Algebra - 8

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- Co-ordinate Geometry
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QA - 24

CEX-Q-0225/18

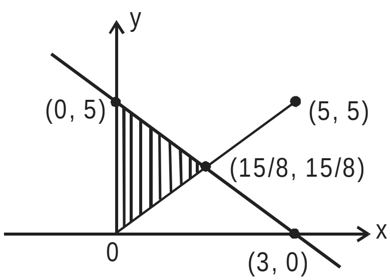
Number of questions : 30

Co-ordinate Geometry

1. Two points A(3,7) and B(- 5, 13) are located on a coordinate plane. Find
   (1) The distance between A,B
   (2) The mid-point of AB
   (3) Coordinates of C such that AC : CB is 1: 2
   (4) Equation of the line passing through A,B

2. The points Q(1; – 1), R(– 1; 0) and S(0; 1) are three vertices of a parallelogram. Write all the possible coordinates of the fourth vertex of the parallelogram.

3. Identify the shaded region from the following given choices.



(1) $x \ge 0, y \ge x$ and $5x + 3y - 15 \le 0$

(2) $x \ge 0, y \ge x$ and $5x + 3y - 15 \ge 0$

(3) $x \ge 0, y \le x$ and $5x + 3y - 15 \le 0$

(4) $x \ge 0, y \le x$ and $5x + 3y - 15 \ge 0$

4. Find the following equations
   (1) Equation of the line passing through (2,1) and having a slope of – 3
   (2) Equation of the line passing through (1,2) and perpendicular to (1)

5. Find the condition under which the straight lines $x + 2y - 9 = 0$, $3x + 5y - 5 = 0$ and $ax + by - 1 = 0$ are concurrent.
   (1) $35a + 22b = 1$ (2) $35a - 22b = -1$
   (3) $35a + 22b = 0$ (4) $22a + 35b = -1$

6. A line L passes through the points (2, 4) and (3, –2). The line L is rotated by 90° in the clockwise direction about the point where the line cuts the X-axis. What is the equation of the new line L?
   (1) $4x - 3y = 9$ (2) $3x + 18y = 4$
   (3) $3x - 18y = 8$ (4) $4x - 9y = 3$

7. What is the perpendicular distance between the two parallel lines $3x + 4y + 3 = 0$ and $3x + 4y + 12 = 0$?
   (1) $\frac{11}{5}$ units (2) 2 units
   (3) 3 units (4) $\frac{9}{5}$ units

8. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$. then which one of the following is a vertex of this rhombus?

(1) $(-3, -8)$ (2) $\left(\frac{1}{3}, \frac{8}{3}\right)$
(3) $\left(\frac{10}{3}, \frac{7}{3}\right)$ (4) $(-3, -9)$

9. Let k be an integer such that the triangle with vertices $(-2k, k)$, (k, k) and $(k, -3k)$ has area 24 sq. unit. Then the orthocentre of this triangle is at the point

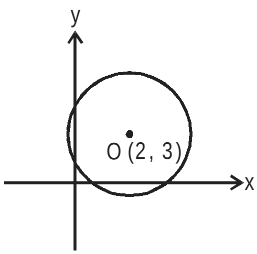
(1) $(1, -1)$ (2) $\left(1, -\frac{3}{4}\right)$
(3) $(2, 2)$ (4) $\left(2, -\frac{1}{2}\right)$

10. The four distinct points $(-a, -b)$, $(0, 0)$, (a, b) and (a^2, ab) are
(1) vertices of parallelogram
(2) vertices of rectangle
(3) collinear
(4) lying on a circle

11. Point P is on the line $y = 5x + 3$. The coordinates of point Q are $(3, -2)$. If M is the midpoint of PQ, then M must lie on the line

(1) $y = \frac{5}{2}x - \frac{7}{2}$ (2) $y = 5x + 1$
(3) $y = -\frac{1}{5}x - \frac{7}{5}$ (4) $y = \frac{5}{2}x + \frac{1}{2}$

12. Which of the following can be the possible equation of the circle shown in the following diagram with O as the center?



(1) $x^2 + y^2 - 4x - 6y + 10 = 0$
(2) $x^2 + y^2 - 4x - 6y + 1 = 0$
(3) $x^2 + y^2 - 4x - 6y - 3 = 0$
(4) $x^2 + y^2 - 4x - 6y + 6 = 0$

13. What is the shortest distance between two circles, the first having centre A(5,3) and radius 12, and the other with centre B(2, –1) and radius 6?
(1) 1 (2) 2
(3) 3 (4) 4

14. Find the minimum possible distance between a point on the circle $(x-1)^2 + y^2 = 9$ and the line $8x + 6y - 48 = 0$.
(1) 0 unit (2) 1 unit
(3) 2 units (4) 3 units

15. Find the number of Lattice points (points with integer coordinates)
(1) On the Triangle
(2) Inside the triangle formed by the points
(i) A(0,0) B(40,0) c) (0,40)
(ii) A(0,0) B(40,0) c) (0,60)

Graphs

16. Find the area enclosed by the graphs
(A) $|x| = 4$; $|y| = 3$
(B) $|x - 3| = 4$; $|y + 3| = 5$
(C) $|x + y| = 3$; $|x - y| = 5$

17. Find the area of the region bounded by the curves $|x| = 3$, $y = |x|$ and $y = -|x|$.
(1) 8 square units (2) 9 square units
(3) 12 square units (4) 18 square units

18. In the X-Y plane, find the area of the region bounded by the graph $|x + 2y| + |x - 2y| = 6$
(1) 18 sq. unit (2) 16 sq. unit
(3) 12 sq. unit (4) 20 sq. unit

19. Find the area of the region in the first quadrant bounded by the co-ordinate axes, $4x + 5y \le 12$ and $2x + 3y \ge 3$.
(1) 2.85 sq. unit (2) 1.95 sq. unit
(3) 3.35 sq. unit (4) 2.45 sq. unit

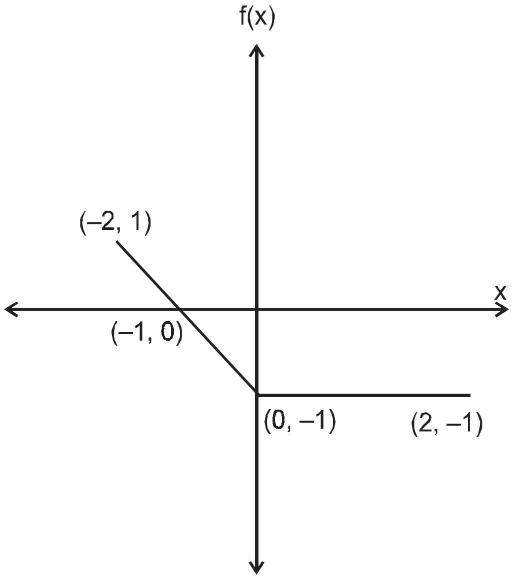
20. If the following graph represents f(x) then draw the graphs of

i. $g(x) = -f(x)$

ii. $g(x) = f(-x)$

iii. $g(x) = |f(x)|$

iv. $g(x) = -(f(-x))$



21. Determine the values of a for which the point (a, a^2) lies inside the triangle formed by the lines:

$2x + 3y = 1$, $x + 2y = 3$ and $5x - 6y = 1$

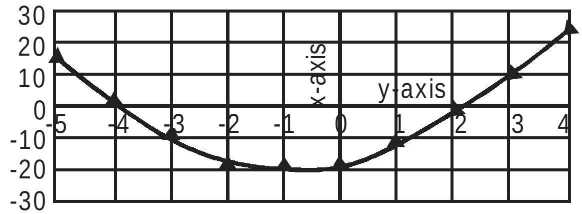
(1) (–3, –1) U (1/2, 1)

(2) $(-\infty, 1/3)$ U $(1/2, \infty)$

(3) (–3/2, –1) U (1/2, 1)

(4) $(-\infty, 1)$ U (1/3, 6)

22. Find the equation of the graph shown below.



(1) $y = 3x - 4$
(2) $y = 2x^2 - 40$
(3) $x = 2y^2 - 40$
(4) $x = 2y^2 + 3y - 19$

23. How many straight lines passing through the point (0, 30) and having negative integeral slope are possible?

(1) 1 (2) 8
(3) 16 (4) infinite

24. When the curves $y = \log_{10}x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for values $x \geq 1$?

(1) Never (2) Once
(3) Twice (4) More than twice

Challenging

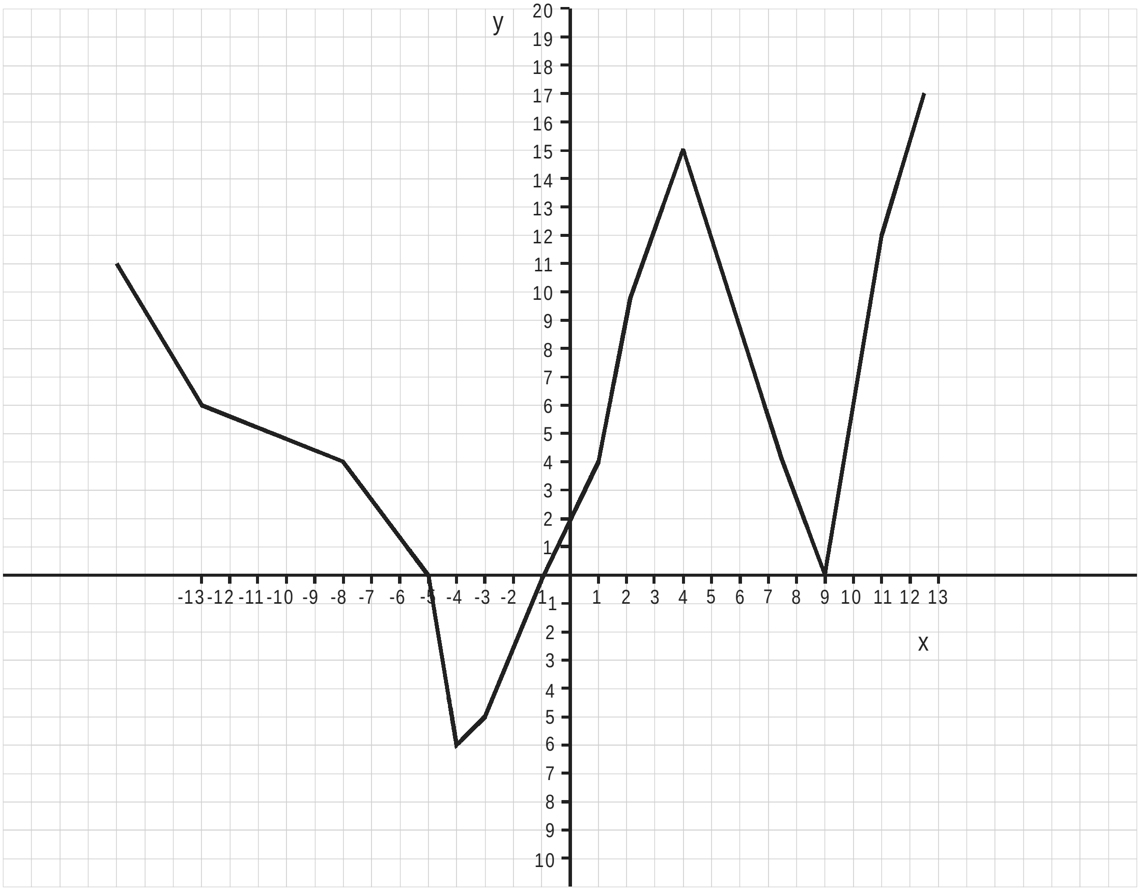
25. A lattice point is a point (x, y) where both x and y are integers. For how many different integer values of k will the two lines $kx - 5y + 7 = 0$ and $k^2x - 5y + 1 = 0$ intersect at a lattice point?

(1) 1 (2) 2
(3) 3 (4) 4

26. Let O denote the origin and A ; B denote respectively the points (–10 ; 0) and (7 ; 0) on the x-axis. For how many points P on the y-axis will the lengths of all the line segments PA, PO and PB be positive integers?

(1) infinite (2) 4
(3) 2 (4) 0

27. The figure below shows the graph of a function f(x). How many solutions does the equation f(f(x)) = 15 have?



(1) 5 (2) 6 (3) 7 (4) 8

28. The coordinates of P and Q are (0, 4) and (a, 6), respectively. R is the midpoint of PQ. The perpendicular bisector of PQ cuts X-axis at point S (b, 0). For how many integer value (s) of a, is b an integer?
(1) 3 (2) 2 (3) 4 (4) 1

29. We have five points A = (7, 4), B = (–10, 0), C = (–10, 3), D = (0, 10) and E = (7, 7). Every second, all the points move by halving their abscissas and by doubling their ordinates. This process continues for 500 years. After 500 years, which two points are closest?
(1) A and B (2) B and C (3) A and E (4) A and C

30. In a rectangular coordinate system, points L, M, N and O are represented by the coordinates (–5, 0), (1, –1), (0, 5), and (–1, 5) respectively. Consider a variable point P in the same plane. The minimum value of PL + PM + PN + PO is
(1) $1+\sqrt{37}$ (2) $5\sqrt{2}+2\sqrt{10}$ (3) $\sqrt{41}+\sqrt{37}$ (4) $\sqrt{41}+1$

QA - 24 : Algebra - 8
Answers and Explanations

1	–	**2**	–	**3**	1	**4**	–	**5**	2	**6**	3	**7**	4	**8**	3	**9**	3	**10**	3
11	4	**12**	2	**13**	1	**14**	2	**15**	–	**16**	–	**17**	4	**18**	1	**19**	1	**20**	–
21	3	**22**	4	**23**	2	**24**	2	**25**	2	**26**	3	**27**	3	**28**	3	**29**	4	**30**	2

1. (1) Distance between A and B

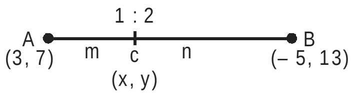
$= \sqrt{\{3-(-5)\}^2 + (7-13)^2} = \sqrt{64+36} = 10$ unit.

(2) The mid-point of AB

$X = \frac{3+(-5)}{2} = -1$

$Y = \frac{7+13}{2} = 10$ which is (–1, 10)

(3)



$X = \frac{1(-5)+2(3)}{1+2} = \frac{1}{3}$

$Y = \frac{1(13)+2(7)}{1+2} = 9$

So, $C \equiv \left(\frac{1}{3}, 9\right)$

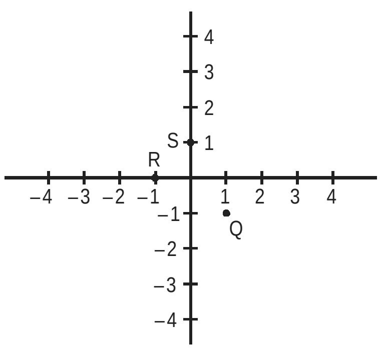
(4) Equation of line through AB

$y - 7 = \frac{13-7}{-5-3}(x-3)$

– 8y + 56 = 6x – 18

6x + 8y = 74.

2.



Possible points of P is (0, – 2), (2, 0), (–2, 2)

Note here, the points must be laying on the line parallel to either RQ or SR.

3. 1 Equation of the line passing through (0, 5) and (3, 0) is 5x + 3y – 15 = 0

The region below the line can be represented by

$5x + 3y - 15 \le 0$ ...(i)

The line passes through (0, 0) and (5, 5) is y = x

For the region above that y ≥ x ... (ii)

The right-hand side region of Y-axis is represented by x ≥ 0 ... (iii)

Hence, total shaded region is represented by the combination of equations (i), (ii) and (iii).

4. (1) Required equation of line is

y – 1 = – 3 (x – 2)

y – 1 = – 3x + 6

3x + y = 7

(2) Slope of line perpendicular to y = – 3x + 7 is $\frac{1}{3}$.

So, required equation of line is

$y - 2 = \frac{1}{3}(x - 1)$

3y – 6 = x – 1

3y = x + 5.

5. 2 It is given that

x + 2y = 9 ...(i)

3x + 5y = 5 ...(ii)

3 × (i) – (ii) gives

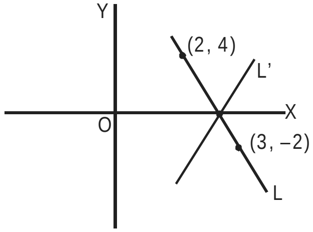
y = 22

Therefore, equation (i) gives x = 9 – 44 = –35.

Since the lines are concurrent the above values of x and y should satisfy ax + by = 1

∴ 22b – 35a = 1

6. 3



The equation of the line L is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

i.e. $y - 4 = \frac{-2-4}{3-2}(x-2)$

or y – 4 = –6 (x –2) or 6x + y = 16

So, the co-ordinate of the point where the line L cuts the X-axis is $\left(\frac{8}{3}, 0\right)$.

New line L' is perpendicular to the line L.

So, $m_1 m_2 = -1$, Here $m_1 = -6$

$\therefore m_2 = \frac{1}{6}$ (where m_1 is slope of line L and m_2 is the slope of the line perpendicular to L)

The equation of new line L' is $y = \frac{1}{6}x + c$.

But this is passing through $\left(\frac{8}{3}, 0\right)$.

So, $0 = \frac{1}{6} \times \frac{8}{3} + c \Rightarrow c = \frac{-4}{9}$

Hence, the equation of new line is 3x – 18y = 8.

7. 4 3x + 4y + 3 = 0 …(i)

3x + 4y + 12 = 0 …(ii)

Put y = 0 in equation (i),

$\therefore$ x = –1

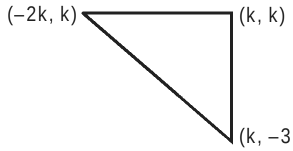
$\therefore$ (–1, 0) lies on the line (i) perpendicular distance between lines (i) and (ii) is same as perpendicular distance of (–1, 0) from the line (ii)

$\therefore$ Required distance = $\left|\frac{3(-1)+4(0)+12}{\sqrt{3^2+4^2}}\right| = \left|\frac{9}{\sqrt{25}}\right| = \frac{9}{5}$.

8. 3 The vertex of rhombus must lie on the line x – y + 1 = 0 or 7x – y – 5 = 0.

Only option (3) satisfy the given condition.

9. 3 By careful observation we can say that the given triangle is a right-angled triangle as shown below.



Area of this triangle = $\frac{1}{2} \times 3k \times 4k = 24$

$\Rightarrow$ k = 2

So, the orthocentre would be the vertex at which right-angle is form, which is (k, k) ≡ (2, 2).

10. 3 Let ≡ (–a, –b), B ≡ (0, 0), C ≡ (a, b) and D ≡ (a^2, ab)

Here, AB = $\sqrt{a^2 + b^2}$

and BC = $\sqrt{a^2 + b^2}$

AC = $2\sqrt{a^2 + b^2}$

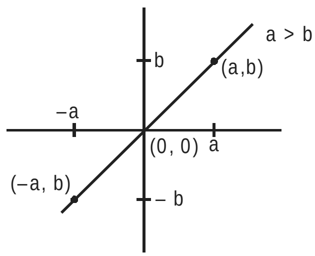
So, A, B and C must be collinear as AB + BC = AC.

$\Rightarrow$ Option (1), (2) and (4) are incorrect.

Hence, option (3) is correct.

Alternative method:

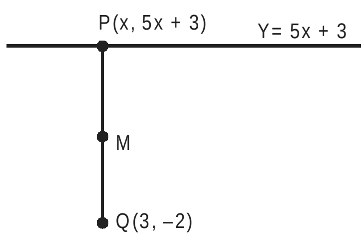
The points A(–a –b), B(0,0) and C(a, b) lie on the same line.



Thus, Option (1), (2) and (4) are incorrect.

Hence, option (3) is correct.

11. 4 The co-ordinates of point P can be given as, P(x, 5x + 3)



Since, M is the mid-point of PQ, the co-ordinates of M is given by

X = $\frac{x+3}{2}$ and Y = $\frac{5x+3-2}{2}$

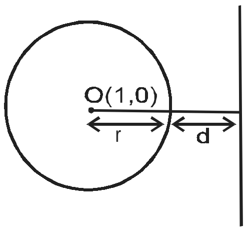
Thus, $Y = \frac{5}{2}x + \frac{1}{2}$

This is the required equation of line.

12. 2 The circle $x^2 + y^2 + 2gx + 2fy + c = 0$ has x intercept if $f^2 > c$ and has y intercept if $g^2 > c$.
To keep the origin outside the circle c > 0.
All the conditions are satisfied by option (2).

13. 1 Here, distance between the centres
$= \sqrt{(5-2)^2 + (3-(-1))^2} = 5$, which is less than the radius of smaller circle.
So, shortest distance between the circles = radius of larger circle – (radius of smaller circle + distance between their centres) = 12 – (6 + 5) = 1 unit.

14. 2



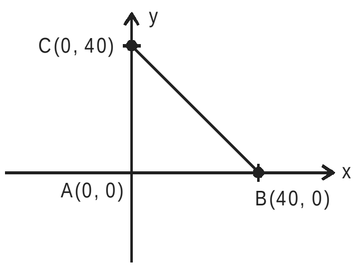
The shortest distance will be the difference between the distance between the centre and the line and the radius.
Centre of the circle is at O(1, 0) and r = 3 units.

$$\Rightarrow r + d = \left|\frac{[8(1) + 6(0) - 48]}{\sqrt{8^2 + 6^2}}\right|$$

$\Rightarrow$ r + d = 4 units

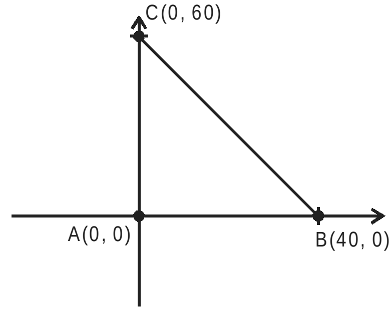
$\Rightarrow$ d = 4 – r = 1 unit.

15. (1)
(i) On the triangle



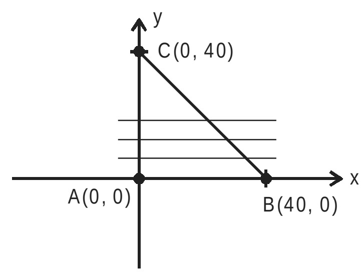
No. of lattice points on AB = 41
No. of lattice points on AC = 41
Now, the equation of line BC is given by x + y = 40.
So, number of whole number solutions of this equation will give the number of points laying on the line BC, which is 41. But the vertices A, B and C are getting counted twice, so, number of points laying on the triangle = 41 + 41 + 41 – 3 = 120.

(ii) Equation of line BC is given by
2y + 3x = 120
2y = 3 (40 – x)



Here, R.H.S. is a multiple of 3.
So, LHS must be a multiple of 3
Let y = 3b
Similarly, x must be a multiple of 2
let x = 2a
So, a + b = 20
Now, number of whole number solutions of this equation gives the number of integral points laying on the line BC, which is 21.
Number of integral points laying on the line AB = 40 (except B)
Number of integral points laying on the line AC = 60 (except C)
But point A is getting counted twice.
So, required points = 21 + 40 + 60 – 1 = 120.

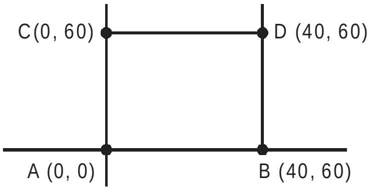
(2) In side the triangle
(i) We will count the number of points laying on the lines y = 1, y = 2, y = 3, y = 39 and also inside the triangle.



Number of points on y = 1 is 38
Number of points on y = 2 is 37 and so on
So, number of point inside the triangle ABC
= 1 + 2 + 3 + ... + 37 + 38

$= \frac{38}{2}[1 + 38] = 19 \times 39 = 741.$

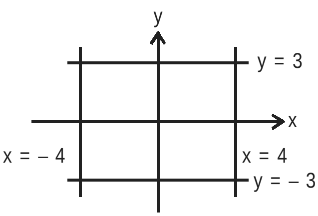
(ii) To solve this, will draw a rectangle as shown below.



Now, total number of points inside the rectangle
= (41 – 2) (61 – 2)
= 39 × 59 = 2301
Out of these points, 19 lie on the diagonal BC (inside the rectangle, as solved earlier)
Thus, number of points inside

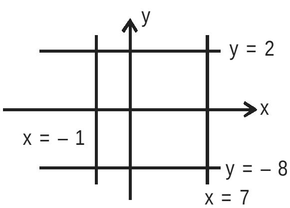
$\Delta ABC = \dfrac{2301-19}{2} = 1141.$

16. (A) The graph of the given functions is given as
$|x| = 4 \quad \Rightarrow \quad x = \pm 4$
$|y| = 3 \quad \Rightarrow \quad y = \pm 3$



So, required area = 8 × 6 = 48 sq unit.

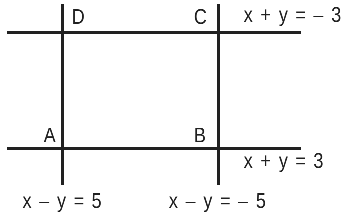
(B) $|x - 3| = 4 \quad \Rightarrow x = 7$ and $x = -1$,
$|y + 3| = 5 \quad \Rightarrow y = 2$ and $y = -8$



So, required area = 8 × 10 = 80 sq. unit.

(C) Here,
$|x + y| = 3 \quad \Rightarrow \quad x + y = \pm 3$
and $|x - y| = 5 \quad \Rightarrow \quad x - y = \pm 5$
The first two lines are parallel to each other, and other two lines are parallel to each other.

So, the possible graph can be drawn as



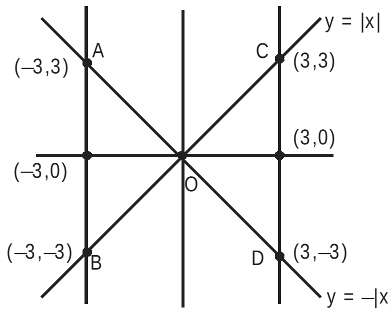
Since, distance between two parallel lines is given by

$$\left|\frac{C_1 - C_2}{\sqrt{a^2 + b^2}}\right|$$

So, $AB = \dfrac{5-(-5)}{\sqrt{2}} = \dfrac{10}{\sqrt{2}}$ and $BC = \left|\dfrac{3-(-3)}{\sqrt{2}}\right| = \dfrac{6}{\sqrt{2}}$

Thus, area of ABCD $= \dfrac{10}{\sqrt{2}} \times \dfrac{6}{\sqrt{2}} = 30$ sq. unit.

17. 4 Given curves can be drawn as



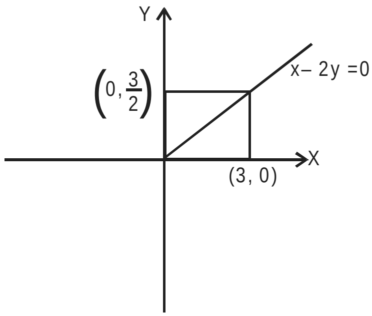
The area generated by the three curves is the sum of area of the triangles OAB and OCD.

$$= 2\left\{\frac{1}{2} \times (6) \times 3\right\} = 18 \text{ square units}$$

18. 1 $|x + 2y| + |x - 2y| = 6$
On replacing x by –x and y by –y in the above equation, we get the same equation. This implies that the curve is symmetric in the quadrants of X-Y plane
In the first quadrant where $x, y > 0$
$|x + 2y| + |x - 2y| = 6$

$$= \begin{cases} (x+2y) + (-x+2y) = 6 \ ; 2y > x \\ (x+2y) + (x-2y) = 6 \ ; 2y < x \end{cases}$$

$$= \begin{cases} y = \dfrac{3}{2} \ ; \quad 2y > x \\ x = 3 \ ; \quad 2y < x \end{cases}$$

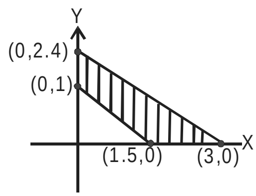


Area in the first quadrant $= \frac{3}{2} \times 3 = \frac{9}{2}$ square unit

Total area of |x + 2y| + |x – 2y| = 6

$= 4 \times \frac{9}{2} = 18$ square unit.

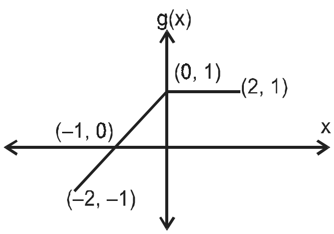
19. 1



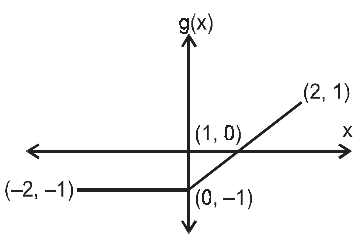
Area of the shaded region

$= \frac{1}{2}\left(3 \times \frac{12}{5} - 1 \times \frac{3}{2}\right) = 2.85$ sq. unit.

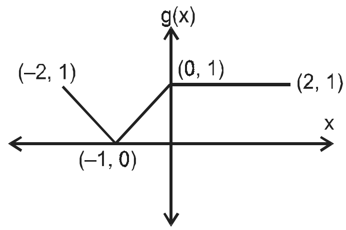
20. I.



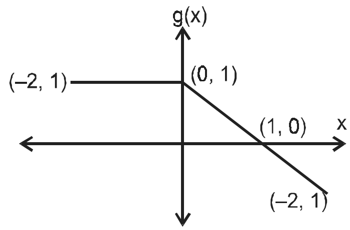
II.



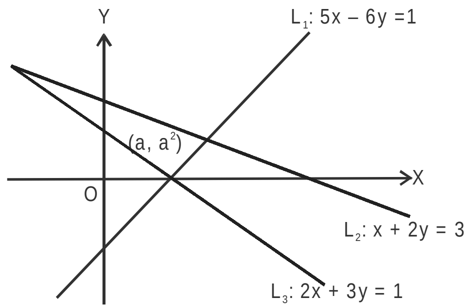
III.



IV.



21. 3



For line L_1: 2x + 3y = 1, origin O and point (a, a^2) are on the opposite sides of the line, so their signs are opposite.

$L_1(0, 0) : 2.0 + 3.0 - 1 = -1 < 0$

$\therefore L_1(a, a^2) = 2a + 3a^2 - 1 > 0$

$\Rightarrow 3a^2 + 3a - a - 1 > 0$

$\Rightarrow (a + 1)(3a - 1) > 0$

$\Rightarrow a < -1 \text{ or } a > \frac{1}{3}$ ... (i)

For line L_2: x + 2y = 3, $L_2(0, 0)$ has same sign as $L_2(a, a^2)$

$L_2(0, 0) : 0 + 2.0 - 3 = -3 < 0$

$\therefore L_2(a, a^2) : a + 2a^2 - 3 < 0$

$\Rightarrow 2a^2 + 3a - a - 3 < 0$

$\Rightarrow (2a + 3)(a - 1) < 0$

$\Rightarrow -\frac{3}{2} < a < 1$ ... (ii)

For line L_3: 5x – 6y = 1, $L_3(0, 0)$ has same sign as $L_3(a, a^2)$

$L_3(0, 0): 5 \times 0 - 3 \times 0 - 1 = -1 < 0$

$\therefore L_3(a, a^2) = 5a - 6a^2 - 1 < 0 \Rightarrow 6a^2 - 1 < 0$

$\Rightarrow 6a^2 - 5a + 1 > 0$

$\Rightarrow (3a - 1)(2a - 1) > 0$

$\Rightarrow a < \frac{1}{3}$ or $a > \frac{1}{2}$ ... (iii)

From (i), (ii) and (iii), we get

$a \in \left(-\frac{3}{2}, -1\right) \cup \left(\frac{1}{2}, 1\right)$.

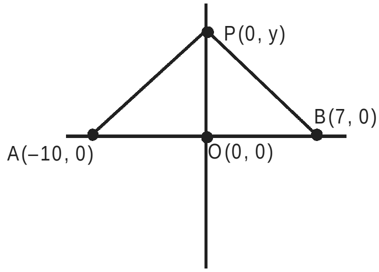
22. 4 In the graph, y-axis is horizontal and x-axis is vertical (opposite of convention). The graph cuts y-axis (x = 0) at two places. Hence, it will be quadratic equation for y. At y = 0 (x-axis), the value of x is near to – 20 (from the graph).
Possible equation is $x = 2y^2 + 3y - 19$

23. 2 The negative slope of a straight line passing through (0, 30) is given by $\frac{-30}{p}$, where p is a positive number.
The number of possible values of p will be the number of possible straight lines.
So, p = 1, 2, 3, 5, 6, 10, 15, 30
i.e. 8 straight lines are possible.

24. 2 For the curves to intersect, $\log_{10} x = x^{-1}$
Thus, $\log_{10} x = \frac{1}{x}$ or $x^x = 10$
This is possible for only one value of x such that 2 < x < 3.

25. 2 $kx - 5y + 7 = 0 \Rightarrow 5y = kx + 7$
and $k^2x - 5y + 1 = 0 \Rightarrow 5y = k^2x + 1$
On equating these two equations, we get
$kx + 7 = k^2x + 1$
$\Rightarrow x(k^2 - k) = 6 = 1 \times 6 = 2 \times 3 = 3 \times 2 = 6 \times 1$
When $x(k^2 - k) = 1 \times 6$
then x = 1 and $k^2 - k = 6 \Rightarrow k = 3, -2$
Since, $y = \frac{kx + 7}{5} = \frac{k + 7}{5}$
$\Rightarrow y = 2$ or 1
When $x(k^2 - k) = 2 \times 3$
then x = 2 and $k^2 - k = 3 \Rightarrow k = \frac{1 \pm \sqrt{13}}{2}$, k is not an integer.
when $x(k^2 - k) = 3 \times 2$
then x = 3 and $k^2 - k = 2 \Rightarrow k = 2, -1$
Since, $y = \frac{3k + 7}{5} \Rightarrow$ y is not integer for any value of k.
When $x(k^2 - k) = 6 \times 1 \Rightarrow x = 6$ and $k^2 - k = 1$
$\Rightarrow k = \frac{1 \pm \sqrt{5}}{2}$, which is not an integer.
Thus, only two values of k are possible.

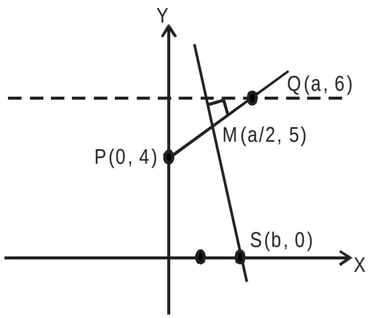
26. 3 Here, any integral value of y, length OP will be integer.



Now, $AP = \sqrt{10^2 + y^2} \Rightarrow AP^2 = 10^2 + y^2$
and $BP = \sqrt{7^2 + y^2} \Rightarrow BP^2 = 7^2 + y^2$
Here, by careful observation, we can say that we need to find the Pythagorean triplet, which satisfy the above two equations.
So, y must be 24.
Which will give AP = 26 and BP = 25
Thus, possible values of P are (0, 24) and (24, 0)
i.e. two points are possible.

27. 3 We are to find the solutions for f(f(x)) = 15.
From the graph, f(4) = 15 and f(12) = 15.
The required solutions will be those values of x for which f(x) = 4 and f(x) = 12.
From the graph, the value of function f(x) is 4 at four different values of x, i.e. –8, 1, 7.5 and 10.
The value of the function f(x) is 12 at three different points, i.e. 3, 5 and 11.
Hence, the given equation has 7 solutions.

28. 3



The co-ordinates of the point M are
$\left(\frac{a+0}{2}, \frac{6+4}{2}\right)$ or $\left(\frac{a}{2}, 5\right)$
Slope of the straight line PQ is $\frac{6-4}{a-0} = \frac{2}{a}$
$\Rightarrow$ Slope of the straight line MS = $-\frac{a}{2}$
$\Rightarrow$ Equation of the straight line MS is $\frac{y-5}{x-\frac{a}{2}} = -\frac{a}{2}$
$\Rightarrow y + \frac{a}{2}x = 5 + \frac{a^2}{4}$

As point S(b, 0) lies on it, we must have;

$$0+\frac{a}{2}\times(b)=5+\frac{a^2}{4} \Rightarrow b=\left(\frac{10}{a}+\frac{a}{2}\right)$$

For a = 2, –2, 10 and –10; b is an integer.

29. 4 Co-ordinates of points after halving their abscissas and doubling their ordinates infinite times:

A (0, High)
B (0, 0)
C (0, High)
D (0, High)
E (0, High)

Now, the closest point would be the points which had the value of their ordinates closest at the beginning.

Hence, the points A and C should be closest.

30. 2 (PL + PN) will be minimum if P lies on LN and (PM + PO) will be minimum if P lies on OM.

$\Rightarrow$ PL + PM + PN + PO will be minimum if P is the point of intersection of the diagonals of quadrilateral LMNO.

$$LN = \sqrt{(-5-0)^2+(0-5)^2} = 5\sqrt{2}$$

$$MO = \sqrt{(1-(-1)^2+(-1-5)^2} = 2\sqrt{10}$$

$$\Rightarrow PL + PM + PN + PO = LN + MO = 5\sqrt{2}+2\sqrt{10}$$