

**CAT**apult Courseware

## **Module 3**

Practice Exercise Solutions

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## Practice Exercise Solutions

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## Practice Exercise Solutions

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# Quantitative Ability



## QA-3.1 | TRIANGLES-I

### PRACTICE EXERCISE

- Measure of the required angle =  $90 - (90 - x) = x^\circ$
- If the complement is  $x^\circ$ , the angle =  $2x^\circ$   
 $2x + x = 90^\circ \quad \therefore x = 30^\circ$   
 $\therefore$  The angle is  $2 \times 30 = 60^\circ$
- If  $m\angle EOB$  then  $m\angle EOD = a$ ; If  $m\angle AOC = b$ , then  $m\angle COD = b$   
 $\therefore 2a + 2b = 180^\circ$   
 $\therefore a + b = 90^\circ$   
 $\therefore m\angle EOC = m\angle EOD + m\angle DOC = a + b = 90^\circ$
- $b + 60^\circ + 2b = 180^\circ$   
 $3b = 120^\circ$   
 $b = 40^\circ$   
also,  $b = 8a$  [vertically opposite angles]  
 $\therefore 8a = 40^\circ \Rightarrow a = 5^\circ$   
also,  $8a + 7c = 180^\circ$   
 $40^\circ + 7c = 180^\circ \Rightarrow 7c = 140^\circ \Rightarrow c = 20^\circ$
- $\angle APC \cong \angle RCB$  ... (corresponding angles)  
 $\therefore m\angle RCB = 70^\circ$   
 $m\angle DRC + m\angle RCB = 180^\circ$  ... (interior angles)  
 $\therefore m\angle CRD = 180^\circ - 70^\circ = 110^\circ$
- PA, QB, RC and SD are perpendicular to AD.  
Hence, they are parallel. So, the intercepts are proportional.  
Let  $PQ = x$   
 $\therefore \frac{AB}{BD} = \frac{PQ}{QS}$   
 $\therefore \frac{60}{210} = \frac{x}{360 - x} \Rightarrow \frac{2}{7} = \frac{x}{360 - x}$   
 $\therefore x = \frac{720}{9} = 80$   
 $\therefore PQ = 80 \Rightarrow QS = 360 - 80 = 280$   
Again,  $\frac{BC}{CD} = \frac{QR}{RS}$   
Let  $QR = y$   
 $\therefore \frac{90}{120} = \frac{y}{280 - y} \Rightarrow \frac{3}{4} = \frac{y}{280 - y}$   
 $\therefore 7y = 280 \times 3 \Rightarrow y = 120$   
 $\therefore QR = 120 \Rightarrow SR = 280 - 120 = 160$
- If the interior angle is  $x$ , exterior angle is  $= 2x$ .  
 $3x = 180^\circ \quad \therefore x = 60^\circ$   
 $\therefore$  Exterior angle  $= 120^\circ$   
 $\therefore$  Sum of the other two angles of the triangle  $= 120^\circ$   
(Exterior angle = Sum of two remote interior angles).
- $m\angle AQR + m\angle CRQ = 180^\circ$   
 $2a + 2b = 180^\circ$   
 $a + b = 90^\circ$   
In  $\triangle PQR$ ,  $m\angle QPR + a + b = 180^\circ$   
...(sum of all angles of a  $\triangle$  is  $180^\circ$ )  
 $m\angle QPR = 180 - 90 = 90^\circ$ .
- $m\angle AXB = m\angle XAB = 30^\circ$  ... (Isosceles triangle)  
 $m\angle ABC = 30 + 30 = 60^\circ$  ... (Exterior angle)  
 $m\angle CYA = m\angle YAC = 40^\circ$  ... (Isosceles triangle)  
 $m\angle ACB = 40 + 40 = 80^\circ$  ... (Exterior angle)  
 $m\angle BAC = 180 - 2 \times (30 + 40) = 40^\circ$  ... (Sum of all angles of a  $\triangle$  is  $180^\circ$ )  
 $m\angle XAY = 180 - (30 + 40) = 110^\circ$
- Exterior angle  $= 150^\circ$   
Corresponding interior angle  $= 180 - 150 = 30^\circ$   
Now this can be one of the equal angles, then its unequal angle  
 $= 180 - 2 \times 30 = 180 - 60 = 120^\circ$ .  
 $\therefore$  Required ratio  $= 30 : 120 = 1 : 4$   
If  $30^\circ$  is the unequal angle. Then the equal angle will be  $\frac{180 - 30}{2} = 75^\circ$ .  
 $\therefore$  Required ratio  $= 75 : 30 = 5 : 2$ . Hence, (3).
- Let the vertex angle be  $\theta$  and base angle be  $b$ .  
 $\therefore \theta + 2b = 180^\circ$  ----- (i)  
 $1.2\theta + 2(0.75b) = 180^\circ$   
 $1.2\theta + 1.5b = 180^\circ$  ----- (ii)  
Multiplying (i) by 1.5 and (ii) by 2  
 $1.5\theta + 3b = 270$   
 $2.4\theta + 3b = 360$   
 $- \quad - \quad = \quad -$   
 $0.9\theta = 90$   


---

 $\theta = 100^\circ$   
Hence, (3).

12. The 3<sup>rd</sup> side of any triangle has to be greater than that difference of two other sides. Thus, 3 cannot be the 3<sup>rd</sup> side of a triangle, where two sides are 7 and 10. Hence, (1).

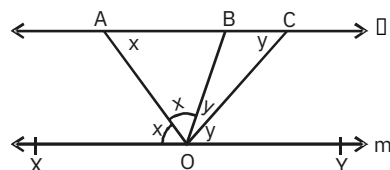
13. Here  $20^2 = 400$

$$10^2 = 100$$

$$12^2 = 144$$

We can see  $20^2 > 10^2 + 12^2$ . Hence, (3).

- 14.



$m\angle AOX = m\angle OAB = x$  ... (alternate angles)

$m\angle COY = m\angle OCB = y$  ... (alternate angles)

$\therefore \ell(AB) = \ell(OB) = (BC) = 5 \text{ cm}$ .

Hence, (2).

15. Out of the given options only option (3) is pythagoras triplet.  $(34)^2 = (16)^2 + (30)^2$ . Hence, (3).

16. By ASA test  $\triangle ABC \cong \triangle DAC$

Thus,  $AB = DA$ . Hence, (3).

17. For  $\triangle ADC$  and  $\triangle AEB$ ,

$\angle A$  is common and  $\angle ADC = \angle AEB = \frac{\pi}{2}$ ,

hence  $\triangle ADC$  and  $\triangle AEB$  are similar.

Hence (1) is true.

$\angle DOB = \angle EOC$  and  $\angle BDO = \angle OEC = \frac{\pi}{2}$ , hence  $\triangle BOD$  and  $\triangle COE$  are similar.

Hence (2) is true.

$\triangle ADC$  and  $\triangle AEB$  are similar hence

$$\frac{AD}{AE} = \frac{AC}{AB} \Rightarrow AD \times AB = AC \times AE.$$

Hence (3) is true.

Also as  $\triangle BOD \sim \triangle COE$

$$\frac{BD}{CE} = \frac{BO}{CO} \therefore BD \times OC = BO \times CE$$

But  $OC \neq OE$ .

Hence, (4).

18. Consider  $\triangle PQT$ ,  $\triangle QRU$  and  $\triangle RPS$

$$\ell(QT) = \ell(RU) = \ell(PS)$$

$$\ell(PQ) = \ell(QR) = \ell(RS) \text{ and}$$

$$m\angle Q = m\angle R = m\angle P = 60^\circ$$

$$\therefore \triangle PQT \cong \triangle QRU \cong \triangle RPS$$

$$\ell(PT) = \ell(QU) = \ell(SR) \quad \dots (i)$$

Also in  $\triangle PSV$ ,  $\triangle QTW$  and  $\triangle XUR$

$$\angle P = \angle Q = \angle R$$

$$\ell(PS) = \ell(QT) = \ell(RU) \text{ and}$$

$$\angle S = \angle T = \angle U$$

$$\therefore \triangle PSV \cong \triangle QTW \cong \triangle XUR$$

$$\therefore \ell(PV) = \ell(QW) = \ell(XR) \quad \dots (ii)$$

$$\text{and } \ell(SV) = \ell(TW) = \ell(UX) \quad \dots (iii)$$

From (i), (ii) and (iii),

$$\ell(SR) - \ell(SV) - \ell(XR)$$

$$= \ell(QU) - \ell(QW) - \ell(XU)$$

$$= \ell(PT) - \ell(PV) - \ell(TW)$$

$$\therefore \ell(VX) = \ell(WX) = \ell(VW)$$

$\triangle VWX$  is an equilateral triangle.

Hence, (1).

Alternatively;

Consider  $\triangle PQT$ ,  $\triangle QRU$  and  $\triangle RPS$

$$\ell(QT) = \ell(RU) = \ell(PS)$$

$$\ell(PQ) = \ell(QR) = \ell(RS) \text{ and}$$

$$m\angle Q = m\angle R = m\angle P = 60^\circ$$

$$\therefore \triangle PQT \cong \triangle QRU \cong \triangle RPS$$

$$\text{Let } m\angle SRP = m\angle TPQ = x$$

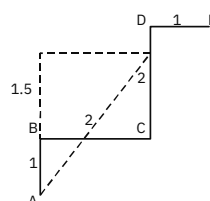
Hence,  $\angle WVX = (60 - x) + x$  (Being exterior angle of  $\triangle VPR$ ) =  $60^\circ$

Similarly,  $\angle W$  and  $\angle X$  are each  $60^\circ$

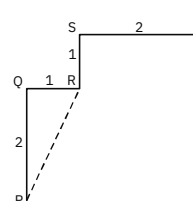
$\therefore \triangle VWX$  is an equilateral triangle.

- 19.

For Amit,



For Sumit,



Let  $S_A$  and  $S_S$  be speeds of Amit and Sumit.  $D_A$  and  $D_S$  be the distances covered by them respectively.

Total distance to be covered is 6 km.

$$S_A : S_S = 3 : 2$$

$$\frac{D_A}{D_S} = \frac{3}{2} \Rightarrow \frac{D_A}{3} = \frac{3}{2}$$

$$\therefore D_A = 4.5 \text{ km.}$$



For Amit,

Distance from starting point

$$= \sqrt{2.5^2 + 2^2} \approx 3.2 \text{ units}$$

For Sumit

Distance from starting point =  $\sqrt{5}$  units.

Required ratio =  $3.2 : \sqrt{5}$

Hence, (2).

20. I. In  $\Delta PQS$   
 $PS^2 = PQ^2 - QS^2 = a^2 - c^2$  ----- (i)  
 In  $\Delta PSR$   
 $PS^2 = PR^2 - SR^2 = b^2 - d^2$  ----- (ii)  
 From (i) and (ii)  
 $a^2 - c^2 = b^2 - d^2$   
 $\therefore a^2 - b^2 = c^2 - d^2$   
 $\therefore (a - b)(a + b) = (c + d)(c - d)$ .  
 Thus, I is true.  
 II.  $(QR)^2 = (m^2 + n^2)^2 = m^4 + n^4 + 2m^2n^2$   
 $(PR)^2 = (2mn)^2 = 4m^2n^2$   
 $(PQ)^2 = (m^2 - n^2)^2 = m^4 + n^4 - 2m^2n^2$   
 Thus we have  $QR^2 - PQ^2$   
 $= m^4 + n^4 + 2m^2n^2 - (m^4 + n^4 - 2m^2n^2)$   
 $= 4m^2n^2 = (2mn)^2 = PR^2$   
 Thus  $\Delta PQR$  is a right angle triangle, right angle at P, with QR as hypotenuse.  
 Thus  $\angle QPR = 90^\circ$ . Thus, II is true. Hence, (3).
21. Using the properties of parallel lines, we can conclude that  $\frac{BD}{DF} = \frac{AE}{EG}$  and  $\frac{CD}{DF} = \frac{CE}{EG}$ .  
 Subtracting the two equations, we get  
 $\frac{BD - CD}{DF} = \frac{AE - CE}{EG}$ , that is,  $\frac{BC}{DF} = \frac{AC}{EG}$   
 $\therefore EG = \frac{12 \times 5}{4} = 15$
22.  $(3x + 5) + (2x + 25) + (x + 30) = 180 - \angle A + 180 - \angle B + 180 - \angle C = 540 - (\angle A + \angle B + \angle C)$   
 $= 540 - 180 = 360^\circ$   
 $\therefore 6x + 60 = 360$   
 $\therefore x = 50$   
 $\therefore \angle CBA + \angle CAB = x + 30 = 80^\circ$   
 $\therefore$  The required answer is 80.

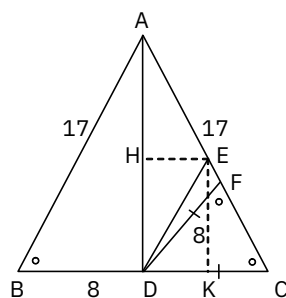
## QA-3.2 | TRIANGLES-II

### PRACTICE EXERCISE

For question 1:

Since ABC is an isosceles triangle with  $AB = AC$ , we can show that ABD and ACD are congruent. Hence  $BD = 8$ .

By the Theorem of Pythagoras,  $AD = \sqrt{17^2 - 8^2} = 15$ .



a. Using Apollonius Theorem

$$AD^2 + DC^2 = 2(DE^2 + DC^2)$$

$$\Rightarrow 15^2 + 8^2 = 2(DE^2 + 8.5^2)$$

Solving we get  $DE = 8.5$

We can show that EDC will be similar to ABC.

Since  $DC = \frac{1}{2} BC$ , DE (and EC) will be  $\frac{1}{2} AB$  or  $AC = 8.5$  units. Now consider  $\triangle ABC$  and  $\triangle DFC$ .

$\angle ABC = \angle ACB = \angle DCF = \angle DFC$  and hence the two triangles are similar. Now  $\frac{BC}{AB} = \frac{16}{17}$ . Hence

$\frac{FC}{DF}$  should also be  $\frac{16}{17}$ . But  $DF = 8$ . Hence  $FC$

$$= \frac{16}{17} \times 8 = \frac{128}{17} \approx 7.53. \text{ Hence EF will be } EC$$

$$- FC = 8.5 - 7.53 = 0.97 \text{ units}$$

b. If we drop a perpendicular EH to AD,  $AHE \sim ADC$  and hence  $\frac{EH}{CD} = \frac{EA}{CA} = \frac{1}{2}$ . But  $CD = 8$ , hence  $EH = 4$

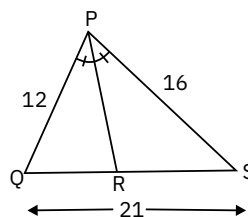
If we drop a perpendicular EK to BC,  $EKC \sim ADC$

and hence  $\frac{EK}{AD} = \frac{EC}{AC} = \frac{1}{2}$ . But  $AD = 15$ , hence  $EK = 7.5$

c. An equilateral triangle of side 16 will have height  $\frac{\sqrt{3}}{2} \times 16 \approx 1.732 \times 8 \approx 13.85$  units.  $AD = 15$  units and hence the excess height is  $15 - 13.85 = 1.15$  units

d. In  $\triangle ADC$ , DE is a median. Hence the two triangles will have equal areas.

2.



$$\frac{PQ}{PS} = \frac{QR}{RS} \dots (\text{angle bisector theorem})$$

$$\frac{12}{16} = \frac{x}{21-x}$$

$$\frac{3}{4} = \frac{x}{21-x}; 63 - 3x = 4x$$

$$\therefore x = 9$$

$\therefore QR = 9$  and  $RS = 21 - 9 = 12$ . Hence, [1].

$$3. \frac{A(\Delta_1)}{A(\Delta_2)} = \frac{b_1 h_1}{b_2 h_2}$$

$$\frac{25}{30} = \frac{b_1}{b_2} \dots (\text{as } h_1 = h_2) \therefore \frac{b_1}{b_2} = \frac{5}{6}$$

$\therefore$  Ratio of bases = 5 : 6. Hence, (4).

4. Using Apollonius Theorem

$$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{AB^2}{PQ^2} \dots (\text{Areas of similar triangles})$$

$$\frac{A(\triangle ABC)}{175} = \left(\frac{2}{5}\right)^2$$

$$A(\triangle ABC) = \frac{4}{25} \times 175 = 28 \text{ sq. cm. Hence, (3).}$$

$$5. (DE)^2 + (DF)^2 = 2[(DP)^2 + (PF)^2]$$

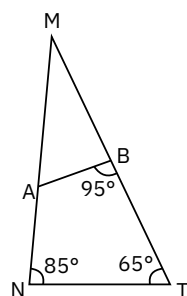
$$7^2 + 9^2 = 2[(DP)^2 + 4^2]$$

$$49 + 81 = 2[(DP)^2 + 16]$$

$$(DP)^2 = 49 \therefore DP = 7$$

$$GP = \frac{1}{3} \times DP = \frac{1}{3} \times 7 = \frac{7}{3}. \text{ Hence, (2).}$$

6.

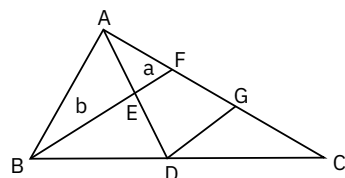


$m\angle MBA = 180^\circ - 95^\circ = 85^\circ$   
 $\therefore \angle MBA \cong \angle MNT$   
 $\angle AMB \cong \angle TMN$   
 ...(same angles with different names)  
 $\therefore \triangle MBA \sim \triangle MNT$  ... (AA test for similarity)  
 $\frac{MB}{MN} = \frac{AB}{NT}$  ... (proportional sides)  
 $\frac{10}{MN} = \frac{5}{9} \therefore MN = \frac{90}{5} = 18$ . Hence, (4).

7.  $FB \parallel CD$ . Hence  $\triangle EFB$  and  $\triangle ECD$  are similar.

$\Rightarrow \frac{FE}{EC} = \frac{y}{DC} \Rightarrow y = \frac{6 \times 22}{12} = 11$   
 $BF \parallel AG$ . Hence  $\triangle BFC$  and  $\triangle AGC$  are similar.  
 $\Rightarrow \frac{FC}{FC+x} = \frac{FB}{GA} \Rightarrow \frac{18}{18+x} = \frac{11}{20}$   
 $\Rightarrow \frac{360}{11} = 18 + x \Rightarrow x = \frac{162}{11}$   
 $\Rightarrow xy = 162$ .  
 Hence, (1).

8.



Drawing  $DG$  parallel to  $EF$ ,  
 $\triangle AEF$  and  $\triangle ADG$  are similar as  $DG$  is parallel to  $EF$ .  
 $\frac{AE}{AD} = \frac{EF}{DG} = \frac{AF}{AG}$   
 $E$  is midpoint of  $AD$   
 $\therefore \frac{AE}{ED} = \frac{AF}{FG} = 1$   
 $\Rightarrow AF = FG \Rightarrow \frac{AG}{AF} = \frac{AD}{AE} = \frac{DG}{EF} = \frac{2}{1} \Rightarrow DG = 2EF$   
 or  $DG = 2a$  ... (1)

Also  $\triangle FBC$  and  $\triangle GDC$  are similar because  $BE$  and  $DG$  are parallel:

Now given  $D$  is midpoint of  $BC$

$$\Rightarrow \frac{BC}{DC} = \frac{FB}{DG} = 2 \Rightarrow FB = 2DG = 4a \quad \dots (2)$$

$$\Rightarrow BF = FB - EF = 4a - a = 3a = b$$

$$\Rightarrow \frac{a}{b} = \frac{1}{3}.$$

Hence, (1).

9.  $\frac{\ell(BC)}{\ell(AC)} = \frac{16}{12} = \frac{4}{3}$

and  $\frac{\ell(AC)}{\ell(DC)} = \frac{12}{9} = \frac{4}{3}$

$\therefore \triangle ADC \sim \triangle ABC$  ... (by S-A-S test)

$$\Rightarrow \frac{\ell(AB)}{\ell(AD)} = \frac{4}{3}$$

$$\Rightarrow \ell(AD) = \frac{3}{4} \times 8 = 6 \text{ units}$$

$$\therefore \text{The required ratio} = \frac{(7+8+6)}{(12+6+9)} = \frac{7}{9}. \text{ Hence, (1).}$$

10.  $\triangle ABC$  is an isosceles triangle.

Hence,  $BD$  is the median from  $B$  to  $AC$ .

$$\therefore CD = AD \Rightarrow y = 2x.$$

$\therefore$  Three sides of the triangle are  $x$ ,  $x$  and  $4x$ .  
 Such a triangle cannot be formed. Hence, [4].

11. Let  $\ell$  be the length of the ladder.

$$\therefore \ell(AB) = \ell(A'B') = \ell$$

$$\therefore \ell(OB) = \frac{\ell}{\sqrt{2}}$$

$$\ell(OB') = \frac{\ell}{\sqrt{2}} - x = \frac{\ell}{2}$$

$$\therefore x = \frac{\ell}{\sqrt{2}} - \frac{\ell}{2} = \frac{\ell}{2\sqrt{2}}(2 - \sqrt{2})$$

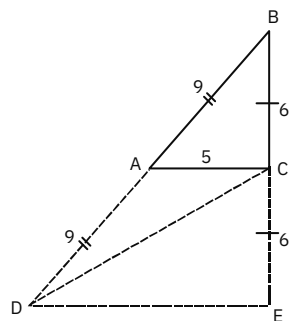
$$\ell(AA') = y = \ell(A'O) - \ell(OA) = \frac{\sqrt{3}\ell}{2} - \frac{\ell}{\sqrt{2}}$$

$$\therefore \frac{x}{y} = \frac{\frac{\ell}{2\sqrt{2}}(2 - \sqrt{2})}{\frac{\ell}{2\sqrt{2}}(\sqrt{6} - 2)} = \frac{2 - \sqrt{2}}{\sqrt{6} - 2} = \frac{\sqrt{2} - 1}{\sqrt{3} - \sqrt{2}}$$

Hence, (1).

12. D is orthocentre of  $\triangle AEC$   
 $\therefore m\angle AEC = 180 - m\angle ADC$   
 $= 180 - (180 - m\angle ADB)$   
 $= m\angle ADB$   
 $\therefore m\angle AEC = 55^\circ$ . Hence, (2).
13.  $(PQ)^2 + (PR)^2 = 2[(PX)^2 + (XR)^2]$   
 .... Appollonius theorem.  
 $(7.5)^2 + (6.5)^2 = 2[(PX)^2 + (4)^2]$   
 $56.25 + 42.25 = 2[(PX)^2 + 16]$   
 $2(PX)^2 = 66.5$   
 $(PX)^2 = 33.25$  or  $PX \approx 5.7$   
 $GX = \frac{1}{3}(5.7) = 1.9$ . Hence, (3).
14. Given the perimeter, an equilateral triangle encloses the maximum area.  
 $\therefore$  Side of the equilateral triangle  $= \frac{12}{3} = 4$  cm.  
 $\therefore$  Area  $= \frac{\sqrt{3}}{4} \times 4^2 = 4\sqrt{3}$  cm<sup>2</sup>. Hence, (1).
15. Since BE is parallel to CD.  
 $\triangle ABE$  and  $\triangle ACD$  are similar.  
 $\frac{AC}{AB} = \frac{CD}{BE} \Rightarrow BE = 9.66 \times \frac{3}{5} \approx 5.8$   
 Hence desired area  $= \frac{1}{2} AB \cdot BE \cdot \sin \angle ABE$   
 $= \frac{1}{2} \times 3 \times 5.8 \times \sin 45^\circ = 6.15$ .  
 Hence, (3).

16.

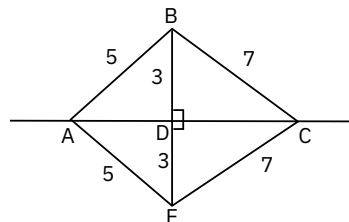


$\triangle BAC \sim \triangle BDE$  (by S-A-S test)  
 $\therefore \frac{\ell(AC)}{\ell(DE)} = \frac{\ell(BC)}{\ell(BE)}$  i.e.,  $\frac{5}{\ell(DE)} = \frac{6}{12}$   
 $\therefore \ell(DE) = \frac{5 \times 12}{6} = 10$  units.  
 C is the mid-point of BE.  
 $\therefore A(\triangle DCE) = \frac{1}{2} \times A(\triangle DBE)$

$$= \frac{1}{2} \times \sqrt{20 \times (10) \times (8) \times (2)}$$

$$= \frac{1}{2} \times 40\sqrt{2} = 20\sqrt{2} \text{ sq. units. Hence, (1).}$$

17.



Area of the above figure  $= 2 A(\triangle ABC)$

$$AD = \sqrt{5^2 - 3^2} = 4$$

$$DC = \sqrt{7^2 - 3^2} = \sqrt{49 - 9} = \sqrt{40} = 2\sqrt{10}$$

$$\therefore AC = 4 + 2\sqrt{10}$$

$$\text{Area of } \triangle ABC = \frac{1}{2} \times BD \times AC$$

$$= \frac{1}{2} \times 3 \times (4 + 2\sqrt{10}) = 3(2 + \sqrt{10})$$

$$\text{Area of figure} = 2 \times 3(2 + \sqrt{10})$$

$$= 6(2 + \sqrt{10}) \text{ sq. units. Hence, (2).}$$

18. Consider all isosceles triangles with integer sides and perimeter 20. The sum of any two sides of a triangle is greater than the third side. Using this property, there are only 4 possible combinations of sides: (6,6,8), (7,7,6), (8,8,4) and (9,9,2).

Hence (2).

19. From the sides of  $\triangle ABC$ , we can conclude that it is a 30-60-90 triangle.

Similarly,  $\triangle ADB$ ,  $\triangle BED$ ,  $\triangle DFE$ ,  $\triangle EFC$  and  $\triangle BDC$  are also 30-60-90 triangles.

$$\text{Now, } BC = \frac{8}{\sqrt{3}}.$$

Let  $EF = a$ .

$$\therefore FC = \sqrt{3}a \text{ and } EC = 2a$$

$$\text{In } \triangle DEC, DE = \frac{EC}{\sqrt{3}} = \frac{2a}{\sqrt{3}}$$

$$\text{In } \triangle BED, BE = \frac{DE}{\sqrt{3}} = \frac{2a}{\sqrt{3}}$$

$$\text{Now, } BC = BE + EC$$

$$\therefore \frac{8}{\sqrt{3}} = \frac{2a}{\sqrt{3}} + 2a = \frac{8a}{3}$$

$$\therefore a = \sqrt{3}$$

Hence (3).

20. In  $\triangle VUS$  and  $\triangle VRP$ ,  $\angle VUS = \angle VRP = 60^\circ$  and  $\angle SVU = \angle PVR$  (vertically opposite angles).

$\therefore \triangle VUS$  is similar to  $\triangle VRP$ .

$$\therefore \frac{VS}{VP} = \frac{US}{RP} = \frac{TU}{QR} = \frac{1}{3}$$

$$\therefore VP = 3 \times 3 = 9$$

$$\therefore PS = 3 + 9 = 12$$

Therefore the required answer is 12.

21.  $\angle BAC + \angle BCA = 90^\circ$

$$\therefore \angle IAC + \angle ICA = 45^\circ$$

$$\therefore \angle AIC = 180 - 45 = 135^\circ$$

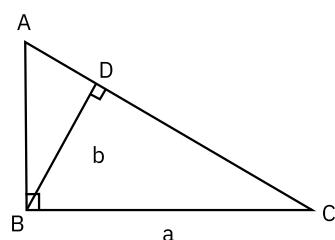
$$\therefore \angle AIB = 360 - 135 - 125 = 100^\circ$$

$$\angle IBA = 45^\circ$$

$$\therefore \angle BAI = 180 - 100 - 45 = 35^\circ$$

Therefore, the required answer is  $35^\circ$ .

22. The triangle can be represented as follows:



$$\frac{1}{2} \times AB \times a = R$$

$$\therefore AB = \frac{2R}{a}$$

Now,  $\triangle ADB \sim \triangle ABC$  (Since  $\angle A$  is common and  $\angle ADB = \angle ABC = 90^\circ$ )

$$\therefore \frac{\text{Area of } \triangle ADB}{\text{Area of } \triangle ABC} = \frac{DB^2}{BC^2} = \frac{b^2}{a^2}$$

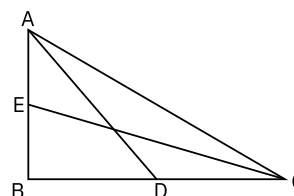
Also,  $\triangle CDB \sim \triangle CBA$  (Since  $\angle C$  is common and  $\angle CDB = \angle CBA = 90^\circ$ )

$$\therefore \frac{\text{Area of } \triangle CDB}{\text{Area of } \triangle CBA} = \frac{DB^2}{BA^2} = \frac{a^2 b^2}{4R^2}$$

$$\therefore \frac{\text{Area of } \triangle ADB}{\text{Area of } \triangle CDB} = \frac{b^2}{a^2} \times \frac{4R^2}{a^2 b^2} = \frac{4R^2}{a^4}$$

Hence (2).

23. We can draw the figure as follows:



$$AD^2 = AB^2 + BD^2 = AB^2 + \frac{1}{4} BC^2$$

$$CE^2 = BC^2 + BE^2 = BC^2 + \frac{1}{4} AB^2$$

$$\therefore AD^2 + CE^2 = \frac{5}{4} (AB^2 + BC^2) = \frac{5}{4} (AC^2)$$

$$\therefore AD^2 + 225 = \frac{5}{4} \times 256 = 320$$

$$\therefore AD = \sqrt{95}$$

Hence [2].

24.  $\angle ABC = \angle ACB$  implies  $AB = AC = 8$

$$\angle ABC = \angle DAB \text{ implies } AD = BD$$

Now,  $\triangle ABC \sim \triangle DAB$  (AA similarity)

$$\therefore \frac{BC}{AB} = \frac{AC}{DB}$$

$$\therefore BC \times (BC + CD) = AB \times AC$$

$$\therefore 4 \times (4 + CD) = 8 \times 8$$

$$\therefore CD = 12$$

Hence, (3).

## QA-3.3 | QUADRILATERALS & POLYGONS

### PRACTICE EXERCISE 1

- $A(\triangle ACD) = \frac{\sqrt{3}}{4} \times (20)^2 = 100\sqrt{3} \approx 173$   
 $(AB)^2 = (AC)^2 - (BC)^2 = 400 - 144 = 256$   
 $\therefore AB = 16$   
 $\therefore A(\triangle ABC) = \frac{1}{2} \times 16 \times 12 = 96$   
 $\therefore A(\square ABCD) = 173 + 96 = 269 \text{ sq. units. Hence, (3).}$
- $(\text{Side})^2 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2 + \left(\frac{1}{2} \times \text{other diagonal}\right)^2$   
 $13^2 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2 + \left(\frac{1}{2} \times 24\right)^2$   
 $169 - 144 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2$   
 $25 = \left(\frac{1}{2} \times \text{one diagonal}\right)^2 \Rightarrow 5 = \frac{1}{2} \times \text{diagonal}$   
 $\therefore \text{diagonal} = 10$   
 $\therefore \text{Area} = \frac{1}{2} \times 10 \times 24 = 120 \text{ sq. cm. Hence, (4).}$
- $3^2 + 4^2 = 5^2$   
 $\therefore BO \perp AC$   
 $\therefore \text{The parallelogram is a rhombus.}$   
 $\therefore \text{Area} = \frac{1}{2} \times 6 \times 8 = 24 \text{ sq. cm. Hence, (3).}$
- The parallelogram ABCD and  $\triangle BCE$  lies between the same parallel lines AB and DE and has bases of equal length.

 $\therefore A(\triangle BCE) = \frac{1}{2} A(\square ABCD)$   
 $= \frac{1}{2} \times 16 = 8 \text{ sq. cm. Hence, (1).}$
- Given AB, FC and ED are parallel.

 $ED = 2AB$   
 $\frac{BC}{CD} = \frac{2}{3}$   
 $\triangle AEB$  and  $\triangle FEG$  are similar:  
 $\Rightarrow \frac{FG}{AB} = \frac{EF}{EA} \quad - (1)$   
 Also  $\frac{AF}{FE} = \frac{BC}{CD} = \frac{2}{3}$   
 $\Rightarrow \frac{AF + FE}{FE} = \frac{2 + 3}{3}$   
 $\Rightarrow \frac{AE}{FE} = \frac{5}{3}$   
 $\Rightarrow \frac{EF}{EA} = \frac{3}{5} \quad - (2)$

$$(1) \text{ and } (2) \Rightarrow \frac{FG}{AB} = \frac{3}{5} = - (3)$$

$\triangle BCG$  and  $\triangle BDE$  are similar:

$$\Rightarrow \frac{BC}{BD} = \frac{GC}{ED}$$

$$\Rightarrow \frac{GC}{2AB} = \frac{2}{5}$$

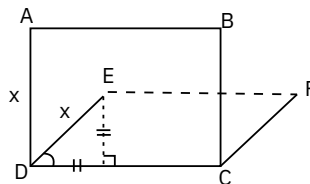
$$\Rightarrow \frac{GC}{AB} = \frac{4}{5} \quad - (4)$$

(3) and (4):

$$\Rightarrow FG + GC \Rightarrow \frac{3}{5} AB + \frac{4}{5} AB$$

$$\Rightarrow \frac{FC}{AB} = \frac{7}{5}. \text{ Hence, (2).}$$

6.



Let the length of the base be x.

Then area of square =  $x^2$

$$\because \angle EDG = 45^\circ, EG = DG \text{ and } (ED)^2 = 2(EG)^2$$

$$\therefore EG = \frac{x}{\sqrt{2}}$$

$$\text{Area of rhombus} = DC \times EG = x \times \frac{x}{\sqrt{2}} = \frac{x^2}{\sqrt{2}}$$

$$\text{Thus required ratio} = x^2 : \frac{x^2}{\sqrt{2}} = \sqrt{2} : 1.$$

Hence, (3).

7.

Area of parallelogram = Base  $\times$  Height

$$= SR \times PO = QR \times ST$$

$$\therefore 10 \times 4 = 5 \times ST \quad \therefore ST = 8$$

$\triangle STR$  is a right triangle

$$\therefore RT = \sqrt{10^2 - 8^2} = \sqrt{100 - 64} = \sqrt{36} = 6.$$

$$\text{Area of } \triangle STR = \frac{1}{2} \times ST \times TR$$

$$= \frac{1}{2} \times 8 \times 6 = 24.$$

Area of  $\square PQTS = 24 + 40 = 64$ . Hence, (2).

8.

Sum of all angles =  $90(2n - 4)$

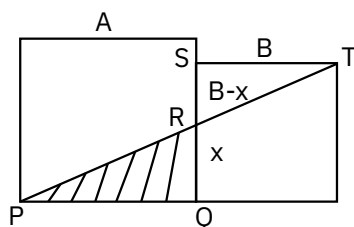
$$2340 = 90(2n - 4)$$

$$\therefore 2n - 4 = 26$$

$$2n = 30 \Rightarrow n = 15. \text{ Hence, (3).}$$

9.  $\triangle ABE \sim \triangle DFE \Rightarrow \frac{\ell(AB)}{\ell(DF)} = \frac{\ell(BE)}{\ell(EF)} = \frac{5}{7}$   
 $\Rightarrow \ell(FD) = 14$  units  
 $\ell(BC) = 14$  units  
 and  $\ell(DC) = 12$  units  
 $(\therefore \square BCDF \text{ is a parallelogram})$   
 Let area of  $\triangle ABE$  be  $x$  sq. units and that of  $\triangle ADC$  be  $y$  sq. units.  
 $\therefore \frac{x}{y} = \frac{(5)^2}{(12)^2} = \frac{25}{144}$  (as  $\triangle ABE \sim \triangle ADC$ )  
 $\therefore x = \frac{25}{144}$   
 $A(\square BCDE) = A(\triangle ADC) - A(\triangle ABE)$   
 $\therefore y - x = 238$   
 $\Rightarrow y - \frac{25}{144}y = 238 \Rightarrow \frac{119}{144}y = 238$   
 $\Rightarrow y = 288$  sq. units  
 $\Rightarrow x = 288 - 238 = 50$  sq. units.  
 Hence, (3).

10.



We need to find out the area of shaded region.

$$A(\triangle PQR) = \frac{1}{2} \times A \times x$$

Now,  $\triangle PQR \sim \triangle TSR$  (AA test of similarity).

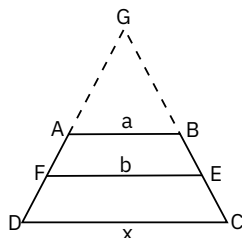
$$\therefore \frac{\ell(QR)}{\ell(SR)} = \frac{\ell(PQ)}{\ell(TS)} \Rightarrow \frac{x}{B-x} = \frac{A}{B}$$

$$\text{or } Bx = \ell(AB) - Ax \text{ or } x(A+B) = AB$$

$$\Rightarrow x = \frac{AB}{A+B}; \text{ Area} = \frac{A^2 B}{2(A+B)} \text{ sq. units.}$$

Hence, (2).

11.



Extending non parallel sides we get similar triangles GAB, GFE, GDC where FE is a line segment that divides the area of trapezium ABCD into two

equal parts.

$$\text{Now, } (\triangle GAB) = ka^2,$$

$$A(\triangle GFE) = kb^2 \text{ and}$$

$$A(\triangle GDC) = kx^2 \text{ for some constant } k \neq 0$$

Now, as per the hypothesis,

$$k(b^2 - a^2) = k(x^2 - b^2)$$

$$\therefore x^2 = 2b^2 - a^2$$

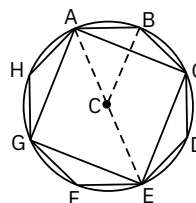
$$\therefore x = \sqrt{2b^2 - a^2} \text{ . Hence, (1).}$$

Alternatively,

Consider a quadrilateral ABCD with F and E as midpoints of AD and BC respectively, then all the conditions given in the question are satisfied.

Also in that case  $a = b = x$  which satisfies only option (1). Hence, the correct option is (1).

12.



Let the radius of the circle which inscribes the octagon be  $r$  units.

Let ABCDEFGH be the octagon as shown in the figure.

The quadrilateral formed by joining the alternate vertices is a square.

$$\text{In } \triangle OAB, m\angle AOB = \frac{360}{8} = 45^\circ$$

$$\therefore A(\triangle OAB) = \frac{1}{2} \times r^2 \times \sin 45^\circ$$

$$= \frac{r^2}{2\sqrt{2}} \text{ sq. units}$$

$$\text{Area of octagon} = 8 \times A(\triangle OAB)$$

$$= 8 \times \frac{r^2}{2\sqrt{2}}$$

$$= 2\sqrt{2} r^2 \text{ sq. units}$$

In  $\triangle ACE$ ,  $\ell(AC) = \ell(CE)$  and

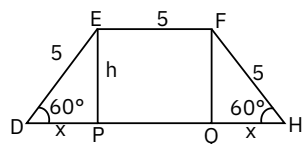
$$m\angle ACE = 90^\circ$$

$$\therefore \ell(AC) = \frac{\ell(AE)}{\sqrt{2}} = \frac{2r}{\sqrt{2}} = \sqrt{2} r \text{ units}$$

$$\therefore A(\square ACEG) = 2r^2 \text{ sq. units}$$

$$\therefore \text{Required ratio} = \frac{2r^2}{2\sqrt{2}r^2} = \frac{1}{\sqrt{2}} \text{ . Hence, (1).}$$

13.



Since  $EG \parallel DC$  and  $\ell(BG) = \ell(GC)$   
 $\Rightarrow \ell(AE) = \ell(ED)$  ..... (isosceles trapezium)

Since  $AE \parallel BF$

$\Rightarrow \square ABFE$  is a parallelogram

$\Rightarrow \ell(EF) = \ell(AB) = 5$  units

Since  $m\angle AEG = 60^\circ$

$\Rightarrow m\angle ADC = m\angle AEG$

$= m\angle BFG = m\angle BGF = m\angle BCD = 60^\circ$

$\Rightarrow \triangle BGF$  is an equilateral triangle.

$\square EFHD$  is trapezium

In  $\triangle EPD$ ,  $m\angle DEP = 30^\circ$ ,  $m\angle EDP = 60^\circ$

$\therefore x = \frac{5}{2}$  units and  $h = \frac{5\sqrt{3}}{2}$  units

Hence, base of (trapezium  $EFHD$ )

$= 5 + 2.5 + 2.5 = 10$  units

$$A(\square EFHD) = \frac{1}{2} \times (5 + 10) \times \frac{5\sqrt{3}}{2}$$

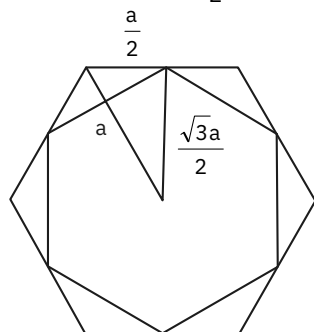
$$= \frac{75\sqrt{3}}{4} \text{ sq. units}$$

$$A(\triangle BGF) = \frac{\sqrt{3}}{4} \text{ sq. units}$$

$$\text{Hence, required area} = \frac{75\sqrt{3}}{4} + \frac{25\sqrt{3}}{4} = \frac{100\sqrt{3}}{4}$$

$= 25\sqrt{3}$  sq. units. Hence, (1).

14. Let the length of each side of the larger hexagon be 'a'. Then, the length of each side of the smaller hexagon will be  $\frac{\sqrt{3}a}{2}$ .



When side is 'a', area of hexagon is  $\frac{3\sqrt{3}a^2}{2}$ .

Therefore, area of the smaller hexagon

$$= \frac{3\sqrt{3}}{2} \times \frac{3a^2}{4} = \frac{9\sqrt{3}a^2}{8}$$

$$\therefore \text{Required percentage} = \frac{\frac{9\sqrt{3}a^2}{8}}{\frac{3\sqrt{3}a^2}{2}} \times 100 = 75\%$$

15.  $\triangle PMQ$  is similar to  $\triangle RMB$  using A-A-A test

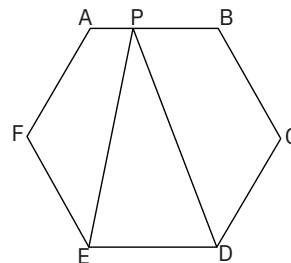
$$\therefore \frac{RM}{PM} = \frac{RB}{PQ} = \frac{4}{5}$$

Hence  $A(\triangle RMQ) : A(\triangle PMQ) = 4:5$

$A(\triangle PQR) = 126/2 = 63$ . Hence  $A(\triangle RMQ) = (4/9)(63) = 28$ .

16.  $AB = 3.5$  units and  $AD = \frac{3\sqrt{3}}{2}$  units as length and height of each triangle is 1 and respectively.  
Hence the area of the rectangle is  $\frac{21\sqrt{3}}{4}$  square units. Hence, (1).

17. We have,



If each side of the hexagon = a, the area of the hexagon =  $\frac{3\sqrt{3}}{2}a^2$

The area of the triangle =  $\frac{1}{2} \times \ell(ED) \times \text{Height}$

Height of the triangle = Distance between two parallel sides of the hexagon =  $2 \times \frac{\sqrt{3}}{2}a = \sqrt{3}a$

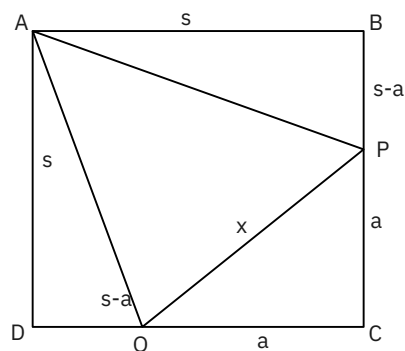
$$\therefore A(\triangle PED) = \frac{1}{2} \times a \times \sqrt{3}a = \frac{\sqrt{3}}{2}a^2$$

$$\text{Therefore required ratio of areas} = \frac{\frac{3\sqrt{3}}{2}a^2}{\frac{\sqrt{3}}{2}a^2} = 3$$

: 1. Hence (2).



18. Suppose 's' is the side of the square and 'x' is the side of the equilateral triangle, as shown below



Triangles ABP and ADQ are congruent using Hypotenuse-Side test. Therefore lengths of the sides are as shown above in the diagram.

$$\text{We have, } x = \sqrt{2}a = \sqrt{s^2 + (s-a)^2}$$

$$\therefore x^2 = 2a^2 = s^2 + s^2 - 2as + a^2$$

$$\therefore x^2 = 2a^2 = 2s^2 - 2as + a^2$$

$$\therefore 2s^2 - 2as - a^2 = 0$$

$$\therefore 2\left(\frac{s}{a}\right)^2 - 2\left(\frac{s}{a}\right) - 1 = 0$$

$$\therefore \frac{s}{a} = \frac{2 \pm \sqrt{4+8}}{4} = \frac{2 \pm \sqrt{12}}{4} = \frac{1 \pm \sqrt{3}}{2}$$

$$\text{Since } \frac{s}{a} > 0, \frac{s}{a} \neq \frac{1-\sqrt{3}}{2} \text{ or } \frac{s}{a} = \frac{1+\sqrt{3}}{2}$$

$$\therefore a = \frac{2s}{1+\sqrt{3}}$$

$$\therefore x = \sqrt{2}a = \frac{2\sqrt{2}}{1+\sqrt{3}}s$$

$$\therefore \frac{x}{s} = \frac{2\sqrt{2}}{1+\sqrt{3}}$$

The ratio of areas of square and the equilateral triangle

$$\begin{aligned} \frac{\frac{s^2}{4}}{\frac{\sqrt{3}}{4}x^2} &= \frac{4}{\sqrt{3}} \frac{s^2}{x^2} = \frac{4}{\sqrt{3}} \times \frac{(1+\sqrt{3})^2}{(2\sqrt{2})^2} \\ &= \frac{4}{\sqrt{3}} \times \frac{4+2\sqrt{3}}{8} = \frac{4+2\sqrt{3}}{2\sqrt{3}} = \frac{2+\sqrt{3}}{\sqrt{3}}. \text{ Hence (4)} \end{aligned}$$

19. We know that AB = 2 and BC = 1.

$$\therefore EF = 2 \text{ and } EH = 4; IL = 4 \text{ and } IJ = 8; PO = 8 \text{ and } MP = 16; QT = 16 \text{ and } TS = 32.$$

$$PL = 0.5 \times (MP - IL) = 6$$

$$PT = 0.5 \times (TS - PO) = 12$$

$$\therefore IT = \sqrt{PT^2 + IP^2} = \sqrt{12^2 + 10^2} = \sqrt{244} = 2\sqrt{61}$$

Hence (2).

20. Since the sides of the squares differ by 2 units, let the sides be  $a - 3$ ,  $a - 1$ ,  $a + 1$  and  $a + 3$ .

$$\therefore (a + 3)^2 - (a + 1)^2 + (a - 1)^2 - (a - 3)^2 = 64$$

$$\therefore 4a + 8 + 4a - 8 = 64$$

$$\therefore a = 8$$

$$\therefore \text{The sides of the squares are } 5, 7, 9 \text{ and } 11.$$

$$\therefore \text{The required answer} = 11^2 - 5^2 = 121 - 25 = 96.$$

## PRACTICE EXERCISE 2

1. The diagonals of a parallelogram bisect each other.

$$\therefore BO = 13$$

$$(AB)^2 + (BC)^2 = 2[(BO)^2 + (OC)^2] \text{ (By Apollonius theorem)}$$

$$17^2 + 11^2 = 2(13^2 + OC^2)$$

$$289 + 121 = 2(169 + OC^2)$$

$$OC^2 = 36 \therefore OC = 6 \therefore AC = 2 \times 6 = 12.$$

Hence, (1).

2.  $\angle CDO \cong \angle OBA$  ... (alternate angles)

$$\angle DOC \cong \angle AOB \text{ ... (vertically opposite angles)}$$

$$\triangle DOC \sim \triangle BOA \text{ ... (AA test for similarity)}$$

$$\frac{A(\triangle DOC)}{A(\triangle BOA)} = \frac{(CD)^2}{(AB)^2} = \left(\frac{CD}{AB}\right)^2$$

$$\therefore \frac{6}{A(\triangle BOA)} = \left(\frac{1}{3}\right)^2$$

$$\therefore A(\triangle BOA) = 6 \times 9 = 54 \text{ sq. cm. Hence, (1).}$$

3. Area of the gravel path =  $(50 \times 42) - (38 \times 30)$

$$= 2100 - 1140 = 960$$

$$\therefore \text{Total cost of gravelling} = 960 \times 10 = \text{Rs. } 9600.$$

Hence, (4).

4. Side of a hexagon =  $\frac{36}{6} = 6$  cm.  
Area =  $\frac{3\sqrt{3}}{2} \times 6^2 = 54\sqrt{3}$  sq. cm. Hence, (4).

5. K is midpoint of DC.

$$\therefore \ell(KC) = \frac{1}{2} \times \ell(DC)$$
$$A(\square AMCK) = \frac{1}{2} \times A(\square ABCD)$$
$$= \frac{1}{2} \times 3000 = 1500 \text{ sq. units}$$
$$A(\square PQRS) = [A(\square AMCK) - A(\square AMQP) + A(\square SRCK)]$$
$$= 1500 - (513 + 388)$$
$$= 599 \text{ sq. units.}$$

Hence, (1).

Alternatively,

Join AC. Since M, K are the midpoints of AB, CD respectively,

$$\text{then } A(\Delta\text{ACK}) = \frac{A(\Delta\text{ACD})}{2}$$

$$\text{and } A(\Delta CAM) = \frac{A(\Delta CAB)}{2}.$$

Hence,  $A(\square AMCK) = A(\Delta ACK) + A(\Delta CAM)$

$$= \frac{A(\triangle ACD)}{2} + \frac{A(\triangle CAB)}{2} = \frac{A(\square ABCD)}{2}$$

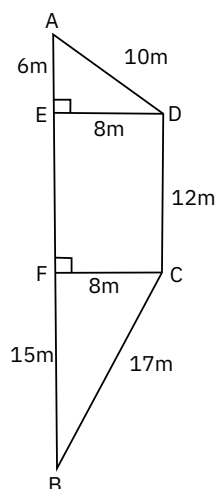
$$\text{Thus } A(\square PQRS) = A(\square AMCK) - A(\square AMQP) - A(\square CKSR)$$

$$= \frac{A(\square ABCD)}{2} - A(\square AMQP) - A(\square CKSR)$$

$$= \frac{3000}{2} - 513 - 388 = 599 \text{ sq. units.}$$

So, area of quadrilateral PQRS is 599 sq. units

6.



Both parallel zebra crossings ED and FC are perpendicular to AB.

Using Pythagoras theorem,

$$\therefore \ell(\text{ED}) = 8 \text{ m}$$

$$\therefore \ell(\text{FC}) = 8 \text{ m.}$$

$$\therefore \ell(\text{FB})^2 = 17^2 - 8^2 = 225$$

$$\ell(\text{FB}) = 15 \text{ m}$$

$\ell(\text{DC}) = \ell(\text{EF}) = 12 \text{ m}$  (parallel edges between two zebra crossings)

$\therefore \square EDCF$  is a rectangle.

$$\ell(\text{AB}) = 6 + 12 + 15 = 33 \text{ m.}$$

$$A(\square EDCF) = 12 \times 8 = 96 \text{ m}^2.$$

Hence, (1).

7. Consider  $\Delta ADE$  and  $\Delta CDF$

$AD \cong CD$  ..... sides of square.

$\angle DAE \cong \angle DCF = 90^\circ$  ..... angles of square.

$$DE \cong DF \dots \text{given.}$$

Thus  $\triangle ADE \cong \triangle CDF$  ..... Hypotenuse – side test.

Thus  $AE = CF$  ..... (2) is true.

Also  $AB = BC$

$$\therefore AB - AE = BC - CF \Rightarrow BE = BF \dots (1) \text{ is true.}$$

Now, Area of  $\triangle ADE$  = Area of DCF as they are congruent.

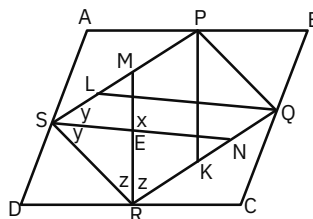
$$\text{Area of } \square ABCD = \text{Area of } \square DEBF + \text{Area of } \triangle ADE + \text{Area of } \triangle DCF$$

$$= \text{Area of } \square DEBF + 2 \text{ Area of } \triangle ADE.$$

Thus, (3) is true.

But (4) may or may not be true. Hence, (4).

8.



□ PQRS is a parallelogram and □ ABCD formed by the angle bisectors of  $\angle P$ ,  $\angle Q$ ,  $\angle R$ ,  $\angle S$  is a rectangle.

In  $\Delta$ SER

$$(180^\circ - x) + y + z = 180^\circ$$

$$\Rightarrow y + z = 90^\circ [\because x = 90^\circ]$$

$$\ell(\text{RM}) = \ell(\text{SR})$$

$$\begin{aligned}\Rightarrow \angle RMS &= 2y \\ \therefore 2y + 2y + z &= 180^\circ \\ \Rightarrow 2y + y + (y + z) &= 180^\circ \\ \Rightarrow 3y &= 90^\circ \\ \Rightarrow y &= 30^\circ \\ \therefore z &= 90^\circ - 30^\circ = 60^\circ \\ \text{i.e. } m\angle SRM = z &= 60^\circ. \text{ Hence, (1).}\end{aligned}$$

9. Sum of interior angles of a polygon  
 $= (2n - 4) \cdot \pi/2$   
 Value of interior angle of a polygon  
 $= \frac{(2n-4)}{n} \pi/2$   
 Let the sides of polygons be  $5x$  &  $3x$  respectively.  

$$\therefore \left[ \frac{2(5x)-4}{5x} - \frac{2(3x)-4}{3x} \right] \times 90 = 8$$

$$\Rightarrow \left[ \frac{10x-4}{5x} - \frac{6x-4}{3x} \right] = \frac{8}{90}$$

$$\Rightarrow \frac{30x-12-30x+20}{15x} = \frac{8}{90}$$

$$\Rightarrow \frac{8}{x} = \frac{8}{6}$$

$$x = 6$$

Hence number of sides of polygons will be 30 and 18. Hence, (2).

10.  $\triangle ABF \sim \triangle HCF$  by A-A test  
 $\therefore \ell(CF) : \ell(BC) = 1 : 3$   
 $\therefore BC$  has to be a multiple of 3  
 $\triangle HCF \sim \triangle HDA$   
 $\therefore \ell(HC) : \ell(HD) = 1 : 3$   
 Let  $\ell(HC)$  be  $x$  cm  
 $\therefore \ell(DC) = 4x$   
 $\therefore DC$  has to be a multiple of 4  
 $\therefore$  Side of the square should be a multiple of 12 and area should be a multiple of 144.  
 Only option (3) satisfies the condition.  
 Hence, (3).  
 Alternatively,  
 $\triangle HCF$  is right angled .  
 $\ell(HF) = 5$  cm  
 $\ell(HF), \ell(FC), \ell(HC)$  form a pythagorian triplet we can assume.  
 $\ell(HC)$  or  $\ell(CF) = 3$  cm or 4 cm.

Also,  $\triangle ABF$  is right angled .

$$\ell(AF) = 5 + 15 = 20 \text{ cm.}$$

$$\therefore \ell(AB) \text{ or } \ell(BF) \text{ is } 12 \text{ cm or } 16 \text{ cm.}$$

For  $BF$  to be 16 cm,  $\ell(CF)$  can be 4 cm.

$$\therefore \text{Side of } \square ABCD \text{ is } 12 \text{ cm.}$$

$$\therefore A(\square ABCD) = 144 \text{ cm}^2$$

11. As  $\square ABCD$  is a parallelogram and  $DE$  is a transversal,

$$m\angle EDC = m\angle AED = \theta$$

$$\ell(AD) = \ell(BC) = 4 \text{ cm}$$

$$\therefore m\angle AED = m\angle ADE$$

$$\therefore \ell(AD) = \ell(AE) = 4 \text{ cm}$$

$$\text{But } \ell(AB) = 8 \text{ cm}$$

$$\therefore \ell(EB) = 4 \text{ cm}$$

$$\therefore \triangle CEB \text{ is an equilateral triangle.}$$

$$m\angle BEC = m\angle EBC = m\angle ECB = 60^\circ$$

In parallelogram,

$$m\angle EBC = m\angle ADC$$

$$\therefore m\angle ADE = m\angle EDC = 30^\circ$$

$$m\angle ECD = 60^\circ$$

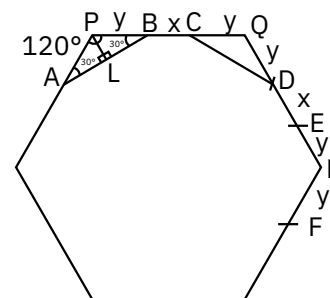
$$\therefore m\angle DEC = 90^\circ$$

$$\therefore \ell(ED) = \frac{\sqrt{3}}{2} \times \ell(CD)$$

$$= \frac{\sqrt{3}}{2} \times 8 = 4\sqrt{3} \text{ cm}$$

Hence, (2).

- 12.



$\triangle APB$  is an isosceles triangle.

$\therefore$  Perpendicular to the base is median to the base and also the angle bisector of  $\angle APB$

In  $\triangle PLB$ ,  $m\angle LPB = 60^\circ$ ,  $m\angle L = 90^\circ$

$$\text{Let } PB = y \text{ and } AB = x \quad \therefore LB = \frac{x}{2}$$

By  $30^\circ$ - $60^\circ$ - $90^\circ$  theorem,

$$\frac{x}{2} = \frac{\sqrt{3}}{2} y \Rightarrow y = \frac{x}{\sqrt{3}}$$

Since ABCDEF is a regular polygon  $AB = BC = CD = DE = EF = \dots$  and hence,  $BC = x$ .

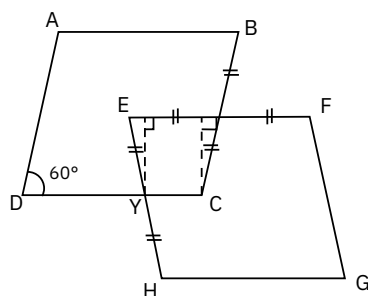
$$\text{Also } x + \frac{x}{\sqrt{3}} + \frac{x}{\sqrt{3}} = 10$$

$$\therefore x \left(1 + \frac{2}{\sqrt{3}}\right) = 10 \quad \therefore x = \frac{10}{\left(1 + \frac{2}{\sqrt{3}}\right)}$$

Perimeter of new 12 sided polygon

$$= 12 \times \frac{10 \times \sqrt{3}}{(\sqrt{3} + 2)} = \frac{120\sqrt{3}}{(2 + \sqrt{3})}. \text{ Hence, (2).}$$

13.



There are two possibilities:

Case I:

$$\ell(BX) = \ell(CX) = \ell(EX) = \ell(FX)$$

$$= \frac{10}{2} = 5 \text{ cm}$$

Let  $m\angle ADC = 60^\circ$

$$\therefore m\angle ABC = m\angle CXE = 60^\circ \text{ (Corresponding angles)}$$

$$\text{In } \triangle CNX, \ell(NX) = \cos 60^\circ \times \ell(CX)$$

$$= \frac{1}{2} \times 5 = \frac{5}{2} \times 5 = \text{cm}$$

Similarly, in  $\triangle YME$ ,  $\ell(ME)$

$$= \cos 60^\circ \times \ell(EY) = \frac{5}{2} \text{ cm}$$

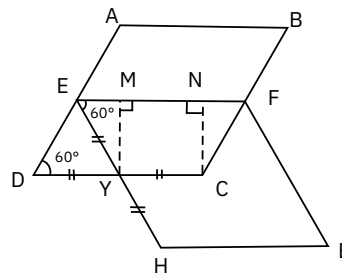
But M, E, N and X are collinear with

$$\ell(EX) = 5 \text{ cm}$$

$\therefore$  Points M and N coincide with each other.

$\therefore$  Points Y and C coincide as well.

i.e., In this case, a quadrangle is not formed.



Case II:

$$\ell(DY) = \ell(CY) = \ell(EY) = \ell(HY)$$

$$= \frac{10}{2} = 5 \text{ cm}$$

$$m\angle ADC = 60^\circ$$

$$m\angle ABC = m\angle CFN = 60^\circ \text{ (Corresponding angles)}$$

$$m\angle YEM = 60^\circ$$

In  $\triangle MEY$ ,

$$\ell(ME) = \ell(EY) \times \cos 60^\circ = 5 \times \frac{1}{2} = \frac{5}{2} \text{ cm}$$

$$\ell(MY) = \ell(EY) \times \sin 60^\circ = 5 \times \frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2} \text{ cm}$$

$$\text{In } \triangle CNF, \ell(NF) = \frac{\ell(CN)}{\tan 60^\circ} = \frac{5\sqrt{3}}{2\sqrt{3}} = \frac{5}{2} \text{ cm}$$

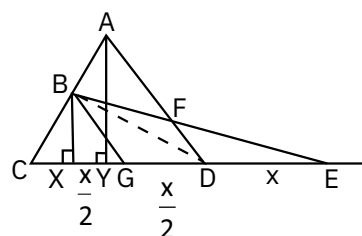
$$\therefore \ell(EF) = \frac{5}{2} + 5 + \frac{5}{2} = 10 \text{ cm.}$$

$$\therefore A(\square EFCY) = \frac{1}{2} \ell(EF) + \ell(CY)\ell(CN)$$

$$= \frac{1}{2} (10 + 5) \times \frac{5\sqrt{3}}{2} = \frac{75\sqrt{3}}{4} \text{ sq. cm.}$$

Hence, (3).

14.



In  $\triangle ACD$ , B is the midpoint of AC and  $BG \parallel AD$ .

$\Rightarrow$  G is the midpoint of CD.

Let BX and AY be the perpendiculars on CE.

Then  $\triangle BXC \sim \triangle AYC$

$$\Rightarrow \ell(BX) : \ell(AY) = 1 : 2$$

$$\therefore \frac{A(\triangle ACD)}{A(\triangle EBC)} = \frac{\ell(AY) \times \ell(CD)}{\ell(BX) \times \ell(CE)} = \frac{2 \times 1}{1 \times 2} = 1$$

$$\begin{aligned}\therefore A(\triangle BDG) &= \frac{1}{2} A(\triangle BDC) \\ &= \frac{1}{2} \times \frac{1}{2} \times A(\triangle ACD) \\ &= \frac{1}{4} \times 40 \\ &= 10 \text{ sq. units}\end{aligned}$$

In  $\triangle EBG$ ,  $FD \parallel BG$  and

$$\ell(GD) : \ell(DE) = 1 : 2$$

$$\Rightarrow \ell(BF) : \ell(FE) = 1 : 2$$

$$\ell(BF) : \ell(BE) = 1 : 3$$

The height of  $\triangle BFD$  is same as that of  $\triangle BEG$  with respect to base  $BF$  and  $BE$  respectively.

$$\frac{A(\triangle BFD)}{A(\triangle BED)} = \frac{\ell(BF)}{\ell(BE)} = \frac{1}{3}$$

$$\begin{aligned}\therefore A(\triangle BFD) &= \frac{1}{3} \times A(\triangle BED) \\ &= \frac{1}{3} \times \frac{1}{2} \times A(\triangle EBC) \\ &= \frac{1}{6} \times 40 \\ &= \frac{20}{3} \text{ sq. units.}\end{aligned}$$

$$\therefore \text{Required area} = 10 + \frac{20}{3} = \frac{50}{3} \text{ sq. units}$$

Hence, (2).

Alternatively,

$$A(\triangle BCG) = \frac{1}{4} \times A(\triangle ACD) = 10 \text{ sq. units}$$

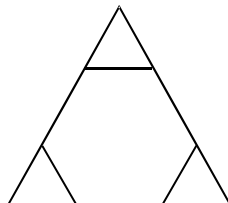
( $B$  is midpoint of  $AC$  and  $BG \parallel AD$  by converse of midpoint theorem)

$$\text{Required region} < A(\triangle ACD) - A(\triangle BCG)$$

$$\text{Required area} < 30 \text{ sq. units}$$

Only option (2) is less than 30 sq. units.

15.



From the figure, we can say that the largest inscribed regular hexagon along with its diagonals divide the equilateral triangle into 9 equal portions.

$$\therefore \text{Area of the hexagon} = \frac{6}{9} \times \text{Area of the equilateral triangle}$$

$$= \frac{6}{9} \times \frac{\sqrt{3}}{4} \times 9^2 = \frac{27\sqrt{3}}{2}. \text{ Hence, (3).}$$

$$16. \text{ Area of rhombus} = \frac{d_1 d_2}{2}$$

Let the diagonals of  $ABCD$  be  $d_1$  and  $d_2$ . Let the diagonals of  $PQRS$  be  $d_1'$  and  $d_2'$ .

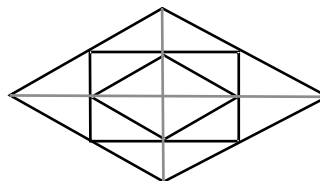
By observing the three ratios, let  $d_1 = 12x$  and  $d_1' = 20x$

$$\text{Then } d_2 = \left(\frac{12x}{4}\right) \times 7 = 21x \text{ and } d_2' = \left(\frac{20x}{4}\right) \times 5 = 25x$$

$$\text{Hence } A(ABCD) : A(PQRS) = \frac{(12x)(21x)}{2} : \frac{(20x)(25x)}{2} = 63 : 125$$

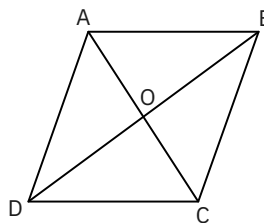
Hence (3).

17. If we draw the diagonals of the rhombus, they divide the figure into 16 equal parts, each whose area is the area of  $\triangle ELI$ .



$$\therefore \text{Area of rhombus } ABCD = 16 \times 2.25 = 36 \text{ cm}^2$$

18.



Rhombus  $ABCD$  is as shown in the diagram. As  $\angle DAB = 120^\circ$ ,  $\angle ABC = 60^\circ$

We know that the diagonal  $AC$  bisects  $\angle DAB$ , hence  $\angle OAB = 60^\circ$ .

As diagonals of rhombus are perpendicular bisectors of each other,  $\angle AOB = 90^\circ$ .

Thus,  $\triangle AOB$  is a 30-60-90 triangle with  $AB = x$ .

$$\text{Thus, } AO = \frac{x}{2} \text{ and } OB = \frac{\sqrt{3}x}{2} \Rightarrow DB = \sqrt{3}x.$$

Now consider rectangle  $PQRS$ . Thus,  $PR = \sqrt{3}x$ .

$$\text{We know that } PQ^2 + QR^2 = PR^2 = 3x^2.$$

Let the smaller side be  $y$  and the bigger side will hence be  $3y$ . Thus,  $10y^2 = 3x^2$ .

$$\text{Thus, } y = \sqrt{\frac{3}{10}} \times x. \text{ Hence (2).}$$

19. Suppose 'a' is the side of the hexagon. Therefore the area of the hexagon

$$= \frac{3\sqrt{3}}{2}a^2 = 1.5\sqrt{3}a^2$$

ACDF is a rectangle such that AF = a; AC = FD =

$$2 \times \frac{\sqrt{3}}{2} \times a = \sqrt{3}a$$

$$\therefore A(ACDF) = \sqrt{3}a^2$$

Therefore the area of the shaded region

$$= 1.5\sqrt{3}a^2 - \sqrt{3}a^2 = 0.5\sqrt{3}a^2$$

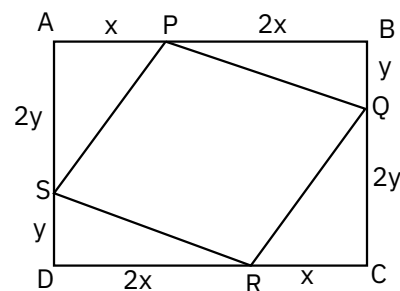
$$\text{Required ratio} = \frac{0.5\sqrt{3}a^2}{1.5\sqrt{3}a^2} = \frac{1}{3}$$

Hence (2).

20.  $\angle BOC = 180^\circ - (\angle OBC + \angle OCB) = 180^\circ - \frac{1}{2}(\angle ABC + \angle BCD)$   
 $= 180^\circ - \frac{1}{2}(\angle EAD + \angle ADF) = 180^\circ - \frac{1}{2}\angle EAD - \frac{1}{2}\angle ADF$   
 $= 180^\circ - (\angle PAD + \angle PDA) = \angle APD$ . Hence (1).

21. There are 3 cases possible.

Case 1:

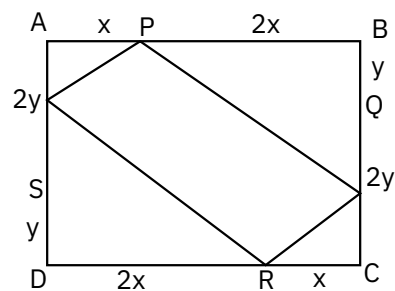


In this case, area of quadrilateral PQRS =  $(3x \times$

$$3y) - \frac{1}{2}(2xy + 2xy + 2xy + 2xy) = 5xy$$

$$\therefore \text{Required ratio} = 5 : 9$$

Case 2:

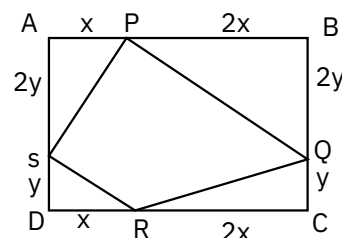


In this case, area of quadrilateral PQRS =  $(3x \times$

$$3y) - \frac{1}{2}(xy + 4xy + xy + 4xy) = 4xy$$

$$\therefore \text{Required ratio} = 4 : 9$$

Case 3:



In this case, area of quadrilateral PQRS =  $(3x \times$

$$3y) - \frac{1}{2}(2xy + 4xy + 2xy + xy) = 4.5xy$$

$$\therefore \text{Required ratio} = 4.5 : 9 = 1 : 2$$

Hence (1).

## QA-3.4 | CIRCLES-I

### PRACTICE EXERCISE

1. Let the diameter of the front wheel be  $d$  m.  
Distance travelled by rear wheel = Distance travelled by front wheel.

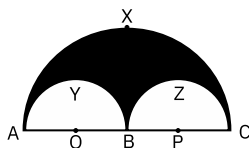
$$80 \times \pi \times 1.5 = 240 \times \pi \times d$$

$$d = \frac{80 \times 1.5}{240} = 0.5 \text{ m.}$$

Hence, (3).

2. A(Shaded region) = [A(semicircle AXC)] - [A(semicircle AYB)] - [A(semicircle BZC)]

$$= \frac{1}{2} \times \pi \times 7^2 - \frac{1}{2} \times \pi \times (3.5)^2 - \frac{1}{2} \times \pi \times (3.5)^2$$

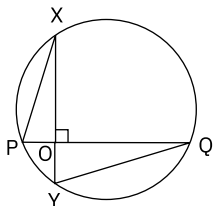


$$= \frac{1}{2} \pi (49 - 12.25 - 12.25)$$

$$= \frac{1}{2} \times \frac{22}{7} \times 24.5 = 38.5 \text{ sq. cm.}$$

Hence, (4).

3.



$$\angle XOQ = \frac{1}{2} [m(\text{arc PY}) + m(\text{arc XQ})]$$

$$90^\circ = \frac{1}{2} [m(\text{arc PY}) + m(\text{arc XQ})]$$

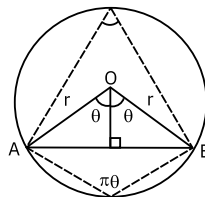
$$m(\text{arc PY}) + m(\text{arc XQ}) = 180^\circ$$

$$2(m\angle PQY) + 2(m\angle QPX) = 180^\circ$$

$$\therefore m\angle PQY + m\angle QPX = 90^\circ.$$

Hence, (2).

4.



Let AB be the chord and O be the centre of circle with radius  $r$ .

$$\text{Then } \sin \theta = \frac{(\sqrt{3}/2)r}{r} = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \theta = 60^\circ$$

$$\Rightarrow \text{Angle subtended at centre} = 120^\circ$$

Hence, (2).

5. Initial area =  $\pi R^2 = 154 \text{ cm}^2$

$$\therefore \frac{22}{7} \times R^2 = 154 \therefore R = 7 \text{ cm.}$$

$$\therefore \text{Original length of the wire} = 2\pi R$$

$$= 2 \times \frac{22}{7} \times 7 = 44 \text{ cm.}$$

$$\text{Now, new area} = \pi r^2 = 38.5 \text{ cm}^2$$

$$\therefore r = \frac{7}{2} \text{ cm.}$$

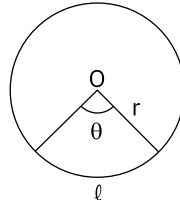
$$\therefore \text{New length of the wire} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times \frac{7}{2} = 22 \text{ cm.}$$

$$\therefore \text{length of the cut portion} = 22 \text{ cm.}$$

Hence, (3).

6.



Let  $\ell$  be the length of the minor arc of a circle with radius  $r$ .

$$\frac{\text{length of arc}}{\text{circumference of the circle}} = \frac{\text{area of the sector}}{\text{area of the circle}}$$

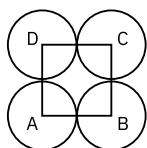
$$\therefore \frac{\ell}{2\pi r} = \frac{A}{\pi r^2} \Rightarrow A = \frac{r\ell}{2} = \frac{7 \times 14}{2} = 49$$

Now,  $A = 49$  sq. units and  $r = 14$  units.





13.



The figure formed by the coins is as shown.  
The maximum distance between the two centres is  $\ell(AC)$  or  $\ell(DB)$ .

So, in the square,  $\ell(AC) = \sqrt{392}$  cm

Then,  $\ell(AB) = 14$  cm.

radius of each coin = 7 cm

$\therefore$  Area not covered by coin

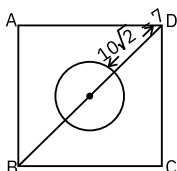
$= A(\square ABCD) - 4 \times \text{area of sector of circle}$

$$= (14)^2 - \left( 4 \times \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 \right)$$

$$= 42 \text{ cm}^2.$$

Hence, (2).

14.



If the radius of the swimming pool is  $r$  units, we have

$$\therefore \pi r^2 = 154 \Rightarrow r = 7 \text{ units.}$$

Now, the shortest distance from a corner, say D, to the swimming pool has to be along the diagonal BD.

$$\therefore \ell(BD) = (10\sqrt{2} - 7) + 7 + 7 + (10\sqrt{2} - 7)$$

$$= 20\sqrt{2} \text{ units.}$$

Then,  $\ell(AB) = 20$  units.

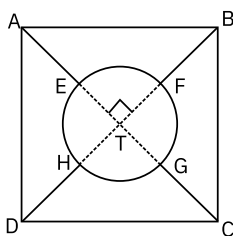
( $\square ABCD$  is square)

So, the required area

$$= (20)^2 - 154 = 246 \text{ sq. units.}$$

Hence, (3).

15.



$$\text{Area of } \square ABCD = 11 \times 11 \text{ cm}^2$$

$$\text{Area of circle} = \frac{11 \times 11}{5} \text{ cm}^2$$

$$\pi r^2 = \frac{11 \times 11}{5} \text{ cm}^2$$

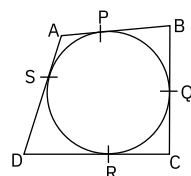
$$r^2 = \frac{11 \times 11}{5\pi} \text{ cm}^2$$

$$r = \frac{11}{\sqrt{5\pi}} \text{ cm.}$$

$$\text{Hence, radius of circle} = \frac{11}{\sqrt{5\pi}} \text{ cm.}$$

Hence, (1).

16.



$$\ell(AP) = \ell(AS), \ell(BP) = \ell(BQ),$$

$\ell(CQ) = \ell(CR)$  and  $\ell(SD) = \ell(RD)$  (Tangents from the same external point to the circle are equal in length).

Then  $\ell(AD) + \ell(CB)$

$$= \ell(AS) + \ell(SD) + \ell(BQ) + \ell(QC)$$

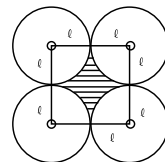
$$= \ell(AP) + \ell(RD) + \ell(BP) + \ell(CR)$$

$$= \ell(AP) + \ell(BP) + \ell(DR) + \ell(RC)$$

$$= \ell(AB) + \ell(CD).$$

Hence, (1).

17.



It is clear from the figure that the shaded area cannot be accessed by any dog

$$= (2\ell)^2 - \pi\ell^2 = (4 - \pi)\ell^2 \text{ sq. units}$$

Hence, (1).

18. Let the radius of the circle be ' $r$ '. Therefore, the length

of major arc AB will be  $\frac{4}{3}\pi r$  and that of minor arc AB

$$= \frac{2}{3}\pi r. \text{ Therefore, length of arc AC or BC} = \frac{1}{3}\pi r$$

The shortest distance will be along O-A-C or O-B-C,

$$\text{that is, } r + \frac{1}{3}\pi r.$$

The longest distance will be along O-A-B-C or O-B-

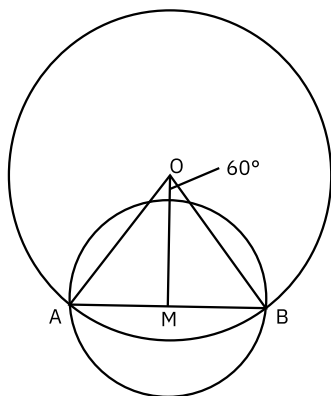
$$\text{A-C, that is, } r + \frac{5}{3}\pi r.$$

$$\therefore \text{Required ratio} = \frac{r + \frac{1}{3} \times \frac{22}{7} \times r}{r + \frac{5}{3} \times \frac{22}{7} \times r} = \frac{1 + \frac{22}{21}}{1 + \frac{110}{21}}$$

= 43 : 131

Hence (4).

19.



Consider the circle with centre O. Chord AB of length 20 cm subtends an angle of  $60^\circ$  at O. Another circle is drawn with AB as diameter. OM is drawn perpendicular to AB.

AM = MB = 10 cm

In  $\triangle OMB$ ,  $\angle BOM = 30^\circ$ . Also, it is a right triangle.

$\therefore$  Radius of the larger circle = Hypotenuse of  $\triangle OMB$  = 20 cm.

Radius of the smaller circle = 10 cm

Now, the area common to both the circles = Area of semicircle of the smaller circle + Area of minor segment AB = Area of semicircle of the smaller circle + [Area of sector OAB – Area of  $\triangle OAB$ ]

$$= \frac{1}{2} \times \pi \times 10^2 + \frac{\theta}{360} \times \pi \times 20^2 - \frac{1}{2} \times \frac{\sqrt{3}}{2} \times 20^2$$

$$= 50\pi + \frac{200}{3}\pi - 100\sqrt{3}$$

$$= \frac{350}{3} \times \frac{22}{7} - 173 = \frac{1100 - 519}{3} = \frac{581}{3} = 193.$$

67  $\text{cm}^2$

Hence, (3).

20. In  $\triangle ABC$ ,  $AB = AC$  and  $\angle BAC = 90^\circ$ .

$$\therefore \text{Area of } \triangle ABC = \frac{1}{2} \times AB^2 = 50$$

$$\therefore AB = 10.$$

$$\therefore BC = 10\sqrt{2}$$

$$\therefore OB = OC = \text{Radius of the circle} = 5\sqrt{2}$$

In  $\triangle POQ$ ,  $PO = QO = \text{Radius} = 5\sqrt{2}$  and  $\angle POQ = 90^\circ$ .

$$\therefore \text{Area of } \triangle POQ = \frac{1}{2} \times AB^2 = \frac{1}{2} \times 5\sqrt{2} \times 5\sqrt{2} =$$

25 square units.

$\therefore$  The required answer is 25.

## QA-3.5 | CIRCLES-II

## PRACTICE EXERCISE - 1

1. Let  $m(\text{arc PCT}) = m(\text{arc TDR}) = x$   
 $m(\text{arc PAS}) = m(\text{arc SBQ}) = y$   
 $m(\text{arc QER}) = 100^\circ$   
 $\therefore 2x + 2y + 100 = 360^\circ$   
 $x + y = 130^\circ$  ..... (i)  
 ST and PR are two secants intersecting inside the circle.  
 $m\angle SVR$

$$= \frac{1}{2} [m(\text{arc SQR}) + m(\text{arc PCT})]$$

$$= \frac{1}{2} [m(\text{arc SBQ}) + m(\text{arc QER}) + m(\text{arc PCT})]$$

$$= \frac{1}{2} [y + 100 + x]$$

$$= \frac{1}{2} (130 + 100)$$

$$= 115^\circ. \text{ Hence, (3).}$$

2.  $\ell(\text{FG}) \times \ell(\text{BG}) = \ell(\text{EG}) \times \ell(\text{AG})$   
 $\Rightarrow \ell(\text{EG}) = \frac{24 \times 5}{8} = 15 \text{ cm}$   
 $\ell(\text{ED}) \times \ell(\text{AD}) = \ell(\text{CD}) \times \ell(\text{BD})$   
 $\ell(\text{BD}) = \frac{7 \times (7 + 15 + 8)}{5} = 42 \text{ cm}$   
 $\Rightarrow \ell(\text{BC}) = 42 - 5 = 37 \text{ cm. Hence, (4).}$

3. Let O be the centre of the hexagon  
 $\triangle OAB$  is an equilateral triangle and OM is height of  $\triangle OAB$ , which is half the length of diagonal MN of  $\square \text{FMCN}$ .

$$\ell(\text{OM}) = \frac{\sqrt{3}}{2} a \text{ or length of diagonal MN} = \sqrt{3} a$$

$$\triangle OFA \text{ is an equilateral triangle so } \ell(\text{OF}) = a.$$

Therefore length of diagonal FC = 2a

$\square \text{FMCN}$  is a parallelogram because diagonals bisect each other.

Further diagonals of  $\square \text{FMCN}$  are perpendicular to each other. So  $\square \text{FMCN}$  is a rhombus.

$$\therefore \text{Area of quadrilateral FMCN} = \frac{1}{2} \times 2a \times \sqrt{3} a = \sqrt{3} a^2$$

$$\text{Each side of } \square \text{FMCN} = \sqrt{\left(\frac{\sqrt{3}}{2} a\right)^2 + \left(\frac{2a}{2}\right)^2}$$

$$= \sqrt{\frac{3}{4} a^2 + a^2}$$

$$= \frac{\sqrt{7}}{2} a$$

$$\therefore \text{Diameter of circle} = \frac{\sqrt{7}}{2} a$$

$$\therefore \text{Area of circle} = \pi \times \frac{7}{16} a^2 = \frac{7}{16} \pi a^2$$

$$\therefore \text{Area of shaded region} = \left(\sqrt{3} - \frac{7}{16} \pi\right) a^2$$

Hence, (3).

4. Diameter of circle =  $(\sqrt{2} \times a)$

$$\therefore \text{Radius } r = \frac{a}{\sqrt{2}}$$

Chord AD = Chord BC = Chord DC

$$\text{Area of segment AEB} = \frac{1}{4} (\text{Area of circle} - \text{Area of square})$$

$$= \frac{1}{4} \left( \pi \frac{a^2}{2} - a^2 \right) = \frac{a^2}{4} \left( \frac{\pi}{2} - 1 \right) = \frac{a^2}{8} (\pi - 2)$$

Now to find out the area of shaded region we have to find area of  $\triangle EGH$

Now,

$\triangle FID$  is right angle triangle

$$\therefore \ell(\text{FI})^2 = \ell(\text{DF})^2 - \ell(\text{DI})^2$$

$$\Rightarrow \frac{a^2}{2} - \frac{a^2}{4} = \frac{a^2}{4}$$

$$\therefore \ell(\text{FI}) = \frac{a}{2}$$

$$\text{Hence height of } \triangle DEC = \ell(\text{FI}) + \ell(\text{FE})$$

$$\therefore \ell(\text{EI}) = \frac{a}{2} + \frac{a}{\sqrt{2}} = \frac{(\sqrt{2} + 1)a}{2}$$

and height of  $\triangle EGH$

$$= \ell(\text{EJ}) = \ell(\text{EI}) - \ell(\text{IJ}) = \frac{(\sqrt{2} - 1)a}{2}$$

$\triangle GEH \sim \triangle DEC$  since,  $AB \parallel DC$  and  $ED$  and  $EC$  are transversals.

$$\therefore \frac{\ell(\text{GH})}{\ell(\text{DC})} = \frac{\ell(\text{EJ})}{\ell(\text{EI})}$$

$$\therefore \text{GH} = \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \times a$$

Hence the area of  $\triangle EGH$

$$= \frac{1}{2} \times \ell(\text{EJ}) \times \ell(\text{GH})$$

$$= \frac{1}{2} \times \frac{(\sqrt{2} - 1)}{2} \times a \times \frac{(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \times a = \frac{1}{40} a^2$$

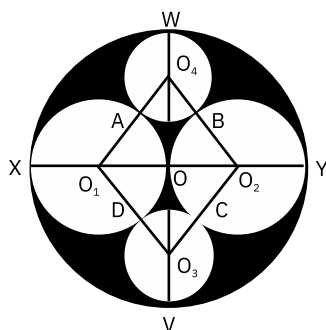
$\therefore$  Area of shaded region

$$= \frac{a^2}{8} (\pi - 2) - \frac{1}{40} a^2$$

$$= \frac{a^2}{40} (5(\pi - 2) - 1)$$

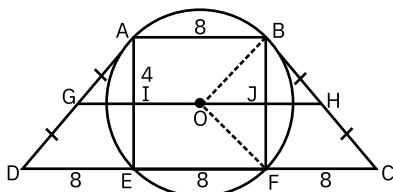


9.



Let  $\ell(AO_4) = x$   
 $\ell(VW) = \ell(XY) = 4 \times 7 = 28$  cm  
 $\ell(O_4O_1) = 7 + x$ ,  $\ell(OO_1) = 7$  cm  
 $\ell(OO_4) = \ell(WO) - \ell(WO_4) = 14 - x$   
 $\Delta OO_4O_1$  is a right angled at O.  
 $(O_4O_1)^2 = (OO_1)^2 + (OO_4)^2$   
 $(x + 7)^2 = 49 + (14 - x)^2$   
 $x^2 + 14x + 49 = 49 + 196 - 28x + x^2$   
 $42x = 196$   
 $x = \frac{14}{3}$  cm  
 Area of shaded region =  $\pi(14)^2 - 2\pi(7)^2 - 2\pi\left(\frac{14}{3}\right)^2$   
 $= \pi(196 - 98 - 43.6)$   
 $= 54.4\pi$  cm<sup>2</sup>. Hence, (1).

10.



When we inscribe a square in a circle; 4 congruent segments viz segment AB, segment BF, segment FE, segment EA, are formed.  
 Required area = Area of the circle - [Area segment (AB) + Area segment (EF)]  
 By geometry; radius of circle =  $4\sqrt{2}$   
 $\therefore$  Required area  
 $= \pi(4\sqrt{2})^2 - 2\left[\frac{90}{360} \times \pi(4\sqrt{2})^2 - 2 \times \frac{1}{2} \times (4)^2\right]$   
 ( $\because$  the segments are congruent)  
 $= 32\pi - 2\left[\left(\frac{32}{4}\pi\right) - 16\right] = 32\pi - \frac{32\pi}{2} + 32$   
 $= 16\pi + 32 = 16(\pi + 2)$  cm<sup>2</sup>.  
 Hence, (2).  
 Alternatively,  
 Radius of the circle =  $4\sqrt{2}$

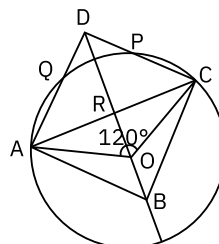
Segment AB = Segment AE = Segment FB

$$= \frac{1}{4} [\text{Area of the circle} - \text{Area of the square}]$$

$$= \frac{1}{4} [\pi(4\sqrt{2})^2 - 8 \times 8] = 8\pi - 16$$

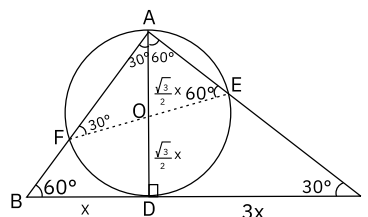
Required area = Area of the circle - [A(Segment AB) + A(Segment EF)]  
 $= 32\pi - 2(8\pi - 16) = 16\pi + 32 = 16(\pi + 2)$  cm<sup>2</sup>.

11.



Let side of square ABCD be 'a', then  
 $\ell(RC) = \frac{a}{\sqrt{2}}$   
 $\Delta CRO$  is a  $30^\circ - 60^\circ - 90^\circ \Delta$ .  
 $\ell(OC) = \frac{2}{\sqrt{3}} \times \ell(RC)$   
 $= \frac{2}{\sqrt{3}} \times \frac{a}{\sqrt{2}} = \sqrt{\frac{2}{3}} a$   
 i.e., Radius of circle =  $r = \sqrt{\frac{2}{3}} a$   
 $m\angle ADC = 90^\circ$   
 $\ell(AD) = \ell(CD)$  ..... (sides of square)  
 $\therefore m\angle DAC = m\angle DCA = 45^\circ$   
 $\therefore$  in  $\Delta POC$ , ( $\ell(OP) = \ell(OC)$ )  
 $m\angle OPC = m\angle OCP = 45^\circ + 30^\circ = 75^\circ$   
 $\Rightarrow m\angle POC = 30^\circ$   
 Similarly,  $m\angle QOA = 30^\circ$   
 $\Rightarrow m\angle POQ = 120^\circ - 30^\circ - 30^\circ = 60^\circ$   
 $\therefore \Delta POQ$  is an equilateral triangle  
 $\Rightarrow \ell(PQ) = \ell(OP) = r = \sqrt{\frac{2}{3}} a$   
 In right angled  $\Delta PDQ$ ,  
 $\ell(DP) = \frac{r}{\sqrt{2}} = \frac{a}{\sqrt{3}}$   
 $\ell(DP) : \ell(PC) = \frac{a}{\sqrt{3}} : a - \frac{a}{\sqrt{3}}$   
 $= 1 : (\sqrt{3} - 1)$ . Hence, (3).

12.



Let  $\ell(AD)$ ,  $\ell(BD)$  and  $\ell(DC)$  be  $\sqrt{3}x$ ,  $x$  and  $3x$  units respectively.

In right angled  $\triangle ADB$ ,

$$\ell(AD) = \sqrt{3} \times \text{units}$$

$$\ell(BD) = x \text{ units}$$

$$\therefore \ell(AD) = 2x \text{ units}$$

$$m\angle BAD = 30^\circ \text{ and } m\angle DBA = 60^\circ$$

Similarly, In  $\triangle ADC$ ,  $\ell(AD) = \sqrt{3} \times \text{units}$ ,

$$\ell(DC) = 3x \text{ units}$$

$$\therefore \ell(AC) = 2\sqrt{3} \times$$

$$\Rightarrow m\angle CAD = 60^\circ$$

$$\Rightarrow m\angle ABD = 60^\circ \text{ and } m\angle ACD = 30^\circ$$

In  $\triangle ABC$ ,

$$m\angle BAC = 180^\circ - (m\angle ABC + m\angle ACB) = 90^\circ$$

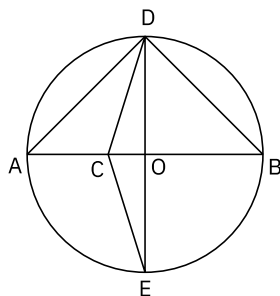
$\therefore EF$  is diameter (diameter subtends an angle of  $90^\circ$  at any point on the circle)

$$\therefore \ell(AO) = \ell(OD) = \frac{\sqrt{3}}{2} \times \text{units}$$

Thus,  $\triangle ABC \sim \triangle AEF$

$$\therefore \frac{A(\triangle AEF)}{A(\triangle ABC)} = \frac{\ell(EF)^2}{\ell(BC)^2} = \frac{(\sqrt{3}x)^2}{(4x)^2} = \frac{3}{16} \text{ . Hence, (1).}$$

13. Let 'O' be the center of the circle



Point C divides diameter AB in the ratio 1 : 2

$$\therefore AC = \frac{2r}{3} \text{ and } CB = \frac{4r}{3}$$

Since O is the center of the circle,

$$OC = r - AC$$

$$\therefore OC = \frac{r}{3}$$

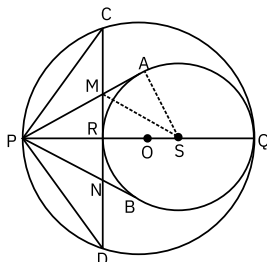
$$A(\triangle DCE) = \frac{1}{2} \times DE \times OC$$

$$= \frac{1}{2} \times 2r \times \frac{r}{3} = \frac{r^2}{3}$$

$$A(\triangle ABD) = \frac{1}{2} \times 2r \times r = r^2$$

Required ratio = 1 : 3. Hence, (4).

- 14.



Let the radius of the smaller circle be  $RS = SQ = r$ .

$$\therefore PQ = 3r \text{ and } OP = \frac{3}{2}r.$$

$\triangle SAP$  is a right-angled triangle with  $SA = r$  and  $SP = 2r$ . Therefore, it is a 30-60-90 triangle and  $\angle ASP = 60^\circ$ .

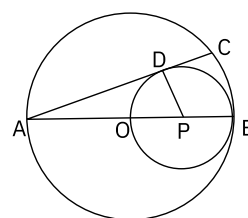
$$\therefore \angle MSR = 30^\circ$$

$$\text{Since } RS = r, \text{ so } MR = \frac{1}{\sqrt{3}}r. \text{ Similarly, } RN = \frac{1}{\sqrt{3}}r.$$

$\therefore$  Ratio of the areas of  $\triangle PMN$  and the smaller circle =

$$\frac{\frac{1}{2} \times \frac{2}{\sqrt{3}}r \times r}{\pi \times r^2} = \frac{1}{\sqrt{3}\pi} \text{ . Hence (4).}$$

- 15.

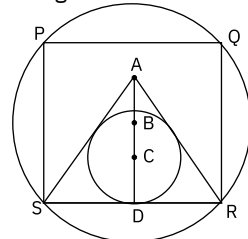


Let A-O-B be the diameter, hence  $OA = 6 \text{ cm}$  and  $OP = 3 \text{ cm} = PD$ . Therefore,  $AD = \sqrt{9^2 - 3^2} = 6\sqrt{2} \text{ cm}$ . Since  $\triangle ADP$  and  $\triangle ACB$  are similar, we have,

$$\frac{AC}{AD} = \frac{AB}{AP}; AC = \frac{12}{9} \times 6\sqrt{2} = 8\sqrt{2} \text{ cm.}$$

Hence, (3).

16. The given data can be represented as follows.



The outer circle is drawn with centre B. Let its radius be denoted as 'r' for now.

The largest square PQRS will have the length of its diagonal as  $2r$ .

$$\therefore \text{The length of each side of the square} = \frac{2}{\sqrt{2}}r = \sqrt{2}r$$

The equilateral triangle inside this square will have each side of length  $\sqrt{2}r$ .

$$\therefore \text{The length of the median AD will be } \frac{\sqrt{3}}{2} \times \sqrt{2}r = \frac{\sqrt{3}}{\sqrt{2}}r$$

Now, the largest circle inside the equilateral triangle will be its incircle, whose centre is same as that cen-

triod of the triangle which divides the median is the ratio 2 : 1.

$$\text{The radius of the circle, } CD = \frac{1}{3} \times \frac{\sqrt{3}}{\sqrt{2}} r = \frac{1}{\sqrt{6}} r$$

Also, the length of BD will be half that of the side of the square =  $\frac{1}{\sqrt{2}} r$

Required distance between the two centres = BC =

$$BD - CD = \frac{1}{\sqrt{2}} r - \frac{1}{\sqrt{6}} r = \frac{\sqrt{3} - 1}{\sqrt{6}} r = \sqrt{3} - 1$$

Hence (3).

17. Using Pythagoras Theorem,

$$l(OP) = \sqrt{9^2 + 12^2} = 15 \text{ cm}$$

OT is a radius of the circle.

Therefore  $l(TP) = 15 - 9 = 6 \text{ cm}$ .

Consider  $\triangle OQP$  and  $\triangle MTP$ .

$\angle OQP \cong \angle MTP$  as both are right angles.

$\angle OPQ \cong \angle MPT$  as it is a common angle to both triangles.

$\therefore \triangle OQP \sim \triangle MTP$  using A-A test.

$$\therefore \frac{OQ}{MT} = \frac{OP}{MP} = \frac{QP}{TP}$$

$$\therefore \frac{9}{MT} = \frac{15}{MP} = \frac{12}{6}$$

$$\therefore MT = 4.5 \text{ cm}$$

Hence (2).

18. Since a circle passes through the points A, B, C, D, E, F, G and H, the diameter of the circle will be same as the diagonals of both the rectangles. Let the length of the diagonals of both the rectangles be 'd'.

$$\therefore BC = \sqrt{d^2 - 15^2} = \sqrt{d^2 - 225} \text{ and } EF = \sqrt{d^2 - 7^2} = \sqrt{d^2 - 49}$$

Ratio of their areas is known.

$$\therefore \frac{15 \times \sqrt{d^2 - 15^2}}{7 \times \sqrt{d^2 - 49}} = \frac{75}{42}$$

$$\therefore \frac{\sqrt{d^2 - 225}}{\sqrt{d^2 - 49}} = \frac{5}{6}$$

$$\therefore \frac{d^2 - 225}{d^2 - 49} = \frac{25}{36}$$

$$\therefore 36d^2 - 8100 = 25d^2 - 1225$$

$$\therefore d^2 = \frac{6875}{11} = 625$$

$$\therefore \text{Required area} = \pi \frac{d^2}{4} = \frac{625}{4} \pi$$

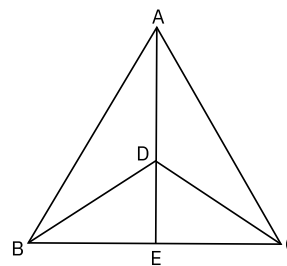
Hence, (1).

19. Since  $m\angle BCE = 76^\circ$ ,  $m\angle BCD = 180^\circ - 76^\circ = 104^\circ$ . That means opposite angles  $\angle DAB$  and  $\angle BCD$  are supplementary. Therefore, quadrilateral ABCD is a cyclic quadrilateral.

Point P is equidistant from vertices A, B and D of the cyclic quadrilateral. That means point P is the center of the circle in which the quadrilateral has been inscribed.

$\therefore l(PC) = l(PA) = 4 \text{ cm}$ . Hence (3).

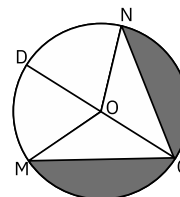
20. Consider  $\triangle ABC$ .



Since  $BC = 14\sqrt{3}$ ,  $EC = 7\sqrt{3}$ .

$\therefore$  Using trigonometry, we get  $CD = \frac{EC}{\cos 30^\circ} = 14$

Consider the circle.



$\angle MCN = \angle MOD = \angle DON = 60^\circ$  and  $\angle COM = \angle CON = 120^\circ$

The shaded segments CM and CN are congruent, so calculating once is sufficient.

$$\text{Area of any one of the segments} = \frac{1}{2} \times 7^2 \times \frac{2\pi}{3} -$$

$$\frac{1}{2} \times 7^2 \times \sin\left(\frac{2\pi}{3}\right) = \frac{49}{2} \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

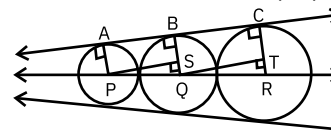
$$\therefore \text{Area of the shaded region} = 49 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$$

Hence, (4).

## PRACTICE EXERCISE - 2

1. Radii of five circles which are adjacent to one another are in continued proportion.

*Proof for reference:* To show that radii of circles between two lines are in continued proportion.



The centres P, Q and R of the three circles, are collinear. This line passing through P, Q and R bisects the angle formed by the two tangents.

PA || BQ || CR (lines perpendicular to line AC).

∴ m∠PQS = m∠QRT.

Also, m∠PSQ = m∠QTR = 90°

Then ΔPQS ~ ΔQRT.

Let x, y and z be the radii from the smallest to the largest circles respectively.

So QS = y - x; RT = z - y

$$\therefore \frac{QS}{PQ} = \frac{RT}{QR} \Rightarrow \frac{y-x}{x+y} = \frac{z-y}{y+z}$$

$$\Rightarrow y^2 = xz \Rightarrow \frac{y}{x} = \frac{z}{y}$$

Thus x, y and z are in continued proportion.

So if k is constant of proportionality, then radius of 5<sup>th</sup> circle = r<sub>5</sub> = k<sup>4</sup>r<sub>1</sub>

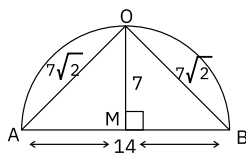
$$\therefore \frac{r_5}{r_1} = k^4 \Rightarrow k = \left(\frac{r_5}{r_1}\right)^{\frac{1}{4}}$$

∴ The middle radius

$$= k^2 r_1 = \left(\frac{r_5}{r_1}\right)^{\frac{2}{4}} \times r_1 = \left(\frac{18}{8}\right)^{\frac{1}{2}} \times 8$$

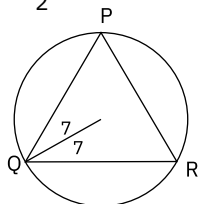
= 12 units. Hence, (3).

2.



Area of largest triangle = Area of ΔAOB

$$= \frac{1}{2} \times 7 \times 14 = 49 \text{ cm}^2.$$



The largest triangle that can be inscribed in a circle is an equilateral triangle.

$$\therefore 7 = \frac{a}{\sqrt{3}}, \text{ where } a \text{ is the side of the triangle.}$$

$$\therefore a = 7\sqrt{3}$$

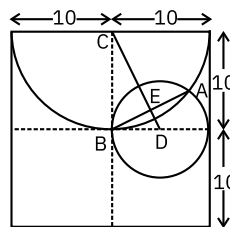
∴ Area of the triangle PQR

$$= \frac{\sqrt{3}}{4} (7\sqrt{3})^2 = \frac{49 \times 3\sqrt{3}}{4}$$

$$\therefore \text{The required ratio} = \frac{49 \times 3\sqrt{3}}{49 \times 4} = 3\sqrt{3} : 4$$

Hence, (4).

3.



From the figure the 'central' full circle and the semi-circle cut each other orthogonally where  $\ell(AE) = \ell$

Let C and D be the two centres.

In right angled ΔACD,

$$\ell(CD) = \sqrt{5^2 + 10^2} = 5\sqrt{5} \text{ units}$$

$$A(\Delta ACD) = \frac{1}{2} \times \ell(CA) \times \ell(AD)$$

$$= \frac{1}{2} \times \ell(AE) \times \ell(CD) \dots (i)$$

$$\therefore \frac{1}{2} \times 10 \times 5 = \frac{1}{2} \times \ell(AE) \times 5\sqrt{5}$$

$$\Rightarrow \ell(AE) = \frac{10 \times 5}{5\sqrt{5}} = 2\sqrt{5} \text{ units}$$

By symmetry,  $\ell(AB)$

$$= 2\ell(AE) = 4\sqrt{5} = \ell$$

Hence, required area =  $\pi \ell^2$

$$= \pi \times (4\sqrt{5})^2 = 16 \times 5\pi = 80\pi \text{ sq. units.}$$

Hence, (2).

4. Let 'r' be the radius of the inner ring road. Therefore, the radius of the outer ring road = (2r)

$$\text{The length of the minor arc } L_3L_2 = \frac{4\pi r}{4} = \pi r.$$

Speed on the outer ring road is 60π kmph.

$$\therefore \text{Time taken on the outer ring road is } \frac{\pi r}{60\pi} \text{ hrs.}$$

$$\text{Similarly, time taken on the chord road is } \frac{\sqrt{5}r}{30\sqrt{5}} \text{ hrs.}$$

$$\text{Thus, } \frac{\pi r}{60\pi} + \frac{\sqrt{5}r}{30\sqrt{5}} = 3 \text{ hours.}$$

$$\therefore \frac{r}{60} + \frac{r}{30} = 3 \Rightarrow r = 60 \text{ km.}$$

$$\therefore 2r = 120 \text{ km. Hence, (5).}$$

5. Total time taken is given by:

$$\frac{\sqrt{5}r}{30\sqrt{5}} + \frac{\pi r}{40\pi} = \frac{r}{30} + \frac{r}{40} = \frac{7r}{120}$$

$$\Rightarrow \text{Total time taken} = \frac{7 \times 60}{120} = 3.5 \text{ hours}$$

= 210 minutes. Hence, (4).

6. The rest of the dimensions can be deduced from the given values of sides and angles.

For maximum grazing area the ropes used have to be 24m at A, 28m at B, 16m at C and 12m at D.



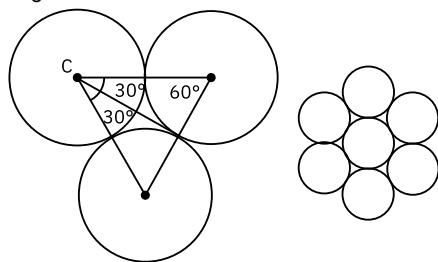
Therefore the total area grazed

$$\begin{aligned}
 &= \left[ (24)^2 \times \frac{135}{360} + (28)^2 \times \frac{90}{360} + (16)^2 \times \frac{45}{360} + (12)^2 \times \frac{90}{360} \right] \pi \\
 &= \left[ (24)^2 \left( \frac{3}{8} \right) + (28)^2 \left( \frac{1}{4} \right) + (16)^2 \left( \frac{1}{8} \right) + (12)^2 \left( \frac{1}{4} \right) \right] \pi \\
 &= [216 + 196 + 32 + 36] \pi \\
 &= 480\pi \text{ sq. units} \\
 \text{Area of the field} &= \text{Area of the rectangle} + \text{Area } (\triangle ABC) \\
 &\quad + \text{Area } (\triangle BFC) \\
 &= (42 \times 38) + \frac{1}{2} (42^2 + 80^2) \\
 &= 1596 + \frac{1}{2} (1764 + 6400) \\
 &= 5678 \text{ m}^2 \\
 \therefore \text{Ungrazed area} &= (5678 - 480\pi) \text{ m}^2. \text{ Hence, (2).}
 \end{aligned}$$

7. Let the side of square be a units.  
The diameter of the inscribed circle is equal to the side of the square, i.e., a units.  
The diameter of the outer circle is equal to the diagonal of the square, i.e.,  $a\sqrt{2}$  units.

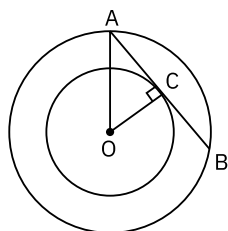
The required ratio =  $\frac{a^2}{(a\sqrt{2})^2} = \frac{1}{2}$ . Hence, (4).

8. The questions can be solved by common sense. Mathematically, considering the tangent to the outer coin from the centre of the inner coin, a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle is formed. So each coin subtends a total angle of  $60^\circ$  at the centre of the inner coin.  
Hence, the number of coins that can be placed is =  $\frac{360}{60} = 6$ .



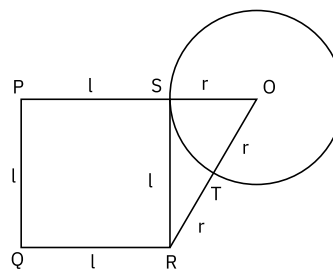
Hence, (2).

- 9.



Since,  $AC = BC$ , the length of  $AC = \frac{6}{2} = 3$  m.  
If the centre of the circle is O, then  $OC^2 + AC^2 = OA^2$   
i.e.,  $OC^2 + 3^2 = OA^2$  and OC, AC and OA are Pythagorean triplets.  
So,  $OC = 4$  m and  $OA = 5$  m. Hence, (1).

- 10.



$TR = OS$ , which is also equal to  $OT$  (radius of the same circle) =  $r$  (say).

Area of the circle =  $\pi r^2$

It can be seen that:  $RO = RT + TO = 2r$

$RS$  is a tangent, hence angle  $RSO = 90^\circ$ .

Thus,  $RS = \sqrt{(RO)^2 - (SO)^2} = \sqrt{(2r)^2 - (r)^2} = r\sqrt{3}$

Thus, area of the square PQRS =  $(r\sqrt{3})^2 = 3r^2$

$\therefore$  The ratio of the area of the circle to the area of the

square =  $\frac{\pi r^2}{3r^2} = \frac{\pi}{3}$ . Hence, (1).

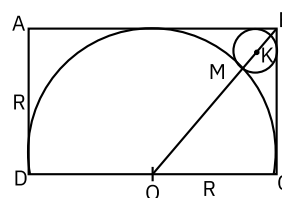
11.  $\angle ACD = \angle ABD$  ( $\because$  Angles formed by the same segment AD)  
 $\angle CAD = \angle CBD = 30^\circ$  ( $\because$  Angles formed by the same segment CD)  
 $\angle CBA = 70^\circ$   
So,  $\angle ABD = \angle CBA - \angle CBD = 70^\circ - 30^\circ = 40^\circ$   
So,  $\angle ACD = \angle ABD = 40^\circ$ . Hence, (1).

12. Area of the circular sheet =  $\pi \times 20^2 = 400\pi$  sq.cm  
Area of one cut circle =  $\pi \times 5^2 = 25\pi$  sq.cm  
Thus, area of four such cut circles =  $4 \times 25\pi = 100\pi$  sq.cm.  
Area of the uncut portion =  $400\pi - 100\pi = 300\pi$  sq.cm.  
Hence, the ratio of uncut to the cut portion =  $300\pi : 100\pi = 3 : 1$ . Hence, (3).

13. In the given triangle ABC:  
Diameter =  $AB = 2 \times 6.5 = 13$  cm,  $AC = 5$  cm  
Angle  $ACB = 90^\circ$  (Since angle in a semi-circle is always a right angle)  
 $\therefore AB^2 = AC^2 + BC^2$   
 $\therefore 13^2 = 5^2 + BC^2$   
 $\therefore BC = 12$  cm

Area of triangle ABC =  $\left( \frac{1}{2} \right) \times 12 \times 5 = 30$  sq.cm  
Hence, (2).

- 14.



In the given figure, let  $AD = BC = R$  and  $AB = DC = 2R$ .

Also, let O be the mid-point of line DC, then, DO = OC = R

figure

Let the line OB intersect the semi-circle at point M.

Then OM = R

In  $\triangle BOC$ ,  $BO^2 = OC^2 + BC^2$

$$\therefore BO = R\sqrt{2}$$

$$\text{Thus, } MB = OB - OM = R\sqrt{2} - R = R(\sqrt{2} - 1)$$

Let the radius of the smaller circle be r.

Then on similar lines,

$$KB = r(\sqrt{2} - 1)$$

$$MB = 2r + KB$$

$$R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$$

$$\therefore \frac{r}{R} = \frac{\sqrt{2} - 1}{\sqrt{2} + 1} = 3 - 2\sqrt{2}$$

$\therefore$  The ratio of areas is

$$\frac{r^2}{R^2} = (3 - 2\sqrt{2})^2. \text{ Hence, (4).}$$

15. In the given figure, angle CBA = angle CDA ... (Since angles in the same segment are equal).

Thus, in the triangles CBE and ADE,

$$\angle CBE = \angle ADE$$

$$\angle BEC = \angle DEA \text{ ... (Vertically opposite angles are equal).}$$

Thus, the 3rd pair of angles,  $\angle BCE$  and  $\angle DAE$  are also equal.

Thus, the two triangles BCE and DAE are similar.

$$AD = 24 \text{ cm} = 2 \times BC$$

$$\therefore \frac{BC}{AD} = \frac{1}{2}$$

Thus, the areas of triangles CBE and ADE are in the ratio of  $1^2 : 2^2 = 1 : 4$ . Hence, (1).

16. In the given figure, angle ACB = angle ADB =  $90^\circ$  (Since angle in a circle with diameter as a side is always a right angle).

Also, AB = 15 cm, AC = 12 cm and BD = 9 cm

(i) In triangle ACB,

$$AC^2 + BC^2 = AB^2$$

$$\therefore 12^2 + BC^2 = 15^2$$

$$\therefore BC = 9 \text{ cm}$$

$$\text{Thus, area of triangle ACB} = \frac{1}{2} \times 12 \times 9 = 54 \text{ sq.cm.}$$

(ii) In triangle ADB,

$$AD^2 + DB^2 = AB^2$$

$$\therefore AD^2 + 9^2 = 15^2$$

$$\therefore AD = 12 \text{ cm}$$

$$\text{Thus, area of triangle ADB} = \frac{1}{2} \times 12 \times 9 = 54 \text{ sq.cm.}$$

The area of the quadrilateral ACBD = Area of triangle ACB + Area of triangle ADB  
= 54 + 54 = 108 sq.cm. Hence, (4).

17. Let the radii and the area of the 2 circles be denoted by  $r_1$ ,  $r_2$ ,  $A_1$  and  $A_2$  respectively.

$$r_1 + r_2 = 15 \quad \dots(i)$$

$$A_1 + A_2 = 153\pi = \pi(r_1)^2 + \pi(r_2)^2$$

$$\therefore (r_1)^2 + (r_2)^2 = 153 \quad \dots(ii)$$

Squaring (i), we get  $(r_1)^2 + (r_2)^2 + 2r_1r_2 = 225$

$$\therefore 2r_1r_2 = 225 - 153 = 72$$

$$\therefore r_1r_2 = \frac{72}{2} = 36$$

$$(r_1)^2 + (r_2)^2 - 2r_1r_2 = (r_1 - r_2)^2 = 153 - 72 = 81$$

$$\therefore (r_1 - r_2)^2 = 9 \quad \dots(iii)$$

Solving (i) and (iii) simultaneously, we get:

$$r_1 = 12, r_2 = 3$$

Thus,  $r_1 : r_2 = 4 : 1$ . Hence, (1).

18. In the given figure: AB = 5, CD = 10 and EF = 12 cm. Also, the radii of the 3 circles = PB = PE = RA = RD = QC = QF = 20 cm

$$\text{Side PR} = PB + RA - AB = 20 + 20 - 5 = 35 \text{ cm}$$

$$\text{Side QR} = QC + RD - CD = 20 + 20 - 10 = 30 \text{ cm}$$

$$\text{Side PQ} = PE + QF - EF = 20 + 20 - 12 = 28 \text{ cm}$$

Thus, perimeter of the triangle PQR = PR + QR + PQ = 35 + 30 + 28 = 93 cm. Hence, (3).

19. Let area, circumference and the radius of each coin be denoted by a, c and r respectively.

Then,  $a = \pi r^2$  and  $c = 2\pi r$

$$\frac{a}{c} = \frac{c}{a}$$

$$\therefore a^2 = c^2 \quad \therefore a = c$$

$$\therefore \pi r^2 = 2\pi r \quad \therefore r = 2$$

Thus, the area of the 4 coins =  $4a = 4\pi r^2 = 4\pi(2)^2 = 16\pi$

Side of the square = Twice the diameter of the coin

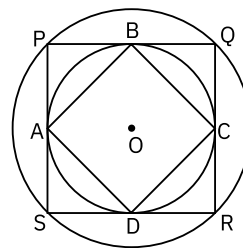
= Four times the radius of the coin

$$= 4r = 8$$

Thus, area of the square =  $8^2 = 64$  sq. units

Hence, the area not occupied by the coins in the square ABCD = Difference in the area of the square and the area of the four coins =  $64 - 16\pi = 16(4 - \pi)$  sq. units. Hence, (3).

- 20.



Let the radius of the inner circle be 1 cm, then AO = BO = 1 cm

$$AB = \sqrt{(AO)^2 + (OB)^2} = \sqrt{2} = BC = CD = DA$$

$$\text{Perimeter of ABCD} = 4\sqrt{2} \text{ cm}$$

$$PQ = AO + OC = 1 + 1 = 2 \text{ cm}$$

$$PB = \frac{PQ}{2} = \frac{2}{2} = 1 \text{ cm}$$

$$\therefore PO = \sqrt{(PA)^2 + (PB)^2} = \sqrt{2} = r$$

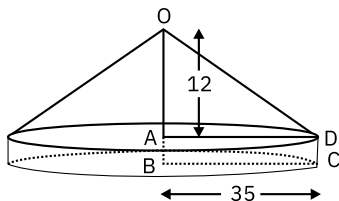
$$\text{Thus, the required ratio} = \frac{2\sqrt{2}\pi}{4\sqrt{2}} = \frac{\pi}{2}$$

Hence, (3).

## QA-3.6 | 3-DIMENSIONAL FIGURES AND MENSURATION

### PRACTICE EXERCISE - 1

- Length of the longest rod =  $\sqrt{\ell^2 + b^2 + h^2}$   
 $= \sqrt{6^2 + 3^2 + 2^2} = \sqrt{36 + 9 + 4}$   
 $= \sqrt{49} = 7 \text{ cm. Hence, (3).}$
- $OD^2 = OA^2 + AD^2 = 12^2 + 35^2$   
 $= 144 + 1225$   
 $= 1369$   
 $OD = 37 \text{ m.}$   
Total surface area = Curved surface area of cone + curved surface area of cylinder  
 $= \pi r \ell + 2\pi rh$   
 $= \pi r(\ell + 2h) = \frac{22}{7} \times 35(37 + 6) = 4730 \text{ sq.m.}$   
 $\therefore$  Total cost = Rs.47300. Hence, (1).
- Total surface area of chimney = Inner curved surface area + Outer curved surface area  
 $= 2\pi \times 2 \times 56 + 2\pi \times 3 \times 56$   
 $= 2 \times \frac{22}{7} \times 56(2 + 3)$   
 $= 2 \times \frac{22}{7} \times 56 \times 5 = 1760 \text{ sq.m.}$   
 $\therefore$  Total cost =  $20 \times 1760 = \text{Rs.}35200$ . Hence, (3).
- Volume of water flowing out in 1 minute =  $\pi r^2 h$   
 $= \pi \times (0.25)^2 \times \pi \times 62.5 \text{ cu. cm.}$   
Time taken to fill the conical tank  
 $= \frac{\text{Volume of conical tank}}{\text{Volume of water flowing out in one minute}}$   
 $= \frac{\frac{1}{3} \pi \times 20 \times 20 \times 24}{\pi \times 62.5} = 51.2 \text{ minutes. Hence, (2).}$
- New total surface area is 1.44 times the old total surface area. Since total surface area is a function of the square of the length of each side, the new length of each side will be  $\sqrt{1.44} = 1.2$  times the old length. Thus, the new volume of the cube will be  $(1.2)^3 = 1.728$  times the original volume. Therefore, the percentage increase will be 0.728 or 72.8%. Hence, (4).
- Volume of sphere =  $\frac{4}{3} \pi \times R^3$   
Radius of bigger spherical ball is 56 m  
radius of small balls is 14 m.  
Number of balls that can be made of radius 14 m each  
are  $\frac{\frac{4}{3} \pi \times R^3}{\frac{4}{3} \pi \times \left(\frac{R}{4}\right)^3} = 64$ .



Surface area of original sphere =  $4\pi R^2 = 4\pi(56)^2$

Total surface area of smaller spheres

$$= 64 \times 4\pi R^2 = 64 \times 4\pi(14)^2$$

$$\frac{\text{total surface area of small spheres}}{\text{surface area of bigger sphere}} = 4$$

Let, old area = A,

old rate of painting per  $\text{m}^2 = X$

original painting cost =  $I = AX$ ,

New area =  $4A$ , new rate =  $\frac{3}{8}X$  and new painting cost  
 $= 4A \times \frac{3}{8}X = 1.5AX = 1.5I$ .

Hence, painting cost increases by 50%. Hence, (2).

7. Capacity of the bowl

$$= \frac{2}{3} \pi r^3 = \frac{2}{3} \times \pi \times 33 = 22\pi \text{ inch}^3$$

$$\therefore \text{Remaining volume} = 22\pi - 18\pi$$

$$= 4\pi \text{ inch}^3.$$

If  $n$  Manchurian balls occupy  $4\pi \text{ inch}^3$ ,  $n \times \frac{4}{3} \pi \times (1)^3$   
 $= 4\pi$

$\therefore n = 3$ . Hence, (3).

8.  $4 \times \text{Area} \times \text{Length} = \text{Volume}$ .

$$(4 \times 250 \times \text{Length}) \text{ cm}^3 = 1\text{m}^3 = 1000000 \text{ cm}^3$$

$$\therefore \text{Length} = 1000 \text{ cm} = 10 \text{ m. Hence, (4).}$$

9.  $h_{\text{cone}} = h_{\text{cyl}} = d = 2r$

$$(i) \text{ Cone: Volume} = \left(\frac{1}{3} \pi\right) r^2 h = \frac{\pi}{3} r^2 (2r) = \frac{2\pi}{3} r^3$$

$$(ii) \text{ Cylinder: Volume} = \pi r^2 h = \pi r^2 (2r) = 2\pi r^3$$

$$(iii) \text{ Hemisphere: Volume} = \frac{\left(\frac{4}{3}\right) \pi r^3}{2} = \frac{2\pi}{3} r^3$$

$$\text{Thus, required ratio} = \frac{2\pi}{3} r^3 : 2\pi r^3 : \frac{2\pi}{3} r^3 = 1 : 3 : 1.$$

Hence, (1).

10. Volume of big cone =  $\left(\frac{1}{3} \pi\right) r^2 h$

This big cone is cut by a plane parallel to the base and at a distance of  $(h/3)$  from the base,

Thus, resulting in a smaller cone of height  $\frac{2h}{3}$  and base radius  $\frac{2r}{3}$

$$\text{The volume of the resulting smaller cone} = \left(\frac{1}{3} \pi\right) \left(\frac{2r}{3}\right)^2 \frac{2h}{3} = \left(\frac{1}{3} \pi\right) r^2 h \times \frac{8}{27}$$

$$\text{Thus, the volume of the frustum of the cone} = \left(\frac{1}{3} \pi\right) r^2 h \left(1 - \frac{8}{27}\right) = \left(\frac{1}{3} \pi\right) r^2 h \times \frac{19}{27}$$

$$\text{Hence, the required ratio} = \frac{\left(\frac{1}{3}\pi\right)r^2 h \times \frac{8}{27}}{\left(\frac{1}{3}\pi\right)r^2 h \times \frac{19}{27}} = 8:19$$

Hence, (2).

11. Given dimensions of the box are for the outer part. Since the box is painted on the inside, the inner dimensions have to be found out.

It can be observed that for the dimensions of length and the width, the box thickness needs to be removed twice from the outer dimension to get the corresponding inner dimension.

Thus, the inside dimensions of the box are:

Length (l) = 21 - 2 × 0.5 = 20 cms

Width (w) = 11 - 2 × 0.5 = 10 cms

As the box is open at the top, we subtract the thickness only once from the height.

Height (h) = 6 - 0.5 = 5.5 cms

Area painted = Four walls + Floor = 2lh + 2bh + lb

= (2 × 20 × 5.5) + (2 × 10 × 5.5) + (20 × 10)

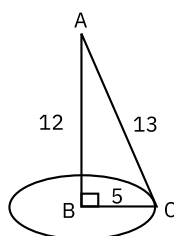
= 220 + 110 + 200 = 530 sq.cms

Total cost = Rs. 70

Thus, cost per square cm =  $\left(\frac{70}{530}\right) \approx 0.13$ . Hence, (3).

12. Let each side of the larger cube be of length 4 units.  
 $\therefore$  Total area painted red =  $6 \times 4^2 = 96$  square units.  
 Now, after cutting for the first time, there are 8 cubes each of side 2 units such that each cube has 3 sides painted and 3 unpainted. So, the total area that will be painted blue =  $8 \times 3 \times 2^2 = 96$  square units.  
 After cutting for the second time, there are 64 cubes each of side 1 unit such that each cube has 3 sides painted and 3 unpainted. So, the total area that will be painted green =  $64 \times 3 \times 1^2 = 192$  square units.  
 $\therefore$  Required ratio = 1 : 1 : 2. Hence, (4).

13. 5, 12 and 13 are Pythagorean triplet.

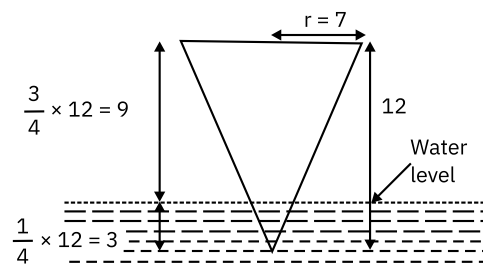


$\therefore$  13 is the slant height of the cone generated.

Total surface area =  $\pi(r + l)$

=  $\pi \times 5(5 + 13) = 90\pi$ . Hence, (1).

14.



Volume of full cone

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12 = 616 \text{ cm}^3$$

Volume of immersed portion

$$= \frac{1}{3} \times \frac{22}{7} \times r_1 \times r_1 \times 3.$$

$$h = 12 \Rightarrow r = 7 \text{ then } h_1 = 3$$

$$\Rightarrow r_1 = \frac{7}{4}.$$

$$\therefore \text{Volume of immersed portion} = \frac{1}{3} \times \frac{22}{7} \times \frac{7}{4} \times \frac{7}{4} \times 3 = \frac{77}{8} \text{ cm}^3.$$

$\therefore$  Volume of dry portion of the cone

$$= 616 - \frac{77}{8} = \frac{4851}{8} \text{ cm}^3. \text{ Hence, (1).}$$

15. The upper surface area of the swimming pool will be  $\frac{2112}{8} = 264 \text{ m}^2$

$\therefore$  Total area of the playground = 264 + 3586 = 3850  $\text{m}^2$

$$\therefore \text{Radius} = \sqrt{\frac{3850 \times 7}{2}} = 35 \text{ m}$$

16. Let the outer radius of the bowl, R = 6 cm

$\therefore$  Its inner radius, r = 6 - 2 = 4 cm

Outer radius of the spherical ball = R

Let its inner radius be  $r_1$ .

$$\therefore \frac{2}{3} \times \pi \times (R^3 - r^3) = \frac{4}{3} \times \pi \times (R^3 - r_1^3)$$

$$\therefore r_1^3 = \frac{1}{2} \times (R^3 + r^3) = \frac{1}{2} \times (6^3 + 4^3) = 140, \text{ which is}$$

greater than  $5.1^3$  but less than  $5.2^3$ .

$\therefore r_1$  will lie between 5.1 and 5.2.

$\therefore$  The thickness of the ball will be between 0.8 cm and 0.9 cm.

$\therefore$  The thickness reduces by 1.1 cm to 1.2 cm from 2 cm.

$\therefore$  The percentage reduction is between  $\frac{1.1}{2} \times 100$  and  $\frac{1.2}{2} \times 100$ , that is, between 55% and 60%.

Hence, (3).

17. Consider the triangular base of the prism.

Let AC = x. Therefore, base = 2x and height =  $\sqrt{3} \times$

Also, CC' = x

$$\therefore \text{Volume of the prism} = \text{Area of base} \times \text{Height} = \frac{1}{2} \times 2x \times \sqrt{3} \times x \times x = \sqrt{3} x^3$$

$$= 27\sqrt{3}$$

$$\therefore x = 3$$

$\therefore$  Area of the rectangular sheet =  $3 \times (3 + 6) = 27$  square units. Hence, (3).

18. Consider the height and base of the original pyramid as  $h_1$  and  $b_1$  respectively.

Thus, volume of original pyramid is  $\frac{1}{3}b_1^2h_1$ .

Similarly, consider the height and base of the new pyramid as  $h_2$  and  $b_2$  respectively.

Thus, volume of the new pyramid will be:  $\frac{1}{3}b_2^2h_2$ .

Now the ratio of volume of original pyramid: volume

$$\text{of new pyramid is: } \frac{\frac{1}{3}b_1^2h_1}{\frac{1}{3}b_2^2h_2} = \frac{8}{1}$$

As the ratio of base and height is equal,  $\frac{b_1}{b_2} = \frac{h_1}{h_2} = \frac{2}{1}$ . Thus,  $b_1 = 2b_2$  and  $h_1 = 2h_2$ .

Now as the base side and the height are in the same ratio, the ratio of slant height will also be the same. Thus, if the slant height of the original and new pyra-

mid is  $l_1$  and  $l_2$  respectively, then  $\frac{l_1}{l_2} = \frac{2}{1}$ .

Lateral surface area of the pyramid is the sum of the area of the four triangles.

Thus, the ratio of the lateral surface area of the orig-

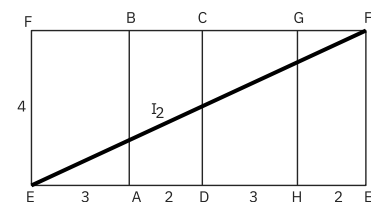
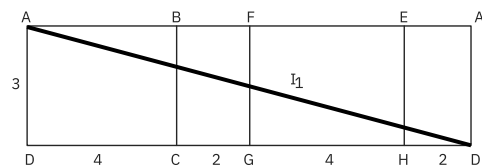
$$\text{inal pyramid to new pyramid will be: } \frac{\frac{1}{2} \times x_1 \times l_1}{\frac{1}{2} \times x_2 \times l_2} = \frac{4}{1}$$

The area of part is 2 is given by: Area of original pyramid – area of new pyramid =  $4 - 1 = 3$ .

Hence, the required ratio will be:  $\frac{1}{3}$ . Hence, (3).

19. Let  $l_1$  be the length of string 1 and  $l_2$  be the length of string 2.

If we open up the faces of the cuboid around which the strings are wound, we get the following.



$$\therefore l_1 = \sqrt{3^2 + 12^2} = \sqrt{153} \text{ and } l_2 = \sqrt{4^2 + 10^2} = \sqrt{116}$$

$$\therefore l_1 : l_2 = \sqrt{\frac{153}{116}}. \text{ Hence, (3).}$$

20. Let the outer radius of both the cylindrical and hemispherical part be 'R' and the inner radius of both the cylindrical and hemispherical part be 'r'. Let the height of the cylindrical part be 'h'.

$$\therefore 2\pi Rh + 2\pi R^2 = 2013 \dots (I)$$

$$\text{Also, } 2\pi rh + 2\pi r^2 = 1188 \dots (II)$$

$$\text{From (I), } 2\pi R(h + R) = 2013$$

$$\therefore 2 \times \frac{22}{7} \times R \times 30.5 = 2013$$

$$\therefore R = 10.5 \text{ cm}$$

$$\therefore h = 30.5 - 10.5 = 20 \text{ cm}$$

$$\text{From (II), } 2\pi r(h + r) = 1188$$

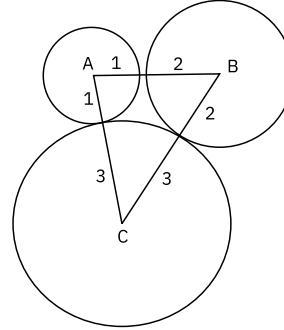
$$\therefore 2 \times \frac{22}{7} \times r \times (r + 20) = 1188$$

$$\therefore r \times (r + 20) = 189 = 7 \times 27$$

$$\therefore r = 7 \text{ cm}$$

$$\therefore \text{Required thickness} = 10.5 - 7 = 3.5 \text{ cm. Hence, (3).}$$

21. Consider the top view of the arrangement of the three cones (A, B and C are the centres of the bases).



The sides of the  $\triangle ABC$  are 3, 4 and 5. Therefore,  $\triangle ABC$  is a right triangle, right-angled at A.

The circle passing through the tips of the cones will be identical to the circum-circle of  $\triangle ABC$ .

The circum-circle of  $\triangle ABC$  will have radius as half of the hypotenuse, which is  $0.5 \times 5 = 2.5$ .

$$\therefore \text{Required circumference} = 2 \times \pi \times 2.5 = 5\pi$$

Hence, (3).

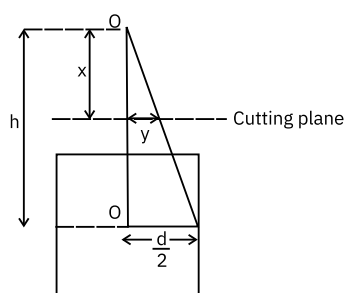
## PRACTICE EXERCISE - 2

1. The total surface areas of the blue-coloured and the red-coloured spheres are in the ratio 9:1. The total surface areas are directly proportional to the square of the ratio of the radii of the two spheres. Therefore, the ratio of the radius of blue-coloured sphere to the radius of red-coloured sphere is 3:1.

The volumes of the two spheres are directly proportional to the cube of the ratio of their radii. Therefore, the volume of the blue-coloured sphere is 27 times that of the red-coloured sphere. Therefore, the volume of the blue-coloured sphere is more than that of the red-coloured sphere by 2600%.

Therefore, the required answer is 2600.

2. Volume of the slab = Volume of the cylindrical rod  
If the length of the rod is L inches, then we have  
 $8 \times 11 \times 2 = L \times \pi \times \left(\frac{8}{2}\right)^2$   
 $\therefore L = 3.5$  inches. Hence, (2).
3. The diameter of the cone and the ball may be equal, but the cone will not go down upto the diameter of the ball due to its slant sides.  
So, more than 50% of the sphere will be outside the cone. Hence, (3).
4. Let 'd' be the base side and 'h' be the height of the pyramid.  
 $\therefore d^2 = 49$   
 $\therefore d = 7$  cm.



$\therefore$  Using similarity,

$$\frac{x}{y} = \frac{h}{\frac{d}{2}}$$

$$\therefore y = \frac{dx}{2h} \quad \dots (i)$$

$\therefore$  Volume of the upper part of the pyramid =  $\frac{1}{3} \times$  base area  $\times$  height

$$= \frac{1}{3} (2y)^2 x = \frac{1}{3} \left(\frac{dx}{h}\right)^2 x$$

$$= \frac{d^2 x^3}{3h^2}$$

Let, the pyramid be cut by a plane which is at a distance of x cm from the top.

The upper part is melted to form a sphere of radius r.

But the diameter of sphere  $2r = 2y$

$\therefore r = y$

$$\therefore \frac{4}{3} \pi r^3 = \frac{d^2 x^3}{3h^2}$$

$$\therefore \frac{4}{3} \pi (y^3) = \frac{d^2 x^3}{3h^2}$$

$$\therefore \frac{4}{3} \pi \left(\frac{dx}{2h}\right)^3 = \frac{d^2 x^3}{3h^2}$$

$$\therefore \frac{\pi}{2} \times \frac{d}{h} = 1$$

$$h = \frac{\pi}{2} d$$

$$h = \frac{22}{7 \times 2} \times 7$$

$\therefore h = 11$  cm. Hence, (3).

5. Volume of air to be changed in 1 hour = (Area of opening)  $\times$  (Flow rate per hour)

$$\therefore \frac{1}{5} \times 24 \times 20 \times 14$$

$$= \frac{4}{3} \times \frac{5}{4} \times \text{Flow rate per hour.}$$

$\therefore$  Flow rate per min

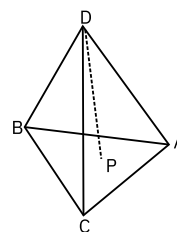
$$= \frac{1}{5} \times \frac{24 \times 20 \times 14 \times 3 \times 4}{4 \times 5 \times 60}$$

$$= \frac{24 \times 14}{25}$$

= 13.44 m/min. Hence, (4).

6. Volume occupied by the ball =  $\frac{4}{3} \pi \times 3^3 = 36\pi \text{ cm}^3$ .  
 $\therefore$  Rise in water level expected =  $36\pi \text{ cm}^3$ .  
Volume available for rise =  $\pi \times 6^2 \times 0.5 = 18\pi \text{ cm}^3$ .  
 $\therefore$  Volume of spilt water =  $36\pi - 18\pi = 18\pi \text{ cm}^3$ .  
Hence, (1).

7.



Let ABCD be a tetrahedron

Let P be the centroid of  $\triangle ABC$ .

$$\therefore l(AP) = \frac{\sqrt{3}}{2} \times \frac{2}{3} \times 12 = 4\sqrt{3} \text{ cm}$$

In  $\triangle APD$ ,

$$l(DP)^2 = l(AD)^2 - l(AP)^2$$

$$= 144 - 48 = 96$$

$$l(DP) = 4\sqrt{6} \text{ cm.}$$

$$\text{Radius of sphere} = \frac{2}{3} \times l(DP)$$

$$= \frac{2}{3} \times 4\sqrt{6} = 8\sqrt{\frac{2}{3}} \text{ cm} \approx 6.5 \text{ cm.}$$

Hence, (4).

Alternatively,

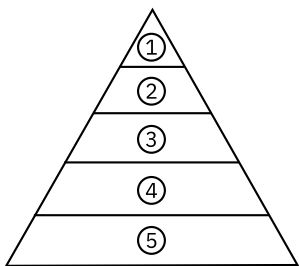
The radius of the sphere is definitely less than the radius of the circumcircle of any face of tetrahedron.

Here, AP is radius of circumcircle of  $\triangle ABC$ .

$$\therefore l(AP) = 4\sqrt{3} \approx 6.9 \text{ cm}$$

Only option (4) is less than 6.9 cm.

8.



As the question is about any right circular cone, and all the options are numerical values, the answer holds good for any set of measurement of  $r$  and  $h$ . Consider the right circular cone whose section is as shown above.

Let the total height be 5 units and base radius be 5 units.

$\therefore$  Volume of the third piece

$$= \frac{\pi}{3} \times 1(3^2 + 2^2 + 3 \times 2).$$

Similarly, volume of the fifth piece =  $\frac{\pi}{3} \times 1(5^2 + 4^2 + 5 \times 4)$

$$\therefore \text{Required ratio} = \frac{[3^2 + 2^2 + 3 \times 2] \frac{\pi}{3}}{[5^2 + 4^2 + 5 \times 4] \frac{\pi}{3}} = \frac{19}{61}.$$

Hence, (3).

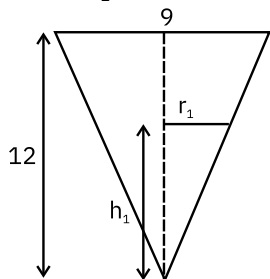
9. Volume of water in frustum = volume of water in inverted cone (in a shape of cone)

$$\frac{\pi h}{3} [R^2 + r^2 + Rr] = \frac{1}{3} \times \pi \times r_1^2 \times h_1$$

where  $h = 8$  cm,  $R = 9$  cm,  $r = 3$  cm

$r_1$  and  $h_1$  are the radius and height respectively of the 'cone' containing water. When the original cone is inverted.

$$\frac{9}{12} = \frac{r_1}{h_1} \Rightarrow h_1 = \frac{4r_1}{3}$$



$$\therefore h[9^2 + 3^2 + 27] = r_1^2 \times \frac{4r_1}{3}$$

$$\Rightarrow 8 \times 117 = r_1^3 \times \frac{4}{3}$$

$$\Rightarrow r_1^3 = 2 \times 13 \times 9 \times 3 \Rightarrow r_1 = 3 \sqrt[3]{26}$$

$$\therefore h_1 = 4 \sqrt[3]{26} \text{ cm. Hence, (4).}$$

10. Total volume = Volume of the cylinder + Volume of

$$\text{cone} = \pi r^2 h_1 + \frac{1}{3} \pi r^2 h_2 = \frac{1}{3} \pi r^2 (3h_1 + h_2)$$

$$= \frac{1}{3} \times \frac{22}{7} \times 64 \times (3 \times 240 + 36) = 50688 \text{ cm}^3$$

$$\text{Hence, weight of the pillar} = 50688 \times \frac{7.8}{1000}$$

$$\approx \frac{50000 \times 8}{1000} \approx 400 \text{ kg, which is closest to option (4).}$$

Hence, (4).

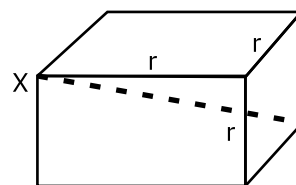
11. Let,  $R$  be the radius of the showpiece.

$$\therefore R = 5 \text{ cm.}$$

Let  $r$  be the radius of the rubber sphere.

Let  $x$  be the body diagonal of the cubic cardboard box.

$$\therefore x = 12\sqrt{3} \text{ cm (body diagonal of cuboid)}$$

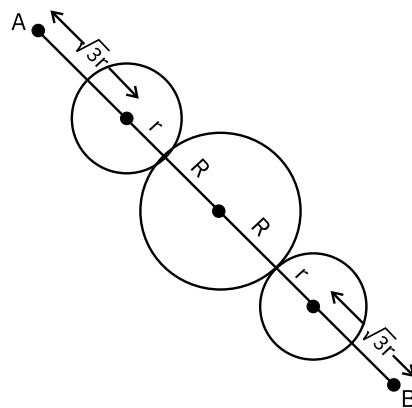


Rubber balls are at a corner of the box.

$\therefore$  Their centres are at a distance  $r$  from the sides of the box which they touch.

Let  $x$  be the centre of corner rubber ball. The rubber ball touches three sides of cuboid. Thus, considering  $x$ , corner and points at which ball touches sides of cuboid forms another cube. Thus, distance between the centre of the rubber ball and corner = body diagonal of cube of side  $r = \sqrt{3} r$ .

Let  $AB$  be the body diagonal of the cube.



$$\therefore \ell(AB) = 2R + 2r + 2\sqrt{3}r = 12\sqrt{3}$$

$$\therefore r = \frac{12\sqrt{3} - 10}{2(1 + \sqrt{3})} = \frac{6\sqrt{3} - 5}{1 + \sqrt{3}} \text{ cm.}$$

Hence, (1).

12. Let, the length of the side and the height of the hexagonal prism be  $a_1$  and  $h_1$  respectively.

Let the length of the side and the height of the hexagonal pyramid be  $a_2$  and  $h_2$  respectively.

From the given condition we have,

$$\frac{\frac{1}{3} \times \frac{3\sqrt{3}}{2} a_2^2 \times h_2}{\frac{3\sqrt{3}}{2} \times a_1^2 \times h_1} = \frac{4}{25} \quad \dots (i)$$

$$\frac{\frac{1}{3} \times \frac{3\sqrt{3}}{2} a_2^2 \times h_2}{\frac{3\sqrt{3}}{2} \times a_1^2 \times h_1} = \frac{9}{100} \quad \dots (ii)$$

Dividing equation (i) and (ii) we get,

$$\frac{h_2^2}{h_1^2} = \frac{16}{9} \Rightarrow \frac{h_2}{h_1} = \frac{4}{3} \quad \dots (iii)$$

Substituting (iii) in (i) we get,

$$\frac{a_2}{a_1} = \frac{3}{5} \quad \dots (iv)$$

∴ Ratio of volumes if sides are interchanged =

$$\frac{\frac{1}{3} \times \frac{3\sqrt{3}}{2} a_1^2 \times h_2}{\frac{3\sqrt{3}}{2} \times a_2^2 \times h_1}$$

$$= \frac{1}{3} \times \frac{25}{9} \times \frac{4}{3}$$

(From (iii) and (iv))

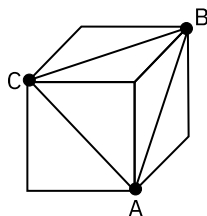
Required ratio = 100 : 81. Hence, (2).

13. Let 'a' be the side length of the cube.  
In the diagram,  $\triangle ABC$  is the largest equilateral triangle that can perfectly fit inside the cube.  
AB, BC and AC are the face diagonals of the cube.

$$\therefore \text{Side of } \triangle ABC = \sqrt{2} a$$

$$\therefore \text{Area of } \triangle ABC = \frac{\sqrt{3}}{4} (\sqrt{2} a)^2 = 50\sqrt{3}$$

$$\therefore a^2 = 100 \quad \therefore a = 10 \text{ cm.}$$



Hence, (4).

14. Let the second largest side be x.

Hence, the triplet is 8, x, x + 2

$$\therefore 8^2 + x^2 = (x + 2)^2$$

$$8^2 + x^2 = x^2 + 4x + 4$$

$$4x + 4 = 64$$

$$x = 16 - 1 = 15$$

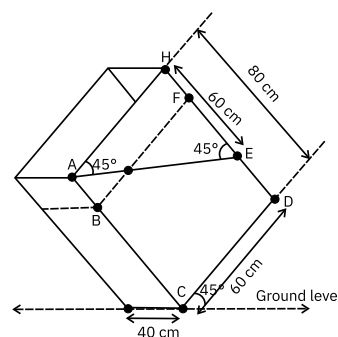
Now obviously, volume generated will be maximum if the larger side serves as the base radius of a cone, which actually is the volume generated in the above process.

$$\text{Hence, maximum volume} = \frac{1}{3} \pi (15)^2 \times 8 \\ = \pi \times 15 \times 5 \times 8 = 600\pi \text{ sq.cm. Hence, (1).}$$

15. The farthest point from A will be on the top surface and diagonally opposite to A.

From the coordinates of A, B and C, we can deduce that the length of the side of the base is 2 units and the height of the figure is 4 units. Also, since it's a regular hexagon, the distance between two parallel sides will be  $2\sqrt{3}$ . Therefore, the required coordinates will be  $(3, 2\sqrt{3}, 4)$ . Hence (1).

- 16.



Total volume of water in the tank initially = 32000  $\text{cm}^3$  + volume of water in the tank after tilting.

Now volume of water in the tank after tilting = Total volume of cuboid – volume of triangular prism with base AHE.

Now  $\triangle AHE$  is a  $45^\circ$ - $45^\circ$ - $90^\circ$  triangle.

$$\therefore \ell(AH) = \ell(HE) = 60 \text{ cm}$$

$$\therefore \text{Volume of prism} = A(\triangle AHE) \times 40$$

$$= \frac{1}{2} \times 60 \times 60 \times 40$$

$$= 72000 \text{ cm}^3.$$

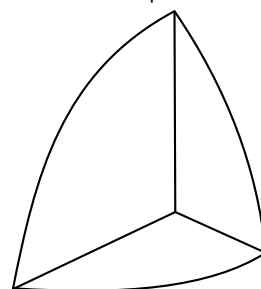
$$\therefore \text{Volume of water in the tank after tilting} = 192000 - 72000 = 120000 \text{ cm}^3$$

$$\therefore \text{Total volume of water in the tank initially} = 152000 \text{ cm}^3$$

$$\therefore \text{Height of water in the tank}$$

$$= \frac{152000}{60 \times 40} \text{ cm} = 63.33 \text{ cm. Hence, (3).}$$

17. Each of the 8 parts will look like this:



The curved surface will be one-eighth of the total curved surface area of the original sphere.



Each part consists of 3 flat surfaces in the form of a sector of a circle with radius = radius of the original sphere and central angle =  $90^\circ$ .

If the radius of the original sphere is 'r', then the re-

$$\text{quired ratio is } \frac{\frac{1}{8} \times 4\pi r^2}{3 \times \frac{90}{360} \times \pi r^2} = \frac{2}{3}$$

Hence, (1).

18. The diameter of the first sphere is 20 cm. The diameter of the second sphere is 10 cm. Each of the other spheres has diameter equal to half the diameter of the earlier sphere. Therefore the diameters of the spheres forms a GP with  $a = 20$  and  $r = \frac{1}{2}$ .

If there were infinite spheres, the sum of their diam-

$$\text{eters} = \frac{20}{1 - \frac{1}{2}} = 40 \text{ cm, which is equal to the height}$$

of the cylinder. Therefore there are infinite number of spheres in the tower of spheres.

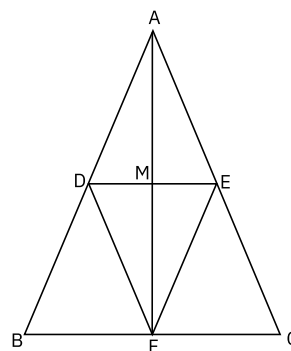
Therefore the sum of the surface areas of all the

$$\text{spheres} = 4\pi(10^2 + 5^2 + \dots) = 4\pi \times \frac{100}{1 - \frac{1}{4}} = 4\pi \times 100 \times \frac{4}{3} = \frac{1600}{3} \pi. \text{ Hence, (1).}$$

19. Let the side of the cube be 'x' units.  
The total surface area of the three cuboids will be  $2(x^2 + 2ax)$ ,  $2(x^2 + 2bx)$  and  $2(x^2 + 2cx)$ .  
Their ratio is given as 13 : 17 : 15.  
 $\therefore (x + 2a) : (x + 2b) : (x + 2c) = 13 : 17 : 15$   
Let  $(x + 2a) = 13m$ ,  $(x + 2b) = 17m$  and  $(x + 2c) = 15m$ .  
Adding them, we get  $3x + 2(a + b + c) = 45m$   
 $\therefore 5x = 45m$  i.e.  $x = 9m$ .  
 $\therefore x$  has to be a multiple of 9. Hence, (4).

20. Let the radius of the smaller spheres be r.  
 $\therefore$  Diameter of the larger sphere =  $2r + 2r + 2r = 6r$   
 $\therefore$  Radius of the larger sphere =  $3r$ .  
Volume of each of the smaller spheres =  $\frac{4}{3} \pi r^3 = V$  (say)  
 $\therefore$  Volume of the larger sphere =  $\frac{4}{3} \pi (3r)^3 = 27V$   
 $\therefore$  Volume of air inside the larger sphere =  $27V - 7V = 20V$   
 $\therefore$  Required ratio = 7 : 20. Hence, (3).

21. The two cones can be represented as follows:



Here,

ME = Radius of the smaller cone = 21 cm

FG = Radius of the larger cone = 35 cm

AF = Height of the larger cone = 45 cm

Using similarity of triangles,  $\frac{AM}{45} = \frac{21}{35}$

$\therefore AM = 27$  cm

$\therefore$  Height of the smaller cone =  $45 - 27 = 18$  cm

$\therefore$  Required volume = Volume of larger cone - Volume of smaller cone

$$= \frac{1}{3} \times \frac{22}{7} \times 35^2 \times 45 - \frac{1}{3} \times \frac{22}{7} \times 21^2 \times 18 = 49434 \text{ cm}^3$$

Therefore, the required answer is 49434.

## QA-3.7 | TIME, SPEED AND DISTANCE

### PRACTICE EXERCISE 1

- The time in each leg is same so we can take the Arithmetic Mean = 60 kmph
- Distance that Q covers in 4 hours is what P would cover in 4.8. Hence if Q travels for 4.5 hours, P would take  $4.5 \times 4.8/4 = 5.4$  hours
- $D \times \frac{3}{7} + D \times \frac{5}{14} + 9 = D$ . Hence on solving,  $D = 42$  km
- If L is the length of train in km. and x the speed of the cyclist, then,  
 (i)  $400L = 42 - x$  (ii)  $720L = 42 + x$   
 On adding, we get:  
 $1120L = 84$   
 $L = \frac{84 \times 1000}{1120}$  m;  $L = 75$  m. Hence, (1).
- In each case, the train has to travel its own length, to pass each man.  
 If x km/hr. is the speed of the train, then,  
 Length = Relative speed  $\times$  Time  
 $= (x - 2) \times \frac{9}{60 \times 60} = (x - 4) \times \frac{10}{60 \times 60}$   
 $\therefore 9x - 18 = 10x - 40 \quad \therefore x = 22$  km/hr.  
 $\therefore$  Length =  $(22 - 2) \times \frac{9}{3600} = \frac{20 \times 9}{3600}$  km  
 $= 50$  m. Hence, (4).
- The required distance  
 $= 300 - 4 \times \frac{5}{18} \times 4 \times 60$   
 $= 33.33$  m.  
 Hence, (3).
- $\frac{13}{\text{Speed upstream}} = 5$  and  $\frac{28}{\text{Speed downstream}} = 5$   
 $\therefore$  Speed upstream =  $\frac{13}{5}$  km/hr.; and Speed downstream =  $\frac{28}{5}$  km/hr.  
 $\therefore$  Speed of river =  $\frac{1}{2} \left( \frac{28}{5} - \frac{13}{5} \right) = \frac{1}{2} \times \frac{15}{5} = 1.5$  km/hr. Hence, (2).

- Let  $AB = BC = x$  km.

Considering the options,

$$1) \text{ time required} = \frac{x}{10} + \frac{x}{10} = \frac{x}{5} = \frac{6x}{30}$$

$$2) \text{ time required} = \frac{x}{5} + \frac{x}{15} = \frac{4x}{15} = \frac{8x}{30}$$

$$3) \text{ time required} = \frac{x}{15} + \frac{x}{10} = \frac{5x}{30}$$

$$4) \text{ time required} = \frac{x}{10} + \frac{x}{5} = \frac{3x}{10} = \frac{9x}{30}$$

Hence, (4).

- The total distance to be covered by one train to completely overtake the other = Sum of the lengths of the 2 trains =  $120 + 120 = 240$  metres

Let the speed of the faster train be x mps and that of the slower train, y mps.

In the first case, Relative Speed =  $x - y$  (since both are moving in the same direction)

$$\text{Time taken} = \frac{\text{Distance}}{\text{Relative Speed}}$$

$$\therefore x - y = \frac{240}{15} = 16 \text{ mps} \quad \text{--- (i)}$$

In the second case, the slower train moves at half its speed.

$$\therefore \text{Relative Speed} = x - 0.5y$$

$$\Rightarrow x - 0.5y = \frac{240}{10} = 24 \text{ mps} \quad \text{--- (ii)}$$

Solving (i) and (ii), we get  $x = 32$  and  $y = 16$ .

$\therefore$  The speeds of the faster and the slower train are 32 mps and 16 mps respectively. Hence, (2).

- Since train A takes one hour more, train B is faster.

Let speed of train A be x km/hr.

Speed of train B be  $x + 10$  km/hr.

Let the distance between P1 and P2 be 'd' km.

$$\text{Then } \frac{d}{x} - \frac{d}{x+10} = 1 \quad \text{..... (i)}$$

Let them meet t hours after starting.

$$\text{then } xt + \frac{200}{9} = t(x + 10)$$

$$\therefore xt + \frac{200}{9} = xt + 10t$$

$$\therefore t = \frac{200}{9} \div \frac{1}{10} = \frac{20}{9}$$

Also when they meet, total distance travelled is d km.

$$\therefore xt + (x + 10)t = d$$

$$\therefore (2x + 10) \times \frac{20}{9} = d \quad \dots (ii)$$

Solving (i) and (ii)

$$d \left( \frac{1}{x} - \frac{1}{x+10} \right) = 1$$

$$\Rightarrow (2x + 10) \times \frac{20}{9} \left( \frac{10}{x(x+10)} \right) = 1$$

$$\Rightarrow 400x + 2000 = 9x^2 + 90x$$

$$\Rightarrow 9x^2 - 310x - 2000 = 0$$

$$\Rightarrow (9x + 50)(x - 40) = 0$$

$x = \frac{-50}{9}$  not possible, as speed cannot be negative.

$$\therefore x = 40$$

$$\therefore d = (2 \times 40 + 10) \times \frac{20}{9} = 200 \text{ km. Hence, (2).}$$

11. If they meet  $t$  hrs after 7 a.m.,

$$10 \times \frac{18}{5} \times t + 10 \times \frac{18}{5} \times \frac{4}{3} \left( t - \frac{1}{3} \right) = 68$$

$$\therefore 36t + 48t - 16 = 68$$

$$\therefore t = 1 \text{ hr.}$$

They meet at a distance of  $10 \times 60 \times 60 = 36000$  m i.e., 36 km from A. Hence, (2).

12. Let the total distance between points A and B be  $d$  km and let the original speed of the car be  $x$  kmph.

Time taken to cover distance  $d$  with speed  $x = d/x$

As per conditions in the question,

$$\frac{18}{x} + \frac{d-18}{\frac{4x}{5}} = \frac{d}{x} + \frac{45}{60}$$

$$0.25d - 4.5 = 0.75x \quad \dots (i)$$

$$\frac{30}{x} + \frac{d-30}{\frac{4x}{5}} = \frac{d}{x} + \frac{36}{60}$$

$$0.25d - 7.5 = 0.6x \quad \dots (ii)$$

Solving (i) and (ii) simultaneously,

$$x = 20 \text{ and } d = 78$$

Hence, (4).

13. Let the lengths and speeds of the two trains be denoted by  $L_1$  and  $L_2$ ,  $s_1$  and  $s_2$  respectively.

Here,  $s_1 = 60$  km/hr and  $s_2 = 50$  km/hr.

(i) Consider the two trains moving in opposite directions:

$$\text{Relative speed} = s_1 + s_2 = 60 + 50 = 110 \text{ km/hr}$$

$$= 110 \times \left( \frac{5}{18} \right) \text{ m/s}$$

$$\text{Time taken to cross} = t = \frac{L_1 + L_2}{s_1 + s_2} = 5 \text{ s}$$

$$\therefore (L_1 + L_2) = 5 \times 110 \times \left( \frac{5}{18} \right) = 152.78 \text{ m}$$

(ii) Consider the two trains moving in the same direction:

$$\text{Relative speed} = s_1 - s_2 = 60 - 50 = 10 \text{ km/hr}$$

$$= 10 \times \left( \frac{5}{18} \right) \text{ m/s}$$

The length to be covered by the 1<sup>st</sup> train if a passenger in it has to completely cross the 2<sup>nd</sup> train = Length of the 2<sup>nd</sup> train =  $L_2$ .

$$\text{Time taken to cross} = t = \frac{L_2}{s_1 - s_2} = 18 \text{ secs}$$

$$\therefore L_2 = 18 \times 10 \times \left( \frac{5}{18} \right) = 50 \text{ m}$$

$$\text{Thus, } L_1 = 152.78 - 50 = 102.78 \text{ m}$$

Hence, (3).

14. (i) From A to B, 12 km

Waiting time at B =  $x$

(ii) From B to C, 12 km

$$\text{Travel time} = t_{bc} = \frac{12}{2x} = \frac{6}{x}$$

Waiting time at C =  $2x$

(iii) From C to D, 12 km

$$\text{Travel time} = t_{cd} = \frac{12}{4x} = \frac{3}{x}$$

Total travel and wait time = 16 hrs

$$\therefore \frac{12}{x} + x + \frac{6}{x} + 2x + \frac{3}{x} = 16$$

$$\therefore \frac{21}{x} + 3x = 16$$

Solving the above equation for  $x$ , we get  $x = 3$

$$\text{or } x = \frac{7}{3}$$

Therefore, the man could have rested at B for 3 hours.

Hence, (1).

15. Let speeds of persons starting with A and B be denoted by  $v_a$  and  $v_b$  respectively.

Here,  $v_a = 4$  km/hr and  $v_b = 1.5 + 0.5t$  km/hr, where  $t$  is the number of hours of journey required for A and B to meet.

Thus, the relative speed of A and B =  $5.5 + 0.5t$

Distance travelled in  $t$  hours = 72 km

$$= \sum_0^t (5.5 + 0.5t) = 5.5t + \left\{ \frac{(t)(t+1)}{4} \right\} = 72$$

Solving for  $t$ , we get  $t = 9$  hrs.

In 9 hrs, by the time they meet, person from A would have travelled  $(9 \times 4) = 36$  km, which is exactly mid-way between A and B.

Hence, (4).

16. Suppose the speed of the train is  $S$  and the time taken in hours is  $T$ .

$$\therefore ST = (S + 8)(T - 3) = (S - 10)(T + 6)$$

$$\therefore ST = ST - 3S + 8T - 24 = ST + 6S - 10T - 60$$

$$\therefore 3S - 8T = -24$$

$$-6S + 10T = -60$$

Solving the two equations, we get  $S = 40$  and  $T = 18$

Therefore the required distance is  $18 \times 40 = 720$

Hence (2).

17. Speed of the boat downstream =  $\frac{24}{3} = 8$  km/hr

Suppose the speed of the boat in still water = ' $x$ ' km/hr.

$$\therefore \text{Speed of the river} = \frac{x}{3} \text{ km/hr}$$

$$\therefore \text{Speed of the boat downstream} = x + \frac{x}{3} = \frac{4x}{3} = 8$$

$$\therefore x = 6$$

Therefore the speed of the boat in still water = 6 km/hr and the speed of the river = 2 km/hr.

Therefore the speed of the boat upstream =  $6 - 2 = 4$  km/hr.

$$\text{Time taken to cover the required distance} = \frac{24}{4} = 6 \text{ hours.}$$

Hence (2).

18. Let speed of boat in still water be ' $x$ ' kmph.

Let speed of water be ' $y$ ' kmph. Now going downstream the speed of boat will be ' $x + y$ ' kmph.

Going upstream speed of boat will be ' $x - y$ ' kmph.

$$\frac{120}{x+y} = 4$$

$$\text{Given } x = 23$$

$$\frac{120}{23+y} = 4$$

$$120 = 92 + 4y \Rightarrow y = 7 \text{ kmph}$$

So speed of boat upstream will be  $23 - 7 = 16$  kmph

$$\text{To go upstream, time taken} = \frac{120}{16} =$$

7 hours 30 minutes

- 19.



Let M be the initial position of the unconscious man. Let L be the initial position of the lifeguards. Let A be the point where the life guards pick up the man. Life guards start bringing the man to consciousness at point A, while the boat is moving with water current at 1 m/s. For 14 seconds. The man regains consciousness when boat is at point B.

$$\therefore AB = 1 \times 14 = 14\text{m.}$$

$$\text{If } AL = d, BL = d - 14$$

Speed of the boat while covering distance BL is  $25 + 1 = 26$  m/s, which is covered in 41 seconds.

$$\therefore BL = 26 \times 41 = 1066$$

$$\therefore d = 1066 + 14 = 1080$$

Speed of the boat while covering distance LA is  $25 - 1 = 24$  m/s

$$\text{Time taken by the boat to cover the distance LA} = \frac{1080}{24} = 45 \text{ seconds.}$$

The man floats on water from point M to A for 45 seconds at 1 m/s

$$\therefore MA = 45\text{m}$$

$$\therefore \text{Distance at which the lifeguards spot the man} = 1080 + 45 = 1125\text{m}$$

20. Ajay's speed = 6 km/hr =  $6 \times \frac{5}{18} = \frac{5}{3}$  m/s = 100 m/min.

Vijay's speed = 18 km/hr =  $18 \times \frac{5}{18} = 5$  m/s = 300 m/min.

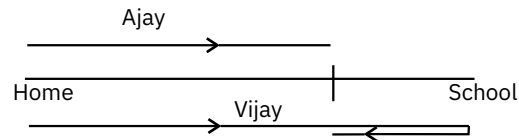
Vijay rode for total 15 minutes.

Therefore, the distance covered by Vijay =  $300 \times 15 = 4500$  m.

Ajay's speed is 1/3rd of Vijay's speed.

Therefore, the distance covered by Ajay =  $\frac{4500}{3} = 1500$  m.

We have the following situation:



It can be seen that the sum of the distances covered by Ajay and Vijay is two times the total distance between their home and the school.

Therefore, the distance between the Home and the School =  $\frac{6000}{2} = 3000$  m.

Therefore, the required distance is 3000 m.

## PRACTICE EXERCISE 2

1. The time in each leg is same so we can take the Arithmetic Mean = 5 kmph

2. Ratio of times of Jenish and Jinesh will be 2 : 3 and hence since the difference in times is 20 minutes, the times must be 40 minutes and 60 minutes. If Jinesh walks at twice his usual speed, he will take half his usual time and hence will reach in 30 minutes

3. Let the required distance be x km.

Kim's normal speed =  $\frac{x}{4}$  kmph.

$$\text{Now } x = \left(\frac{x}{4} - 3\right) \times \left(4 + \frac{40}{60}\right)$$

$$\Rightarrow x = \left(\frac{x}{4} - 3\right) \times \left(4 + \frac{2}{3}\right)$$

$x = 84$ . Hence, (2).

4. In 3 hours, Kim travels  $\frac{84}{4} \times 3 = 63$  km.

$\therefore$  Distance remaining =  $84 - 63 = 21$  km.

As she is 2 hours late, she takes  $2 + 1 = 3$  hours to walk.

$\therefore$  Speed =  $\frac{21}{3} = 7$  kmph. Hence, (4).

5. Average speed is the harmonic mean of 100, 120, 80 and 120.

$$\therefore \text{Average speed} = \frac{4}{\frac{1}{100} + \frac{1}{120} + \frac{1}{80} + \frac{1}{120}}$$

$\approx 102.13$ . Hence, (2).

6. Their speeds are  $60 \times \frac{22}{15}$  ft./sec. = 88 ft/sec.  
and  $45 \times \frac{22}{15}$  ft./sec. = 66 ft/sec.

$\therefore$  Their relative speed =  $88 + 66$

= 154 ft./sec.

Distance to be covered =  $(88 + 132)$  yds.

= 220 yard =  $220 \times 3$  ft. = 660 ft.

- $\therefore$  Time taken to pass  $\frac{660}{154} = 4\frac{2}{7}$  = secs.
- Hence, (3).
7. The required time =  $150 \div \frac{20 \times 5}{18} = 27$  second.
- (Note that the speed of train in this case is immaterial)
- Hence, (1).
8. Let the policeman's speed be  $x$  m/s.
- $\therefore$  Relative speed =  $(x - 10)$  m/sec.
- Now  $(80 - 40) = (x - 10) \times 8$
- $\therefore x = 15$ . Hence, (3).
9. Speed of the river = 2 mph
- Speed in upstream =  $\frac{9}{3} = 3$  mph
- $\therefore$  My speed in still water = 5 mph
- $\therefore$  My speed down the stream = 7 mph
- $\therefore$  The time required to return =  $\frac{9}{7} = 1\frac{2}{7}$  hr.
- Alternatively,
- If my speed in still water is  $x$  mph, then  $\frac{9}{x-2}$
- $\therefore x = 5$
- $\therefore$  The time required to return  $\frac{9}{7} = 1\frac{2}{7}$  hrs.
- Hence, (1).
10. Let distance between Mumbai and Calcutta be  $d$  km.
- 
- When they meet each other for the first time, they together travel a total distance of  $d$  km.
- When they meet second time, they together travel a total distance of  $3d$  km. So, the two trains will cross each other after  $18 \times 3 = 54$  hours from the starting point. Hence, (3).

11. Time taken for the arrow to travel 300m =  $\frac{300}{30}$
- = 10 second.
- Now the deer has to run more than  $300 - 200$
- = 100m in 10 seconds.
- $\therefore$  Its speed should be more than  $\frac{100}{10} = 10$  m/sec.
- Hence, (1).
12. Let the rowing speed be  $y$  and the speed of the stream be  $x$ .
- $\frac{12}{y-x} - \frac{12}{y+x} = 4$  i.e.,  $\frac{24x}{y^2 - x^2} = 4$  ... (i)
- and  $\frac{12}{2y-x} - \frac{12}{2y+x} = \frac{1}{2}$
- i.e.,  $\frac{24x}{4y^2 - x^2} = \frac{1}{2}$  ... (ii)
- Dividing (i) by (ii) we get,
- $\frac{4y^2 - x^2}{y^2 - x^2} = 8$
- $\therefore 4y^2 - x^2 = 8y^2 - 8x^2$
- $\therefore 4y^2 = 7x^2$
- Substituting in (ii),
- $\frac{24x}{7x^2 - x^2} = \frac{1}{2}$
- $\therefore \frac{24x}{6x^2} = \frac{1}{2}$
- $\therefore x = 8$ . Hence, (2).
13. The two equations are  $T_1 - T_2 = \frac{1}{2} + \frac{1}{2}$
- = 1 .... (i)
- Where  $T_1$  and  $T_2$  are the time taken by trains A and B to cover the whole distance
- Also,  $= \frac{2}{T_1} + \frac{1.5}{T_2} = \frac{11}{30}$
- Solving equations (i) and (ii) we get,
- $T_1 = 10$  hrs,  $T_2 = 9$  hrs. Hence, (3).

14. Let  $t$  be the time in hours after 5.00 am when the trains meet

$$\therefore \text{Distance covered} = 100 \times t = 120 \times (t - 2)$$

$$\therefore t = 12 \text{ hours.}$$

i.e. the trains meet at 5 pm. Till then the man cycles for 17 hours.

$$\therefore \text{Distance covered by the man}$$

$$= 5 \times 17 = 85 \text{ km. Hence, (2).}$$

15. The time taken by the car in both the directions is the same. As Neera reaches 10 minutes earlier than usual, the car saves 10 minutes of total travel, and 5 minutes of travel in each direction.

$\therefore$  The car which usually meets Neera at 6:00 pm, meets her at 5:55 pm.

$\therefore$  Neera walks for 55 minutes.

Hence, (4).

16. Consider Option 1: 'Average speed downhill is greater than that of uphill.'

In both cases, same distance was covered in the same duration of time. Thus, average speed uphill and downhill is the same. Hence, the given statement is false.

Consider Option 2: 'At noon, I was at the same spot on both the days.'

Since speeds could vary within the journey, it is not necessary that the person is at the same spot on both the days at noon. Hence, the given statement is false.

Consider Option 3: 'There must be a point which I reached at the same time on both the days.'

Assume that two people start walking at 6:00 a.m., one from the top of the hill and one from the bottom of the hill and reach their respective destinations at 6:00 p.m.

It is obvious that they will meet at some point, which means that they will reach the same point at the same time. Extending the same logic, there has to be a point at which both reached at the same time on both the days.

Thus option 3 is true and option 4 is false.

Hence, (3).

17. Time taken by the bus to reach its destination =  $27/18 = 1.5$  hours

$$\text{Return speed} = 18 \times 1.5 = 27 \text{ mph.}$$

$$\therefore \text{Time taken to return to the bus stand} = 27/27 = 1 \text{ hour}$$

$$\text{Waiting time} = 30 \text{ minutes} = 0.5 \text{ hours.}$$

$$\therefore \text{Total time to return to bus stand} = 1.5 + 0.5 + 1 = 3 \text{ hours}$$

Since the bus started at 8 a.m., it will return at  $8 + 3 = 11$  a.m.

Hence, (2).

18. Let the original speed and original time taken by the car be 's' and 't' respectively.

We have the following:

Speed	s	s + 20	s + 40
Time	t	t - 2	t - 3

Thus the three speeds are in AP. Therefore the times taken will be in HP.

$$\therefore t - 2 = \frac{2t(t - 3)}{2t - 3}$$

$$\therefore 2t^2 - 7t + 6 = 2t^2 - 6t$$

$$\therefore t = 6$$

Also,  $st = (s + 20)(t - 2)$ . Using the value of  $t = 6$ , we get  $s = 40$ .

$$\therefore \text{Speed of the car if it reaches 2 hours late} = \frac{40 \times 6}{(6 + 2)} = 30 \text{ km/h.}$$

19. Here, the speeds of the hare and the tortoise, and the time that the hare slept are immaterial.

Before going to sleep, the hare gained a relative distance of 35 m over the tortoise.

After waking from sleep, the hare gained a relative distance of  $35 - 7 = 28$  m over the tortoise, although he was behind the tortoise.

Since their speeds are uniform throughout, the ratio of the times will be same as the ratio of the relative distances, that is 35 : 28 or 5 : 4.

Hence (2).

20. Let the speed of the ship be  $x$  km/h and the speed of the stream be  $y$  km/h.

Therefore, upstream speed is  $(x - y)$  km/h and downstream speed is  $(x + y)$  km/h. Since the distance travelled in both cases is the same, we get

$$\frac{(x - y)}{(x + y)} = \frac{0.5}{1.5} = \frac{1}{3}$$

Solving this, we get  $3x - 3y = x + y \Rightarrow x = 2y$ .

The distance covered downstream will hence be  $\frac{(x + y)}{2} = \frac{3y}{2}$  or  $\frac{3x}{4}$ .

Suppose if the speed of the boat reduces by  $a\%$ .

New speed will hence be  $\left(\frac{100 - a}{100}\right)x = 2\left(\frac{100 - a}{100}\right)y$

Downstream speed has to be  $x + y$  or  $3y$ .

Hence to compensate for the decrease in the speed of the ship, stream speed should be:

$$3y - 2\left(\frac{100 - a}{100}\right)y = \left(\frac{100 + 2a}{100}\right)y$$

Thus, it has to increase by  $2a\%$ .

Hence the ratio of increase of stream speed:  
Decrease in boat speed =  $2a : a = 2 : 1$ .

Hence, (2).



## QA-3.8 | WORK, PIPES AND CISTERN

### PRACTICE EXERCISE - 1

- If B can do the job in  $3x$  days, A can do the same job in  $x$  days.  
 $\therefore 3x - x = 60 \quad x = 30$   
 $\therefore$  B takes 90 days and A takes 30 days.  
 $\therefore$  Work done by them in 1 day =  $\frac{1}{90} + \frac{1}{30} = \frac{2}{45}$  th part.  
 $\therefore$  Time taken for doing the work together =  $\frac{45}{2}$  =  $22\frac{1}{2}$  days. Hence, (3).
- In one hour, two pipes will fill  $\frac{1}{10}$  th and  $\frac{1}{12}$  th of the cistern and the third pipe will empty  $\frac{1}{15}$  th of the cistern.  
 $\therefore$  In one hour  $\frac{1}{10} + \frac{1}{12} - \frac{1}{15} = \frac{6+5-4}{60} = \frac{7}{60}$  of the cistern will get filled.  
 $\therefore$  Total time taken  $\frac{60}{7} = 8\frac{4}{7}$  hours. Hence, (1).
- If fixed interval be  $x$   
 $\frac{5}{10} + \frac{5-x}{15} + \frac{5-2x}{20} = 1$   
 $\therefore (5)6 + (5-x)4 + (5-2x)3 = 60$   
 $\therefore 30 + 20 - 4x + 15 - 6x = 60$   
 $x = \frac{1}{2}$  hour. Hence, (1).
- Work done by A in 1 day =  $\frac{1}{18}$  th  
 Work done by B in 1 day =  $\frac{1}{16}$  th  
 A and B work for 5 days, A works for 3 days and B works for remaining days (say  $x$ ).  
 $5\left(\frac{1}{18} + \frac{1}{16}\right) + 3\left(\frac{1}{18}\right) + x\left(\frac{1}{16}\right) = 1$   
 $\frac{5}{18} + \frac{5}{16} + \frac{3}{18} + \frac{x}{16} = 1$   
 $\frac{8}{18} + \frac{5+x}{16} = 1$   
 $\frac{4}{9} + \frac{5+x}{16} = 1 \Rightarrow 64 + 45 + 9x = 144$   
 $9x = 35 \Rightarrow x = \frac{35}{9} = 3\frac{8}{9}$   
 Total time taken =  $8 + 3\frac{8}{9} = 11\frac{8}{9}$  days.  
 Hence, (3).
- Work done by B and C in 1 day =  $\frac{1}{10} + \frac{1}{12} = \frac{11}{60}$ .  
 Also (for the same job) work done A in 1 day =  $\frac{1}{7}$   
 $\therefore$  Ratio of the time taken by B and C together to that taken by A =  $60 : 77$ .

- Time taken by B and C together to paint the larger wall =  $\frac{11 \times 60}{77} \approx 8.57$  days  $\approx 9$ . Hence, (1).
- Let us assume that 1 man takes  $x$  days to finish the work and 1 woman takes  $y$  days to finish the work.  
 $\therefore$  For 1 day, 1 man will finish  $\frac{1}{x}$  of the work and 1 woman will finish  $\frac{1}{y}$  work.  
 As per data provided  
 $\frac{8}{x} + \frac{3}{y} = \frac{1}{8} \quad \dots (i)$   
 and  $\frac{2}{x} + \frac{12}{y} = \frac{1}{8} \quad \dots (ii)$   
 from (i) and (ii)  
 $\frac{8}{x} + \frac{3}{y} = \frac{2}{x} + \frac{12}{y}$   
 $\therefore \frac{8}{x} - \frac{2}{x} = \frac{12}{y} - \frac{3}{y}$   
 $\therefore \frac{6}{x} = \frac{9}{y}$   
 $\frac{2}{x} = \frac{3}{y}$   
 i.e. work done by 2 men is equivalent to work done by 3 women.  
 OR  
 $8x + 3y = 2x + 12y$   
 $6x = 9y$   
 $\therefore 2x = 3y$   
 $\therefore$  Work done by 6 men and 6 women is same as work done by 6 men + 3 women + 3 women  
 $= 6 \text{ men} + 2 \text{ men} + 3 \text{ women}$   
 $= 8 \text{ men} + 3 \text{ women}$  or work done by 6 men and 6 women is equivalent.  
 It will take 8 days for 6 men and 6 women to finish the work. Hence, (2).
- Let  $x$  be the estimated number of days to complete the project. Then,  $\frac{1}{4}$  th of the work is done in  $\frac{y}{3}$  days by 16 workers. Remaining  $\frac{3}{4}$  th completed is  $\frac{y}{3}$  days.  

Number of workers	Days	Amount of work
16	$\frac{y}{3}$	$\frac{1}{4}$
?	$\frac{y}{3}$	$\frac{3}{4}$

 Total number of workers  
 $= 16 \times \frac{\frac{3}{4}}{\frac{1}{4}} = 48$   
 $(\because \text{The number of days are same})$   
 $\therefore$  The number of extra workers  
 $= 48 - 16 = 32$ . Hence, (3).

8. Let the work done by the man, the woman and the boy in same fixed time be  $m$ ,  $w$  and  $b$  respectively.

$$5b = 3w \text{ and } 7w = 5m$$

$$\Rightarrow 35b = 21w = 15m$$

$$\text{Let } 35b = 21w = 15m = k$$

Then earnings of the man

$$= \frac{\frac{26k}{15}}{\frac{26k}{15} + \frac{22k}{21} + \frac{21k}{35}} \times \text{Rs.}148.39$$

$$= \frac{182}{182 + 110 + 63} \times \text{Rs.}148.39$$

$$= \frac{182}{355} \times \text{Rs.}148.39$$

$$= \text{Rs.}76.076. \text{ Hence, (1).}$$

**Answers to questions 9 and 10:**

Let the work done by Sita in 1 day be  $x$

then work done by Gita in 1 day =  $\frac{4}{5}x$

Work done by Rita in 1 day =  $\frac{3}{5}x$

Total work done in 1 day if all of them work together

$$= x + \frac{4}{5}x + \frac{3}{5}x = \frac{12x}{5}$$

Work done by three of them in 25 days

$$= \frac{12x}{5} \times 25 = 60x$$

$$\therefore \text{Rita requires} = 60x \times \frac{5}{3x} = 100 \text{ days.}$$

$$\text{and Gita requires} = 60x \div \frac{4x}{5} = 60x \times \frac{5}{4x} = 75 \text{ days}$$

$$9-(3) \quad 10-(2)$$

11. Let the fourth pipe take ' $t$ ' hours (alone) to fill the pond.

$$\text{Now } \frac{2}{4} + \frac{2}{6} + \frac{(2-1)}{8} + \frac{(2-1)}{t} = 1$$

$$\therefore \frac{1}{t} = \frac{1}{24} \Rightarrow t = 24.$$

Hence, (1).

12. Let  $V$  be the volume.

In one minute, A and B can fill  $\frac{V}{15}$  and  $\frac{V}{20}$  of the cistern respectively.

Also, C can carry off 10 litres in one minute.

Now, we have,

$$V + 120 \left( \frac{V}{15} + \frac{V}{20} - 10 \right) = 0$$

$$\therefore V + (8V + 6V - 1200) = 0$$

$$\therefore 15V = 1200$$

$$V = 80 \text{ litres. Hence, (1).}$$

13. Fewer days, more men; more length, more men.

Men	Days	Length of Wall
56	27	448
$x$	23	552

$$\therefore x = 56 \times \frac{27}{23} \times \frac{552}{448} = 81$$

$$\therefore \text{Extra men required} = 81 - 56 = 25. \text{ Hence, (2).}$$

14. Let the every day supply of water be  $x$  gallons.

$$(x + 7.5) \times 150 = (x + 15) \times 100$$

$$\therefore (x + 7.5) \times 3 = (x + 15) \times 2$$

$$\therefore 3x + 22.5 = 2x + 30$$

$$\therefore x = 7.5$$

$$\text{So, the total quantity of water supply} = (7.5 + 7.5) \times 150 = 2250 \text{ gallons. Hence, (1).}$$

15. The amount of food consumed by 200 soldiers in 200 days = 10 tonnes

$$= 10 \times 1000 = 10000 \text{ kg}$$

The amount of food consumed by 200 soldiers in 20

$$\text{days} = \frac{20 \times 10000}{200} = 1000 \text{ kg}$$

The amount of food consumed by 20 soldiers in 20

$$\text{days} = \frac{20 \times 1000}{200} = 100 \text{ kg. Hence, (3).}$$

16. Let the number of workers be  $n$

The number of workers decreases by 1 everyday

$\therefore$  On the second day, the number of workers =  $(n - 1)$

On the third day, the number of workers =  $(n - 2)$

Similarly, on the  $n$ th day, the number of workers =  $[n - (n - 1)] = 1$  worker

So, the job is completed in  $n$  days

Let the amount of work done by each worker each day be 1 unit

$\therefore$  Work done on the first day =  $n$  units

Work done on the second day =  $(n - 1)$  units

Work done on the  $n$ th day = 1 unit

The total amount of work =  $n + (n - 1) + \dots + 1 =$

$$\frac{n(n+1)}{2} \text{ units} \dots (i)$$

If all the  $n$  workers worked together,

$$\text{Number of days} = \frac{2}{3}n$$

$$\therefore \text{The total amount of work} = \frac{2}{3}n^2 \text{ units}$$

$$\text{From (i) and (ii), } \frac{2}{3}n^2 = \frac{n(n+1)}{2}$$

$$\therefore n = 3. \text{ Hence, (2).}$$

17. LCM of 8, 12 and 16 is 48. Suppose total work required to be done is 48 units.

Therefore P completes 6 units, Q completes 4 units and R completes 3 units everyday.

If P+Q work together, they complete 10 units. If P+R work together, they complete 9 units. Similarly if Q+R work together, they complete 7 units.

Since no worker works on more than two consecutive days, they work in the order P+Q, P+R and Q+R (or some other combination thereof). In first three days, total 26 units of work is completed (10+9+7) and 22 units is still remaining.

Since the sum of any two of the three numbers (10, 9 and 7) taken together is less than 22, the work will not be completed before 5 days.

If P+Q work together on the sixth day, P+R and R+Q work on the previous two days and complete 16 units. Therefore P+Q have to finish 6 units' work. The time taken by them is:  $\frac{6}{10} = \frac{3}{5}$ .

If P+R work together on the sixth day, P+Q and R+Q work on the previous two days and complete 17 units. Therefore P+R have to finish 5 units' work. The time taken by them is:  $\frac{5}{9}$ .

If Q+R work together on the sixth day, P+R and P+Q work on the previous two days and complete 19 units. Therefore Q+R have to finish 3 units' work. The time taken by them is:  $\frac{3}{7}$ .

Therefore the minimum number of days taken to complete the work =  $5\frac{3}{7}$ . Hence (4).

18. Let the total work done by 1 man in 1 day be 'm' units and the total work done by 1 woman in 1 day be 'w' units.

Let the number of days required by groups A, B and C be 6x, 4x and 3x respectively.

$$\therefore (m + 3w) \times 6x = (2m + 3w) \times 4x$$

$$\therefore 6mx + 18wx = 8mx + 12wx$$

$$\therefore m = 3w$$

Also, total work =  $18wx + 18wx = 36wx$ .

Since group C takes 3x days to finish the job, in one day it will complete 12w work.

$\therefore$  There are 3 possibilities of the number of men and women in group C: (3 men and 3 women), (2 men and 6 women) and (1 man and 9 women)

$\therefore$  There are 3 possibilities of the total number of men and women: (6 men and 9 women), (5 men and 12 women) and (4 men and 15 women)

$\therefore$  There are 3 possibilities of the total work done in one day by all men and all women: (18w and 9w), (15w and 12w) and (12w and 15w)

$\therefore$  There are 3 possibilities of the ratio of the number of days required by all men and all women:

$$\left(\frac{1}{18} : \frac{1}{9}\right), \left(\frac{1}{15} : \frac{1}{12}\right) \text{ and } \left(\frac{1}{12} : \frac{1}{15}\right) \text{ or } (1 : 2),$$

(4 : 5) and (5 : 4). Hence (3).

19. LCM of 8 and 12 is 24. Let the quantum of total work to be completed be 24 units. Therefore Hitler com-

pletes  $\frac{24}{8} = 3$  units per day while Mussolini com-

pletes  $\frac{24}{12} = 2$  units per day.

After 't' days,

Work completed by Hitler = 3t and work completed by Mussolini = 2t.

Therefore work remaining to be done by Hitler =  $24 - 3t$  and work remaining to be done by Mussolini =  $24 - 2t$

$$\therefore 24 - 2t = 2(24 - 3t)$$

$$\therefore 24 - 2t = 48 - 6t \quad \therefore t = 6$$

Therefore, the required answer is 6.

20. At the end of 5<sup>th</sup> day, original assignment required A to complete the work in 15 more days. But now the requirement is that he should complete the work in 10 more days.

$\therefore$  A and B complete the work in 10 days that is completed by A alone in 15 days.

$$\therefore \frac{\text{work done by A and B in one day}}{\text{work done by A alone in one day}} = \frac{15}{10} = \frac{3}{2}$$

$\therefore$  Work done by B in one day

$$= \frac{\text{work done by A in one day}}{2}$$

Suppose work done by A in one day = 8 units and work done by B in one day = 4 units.

At the end of 10<sup>th</sup> day, A and B were originally required to complete the work in 5 more days. But now the requirement is that they should complete the work in 3 more days.

$\therefore$  A, B and C complete the work in 3 days that is completed by A and B in 5 days.

$\therefore$  work done by A and B in 5 days =  $5(8 + 4) = 60$  units.

$\therefore$  Work done by A, B and C in one day =  $\frac{60}{3} = 20$  units.

$\therefore$  Work done by C in one day =  $20 - (8 + 4) = 8$  units. A completes the work alone in 20 days.

$\therefore$  Total quantum of work =  $20 \times 8 = 160$  units.

$\therefore$  Time taken by A, B and C while working together =  $\frac{160}{20} = 8$  days.

21. A takes 15 days to finish the work and B takes 10 days to finish the work. Since both A and B work till the work is finished, their earnings are in the ratio 10 : 15 or 2 : 3.

If earnings of A and B are 2x and 3x respectively, earning of C is  $2x - 1300$ .

$$\therefore 2x + 3x + (2x - 1300) = 12000 \text{ or } x = 1900$$

$$\therefore \text{Earning of B} = 3x = 3 \times 1900 = 5700.$$

### PRACTICE EXERCISE - 2

1. Let 1 man take m days to complete the work.  
Let 1 woman take w days to complete the work.  
Let 1 boy take b days to complete the work.

$$\therefore \frac{3}{w} + \frac{4}{b} = \frac{2}{5} \quad \dots (i)$$

$$\frac{4}{m} + \frac{2}{b} = \frac{1}{2} \quad \dots (ii)$$

$$\frac{2}{m} + \frac{3}{w} = \frac{2}{5} \quad \dots (iii)$$

Adding (i), (ii) and (iii),

$$6\left(\frac{1}{m} + \frac{1}{b} + \frac{1}{w}\right) = \frac{4+5+4}{10}$$

$$\therefore \frac{1}{m} + \frac{1}{b} + \frac{1}{w} = \frac{13}{60}$$

$\therefore$  1 man + 1 woman + 1 boy will take  $\frac{60}{13}$  days working at their normal efficiency.

When they all are working at double their respective efficiencies, they require  $\frac{60}{13 \times 2} = \frac{30}{13} = 2\frac{4}{13}$  days. Hence, (2).

2. Let the earnings of the man, the woman and the boy be m, w and b respectively.

	Man m	Woman w	Boy b
Wages			
Ratio of their work	$\frac{16}{3}$	$\frac{14}{4}$	$\frac{10}{5}$

$$\therefore \frac{3m}{16} = \frac{2w}{7} = \frac{b}{2} = \frac{(m+b+m)}{\left(\frac{16}{3} + \frac{7}{2} + 2\right)}$$

$$= \frac{121.55}{65} = 11.22 \quad \therefore m = 59.84. \text{ Hence, (3).}$$

3. A, B and C finish  $\left(\frac{1}{9}\right)^{\text{th}}$ ,  $\left(\frac{1}{12}\right)^{\text{th}}$  and  $\left(\frac{1}{15}\right)^{\text{th}}$  work respectively, in a day.

$$\therefore \text{In 3 days, they will finish} = \frac{1}{9} + \frac{1}{12} + \frac{1}{15}$$

$$= \frac{20+15+12}{180} = \frac{47}{180} \text{ work.}$$

$$\therefore \text{In 9 days, } \frac{47 \times 3}{180} = \left(\frac{141}{180}\right)^{\text{th}} \text{ of the work is finished.}$$

$$\therefore \left(\frac{39}{180}\right)^{\text{th}} \text{ of the work is remaining.}$$

$$\text{A does } \left(\frac{20}{180}\right)^{\text{th}} \text{ of work.}$$

$$\text{From the remaining } \left(\frac{19}{180}\right)^{\text{th}} \text{ work, B does } \left(\frac{15}{180}\right)^{\text{th}} \text{ of the work.}$$

$$\therefore \text{C does } \left(\frac{4}{180}\right)^{\text{th}} \text{ of the work.}$$

$$\therefore \text{Work done by C} = 3\left(\frac{1}{15}\right) + \frac{4}{180} = \frac{40}{180} = \frac{2}{9}.$$

Hence, (1).

4. It is known that 4 men in the latter case do as much work as 6 men in the former case. So, we can say that 36 men in the latter case do as much work as 54 men in the former case. Hence, we can now say that since  $18 \times 7.5 \times 10$  man hours are required to manufacture 20 shoes.  
 $\therefore 2 \times 18 \times 7.5 \times 10$  man hours are required to manufacture 40 shoes.  
 Now, there are 54 men working 6 hours a day for say 'd' days.  
 i.e.,  $54 \times 6 \times d = 2 \times 18 \times 7.5 \times 10$   
 $\therefore d = \frac{25}{3}$  days. Hence, (2).

5. If x is the time taken to fill the first 65% of the tank and y is the time taken to fill the remaining 35% then we have the following two equations.

$$\frac{x}{5} - \frac{x-1}{6} = 0.65 \quad \dots (i)$$

$$\frac{y}{5} - \frac{y}{6} + \frac{y}{7} - \frac{y}{8} = 0.35 \quad \dots (ii)$$

Since we want the total time required, our answer is x + y. This makes it tough to guess from the options as x and y belong to two separate equations.

Solving equation (i) we get,

$$\frac{x}{5} - \frac{x}{6} + \frac{1}{12} = \frac{13}{20}$$

$$\therefore \frac{x}{30} = \frac{13}{20} - \frac{1}{12} = \frac{34}{60} = \frac{17}{30} \quad \therefore x = 17$$

$$\text{Solving equation (ii), we get } y = \frac{294}{43}.$$

$$\text{Hence our final answer} = 17 + \frac{294}{43} = \frac{1025}{43}$$

Hence, (1).

6. Let the entire time taken be 'n' hours.

In 1 hour 'A' can fill  $\frac{1}{4}$  th of the tank

$$\therefore \text{In 'n' hours} > \frac{n}{4}$$

Hence we get,

$$\text{total job} = \frac{n}{4} - \frac{n-1}{5} + \frac{n-2}{6} - \frac{n-3}{7} = 1$$

$$105n - 84n + 84 + 70n - 140 - 60n + 180 = 420$$

$$31n = 296$$

$$n = \frac{296}{31}. \text{ Hence, (2).}$$

7. Notice that the water that goes into  $T_{10}$  and U is exactly the same in every case. Now consider two adjacent tanks  $T_i$  and  $T_{i+1}$ . Till  $T_i$  gets filled, the rate at which water goes into  $T_{i+1}$  is half that which goes into  $T_i$ . Hence, when  $T_i$  gets filled  $T_{i+1}$  will be half filled. After  $T_i$  gets filled, the water goes into  $T_{i+1}$  at the rate of 50 litres/min. So it takes 1 hour more to fill  $T_{i+1}$  than it takes to fill  $T_i$ . Also it takes 2 hours to fill  $T_1$ . Combining these two, we get that it takes 11 hours to fill  $T_{10}$  as well as U. Hence, (3).

Alternatively,

When tank U is filled completely all tanks are filled completely.

$$\therefore \text{Time taken} = \frac{6000 \times 11}{100}$$

$$= 660 \text{ min}$$

$$= 11 \text{ hours. Hence, (3).}$$

8. Given that the third pipe fills the tank in 3 hrs. Hence, the second pipe will fill the tank in 6 hrs and the first pipe will fill the tank in 9 hrs.

In the first three hours, work done by three pipes is

$$\frac{1}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6} + \frac{1}{9} + \frac{1}{9} - \frac{1}{3} - \frac{1}{6} - \frac{1}{9}$$

$$= \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

In the fourth hour,  $\frac{1}{6} - \frac{1}{9} + \frac{1}{3} = \frac{3-2+6}{18} = \frac{7}{18}$  th part will be filled.

∴ In first four hours,  $\left(\frac{11}{18} + \frac{7}{18} = 1\right)$  tank is full. Hence, (2).

9.	Men	Women	Children
Work	3	2	1
Numbers	20	30	36

Ratio of wages =  $(3 \times 20) : (2 \times 30) : (1 \times 36)$   
= 5 : 5 : 3

Total wages of 20 men =  $\frac{5}{13} \times 780 = \text{Rs.}300$   
∴ Wages of a man = Rs.15.

Similarly, wages of 30 women  $\frac{5}{13} \times 780 = \text{Rs.}300$   
wages of a woman = Rs.10

and wages of 36 children  $\frac{3}{13} \times 780 = \text{Rs.}180$   
wages of a child = Rs.5

∴ Total wages of 15 men, 21 women and 30 children  
=  $15 \times 15 + 21 \times 10 + 30 \times 5 = \text{Rs.}585$

∴ Total wages for 2 weeks = Rs.1170. Hence, (3).

10. I. Work done by A, B and C together in 3 days

$$\frac{1}{8} + \frac{1}{12} + \frac{1}{16} = \frac{13}{48} \text{ th of the work.}$$

Thus, in 9 days the total work done will be

$$3 \times \frac{13}{48} = \frac{39}{48} \text{ th of the work}$$

∴ Work remaining  $1 - \frac{39}{48} = \frac{9}{48}$  th of the work is left

of which A will finish  $\frac{6}{48}$  th of work on 10th day

and B will finish remaining work on 11th day.  
Thus, I is false.

- II. Work done by A, B, C together in 9 days =  $\frac{39}{48}$ .

Work left =  $1 - \frac{39}{48} = \frac{9}{48}$  th of the whole.

On the 10th day B will finish  $\frac{1}{12}$  th of the whole.

∴ Work left =  $\frac{9}{48} - \frac{1}{12} = \frac{5}{48}$  th of the whole.

On 11th day C will finish  $\frac{1}{16}$  th of the work

∴ Work left =  $\frac{5}{48} - \frac{1}{16} = \frac{2}{48}$  th of the whole

which will be done by A on the 12th day. Thus,  
work will be finished on the 12th day.

Thus, II is false.

- III. Work done by A, B and C in 9 days =  $\frac{39}{48}$ .

Work left after 9 days =  $1 - \frac{39}{48} = \frac{9}{48}$  th of the whole.

On the tenth day  $\frac{1}{16}$  th will be finished by C.

∴ Work left =  $\frac{9}{48} - \frac{1}{16} = \frac{6}{48}$  th of the whole,

which will be finished by A on the eleventh day.

Thus, III is true. Hence, (3).

11. In a cycle of 3 hours; pipes P, Q and R are working as inlet pipes for 2 hours each and they are working as outlet pipes for an hour each. So part of tank filled in 3 hours.

$$= 2 \times \left(\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right) - \left(\frac{1}{8} + \frac{1}{10} + \frac{1}{12}\right)$$

$$= \frac{1}{8} + \frac{1}{10} + \frac{1}{12}$$

$$= \frac{15 + 12 + 10}{120} = \frac{37}{120}$$

In 9 hours, tank will be filled by  $\frac{111}{120}$  or  $\frac{37}{40}$  th of

the capacity of the tank. So,  $\frac{3}{40}$  of the tank has to be

filled after 9 hours. In the 10th hour  $\frac{1}{8} + \frac{1}{12} - \frac{1}{10}$

or  $\frac{13}{120}$  of the tank will be filled.

So, in the 10th hour tank will be filled after  $\frac{3}{40} \div$

$$\frac{13}{120} \Rightarrow \frac{3}{40} \times \frac{120}{13} = \frac{9}{13} \text{ th of an hour.}$$

So tank will be filled in  $9\frac{9}{13}$  hours.

Thus, the tank will be filled in the 10th hour.

Hence, (3).

12. Since 5 tanks of same efficiency fill the boiler in 2 hours each tank individually takes  $2 \times 5 = 10$  hours to fill it.

The tank with half the efficiency will take  $10 \times 2 = 20$  hours.

In one hour, the fraction of boiler getting filled when 4 fully efficient pipes with two as inlet and two as outlet and the 5th acts at 50% efficiency as an inlet:

$$\frac{1}{10} + \frac{1}{10} + \frac{1}{20} - \frac{1}{10} - \frac{1}{10} = \frac{1}{20}$$

Hence it will take 20 hours to fill the boiler tank.

Hence, (3).

13. Here the fraction of tank filled in 1 hour.

$$\frac{1}{10} + \frac{1}{10} + \frac{1}{20} = \frac{5}{20} = \frac{1}{4} \text{ th of the tank}$$

If they are opened alternately then tank gets filled in  $4 \times 3 = 12$  hours

Hence half of the tank gets filled in  $\frac{12}{2} = 6$  hours.  
Hence, (3).

14. Tap A can fill 4 buckets in 24 minutes

∴ In 2 hrs or 120 mins, it can fill  $\frac{120 \times 4}{24} = 20$  buckets

Tap B can fill 8 buckets in 60 minutes

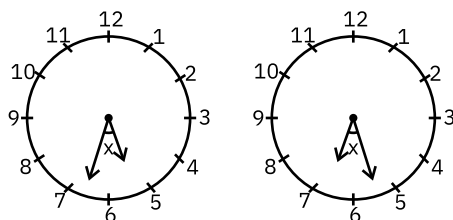
- ∴ In 2 hrs or 120 mins, it can fill  $\frac{120 \times 8}{60} = 16$  buckets  
 Tap C can fill 2 buckets in 20 minutes
- ∴ In 2 hrs or 120 mins, it can fill  $\frac{120 \times 2}{20} = 12$  buckets  
 Thus, in 2 hrs, taps A, B and C can fill  $20 + 16 + 12 = 48$  buckets =  $48 \times 5 = 240$  litres. Hence, (2).
15. Let the capacity of the tank be 24 units  
 The rate at which the leak, leaks the water = 3 units/hr  
 When both the tap and the leak were open, the tank is emptied in 12 hours  
 In 12 hours, the leak would have emptied 36 units of water  
 In 12 hours, the tap would have filled in  $12 \times 6 = 72$  litres of water  
 So, 24 units + 72 litres = 36 units  
 ∴ 12 units of water = 72 litres  
 The capacity of the tank is 24 units, i.e. 144 litres  
 Hence, (3).
16. Every day, let the amount of work done by a man, a woman and a child be 6 units, 3 units and 1 unit respectively  
 In all, 39 people are hired with the men, women and children ratio being 6: 5:2,  
 ∴ Number of men =  $\left(\frac{6}{6+5+2}\right) \times 39 = 18$   
 ∴ Number of women =  $\left(\frac{3}{6+5+2}\right) \times 39 = 15$   
 ∴ Number of children =  $\left(\frac{1}{6+5+2}\right) \times 39 = 6$   
 ∴ In one day, the amount of work done =  $18 \times 6 + 15 \times 3 + 6 \times 1 = 159$  units  
 i.e. The wages paid for 159 units of work = Rs 1113  
 ∴ Wages paid for one unit of work = wages of a child per day =  $\frac{1113}{159} = \text{Rs.}7$ . Hence, (3).
17. 2 women working alone can complete the entire job in 12.5 days.  
 ∴ 1 woman working alone can complete the entire job in 25 days.  
 5 men working alone can complete the entire job in 3 days  
 ∴ 1 man working alone can complete the entire job in 15 days.  
 ∴ The ratio of the efficiency of 1 man and 1 woman = 5 : 3  
 Let the total work done by 1 man in 1 day be 5 units and the total work done by 1 woman in 1 day be 3 units.  
 Total work of the entire job = 75 units.  
 Let the number of days that the men and women did not work be 5x and 3x respectively.  
 ∴ The number of days that the men and women worked will be  $(11 - 5x)$  and  $(11 - 3x)$  respectively.  
 ∴  $3 \times 5 \times (11 - 5x) + 4 \times 3 \times (11 - 3x) = 75$   
 Solving this equation, we get  $x = 2$ .  
 ∴ The ratio of the number of days that the men and women worked = 1 : 5. Hence, (2).
18. Let C complete 5x units of work in one day. Then B will complete 4x units of work and A will complete 6x units of work in one day.  
 Total work done by A and B together in one day will be 10x units and the total work done by B and C together in one day will be 9x units.  
 Let the number of days required by A and B together to complete the work be 'd'.  
 ∴  $10x \times d = 9x \times (d + 1)$   
 ∴  $d = 9$   
 ∴ Total work = 90x units  
 Quantum of work completed by C alone in 9 days = 45x  
 ∴ Required percentage =  $\frac{45x}{90x} \times 100 = 50\%$ . Hence, (2).
19. 5 men can mow 500 m<sup>2</sup> of a lawn in 10 days.  
 ∴ 1 man can mow 100 m<sup>2</sup> of the lawn in 10 days.  
 15 women can mow 1500 m<sup>2</sup> of the lawn in 20 days.  
 ∴ 1 woman can mow 100 m<sup>2</sup> of the lawn in 20 days.  
 Now, 6 men can mow 600 m<sup>2</sup> of the lawn in 10 days. (I)  
 Also, 2 women can mow 200 m<sup>2</sup> of the lawn in 20 days.  
 That is, 2 women can mow 100 m<sup>2</sup> of the lawn in 10 days. (II)  
 From (I) and (II), 6 men and 2 women can mow 700 m<sup>2</sup> of the lawn in 10 days.  
 Therefore, the number of days required for 6 men and 2 women to mow 2100 m<sup>2</sup> of the lawn =  $10 \times 3 = 30$  days.
20. Jericho finished half the work in 16 days. Therefore, if he had continued to work in the same rate, he would have finished in total 32 days.  
 Therefore, he finished  $\frac{1}{32}$  of the work each day up to the 16<sup>th</sup> day.  
 Total  $\frac{1}{2}$  of the work is pending.  
 Let's calculate starting from the 17<sup>th</sup> day.  
 On the 17<sup>th</sup> day, he would have completed  $\frac{1}{16}$  of the work (less than  $\frac{1}{2}$ ).  
 On the 18<sup>th</sup> day, he would have completed  $\frac{1}{16} + \frac{1}{8} = \frac{3}{16}$  of the work (less than  $\frac{1}{2}$ ).  
 On the 19<sup>th</sup> day, he would have completed  $\frac{3}{16} + \frac{1}{4} = \frac{7}{16}$  of the work (less than  $\frac{1}{2}$ ).  
 On the 20<sup>th</sup> day, he would have completed  $\frac{7}{16} + \frac{1}{2} = \frac{15}{16}$  of the work (more than  $\frac{1}{2}$ ).  
 ∴ Jericho would have completed the work on the 20<sup>th</sup> day.  
 Therefore, the required answer is 20.

## QA-3.9 | TSD APPLICATIONS-I

### PRACTICE EXERCISE

- Time taken is same =  $t$ .  
Hence,  $45t + 54t = 1980$   
 $99t = 1980$   
 $t = 20$  min  
Distance between two points =  $45 \times 20 = 900$  m.  
or  $54 \times 20 = 1080$  m.  
Hence, (1).
- A beats B by 60 seconds and B beats C by 30 seconds,  
 $\therefore$  A beats C by  $60 + 30 = 90$  seconds or 375 metres  
 $\therefore$  C runs 1000 metres in  $\frac{1000 \times 90}{375}$   
 $= 240$  seconds = 4 minutes.  
 $\therefore$  B runs 1000 metres in  $4 - \frac{1}{2} = 3\frac{1}{2}$  minutes.  
A runs 1000 metres in  $3\frac{1}{2} - 1 = 2\frac{1}{2}$ .  
Hence, (1).
- When A runs 5 rounds, B runs 4 rounds (Ratio of speeds)  
 $\therefore$  A passes B each time A has run 5 rounds or  
 $5 \times \frac{1}{4} = \frac{5}{4}$  km.  $= 1\frac{1}{4}$  km.  
 $1\frac{1}{4}$  km is contained in 4 km. 3 times.  
 $\therefore$  A passes B thrice.  
Hence, (3).
- The time taken for 1 round by them is 192 seconds and 210 seconds respectively.  
The LCM is 6720 seconds = 1 hour 52 minutes.  
 $\therefore$  They will meet again at the starting point at 4:52 p.m. Hence, (1).
- The watch gains 1 minute in 15 hours.  
 $\therefore$  It will gain  $\frac{24}{15}$  minutes in 24 hours; i.e. 1.6 minutes in 24 hours.  
Hence, (1).

6.



When the hour hand moves 1 minute space, the minute hand moves 12 minute spaces.

- $\therefore$  When the hour hand moves  $x$  minutes, the minute hand moves  $12x$  minutes.  
Minute hand would have moved 60 minutes to take the original position. But, since the places have interchanged, it will cover only  $(60 - x)$  minutes.  
 $\therefore 12x = 60 - x \therefore x = \frac{60}{13}$   
Interchangeable positions occur when the original interval between the hands is  $4\frac{8}{13}$  minute spaces or a multiple of this.  
The hour hand moves through  $\frac{60}{13}$  minute spaces.  
At 5 o'clock, the minute hand is 25 minutes behind - time.  
It has to gain  $25 + \frac{60}{13} = \frac{385}{13}$  minutes.  $\frac{385}{13}$  minutes will be gained in:  
 $\frac{385}{13} \times \frac{60}{55} = \frac{420}{13} = 32\frac{4}{13}$  minutes.  
 $\therefore$  The man went out at  $32\frac{4}{13}$  minutes past 5 o'clock.  
Hence, (1).
- When they are in the same straight line, in opposite directions, the minute hand and hour hand are 30 minute spaces apart. At 7 p.m., they are 25 minute spaces apart.  
 $\therefore$  The minute hand has to gain only  $(30 - 25) = 5$  minute spaces.  
i.e.,  $\frac{60}{55} \times 5 = \frac{60}{11} = 5\frac{5}{11}$  minutes.  
 $\therefore$  The hands are in the same straight line at  $5\frac{5}{11}$  minutes past 7.  
Hence, (1).
  - At 5 o'clock the two hands are 25 minute spaces apart.  
(i) When the minute hand is 3 minute spaces behind the hour hand, it has to gain  $(25 - 3 = 22)$  minute spaces.  
i.e.,  $\left(\frac{60}{55} \times 22\right) = 24$  minutes. i.e., at 24 minutes past 5.  
(ii) When the minute hand is 3 minute spaces ahead of the hour hand it has to gain  $(25 + 3 = 28)$  minute spaces.  
i.e.,  $\left(\frac{12}{11} \times 28\right) = 30\frac{6}{11}$  minutes past 5.  
Hence, (2).
  - In 24 hours, it will lose  $24 \times 5 = 120$  seconds or 2 minutes.  
 $\therefore$  It will show 11:58 a.m. on Monday noon.  
Hence, (2).

10. Let us calculate the day on 1<sup>st</sup> March, 1952.  
1<sup>st</sup> March 1952 means (1951 complete year + first 2 months of the year 1952 + 1 day of March).  
1600 years give no odd days.  
300 years give 1 odd day.  
51 years give (51 + 12) 0 odd day.  
∴ 1951 years give 1 odd day.  
From 1<sup>st</sup> January to 1<sup>st</sup> March there are 5 odd days.  
Jan → 3 odd days  
Feb → 1 odd day  
March → 1 odd day  
∴ Total number of odd days = 1 + 5 = 6 odd days.  
∴ 1<sup>st</sup> March 1952 is a Saturday.  
∴ 3<sup>rd</sup> March 1952 is a Monday.  
∴ Ram was born on 3<sup>rd</sup> March 1952.  
Hence, (2).
11. Let us first calculate the day on 22<sup>nd</sup> February 1998.  
1600 years → 0 odd day  
300 years → 1 odd day  
97 years (24 + 73) = (24 × 2 + 73) → 2 odd days  
∴ 1997 gives 3 odd days  
January → 3 odd days  
February → 22 = 1 odd day  
∴ Total number of odd days = 3 + 3 + 1 = 7 = 0 odd day  
∴ 22<sup>nd</sup> February 1998 is Sunday ... (1)  
∴ Option (4) is false.  
∴ 22<sup>nd</sup> February 1999 is Monday.  
∴ 27<sup>th</sup> February 1999 is Saturday.  
∴ (2) is true.  
Hence, (2).
12. Let the speed of A, B, C be  $V_A, V_B, V_C$  respectively.  
Then  $\frac{V_A}{V_B} = \frac{10 - \frac{1}{5}}{9 - \frac{1}{4}} = \frac{49}{44}$  ... (1)  
and  $\frac{V_A}{V_B} = \frac{12 - \frac{1}{2}}{11 - \frac{1}{2}} = \frac{23}{21}$  ... (2)  
 $V_C = \frac{20 \times 2 \times \pi \times 50}{30}$  meter/minutes ... (3)  
using (1), (2) and (3) we have  
 $V_A = \frac{49}{44} \times \frac{23}{21} \times \frac{20 \times 2 \times 22 \times 50}{30 \times 7}$  meter/minutes  
 $= \frac{7 \times 23 \times 2 \times 50}{21 \times 3}$  meter/minutes  
 $= 255.5$  meter/minutes  
 $\approx 256$  meter/minute.  
Hence, (3).
13. Normal time taken by the minute and hour hands to coincide is  $65 \frac{5}{11}$  minutes  
∴ Time lost by the clock  
 $= \left( 66 \frac{9}{11} - 65 \frac{5}{11} \right) \times 22$

$$= \left( \frac{726 + 9}{11} - \frac{715 + 5}{11} \right) \times 22$$

$$= \frac{735 - 720}{11} \times 22 = 30 \text{ minutes.}$$

Hence, (2).

14. A overlaps B in the middle of the 6<sup>th</sup> round. So, when A runs  $5\frac{1}{2}$  rounds, B runs only  $4\frac{1}{2}$  rounds.

$$\therefore \frac{\text{A's Speed}}{\text{B's Speed}} = \frac{5\frac{1}{2}}{4\frac{1}{2}} = \frac{11}{9}$$

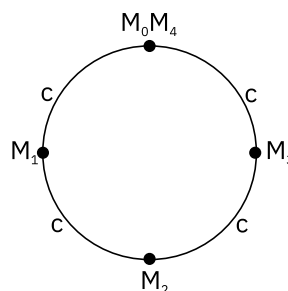
∴ When A runs 11 km B runs 9 km.

$$\text{When A runs 2 km, B runs } 2 \times \frac{9}{11} = \frac{18}{11} \text{ km.}$$

$$\therefore \text{A wins by } \left( 2 - \frac{18}{11} \right) = \frac{4}{11} \text{ km.}$$

Hence, (3).

15. Let the circumference of the circular course be  $4c$  km. As A's speed is thrice B's, before they meet the first time at  $M_1$ , A travels  $3c$  km and B travels  $c$  km.



Then they interchange speeds and directions and meet at  $M_2, M_3$  and  $M_4$  as shown.

∴ The meeting point  $M_4$  and the initial point will coincide.  
Hence, (4).

16. The shortest distance between  $M_1$  and  $M_2$  will be one fourth of the circumference of the circle  
 $\frac{1}{4} \times 2 \times \frac{22}{7} \times 7 = 11$  km.  
Hence, (1).
17. The shortest distance between  $M_1$  and  $M_3$  will be half of the circumference of the circle  
 $\frac{44}{2} = 22$  km  
Hence, (1).
18. The total distance travelled by A till the third meeting point is  $= 33 + 11 + 33 = 77$  km.  
Hence, (1).



19. In a perfect watch, the minute hand crosses the hour hand after every  $\frac{720}{11}$  minutes.

So, in a perfect watch, the minute hand crosses the hour hand for the third time after  $\frac{2160}{11}$  minutes, i.e. after  $196\frac{4}{11}$  minutes.

It is given that in this particular watch, the minute hand crosses the hour hand for the third time after 3 hrs. 18 min. 15 sec., i.e., after 198.25 minutes.

So, the time lost in  $196\frac{4}{11}$  min.

So, the time lost in 24 hrs is =  $\left(198.25 - 196\frac{4}{11}\right)$  minutes.  
= 13.833 minutes, or 13 minutes 50 seconds  
Hence, (2).

20. Given: 1 mile = 1600 m.

(i) Akshay can be given a start of 128 m by Bhairav in a mile race:

When Bhairav covers 1600 m, then Akshay covers  $(1600 - 128) = 1472$  m

(ii) Chinmay can be given a start of 4 m by Bhairav in a 100 m race:

When Bhairav covers 100 m, then Chinmay covers  $(100 - 4) = 96$  m

Thus, when Bhairav covers 1600 m, then Chinmay covers  $96 \times 16 = 1536$  m and Akshay covers 1472 m. Hence, when Chinmay covers 2400 m (1.5 mile), then Akshay covers

$$\frac{1472 \times 2400}{1536} = m$$

Thus, Akshay can be given a start of

$$2400 - 2300 = 100 \text{ m} = \frac{1}{16} \text{ th mile by Chinmay.}$$

Hence, (4).

21. Suppose the circumference of the circular path = 99a metres (LCM of 9 and 11). Therefore, Raghu will complete one round in  $\frac{99}{9}a = 11a$  seconds while Ram

will complete one round in  $\frac{99}{11}a = 9a$  seconds. They

will meet at the starting point for the first time after 99a seconds (LCM of 9a and 11a). That means when they meet for the first time, Ram would have completed 11 rounds while Raghu would have completed 9 rounds. This means that Ram has gained two rounds over Raghu till the time they meet at the starting point for the first time. Therefore, Ram would have overtaken Raghu only once in between.

Hence, (1).

22. When Abbas ran 100 metres, let the time taken be 't' seconds.

Mustaan ran 96 metres and took time 't + 19' seconds.

$$\therefore \frac{100}{t} : \frac{96}{t+19} = 6 : 5$$

$$\therefore \frac{t+19}{t} = \frac{144}{125}$$

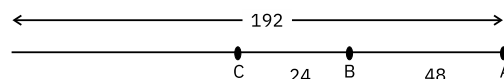
$$\therefore t = 125 \text{ seconds.}$$

$$\therefore \text{Abbas's speed} = \frac{100}{125} = 0.8 \text{ m/s and Mustaan's speed} = \frac{96}{144} = 0.66 \text{ m/s}$$

$\therefore$  Mustaan will take 150 seconds to walk 100 metres. When given 30 seconds headstart, Mustaan will take another 120 seconds to finish the race. But Abbas will finish the race in 125 seconds. Therefore, Mustaan will win by 5 seconds.

Hence (2).

- 23.



When A runs 192 m, B runs  $192 - 48 = 144$  m and C runs  $192 - 72 = 120$  m.

$$\therefore \text{Ratio of speeds of B and C} = \frac{144}{120} = \frac{6}{5}.$$

$$\therefore \text{When B runs 192m, the distance run by C} = 192 \times \frac{5}{6} = 160 \text{ m.}$$

$$\therefore \text{Required answer} = 192 - 160 = 32 \text{ m.}$$

Hence, (2).

24. Suppose the distance between A and B is 'd'. Therefore, the ratio of the speeds of Karan and Arjun is given by

$$\frac{d}{d-20} = \frac{d+10}{d-12}$$

$$\therefore d^2 - 12d = d^2 - 10d - 200$$

$$\therefore d = 100$$

Therefore, the distance between A and B is 100 m.

## QA-3.10 | TSD APPLICATIONS-II

### PRACTICE EXERCISE

#### Answers to questions 1 and 2:

Let the speed of the plane be  $x$  km/hr

Then its speed from B to A =  $(x - 50)$  km/hr

and its speed from A to B =  $(x + 50)$  km/hr

The flight starts from city B (8.00 a.m.) and arrives at city A (8.00 p.m.) after halting for 1 hour in city A

$\therefore$  Total time taken = 11 hours

$$\text{i.e., } \frac{3000}{x + 50} + \frac{3000}{x - 50} = 11$$

$$\Rightarrow \frac{2x}{x^2 - 2500} = \frac{11}{3000}$$

$$\Rightarrow 11x^2 - 6000x - 27500 = 0$$

Solving the above we get,  $x = 550$

$$\therefore \text{Time taken from B to A} = \frac{3000}{500} = 6$$

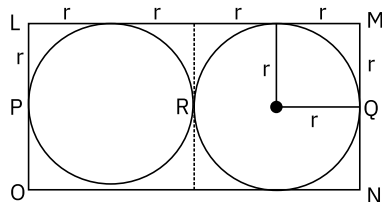
Flight reaches A when local time in B is 2.00 p.m. which is same as local time 3.00 p.m. in A

$\therefore$  Required time difference = 1 hour

1-(2)

2-(4).

3. The circular and rectangular tracks are shown below:



Let 'A' and 'B' start from P.

The two circles are of radius ' $r$ '.

Length of the rectangle =  $4r$

Breadth of the rectangle =  $2r$

Let the time taken by 'A' and 'B' to reach P again be ' $t$ ' min.

'A' covers  $2(4r + 2r) = 12r$  in  $t$  minutes

$$\therefore \text{A's speed} = \frac{12r}{t}$$

B covers  $2\pi r + 2\pi r = 4\pi r$  in  $t$  minutes

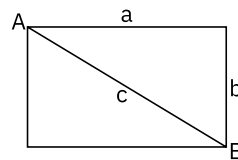
$$\therefore \text{B's speed} = \frac{4\pi r}{t}$$

$$\frac{\text{B's speed}}{\text{A's speed}} = \frac{4\pi r/t}{12r/t} = \frac{\pi}{3} \approx 1.047$$

$\therefore$  B's speed  $\approx 0.47 \times 100 = 4.7\%$  more than A.

Hence, (4).

4. Let the rectangular track be as follows:



Let speed of X, Y and Z be  $V_x$ ,  $V_y$  and  $V_z$  respectively.

$$\Rightarrow a + b = c + 12V_x$$

$$\text{and } a + b = c + 4V_y$$

$$\Rightarrow 3V_x = V_y$$

$$\text{Now, average speed} = V_z = \frac{3}{2} V_x$$

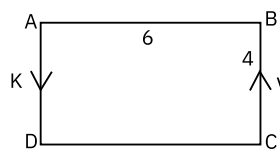
$$\therefore \frac{3}{2} V_x = \frac{9000}{10 \times 60}$$

$$\therefore V_x = 10 \text{ m/s.}$$

$$\therefore \text{Speed of Y} = V_y = 30 \text{ m/s.}$$

Hence, [5].

#### Answers to questions 5 and 6:



At 8:50 am, Kedar would have cycled  $6 \times \frac{50}{60} = 5$  km.  
 $\therefore$  He will be 5 km away from Vishnu.

$\therefore$  He will overtake Vishnu in  $\frac{5}{6-1} = 1$  hr. after 8:50 a.m.,

i.e., at 9:50 a.m.

$\therefore$  At 10:50 am, i.e., 1 hr. after 9:50 a.m. they will be 5 km apart.

$$5 - (3) \quad 6 - (1)$$

7. Let  $AB = BC = CD = DE = x$  m

$$\therefore \frac{4x}{10} = \frac{1.2 \times 1000}{60} = 20 \text{ m/min.}$$

$$\Rightarrow 4x = 200 \text{ m}$$

$$\Rightarrow x = 50 \text{ m}$$

$\therefore AC = BD$ ,  $\square ACDB$  is a rhombus.

As  $AB = AC = BD = CD = BD$ ,  $\triangle ABC$  and  $\triangle CBD$  are equilateral triangles.

$$\therefore AD = 2 \times \text{height of } \triangle CBD = 2 \times \frac{\sqrt{3}}{2} \times CD = 2 \times \frac{\sqrt{3}}{2} \times 50 = 50\sqrt{3} \text{ m. Hence, (2).}$$

8. Total distance that K has to cover

$$= AD + DE = (50\sqrt{3} + 50) \text{ m}$$

$$\therefore \text{Time taken} = \frac{(50\sqrt{3} + 50)}{20} \text{ m/min}$$

$\therefore$  Time taken =  $\frac{50(1 + \sqrt{3})}{20} \approx \frac{5}{2}(1 + 1.7)$   
 $(\because \sqrt{3} \approx 1.7)$   
 $= \frac{5}{2} \times 2.7 = \frac{13.5}{2} = 6.75$   
 $\therefore$  Time taken is approximately 7 minutes.  
 Hence, (2).

9. In two minutes the wolf will cover  $2 \times 20 = 40$  metres. Now, the distance between the wolf and rabbit =  $100 - 40 = 60$  meters. Now, since they both should reach the burrow simultaneously the rabbit will run 40 metres whereas, the wolf has to run  $60 + 40 = 100$ m. Time taken by the wolf to run 100m =  $\frac{100}{20} = 5$  minutes.

The rabbit has to run 40 metres in 5 minutes

$\therefore$  Its speed =  $\frac{40}{5} = 8$  m per minute

As the rabbit covers 1 metre in 3 leaps, its speed will be  $8 \times 3 = 24$  leaps per minute.

Hence, (1).

10. If Amy and Pamy took 'a' and 'b' seconds to reach their destinations after meeting each other, the time "t" taken by them to meet is given by  $t = \sqrt{ab}$ . Therefore, the required answer is  $t = \sqrt{6.4 \times 10} = 8$  seconds.

Hence (1).

11. Let A and B's individual speeds be 'a' m/s and 'b' m/s respectively.

$$\therefore \frac{200}{b} = \frac{200}{a} + \frac{20}{3} \Rightarrow b = \frac{30a}{a + 30}$$

$$\text{Also, } \frac{200}{a + b} = \frac{100}{a} + \frac{4}{3}$$

Substituting the value of b, we get  $a = 15$  m/s.

Hence, (3).

12. Suppose Basanti's speed = b km/hr and the speed of the train = t km/hr.

The relative speed of the train going from Pune to Baramati with respect to Basanti =  $t - b$

The relative speed of the train going from Baramati to Pune with respect to Basanti =  $t + b$

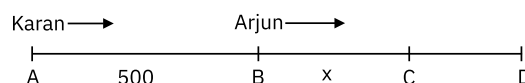
The speed of the train going in either direction = t

Since  $t - b$ , t and  $t + b$  are in AP, the times taken will be in HP.

Therefore the time interval between the two consecutive trains =  $\frac{2 \times 6 \times 8}{6 + 8} = \frac{48}{7} = 6\frac{6}{7}$  minutes.

$$\text{Hence, (2).}$$

13. The situation is as follows:



C is the meeting point and D is the point where Arjun reaches by the time Karan has returned to point A. Therefore,  $l(BD) = 1200$ .

Arjun has run 'x' by the time Karan has run '500 + x' while Arjun has run 1200 by the time Karan has run  $2(500 + x)$ .

Therefore,

$$\text{we have } \frac{x}{x + 500} = \frac{1200}{2(x + 500)} = \frac{600}{x + 500}$$

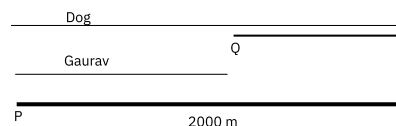
$$\therefore x = 600$$

Therefore, the required answer is  $2 \times (500 + 600) = 2200$ .

14. Since the speed of the dog is twice the speed of Gaurav, he meets Gaurav at point P, which is at a distance 1000 m from point A, as shown below. By that time, the dog has run 2000 m in the direction opposite to Gaurav's direction.



Now consider the following situation-



The dog meets Gaurav at point Q. Since the speed of the dog is two times that of Gaurav, we have,

$$\frac{2000 + BQ}{2000 - BQ} = \frac{2}{1}$$

$$\therefore BQ = \frac{2000}{3} \text{ and } PQ = \frac{4000}{3}$$

Therefore the distance run by the dog in the direction opposite to Gaurav = half the distance run by Gaurav in the direction A to B till they meet for the second time. Same story continues in the subsequent meetings till the time Gaurav reaches point B.

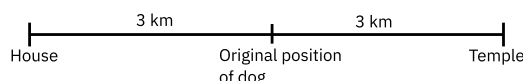
Therefore total distance run by the dog in the opposite direction =  $2000 + \frac{2000}{2} = 3000$ m

15. Total time taken by Rajesh to reach the temple =  $\frac{6}{6} =$

1 hour. The dog was running for 1 hour. Therefore the total distance covered by the dog = 11 km.

When Rajesh reached the temple, the dog also reached there with him. Initially the dog was at a distance 3 km from the temple. That means the dog found himself at a distance 3 km away from his original position after running 11 km.

The situation is as follows:



The dog finds himself at the temple (i.e. at a distance of 3 km from his starting point) after having run a total distance of 11 km. We can look upon this situation as if the dog ran 3 km and reached the temple, ran additional 4 km further away from the temple in his original direction and 4 km back opposite his original direction. Therefore, the distance run by the dog in the direction opposite to Rajesh = 4 km. Hence, (2).

16. Suppose the speed of the escalator is 'x' steps/second.

When Sachin is climbing up the escalator that is going up, the escalator will take  $20x$  steps in 20 seconds, while Sachin takes 120 steps. Therefore, the total number of steps =  $20x + 120$

When Sachin is climbing down the escalator that is going up, the escalator will take  $40x$  steps in 40 seconds, while Sachin takes 240 steps. Therefore, the total number of steps =  $240 - 40x$ .

$$\therefore 20x + 120 = 240 - 40x$$

$$\therefore x = 2 \text{ steps/second}$$

Therefore, the number of steps required on a stationary escalator =  $20 \times 2 + 120 = 160$ .

Hence, (3).

17. Time required to travel from A to C =  $\frac{250\sqrt{3}}{70}$   
= 6.1 hrs

= 6 hrs 11 mins

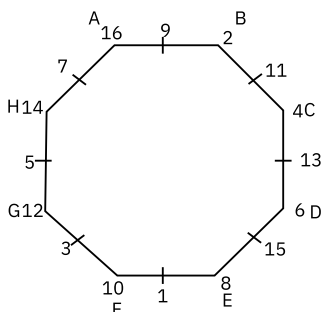
$$\text{Time required to travel from B to C} = \frac{250}{50} = 5 \text{ hrs}$$

$\therefore$  Rahim should reach at C before (8:00 + 5 hrs - 15 minutes) i.e., 12:45 p.m.

$\therefore$  Rahim should leave place A before (12:45 - 6 hrs 11 mins) i.e., 6:34 a.m. Hence, (2).

18. Let each side of the octagon be of length x. Therefore, the total length of the track will be  $8x$ .

Since their speeds are in the ratio 9 : 7, they will travel distances  $4.5x$  and  $3.5x$  every time they meet. Their various meeting point can be represented as follows:



The 12<sup>th</sup> meeting will be at G.

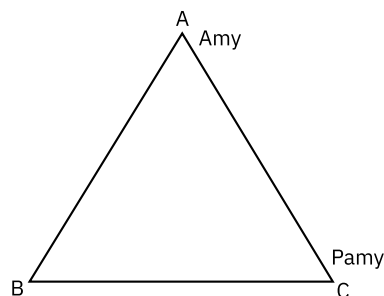
The 15<sup>th</sup> meeting will be between D and E.

The 20<sup>th</sup> meeting will be at C.

The 24<sup>th</sup> meeting will be at E.

Hence (4).

- 19.



Amy and Pamy will meet for the first time after together they run total distance equal to the side of the triangle (say s). From that point onwards, their each next meeting will be after they run additional total distance equal to the perimeter of the triangle (3s).

Therefore, the total distance covered by the two cats when they meet for the third time =  $s + 2 \times 3s = 7s$

If they take 12 seconds to run a total distance of s, they will take  $12 \times 7 = 84$  seconds to run a total distance of  $7s$  together.

Therefore, the required answer is 84.

20. Suppose the speed of the escalator = x steps/second.

$\therefore$  In 28 seconds, the escalator will supply  $28x$  steps while in 16 seconds, the escalator will supply  $16x$  steps.

$$\therefore 28x + 35 = 16x + 44$$

$$\therefore 12x = 9$$

$$\therefore x = \frac{3}{4}$$

Therefore, the number of steps on the escalator = 28

$$\times \frac{3}{4} + 35 = 16 \times \frac{3}{4} + 44 = 56$$

Therefore, the required answer is 56.

## QA-3.11 | TRIGONOMETRY

### PRACTICE EXERCISE

1.  $\cot\theta + \tan\theta = p$

$$\frac{1}{\tan\theta} + \tan\theta = p$$

$$\frac{1 + \tan^2\theta}{\tan\theta} = p$$

$$\sec^2\theta = p \tan\theta$$

$$p \tan\theta \cdot \cos^2\theta = 1 \quad (2) \text{ is true}$$

$$p \frac{\sin\theta}{\cos\theta} \cos^2\theta = 1$$

$$p \sin\theta \cos\theta = 1$$

$$p \sqrt{1 - \sin^2\theta} \sin\theta = 1$$

$$p \sqrt{\sin^2\theta - \sin^4\theta} = 1 \quad (1) \text{ is true}$$

Hence, (3).

Alternatively,

$$p \sqrt{\sin^2\theta - \sin^4\theta} = p \sqrt{\sin^2\theta \cos^2\theta}$$

$$= \sin\theta \cos\theta (\cot\theta + \tan\theta)$$

$$= \cos^2\theta + \sin^2\theta = 1$$

$$p \tan\theta \cos^2\theta = (\cot\theta + \tan\theta) \times \tan\theta \cos^2\theta$$

$$= \cos^2\theta + \tan^2\theta \cos^2\theta$$

$$= \cos^2\theta + \sin^2\theta = 1.$$

Hence, (3).

2. Since  $\theta$  &  $\phi$  are supplementary  
hence  $\theta + \phi = \pi$

$$\Rightarrow \tan\theta = \tan(\pi - \phi) = -\tan\phi$$

$$\Rightarrow \sin\theta = \sin(\pi - \phi) = \sin\phi$$

$$\Rightarrow \cos\theta = \cos(\pi - \phi) = -\cos\phi$$

$$\tan^2\theta - \sec^2\phi = \tan^2\phi - \sec^2\phi = -1$$

$$\sin^2\theta + \cos^2\phi = \sin^2\phi + \cos^2\phi = 1$$

$$\sin^2\theta - \cos^2\phi = \sin^2\phi - \cos^2\phi$$

$$\cos^2\theta - \sin^2\phi = \cos^2\phi - \sin^2\phi$$

$$-\sin^2\theta - \cos^2\phi = -\sin^2\phi - \cos^2\phi = -1.$$

Hence, (4).

3. We know that  $\sin(90 \pm \theta) = \cos\theta$ ,  
 $\cos(90 \pm \theta) = \mp \sin\theta$

Hence, given expression becomes

$$[\sin\theta \cos\theta + (\cos\theta)(-\sin\theta)]^2 - \sin\theta \cos\theta + (\cos\theta)(+\sin\theta)$$

$$= 0^2 - 0 = 0. \text{ Hence, (4).}$$

4.  $\operatorname{cosec}\theta - \sin\theta = a \Rightarrow \frac{1}{\sin\theta} - \sin\theta = a$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = a \quad \dots (1)$$

$$\sec\theta - \cos\theta = b \Rightarrow \frac{1}{\cos\theta} - \cos\theta = b$$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = b \quad \dots (2)$$

$$\tan\theta + \cot\theta = c \Rightarrow \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = c$$

$$\Rightarrow \frac{1}{\sin\theta \cos\theta} = c \quad \dots (3)$$

$$(1), (2) \& (3) \Rightarrow abc = 1 \quad (1) \text{ is true.}$$

$$(1) \& (2) \Rightarrow ab = \sin\theta \cos\theta \quad (3) \text{ is true.}$$

$$(1) \& (2) \Rightarrow a^2 + ab^2 = \cos^3\theta + \sin^3\theta \neq 1$$

(2) is not true.

We have already proved  $abc = 1$ .

$$\text{LHS} \Rightarrow \frac{1 + \sin\theta + \cos\theta}{ab} - (\sin\theta + \cos\theta)$$

$$\Rightarrow \frac{abc + \sin\theta + \cos\theta}{ab} - \sin\theta - \cos\theta$$

$$\Rightarrow \frac{abc}{ab} + \frac{\sin\theta}{ab} + \frac{\cos\theta}{ab} - \sin\theta - \cos\theta$$

we have already proved  $ab = \sin\theta \cos\theta$

$$\Rightarrow c + \frac{\sin\theta}{\sin\theta \cos\theta} + \frac{\cos\theta}{\sin\theta \cos\theta} - \sin\theta - \cos\theta$$

$$\Rightarrow c + \frac{1}{\cos\theta} + \frac{1}{\sin\theta} - \sin\theta - \cos\theta$$

$$\Rightarrow c + \sec\theta + \operatorname{cosec}\theta - \sin\theta - \cos\theta$$

$$\Rightarrow c + (\sec\theta - \cos\theta) + (\operatorname{cosec}\theta - \sin\theta)$$

$$\Rightarrow c + a + b = \text{RHS}$$

Here statement (4) is also true.

Hence, (2).

5.  $\operatorname{cosec}\theta - \sin\theta = p$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = p$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} = p \quad \dots (1)$$

$$\sec\theta - \cos\theta = q$$

$$\Rightarrow \frac{1}{\cos\theta} - \cos\theta = q$$

$$\Rightarrow \frac{\sin^2\theta}{\cos\theta} = q \quad \dots (2)$$

$\therefore pq = \sin\theta\cos\theta$ . Hence (2) is not true.

$$\frac{p}{q} = \frac{\cos^2\theta}{\sin\theta} \times \frac{\cos\theta}{\sin^2\theta} = \frac{\cos^3\theta}{\sin^3\theta}$$

Hence (4) is not true

$$p^2q^2 + pq^2$$

$$\Rightarrow pq^2[p + 1]$$

$$\Rightarrow \frac{\cos^2\theta}{\sin\theta} \frac{\sin^4\theta}{\cos^2\theta} \left[ \frac{\cos^2\theta}{\sin\theta} + 1 \right]$$

$$\Rightarrow \sin^3\theta \left[ \frac{\cos^2\theta}{\sin\theta} + 1 \right]$$

$$\Rightarrow \sin^2\theta\cos^2\theta + \sin^3\theta$$

Hence (1) is not true

$$(p^2q)^{2/3} + (pq^2)^{2/3} = (\cos^3\theta)^{2/3} + (\sin^3\theta)^{2/3} \\ = \cos^2\theta + \sin^2\theta = 1$$

Hence, (3).

6. We have,

$$\operatorname{cosec}^2x + \sec^2x = \frac{1}{\sin^2x} + \frac{1}{\cos^2x} \\ = \frac{\sin^2x + \cos^2x}{\sin^2x\cos^2x} = \frac{1}{\sin^2x\cos^2x}$$

Consider  $\frac{1}{\sin^2x}$  and  $\frac{1}{\cos^2x}$ .

$$AM = \frac{\frac{1}{\sin^2x} + \frac{1}{\cos^2x}}{2} = \frac{\sin^2x + \cos^2x}{2\sin^2x\cos^2x} = \frac{1}{2\sin^2x\cos^2x}$$

$$GM = \sqrt{\frac{1}{\sin^2x} \times \frac{1}{\cos^2x}} = \frac{1}{\sin x \cos x}$$

Since  $AM \geq GM$

$$\frac{1}{\sin^2x\cos^2x} > \frac{1}{\sin x \cos x}$$

$$\therefore \frac{1}{\sin x \cos x} \geq 2$$

$$\therefore \frac{1}{\sin^2x\cos^2x} \geq 4$$

Therefore the minimum value of  $\frac{1}{\sin^2x\cos^2x}$  is 4 or the minimum value of  $\operatorname{cosec}^2x + \sec^2x$  is 4.

In the options, first option is  $2\sqrt{3}$ , which is less than

4. All other options are greater than 4.

Hence (1).

$$7. \quad 4\sin A + 3\cos A = 5 \left( \frac{4}{5} \sin A + \frac{3}{5} \cos A \right)$$

$$\text{If } \frac{4}{5} = \cos B, \text{ then } \frac{3}{5} = \sin B$$

$$\therefore 4 \sin A + 3 \cos A = 5 (\cos B \sin A + \sin B \cos A) = 5 \sin (A + B)$$

The maximum value that  $\sin (A + B)$  can take is 1.

Therefore, the required answer is 5.

8. Value of  $\sin(30^\circ) = 1/2$ , Hence, we need to find the maximum value of  $(\cos x^\circ - \sin x^\circ)$  to solve the question

As value of  $\cos x^\circ$  decreases as angle moves from 0 to 90 and value of  $\sin x^\circ$  increases, highest possible value of  $\cos x^\circ - \sin x^\circ$  will be at  $5^\circ$  as it is closer to zero.

The minimum value will be at  $15^\circ$ , in the given range of:  $5^\circ < x^\circ < 15^\circ$ .

$\cos 15^\circ - \sin 15^\circ$  can be written as:  $\cos(45^\circ - 30^\circ) - \sin(45^\circ - 30^\circ)$

Applying the formula for  $\cos(A-B)$  and  $\sin(A-B)$

$$(\cos 45^\circ \times \cos 30^\circ + \sin 45^\circ \sin 30^\circ) - (\sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ)$$

$$\rightarrow \left( \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} - \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2} \right)$$

$$\rightarrow \left( \frac{1}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} \right)$$

$$\rightarrow \left( \frac{2}{2\sqrt{2}} \right)$$

$$\rightarrow \frac{1}{\sqrt{2}}$$

$$\rightarrow \frac{1}{1.42}$$

$$\rightarrow .71$$

This means that 0.71 is the minimum value of the expression  $(\cos x^\circ - \sin x^\circ)$  in the range of  $x$  given.

Hence, at  $15^\circ$ , the value of  $(\sin 30^\circ + \cos x^\circ - \sin x^\circ)$  would be  $(0.5 + 7.1)$ , which is greater than 1.

So, Option (5) is the correct answer.

Hence, (5).

$$9. \quad \frac{12}{\sin 30} = \frac{12}{\frac{1}{2}} = 24$$

Then by the sine rule,

$$\frac{10}{\sin A} = \frac{12}{\sin C} = 24 \text{ i.e.}$$

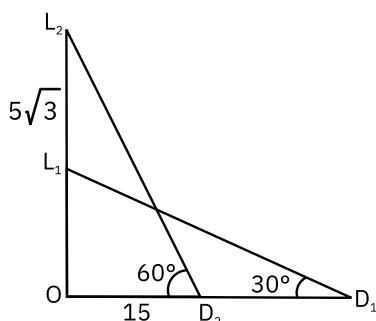
$$\sin A = \frac{10}{24} = \frac{5}{12}$$

$$\text{Also } \frac{16}{\sin B} = 24 \therefore \sin B = \frac{8}{12}$$

$$\therefore \sin B - \sin A = \frac{8}{12} - \frac{5}{12} = 0.25.$$

Hence, (4).

10.



Let  $OL_2$  be the wall.

$D_1$  and  $D_2$  be the positions of the dog in the two instances and  $L_1$  and  $L_2$  be the two positions of the lizard.

Required  $D_1D_2$

$$\tan 60^\circ = \frac{OL_2}{OD_2} = \frac{OL_2}{15}$$

$$\sqrt{3} = \frac{OL_2}{15} \Rightarrow OL_2 = 15\sqrt{3}$$

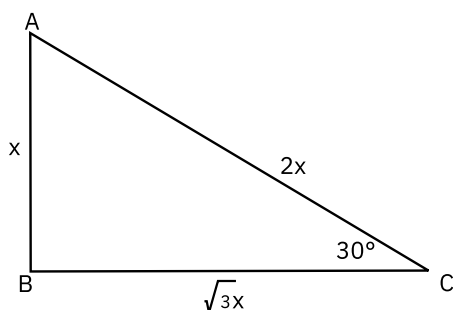
$$OL_1 = OL_2 - L_1L_2 = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3}$$

$$\tan 30^\circ = \frac{OL_1}{OD_1} \therefore \frac{1}{\sqrt{3}} = \frac{10\sqrt{3}}{OD_1} \Rightarrow OD_1 = 30$$

$$D_1D_2 = OD_1 - OD_2 = 30 - 15 = 15 \text{ m.}$$

Hence, (2).

11.



Let  $AB$  be the building and  $B$  and  $C$  be the initial positions of the pigeon and crow respectively.

Let the time taken by the pigeon to fly ' $x$ ' distance be ' $t$ ' seconds. Therefore, the time taken by the crow to fly ' $2x$ ' distance is ' $t + 20$ ' seconds. Since their speeds

the same,  $\frac{x}{t} = \frac{2x}{t + 20}$ . Therefore,  $t = 20$ . When they

fly along  $BC$  towards each other, since their speeds are the same, they will meet at the midpoint of  $BC$ . Therefore, distance travelled by the pigeon will be

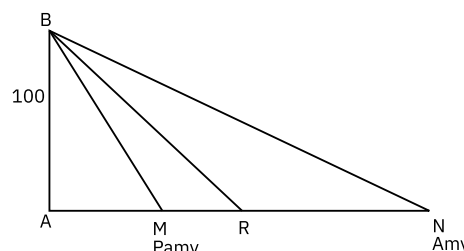
$\frac{\sqrt{3}}{2}x$ . If the pigeon takes 20 seconds to travel ' $x$ '

distance, then time taken to travel  $\frac{\sqrt{3}}{2}x$  will be

$$\frac{\frac{\sqrt{3}}{2}x}{x} \times 20 = 10\sqrt{3} \text{ seconds.}$$

Hence (4).

12. As shown in the figure below, Amy is standing at point  $N$  and Pamy is standing at point  $M$ .



Angle of elevation of point  $B$  from point  $N = 30^\circ$ .

Therefore,  $\ell(AN) = 100\sqrt{3}$  m.

Angle of elevation of point  $B$  from point  $M = 60^\circ$ .

Therefore,  $\ell(AM) = \frac{100}{\sqrt{3}}$  m.

Angle of elevation of point  $B$  from point  $R = 45^\circ$ . Therefore,  $\ell(AR) = 100$  m.

The distance run by Amy =  $100\sqrt{3} - 100$  and the distance run by Pamy =  $100 - \frac{100}{\sqrt{3}} = \frac{100\sqrt{3} - 100}{\sqrt{3}}$

Therefore, the ratio of Amy's speed to Pamy's speed

$$= \sqrt{3} : 1$$

The distance between the foot of the tower and

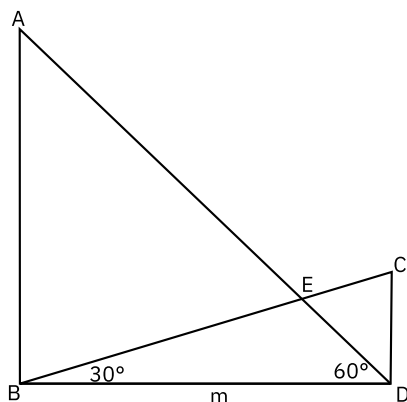
Pamy's position =  $\frac{100}{\sqrt{3}}$  m.

Therefore, the distance run by Amy when Pamy runs

$\frac{100}{\sqrt{3}}$  is 100 m.

Therefore, Amy will be at a distance  $100\sqrt{3} - 100 = 100(\sqrt{3} - 1)$  m. Hence (4).

13. The situation can be represented diagrammatically as follows:



Since they meet along their flights, they have to meet at E (at the same instant).

Let  $BD = m$ . Then, using trigonometric ratios, we get the following:

$$AB = \sqrt{3} \text{ m}; AD = 2m; CD = \frac{1}{\sqrt{3}} \text{ m}; BC = \frac{2}{\sqrt{3}} \text{ m}$$

$$\text{Also, } \angle BED = 180 - 30 - 60 = 90^\circ$$

$\therefore \triangle CED$  and  $\triangle AEB$  are  $30^\circ$ - $60^\circ$ - $90^\circ$  triangles

$$\therefore DE = \frac{1}{2} \text{ m and } BE = \frac{\sqrt{3}}{2} \text{ m}$$

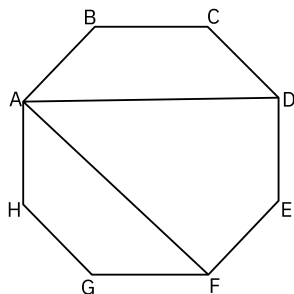
$\therefore$  The distances flown by the eagle and the kite are AE and EC respectively, that is,  $(AD - DE)$  and  $(BC - BE)$

$$\text{respectively, that is, } 2m - \frac{1}{2} \text{ m and } \frac{2}{\sqrt{3}} \text{ m} - \frac{\sqrt{3}}{2} \text{ m}$$

$$\text{respectively, that is, } \frac{3}{2} \text{ m and } \frac{1}{2\sqrt{3}} \text{ m respectively.}$$

Since the time taken for both the birds to fly their respective distances is the same, the required ratio of their speeds =  $3\sqrt{3} : 1$ . Hence (4).

- 14.



Since it is a regular octagon, therefore  $AD = AF$ . Let each be equal to  $x$ .

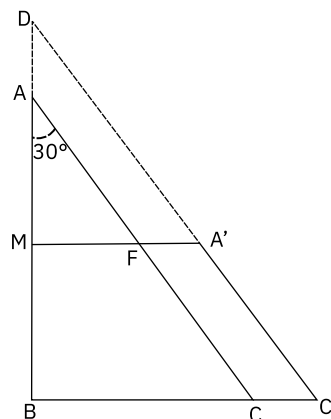
$$\therefore \text{The height of the tower at D} = h_1 = x \times \tan 30^\circ =$$

$$\frac{x}{\sqrt{3}}$$

Also, the height of the tower at F =  $h_2 = x \times \tan 45^\circ = x$

$$\therefore \text{Required ratio} = \frac{1}{\sqrt{3}}. \text{ Hence, (3).}$$

15. We can represent the given situation using the following diagram:



AB is the pole where M is its midpoint. BC is the shadow before bending and  $BC'$  is the shadow after bending. Thus,  $AM = MB = A'M = 10$ .

$$\text{Using the rules of trigonometry in } \triangle ABC, BC = AB \tan 30^\circ = \frac{20}{\sqrt{3}}$$

$$\text{Similarly, in } \triangle DMA', DM = \frac{MA'}{\tan 30^\circ} = 10\sqrt{3}.$$

$$\text{In } \triangle DBC', BC' = DB \tan 30^\circ = \frac{10 + 10\sqrt{3}}{\sqrt{3}}$$

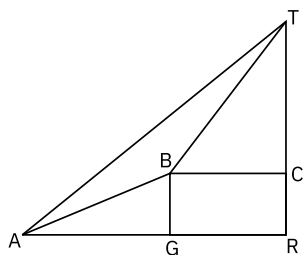
$$\therefore \text{Required answer} = CC' = \frac{10 + 10\sqrt{3}}{\sqrt{3}} - \frac{20}{\sqrt{3}} =$$

$$\frac{10\sqrt{3} - 10}{\sqrt{3}} = 10 \left( \frac{\sqrt{3} - 1}{\sqrt{3}} \right)$$

Hence (1).



16.



Let BG be the building of height  $h_1$  and TR be the tower of height  $H$ . Let TC be  $h_2$ .

$$\therefore h_1 + h_2 = H$$

$$\text{Now, } AG = \frac{BG}{\tan 30^\circ} = \sqrt{3} h_1$$

$$\text{Also, } GR = BC = \frac{TC}{\tan 60^\circ} = \frac{h_2}{\sqrt{3}}$$

$$\text{Since } \angle TAR = 45^\circ, TR = AR = H$$

$$\therefore h_1 + h_2 = \sqrt{3} h_1 + \frac{h_2}{\sqrt{3}}$$

$$\therefore (\sqrt{3} - 1) h_1 = \left(1 - \frac{1}{\sqrt{3}}\right) h_2$$

$$\therefore \sqrt{3} h_1 = h_2$$

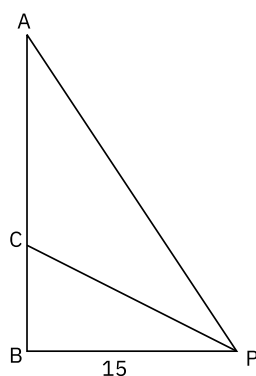
$$\therefore \text{Required ratio} = AG : GR = \sqrt{3} h_1 : \frac{h_2}{\sqrt{3}} = \sqrt{3}$$

$$h_1 : h_2 = \sqrt{3} : 1$$

Hence (1).

17. Let  $\angle BPC = \angle CPA = \alpha$  and  $\angle BPA = \beta$

We can draw a diagram as follows:



$$\tan \alpha = \frac{CB}{15} = \frac{1}{3}$$

$$\therefore CB = 5$$

$$\therefore CP = \sqrt{5^2 + 15^2} = 5\sqrt{10}$$

$$\therefore \sin \alpha = \frac{1}{\sqrt{10}} \text{ \& } \cos \alpha = \frac{3}{\sqrt{10}}$$

$$\therefore \sin \beta = \sin 2\alpha = 2 \cdot \sin \alpha \cdot \cos \alpha = 2 \times \frac{1}{\sqrt{10}} \times \frac{3}{\sqrt{10}} = \frac{3}{5}$$

$$\therefore \cos \beta = \sqrt{1 - \left(\frac{3}{5}\right)^2} = \frac{4}{5}$$

$$\therefore \frac{15}{AP} = \frac{4}{5}$$

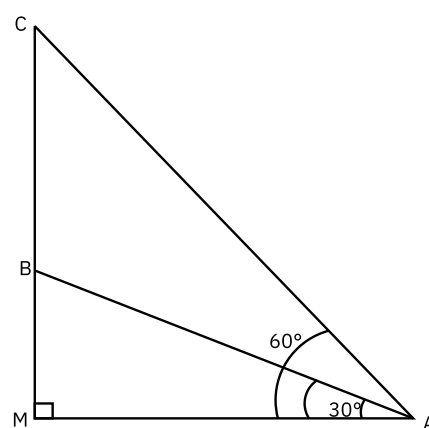
$$\therefore AP = \frac{75}{4}$$

$$\therefore \frac{AB}{\frac{75}{4}} = \frac{3}{5}$$

$$\therefore AB = 11.25$$

Hence (1).

18. The situation can be represented as follows:



Let  $BM = x$ .

$$\therefore AM = \frac{x}{\tan 30^\circ} = \sqrt{3} x$$

$$\therefore CM = AM \times \tan 60^\circ = 3x$$

$$\therefore BC = 3x - x = 2x$$

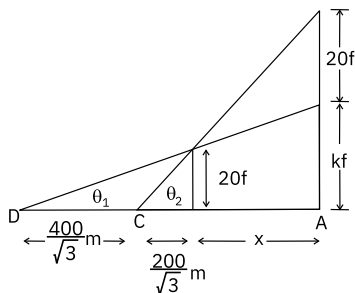
$\therefore$  The distances travelled by the insect and the lizard are  $2x$  and  $3x + \sqrt{3} x$ .

Since the time taken to travel the individual distances is the same in each case, required ratio of speeds =

$$\frac{2}{3 + \sqrt{3}}$$

Hence (4).

19. Let  $f$  be the height of each floor. Let  $(k + 20)$  be the number of floors of building A.



$$\ell(\text{CB}) = \ell(\text{DB}) - \frac{400\sqrt{3}}{3} = \frac{200}{\sqrt{3}}$$

$$\Rightarrow \frac{1}{10\sqrt{3}} = \frac{k}{x + 200\sqrt{3}} \quad \dots (i)$$

$$\begin{aligned}\tan\theta_2 &= \frac{20f}{\frac{200}{\sqrt{3}}} = \frac{20f + kf}{x + \frac{200}{\sqrt{3}}} \\ \Rightarrow \frac{\sqrt{3}}{10} &= \frac{20 + k}{x + \frac{200}{\sqrt{3}}} \quad \dots (ii)\end{aligned}$$

From (i),  $x + 200\sqrt{3} = 10\sqrt{3}k$  ... (iii)

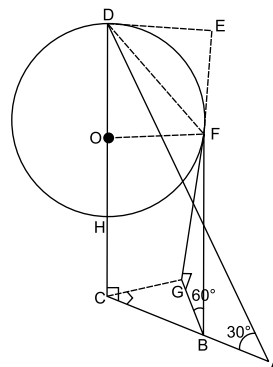
From (ii),  $\sqrt{3} x = 10k$  ... (iv)

Substituting in (iii), we get,

$$x + 200\sqrt{3} = 3x \Rightarrow x = 100\sqrt{3} \text{ m}$$

Hence, (1).

- 20.



Let A be the initial position of the person.

AC is the horizontal distance of the person from the bottom of the tower.

Let  $D$  be the initial position of the minute hand at 5:00 pm and  $F$  be the position of the minute hand at 5:10 pm.

$\triangle ACD$  is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle

Given  $AC = 1800$  m

$$\therefore DC = \frac{1800}{\sqrt{3}}$$
$$= 600\sqrt{3} \text{ m}$$

$$DO = \text{length of minute hand} = 200\sqrt{3} \text{ m}$$

In  $\Delta DOF$ ,  $\angle DOF = 60^\circ$  ( $\because$  it covers 10 minutes or  $\frac{10}{60}$  of the circumference of the clock)

Also  $\therefore DO = OF$  (both are radii or length of the minute hand)

$$\angle ODF = \angle OFD = \frac{180 - 60}{2} = 60^\circ$$

$\therefore \Delta DOF$  is an equilateral triangle

$$\text{So DF} = 200\sqrt{3} \text{ m}$$

Now in  $\triangle DEF \because EF \parallel DO, \angle DEF = 90^\circ$

Also  $DE \perp OD$  ( $\because$  Tangent perpendicularity Theorem)

So  $\angle FDE = 30^\circ$  ( $\because \angle ODF + \angle FDE = 90^\circ$ )

$$EF = 100\sqrt{3} \text{ m and } DE = 300 \text{ m}$$

$$\therefore EG = DC \Rightarrow FG = 600\sqrt{3} - 100\sqrt{3} = 500\sqrt{3}$$

sqm

Now FGB is a  $30^\circ$ - $60^\circ$ - $90^\circ$  triangle

$$\therefore GB = 500 \text{ m}$$

Also in  $\triangle CGB$ ,  $CG = DE = 300$  m

$$CB = \sqrt{500^2 - 300^2} = \sqrt{1600} = 400 \text{ m}$$

Distance travelled by person =  $AB = AC - BC = 1400 \text{ m}$

$$\text{Speed of person} = \frac{1400}{600} \times \frac{18}{5} \Rightarrow \frac{42}{5} = 8.4 \text{ km/hr}$$

Hence, (4).

# **Data Interpretation & Logical Reasoning**



## DI-3.1 | DATA INTERPRETATION MISCELLANEOUS SETS

### PRACTICE EXERCISE-1

**Answers to questions 1 to 4:**

From statements (3) and (4), we get

Name	Speed	Stamina	Strength	Total
Anil	3			
Bhaskar				
Chandrakant	1			
Dattu			3	
Eknath		1		

From statement (6), sum of Chandrakant's and Eknath's scores in Strength is 6. There are two possible ways in which this can happen—(1+5) or (2+4). However, Chandrakant and Eknath both have scored 1 point in one other criterion. Therefore only possible way to have sum of Chandrakant's and Eknath's scores equal to 6 is (2+4).

Dattu's scores in Speed, Stamina and Strength are in AP. He has scored 3 in Strength. Therefore his scores in Speed, Stamina and Strength can be (1,2,3) or (5,4,3). Dattu can't score 1 in Speed as Chandrakant has already scored 1 in Speed. Therefore Dattu scored 5, 4 and 3 in Speed, Stamina and Strength respectively.

Anil's scores in Speed, Stamina and Strength are in AP. He has scored 3 in Speed. Therefore his scores in Speed, Stamina and Strength can be (3,4,5) or (3,2,1). Anil can't score 4 in Stamina as Dattu has scored 4 in Stamina. Therefore Anil scored 3, 2 and 1 in Speed, Stamina and Strength respectively.

Therefore we have,

Name	Speed	Stamina	Strength	Total
Anil	3	2	1	
Bhaskar				
Chandrakant	1		2/4	
Dattu	5	4	3	
Eknath		1	4/2	

Rest of the entries in the table can be filled as follows:

Name	Speed	Stamina	Strength	Total
Anil	3	2	1	6
Bhaskar	2/4	3	5	10/12
Chandrakant	1		2/4	
Dattu	5	4	3	
Eknath		1	4/2	

This means that Dattu was definitely selected. If both Bhaskar and Chandrakant scored 10, rigorous stress test would have become necessary. But from statement (7), we can see that the stress test was not required. Therefore Bhaskar scored 12 and Chandrakant scored 8.

Therefore the final table is as follows:

Name	Speed	Stamina	Strength	Total
Anil	3	2	1	6
Bhaskar	4	3	5	12
Chandrakant	1	5	2	8
Dattu	5	4	3	12
Eknath	2	1	4	7

Now all the questions can be answered.

1. Therefore 6.
2. Therefore 2.
3. Therefore 1.
4. Scores of all the five athletes in all the three criteria can be uniquely determined. Therefore required answer is 5.

#### Answers to questions 5 to 8:

From Graph 1, we can see that Anup scored 87 in S3. Therefore from table 1, we can see that Anup is A3 and Physics is S3.

From Graph 1, we can see that Bhushan scored 94 in S1. Therefore from table 1, we can see that Bhushan is A5 and Mathematics is S1.

From Graph 1, we can see that Chitra scored 98 in S2. Therefore from table 1, we can see that Chitra is A1 and Biology is S2.

From Graph 1, we can see that Dennis scored 88 in S3. Therefore from table 1, we can see that Dennis is A4 and Physics is S3.

Therefore A2 is Ekta and S4 is Chemistry.

Therefore so far we have,

	Anup	Bhushan	Chitra	Dennis	Ekta	Total
Physics	87	90		88	78	434
Chemistry					82	434
Mathematics	86	94	93			447
Biology			98	99		429
Total	327		378			

$$\therefore \text{Chitra's score in Physics} = 434 - (87 + 90 + 88 + 78) = 91$$

$$\therefore \text{Chitra's score in Chemistry} = 378 - (93 + 91 + 98) = 96$$

Using this and the information given in the question, we get the following

	Anup	Bhushan	Chitra	Dennis	Ekta	Total
Physics	87	90	91	88	78	434
Chemistry	d	a+17	96	b	82	434
Mathematics	86	94	93	b+2	b+6	447
Biology	c	a	98	99	c+11	429
Total	327		378			

Total in Mathematics is 447.

$$\therefore 86 + 94 + 93 + (b + 2) + (b + 6) = 447$$

$$\therefore 281 + 2b = 447$$

$$\therefore 2b = 166$$

$$\therefore b = 83$$

Total in Chemistry is 434

$$\therefore d + (a + 17) + 96 + b + 82 = 434$$

$$\therefore a + b + d + 195 = 434$$

$$\therefore a + 83 + d = 239$$

$$\therefore a + d = 156 \quad \text{..... (I)}$$

Anup's total score is 327.

$$\therefore 86 + 87 + d + c = 327$$

$$\therefore c + d = 154 \quad \text{..... (II)}$$

Total in Biology is 429

$$\therefore a + c + 98 + 99 + (c + 9) = 429$$

$$\therefore a + 2c + 208 = 429$$

$$\therefore a + 2c = 221 \quad \text{..... (III)}$$

From equations (I) and (II)

$$a - c = 2 \quad \text{..... (IV)}$$

Solving equations (III) and (IV) simultaneously, we get  $c = 73$ ,  $a = 75$ ,  $d = 81$ .

Therefore we get

	Anup	Bhushan	Chitra	Dennis	Ekta	Total
Physics	87	90	91	88	78	434
Chemistry	81	92	96	83	82	434
Mathematics	86	94	93	85	89	447
Biology	73	75	98	99	84	429
Total	327	351	378	355	333	

Now all the questions can be answered.

5. Hence, (2).

6. Hence (1).

7. Hence (4).

8.  $\frac{333}{400} \times 100 = 83.25\%$ . Hence, (1).

**Answers to questions 9 to 12:**

We can make the table shown below for the various customers:

C1	Cust	A (08:53)	F (09:19)	I (09:30)	L (09:51)	
	In	9:00	9:19	9:36	9:51	
	Out	9:15	9:36	9:47	10:03	
C2	Cust	B (08:55)	D (09:09)	H (09:29)	K (09:45)	M (09:59)
	In	9:00	9:12	9:29	9:45	9:59
	Out	9:12	9:29	9:45	9:56	10:10
C3	Cust	C (08:59)	E (09:13)	G (09:23)	J (09:37)	
	In	9:00	9:13	9:25	9:37	
	Out	9:13	9:25	9:36	9:54	

9. From the above table we can see that the only customers who had to wait were A (7 min), B (5 min), C (1 min), D (3 min), G (2 min), and I (6 min). Hence the total waiting time was 24 minutes.
10. From the table we can see that the slack for C1 was 4 (between A and F) + 4 (between I and L) + 27 (after L till 10:30) = 35 minutes. Similarly that for C2 is 3 (between K and M) + 20 (after M till 10:30) = 23 minutes, and that for C3 is 1 (between G and J) + 36 (after J till 10:30) = 37 minutes. In total, therefore, the slack time for all the consultants will be 35 + 23 + 37 = 95 minutes.  
 Alternatively, adding up the total consulting time of all the customers we find it comes to 175 minutes. The total available consulting time is  $90 \times 3 = 270$  minutes. So the slack time will be  $270 - 175 = 95$  minutes.
11. From the table we can see that C1 advised 4 customers (A, F, I and L)
12. The total consulting time of all the customers is 175 minutes (as seen earlier). The total consulting time of two consultants is 180 minutes. So the slack time will be 5 minutes.



**Answers to questions 13 to 16:**

We can build the table as follows:

Year	Number of employees on 1 <sup>st</sup> January	Number of new employees joining on 30 <sup>th</sup> June	Number of employees on 1 <sup>st</sup> July	Number of employees leaving on 31 <sup>st</sup> December	Number of employees at the end of 31 <sup>st</sup> December
2009	1000	50% of 1000 = 500	1000 + 500 = 1500	20% of 1500 = 300	1500 – 300 = 1200
2010	1200	40% of 1200 = 480	1200 + 480 = 1680	25% of 1680 = 420	1680 – 420 = 1260
2011	1260	50% of 1260 = 630	1260 + 630 = 1890	20% of 1890 = 378	1890 – 378 = 1512
2012	1512	25% of 1512 = 378	1512 + 378 = 1890	$\frac{1}{9} \times 1890 = 210$	1890 – 210 = 1680
2013	1680	40% of 1680 = 672	1680 + 672 = 2352	25% of 2352 = 588	2352 – 588 = 1764
2014	1764	25% of 1764 = 441	1764 + 441 = 2205	60% of 2205 = 1323	2205 – 1323 = 882
2015	882	$\frac{4}{9} \times 882 = 392$	882 + 392 = 1274	$\frac{1}{14} \times 1274 = 91$	1274 – 91 = 1183

Now all the questions can be answered:

13. The number of employees joining on 30th June of 2011 and 2013 was more than 500. Therefore, the required answer is 2. Hence, (3).
14. The number of employees on 1st July of 2011 and 2012 was equal (1890). Hence, (2).
15. The number of employees was highest on 1st January 2014 (1764). Hence, (3).
16. The required ratio is  $\frac{1680}{2205} = \frac{16}{21}$  : Hence, (4).

## PRACTICE EXERCISE-2

1. Team 1 utilized all its resources as it completed the untruncated inning. So  $R_1 = 100$ . Team 2 has only 30 overs to play. Before it starts batting, it has lost 0 wickets. Using the table, the resources available to Team 2 at its disposal = 75.1

$$\therefore R_1 > R_2$$

$$\therefore T = \left( S \times \frac{R_2}{R_1} \right) + 1 = 240 \times \frac{75.1}{100} + 1 = 181.24 \text{ or } 181.$$

Hence (4).

2. Team 1 had 10 overs left with 5 wickets remaining when rain interrupted the play. Thus, resources unutilised = 26.1. Hence, resources used by Team 1,  $R_1 = 100 - 26.1 = 73.9$ .

Team 2 has only 25 overs left with 0 wickets lost, thus resources available to Team 2,  $R_2 = 66.5$ .

$$\therefore R_1 > R_2$$

$$\text{Therefore, } T = S \times \frac{R_2}{R_1} + 1 = 220 \times \left( \frac{66.5}{73.9} \right) + 1 = 198.97 \text{ or } 198. \text{ Hence, (1).}$$

3. Team 1 had 5 overs left with 8 wickets lost, thus resources unutilised = 16.8

Hence, resources used by Team 1,  $R_1 = 83.2$

Team 2 has 40 overs left with 0 wickets lost, thus the resources to be utilised,  $R_2 = 89.3$

$$\therefore R_2 > R_1$$

$$G_{50} = 280 \times \left( \frac{100}{83.2} \right) = 336.54 \text{ or } 336$$

$$T = S + (R_2 - R_1) \times \frac{G_{50}}{100} + 1$$

$$\therefore T = 280 + (89.3 - 83.2) \times \frac{336}{100} + 1 = 301.496 \text{ or } 301.$$

Hence, (4).

4. Australia batted for full 50 overs. Therefore  $R_1 = 100$ .

Out of 50 overs in South African inning, 45 are completed. So, South Africa has 5 more overs left with the loss of 4 wickets. Therefore, using the table, resources unutilized by South Africa = 16.1. Therefore,  $R_2 = 100 - 16.1 = 83.9$

$$\therefore R_1 > R_2$$

Therefore, the target score for South Africa :

$$= \left( S \times \frac{R_2}{R_1} \right) + 1 = \left( 356 \times \frac{83.9}{100} \right) + 1 = 299.684 \text{ or } 299.$$

Since South Africa had scored only 292 runs, Australia won the match.

Hence (1).

**Answers to questions 5 to 8:**

Each worker works for 8 hours everyday. If the number of workers changes by 1 with respect to the previous day, we have the following possibilities:

Possibility	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total worker-days
1	4	5	4	5	6	5	29
2	4	5	4	5	4	5	27
3	4	5	6	5	6	5	31
4	4	5	6	5	4	5	29
5	4	3	4	5	4	5	25
6	4	3	4	5	6	5	27
7	4	5	6	7	6	5	33
8	4	5	4	3	4	5	25
9	4	3	2	3	4	5	21
10	4	3	4	3	4	5	23

5. It can be seen that the minimum number of worker-days required to finish the order is 21. Therefore, the minimum number of workers is 21 if the order is to be completed in one day.

Therefore, the required answer is 21.

6. Each worker gets paid Rs. 100 per hour or Rs. 800 per day. Therefore, total number of worker days =  $\frac{21600}{800} = 27$ . Therefore, we have the following possibilities:

Possibility	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total worker-days
2	4	5	4	5	4	5	27
6	4	3	4	5	6	5	27

In either case, the number of workers working on the 3rd day is 4.

Therefore, the required answer is 4.

7. If only 4 workers worked on this order on day 5, we have the following possibilities:

Possibility	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total worker-days
2	4	5	4	5	4	5	27
4	4	5	6	5	4	5	29
5	4	3	4	5	4	5	25
8	4	5	4	3	4	5	25
9	4	3	2	3	4	5	21
10	4	3	4	3	4	5	23

It can be seen that the maximum number of worker days is 29.

Therefore, the maximum amount that the company can spend for wages =  $29 \times 8 \times 100 = 23200$ .

Therefore, the required answer is 23200.

8. If only 2 workers work on this order on some day, we have the following possibility:

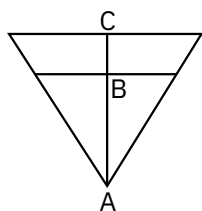
Possibility	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Total worker-days
9	4	3	2	3	4	5	21

That means the total number of worker days required = 21. Therefore, if 7 workers worked on the order everyday, the number of days required =  $\frac{21}{7} = 3$ .

Therefore, the required answer is 3.

### Answers to questions 9 to 11:

Consider each equilateral triangular region.



Since  $AB : AC = 2 : 3$ , the ratio of the area of the smaller equilateral triangle to the area of the larger equilateral triangle will be  $4 : 9$ .

Let the area of each smaller equilateral triangular region be  $8x$ .

$\therefore$  The area of the larger equilateral triangular region will be  $18x$ .

$\therefore$  The area of each trapezoidal region will be  $18 - 8 = 10x$ .

Total area of the farmland will be  $18x \times 6 = 108x$

$\therefore$  Using this and the shading pattern in the figure, we can calculate the total farmland allotted to each son as follows:

Son A:  $4x + 4x + 4x + 5x + 5x + 5x = 27x$

Son B:  $4x + 4x + 4x + 5x + 5x = 22x$

Son C:  $4x + 4x + 5x + 5x + 5x = 23x$

Son D:  $4x + 4x + 4x + 5x = 17x$

Son E:  $4x + 5x + 5x + 5x = 19x$

Natwar received the least total area of the farm, so he received  $17x$ .

The total farmland received by Lallan, Janak and Manoj form an AP in that order. The only AP possible is  $19x-23x-27x$ . Therefore, Janak received  $23x$ . Therefore, Kishan received  $22x$ .

9. Required percentage =  $\frac{23x + 22x}{108x} \times 100 = 41.67\%$ . Hence, (3).

10. Natwar and Lallan together received either  $36x$  ( $17x + 19x$ ) or  $44x$  ( $17x + 27x$ ).

For  $36x$ , required percentage =  $\frac{36x}{108x} \times 100 = 33.33\%$   
Hence, (3).

11. Difference between the total farmland assigned to Janak and Natwar =  $23x - 17x = 6x$   
Difference between the total farmland assigned to Kishan and Natwar =  $22x - 17x = 5x$   
Difference between the total farmland assigned to Kishan and Lallan =  $22x - 19x = 3x$   
Difference between the total farmland assigned to Lallan and Manoj =  $27x - 19x = 8x$   
Hence, (4).

**Answers to questions 12 to 15:**

The total number of participants from these five schools in Physics, Chemistry, Biology, Mathematics, History and Civics competitions are 10, 11, 10, 13, 11 and 13 respectively. Therefore, total 5, 6, 5, 8, 6 and 8 students got -1 points in these competitions.

We can fill the following table as follows: (In the table, the numbers indicate the points scored by different students in different competitions. For example, the three students from School A scored 4, -1 and -1 points in Mathematics competition).

	Physics	Chemistry	Biology	Mathematics	History	Civics
School A	3	1,-1,-1	4,-1	4,-1,-1	5,-1	2,-1,-1
School B	-1, -1	5	3,-1	-1,-1	4,2,-1	-1,-1,-1,-1
School C	5,1,-1,-1	2,3,-1	2,-1	3,-1	-1,-1,-1	1
School D	2	-1,-1	5,1,-1	2,-1,-1,-1	1,-1	4
School E	4,-1	4,-1	-1	5,1	3	5,3,-1,-1

Now all the questions can be answered:

12. The success ratios of schools A, B, C, D and E are  $\frac{6}{14}$ ,  $\frac{4}{14}$ ,  $\frac{7}{15}$ ,  $\frac{6}{13}$  and  $\frac{7}{12}$  respectively.  
Therefore, the success ratio of school E is the highest.  
Hence, (4)
13. It can be seen that the points scored by all the students in all the competitions can be uniquely determined.  
Hence, (1)
14. It can be seen that school E got first rank in Mathematics and Civics competitions.  
Hence, (3).
15. It can be seen that total 8 students did not secure any rank in the different competitions.  
Hence, (2)

### PRACTICE EXERCISE-3

#### Answers to questions 1 to 4:

The number of people who visited the library in both the slots = 45.

Since the number of people who visited in the morning slot was one less than the number of people who visited in the evening slot, the number of people who visited only in the morning slot = 67 and the number of people who visited only in the evening slot = 68.

Using the information given in the question, we have the following:

Morning 67	Both 45	Evening 68
Only science: 5a	Only science: 15 – a	Only science: d
Both science & literature: b	Both science & literature: a	Both science & literature: 3b
Only literature: c	Only literature: 30	Only literature: c

Now,  $c = 18b$ . Therefore,  $5a + b + c = 67$  or  $5a + 19b = 67$ . The only natural number solution to this equation is  $b = 3$  and  $a = 2$ .

Also,  $21b = 30 + d$ . Therefore,  $d = 33$ .

Now we have the following:

Morning 67	Both 45	Evening 68
Only science: 10	Only science: 13	Only science: 33
Both science & literature: 3	Both science & literature: 2	Both science & literature: 9
Only literature: 54	Only literature: 30	Only literature: 26

Now all the questions can be answered.

1. Hence, (1).
2. Hence, (3).
3. Hence, (2).
4. Hence, (1).
5. Hence, (4).

**Answers to questions 6 to 9:**

The given information can be expressed as follows:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Total Students	283	284	286	284	282	284		283	276	273	278	279
New Students					95				91			

Note that the total number of students in any month is the sum of the new students for that month and the previous two months. So, for example, since the total in Mar is 286, we can conclude that the total number of students joining in Jan, Feb and Mar must have been 286.

Now, we can see from the table that  $\text{Mar} + \text{Apr} + \text{May} = 282$ . Also  $\text{Feb} + \text{Mar} + \text{Apr} = 284$ . So  $\text{Feb} = \text{May} + 2$ . But  $\text{May} = 95$ , so  $\text{Feb} = 97$ . In a similar manner we can draw the following conclusions:

$\text{Jul} + \text{Aug} + \text{Sep} = 276$ , while  $\text{Jun} + \text{Jul} + \text{Aug} = 283$ . So  $\text{Jun} = \text{Sep} + 7 = 98$ .

$\text{Mar} + \text{Apr} + \text{May} = 282$ , while  $\text{Apr} + \text{May} + \text{Jun} = 284$ . So  $\text{Mar} = \text{Jun} - 2 = 96$

$\text{Sep} + \text{Oct} + \text{Nov} = 278$ , while  $\text{Oct} + \text{Nov} + \text{Dec} = 279$ . So  $\text{Dec} = \text{Sep} + 1 = 92$

Now we know that  $\text{Feb} = 97$ ,  $\text{Mar} = 96$ ,  $\text{Jan} + \text{Feb} + \text{Mar} = 286$  and  $\text{Feb} + \text{Mar} + \text{Apr} = 284$ ; from this we can say that  $\text{Jan} = 93$  and  $\text{Apr} = 91$ .

At this point the table looks like this:

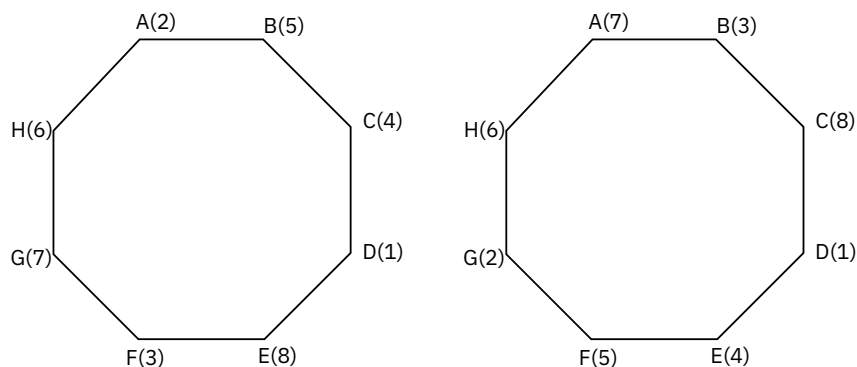
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Total Students	283	284	286	284	282	284		283	276	273	278	279
New Students	93	97	96	91	95	98			91			92

Now all the questions can be answered.

6. The number of new students who joined Speed Infotech in February, April, June and December are 97, 91, 98 and 92 respectively. Hence (3)
7. The total number of students joining in Jan + Feb + Mar = 286, in Apr + May + Jun 284, in Jul + Aug + Sep = 276 and in Oct + Nov + Dec is 279. So the total joining over the 12 months is  $286 + 284 + 276 + 279 = 1125$  and the average is  $1125/12 = 93.75$ . Hence (2)
8. If 97 students join in November, then as above we can fill up the remaining values:  
 $\text{Oct} + \text{Nov} + \text{Dec} = 279$ . So  $\text{Oct} = 90$   
 $\text{Aug} + \text{Sep} + \text{Oct} = 273$ . So  $\text{Aug} = 92$   
 $\text{Jul} + \text{Aug} + \text{Sep} = 276$ . So  $\text{Jul} = 93$   
Hence the students studying during July = May + Jun + Jul = 286. Hence (1)
9. We have,  
 $\text{Dec}_{14} + \text{Jan}_{15} + \text{Feb}_{15} = 284$   
Since  $\text{Feb}_{15} = 97$  and  $\text{Jan}_{15} = 93$ ,  $\text{Dec}_{14} = 284 - 97 - 93 = 94$   
Hence (2).

#### Answers to questions 10 to 13:

Using the information given, we can generate the following two possibilities:



Now all the questions can be answered.

10. It can be seen that attractions 6 and 8 are along one middle diagonal. Therefore, the distance between them is 24.14 units. Hence, (3).
11. Hence, (3).



12. It can be seen that attractions 4 and 7 are along one longest diagonal. One can travel from 4 to 7 through one intermediate attraction in the following ways:
- i. First go to neighbouring attraction (either 5 or 1) and then go along a middle diagonal to reach 7. Total distance =  $10 + 24.14 = 34.14$  units
  - ii. First go to an attraction via one of the shortest diagonals (either to 8 or 2) and then travel to 7 via another shortest diagonal. Total distance =  $18.48 + 18.48 = 36.96$  units
  - iii. First go to an attraction via one of the middle diagonals (either to 6 or 3) and then travel to 7 along the side of the octagon. Total distance =  $24.14 + 10 = 34.14$  units
- Hence, (1).
13. Hence, (4).

**Answers to questions 14 to 17:**

14. Suppose the cruising speed of the plane is 's' kmph. Therefore effective speed of the plane while going from West to East is 's - 100' kmph while the effective speed of the plane while going from East to West is 's + 100' kmph.
- Time at B is ahead of the time at A. Consider a plane taking off from A at 0800 hours and reaching B at 1130 hours and a plane taking off from B at 0800 hours and reaching A at 0900 hours. Total distance = 2000 km. Suppose the time difference between A and B is 't' hours. Therefore effective flying time of the plane going from A to B: (3.5 - t) hours and the effective flying time of the plane going from B to A: hours.
- $$\therefore (s - 100)(3.5 - t) = (s + 100)(t + 1) = 2000$$
- $$\therefore 3.5s - 350 - st + 100t = st + 100t + s + 100 = 2000$$
- $$\therefore 3.5s - 350 - st = st + s + 100$$
- $$\therefore 2st - 2.5s = -450$$
- $$\therefore s(2t - 2.5) = -450$$
- $$\therefore s = \frac{450}{2.5 - 2t}$$
- It can be seen that to ensure that the denominator is positive,  $t < 1.25$ . Since the time difference between any two pairs of cities is a multiple of 30 minutes,  $t = 0.5$  or 1 hours.
- Using the options, it can be seen that  $t = 1$  hour and  $s = 900$  kmph.
- Hence, (2).
15. It can be seen that the plane taking off from A at 0800 hours reaches B at 1130 hours and the plane taking off from B at 0800 hours reaches C at 1500 hours. Therefore the plane taking off from B at 1130 hours will reach C at 1830 hours. Similarly the plane taking off from C at 0800 hours reaches D at 1200 hours. Therefore the plane taking off from C at 1830 hours reaches D at 2230 hours.
- Therefore if a plane takes off from A at 0800 hours, it will reach D at 2230 hours local time. Total distance between A and D is 8000 km. From the explanatory answer to the previous question, it can be seen that the cruising speed of the planes is 900 kmph. Therefore the effective speed of the plane while going from west to east is  $900 - 100 = 800$  kmph.
- Therefore the flying time between A and D =  $\frac{8000}{800} = 10$  hours.
- If 't' is the time difference between A and D, the flying time =  $14.5 - t = 10$  hours or  $t = 4.5$  hours.
- Hence, (3).

16. Total distance between A and D = 8000 km.

From the explanatory answer to the previous two questions, the effective speed of the plane going from A to D = 800 kmph while the effective speed of the plane going from D to A = 1000 kmph.

Therefore we have,

$$\frac{AP}{PD} = \frac{800}{1000} = \frac{4}{5}$$

$$\text{Therefore } \ell(AP) = \frac{4}{9} \times 8000 = \frac{32000}{9} = 3000 + \frac{5000}{9} \text{ km}$$

$$\text{Therefore point P is located at a distance of } 1000 + \frac{500}{9} = 1555.55 \text{ km}$$

Hence, (4).

17. Total distance between A and D = 8000 km.

From the explanatory answer to the previous questions, the effective speed of the plane going from D to A = 1000 kmph. Therefore the flying time =  $\frac{8000}{1000} = 8$  hours.

From the explanatory answer to the 2<sup>nd</sup> question, the time difference between A and D is 4.5 hours.

Therefore the local time at A when the plane reaches A = (0800 + 0800 – 0430) = 1130 hours.

Hence, (1).

**LR-3.1 | TOURNAMENTS****PRACTICE EXERCISE 1****Answers to questions 1 to 4:**

The important point to note is that if the number of serve changes is even, then the player who serves first wins the match, while if the number of serve changes is odd, the opponent wins the match. If the number of serve changes is say 0, the opponent does not get any opportunity to score points and the player who serves first wins the match. Similar argument can be made if the number of serve changes is 2, 4, 6 or any other even number. On the other hand, if the number of serve changes is 1, the player who served first cannot get any opportunity to win the match and the opponent will continue to serve till he wins the match. Similar argument can be made if the number of serve changes is 3, 5, 7 or any other odd number.

1. Since E lost all the matches and the number of serve changes in all the matches played by E is even, E did not win the toss in a single match.

Therefore the required answer is 0.

2. B won the toss in the match against E. The number of serve changes in that match is 2. Therefore initially B started serving, which was broken by E (first serve change). Then E started serving, which was broken by B (second serve change). On each service change, the player who breaks the service wins 2 points. Therefore the points scored by both the players is of the form  $3a+2$ , where 'a' is the natural number. Also B won the match against E because the number of serve changes is even. Therefore B scored at least 26 points against E. The number 26 is of the form  $'3a+2'$ . Therefore B scored 26 points against E.

Therefore the required answer is 26.

3. The winner scores at least 26 points. If the winner scores 3 points when he is on 25, he will score 28 points and win the match. Therefore it is not possible for the winner to score more than 28 points.

Therefore the required answer is 28.

4. The number of serve changes in the match between D and A is 9, an odd number. Also D scored 28 points, which means D won the match. Since the number of serve changes is odd, the player who served first lost the match. Therefore A won the toss and served first. This means that D broke A's serve 5 times. Each service break yields 2 points. Therefore D got 10 points by breaking A's service. Therefore D scored  $28 - 10 = 18$  points by service, which was not broken. Therefore the number of times D served in the match against A is  $\frac{18}{3} = 6$ .

Therefore the required answer is 6.

**Answers to questions 5 to 8:**

We have,

	Game I	Game II	Game III	Total
Australia	5		1	
Bangladesh		0		
Canada				
Denmark		3		11

Denmark scored 8 points from Game I and Game III together. That means Denmark scored 5 points in one game and 3 points in the other game. Since Australia scored 5 in Game I, Denmark must have scored 3 points in Game I and 5 points in Game III. Therefore,

	Game I	Game II	Game III	Total
Australia	5		1	
Bangladesh		0		
Canada				
Denmark	3	3	5	11

Bangladesh and Canada would have scored 1 & 0 in Game I, Australia and Canada would have scored 5 & 1 in Game II while Bangladesh and Canada would have scored 3 & 0 in Game III. Therefore we have,

	Game I	Game II	Game III	Total
Australia	5	5 or 1	1	11 or 7
Bangladesh	1 or 0	0	3 or 0	4/1/3/0
Canada	0 or 1	1 or 5	0 or 3	0/3/1/4 Plus 1 or 5
Denmark	3	3	5	11

5. If there was a tie for the first position, Australia must have scored 11 points. Therefore we have,

	Game I	Game II	Game III	Total
Australia	5	5	1	11
Bangladesh	1 or 0	0	3 or 0	4/1/3/0
Canada	0 or 1	1	0 or 3	1/4/2/5
Denmark	3	3	5	11

Therefore, Canada could not score 3 points.

Hence, (3).

6. If Canada scored exactly 0 in two games, we have

	Game I	Game II	Game III	Total
Australia	5	5 or 1	1	11 or 7
Bangladesh	1	0	3	4
Canada	0	1 or 5	0	1/5
Denmark	3	3	5	11

If Australia scored 5 points and Canada scored 1 point in Game II, Canada would be ranked 4th. However, if Australia scored 1 point and Canada scored 5 points in Game II, Bangladesh would be ranked 4th. Hence, (4).

7. If Bangladesh scored 0 in all games, we have

	Game I	Game II	Game III	Total
Australia	5	5 or 1	1	11 or 7
Bangladesh	0	0	0	0
Canada	1	1 or 5	3	5 or 9
Denmark	3	3	5	11

Hence, (4).

8. If Bangladesh scored total 4, we have

	Game I	Game II	Game III	Total
Australia	5	5 or 1	1	11 or 7
Bangladesh	1	0	3	4
Canada	0	1 or 5	0	1 or 5
Denmark	3	3	5	11

Hence, (1).

#### Answers to questions 9 to 12:

From the table, we can conclude that Anna has to score 4 points in Round 4 by hitting Region 4. That means she hit Region 3 in Round 2.

For Round 1, Bella cannot be the person who scored 4 points because her total points at the end of four rounds are only 8. Therefore, Chelsea scored 4 points in Round 1 by hitting Region 1. Similarly, she scored 4 points in Round 3 as well by hitting Region 3.

Chelsea can score 2 points in Round 4 if she hits Region 2. That means she hit Region 4 in Round 2 and scored 2 points in that round.

Diana can score 3 points in Round 4 only if she hits Region 3. That means she hit Region 2 in Round 2 and scored 4 points in that round.

So far, we have concluded the following data:

	Round 1	Round 2	Round 3	Round 4	Total
Anna	3 (2)	3 (3)	2 (1)	4 (4)	12
Bella		3 (0)		2 (2)	8
Chelsea	4 (1)	2 (4)	4 (3)	2 (2)	12
Diana	1 (4)	4 (2)	2 (1)	3 (3)	10

Bella scored a total of 3 points in rounds 1 and 3 combined. So, she scored 1 and 2 points in these rounds. The lowest points that Round 3 fetches are 2 points, so Bella cannot score 1 point in Round 3. Therefore, Bella scored 1 point in Round 1 by hitting Region 4 and 2 points in Round 3 by hitting Region 1. This means that she hit Region 3 in Round 2. So, we get the final table as follows:

	Round 1	Round 2	Round 3	Round 4	Total
Anna	3 (2)	3 (3)	2 (1)	4 (4)	12
Bella	1 (4)	3 (3)	2 (1)	2 (2)	8
Chelsea	4 (1)	2 (4)	4 (3)	2 (2)	12
Diana	1 (4)	4 (2)	2 (1)	3 (3)	10

Using this, all the questions can be answered.

9. Therefore, the required answer is 12.

10. Therefore, the required answer is 2.

11. Therefore, the required answer is 12.

12. Therefore, the required answer is 1.

**Answers to questions 13 to 16:**

13. If there are no upsets in the tournament, then the teams A1, B1, C1 and D1 will reach the finals.

Hence, (2)

14. If teams A1, A2, ..., A8 reach the third round, it means that they won their matches in the first two rounds.

In the first round, the teams A5, A6, A7 and A8 won their respective matches against the teams that are ranked higher. Therefore, in all, at least 4 upsets took place in the first round.

In the second round, the teams A3 and A7 are ranked third while the teams A4 and A8 are ranked fourth. They win their respective matches. Therefore, at least 4 upsets took place in the second round.

Therefore, the minimum number of upsets in the first two rounds =  $4 + 4 = 8$ .

Hence, (2).

15. If all matches in the first three rounds end in upset, this situation will arise. Therefore the maximum number of upsets =  $16 + 8 + 4 = 28$ .

Hence, (2).

16. If the first round matches are won by even-ranked teams, the teams that reach the second round are A2, A4, A6, A8, B2, B4, B6, B8, C2, C4, C6, C8, D2, D4, D6 and D8. Similarly, if the second round matches are won by even-ranked teams, the teams that reach the third round are A4, A8, B4, B8, C4, C8, D4 and D8. If the odd-ranked teams won the third round, the teams that reach the fourth round are A4, B4, C4 and D4. Hence, (2).

## PRACTICE EXERCISE 2

## Answers to questions 1 to 4:

In each round, the total points scored by all the contestants will be  $5 + 4 + 3 + 2 + 1 + 0 = 15$ . So after 3 rounds, the total points of all should be 45. The 5 known scores add up to 37, and hence F must have scored total 8 points in the first 3 rounds. Similarly, the total score for 6 rounds should be 90, and the 5 known scores add up to 73, and hence E must have scored 17 points in the 6 rounds.

Knowing the total in 6 rounds, and the total in the first 3 rounds, we can figure out each contestant's score in rounds 4-6 (for example, A will have  $10 - 4 = 6$  points). At this point the table looks like this:

	Rounds 1 – 3	Total for 1-3	Rounds 4 – 6	Total for 4-6	Overall Total
A		4		6	10
B		11		4	15
C		7		11	18
D		3		8	11
E		12		5	17
F		8		11	19

Now let us focus on the first 3 rounds. Each contestant has 3 different scores between 0 and 5. So for E, the only way we can get 12 is  $5 + 4 + 3$ , for B, the only way we can get 11 is  $5 + 4 + 2$ , for D, the only way we can get 3 is  $2 + 1 + 0$  and for A the only way we can get 4 is  $3 + 1 + 0$ . This leaves the scores 0, 1, 2, 3, 4, 5 once each to be divided among C and F. Now C got 5 points in the 2nd round, so the only way we can achieve this is  $5 + 2 + 0$  for C and  $4 + 3 + 1$  for F. The table now looks like this:

	Rounds 1 – 3	Total for 1-3	Rounds 4 – 6	Total for 4-6	Overall Total
A	$3 + 1 + 0$	4		6	10
B	$5 + 4 + 2$	11		4	15
C	$5 + 2 + 0$	7		11	18
D	$2 + 1 + 0$	3		8	11
E	$5 + 4 + 3$	12		5	17
F	$4 + 3 + 1$	8		11	19

For the next set of 3 rounds, we can again see that both C and F have scored 11 (which can only be done as  $5 + 4 + 2$ ) and B has 4 (which is  $3 + 1 + 0$ ). A finished first in round 6 so he must have  $5 + 1 + 0$ . The remaining unallocated scores are 0, 1, 2, 3, 3, 4. Now E can get 5 points as  $4 + 1 + 0$  or  $3 + 2 + 0$ . But if he gets  $4 + 1 + 0$ , then D will have the same score (3) in two rounds. So E must be  $3 + 2 + 0$  and D must be  $4 + 3 + 1$ . The final table, therefore, will look like this:

	Rounds 1 – 3	Total for 1-3	Rounds 4 – 6	Total for 4-6	Overall Total
A	$3 + 1 + 0$	4	$5 + 1 + 0$	6	10
B	$5 + 4 + 2$	11	$3 + 1 + 0$	4	15
C	$5 + 2 + 0$	7	$5 + 4 + 2$	11	18
D	$2 + 1 + 0$	3	$4 + 3 + 1$	8	11
E	$5 + 4 + 3$	12	$3 + 2 + 0$	5	17
F	$4 + 3 + 1$	8	$5 + 4 + 2$	11	19

Now all the questions can be answered:

1. The highest scorer is F, with 19. Hence, (1).
2. A achieved 0 and 1 twice, C achieved 2 and 5 twice, D achieved 1 twice, E achieved 3 twice and F achieved 4 twice. So all the scores were achieved twice by some participant or other. Hence, (4).
3. We can see that only D did not finish first in a single round. Hence, (4).
4. We can see that B is the only such person. Hence, (3).

**Answers to questions 5 to 8:**

Suppose both Prakash and Prashant start with \$x on Monday, we have the following cases,

	Number of participants on						Prashant won on	Prakash won on	Final amount with Prashant	Final amount with Prakash
	Mon	Tue	Wed	Thu	Fri	Sat				
1	4	5	6	7	6	5	Tue, Wed, Thu	Sat	x+120	x-10
2	4	5	6	5	6	5	Tue, Wed, Fri	-	x+110	x-60
3	4	5	4	5	6	5	Tue, Thu, Fri	-	x+100	x-60
4	4	3	4	5	6	5	Wed, Thu, Fri	-	x+90	x-60
5	4	5	6	5	4	5	Tue, Wed, Sat	Fri	x+100	x-20
6	4	5	4	5	4	5	Tue, Thu, Sat	-	x+90	x-60
7	4	3	4	5	4	5	Wed, Thu, Sat	-	x+80	x-60
8	4	5	4	3	4	5	Tue, Fri, Sat	Thu	x+80	x-30
9	4	3	4	3	4	5	Wed, Fri, Sat	-	x+70	x-60
10	4	3	2	3	4	5	Thu, Fri, Sat	Wed	x+60	x-40

Now all the questions can be answered.

5. It can be seen that Prashat definitely won on 3 days.  
Therefore the required answer is 3.
6. It can be seen that the maximum difference between the final amounts with Prashant and Prakash can be  $(x + 110) - (x - 60) = 170$ .  
Therefore the required answer is 170.



7. If the difference between the final amounts with Prashant and Prakash on Saturday was \$150, we have the following two cases:

	Number of participants on									
	Mon	Tue	Wed	Thu	Fri	Sat	Prashant won on	Prakash won on	Final amount with Prashant	Final amount with Prakash
4	4	3	4	5	6	5	Wed, Thu, Fri	-	x+90	x-60
6	4	5	4	5	4	5	Tue, Thu, Sat	-	x+90	x-60

Therefore the number of players on Wednesday was 4.

Therefore the required answer is 4.

8. If 7 players participated on Thursday, we have the following case

	Number of participants on									
	Mon	Tue	Wed	Thu	Fri	Sat	Prashant won on	Prakash won on	Final amount with Prashant	Final amount with Prakash
1	4	5	6	7	6	5	Tue, Wed, Thu	Sat	x+120	x-10

Therefore the required answer is 130.

#### Answers to questions 9 to 12:

Each of the six players played against 4 others. Therefore the total number of matches played =  $\frac{6 \times 4}{2} = 12$ . Therefore the number of matches played on each day =  $\frac{12}{4} = 3$ . The number of matches played by each player = 4.

Consider the 1st day. B played 2 matches and E did not play a single match on the 1st day. Therefore A, C, D and F played one match each on the 1st day. C played with D on the 1st day. Therefore the 2 matches that B played on the 1st day must have been against A and F. Therefore the following matches were played on the 1st day: C vs. D, B vs. A and B vs. F.

This way we can continue to fill the following table for all the 4 days. We can also figure out the one player with whom each of the six players did not play.

	A	B	C	D	E	F
A	X	day 1	day 3	day 2	No	day 4
B	day 1	X	No	day 4	day 4	day 1
C	day 3	No	X	day 1	day 2	day 3
D	day 2	day 4	day 1	X	day 3	No
E	No	day 4	day 2	day 3	X	day 2
F	day 4	day 1	day 3	No	day 2	X

Now all the questions can be answered.

9. Therefore the required answer is 6.

10. Therefore the required answer is 1.

11. A, D and F played one match everyday. Therefore the required answer is 3.

12. Therefore the required answer is 0.

**Answers for questions 13 to 17:**

The table of wins and cumulative spread for each team after 4 rounds is as follows:

	A	B	C	D	E	F	Wins	Margin
A	–	230-240	250-210	210-260		270-190	2	60
B	240-230	–	260-190	150-220	180-230		2	-40
C	210-250	190-260	–		160-180	140-280	0	-270
D	260-210	220-150		–	230-210	240-180	4	200
E		230-180	180-160	210-230	–	290-190	3	150
F	190-270		280-140	180-240	190-290	–	1	-100

The matches remaining are A v/s E, B v/s F and C v/s D. Note that D and E have already qualified for the semi-finals, while C is already out of contention.

13. If A beats E, A reaches 3 wins and qualifies. So it is now a straight play-off between B and F for the final place. F is currently 1 win and 60 points behind B, so a win by exactly 30 points will tie the two, while a win by more than 30 points will put F ahead. Since we wish to guarantee F qualifying, the minimum margin will be 40 points. Hence (2)
14. For F to make the semi-finals it has to do so at the expense of either A or B. But F definitely has to beat B. Hence (I) is true. Now, either F should beat B by a margin sufficient to go ahead of B on points (a win by at least 40, as seen in the earlier question) or else E should beat A by a huge margin so that A goes below F and B. But even if A beats E, F could still reach the semi-finals so (III) is not necessary. (II) can be easily eliminated as the result of the C v/s D match will have no effect on qualifying. Hence (1)
15. For A to get eliminated, A must lose to E in the last qualifier, which gives E 4 wins till this point. E must also win the remaining two matches (semi-final and final) to win the tournament. So E must be on 6 wins for sure. The only other team which can have 6 wins is D – if they win the last league match against C, and also win their semi-final, losing to E in the final, they would also reach 6. Hence (4)
16. If A wins against E by 10 points then the top of the group is D (with 4 or 5 wins, depending on the result of the match with C). E is second with 3 wins and 140 points, while A is currently third with 3 wins and 70 points. Now since 1st place plays 4th place, for B to not play D, B has to at least go above A. Hence B (currently at 2 wins and -40 points) must win by at least 120 points to definitely not face D. Hence (5)
17. For all the teams to have a different number of wins, they must have 5, 4, 3, 2, 1 and 0 wins. Now a little trial and error will quickly establish that only D can reach 5. After this, only E can reach 4. Hence both D and E must win their matches, and their opponents C and A must remain on 0 and 2 wins respectively. So B must beat F to reach 3 wins, while F remains on 1 win. Hence the final order is D, E, B, A, F, C, which gives the semi-final line-up as D v/s A and E v/s B. The only final combination possible of the options is therefore D v/s E. Hence (4).

## LR-3.2 | PUZZLES

### PRACTICE EXERCISE 1

#### Answers for questions 1 to 4:

Using their statements, let us make assumptions and see what their statements infer.

Assumption	Inferences	Conclusion
A is honest.	- B (6 <sup>th</sup> ) is a liar. - C did not come 4 <sup>th</sup> .	Nothing decisive.
B is honest.	- C (4 <sup>th</sup> ) is a liar. - A (who said B is 6 <sup>th</sup> and hence a liar) is a liar. - E (who said A is 3 <sup>rd</sup> and hence honest) is a liar. - D (who said E is 1 <sup>st</sup> and hence honest) is a liar.	Since 4 liars are not possible, this assumption is wrong and hence, B is a liar.
C is honest.	- F (5 <sup>th</sup> ) is a liar. - D did not come 2 <sup>nd</sup> .	Nothing decisive.
D is honest.	- E (1 <sup>st</sup> ) is honest. - A (3 <sup>rd</sup> ) is honest. - So the 3 <sup>rd</sup> honest person has to be D who came 2 <sup>nd</sup> .	This means that all others, including F, are liars. But F said D came 2 <sup>nd</sup> , which is a true statement. Hence, our assumption is wrong and D is a liar.
E is honest.	- A (3 <sup>rd</sup> ) is honest. - B (6 <sup>th</sup> ) is a liar.	Nothing decisive.
F is honest.	- D (2 <sup>nd</sup> ) is honest.	But we already proved that D is a liar. Therefore, F also is a liar.

Thus, the honest people are A, C and E and the liars are B, D and F. Using this conclusion, their correct positions are as follows:

Position	1	2	3	4	5	6
Person	C	E	A	D	F	B

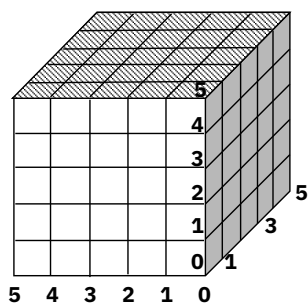
Using this, all the questions can be answered.

- Hence, (2).
- Hence, (4)
- Only the 2<sup>nd</sup> option is a true statement. Hence, (2).
- Only the 1<sup>st</sup> option is a false statement. Hence, (1).

**Answers to questions 5 to 8:**

5. The number of cubes of the smallest size  $1 \times 1 \times 1 = 125$ . Similarly, the number of cubes of size  $2 \times 2 \times 2$  will be 64, of size  $3 \times 3 \times 3$  will be 27, of size  $4 \times 4 \times 4$  will be 8 and of size  $5 \times 5 \times 5$  will be just 1. So overall we will have  $1 + 8 + 27 + 64 + 125 = 225$  cubes.

6. We have to choose a length (let us define this as the dimension parallel to X-axis in the adjoining figure), breadth (parallel to Y-axis) and height (parallel to Z-axis) for each cuboid. To choose the length, we can choose any two X co-ordinates from 0 to 5 (which can be done in  ${}^6C_2 = 15$  ways). Similarly we can choose a breadth and a height in 15 ways each. So overall there will be  $15 \times 15 \times 15 = 3375$  cuboids possible.



7. The number of cubes of the smallest size  $1 \times 1 \times 1 = 6 \times 5 \times 4 = 120$ . Similarly, the number of cubes of size  $2 \times 2 \times 2$  will be  $5 \times 4 \times 3 = 60$ , of size  $3 \times 3 \times 3$  will be  $4 \times 3 \times 2 = 24$  and of size  $4 \times 4 \times 4$  will be  $3 \times 2 \times 1 = 6$ . So overall we will have  $120 + 60 + 24 + 6 = 210$  cubes.
8. We can choose a length in  ${}^7C_2 = 21$  ways, a breadth in  ${}^6C_2 = 15$  ways and a height in  ${}^5C_2 = 10$  ways. So overall we will have  $21 \times 15 \times 10 = 3150$  cuboids.

**Answers to questions 9 to 11:**

Depending on the numbers on the two dice, we get the following movements in the coin:

Numbers on the dice	Movement of the coin	Comment
1 & 1	1	Rule 1
1 & 2	0	Rule 6
1 & 3	3	Rule 1
1 & 4	0	Rule 6
1 & 5	5	Rule 1
1 & 6	0	Rule 1
2 & 2	8	Rule 2: 5, Rule 3: 8. Hence 8
2 & 3	11	Rule 3
2 & 4	8	Rule 2: 7, Rule 5: 8. Hence 8
2 & 5	17	Rule 3
2 & 6	12	Rule 2: 9, Rule 5: 12. Hence 12
3 & 3	15	Rule 1: 5, Rule 3: 15. Hence 15
3 & 4	12	Rule 5
3 & 5	23	Rule 1: 7, Rule 3: 23. Hence 23
3 & 6	18	Rule 5
4 & 4	9	Rule 2: 9, Rule 4: 8. Hence 9
4 & 5	20	Rule 5
4 & 6	14	Rule 2: 11, Rule 4: 14. Hence 14
5 & 5	35	Rule 1: 9, Rule 3: 35. Hence 35
5 & 6	30	Rule 5
6 & 6	24	Rule 2: 11, Rule 4: 24. Hence 24

Thus different possible movements in the coin are 0, 1, 3, 5, 8, 9, 11, 12, 14, 15, 17, 18, 20, 23, 24, 30 and 35.

9. It can be seen that if the two numbers on the dice are 6 & 6, the coin will move by 24 and it will directly move to 25.  
Hence, (1).
10. It can be seen that if the numbers on the dice are 2 & 2 or 2 & 4 (in no particular order), the coin will move by 8 in one step. Therefore, the numbers on the dice can be 2,2 or 2,4 or 4,2. Hence, (3).
11. As seen earlier, the different possible movements in the coin are 0, 1, 3, 5, 8, 9, 11, 12, 14, 15, 17, 18, 20, 23, 24, 30 and 35. If the coin is currently at 1, accordingly the different squares it can move to are: 1, 2, 4, 6, 9, 10, 12, 13, 15, 16, 18, 19, 21, 24, 25, 19 and 14 respectively.  
Therefore, the different unique squares the coin can move to are 1, 2, 4, 6, 9, 10, 12, 13, 14, 15, 16, 18, 19, 21, 24 and 25 (total 16). Hence (3).

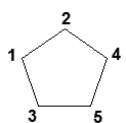
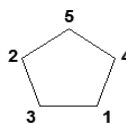
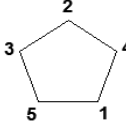
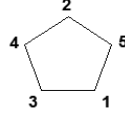
**Answers to questions 12 to 15:**

The important thing to note is that bulb 5 has to be the last bulb to be switched on; otherwise it will lead to short circuit.

Start by placing bulb 1 arbitrarily in any position. The second bulb that will be switched on can be 2, 3 or 4.

If the second bulb is 2, the third bulb that will be switched on can be 3 or 4. If the second bulb is 3, the third bulb that will be switched on can be 2 or 4. If the second bulb is 4, the third bulb that will be switched on can be 2 or 3. For each combination, check whether all the bulbs get switched on.

This way, if we check for each combination, we get exactly one order of bulbs for each case when the first bulb to be switched on is bulb 1, 2, 3 or 4, as shown below.

Arrangement	1 <sup>st</sup> Bulb	2 <sup>nd</sup> Bulb	3 <sup>rd</sup> Bulb	4 <sup>th</sup> Bulb	5 <sup>th</sup> Bulb
	1	3	4	2	5
	2	1	3	4	5
	3	4	2	1	5
	4	2	1	3	5

Using this, all the questions can be answered.

12. It can be seen that this situation corresponds to case 4. Hence, (2)
13. It can be seen that this situation corresponds to case 1. Hence, (3)
14. It can be seen that this situation corresponds to case 1 or 3. Hence, (4)
15. It can be seen that the user can choose to switch on either bulbs 1 or 2 or 3 or 4 first and get all the five bulbs switched on, but not bulb 5. Hence, (2).

## PRACTICE EXERCISE 2

## Answers to questions 1 to 4:

1. Odd multiples of 7 will be present in every surface except 2 & 5.

It can be observed that the red part is always the  $(4k + 1)$ th part of a surface (where  $k$  is an integer). Thus, we observe the following:

Surface	Multiples of 7	Multiples in red part	Parameter X
1	7, 21, 35, 49, 63	49	1
3	7	----	0
4	7, 14, 28, 35, 49, 56, 70, 77, 91	----	0
6	7, 21, 35	21	1

Therefore the sum of X for the six surfaces =  $1 + 1 = 2$ .

2. We observe the following numbers placed in yellow part:

Surface	Prime numbers in yellow part	Parameter Y
1	7, 23, 31, 47, 71	5
2	----	0
3	All 9 prime numbers are placed in yellow part	9
4	----	0
5	----	0
6	----	0

Therefore the sum of Y for the six surfaces =  $5 + 9 = 14$ .

3. The following table gives the colours on which both the numbers lie on each surface:

Surface	Colour of 7	Colour of 21
1	Yellow	Green
3	Yellow	-
4	Green	-
6	Green	Red

Thus, 7 & 21 both are placed in two different coloured parts. Therefore, the required answer is 0.

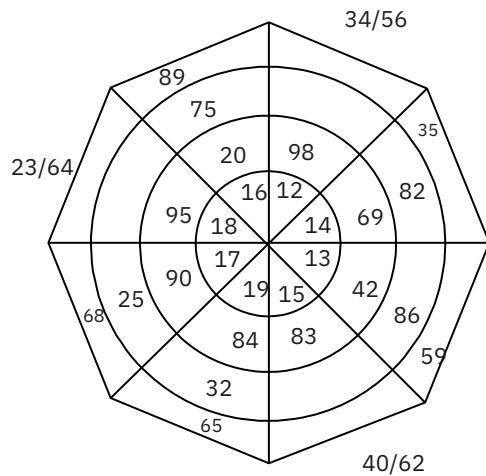
4. All letters are on surface 5. As there are 26 letters and 36 parts.

Hence A will occupy 1st & 27th part, B will occupy 2nd & 28th part and so on.

Hence, A's colours will be Red & Green. Similarly, for all other letters placed in parts 27 to 36, there will be two different coloured parts. Hence, there will be 10 letters that will be placed in more than one different coloured part.

**Answers to questions 5 to 8:**

Based on the statements, we get the following diagram:



The numbers, whose positions cannot be uniquely determined, are mentioned outside the figure.

Now all the questions can be answered.

5. Hence, (3).
6. Hence, (2).
7. Hence, (1).
8. Hence, (2).

**Answers to questions 9 to 11:**

Using the information given, we have the following

Transaction Number	Giver	Taker	Number of coins
1	D	B	2
2	G	D	1
3	A	H	5
4	C	F	3
5		H	

We also have the following

	A	B	C	D	E	F	G	H
Giver	Yes		Yes	Yes			Yes	
Taker		Yes		Yes		Yes		Yes

The only friend who is involved in both giving and taking is D. Also exactly 3 friends did not give any coins while exactly 4 friends did not receive any coins. Therefore we have the following

	A	B	C	D	E	F	G	H
Giver	Yes	No	Yes	Yes	Yes	No	Yes	No
Taker	No	Yes	No	Yes	No	Yes	No	Yes



Therefore in the 5th transaction, E gave coins to H and the number of coins given was 4. Therefore we have the following:

Transaction Number	Giver	Taker	Number of coins
1	D	B	2
2	G	D	1
3	A	H	5
4	C	F	3
5	E	H	4

Therefore the number of coins with the friends at the end is as follows:

A :  $25 - 5 = 20$

B :  $25 + 2 = 27$

C :  $25 - 3 = 22$

D :  $25 - 2 + 1 = 24$

E :  $25 - 4 = 21$

F :  $25 + 3 = 28$

G :  $25 - 1 = 24$

H :  $25 + 5 + 4 = 34$

Now all the questions can be answered.

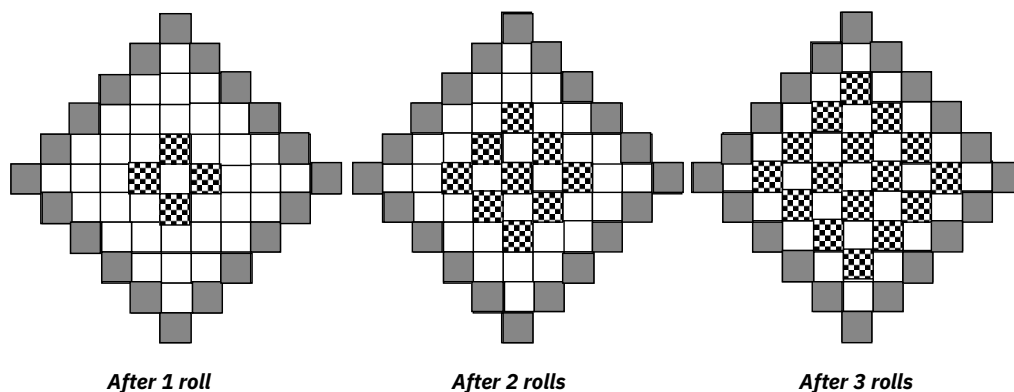
9. Hence, (4).

10. Hence, (1).

11. Hence, (4).

#### Answers to questions 12 to 19:

12. Let us see what can be the possible positions after 1 roll, 2 rolls, etc.

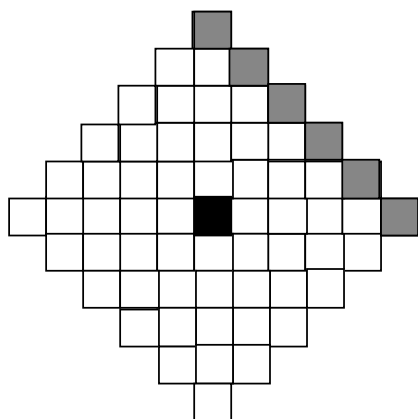


Note that after one roll the number of possible positions is 4. After two rolls, it becomes 9 (including the original position). After 3 rolls, there are 16 possible final positions. In general, we see that after  $n$  rolls ( $n < 5$ ), there are  $(n+1)^2$  final positions possible. So after 4 rolls, the coin could be on any of 25 possible final positions. Hence (3)

13. The minimum number of moves to reach a border square (and achieve the objective) is 5. If the border has not been reached in 5 moves, then the coin must have taken at least one step in the reverse direction (away from the border) and hence needs one more step to compensate for that. Hence the next possible number of steps which could reach the border would be 7. The subsequent values could be 9, 11, etc (i.e. any odd number greater than or equal to 5). 64 being even, reaching the boundary in 64 moves is not possible. Hence (2)

Alternatively, we can notice a pattern from the solution to the previous question: a particular square can be landed on either after an even number of moves or after an odd number of moves. Now the squares at the border, which are 5 steps away from the centre, can only be reached after an odd number of moves. Hence (2)

14.



If Shravya reaches the objective in 5 moves, that is the absolute minimum possible. So the coin has to keep moving towards the same boundary. For example, to reach any of the squares on the North-East (NE) boundary, as shown in the figure alongside, the coin has to move either North (N) or East (E) in every single move. This means that there are  $2^5 = 32$  sequences of rolls which could reach her to one of these squares. Similarly there would be 32 ways each to reach the squares on the NW, SE and SW boundaries. However, 4 sequences of moves (NNNNN, EEEEE, SSSSS and WWWW) have been counted twice in the above reckoning and hence we will have  $32 \times 4 - 4 = 124$  possible sequences. Hence (1).

15. For the coin to have arrived back at the central square, the moves N and S must cancel out each other, and so must the moves E and W. The possible cases are

NNSSS (which could occur in  $\frac{6!}{3!3!} = 20$  ways)

NNSSEW (which could occur in  $\frac{6!}{2!2!} = 180$  ways)

NSEWW (which could occur in  $\frac{6!}{2!2!} = 180$  ways)

EEWW (which could occur in  $\frac{6!}{3!3!} = 20$  ways)

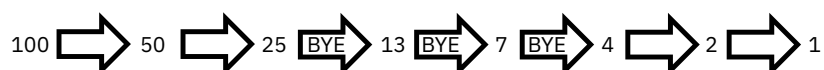
In total, therefore, we would have 400 ways. Hence, (4).

16. The number of cubes with none of the six faces painted =  $(4 - 2)^3 = 8$ .  
Therefore the required answer is 8. Hence, (2).
17. We are looking for the number of cubes with three of their faces painted, one face being red and the other two faces being blue. It can be seen that total 8 cubes have three of their faces painted. These 8 cubes are located along the 8 vertices of the cube. It can be seen that the 4 cubes located along the top 4 vertices have one face painted in red and two faces painted in blue.  
Therefore the required answer is 4. Hence, (1).
18. We are looking for the number of cubes with three of their faces painted, one face being red and the other two faces being blue. It can be seen that total 8 cubes have three of their faces painted. These 8 cubes are located along the 8 vertices of the cube. It can be seen that the 4 cubes located along the bottom 4 vertices have one face painted in blue and two faces painted in red.  
Therefore the required answer is 4. Hence, (1).
19. It can be seen that there are 8 cubes that have only two faces painted, using the same colour (red or blue).  
Therefore the required answer is 8. Hence, (2).

#### Answers for questions 20 to 23:

If the tournament had only one round we could have had a maximum of 2 participants. In two rounds, we would be able to accommodate up to 4 participants, in three rounds up to 8, and so on. In short, in  $n$  rounds we could have a maximum of  $2^n$  participants. Now we know that Shashank has played 7 matches (up to and including the final!) and has not received any byes; thus we know that the tournament must have had 7 rounds. Thus there must be more than 26 but not more than 27 i.e. between 65 and 128 participants.

Given the number of players, we can figure out in which rounds there were byes. If there are an odd number of players at the start of a round, there would be a bye in that round, while if there are an even number of players, there would be no need for a bye. For example, if there were 100 players, we could proceed as follows:



Similarly given the rounds in which byes happened, we could work backward to figure out the number of players (for example, if there were  $x$  participants after a round, there must have been  $2x$  participants before the round if there was no bye in the round, while there must have been  $2x-1$  participants if there was a bye). For example, if we say there were byes only in rounds 1, 3 and 4, then we could find the original number of players as follows, starting from the end:

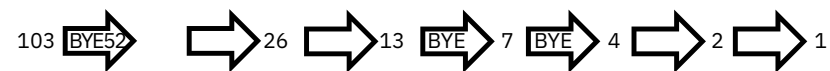
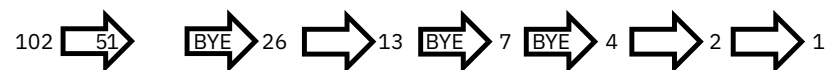
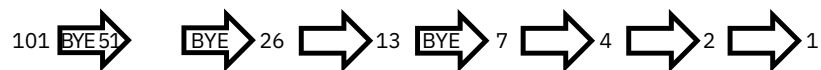


20. If there were 120 players, then the following would be the sequence of events:



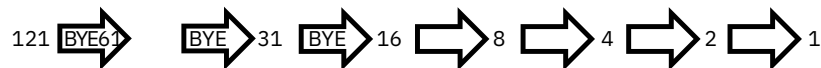
So a participant would have received a bye only in Round 4. Hence (2)

21. The simplest way to deal with the is to check each option to see which one gives us 2 byes:



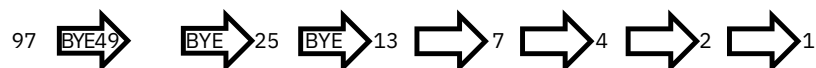
Hence (4).

22. We can work backward to find out the initial number of people:



Thus there were 121 participants in the tournament. To arrive at a winner, 120 participants must get knocked out. In every match, one person gets knocked out. Thus there must have been 120 matches. Out of those, 7 involved Shashank, so the remaining 113 matches must not have involved Shashank. Hence (1).

23. Let us find the number of people involved in the two extreme cases, where the 5 byes happen as early as possible (rounds 1, 2, 3, 4 and 5) and as late as possible (rounds 2, 3, 4, 5 and 6):



Hence K lies somewhere between 66 and 97 inclusive. Hence (2)

# Verbal Ability



## VA-3.1 | SUBJECT-VERB AGREEMENT AND PRONOUNS

### PRACTICE EXERCISE 1

Qs. 1 to 6: When a pronoun performs the function of a subject (doer) in a sentence, it should be in the subjective form. The subjective forms of the various personal pronouns are: *I, we, you, he, she, they, it, who, whoever*. In sentence 1, the missing pronoun denotes a person who goes for a movie (i.e. is the subject of the verb 'goes'). So, it should be subjective (i.e., 'I'). In sentence 2, the missing pronoun denotes the person who is the protagonist of a movie. It acts as the subjective complement (i.e. completes the subject). So, it should be subjective i.e., 'he'. In sentence 3, the missing pronoun denotes a person who is making a pact. So, it should be subjective. In sentences 4 and 5, two subjects are being compared, in terms of their height and score. A subject is not being compared to an object. So, in both cases, subjective pronouns should be used. In informal English, it is acceptable to use the objective pronouns in these cases of comparison. In sentence 6, though the missing pronoun comes after the verb 'was', note that it denotes a subject (a person who was calling incessantly). So, it should be subjective.

Qs. 7 to 15: When a pronoun performs the function of an object (receiver; on whom or for whom an action is performed) in a sentence, it should be in the objective form. The objective forms of the various personal pronouns are: *me, us, you, him, her, them, it, whom, whomever*. In sentence 7, the missing pronoun denotes a person who did not perform any action, rather simply received money. So, it should be objective i.e., 'him'. In sentence 8, though there is a possible future action (deciding), the missing pronoun will be treated as the object of the preposition 'to'. So, 'me' will be used here. In sentence 9, the pronoun is supposed to replace the noun 'voters', which is the object of the sentence. Therefore, the pronoun should be in the objective form, i.e., 'us'. In sentence 10, the pronoun is the object of the verb 'let'; therefore, it should be in the objective form, i.e., 'me' and not 'I'. In sentence 11, the missing pronoun is the object of the verb 'gifted', and thus should be objective. In sentence 12, the missing pronoun is

the object of the verb 'joined', and thus should be objective. In sentence 13, the missing pronoun is the object of the preposition 'for', and thus should be objective. In sentence 14, the missing pronoun denotes the subject (teachers who feel proud) and not an object. So, it should be subjective, not objective. In sentence 15, though there is a possible future action (climbing Mt. Everest), the missing pronoun will be treated as the object of the preposition 'for'.

Qs. 16 to 21: A pronoun must agree with its antecedent in person, number and gender. An antecedent is a noun or pronoun mentioned earlier in the text, to which the subsequent pronoun refers. A pronoun should refer clearly to one clear noun coming before it. So, it is necessary to identify the antecedent correctly and then use a suitable pronoun to refer to it subsequently. In sentence 16, the antecedent is 'a student' i.e. singular. So the pronoun following it must also be singular i.e., 'his or her'. In sentence 17, the antecedent is the plural word 'crises'. So, the pronoun following it should be 'themselves'. We have to carefully read a sentence that has multiple nouns, used as subjects or objects, to figure out what is the actual antecedent of the pronoun in question. For example, in sentence 18, the subject of 'must work' is 'leader' but the pronoun is referring to the interest of the 'masses' and hence should be plural. Remember that a phrase or clause between an antecedent and the pronoun following it does not change the number of the antecedent. So, in sentence 19, we will ignore 'of wolves' and the singular antecedent 'pack' will be followed by the singular pronoun 'its'. The same logic can be applied to sentence 20. Sometimes it can be tricky to figure out which subjects are singular and which are plural. So, you must read up sufficient material and hone your skills. Though 'spectacles' is a plural word, when it is preceded by 'the pair of', it is treated as singular. In sentence 21, the subject is the singular word 'parent'. So, the first two blanks must be filled with singular pronouns. The last blank is referring to the child, not the parent. Since 'child' is singular, the pronoun will be 'its'.

Qs. 22 to 30: Indefinite pronouns such as *each, each one, everyone, everybody, everything, either, neither, one, no one, nobody, nothing, someone, somebody, something, anybody, anyone, anything*, etc. are always treated as singular. Indefinite pronouns such as *several, few, both, many*, etc. are plural. 'Many a' is singular. Some indefinite pronouns that are modified by a prepositional phrase may be either singular or plural. *Some, any, none, all, most* belong to this category. If the object of the preposition is plural, the antecedent is treated as plural. If the object of the preposition is singular/uncountable, the antecedent is treated as singular. In sentence 22, the antecedent is the plural pronoun 'all (the members)', therefore, the pronoun should also be plural, i.e., 'their'. In sentence 23, the antecedent is the uncountable entity 'all the hype'. So, the subsequent pronoun should be 'its'. In sentence 24, the antecedent is the singular pronoun 'each (female character)', therefore, the singular pronoun 'her' should be used in the blank. In sentences 25, 26 and 27, the antecedents are singular (*somebody, everybody, each one*) so the pronouns should also be singular (*his or her*). In sentence 28, 'some' is not qualified by a prepositional phrase, so it is plural by default. So, the pronoun following it should be plural (their). In sentence 29, 'some' is qualified by the singular noun 'coffee'. So, it will be treated as singular. Thus, the pronoun that follows it will be singular (its). 'Many', in sentence 30, is plural.

31. There is ambiguity in the use of the pronoun 'him'. From the sentence, it is not clear whether 'him' refers to the 'principal' or the 'teacher'. In such cases, it is better to use the actual noun instead of an ambiguous pronoun.
32. The pronoun 'where' is mainly used to refer to a place. Here the antecedent of the pronoun is 'a coded sign language'; therefore, the appropriate pronoun would be 'which'.
33. There is ambiguity with respect to the use of the pronoun 'him'. It is not clear from the sentence whether 'him' refers to Rohan or Aryan. A pronoun should have a very clear and distinct antecedent.
34. There is pronoun-antecedent ambiguity here. It is not clear whether 'his friend' refers to Ravi's friend or his son's friend.
35. When two things are being compared, the pronoun should replace the things that are being compared and not any other noun. The sentence incorrectly

compares the ethics prevalent in developed countries with the third world countries. So, the last part of the correct sentence should read: 'better than those of the third world countries' or 'better than their ethics'.

Qs. 36 to 38: A verb should agree with its subject in number and person. In sentence 36, the subject is the singular word 'rhythm', hence the singular verb 'is'. In sentence 37, the subject 'all the cats' is plural, hence the verb is also plural, i.e., 'are'. In sentence 38, 'I' is in the first person singular; hence, the corresponding verb should be 'am'. In the second blank of the same sentence, the appropriate verb would be 'are' because it has to agree with 'they', which is a plural pronoun and in the third person.

Qs. 39 to 41: The verb should agree in number with its proper subject and not just with any noun/pronoun placed near it. This kind of error of proximity is to be avoided. In sentence 39, the actual subject is the singular word 'suffering' and not the plural noun 'saints', hence the verb 'is'. Similarly in sentences 40 and 41, the subjects are 'quality' and 'termination' respectively and both of them are singular nouns. Thus, the corresponding verbs in each case would be singular.

Qs. 42 to 44: Some nouns that are plural in form but singular in meaning, take the singular verb. 'Statistics', 'news' and 'conjunctivitis' are singular words, though they look like plural words because they end with an '-s'. Therefore, the correct corresponding verbs would be 'is', 'was' and 'causes'.

Qs. 45 & 46: When a plural noun is a proper name for some single object or some collective unit, it is followed by a singular verb. Netherlands is the name of a country, hence it should be followed by a singular verb. Similarly, *Memoirs of a Geisha* is the name of a book, so the corresponding verb should also be singular.

Qs. 47 to 50: Two or more singular subjects when connected by 'and' take a plural verb. So, 'have' would be correct in sentence 47. If two singular nouns connected by 'and' refer to the same person or thing, the verb must be singular. In such cases, the article is used only once before the nouns. So, in sentence 48, 'has' would be correct. If different subjects are referred to, by using separate articles before each noun, the corresponding verb becomes plural. In sentence 49, actor and director



are two different persons; hence the verb should be 'were'. If two subjects together express one idea, the verb must be singular. So, 'is' would be correct in sentence 50.

Qs. 51 to 53: Subjects joined to others by 'as well as', 'together with', 'along with', 'in addition to' etc., are parenthetical and do not affect the number of the verb. The verb is matched with only the first subject. So, in sentences 51 and 53, the verb should be 'is', to match with the singular subject 'teacher' and 'he' respectively. In sentence 52, the main subject is the plural word 'directors', so the verb should be 'have'.

Qs. 54 to 57: If singular subjects are preceded by 'each' or 'every', the corresponding verb is singular. So, sentences 54 and 57 should take the singular verb. Two nouns qualified by 'each' and 'every', even though connected by 'and', require a singular verb. Therefore, sentences 55 and 56 would also take singular verbs.

Qs. 58 to 61: Subjects with 'either', 'neither', 'many a' indicate a singular number. They are always followed by a singular verb. So, in sentences 58 and 60, 'is' should be used. In sentence 59, the correct verb would be 'goes'. The expression 'more than one' precedes a singular noun. Hence the subject is considered singular and the corresponding verb is also singular. Therefore, in sentence 61, 'is' would be the correct choice.

Qs. 62 & 63: Two or more singular subjects connected by 'or' or 'nor' require a singular verb. So, 'originates' and 'is' are the correct words for the sentences respectively.

## PRACTICE EXERCISE 2

Qs. 1 & 2. In sentence 1, the missing pronoun denotes a person who received a loan; thus it should be objective. Sentence 2 can be interpreted in two ways: Megha loves you more than I love you / Megha loves you more than Megha loves me. So, either form of the personal pronoun will do.

Qs. 3 to 10: In sentence 3, the missing pronoun is the subject of the verb 'watch'. So it should be subjective. In sentence 4, the missing pronoun is the object of the verb 'like', whose subject is 'you'. So, the answer will be 'whom'. Similarly, in sentence

5, the missing pronoun is the indirect object of the verb phrase 'hand over'. So, the answer will be 'whom'. In sentence 6, the missing pronoun is the object of the verb 'met'. So, the answer will be 'whom'. In sentence 7, the missing pronoun is the subject of the verb 'sit'. So it should be subjective. In sentence 8, the missing pronoun is the object of the verb 'ask'. So it should be objective. In sentence 9, the missing pronoun is the object of the preposition 'for'. So, it should be objective. In sentence 10, the missing pronoun qualifies the noun 'actor', so it should be 'whichever', not a personal pronoun.

Qs. 11 & 12: Gerunds (verbs in the 'ing' form that can be used as nouns too) should be preceded by possessive pronouns. The possessive forms of the various personal pronouns are: *my, our, your, his, her, their, its, whose*. Sentences 11 and 12 should use the pronouns 'my' and 'their' respectively.

Qs. 13 to 15: Sentence 13 can be interpreted in two ways: Riti looks out for Rahini and Rahini looks out for Riti / Riti and Rahini look out for some other people. Sentence 14 can also be interpreted in two ways: Smitha insisted on driving some other girl to the mall / Smitha did not want anyone else to drive. In the latter case, the emphatic pronoun 'herself' should be used. In sentence 15, the reflexive pronoun 'herself' should be used as Shefali did not hurt anyone else.

Qs. 16 to 20: 'Many a', in sentence 16, will be treated as singular. 'Nobody' (in sentence 17) is singular while 'both' and 'few' (in sentences 18 and 19) are plural. 'None' (in sentence 20) could be either singular or plural (as it can be treated as 'no one' or 'all'). As there is no clue hidden in the sentence, the blank can be filled with either option.

Qs. 21 to 23: When a pronoun stands for a collective noun, its number depends on the number implied by the collective noun. If the collective noun is viewed as a single unit, i.e., if the members constituting the group act together as one, a singular pronoun is used to replace the noun. If the collective noun indicates separate individuals constituting the group, a plural pronoun should be used. In sentence 21, the team members have not acted as a single team; rather, they have indicated their personal preferences. So, the plural pronoun 'their' has to be used here. The same is true for sentence 23: the members of the committee will consider the resolutions separately and give their individual

- opinions. From sentence 22, it is clear that the 'crew' as a single unit has failed to perform; hence a singular pronoun has to be used.
- Qs. 24 to 26: Compound subjects joined by 'and' take a plural pronoun, unless they both refer to the same person or idea or things that together make up one entity. Compound subjects joined by 'or/nor' take a pronoun that agrees with the antecedent closer to the pronoun. In sentence 24, two pronouns are combined using 'and', so the antecedent will be considered plural. In sentence 25, the antecedent of the pronoun is the proper noun 'Jai', as it is closer to the pronoun. Since, it is singular, the pronoun will also be singular. In sentence 26, the closer noun is 'friends', so the pronoun will be plural.
- Qs. 27 and 28: Titles of single entities (books, organizations, countries, etc.) take a singular pronoun. Sentence 27 will take a singular pronoun as the United States is one country. Similarly, sentence 28 will take a singular pronoun as one book is being referred to.
- Qs. 29 and 30: 'The number of' (followed by a noun) is treated as a singular antecedent and 'a number of' (followed by a noun) is treated as plural. So, sentence 29 should use 'its' and sentence 30 should use 'their'.
31. 'News' is a singular antecedent and must be followed by 'its', not 'their'.
32. The expression 'a number of' is a plural idea; therefore, the pronouns replacing the subject should also be plural. Thus, 'he' should be replaced by 'they' and 'his experience' should be replaced by 'their experiences'.
33. The determiner 'many a' before a noun or a series of nouns requires a singular referent. So, in the given sentence, the plural pronoun 'their' should be replaced by 'her'.
34. When two things are being compared, the pronoun should replace the thing that is actually being compared. Here, the geographic location of one centre is being compared to the geographic locations in other countries in higher latitudes. Therefore, the last part of the sentence should read: 'than those of the centres of other countries in higher latitudes'.
35. When two entities are joined by 'or/nor', the pronoun must agree with the noun closer to it. Here, the noun closer to the pronoun is 'employees', so 'his' should be replaced by 'their'.
36. Two or more singular subjects connected by 'nor' require a singular verb. So, 'makes' fits the blank correctly.
- Qs. 37 & 38: When one of the subjects joined by 'or' or 'nor' is plural, the verb must be plural and the plural subject should be placed nearest to the verb. Therefore, in sentence 37, the appropriate word would be 'have'. When the subjects joined by 'or' or 'nor' are of different persons, the verb agrees with the person of the nearer subject. In sentence 38, 'I' is in first person; the appropriate word for the blank would be 'am'.
- Qs. 39 to 41: Indefinite and distributive pronouns such as *anyone*, *everybody*, *everyone*, *nobody*, etc. are always singular and require a singular verb.
- Qs. 42 to 44: When a plural noun denotes some specific amount or quantity, which is considered as a whole, the corresponding verb is generally singular. Therefore, sentences 42 and 43 will require 'is' to make them complete grammatically. Some nouns (such as 'dozen'), which are singular in form but plural in meaning, take a plural verb. So, sentence 44 would require 'cost' to make it correct as it is more than one dozen.
- Qs. 45 to 49: Some nouns are used only in the plural. They may indicate a pair, e.g. trousers, scissors, spectacles, etc. 'Means', in the sense of income, is always considered a plural noun. Some other nouns, which are always used as plural, are: assets, nuptials, premises, annals, tidings, proceeds, etc. Therefore, plural verbs have to be selected for sentences 45 to 49.
- Qs. 50 & 51: 'None' is considered singular or plural as the context may require. In 50 and 51, either of the verbs can be selected. In 50, for instance, 'none of them *has* escaped' is another way of saying that 'not a single miscreant has escaped', while 'none of them *have* escaped' is another way of saying that 'no miscreants have escaped'. In other words, the two forms are slightly different ways of saying the same thing, and both are grammatically valid.
- Qs. 52 & 53: A collective noun takes a singular verb when the collection is thought of as one whole, as in sentence 52. When a collective noun is the subject of a sentence but the focus is more on the individuals who compose the group, the cor-

responding verb is plural. In sentence 53, 'team' refers to the individual members of the team, so the verb should be plural. (Hint: later on in the sentence, 'team' is referred to as 'they'.)

- Qs. 54 & 55: If a sentence compounds a positive and a negative subject, the verb should agree with the positive subject. In sentence 54, the positive subject is the singular word 'landlord', whereas the negative subject is the plural word 'tenants'. So, the verb should agree with the singular subject and be 'decides'. Similarly, in sentence 55, the correct word would be 'is', in agreement with the singular subject 'team leader'.
- Qs. 56 & 57: Fractional expressions such as 'a percentage of', 'a majority of', 'half of', etc. are sometimes singular and sometimes plural, depending on the nouns they qualify. Therefore, their corresponding verbs also vary. In sentence 56, the reference is to the 'actual consumers', i.e., a plural entity; hence the verb should also be plural. On the other hand, in sentence 57, the noun that is qualified by the expression 'a large percentage of' is the singular word 'generation'. Thus, the corresponding verb should also be singular, i.e., 'was'.
- Qs. 58 & 59: In constructions of the type 'one of [plural noun/pronoun] that (or 'who' or 'which')', the antecedent of 'that' is considered to be the plural noun/pronoun, and should therefore be followed by a plural verb. In sentence 58, 'memories', a plural noun, is the subject of the verb 'stand'. So, the corresponding verb should also be plural. Note: the verb 'stand' refers to all the memories that stand out; one of them *is* related to what happened to the author in the fourth grade. On the other hand, in sentence 59, the subject of the missing verb is 'one', so it should be singular, i.e. 'is'. Note: here the missing verb is related to the *one* problem that has been encountered.
- Qs. 60 & 61: In sentences beginning with 'here' or 'there', the actual subject follows, not precedes, the verb. In sentence 60, 'way' is the subject and in sentence 61, 'leaders' is the subject. So, 'is' is appropriate for 60 and 'come' is apt for 61.
- Qs. 62 & 63: Subjects don't always come before verbs in questions. Make sure you accurately identify the subject before deciding on the proper verb form to use. In sentence 62, the subject is 'dog' and in sentence 63, the subject is 'pages'. So, the singular verb has to be chosen for the former and the plural verb for the latter.

## VA-3.2 ANALOGY, FIB, CLOZE AND CONFUSABLE WORDS

### PRACTICE EXERCISE 1

1. Sentence (2) should read 'The sun is about to set on her illustrious career'. When the 'sun sets on something', it means 'something is about to end'. Hence, (2).
2. Sentence (1) should read 'In Africa we have to lay hands on only one interest, the agricultural interest.' 'To lay hands on something' is an idiomatic expression, which means 'to find something'. Hence, (1).
3. In sentence (1) the word should be spelt as 'break-away'. It has been wrongly split up. 'Breakaway' means 'separation from a group'. Hence, (1).
4. Sentence (2) should read 'fell afoul of the law' and not 'fell afoul with the law'. Hence, (2).
5. Sentence (4) should read 'She is so bound up in her own anxieties that she is inattentive to our needs'. 'Held up' means 'to get delayed' and is inappropriate for the context. 'Bound up' means 'inseparably connected with' and appropriately fits the context. Hence, (4).
6. Sentence (1) is incorrect. It should read, 'The reports bear out all my initial fears and misgivings.' This means the reports corroborate the initial misgivings. 'Bear up' means 'to endure'. Hence, (1).
7. Sentence (4) should read, 'I couldn't attend your wedding because I was called away by the army.' The given sentence makes no sense. 'Call off' means 'to cancel'. 'Call away' means 'to summon'. Hence, (4).
8. Sentence (4) should read, 'He has cut himself off from the rest of the world.' This would mean that he has become a recluse. Hence, (4).
9. Sentence (3) should read, 'He knocked about quite a few cities before settling down in Mumbai.' 'Knocked about' means 'wandered about'. Hence, (3).
10. Sentence (4) should be, 'As an infant I always picked at my mother's earrings'. 'Pick at' means 'to grasp at or touch' whereas 'pick out' means 'to choose'. Hence, (4).
11. The answer is option (C). Option (A), 'clarifications', is incorrect. There is nothing in the following sentences to imply that emotions have to be 'clarified' or explained – instead it is about the range of emotions around love. Reject option (A). Option (B) is incorrect. The cinematic depictions of love are said to be changing from the ones focused on romance – there is no implication of 'introspection' or self-analysis of specific emotions. Eliminate option (B). Option (D), 'betrayals' is incorrect. The paragraph implies that stories reveal, not betray, emotions. Reject option (D). The passage is about the current dominant position of romance in society, having changed from what it was a few centuries ago. That stories reveal/represent the different emotions in us is what the first sentence should convey. The apt word for expressing this would be 'portrayals' since it refers to the description of someone or something in a particular way. Hence, (C).
12. The answer is option (D). Option (A), 'appraise', is incorrect. If the stories shape our expectations, we will want to live by them, not analyse or appraise them. Option (B), 'test', is also incorrect as we want our expectations to be idealised into the form of love as portrayed on the silver screen and not test these expectations. Reject. Option (C), 'resist', is incorrect as we want to lead these feelings of love into its idealised form. One cannot achieve it by resisting these feelings. Reject. Our expectations are shaped by these stories and so these expectations would be what we would want to live by or 'abide' (which means 'to accept or act in accordance with'). Hence, (D).
13. The answer is option (B). Option (A), 'rationalise' is incorrect - we don't rationalise our feelings into an exalted form. Option (C) is incorrect - we cannot 'coerce' our feelings. Reject. Option (D) is also incorrect – we cannot coax or 'cajole' our feelings into an exalted form. Eliminate. The word 'idealise', which means to 'regard or represent as perfect or better than in reality' is the best fit. Hence, (B).
14. The answer is option (A). Option (B) is a grammatically incorrect choice. 'Germane' means 'relevant to a subject under consideration' but in usage, it

- is always followed by 'to'. Reject. Option (C) is incorrect as the position of romance in society is being compared between two time periods and it cannot be said to be 'urbane' i.e., 'having the polish of sophisticated social life in major cities'. Reject. Option (D) is incorrect as romance is not said to be an 'essential' part of culture even today. It is said to enjoy relatively more importance than it ever has. Eliminate it. The paragraph implies that romantic love has an unusual position of importance in the modern context whereas it held a peripheral one in the past. To emphasise its changed position as compared to the past, 'dominant' is the most appropriate word from among the options. Hence, (A).
15. The answer is option (C). Option (A) is incorrect – the paragraph does not find the position of love in society today a 'necessary' one. Eliminate. Option (B) is incorrect. It fits grammatically but is an inferior option to (C) because love is being observed in the cultural context specifically. Refer the previous sentence which mentions the 'cultural imaginary'. The nature of the acclaim that romantic love has acquired today in contrast to its peripheral position in the past should be conveyed by the sentence in terms of its changed social status. Among the given options therefore, it is 'cultural' that fits the blank most appropriately. Option (D), 'political', is incorrect as it has no basis in the paragraph. Hence, (C).
  16. Fascist politics (radical right-wing, authoritarian ultranationalism characterized by dictatorial power) cannot 'encourage' or 'cultivate' relations of mutual respect between citizens. 'Retract' (meaning 'withdraw') makes no sense in the given context. Only 'destroy' fits in the given context. Hence, (C).
  17. Citizens are the 'foundation' of a democracy and not its 'origins'. 'Bane' and 'heritage' are inappropriate in the given context. Hence, (B).
  18. 'Faith' is both grammatically and contextually correct. The presence of 'in' after the blank makes options (B), (C) and (D) incorrect ('hatred of', 'dedication to' and 'worship of' would be the correct usage). Hence, (A).
  19. Only 'successful' makes sense in the blank. 'Positive', 'progressive' and 'optimistic' cannot be associated with fascism. Hence, (A).
  20. The gist of the paragraph is that the fascist politics destroys the relations of mutual respect between citizens and replaces it with faith in one figure alone. Thus, only 'trustworthy' is the correct option. The other options do not make sense in the given context. Hence, (C).
  21. If one reads the entire paragraph, one can understand that it is about firing people. Thus only option (3) correctly fits the first blank. Though 'dismissing' also fits the first blank' however, in large corporations 'dismissal' would imply that the employee is not suited for the job for some reason like inefficiency or indiscipline. Hence dismissal is not "common" in large corporation. Hiring and firing is common. 'Punishing' is incorrect in this case as the paragraph is only about removing a person from the job and not admonishing (which means 'scolding') or punishing him about a mistake. Hence, (3).
  22. The phrase 'using a gun or switch blade' implies the use of threat to solve or settle an argument. Thus only 'resolve' which means 'to settle or solve' correctly fits the second blank. Threat does not 'thwart' ('to oppose successfully') or defeat or close an argument. Thus the other options get eliminated. A threat, however, can resolve (break up or dissolve) an argument instantly and perhaps temporarily. In the context 'firing' resolves certain issues in large corporations instantly. Hence, (1).
  23. As per the last sentence, the important learning of the managers while firing a person is to remember that the fired employee is a 'failed product' and to try to understand how such a product was allowed to enter the organisation. 'Derived' means 'obtained from a source' and is incorrect in the given context as an employee cannot be 'derived' from somewhere. 'Engineered' means 'designed or created using engineering methods'. An employee cannot be 'created or designed'. Thus option (2) can be negated. An employee cannot be 'produced' as well. Hence, (4).
  24. Collective nouns can be considered singular or plural depending on the context. When people do one thing 'in unison' with the other members of their group, that noun becomes singular. As a result, you must use singular verbs and pronouns with it. However, when the members of a group are 'acting as individuals', the collective noun is plural and requires plural verbs and pronouns.

If there are two verbs in the sentence, one also needs to identify the principal verb in the sentence. All other verbs in any phrase modifying the noun will take the number which is assigned to the noun by the connotation of the principal verb. In the first sentence, 'is (A) / are at sixes and sevens', is the principal verb and pertains to the disagreement among the individual members of the council. Here, the council members are acting as individuals, that is, they are 'at sixes and sevens', meaning, 'they do not agree'. Hence, the council, in this context, will be a plural noun. Once identified as a plural noun, verbs in any phrase modifying the plural noun will be plural. '...that was/were elected last March' is a modifying phrase. The verb will stand in agreement with the plural noun. Therefore, even 'that was/were elected last March' would take a plural verb. Thus the correct order is BBBBA. Hence, (3).

25. 'Further' in sentence 1 means 'to a great degree or extent' while 'farther' denotes distance. In this case, 'further' fits the blank correctly as it is not the distance but rather the extent of pushing oneself further that is discussed. 'Historic' means 'significant' whereas 'historical' means 'connected with the past'. Thus 'historic' fits the second sentence. 'Mistrust' and 'distrust' are synonymous, but the latter is slightly stronger than the former. If you are sure that someone is acting dishonestly or cannot be relied on, you are more likely to say you 'distrust' them. If you are expressing doubts and suspicions, on the other hand, you would probably use 'mistrust'. Also, distrust is always used with 'of' and mistrust with 'for'. Thus 'mistrust' fits the third sentence. When we refer to something connected with facts rather than things that have been invented or guessed we use 'true'. 'Real' is used to refer to something that exists and is not imagined. Thus 'true' is a better fit for the fourth sentence. 'Compliment' means 'praise' whereas 'complement' is 'something that completes'. The 'compliment' is backhanded means the compliment is indirect or ambiguous. So the correct order is ABBBA. Hence, (2).
26. The answer is option (2), BBBAB. 'Regretfully' means 'feeling or showing regret' while 'regrettably' denotes 'something undesirable or unwelcome'. Since the speaker is feeling sorry for declining the invitation, 'regretfully' completes the sentence meaningfully. 'Sensual' means 'depending on the senses' whereas 'sensuous' means 'aesthetically

pleasing'. Since the second sentence is talking about the quality of her paintings, 'sensuous' is a better fit in the second sentence. 'Besides' means 'in addition to'. The usage 'to be beside yourself with something' means 'you are unable to control yourself because of the strength of emotion (here, rage) you are feeling'. 'Stationary' means 'motionless' whereas 'stationery' stands for 'materials used for writing'. So 'stationary' fits the fourth sentence. 'Above' shows 'at or to a higher place or position than something'. 'Over' denotes 'movement or position beyond something stated or implied'. Thus 'above' fits the last sentence correctly. Thus the correct order is BBBAB. Hence, (2).

27. 'Silver bullets' refers to 'something that acts as a magical weapon; a very effective (almost magical) remedy; especially one that instantly solves a long-standing problem'. In option (1), while 'hope' may fit in the first blank grammatically, it contradicts the logical tenor of the sentence. Since 'silver bullets' or cures have not been found, there is no reason for a positive word like 'hope'. 'Scientific' expectations – for the second blank – makes little sense. Eliminate option (1). Similarly, in option (2), while 'optimism' may be apt for the first blank grammatically, it contradicts the sentence logically - since 'silver bullets' have not been found, there is no room for optimism. Eliminate option (2) and (5). Between options (3) and (4), we choose option (4) since 'pessimism' is a more apt word than 'diffidence' in the context. 'Diffidence' means 'a lack of self-confidence' and is not suitable contextually. If 'silver bullets' or cures have not been found, expectations about them can be termed 'unrealistic' and 'pessimism' can arise. Moreover, option (4) aptly fits the context of the second sentence. Hence, (4).
28. 'Dissent' is the clue in the sentence to decide the operative idea; the next clue is 'savage'. Putting together these clues helps us decide option (3) as the best choice to get the most meaningful sentence. 'Totalitarian' regimes would treat dissent with savage 'reprisal'. Moreover, option (3) aptly fits the context of the second sentence. 'Democratic' or 'liberal' regimes would not have savage reactions to dissent. Eliminate options (1), (4) and (5). 'All ancient regimes' would not treat dissent with savage rituals. Eliminate option (2). Hence, (3).

29. As per the sentence, the term 'intelligent design' was first explained by William Paley. Now, proponents of intelligent design are again using this term by relating it to molecular biology. Thus 'destroying' is incorrect in the given context. 'Testing' is also incorrect as they are not testing this term but applying it to some other theory. 'Resurrecting' means 'bringing it back into use' and correctly fits the given sentence. 'Questioning' is also incorrect in the given context. Hence, (3).
30. Seeing so many beggars can be 'irritating, disgusting or distressing' for anybody. However, since the sentence specifies 'a sensitive traveller', only 'distressing' which means 'causing great pain or anxiety' correctly fits the highlighted word. Hence, (4).
31. Since the note is very short, only 'terse' which means 'curt or brusque' correctly fits the highlighted word. Hence, (4).
32. 'Reports, stories and books' are not specifically written for fools. 'Reports' can be written for a specific purpose while 'stories and books' are written for people in general. Thus none of them fit the first blank. 'Obituaries' mean 'published announcements of death' while 'fading away' means 'dying' or 'vanishing slowly'. Thus the given sentence means that though 'obituaries' are written not only for people but also for practises, organisations and institutions, these institutions do not seem to be dying. Hence, (4).
33. As per the sentence, the image of Darwin comes across as someone who possesses the attributes of a world class thinker as well as a head of the household. Thus 'figures' is incorrect in the second blank. This negates option (1). 'Arises' means 'wakes up' or 'comes into being'. Since the sentence implies the image of Darwin became apparent from certain things, 'arises' is incorrect to fit the first blank. Thus option (2) can also be negated. 'Appeared' is in the past tense while the rest of the sentence is in the present tense. Thus option (4) is grammatically incorrect. Hence, (3).
34. One can know the reaction of a person to a particular event by looking at the emotions or expressions on that person's face. Since the absence of make-up or scars does not influence the emotions of a person's face, options (1) and (4) can be negated. 'Diagnose' means 'to determine the identity of (a disease, illness, etc.) by a medical examination'. Since the sentence does not suggest that she was doing a medical exam, option (3) can also be negated. Both 'expression' and 'ascertain' correctly fit the given sentence as it implies that since there was no emotion on her face, one couldn't know if she appreciated what had happened. Hence, (2).
35. An 'affair' (i.e. 'a particular action or event') of the labour movement cannot be 'weird' (which means 'involving or suggesting the supernatural'). Thus option (1) can be negated. Any activity of the British labour movement cannot be 'moving' which means 'evoking strong feelings' or 'capable of movement'. Thus option (2) is incorrect in the given context. Both 'experience' and 'significant' correctly fit the two blanks as an experience of the British labour movement can be important. Retain option (3). The 'atmosphere' (which means 'a special mood or character associated with a place') of a labour party movement cannot be 'gloomy' (which means 'sad'). Hence, (3).
36. 'Specious, deceitful, fallacious and deceptive' all mean 'tending to deceive or misleading'. But 'credible' means 'believable or trustworthy'. Hence, (3).
37. 'Obviate, forestall and preclude' all mean to 'prevent'. Thus they can be eliminated. 'Bolster' means 'to support or uphold' and is incorrect in the given sentence. Hence, (4).
38. 'Discarded, obliterated and unfashionable' all are similar in meaning to 'disuse' which means 'not in use or practice. However, 'prevalent' means 'widespread' or 'generally used' and is incorrect in the given sentence. Hence, (1).
39. 'Parsimonious' means 'frugal or stringy'. Thus option (1) can be negated. 'Penurious and thrifty' mean both mean 'stringy' and are thus the synonyms of the given word and can be negated. 'Altruistic' means 'devoted to the welfare of others' and does not fit the given sentence. Hence, (4).
40. 'Facetious' means 'not meant to be taken seriously or literally' or 'humorous'. Thus 'jovial, jocular and joking' all are synonyms of the given word and can be negated. However, 'jovian' means 'of or relating to the Roman god Jupiter or the planet Jupiter'. Hence, (1).

41. A 'doctor' cures or treats 'diseases'. In option (1), a 'policeman' apprehends 'criminals', he does not rehabilitate them. Thus, this option is not the answer. In option (2), a 'dentist' drills 'teeth', thus not the answer. In option (3), a 'politician' is elected by the 'electorate', thus not the answer. In option (4), a 'teacher' cures or treats 'ignorance' by imparting learning. Although this might not be the exact relationship, the objective is similar. Hence, (4).
42. 'Ornithology' is the study of 'birds'. In option (1), 'zoology' is the study of 'animals', however the order of the words has been reversed, thus not the answer. In option (2), 'galaxy' is a collection of 'stars', thus not the answer. In option (3), 'archaeology' deals with 'antiquity', which means extreme old times, but the order has been reversed. In option (4), 'botany' is the study of 'plants', thus it is the correct answer option. Hence, (4).
43. 'Food' serves as an antidote to 'hunger' or it absolves hunger. In option (1), 'sleep' serves as an antidote to 'weariness' or absolves weariness. In option (2), we 'slumber' at 'night', slumber means being asleep - does not have the same relationship as mentioned in the defining sentence. In option (3), we 'dream' while we are 'asleep' - It does not share the same relationship as the capitalized pair. In option (4), 'drink' serves as an antidote to 'thirst' or absolves 'thirst' but the words are not in the same order, thus not the answer. Hence, (1).
44. A 'dart' is a smaller version of a 'spear', both are used as weapons. In option (1), 'sword' is not a smaller version of a 'knife', though a similar relationship has been expressed, the words are in reverse order. In option (2), a 'window' is more or less a smaller version of a 'door', but the objective of both is not the same. A door serves as an entry for people into a place while a window serves as an entry for light and air, though door allows air and light too. Actually this is a close contender. In option (3), a 'molehill' is not a smaller version of a 'mountain'. And a molehill is not a naturally formed heap, but formed when ants or insects dig into the ground. In option (4), a 'gun' is a smaller version of a 'cannon' and both are used as weapons. Hence, (4).
45. A 'building' is made up of 'bricks'. In option (1), addition of 'lime' to 'cement' mortar makes it thicker. Hence, it is not relevant to the defining sentence. In option (2), mixed 'asphalt' and crushed gravel or sand is used especially for paving 'highways'. In other words, highways are made of asphalt. In option (3), a 'bridge' is a structure that allows people or vehicles to cross the 'river'. Hence, it is irrelevant to the defining sentence. In option (4), a 'road' is an open way (generally public) for travel or transportation of 'vehicles'. Hence, it is incorrect. Hence, (2).
46. Option (4) is incorrect; the correct usage is 'got off the mark' not 'got on the mark'. Hence, (4).
47. Option (2) is incorrect; the correct usage is 'take it easy' not 'take off easy'. Hence, (2).
48. Option (3) is incorrect; the correct usage is 'home and dry' not 'home to dry'. Hence, (3).
49. Option (4) is incorrect; the correct usage is 'at a loose end' not just 'a loose end'. Hence, (4).
50. Option (1) is incorrect; 'greenhorn' is a single word meaning 'inexperienced or immature'. Hence, (1).
51. An 'entomologist' studies 'insects'. Similarly, a 'philologist' studies literary 'texts'. A 'semanticist' studies the 'relationship between symbols, words and their meaning'. A 'linguist' studies the 'science of languages'. A 'geologist' studies the 'physical surface of the Earth'. Hence, (4).
52. The relationship is one of degree. 'To jog' is 'to run slowly' while 'to sprint' is 'to run fast'. Similarly, 'trot' is 'a horse's slow movement', while a 'gallop' is used to refer to 'a horse running fast'. Hence, (1).
53. 'To tether' is 'to fasten by rope or a chain to some object'. Animals are usually tethered in order to confine them to a particular area. Similarly, 'to moor' is 'to secure in a particular place by cables and anchors'. Ships are usually moored in order to confine them to a particular area. All the rest of the options refer to objects and where they are kept. Hence, (3).
54. The relationship is the have-not link. A 'gauche' person is clumsy and lacks social skills or 'tact'. Similarly, a 'novice' lacks 'experience'. 'Adroit' means 'having skill or finesse'. 'Morbid' means 'sickly or unhealthy'. 'Conceited' is 'having a very high opinion of oneself'. Hence, (3).
55. A 'pugilist' is a 'boxer'; he 'punches'. Similarly, the main action of a 'wrestler' is to 'grapple'. A 'skier' skis; an 'oarsman' rows; an 'athlete' runs, leaps, jumps or sprints. Hence, (4).



56. Sentence (4) should read, 'Difficult situations brought out the best in him', meaning, 'he thrives in difficult situations and they suit him'. Hence, (4).
57. Sentence (4) should read, 'I don't want our private problems to carry over into our professional life'. Hence, (4).
58. The correct idiomatic expression is 'turn in for the night', which means 'to go to bed'. Hence, (2).
59. In (1), there is an incorrect expression. The correct phrase is 'do well by' which means 'treat generously'. Hence, (1).
60. In (4), the correct phrasal verb should be 'speak up' (meaning 'talk loudly') not 'cry up' (meaning 'praise'). Hence, (4).
6. As per the sentence, the US' war on terrorism may indicate trouble in the Gulf. 'Portends' means 'indicates in advance' or 'bodes'. Hence, (4).
7. As per the sentence, when she was shown a video tape of her meeting, she started speaking evasively. 'Prevaricating' means 'speaking evasively with an intent to deceive'. Hence, (1).
8. As per the sentence, the crowd became impatient and restless when the minister did not appear even by 10 p.m. 'Restive' means 'restless or uneasy'. Hence, (3).
9. 'Ostensible' means 'apparent'. Hence, (1).
10. Since King's is mentioned to be the subsidiary of M&S, it does not make sense for the company to try and 'acquire' its own subsidiary. So we rule out option (1). 'Concede' means 'to acknowledge, often reluctantly, as being true'. This fits the mood of the sentence better than 'announced'. Thus option (3) can be negated. 'Ratified' means 'confirmed by formal sanction' and does not make sense in the first blank as a company cannot make a formal sanction of a defeat. Hence, (2).

## PRACTICE EXERCISE 2

1. Sentence (4) should read 'of his own accord' and not 'on his own account'. Hence, (4).
2. Among all the possible combinations, only the combination A-E, B-F, C-G and D-H, viz. option (4) makes sense. Hence, (4).
3. We find that only (3) matches all the definitions to the usages correctly. Hence, (3).
4. Read all the sentences carefully. In sentence (1), the word 'shoot' is used to mean 'a young branch'; this usage in this context is correct. In sentence (2), 'shoot' is used as a verb meaning 'to run quickly'. This too is a perfectly correct usage. Sentence (3) is somewhat trickier, as 'shoot' is applied to the action of both cameras and guns. One needs to ensure that it is correct in both contexts. However, 'shoot' can mean both 'to hit with a missile from a weapon' and 'to make a film or photograph of something;' i.e., it applies to both guns and cameras. So sentence (3) is correct. Sentence (4) is clearly an incorrect usage of 'shoot'—and not only because it is the only remaining one. The correct expression should be 'gun shot wound' not 'gun shoot wound'. Hence, (4).
5. As per the sentence, the police officer's conduct was 'partisan' (which means 'partial to a specific party, person, etc.'). Thus he would be criticized for his behaviour. 'Opprobrium' means 'the state of being scornfully criticized'. Hence, (1).
11. As per the sentence, the tendency to develop friendship networks outside college masks signals of maladjustment i.e. it becomes difficult to understand the signals of maladjustment to college culture. Thus 'treatment or prevention' can be negated in the first blank as one can only 'treat or prevent' something that is apparent or visible. This negates options (1) and (4). 'Facilitated', which means 'makes it easier', is also incorrect in the second blank as the signals of maladjustment are masked. Thus option (2) can also be negated. Hence, (3).
12. As per the sentence, Spain had different regions and the views about a common language within each region were also different. This 'competing' can be negated as the regions were not competing for a common language. This negates option (4). 'Divergent' means 'differing or deviating'. Since the regions of a common country cannot be 'deviating', option (3) can also be negated. One of the meanings of 'distinct' is 'unique' and thus is incorrect in the context of the given sentence as it would mean 'the unique regions of Spain all have unique cultures'. Thus option (2) can also be negated. 'Different' and 'discrete' (which means 'separate') correctly fit both the blanks. Hence, (1).

13. As per the given sentence, expert professionals are unhappy to train foreigners as these foreign students replace the teachers who then have to work for low pay or become unemployed. Thus the expert professionals do not 'welcome' the foreign students for training. This negates option (3). 'Are' is grammatically incorrect in the given sentence. Thus option (4) can also be negated. 'Challenging' is incorrect in the second blank as only challenging will not lead to unemployment or reduced pay packages. Thus only 'resent' and 'replacing' fit both the blanks correctly. Hence, (1).
14. As per the given sentence, if a company does not give rewards, it encourages negative behaviour rather than encouraging a genuine interest in working better. 'Giving' or 'bestowing' rewards will not encourage negative behaviour. Thus options (1) and (2) can be negated. 'Discrediting' means 'destroying the confidence in something' and cannot fit the second blank as a company cannot discredit a genuine interest in working better. 'Withholding' rewards can encourage negative behaviour instead of 'fostering' (which means 'encouraging') a genuine interest in doing the work well. Hence, (4).
15. 'Cryptic' means 'puzzling or based on a code'. 'Ambiguous' is 'vague, by accident or intent'. 'Apocryphal' means 'of doubtful authorship or authenticity'. 'Perplexing' means 'complicated or confusing'. The phrase, 'other equally cryptic works' in the latter part of the sentence gives us a clue that 'cryptic' would be the correct choice for the first blank. However, since the other words for the first blank among the given options have similar meanings we will consider all the words to be suitable and look for our answer from the second blank. 'Phaneritic' is 'related to rocks that have visible crystals'. It is completely unrelated to the context. This eliminates option (3). 'Esoteric' means 'understood only by a select few'. Since the 'symbols, diagrams and textual imagery' contain multiple layers of meanings, 'allegories and references to other equally cryptic works, they must be laboriously decoded in order to discover their real or true meaning.' Thus, the second blank cannot have a meaning that is 'inferred', or understood by only a select few. Eliminate options (1) and (4). Moreover, the word 'plain' implies that the meaning is already clear and straightforward - i.e. it would not have to be 'decoded'. This eliminates option (5). Hence, (2).
16. The 'universe' is a 'cosmic' body. In option (1), the 'earth' is a 'terrestrial' body. In option (2), the 'Moon' is a 'lunar' body. In option (4), a 'ring' is an 'annular' body. In option (3), 'youth' is 'not' a 'connubial' body. 'Connubial' means, 'of, or relating to the relationship between husband and wife.' Hence, (3).
17. 'Canine' 'pertains' to 'dogs'. Though it is not a synonym to dog, but canine refers to dogs. In option (1), 'feline' 'pertains' to 'cats'. In option (3), 'serpentine' 'pertains' to 'snakes'. 'Serpentine' means 'snakelike'. In option (4), 'vulpine' pertains to 'fox'. 'Vulpine' means 'fox like characteristics'. In option (2), 'aquiline' pertains to 'a parrot's beak' and not the bird as a whole. This is also close as an answer choice, but it is not as holistic as the other options. Hence, (2).
18. 'Xenophobia' is fear of 'foreigners'. Option (1) is the answer as 'claustrophobia' is 'fear of closed space'. Thus this option does not have a similar relationship as the capitalized pair. In option (2), 'anglophobia' is a fear or dislike for the 'British' people. In option (3), 'bibliophobia' is fear of 'books'. In option (4), 'hemophobia' is fear of 'blood'. Hence, (1).
19. A 'reaction' is a result of an 'action'. Options (1),(3) and (4) do not fit the relationship 'is a result of'; as 'extrovert' is not a result of 'introvert', 'doctor' is not a result of 'diseased' and 'treatment' is not a result of 'death'. Hence, (2).
20. 'Malapropism' is the misuse of 'words'. Option (1) is the correct answer. 'Anachronism' is the misplaced 'time' of an event. Option (2) is a close contender; 'ellipsis' means 'omission of a word or a part of a sentence'. However, this cannot be interpreted as the misuse of a sentence. Option (3) is incorrect as there is no word as 'jinjanthropism'. In option (4), 'catechism' means 'a series of questions asked' and has no relationship to 'religion'. Hence, (1).
21. The adjective form of the noun 'salt' is 'salty'. Anything that contains or is filled with salt is referred to as salty. Option (1) is the correct answer. 'Sweet' is filled with 'sugar'. In option (2) 'bovine' refers to a 'cow' but since the given pair refers to taste, it can be negated. In option (3), 'bitter' and 'sour' are synonyms and adjectives. Hence this option is incorrect. In option (4), 'taste' and 'flavour' are near synonyms and nouns, hence this option is incorrect. Hence, (1).

22. 'Divide' is to break into parts while 'unite' is to bring two or more parts together, hence they are antonyms. We are looking for a similar relationship. In option (1), 'split' and 'apart' are synonyms; hence this option is incorrect. In option (2), 'marriage' is the act of two people coming together while 'divorce' is the act of two married people breaking apart. Though these two words are near antonyms, the relationship is inverted. Option (3) is the correct answer, 'fission' is the process of a bigger nucleus splitting into smaller nuclei, while 'fusion' is the process of two nuclei merging to form one nucleus. In option (4), 'chasm' and 'gap' are synonyms; hence this option is incorrect. Hence, (3).
23. 'Crime' leads to 'punishment'. We are looking for a similar relationship. In option (1), 'lawyer' and 'judge' are two related professions, hence this option is incorrect. In option (2), a 'court' is a type of 'room', hence this option is incorrect. In option (3), 'accused' and 'defendant' are synonyms, hence this option is incorrect. Option (4) is the correct answer. 'Homicide' (killing of a human by another human) leads to 'penalty'. Hence, (4).
24. 'Nuts and bolts' refer to detailed information about how something works. We are looking for a similar relationship. Option (1) is the correct answer. 'Nitty-gritty' refers to 'an important part about some idea or experience'. In option (2), 'feet' could be 'bare', hence this option is incorrect. In option (3), 'naked' and 'clothed' are antonyms, hence this option is incorrect. In option (4), 'hard' and 'soft' are antonyms, hence this option is incorrect. Hence, (1).
25. 'Just' means 'precise or fair' indicating a positive sense while 'arbitrary' means 'randomly picked', hence they are antonyms. We are looking for a similar relationship. Option (1) is the correct answer, 'order' (indicates a positive sense) and 'chaos' are antonyms. In option (2), although 'bare' and 'clothed' are antonyms, bare or clothed do not indicate any positive sense, hence this option is incorrect. In option (3), 'hope' and 'surprise' are not antonyms, hence this option is incorrect. In option (4), although 'improper' and 'proper' are antonyms, this is not the most appropriate choice since they share an inverse relation as compared to the capitalized pair. Hence this option is incorrect. Hence, (1).
26. 'Horology' is the science of measuring 'time'. Similarly, 'speleology' is the exploration and study of 'caves'. 'Entomology' is the branch of zoology 'dealing with insects'. 'Meteorology' is the 'study of the atmosphere, climate, and weather' and not of 'meteors' or 'small celestial bodies traveling very quickly through space'. 'Psephology' is the study of 'elections'. Hence, (1).
27. An 'oenophile' is 'a person who enjoys wines, as a connoisseur'. Similarly, a 'gourmet' is 'one who enjoys and knows a lot about good food'. A 'harpsichord' is a 'keyboard musical instrument, a precursor of the piano'. An 'equestrian' is 'one who rides a horse or performs on horseback', but not necessarily a connoisseur of fine 'horses'. A 'technician' is 'one whose occupation requires training in a specific technical process', but not necessarily one who understands the 'science' related to that process. Hence, (2).
28. The relationship between the two words in the stem is that of degree. When two things are 'disunited', they are separated or divided; when they are 'irreconcilable', they are incapable of being brought together again. 'Ravenous' in option (1) means extremely 'hungry', so this pair shows a similar relationship. 'Comatose' means 'unconscious'. 'Fatal' means 'capable of causing death' while 'viable' means 'capable of life or normal growth and development'. 'Slothful' means 'lazy'. Hence, (1).
29. 'Infected' people - i.e. those contaminated with a contagious disease - may need to be 'quarantined' - i.e. put in a strict isolation imposed to prevent the spread of disease. Similarly, when people are seriously 'ill', they might be 'hospitalized' - i.e. placed in a hospital for treatment, care, or observation. 'Sedated' means 'calmed or relieved by means of a drug' and 'anesthetize' means 'render physically insensible by means of drugs'. 'Rusted' means 'corroded by way of being 'oxidized' or 'combined with oxygen'. One way of acquiring a 'tan' or the brown colour imparted to the skin by exposure to ultraviolet rays is to get 'sunned' or exposed to the sun's rays. Hence, (3).
30. The relationship is one of 'defining characteristic'. A 'burlesque' is an artistic composition that, for the sake of laughter, 'vulgarizes' lofty material. The only option that bears the same relationship is (3): a 'tirade' is 'a long, angry or violent speech,

- one that usually 'denunciates' (condemns) someone or something'. A 'dirge' or funeral song does not necessarily 'eulogize' or praise; it laments. A 'caricature', or a picture or description ludicrously exaggerating the peculiarities or defects of persons or things, does not 'berate' or rebuke. A 'limerick' is a light humorous, nonsensical, or bawdy verse of five lines; it does not 'criticize', it entertains. Hence, (3).
31. 'Ostracism' means a forcible exclusion from 'society'. The only option that bears the same relationship is option (2): 'expulsion' is a forcible removal from 'school'. All the other options have words that indicate a wilful movement from a place or situation. Hence, (2).
32. The relationship is one of 'have-not'. A 'glutton', a person who eats or drinks excessively, is not 'abstemious', or having forbearance from any indulgence of appetite. The only option that bears the same relationship is option (1): a 'virago', or domineering, ill-tempered woman, is not 'affable' or likeable. A 'misanthrope' is 'a hater of humankind'. 'Dulcet' means 'melodious'. An 'ensemble' is 'a coordinated outfit or costume', while 'eclectic' means 'selecting from various sources'. Hence, (1).
33. 'Somatic' means pertaining to the 'body'. The only other pair exhibiting the same relation is option (3): 'noetic' means pertaining to the 'mind'. Hence, (3).
34. The relationship is one of 'defining characteristic'. A 'toady', a person who flatters or defers to others for self-serving reasons, is characterized as 'obsequious' or full of servile compliance. The only pair to display a similar relationship is option (2), as a 'seditionist', one who rebels against the authority of a state, is characterized as 'factious' or mutinous. Hence, (2).
35. A 'sword' is contained in a 'scabbard'. The only other pair that shares a similar relationship is option (2): 'pins', when not in use, are stuck into a 'cushion'. Hence, (2).
36. Just as a 'collocutor' is a participant in a 'conversation', a 'thespian', or actor or actress, is a participant in a 'play'. Hence, (2).
37. As per the sentence, this book is useful for someone who wants to make important decisions. 'Primer' means 'an elementary book for teaching children to read'. Thus option (1) can be negated. 'Tract' means 'a brief pamphlet for general distribution, usually on a religious or political topic' and is incorrect in the given context. 'Under' is grammatically incorrect in the second blank as nobody is under important decisions. Thus only 'guide' and 'facing' correctly fit both the blanks. Hence, (4).
38. As per the sentence, physicians can help paralysed people to move by bypassing the damaged nerves. 'Impediments' means 'obstacles' and is incorrect in the first blank. Physicians do not need a reason to help paralysed people. Also, there needs to be an article preceding 'reason' to make the sentence grammatically correct. Thus option (3) can be negated. 'Detrimental' means 'harmful'. Since the nerves once controlled their muscles, they cannot be harmful. This negates option (1). Only 'ways' and 'damaged' correctly fit both the blanks. Hence, (2).
39. As per the sentence, there is no limit of choices and hardly any constraints for internet users. Thus only option (3) correctly fits both the blanks. Hence, (3).
40. 'Lying' 'under oath' is 'perjury'. In option (1) making a 'statement' 'under oath' is 'testimony'. In option (2), 'seeing' in general is a part of 'observation' not necessarily 'under oath'. In option (3), 'taking' without 'permission' is 'stealing'. In option (4), 'eating' in general is 'dining' and has nothing to do with 'under oath'. Hence, (1).
41. 'Prehistoric' and 'medieval' are periods or points in history or refer to a timeline. In option (1), 'Akbar' and 'British' are not time periods or timelines. In option (2), 'present' and 'future' are time periods or points on a timeline, thus it is the answer. In option (3) are two people from history and have no similarity to the relationship mentioned in the defining sentence. In option (4) an 'elephant' is a 'colossus' animal. The two words are synonyms and do not have the same relationship as the one defined in the sentence. Hence, (2).
42. 'Loud' and 'stentorian' are synonyms. In options (1), (2) and (3) the pair of words are not synonyms, whereas in option (4) 'resplendent' means 'bright, splendid or beautiful'. Hence, (4).
43. 'Limpid' is the opposite of 'murky'. 'Limpid' means 'bright' and 'murky' means 'gloomy and clouded'. In option (1), 'dazed' and 'clouded' mean the same as 'dim and gloomy'. In option (2), 'obscure' and 'vague' mean the same as 'unclear or lacking

- clarity'. In option (3), 'bright' and 'gloomy' are antonyms and this pair has the same relationship as depicted in the defining sentence. In option (4), 'nebulous' means 'dim' or 'lacking clarity', thus not the opposite. Hence, (3).
44. 'Alleviate' is 'an act of making things easier'. In option (1), 'hint' and 'allocate' have no relationship at all. In option (2), 'revolt' and 'repudiation' mean the same and are acts of rebellion. In option (3), 'collapse' and 'rise' are antonyms. In option (4), 'interrogate' is 'an act' of asking 'questions'. Therefore, it is the correct answer. Hence, (4).
45. 'Secret' and 'clandestine' are synonyms. In option (1), 'overt' means 'open and visible' and 'furtive' means 'secretive behaviour'. Therefore, they are not synonyms. In option (2), 'covert' and 'stealthy' mean the same as 'secretive' and are also synonyms. In option (3), 'open' is an antonym to the word 'closed'. In option (4), 'news' and 'rumour' are both information but do not have a similar relationship to the capitalized pair. Hence, (2).
46. A 'drama' is 'viewed by' an 'audience'. In option (1), 'brawl' means 'a fight' and 'vagabond' means 'one without a fixed position' or 'moving around constantly'. In option (2), a 'game' is 'viewed by' 'spectators', thus it is the answer. In option (3), 'art' and 'critics' do not have the same relationship as the capitalized pair. In option (4), 'movie' and 'actors' do not share the same relationship as a movie is not 'viewed by' actors. Hence, (2).
47. A 'building' consists of 'storeys' or 'storeys' constitute a 'building'. In option (1), a 'book' consists of 'chapters'. In option (2), a 'sentence' may or may not consist of an 'adjective'. Secondly, 'adjectives' do not constitute a 'sentence'. In option (3), a 'tree' consists of 'stems' but a 'stem' does not constitute a 'tree'. In option (4), an 'elephant' does not consist of 'tusks' and 'tusks' do not constitute an 'elephant'. Hence, (1).
48. 'Germane' and 'pertinent' are synonyms. The pair of words in options (1), (2) and (3) are synonyms. In option (4) the words 'excited' and 'serene' are not synonyms in their relationship. Therefore, it is the correct answer. Hence, (4).
49. A 'sail' helps a 'ship' to navigate. In this instance, sail is not a verb but a noun which means 'a fairly large piece of fabric used on ships'. In option (1), 'propeller' helps fly an 'aeroplane'. In option (3), 'hydrogen' helps fly a 'balloon'. In option (4), an 'accelerator' helps run or drive a 'car'. In option (2), a 'radar' is not used to run or navigate a 'satellite'. Thus option (2) is the answer. Hence, (2).
50. The word 'economical' either refers to 'something that is cheap and low-cost', or 'someone that is frugal'. It makes little sense in the given context. 'Pecuniary' means 'something relating to or consisting of money'. The word 'penurious' refers to 'poverty'. It does not fit with the context. 'Accounting' means 'the process or work of keeping financial accounts'. Between 'accounting' and 'economic', the latter is the correct choice. Hence, (C).
51. Large economic gains can only be realized when the population is 'larger'. 'Smaller' population cannot contribute to economic gains. 'Contended' and 'more active' do not make any sense here. Hence, (B).
52. Only 'dependent' can fit in the blank. The other options make no sense in the given context. Hence, (C).

### PRACTICE EXERCISE 3

1. Only 'adaptation' and 'acclimation' are correct since the sentence talks about "getting used to new technologies". However only 'typical' is correct for the second half. Hence, (4).
2. A leopard is likely to become a man-eater when there is a scarcity of herbivores. A 'scavenger' means a hunter and fits in best in the first blank. Hence, (1).
3. Be careful about the use of 'but' in the sentence. It indicates that the word in the second blank should have a negative connotation. Between 'limits' and 'eliminates', the former is a better fit because 'eliminates' is too extreme a word in this context. Hence, (3).
4. Since the marketplace of ideas has a wide range, we can call it a highly diversified one. Whenever there is an argument, there is an intention to convince the person on the opposite side. Only option (2) has words that are appropriate for both the blanks. Hence, (2).
5. We need a word synonymous with 'enthusiasm' to fill this blank. This is clear from 'so is its enthusiasm for the internet'. 'Phobia' is 'fear', 'difficulty' and

- 'destruction' are clearly unsuitable. Only 'obsession' meets the requirement for this blank. Hence, (2).
6. This is an 'initiative' on the part of the government. 'Aggression' is completely out of the purview of the context. Hence, (1).
  7. The clue for this blank is in the relation of the two figures (70% and 80%) quoted in the line. 'Extend' and 'arrive' do not fit in the blank because there is no appropriate preposition to follow them. We need a word that means 'touch'. 'Reach' is the better option as 'attain' gives a sense of completion. Hence, (4).
  8. The passage states that the government is not likely to achieve its targets of government business accessed through the internet. This indicates that the word for this blank should mean 'disconcerting'. Only 'embarrassing' of the given options meets this requirement. Hence, (1).
  9. For this blank we need a word that is antonymous to 'grown'. 'Vanished' and 'increased' are clearly not compatible with the contextual requirement. 'Augment' is opposite to what this sentence conveys. Hence, (1).
  10. If you have a knowledge of the Indian roads, you should be able to mark option (1). Hence, (1).
  11. Read the sentence carefully – the comparison is between 'roads' and 'wealth'. Hence, (4).
  12. A 'well developed' India cannot have 'bad roads', thus option (1) is out. Option (4) does not make sense, option (3) does not provide the necessary contradiction which is implied by the conjunction 'but'. Hence, (2).
  13. Highways are meant for the fast movement of traffic but in India it does not serve that purpose. Therefore, 'so-called' fits the best. Hence, (3).
  14. Options (1) and (3) are absurd, one does not 'operate' a road. Hence, (2).
  15. 'Coming face to face' is not the same as passing or colliding, thus option (1) and (2) are ruled out. 'Stand' in a very special manner sounds ridiculous. One comes face to face to talk or communicate. Hence, (3).
  16. 'Weaving back and forth' would mean 'repetition of the same idea'. 'Yet' brings in an opposition, therefore 'strangely' should follow, as it conveys the meaning that in spite of weaving back and forth there is no re-writing or dithering over adjectives. Hence, (2).
  17. The accepted belief was that 'flopping on your bed after meals leads to weight gain'. However the fact that is not so is 'good news'. Hence, (4).
  18. 'Need of the hour' is an idiomatic phrase which conveys the meaning: 'that which is essential'. Hence, (1).
  19. The growth pattern is studied to estimate or outline the future prospects of the company. Thus option (1) is the most logical fit. Hence, (1).
  20. A property cannot be 'apparently' or 'tightly' insured. Also, 'sufficiently' insured is a wrong usage. The correct word to be used here is 'fully'. Hence, (1).
  21. 'Insurmountable' means 'something which is very difficult to handle'. From the given options, the right combination of word with 'insurmountable' is 'appeared'. Hence, (3).
  22. In this case, the adverb form of the missing word is to be used with the verb 'intervene'. 'Military' and 'militia' are nouns and 'militant' is an adjective. Hence, (2).
  23. All the options, except (1), are not used with the word 'mild'. Hence, the right answer is 'quite mild'. Hence, (1).
  24. 'Flied' and 'flead' are grammatically incorrect words. 'Flew' is not used with respect to 'people'. Hence, only option (4) fits in the blank correctly. Hence, (4).
  25. 'Irritated' is followed by 'by' and not 'of'. When the judge told him to stop lying, it indicates that they were 'sceptical' of his statements. If you knew the meaning of 'prevaricating' (not giving direct answers in order to hide the truth) you could have easily solved this. Hence, (2).
  26. 'Infraction' means 'violation of rule' (not paying tax is considered a violation of rule). Options (1) and (2) do not fit in and 'insurgency' is 'an extreme situation where one fights against the government'. Hence, (3).
  27. 'The series of bomb blasts' indicates that this was an attack. 'Barbaric' which means 'cruel' would most appropriately describe the attack. All other options are unrelated. For example, 'lavish' means 'to give in abundance' and 'deriding' means 'to act or treat with contempt'. Hence, (3).

28. The idea conveyed by the sentence is that he praised all his students in a warm way. The adverbs 'forcefully' and 'fervently' are not used with 'praise'. 'Incredibly' which means 'difficult to believe', has a negative connotation. Hence, (2).
29. 'Oil' is uncountable, so 'few' cannot be used. 'Tiny amount' is also a wrong usage. 'Sporadic' refers to 'an event that takes place at intervals'. Only 'negligible' is correct in the context and indicates the meagre amount in the vial. Hence, (1).
30. The relationship is one of degree. A 'fissure' is 'a long, narrow crack or opening', while a 'chasm' is 'a yawning fissure' or 'deep, wide opening'. Similarly, a 'torrent', or rushing, violent, or abundant and unceasing stream, is an extremely magnified version of a 'trickle', or small, gentle stream. A 'ridge' and 'range' are both 'a line of hills or mountains'. A 'coast' and 'plain' are two different types of geographical areas. 'Heavy, drenching rain' is referred to as a 'downpour' or a 'deluge'. Hence, (4).
31. When one 'abbreviates' something, one makes it 'brief' or shorter than the original version. Similarly, when one 'aggravates' a problem, one makes it more serious or 'grave' than it used to be. To 'vindicate' something is 'to uphold or justify it by argument or evidence' which has nothing with being 'vindictive' or 'showing ill will and a desire to hurt, motivated by spite'. To 'instantiate' a statement is 'to provide concrete evidence in support of it', which has nothing to do with being 'instant' or 'immediate'. To 'supplicate' is 'to ask for humbly or earnestly' which has nothing to do with being 'supple' or 'bending readily without breaking or becoming deformed'. Hence, (3).
32. The relationship is one of degree. To 'sprint' is to 'run' very fast. Only option (4) bears the same relationship: to 'loathe' is to 'dislike' excessively. Hence, (4).
33. The relationship is one of 'type of'. A 'gun' is a type of 'arm' or weapon. Only option (1) bears the same relationship: a 'shirt' is a type of 'garment'. Hence, (1).
34. When one makes 'food' impure by adding another substance to it, one is said to 'adulterate'. It. Similarly, when 'water' is made impure by other substances getting mixed in it, it is said to be 'contaminated'. None of the other options have the idea of impurity. Hence, (4).
35. An 'imposter' or 'one' who assumes false identity or title for the purpose of deception' is one who fraudulently manipulates 'identity' by posing as someone else. Similarly, a 'forger' is 'one who makes or imitates something, especially a 'document', falsely especially with intent to defraud. A 'counterfeiter' makes a fraudulent replica, of paper money and not 'bullions' or 'uncoined gold or silver in bars or ingots'. A 'martyr' is 'a person who sacrifices life itself for the sake of principle or religion'. A 'militant' is 'a fighting, warring, or aggressive person'. Hence, (2).
36. The relationship is one of cause and effect. Just as consuming an 'intoxicant', or liquor or alcohol, leads to 'inebriation' or intoxication, a 'disinfectant' causes 'germicide', i.e. the killing of germs. Option (1) is wrong, as a 'preservative' does not cause 'amalgamation' or consolidation. Nor does an 'appetizer' cause 'digestion', or a 'lubricant' cause 'mechanization'. Hence, (4).
37. The relationship is one of 'type of'. Just as 'infiltration' (a process in which individuals or small groups penetrate an area without detection) is a type of activity associated with 'espionage' (spying), 'arson' (malicious burning to destroy property) is a type of activity associated with 'sabotage' (destruction of property or obstruction of normal operations). 'Suspension' is not a type of 'camouflage' (the concealment of something by making it indistinguishable from its surroundings). 'Affiliation' (social or business association) is not a type of 'administration'. 'Orientation' (adaptation to a new environment, situation, custom, or set of ideas) is not a type of 'technology'. Hence, (1).
38. The idea of 'physical relations' in A is best conveyed by 'exceed its banks' in H. This leaves us with options (1) and (2). 'Beyond the comprehension of' fits well with 'exceeds our finite minds'. So C-E. Hence, (1).
39. The sentence E states 'we see smoke and infer fire'. It can't be that we guess ('surmise') fire, 'point out' fire or 'hint' fire. But 'derive or imply' fire fits this sentence. So, A-E. Hence, (4).
40. 'Of soft and loamy consistency' is best conveyed by the 'mellow soil'. Thus C-H. This negates options (2) and (4). 'Freed from the rashness of youth' fits well with 'mellowed with age'. Thus B-E. Hence, (3).

41. 'Removal of something distressing' is best conveyed by 'taking off the tight shoes'. Thus A-F. This negates options (3) and (4). 'Release from the performance of duty' fits well with 'relief of a sentry after the morning shift'. Thus D-E. Hence, (2).
42. 'Remove a stigma from the name of' is best conveyed by 'attempt to purge himself of a charge of heresy'. Thus A-F. This negates options (1) and (4). 'Make clean' fits well with 'purge water by distillation'. Thus B-H. Hence, (4).
43. 'Vessel of standard capacity' is best conveyed by 'used a measure to take out one litre of oil'. Thus B-H. This negates options (1) and (2). 'Ascertain extent or quantity' fits well with 'asked to measure each item'. Thus D-F. Hence, (3).
44. 'Limiting value' is best conveyed by 'strains the bounds of credulity'. Thus B-G. This negates options (1) and (3). 'Move in a specified direction' fits well with 'bound for a career in law'. Thus C-H. Hence, (2).
45. 'Thing or person worth trapping' is best conveyed by 'Prasad was a good catch'. Thus D-E. 'Deception' fits well with 'where is the catch'. Thus C-F. Hence, (4).
46. 'Stock or sell' is best conveyed by 'deals in cards'. Thus B-G. 'Give out to a number of people' fits well with 'dealing the cards'. Thus C-E. Hence, (2).
47. 'Give new direction to' is best conveyed by 'turn his face to the left'. Thus A-G. This negates option (1). 'Change in form' fits well with 'turned into a museum'. Thus C-H. Hence, (4).
48. In option (4), the correct collective noun is 'bunch of boy-scouts' and not 'bundle of boy-scouts'. All the other sentences use the given word correctly. In option (1) it means the baby has brought 'a lot of joy' for the family. In option (2), it means 'a number of things considered together' while in option (3) it means 'a great deal of money'. Hence, (4).
49. In option (2), 'distinct' means 'clearly visible'. In option (3) it means 'different in nature or quality' while in option (4) it means 'unquestionably exceptional or notable'. However, 'distinct' does not make sense in option (1) as the word is used in the sense of 'certain' or 'clear'. Hence, (1).
50. In option (2) 'implication' means 'the act of indicating that one or more persons may be involved, as in a crime'. In option (3), the given word means 'significance or meaning' while in option (4) it means 'something suggested as naturally to be inferred or understood'. However, 'implication' is incorrectly used in option (1). Hence, (1).
51. In option (1), 'host' means 'throw a party' and is correctly used. In option (2), 'host' means 'an area in which particular plants or animals are found' and is correctly used in the given sentence. 'Host also means 'an animal or plant on which or in which a parasitic organism lives'. In case of option (4), it means the computer in which the virus has infected. Thus option (4) is also correct. However, 'the host' is incorrect usage of the given word in option (3). Hence, (3).
52. In option (1), 'sort' means 'a particular kind or variety' and is correctly used. In option (3), 'of a sort' means 'of an indefinite kind' while in option (4), it means 'to separate'. However, in option (2), the correct phrase is 'of all sorts' which means 'many varieties of something' and not 'of all sort'. Hence, (2).
53. In option (1), 'help' means 'remedy' while in option (2) it means 'refrain from' or 'avoid'. In option (4), the given word means 'give assistance or support' and is correctly used. 'Help himself' means 'to take something for (oneself) without permission'. However, since the speaker has already given permission to Ranjit to take the beer from the fridge, the idiom 'help oneself' is incorrectly used in it. Hence, (3).
54. In option (1), the idiom 'on the paper' is incorrect. The correct idiom is 'on paper' which means 'in theory rather than in practice'. In option (3), the idiom 'paper over' means 'to conceal (something controversial or unpleasant)' while in option (4), 'paper' means 'an essay or article on a particular topic'. Hence, (1).
55. In option (2), 'service lift' means 'lift of or used by servants, delivery people, etc.' and is correct in the given context. In option (3), 'service' means 'to meet interest and capital payments on (debt)' while in option (4) it means 'in the occupation of service (usually military)'. However, 'service' is incorrectly used in option (1) as it needs the verb form 'serve' to make it meaningfully complete. Hence, (1).



56. In option (1), 'beyond all reason' means 'to a foolishly excessive degree'. In option (2), 'reason' means 'a cause or explanation' while in option (3) it means 'explanation of a belief or action'. However, in option (4), the correct idiom is 'listen to reason' which means 'be persuaded to act sensibly' and not 'listen to a reason'. Hence, (4).
57. As per the sentence, a reader becomes conscious of the absence of punctuation only when it is important. 'Obtrude' means 'to thrust forth' and is incorrect in the context of punctuation. Thus option (1) can be negated. 'Enjoins' means 'prescribes a course of action with some authority' and cannot be used in the given context. Punctuation cannot 'conceal' (which means 'hide') itself. Thus option (3) is also incorrect. Only option (4) implies that the reader only becomes aware of punctuation being 'effaced' (which means 'erased') only when it is 'counted' (which means 'to have importance'). Hence, (4).
58. Demand can only be 'assessed, stimulated or restrained'. It cannot be 'outstripped' (which means 'excelled'). Thus option (2) can be negated. 'Budget deficits' may be 'minimized or controlled' but not 'concealed'. Thus option (4) can also be negated. If one needs to assess demand, there is no need to minimize budget deficit. However, demand can be 'stimulated' by 'controlling' budget deficits. Hence, (3).
59. As per the sentence, the Athenians could sit at home and think or travel the world. 'Temerity' which means 'rashness' is incorrect in the first blank. This negates option (4). If one is travelling the world, one cannot 'ignore or suffer' it. This negates options (2) and (3). Thus only option (1) implies that the Athenians had the leisure to sit at home and think or explore the world while they travelled. Hence, (1).
60. 'Grovel' means 'to humble or abase oneself, as in showing respect'. This implies that she had no respect for men. Thus both the words should have a similar connotation. This negates options (1) and (4) because she did not 'admire' men nor does 'fidelity' fit the given context. 'Temperance' means 'self-control' while 'impertinence' means 'disrespectful behaviour'. Since they are not similar in meaning, option (3) can also be negated. 'Indifference and contempt' mean 'lack of care or concern' and 'the attitude of a person towards

a person or thing that he considers worthless or despicable' respectively. Both the words correctly fit the blanks. Hence, (2).

### PRACTICE EXERCISE 4

1. 'Get through' implies 'to deal with an unpleasant experience'. Hence, (1).
2. 'Are' is the most fitting option (A)s the sentence is in the present tense. Hence, (4).
3. 'Had come' is the correct option (B)ecause of the word 'yesterday'. Hence, (2).
4. The demand has to be met by adequate supply, thus only (4) fits in with the theme. Hence, (4).
5. Experiments can be explained in various ways. Hence, (4).
6. Cliches can be both 'objective' and 'subjective'. Cliches are 'oft repeated sentences', the fact that they are so warrants some degree of truth. Hence, (3).
7. The word trick should give you the clue, only the word 'illusions' fits in with 'trick'. Hence, (4).
8. If you know the meaning of 'optimists' – those who see the positive side of life – you should easily get the answer correct. Hence, (2).
9. 'Tie down' means 'to restrict somebody's freedom'. 'Tie up' means 'to attach or to close something'. 'Tie in' means 'to link something or arrange for something to happen, at the same time as something else'. 'Tie off' means 'to close something with a string or thread'. Hence, (3).
10. 'Panned out' means 'turned out well i.e., to be successful'. 'Put out' does not fit in. 'Pulled down' means 'demolished'. Thus, options (1), (2) and (3) do not fit in. 'Petered out' which means 'gradually dwindle' is the correct usage here. Hence, (4).
11. 'Non-literally' is clearly out of context. 'Emblematically' may also be ruled out. 'The red carpet was out for me'—this is clearly a symbolic inference. Hence, 'symbolically' and 'metaphorically' are plausible. Of the remaining options for the second blank, 'unexpected' scores over the others. 'startling' and 'abrupt' do not qualify 'pleasure'. 'Surprising' and 'pleasure' are contextually incorrect. Hence, (4).

12. 'Deviance' refers to 'a state or condition markedly different from the norm, that is generally not usual' and is considered to be unacceptable. Since biomedicine tries to bring about the well-being of man, 'integrates' would be the appropriate word. 'Mutation' is incorrect in the context of the sentence, as it means 'a change in the nature or form of something' and cannot be applied to 'sickness'. In (3), 'malady' is synonymous to sickness, hence a repetition. 'Incorporates' is also incorrect in the contextual usage. 'Dexterity' means 'being skilled or adept at a particular thing' and also does not fit in contextually. 'Enmeshes' means 'getting entangled' and is too ambiguous. Hence, (1).
13. Neither 'naturalness' nor 'ambit' fit in appropriately in the sentence. The same is the case with options (3) and (4). 'Entirety' means 'all' as such it does not convey the idea of 'completeness' which 'wholeness' conveys. 'Ensemble' is 'a group or set', which does not convey the idea of 'wholeness'. Ayurveda addresses a host of problems plaguing the human body and the mind and looks at such problems in complete detail. Thus, option (1) has the most appropriate combination of words. Hence, (1).
14. One cannot say that a given social order is 'angered', and 'enabled' is also vague. Though 'challenged' in (2) is correct, 'reprobated' does not fit in contextually, since it means 'behaviour which is immoral or inappropriate'. The options in (3) are most appropriate; the comedies mocked the social order, bringing back the existing conditions. 'Amplified' meaning 'expanded' makes no sense. Hence, (3).
15. The phrase 'have to live with' indicates that the situation that we are talking of is imminent. On this basis, we can negate options (2) and (3). For the second blank, we need a word that indicates some kind of secrecy. Option (4) cannot be the correct answer because though 'clandestine' also means secret, 'on the clandestine' is a wrong expression. 'On the sly', which indicates 'an activity that is performed in a way to escape notice', is idiomatically correct. Hence, (1).
16. 'Organized and orchestrated' will be a redundant expression here because 'orchestrate' itself means 'to organize an event very carefully'. 'Manoeuvre' is used in the context of 'some clever plan which gives somebody an advantage'. There is no hint of any combination of elements so 'amalgamated' is also negated. The idea is that the meeting has been carefully designed. The only word that fits the context is 'scripted'. Hence, (2).
17. If we read the first part of the sentence, we notice that the tone is negative. The unresponsiveness of the defendants is one of the reasons why Mr. Chen like other petitioners has moved to Beijing. Option (1) is negated because a court proceeding is normally serious and the graveness will not influence the petitioner to move to a different place for better and faster judicial proceeding. Option (3) is ruled out because if the judicial procedure was swift and the prospect of victory was bright, the petitioners would not have moved to other places. In case of option (4) also, 'glaring' does not fit in. Hence, (2).
18. If we read the second part of the sentence, we realize that the executives have to perform under constraints of time. So option (1) cannot be the correct answer. 'Constrained time' is a wrong expression and 'paucity' should be followed by 'of'. So options (2) and (3) are ruled out. The second blank should have a word that means the executives themselves evaluate the deal and finalize things. Hence, (4).
19. Since in the same sentence 'hunger' as well as 'continuous' have been used, one can assume that 'insatiable' is correct. Moreover, the need for continuous affirmation is implied. Hence, (2).
20. Although the first words of all the options fit in, only 'fanned' fits the latter part of the sentence. Hence, (3).
21. The relationship is one of 'have-not'. Just as a person who is 'infallible' is one who is not capable of being 'defeated', a person who is 'incorrigible' is one who is not capable of being 'reformed'. Hence, (1).
22. 'Assets' can 'accumulate', i.e. collect or gather. Similarly, 'mobs' can 'congregate', i.e. come together or gather. None of the other pairs have a similar relationship. Hence, (2).
23. Just as a 'minister', a member of a church's leadership, can be 'defrocked', i.e. stripped of priestly privileges or functions for violating the tenets of a particular church, a 'president' can be 'ousted' or removed from office for violating the constitution. A 'moderator' or one who presides over a meeting

- can 'screen' or examine systematically in order to determine the suitability of the statements of the participants. An 'arbitrator' is a person who 'settles' how a dispute or quarrel should be resolved. An 'executor' is a person designated to carry out or 'implement' the provisions in a will. Hence, (3).
24. Just as the job of a 'censor', i.e. one who supervises the manners or morality of others, is to 'expurgate' or remove unacceptable material from a book, film, etc., the job of an 'actuary', i.e. a person who computes insurance premium rates, dividends, etc., is to 'assess' or estimate risk. A 'contractor' is 'one who performs work at a certain price or rate'; to 'depreciate' is 'to reduce the price or value of'. An 'undertaker' is 'one who arranges for the burial or cremation of the dead'; to 'propose' is 'to offer or suggest'. A 'surveyor' is 'one who conducts a detailed inspection or investigation of something'; to 'preview' is 'to view or show beforehand or in advance'. Hence, (2).
25. Just as a 'treaty' or a formal agreement between two or more states is 'abrogated' or nullified when it is not obeyed, a 'contract' or agreement between two or more parties enforceable by law is 'breached' or broken when one or both of the parties does not follow the provisions specified in it. People 'bequeath' or hand down a 'legacy' or gift of property in a will. Cautious investors 'balk' or refuse to go on at any investment that carries a high 'risk'. Doctors 'cauterize' or burn with a chemical or hot metal a 'wound' to prevent it from becoming infected. Hence, (4).
26. 'Aquiline' is the adjective derived from the noun 'eagle' and it means eagle-like. The only pair exhibiting a similar relationship is (1): 'saurian' is an adjective derived from the noun 'lizard' and means lizard-like. 'Rangy' is an adjective meaning having long slender limbs, but is not derived from 'wolf'. 'Terrestrial' means of, relating to, or composed of land, but is not an adjective derived from 'insect'. 'Crustacean' is an adjective meaning having a crust or shell, but is not derived from 'spider'. Hence, (1).
27. 'Property' is 'mortgaged' or put up as security or collateral to obtain a loan. Similarly, 'jewellery' is 'pawned' or deposited as security for the payment of money borrowed to obtain a loan. To 'ingrain' is 'to implant or fix deeply and firmly' and has nothing to do with a 'harvest'. A 'franchise' is 'a business established under an authorization to sell or distribute a company's goods or services in a particular area'. To 'embroil' is 'to force into some kind of situation, condition, or course of action' and is not related to a 'surcharge' or 'an extra amount of money charged over and above the regular price or fee'. Hence, (3).
28. The relationship is one of defining characteristic. Just as an action that is 'impromptu', or 'having little or no preparation or forethought', is defined by 'spontaneity', or 'the quality of acting from native feeling, without constraint or external force', an action that is 'hesitant', or 'lacking readiness', is defined by 'indecision', or 'inability to decide'. An action that is 'glamorous', or 'having an air of allure, romance and excitement', is not defined by 'exactitude', or 'the quality of being exact'. An action that is 'perceptive', or 'marked by deep understanding,' is not defined by 'conscience', or 'the ethical or moral principles that govern a person's thoughts and actions'. An action that 'is 'peripheral', or 'on or near an edge', is not defined by 'centrality', or 'the tendency to be or remain at the centre'. Hence, (4).
29. The first phrase in each pair is a Latin phrase commonly used in English; the second part gives its purported meaning. So 'sine qua non' means 'essential condition or requirement'. Similarly, 'persona non grata' means 'unwelcome person'. However, 'carpe diem' means 'seize the day', 'caveat emptor' means 'let the buyer beware', and 'alma mater' refers to one's school or college. Therefore, these three are incorrect, and only option (4) is correct. Hence, (4).
30. In option (1), 'fallout' means 'the settling to the ground of airborne particles ejected into the atmosphere from the earth by explosions, especially nuclear explosions'. In options (2) and (3), the given word means 'an unexpected or incidental effect or outcome'. However, in option (4), 'fallout' is incorrectly used. We need to use 'failure' to make the sentence meaningful. Hence, (4).
31. In option (2), the idiom 'at first hand' means 'directly from the origin, without intervention or intermediary'. In option (3), the idiom 'out of hand' means 'out of control' while 'lend me a hand' means 'be of assistance'. However, in option (1), the correct idiom is 'have my hands full' which means 'be extremely busy' and not 'have my hand full'. Hence, (1).

32. In option (1), 'near miss' means 'an attempt that falls just short of success'. 'Near' in option (3) means 'almost' while in option (4) it means 'close to' or 'verging on'. However, 'near friend' is incorrect as it does not mean 'close friend'. Hence, (2).
33. Option (1) does not fit in any sentence. In option (2) 'accident' fits into sentence B as it mentions a smashed arm which is true in case of an accident. Option (3) does not fit in any sentence. In option (4) 'calamity' fits into sentence C as these are the ones without warning. The word in option (4) also fits into sentence D, where dogs can sense a calamity. Hence, (4).
34. The word 'dejection' can fit into sentences B and D. Though its usage in sentence A is not grammatically incorrect, it is not appropriate to the context of the sentence since 'dejection' is a state of mind rather than a prolonged feeling. It is more likely that the men had fought a feeling that was more persistent than 'dejection'. The word 'lonely' or option (2) fits into sentence C as it is natural for a deeply hurt person to feel lonely. The word 'trouble' or option (3) fits into sentence B as trouble or problems does not afflict all adults as they learn to deal with them with age. The word 'depression' or option (4) fits into sentence A, here depression and emotional withdrawal are two different things showing diverse experiences equipping one to help Jim. 'Depression' also fits into sentence B, as it is something that afflicts all adults at some point of time. Sentence D discusses about a feeling hard to survive, which depression suits best. Therefore, the word 'depression' fits into three sentences and is the correct answer. Hence, (4).
35. The word 'advantage' or option (1) fits into sentence C as one can have an advantage over the other as opposed to any other options given. In option (2) 'privilege' fits into sentence A as it means 'a right reserved for a special group like rich people' and it fits into sentence D too as it means benefit or exclusivity. Option (3) 'right' fits into sentence B as it refers to 'basic rights of a person'. Hence, (2).
36. The word 'illusion' or option (1) does not fit into any sentence. The word 'imagination' or option (2) fits into sentence A as it is the only positive word among the choices and power is used with positive words rather than negative. Sentence D also takes 'imagination' as the sentence speaks about a positive thing which is creative. The word 'hallucination' or option (3) fits into sentence B as it is common for people with high fever to have hallucinations. The word 'misunderstanding' or option (4) fits into sentence C as a misunderstanding can lead to enmity and no other option (B)est fits the context of the sentence. Hence, (2).
37. The word 'state' or option (1) fits into sentences A and D, where it means 'the region or declared boundary under administration' which is 'state'. It is also appropriate in sentence B where it means 'a current condition'. In option (2) 'nation' does not fit in sentence A as an individual should live for a state not a nation. Option (3) 'government' does not fit into any sentence. The word 'conditions' or option (4) fits into sentence C where it means 'a certain criterion or condition earlier set have now been laid down'. Hence, (1).
38. 'Stratosphere' is one of the different layers or a particular type of 'atmosphere'. We are looking for a similar relationship. In option (1), a 'nimbus' is a type of 'cloud'; here they share an inverse relation to the capitalized pair. In option (2), 'calibration' is an act of checking or adjusting the accuracy of an 'instrument'. Option (3), is the correct answer as a 'jet' is a type of 'aircraft', thus the correct answer choice. In option (4), 'rain' is of a short duration while 'climate' refers to the weather of a place over a long period, thus they do not share a similar relationship. Hence, (3).
39. A 'gourmet' is 'someone who appreciates or loves food'. We are looking for a similar relationship. In option (1), a 'critic' need not necessarily appreciate a 'book', in fact the critic could pass harsh judgements on a book. Option (2) is the correct answer. A 'connoisseur' appreciates 'art', thus the correct answer choice. In option (3), a 'fan' loves a 'sport' but need not necessarily appreciate all those who play it. Thus they do not share a similar relationship, hence this option is incorrect. In option (4), 'craft' and 'skill' are synonyms, thus they do not share a similar relationship. Hence this option is incorrect. Hence, (2).
40. 'North' and 'South' are antonyms. We are looking for a similar relationship. Option (1) is the correct answer. 'Black' and 'white' are antonyms, thus the correct answer choice. In option (2), 'yellow' and

- 'orange' are colours, hence this option is incorrect. In option (3), 'red' and 'maroon' are merely two different shades of colours, hence this option is incorrect. In option (4), 'red' and 'indigo' are merely two different colours, hence this option is incorrect. Hence, (1).
41. A 'brand' is a name of a 'product'. We are looking for a similar relationship. In option (1), 'dalda' and 'rath' are two different brands of vanaspati, hence this option is incorrect. In option (2), an 'aircraft' is a type of flying 'machine'. In option (3), 'Reebok' refers to a brand of 'shoes', but they share an inverse relation to the capitalized pair. Hence this option is incorrect. Option (4) is the correct answer. A 'Ferrari' is a name of a particular 'sports car', thus the brand product relation is established. Hence, (4).
42. Shortage of rainfall ('drought') leads to shortage of food ('famine'). We are looking for a similar relationship. In option (1), 'poverty' and 'plenty' are antonyms, hence this option is incorrect. In option (2), a 'fluid' sometimes flows in drops (or 'drips'), hence this option is incorrect. In option (3), a 'camera' usually makes use of a 'film', hence this option is incorrect. Option (4), is the correct answer. 'Training' leads to development of a 'skill'. Hence, (4).
43. 'Anterior' is a position opposite of 'posterior'. 'Anterior' is a position in the 'front' and 'posterior' is a position at the 'back or rear'. Options (1), (2) and (3) do not exhibit the same relationship as the capitalized pair, although they are opposites. Option (4) is the answer. 'Front' and 'rear' mean exactly the same as the capitalized pair. Hence, (4).
44. A 'barge' is a kind of 'vessel'. Option (1) is the correct answer choice as a 'shovel' is a kind of implement. In option (2), 'anthology' is a collection of passages and is not necessarily a kind of 'book'. In option (3), 'rim' and 'edge' do not share the relationship 'kind of'. In option (4), 'training' is one form of 'preparation' and not necessarily a kind of preparation. Hence, (1).
45. 'Love' and 'affection' are synonyms. In option (1), 'happiness' and 'joy' are synonyms. In option (2), 'amity' is used in the context of friendship, to show cordiality while 'harmony' is used to denote a state of co-existence. In option (3), an 'enemy' is a person whereas 'hatred' is an emotion. In option (4), 'sorrow' means 'grief or sadness' while 'misery' means 'great emotional or mental distress'. The two words are not synonyms. Hence, (1).
46. 'Paradigm' and 'pattern' are synonyms. In options (1), (3) and (4), the pair of words are not synonyms. In option (2), 'method' and 'system' are synonyms. Hence, (2).
47. 'Peel' and 'peal' sound alike. They are homophones. In option (1), 'coat' and 'ring' do not sound the same. Therefore, this is not the correct answer. In option (2), 'laugh' and 'bell' do not sound alike. Therefore, this is not the correct answer. In option (3), 'rain' and 'reign' sound alike. Therefore, this is the correct answer. In option (4), 'brain' and 'cranium' do not sound alike. Therefore, this is not the correct answer. Hence, (3).

### PRACTICE EXERCISE 5

1. 'To walk somebody through' means 'to familiarize one with a process step by step'. Hence, (2).
2. 'Sprucing up' is the correct usage and here means 'gearing up for the interview'. Hence, (3).
3. Two contrast settings are depicted, modern skyscrapers existing alongside the rustic country houses. Modernity existing simultaneously with rusticity. Hence, 'contrasting starkly' fits in best in the given context. Hence, (4).
4. 'Acquisition of learning and innovation' cannot have a negative effect. So it cannot 'put in reverse gear' periods of economic wealth and social cohesion. Again, if there is learning and innovation, there will be no need 'to fall back upon' periods of economic wealth and social cohesion. So option (2) can be eliminated. 'To embark' would be wrong usage here. 'To leapfrog' means 'to get to a higher position by missing out some stages'. Hence, (3).
5. 'Salacious' (lewd) and 'squalid' (dirty or repulsive) cannot be used to describe 'geometry', so both options (1) and (4) can be ruled out. 'Scabrous' (having a rough surface) and 'serrated' (jagged) can both fit into the first blank. 'Tousled' (messy or tangled) is a description that might have fitted among the last three adjectives, but it does not fit between 'pitted' and 'broken up'. On the other hand, 'pocked' (full of holes) fits perfectly well

- between these two. Hence, (2).
6. The description 'light-hearted' suggests that the word in the first blank cannot be 'atrocious' or 'iniquity', both of which refer to 'highly wicked acts'. 'Infraction', meaning 'breaking a rule or law', and 'peccadillo', meaning 'minor offense', fits better in this context. However, the word 'chide' cannot fit into the second blank, as it is a verb, while the article 'a' before it implies that only a noun can fit into this blank. 'Reprimand', meaning an 'expression of disapproval, especially a formal one', fits best in this blank. Hence, (4).
  7. 'Obtuse' (dull, rounded or mentally slow) cannot be used to describe 'aspect of nature', so option (3) can be ruled out at once. 'Clever or ...' before the second blank suggests that the word in that blank should be similar in meaning to 'clever'. So 'sanctimonious' (falsely pious) and 'ingenuous' (sincere or naïve) do not fit into this blank. Thus options (2) and (4) can be eliminated. Only the words in option (1) – 'recondite' (profound or difficult to understand) and 'ingenious' (cleverly inventive or resourceful) – fit both blanks. Hence, (1).
  8. 'Discreet' means 'prudent or circumspect'. 'Discrete' means 'separate or distinct'. In the first sentence we should use discreet. A bored person is 'uninterested'. 'Disinterested' means 'objective or neutral'. The correct option for the second sentence is 'uninterested'. 'Restive' means 'restless or impatient', 'restful' means 'peaceful or tranquil'. So, 'restive' is appropriate for the third sentence. 'Dispassionate' is the opposite of 'passionate' and means 'free from or unaffected by passion'. Thus, in the fourth sentence we should use 'impassioned'. The correct sequence is AAAA. Hence, (3).
  9. People who 'dissemble' are being dishonest, trying to hide what they are really up to. When you 'disassemble' something, you take it apart. In the first sentence 'dissemble' is the correct word. 'Official' means 'legitimate or correct'. 'Officious' means 'intrusive or overzealous'. Since his concern has annoyed the speaker, we can infer that a word with a negative connotation is required in the sentence. So, 'officious' is the appropriate word for the second sentence. 'Reputable' means 'worthy of respect or distinguished'; 'reputed' means 'regarded or considered to be something'. When you say that an institute is 'reputable', what you mean is that it has a good reputation, and therefore can be trusted. When you say something is 'reputed', you are basing it on hearsay. Thus, 'reputable' is the correct word for the third sentence. 'Distinct' means 'unique', something which stands out, like the scar described in this sentence. 'Clear' means 'easy to understand or simple'. So, distinct is the correct word for the fourth sentence. The correct sequence is ABAA. Hence, (3).
  10. 'Earthy' means 'crude or offensive', 'earthly' means 'related to or belonging to the earth'. In the first sentence, 'earthy' is the correct word. 'Economic' means 'related to the economy or money in general', while 'economical' is 'avoiding waste or extravagance' 'thrifty'. In the second sentence, the word should be 'economical', referring to a tighter spending policy. A 'feint' is a manoeuvre designed to divert the opponent's attention from the real centre of attack'. On the other hand, 'faint' means 'to lose consciousness'. In the third sentence, the correct word should be 'feint'. 'Emigrate' means 'to move out of a country' and is used as 'emigrate from' and 'immigrate' means 'to move into a country' and is used as 'immigrate to'. In the fourth sentence, we should use immigrate as the students they emigrated from India but immigrated to the US. The correct sequence is AAAB. Hence, (4).
  11. 'Exorcise' means 'to free of evil spirits or to cast away something unpleasant'. One 'exorcises' evil spirits using an exorcist; but when one gives one's body a workout, it is 'exercise'. Therefore, 'exorcise' is the appropriate word for the first sentence. 'Forbidding' means 'preventing someone from doing something'. 'Foreboding' means 'an ominous sign or a feeling of evil to come'. So, in the second sentence, 'foreboding' would be the appropriate word. 'Fortuitous' events happen by chance; they need not be 'fortunate' events, only random ones. 'Fortunate' events are chance events, which happen due to good luck. 'Fortunate' is the appropriate word for the third sentence. You 'capture' something, when you grasp it. Here we need a synonym of 'mesmerise', which means 'to enchant or captivate somebody'. So, 'captivated' is the appropriate word for the last sentence. Thus, the correct sequence is ABBA. Hence, (2).
  12. 'Drastic' and 'dramatic' are similar in meaning, but 'drastic' has a negative connotation and should be used only when the situation being described is

- dangerous or harmful. Here improvement is being described, hence, we should use dramatic. 'Wont' means 'habit or practice'; 'want' means 'desire'. In this particular case, the more appropriate word is 'wont' and not desire. If we say our opinions 'varied', it means 'they were changing all the time'. The sentence indicates that the opinions clashed. Hence, 'differed' would be the appropriate word here. The word 'device' is a noun and refers to a type of equipment. 'Devise' is a verb, which means 'to plan or design something'. So, 'devise' is appropriate for the last sentence. Thus, the correct sequence is BABB. Hence, (1).
13. 'Ceremonial' has to do with a literal ceremony. 'Ceremonious' describes formal behaviour, which has nothing to do with an actual ceremony. Here, we should use 'ceremonial'. 'Childish' and 'child-like' are sometimes similar but 'child-like' is used to describe affable qualities while 'childish' is used in a negative way. Here we should use 'childish'. A quiet, reserved person is 'demure'. To 'demur' is 'to refuse'. Hence, we should use 'demur'. 'Classic' describes things that are outstanding examples of their kind. 'Classical' refers to 'ancient art'. Here we should use 'classic'. The correct sequence is AABA. Hence, (2).
14. 'To bate' is 'to moderate'. 'To bait' is 'to lure'. Hence we should use 'baited'. 'To depreciate' something is 'to actually make it worse', whereas to 'deprecate' something is simply 'to speak or think of it' in a manner that demonstrates one's low opinion of it. When actual value is considered, we should use 'depreciate'. When something is 'alluded' to, it is 'referred' to. When something is 'hard to find', it is 'elusive'. Here, we should use 'eluded'. In the fourth sentence, the connotation is that of 'every other' and this can only be replaced by 'alternate' and not 'alternative', which means 'a choice between two or more things'. The correct sequence is ABAA. Hence, (4).
15. 'Apposite' means 'appropriate to the situation'; 'opposite' means 'on the other side'. In this case, the correct word is 'apposite'. 'Enormity' and 'enormousness' are at times interchangeable but 'enormity' is usually used to describe negative situations. Since 'marvel' is mentioned here, we have to use 'enormousness'. 'Complacent' means 'pleased or self-satisfied', whereas 'complaisant' means 'obliging'. Here we should use 'complaisant'. An 'altar' is 'a platform at the front of a church or in a temple at which religious rites are performed'; to 'alter' something is 'to change it'. So, we should use alter here. Hence the correct sequence is ABBA. Hence, (3).
16. 'Archaic' means 'old or ancient', while 'arcane' means 'mysterious'. Here, we should use 'archaic'. 'Rout' means 'comprehensive defeat'; 'route' means 'a way to reach one point from another'. In this situation, 'rout' is the correct word. 'Practice' is the noun form and 'practise' is the verb form; so in this case 'practise' should be used. 'Farther' refers to physical distance, while 'further' means 'in addition to'. Hence we should use 'further'. Hence, the correct sequence is AABB. Hence, (1).
17. 'Liable' and 'likely' can be used interchangeably but 'liable' has negative connotations. Here, we should use 'liable'. To be 'fearful' is 'to be afraid'. To be 'fearsome' is 'to cause fear in others'. Here, we should use fearsome'. 'Pour' refers to 'transferring a liquid' and to 'pore' is 'to be absorbed in the reading or study of something'. In this case 'pored' is appropriate. When we take over a post, we 'assume' it. Hence, the correct sequence is ABBA. Hence, (4).

WORD	MEANING
Epitaph	An inscription on a tomb
Epigram	A short, witty saying
Ingenious	Cleverly inventive or resourceful
Ingenuous	Candid or artless
Eagerly	Expecting impatiently (usually something pleasant)
Anxiously	With anxiety or apprehension (usually something not pleasant)
Click	Snap; striking with a noise
Clique	An exclusive circle of people
Corroborated	Confirmed or strengthened a testimony
Collaborated	Worked together
Alternately	In alternating sequence
Alternatively	As an alternative to
Duel	A fight, a struggle
Dual	Double, two of something
Horde	A crowd or throng
Hoard	Save up
Flair	A natural talent
Flare	Sudden burst of flame

Coarse	Rough (vulgar, when referring to language)
Course	Path over which something moves
Concurrent	Happening at the same time
Consecutive	Successive; following one another
Contemptuous	Describes a person who feels contempt towards something
Contemptible	Describes a person who is worthy of contempt

18. Refer to the table. Hence, (3).

19. Refer to the table. Hence, (4).

20. 'Exile' is a coercive form of 'emigration'. Similarly 'incarceration' is a coercive form of 'residence'. To 'usurp', or 'seize and hold by force and without legal authority', is not a coercive form of 'conspiring', or 'planning together secretly to commit an illegal or wrongful act'. To 'excommunicate', or 'exclude from a group or membership by decree', is not a coercive form of 'bedeviling' or 'confusing'. To 'eradicate', or 'destroy completely', is not a coercive form of 'transpiring', or 'becoming known'. Hence, (3).

21. The relationship is one of 'type'. A 'sonnet' is a type of a 'poem'. Similarly, 'flamenco' is a type of Spanish 'dance'. 'Burlesque' in option (3) is 'a type of a play or written piece which is a parody or a vulgar caricature of something'. Hence, (2).

22. 'Acoustic' means pertaining to 'sound'. Similarly, 'histrionic' means pertaining to 'acting'. 'Logistic' pertains to the details of an operation. 'Seismic' pertains to an earthquake. 'Vernal' pertains to the season of spring, not 'seasons' in general. Hence, (3).

23. The relationship is one of 'defining characteristic'. Anything that is 'abhorrent' 'repulses', or drives back with disgust. Similarly, anything that is 'cogent' 'convinces'. 'Incoherent' means 'order'. 'Threatening' is something that might cause 'hurt'. 'Articulate' speech is clear or distinct, with appropriately chosen words. Hence, (4).

24. The relationship is one of 'type'. A 'paean' is a song of 'praise'. Similarly, a 'dirge' is 'a song of mourning'. An 'encomium' is 'a formal expression of high praise'. A 'eulogy' is 'a written composition

or speech in praise of someone or something'. A 'diatribe' is 'a heavy criticism or attack'. Hence, (2).

25. The relationship is the 'have-not' link. An 'apparition' is a ghost-like figure; it lacks 'reality'. Similarly, a 'renegade' is 'one who deserts a party or a faith for another one'; thus, a 'renegade' lacks 'loyalty'. An 'ingrate' is 'an ungrateful person'. 'Bombast' refers to 'a grand, pompous, speech'. 'Gallant' means 'brave and chivalrous'. Hence, (3).

26. The analogy is based on association of certain types of behaviour associated with certain professions. Just as we associate a 'diplomat' with having 'tact', 'sober' behaviour is expected of a 'judge'. Hence, (4).

27. 'Fair' and 'fare' have no relationship except that of being similar sounding words. So, the correct option is 'Hair: Hare'. Hence, (3).

28. 'Galley' is the part of a 'ship' where food is prepared. Similarly, the 'kitchen' is that part of the 'house' where food is prepared. A hotel's food may be prepared in its restaurant, but more precisely in the restaurant's kitchen. Hence, (2).

29. A 'lethargic' person needs to be 'stimulated', while a 'sceptical' person needs to be 'convinced'. Hence, (1).

30. 'Stagger' is a form of 'arrangement', just as 'canter' is a form of 'movement'. Hence, (1).

31. A 'savant' is wise - so 'obtuse' would be a quality that is definitely not part of a 'savant'. Similarly, an 'athlete' needs to be energetic; he/she cannot be 'sluggish'. Hence, (4).

32. 'Chantry' is a part of a 'church'. Similarly, 'bedroom' is a part of a 'house'. Hence, (1).

33. 'Facade' and 'veneer' both refer to a superficial aspect; similarly 'parade' and 'march' are synonyms. Hence, (2).

34. 'Heathen' and 'infidel' are synonyms, and so are 'imbecilic' and 'foolish'. Hence (1).

35. 'Purée' is a 'culinary' term, just as 'affidavit' is a 'legal' term. Hence, (2).

36. 'Liquid' is a state of matter. Any liquid when converted gets into 'gaseous' matter. In option (1), 'serum' is liquid and 'fume' is gaseous. In option (2), 'humid' and 'arid' means 'dry condition' so we can negate this answer option. In option (3)



- 'thaw' means 'change from solid to liquid' and 'distil' means 'purification'. In option (4), 'smoke' and 'cloud' are both in gaseous form; therefore we can negate this answer option as well. Hence, (1).
37. 'Fission' is opposite of 'fusion', but this is not enough to answer the question; here 'fission' means 'dividing' and 'fusion' means 'combining or a union'. Therefore, we are looking for an answer choice with the meaning divide: union. Although options (1), (3) and (4) are opposites, they do not satisfy the relation 'divide : union'. Option (2) is the answer as 'separation': 'togetherness' is a literal translation of 'divide: union'. Hence, (2).
38. 'Doubt' is without 'faith' or lack of faith. Option (1) is the correct answer option (A)s the option shares the same relationship as 'without'. An 'atheist' is 'one without religion'. In option (2), 'sceptic' is 'one who questions the commonly held beliefs', 'pious' means 'showing reverence to a deity or god'. Though a close contender this does not hold the same relationship as 'without.' In option (3), an 'iconoclast' is 'one who destroys the idols' but not one 'without' an idol. In option (4), an 'apostate' is 'one who is not faithful' and the second word 'state' has no relevance or relationship to the first word. Hence, (1).
39. 'Building consists of bricks', but this is not sufficient to answer the question. Furthermore, buildings consist of mortar and steel. In essence we are looking for the relationship 'consists of, among others'. In option (1), 'dictionary' consists of 'words', among others such as meanings, pictures and illustrations. In option (2), 'letters' consist of 'alphabets' is a closest contender. However, it does not satisfy the relationship 'among others'. In option (3), the relationship is in reverse order, also it does not satisfy the 'among others' relationship. In option (4), an 'idiom' is a manner of speaking a 'language' and has no relevance with respect to the relation 'consists of, among other' Hence, (1).
40. 'Dulcet' is the opposite of 'raucous'. 'Dulcet' means 'extremely pleasant to hear' and 'raucous' means 'extremely loud or harsh to hear'. Therefore, we are looking for a (+,-) pair (the positive word coming first and the word with the negative connotation coming second). Option (1) does not have words that are opposite in meaning. Although option (2) contains words that are opposite in meaning, they do not constitute a (+,-) pair in the same order. In option (3), 'palliative' and 'exacerbate' are opposite in meaning; 'palliative' means 'something which relieves' and 'exacerbate' means 'making worse'. They constitute a (+,-) pair in the same order. Although option (4) has words that are opposite in meaning, they do not constitute a (+,-) pair as mentioned in the defining sentence. Hence, (3).
41. A 'doggerel' is written by a 'poet'. In option (1), a 'symphony' is created or composed by a 'composer', This comes close as the answer choice but is not the best choice as it is inversed. In option (2), 'predictions' are made by an 'astrologer'. This option (D)oes not have the same relation as depicted in the defining sentence. In option (3), 'vintner' is a seller of 'wines'. This does not have the same meaning as 'written by'. In option (4), 'pulp-fiction' – though not a defined word but it has a close bearing to the meaning of a soft or rich fiction – a literature written by a novelist. Thus, it has the same relation as the defining sentence. Hence, (4).
42. 'Conclusions' are 'based on' a 'premise.' In option (1), 'inferences' are based on evidence and not 'assumptions'. Also, the relationship is inversed. In option (2), 'theories' are 'based on' 'hypothesis'. This is the correct answer choice. In option (3), 'knowledge' and 'ideas' do not share the same relation as 'based on'. In option (4), a 'brand' cannot be said to be 'based on' marketing. However, marketing could be used to build brands. Hence, (2).
43. The answer is ABBA. A 'boor' is 'someone with poor manners' while a 'bore' is 'a dull or uninteresting person'. 'Palette' refers to 'an artists' board for mixing colours' whereas 'palate' refers to 'a person's ability to distinguish between and appreciate different flavours'. 'Peaked' is 'to reach the highest level' whereas 'piqued' refers to 'arousing of interest or curiosity'. 'Bole' refers to 'the trunk of a tree' whereas 'boll' refers to 'the rounded seed capsule of plants such as cotton or flax'. Hence, ABBA.
44. The answer is ABBA. 'Pared' means 'narrowed' as opposed to 'paired' which means 'along with'. 'Islets' refer to 'small islands' whereas 'eyelets' refer to 'small round holes in leather or cloth

- for threading a lace, string, or rope through'. To be 'complacent' is 'to be smug or unbothered' whereas to be 'complaisant' is 'to be willing to please others' or 'to accept what they do or say without protest'. 'Coarse' refers to 'something rough' whereas 'course' refers to 'the track or the field used in games'. Hence, (ABBB).
45. The answer is BAAA. A 'canon' refers to 'a general rule or principle by which something is judged'. One of the meanings of 'cannon' is 'to collide with something forcefully or at an angle'. 'Caret' refers to 'a proof-reader's mark (^) placed below a line of text to indicate a proposed insertion or correction' whereas 'carat' refers to 'a unit of weight for precious stones and pearls, equivalent to 200 milligrams'. A 'canvas' is 'a strong coarse unbleached cloth used to make sails, tents etc. and as a surface for oil painting' whereas 'to canvass' is 'to solicit votes'. A 'tic' refers to 'a habitual spasmodic contraction of the muscles, most often in the face' whereas a 'tick' refers to 'a mark (✓) used to indicate that an item in a list or text is correct or has been chosen, checked, or dealt with'. Hence, (BAAA).

## PRACTICE EXERCISE 6

- Sentence (2) should be, 'Her style of management doesn't work for me'. This means 'I don't like her style'. Hence, (2)
- The error is in sentence (1). 'Throwaway' is a single word and means 'made to be destroyed after use'. Hence, (1).
- The correct phrase in (1) should be 'roped off', which means to 'cordon off or separate by the means of ropes'. Hence, (1).
- Sentence (1) should be 'Her insensitive question put me on the spot'. The phrase means 'to cause someone difficulty or make them embarrassed by forcing them at that moment to make a difficult decision or answer an embarrassing question'. Hence, (1).
- Sentence (1) should be, 'I can't come because I have to look after my sick mother'. This means 'I have to take care of her'. 'Look down upon' means 'have contempt for'. Hence, (1).
- Students would be surprised if the statements of a 'learned' professor are 'refuted' or disproved. 'Decry' is too strong a word in this context. Hence, (2).
- A reflexive pronoun is correctly used as an object when the subject and object both refer to the same person(s) or thing(s). Options (2),(3) and (4) are incorrect usages. Hence, (1).
- 'Come of age' is the correct usage and indicates 'attainment of a state of maturity'. Hence, (3).
- 'Flowed down' is the correct expression which means 'Hung loosely and freely'. Hence, (3).
- 'Get into' is the correct usage in the given context and means 'to make a foray'. Hence, (3).
- Earlier women were not allowed to offer prayers – if you know that your choice becomes clear. Your 'shadow word' should have been 'allowing' which is closest in meaning to 'permitting'. Hence, (2).
- Options (2) and (4) sound ridiculous. 'Worldwide leader in showrooms' does not fit in either. Option (1) fits in both the blanks the best. Hence, (1).
- Only 'just' fits in with 'on the verge'. Hence, (4).
- Only (1) fits in with the theme of the sentence. Hence, (1).
- Only 'concupiscence' and 'motive' make sense for the first half. But only 'incitement' agrees in the second half with 'stimulus'. Hence, (2).
- Only 'dichotomy' is correct since the sentence says this division into two usually contradictory parts or opinions is wrong, not that the argument or choice per say is wrong. 'Divaricator' is irrelevant. Hence, (4).
- Only 'hypothesis' and 'thesis' are correct for the second part of the sentence. However 'methodology' is more in agreement than 'mode'. Hence, (3).
- Only 'pathological' agrees with 'antisocial behaviour' and only 'redeeming' is the correct option for the second half of the sentence. Hence, (2).
- The given statement is trying to connect high prices to the goods. Logically, high prices are noticed, especially, when there is a shortage of goods Option (1) is correct since 'accompaniment' means 'at the same time' and 'dearth' means 'shortage or less availability of goods'. Option (2) is incorrect since 'high prices' are the effect and

- not the cause of 'scarcity of goods'. Option (3) is incorrect since 'destitution' means without money, thus its usage here is not appropriate. Option (4) is incorrect since 'glut' refers to 'excess of goods', which is logically incorrect in the given sentence. Hence, (1).
20. The given statement has the phrase 'should not have ... to talk', here it is clear that this blank has to be filled with a verb in the past tense. Among the given options, only 'ventured' is suitable. Hence, (3).
21. 'Car' travels on a 'road'. We are looking for a similar relationship. 'Electricity' travels through a 'cable'. Thus, option (1) is the correct answer. In option (2), 'ink' and 'pencil' are not related in the same manner. Hence, this option is incorrect. In option (3), 'bomb' and 'missile' are two destructive weapons. They do not share a similar relationship as the capitalized pair. Hence, this option is incorrect. In option (4), a 'bird' can 'fly', but the relationship is not similar. Hence, this option is incorrect. Hence, (1).
22. 'Foresight' and 'farsightedness' are synonymous nouns. We are looking for a similar relationship. In option (1), though 'long' and 'lengthy' are synonyms, both are adjectives, thus they do not share a similar relationship. In option (2), though 'further' and 'farther' are synonyms, both are adverbs, thus they do not share a similar relationship. In option (3), though 'short' and 'dwarf' are synonyms, both are adjectives, thus they do not share a similar relationship. Option (4) is the correct answer. 'Thinker' and 'visionary' are synonymous nouns. Hence, (4).
23. A 'fleet' is a logical section of a 'navy'. We are looking for a similar relationship. Option (1), is the correct answer. A 'chapter' is a logical section of a 'book', thus the correct answer choice. In all the other options, the relationship is inverted. Hence, (1).
24. Many 'feathers' make up a portion of the 'wing'. We are looking for a similar relationship. In option (1), 'down' means 'a soft feather', but many downs alone do not make up a 'goose', thus they do not share a similar relationship. In option (2), the word 'many' cannot be used with 'cotton' since it is an uncountable noun. In option (3), many 'subheadings' do not make up the main 'heading', thus they do not share a similar relationship. Option (4), is the correct answer. A 'wall' is made up of many 'bricks'. Hence, (4).
25. 'Sugar' is put into 'tea'. We are looking for a similar relationship. In option (1), 'paper' is edited by the 'editor', thus they do not share a similar relationship. In option (2), 'weapon' is used by a 'murderer', thus they do not share a similar relationship. Option (3) is the correct answer; 'button' is put into a 'buttonhole'. In option (4), 'umbrella' is used in the 'rain', thus they do not share a similar relationship as the capitalized pair. Hence, (3).
26. A 'symphony' is 'created' by a 'composer'. We are looking for a similar relationship. In option (1), 'rain' is not 'created' by 'flood'. Rain and flood might have a bearing but the relationship is not the same as the capitalized pair. In option (2), 'light' is not 'created' by a 'switch'. Switch is a gadget or an implement to turn on and off the bulb, thus irrelevant. Option (3), is the correct answer as a 'novel' is 'created' by an 'author'. In option (4), a 'song' is not 'created' by 'music'. Song is a form of music, thus not the answer. Hence, (3).
27. Trunk is the main stem of the tree, or is the foundation for the tree to grow. In option (1), 'ink' is neither the stem of 'pen' nor the foundation for a pen, thus not the answer. In option (2), a 'chassis' is the stem or foundation for a 'car', upon which the body of the car is built. In option (3), 'log' may not be the foundation for a 'cabin'. A cabin may be made up of logs but the relationship is not similar. In option (4), 'arm' is a part of the 'body'. However, it is not the foundation or the main organ of the body. Many parts collective make the body and this pair does not share the same relationship as the capitalized pair. Hence, (2).
28. An 'arc' is a 'continuous portion of' a 'circle'. In option (1), a 'segment' is a continuous portion of a 'line', thus the answer. In option (2), a 'whole' is not a continuous portion of a 'part'. In option (3), a 'fraction' is not a continuous portion of a 'percentage'. In option (4), 'big' is not a continuous portion of 'small'. Hence, (1).
29. A 'cow' is a member of a 'herd'. In option (1), 'grass' is not a member of the 'green' as grass could be green in colour only. In option (2), a 'child' is the member of a 'family', thus the answer. In option (3), 'bald' is not a member of 'hair', however baldness is a disorder related to hair. In option (4), a 'jockey' rides a horse'. The

- relationship is not similar to the capitalized pair. Hence, (2).
30. 'Wool' is a fabric made from the hair of sheep, whereas 'acrylic' is synthetic in its origin. In option (1), 'rayon' is synthetic 'silk' like fabric. The order here is reversed, thus it is incorrect. In option (2), 'plastic' is synthetic or semi-synthetic material. 'Rubber' is an elastic material and it is not synthetic, thus this option is irrelevant. Option (3), 'winter' is the coldest season of the year and 'spring' is the season of growth. It is not relevant as the answer. In option (4), 'cotton' is a soft silky fiber made from the cotton plant, whereas 'terylene' is a synthetic polyester fabric. Hence this pair shows the same relationship as the capitalized pair. Hence, (4).
31. 'Rent' has to be paid if something is taken on a 'lease'. We are looking for a dissimilar relationship. In option (1), 'interest' has to be paid if an amount is 'borrowed', thus they share a similar relationship. In option (2), 'salary' has to be paid if someone is 'employed', thus they share a similar relationship. In option (3), 'price' has to be paid if an amount is 'bought', thus they share a similar relationship. Option (4), is the correct answer, 'tax' is not paid if you need to 'govern' something, thus they do not share a similar relationship as the capitalized pair. Hence, (4).
32. 'Temperature' is used to measure or denote the amount of 'heat'. We are looking for a dissimilar relationship. In option (1), 'votes' are used to measure 'popularity', thus they share a similar relationship. In option (2), 'IQ' is used to measure 'intelligence', thus they share a similar relationship. In option (3), 'ohms' is used to measure the amount of 'resistance', thus they share a similar relationship. Option (4) is the correct answer; 'speed' is not used to measure 'distance', thus they do not share a similar relationship as the capitalized pair. Hence, (4).
33. A 'progressive' person brings about 'progress'. We are looking for a dissimilar relationship. In option (1), a 'reformist' brings about 'reform', thus they share a similar relationship. Option (2) is the correct answer as a 'sympathizer' just shows 'sympathy'; he does not bring about sympathy. Thus they do not share a similar relationship. In option (3), a 'revolutionary' brings about 'revolution'; thus they share a similar relationship. In option (4), a 'terrorist' brings about 'terror'; thus they share a similar relationship. Hence, (2).
34. 'Stubborn' and 'adaptable' are exact antonyms. We are looking for a dissimilar relationship. In option (1), 'stupid' and 'bright' are exact antonyms, thus they share a similar relationship. Option (2) is the correct answer as 'moral' and 'amoral' are not exact antonyms, thus they do not share a similar relationship as the main pair. In option (3), 'inherent' and 'extraneous' are exact antonyms, thus they share a similar relationship. In option (4), 'friend' and 'enemy' are exact antonyms, thus they share a similar relationship. Hence, (2).
35. The use of a 'clip' is to hold 'papers' together. We are looking for a dissimilar relationship. In option (1), the use of a 'thread' is to hold 'beads' together, thus they share a similar relationship. In option (2), the use of 'cement' is to hold the 'bricks' together, thus they share a similar relationship. In option (3), the use of a 'ribbon' is to hold the 'hair' together, thus they share a similar relationship. Option (4) is the correct answer; the use of a bag is to carry vegetables, not to hold vegetables together; thus they do not share a similar relationship as the capitalized pair. Hence, (4).
36. The sentence states that there is a lot of unemployment. Thus only 'falling' and 'shrinking' can fit in the first blank. Options (2) and (3) can be eliminated. As only high expertise jobs will be hit, the effect of AI is unlikely to be 'disruptive' or 'of any significance'. So, option (4) is eliminated. Hence, (1).
37. All the options are appropriate for the first blank. However, only 'jeopardy' is grammatically correct in the second blank. Hence, (3).
38. 'Pertinacity' means 'persistent determination' over one's viewpoints. 'Prolixity' means 'to extend to great, unnecessary, or tedious length'. 'Brevity' represents 'the concise and exact use of words in writing or speech'. All the words can fit in the first blank. However, for consistence of logic, the second blank should also contain a synonymous word of the first blank. Thus options (1) and (3) containing antonymous words are eliminated. 'Circumlocution' means 'the use of many words where fewer would do, especially in a deliberate attempt to be vague or evasive'. 'Adroitness' means 'cleverness or skill'. It would be imprudent for a

- teacher to avoid brevity and berate students for being clever. Thus option (4) is also eliminated. Thus 'prolixity' and 'circumlocution' are the two words that fit in the two blanks. Hence, (2).
39. 'Obtuse' means 'to be annoyingly insensitive or slow to understand'. 'Prosaic' refers to 'something or someone that is commonplace or unromantic'. 'Taciturn' refers to 'a person who is reserved or uncommunicative in speech'. 'Implacable' means 'someone who is unable to be appeased or placated'. All the four words can fit in the first blank. However, the word in the second blank should present a contradiction. Thus only option (3) can fit in the blank. Hence, (3).
40. If a historian's work has led to the preservation of facts in the country's history, religion, antiquities and language, it (the work) can be said to long-lasting or enduring. Thus 'abiding' makes sense in the first blank. 'Evanescent' (which means 'brief, fleeting'), 'eternal' (which means 'timeless') or 'eminent' (which means 'famous') are incorrect as they are grammatically or contextually inappropriate. If the historian did not work on preserving the facts about the country's history, they might have been unknown to the world. Hence, 'perished' is the correct option. 'Elevated' or 'expired' do not make sense. Facts cannot 'languish' for lack of exposure. Hence, (3).
41. A garden can only be a 'repository' or 'a place where something, especially a natural resource, is found in significant quantities'. 'Collective' (a cooperative enterprise), 'conservatoire' (a college for the study of classical music) and 'fountainhead' (an original source of something) do not make sense in the first blank. Thus the first three options are eliminated. 'Specimens' is the correct choice for the second blank. Hence, (4).
42. A confusion can only arise if the usage of words or idioms leads to being open to more than one interpretation. Thus, 'ambiguous' is correct in the first blank. Word and idiom usage cannot be 'absurd' (wildly unreasonable, illogical, or inappropriate). Also, there can hardly be any confusion over the intentional use of 'stimulating' or 'profound' words. Hence options (2), (3) and (4) are eliminated. Hence, (1).

## VA-3.3 | TENSES AND SUBJUNCTIVE MOOD

### PRACTICE EXERCISE 1

1. is writing
2. spend, stays
3. needs, getting
4. are, singing
5. is, have solved
6. is working
7. are playing
8. coming, are going
9. will finish
10. is dreaming
11. plays
12. fall
13. going, think
14. are
15. hold
16. is
17. live, is working
18. like, am trying
19. are going, come
20. has
21. looks
22. has
23. has
24. have found
25. purchased, read
26. had written
27. travelled, was getting
28. will be meeting, comes
29. shall complete
30. saves
31. The sentence indicates a supposition: if you touch, hence, it is in imperative mood.
32. The sentence indicates a condition that is imaginary and probabilistic, hence, it is in the subjunctive mood.
33. Here the wish of the speaker is being conveyed. So the sentence is in the subjunctive mood.
34. In this sentence the speaker expresses a supposition that is not present in the real scenario. Hence, it is in subjunctive mood.
35. Here the sentence can indicate an order or an advice. Hence, it is in the imperative mood.
36. After the verb *wish* the past subjunctive is used to indicate an imaginary situation. So the past form *could* should be used here.
37. In subjunctive mood, in case of the verb *to be* the past form is *were* for all numbers and persons. So the correct form is *were*.
38. There is no error in the sentence because in the subjunctive mood in the case of the verb *to be*, the present form is *be* for all numbers and persons.
39. There is no error in the sentence. The past subjunctive is used after *as if* to indicate unreality or improbability.
40. In this sentence the base form of the verb should be used to indicate the present subjunctive mood. Moreover, *should* is implied in the sentence, so the correct verb should be *wins*.
41. The correct form is *goes*. This is a situation, which is always true if the condition is fulfilled, i.e., it is indicative of a habitual result. So the verbs in both the principal and the *if* clause are denoted by the present tense.
42. The correct sentence is: If I had not seen him before, I would not have recognised him. This sentence refers to a past situation with hypothetical result. The *if* clause should be in the past perfect tense and the main clause should be in the perfect conditional.
43. *If I were the Principal* because the *to be* form of the verb in the first person is always conjugated as *were* in the second conditional.
44. The correct form is *will pass*. This is an example of Conditional 1 where if the *if* clause is in the present tense; the verb in the main clause is in the simple future.

45. This sentence states a situation, which happens if a condition is fulfilled. In such cases the verb in the principal clause is in simple present and the if clause in the simple future. Hence, *will bring*.
46. (3) is the correct answer. This is an example of a past conditional because it refers to a past situation with hypothetical results. The condition cannot be fulfilled because the time is past. The *if* clause should be in the past perfect tense. Hence, (3).
47. (1) is the correct answer. Here the situation arises if a particular condition is met. This type of conditional is formed by the use of the present simple in the *if clause + will + verb* (base form) in the part that indicates the result. Hence, (1).
48. The correct answer is (3). This sentence indicates past conditional because it refers to a past situation and the condition cannot be fulfilled now because the time is past. The *if* clause must be in the past perfect tense and the main clause must be in the perfect conditional. Hence, (3).
49. The correct answer is (2). This sentence refers to an unreal or improbable situation. It provides an imaginary result for a given situation. Here *would* should be followed by the base form of the verb. Hence, (2).
50. (1) is the correct answer. (4) unnecessarily changes the structure of the sentence. This is a type of situation which occurs when a particular condition is met. *Unless* implies *if ...not* and conveys the idea properly. Hence, (1).

## PRACTICE EXERCISE 2

- Qs. 1 to 4: The present perfect tense is used to denote an action that began at some time in the past and is continuing up to the present moment. Sentences 1 and 2 are such examples. Sentence 3 indicates that she has finally achieved the unattainable; hence she can now be satisfied. In such cases where a past event is described but its effect in the present is more important than the action itself, the present perfect tense is used. The present perfect tense is not used with words denoting the past time (i.e. yesterday). So, in sentence 4, 'updated' should be used.
- Qs. 5 and 6: The present perfect continuous tense is used for an action that began at some time in the past and is still continuing. In sentence 5, the stealing of money, which started in the past, is still continuing. The same is the case with the project work, in sentence 6, which is still going on.
- Qs. 7 to 9: The simple past tense is used to indicate an action that occurred in the past, as in sentences 7 and 8. The simple past tense is also used to refer to past habits, as in sentence 9.
- Qs. 10 to 12: In these sentences the past continuous tense should be used. This tense form is used to denote an action going on at some time in the past, as in sentences 10 and 11. Sometimes, in the same sentence, the past continuous and the simple past tense can be used when the reference is to a new action that happened in the middle of a longer action. The simple past tense is used for the shorter or the new action. In sentence 12, our meeting him is the new action and his shooting is the longer action. So, the blank should be filled with 'was still shooting'.
- Qs. 13 and 14: In these sentences, the past perfect tense should be used. It is used in cases where two actions, both in the past, need to be mentioned. For the action that completed first, the past perfect tense (i.e., had + past participle) is used. In sentence 13, his committing suicide happened before his being saved from the death sentence. So, the correct sentence should read: 'He was saved from the death sentence but he had already committed suicide'. In sentence 14, 'had done' would be appropriate because the rest of the sentence is in the past tense.
- Qs. 15 and 16: In these sentences we are referring to actions that began before a certain point in the past and continued up to that time (which is also in the past). So, in sentence 15, the correct phrase would be 'had been trying' and in sentence 16 'had already been teaching'.
- Qs. 17 and 18: The sentences indicate events that are yet to take place. Therefore, we should use the simple future tense. In sentence 17, 'will meet' is appropriate. In sentence 18 also, a future event is being referred to. The present continuous tense (am going) can also be used to describe a future event.
- Qs. 19 and 20: The future continuous tense is used to represent an action that is expected to go on at some time in the future. So, in sentence 19, the correct phrase would be 'will be doing'. Sentence 20 represents an action that would be in progress over a period of time in the future and will end at some point of time in the future. So, in sentence

- 20, the future perfect continuous tense should be used, i.e., 'shall have been working'.
- Q.21: This sentence is an example of the future perfect tense, which indicates the completion of an action by a certain time in the future. So, 'will have left' would be correct here.
- Qs. 22 & 23: If the main clause is in the past tense, the subordinate clause is also denoted by the past or past perfect tense. So, in sentence 22, 'was' would be correct. If there is a distinct difference in the time frame of two actions in a sentence, they can be denoted by a different tense. So, in sentence 23, 'is' would be appropriate.
- Qs. 24 & 25: Sentence 24 describes two events that are going to take place in the future, one preceding the other. Normally in such scenarios the future perfect tense is used. But the future perfect tense is not used in clauses beginning with time expressions such as 'when', 'before', 'unless', etc. So in sentence 24 the correct option would be 'finish'. We require the future perfect tense in sentence 25 because it states two events that will take place in the future – she will learn French before moving to Paris. Therefore, the former action (that is supposed to complete first) should be stated using the future perfect tense, i.e., 'Will she have learnt'.
- Qs. 26 & 27: The past subjunctive form is used after the verb 'wish' to indicate a situation, which is unreal, imaginary, or contrary to fact. The verb in the past subjunctive is always used in the past tense and the 'to be' verb is always expressed as 'were' even for singular subjects, e.g. I wish I were rich.
26. I wish this bag *belonged* to me.
27. The given sentence is correct.
- Qs. 28 & 29: The past subjunctive is used after 'if' (conditional clause) to express improbability or unreality in the present. The verb 'to be', when used in such unreal situations, is always in the 'were' form. In the result part, 'would' should be used.
28. If I *were* the owner of this car I would get rid of the clear lamps and put in the stock lamps.
29. It occurred to him that if he *were* the gatekeeper of hell, he *would* have plenty of opportunities to turn the key.
- Qs. 30 to 32: The past subjunctive is used after 'as if' and 'as though' to indicate unreality.
30. The man was limping on towards the marshland, as if he *were* the pirate come to life, and come down, and going back to hook himself up again.
- The sentence structure 'would rather + subject + past subjunctive' is used to indicate preference.
31. I would rather you *went* for the movie. (i.e. I would prefer you to go for the movie.)
- The past subjunctive is used in expressions following 'it is time' or 'it is high time', when it implies that it is already late.
32. It is high time that she *consulted* a doctor.
- Qs. 33 to 36: These are cases of real conditionals, i.e., real or possible situations. The situations take place if a certain condition is met. In this case, the 'if' clause is in the present tense and the clause denoting the result is in the future tense or present tense depending on the context. Sentences 33 and 34 are cases where the result is always true if the condition is fulfilled, and is indicative of habitual action. So, the verb in both the principal and the subordinate clauses should be in the present tense. The format is 'if/when + simple present'...'simple present'.
33. If we change anything in the calculation, we ~~will~~ get a different result.
34. Whenever we call him he says he *is* busy.
- Sentences 35 and 36 should have the format: 'if + simple present'...'simple future'.
35. If they declare a holiday, I *will plan* my trip to my uncle's house.
36. The given sentence is correct.
- Qs. 37 to 40: These are cases of unreal conditionals, i.e., they denote impossible or improbable situations. They provide an imaginary result for an imagined situation. The format is 'if + simple past' and the result part should have 'would + base form of the verb'.
37. The given sentence is correct.
38. If she had the looks, she *would* be a successful model.
- When the verb 'to be' is used in such unreal situations, it is always in the 'were' form.
39. If she *were* a writer, she would ~~have~~ beautifully ~~penned~~ her experiences.
- When the verb 'to be' is used in such unreal situations, it is always in the 'were' form.



40. Had I obtained admission to the elite club, I *would/could have established good contacts* with successful people.
- Past conditionals denote past situations with hypothetical results. The auxiliary verb 'had' can also be used to denote the imaginary condition. The format would be: 'Had + simple past'...'would/could have + past participle'.
41. The sentence formed by parts D and E is a case of past conditional. It denotes a hypothetical situation in the past with a hypothetical result: If they knew this business would be successful, they would have thought of a better name. This type of conditional provides an imaginary result for a given situation in the past. Such constructions use the auxiliary 'had' to indicate the occurrence of a condition in the past and the 'would + have + past participle' in the result clause. Therefore, D and E should be rewritten as: 'In retrospect, had we known that this would eventually develop into a business on its own, we would have put more thought into selecting a more appropriate name'. Hence, (4).
42. The excerpt describes an event in the past. The same tense form has to be maintained throughout. Parts C, D and E taken together actually state an imaginary situation in the past which would have yielded an imaginary result. Therefore, the sentence should be rewritten in the past conditional format: 'He knew well that had he proposed the measure as it is now before us, and had he shown it to his party first, they would have started back from it in horror.' Hence, (3).
43. Only part E is erroneous. The past subjunctive form is used after the verb 'wish' to indicate a situation, which is imaginary or unreal. E means that the speaker is always sad when he cannot go back home and then he longs for the company of Michael who cannot be with him in reality. Here 'were' should be used despite the fact that the corresponding subject is singular. E should be rewritten as: 'I always get sad when this happens, and I wish Michael were here'. Hence, (4).
44. In part C, there is an incorrect word form. 'Naïveté' is a noun, whereas here we require the adjective form 'naïve' because part C describes certain qualities of Hannah. In E, the present subjunctive form is required because of the clause with the verb 'insisted'. In case of clauses with verbs like wish, advise, insist, resolve, command, etc., the preferred form is the present subjunctive. So, E should be rephrased as: 'he loved her, even when he insisted that they marry. Hence, (3).
45. There is an error in part B. It states an unreal situation (indicated by the use of the expression 'as if'). Hence, we should use the subjunctive form here and B should be rewritten as: 'Today when she met him, it was almost as if she were...' Hence, (2).
46. The excerpt states a scenario where the fulfilment of a condition results in another event. It is indicative of a habitual action: if the condition is fulfilled, the result will always take place. In such cases, we use the form 'if + simple present' in the condition part and the simple present tense in the result part. Therefore, part C should be rewritten as: 'If something thwarts him...' and E should be rephrased as: 'If he is annoyed, his tongue is apt to be sharp'. Hence, (4).
47. There is an error in part B. The conditional clause actually states a situation that is unreal or improbable and provides an imaginary result for the imagined situation. This is a case of an unreal conditional and the sentence structure should be: 'if + simple past' in the condition part and 'would/could + verb' in the result part. The verb 'to be', when used in such unreal situations, is always in the 'were' form. Thus, part B should be written as: '... If she were in any way anaemic'. Hence, (2).
48. There is an error in part D. The present subjunctive form is used in sentences with verbs expressing desire, suggestion, intention, command, etc. and followed by a noun clause. Thus, part D should be rewritten as: 'In answer to an implied suggestion that he stop philosophizing'. Hence, (3).
49. The errors are in parts C and D. These parts express a hypothetical situation and the imaginary result of such a situation. So, we need the subjunctive form here. The parts should be rewritten as: 'Were he the author of the book, he could easily omit the offending words' or 'Had he been the author of the book, he could have easily omitted the offending words.' Please note that the plural past form of the 'to be' verb, i.e., 'were' should be used irrespective of the number and person of the subject. Hence, (3).

50. The error is in part B. This is an example of a present unreal conditional, which talks about what Arthur and Brenda would generally do in the imaginary situation. The format would be 'If + simple past' in the conditional clause and 'would + verb in the base form' in the result part. So, B should be rephrased as: 'if they were to have the means, they would never become...'. Hence, (2).

## VA-3.4 | SENTENCE STRUCTURE, MODIFIERS AND PARALLELISM

### PRACTICE EXERCISE 1

**Directions:** Given below is a set of sentences. Read each sentence and

Mark (1), if it is a Simple Sentence.

Mark (2), if it is Compound Sentence.

Mark (3), if it is a Complex Sentence.

Mark (4), if it is a Compound-Complex Sentence.

1. It is a compound sentence. Independent Clauses – ‘I tried to catch the 10 a.m. flight to Delhi’ and ‘my friend tried to catch the 10.15 a.m. flight to Chennai’; Coordinator – ‘and’
2. It is a complex sentence. ‘We proceeded for lunch’ is the independent clause, ‘we finished our presentation’ is the dependent clause and ‘after’ is the subordinator.
3. It is a complex sentence. ‘He forgot to give his research guide the synopsis of his study’ is the independent clause, ‘he submitted his dissertation’ is the dependent clause and ‘when’ is the subordinator.
4. It is a complex-compound sentence. The independent clauses are ‘the servant lived in the outhouse’ and ‘the land-owner lived inside the mansion’. The dependent clause is ‘who knew he was superior’.
5. It is a complex-compound sentence. The independent clauses are ‘he finished his work’ and ‘rushed to the drawing room to switch on the TV’ and the dependent clause is ‘he heard his mother calling’.
6. It is a simple sentence.
7. It is a complex-compound sentence. The independent clauses are ‘she watched a new comedy film’ and ‘she enjoyed it thoroughly’ and the dependent clause is ‘Jaya prefers watching social drama’.
8. It is a simple sentence.
9. It is a compound sentence. Independent Clauses – ‘visited an online seller’ and ‘I could not complete my transaction’; Coordinator – ‘but’
10. It is a complex sentence. ‘He returned the book’ is the independent clause, ‘he noticed the missing pages’ is the dependent clause and ‘after’ is the subordinator. Note that when the independent clause begins the sentence with subordinators in

the middle, no comma is required.

**Directions:** Identify the modifier errors in the following sentences and correct them.

11. There is a dangling modifier here. It is not clear who flashed the Rs.100 note at her kid brother, as the subject is missing from the sentence. The correct way to phrase it will be ‘Reaching into her wallet, she flashed the Rs.100 note at her kid brother.’
12. The sentence does not have a clear subject that the modifier ‘while watering the plants’ can qualify. This is a case of dangling modifier. The subject should be introduced in the sentence and placed as close as possible to the modifier. So, the correct sentence should read ‘While watering the plants, he splashed water all over.’
13. There is a dangling modifier here. It does not become clear immediately who was making the arrangements. So the revised sentence, which will be clearly understood, is ‘After months of making arrangements, we finally presented our fashion show.’
14. There is a dangling modifier here. Who is returning home after a hectic day at the clinic is not clear from the sentence. The revised sentence should be ‘Returning from a hectic day at the clinic, you should be able to relax at home.’
15. There is a misplaced modifier here. The sentence makes it seem that Shwetha was smashed flat by a truck but, yet, was able to take what was left of the doll. The corrected sentence should read ‘Shwetha gloomily took what was left of the doll which was smashed flat by a passing truck.’
16. The sentence has a misplaced modifier. ‘Six earrings’ cannot strive to get attention; the action can only be done by the subject Sheila. So, the correct sentence should read ‘To gain attention, Sheila wore six earrings.’
17. The sentence contains a squinting modifier: ‘recently’. The corrected sentences can be ‘The advertising firm that had recently been in the news declared bankruptcy’ or ‘The advertising firm that had been in the news declared bankruptcy recently’.
18. This sentence exhibits an error of dangling modifier. The subject that the clause ‘as her teachers had

- recommended') qualifies is missing. The correct sentence should read 'As her teachers had recommended, she started to go for music classes and thoroughly enjoyed the experience.'
19. In this sentence there is an error of misplaced modifier: she is in hurry and not her certificates. So, the correct sentence should read 'In a hurry, she forgot to collect her certificates from her school.'
20. In this sentence the modifying phrase 'that had brought bad luck to the ship' is misplaced. Instead of qualifying the weapon 'crossbow', the phrase should qualify the 'albatross' that was killed. Hence, the correct sentence should read 'In *The Rime of the Ancient Mariner*, the mariner used a crossbow to shoot the albatross that had brought bad luck to the ship.'

**Directions:** Choose the most logical and grammatically correct option from the four choices.

21. It is not clear from option (1) who failed to attend court; so this is a case of dangling modifier. Option (2) does not place the subject next to the modifying phrase ('after failing to attend court several times'). Option (3) solves both problems. Hence, (3).
22. The modifier 'on his notebook' should follow 'his homework', as that is the phrase it should modify. Hence, (3).
23. There is a dangling modifier in option (1). The person who is 'out of breath' is not mentioned at all. Both options (2) and (3) are correct as they contain a subject at the appropriate location. Hence, (4).
24. There is a misplaced modifier in options (2), (3) and (4). It is not possible to write for over 100 years. The modifier 'for over 100 years' should be placed immediately next to the part it modifies. So, option (1) is the best option. Hence, (1).
25. In option (1), 'either' has been placed incorrectly. 'Either' should be placed before the word that it is supposed to modify. The choice is not between 'visiting' and another action or between 'we' and other visitors. The choice is between two people and places. So, only option (3) is correct. Hence, (3).

26. There is a dangling modifier in options (1) and (2). The modifier should qualify a person who has done the action of reading the story, but the subject is not mentioned at all. Hence the modifier dangles. Option (3) also does not place the subject next to the modifying phrase ('after reading the original story'). The correct sentence should read 'After reading the original story, I find the movie based on it unconvincing.' Hence, (4).
27. In options (1) and (2) 'almost' has been placed incorrectly. One cannot 'almost' research and there cannot be an 'almost' project. The modifier 'almost' should modify the time frame and not the action itself. Thus option (3) is correct. Hence, (3).
28. There is a squinting modifier in option (1). The position of 'eventually' is causing confusion. It is not clear which action 'eventually' applies to. Both options (2) and (3) are correct, though they convey different ideas. Hence, (4).

**Directions:** In each question, there are five sentences or parts of sentences that form a paragraph. Identify the sentence(s) or part(s) of sentence(s) that is/are incorrect in terms of grammar and usage. Then, choose the most appropriate option.

29. When a comparison is made between two objects, both of which are considered to be equal in quality or quantity, 'as ... as' or 'not as ... as' is used. Therefore, in part E, 'like' should be replaced by the conjunction 'as'. Hence, (3).
30. A sentence that uses a string of clauses must use the same grammatical form in each clause for the sake of parallelism. Part C is not aligned to parts A and B where the clauses start with 'where'. So, part C should be reframed as 'and where love is scorned as illusion'. In parts D and E, the parenthetical part states some of the attributes of the work of fiction. Since they are of similar content, they should be expressed in the same part of speech and form. 'Romantic' and 'suspenseful' are adjectives whereas 'evokes eroticism' is a verb phrase; therefore, part E should read 'erotic, suspenseful – and completely unforgettable'. Hence, (3).
31. In a sentence, gerunds, participles or infinitives should be used in a uniform manner to denote similar content or function. Since parts C and D provide a list of duties associated with a woman,

the gerund form should be used throughout. So, part D should be reframed as 'and taking care of children...' to make it parallel to 'cooking, cleaning, sewing'. There is an error in part E: 'unsuitable' should be followed by the preposition 'for' and 'incapable' should be followed by 'of'. The appropriate prepositions should be used after the words to make the parts parallel and grammatically correct. Hence, (3).

32. Combining parts B, C, D and E we get to know what the heath is capable of doing. Information of similar content should be stated in similar grammatical form. The auxiliary 'could' in part B is common for the verbs in each of the subsequent phrases. In part C, 'anticipating' should be changed to 'anticipate', making it parallel to 'retard' and 'sadden'. Similarly, in part D, 'intensified' should be changed to 'intensify'. Hence, (3).

**Directions:** *In each of the following sentences, a part has been underlined. Beneath each sentence, four different ways of phrasing the underlined part have been indicated. Choose the best alternative from among the four.*

33. The underlined part states the conditions under which vegetation near water bodies flourishes. Since all parameters serve the same purpose, they should be parallel. In option (1), the second part (which talks of purity of water) is not parallel to the other two parts. In option (3), the possessive form 'water's' is not required. Option (4) distorts the information and incorrectly uses the abbreviation of 'it is' and renders the sentence meaningless. Only option (2) gives us a grammatically correct and parallel sentence. Hence, (2).
34. In order to maintain a parallel structure in a sentence, we should ensure that conjunctions join words, phrases or clauses that are of similar grammatical form. 'Literary sensation' and 'run-away bestseller' are phrases that are parallel and include one adjective and one noun. In options (2) and (3) this parallelism is affected because of the use of the noun form 'literature'. Moreover, in option (2), the possessive form 'country's' is replaced by the plural 'countries', which make the sentence incorrect. In option (3), 'seamless authentic' is incorrect because both are adjectives, whereas the part after the conjunction 'and' has an adjective-noun combination. Though option (1) does not have these errors, it is incorrect

because 'lyrics that are exquisite' is not parallel to 'seamless authenticity'. Only option (4) gives a grammatically correct sentence. Hence, (4).

## PRACTICE EXERCISE 2

**Directions:** *Identify the modifier errors in the following sentences and correct them.*

1. The sentence contains a squinting modifier. It is not clear whether the book was used to hit the boy or whether the boy was carrying a book. The corrected sentences could read 'The teacher used a book to hit the boy' or 'The teacher hit the boy who was carrying a book'.
2. The modifier 'as an actress' qualifies the pronoun 'she' and not 'the desire'. This is a case of misplaced modifier. The correct sentence should read, 'As an actress, she always had the desire to scale new creative heights.'
3. The modifier 'with all modern facilities', which should ideally modify the 'gymnasium', is misplaced in the sentence and incorrectly conveys that the employees come with all modern facilities. The correct sentence should read 'The company is planning to set up a gymnasium with all modern facilities, within the office premises, for its employees.'
4. This is a case of dangling modifier because the sentence does not have a subject. We do not know who was staggering like a drunkard. The correct sentence should read 'Staggering like a drunkard, he could not haul his aching body upright.'
5. Here the modifier 'that enjoy excellent goodwill' is incorrectly placed after 'India', which gives the impression that India and not 'our products' enjoys goodwill. The correct sentence should read 'Our products that enjoy an excellent goodwill are marketed throughout India.'

**Directions:** *Read the following sentences carefully and identify whether the following sentences have been constructed correctly, and make corrections where necessary.*

6. This sentence is correct.
- 7 & 8: When there is a comparison between two elements in a sentence, the elements being compared should be actually comparable and be parallel in structure. In 7, the 'climate in New Zealand' should be compared to 'the climate in any other country'

but the current structure of the sentence implies that the comparison is between 'climate in New Zealand' and 'any other country'. Similarly, in 8, the format of the new journal can be compared only to the format used for its predecessor. So, the sentences should be reframed to 'The climate in New Zealand is better than that of any country in the world; yet vast areas of good land are lying idle, growing weeds and rubbish' and 'The format of the new journal is more pleasing than that of its predecessor and should appeal to a wider public'.

9 to 11: In a sentence, gerunds, participles or infinitives should be used in a uniform manner to denote similar content or function. So, 9 should have the gerund form across: 'playing cricket, watching movies and reading business magazines'. If we state a list of things, all the elements of the list must be in the same grammatical form. The forms / parts of speech of words should not be mixed. Sentence 10 is correct because all the items in the list are comparative adjectives: 'safer', 'stronger' and 'more secure' (since 'secure' does not have an '-er' form). In sentence 11, we should have the adverb form throughout; so the sentence should be reframed as 'Though she was in a hurry, she completed the work at hand quickly, accurately and thoroughly.'

12 & 13: In any sentence, the tense forms should also be parallel. Only if there are distinctly different time frames should we change the tenses. In sentence 12, the tense of the third verb has been unnecessarily changed. The sentence should be reframed as 'He was so overpowered by his kidnapper that he did not have the strength to shout, to strike a blow or to make an attempt to free himself.' Similarly, in sentence 13, the use of the past continuous tense in the last clause is incorrect. It should be changed to '...and waited eagerly for the train to arrive.'

14 to 16: While using conjunctions in a sentence, we should ensure that words, phrases or clauses that are thus joined are of the same grammatical form, i.e., they join nouns to nouns, adverbs to adverbs, verbs to verbs, phrases to phrases, clauses to clauses, etc. In sentence 14, 'either' is followed by the pronoun 'you', whereas 'or' is followed by the verb 'attend'. So, sentence 14 should be reframed as 'You can either go for the picnic or attend the class.' In sentence 15, the phrase 'diverse

landscape' is not parallel to 'heritage that is rich'. So, the sentence should be reframed as 'India is a land of diverse landscape and rich heritage.' In sentence 16, the gerund form 'dancing' should be used to make it parallel to 'swimming'.

17. When a list is stated after a colon, we should ensure that all the elements in the list are of the same structural form. So, sentence 17 should be rewritten as 'One is still fascinated by the film for these reasons: striking imagery, gripping scenes, bright dialogue and exquisitely drawn characters.'
18. If two words in a sentence take different prepositions after them, we should not use a common preposition thinking it would make the parts parallel. This mistake can be seen in sentence 18. 'Worried' takes the preposition 'about' and 'anxious' should be followed by 'for'. So, the sentence should read 'After her husband's death she was worried about and anxious for her daughter's future.'
19. A parallel structure that begins with clauses must continue with clauses. Changing to another pattern or changing the voice of the verb (from active to passive or vice versa) will break the uniformity. Sentence 19 lacks the uniformity of clauses; hence it should be rewritten as 'On the eve of the final examination, the education counsellor told the students that they should eat a light meal, sleep properly and not panic unnecessarily.'
20. Sentence 20 mixes the active and the passive voice, so it should be rephrased as 'I expected that she would not divulge the secret to my family members but she revealed it to my brother.'

**Directions:** *Identify the phrases and name them.*

21. by the seaside — prepositional phrase
22. with immense depth — adjective phrase
23. to lead his followers — noun phrase
24. in an evasive manner — adverb phrase
25. in haste — adverb phrase

**Directions:** *Identify the main clause and the subordinate clause and also state the kind of subordinate clause.*

26. Many of us are made to believe (Main clause); that dinosaurs exist (Subordinate noun clause).
27. They ran (Main clause); wherever they could (Subordinate adverb clause of place).

28. We have no clue (Main clause); how the promoter got the contract (Subordinate adverb clause of reason).
29. The hand rules the world (Main clause); that rocks the cradle (Subordinate adjective clause).
30. He is quite sure (Main clause); that he will win the chess competition again this time (Subordinate noun clause).

**Directions:** *Identify the errors of modifiers in the sentences and write the correct form.*

31. By his sister is a misplaced phrase. This reads as if his sister selected him for the course. This is not the idea. By shifting the modifying phrase appropriately the error can be rectified as: He was informed by his sister that he had been selected for the course.
32. Who took the stroll? Note that in the given expression the subject is missing, thereby giving rise to ambiguity. The correct expression is: After the completion of dinner, a long stroll was taken by him/her/us.
33. Who debated? Who reached at a decision? The given sentence fails to relate the subject to the action taken. The correct expression is: After debating over the issue for hours, a decision was reached at by us or After debating over the issue for hours, we reached at a decision.
34. The misplaced phrase climbing up the hill conveys wrong meaning that daffodils are climbing up the hill! It is actually the trekkers who were climbing up the hill. Hence, climbing up the hill, the trekkers were thrilled to see a field of daffodils.
35. It reads as if the dog was going to the market! The modifying phrase on his way to market may be positioned as: On his way to the market, he noticed a dog gnawing a bone.

**Directions:** *Identify the errors in each sentence and correct them.*

36. Can your dog run as fast as mine? or  
Can your dog run as fast as my dog?
37. His horse is stronger than Zoya's horse. or  
His horse is stronger than Zoya's.
38. Our bungalow seems to be as modern as yours.

39. Kedar's performance this year has been better than that of Amit. or Kedar's performance this year has been better than Amit's.
40. The journey was both long and very tiring.
41. This is not a time for words but for action.
42. You must either accept my fees or give me an explanation for not doing so.
43. I believed him because he said he was going to return my money.
44. Tarun was not just good at playing cricket but also played golf or Tarun is not just good at playing cricket but plays golf as well.
45. I like to officiate basketball than to play basketball or I like officiating basketball than playing basketball.

### PRACTICE EXERCISE 3

**Directions:** *Identify the phrases and name them.*

1. watching television — noun phrase
2. to end the work in time — adverb phrase
3. of cowardice — adjective phrase
4. atop the table — prepositional phrase
5. of wisdom — adjective phrase

**Directions:** *Identify the main clause and the subordinate clause and also state the kind of subordinate clause.*

6. She is much younger (Main clause); than I thought (Subordinate adverb clause of comparison).
7. She was so exhausted (Main clause); she could hardly travel further (Subordinate adverb clause of reason).
8. We should respect people (Main clause); who die fighting for their motherland (Subordinate adjective clause).
9. There were so many members in the parliament (Main clause); that there were no seats available (Subordinate adverb clause of consequence).
10. there is a way (Main clause); When there is a will (Subordinate adverb clause of condition).
11. She preferred only those dresses (Main clause); that were either blue or white (Subordinate adjective clause).

12. Do (Main clause); what I ask you to do (Subordinate noun clause).
13. She managed to reach office on time (Main clause); though she left home late (Subordinate adverb clause of concession).
14. Everyone knows (Main clause); that life is not a bed of roses (Subordinate noun clause).
15. One should concentrate more (Main clause); while working on a time-bound project (Subordinate adverb clause of time).

**Directions:** *Identify the errors of modifiers in the sentences and write the correct form.*

16. In the given expression, placing the biting characteristic of the dog after adjective 'insane' changes the meaning of the sentence; it indicates that the dog which is insane bites. The correct expression is: The dog that bites is insane.
17. Can you imagine a 4000-year-old museum? The clause 'that is 4000 years old' has to actually modify Indian crown. The correct sentence may be written as under: There is a 4000-year old Indian crown in our city museum.
18. Did the leader not believe his own promises? Here the modifying phrase is 'without believing a word of his promises' which can be placed appropriately as under to mean the party men did not believe the leader's words: Without believing a word of his promises, the disgruntled party men listened to what their leader said.
19. In the given sentence, the subject is missing. The correct expression is: It being a moonlit night, we decided to see the Taj Mahal.
20. Who was practising in the auditorium? As there is no subject for this action present in the sentence, the phrase 'while practising in the auditorium' wrongly modifies the word 'announcement'. By introducing a subject for the verb 'practising' we can rectify the error: While 'practising' in the auditorium, we received the announcement from the coach.
21. As the verb 'to complete' does not have a subject, the phrase 'to complete his project on time' tends to modify (John's) holidays wrongly. (Holidays cannot complete a project!) We may get correct sentences as under: By introducing the subject John: So that John could complete his project on time, his holidays were spent in the computer lab.

By making John the object of the modifying phrase 'to complete his project on time': To complete his project on time, John spent his holidays in the computer lab.

22. There is no subject for the verb 'watching'; hence, the phrase 'watching the movie alone' modifies wrongly the noun 'explosion'. ('The explosion' was watching the movie!) We may correct the sentence, by inserting a subject for the verb 'watching' as under: While I was watching the movie alone, the explosion made me jump up. Alternatively, the clause 'the explosion made me jump up' may be changed into active voice: Watching the movie alone, I jumped up when I heard the explosion.
23. Who has to practise to become a prolific batsman? There is no subject in the sentence. The error may be corrected as under: For you to become a prolific batsman, regular practice is of utmost importance.
24. Microphone does not perform on stage! Hence, introduce an object to the modifying phrase 'when performing on stage' as under: When performing on stage, you should not place the microphone too near the speaker cones.
25. Note that in the given sentence the subject is missing. Who described and illustrated is ambiguous. Thus, the correct sentence is: While describing diverse clausal usages, he/she also illustrated how to construct correct sentences.

**Directions:** *Identify the errors in each sentence and correct them.*

26. Clara likes singing carols and playing the mandolin or Clara likes to sing carols and to play the mandolin.
27. Jack loves answering queries and filling out admission forms for us or Jack loves to answer queries and to fill out admission forms for us.
28. Purgery, forgery and fraud will result in suspension or even rustication from this college.
29. The doors of the castle were not only too long but also too creaky.
30. Krishna could not convince me that giving is as much a joy as receiving.
31. Shalin believes that Bines is innocent and that Verza's book doesn't characterise Bines properly.
32. Shonn was advised to either undergo a surgery or



- to use a wheelchair.
33. I enjoy swimming, running and jogging.
  34. One cannot always be worried about or terrified of the difficulties in life.
  35. The manager taught his client how to stand, how to cry and how to talk with fans.
  36. You can join either the army or the air-force.
  37. The revamping of the office is neither simple nor cheap.
  38. When I was in London, I learned the piano and the guitar.
  39. Father supports my view because first it is simple and second it is unique.
  40. We were asked either to change our flight or take a train.
  41. One should look at the human constitution not as a bundle of faculties but as a cycle of activities.
  42. Community participation is necessary to build a self-reliant economy and to develop a local participatory infrastructure.
  43. My research project is more interesting than his.
  44. He considered mailing, talking, writing and social networking to be the best modes of communication.
  45. The pilot stated that the weather was not turbulent, that the climate was o.k., and that the flight was on schedule.

## VA-3.5 PREPOSITIONS, PHRASAL VERBS AND IDIOMS

### PRACTICE EXERCISE 1

1. Sometimes, you will notice that the prepositions change as per the context. This is one such case: here both the prepositions can be used depending on the context. If the connotation were that I would return at the end of one hour, 'in' would be appropriate. If I were planning to return before the end of one hour, then 'within' would be appropriate.
- 2 & 3. 'Beside' and 'besides' is another confusing pair. 'Beside' means 'by the side of', while 'besides' means 'in addition to'. In 2, 'beside' would be the correct answer as it means that 'he sat by his side'. In 3, 'besides' would be the appropriate preposition; the connotation is that apart from his mother, he had to care for his children. So, 'besides' would be appropriate here.
- 4 & 5. The preposition 'till' is used with reference to time, whereas, 'to' is used in case of place. So, in 4, 'to' would be correct and in 5, 'till' should be used.
- 6 & 7. The prepositions 'in', 'at', 'into', 'on' are used in the context of location. 'Into' is used in speaking of things in motion; so, '...she jumped into the well...'. The preposition 'at' is used to refer to a very specific position, hence 'sitting at the table'. The correct preposition for the second blank in 7 is 'in'.
- 8 to 13. Sometimes the use of prepositions is unnecessary. So no prepositions are needed in sentences 8, 9, and 10. In 11, we should not use any preposition in the blank because it will be redundant. Similarly, the verbs 'discuss', 'stress', etc, used as transitive verbs, do not take any preposition. We do not use any prepositions with words like 'downtown', 'outskirts', 'downstairs', 'home', etc. So, no prepositions should be used in 11 and 12. In 13, the pronoun 'that' is the object of a preposition, hence the preposition 'to' should be used.
- 14 to 19. Certain words take different prepositions in different contexts. One has 'affection for' something but one is 'affectionate to' someone. Similarly, one is 'liable for' someone else's faults but 'liable to' a fine. One 'agrees to' a proposal, 'agrees on' a price and 'agrees with' someone.
20. 'Encounter' is followed by the preposition 'with' and means 'to come upon' or 'meet with'. 'Correspond to' is the appropriate phrase when the reference is to a thing. 'Correspond with' is used in case of a person.
21. One 'argues' with someone and not 'to' someone.
22. Certain words take fixed prepositions, like 'man of honour', 'man of steel', etc. Similarly, one 'sticks to his convictions', 'sticks to his/her principle', etc.
23. The preposition 'across' is incorrect in sentence A because it means 'from one side to the other'; whereas, the sentence implies recognizing differences in two things. Thus, 'between' would be the appropriate preposition here. In sentence C, the preposition 'from' after 'independent' is incorrect; it should be followed by the preposition 'of'. Hence, (1).
24. The possessive pronoun 'its' has been incorrectly used in place of 'it's' (the abbreviated form of 'it is') in sentence C. In sentence D, 'tweak it in' is an incorrect expression; it should read 'tweak it into' which means 'to make minor adjustments to'. Hence, (2).
25. There is a prepositional error in B: the word 'aware' or 'unaware' is followed by the preposition 'of' and not 'about'. 'Unaware of' means 'not understanding' or 'realizing something'. In C, the preposition 'beside' is inappropriate. Here we require the adverbial usage, 'besides', which means 'moreover or furthermore'. Hence, (2).
26. The preposition 'amid' is incorrect in the context of sentence A because it means 'surrounded' and makes no sense in the context. Here, the appropriate preposition would be 'between', which in this context means 'involving or concerning'. The phrase 'come around' means 'to recover consciousness' or 'change one's opinion'. The connotation here is that of returning to a former position or state; therefore, 'come back to' would be the appropriate phrase. Hence, (1).
27. One provides a solution 'to' something; therefore, the appropriate preposition for the first blank would be 'to'. For the second blank, the correct preposition would be 'of', which here indicates a specific identity. For the third blank, the appropriate

preposition would be 'with'; 'to be suffused with something' means 'to be overspread with'. 'To pave the way for' is the correct expression, which means 'to prepare for and facilitate the entrance of something'. So, 'for' would be the appropriate preposition for the fourth blank. The fifth blank should have the preposition 'up'. To 'light up' something means 'to brighten something'. In the sixth blank, the correct preposition would be 'for' which indicates 'to belong to someone'. Thus, the correct sequence is: to, of, with, for, up, for. Hence, (1).

28. One studies a species 'in' its native element. Therefore, options (2) and (4) can be eliminated. Option (3) is incorrect as the preposition 'from' would be incorrect for this blank. The use of 'by' is correct for the last blank- '...crushing the flesh by means of ...' This eliminates option (5). Option (1) uses appropriate prepositions. Hence, (1).

## PRACTICE EXERCISE 2

1. 'Relevant' is followed by the preposition 'to' and means 'pertinent'. 'Take back' in the sense of return is followed by the preposition 'to'. For the third blank, 'of' would be the correct preposition as 'interpretation of' something means 'an elucidation or explanation on something'. Since the Pandavas have lost their father's kingdom, 'cheated out of' would be correct here. Though 'quest for' is a valid expression, here the word should be followed by the preposition 'to' as the word following the blank is in the base form. The appropriate preposition for the sixth blank is 'through', which means during the whole period of. Thus, the correct sequence of prepositions would be: to, to, of, of, to, through. Hence, (4).
2. 'To come across' best expresses the idea of finding something by chance. Hence, 'came across'.
3. 'To make up' means 'to invent a false story'. Hence, 'made up'.
4. 'To blow up' means 'to suddenly become angry'. Hence, 'blew up'.
5. 'To take up' means 'to start a new job or activity'. Hence, 'take up'.
6. 'To take back something' means 'to admit that something one said was wrong'. Hence, 'take back'.

7. to, on, on, by, in, on
8. to, to, of, at, to/on, to, on
9. in, in, into, on
10. on, on, in/at, at, to, at, at/on
11. to, \_\_\_\_, \_\_\_\_
12. in, by, on
13. of, to
14. from, of
15. 'Acceded to' is the correct usage.
16. The correct preposition here is 'at' and not 'in'.
17. The correct prepositional usage is 'engaged to'.
18. 'Deprived me of' is the correct usage.
19. Preposition 'for' should be used after 'asked'.
20. The noun sequel takes the preposition 'to' and not 'of'.
21. Usage of preposition 'of' after 'because' is redundant.

## VA-3.6 PUNCTUATION AND GRAMMAR REVIEW

### PRACTICE EXERCISE 1

1. The underlined clause is the adverb clause of place as it denotes the position *where you can stand*. It does the work of an adverb. Hence, (1).
2. The underlined clause is an adjective clause, which modifies the noun *bag*. Hence, (2).
3. The underlined clause is the object of the verb *expect*, and so does the work of a noun. Hence, (1).
4. The underlined clause *till I return* is an adverb clause of time. Hence, (3).
5. The underlined clause is a noun clause because it is the object of the verb *know*. Hence, (1).
6. The underlined clause does the work of an adverb clause of place. It modifies the verb *followed*. Hence, (4).
7. It is a noun clause because the underlined part is the object of the verb *fears*. Hence, (1).
8. The clause *that sounded fictitious* modifies the noun *story*. Hence, (3).
9. *Why he betrayed his country* is a noun clause. If we ask the question *what* to the verb in the main clause, we get the underlined part as the answer. Hence, (2).
10. *Why he committed the crime* is an adjective clause. It tells you something more about the noun, *reason*. Hence, (1).
11. The underlined part does not indicate any specific time. So option (2) is negated. There is no comparison in the sentence, so option (1) is also out. Adverb clauses of condition are usually introduced by *if*, *whether*, *unless*, etc. In this sentence, a supposition is indicated because of the words *even if*. Adverb clauses of supposition are introduced by subordinating conjunctions *though*, *although* and *even if*. Hence, (4).
12. This clause is introduced by the subordinating conjunction *where*. It denotes place. Hence, (2).
13. The noun clause *what he did* is the object of the preposition *in* in this sentence. Hence, (2).
14. The clause qualifies the noun *author* and answers the question *which author?*. Hence it is an adjective clause. Hence, (3).
15. The given sentence is an example of a complex sentence. The clause, *that some day her son would come back* contains a subject and a predicate. This clause is the object of the verb *believed* and so does the work of a noun. Therefore, it is a noun clause. Hence, (2).
16. The adjective *major* cannot have *more* before it. *Major* is used in the positive degree only. Hence, (1).
17. *Herd* is a collective noun and takes a singular verb. Hence, *was passing* is the correct usage. Hence, (4).
18. Both the subjects are singular; hence, takes a singular verb – *has done*. Hence, (3).
19. *A few* means *some*. *Few* means *hardly any*. Hence, (1).
20. The definite article *the* is used before the names of oceans, seas, deserts, etc. Hence, *the Sahara*. Hence, (2).
21. *The* is not required before *computers* because here, a plural countable noun is used in the general sense. Hence, (2).
22. *Here was murdered* is appropriate as the first half of the sentence has a past form of the verb – *conquered*. Hence, (2).
23. There is an error of dangling modifier. A subject is required to make the sentence grammatically correct. Only option (3) rectifies the error and gives us a grammatically correct sentence. Hence, (3).
24. The given sentence is not in the past tense because of the word *says* and, hence, we can negate option (2). In option (3), *...reached into an agreement* is an incorrect usage. Also, in option (4), *... talks at Geneva* is grammatically incorrect. It is used when we indicate a specific location like *at the station*. Only option (1) has proper usage of preposition and also uniformity in the tense. Hence, (1).
25. In the given sentence we need not use the continuous form *bribing* because we are not stating any continuous activity. Here we require the infinitive form *to bribe*. Option (2) changes the meaning of the sentence. (4) incorrectly uses the verb form *offend* in place of the noun form *offence*. Only option (3) gives us a grammatically correct sentence. Hence, (3).
26. This sentence tests the usage of phrasal verbs. Option (1) is out because *ruled out* means *excluded* or *considered irrelevant*. Option (3) is wrong since *lay a rule* means *to formulate a rule*. In this context, it should be *laid down a rule*. Option (4) has *as a rule* which means *usually*, whereas *ruled in* in option (2) means *gave an official decision about something; pronounced*. Hence, (2).
27. Here the climate of Europe is being compared to the climate of Uganda. Hence, (1).
28. The relative pronoun *who* should be placed next to *sportsman*. Hence, (3).

29. To denote a routine action, we use the simple present tense. So, *I get up*. Hence, (3).
30. When places like *school*, *hospital*, etc., are visited for a primary purpose, we need not use a definite article. Hence, (3).
31. In part B, the pronoun 'that' introduces a non-restrictive clause, one that is not essential to the meaning of the sentence. Here we should use 'which' to introduce such a clause. Hence, (4).
32. In part A, there is a missing a pronoun. 'One of ... cases' is the subject and we need a relative pronoun 'that' before 'of' to make the sentence correct. In part B, there is an incorrect pronoun. Since the reference is to the person 'Don Piper', we require the personal pronoun 'who' in place of 'which'. Hence, (1).
33. The error is in part D. The reference is to 'only one of his designs', which is a singular idea. Therefore, we should use a singular pronoun in place of 'they'. Part D should read: 'it is the only one of ...'. Hence, (2).
34. The error is in part C: the relative pronoun itself fulfils the purpose of a conjunction; hence no conjunction should be placed before it. So, in C, instead of 'and which', only 'which' should be used. There is another pronoun error in part D. The antecedent is the singular word 'bamboo'. 'Even now it is' would be the correct expression in part D. Hence, (3).
35. There is an error in part C. In a sentence, when a part of the antecedent is 'same', the consequent should be 'that' or 'as'. So, the correct expression would be 'the same man that I had met'. Hence, (2).

## PRACTICE EXERCISE 2

1. *The talents* is used as an instrument (as opposed to agent) and so needs the preposition *with* not *by*. Options (2) and (3) are ruled out for splitting the infinitive *to make*. Hence, (4).
2. Only option (4) uses the correct verb form *called for* which conveys the idiomatic meaning *demanded*. Hence, (4).
3. Both options (1) and (4) have a parallel construction error. The preposition *of* should be either used before each component or used only before the first component. The phrase *charged with hail and snow* qualifies the word *clouds*, so the conjunction *and* is wrongly placed in option (3). Hence, (2).
4. The preposition *at* is used when speaking of small towns and villages. The correct preposition to be used in respect of a large city like New York is *in*. This rules out options (1) and (2). Of the remaining, we rule out option (3) for wrongly using the participle *visiting* instead of the infinitive *to visit*. Hence, (4).
5. *Besides* means *in addition to*. From the sentence, we can infer that Sonia was by the side of her friend. Hence, *beside* is the correct word. This rules out options (2) and (4). The preposition *to* finds use when the reference is to a place, as for example, *to the end of the street*. In the present context, the reference is to time. Hence, the preposition *till* is in order. This rules out option (1), from the remaining options. Hence, (3).
6. We rule out options (1) and (3) for making wrong use of the preposition *to*. The adjective *possible* should appear after the noun it describes. Of the remaining, we rule out option (2), which uses the adjective *possible* before the noun *hotel*. Hence, (4).
7. In option (1), there is a subject-verb mismatch: *memorials* is plural so the verb should be *testify*. *Testify for* in option (3) is wrong usage. In option (4), *build* is the incorrect form of the verb and *testify* is used here without a preposition. Only option (2) gives us a grammatically correct sentence. Hence, (2).
8. Options (1) and (3) are ruled out for making wrong use of the prepositions *through* and *over*. The conjunction *but* is used to suggest a contrast. This is not conveyed by *since*; thus ruling out option (2). Hence, (4).
9. Option (1) uses a wrong tense form — *before being opened*. Option (4) is also wrong because of the expression while *it was being opened*. Option (3) is verbose. Only option (2) gives us a structurally correct sentence. Hence, (2).
10. *To estimate* means *to calculate the price of something*. But the idea here is that her thesis was greatly admired by people. So *esteemed* is the correct word. Option (3) has inconsistency in the time frame and option (4) has a subject-verb mismatch. Hence, (1).
11. Option (2) has a subject-verb disagreement. Option (3) is verbose and option (4) completely distorts the meaning. Only option (1) takes care of the subject-verb agreement and highlights the supposition. Hence, (1).
12. In option (4), *inside* is a wrong word to be used in this context. In option (3), there is a subject-verb mismatch and option (1) uses a wrong expression *non-discreet*. Hence, (2).
13. *The theory* is a universal truth; therefore, we require the finite verb *is*. *While vaccine trials* is incorrect — it should be *during vaccine trials*. Hence, (2).

14. Option (1) has two mistakes: *at least* should come before *some part* and the preposition used in relation to *access* should be *to* not *with*. Option (3) changes the meaning of the sentence, making it nonsensical. Option (4) again has the mistake of using the preposition *for* for the noun *access*. Hence, (2).
15. *Besides* implies *over and above*, whereas *beside* means *by the side of*. So, options (2) and (3) are negated. Option (4) is verbose. Only the first option is correct. Hence, (1).
16. The relative pronoun *whom* finds use only as an object in a sentence. We rule out option (4) which uses *whom* to qualify the subject *friend*. In option (1), the tense is unnecessarily changed. Option (2) is wrong because of the use of *myself* because we are not emphasising anything. The grammatically correct option is (3). Hence, (3).
17. Options (1) and (2) are wrong as *volte-face* must be followed by the preposition *on* and *not* of or *for* in this context. Options (1) and (4) are wrong because the correct expression in this case is *close on the heels*, not *to the heels*. Only option (3) is grammatically correct. Hence, (3).
18. The verb *insist* indicates the subjunctive mood. Hence, the base form of the verb follows it. Also, *proceedings* is followed by *against*. Hence, (1).
19. *Equal to ... better than* is the correct phrase. Hence, (2).
20. The phrase *together with* when joined to a singular subject, i.e., *rate* takes the verb in the singular. Hence, (2).
21. The action, in this sentence, began at sometime in the past and is still continuing. So we require the present perfect continuous tense. Hence, (3).
22. The two actions happened in the past. *Had finished* represents the first completed action. Hence, (2).
23. There is no error in the sentence. Hence, (4).
24. Here *through* is the correct preposition and indicates *the whole period*. Hence, (1).
25. *Accountable to somebody* is the correct expression. Hence, (2).
26. Here we are expressing a past action whose time is not given and not definite. So, we require the present perfect tense here. Hence, (1).
27. *Heavier* itself is the comparative form. So, *more* is redundant here. The correct answer should be *heavier*. Hence, (3).
28. The correct preposition in this context would be *in*. Hence, (4).
29. Option (3) is the correct usage in this context. Hence, (3).
30. *Spends* suggests that it is continuing to do so. Between options (2) and (3), the latter is correct because *on any other industry* would mean spending money *on an industry*. Only option (3) makes the comparison clear. Hence, (3).
31. The error is in part D: the relative pronoun should be in the accusative case as it is the object of the verb 'admired'. Therefore, 'whom' should be used in place of 'who'. In E, the nominative form 'who' is correct because here it is the subject that performs the action. Hence, (3).
32. The error is in part C. 'That' is introducing a non-restrictive clause: the information provided after 'food lovers' is not essential to the sentence. Therefore, 'who' should be used in place of 'that'. Hence, (3).
33. There is an incorrect word form in part B. 'It's' is not a pronoun; it is the abbreviated form of 'it is'. Here, the context demands that we use a possessive pronoun, i.e., 'its'. The second sentence in this excerpt makes an incorrect comparison. The comparison is between the condition of roads in a developing nation and those in war ravaged countries. Therefore, an appropriate pronoun should be inserted in E and that part should read: 'our roads are worse than those of war ravaged nations'. Hence, (2).
34. There are errors in parts B, C and E. A collective noun such as jury, team, committee, etc. can take either a singular or a plural pronoun, depending on the context. Since the jury was divided, it implies that the individuals constituting the jury did not work in sync and behaved as separate individuals. In this context we require a plural pronoun, i.e., 'they' in place of 'it'. There is a subject-verb mismatch in part B. Since we are talking of the jury that are divided in their opinion, we require the plural verb 'were' after the collective noun 'jury'. In part E 'complained about', which means 'to generally say that something is not satisfactory,' should be the correct phrase, not 'complained of', which means 'to tell other people that something is making you feel ill'. Hence, (4).
35. In part E, the pronoun 'their' is incorrectly used. The word 'none' can be construed both as singular and plural. The use of 'has' in parts C and D indicates that 'none of the measures' is a singular idea because the corresponding verbs are also singular. Therefore, the pronoun in part E should be 'its' and not 'their'. The sentence implies that not a single measure has the required potential for its use with older adults. Hence, (4).

### PRACTICE EXERCISE 3

1. Verbs of understanding and perception take the simple present — so, *seems* should be used. Hence, (3).
2. There is an error of a dangling modifier. Option (1) has a tense error. Option (2) is ambiguous. Option (4) is not grammatically correct. Only option (3) rectifies the error of dangling modifier and places the subject *he* in the appropriate place. Hence, (3).
3. *Isn't that so?* is the appropriate question tag in this context. In options (1) and (3), the tense forms have been jumbled. Option (2) gives a completely different meaning to the sentence. Hence, (4).
4. Option (3) is the correct usage in this context because it goes with the tense form of the other verbs in the sentence. Hence, (3).
5. Verbs of understanding and perception take the simple present form — *seems*. Hence, (3).
6. *We usually have lunch* is grammatically correct because it indicates a habitual action. Hence, (3).
7. *Died of* is the appropriate verb preposition combination. Hence, (3).
8. There is an error in tense — *was murdered* is the correct usage because the happening is in the past. Hence, (2).
9. There are two ways to correct this sentence: first, introduce relative pronoun *which* before *threw*. This would make the rest of the sentence a clause. The other: turning *threw* into a present participle *throwing* that turns the rest of the sentence into a phrase. This option is provided in option (3). Hence, (3).
10. The correlative conjunctions always come in fixed pairs. *Not only ..... but also* is one of them. Only option (1) uses the conjunctions in the proper order. Hence, (1).
11. The first *and* should be followed by the verb *is* to complete the compound sentence correctly (judgement is such an excellent activity) in the correct tense. Hence, (4).
12. An adverb of manner should not be put between the verb and the object, so options (1) and (2) are out. What is taken in a robbery is always stolen, not robbed. So options (1) and (4) are out. Hence, (3).
13. In this case, the adverb of time should immediately precede the verb it modifies. Hence, (3).
14. According to the rules of parallel construction, parallel parts of the sentence must have the same verbal structure. The sentence begins with *to go ...* and should continue with *to gamble ... to dance ... to flirt...* Hence, (4).
15. The sentence begins in present tense, therefore, *the route it selects ... is riddled*. This narrows down our options to (3) and (4). Note, however, that the 19th century refers to the past and, therefore, *the same weaknesses that plagued the ...* Hence, (4).
16. The comparison is between the hood of the cobra and the hood of the non-poisonous variety, not between one hood and another snake. Hence, (4).
17. Only option (3) uses the correct pair of correlative conjunctions — *not only ...but also...* Hence, (3).
18. Here the verb *has* should not be repeated before *lots* and *many*. Only when a different verb is required in each phrase, should the verb be provided. Hence, (3).
19. Option (1) is wrong because it does not indicate the possession. Apostrophe after both the nouns (Kalidasa and poet) is redundant in indicating the possession. In these cases, the apostrophe is used after the second noun and not after the first. So the correct sentence should be: That is Kalidasa, the poet's house. Option (4) is wrong because the plural form of 'poet' is not required here; we are referring to one poet. Hence, (3).
20. The first part of sentence C is an unreal condition (i.e., it implies that childbearing is no longer compulsory for women), so the subjunctive mood is correctly used, but the second part is stating a historical fact, so the indicative, not the subjunctive mood is required. Sentence A is also wrong, as the verb *constitute* does not take a preposition after it. Only sentences B and D are grammatically correct. Hence, (3).
21. The Royal Order of Adverbs is violated in sentence A — the adverb phrase of place, *in a virtual reality*, should precede, not succeed, the adverb phrase of time, *in the far future*. Sentence B wrongly expresses an unreal conditional *if the virtual reality was good enough* in the indicative mood instead of the subjunctive. Only sentences C and D are correct. Hence, (4).
22. Sentence C fails to follow the rule of the Royal Order of Adjectives, as the observation *calm* should precede the colour adjective *blue*. All the other sentences are grammatically correct. Hence, (3).

23. Both sentences B and C violate the rule of the Royal Order of Adjectives. The correct order in B should be observation *shimmering*, then colour *aquamarine*, and in C, colour *white*, then material *ivory*, not the other way around. Sentences A and D are correct, as they follow the rule. Hence, (1).
24. Sentences A, B and D are correct. In C, there is an error with respect to the use of the conditional. This is an example of an unreal or improbable situation and provides an imaginary result for the situation. So we require the subjunctive past form in the *if* clause. Moreover, the second part of the sentence that has the *could + base form of the verb*, gives us a clue. The correct sentence should begin as: *If vaccinations were stopped...* Hence, (4).
25. Sentence A has an error pertaining to the Royal Order of Adjectives, according to which the adjective indicating observation (heavy) should precede the one indicating material (gold). Sentence B is correct in all respects. In sentence C, there is subject-verb agreement error. *Everybody* should be followed by a singular verb and not a plural one. Sentence D expresses a supposition that is assumed as a fact, hence, in the indicative mood. The latter part of the sentence gives us a clue that the verb in the *if* clause should also be in the present form — *we try*. So sentences A, C and D are incorrect. Hence, (3).
26. Sentences A and D are grammatically correct. In sentence B, the correct expression should be *off the kerb*, which implies that the activists were displaced from the pavements into the streets. There is another error in this sentence — we require the possessive form: *activist's* and not just the plural form. In sentence C, there is an error in the subjunctive form of the verb. In sentences with verbs expressing command or insistence, the verbs in the present subjunctive are in their base forms. So the correct expression in C should be: *He insisted that they move...* Hence, (2).
27. Sentences A and B are grammatically correct. Sentence C denotes an unreal or impossible situation with hypothetical results. So we require the present unreal conditional form here. So the correct sentence should begin as: *If he were the reason...* In sentence D, the reflexive pronoun (himself) and the possessive pronoun (him) have been mixed in the same sentence, which results in a parallelism error. The form has to be uniform in order to make the sentence correct. Hence, (1).
28. Options (1), (2) and (3) are negated because the modifier is misplaced, giving a wrong meaning to the sentence. Only option (4) is grammatically correct. Hence, (4).
29. The error here is of misplaced modifier where the word *scarcely* is misplaced in options (1), (3) and (4) which results in ambiguity. Only option (2) places the subject in the appropriate place. Hence, (2).

## PRACTICE EXERCISE 4

- Only option (2) agrees with the tense of the sentence. Hence, (2).
- According to the rules of parallel construction, if both clauses of a sentence refer to the same time frame, they should be in the same tense. Among the given options, only option (1) follows this rule, i.e., both the verbs are in the present tense. Hence, (1).
- It's clear* is in the present tense, so we have, *makes it*. However, *next few weeks* refers to the future, therefore, *he will still be*. Hence, (4).
- An article or a preposition applying to all the members of a series must either be used only before the first term or else be repeated before each term. Options (1), (3) and (4) are incorrect because they do not conform to this rule of parallel construction. Hence, (2).
- Superior* takes the preposition *to* in the comparative form. Here, *ape's* implies *intelligence of apes*. Therefore, there is no error in the sentence. Hence, (4).
- The right answer is option (2) because it makes use of the correct tense form. In option (1), there is a mix of past and present tenses. Option (3), too, has the same error. Option (4) is wrong as it uses the gerund *eliminating*. *Clear up* means *to make something neat and clean*. Only option (2) gives us a sentence where the tense form is uniform. Hence, (2).
- The idiomatic phrase *on the altar* and the expression *another one* in option (3) are the correct usages. Through option (4) apparently seems correct, the verb *is* has been wrongly repeated in the sentence. Hence, (3).
- Usually keep visitors* is the correct usage. *Usually* is an adverb of frequency and gives the meaning of *how often*. The adverb should be followed by the verb *keep*. *To a minimum* is the correct usage and not *at a minimum*. *100 officials* is the correct sequence of adjective and noun. *Descend on/upon* means to visit something in large number. *Media*



- takes *have* as a verb. Hence, (4).
9. The Royal Order of Adjectives follows this sequence: observation (*new*), size (*large*), shape (*box-shaped*), colour (*blue*), material (*metal*), purpose (*racing*) and noun (*machine*). Hence, (3).
  10. According to the Royal Order of Adverbs, the sequence should be: manner (*tirelessly*), place (*at home*), frequency (*twice a day*), time (*after his meals*) and purpose (*to improve his scores*). Hence, (1).
  11. Option (1) has a tense mismatch and it also uses the wrong form *latter*. Option (2) corrects the subject-verb agreement error. Option (3) changes the meaning of the text, while option (4) unnecessarily introduces a passive construction. Hence, (2).
  12. While all the others are grammatically incorrect, option (2) rectifies the error of the modifier by placing the noun, *H. T. Parekh* immediately after it. In option (4) the emphasis shifts from the time that is being referred to. Hence, (2).
  13. The error is one of wrong preposition. It should be *for* not *on* me. Hence, (3).
  14. *Took leave of her ...* is the correct phrase. Hence, (3).
  15. Only option (4) conveys the meaning clearly in the correct tense. *Not only ... but also is* the correct conjunction pair. Hence, (4).
  16. Correlative expressions such as *either ... or, both ... and, not only ... but also ...* etc., should be followed by the same grammatical construction. Hence, option (1) and (3) are out. Option (4) is incorrect since a semicolon is used to separate a phrase that is integral to the sentence's meaning. Hence, (2).
  17. Options (2) and (4) unnecessarily change the verb form to the past tense. The sentence conveys that the government is yet to take the step. An investment can be made *over a period of ten years*. Only option (3) is correct. Hence, (3).
  18. In part B, the word 'home' should be in plural, i.e., 'homes' since millions of people cannot share a single 'home'. In part D, there is a missing article. The correct phrase would be 'unable to reach the hundreds'. In part E, the verb should be the singular 'has' because the subject 'death count' is a singular idea. Only parts A and C are correct. Hence, (3).
  19. 'Most' conveys a plural idea: 'almost all'. It is followed by a plural noun. So, the corresponding verb should also be plural. In part A, since 'most of us' is the subject, the verb should be 'consider'. The other parts are error free. Hence, (3).
  20. There is a very obvious error in part C. The subject 'governments' is plural but the corresponding verb is singular. Therefore, the correct expression should be 'governments have come to rely'. There is another subject-verb mismatch in part E. The subject of the verb 'runs' is the plural noun 'forty-three states and every Canadian province'. Had the subject been only 'every Canadian province', 'runs' would have been appropriate. But in the current case, the appropriate verb would be 'run'. Parts A, B and D are correct. Hence, (4).
  21. In part B there is a subject-verb disagreement. The subject of the verb 'were' is the singular noun 'use' and not 'native tongue and dialectical forms', though the latter phrase is placed closer to the verb than the actual subject. Here we should be guided not by proximity but by the actual context. Thus, the appropriate verb in B should be 'was'. There is another subject-verb agreement error in part D. In constructions of the type 'one of + plural noun + who' the subject of the verb is the plural noun and not the singular 'one'. Therefore, in such cases the plural verb has to be used after the pronoun 'who'. Parts A, C and E are correct. Hence, (1).
  22. The determiner 'more than one' generally precedes a singular noun and conveys a singular idea. Therefore, the corresponding verb should also be singular. In part A, the verb is the plural 'are' placed before the subject. It should be changed to 'is' to make the sentence correct. The other parts are correct. Hence, (4).
  23. In case of singular subjects linked by 'as well as', the verb is matched with the first subject. The phrase 'as well as' actually modifies the subject but does not compound the same. The actual subject of the verb in part B is the singular word 'performance'. So, the verb should be singular 'depends' and not the plural form as given. Part E also has a subject-verb disagreement: the subject of the verb 'are' is the singular word 'notion' and not 'perturbations' though the latter appears close to the noun. Therefore, the verb should be changed to 'is'. Parts A, C and D are error free. Hence, (3).

24. There are subject-verb agreement errors in parts A, C and E. In part A, the subject is singular 'Oliver', so the verb should also be singular, i.e., 'finds'. When one of the subjects joined by 'neither ... nor' is singular and the other plural, and the plural subject is placed near the verb, the verb matches the nearest subject. In part C, the plural subject 'boys' is closer to the verb; therefore, the appropriate verb should be 'are' and not 'is'. Part E should have the singular verb 'is' in place of 'are' because the subject of the verb is the singular word 'life'. Parts B and D are correct. Hence, (1).
25. Sentence B is wrong because 'making them to break apart' is incorrect usage. The correct expression should be 'causing them to break apart'. In sentence C, there is a subject-verb agreement error. The determiner 'many a' is followed by a singular noun as it indicates each of a considerable number. Therefore, the verb should also be singular. The correct sentence would read: 'Many an offending chemical has now been banned'. Sentence D should read 'It will still be several decades before .....' or 'It will still take several decades for these substances to disappear from the atmosphere.' Hence, (3).
26. In sentence C, the clause 'that has emerged' actually qualifies the plural noun 'methods' and not 'one'. Therefore, the verb should be plural, i.e., 'that have emerged'. In sentence D, the verb should be 'focuses' because its subject is the singular word 'treatment'. Sentences A and B are correct. Hence, (4).
27. In sentence A, the phrase 'corporate world' is singular because it refers to all the corporate companies taken together as a single unit. So, the corresponding verb should be singular, i.e., 'wants'. In sentence B, again, there is a subject-verb mismatch: 'most' followed by a plural noun indicates in the majority of instances, hence it has a plural connotation and should take a plural verb. So, sentence B should read: 'No wonder most people think women ...'. Fractional expressions like 'a percentage of', 'a majority of', etc. are sometimes singular and sometimes plural, depending on the meaning. Here the expression '20 per cent of India's corporate workforce' is preceded by 'no more than'. Cumulatively they indicate a singular idea. Thus, the verb should be singular, i.e., 'comprises'. Only sentence C is grammatically correct. Hence, (3).
28. 'Eugenics' is the name of a subject of study and it refers to a single unit, though it looks like a plural word. The same is true for the names of subjects like linguistics, economics, civics, etc. The corresponding verb should, therefore be singular, i.e., 'studies'. In sentence B, there is a subject-verb disagreement. The verb (are) should be singular as the subject is the singular 'idea' and not 'eugenics', though the latter appears closer to the verb in the sentence. Sentences C and D are correct. Hence, (4).
29. The indefinite pronouns like 'anyone', 'everyone', 'someone', etc are always considered singular, therefore, they always take singular verbs. In sentence A, the subject is 'anyone'; hence, the corresponding verbs should be 'has' and 'knows'. Sentences B, C and D are grammatically correct. Hence, (2).
30. In sentence B, there is a subject-verb disagreement. 'Neither' implies not either of the two available options and it implies a singular unit. Therefore, a subject qualified by 'neither' is followed by singular verb. 'Transgresses' would be the correct word in sentence B. In sentence D, the subject of the verb 'entail' is the singular idea, 'the presence of a barrier'; therefore, the verb should be in the singular form, i.e., 'entails'. Moreover, the phrase 'the seas being merging' is not parallel to 'the presence of a barrier' due to which the sentence seems quite clumsy. So this part should be reframed as: 'the idea of the merging of the seas and the presence of a barrier at the same time.' Sentences A and C are grammatically correct. Hence, (3).
31. In sentence B, the subject of the verb is the singular word 'reliance' and not 'calculations'. Thus, the verb should be changed to 'was'. There is a subject-verb disagreement in sentence C. The phrase introduced by 'as well as' or 'along with' will modify the earlier word (the first subject), but it does not compound the subjects. Therefore, the verb is matched with the first subject. Here, the subject is 'society'; hence the corresponding verbs should be 'involves' and 'requires' to make the sentence correct. Sentences A and D are correct. Hence, (1).
32. In sentence B, the subject is the singular entity 'society' and the clause 'in which ... high levels of literacy' is the modifier. The corresponding verb should, thus, be singular, i.e., 'is' in place of 'are'. The determiner 'most' means in the greatest

- quantity or number, hence it conveys a plural idea. Therefore, the corresponding verb should also be plural, i.e., 'say' in place of 'says' to make sentence D correct. Hence, (4).
33. The subject of the verb is not 'large business/es' but the gerund 'running'. Therefore, the verb should be singular, i.e., 'consists'. On this ground we can eliminate options (1) and (4). Option (3) distorts the meaning of the sentence and shows another subject-verb mismatch. The singular pronoun 'it' should be followed by the singular verb 'costs'. Only option (2) is correct in all respects. Hence, (2).
34. When a sentence compounds a positive and a negative subject and one is plural, the other singular, the verb should agree with the positive subject. Here the positive subject is 'the speaker' and the negative subject is 'his ideas', therefore, the verb should be singular as it would agree with the first subject (the speaker). Thus, option (1) is eliminated. In option (2), the subject is not compounded, hence the verb should agree with the immediate subject which is plural (ideas). In option (4), there is an incorrect word form – the possessive form 'its' is incorrectly used in place of the abbreviated form 'it's'. Moreover, the subject verb disagreement is still present in option (4). Only option (3) gives us a grammatically correct sentence. Hence, (3).
35. Indefinite pronouns like 'everyone', 'somebody', etc are always considered singular. Therefore, the corresponding verb in this sentence should be 'has' and not 'have'. Options (2) and (3) can be eliminated on this ground. In option (4) there are two errors. Firstly, 'everyone' has a singular connotation so, 'their' cannot be used to substitute this pronoun. Secondly, the clause 'Accepting that ...own problems' acts as the subject of the verb 'are'. Since the subject clause is singular, the verb should also be singular, i.e., 'is' and not 'are'. Only option (1) is a grammatically correct sentence. Hence, (1).
36. When a plural noun is a proper name of some single object or some collective unit, it is followed by a singular verb. 'United States' is one such example and hence the corresponding verb should be 'has'. So, options (1) and (3) are negated. Option (2) has another subject-verb disagreement: the clause 'The idea that US economy' is the actual subject of the verb 'have'. Since 'the idea' is singular, the verb should be 'has' and not 'have'. Only option (4) gives a grammatically correct sentence. Hence, (4).
37. The sentence in option (1) suffers from subject-verb disagreement. The subject of the verb 'suffers' is the plural phrase 'international-relations experts', so the verb should be 'suffer' and not 'suffers'. In option (3), apart from this mistake, we find an inappropriate word 'informations'. The word 'information' is used both as a singular and a plural noun. Moreover, the corresponding verb 'are' is also incorrect in the context where a particular segment of 'information' is being referred. In option (4), the plural verb 'suffer' does not agree with the singular subject 'expert'. Only option (2) gives a grammatically correct sentence. Hence, (2).

### PRACTICE EXERCISE 5

1. 'Felicitate' means 'congratulate' and is incorrect in the part (3). In the third part of the sentence it should have been 'a network to facilitate contacts' as the fourth part of the sentence is 'between small and medium enterprises'. Hence, (3).
2. In the fourth part of the sentence, the definite article 'the' before 'public sector' is missing. Hence, (4).
3. In the first part of the sentence, 'The Indian government's choice' is singular. Therefore the verb 'stem' in the third part of the sentence should have been written as 'stems'. Hence, (3).
4. 'Many of money' is incorrect usage. It can be corrected with 'most of money'. Therefore, the fourth part should have been written as 'most of it must be a miser'. Hence, (4).
5. The second part of the sentence contains a tense error. The sentence should have been 'position, you would have definitely shown', instead of 'position, you were definitely'. Hence, (2).
6. The error is in the third part of the sentence. Instead of 'require', it should be 'requires' as 'position' is the singular subject for the verb. Hence, (3).
7. The error is in the fourth part of the sentence. Since, 'the choice of either extreme positions' is singular, the fourth part of the sentence should have been 'positions inevitably invites criticism', instead of 'positions inevitably invite criticism'. Hence, (4).

8. The error is in the third part of the sentence. It should have been 'test matches, decided', instead of 'test matches, he then decided'. Hence, (3).
9. The first part of the sentence is erroneous with the word 'of' instead of 'off'. Hence, (1).
10. In the third part of the sentence, the word 'minority' is singular. Hence, the third part of this sentence should have been 'minority which has been', instead of 'minority which have been'. Hence, (3).
11. There is a redundancy error in the third part of the sentence. The word 'aspiring' means 'desiring or striving for recognition or advancement', which is synonymous with 'ambitious'. Therefore, either word would have been sufficient in the sentence. Hence, (3).
12. The error is in the second part. Instead of 'whom most people thought', it should have been 'that most people thought'. Hence, (2).
13. The error is in the third part of the statement. The third part should have been 'in the front side of it' instead of 'on the front side off it'. Hence, (3).
14. Since the phrase in the third part of the sentence is 'I am planning', the fourth part should have been 'to go there tomorrow' instead of 'I would go there tomorrow'. Therefore, the error is evident and it is in the fourth part of the sentence. Hence, (4).
15. The error is in the third part. It should have been 'I found that it was none other' instead of 'I found that it was not other'. Hence, (3).
16. The second part of the sentence contains an error in parallelism. It should have been 'go to the bank' instead of 'to go to the bank'. Hence, (2).
17. The given sentence is in the simple present tense. Hence, the phrase 'are not seeing it' in part (2) of the sentence needs to be replaced by 'do not see it' to keep the tense consistent throughout the sentence. Hence, (2).
18. In part (4) of the sentence, the usage of the word 'other' is incorrect. It needs to be replaced by the word 'others'. Hence, (4).
19. The usage of the comma in the part (1) of the sentence is unnecessary. Hence, (1).
20. The preposition 'of' in part (2) has been incorrectly used instead of 'from'. Hence, (2).
21. In part (4) of the sentence, the word 'asked' is incorrectly used. The correct formation is 'if you had asked for them'. Hence, (4).
22. In part (4), there should be a comma before 'did you'. Hence, (4).
23. In part (1) it should be either 'Are you invited' or 'Have you been invited'. Hence, (1).
24. Part (4) is incorrect because of the word 'assure', which should be changed to 'ensure'. Hence, (4).
25. Part (3) is incorrect. The word 'who' should be changed to 'whom'. Hence, (3).
26. Part (3) has an error in pronoun consistency. 'One has to decide....one should choose' is the correct structure. Hence, (3).
27. Part (3) has to be changed to 'should be made responsible for the student's conduct' because Part (1) says 'teachers insist that' which leads the rest of the sentence to be a 'what ought to be' and 'is' in part (3) makes it a simple present statement rather than 'what should be'. Hence, (3).
28. Part (3) makes a reference to voters as 'man and woman'- singular. Thus, it makes the pronoun 'their' as an incorrect one. It should be 'his and her choice respectively'. Hence, (4).
29. Part (2) is incorrect: 'frequently' should either be modifying a verb, which is missing, or be in the verb form itself, i.e. 'frequent'. Hence, (2).
30. The usage of the verb 'tends' in part (3) is incorrect – it should be in the plural form – i.e. 'tend' – as the subject is the plural noun 'people' not the singular noun 'language'. Hence, (3).
31. The 'it' in part (2) is redundant and therefore incorrect, as the object of 'deem' is 'something' in part (1). The other parts of the sentence are grammatically correct. Hence, (2).