Algebra - 1

Career Launcher

Contents

- Polynomials
- Linear Equations

QA - 17

CEX-Q-0218/18

Number of questions : 30

Polynomials

1. If x = 1, 2, –3 are three of the roots of the polynomial $P(x) = Ax^4 + Bx^3 + cx^2 + Dx + E$, then how many of the following statement(s) is(are) always true?
   A. P(2) = 0
   B. P(x) = (x – 1) (x – 2) (x + 3) is the polynomial.
   C. There is a fourth root, which could be either real or imaginary.
   D. If E = 12 and A = 2, then the fourth root is –1.

   (1) One (2) Two
   (3) three (4) Four

2. If f(x) denotes $x^3 + 3x^2 + 3x$, find f(x + 1).
   (1) $x^3 + 4x^2 + 5$
   (2) $2x^3 + 6x^2 + 3$
   (3) $3x^3 + 12x^2 + 4x + 1$
   (4) $x^3 + 6x^2 + 12x + 7$

3. (x – 3) is a factor of $f(x) = x^3 – 5x^2 + px + 9$. If all the roots are integers, then
   A. What is the value of p?
   B. The roots of f(x) are ___, ___ and ____.

4. (x + 4) is a factor of $f(x) = 3x^3 + px^2 – 30x + 24$.
   A. Find P.
   B. Solve for f(x) = 0, if all the roots are integers.

5. If (x + a) is a factor of $x^3 + ax^2 + ax + 4$. Then the values "a" can take are
   (1) 2 (2) –2
   (3) Either of them (4) Neither of them

6. If a,b,c are real numbers such that $a^2 + b^2 + 2c^2 – 4a + 2c – 2bc + 5 = 0$, then what is the value of a + b – c?
   (1) 0 (2) 2
   (3) –2 (4) 4

7. If $x^4 + \frac{1}{x^4} = 47$ (x > 0), then find the value of $x^3 + \frac{1}{x^3}$.
   (1) 18 (2) 27
   (3) 9 (4) 12

8. If a + b + c = 0, where $a \neq b \neq c$,
   then $\frac{a^2}{2a^2 + bc} + \frac{b^2}{2b^2 + ac} + \frac{c^2}{2c^2 + ab} =$
   (1) 0 (2) 1
   (3) –1 (4) abc

9. If $x^2 + 5y^2 + z^2 = 2y(2x + z)$, then which of the following statements is(are) necessarily true?
   A. x = 2y
   B. x = 2z
   C. 2x = z

   (1) Only A (2) B and C
   (3) A and B (4) None of these

10. Simplify:

$$\frac{243^3 + 257^3 + 500^3 - 3 \times 243 \times 257 \times 500}{243^2 + 257^2 + 500^2 - (243 \times 257 + 257 \times 500 + 243 \times 500)}$$

(1) 5000 (2) 1000
(3) 900 (4) 960

11. If $|x^2 + 1| \leq 1$, then find the value of (x + 2) (x + 1) (x) (x – 1) (x – 2) .... 1000 terms
(1) –2 (2) 0
(3) –1 (4) > 0

Linear Equations

12. The sum of the digits of a two-digit number is 7. If its unit's digit is 3 less than its ten's digit, then what is the number?
(1) 25 (2) 52
(3) 61 (4) 74

13. In a vehicle showroom, there are only two types of vehicles, three wheeler auto-rickshaws and four wheeler cars. If we count the total number of wheels, there are 550 wheels but 160 vehicles to show. Find the number of auto-rickshaws in the showroom.
(1) 70 (2) 100
(3) 90 (4) 80

14. Shrishti is 2 years older than Alia and Aasma is 6 years younger than Pariksha. If the absolute difference between the ages of Pariksha and Alia is 5 years, then who among the following is the youngest?
(1) Aasma (2) Alia
(3) Pariksha (4) Either (1) or (2)

15. The owner of a local jewellery store hired three watchmen to guard his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each watchman, one at a time. To each he gave $\frac{1}{2}$ of the diamonds he had then, and 2 more besides. He escaped with one diamond. How many did he steal originally?
(1) 40 (2) 36
(3) 25 (4) None of these

16. The cost of 1 dozen apple and 1 kg orange is Rs.20, while the cost of 1 dozen orange and 1 kg apple is Rs.16. Also, the cost of an apple and an orange is Rs.3. If 1 kg of apple and 1 kg of orange have m and n (m and n are integers) of the same kind of fruits respectively, then the minimum value of m will be
(1) 5 (2) 6
(3) 7 (4) Data inconsistent

17. Three friends, returning from a movie, stopped to eat at a restaurant. After dinner, they paid their bill and noticed a bowl of mints at the front counter. Sita took one-third of the mints, but returned four because she had a momentary pang of guilt. Fatima then took one-fourth of what was left but returned three for similar reason. Eswari then took half of the remainder but threw two back into the bowl. The bowl had only 17 mints left when the raid was over. How many mints were originally in the bowl?
(1) 38 (2) 31
(3) 41 (4) None of these

18. Given the system of equations 3x + ky = 1 and 6x + 3y = 2. For which values of k does this system have infinite solutions?
(1) $\frac{3}{2}$ (2) $-\frac{3}{2}$
(3) $\frac{2}{3}$ (4) $-\frac{2}{3}$

19. The number of solutions for the equation 2x + 3y = 40, such that both x and y are natural numbers, is
(1) 20 (2) 13
(3) 6 (4) 8

20. The total number of integers pairs (x, y) satisfying the equation x + y = xy is
(1) 0 (2) 1
(3) 2 (4) None of these

21. A Shopkeeper can buy X number of pencils and Y number of pens in Rs. 1000. Cost price of a pencil is Rs. 30 and that of a pen is Rs. 20. He gifts away a few pens and sells the remaining pens at Rs. 25 each and pencils at Rs. 35 each. If he does not make any profit or loss in this transaction, then which of the following relations can be correct?

A. $X + Y = 25$
B. $X + Y = 34$
C. $X + Y = 45$

(1) Only A (2) A and B
(3) Only C (4) A and C

22. Some children stand in a queue and share a box of chocolates in such a manner that Child 1 takes 100 chocolates plus $\frac{1}{10}$th of whatever remains in the box. Then Child 2 takes 200 chocolates plus $\frac{1}{10}$th of whatever remains, then Child 3 takes 300 chocolates plus $\frac{1}{10}$th of whatever remains, and so on for each child in the queue. It turns out that each child gets the same number of chocolates. Then

(1) there must be exactly 7 children in the queue.
(2) each child must have received 900 chocolates.
(3) the total number of chocolates initially in the box must have been 6300.
(4) both (1) & (2) true

23. f(x) is a polynomial of degree 49 such that $f(1) = 4$, $f(2) =5$, $f(3) = 6$, ... $f(49) = 52$. Find $f(50)$. (k is positive constant)

(1) $53 + k49!$ (2) $53!$
(3) $54!$ (4) $49!$

24. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$x+ 2y - 3z = p$
$2x + 6y - 11z = q$
$x - 2y + 7z = r$

(1) $5p - 2q - r = 0$
(2) $5p + 2q + r = 0$
(3) $5p + 2q - r = 0$
(4) $5p - 2q + r = 0$

25. Every 10 years the Indian Government counts all the people living in the country. Suppose that the director of the census has reported the following data on two neighbouring villages Chota Hazri and Mota Hazri.
Chota Hazri has 4,522 fewer males than Mota Hazri.
Mota Hazri has 4,020 more females than males.
Chota Hazri has twice as many females as males.
Chota Hazri has 2,910 fewer females than Mota Hazri.
What is the total number of males in Chota Hazri?

(1) 11,264 (2) 14,174
(3) 5,632 (4) 10,154

Challenging

26. A test has 50 questions. A student scores 1 mark for a correct answer, $-\frac{1}{3}$ for a wrong answer, and $-\frac{1}{6}$ for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than

(CAT 2003(L))

(1) 6 (2) 12
(3) 3 (4) 9

27. F(x) is a fourth order polynomial with integer coefficients and with no common factor. The roots of F(x) are –2, –1, 1, 2. If p is a prime number greater than 97, then the largest integer that divides F(p) for all values of p is **(XAT 2009)**

(1) 72 (2) 120
(3) 240 (4) 360
(5) None of these

28. The number of distinct solutions of the system of equations given below is

$x^2 + 2y^2 + 4z^2 + 3(xy + yz + zx) = 16$

$y^2 + 2z^2 + 4x^2 + 3(xy + yz + zx) = 16$

$z^2 + 2x^2 + 4y^2 + 3(xy + yz + zx) = 16$

(1) 1 (2) 2
(3) 4 (4) 8

29. In a test of three papers viz. A, B and C, a student attempted 20 questions put together. Papers A, B and C offer 20, 25 and 30 marks respectively for each correct answer, and no negative marks for unattempted or wrong answer. Then the number of ways in which a student can score 520 marks and the maximum number of questions attempted by the student in section A is respectively

(1) 13, 8 (2) 13, 12
(3) 9, 8 (4) 9, 12

30. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs. 250 and Rs. 300 per day respectively. In addition, a male operator gets Rs. 15 per call he answers and female operator gets Rs. 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(1) 15 (2) 14
(3) 12 (4) 10

QA - 17 : Algebra – 1
Answers and Explanations

CEX-Q-0218/18

1	2	**2**	4	**3**	–	**4**	–	**5**	3	**6**	2	**7**	1	**8**	2	**9**	3	**10**	2
11	2	**12**	2	**13**	3	**14**	1	**15**	2	**16**	4	**17**	4	**18**	1	**19**	3	**20**	3
21	3	**22**	2	**23**	1	**24**	1	**25**	3	**26**	3	**27**	4	**28**	4	**29**	3	**30**	4

1. 2 Statement (A), P(2) must be zero. As, 2 is a root of the given polynomial. So it is true.
Statement (B), if 1, 2 and –3 are roots of a polynomial, then it must be in the form of
P(x) = A(x – 1) (x – 2) (x – 3), where A can take any value. But in this option, A = 1, which is not necessarily true or there must be fourth power of x in the polynomial.
Statement (C), the fourth root cannot be imaginary, as imaginary roots exist in pairs. So statement (C) is not correct.
Statement (D), since product of the roots for this polynomial = $\frac{E}{A}$

So, $1 \times 2 \times (-3) \times \text{fourth root} = \frac{12}{2} = 6$

$\Rightarrow$ fourth root = –1
Hence, it is true.
So, statement (A) and statement (D) are true.

2. 4 $f(x) = x^3 + 3x^2 + 3x$

$$f(x+1) = (x+1)^3 + 3(x+1)^2 + 3(x+1)$$
$$= x^3 + 1 + 3x + 3x^2 + 3(x^2 + 1 + 2x) + 3x + 3$$
$$= x^3 + 6x^2 + 12x + 7$$

Alternative method:
We can solve this question by giving value to x. e.g. put x = 1 then
$f(x + 1) = x^3 + 3x^2 + 3x$
$\Rightarrow f(2) = 2^3 + 3(2)^2 + 3 \times 2 = 26$
put x = 1 in the options
Hence, option (4) is the answer.

3. If (x – 3) is a factor then f(3) = 0. So 27 – 45 + 3p + 9 = 0 $\Rightarrow$ p = 3
If all the roots are integers, then the possible combinations are 3, 3, –1 or 3, –3, 1.
Of the two, we find that only the first is possible since sum of the roots = 5.

4. Since (x + 4) is a factor, we put x = –4 in the given expression.
(A) So, $3(-4)^3 + p(-4)^2 - 30(-4) + 24 = 0$
$-192 + 16p + 120 + 24 = 0$
$16p = 48 \Rightarrow p = 3.$
(B) $3x^3 + 3x^2 - 30x + 24 = 3(x + 4)(x - 2)(x - 1)$
Thus, roots are –4, 2 and 1.

5. 3 Using the remainder theorem we can conclude that $(-a)^3 + a(a^2) + a(-a) + 4 = 0.$
$\Rightarrow (a - 2)(a + 2) = 0$
So, a = –2 or 2.

6. 2 We can factor the left hand side into $(a - 2)^2 + (c - b)^2 + (c + 1)^2 = 0$
Because the left hand side consists of a sum of squares and the right hand side is 0, it must be that each square is 0.
This implies that a = 2; c = –1; b = –1.
So, a + b – c = 2 + (–1) – (–1) = 2.

7. 1 Given $x^4 + \frac{1}{x^4} = 47$ or $x^4 + \frac{1}{x^4} + 2 = 49$

or $\left(x^2 + \frac{1}{x^2}\right)^2 = 7^2$ or $x^2 + \frac{1}{x^2} = 7$

or $x^2 + \frac{1}{x^2} + 2 = 9$

or $\left(x + \frac{1}{x}\right)^2 = 3^2$ or $x + \frac{1}{x} = 3$

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right)$$
$$= 3^3 - 3 \times 3 = 18.$$

8. 2 The given expression is symmetrical i.e. if we replace a, b and c with each other, the expression remains unchanged.
So, a = b = c
Hence, the given expression can be re-written as

$$\frac{a^2}{2a^2+a^2}+\frac{a^2}{2a^2+a^2}+\frac{a^2}{2a^2+a^2}=\frac{1}{3}+\frac{1}{3}+\frac{1}{3}=1$$

Alternative method:
We assume some values of a, b and c such that $a + b + c = 0$ and $a \neq b \neq c$, and find the value of the expression that is given. So let a = 1, b = –1 and c = 0.

So we find that $\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$

$= \frac{1}{2}+\frac{1}{2}+0 = 1.$

9. 3 $x^2+5y^2+z^2=4yx+2yz$

$(x^2+4y^2-4yx)+z^2+y^2-2yz=0$

$(x-2y)^2+(z-y)^2=0$

It can be true only if x = 2y and z = y

Alternative method:
Option (1) cannot be the answer as L.H.S. has z^2 and R.H.S has z
Option (2) cannot be the answer as they are contradictory conditions.
So, option (3) is the only possibility.

10. 2 $a^3 + b^3 + c^3 - 3abc$
$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$\therefore \frac{(a^3+b^3+c^3-3abc)}{(a^2+b^2+c^2-ab-bc-ca)}=a+b+c$$

Here, a = 243, b = 257, c = 500

$\therefore a+b+c=243+257+500=1000$

11. 2 Since x^2 cannot be less than 0 for any value of x, so $|x^2+1|\leq 1$ will hold only when x = 0.

Hence, $(x+2)(x+1)(x)(x-1)(x-2)\cdots=0$

12. 2 Let the two-digit number be 10y + x.
So, x + y = 7 and y = x + 3
On solving these equations we get,
x = 2 and y = 5
⇒ Required number = 52.

Alternative method:
Use the options and apply the given conditions.

13. 3 Let, the number of cars = x and number of auto-rickshaws = y
So, x + y = 160 … (i)
and 4x + 3y = 550 … (ii)
On multiplying equation (i) with 4 and subtracting equation (ii), we get y = 90
Thus, there are 90 auto-rickshaws.

14. 1 Let the age of Alia be x years.
∴ The age of Shrishti = (x + 2) years
Now, the age of Pariksha can be either (x + 5) or (x – 5) years.
Case I: The age of Pariksha is (x + 5) years.
The age of Aasma = (x + 5 – 6) = (x – 1) years.
Case II: The age of Pariksha is (x – 5) years.
The age of Aasma = (x – 5 – 6) = (x – 11) years.
Hence, in both the cases Aasma is the youngest person.

15. 2 Since thief escaped with 1 diamond,
Before 3rd watchman he had (1 + 2) × 2 = 6
Before 2nd watchman he had (6 + 2) × 2 = 16
Before 1st watchman he had (16 + 2) × 2 = 36.

Alternative method:
Use the options and try.

16. 4 According to the information given in the question,
Cost of 1 dozen apple + cost of 1 kg orange = Rs.20
Cost of 1 dozen orange + cost of 1 kg apple = Rs.16
Cost of 1 dozen orange + cost of 1 dozen apple
= 3 × 12 = Rs.36
∴ Cost of 1 kg orange + cost of 1 kg apple = 0
Which is not possible. Hence, the data is inconsistent.

17. 4 Let there be x mints originally in the bowl.

Sita took $\frac{1}{3}$, but returned 4. So now the bowl has

$\frac{2}{3}x+4$ mints.

Fatima took $\frac{1}{4}$ of the remainder, but returned 3.

So the bowl now has $\frac{3}{4}\left(\frac{2}{3}x+4\right)+3$ mints.

Eshwari took half of remainder that is

$$\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x+4\right)+3\right]$$

She returns 2, so the bowl now has

$$\frac{1}{2}\left[\frac{3}{4}\left(\frac{2}{3}x+4\right)+3\right]+2=17 \Rightarrow x = 48$$

Short cut:
Since Sita was the first person to pick and she picks up $\frac{1}{3}$ of the mint, but if you see the options, none of the option is a multiple of 3.

18. 1 3x + ky = 1 and 6x + 3y = 2 shall have infinite solutions

if $\frac{3}{6}=\frac{k}{3}=\frac{1}{2}$ or k = $\frac{3}{2}$

19. 3 2x and 40 are even numbers. Therefore, 3y must be an even number. There are 6 even multiples of 3 from 0 to 40. Hence, there are 6 values of (x, y) that satisfy the equations.
They are (17, 2), (14, 4), (11, 6), (8, 8), (5, 10), (2, 12).

20. 3 Given equation is x + y = xy
⇒ xy – x – y + 1 = 1
⇒ (x – 1)(y – 1) = 1
x – 1 = 1 & y – 1 = 1 or x – 1 = –1 & y – 1 = –1
Clearly (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

21. 3 Given that 30X + 20Y = 1000
⇒ 3X + 2Y = 100 ...(i)
Let K pens be gifted away.
⇒ 35X + 25(Y – K) = 1000
7X + 5Y – 5K = 200 ...(ii)
2(3X + 2Y) + X + Y – 5K = 200
200 + X + Y – 5K = 200
⇒ X + Y = 5K

A. X + Y = 25 and 3X + 2Y = 100 gives Y = – 25 (not possible).
B. Not possible as (X + Y) has to be a multiple of 5.
C. X + Y = 45 and 3x + 2Y = 100, X = 10 and Y = 35.

Therefore, only C can be correct.

22. 2 Let the total number of chocolates in the box be x.
Number of chocolates taken by Child 1
= 100 + 0.1(x – 100) = 90 + 0.1x
Number of chocolates taken by Child 2
= 200 + 0.1(x – (290 + 0.1x) = 171 + 0.09x.
As each child receives the same number of chocolates, therefore 90 + 0.1x = 171 + 0.09x
Therefore, the value of x = 8100.
Number of chocolates received by each child
= 90 + 0.1x = 900.

Alternative method:
Use the options to solve the question.
e.g. if we pick option (2) which says each child must have received 900 chocolates.
Using this in the condition given in the question for child 1.

$900 = 100 + \frac{1}{10}(T-100)$ {where T is total number of chocolate}
⇒ T = 8100
Trying this in the conditions it satisfies.
So, option (2) is correct and option (1) & (3) cannot be correct simultaneously.

23. 1 From given data, we can say that
f(x) = k(x – 1) (x – 2) (x – 3) ......(x – 49) + x + 3
So, f(50) = k49! + 53.

24. 1 It is given that $p+q+r \neq 0$, if we consider the first option, and multiply the first equation by 5, second by –2 and third by –1, we see that the coefficients of x, y and z all add up-to zero.
Thus, 5p – 2q – r = 0
No other option satisfies this.

25. 3 Let x be the number of males in Mota Hazri.

	Chota Hazri	Mota Hazri
Males	x – 4522	x
Females	2(x – 4522)	x + 4020

x + 4020 – 2(x – 4522) = 2910 ⇒ x = 10154
∴ Number of males in Chota Hazri = 10154 – 4522
= 5632

26. 3 Let the number of questions answered correctly be x, that of answered wrongly be y and that of left unattempted be z.
Thus, x + y + z = 50 …(i)

And $x-\frac{y}{3}-\frac{z}{6}=32$

The second equation can be written as,
6x – 2y – z = 192 …(ii)
Adding the two equations, we get,

$7x - y = 242$ or $x = \frac{242+y}{7}$

Since x and y are both integers, the minimum value of y must be 3.

27. 4 Given that $F(x) = (x+2)(x+1)(x-1)(x-2)$

Putting x = P, we have

$F(P) = (P+2)(P+1)(P-1)(P-2)$

Now P is in the form $6K \pm 1$ where K is a positive integer.

$F(6K+1) = (6K+3)(6K+2)(6K)(6K-1)$

$= (36)(2K+1)(3K+1)(K)(6K-1)$ ...(i)

$F(6K-1) = (6K+1)(6K+2)(6K)(6K-3)$

$= 36(6K+1)(3K+1)(K)(3K-1)$ ...(ii)

Please note that the value of $K \geq 17$ and expression F (6K + 1) and F (6K – 1) always bear the factor 10. Hence, 360 is the correct choice.

28. 4 All the three equations are symmetrical. So, the solutions exist for the conditions x = y = z, x = y = –z, x = –y = z and –x = y = z.

Case I:
When x = y = z
then all the three equations give x = y = z = ±1
i.e. two solutions

Case II:
When x = y = –z
then all the three equations give x = y = –z = ±2
i.e. x = y = –z = 2 ⇒ (x, y, z) = (2, 2, –2)
or x = y = –z = –2 ⇒ (x, y, z) = (–2, –2, 2)
i.e. two solutions

Case III:
When x = –y = z
then all the three equations give x = –y = z = ±2
i.e. x = –y = z = 2 ⇒ (x, y, z) = (2, –2, 2)
or x = –y = z = –2 ⇒ (x, y, z) = (–2, 2, –2)
i.e. two solutions

Case IV:
When –x = y = z
then all the three equations give –x = y = z = ±2
i.e. –x = y = z = 2 ⇒ (x, y, z) = (–2, 2, 2)
or –x = y = z = –2 ⇒ (x, y, z) = (2, –2, –2)
i.e. two solutions

Hence, total 8 solutions are possible.

29. 3 Let the student attempted a, b and c number of questions in paper A, B and C respectively.

So, a + b + c = 20 ...(i)
and 20a + 25b+ 30c = 520 ...(ii)

On multiplying equation (i) by 20 and subtracting it from equation (ii), we get
5b + 10c = 120
⇒ b + 2c = 24

So,

$$\begin{array}{l} c \quad b \\ \left.\begin{array}{l} 0 — 24 \\ 1 — 22 \\ 2 — 20 \\ 3 — 18 \end{array}\right\} \text{rejected} \\ 4 — 16 \\ \vdots \quad \vdots \\ 10 — 4 \\ 11 — 2 \\ 12 — 0 \end{array}$$

(We don't need to write all the values, as c will increase in the step of 1, so b will decrease in the steps of 2)
But here a + b + c = 20, so b cannot be more than 16.
i.e. there are 9 solutions.
Hence, the student can score 520 marks in 9 ways.
Now, the values that a can take are 0, 1, 2, 3, 4, 5, 6, 7, 8. So, the maximum number of questions attempted by the student in section A is 8.

30. 4 There are two equations to be formed 40 m + 50 f = 1000
250 m + 300 f + 40 × 15 m + 50 × 10 f = A
850 m + 800 f = A
m and f are the number of males and females A is amount paid by the employer.
Then the possible values of f = 8, 9, 10, 11, 12
If f = 8
m = 15
If f = 9, 10, 11 then m will not be an integer while f = 12 then m will be 10.
By putting f = 8 and m = 15, A = 18800. When f = 12 and m = 10 then A = 18100
Therefore the number of males will be 10.