Geometry - 3

Career Launcher

QA - 27

CEX-Q-0228/18

Contents

- Polygon
- Quadrilateral

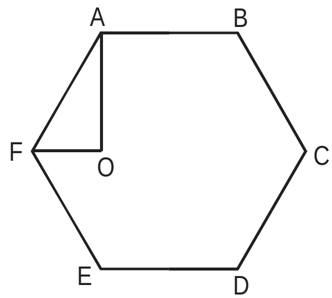
Number of Questions : 25

1. If the sides of a regular octagon are extended to form a star shaped figure, then find the angle formed at each vertex of the star shaped figure.
(1) 90° (2) 135°
(3) 45° (4) 75°

2. If PQRSTUVW is a regular octagon, then what is the measure of $(\angle WPQ - \angle PVQ)$?
(1) 135° (2) 122.5°
(3) 115° (4) 112.5°

3. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be
(1) 20 (2) 0
(3) 21 (4) 22

4. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



(1) $\frac{1}{12}$ (2) $\frac{1}{6}$
(3) $\frac{1}{24}$ (4) $\frac{1}{18}$

[CAT 2003 (L)]

5. In a regular hexagon ABCDEF, O is the meeting point of the larger diagonals of this regular hexagon. Find (area of the triangle AOF): (area of triangle AED):(area of triangle AEC):(area of triangle FEC)?
(1) 1 : 2 : 3 : 3 (2) 1 : 2 : 3 : 2
(3) 1 : 3 : 4 : 3 (4) 2 : 2 : 3 : 2

6. The number of sides of two regular polygons is in the ratio 1 : 2 and the difference between their exterior angles is 45 degrees. The number of sides in one of these polygons is
(1) 12 (2) 10
(3) 6 (4) 8

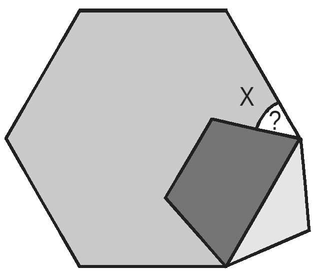
7. In a regular octagon PQRSTUVW, each side is of length 2 cm and XYUT is a square such that points X and Y are inside the octagon. Find the area of hexagon PQRXYW?

(1) $4\sqrt{2}+2$ cm^2 (2) $4\sqrt{3}+2$ cm^2

(3) $4\sqrt{2}+4$ cm^2 (4) $3\sqrt{2}+2$ cm^2

8. Given any (concave or convex) octagon, what's the maximum possible number of acute internal angles?

9. Two vertices of a regular pentagon (smaller polygon) intersect with two vertices of a regular hexagon (larger polygon) as given in the figure below. What is the measure of the angle x?

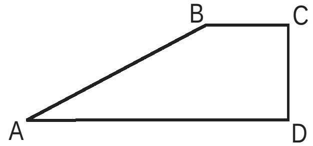


10. A regular polygon of 12 sides is circumscribed by a circle. Find the ratio of the area of polygon and the circle.

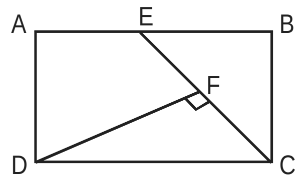
(1) $\frac{8}{3\pi}$ (2) $\frac{5}{2\pi}$

(3) $\frac{3}{\pi}$ (4) None of these

11. In the figure below angles ADC and BCD are right-angles and AB has length 13, AD = 16 and CD = 5. What is the perimeter of quadrilateral ABCD?



12. ABCD is a rectangle. DF = 20 cm, CE = 30 cm. What is the area of rectangle ABCD?



(1) 150 cm^2 (2) 300 cm^2
(3) 600 cm^2 (4) Cannot be determined

13. ABCD is a parallelogram. P is a point on AB such that AP : PB = 3 : 2. Q is a point on CD such that CQ : QD = 7 : 3. If PQ intersects AC at R, then find the ratio AR : AC.
(1) 5 : 11 (2) 1 : 2
(3) 4 : 7 (4) 6 : 13

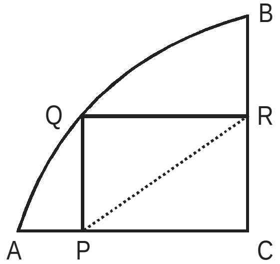
14. ABCD is a rectangle inscribed in a circle of radius 5 cm. One of the sides of the rectangle is 8 cm long. Find the area of the quadrilateral formed by joining the mid-points of the sides of the rectangle.
(1) 48 sq. cm (2) 36 sq. cm
(3) 24 sq. cm (4) 30 sq. cm

15. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$, respectively, then the ratio $\frac{P_1+P_2+P_3+\cdots}{A_1+A_2+A_3+\cdots}$ equals **[CAT 2003 (R)]**

(1) $\frac{2\left(1+\sqrt{2}\right)}{a}$ (2) $\frac{2\left(2-\sqrt{2}\right)}{a}$

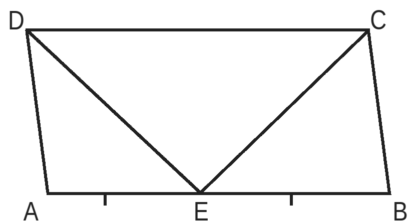
(3) $\frac{2\left(2+\sqrt{2}\right)}{a}$ (4) $\frac{2\left(1+2\sqrt{2}\right)}{a}$

16. The quarter circle has center C & radius = 10. If the perimeter of rectangle CPQR is 26, then the perimeter of APRBQA is

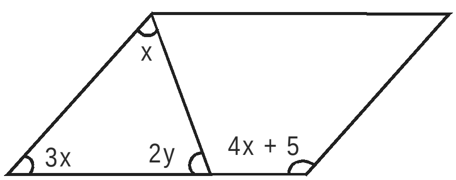


(1) $12+5\pi$ (2) $17+5\pi$
(3) $15+7\pi$ (4) $15+5\pi$

17. ABCD is a parallelogram. E is the midpoint of AB & CE bisect angle BCD. Then angle DEC in degrees is

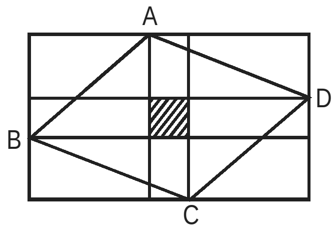


18. In the parallelogram below, what is the value of y?



19. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to 40% of the longer side. Then the ratio of the shorter side to longer side is-
(1) 3 : 4 (2) 5 : 12
(3) 8 : 15 (4) 2 : 3

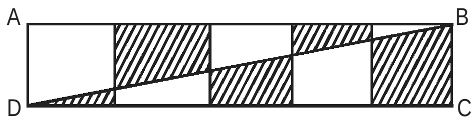
20. Two pairs of parallel lines divide a rectangle into 9 smaller rectangles. The area of original rectangle is A, the area of shaded rectangle in the middle is A_0. The area of quadrilateral ABCD will be-



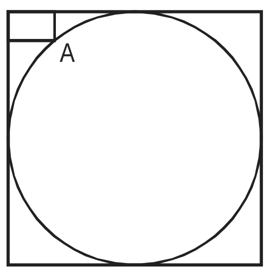
(1) $\dfrac{A-2A_0}{2}$ (2) $\dfrac{A+A_0}{2}$

(3) $\dfrac{A+2A_0}{2}$ (4) $\dfrac{A-A_0}{2}$

21. In the figure, ABCD is a rectangle formed by five squares, each of side 4 cm. BD is a diagonal. Find the area (in cm^2) of shaded region.



22. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



(1) 10 (2) 40
(3) 50 (4) None of these

23. A square is inscribed in a quarter of a circle in such a manner that two of its adjacent vertices lie on the radii at an equal distance from the centre, while the other two vertices lie on the circular arc. If the square has side of length x, then the radius of the circle is.

(1) $\frac{16x}{(\pi+4)}$ (2) $x\sqrt{2.5}$

(3) $x\sqrt{2}$ (4) $\sqrt{\frac{3}{2}}x$

24. ABCD is an isosceles trapezium with BC = AD = 10 units, AB = 2 units and CD = 14 units. The mid-points of the sides of the trapezium are joined to form a quadrilateral PQRS. Find the ratio of the area of the circle inscribed in the quadrilateral PQRS to the area of trapezium ABCD.

(1) $\frac{3\pi}{8}$ (2) $\frac{3\pi}{16}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{8}$

25. Consider a square ABCD with midpoints E, F, G, H of AB, BC, CD and DA respectively. Let L denote the line passing through F and H. Consider points P and Q, on L and inside ABCD, such that the angles APD and BQC both are equal to 120°. What is the ratio of the area of ABQCDP to the remaining area inside ABCD?

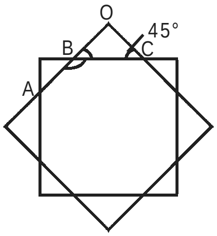
(1) $2+\sqrt{3}$ (2) $\frac{10-3\sqrt{3}}{9}$

(3) $1+\frac{1}{\sqrt{3}}$ (4) $2\sqrt{3}-1$

QA - 27 : Geometry - 3
Answers and Explanations

1	1	**2**	4	**3**	3	**4**	1	**5**	2	**6**	4	**7**	3	**8**	6	**9**	48	**10**	3
11	38	**12**	3	**13**	4	**14**	3	**15**	3	**16**	2	**17**	90°	**18**	40°	**19**	3	**20**	2
21	40	**22**	3	**23**	2	**24**	4	**25**	4										

1. 1



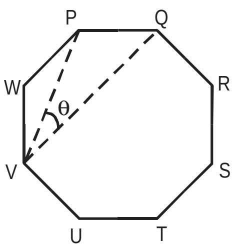
Internal Angle of an octagon $= 180° - \frac{360°}{8} = 135°$

The two base angles of the ΔOBC are (180° – 135°) = 45° each.

So, the internal angle of the star

= $\angle BOC$ = 180° – (45° × 2) = 90°.

2. 4



$\angle WPQ$ = Interior angle of a octagon = 135°

In ΔVWP, $\angle WVP = \angle WPV$

(Angles opposite to equal sides)

and $\angle VWP = 135°$, $\therefore \angle WVP + \angle WPV = 45°$

$\therefore \angle WVP = \frac{45°}{2} = 22.5°$

$\therefore \angle PVQ = 45° - \angle WVP = 45° - 22.5° = 22.5°$

$\therefore \angle WPQ - \angle PVQ = 135° - 22.5° = 112.5°$.

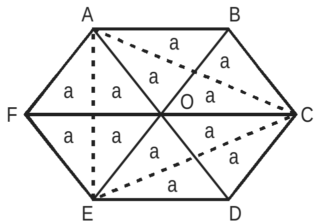
3. 3 In this kind of polygon, the number of convex angles will always be exactly 4 more than the number of concave angles.

NOTE : The number of vertices have to be even. Hence the number of concave and convex corners should add up to an even number. This is true only for the answer choice (3).

4. 1 It is very clear, that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle.

Hence, the required ratio = 1 : 12.

5. 2



The regular hexagon can be divided into 12 triangles of equal area, say a

Area of ΔAOF : Area of ΔAED : Area of ΔAEC : Area of ΔFEC = 2a : 4a : 6a : 4a = 1 : 2 : 3 : 2.

6. 4 Let the number of sides be x and 2x.

Exterior angles will be $\frac{360}{x}$ and $\frac{360}{2x}$ and their difference is given as 45.

So x = 4, thus sides will be 4 & 8.

7. 3 In a regular octagon, each angle is equal to 135.

Area of hexagon = area of rectangle PQYX + 2(Area of triangle PWX)

PX = $2\sqrt{2}$ and XY = 2 units

Area of rectangle PQXY = $4\sqrt{2}$

Area of triangle PWX = 2

Required area = $4\sqrt{2}$ + 4

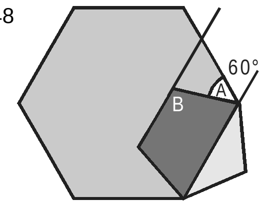
8. 6 We know, for a 'n' sided polygon the sum of all internal angles = $\frac{2 \times (n-2)\pi}{2}$.

So for an octagon this sum = $\frac{2 \times (8-2)\pi}{2} = 6\pi$. .

Now if, 8 angles are acute, then the sum will be < $\left\{\frac{8 \times \pi}{2}\right\} = 4\pi$.

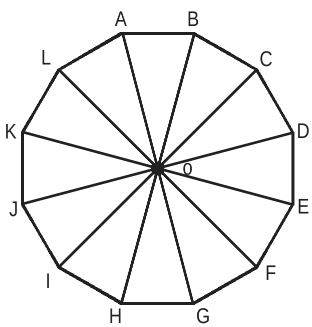
So, it cannot be true.
If 7 angles are acute then the sum of those 7 angles is < $\left\{\frac{7\times\pi}{2}\right\} = 3.5\pi$.
Then, the rest angle must be greater than $(6\pi - 3.5\pi) = 2.5\pi$ which can't exist.
If 6 angles are acute, then the sum of those 6 angles is < $\left\{\frac{6\times\pi}{2}\right\} = 3\pi$ and the sum of other 2 angles > $(6\pi - 3\pi) = 3\pi$.
So it can exist.
Hence the answer is 6.

9. 48



Interior angle of regular hexagon is $4 \times \frac{180}{6} = 120$ and its external angle will be 60. On the other hand B is an interior angle of a regular pentagon, so B is $3 \times \frac{180}{5} = 108$.
By parallel lines property B = A + 60 and A = B – 60 = 108 – 60 = 48.

10. 3



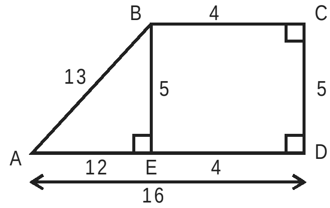
A regular polygon ABCDEFGHIJKL of 12 sides can be divided into 12 congruent triangles.
Let O be the centre of circle circumscribing the polygon.

Hence, Area of polygon = $12 \times \frac{1}{2} \times r^2 \times \sin 30° = 3r^2$

Area of circle = πr^2

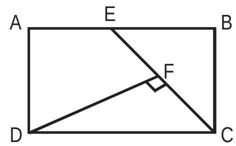
Ratio of area of polygon to circle = $\frac{3r^2}{\pi r^2} = \frac{3}{\pi}$

11. 38



BE is $\perp^{ar}$ to AD
BE = CD = 5
AE = $\sqrt{13^2 - 5^2} = \sqrt{144} = 12$
So, ED = 4
Also, ED = BC = 4
Therefore, perimeter of quadrilateral ABCD = 13 + 4 + 5 +16 = 38

12. 3 Construction: Join DE



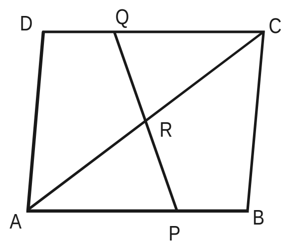
Area of $\Delta CED = \frac{1}{2} \times CE \times DF = \frac{1}{2} \times 30 \times 20$

= 300 sq. cm

Area of $\Delta CED = \frac{1}{2}$ [Area of rectangle ABCD]

Hence, area of rectangle ABCD = 600 cm^2

13. 4



In ΔCRQ and ΔAPR,
$\angle CRQ = \angle ARP$ (Vertically opposite angle)
$\angle RQC = \angle RPA$ (Alternate angles)
$\therefore$ ΔCRQ is similar to ΔARP

$\therefore \frac{AR}{RC} = \frac{AP}{CQ}$

$\frac{AP}{PB} = \frac{3}{2}$

$\therefore AP = \frac{3}{5} AB$

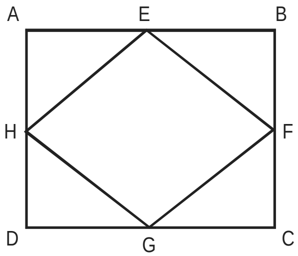
$\frac{CQ}{QD} = \frac{7}{3}$

$$\therefore CQ = \frac{7}{10}CD = \frac{7}{10}AB$$

$$\therefore \frac{AR}{RC} = \frac{AP}{CQ} = \frac{\frac{3}{5}AB}{\frac{7}{10}AB} = \frac{6}{7}$$

$$\therefore \frac{AR}{AC} = \frac{AR}{AR+RC} = \frac{6}{13}$$

14. 3



Now, since the diameter of the circle is 10 cm, it will be the diagonal of the rectangle. Thus, the sides of the rectangle would be 8 cm and 6 cm. EFGH will be a rhombus, the length of whose diagonals are 6 cm and 8 cm.
Thus, the area is given by

$$\frac{1}{2}\times d_1\times d_2 = \frac{1}{2}\times 8\times 6 = 24 \text{ sq. cm}$$

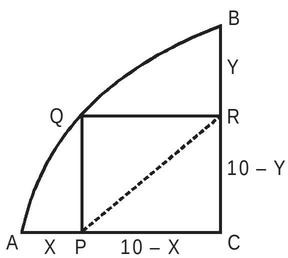
15. 3

$$\frac{P+\frac{P}{\sqrt{2}}+\cdots\infty}{A+\frac{A}{2}+\cdots\infty} = \frac{\frac{P}{1-\frac{1}{\sqrt{2}}}}{2A} = \frac{P\sqrt{2}}{\left(\sqrt{2}-1\right)}\times\frac{1}{2A}$$

$$= \frac{\sqrt{2}P\ (\sqrt{2}+1)}{2A} = \frac{\sqrt{2}\times 4a\left(\sqrt{2}+1\right)}{2\times a^2}$$

$$= \frac{\sqrt{2}\times 2\left(\sqrt{2}+1\right)}{a} = \frac{2\left(2+\sqrt{2}\right)}{a}$$

16. 2

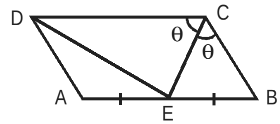


Let AP & BR be X & Y respectively.
2(10 – X) + 2(10 – Y) = 26
(Perimeter of rectangle CPQR)
X + Y = 7
PR = CQ = 10 (diagonal of rectangle)

Perimeter of APRBQA is
= AP + PR + RB + Arc AB

$$= X + 10 + Y + \frac{2\pi\times 10}{4} = 17 + 5\pi$$

17. 90°

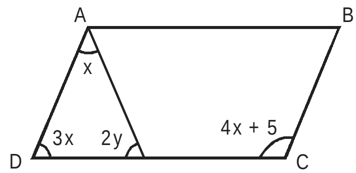


$\angle DCE = \angle CEB = \theta$ (CD || AB)
$\angle EBC = 180 - 2\theta$, EB = CB (Isosceles ΔCEB)
Hence, AD = AE
$\angle EAD = 2\theta$ ($\angle EAD + \angle EBC = 180$)

Now, $\angle AED = \angle ADE = \frac{180-2\theta}{2} = 90-\theta$

Therefore, $\angle DEC = 180 - (90 - \theta) = 90°$

18. 40°



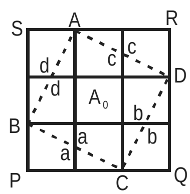
3x + 4x + 5 = 180° (AD || BC)
x = 25°
Now, x + 3x + 2y = 180° (Sum of the angles of D)
y = 40°

19. 3 Let a and b be the sides of rectangle.

than $\sqrt{a^2+b^2} = (a+b) - \frac{2}{5}a$

$$\Rightarrow \frac{16a}{5} = 6b \quad \Rightarrow \frac{b}{a} = \frac{8}{15}.$$

20. 2



Let 2a, 2b, 2c and 2d are the areas of four other rectangles as shown in the figure above. Since AB, BC, CD and AD are the diagonals of these rectangles, so it will divide them into two equals parts.
Now, $2a + 2b + 2c + 2d + A_0 = A$

$$a+b+c+d = \frac{A-A_0}{2}$$

Here, area of quadrilateral ABCD = $a + b + c + d + A_0$

$$= \frac{A+A_0}{2}.$$

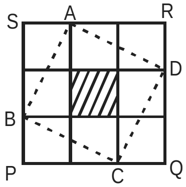
Alternative method:

We will try to make symmetrical figure by making all the rectangles as squares as shown below.

Let the area of each square be 4 sq. unit.

Then A_0 = 4 sq. unit and A = 36 sq. unit

Now, will reject the area of triangles ABS. BCP, CDQ and ADR from the larger square.



$\Rightarrow$ Area of quadrilateral ABCD = $36 - 4 \times \frac{1}{2} \times 4 \times 2$

= 20 sq. unit

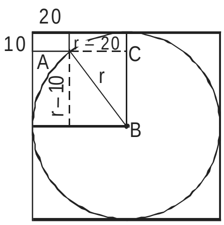
Here, 20 = $\frac{36+4}{2} = \frac{A + A_0}{2}$

i.e. only option (2) satisfy the given condition.

21. 40 By careful observation we can say that the shaded regions of first and last squares add up to form a single square, same is for second square from left and second square from right.

$\Rightarrow$ Area of shaded region = $\frac{4 \times 4 \times 4 \times 5}{2} = 40\,\text{cm}^2$.

22. 3

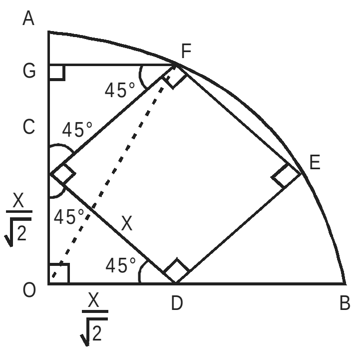


Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have $(r-10)^2 + (r-20)^2 = r^2$

i.e. $r^2 - 60r + 500 = 0$. Thus r = 10 or 50.

It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

23. 2



Let O be center CD = x

OC = OD = $\frac{x}{\sqrt{2}}$ (OCD is an isosceles Δ)

Drop a $\perp$ar from F to AO

ΔGCF will be an isosceles Δ.

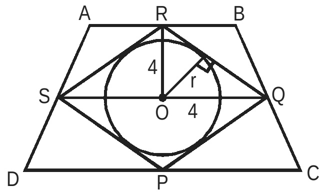
GC = GF = $\frac{x}{\sqrt{2}}$

In ΔOGF

Radius = OF = $\sqrt{GO^2 + GF^2} = \sqrt{\left(\sqrt{2}x\right)^2 + \left(\frac{x}{\sqrt{2}}\right)^2}$

$= \sqrt{2x^2 + \frac{x^2}{2}} = \sqrt{\frac{5x^2}{2}} = \sqrt{2.5}\,x$

24. 4 Here SQ = 8 unit and height of trapesium = 8 unit. and RQ = $4\sqrt{2}$ unit

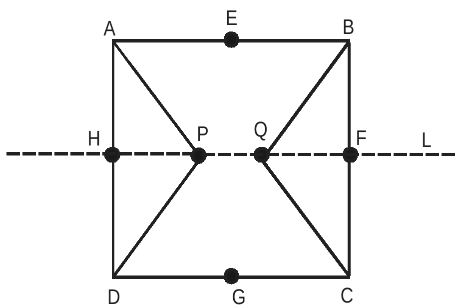


Again, 4 × 4 = $4\sqrt{2} \times r$

$\Rightarrow r = 2\sqrt{2}$ unit

Now, required ratio = $\frac{\pi(2\sqrt{2})^2}{\frac{1}{2} \times (14+2) \times 8} = \frac{16\pi}{16 \times 8} = \frac{\pi}{8}$.

25. 4



Let BF = $\sqrt{3}$ unit. So, area of square ABCD

= $\left(2\sqrt{3}\right)^2$ = 12 square unit.

So, QF = 1 unit

Thus, area of triangle BQF = $\frac{1}{2} \times \sqrt{3} \times 1 = \frac{\sqrt{3}}{2}$ sq. unit.

So, required ratio $= \frac{12 - 4 \times \frac{\sqrt{3}}{2}}{4 \times \frac{\sqrt{3}}{2}} = 2\sqrt{3} - 1$.