Numbers - 4

Career Launcher

QA - 14

CEX-Q-0215/18

Contents

- Divisibility Rule
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Number of Questions : 30

Divisibility Rule

1. Let $N = 72^3 - 55^3 - 17^3$. N is divisible by **[CAT 2000]**
   (1) both 7 and 13 (2) both 3 and 13
   (3) both 17 and 7 (4) both 3 and 17

2. What least value must be assigned to 'a' such that the number '235a791' is divisible by 11?
   (1) 3 (2) 2
   (3) 4 (4) 6

3. Find single digit natural number 'x' if 8x01 is divisible by 9.
   (1) 1 (2) 4
   (3) 3 (4) 9

4. $2^{3n} - 1$ for all natural numbers 'n', is always divisible by
   (1) 2 (2) 3
   (3) 5 (4) 7

5. If n is any positive integer, then $n^3 - n$ is
   (1) always divisible by 12
   (2) never divisible by 12
   (3) always divisible by 6
   (4) never divisible by 6

6. If n is a positive integer, then $(4^{5n} - 5^{4n})$ is always divisible by
   (1) 17 (2) 113
   (3) 133 (4) 13

7. How many three-digit numbers are there, such that
   I. all are divisible by 55,
   II. the difference between the digits in the hundred's and ten's place is one more than the digit in the unit's place?

   (1) 5 (2) 8
   (3) 3 (4) 4

8. A is a 4-digit positive integer and B is the 4-digit positive integer formed by reversing the digits of A. If A – B is greater than zero and is divisible by 45, then find the number of possible values of A.
   (1) 248 (2) 400
   (3) 512 (4) 805

9. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by **[CAT 2002]**
   (1) 13
   (2) 127
   (3) 559
   (4) More than one of the above

10. Let $N = (666)^2 + (aaa)^2$, where 'N' is a natural number and 'aaa' is a three-digit natural number. 'N' is definitely divisible by which of the following numbers?
    (1) 222 (2) 444
    (3) 12321 (4) 666

11. If n is natural number greater than 2, then $n^5 - 5n^3 + 4n$ is always divisible by which of the following numbers?

(1) 36 (2) 25
(3) 124 (4) 120

12. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers?

(1) 21 (2) 25
(3) 41 (4) 67

13. Let k be a positive integer such that k – 4 is divisible by 5. The smallest integer n > 2, such that $n^2 + k$ is divisible by 5, is

(1) 4 (2) 19
(3) 24 (4) 5

14. Every integer of the form $(n^3 - n)(n - 2)$, (for n = 3, 4, 5, …) is

(1) divisible by 6 but not always divisible by 12
(2) divisible by 12 but not always divisible by 24
(3) divisible by 24 but not always divisible by 48
(4) divisible by 9

15. If X is the smallest number that is divisible by both 6 and 5, then find the maximum possible power of 10 that would completely divide the product of the first 20 multiples of X.

(1) 23 (2) 24
(3) 25 (4) 26

16. If $V = 3^{2n+2} - 8n - 9$, where n is a natural number, then V is always divisible by

(1) 128 (2) 192
(3) 64 (4) 56

17. The numbers from 1 to m are written one after another as follows 1234567…….m. The resulting number is divisible by 3 if m is of the form (n is a natural number):

I. 3n
II. 3n – 1
III. 3n – 2

(1) I only (2) I or II only
(3) I or III only (4) II or III only

18. Let X be a two digit number such that both X and X^2 end with the same digit and none of the digits in X equals zero. When the digits of X are written in the reversed order, the square of the new number so obtained has last digit as 6 and is less than 3000. Then the number of distinct possibilities for X is given by

19. The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If B > A and B – A is perfectly divisible by 7, then which of the following is necessarily true?

(1) 100 < A < 299 (2) 107 < A < 300
(3) 112 < A < 311 (4) 118 < A < 317

20. A, B, C and D remark the following about a natural number.
A says, "It is a two-digit number."
B says, "It is a divisor of 120."
C says, "It is not 120."
D says, "It is divisible by 30."

If at least one of A, B, C and D are lying, which of the following is not a possibility of the only person lying?

(1) A and C (2) B
(3) C (4) D

21. When a two-digit number is divided by the sum of its digits, the result is 4. If the digits are reversed, the new number is 6 less than twice the original. The number is

Cyclicity

22. What will be the last digit of $2^{3^{4^5}} \times 3^{15^{3^5}}$?

(1) 6 (2) 4
(3) 2 (4) 8

23. The unit's digit of $1^5 + 3^5 + 5^5 + 7^5 + \ldots\ldots + 99^5$ is

(1) 0 (2) 5
(3) 3 (4) 9

24. The last digit of the number 13^{24} is

25. What is the 12th digit from the left of $(6007)^3$?

(1) 0 (2) 1
(3) 2 (4) 3

26. What are the last two digits of 7^{2008}?

(1) 21 (2) 61
(3) 01 (4) 41

27. What are the last two digits of 475^{23} ?

28. The rightmost non-zero digits of the number 30^{2720} is

(1) 1 (2) 3
(3) 7 (4) 9

29. Find the unit digit of the product $2^{2010} \times 6^{2011} \times 7^{2012} \times 8^{2013}$.

30. If P and Q are natural numbers less than 10 and are not necessarily distinct, how many ordered pairs (P, Q) are there such that the unit's digits of P^P and Q^Q are the same?

(1) 12 (2) 15
(3) 10 (4) 14

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QA - 14 : Numbers - 4
Answers and Explanations

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1	4	**2**	1	**3**	4	**4**	4	**5**	3	**6**	3	**7**	4	**8**	4	**9**	4	**10**	3
11	4	**12**	3	**13**	1	**14**	3	**15**	2	**16**	3	**17**	2	**18**	–	**19**	2	**20**	3
21	–	**22**	2	**23**	1	**24**	–	**25**	4	**26**	3	**27**	–	**28**	1	**29**	–	**30**	2

1. 4 We know that if a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$.
$\therefore 72^3 - 55^3 - 17^3 = 3 \times 72 \times (-55) \times (-17)$
The above number is divisible by both 3 and 17.
Hence, option (4) is the correct answer.

2. 1 A number is divisible by 11, if the difference of the sum of digits at odd places and sum of digits at even places is either 0 or a multiple of 11.
$\Rightarrow |2 + 5 + 7 + 1 - (3 + a + 9)|$ is either 0 or a multiple of 11
$\Rightarrow |3 - a|$ is either 0 or a multiple of 11
$\therefore$ The value of 'a' is 3.

3. 4 For a number to be divisible by 9, sum of the digits of the number has to be divisible by 9.
Here, 8 + x + 0 + 1 = 9 + x.
Hence, x is a multiple of 9
Since x is a natural number, x = 9.

4. 4 $2^{3n} - 1 = (2^3)^n - 1 = (8)^n - 1$
$(x)^n - 1$ is always divisible by x – 1.
Therefore, $2^{3n} - 1 = (8)^n - 1$ is divisible by 8 – 1 = 7

5. 3 $n^3 - n = n(n^2 - 1) = (n - 1)\,n(n + 1)$ = Product of three consecutive numbers. So at least one number is even, and one number is a multiple of 3. So the product is divisible by 6.

6. 3 $4^{5n} - 5^{4n} = 1024^n - 625^n$
The above expression is always divisible by (1024 – 625) i.e. 399, which is divisible by 133.
Hence, option (3) is the correct answer.

7. 4 Out of the three-digit numbers that are divisible by 55, there are four numbers viz. 605, 715, 825 and 935 that satisfy the given condition.

8. 4 A can be written as
1000a + 100b + 10c + d
B can be written as
1000d + 100c + 10b + a
So A – B > 0
$\Rightarrow$ (1000a + 100b + 10c + d) – (1000d + 100c + 10b + a) > 0
$\Rightarrow$ 999a + 90b – 90c – 999d > 0
This has to be divisible by 45.
Then, it can be written as
(990 + 9)a + 90b – 90c – (990 + 9)d
Now 9a – 9d has to be divisible by 45.
So a – d is 5 or 0
Case I:
When a – d = 5,
a = 6, 7, 8 and 9 and corresponding value of
d = 1, 2, 3 and 4
The number of values of A is 4 × 10 × 10 i.e. 400.
Case II:
When a – d = 0, b > c
Then number of values of A is 9(9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1) i.e. 9 × 45 or 405
Hence number of values of A is 400 i.e. 805.

9. 4 $(a^n - b^n)$ is always divisible by (a + b) if n is an even number. Thus, $(7^{6n} - 6^{6n})$ is divisible by (7 + 6) i.e. 13.
It can also be written as $(343)^{2n} - (216)^{2n}$.
or $(343^n + 216^n)\,(343^n - 216^n)$
So, it can also be divisible by 127 and 559.

10. 3 We can write
$N = (111)^2\,(6^2 + a^2)$
Since we do not know what is a, the number N is definitely divisible by $(111)^2 = 12321$.

11. 4 The expression $(n^5 - 5n^3 + 4n)$ can be written as (n – 2)(n – 1)(n) (n + 1)(n + 2) which is nothing but the product of five consecutive numbers. Therefore the expression is divisible by 1 × 2 × 3 × 4 × 5 = 120.

12. 3 The sum of four consecutive two-digit odd numbers lies between 56 and 384. Since the sum of four two-digit numbers gives a perfect square on dividing by ten, the number has to be of the form $10a^2$, where a is an even natural number. The possible values of a are 4 and 6. Accordingly, the numbers are 37, 39, 41 and 43 or 87, 89, 91 and 93.
$\therefore$ 41 is a possible value of one of the odd numbers.

13. 1 If k – 4 is divisible by 5, then the smallest k = 9.
Since $k + n^2$ is divisible by 5, the lease possible value that n can take is 4.

14. 3 The expression can be expanded and written as (n – 2) (n – 1) n (n + 1)
The product of 4 consecutive terms is divisible by 24 but not always divisible by 48.

15. 2 Product of first 20 multiples of 30 can be written as $(30)^{20}$ [20!]. $(30)^{20}$ has 20 zeroes and 20! has 4 zeroes. Hence total number of zeroes is 24.

16. 3 $3^{2n+2} - 8n - 9 = 9 \times 3^{2n} - 8n - 9$
$= 9 \times 9^n - 8n - 9 = 9[(1 + 8)^n] - 8n - 9$
$= 9\,[1 + 8n + {}^nC_2 \times 8^2 + {}^nC_3 8^3 \ldots + 8^n] - 8n - 9$
$= 64n + 9({}^nC_2 \times 8^2 + {}^nC_3 8^3 \ldots + 8^n)$
$= 64[n + 9 \times ({}^nC_2 + {}^nC_3 8 \ldots + 8^{n-2})] = 64K.$

17. 2 Sum of the first two natural number in any three consecutive natural number, which digits sum are in the form of 3k + 1, 3k + 2 and 3k + 3 or sum of all the three such numbers would be divisible by 3.
Hence m must be either of the form 3n or 3n – 1

18. Let X = AB
Now, as per the information given in the questions the value of B could be 1, 5 or 6.
The square of the number obtained by reversing the digits of AB has last digit as 6, which implies that the value of A is 6 or 4.
So possible values of AB from the above conclusion could be 61, 65, 66, 41, 45 and 46, but the square of the number BA should also be less than 3000.
So, number of distinct possibilities for X could be 61, 41 and 45.

19. 2 Let A = abc, then B = cba
Given, B > A which implies c > a … (1)
as B – A = (100c + 10b + a) – (100a + 10b + 1)
B – A = 100(c – a) + (a – c)
B – A = 99(c – a) and (B – A) is divisible by 7.
As 99 is not divisible by 7 (no factor like 7 or 7^2), (c – a) must be divisible by 7 {i.e., (c – a) must be 7, 7^2 etc.}. As c & a are single digits. (c – a) must be 7 only, the possible values (c, a) {with c > a} are (9, 2) & (8, 1), with this we can write A as
$A : abc \equiv 1b8$ or $2b9$
as b can take values from 0 to 9, the smallest & largest possible value of are:
$A_{min} = 108$ and $A_{max} = 299$.

20. 3 Work with options
If A and C are the only persons lying, then B and D say the truth. Thus the number has to be one of 30, 60, 90 or 120. Since A lies, the number has to be 120.
It also is in agreement with C lying. Thus A and C could be the only persons lying.
The only person lying could be B as they could have thought of the number as 30, 60 or 90 and all of A, C and D would be speaking the truth.
Similarly, D could be the only person lying as any two-digit divisor of 120 would make A, B and C's statement true.
Could C alone lie? If C lies that means the number would be 120 which would make A's statement false. Thus C alone lying is not a possibility.

21. Let the two-digit number be 10a + b.
According to the question,
10a + b = 4(a + b) … (i)
10b + a = 2(10a + b) – 6
⇒ 19a – 8b = 6 … (ii)
From equation (i) and (ii), we get
a = 2, b = 4.
Hence, the number is 24.

22. 2 3^{4^5} when divided by 4 gives remainder 1.

Units digit of $2^{3^{4^5}}$ is 2.
Cyclicity of 3 is 4:

$$\frac{15^{3^5}}{4} = \frac{(16-1)^{3^5}}{4}$$

15^{3^5} when divided by 4 given a remainder –1 or 3.

∴ Unit digit of $3^{15^{3^5}} = 7$.

Unit digit of $2^{3^{4^5}} \times 3^{15^{3^5}} = 4$.

23. 1 In the first 5 numbers, the sum of all numbers ending in the 1, 3, 5, 7 and 9 has unit's digit 5. Similarly, in the next 5 numbers, the sum ends in 5. A similar pattern is followed by subsequent sets of 5 numbers. Hence, the sum of 10 sets of numbers each ending in a 5 is 0.

24. 13^2 ends in 9, 13^3 ends in 7, 13^4 ends in 1, thus $(13^4)^6$ ends in 1

25. 4 To simplify the problem, lets say
$6007 \approx 6000$
$6000^2 = 36000000$ (8 digits)
$6000^3 = 216000000000$ (12 digits)
Similarly, $(6007)^3$ will also contain 12 digits
∴ The 12th digit will be the last digit of $(6007)^3$ and it will depend upon '7'
$7^3 = 343$
∴ The 12th digit in $(6007)^3$ will be 3.

26. 3 The last two digits of any number in the form of 7^{4n} will always be equal to 01.
For example $7^4 = 2401$ and $7^8 = 5764801$.
Hence, option (3) is the correct choice.

27. 475^{23} can be written as $(500 - 25)^{23}$
Every term has atleast 2 zeroes and the last term is negative and ends in 25. Hence the number must end in 75.

28. 1 $((30)^4)^{680} = (810000)^{680}$
Hence the right most non-zero digit is 1.

29. Unit digit of the 2^{2010}, 6^{2011}, 7^{2012} and 8^{2013} is 4, 6, 1 and 8 respectively.
∴ Required unit digit = 4 × 6 × 1 × 8 = 2.

30. 2 Following are the two cases that are possible.
Case I:
P = Q.
There are 9 such ordered pairs (P, Q).
Case II:
P = 4, 6 or 8 and Q = 4, 6 or 8.
There are 6 such ordered pairs (P, Q).
Therefore, there are 9 + 6 = 15 such ordered pairs (P, Q).