QA - 7 : Modern Maths

Career Launcher

Workshop

Number of Questions : 20

WSP–0007/18

1. The angle of elevation of an aeroplane changes from 60° to 30° in 10 minutes from a point on the ground. Find the speed (in kmph) of the aeroplane if it is flying horizontally at an altitude of 84 km.

(1) 576 (2) 582
(3) 588 (4) 579

2. The ratio of the sum of odd factors to that of even factors of a natural number is 1: 62. If one of factors of the number is chosen randomly, what is the probability that the chosen factor is an even natural number?

(a) $\frac{1}{7}$ (b) $\frac{6}{7}$
(c) $\frac{1}{6}$ (d) $\frac{5}{6}$

3. If a, b, c are three distinct numbers chosen from numbers 1 to 20, in how many ways can we choose the numbers such that a > b > c and also abc > a + b + c?

(1) 1139 (2) 1140
(3) 570 (4) $^{20}C_3 \times 3!$

4. If there are 10 stations between stations A and B and the train has to stop at station number 3 but will not stop at station number 7, then how many ways are possible to cover the journey from A to B?

(1) 2048 (2) 1024
(3) 512 (4) 256

5. From a chess board in how many ways can you choose 2 black boxes such that no two of them either belong to same row or same column?

(1) 496 (2) 480
(3) 800 (4) 400

6. If 5 boys and 5 girls are to be arranged in a row then what is the probability that girls and boys are alternate? Given that no two girls are together.

(1) $\frac{^6P_5}{5!6!}$ (2) $\frac{1}{5}$
(3) $\frac{2}{5}$ (4) $\frac{1}{3}$

7. There are 100 students in a class who appeared for 3 exams viz. English, Math and Reasoning and 15 of them failed in all 3 subjects. 20 of them passed in all 3 subjects. If 30 of them failed in exactly one subject, then how many of them failed in at least 2 subjects?

(1) 15 (2) 35
(3) 50 (4) 70

8. In a class, 30 like English, 40 like Maths and 50 like Reasoning. 50 like exactly 2 subjects. How many like all the three subjects if 20 like exactly one subject?

(1) 0 (2) 20
(3) 30 (4) 50

9. There are 150 students in a class 100 of them like at least 2 subjects of the three, 10 of them like none of the subjects. How many students like at most 1 subject?
(1) 0 (2) 10
(3) 40 (4) 50

10. In a group of sports lovers, 30 like Cricket, 40 like Football and 50 like Badminton. Find the maximum number of students who may like all the three sports, given that 50 students like at least 2 games.
(1) 0 (2) 20
(3) 30 (4) 50

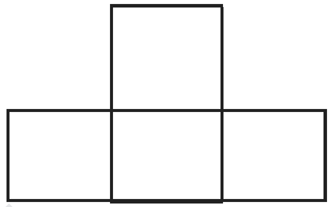
11. In how many ways can 20 identical chocolates be distributed among 4 children such that each child receives at least one chocolate and none of them receives 17 chocolates?
(1) 965 (2) 969
(3) ${}^{23}C_3$ (4) ${}^{23}C_3 - 4$

12. If five different addressed letters are to be put in five different addressed envelopes. In how many ways can you put them if at most 2 of them can go into wrong envelopes?
(1) 11 (2) 10
(3) 60 (4) 59

13. If 5 identical balls are to be put into 5 boxes such that 2 boxes are of type-A and 3 of them are of type-B. In how many ways can this be done.
(1) 120 (2) 10
(3) ${}^{9}C_4$ (4) 25

14. An eight-digit number is formed by using the digits 1 through 8, without repetition. What is the probability that the number thus formed is divisible by 11?
(a) $\frac{3}{35}$ (b) $\frac{2}{35}$
(c) $\frac{4}{35}$ (d) $\frac{6}{35}$

15. The following tile consists of four identical tiles of unit area each.



If the tiles can be placed in any orientation as long as no two tiles overlap each other, then what is the minimum possible number of such tiles required to obtain a rectangular figure?
(a) 6 (b) 4
(c) 3 (d) 5

16. What is the minimum value of $\sin^2\theta + \cos^2\theta + \tan^2\theta + \cot^2\theta + \sec^2\theta + \operatorname{cosec}^2\theta$?
(1) 6 (2) 7
(3) 8 (4) 9

17. If the angles of elevation of a pole from 2 points 120 m apart and on either side of the pole are 30° and 60°, find the height of pole
(1) 30 (2) 40
(3) $30\sqrt{3}$ (4) $40\sqrt{3}$

18. On a computer screen of size 20 cm × 30 cm, as a screen saver, a circle of radius 7 cm is dancing at random. What is the probability that a point chosen at random is not covered by the circle?
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{1}{7}$ (4) $\frac{7}{100}$

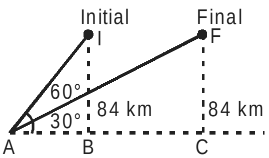
19. The number of solutions of x – 10 sinx = 0 are
(1) 1 (2) 3
(3) 5 (4) 7

20. In how many ways can 3 identical colours be painted on 6 faces of a cube such that same colour comes on opposite faces?
(1) 1 (2) 6!
(3) $\frac{6!}{2}$ (4) 3! × 2

Answers and Explanations

1	2	**2**	4	**3**	1	**4**	4	**5**	4	**6**	4	**7**	3	**8**	1	**9**	4	**10**	2
11	1	**12**	1	**13**	4	**14**	3	**15**	2	**16**	2	**17**	3	**18**	4	**19**	4	**20**	1

1. 2



AB = $\frac{84}{\sqrt{3}}$ and AC = $84\sqrt{3}$

$\Rightarrow$ BC = $\frac{168}{\sqrt{3}} \approx 97$ km in 10 minutes

$\Rightarrow$ 582 km in 60 minutes i.e., 582 kmph.

2. 4 Given,

$$\frac{\text{Sum of odd factors}}{\text{Sum of even factors}} = \frac{1}{62}$$

Let the sum of odd factors be x.
$\therefore$ Sum of all the even factors = 62x
Sum of all the factors = 63x = $(1 + 2 + 2^2 + 2^3 + 2^4 + 2^5)$x
Let x = $n_1 + n_2 + n_3 + \ldots$, where $n_1, n_2, n_3, \ldots$ are odd natural numbers.
63x = $(1 + 2 + 2^2 + 2^3 + 2^4 + 2^5) \times (n_1 + n_2 + n_3 + \ldots)$
Since 1 out of the six number in the first bracket is an odd natural number and all the numbers in the second bracket are odd natural numbers, one-six of the total terms in the product of the above number will be odd natural numbers and the rest of the terms will be even natural numbers. It can also be noted that all the terms together in the product will be equal to the number of factors of the number.

Hence, the probability = $\frac{5}{6}$.

3. 1 Three numbers out of 20 numbers can be chosen in $^{20}C_3$ ways and these will automatically satisfy the condition a > b > c. But 1, 2, 3 is the only possible case which will not satisfy abc > a + b + c.
So, possible ways = $^{20}C_3 - 1 = 1140 - 1 = 1139$

4. 4 A train either can stop or can't stop at any particular station.
So, according to the given scenario.

A 1st 2nd 3rd 4th 5th 6th 7th 8th 9th 10th B

↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑ ↑

2 × 2 × 1 × 2 × 2 × 2 × 1 × 2 × 2 × 2
ways

= 2^8 = 256 ways

5. 4 First black square:
The total number of squares = 8 × 8 = 64
Half are black and half are white.
So, for 1st black square, 32 options
Now, second black square (Since, we cannot choose the 2nd black box from same row or same column) can be selected in 25 ways.
$\Rightarrow$ Total = 32 × 25 ways (ordered pairs)
Since, the order of the squares doesn't matter

So, required answer = $\frac{32 \times 25}{2!} = 400$ ways

6. 4 Total number of possible cases (no two girls are together) = 5! × 6!
Total number of favourable cases (girls and boys are alternate) = 5! × 5! × 2

So, probability $= \frac{5! \times 5! \times 2}{5! \times 6!} = \frac{1}{3}$

7. 3 We will number students from 1 to 100 and by eliminating cases, we can get the required answer, which is 50 students.

8. 1 Since 50 like exactly two subjects, we will assume that out of 50 students, who like reasoning, 30 like English as well. Now, the rest 20 students who like reasoning, also like maths. Thus, there would be 20 students (exactly) which will like only maths (exactly one subject). Thus, there is no one who will like all the three subjects.

9. 4 Number of students who like none = 10
So, left = 140, out of which 100 like at most 2 subjects. Thus, remaining 40 would have been liking only one subject (to maximise).
So, total students liking at most one subject = student who do not like any one subject + students who like exactly one subject = 10 + 40 = 50 students.

10. 2 As, it is given that at least 50 students like at least 2-games. We will suppose that 40 students who like Football, also like Badminton and rest 10 students who like Badminton, like Cricket. Thus, there would be 20 students left who like Cricket and we will assume that these 20 students like all three games.
i.e., 20 is the maximum number of students who may like all the three games.

11. 1 We will initially give one chocolate to each child.
So we will be left with 16 chocolates.
So a + b + c + d = 16
$\Rightarrow$ $^{19}C_3$ = 969 ways.
But we will have to remove those cases in which a or b or c or d would get 16 alone (i.e. 17 total, as we have already distributed one to each child).
thus 4 cases will be deducted
i.e. 969 – 4 = 965 ways.

12. 1 0 wrong $\Rightarrow$ all right = 1 way
Or 1 wrong $\Rightarrow$ not possible (because if one would be wrong, other has to be wrong)
Or 2 wrong $\Rightarrow$ ${}^5C_2$ = 10 ways
So, total = 11 ways

13. 4

Case	Balls	2 Boxes of type A	Balls	3 Boxes of type B
I	5	(5, 0) (4, 1) (2, 3)	0	(0, 0, 0)
II	4	(1, 3), (2, 2), (4, 0)	1	(1, 0, 0)
III	3	(3, 0), (1, 2)	2	(2, 0, 0), (1, 1, 0)
IV	2	(1, 1), (2, 0)	3	(3, 0, 0), (2, 1, 0), (1, 1, 1)
V	1	(1, 0)	4	(4, 0, 0), (1, 3, 0), (1, 2, 1), (2, 2, 0)
VI	0	(0, 0)	5	(5, 0, 0), (4, 1, 0), (2, 3, 0), (1, 2, 2), (1, 3, 1)

Total:- 3 + 3 + 4 + 6 + 4 + 5 = 25

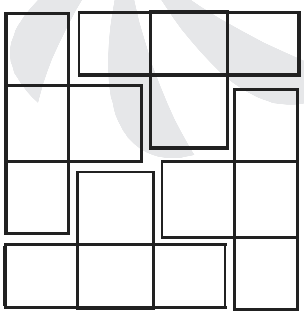
14. 3 Let the 8-digit number be abcdefgh and the sum of all digits is 36.
The number is a multiple of 11.

$$|(a+c+e+g)-(b+d+f+h)| = 0, 11, 22 \text{ or } 33$$

Let (a + c + e + g) = k_1 and (b + d + f + h) = k_2.
$\therefore$ $|k_1 - k_2|$ = 0, 11, 22 or 33 … (i)
Also, $k_1 + k_2$ = 36 … (ii)
Out of the possible pairs of equations from (i) and (ii), only $k_1 + k_2 = 36$ and $k_1 - k_2 = 0$ give an integral solution.
$\therefore$ a + c + e + g = b + d + f + h = 18
The possible values of quadruplet (a, c, e, g) are (1, 2, 7, 8), (1, 3, 6, 8), (1, 4, 5, 8), (1, 4, 6, 7), (2, 3, 6, 7), (2, 3, 5, 8), (2, 4, 5, 7) and (3, 4, 5, 6) and thus, quadruplet (a, c, e, g) has 8 values.
Similarly, quadruplet (b, d, f, h) also has 8 values.
The 4 numbers, a, c, e and g, can be rearranged in 4! ways and the same is true for b, d, f and h.

Hence, the required probability = $\dfrac{8 \times 4! \times 4!}{8!} = \dfrac{4}{35}$.

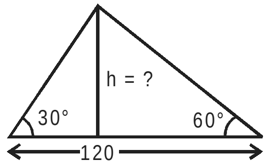
15. 2 One of the possible combinations out of the choices.



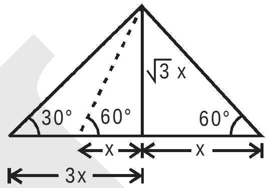
So, dimension of the rectangle is (4 × 4) and the minimum numbers of blocks required is 4.

16. 2 At $\theta = 45°$, expression would attain maximum value which is 7.

17. 3

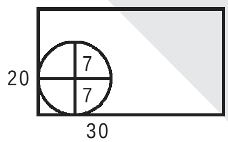


$\Rightarrow$



Now, 4x = 120 m $\Rightarrow$ x = 30 m
Now, height = $\sqrt{3} \times 30$m.

18. 4

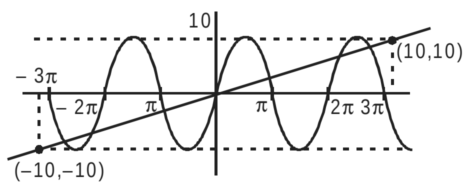


Since, the circle is dancing, all the areas would be covered by the circle except the corners.

So, area left at the corner = $\left\{7^2 - \dfrac{\pi}{4} \times 7^2\right\} \times 4$ = 42 square units.

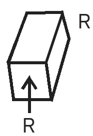
So, probability = $\dfrac{42}{20 \times 30} = \dfrac{7}{100}$.

19. 4 x – 10.sinx = 0 $\Rightarrow$ x = 10. sinx
It can be treated individually as Y = X and Y = 10.sinx as shown below:



Thus, 7 possible solutions.

20. 1



There is only 1 way.
Place any colour on any face.
The opposite will be same colour.
Or in other words, cube will look identical from all the sides.