero.

1) Given number : 106.
   Step 1: Difference between 100 and 106 is 6.
   Step 2: 106 + 6 = 112
   Step 3: $6^2 = 36$.
   $\therefore$ Required square is 11236.
2) Given number : 113.
   Step 1: Difference between 113 and 100 is 13.
   Step 2: 113 + 13 = 126
   Step 3: $13^2 = 169$
   $\therefore$ 1 is carried forward.
   $\therefore$ Required square is (126 + 1)69 or 12769.

Exercise 2.1.D

Calculate squares of the following numbers

1) 103 2) 107 3) 109 4) 114 5) 117

E) Square of a number close to 50 but smaller than 50

Process is as follows:

i) Calculate the difference between given number and 50.
ii) Subtract the difference from 25.
iii) Annex square of the difference to the number obtained in step (ii).

1) Given number : 47
   Step 1: Difference between 50 and 47 is 3.
   Step 2: 25 − 3 = 22
   Step 3: $3^2 = 9$
   $\therefore$ Required square = 2209
2) Given number = 38.
   Step 1: Difference between 50 and 38 is 12.
   Step 2: 25 − 12 = 13
   Step 3: $12^2 = 144$. 1 is carried forward.
   $\therefore$ Required square = (13 + 1)44 = 1444.

Exercise 2.1.E

Calculate squares of the following numbers

1) 44 2) 39 3) 47 4) 36 5) 42

F) Square of a number close to 50 but greater than 50

Following is the process:

i) Calculate difference between 30 and given number.
ii) Add the difference to 25.
iii) Annex the square of the difference to the number obtained in step (ii).

Examples:

1) Given number : 53
   Step 1: Difference between 40 and 53 is 3.
   Step 2: 25 + 3 = 28
   Step 3: $3^2 = 9$
   $\therefore$ Required square is 2809
2) Given number = 62.

Step 1: Difference between 50 and 62 is 12.
Step 2: $12^2 = 144$
Step 3: 1 is carried forward.
$\therefore$ Required square is (37 + 1)44 = 3844.

Exercise 2.1.F

Calculate squares of the following numbers

1) 52 2) 56 3) 58 4) 63 5) 57

G) Generalized method of calculation of a square of a two-digit number

Suppose given two digit number is (ab) or 10a + b.
Using algebraic formula, $(10a + b)^2 = 100a^2 + 20ab + b^2$
This can be denoted as $a^2/2ab/b^2$ where a^2 is the digit in hundreds place, 2ab is the digit in tens place and b^2 is the digit in units place. If any number so obtained has more than one digit, only rightmost digit is retained and the remaining digit is carried forward to the left.

Examples:

1) Given number : 69.
$\therefore$ $(69)^2 = 6^2/2 \times 6 \times 9/9^2$ or 36/108/81.
Carrying forward –
$(69)^2 = 36/(108 + 8)/1 = 36/116/1 = (36 + 11)/6/1$ or 4761
2) Given number : 83.
$(83)^2 = 8^2/2 \times 8 \times 3/3^2$
$(83)^2 = 64/48/9 = (64 + 4)/8/9 = 6889$

Exercise 2.1.G

Calculate squares of the following numbers

1) 82 2) 73 3) 68 4) 39 5) 79

H) Generalized method of calculation of a square of a three digit number

Suppose the given three digit number is (abc) of 100a + 10b + c
Using algebraic formula, $(100a + 10b + c)^2 = 10000a^2 + 100b^2 + c^2 + 2000ab + 200ac + 20bc$.
This can be expressed as $a^2/2ab/(b^2 + 2ac)/2bc/c^2$, where

a^2	:	Digit in ten thousands place.
2ab	:	Digit in thousands place.
$b^2 + 2ac$	:	Digit in hundreds place.
2bc	:	Digit in tens place.
c^2	:	Digit in units place.

If any number so obtained has more than one digit, only rightmost digit is retained and number formed by remaining digits is carried forward to the left part.

Examples:

1) Given number : 379
$\therefore$ $(379)^2 = 3^2/2 \times 3 \times 7/7^2 + 2 \times 3 \times 9/2 \times 7 \times 9/9^2$
= 9/42/103/126/81
= 9/42/103/(126 + 8)/1
= 9/42/103/134/1
= 9/42/(103 + 13)/4/1
= 9/42/116/4/1
= 9/(42 + 11)/6/4/1

= 9/53/6/4/1
= (9 + 5)/3/6/4/1
= 143641

2) Given number : 843.
$\therefore$ $(843)^2 = 8^2/(2 \times 8 \times 4)/(4^2 + 2 \times 8 \times 3)/(2 \times 4 \times 3)/3^2$
$\therefore$ $(843)^2$ = 64/64/64/24/9
$\therefore$ $(843)^2$ = 64/64/(64 + 2)/4/9
$\therefore$ $(843)^2$ = 64/64/66/4/9
$\therefore$ $(843)^2$ = 64/(64 + 6)/6/4/9
$\therefore$ $(843)^2$ = 64/70/6/4/9
$\therefore$ $(843)^2$ = (64 + 7)/0/6/4/9
$\therefore$ $(843)^2$ = 710649
Using techniques outlined above, you can calculate square of any number up-to 999.

Exercise 2.1.H

Calculate squares of the following numbers

1) 218	2) 447	3) 832	4) 392	5) 507
6) 622	7) 368	8) 482	9) 917	10) 772

Students are strongly advised to practice these short-cut methods of calculating squares. This will help in speedy calculations in Problem Solving as well as Data Interpretation questions.

2.2. Square Roots: Division method

Finding the square root of a number is also often useful. In some situations we may need to find the exact square root, while in others an approximate value will suffice, so let us examine both cases:

Finding Exact Square Roots: Suppose we wish to find the square root of 65536. We start by splitting the number in blocks of 2 digits, starting from the decimal point, as **6 55 36**:

Step 1: Now considering the first block i.e. 6, let us look for the largest number "a" such that a x a d" 6. This gives us a = 2. Subtracting $2 \times 2 = 4$ from 6 we get 2. Now we bring down the next block of two digits i.e. 55, giving us 255.

a	**6 55 36**
a	

Step 2: Meanwhile, we write 2 + 2 = 4 on the left. Now we need to add the largest digit "b" such that 4b x b $\leq$ 255. We find that b = 5 is the largest possible and so we subtract 45 x 5 = 225 from 255, leaving 30. Bringing down the next and last block of digits i.e. 36, we get 3036.

2	**6 55 36**
2	**− 4**
4 b	**2 55**
b	

Step 3: On the left, we write 45 + 5 = 0. Now we need to add the largest digit "c" such that $50c \times c \leq 3036$. We find that c = 6 is the largest possible and so we subtract $506 \times 6 = 3036$ from 3036, leaving 0.

2	**6 55 36**
2	**− 4**
4 5	**2 55**
5	**− 2 25**
	5 0 c
	30 36

Step 4: Once we arrive at a zero, we note all the digits which we had introduced i.e. a, b and c to get the answer 256. So $\sqrt{65536}$ = 256.

In similar fashion the square roots of a few other numbers are demonstrated below:

1	1 12 36
1	−1
20	0 12
0	−0
206	12 36
6	−12 36
	0

$\sqrt{11236}$ = 106

8	77 44
8	−64
168	13 44
8	−13 44
	0

$\sqrt{7744}$ = 88

1	1 46 41
1	−1
2 2	0 46
2	−44
2 4 1	2 41
1	−2 41
	0

$\sqrt{14641}$ = 121

2	5 47 56
2	−4
43	1 47
3	−1 29
464	18 56
4	−18 56
	0

$\sqrt{54756}$ = 234

In fact, the same approach can be used to find the square roots of numbers which are not perfect squares as well; we can just keep pairs of digits after the decimal point. For example, the square root of 2, 3.2, 0.5184 and 13 are computed below to a couple of decimal places:

1.	2. 00 00	1.	3. 20 00	0.	0. 51 84	3.	13. 00 00
1	−1	1	−1	0	−0	3	−9
24	1 00	27	2 20	07	0 51	66	4 00
4	−96	7	−1 89	7	−49	6	-3 96
281	4 00	348	31 00	142	2 84	720	4 00
1	−2 81	8	−27 84	2	−2 84	0	−0
	1 29	356.	3 16		0	720	4 00

$\sqrt{2}$ = 1.41…. $\sqrt{3.2}$ = 1.78… $\sqrt{0.5184}$ = 0.72 $\sqrt{13}$ = 3.60…

Finding Approximate Square Roots: Sometimes it is sufficient to be able to estimate roughly the value of a square root. This can be done by looking for the nearest perfect squares on either side of the number, and extrapolating linearly in between.

For example if we wish to estimate the square root of 234 we can observe $225 < 234 < 256$ and note that 234 is closer to 225 than to 256; thus we can say that the square root of 234 must lie between 15 and 16, and closer to 15, perhaps around 15.3 *(the actual answer is around 15.29)*.

Similarly if we wish to find the square root of 666 we can say $625 < 666 < 676$ and hence the answer must be between 25 and 26 and much closer to 26, say 25.8 *(the actual value is around 25.81)*

This can also be modified as required; for example if we want the square root of 7 we could say $4 < 7 < 9$ and hence between 2 and 3, but we could get more accurate than that by treating 7 as $\frac{700}{100}$ which would mean that $\sqrt{7} = \frac{\sqrt{700}}{\sqrt{100}} = \frac{\sqrt{700}}{\sqrt{10}}$. Now 700 lies between 676 and 729 and so $\sqrt{700}$ must lie almost midway between 26 and 27, say 26.5 Thus $\sqrt{7}$ must be around 2.65 *(the actual value is around 2.645)*

Similarly if we want the square root of 10, we can find $\sqrt{1000}$ by noting that 1000 lies between 961 and 1024 and hence $\sqrt{1000}$ lies between 31 and 32 and closer to 32, perhaps around 31.7. Thus $\sqrt{10}$ must be around 3.17 *(the actual value is around 3.162)*

It is worth remembering the approximate square roots of some small numbers, to save time:

$\sqrt{2} \approx 1.414$ $\sqrt{3} \approx 1.732$ $\sqrt{5} \approx 2.236$ $\sqrt{6} \approx 2.45$ $\sqrt{7} \approx 2.645$ $\sqrt{8} \approx 2.828$

$\sqrt{10} \approx 3.162$ $\sqrt{11} \approx 3.316$ $\sqrt{12} \approx 3.464$ $\sqrt{13} \approx 3.6$ $\sqrt{14} \approx 3.741$ $\sqrt{15} \approx 3.873$

EXERCISE 2.2.A

DIRECTIONS for questions: Compute the square roots of the following numbers:

1) 9604	2) 163216	3) 60025	4) 3249
5) 1156	6) 676	7) 5041	8) 8464
9) 17161	10) 4761	11) 2116	12) 44100
13) 24025	14) 8281	15) 841	16) 65536
17) 4489	18) 16384	19) 25921	20) 1089

EXERCISE 2.2.B

DIRECTIONS for questions: Compute the approximate square roots of the following numbers:

1) 145	2) 410	3) 95	4) 646
5) 472	6) 341	7) 156	8) 128
9) 746	10) 424	11) 652	12) 378
13) 999	14) 1223	15) 600	16) 420
17) 219	18) 185	19) 692	20) 786

2.3. Square roots: Alternate method for calculating approximate answer

Suppose you have to calculate square root of a number such as 130, which is not a perfect square. You can apply the following steps to calculate square roots of such numbers—

<u>Step I:</u> Identify the largest perfect square, which is smaller than the given number. In this case, the largest perfect square, which is smaller than 130 is 121. If we call that number as a, a = 121.

<u>Step II:</u> Calculate the difference between the given number and number 'a'. In this case, difference = 130 − 121 = 9

<u>Step III:</u> Required square root = $\sqrt{a} + \dfrac{\text{Difference calculated in step II}}{2\sqrt{a}}$

<u>Example</u>

1) Calculate $\sqrt{378}$

<u>Step I:</u> a = 361

<u>Step II:</u> Difference = 378 − 361 = 17

<u>Step III:</u> $\sqrt{378} = \sqrt{361} + \dfrac{17}{2 \times \sqrt{361}} = 19 + \dfrac{17}{2 \times 19} = 19 + \dfrac{17}{38} = 19.4473$

Accurate answer is 19.4422

2) Calculate $\sqrt{798}$

<u>Step I:</u> a = 784

<u>Step II:</u> Difference = 798 − 784 = 14

<u>Step III:</u> $\sqrt{798} = \sqrt{784} + \dfrac{14}{2 \times \sqrt{784}} = 28 + \dfrac{14}{2 \times 28} = 28.25$

Accurate answer is 28.2489

3) Calculate $\sqrt{1489}$

<u>Step I:</u> a = 1444

<u>Step II:</u> Difference = 1489 − 1444 = 45

<u>Step III:</u> $\sqrt{1489} = \sqrt{1444} + \dfrac{45}{2 \times \sqrt{1444}} = 38 + \dfrac{45}{2 \times 38} = 38 + \dfrac{45}{76}$

$\dfrac{45}{76} \approx 0.6$

Accurate answer is 38.5876

4) Calculate $\sqrt{5.1}$

When you want to calculate square root of a small number, it is advisable to calculate the square root of 100 times the number and dividing the square root so obtained by 10, in order to get better accuracy.

$\therefore \sqrt{5.1} = \dfrac{\sqrt{510}}{10}$

Step I: a = 484
Step II: Difference = 510 − 484 = 26

Step III: $\sqrt{510} = \sqrt{484} + \frac{26}{2 \times \sqrt{484}} = 22 + \frac{26}{2 \times 22} = 22 + \frac{13}{22} = 22.5909$

$\therefore \sqrt{5.1} = \frac{\sqrt{510}}{10} = \frac{22.5909}{10} = 2.25909$

As you can see, this method gives square roots to reasonable degree of accuracy. Accurate answer is 2.25831

EXERCISE 2.3

DIRECTIONS for questions: Calculate approximate square root of the following numbers (Calculate upto one decimal point)

1) 154	2) 326	3) 569	4) 4984
5) 1179	6) 5253	7) 2152	8) 7011
9) 4375	10) 3838	11) 6163	12) 7128
13) 8014	14) 8984	15) 9437	

ANSWERS

Exercise 2.1.A:

1. 1225	2. 4225	3. 7225	4. 13225	5. 15625

Exercise 2.1.B:

1. 961	2. 2601	3. 3721	4. 8281	5. 4641

Exercise 2.1.C:

1. 9604	2. 9409	3. 8644	4. 7569	5. 7056

Exercise 2.1.D:

1. 10609	2. 11449	3. 11881	4. 12996	5. 13689

Exercise 2.1.E:

1. 1936	2. 1521	3. 2209	4. 1296	5. 1764

Exercise 2.1.F:

1. 2704	2. 3136	3. 3364	4. 3969	5. 3249

Exercise 2.1.G:

1. 6724	2. 5329	3. 4624	4. 1521	5. 6241

Exercise 2.1.H:

1. 47524	2. 199809	3. 692224	4. 153664	5. 257049
6. 386884	7. 135424	8. 232324	9. 840889	10. 595984

Exercise 2.2.A:

1. 98	2. 404	3. 245	4. 57	5. 34
6. 26	7. 71	8. 92	9. 131	10. 69
11. 46	12. 210	13. 155	14. 91	15. 29
16. 256	17. 67	18. 128	19. 161	20. 33

Exercise 2.2.B:

1. 12.04	2. 20.25	3. 9.75	4. 25.42	5. 21.72
6. 18.47	7. 12.49	8. 11.31	9. 27.31	10. 20.59
11. 25.53	12. 19.44	13. 31.61	14. 34.97	15. 24.49
16. 20.49	17. 14.80	18. 13.60	19. 26.31	20. 28.04

Exercise 2.3:

1. 12.4	2. 18.1	3. 23.9	4. 70.6	5. 34.33
6. 72.5	7. 46.4	8. 83.7	9. 66.1	10. 61.9
11. 78.5	12. 84.4	13. 89.6	14. 94.8	15. 97.1

3 MISCELLANEOUS CALCULATIONS

3.1. Averages:

An **average** could be considered as a representative number for a set of values. The most commonly used types of averages are the Arithmetic Mean, the Geometric Mean and the Harmonic Mean.

For a set of values $a_1, a_2, a_3 \ldots a_n$ the **Arithmetic Mean (AM)** could be defined as a number K such that if each of the numbers $a_1, a_2, a_3 \ldots a_n$ were to be replaced by K, the ***sum*** would still be the same. That is, the AM is an ***additive*** mean.

$a_1 + a_2 + a_3 + \ldots + a_n = K + K + K \ldots$ n times $= n \times K$. Hence $AM = \dfrac{a_3 + \ldots + a_n}{n}$

For a set of values $a_1, a_2, a_3 \ldots a_n$ the **Geometric Mean (GM)** could be defined as a number K such that if each of the numbers $a_1, a_2, a_3 \ldots a_n$ were to be replaced by K, the ***product*** would still be the same. That is, the GM is a ***multiplicative*** mean.

$a_1 \times a_2 \times a_3 \times \ldots \times a_n = K \times K \times K \ldots$ n times $= K^n$. Hence $GM = \sqrt[n]{a_1 \times a_2 \times a_3 \times \ldots \times a_n}$

For a set of values $a_1, a_2, a_3 \ldots a_n$ the **Harmonic Mean (HM)** could be defined as a number K such that if each of the numbers $a_1, a_2, a_3 \ldots a_n$ were to be replaced by K, the ***sum of the reciprocals*** would still be the same. That is, the HM is a ***reciprocative*** mean.

$$\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + \ldots + \frac{1}{a_n} = \frac{1}{K} + \frac{1}{K} + \frac{1}{K} + \ldots + \frac{1}{K} = \frac{n}{K}.$$

Hence $HM = \dfrac{n}{\dfrac{1}{a_1} + \dfrac{1}{a_2} + \dfrac{1}{a_3} + \ldots + \dfrac{1}{a_n}}$

Tip: For two numbers a and b, we can show that their Harmonic Mean will be

The Assumed Average approach to finding an Arithmetic Mean

749	= 760	**–11**
786	= 760	**+26**
774	= 760	**+14**
769	= 760	**+9**
762	= 760	**+2**
Total	**= 760 × 5**	**+40**

∴ A.M. = 760 + = 768

If we want to find the average of a set of large numbers which are very close together, it often proves simpler to take a suitable round number (close to the given numbers) and measure only the deviations from there.

For example if we wish to average 749, 786, 774, 769 and 762, rather than adding up all the numbers, we could assume a reasonable starting value (say 760) and find the deviations from there (which are -11, +26, +14, +9 and +2). These deviations add up to +40 for 5 numbers, which means the average deviation per number is i.e. 8. Thus the required average will be 760 + 8 = **768.**

EXERCISE 3.1

1) Find the Arithmetic Mean of the first 15 natural numbers

2) Find the Arithmetic Mean of the first 7 perfect squares

3) Find the Geometric Mean of 25 and 49

4) Find the Geometric Mean of 4, 6 and 9

5) Find the Harmonic Mean of 2 and 6

6) Find the Harmonic Mean of 2, 3 and 6

7) Find the Harmonic Mean of 30 and 70

8) Find the difference between the Arithmetic and Harmonic Means of 20 and 30

9) Find the Geometric Mean of 64, 125 and 343

10) Find the Arithmetic Mean of 125, 136, 145, 154 and 162

11) Find the Arithmetic Mean of 589, 624, 613, 598 and 592

12) Find the Arithmetic Mean of 7561, 7546, 7572, 7558 and 7539

13) If the Arithmetic Mean of 67, 64, 65, 71 and x is 67, find x

14) If the Arithmetic Mean of 3586, 3575, 3578, 3550 and z is 3572, find z

15) If the Arithmetic Mean of a, b, c and d is 10, find the Arithmetic Mean of 2a, 2b, 2c and 2d

16) If the Arithmetic Mean of a, b, c and d is 6, find the Arithmetic Mean of a + b, b + c, c + d and d + a

17) If two distinct natural numbers have a Geometric Mean of 7, then find their Arithmetic Mean

18) If the Geometric Mean of a, b, c and d is 8, find the Geometric Mean of 2a, 2b, 2c and 2d

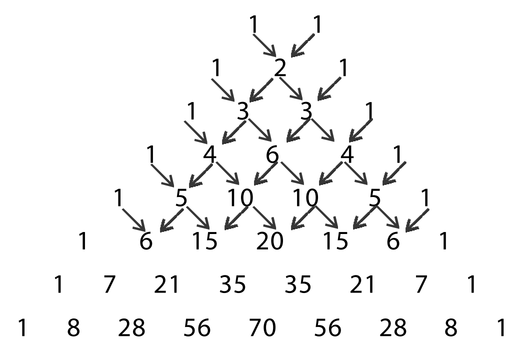
19) If the Geometric Mean of a, b, c and d is 5, find the Geometric Mean of a^2, b^2, c^2 and d^2

20) If the Geometric Mean of ab and cd is 9, find the Geometric Mean of ad, bc, ac and bd

3.2 Pascal's Triangle - Combinations

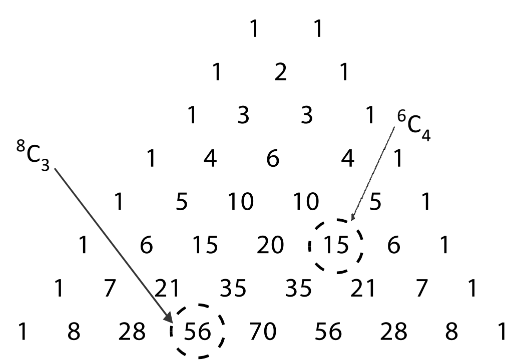
We know from the theory of combinatorics that the number of ways of selecting r out of n distinct objects, which in short is called ${}^nC_r$, can be found using the formula ${}^nC_r = \dfrac{n!}{r!(n-r)!}$. If we need to find a single ${}^nC_r$ value, then this doesn't take too much time. But on many occasions we are required to quickly find several ${}^nC_r$ values. In such a case, we can speed up the process by using Pascal's Triangle.

Pascal's Triangle is an infinite triangular array of numbers, starting with two '1's in the first row, and with each subsequent row containing one number more than the previous one. Each row starts and ends with a 1, and each number in between is computed by adding the two numbers diagonally above in the previous row as shown in the adjacent figure.



One of the most useful properties of Pascal's triangle is the fact that the elements can directly give us ${}^nC_r$ values. Specifically, the elements of the n^{th} row, read in order, will give us ${}^nC_0,\ {}^nC_1,\ {}^nC_2 \dots {}^nC_n$.

For example if we need ${}^6C_4$, we can go to the 6^{th} row and take the 5^{th} value (remember, we start from ${}^6C_0$!). Thus ${}^6C_4 = 15$.



In a similar manner we can find, say, ${}^8C_3 = 56$

From Pascal's triangle, we can also derive* several properties of ${}^nC_r$:

1) ${}^nC_0 = {}^nC_n = 1$ 2) ${}^nC_1 = {}^nC_{n-1} = n$
3) ${}^nC_r = {}^nC_{n-r}$ 4) ${}^nC_r = {}^{n-1}C_{r-1} + {}^{n-1}C_r$
5) ${}^nC_0 + {}^nC_1 + {}^nC_2 + \dots + {}^nC_{n-1} + {}^nC_n = 2^n$

(*The proof is left as an exercise for the reader)

Since Binomial expansion can be written using ${}^nC_r$ values, Pascal's triangle can also help to quickly work out basic binomial expansions as follows:

In general, we know that

$(a + b)^n = {}^nC_0\, a^n + {}^nC_1\, a^{n-1}b + {}^nC_2\, a^{n-2}b^2 + \dots + {}^nC_{n-1}\, ab^{n-1} + {}^nC_n\, b^n$

But since ${}^nC_0, {}^nC_1, {}^nC_2 \dots {}^nC_n$ are directly given by the n^{th} row of the triangle, we can quickly write the expansion using this table. For example for $(a + b)^5$ we could look at the 5^{th} row of the triangle and write:

$(a + b)^5 = 1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$

Similarly $(a + b)^7 = 1a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 35a^3b^4 + 21a^2b^5 + 7ab^6 + 1b^7$

Pascal's Triangle - Powers of 11

Using Pascal's triangle, we can also easily find out powers of 11; the value of 11^n can be obtained directly by reading off the n^{th} row. If a number in the triangle is greater than 10, however, it needs to be carried over.

$$\begin{array}{c} 1 \quad 1 \\ 1 \quad 2 \quad 1 \\ 1 \quad 3 \quad 3 \quad 1 \\ 1 \quad 4 \quad 6 \quad 4 \quad 1 \\ 1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1 \\ \boxed{1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6} \quad 1 \\ 1 \quad 7 \quad 21 \quad 35 \quad 35 \quad 21 \quad 7 \quad 1 \\ 1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \end{array}$$

For example, 11^6 is computed by taking the 6^{th} row i.e. 1 6 15 20 15 6 1 and adding up as shown alongside; thus 11^6 = 1771561.

This is particularly useful when calculating 10% compound interest (a fairly common question) as we need to calculate powers of $\left(1+\frac{10}{100}\right)^n = 1.1^n$

$$\begin{array}{ccccccc} 1 & - & - & - & - & - & - \\ & 6 & - & - & - & - & - \\ & 1 & 5 & - & - & - & - \\ & & 2 & 0 & - & - & - \\ & & & 1 & 5 & - & - \\ & & & & & 6 & - \\ & & & & & & 1 \\ \hline 1 & 7 & 7 & 1 & 5 & 6 & 1 \end{array}$$

So in 6 years at 10%, the C.I. earned is 77.1561%

EXERCISE 3.2

Directions for the questions: Compute the following.

1) $^6C_2$
2) $^7C_5$
3) $^9C_8$
4) $^6C_3$
5) $^7C_3$
6) $^9C_5$
7) $^6C_5$
8) $^8C_6$
9) $^{10}C_5$
10) $^5C_3$
11) $^8C_4$
12) $^{10}C_6$
13) $^5C_4$
14) $^9C_2$
15) $^{10}C_3$
16) 11^5
17) 11^3
18) 1.1^7
19) 1.1^6
20) 11^4
21) $^5C_2 + {}^6C_2 + {}^7C_2 + {}^8C_2$
22) $^6C_0 + {}^6C_1 + {}^6C_2 + {}^6C_3 + {}^6C_4 + {}^6C_5 + {}^6C_6$
23) $^7C_3 - {}^6C_3$
24) $^9C_5 + {}^9C_6 - {}^{10}C_6$
25) $^9C_0 + {}^9C_1 + {}^9C_2 + {}^9C_3 + {}^9C_4$
26) $^6C_4 \times {}^8C_1$
27) $^8C_4 \div {}^5C_3$
28) $^5C_2 \times {}^6C_2$
29) $^5C_3 \div {}^6C_4$
30) $^8C_2 \div {}^7C_3$

3.3 Approximating Powers of Numbers close to 1:

Often, the exam requires us to estimate values such as $(1.08)^6$ or $(1.03)^9$. (For example, if we have to calculate Compound Interest at 7% per annum for 8 years, the multiplying factor for the amount would be $(1.07)^8$. Other similar questions which would need this kind of calculation would include population growth, share value growth, depreciation etc. Let's see how to approach such numbers:
Suppose we need to calculate the value of $(1.05)^9$. Let us write it as $(1 + 0.05)^9$. Now from binomial expansion
$(a+b)^n = a^n + {}^nC_1 \times a^{n-1} \times b + {}^nC_2 \times a^{n-2} \times b^2 + {}^nC_3 \times a^{n-3} \times b^3 + \ldots$
And hence we can write
$(1+x)^n = 1^n + {}^nC_1 \times 1^{n-1} \times x + {}^nC_2 \times 1^{n-2} \times x^2 + {}^nC_3 \times 1^{n-3} \times x^3 + \ldots$
$= 1 + {}^nC_1 \times x + {}^nC_2 \times x^2 + {}^nC_3 \times x^3 + \ldots$
But if x is very small (compared to 1), then the later terms become negligible. It turns out that if we take the first 3 terms (i.e. up till the term containing x^2), the answer will be close enough for practical purposes. Also, ${}^nC_1 = n$ and ${}^nC_2 = \frac{n(n-1)}{2}$
Hence we can approximate such a term as follows:

$$(1+x)^n \approx 1 + n \times x + \frac{n(n-1)}{2} \times x^2$$

Hence $\mathbf{(1.05)^9} \approx 1 + 9 \times 0.05 + \frac{9(9-1)}{2} \times 0.0025 \approx 1 + 0.45 + 0.0900 \approx$ **1.54** *(the exact value is 1.5513..)*
Let us try to estimate the value of $(1.06)^5$ by a similar approach:

$\mathbf{(1.06)^5} \approx 1 + 5 \times 0.06 + \frac{5(5-1)}{2} \times 0.0036 \approx 1 + 0.30 + 0.0360 \approx$ **1.336** *(the exact value is 1.3382…)*

This technique can be extended to cases of numbers near 10 or 100 as well, as follows:

$\mathbf{(10.8)^4} = (1.08)^4 \times 10^4 \approx (1 + 4 \times 0.08 + \frac{4(4-1)}{2} \times 0.0064) \times 10^4 \approx (1 + 0.32 + 0.0384) \times 10^4 \approx$ **13584**

$\mathbf{(105)^5} = (1.05)^5 \times 100^5 \approx (1 + 5 \times 0.05 + \frac{5(5-1)}{2} \times 0.0025) \times 10^{10} \approx (1 + 0.25 + 0.025) \times 10^{10} \approx \mathbf{1.275 \times 10^{10}}$

We can also find powers of numbers slightly *less* than 1, by making the second term negative:

$\mathbf{(0.98)^5} \approx 1 - 5 \times 0.02 + \frac{5(5-1)}{2} \times 0.0004 \approx 1 - 0.10 + 0.004 \approx$ **0.904**

$\mathbf{(0.95)^6} \approx 1 - 6 \times 0.05 + \frac{6(6-1)}{2} \times 0.0025 \approx 1 - 0.30 + 0.0375 \approx$ **0.7375**

EXERCISE 3.3

1) The value of $(1.09)^4$ is closest to
(a) 1.23 (b) 1.4 (c) 1.47 (d) 1.52

2) The value of $(1.07)^5$ is closest to
(a) 1.4 (b) 1.5 (c) 1.6 (d) 1.3

3) The value of $(0.92)^4$ is closest to
(a) 0.718 (b) 0.682 (c) 0.656 (d) 0.604

4) The valueof $(1.11)^4$ is closest to
(a) 1.46 (b) 1.52 (c) 1.58 (d) 1.63

5) The value of $(1.05)^6$ is closest to
(a) 1.45 (b) 1.51 (c) 1.39 (d) 1.34

6) The value of $(1.06)^8$ is closest to
(a) 1.52 (b) 1.58 (c) 1.67 (d) 1.49

7) The value of $(1.07)^4$ is closest to
(a) 1.28 (b) 1.31 (c) 1.36 (d) 1.39

8) The value of $(1.08)^9$ is closest to
(a) 1.8 (b) 1.88 (c) 2 (d) 2.1

9) The value of $(10.2)^5$ is closest to
(a) 113000 (b) 112000 (c) 110000 (d) 101000

10) The value of $(1.04)^{11}$ is closest to
(a) 1.46 (b) 1.6 (c) 1.53 (d) 1.63

11) The value of $(1.09)^7$ is closest to
(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

12) The value of $(0.97)^7$ is closest to
(a) 0.912 (b) 0.883 (c) 0.851 (d) 0.811

13) The value of $(1.05)^8$ is closest to
(a) 1.48 (b) 1.4 (c) 1.61 (d) 1.57

14) The value of $(10.6)^3$ is closest to
(a) 1234 (b) 1190 (c) 1122 (d) 1313

15) The value of $(1.06)^9$ is closest to
(a) 1.5 (b) 1.54 (c) 1.6 (d) 1.67

16) The value of $(1.1)^6$ is closest to
(a) 1.51 (b) 1.64 (c) 1.76 (d) 1.83

17) The value of $(1.09)^9$ is closest to
(a) 1.99 (b) 2.15 (c) 2.33 (d) 1.85

18) The value of $(1.05)^4$ is closest to
(a) 1.23 (b) 1.32 (c) 1.29 (d) 1.18

19) The value of $(1.07)^8$ is closest to
(a) 1.6 (b) 1.54 (c) 1.63 (d) 1.7

20) The value of $(1.04)^9$ is closest to
(a) 1.36 (b) 1.42 (c) 1.49 (d) 1.56

3.4. Solving Simple Quadratic Equations:

When faced with a quadratic expression, one of the most common requirements is to extract its roots. After studying the theory of quadratic equations, we would know that the roots of a quadratic can be irrational or even complex in many cases. In those cases we would have to use formulaic approaches.

However, quite often we will find that the solution procedure for a problem involves a quadratic expression whose roots are integers. In such cases, applying such a complex and time-consuming formula would be a sub-optimal approach and hence we should be able to look at such a quadratic and extract its roots directly, by observation alone, as follows:

The theory of quadratic equations tells us that if a quadratic expression is of the form $ax^2 + bx + c$ then:

The sum of its roots is given by $\frac{-b}{a}$

The product of its roots is given by $\frac{c}{a}$

So if a quadratic has leading coefficient 1, i.e. if it is of the form $x^2 + bx + c$ then the sum of the roots will be $-b$ while their product will be c. Let us see how we can use this to get answers quickly:

Consider the equation $x^2 + 41x + 180$; the sum of roots should be -41 and their product should be 180. Since the product of the roots is 180 (positive) the roots must both be of the same sign. Since the sum is -41, both must be negative and their sum must be -41. A quick look at possible factors of 180 gives us $180 = 36 \times 5$ and hence the roots will be -36 and -5.

Similarly if we were to consider the equation $x^2 - 41x + 180$; the sum of roots should be 41 and their product should be 180. Since the product of the roots is 180 (positive) the roots must both be of the same sign. Since the sum is 41, both must be positive. Hence the roots will be +36 and +5.

Now if we consider $x^2 + 41x - 180$; the sum of roots should be -41 and their product should be -180. Since the product of the roots is -180 (negative) the roots must be of opposite signs. Since the sum is -41, the larger root must be negative and the difference in magnitude of the roots must be 41. A quick look at possible factors of 180 gives us $180 = 45 \times 4$ and hence the roots will be -45 and +4.

Finally, looking at $x^2 - 41x - 180$; the sum of roots should be 41 and their product should be -180. Applying the approach described above, the roots will be +45 and -4.

Some more examples of the above approach:

$x^2 - 18x - 19$: Look for two factors of 19 whose *difference* is 18 $\Rightarrow$ the roots are -1 and +19

$x^2 - 11x + 10$: Look for two factors of 10 whose *sum* is 11 $\Rightarrow$ the roots are +1 and +10

$x^2 + 16x - 105$: Look for two factors of 105 whose *difference* is 16 $\Rightarrow$ the roots are -21 and +5

$x^2 + 17x + 60$: Look for two factors of 60 whose *sum* is 17 $\Rightarrow$ the roots are -5 and -12

EXERCISE 3.4.A

Directions for the questions: Find the roots of the following quadratic expressions, without writing:

1) $x^2 + 15x + 54$
2) $x^2 + 11x - 102$
3) $x^2 + 12x - 28$
4) $x^2 - 14x$
5) $x^2 - 10x + 9$
6) $x^2 + 11x - 60$
7) $x^2 - 4x - 12$
8) $x^2 + 17x - 60$
9) $x^2 + 16x - 17$
10) $x^2 + 16x + 15$
11) $x^2 - 17x + 16$
12) $x^2 + 19x + 18$
13) $x^2 - 15x - 16$
14) $x^2 + 17x + 16$
15) $x^2 + 8x$
16) $x^2 - 16x + 39$
17) $x^2 - 13x + 42$
18) $x^2 + 10x - 11$
19) $x^2 + 15x + 44$
20) $x^2 - 7x$

EXERCISE 3.4.B

Directions for the questions: Find the roots of the following quadratic expressions, without writing:

1) $x^2 - 16x + 63$
2) $x^2 - 9x + 8$
3) $x^2 + 7x$
4) $x^2 + 4x + 3$
5) $x^2 - 16x$
6) $x^2 + 12x - 108$
7) $x^2 + 15x - 76$
8) $x^2 + 19x - 20$
9) $x^2 - 6x + 9$
10) $x^2 - 12x + 27$
11) $x^2 - 8x + 15$
12) $x^2 + 14x - 120$
13) $x^2 - 8x + 16$
14) $x^2 + 7x + 10$
15) $x^2 + 6x - 7$
16) $x^2 + 16x + 63$
17) $x^2 - 7x + 12$
18) $x^2 + 12x + 11$
19) $x^2 - 17x - 18$
20) $x^2 + 15x - 34$

3.5 Pythagorean Triplets

Pythagorean Triplets are sets of three natural numbers that satisfy the Theorem of Pythagoras i.e. a set (x, y, z) such that $x^2 + y^2 = z^2$. They come in handy quite often, particularly in Geometry-based Word Problems, and therefore it is a good idea to know a few triplets by heart, and to know how to generate a few more if necessary.

It is possible to generate a triplet starting from any natural number greater than 2 *(note that this would not necessarily be the only triplet involving that number; the same number can feature in multiple triplets).* Let us see how to do this:

Starting with an Odd number: Starting with an odd number n, then if we write n^2 as the sum of two consecutive natural numbers a and b, then n, a and b will form a triplet. (Note that a, b = $\frac{n^2 \pm 1}{2}$).

Thus, if we start with 3, we can express $3^2 = 9$ as 4 + 5. Thus 3, 4, 5 is a valid triplet

If we start with 11, we can express $11^2 = 121$ as 60 + 61. Thus 11, 60, 61 is a valid triplet

If we start with 17, we can express $17^2 = 289$ as 144 + 145. Thus 17, 144, 145 is a valid triplet

Starting with an Even number: Starting with an even number n, then if we divide n by 2, square it, and add / subtract 1 to get two numbers a and b, then n, a and b will form a triplet. (Here a, b = $\left(\frac{n}{2}\right)^2 \pm 1$)

Thus, if we start with 8, we can write $4^2 \pm 1$ = 15 and 17. Thus 8, 15, 17 is a valid triplet

If we start with 12, we can write $6^2 \pm 1$ = 35 and 37. Thus 12, 35, 37 is a valid triplet

If we start with 20, we can write $10^2 \pm 1$ = 99 and 101. Thus 20, 99, 101 is a valid triplet

Another useful trick is to figure out the third side when the hypotenuse and one side are given, without actually calculating the squares of the given sides. If we are given the hypotenuse z and one of the other sides, x, and we want to find the third side y, then we need only remember that $x^2 + y^2 = z^2$ and thus $y^2 = z^2 - x^2 = (z + x)(z - x)$.

So for example is we are told that the longest side of a right triangle is 52 and one of the other sides is 20, then we can find the third side as $\sqrt{(52-20)(52+20)} = \sqrt{32 \times 72} = \sqrt{2 \times 16 \times 2 \times 36} = 48$

EXERCISE 3.5

Directions for the questions:Find the third side, given the hypotenuse and one leg of a right triangle

1) 25, 7	2) 15, 17	3) 65, 63	4) 113, 15
5) 26, 24	6) 56, 65	7) 12, 13	8) 80, 82
9) 35, 21	10) 25, 65	11) 40, 50	12) 53, 45
13) 6, 10	14) 78, 72	15) 16, 65	16) 29, 21
17) 16, 34	18) 13, 85		

3.6 Base Systems: Introduction to Base Conversion

Human beings have ten fingers. It is not surprising, then, that several civilisations independently developed number systems based on the number ten. So as we start counting from 1, 2, 3, 4…when we cross 9 we count the new number as 1 ten, and restart the count again as follows 10, 11, 12, 13. Similarly when we cross 99 (i.e. the count of tens has also crossed 9) we count it as 1 "ten-squared" (i.e. one hundred) and so on. Thus our system of counting is based on the number of ones, tens, hundreds, thousands and so on. Hence a number like 786 really implies $7 \times 10^2 + 8 \times 10^1 + 6 \times 10^0$ and a number like 1729 would be $1 \times 10^3 + 7 \times 10^2 + 2 \times 10^1 + 9 \times 10^0$. Note that each of these terms contain the form a^m i.e. $base^{index}$ and since the base is always a power of 10, we call it a "base ten" or decimal system.

A base 9 number, then, would involve powers of 9 instead – for example 786 in base 9 (usually written as $(786)_9$ would work out to $7 \times 9^2 + 8 \times 9^1 + 6 \times 9^0 = 567 + 72 + 6$ which is 645 in base 10. Similarly if we were to convert 143 in base 11 that is to say $(143)_{11}$ to base 10, we would get $1 \times 11^2 + 4 \times 11^1 + 3 \times 11^0 = 121 + 44 + 3 = 168$ in base 10.

Thus to convert from any base to base 10, we use the powers of that base. If we write a 5-digit number $(abcde)_n$, then it will be equivalent to $an^4 + bn^3 + cn^2 + dn^1 + e$ in base 10.

To convert in the reverse direction, i.e. from base 10 to some other base n, we can successively divide the original (base 10) number by n and note down the remainders in the REVERSE order. So if we wish to convert 120 in base 10 to base 7, we could do

$120 \div 7 = 17$ remainder 1
$17 \div 7 = 2$ remainder 3 $\Rightarrow (120)_{10} = (231)_7$
$2 \div 7 = 0$ remainder 2

The logic is straightforward; when we divide 120 by 7, the remainder obtained is the number of units, while the quotient obtained is the number of complete 7s. Now when we divide it again by 7 (effectively dividing the original number by 49), the remainder will be the number of 7s left after dividing by 49 and so on. Thus we are generating, in reverse order, the units (7^0), sevens (7^1), forty-nines (7^2) and so on.

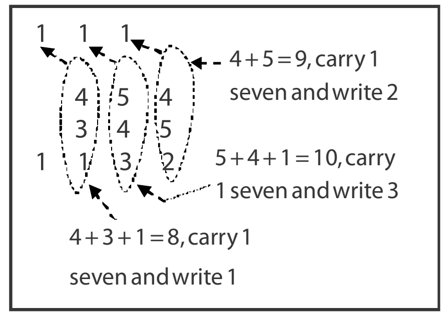
Similarly if we wish to convert 87 in base 10 into base 4, we could do
$85 \div 4 = 21$ remainder 3
$21 \div 4 = 5$ remainder 1 $\Rightarrow (87)_{10} = (1113)_4$
$5 \div 4 = 1$ remainder 1
$1 \div 4 = 0$ remainder 1

An alternative method to convert from base 10 to some other base

Suppose we want to convert, let's say, 100 from base 10 to base 6. First of all, let us note that 100 lies between 36 i.e. 6^2 and 216 i.e. 6^3 it will be a 3 digit number in base 6. So we need to find (abc) such that a is the maximum possible number of occurrences of '36', b is the maximum possible number of occurrences of '6' in the remaining part, and c is the number of 'units' left.
Now, '36' will occur twice, so a will be 2. The balance is 28 (from $100 - 36 \times 2$) in which '6' occurs 4 times, so b will be 4 and the balance 4 will be the number of units i.e. c. Hence $\mathbf{(100)_{10} = (244)_6}$
Similarly if we attempt to convert the number 142 from base 10 into base 5 it will be 125 (1 time) leaving 17 $\Rightarrow$ 25 (0 times) leaving 17 $\Rightarrow$ 5 (3 times) leaving 2 $\Rightarrow$ 1 (2 times) and so $\mathbf{(142)_{10} = (1032)_5}$ and if we try to convert 128 into base 3 it will be 81 (1 time) leaving 47 $\Rightarrow$ 27 (1 times) leaving 20 $\Rightarrow$ 9 (2 times) leaving 2 $\Rightarrow$ 3 (0 times) leaving 2 $\Rightarrow$ 1 (2 times) and hence $\mathbf{(128)_{10} = (11202)_3}$

Addition of two numbers in the same base

Suppose we have to add $(454)_7$ and $(345)_7$. Note that when adding two numbers in base 10, we "carry over" anything extra whenever the sum crosses 10. Why is this? Because in base 10, each successive digit position carries 10 times the value of the one to its right. So in base 7, we should carry over anything extra when the sum crosses 7. The box on the right demonstrates this technique.



Direct conversion between two bases, one of which is a power of the other

Consider a number $(abcdefgh)_2$ which has to be converted to base 8. The number can be expanded as $a \times 2^7 + b \times 2^6 + c \times 2^5 + d \times 2^4 + e \times 2^3 + f \times 2^2 + g \times 2^1 + h \times 2^0$. We can break this up, combining three terms at a time (from the right), as follows: $\mathbf{(a \times 2^7 + b \times 2^6) + (c \times 2^5 + d \times 2^4 + e \times 2^3) + (f \times 2^2 + g \times 2^1 + h \times 2^0)}$ which can be rewritten as $\mathbf{8^2 \times (a \times 2^1 + b \times 2^0) + 8^1 \times (c \times 2^2 + d \times 2^1 + e \times 2^0) + 8^0 \times (f \times 2^2 + g \times 2^1 + h \times 2^0)}$. In other words, we can combine 3 digits at a time from the number in base 2 to get a single base 8 digit.

So $\mathbf{(11010101011)_2}$ would be $\mathbf{(3253)_8}$ while $\mathbf{(1727)_8}$ would be $\mathbf{(1111010111)_2}$

Similarly we could convert from base 2 to base 16, 4 digits at a time: $\mathbf{(11010101011)_2}$ would be $\mathbf{(6AB)_{16}}$ while $\mathbf{(15D9)_{16}}$ would be $\mathbf{(1010111011001)_2}$

(We could also convert from base 3 to base 9 and vice-versa in the same manner, try it for yourself!)

EXERCISE 3.6

Find the value of N in each of the following questions:

1) $(25)_7 = (N)_{10}$
2) $(N)_2 = (25)_{10}$
3) $(101)_9 = (N)_{10}$
4) $(121)_8 = (N)_{10}$
5) $(N)_3 = (144)_{10}$
6) $(125)_6 + (353)_6 = (N)_6$
7) $(568)_9 + (747)_9 = (N)_9$
8) $(10201)_3 = (N)_9$
9) $(1101010110)_2 = (N)_8$
10) $(2535)_8 = (N)_2$

3.7 Some basics of Logarithms

The logarithmic function may be thought of as the inverse of the exponential function (a^x). It may be more formally defined as: The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number. In other words, if $m^b = a$ then we can say that $\log_m a = b$

Let us look at some examples to clarify this:

$\log_5 625 = 4$ ($\because 5^4 = 625$) $\log_2 512 = 9$ ($\because 2^9 = 512$) $\log_{10} 1000 = 3$ ($\because 10^3 = 1000$)
$\log_3 243 = 5$ ($\because 3^5 = 243$) $\log_4 1 = 0$ ($\because 4^0 = 1$) $\log_{99} 9801 = 2$ ($\because 99^2 = 9801$)

$\log_{\frac{1}{2}}\left(\frac{1}{32}\right) = 5$($\because \left(\frac{1}{2}\right)^5 = \frac{1}{32}$) $\log_{\frac{1}{13}}\left(\frac{1}{169}\right) = 2$ ($\because \left(\frac{1}{13}\right)^2 = \frac{1}{169}$) $\log_{\frac{1}{7}}\left(\frac{1}{2401}\right) = 4$ ($\because \left(\frac{1}{7}\right)^4 = \frac{1}{2401}$)

Logarithms can also be fractional….

$\log_{64} 4 = \frac{1}{3}$ ($\because 64^{1/3} = 4$) $\log_{256} 2 = \frac{1}{8}$ ($\because 256^{1/8} = 2$) $\log_{343} 7 = \frac{1}{3}$ ($\because 343^{1/3} = 7$)

…or even negative

$\log_3\left(\frac{1}{9}\right) = -2$ ($\because 3^{-2} = \frac{1}{9}$) $\log_{\frac{1}{2}} 8 = -3$ ($\because \left(\frac{1}{2}\right)^{-3} = 8$) $\log_{\frac{1}{27}} 3 = -\frac{1}{3}$ ($\because \left(\frac{1}{27}\right)^{-\frac{1}{3}} = 3$)

Note, however, that a logarithm is not defined for certain cases:

A logarithm for a negative number or for 0 is not defined

If we try to define something like $\log_2(-3)$, we will realise that it makes no sense – any positive base, raised to any power, will still remain positive. Also if we try to something like $\log_2 0$ we will find that the value will be undefined.

A logarithm is not defined to a base which is negative or 0

If we try to define something like $\log_{(-2)} 8$ we will find that there is no power to which we can raise -2 which will give 8. Also raising a negative number to a fractional value will be problematic.

A logarithm is not defined to base 1

If we try to define something like $\log_1 5$ we will have to solve an equation $1^x = 5$ which is patently absurd! Thus a log to base 1 is not defined.

Thus, in brief, $\log_m a$ is defined only if a > 0 and if m > 0 and m 1

Unspecified Bases

Quite often, in an exam like the CAT, the base will not be specified. In this case, it is usually understood that the base is to be taken as 10. Note that in pure mathematics (including calculus) if the base is not specified, it is assumed to be not 10, but e (where e H” 2.718 is called the base of “natural logarithms”). Let us look at some useful results involving logarithms:

➢ **$\log_n n = 1$ and $\log_n 1 = 0$ for any n** - - - - - **(1)**

This is self-evident and can be proved using the basic definition of a logarithm

➢ **$\log_n xy = \log_n x + \log_n y$** - - - - - **(2)**

➢ **$\log_n\left(\frac{x}{y}\right) = \log_n x - \log_n y$** - - - - - **(3)**

If $\log_n x = a$ and $\log_n y = b$, then $x = n^a$ and $y = n^b$. Now applying the laws of indices $xy = n^{a+b}$ which means that $\log_n xy = a + b = \log_n x + \log_n y$. Similarly $\frac{x}{y} = n^{a-b}$ which means that $\log_n\left(\frac{x}{y}\right) = a - b = \log_n x - \log_n y$

➢ $\mathbf{\log_n x^a = a \log_n x}$ - - - - - **(4)**

We can write $\log_n x^a = \log_n(x * x * x * \ldots$ a times) which, on applying result (2) above, becomes $\log_n x + \log_n x + \log_n x + \ldots$a times $= a \log_n x$

➢ $\mathbf{\log_n \sqrt[a]{x} = \frac{\log_n x}{a}}$ - - - - - **(5)**

Let $\sqrt[a]{x} = y$. Then $x = y^a$. Thus by the result (4) above we can say $\log_n x = a \log_n y$

Thus $\log_n y = \frac{\log_n x}{a}$ but since $y = \sqrt[a]{x}$, this becomes the required $\log_n \sqrt[a]{x} = \frac{\log_n x}{a}$

➢ $\mathbf{\log_b a = \frac{\log_n a}{\log_n b}}$ - - - - - **(6)**

Suppose $\log_b a = c$. Then $a = b^c$. Taking log to base n on both sides $\log_a n = \log_n b^c$
Applying result (4) above, $\log_a n = c * \log_n b$

Hence $c = \frac{\log_n a}{\log_n b}$

➢ $\mathbf{n^{\log_n x} = x}$ - - - - - **(7)**

Let $n^{\log_n x} = y$. Taking log to base n on both sides $\log_n(n^{\log_n x}) = \log_n y$.

Using result 4, $\log_n() = \log_n x * \log_n n$ which, since $\log_n n = 1$, gives us just $\log_n x$.
So since $\log_n x = \log_n y$ we find that y must be equal to x itself.

➢ $\mathbf{n^x = x}$ - - - - - **(8)**

Applying result (4) gives us that $n^x = x$ n. But since $\log_n n = 1$, this will just be x.

➢ $\mathbf{\log_b a * \log_a b = 1}$ **i.e** $\mathbf{\log_b a = \frac{1}{\log_a b}}$ - - - - - **(9)**

This can easily be proved from result 6.

EXERCISE 3.7A

Find the value of the following

1) $\log_9 243$
2) $\log_{125} 25$
3) $\log_{100} 1000$
4) $\log_{343} 7$
5) $\log_{43}\left(\frac{1}{43}\right)$
6) $\log_{32}\left(\frac{1}{2}\right)$
7) $\log_{\frac{3}{5}}\left(\frac{5}{3}\right)$
8) $\log_{\frac{1}{81}}(27)$
9) $\log_{\frac{1}{2}}(0.125)$
10) $\log_{\frac{3}{2}}\left(\frac{8}{27}\right)$

EXERCISE 3.7B

Find the value of X if

1) $\log_X 1331 = 1.5$
2) $\log_4 64 + \log_8 64 = \log_X 243$
3) $\log_{\frac{1}{2}}(X) = \log 3\left(\frac{1}{81}\right)$
4) $\log_9 X + \log_9 X^3 = 2$
5) $\log_9 X + \log_3 X = 6$
6) $\log_{15} 25 + \log_{15} X = 2$
7) $\log_3 25 \times \log_5 27 = X$
8) $\log_{\frac{1}{2}} 15 = \log_x(225)$
9) $\log_{\frac{1}{27}}\left(\frac{1}{3}\right) \times \log_3(X) = 1$
10) $\log_{\frac{1}{2}}\left(\frac{1}{16}\right) + \log_5(X) = 6$

3.8 Comparison of Fractions

Often in Data Interpretation we need to compare two fractions. There are circumstances in which, instead of calculating, we can find the answer easily (and quickly) by applying certain simple rules:

1. If the difference between numerator (N) and denominator (D) of two fractions is the same, then:

a) If they are proper fractions then the larger N and D imply a larger fraction (e.g. $\frac{1}{4} < \frac{2}{5} < \frac{3}{6} < \frac{4}{7}$)

b) If they are improper fractions then the larger N and D imply a smaller fraction (e.g. $\frac{7}{4} < \frac{8}{5} < \frac{9}{6} < \frac{10}{7}$)

Rule of Thumb: Proper fractions behave "properly" (larger N and D leading to larger fraction), while Improper fractions behave "improperly" (smaller N and D leading to larger fraction)

So for example if we want to compare $\frac{363}{351}$ and $\frac{373}{361}$, we can just look at them and say that the difference |N – D| is 12, and both are improper fractions, and hence $\frac{363}{351}$ must be the larger of the two.

2. If the Numerators or the Denominators of the two fractions can be equalised then:

a) If the denominators of two fractions can be equalised, the fraction with the larger numerator is larger (e.g. to compare $\frac{3}{4}$ and $\frac{13}{16}$, we can write them as $\frac{12}{16}$ and $\frac{13}{16}$ and hence $\frac{13}{16}$ is the larger).

b) If the numerators of two fractions can be equalised, the fraction with the smaller denominator is larger (e.g. to compare $\frac{3}{4}$ and $\frac{9}{13}$, we can write them as $\frac{9}{12}$ and $\frac{9}{13}$ and hence $\frac{3}{4}$ is the larger).

Rule of Thumb: *A fraction behaves the same way as its Numerator (increasing or decreasing along with it), but the opposite way as its Denominator.*

So for example if we want to compare $\frac{343}{350}$ and $\frac{679}{700}$, we can make the first fraction $\frac{686}{700}$, and hence say that it must be larger than $\frac{679}{700}$

Rule 3: If neither of the above applies, then a method such as cross-multiplication can be used:

To compare two fractions $\frac{a}{b}$ and $\frac{c}{d}$, we compare 'a × d' and 'b × c'. If 'a × c' is larger then the first fraction is the greater, else the second.

So for example if we want to compare $\frac{3}{209}$ and $\frac{5}{345}$, we can cross-multiply to get 3 × 345 and 5 × 209 i.e. 1035 and 1045 and hence say that the second fraction is the larger of the two.

EXERCISE 3.8

Fill in the blanks with > or < or =

1) $\frac{3}{10}$ $\square$ $\frac{6}{13}$

2) $\frac{26}{9}$ $\square$ $\frac{35}{12}$

3) $\frac{11}{15}$ $\square$ $\frac{18}{25}$

4) $\frac{13}{7}$ $\square$ $\frac{23}{13}$

5) $\frac{43}{41}$ $\square$ $\frac{87}{83}$

6) $\frac{243}{258}$ $\square$ $\frac{353}{368}$

7) $\frac{303}{8}$ $\square$ $\frac{512}{13}$

8) $\frac{236}{225}$ $\square$ $\frac{753}{741}$

9) $\frac{413}{239}$ $\square$ $\frac{469}{450}$

10) $\frac{187}{11}$ $\square$ $\frac{417}{243}$

11) $\frac{127}{255}$ $\square$ $\frac{306}{18}$

12) $\frac{776}{16}$ $\square$ $\frac{151}{297}$

13) $\frac{112}{11}$ $\square$ $\frac{907}{18}$

14) $\frac{112}{11}$ $\square$ $\frac{133}{14}$

15) $\frac{786}{687}$ $\square$ $\frac{686}{587}$

ANSWERS

Exercise 3.1:

1. 8	2. 20	3. 35	4. 6	5. 3
6. 3	7. 42	8. 1	9. 140	10. 144.4
11. 603.2	12. 7555.2	13. 68	14. 3571	15. 20
16. 12	17. 25	18. 16	19. 25	20. 9

Exercise 3.2:

1. 15	2. 21	3. 9	4. 20	5. 35
6. 126	7. 6	8. 28	9. 252	10. 10
11. 70	12. 210	13. 5	14. 36	15. 120
16. 161051	17. 1331	18. 1.9487171	19. 1.771561	20. 14641
21. 74	22. 64	23. 15	24. 0	25. 256
26. 120	27. 7	28. 150	29. 0.5	30. 0.8

Exercise 3.3:

1. b	2. a	3. a	4. b	5. d
6. b	7. b	8. c	9. c	10. c
11. d	12. d	13. a	14. b	15. d
16. c	17. b	18. a	19. d	20. b

Exercise 3.4.A:

1. −6, −9	2. −17, 6	3. −14, 2	4. 0, 14	5. 1, 9
6. −15, 4	7. −2, 6	8. −20, 3	9. −17, 1	10. −15, −1
11. 1, 16	12. −18, −1	13. −1, 16	14. −16, −1	15. −8, 0
16. 3, 13	17. 6, 7	18. −11, 1	19. −11, −4	20. 0, 7

Exercise 3.4.B:

1. 7, 9	2. 1, 8	3. −7, 0	4. −3, −1	5. 0, 1
6. −18, 6	7. −19, 4	8. −20, 1	9. 3, 3	10. 3, 9
11. 3, 5	12. −20, 6	13. 4, 4	14. −5, −2	15. −7, 1
16. −9, −7	17. 3, 4	18. −11, −1	19. −1, 18	20. −17, 2

Exercise 3.5

1. 24	2. 8	3. 16	4. 112	5. 10
6. 33	7. 5	8. 18	9. 28	10. 60
11. 30	12. 28	13. 8	14. 30	15. 63
16. 20	17. 30	18. 84		

Exercise 3.6

1. 19	2. 11001	3. 82	4. 81	5. 12100
6. 522	7. 1426	8. 121	9. 1526	10. 10101011101

Exercise 3.7A

1. 5/2	2. 2/3	3. 3/2	4. 1/3	5. –1
6. –1/5	7. –1	8. –3/4	9. 3	10. –3

Exercise 3.7B

1. 121	2. 3	3. 16	4. 3	5. 81
6. 9	7. 6	8. 1/4	9. 27	10. 25

Exercise 3.8

1. $<$	2. $<$	3. $>$	4. $>$	5. $>$
6. $<$	7. $<$	8. $<$	9. $>$	10. $>$
11. $=$	12. $<$	13. $<$	14. $>$	15. $<$

4 PRACTICE TESTS

In this section, overall 20 practice tests have been provided. Each test contains questions on all the different speed mathematics techniques (describe in this book).

You should start solving the tests after solving the exercises in the preceding three chapters. When you start a test, complete all the problems in the test in the same attempt. Note down the time taken for solving the test. Ideally the time taken for the test reduce as you go on to solve further tests.

After you complete each test, check your answers and find out whether the mistakes in your answers are because of an oversight or because of lack of familiarity with the concepts. If the mistakes are because of oversight, make sure that you do not repeat the mistakes in next test. If the mistakes are because of lack of familarity with the concepts, brush up those concepts before attempting the next test.

Do not solve all the 20 tests at one go. Make sure that you phase out taking the tests so that you will have required touch with the concepts till your actuall CAT. Ideally you should take the last test about two weeks before your CAT.

All The Best!!!

Test 1

Answers

1] $682 + 827 + 196 + 82 + 306 + 782 =$

2] $750 + 553 + 421 + 326 + 994 + 654 =$

3] $49^2 =$

4] $26 \times 6 =$

5] $18 \times 9 =$

6] The value of ${}^{12}P_3$ is

7] The roots of $x^2 + 13x - 14 = 0$ are

8] The roots of $x^2 - 3x - 10 = 0$ are

9] The value of $\log_9 27$ is:

10] The average of 37, 44, 56, 61 and k is 52. Find x

11] Find the HCF of 360, 480 and 540

12] $\frac{4}{11}$ is roughly how much percent?

13] 53.58×44.68 is approximately:

(a) 2394 (b) 2493 (c) 2239 (d) 2199

14] $\frac{311}{807}$ is closest to:

(a) 3.86 (b) 0.358 (c) 3.58 (d) 0.385

15] $\frac{2919}{1033}$ is closest to:

(a) 2.56 (b) 2.82 (c) 2.98 (d) 2.36

16] The value of $(1.09)^5$ is closest to

(a) 1.46 (b) 1.58 (c) 1.61 (d) 1.53

Test 2

Answers

1] $331 + 181 + 618 + 300 + 856 + 998 =$

2] $862 + 984 + 981 + 465 + 643 + 812 =$

3] $65^2 =$

4] $32 \times 8 =$

5] $7 \times 14 =$

6] The value of $^{18}C_2$ is

7] The roots of $x^2 - 14x - 120 = 0$ are

8] The roots of $x^2 + 14x + 49 = 0$ are

9] The value of $\log_3 2 \times \log_{16} 27$ is:

10] The Harmonic Mean of 16 and 24 is:

11] If $\frac{1}{24} + \frac{1}{72} = \frac{1}{x}$, find x

12] 72% of 25 equals:

13] 13.02×72.61 is nearest to:

(a) 919.7 (b) 94.53 (c) 91.95 (d) 945.4

14] $\frac{416}{184}$ is roughly:

(a) 2.96 (b) 2.53 (c) 2.26 (d) 2.14

15] $\frac{5597}{8101}$ is closest to:

(a) 0.583 (b) 0.732 (c) 0.691 (d) 0.667

16] The value of $(1.07)^7$ is closest to

(a) 1.49 (b) 1.59 (c) 1.65 (d) 1.69

Test 3

Answers

1] $917 + 586 + 934 + 779 + 240 + 151 =$

2] $583 + 165 + 495 + 19 + 97 + 725 =$

3] $9999^2 =$

4] $1001 \div 13 =$

5] $99 \times 5 =$

6] The value of ${}^{11}C_3$ is

7] The roots of $x^2 - 11x - 42 = 0$ are

8] The roots of $x^2 + 5x + 4 = 0$ are

9] $[\log_3 80]$ will be *(where [x] is the Greatest Integer less than or equal to x)*:

10] The Geometric Mean of 36, 75 and 80 is x. Find x

11] The fractions $\frac{1}{3}$, $\frac{1}{5}$ and $\frac{1}{8}$ are in what ratio?

12] $\frac{5}{7}$ is roughly how much percent?

13] $\frac{4911}{3110}$ is closest to:

(a) 1.58 (b) 1.69 (c) 2.57 (d) 1.79

14] The value of $(1.02)^7$ is closest to

(a) 1.11 (b) 1.15 (c) 1.21 (d) 1.27

15] 31.33×47.8 is approximately:

(a) 1345 (b) 1497 (c) 1562 (d) 1434

16] $\frac{535}{281}$ is roughly:

(a) 1.76 (b) 1.99 (c) 2.07 (d) 1.9

Test 4

Answers

1] $147 + 451 + 452 + 126 + 614 + 224 =$

2] $384 + 961 + 954 + 228 + 227 + 405 =$

3] $27^2 + 36^2 =$

4] $38 \times 42 =$

5] $12 \times 19 =$

6] The value of ${}^{10}C_4$ is

7] The roots of $x^2 - x - 2 = 0$ are

8] The roots of $x^2 + 6x - 16 = 0$ are

9] The greatest among $\log_3 30$, $\log_4 40$ and $\log_5 50$ is:

10] The average of 10 values is 15 and the average of another 15 values is 10. Then the average of all 25 values is:

11] The number which is 420% of 5 must be

12] If $\frac{1}{24} - \frac{1}{72} = \frac{1}{x}$, find x:

13] The value of $(10.8)^4$ is closest to

(a) 13531 (b) 1354.6 (c) 13584 (d) 1360.9

14] 54.19×53.01 is nearest to:

(a) 2287 (b) 2827 (c) 2728 (d) 2872

15] $\frac{466}{134}$ is closest to:

(a) 3.477 (b) 3.712 (c) 3.691 (d) 3.111

16] $\frac{7203}{1132}$ is roughly:

(a) 6.969 (b) 7.034 (c) 6.363 (d) 6.149

Test 5

Answers

1] $6593 + 7037 + 3474 + 9548 + 5134 + 7751 =$

2] $973 + 188 + 532 + 710 + 721 + 745 =$

3] $102^2 =$

4] $999 \div 27 =$

5] $10101 \times 78 =$

6] The smallest 4 digit number in base 7 will be

7] The roots of $x^2 + 15x + 50 = 0$ are

8] The roots of $x^2 - 3x - 4 = 0$ are

9] $\log_b a^2 \times \log_c b^3 \times \log_a c^4 =$

10] There are 20 notes of Rs 100 and 100 notes of Rs 10. Average value per note is

11] If Rs 2000 is divided in the ratio 4 : 13 : 17, the largest portion will be

12] A decrease of 40% will be offset by a subsequent increase of x%. Find x

13] $\frac{489}{659}$ is closest to:

(a) 0.69 (b) 0.74 (c) 0.64 (d) 0.79

14] The value of $(1.08)^7$ is closest to

(a) 1.62 (b) 1.59 (c) 1.70 (d) 1.77

15] 99.21×20.24 is approximately:

(a) 1997 (b) 2010 (c) 2035 (d) 2123

16] $\frac{3728}{3977}$ is roughly:

(a) 0.888 (b) 0.832 (c) 0.971 (d) 0.937

Test 6

Answers

1] $7096 + 2005 + 4656 + 8012 + 2978 + 2308 =$

2] $994 + 760 + 886 + 158 + 260 + 183 =$

3] $40^2 + 75^2 =$

4] Find the square root of 64009

5] $49 \times 80 =$

6] The value of ${}^8C_8$ is

7] The roots of $x^2 + 14x - 51 = 0$ are

8] The roots of $x^2 + 15x + 36 = 0$ are

9] $\log_6 4 + 2\log_6 3$ is:

10] When a number is increased by 40% and then decreased by 5%, it changes by ___ %

11] Is 90% of 80% of 70% of 2000 less than 1000?

12] If $\frac{1}{54} + \frac{1}{108} = k^2$, find the value of 6k

13] The value of $(1.1)^4$ is closest to

(a) 1353 (b) 1.464 (c) 1.414 (d) 1.512

14] 61.92×66.91 is nearest to:

(a) 3978 (b) 4423 (c) 4342 (d) 4143

15] $\frac{5672}{3603}$ is closest to:

(a) $\frac{\pi}{2}$ (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) $\sqrt{5}$

16] $\frac{941}{175}$ is roughly:

(a) 5.35 (b) 4.99 (c) 5.53 (d) 5.82

Test 7

Answers

1] $824 + 508 + 143 + 536 + 541 + 213 =$

2] $2414 + 6190 + 6921 + 8419 + 5501 + 4148$

3] $71^2 =$

4] Find the square root of 3481

5] $234234 \div 117 =$

6] The value of $^7C_3$ is

7] The roots of x^2 _ $15x - 34 = 0$ are

8] The roots of $x^2 - 15x - 54 = 0$ are

9] If log 2 = 0.3, then log 32 will be

10] If $\frac{2}{25} - \frac{1}{75} = \frac{1}{x}$, find x

11] If two successive discounts of 20% are offered, the overall discount is:

12] Find the HCF of 333, 370 and 444

13] 99.21×20.24 is approximately:

(a) 2008 (b) 1927 (c) 2059 (d) 2195

14] $\frac{631}{907}$ is closest to:

(a) 0.70 (b) 0.658 (c) 0.667 (d) 0.732

15] $\frac{9954}{3097}$ is closest to:

(a) 3.51 (b) 2.92 (c) 3.21 (d) 4.36

16] The value of $(0.93)^5$ is closest to

(a) 0.597 (b) 0.767 (c) 0.699 (d) 0.645

Test 8

Answers

1] $590 + 333 + 782 + 342 + 275 + 668 =$

2] $5843 + 1919 + 9732 + 3998 + 6097 + 2589 =$

3] $97^2 =$

4] Find the fifth root of 243

5] $325 \times 11 =$

6] The base in which $(245)_{10}$ is written as 302 is

7] The roots of $x^2 - x - 12 = 0$ are

8] The roots of $x^2 + 2x - 8 = 0$ are

9] $[\log_4 250]$ will be *(where [x] is the Greatest Integer less than or equal to x)*:

10] If $\frac{1}{36} + \frac{1}{60} = \frac{2}{x}$, find x

11] x% increase followed by 10% decrease equals 17% increase. Find x

12] Find the LCM of 40, 48 and 30

13] 35.79×33.76 is approximately:

(a) 1174 (b) 1158 (c) 1208 (d) 1145

14] $\frac{865}{362}$ is closest to:

(a) 2.57 (b) 2.71 (c) 2.15 (d) 2.39

15] $\frac{6873}{3314}$ is closest to:

(a) 1.93 (b) 2.07 (c) 2.23 (d) 2.41

16] The value of $(1.06)^4$ is closest to

(a) 1.26 (b) 1.19 (c) 1.31 (d) 1.36

Test 9

Answers

1] $782 + 840 + 156 + 970 + 564 + 432 =$

2] $5837 + 6463 + 2407 + 9765 + 7749 + 2434 =$

3] $112^2 =$

4] Find the square root of 5476

5] $1947 \div 11 =$

6] The value of ${}^6P_6$ is

7] The roots of $x^2 - 8x - 9 = 0$ are

8] The roots of $x^2 - 16x + 55 = 0$ are

9] The smallest among $\log_3 100$, $\log_5 600$ and $\log_6 1300$ is:

10] If $\frac{1}{6} - \frac{1}{x} = \frac{1}{7}$, find x

11] Find the HCF of 242 and 243

12] The fractions $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{6}$ are in what ratio?

13] 25.47×88.66 is approximately:

(a) 2344 (b) 2260 (c) 2178 (d) 2069

14] $\frac{613}{132}$ is closest to:

(a) 4.45 (b) 0.472 (c) 5.18 (d) 4.64

15] $\frac{8889}{9468}$ is closest to:

(a) 0.938 (b) 0.889 (c) 0.987 (d) 0.966

16] The value of $(10.3)^5$ is closest to

(a) 123000 (b) 11600 (c) 12300 (d) 116000

Test 10

Answers

1] $992 + 408 + 180 + 366 + 976 + 924 =$

2] $5045 + 2909 + 3125 + 9116 + 5182 + 3080$

3] $103^2 - 97^2 =$

4] Find the square root of 8649

5] $8532 - 983 =$

6] The value of ${}^5P_3$ is

7] The roots of $x^2 + 16x - 105 = 0$ are

8] The roots of $x^2 + 13x + 30 = 0$ are

9] If log 125 = 2.1, then log 25 will be

10] If $\frac{1}{9} + \frac{1}{X} = \frac{1}{8}$, find x

11] A 50% increase followed by a 50% decrease will result in

12] Find the HCF of 15, 21 and 35

13] 93.02×77.94 is approximately:

(a) 7394 (b) 7475 (c) 7334 (d) 7250

14] $\frac{873}{699}$ is closest to:

(a) 1.35 (b) 1.25 (c) 1.07 (d) 1.13

15] $\frac{9102}{9342}$ is closest to:

(a) 0.952 (b) 0.974 (c) 0.917 (d) 0.939

16] The value of $(1.08)^6$ is closest to

(a) 1.46 (b) 1.58 (c) 1.61 (d) 1.49

Test 11

Answers

1] $279 + 254 + 308 + 963 + 436 + 174 =$

2] $4784 + 2790 + 9323 + 4195 + 6894 + 6824$

3] $121^2 =$

4] Find the square root of 4096

5] $53 \times 57 =$

6] The number of 4 digit numbers in base 5 will be

7] The roots of $x^2 + 14x - 72 = 0$ are

8] The roots of $x^2 - 12x - 28 = 0$ are

9] log 8 + 3 log 5 will be equal to

10] $\frac{11}{9}$ is roughly how much percent?

11] If $\frac{1}{5} - \frac{1}{7} = \frac{2}{x}$, find x

12] If a d% rise followed by a d% drop results in a 2.25% drop, then d must be

13] 64.37×61.94 is approximately:

(a) 3789 (b) 3987 (c) 3897 (d) 3879

14] $\frac{296}{881}$ is closest to:

(a) 0.336 (b) 0.393 (c) 0.311 (d) 0.299

15] $\frac{7143}{2402}$ is closest to:

(a) 2.46 (b) 2.97 (c) 3.04 (d) 2.78

16] The value of $(1.05)^7$ is closest to

(a) 1.3 (b) 1.35 (c) 1.4 (d) 1.49

Test 12

Answers

1] $637 + 520 + 535 + 666 + 938 + 155 =$

2] $4258 + 7798 + 6196 + 1800 + 6922 + 7202$

3] $89^2 =$

4] Find the cube root of 1728

5] $177 \times 113 =$

6] The value of ${}^{15}C_2$ is

7] The roots of $x^2 + 16x - 57 = 0$ are

8] The roots of $x^2 - 15x + 54 = 0$ are

9] The value of $\log_8 16$ is:

10] Find z, if 56 is z% of 80

11] 64% of 125 must equal

12] $\frac{7}{12}$ is roughly how much percent?

13] 45.95×41.69 is approximately:

(a) 1977 (b) 2003 (c) 1995 (d) 1916

14] $\frac{345}{763}$ is closest to:

(a) 0.423 (b) 0.478 (c) 0.495 (d) 0.452

15] $\frac{7140}{8893}$ is closest to:

(a) 0.867 (b) 0.835 (c) 0.802 (d) 0.779

16] The value of $(0.96)^6$ is closest to

(a) 0.736 (b) 0.784 (c) 0.656 (d) 0.832

Test 13

Answers

1] $258 + 894 + 321 + 640 + 675 + 426$

2] $1820 + 7790 + 6157 + 7436 + 8110 + 3781$

3] $56^2 - 44^2 =$

4] Find the square root of 11881

5] $10000 - 5607 =$

6] The value of ${}^{10}C_5$ is

7] The roots of $x^2 + 11x + 18 = 0$ are

8] The roots of $x^2 - 14x + 40 = 0$ are

9] If $\log_{10}k = 5.6789$, then how many digits will k have?

10] 70% of 60% of 50 must be

11] Find y, if 75 is 30% of y

12] Find the HCF of 1100, 1210 and 1331

13] 65.52×95.99 is approximately:

(a) 6289 (b) 6403 (c) 6370 (d) 6429

14] $\frac{802}{733}$ is closest to:

(a) 1.09 (b) 1.21 (c) 1.48 (d) 1.17

15] $\frac{1889}{9286}$ is closest to:

(a) 0.203 (b) 0.234 (c) 0.221 (d) 0.197

16] The value of $(1.12)^3$ is closest to

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

Test 14

Answers

1] $750 + 915 + 957 + 421 + 974 + 674 =$

2] $4007 + 3431 + 4543 + 3517 + 1621 + 7720$

3] $202^2 =$

4] Find the ninth root of 512

5] $321 \times 123 =$

6] If a number is written as $(143)_n$, then the smallest value it can take in base 10 is

7] The roots of $x^2 + 15x - 100 = 0$ are

8] The roots of $x^2 - 13x - 68 = 0$ are

9] The value of $\log_5 49 \times \log_{343} 25$ is

10] $\frac{10}{13}$ is roughly how much percent?

11] Find the Arithmetic Mean of 327, 344, 321, 321 and 332

12] Find the Arithmetic Mean of $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{12}$

13] 17.46×30.17 is approximately:

(a) 5268 (b) 534.6 (c) 526.7 (d) 5345

14] $\frac{239}{995}$ is closest to:

(a) 0.232 (b) 0.218 (c) 0.255 (d) 0.24

15] $\frac{4553}{3437}$ is closest to:

(a) 1.32 (b) 1.23 (c) 1.45 (d) 1.11

16] The value of $(10.9)^3$ is closest to

(a) 1225 (b) 1295 (c) 1355 (d) 1385

Test 15

Answers

1] $840 + 203 + 151 + 692 + 868 + 609 =$

2] $7959 + 7659 + 3247 + 2702 + 9305 + 7350$

3] $142^2 - 141^2 =$

4] Find the square root of 4761

5] $1337 \div 191 =$

6] The value of ${}^5C_3 + {}^5C_4 + {}^5C_5$ is

7] The roots of $x^2 - 15x + 56 = 0$ are

8] The roots of $x^2 + 15x + 14 = 0$ are

9] $1 - (\log_{xyz}x + \log_{xyz}y) =$

10] If 54% of n exceeds 45% of n by 360, find n

11] $\frac{8}{11}$ is roughly how much percent?

12] Divide 288 in the ratio 3 : 4 : 5

13] 78.33×52.47 is approximately:

(a) 4110 (b) 4220 (c) 4001 (d) 4212

14] $\frac{521}{713}$ is closest to:

(a) 0.707 (b) 0.776 (c) 0.731 (d) 0.81

15] $\frac{5817}{5596}$ is closest to:

(a) 0.987 (b) 1.111 (c) 1.087 (d) 1.039

16] The value of $(1.07)^9$ is closest to

(a) 1.77 (b) 1.82 (c) 1.89 (d) 1.95

Test 16

Answers

1] $877 + 851 + 475 + 440 + 772 + 425 =$

2] $5213 + 2957 + 1982 + 8224 + 9544 + 1933$

3] $83^2 =$

4] Find the fifth root of 3125

5] $32 \times 62 =$

6] The value of ${}^7C_5$ is

7] The roots of $x^2 + 17x - 18 = 0$ are

8] The roots of $x^2 - 10x - 11 = 0$ are

9] Which is greater: $\frac{\log 2}{3}$ or $\frac{\log 5}{7}$

10] $\frac{7}{16}$ is roughly how much percent?

11] Find the LCM of $\frac{5}{8}$, $\frac{3}{4}$ and $\frac{5}{12}$

12] Find the Median of 327, 344, 321, 321 and 332

13] 76.57×58.24 is approximately:

(a) 4459 (b) 4594 (c) 4549 (d) 4559

14] $\frac{170}{827}$ is closest to:

(a) 0.229 (b) 0.197 (c) 0.205 (d) 0.243

15] $\frac{9392}{9700}$ is closest to:

(a) 0.968 (b) 0.922 (c) 0.945 (d) 1.023

16] The value of $(1.11)^3$ is closest to

(a) 1.36 (b) 1.51 (c) 1.45 (d) 1.32

Test 17

Answers

1] $683 + 547 + 924 + 602 + 482 + 660 =$

2] $6914 + 2772 + 6808 + 3251 + 4711 + 6949$

3] $39^2 =$

4] Find the square root of 6561

5] $91 \times 19 =$

6] The value of ${}^{25}C_{23}$ is

7] The roots of $x^2 - 6x - 16 = 0$ are

8] The roots of $x^2 + 3x = 0$ are

9] If log 8 = 0.9, then log 125 will be

10] $\frac{22}{7}$ is roughly how much percent?

11] Divide 123 in two parts such that the first is 50% of the second

12] Find the Geometric Mean of 21, 72 and 49

13] 30.28×66.06 is approximately:

(a) 2094 (b) 2123 (c) 2089 (d) 2000

14] $\frac{900}{781}$ is closest to:

(a) 1.43 (b) 1.34 (c) 1.51 (d) 1.15

15] $\frac{8320}{6071}$ is closest to:

(a) 1.56 (b) 1.37 (c) 1.73 (d) 1.16

16] The value of $(0.93)^7$ is closest to

(a) 0.666 (b) 0.565 (c) 0.609 (d) 0.544

Test 18

Answers

1] $387 + 525 + 132 + 424 + 974 + 965 =$

2] $3299 + 2675 + 2833 + 6930 + 2918 + 9875$

3] $65^2 - 35^2 =$

4] Find the square root of 12996

5] $2015 \div 31 =$

6] The value of ${}^{21}P_3$ is

7] The roots of $x^2 + 16x + 39 = 0$ are

8] The roots of $x^2 + 1x = 0$ are

9] The smallest among $\log_9 2$, $\log_{25} 3$ and $\log_{49} 4$ is

10] $\frac{9}{16}$ is roughly how much percent?

11] Divide 198 in the ratio 3 : 4 : 7 : 8

12] Find the Mode of 327, 344, 321, 321 and 332

13] 48.92×76.55 is approximately:

(a) 3576 (b) 3657 (c) 3675 (d) 3745

14] $\frac{975}{446}$ is closest to:

(a) 2.03 (b) 1.97 (c) 2.32 (d) 2.18

15] $\frac{1434}{8476}$ is closest to:

(a) 0.148 (b) 0.203 (c) 0.182 (d) 0.169

16] The value of $(1.06)^6$ is closest to

(a) 1.48 (b) 1.59 (c) 1.3 (d) 1.42

Test 19

Answers

1] $836 + 941 + 190 + 180 + 103 + 627 =$

2] $9706 + 8384 + 8297 + 3983 + 8871 + 3919$

3] $92^2 =$

4] Find the fourth root of 2401

5] $222 \times 22 =$

6] The sum of $(254)_6$ and $(435)_6$ is $(k)_6$. Find k

7] The roots of $x^2 + 3x - 18 = 0$ are

8] The roots of $x^2 - 16x - 36 = 0$ are

9] What will be the value of $\log_3(9^7)$

10] Find the Harmonic Mean of $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{12}$

11] The fractions $\frac{1}{4}$, $\frac{1}{6}$ and $\frac{1}{9}$ are in what ratio?

12] Divide 65 in three parts such that the first is 25% of the second

13] 76.62×87.12 is approximately:

(a) 6507 (b) 6593 (c) 6743 (d) 6675

14] $\frac{173}{191}$ is closest to:

(a) 0.906 (b) 0.887 (c) 0.945 (d) 0.973

15] $\frac{5781}{4151}$ is closest to:

(a) 1.56 (b) 1.39 (c) 1.78 (d) 1.26

16] The value of $(1.1)^5$ is closest to

(a) 1.4 (b) 1.5 (c) 1.6 (d) 1.7

Test 20

Answers

1] $891 + 988 + 390 + 321 + 942 + 854 =$

2] $3738 + 2762 + 5129 + 4232 + 1440 + 1991$

3] $1001^2 =$

4] Find the square root of 3969

5] $510510 \div 105 =$

6] The value of ${}^9C_4$ is

7] The roots of $x^2 - 14x - 72 = 0$ are

8] The roots of $x^2 - 16x + 15 = 0$ are

9] $[\log_5 128]$ will be *(where [x] is the Greatest Integer less than or equal to x)*:

10] If $\frac{1}{30} + \frac{1}{15} = \frac{2}{x}$, find x

11] Find k, if an increase of 25% and a decrease of k% neutralize each other

12] The HCF of $\frac{4}{3}$, $\frac{16}{5}$, $\frac{12}{7}$, and $\frac{8}{3}$ is

13] 73.38×25.24 is approximately:
(a) 1852 (b) 1903 (c) 1927 (d) 1776

14] $\frac{500}{285}$ is closest to:
(a) 2.02 (b) 1.75 (c) 1.56 (d) 1.89

15] $\frac{1620}{2671}$ is closest to:
(a) 0.623 (b) 0.635 (c) 0.607 (d) 0.591

16] The value of $(10.4)^4$ is closest to
(a) 1199 (b) 1169 (c) 11999 (d) 11699

ANSWERS

Test 1

1. 2875	2. 3698	3. 2401	4. 156	5. 162
6. 1320	7. −14 & 1	8. −2 & 5	9. 1.5	10. 62
11. 60	12. 36.36%	13. a	14. d	15. b
16. d				

Test 2

1. 3284	2. 4747	3. 4225	4. 256	5. 98
6. 153	7. −6 & 20	8. −7 & −7	9. 0.75	10. 19.2
11. 18	12. 18	13. d	14. c	15. c
16. b				

Test 3

1. 3607	2. 2084	3. 99980001	4. 77	5. 495
6. 165	7. −3 & 14	8. −4 & −1	9. 3	10. 60
11. 40:24:15	12. 71.42%	13. a	14. d	15. b
16. B				

Test 4

1. 2014	2. 3159	3. $45^2 = 2025$	4. 1596	5. 228
6. 210	7. −1 & 2	8. −8 & 2	9. $\log_3 30$	10. 12
11. 21	12. 36	13. c	14. d	15. a
16. c				

Test 5

1. 39537	2. 3869	3. 10404	4. 37	5. 787878
6. $7^3 = 343$	7. −10 & −5	8. −1 & 4	9. 24	10. 25
11. Rs 1000	12. 66.66%	13. b	14. c	15. b
16. d				

Test 6

1. 27055	2. 3241	3. $85^2 = 7225$	4. 253	5. 3920
6. 1	7. −17 & 3	8. −12 & −3	9. 2	10. +33%
11. No, 1008	12. 1	13. b	14. d	15. a
16. a				

Test 7

1. 2765	2. 33593	3. 5041	4. 59	5. 2002
6. 35	7. −2 & 17	8. −3 & 18	9. 1.5	10. 15
11. 36%	12. 37	13. a	14. a	15. c
16. c				

Test 8

1. 2990	2. 30178	3. 9409	4. 3	5. 3575
6. 9	7. −3 & 4	8. −4 & 2	9. 3	10. 45
11. 30	12. 240	13. c	14. d	15. b
16. a				

Test 9

1. 3744	2. 34655	3. 12544	4. 74	5. 177
6. 720	7. –1 & 9	8. 5 & 11	9. $\log_5 600$	10. 42
11. 1	12. 15:12:10	13. b	14. d	15. a
16. d				

Test 10

1. 3846	2. 28457	3. 1200	4. 93	5. 7549
6. 60	7. –21 & 5	8. –10 & –3	9. 1.4	10. 72
11. 25% decrease	12. 1	13. d	14. b	15. b
16. b				

Test 11

1. 2414	2. 34810	3. 14641	4. 64	5. 3021
6. 500	7. –18 & 4	8. –2 & 14	9. 3	10. 122.22
11. 35	12. 15	13. b	14. a	15. b
16. c				

Test 12

1. 3451	2. 34176	3. 7921	4. 12	5. 20001
6. 105	7. –19 & 3	8. 6 & 9	9. 4/3	10. 70
11. 80	12. 58.33	13. d	14. d	15. c
16. b				

Test 13

1. 3214	2. 35094	3. 1200	4. 109	5. 4393
6. 252	7. –9 & –2	8. 4 & 10	9. 6	10. 21
11. 250	12. 11	13. a	14. a	15. a
16. b				

Test 14

1. 4691	2. 24839	3. 40804	4. 2	5. 39483
6. 48 (when n = 5)		7. –20 & 5	8. –4 & 17	9. 4/3
10. 76.9	11. 329	12. 1/4	13. c	14. d
15. a	16. b			

Test 15

1. 3363	2. 38222	3. 285	4. 69	5. 7
6. 16	7. 7 & 8	8. –14 & –1	9. $\log_{xyz} z$	10. 4000
11. 72.72	12. 72, 96, 120	13. a	14. c	15. d
16. b				

Test 16

1. 3840	2. 29853	3. 6889	4. 5	5. 1984
6. 21	7. –18 & 1	8. –1 & 11	9. $\frac{\log 2}{3}$	10. 43.75
11. 15/4	12. 327	13. a	14. c	15. a
16. a				

Test 17

1. 3898	2. 31405	3. 1521	4. 81	5. 1729
6. 300	7. –2 & 8	8. –3 & 0	9. 2.1	10. 314.28
11. 41, 82	12. 42	13. d	14. d	15. b
16. c				

Test 18

1. 3407	2. 28530	3. 3000	4. 114	5. 65
6. 7980	7. –13 & –3	8. –1 & 0	9. $\log_9 2$	10. 56.25
11. 27, 36, 63, 72		12. 321	13. d	14. d
15. d	16. d			

Test 19

1. 2877	2. 43160	3. 8464	4. 7	5. 4884
6. 1133	7. –6 & 3	8. –2 & 18	9. 14	10. 3/20
11. 9:6:4	12. 13, 52	13. d	14. a	15. b
16. c				

Test 20

1. 4386	2. 19292	3. 1002001	4. 63	5. 4862
6. 126	7. –4 & 18	8. 1 & 15	9. 3	10. 20
11. 20%	12. 4/105	13. a	14. b	15. c
16. d				

Notes

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