

**Preface**[Time Speed and Distance](#)[Practice Exercises](#)[Answer key](#)[Explanations](#)

Dear Student,

The journey to achieve success has begun. The CL Educate team brings to you an offering, which incorporates **theme based learning** that revolves around different concepts with **diverse applications**. The outcome is an enriching learning experience.

Our integrated thematic methodology is driven by latest research, undertaken to enhance learning. Numerous practice exercises and tests have been incorporated to reinforce the conviction in one's ability. Our teaching experience coupled with extensive research has lent credence to our conviction that learning is at its best when concept based understanding and applications go hand in hand.

To enhance your learning and assimilation of relevant concepts, our attempt has been to identify the basic concepts (or themes) that are required to solve different questions in MBA entrance examinations. Our class exercises integrate the different types of questions requiring application of these concepts. Each set of concepts along with relevant question types therefore, forms a module. At the end of each module we expect the student to:

- 1) Clearly understand a concept through its repeated application in different question types.
- 2) Quickly and effectively apply the relevant concept to different question types in a time-bound examination scenario.
- 3) Develop long-lasting skills by imbibing each concept that is clearly covered through a module.

Armed with the latest tools for success, along with your diligence and positive attitude, you have begun your march towards success. Have faith in yourself!

The woods are lovely, dark and deep,

But I have promises to keep,

And miles to go before I sleep,

And miles to go before I sleep

(Robert Frost)

How to use this book



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How to use this book

1. Before you enter the class read the topics that are to be covered beforehand. This will help you immensely in understanding the concepts when they are taught in the class.
2. After each class, once again go through the relevant topics very carefully, in order to understand the concepts and relate them to what was taught in the class.
3. Do not directly jump to the practice problems but go through the solved examples first as they will enhance your problem solving skills and help in further clarifying concepts.
4. After you are through with the fundamentals and the solved examples, move on to the unsolved problems given at the end of the book and the practice exercises.
5. Start with the Level - I problems as they are easier. Move to the Level - II problems, if and only if you have completely understood the concept used in every problem in Level - I. Similarly, move to the Level - III problems after you have completed all the problems in Level - II.

Time Speed & Distancee

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Introduction

A perennial favourite of CAT setters, problems on Time, Speed and Distance are always found in good numbers in CAT and other entrance exams. While the other entrance exams typically stick to the "standard" problems of TSD like those involving trains crossing or boats and streams, CAT usually have unusual & innovative problems on TSD. Of late, CAT has a caselet of two to three questions based on TSD of people running in complex Geometry figures.

While students find the variety of problems that appear in TSD a little difficult to digest, the irony is that there is just one formula in the entire chapter, that of speed, which everyone is familiar with. Thus to master TSD, one just needs to recollect the fundas of Ratio and Proportion and see if these can be applied orally and avoid the use of equations.

Learning Objectives

- Proportionality between Time, Speed and Distance
- Average Speed
- Relative Speed
- a. Problems on trains crossing
- b. Problems on boats and streams
- Circular Motion
- Clocks

• Work

Relationship between Time, Speed & Distance

Speed is the rate at which distance is covered and thus the basic relation between Time, Speed and Distance is

$$\text{Speed} = \frac{\text{Distance covered}}{\text{Time taken}}$$

Thus the unit of speed will be either meter/second or kilometre/hour.

$$1 \text{ km/hr} = \frac{1 \text{ km}}{1 \text{ hr}} = \frac{1000 \text{ m}}{3600 \text{ s}} = \frac{5}{18} \text{ m/s}$$

Thus to convert speed given in kmph to m/s, multiply it with $\frac{5}{18}$ and to convert m/s to kmph, multiply with $\frac{18}{5}$.

It would be worthwhile to remember some common speeds given in kmph and which have to be converted to m/s to save time in calculations. Every 18 kmph corresponds to 5 m/s.

Thus $36 \text{ kmph} = 10 \text{ m/s}$, $54 \text{ kmph} = 15 \text{ m/s}$ and $72 \text{ kmph} = 20 \text{ m/s}$.

Example 1:

A person moves from A to B travelling a distance of 10 km in 4 hr. What is his speed?

Solution:

Distance travelled = 10 km. Time taken to travel this distance = 4 hr.

$$\text{Therefore, speed} = \frac{10 \text{ km}}{4 \text{ hr}} = 2.5 \text{ km/hr.}$$

A speed of 2.5 km/hr means the person is able to move 2.5 km every 1 hr. The standard units for speed are km/hr or m/s. The corresponding distances in each case must be in kilometre (km) or in metre (m) respectively and the time in hour (hr) or second (s) respectively.

Example 2:

If this person moving at the speed of 2.5 km/hr travels from Delhi to Sonepat in 1 day and 10 hr (assume he does not stop anywhere). Find the distance between Delhi and Sonepat.

Solution:

Make the units consistent first.

$$\text{Then apply } \text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Distance} = \text{Speed} \times \text{Time} = 2.5 \text{ km/hr} \times (1 \text{ day} + 10 \text{ hr})$$

$$\text{or distance} = (2.5 \text{ km/hr}) \times (34 \text{ hr}) = 85 \text{ km}$$

Example 3:

In the above problem, if a person wants to reach Delhi back in 17 hr, at what speed should he move?

Solution:

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{So, speed} = (85 \text{ km}/17 \text{ hr}) = 5 \text{ km/hr}$$

In the above problem, we had to convert the time from days into hours in order to make the units of all the three consistent.

Example 4:

Shoaib Akhtar has a run-up of 100 m. If the speed at which he runs is 36 km/hr, how much time does he take to complete his run-up?

Solution:

Convert the speed into m/s.

$$\text{So } 36 \text{ km/hr} = 36 \times \frac{5}{18} \text{ m/s} = 10 \text{ m/s.}$$

$$\text{Thus, time taken to complete the run-up} = \frac{100}{10} = 10 \text{ s.}$$

If you compare the time taken to cover the distances in example 2 and 3, we would observe that if the speed increases the time taken reduces to cover the same distance.

Let's formalise certain relationships similar to the earlier.

All these stem from the equation: Speed \times Time = Total distance travelled.

More useful than the formula of speed is the proportionality relationship between Time, Speed and Distance. From the formula we identify the following:

Speed is directly proportional to distance. Thus if twice the distance is covered in same time, speed has become twice

Speed is inversely proportional to time. Thus if speed becomes x times, time taken to cover same distance will become $\frac{1}{x}$ times

Distance is directly proportional to time taken. Thus if time taken at same speed becomes $\frac{1}{x}$, it means distance has become $\frac{1}{x}$ th.

In all the following examples please pay attention to the oral method given using ratios and try to avoid the use of equations. It will help you increase your speed tremendously.

Example 5:

Amitabh sees Uma standing at a distance of 200 m from his position. He increases his speed by 50% and hence takes 20 s now to reach her.

- If he travels at the original speed, how much time will he take?
- What was his original speed (in km/hr)?
- What is his new speed?

Solution:

a. Travelling at 150% of his original speed (i.e. $\frac{3}{2}$ times) his original speed, Amitabh should take $\frac{2}{3}$ of the original time, i.e. 20 s. So his original time is 30 s.

$$\text{b. Original speed} = \frac{\text{Distance}}{\text{Original time}} = \frac{200}{30} \text{ m/s or } \frac{20}{3} \times \frac{18}{5} = 24 \text{ km/hr.}$$

$$\text{c. His new speed is 50\% more} = 24 \times 1.5 = 36 \text{ km/hr.}$$

(Amitabh was running as fast as Shoaib Akhtar!!)

Example 6:

If instead of going at my usual speed of 30 kmph, I go to office at 40 kmph, I reach office 10 minutes early. What is the usual time I take to reach office and what is the distance I travel?

Solution:

The equation is quite simple and can be written as

$$\frac{d}{30} - \frac{d}{40} = \frac{10}{60} \Rightarrow 4d - 3d = 20 \Rightarrow d = 20$$

and regular time taken is $2/3$ of an hour i.e. 40 minutes.

However the problem is too simple to involve any writing. Since both the speeds are given i.e. usual speed and new speed, one should immediately note the ratio of usual speed to new speed is 3 : 4. Thus ratio of usual time to new time will be 4 : 3. And we know the difference in these time is 10 minutes. Thus usual time taken is 40 minutes and new time taken is 30 minutes. Also distance can be found out by multiplying either speed with the respective time taken. This whole process can be done orally and would go a long way in building speed.

Example 7:

Two bicyclists cover the same distance at 15 km/hr and 16 km/hr respectively. Find the distance travelled by each, if one takes 16 min. longer than the other.

Solution:

Let the required distance be x km.

$$\frac{x}{15} - \frac{x}{16} = \frac{16}{60} \text{ or, } 16x - 15x = 64 \text{ or } x = 64$$

Hence, the required distance = 64 km.

Example 8:

Buses take 12 hr to cover the distance of 120 km between A and B. A bus starts from A at 8.00 a.m. and another bus starts from B at 10.00 a.m. on the same day. When do the two buses meet?

Solution:

The distance between A and B is 120 km.

$$\text{Speed of the buses} = \frac{120}{12} = 10 \text{ km/hr}$$

By 10.00 a.m., the bus from A would have covered 20 km.

Hence, the distance between the buses at 10.00 a.m. = 120 - 20 = 100 km

Relative speed of the buses = 20 km/hr.

$$\text{Time taken to meet} = \frac{100}{20} = 5 \text{ hr.}$$

Hence, the buses meet at 3:00 p.m.

Example 9:

A, B and C start swimming in a pool simultaneously from the same end. To complete 10 laps, A takes 10 minutes, B takes 8 minutes and C takes 6 minutes. What is the ratio of speeds of A, B and C.

Solution:

Since all three swim for 10 laps, i.e. distance is same for all three, speed is inversely proportional to the time taken and thus ratio of speeds of A, B and C is

$$\frac{1}{10} : \frac{1}{8} : \frac{1}{6} = 12 : 15 : 20$$

Example 10:

Mr Ghosh arrives at his office 30 min late everyday. On a particular day, he reduces his speed by 25% and hence arrived 50 min late instead.

a. Find how much time would he take to travel to his office if he decides to be on time on a blue moon day?

b. If he has to arrive on time, by what percentage should he increase his speed?

(Assume on each of these days, he starts from his home at the same time.)

Solution:

a. Speed has become $\frac{3}{4}$ of the speed that he travelled with when he was 30 min late.

So time taken to travel should become $\frac{4}{3}$ of the earlier time or $\frac{1}{3}$ of the earlier time extra.

So $\frac{1}{3}$ of the earlier time = (50 - 30) = 20 min.

This implies when he was 30 min late, he was taking $3 \times 20 = 60$ min to travel the distance.

So if he decides to come on time, he would take 30 min to travel.

If too many words are confusing you, the standard equation method could be applied.

Assume normal time taken, if on time, is T hr, distance is D km, speed when 30 min late is S km/hr.

Then $\left(T + \frac{1}{2}\right) \times S = \left(T + \frac{5}{6}\right) (0.75 S)$. Solving for T, we get $T = \frac{1}{2}$ hr = 30 min.

b. To be on time Mr Ghosh takes $\frac{1}{2}$ hr, whereas if he is 30 min late, he takes 1 hr. Thus he needs to double his speed or, in other words, to increase his speed by 100%, if he has to reach on time.

Example 11:

Ram and Shyam are two friends whose homes are 30 km apart. Ram calls up Shyam on the phone and they decide to meet somewhere between their houses. Ram rides at 4 km/hr and Shyam at 6 km/hr, and they start simultaneously from their respective homes.

a. At what distance from Shyam's house will they meet?

b. After how much time will they meet?

Solution:

Method 1:

a. If the two meet at a distance of x km from Shyam's house, then time taken by Ram

$$= \frac{\text{Distance travelled by Ram}}{\text{Speed of Ram}} = \frac{30 - x}{4} \dots (\text{i})$$

$$\text{Similarly, time taken by Shyam} = \frac{x}{6} \dots (\text{ii})$$

Since both start simultaneously from their respective homes, time taken by Ram must be equal to the time taken by Shyam.

So (i) = (ii).

Hence, solving for x , we get $x = 18$ km and $(30 - x) = 12$ km.

Thus, Ram and Shyam meet at a distance which is 12 km from Ram's and 18 km from Shyam's house.

b. The two will meet after $\frac{18}{6}$ hr or $\frac{12}{4}$ hr,

i.e. 3 hr.

Method 2:

They together cover 30 km when they meet. Since they travel for the same amount of time when they meet, the ratio of distances travelled will be equal to the ratio of their speeds.

So the ratio of the distances travelled by Ram and Shyam = 4 : 6 = 2 : 3.

Hence, the distance travelled by Shyam = Distance of the meeting point from Shyam's house = $\left(\frac{3}{5} \times 30\right) = 18$ km.

b. The two will meet after $\frac{18}{6}$ hr or $\frac{12}{4}$ hr, i.e. 3 hr.

We will revisit this problem when we would discuss the concept of relative speed.

Example 12:

A train met with an accident 150 km from station A. It completed the remaining journey at $\frac{5}{6}$ of the previous speed and reached 15 min late at station B. Had the accident taken place 30 km further, it would have been only 7 min late. Find the speed of the train and the distance between the two stations A and B.

Solution:

Equation Method : Let distance between site of accident and station B be x km and let the usual speed of the train be s kmph.

$$\text{Usual time taken from Station A to Station B} = \frac{150+x}{s};$$

$$\text{But because of accident time taken} = \frac{150}{s} + \frac{x}{\frac{5}{6}s}$$

$$\text{Had accident occurred 30 km further, time taken} = \frac{180}{s} + \frac{x-30}{\frac{5}{6}s};$$

We know that

$$\left(\frac{150}{s} + \frac{x}{\frac{5}{6}s} \right) - \left(\frac{150+x}{s} \right) = \frac{15}{60} \dots (i)$$

and

$$\left(\frac{180}{s} + \frac{x-30}{\frac{5}{6}s} \right) - \left(\frac{150+x}{s} \right) = \frac{7}{60} \dots (ii)$$

Subtracting (ii) from (i), we have,

$$\left(\frac{30}{\frac{5}{6}s} \right) - \left(\frac{30}{s} \right) = \frac{8}{60} \Rightarrow \frac{36}{s} - \frac{30}{s} = \frac{8}{60}$$

$$\Rightarrow \frac{6}{s} = \frac{2}{15} \Rightarrow s = 45 \text{ kmph}$$

Substituting s in either (i) or (ii), x can be found out.

This problem is a classic case of why students find TSD difficult, as it involves cumbersome equation. However please read through the following solution to realise why you must strive to use ratio and proportion and not equations and then most questions of TSD will become oral questions.

R & P method :

The above problem can be distilled to just the following two facts :

1. Travelling at $\frac{5}{6}$ th of its speed from the site of accident to station B, the train is late by 15 minutes.
2. The train takes 8 minutes less to cover a distance of 30 kms, if it travels at its usual speed rather than $\frac{5}{6}$ th of its speed.

Hope point 2 is clear. The only difference when the train is late by 15 minutes and when the train is late by 7 minutes is the stretch of 30 kms on which the train travels at its usual speed in latter case and at $\frac{5}{6}$ th of its speed in former case. Thus 8 more minutes are taken to cover the 30 kms when train travels at $\frac{5}{6}$ th speed.

Since train travels at $\frac{5}{6}$ th speed, time taken will be $\frac{6}{5}$ th of former time i.e. time taken will be

$\frac{1}{5}$ th more than former time. But we know that time taken is 8 minutes more. Thus former time taken will be 40 minutes to cover this 30 kms. Thus usual speed is 45 kmph.

Consider point 1, from site of accident to station B, because speed is $\frac{5}{6}$ th, time taken will be $\frac{1}{5}$ th more than the usual time and this increase in time is 15 minutes. Thus usual time is 75 minutes. And at a speed of 45kmph, the distance between accident site and station B will be

$$45 \times \frac{75}{60} = 56.25 \text{ km.}$$

Thus total distance between A and B = $150 + 56.25 = 206.25 \text{ km}$

Example 13:

Mr. Ghosh Babu leaves his house daily at a particular time to go to his office. He reaches office 10 minutes late when he travels at 20 kmph and 5 minutes early when he travels at 30 kmph. At what speed should he travel to reach office on time?

Solution:

Lets assume his usual speed is 20 kmph and new speed is 30 kmph. Thus ratio of usual speed to new speed is $2 : 3$. Thus ratio of usual time taken to new time taken is $3 : 2$ and we know the difference in these times is 15 minutes. Thus time taken at 20 kmph = 45 minute and with this speed he reaches late by 10 minutes. Thus he should take 35 minutes to reach office and the speed should be $20 \times \frac{45}{35} = \frac{180}{7} = 25.71 \text{ kmph.}$

Example 14:

A and B start from point X towards Y and C from point Y towards point X simultaneously at 9 am. If the ratio of speeds of A, B and C is 5 : 3 : 4 and A meets C at 10 am, at what time will A, B and C reach their destination?

Solution:

Let the speeds of A, B and C be $5k$, $3k$ and $4k$ respectively. In 1 hour, A travels $5k$ and C travels $4k$ and since they meet starting from opposite ends, we can say that the distance XY is $5k + 4k = 9k$.

Thus time taken by A to reach destination is $\frac{9k}{5k} = 1.8$ hours i.e. 1 hour 48 minutes

Time taken by B to reach destination is $\frac{9k}{3k} = 3$ hours

And time taken by C to reach destination is $\frac{9k}{4k} = 2.25$ hours i.e. 2 hours 15 minutes.

A, B and C will reach their destinations at 10:48 am, 12 noon and 11:15 am respectively.

Example 15:

A and B start swimming simultaneously from opposite ends, X and Y respectively of a swimming pool of length 200 mts. A crosses B, reaches the opposite end Y and turns back immediately and again meets B at a distance of 120 mts from X. Find the ratio of the speeds of A and B if

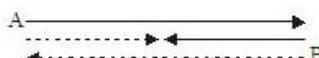
- B has not yet reached the opposite end even once
- B has reached opposite end once and turned back.

Solution:

Since A and B start simultaneously, when they meet, they would have swum for equal amounts of time and thus speed is going to be directly proportional to the distance.

Case i.

When they meet for the second time, A has swum $200 + (200-120) = 280$ mts and B has swum 80 mts and thus the ratio of the speeds is $280 : 80$ i.e. $7 : 2$.

Case ii.

A has swum $200 + (200-120) = 280$ mts and B has swum $200 + 120 = 320$ mts and thus ratio of speed is $280 : 320 = 7 : 8$.

Example 16:

A overtakes (i.e. in the same direction) B at point X at 9 am. A reaches point Y at 10 am and immediately turns back and again meets B at point Z at 10:30 am.

- What is the ratio of speeds of A and B?
- At what time will B reach Y?

Solution:

If distance YZ is k , then distance XY is $2k$. Why? Time taken by A to travel YZ is 30 minutes and to travel XY is 60 minutes. Since his speed is constant, distance is directly proportional to time taken, hence.

Thus in the time in which A travels $2k + k = 3k$, B just travels k and thus ratio of speed of A to speed of B is $3 : 1$.

In 90 minutes (from 9 to 10:30), B travels a distance of k and thus would take a further 90 minutes to reach Y and thus would reach at 12 noon.

Example 17:

In order to travel a certain distance, when the speed reduces by 2 m/s, time taken increases by 10 s and when the speed is reduced by 3 m/s, the time increases by 18 s. Find his normal speed, time and distance.

Solution:

Let V , T and D be the original speed, time and distance respectively.

$$\text{Then } D = VT = (V - 2)(T + 10) = (V - 3)(T + 18)$$

$$\text{Now } VT = (V - 2)(T + 10)$$

$$= VT + 10V - 2T - 20 \text{ or } 5V - T = 10 \dots (\text{i})$$

Similarly,

$$VT = (V - 3)(T + 18) = VT + 18V - 3T - 54$$

$$= 18V - 3T = 54$$

$$6V - T = 18 \dots (\text{ii})$$

Using (i) and (ii), we get $V = 8$ m/s and $T = 30$ s.

Hence, $D = 240$ m.

Example 18:

Consider a square ABCD. X starts from A and travels continuously along the path ABDCA and Y starts from B and continuously moves along path BCADB. If speed of X and Y are equal, how many times would they have met when X has completed 10 laps and reached the starting point A again?

Solution:

When X reaches B, Y would have reached C. Now both would move along the diagonal and would meet at the intersection of the diagonals. Beyond this when X reaches D, Y reaches A. Further when X reaches C, Y would reach D and again after this they would move along the diagonals and would once again meet at the intersection of diagonals. After this they would reach the starting points once again. Thus in every one lap, they meet twice. Thus in 10 laps they would have met 20 times.

Example 19:

A and B start from opposite ends X and Y towards Y and X respectively. They meet on the way at Z and if A and B take 36 and 25 minutes respectively after meeting to reach their ends, what is the ratio of speeds of A and B?

Solution:

This problem can be a landmark problem and thus is useful if we get a standard result for such a problem, which can be built-on later for a more complex problem.

Let t_a and t_b be the time taken by A and B respectively after the meeting to reach their respective ends. Thus in this case $t_a = 36$ and $t_b = 25$.

If A travels distance a and B travels distance b till their meeting, since they started simultaneously, we have $\frac{s_a}{s_b} = \frac{a}{b}$ where s_a and s_b are speed of A and B respectively.

Also after meeting A and B will have to travel distances b and a respectively to reach their ends. Thus we have $t_a = \frac{b}{s_a}$ and $t_b = \frac{a}{s_b}$ i.e. $b = t_a \times s_a$ and $a = t_b \times s_b$.

$$\text{Thus we have } \frac{a}{b} = \frac{t_b \times s_b}{t_a \times s_a}.$$

Substituting in earlier equation we have

$$\frac{s_a}{s_b} = \frac{t_b \times s_b}{t_a \times s_a} \text{ giving us the relation } \frac{s_a}{s_b} = \sqrt{\frac{t_b}{t_a}}.$$

Please note this relation is to be used only when the two persons start simultaneously and the time given is the time taken after meeting to reach respective ends.

$$\text{Thus in this problem ratio of speed of A and B} = \sqrt{\frac{25}{36}} = \frac{5}{6}.$$

Example 20:

A man starts from L to M, another from M to L at the same time. After passing each other, they complete their journey in $3\frac{1}{3}$ and $4\frac{4}{5}$ hr respectively. Find the speed of the second man if the speed of the first is 24 km/hr.

Solution:

$s_1 : s_2 = \sqrt{\frac{b}{a}}$, where a and b are the time taken by 1st man and 2nd man respectively after meeting each other.

$$\sqrt{\left[\frac{24}{5} \times \frac{3}{10} \right]} = \frac{s_1}{s_2} = \sqrt{\left[\frac{36}{25} \right]} = \frac{6}{5}$$

$$\text{Thus, 2nd man's speed} = \frac{5}{6} \times 24 = 20 \text{ km/hr.}$$

Example 21:

In a race of 1 km, A can beat B by 100 mts or by 5 sec. What is speed of A?

Solution:

To revise the understanding of "in a game of n points, A can beat B by m points", revisit the chapter of ratio and proportion.

When A runs 1000 mts, B is 100 mts behind. To cover this 100 mts, B takes 5 seconds and thus speed of B is 20 m/s. Since the ratio of speed of A and B is 10 : 9, A's speed = $200/9 = 22.22$ m/s

Average Speed :

If a person completes a journey travelling stretches at different speeds, the average speed over the entire journey is not necessarily equal to the average of the speeds. We have already seen this in the chapter on weighted averages. What follows here is just a revision.

The average speed is defined as the total distance travelled divided by the total time taken. Suppose a person travels half the distance at a speed of 30 kmph and remaining half the distance at 40 kmph. If the total distance was 2D, the average speed is given by

$$\frac{2D}{\frac{D}{30} + \frac{D}{40}} = \frac{2 \times 30 \times 40}{30 + 40} = 34.28 \text{ kmph and we see that it is not equal to 35 kmph.}$$

However if a person travels for half the time at a speed of 30 kmph and remaining half the time at speed of 40 kmph, the average speed is $\frac{30 \times t + 40 \times t}{2t} = \frac{30 + 40}{2} = 35$ kmph i.e. the average of the speeds.

To understand the difference in the two cases, just recollect that the average speed is the weighted average with the time traveled at the different speeds being the weight and not the distance travelled at the different speeds.

If equal distances are travelled at a speed of u and v , the average speed boils down to $\frac{2uv}{u+v}$.

Example 22:

What is the average speed if a man travels $1/2$ the distance at speed of 40 kmph, $1/3$ the distance at 50 kmph and rest of the distance at 60 kmph?

Solution :

If the total distance is $6D$, the average speed will be

$$\frac{\frac{6D}{40} + \frac{2D}{50} + \frac{D}{60}}{\frac{3D}{40} + \frac{2D}{50} + \frac{D}{60}} = \frac{6D \times 600}{45D + 24D + 10D} = \frac{3600}{79} = 45.57 \text{ kmph}$$

Example 23:

What is the average speed if a man travels $1/2$ the time at speed of 40 kmph, $1/3$ the time at 50 kmph and rest of the time at 60 kmph?

Solution :

In this case the average speed will be the weighted average

$$\frac{1}{2} \times 40 + \frac{1}{3} \times 50 + \frac{1}{6} \times 60 = 46.66 \text{ kmph.}$$

Example 24:

If I travel along the three sides of an equilateral triangle at speeds of 30 kmph, 40 kmph and 50 kmph respectively, what is my average speed for the entire journey?

Solution:

Just to revise, average speed is a weighted average speed (with weights equal to the time and not distance) and defined as total distance travelled divided by total time taken. And it is not necessarily equal to the average of the speeds. To revise the fundamentals of average speed, please visit the chapter on mixtures and the topic on unusual mixtures.

$$\begin{aligned}\text{Average speed} &= \frac{d}{\frac{d}{30} + \frac{d}{40} + \frac{d}{50}} \\ &= \frac{3 \times 30 \times 40 \times 50}{40 \times 50 + 30 \times 50 + 30 \times 40} = \frac{3 \times 30 \times 40 \times 50}{2000 + 1500 + 1200} \\ &= \frac{3 \times 30 \times 40 \times 50}{4700} = \frac{1800}{47} = 38.3 \text{ kmph}\end{aligned}$$

Relative Speed

Relative speed is a phenomena that we observe daily whenever two objects are moving simultaneously.

Suppose you are travelling in a train and there is a second train coming in the opposite direction on a parallel track. It seems the second train is moving much faster than it is actually.

On the other hand if you are travelling on the Mumbai Pune expressway at 100 kmph in your car and there is another car moving besides you, also moving at 100 kmph in the same direction. The cars when seen from the other car seem stationary!

Seeing from the window of a running train, the train appears stationary and the trees appear to be moving in the opposite direction.

All these are examples of relative speed.

Consider two friends Tango and Cash separated by 500 mts. Tango can run at a speed of 3 m/s and Cash can run at a speed of 2 m/s. If they move towards each other, after what time will they meet?

Lets see what happens after 1 second. In one second, Tango would have covered 3 mts and Cash would have covered 2 mts and thus the distance separating them would have reduced by 5 mts. Thus the distance between them would be reducing at 5 meters per sec when they move towards each other. This 5 m/s is called the relative speed of Tango or Cash with respect to the other.

Thus for the entire 500 mts to disappear between them, it would take 100 seconds. In these 100 seconds, Tango has covered 300 mts and Cash 200 mts and both would have met. Please note that 500 mts is the distance covered by both of them together and not by any one of them. Consequently the speed to be taken is the relative speed and not either's speed.

Once they have met, if they yet keep running, in how much time after meeting would a distance of 100 mts separate them?

After 100 second, both, Tango and Cash are together. If they keep running, after 1 sec, they would be separated by a distance of 5 mts (as Tango would have run 3 mts and Cash would have run 2 mts in opposite direction). Thus the relative speed still is 5 m/s and has to be so because their individual speed and direction of running has not changed.

Thus time till they are separated by 100 mts, at the rate of 5 m/s will be 20 seconds.

If two objects are moving in opposite direction (either towards each other or away from each other), each with a speed of U and V respectively, the relative speed of either of them with respect to other is $U + V$

Take the initial position again that of Tango and Cash being separated by 500 mts. If Tango starts chasing Cash i.e. both run in same direction with Tango behind Cash, how

much time would Tango take to catch Cash?

In one second, Tango would run 3 mts closer to Cash but in the same time, Cash would have run 2 mts away and thus in each second, Tango can just get closer by a net 1 mt. This 1 m/s is the relative speed in this case. Thus every second Tango would come closer by 1 meter and to come catch Cash he has to come closer by 500 mts and would thus take 500 seconds.

In 500 seconds, Cash would have run 1000 mts and Tango would have run 1500 mts. Thus Tango would have run 500 mts more than Cash and would thus have covered the initial distance separating them.

If two objects are moving in same direction, each with a speed of U and V respectively, the relative speed of either of them with respect to other is $U - V$ or $V - U$.

Example 25:

During the shooting of a certain movie, Salman is separated from Madhuri by 500 m and is supposed to take Madhuri up in his arms. Madhuri starts running away from Salman at 1.5 m/s, as soon as Salman starts off. If Salman has to catch up with her in 50 s, what should be the speed of Salman?

Solution:

$$\text{Relative speed} = \frac{\text{Distance to be covered}}{\text{Time taken to catch up}} = \frac{500}{50} = 10 \text{ m/s}$$

So Salman's relative speed with respect to Madhuri = 10 m/s.

Hence, Salman's speed = $10 + 1.5 = 11.5$ m/s. (Since he is travelling in the same direction.)

Example 26:

In the movie *Ghulam*, Aamir is able to spot the train when it is 2 km away coming towards him. The train is moving at a constant speed of 36 km/hr. The red handkerchief

is near a pole 400 m away from Aamir in the same direction in which the train is travelling. How fast must Aamir run in order to reach the pole and just be saved from being over run?

Solution:

If he moves at a speed of v m/s, then time taken by him to reach the pole = $\left(\frac{400}{v}\right)$ seconds.

Now in the same time, the train must reach the pole which is $2000 + 400 = 2400$ m away from where it is now. Therefore, the train takes $\frac{2400}{10} = 240$ s to reach the pole.

($36 \text{ km/hr} = 10 \text{ m/s}$) Therefore, in 240 s, Aamir has to cover 400 m distance.

$$\text{So Aamir's speed } v = \frac{400}{240} = \frac{5}{3} \text{ m/s or } 6 \text{ km/hr.}$$

Example 27:

A and B start from X and Y respectively at 9 am and travel towards Y and X respectively at uniform speed. If A reaches Y at 1 pm and B reaches X at 3 pm on same day, then at what time do they meet.

Solution:

Let the distance XY be d . Thus A takes 4 hours (9 am to 1 pm) and B takes 6 hours (9 am to 3 pm) to travel this distance. Thus their speed is $\frac{d}{4}$ and $\frac{d}{6}$ respectively. Thus the problem is now that two friends are separated by distance d and move towards each other at speeds of $\frac{d}{4}$ and $\frac{d}{6}$. After how much time will they meet.

And the answer is they would meet after $\frac{d}{\frac{d}{4} + \frac{d}{6}} = \frac{6 \times 4}{6 + 4} = 2.4$ hours i.e. at 11:24 am (and not 11:40).

Example 28:

In the above example if A left X at 9 am and reached Y at 1 pm and if B left Y at 10 am and reached at X 4 pm, when would they meet?

Solution:

The problem is different just because the starting times is not same and thus we cannot add their speeds to get the relative speed as for the first hour only A is moving and not B. So the solution is to account for the first hour separately and thus make the starting time same.

Again if XY = d , speed of A and B will be $\frac{d}{4}$ and $\frac{d}{6}$ respectively. Thus from 9 am to 10 am,

A would have travelled $\frac{d}{4}$ and thus at 10 am distance separating A and B will be just

$$d - \frac{d}{4} = \frac{3d}{4}. \text{ The time taken to meet from 10 am will thus be } \frac{\frac{3}{4}d}{\left(\frac{d}{4} + \frac{d}{6}\right)} = \frac{3}{4} \times \left(\frac{6 \times 4}{6 + 4}\right) = 1.8$$

hours i.e. they would meet at 11:48.

If you do not want to work with fractions, you could just assume the distance between XY as the product of the time taken i.e. 24 kms in this case. Thus A's speed = 6 kmph and B's speed = 4 kmph. From 9 am to 10 am, A would cover 6 km and thus distance separating them would be just $24 - 6 = 18$ kms and the relative speed will be $6 + 4 = 10$ kmph. Thus they would take $18/10 = 1.8$ hours after 10 am.

Example 29:

A thief escapes from police station A in a jeep and starts driving towards B at a speed of 60 kmph. The police realise the escape after 2 hours and immediately call up the police station at B (400 kms from A) to send a team towards A to nab the thief on way. The team from B travel at speed of 40 kmph. At the same time a team from A leaves for B with a speed of 80 kmph. At what distance from A will the thief be caught? By which team? And after how many hours will the other team reach the spot of capture?

Solution:

When the teams start the chase the thief is at 120 kms from A and 280 kms from B.

The team from A will catch the thief after $\frac{120}{80 - 60} = 6$ hrs whereas the team from B would catch him after $\frac{280}{40 + 60} = 2.8$ hours. Thus the team from B will catch the thief first at a distance of $40 \times 2.8 = 112$ kms from B i.e. 288 kms from A. The team from A will take $\frac{288}{80} = 3.6$ hours to reach the spot i.e. will arrive 0.8 hours i.e. 48 minutes later.

Example 30:

A thief is spotted by a policeman at a distance of 200 m. If the speed of the thief be 10 km/hr and that of the policeman be 12 km/hr, at what distance will the policeman catch the thief?

Solution:

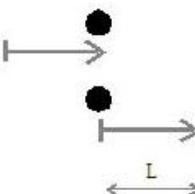
Relative speed of the policeman with respect to the thief = 2 km/hr.

Time taken by the policeman to cover the relative distance of 200 m = $\frac{200}{1000} \times \frac{1}{2}$ hr. = $\frac{1}{10}$ hr

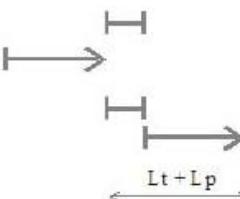
In this time, the policeman covers a distance = $12 \times \frac{1}{10} = 1.2$ km.

Problems on Trains:

Consider that we need to measure the time taken by the train to cross a pole. The stop-watch will be started when the engine will just reach the pole and the stop-watch will be stopped when the guard just clears the pole. Thus in the time taken to cross the pole, the train has travelled a distance equal to its length.



Similarly the following pictures will make it clear that if the object being crossed also has a length (say a bridge or a platform or another train), the distance covered in the time taken to cross will be equal to the length of the train + length of the object being crossed.



However the object being crossed, itself may be moving! Say a train crossing a walking man. Or a train crossing another moving train. In these cases, we have to consider the relative speed. Just to remind - when two objects are moving the relative speed is the sum of the speeds if moving in opposite direction or the difference in the speeds if moving in same direction.

Thus we can come with a general formula which can be useful in any case (the object being crossed having a length or not having and also if it is moving or not moving) as

$$\text{Time taken to cross} = \frac{\text{Sum of lengths of the two objects}}{\text{Relative speed}}$$

Example 31:

A train of length 500 m travelling at 72 km/hr approaches a man standing on the platform. How much time does it take to completely cross him?

Solution:

The distance to be travelled by the train in order to cross the man is the length of the train itself. Hence, the problem boils down to find the time taken by a train moving at 72 km/hr to travel a distance of 500 m. ($72 \text{ km/hr} = 20 \text{ m/s}$)

$$\text{Therefore, time to cross the man} = \frac{(500 \text{ m})}{(20 \text{ m/s})} = 25 \text{ s.}$$

All types of variations to this question can be inserted but the essential idea remains the same.

Example 32:

Refer to example 31, if the man also starts moving in the same direction as the train, at a speed of 10.8 km/hr, then what is the time taken by the train to cross the man?

Solution:

$$\text{Speed of the man} = 10.8 \text{ km/hr} = 3 \text{ m/s.}$$

$$\text{The relative speed of the train} = 20 - 3 = 17 \text{ m/s.}$$

$$\text{Time taken by the train to cross the man} = \frac{500}{17} \text{ s} = 29.41 \text{ s.}$$

Example 33:

In the previous problem, if the man moves in the direction opposite to that of the train, what is the time taken by the train to cross the man?

Solution:

The relative speed will change to $(20 + 3) \text{ m/s} = 23 \text{ m/s.}$

$$\text{The time taken to cross the man} = \frac{500}{23} = 21\frac{17}{23} \text{ s.}$$

For any two objects crossing each other the same method is applicable.

Example 34:

If the length of a train and a platform are 500 m and 700 m respectively, find the time taken by the train moving at 54 km/hr to cross the platform.

Solution:

The total distance that the train needs to travel = Length of the train + Length of the platform.

$$\text{Hence, the time taken by the train} = \left\{ \frac{(500 + 700)}{15} \right\} \text{ s} = 80 \text{ s.} [\text{Since } 54 \text{ km/hr} = 15 \text{ m/s}]$$

Example 35:

Two trains of lengths 400 m and 600 m have the speeds of 36 km/hr and 54 km/hr respectively. In what time will the trains be able to completely pass each other, given that the trains are moving in

- a. the same direction with the faster train approaching the slower,
- b. the opposite direction?

Solution:

a. When the trains are moving in the same direction, the faster train crosses the slower when the tail of the faster just passes the front end of the slower train.

Hence, the distance that the faster train has to cover before crossing the slower train

$$= \text{Sum of the lengths of the trains} = (400 + 600) \text{ m} = 1000 \text{ m.}$$

Relative speed of the faster train with respect to the slower

$$= (54 - 36) \text{ km/hr} = 18 \text{ km/hr} = 5 \text{ m/s.}$$

$$\text{Therefore, time taken for the faster train to cross the slower} = \frac{1000}{5} \text{ s} = 200 \text{ s.}$$

b. The distance travelled by the trains together = Sum of the lengths of the two trains
 $= 400 + 600 = 1000 \text{ m.}$

$$\text{Relative speed of the trains} = (54 + 36) \text{ km/hr} = 90 \text{ km/hr} = 25 \text{ m/s.}$$

$$\text{Therefore, time taken} = \frac{1000}{25} = 40 \text{ s.}$$

Example 36:

A train running at 54 km/hr takes 20 s to cross a platform and 12 s to pass a man walking in the same direction at a speed of 6 km/hr. Find the length of the train and the platform?

Solution:

Let the length of the train = x meters.

Let the length of the platform = y meters.

$$\text{Speed of the train relative to the man} = 48 \text{ km/hr} = \frac{40}{3} \text{ m/s}$$

In passing the man, the train covers its own length with relative speed.

$$\text{Length of train } x = \frac{40}{3} \times 12 = 160 \text{ m.}$$

$$\text{Speed of train} = 54 \text{ km/hr} = 15 \text{ m/s}$$

$$\therefore \frac{x+y}{15} = 20 \text{ or } x+y = 300 \text{ or, } y = 140$$

$$\text{Length of the platform} = 140 \text{ m.}$$

Problems on Boats and Streams:

If B is the speed of boat in still water and R is the speed of any stream or river, if the boat is rowed in the stream, its speed will no longer remain B . For e.g. if $B = 0$, when the boat is on a river or stream, it's speed does not remain 0 but it will move along with the speed of the river. The boat is said to travel Downstream if it is being rowed in the direction of the stream and the boat is said to travel Upstream if it is being rowed against the direction of the stream. If U and D are the upstream and the downstream speeds, then we have

$$D = B + R \text{ and } U = B - R.$$

Further, by adding and subtracting these equations we get,

$$B = \frac{D+U}{2} \text{ and } R = \frac{D-U}{2}$$

Example 37:

The speed of a boat in still water is 10 m/s and the speed of the stream is 6 m/s. If the boat is moving upstream and again downstream, what is the ratio of the time taken to cover a particular stretch of distance in each direction?

Solution:

Method 1: Since the distances travelled in each direction is the same, and the effective speeds are in the ratio $(10 - 6) : (10 + 6) = 1 : 4$, the time taken to travel upstream and downstream would be in the ratio $\frac{1}{1} : \frac{1}{4} = 4 : 1$.

Method 2: Let the distance travelled in one direction be x .

$$\text{Time taken for covering the distance upstream} = \left(\frac{x}{10-6} \right) = \frac{x}{4}.$$

$$\text{Time taken for covering the same distance downstream} = \left(\frac{x}{10+6} \right) = \frac{x}{16}.$$

$$\text{Hence, the ratio of the times} = \frac{x}{4} : \frac{x}{16} = 4 : 1.$$

Example 38:

A boat takes a total of 6 hours to row 8 kms downstream and to return back to the starting point. If speed of the boat is 3 kmph, for how much time was the boat moving downstream and for how much time upstream?

Solution:

$$\text{If } s \text{ is the speed of the stream, we have } \frac{8}{3+s} + \frac{8}{3-s} = 6 \Rightarrow \frac{24 - 8s + 24 + 8s}{9 - s^2} = 6 \\ \Rightarrow 48 = 54 - 6s^2 \Rightarrow 6s^2 = 6 \Rightarrow s = 1 \text{ kmph}$$

Thus time taken downstream = $8/4 = 2$ hours and time taken upstream = $8/2 = 4$ hours.

Example 39:

A boatman rows for 3 hours downstream and then for 3 hours upstream. In this whole process he covers a total distance of 12 kms. If the speed of stream is 1 kmph, for how much more time will he have to row upstream to reach the starting point?

Solution:

Knowing that speed downstream is $B+R$ and speed upstream is $B - R$, and time spent is 3 hours both way, we have

$$3(B+R) + 3(B-R) = 12 \Rightarrow 6B = 12 \Rightarrow B = 2 \text{ kmph}$$

Also distance left to be covered to reach starting point is $3(B+R) - 3(B-R) = 6R = 6 \text{ km}$

Thus time taken to reach the top

$$= \frac{6}{B-R} = \frac{6}{1} = 6 \text{ hours.}$$

Example 40:

A man rows 27 km downstream and 18 km upstream taking 3 hr each time. What is the speed of the current?

Solution:

$$\text{Speed downstream} = \frac{27}{3} = 9 \text{ km/hr}$$

$$\text{Speed upstream} = \frac{18}{3} = 6 \text{ km/hr}$$

$$\text{Velocity of current} = 0.5(9 - 6) = 1.5 \text{ km/hr}$$

Example 41:

A man can row upstream at 7 km/hr and downstream at 10 km/hr. Find his rate in still waters and the rate of the current.

Solution:

$$\text{Rate in still waters} = 0.5(10 + 7) = 8.5 \text{ km/hr}$$

Rate of current = $0.5 (10 - 7) = 1.5 \text{ km/hr}$

Circular Motion

Students find problems on circular motion difficult as it is not possible to visualise the scenario to a great extent. However as soon as one understand the relative distance to be covered for two joggers running on a circular track (either in same direction or in opposite direction), the problems become as simple as linear motion. So just spend some quality time to read the following and you would never have to worry about circular motion again.

Consider A and B, participating in the Grand Prix race, are at the same starting point on a circular track of length (circumference) 500 m. A drives at a speed of 5 m/s whereas B drives at a speed of 3 m/s. If they start simultaneously, after what time will they meet for the first time.

Since it is a race, they would be running in the same direction.

On a circular track, when running in the same direction, every time the faster one covers one entire round more than the slower one, they meet. Or in other words, for them to meet, the faster one has to cover one entire round more than the slower one.

Lets understand this thoroughly as this is the place, most student finds a problem.

If we freeze the situation just a fraction of a second after it has started, we will see that A has taken a minute lead over B. Now, looking from A's view point, we see that for A to catch up with B, he sees B one full round away from him. Thus it is just like the linear case of two friends who are separated by a distance equal to the circumference and moving in same direction. Thus time taken for them to meet = $\frac{\text{distance i.e. circumference}}{\text{relative speed}}$

Lets look at it from another view-point. We know that when A has completed 5 rounds, B would have completed 3 rounds. Thus A would have been at the starting point and so also would B i.e. they would be together. But this is the second time ($5 - 3 = 2$) that they would

be together after start and not the first time. Consider when A has completed 2.5 rounds. B would have completed 1.5 rounds. Thus both of them would have been at the point diametrically opposite to the starting point and thus both would have met even then. Again in this situation we see than A has run $2.5 - 1.5 = 1$ full round more than B. Before this, A would always have run something less than one full round more than B and thus wherever B is, A would have been less than a round ahead of him and thus would never be along with B. Thus A and B meet for the first time when A has run one full round more than B.

Thus, with the values given A and B would meet for the first time after $\frac{500}{5 - 3} = 250 \text{ sec}$

When would A and B be together at the starting point for the first time? This is a question more on LCM rather than TSD. A would be at the start after every 100 sec and B would be at the start after every $\frac{500}{3}$ sec. Thus they would be together at the start for the first time at the LCM of 100 and $\frac{500}{3}$ i.e. 500 sec.

What if A and B would have been running in opposite direction? This case is more easy to understand than the case of same direction. When running in opposite direction, they would meet when together they have covered the entire circle i.e. after distance i.e. circumference.

relative speed

In this case the relative speed would be the sum of the speed as they are running in opposite direction.

Consider the following problems to consolidate the learnings :

A can run one full round of a circular track in 6 minutes and B can run one full round of the circular track in 15 minutes. If both A and B start simultaneously from the same starting point, find

1. when would they meet at the starting point for the first time
2. when would they meet for the first time if they are running in same direction
3. when would they meet for the first time if they are running in opposite directions
4. how many time would they have met in the time B has completed 10 rounds when running in same direction
5. how many time would they have met in the time B has completed 10 rounds when running in opposite direction
1. This is the straightforward case of meeting at the starting point for the first time which is equal to the LCM of 6 and 15 i.e. 30 minutes, irrespective of the directions they are running.

2 & 3.**Method 1:**

If d was the track length, speed of A would have been $\frac{d}{6}$ and that of B would have been $\frac{d}{15}$.

Thus they would meet after $\frac{d}{\frac{d}{6} - \frac{d}{15}} = \frac{15 \times 6}{15 - 6} = 10$ min if running in same direction and after

$\frac{d}{\frac{d}{6} + \frac{d}{15}} = \frac{15 \times 6}{15 + 6} = 4 \frac{2}{7}$ minutes if running in opposite direction

Method 2:

In 30 minutes (LCM of 6 & 15), A would have run 5 rounds and B would have run 2 rounds.

When running in same direction this would mean A having run $5 - 2 = 3$ rounds more than B. Thus to run 1 round more than B, A would have taken just $\frac{30}{3} = 10$ minutes.

When running in opposite direction, this would mean A and B together having run $5 + 2 = 7$ rounds. To run 1 round together, they would take $\frac{30}{7} = 4 \frac{2}{7}$ minutes.

4 & 5.

When B has completed 10 rounds, A would have completed $10 \times \frac{15}{6} = 25$ rounds.

When running in same direction, this would mean A having run 15 rounds more than B and thus would have met 15 times (For every one round that A runs more than B, A meets B)

When running in opposite direction, this would mean A & B together having run 35 rounds and thus would have met 35 times.

Example 42:

Laxman joins Saurav and Sachin and all of them run on a circular track of length 500 m in the same direction from the same point simultaneously. Laxman moves at 3 km/hr, Sachin at 5 km/hr and Saurav at 8 km/hr. When will they all be together again

a. for the first time,

b. for the first time at the starting point?

Solution:

a. Break the problem into two separate cases.

In the first case, Saurav moves at a relative speed of $(8 - 5) = 3$ km/hr with respect to Sachin. At a relative speed of 3 km/hr, he would meet Sachin after every $\frac{500}{3 \times \frac{5}{18}} = 600$ s = 10 min

In the second case, Saurav moves at $(8 - 3)$ km/hr = 5 km/hr with respect to Laxman.

At a relative speed of 5 km/hr, he would meet Laxman after every $\frac{500}{5 \times \frac{5}{18}} = 360$ s = 6 min

Therefore, if all the three have to meet, they would meet after every $\text{LCM}(10, 6) = 30$ min or $\frac{1}{2}$ hr. Hence, they would all meet for the first time after 30 min.

b. If we need to find the time after which all of them would be at the starting point simultaneously for the first time, we should use the same method as in the case involving two people.

At a speed of 8 km/hr, Saurav takes 225 s to complete one circle.

At a speed of 5 km/hr, Sachin takes 360 s to complete one circle.

At a speed of 3 km/hr, Laxman would take 600 s to complete one circle.

Hence, they would meet for the first time at the starting point after $\text{LCM}(225, 360, 600) = 1800$ s = 30 min.

Problems on Clocks

Concept

Clocks follow the principle of relative speed. Take the instance of a typical clock. There are two hands in the clock: the hour and the minute hand. Both are moving in the same direction. The hour hand moves $\left(\frac{1}{2}\right)^\circ$ per minute, whereas the minute hand moves 6° per minute. The minute hand is constantly chasing the hour hand. The relative speed of the minute hand with respect to the hour hand is $5\frac{1}{2}^\circ$ per minute.

As we know the relative speed of hour hand and minute hand, we can calculate the frequency with which both of them coincide continuously.

The relative speed of minute hand with respect to hour hand is $5\frac{1}{2}^\circ$ per minute.

Difference of $5\frac{1}{2}^\circ$ comes in 1 min. So the difference of 360° will come in $\frac{360}{5\frac{1}{2}}$ min = $\frac{720}{11}$ min

= $65\frac{5}{11}$ min, and in a day, hour and minute hands coincide 22 times.

$$\left[\because \frac{\text{Total minutes in a day}}{\text{Time (in minutes for one coincidence)}} = \frac{24 \times 60}{\frac{720}{11}} = 22 \text{ times} \right]$$

The types of questions that you would encounter would be as follows.

- At a particular time what is the angle between the hands of a clock?
- When do the hands of a clock coincide or make some angle between t and $(t + 1)$ hr?
- Problems where two different clocks gain or lose certain time in a particular time period.

Example 43:

At 3:45, what is the (acute) angle between the hands of a clock?

Solution:

At 3 o' clock, the minute hand of a clock would be 90° behind the hour hand.

In 45 min, the minute hand of a clock would move $(45^\circ \times 6) = 270^\circ$ forward.

The hour hand would move $\left(\frac{1}{2} \times 45^\circ\right) = \left(22\frac{1}{2}^\circ\right)$ forward.

Hence, the angle between the hands would be $270^\circ - (90^\circ + 22\frac{1}{2}^\circ) = 157\frac{1}{2}^\circ$.

Example 44:

When do the hands of a clock coincide between 4 and 5?

Solution:

At 4 the minute hand is $4 \times 30^\circ = 120^\circ$ behind the hour hand.

The minute hand takes a lead of $5\left(\frac{1}{2}\right)^\circ$ every minute over the hour hand.

The time it takes to catch up $120^\circ = \frac{120}{11/2} = \frac{240}{11}$ min after 4.

This is when they would coincide.

Example 45:

Two clocks show the same time at 4 p.m. The first clock loses 10 min every 2 hr and the second gains 10 min every hour. When will they both show the same time again?

Solution:

If they had not lost or gained any time, they would both show the same time always. But in this case, the first clock would be behind the second clock by 15 min at the end of 1 hr.

(Since the first clock loses 5 min and the second gains 10 min in 1 hr) They would both show the same time again if they are separated by $12 \text{ hr} = 12 \times 60 \text{ min}$.

Number of hours the first clock takes to be behind the second by 12 hr = $\frac{(12 \times 60)}{15} = 48 \text{ hr}$.

So they would show the same time again after exactly 2 days.

Work

Work problems are similar to TSD problems. Speed is nothing but rate of doing work, distance is the amount of work done and time taken remains as the time taken for the work to be done.

Thus time taken for work to be done = $\frac{\text{Amount of work}}{\text{Rate of doing work}}$

When two or more persons work on the same job, the rate of doing work gets added.

Thus, if Raju and Rina can do a job in 10 days and 15 days independently, how many days would they take to complete the same job working simultaneously?

If total work is W, Raju's rate of working = $\frac{W}{10}$ per day and that of Rina = $\frac{W}{15}$ per day.

Thus when working simultaneously, rate of work done = $\frac{W}{10} + \frac{W}{15}$ and thus time taken

$$= \frac{\frac{W}{10} + \frac{W}{15}}{\frac{W}{10} + \frac{W}{15}} = \frac{15 \times 10}{15 + 10} = 6 \text{ days}$$

In above problem we can work assuming work to be just 1 unit and thus eliminating use of W.

Alternately, assume the total work to be the LCM of the days taken individually i.e LCM of 10 & 15 i.e. 30 units of work.

Thus Raju's rate of working = 3 units per day and Rani's rate of working = 2 units per day. When working simultaneously, $3 + 2 = 5$ units of work is done everyday and thus it would take $\frac{30}{5} = 6$ days.

All problems of work can be solved in either way and both ways take almost the same time as there are exactly the same number of calculations involved. However if you are not comfortable with fraction, the approach using LCM may seem better by you.

Example 46:

Lattoo, Mattoo, Gattoo are working on a job. They take 12, 15 and 18 days respectively to complete the job. If Lattoo and Mattoo start off with the job and work for 4 days together before being joined by Gattoo, when will the job be completed?

Solution:

Method 1:

In the first 4 days, the part of the job completed by Lattoo and Mattoo working together would be $\frac{4}{12} + \frac{4}{15} = \frac{9}{15}$ of the job. This implies $\frac{6}{15}$ of the job is yet to be completed.

They are then joined by Gattoo. Hence, they would all working together do $\frac{37}{180}$ of the job

in a day. To complete the remaining work, they would take $\left(\frac{\frac{6}{15}}{\frac{37}{180}}\right)$ days.

So the total time taken to complete the job = $4 + 1\frac{35}{37}$ days = $5\frac{35}{37}$ days.

The above method is slightly inconvenient because of the fractions that appear in the calculation.

Method 2:

Let the number of units of work to be done is a LCM(12, 15, 18) = 180. So Lattoo does 15 units per day, Mattoo does 12 units per day, and Gattoo does 10 units per day. The first 4 days when Lattoo and Mattoo were working all by themselves, the amount of work done is

$$(12 + 15) \times 4 = 108 \text{ units.}$$

The remaining work = 72 units.

When all the 3 are working together, 37 units of work is done on 1 day. Hence, the rest of the work is completed in $\frac{72}{37}$ days. So total time taken = $5\frac{35}{37}$ days.

Example 47:

A and B are working on a job. A is the builder and B is the demolition man. A takes 10 days to construct a wall completely. B takes 20 days to demolish it completely. How much time do they take to build the wall completely

- a. if they work simultaneously,
- b. if they work on alternate days and A starts the job?

Solution:

a. If they work simultaneously, the part of the job done in 1 day = $\left(\frac{1}{10}\right) - \left(\frac{1}{20}\right) = \frac{1}{20}$.

Hence, they take in total 20 days to complete the job.

b. It is preferable to do this question using the units method.

Suppose the total number of units of job to be completed is 20 m. A builds 2 m every day.

B demolishes 1 m everyday. On the first 2 days, the total length of wall that is built = $(2 - 1) = 1$ m.

Now most students make a mistake of arriving at the conclusion that to build 20 m of the wall, the time required is $20 \times 2 = 40$ days. (Think, if that was the case who was working on the last day!!)

On day two, 1 m of the wall is completed; on day four, 2 m would have been completed; ... so on.

On day 36, 18 m of the wall is completed and the next day it is A who would be doing the job. Hence, he takes 1 day more to complete the rest of the job.

So in total it takes $36 + 1 = 37$ days to completely build the wall.

Example 48:

An inlet pipe alone can fill the whole tank in 10 hr while an outlet pipe can empty the tank in 25 hr. When both the pipes are open, in what time will the tank be filled?

Solution:

Every hour, water added in = $\frac{1}{10}$ of volume of tank. And water leaked out = $\frac{1}{25}$ of volume of tank.

\therefore Every hour, net water added in = $\left(\frac{1}{10} - \frac{1}{25}\right)$ of the volume of the tank = $\frac{3}{50}$.

So time taken to fill the tank completely = $\frac{50}{3} = 16\frac{2}{3}$ hr.

Example 49:

A cistern is filled by an inlet valve in 5 hours. However because of a leak, it takes 30 minutes more to fill up. In how much time will the leak empty the filled cistern independently?

Solution:

Let the leak empty the filled cistern in t hour. Thus rate of filling = $\frac{1}{5}$ th of cistern/hour

and rate of emptying = $\frac{1}{t}$ th of cistern/hour and effective rate = $\frac{1}{5.5}$ th of cistern per hour.

Thus, $\frac{1}{5} - \frac{1}{t} = \frac{10}{55} \Rightarrow \frac{1}{t} = \frac{1}{55} \Rightarrow t = 55$ hours.

Alternately, the amount of water the inlet pipe fills in the extra time i.e. in 0.5 hours is emptied by the leak in the entire 5.5 hours \Rightarrow the leak in 5.5 hours can empty $\frac{1}{10}$ th of the cistern and thus would take 55 hours to empty the filled cistern.

Example 50:

A force of 60 men has food for 28 days. 8 days later reinforcements arrive leaving the number of days the food would last to 15 days. What was the strength of the reinforcement?

Solution:

Originally the food would have lasted for 28 days.

After 8 days the food would have lasted for 20 days.

Let the reinforcement number be x.

The food that would have been consumed by 60 men in 20 days, was consumed by

$(60 + x)$ in 15 days.

$$60 \times 20 = (60 + x) 15$$

$$x = 20.$$

The strength of the reinforcement was 20.

Example 51:

A ship 156 km away from the shore springs a leak which admits $\frac{7}{3}$ tons of water in 6.5 min. A pump throws out 15 tons of water in 1 hour. If 68 tons would suffice to sink the ship, find the average rate of sailing so as to just reach the shore.

Solution:

$$\text{Net volume of water in the ship in 1 min} = \frac{1}{6.5} - \frac{15}{60} = \frac{17}{156} \text{ tons.}$$

$$\text{Time required for 68 tons of water} = 68 \times \frac{156}{17} = 624 \text{ min}$$

$$\text{Speed of ship} = \frac{\text{distance}}{\text{time}} = \frac{156 \text{ km}}{624 \text{ min.}} = \frac{1}{4} \text{ km/min} = 15 \text{ km/hr}$$

Example 52:

A and B can separately do a piece of work in 20 and 15 days respectively. They worked together for 6 days, after which B was replaced by C. If the work was finished in next 4 days, then find the number of days in which C alone could do the work.

Solution:

$$(A + B) \text{ completed} = 6 \left(\frac{1}{20} + \frac{1}{15} \right) = \frac{7}{10} \text{ part of the work in 6 days}$$

$(A + C)$ must have done $= \frac{3}{10}$ part in 4 days; work done by $(A + C)$ in a day $= \frac{3}{40}$ part

$$\text{Work done by A in a day} = \frac{1}{20} \text{ part}$$

$$\therefore \text{Work done by C in a day} = \left(\frac{3}{40} - \frac{1}{20} \right) = \frac{1}{40} \text{ part}$$

Hence, C alone can finish the work in 40 days.

Example 53:

Three men can complete a piece of work in 6 days. Two days after they start, 3 more men joined them. How many days will they take to complete the remaining work (Assume they all have equal efficiency)?

Solution:

$$\text{Work done by 3 men in 2 days} = \frac{1}{6} \times 2 = \frac{1}{3}$$

$$\text{Remaining work} = \left(1 - \frac{1}{3} \right) = \frac{2}{3}$$

$$\text{Work done in a day by 3 men} = \frac{1}{6}$$

$$\text{Work done in a day by 6 men} = \frac{1}{3}$$

$$\frac{2}{3} \text{ work is done by them in } \left(\frac{3 \times 2}{3} \right) = 2 \text{ days.}$$

Alternative Method:

After two days, only four days' work remains.

If 3 more men join the existing 3 men, work will be completed by half the time.

On solving, we get, $x = \frac{7}{5}$ and $y = \frac{2}{5}$

Therefore, the work is completed in 2 days.

(Work done by 2 men + 1 boy) in 1 day = $(2 \times \frac{7}{5} + 1 \times \frac{2}{5}) = \frac{16}{5}$ units.

Example 54:

Two men and 3 boys can do a piece of work in 10 days, while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Number of days taken to do the work = $40 \times \frac{5}{16} = \frac{25}{2}$ days.

Solution:

Let work done by 1 man in 1 day = x part

Let work done by 1 boy in 1 day = y part

$$2x + 3y = \frac{1}{10} \text{ and } 3x + 2y = \frac{1}{8}$$

On solving, we get $x = \frac{7}{200}$ and $y = \frac{1}{100}$

(Work done by 2 men + 1 boy) in 1 day = $2 \times \frac{7}{200} + 1 \times \frac{1}{100} = \frac{2}{25}$ part

Thus, 2 men and 1 boy can finish the work in $\frac{25}{2}$ days.

Alternative Method:

Let the total work be $\text{LCM}(10, 8) = 40$ units.

Let work done by 1 man in 1 day = x units

Let work done by 1 boy in 1 day = y units

$$2x + 3y = 4 \text{ and } 3x + 2y = 5$$

Practice Exercise

2

Introduction

There are 7 practice exercises out of which 2 are of level-1, 3 are of level-2 and 2 are of level 3 apart from the non MCQ Questions to strengthen you fundamentals. While solving the exercises make sure that each and every concept is understood properly.

Problems for Practice (Non MCQ)

Level - I

1. A man walks at 36 km/hr. In what time will he cover one-tenth of a kilometre?
2. The wheel of an engine of 250 cm in circumference makes 10 revolutions in 4 sec. What is the speed of the wheel (in km/hr)?
3. A man rides at the rate of 7 km/hr but stops 10 min to change the horses at the end of every 14th kilometre. How long will he take to go a distance of 98 km?
4. A train starts from X towards Y, which is at a distance of 55 km, at a speed of 40 km/hr. After running a certain distance, it increases its speed to 50 km/hr and reaches Y in 1 hr 15 min after leaving X. After what time did the train change its speed?
5. A man covers a distance at 75 km/hr and returns to the starting point at 50 km/hr. What is his average speed?
6. The driver of a car sees a bus 40 m ahead of him. After 20 s, the bus is 60 m behind. If the speed of the car is 30 km/hr, what is the speed of the bus?
7. A man on a platform notices that a train going in one direction takes 10 s to pass him, and a train of same length going in the opposite direction takes 15 s to pass him. What is

the time taken by the two trains to pass one another if the length of the trains is 200 m each?

8. A man rows a distance downstream in 45 min and the same distance upstream in 75 min. What is the ratio of speed of the stream to the boat in still water?

9. The speeds of three athletes A, B and C, who are racing on a circular track of length 200 m, are 20 m/s, 23 m/s and 27 m/s respectively. They do not rest until all of them meet at a place. When they stop,

a. what is the ratio of the distances travelled by each?

b. what is the time elapsed after the start?

10. A and B start from opposite points of a circular track of 500 m. Their speed is 5 m/s and 3 m/s respectively. When will they meet for the first time if both of them moves in same direction.

11. In question 10, if both of them move in the opposite directions, when will they meet for the second time?

12. A and B can do a job in 20 and 30 days respectively. If both of them work on the job together, when would they finish 75% of it?

13. A and B working together take 20 days; B and C take 15 days; A and C take 18 days to complete a job. How much time would A take to do the same job all alone?

Level - II

14. The difference in times, when a man covers a certain distance at 10 km/hr and 20 km/hr, is 45 min. What would the difference be if the same distance is covered with speeds 25 km/hr and 40 km/hr?

15. Sanju goes to school at 10 km/hr and reaches 6 min late. When he travels at 12 km/hr, he reaches 9 min earlier than the scheduled time. What is the distance of the school from

his house?

16. Two trains start at the same time from A and B and proceed towards B and A at 32 km/hr and 41 km/hr respectively. When they meet, it is found that one train has moved 45 km more than the other. What is the distance between A and B?

17. Two trains start simultaneously from A and B heading towards B and A respectively. If the ratio of the distances covered, when they meet, is 4 : 3, what is the ratio of their speeds?

18. A train leaves A at 40 km/hr. At the same time, another train departs from B at a speed of 60 km/hr. They reach the respective destinations and turn back immediately towards the starting points. Now if they meet at a distance of 200 km from A, what is the distance between A and B?

19. Two persons start from A and B with the speeds of 25 km/hr and 49 km/hr respectively towards each other at the same time. After they cross each other, the person from B covers 145 km to reach A. What is the distance AB?

20. A, who is travelling at 3.5 km/hr, starts 2.5 hr before B who travels at 4.5 km/hr in the same direction as A. In how much time will B overtake A?

21. A boat is moving downstream and reaches its destination in 25 hrs while moving at a speed of 50 km/hr (given speed is in still water). One particular day due to engine problem at mid-point, the ship's speed reduced by 20% of the original and it reached its destination 2.5 hrs late. Find out the speed of the river. [Assume its speed to be uniform]

22. A thief, who had escaped at 7 p.m., was followed by a policeman at 9 p.m. at the rate of 6 km/hr. At what time will the policeman overtake him, supposing the thief runs at 4.5 km/hr?

23. Bhim and Arjun were exercising during their Vanvaas. They start running on a circular track simultaneously and in the same direction. If Bhim takes 4 min to complete one full round, and Arjun takes 7 min to complete one full round

a. after how much time will they meet for the first time?

b. after how much time will they meet for the first time at the starting point?

c. after how much time would they meet for the first time at a point diametrically opposite to the starting point on the track.

24. In question 23, if Bhim gives Arjun a lead of 4 min, when would they meet for the first time?

25. A watch loses 6 min and the other gains 4 min daily. They are set right at 3 p.m. After how many minimum number of days, both of them will show the same time?

26. A person goes between 4 and 5 and comeback between 5 and 6 and realises that the clock hands (minute hand and hour hand) interchange their respective positions. How much time did he remain out of his house?

27. 20 women can finish a job in 20 days. After each day, one woman is replaced by a man or a boy alternatively starting with a man. Man is twice efficient and boy is half efficient as a woman. On which day does the job get completed?

28. 20 women can do a job in 20 days. After each day, a woman is replaced by a man and a man is twice efficient as a woman. On which day does the job get completed?

29. A can do a job in 10 days. B can do the same job in 15 days. C can demolish the same job in 20 days. In how many days does the work get completed, if C starts the work and they work on alternate days as C-A-B-C and so on.

30. A and B can do a job in 10 days. B and C can do the same job in 15 days. If all three together can do the work in 6 days, then in how many days can B complete the whole job?

31. A can do a job in 10 days. Due to problematic relationship with B, his efficiency reduces to 60% and both can finish the same job in 10 days. In how many days can B alone do the job? [Assume B's efficiency remains the same while working alone.]

32. Two inlet pipes take 10 min and 20 min to fill an empty tank. But they take 25 min to fill it because of a leak. How much time would the leak take to empty a full tank?

33. An inlet pipe is filling a tank of capacity 2,000 L. The outlet is discharging water at the rate of 30 L per minute. If both the inlet and the outlet are opened, the tank gets filled in 40 min. How much time would the inlet take to fill the empty tank?

34. A swimming pool is fitted with three pipes with uniform flow. The first pipe can fill the tank in t hours. The second pipe can fill in double of the first's time and the third pipe can fill in arithmetic mean of the first and the second's time. In how much time can first pipe alone fill the tank if together with second and third it can fill the tank in 3 hr?

Level - III

35. Trains are moving from A to B and B to A at a regular interval of 1 hr. They complete their journey in 5 hr. How many trains coming from station B will cross the train coming from station A that started at 10 a.m.? Assume the trains start from both the stations at the same time.

36. In question 35, if there is a time gap of $\frac{1}{2}$ hr between the trains starting from the two respective stations, then how many trains coming from station B will cross a train coming from station A that started at 10 a.m.?

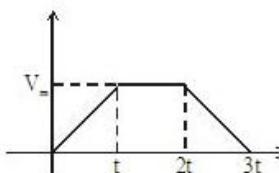
37. In question 35, if the trains starting from B take only 3 hr while trains from A take only 5 hr, then how many trains coming from B will cross a train coming from station A, having left station A at 10 a.m.?

38. In question 35, if the trains from station B start at a regular interval of 2 hr, then how many trains will cross a train coming from station A that started at 10 a.m.?

39. Two truck-drivers are moving towards each other with speeds 36 km/hr and 18 km/hr in a straight line. The faster driver can see the slower driver from a certain distance. Immediately upon seeing the slower driver, he applies the brakes successively

such that the speed of his truck reduces by 1 m/sec in each second. It is given that the slower truck driver is as efficient as the faster one in all respects and both the trucks stop simultaneously. Find out the minimum rate of reduction of speed of the slower truck (in m/sec) to avoid collision and the minimum separation between the two trucks when they notice each other.

40. Munna Bhai travels 280 km from Mudhuvani to Darbhanga, partly by bus and partly by Jeep. Both the vehicles move in the given fashion.



$$V_m = \text{Maximum speed}$$

But their maximum speeds differ by 10 km/hr. Both the vehicles travel for 6 hr each. Find out their respective maximum speeds. There is no fine gap between movement of both vehicles. [Assume that the bus is moving first and faster than the Jeep].

41. What is the average speed of the bus and of the car in question number 40?

42. Two swimmers in a river started off at the same time from two different points along the course of the river separated by 100 m and moved towards each other cross each other, reach opposite ends and reverse their directions to come back to their respective start positions. Speed of one swimmer is 6 m/s and the second is 4 m/s. The ratio of the distances covered by them is 4 : 1 during the first meeting. Find out their first meeting time, river flow speed and second meeting time during the race.

43. There are three inlet pipes whose diameters are 1 m, 2 m and 3 m respectively. If the rate of flow is directly proportional to the square of the diameter, find the time taken to

fill an empty tank when all the three pipes are opened, given that the smallest pipe takes 9 min to fill the tank.

44. There are two taps connected to a tank. Capacity of the tank is 40 L. Tap 1 can fill the tank in 10 hr. Tap 2 can empty the tank in 20 hr. How much time will both the taps take to fill the tank when both are open simultaneously? It is given that water evaporates at the rate of 2.5% of the total capacity of tank in an hour.

Practice Exercise 1 - Level 1

1. At a speed of 48 km/hr, a car can finish a journey in 10 hr. To cover the same distance in 8 hr, the speed must be increased by

- a. 6 km/hr b. 7.5 km/hr c. 8 km/hr d. 12 km/hr e. 15 km/hr

2. Walking at $\frac{3}{4}$ th of his normal speed, a man gets late by 2.5 hr. At his normal speed, what time will he take to cover the same distance?

- a. 7.5 hr b. 3.5 hr c. 3.25 hr d. $\frac{7}{8}$ hr e. 5 hr

3. There is a gap of 200 km between two trains which simultaneously starts towards each other. If they cross each other at a distance of 110 km from one of the stations, then what is the ratio of their speeds?

- a. 11 : 20 b. 9 : 20 c. 11 : 9 d. 17 : 9 e. 31 : 20

4. The speeds of A and B are in the ratio 3 : 4. A takes 30 min more than B to cover the same distance. How much time does A take to cover this distance?

- a. 1.25 hr b. 1.33 hr c. 4 hr d. 2 hr e. 1.5 hr

5. A is twice as fast as B and B is twice as fast as C. The distance covered by C in 54 min, will be covered by A in

- a. 216 min b. 27 min c. 108 min d. 13.5 min e. 36 min

6. The ratio of speeds of A and B is 3 : 4. If B covers a distance in 36 min, A will cover it in

- a. 27 min b. 48 min c. $15\frac{3}{7}$ min d. $36\frac{8}{7}$ min e. None of these

7. A man goes up a hill at 24 km/hr and comes down at 36 km/hr along the same path. What is his average speed?

- a. 30 km/hr b. 28.8 km/hr c. 32.6 km/hr d. 35 km/hr e. 27 km/hr

8. A plane travels 2500 km, 1200 km and 500 km at speeds of 500 km/hr, 400 km/hr and 250 km/hr respectively. Its average speed is

- a. 405 km/hr b. 410 km/hr c. 575 km/hr d. 420 km/hr e. 375 km/hr

9. A 110 m long train is moving at 132 km/hr. How long will it take to cross a platform of 165 m long?

- a. 5 sec b. 7.5 sec c. 10 sec d. 12.5 sec e. 15 sec

10. A train travelling at the rate of 90 km/hr crosses a pole in 10 sec. Its length is

- a. 250 m b. 150 m c. 900 m d. 100 m e. 324 m

11. A 300-m long train crossed a 900-m long platform in 1 min 12 sec. The speed of the train (in km/hr) is

- a. 45 b. 50 c. 54 d. 60 e. 75

12. A can do a piece of work in 7 days working 9 hrs on each day and B can do it in 6 days working 7 hrs on each day. How long will they take to do it working together 8.4 hrs a day?

- a. 3 days b. 4 days c. 4.5 days d. 6 days e. 7.5 days

13. A can do a piece of work in 80 days. He works for 10 days and then B alone finishes the remaining work in 42 days. The two together could complete the work in

- a. 24 days
- b. 25 days
- c. 27 days
- d. 29 days
- e. 30 days

14. A is twice as good as B and together they finish a job in 14 days. A will finish the work alone in

- a. 11 days
- b. 21 days
- c. 28 days
- d. 42 days
- e. 56 days

15. A sum of money is sufficient to pay A's wages for 21 days or B's wages for 28 days. It is then sufficient to pay the wages of both for

- a. 7 days
- b. 12 days
- c. 14 days
- d. 12.25 days
- e. 24.5 days

16. Pipes A, B and C take 20, 15 and 12 min respectively to fill in a cistern. Together they will fill the cistern in

- a. 15 min
- b. 10 min
- c. 12 min
- d. 8 min
- e. 5 min

17. Two pipes can fill in a tank in 10 hr and 12 hr, while a third can empty it in 20 hr. If all three are opened simultaneously, in how much time will the tank be filled in?

- a. 7 hr
- b. 8 hr
- c. 7.5 hr
- d. 8.5 hr
- e. 9 hr

18. A cistern can be filled in 9 hr but it takes 10 hr due to a leak. In how much time will the leak empty the full cistern?

- a. 60 hr
- b. 70 hr
- c. 80 hr
- d. 90 hr
- e. 100 hr

19. Taps A and B can fill in a tank in 12 and 15 min respectively. If both are opened and A is closed after 3 min, how long will it take for B to fill in the tank?

- a. 8 min 15 sec
- b. 7 min 15 sec
- c. 8 min 5 sec
- d. 7 min 45 sec
- e. 8 min 25 sec

20. Working together, A and B can do a piece of work in 72 days, B and C in 120 days and A and C in 90 days. In how many days can A do it alone?

- a. 150 days
- b. 120 days
- c. 100 days
- d. 80 days
- e. 75 days

Practice Exercise 2 - Level 1

1. Tarun can cover a certain distance in 1 hr 24 min by covering $\frac{2}{3}$ rd of the distance at 4 km/hr and the rest at 5 km/hr. The total distance is

- a. 5 km
- b. 5.2 km
- c. 6 km
- d. 8 km
- e. 9.2 km

2. If a train runs at 40 km/hr, it gets late by 11 min, and if it runs at 50 km/hr, it gets late by 5 min. If the train runs at its correct speed, in how much time will it complete its journey?

- a. 13 min
- b. 15 min
- c. 21 min
- d. 19 min
- e. 24 min

3. A man is travelling by car at the rate of 40 km/hr. After every 80 km, he rests for 20 min. How long will he take to cover a distance of 240 km?

- a. 6 hr 40 min
- b. 6 hr
- c. 6 hr 20 min
- d. 7 hr
- e. 6 hr 30 min

4. In a 100 m race, Sujit beats Rishi by 5 m and Rishi beats Praveen by 5 m. By what distance does Sujit beat Praveen?

- a. 10 m
- b. 11 m
- c. 9 m
- d. 9.75 m
- e. 9.5 m

5. Pallavi beats Anuva in a kilometre race by 50 s and Richa by 450 m. If Anuva and Richa run a kilometre race, Anuva wins by 40 s. How much time does Richa take to run a kilometre?

- a. 160 sec
- b. 110 sec
- c. 200 sec
- d. 150 sec
- e. None of these

6. A person is standing on a stair case. He walks down 4 steps, up 3 steps, down 6 steps, up 2 steps, up 9 steps, and down 2 steps. Where is he standing now in relation to the step on which he stood when he started?

- a. 2 steps up
- b. 1 step up
- c. At the same place
- d. 1 step down
- e. 4 steps up

7. A man starts moving at 7 a.m. uphill on a road of length 25 km and reaches the top by travelling at 10 km/hr. The next day he starts the downhill journey at 7 a.m. moving at 15 km/hr. Where will he be at the same point of time as he was the day before?

- a. 15 km from the top of the hill
- b. 15 km from the bottom of the hill
- c. 20 km from the top of the hill
- d. 20 km from the bottom of the hill
- e. He will never be at the same point at the same point of time.

8. A car has to cover 80 km in the available time of 10 hr. If it covers the first half of the journey in $\frac{3}{5}$ th part of the available time, what should be its speed for the remaining journey?

- a. 8 km/hr
- b. 6.4 km/hr
- c. 10 km/hr
- d. 15 km/hr
- e. 20 km/hr

9. Sujit plans to travel 180 km. He travels first 30 km at 45km/hr, next 60 km at 90 km/hr and remaining distance at 75 km/hr. Being a literature student he is unable to calculate his average speed of the entire journey. Can you help him out with the correct answer?

a. 79 km/hr b. $71\frac{1}{19}$ km /hr c. $72\frac{1}{19}$ km /hr

d. 75 km/hr e. None of these

10. A 110-m long train is travelling at the speed of 58 km/hr. What is the time in which it will pass a man walking in the same direction at 4 km/hr?

- a. 6 s
- b. 7.5 s
- c. 7.33 s
- d. 7 s
- e. 7.33 min

11. A 108-m long train moving at a speed of 50 km/hr crosses a 112-m long train coming from the opposite direction, in 6 sec. The speed of the second train is

- a. 82 km/hr
- b. 54 km/hr
- c. 66 km/hr
- d. 56 km/hr
- e. 72 km/hr

12. Two trains travel in opposite directions one at 36 km/hr and the other at 45 km/hr. A man sitting in the slower train passes the faster train in 8 s. The length of the faster train is

- a. 80 m
- b. 180 m
- c. 120 m
- d. 150 m
- e. 110 m

13. A river runs at 2 km/hr. If the time taken by a man to row his boat upstream is twice the time taken by him to row it downstream, then at what speed can he row his boat, in still water?

- a. 5 km/hr
- b. 4 km/hr
- c. 10 km/hr
- d. 6 km/hr
- e. 8 km/hr

14. A train crosses a pole in 15 s and a 100-m long platform, in 25 s. The length of the train is

- a. 200 m
- b. 150 m
- c. 100 m
- d. 50 m
- e. Data insufficient

15. A train speeds past a pole in 15 s and crosses past a platform of 100 m long in 30s. Find the length of the train.

a. 75 m b. 130 m c. 100 m d. 110 m e. Cannot be determined

16. A job is done by 10 men in 20 days and 20 women in 15 days. How many days will it take for 5 men and 10 women to finish the work?

a. $17\frac{1}{2}$ days b. $17\frac{1}{7}$ days c. 17 days d. $17\frac{1}{20}$ days e. $17\frac{1}{15}$ days

17. Working alone, a man can complete a job in as many days as his two sons can, working together. If one of the sons completes the work in 3 hr and the other in 6 hr., then in how many hours can the father complete the work?

a. 1 hr b. 2 hr c. 3 hr d. 4 hr e. 4.5 hr

18. A, B and C can do a job in 15 days. After working with B and C for 5 days, A leaves. Then B and C finish it in 20 days more. In how many days can A do the job alone?

a. 25 days b. 27 days c. 30 days d. 28 days e. 32 days

19. Raghu, Ram and Rahim can finish a project in 22, 33 and 44 days respectively. How soon can they finish the project, if they work jointly?

a. $10\frac{2}{13}$ days b. 12 days c. 15 days d. 17.5 days e. 14.5 days

Practice Exercise 3 - Level 2

1. A man travels 35 km partly at 4 km/hr and partly at 5 km/hr. If he covers the former distance at 5 km/hr, he could cover 2 km more in the same time. The time taken to cover the whole distance at the original rate is

a. $4\frac{1}{2}$ hr b. 7.5 hr c. 7.4 hr d. 9 hr e. 8.2 hr

2. A man takes 7 hr in walking to a certain place and riding back. He would have gained 2 hr by riding both ways. The time he would take to walk both ways is

a. 6 hr b. 7 hr 30 min c. 4 hr 30 min d. 8 hr 30 min e. None of these

3. A, B and C are on a trip. A drives at 50 km/hr for 1 hr. B drives the next 2 hr at 48 km/hr. C drives the next 3 hr at 52 km/hr. They reach their destination exactly after 6 hr. Their mean speed was

a. 52 km/hr b. $50\frac{6}{7}$ km/hr c. $51\frac{1}{3}$ km/hr d. $50\frac{1}{3}$ km/hr e. $52\frac{1}{3}$ km

4. A man can row 13 km upstream or 28 km downstream in 5 hrs. The speed of stream is

a. 1.5 km/hr b. 2 km/hr c. 2.5 km/hr d. 3 km/hr e. 3.5 km/hr

5. If a man rows at 6 km/hr in still water and at 4.5 km/hr against the current, then what is his speed of rowing along the current?

a. 9.5 km/hr b. 5.25 km/hr c. 7 km/hr d. 7.5 km/hr e. 9 km/hr

6. A man sees a train passing over a 1 km long bridge. The length of the train is half that of the bridge. If the train clears the bridge in two min, the speed of the train is

a. 30 km/hr b. 45 km/hr c. 50 km/hr d. 60 km/hr e. 75 km/hr

7. A stream runs at 1 km/hr. A boat goes 35 km upstream and comes back taking a total time of 12 hr. The speed of the boat in still water is

a. 6 km/hr b. 7 km/hr c. 8 km/hr d. 8.5 km/hr e. 9 km/hr

8. Two shots were fired from the same place at an interval of 12 min; but a person in the train approaching the place hears the second shot 10 min after the first. The speed of the train, if speed of sound is 330 m/s, is

a. 65 m/s b. 65 km/hr c. 56 m/s d. 56 km/hr e. None of these

9. A train starts from station P at 7 a.m. and moves at 60 km/hr towards Q. At 1 p.m. another train starts from Q towards P at 80 km/hr. When do the two trains meet, if the distance between P and Q is 1,200 km?

- a. 6 p.m.
- b. 7 p.m.
- c. 8 p.m.
- d. 7.30 pm
- e. None of these

10. In the above problem, at what distance from Q, do the two trains meet?

- a. 360 km
- b. 720 km
- c. 480 km
- d. 520 km
- e. None of these

11. A man can row $\frac{3}{4}$ of a kilometre against a stream in $11\frac{1}{4}$ min and return in $7\frac{1}{2}$ min.

The speed of the man in still water is

- a. 2 km/hr
- b. 3 km/hr
- c. 4 km/hr
- d. 5 km/hr
- e. 5.2 km/hr

12. A boat covers 40 km upstream and 90 km downstream in 5 hr. It can also cover 60 km upstream and 60 km downstream in 5 hr. The speed of the water current is

- a. 4 km/hr
- b. 5 km/hr
- c. 20 km/hr
- d. 25 km/hr
- e. 6 km/hr

13. Working alone, A and B can do a piece of work in 45 and 40 days respectively. They begin working together, but A leaves after some days and B completes the rest of the work in 23 days. For how many days did A work?

- a. 6 days
- b. 8 days
- c. 9 days
- d. 12 days
- e. 13 days

14. A can complete a certain job in 12 days. B is 60% more efficient than A. In how many days can B complete the same job?

- a. 6 days
- b. 6.25 days
- c. 7 days
- d. 7.5 days
- e. 4.8 days

15. Working alone, A and B can do a job in 25 days and 20 days respectively. A started the work alone and after 10 days was joined by B. The total number of days taken by A and B to complete the work is

- a. 12.5 days
- b. 14.22 days
- c. 15 days
- d. 16.66 days
- e. 21.11 days

16. A does half as much work as B does and C does half as much work as A and B do together, in the same time. If C alone can do the work in 40 days, then working together, the three of them will finish the work in

- a. 13 days
- b. 15 days
- c. 20 days
- d. 13.33 days
- e. 12 days

17. A young man can do a job in 10 days, his father takes 15 days while his grandfather finishes it in 20 days. They worked together for 2 days. The young man left the job. His father left the job a day earlier than the completion of the work. The job was completed in

- a. $7\frac{3}{7}$ days
- b. $6\frac{3}{7}$ days
- c. $5\frac{3}{7}$ days
- d. $7\frac{2}{7}$ days
- e. None of these

Practice Exercise 4 - Level 2

1. Golu and Mayank start running simultaneously. Golu runs from point A to point B and Mayank from point B to point A. Golu's speed is $\frac{6}{5}$ of Mayank's speed. After crossing Mayank, if Golu takes $2\frac{1}{2}$ hr to reach B, how much time does Mayank take to reach A after crossing Golu?

- a. 3 hr 6 min
- b. 3 hr 16 min
- c. 3 hr 26 min
- d. 3 hr 36 min
- e. 3 hr 32 min

2. In a running race, Pallavi gives a head start of 350 m to Richa. If the ratio of speeds of Pallavi and Richa is 20 : 13, how far must the winning point be so that Pallavi and Richa finish the race at the same time?

- a. 1 km
- b. 2 km
- c. 3 km
- d. 2.5 km
- e. None of these

3. After travelling 30 min, a train met with an accident and was stopped there for 45 min. Due to the accident, its speed reduced to $\frac{2}{3}$ of its former speed and the train reached its

destination 1 hr 30 min late. Had the accident occurred 60 km after the point it occurred earlier, the train would have reached 30 min earlier. The length of the journey is

- a. 90 km
- b. 120 km
- c. 150 km
- d. 180 km
- e. 130 km

4. Vibhor started the journey with his car from the badlands with tank full of fuel, 12 gallons exactly. However, the moment he started out, the fuel tank sprung a leak. He travelled at 50 mph until he ran out of fuel exactly 4 hr later. Knowing that the car runs 25 miles for each gallon, how much fuel did he lose through the hole?

- a. 4 gallons
- b. 5 gallons
- c. 6 gallons
- d. 3 gallons
- e. 7 gallons

5. Time taken by three Grand Prix contestants to reach a terminal are in the ratio of 6 : 4 : 3. What will be the ratio of the speeds at which they cover the distance?

- a. 4 : 6 : 9
- b. 2 : 3 : 4
- c. 2 : 3 : 8
- d. 3 : 4 : 6
- e. None of these

6. In a race, the speeds of A and B are in the ratio 3 : 4. A takes 30 min more than B to reach the desired destination. The time taken by A to reach the destination is

- a. 1 hr
- b. 90 min
- c. 2 hr
- d. 2.5 hr
- e. 105 min

7. Nupur goes to Kashipur from Bareily at 6 km/hr and returns to Bareily from Kashipur at x km/hr. What is the value of x , if average speed of Nupur in going and coming back is 12 km/hr?

- a. 18 km/hr
- b. 30 km/hr
- c. 90 km/hr
- d. 45 km/hr
- e. Not possible

8. Sujit and Shivku start travelling in the same direction at 8 km/hr and 13 km/hr respectively at the same time. After 4 hr Sujit doubled his speed and Shivku reduced his speed by 1 km/hr and reached the destination together. How long did the entire journey last?

- a. 9 hr
- b. 10 hr
- c. 11 hr
- d. 8 hr
- e. None of these

9. On a circular track of length more than 200 m, Shivku runs twice as fast as Pawan. In a running race, if Shivku gives a lead of 100 m to Pawan, at what distance they will meet for the first time from the starting point? At what distance they will meet for the first time from the starting point if the length of the track is only 150m?

- a. 150 m, 90 m
- b. 250 m, 75 m
- c. 200 m, 50 m
- d. 200 m, 90 m
- e. None of these

10. What is the minimum number of rounds that A must make in order to meet B at the starting point? The speeds of A and B are 4 m/s and 12 m/s respectively.

- a. 1
- b. 12
- c. 3
- d. 4
- e. 2

Direction for questions 11 and 12:

Answer the questions based on the following information.

A and B are racing (in the same direction) on a circular track of length 1 km. In a 500 m race, A can give B a start of 100 m.

11. If both A and B start simultaneously on the circular track, find after how many rounds of A would the two be together for the first time?

- a. 3 rounds
- b. 4 rounds
- c. 5 rounds
- d. 6 rounds
- e. 7 rounds

12. When A meets B for the first time, B has made 3 rounds. What is the lead that B gives to A?

- a. 250 m
- b. 750 m
- c. 400 m
- d. 600 m
- e. Cannot be determined

13. The minute hand of a clock is found to cross the hour hand x minutes past three. Then $x =$

- a. $10\frac{5}{11}$
- b. $21\frac{9}{11}$
- c. $16\frac{4}{11}$
- d. 18
- e. $15\frac{3}{11}$

14. If the output of 3 men is equivalent to that of 4 women who can complete a job in 12 days working together, how much time would 5 women and 3 men together take to complete the same job?

- a. 27 days b. 24 days c. $5\frac{1}{3}$ days d. 7 days e. None of these

15. Three men earn as much as 4 women; 4 women earn as much as 6 boys; and 8 boys earn as much as 10 girls. If a girl earns Rs. 50 a day, then the earning of a man would be

- a. Rs. 115 b. Rs. 135 c. Rs. 125 d. Rs. 150 e. Rs. 120

16. A group of workers decided to finish a work in 10 days but 5 of them could not join the team. If the rest of the crew completed the job in 12 days, the number of members present originally in the team was (Assume all workers have equal efficiency)

- a. 45 b. 30 c. 50 d. 35 e. 32

17. The work done by a man, a woman, and a child are in the ratio 3 : 2 : 1. If daily wages of 20 men, 30 women and 36 children amount to Rs. 78, what will be the wages of 15 men, 21 women and 30 children for 18 weeks?

- a. Rs. 7,371 b. Rs. 8,645 c. Rs. 9,000 d. Rs. 7173 e. None of these

18. A, B and C working together complete a job in 18 days. A and B together work twice as much as C; A and C together work thrice as much as B. A alone can finish the work in

- a. 18 days b. 43.2 days c. 54 days d. 72 days e. 42 days

19. Two workers, A and B, completed a job in 10 days, but A did not work during the last 2 days. In the first 7 days they together completed $\frac{4}{5}$ of the job. How long would A take to do the job alone?

- a. 16 days b. 15 days c. 14 days d. 12 days e. None of these

20. To complete a task in 45 days, a contractor employs 45 people. Upon reviewing the work after 30 days, he notices that only half the task is completed. In order to meet the deadline of 45 days, how many extra people must he employ?

- a. 25 b. 15 c. 60 d. 45 e. 42

21. If 15 men or 24 women or 36 boys can do a work in 12 days working 8 hr a day, how many men must be associated with 12 women and 6 boys to do another work, $2\frac{1}{4}$ times as great in 30 days working 6 hr per day?

- a. 10 b. 15 c. 8 d. 12 e. None of these

22. A and B can do a work in 10 days; B and C in 15 days; A and C in 25 days. They all worked together for 4 days. A then leaves, and B and C go on together for 5 days more and then B leaves. In how many more days will C complete the remaining work?

- a. 76 b. 57 c. 68 d. 72 e. None of these

23. A does a work in 6 days, B in 8 days, and C in 10 days. A worked alone for 2 days before leaving and then B and C worked together for 2 days and after that B left. The remaining work was finished by C alone. How long did it take to finish the work?

- a. $6\frac{1}{6}$ days b. $2\frac{1}{6}$ days c. $4\frac{1}{6}$ days d. 7.5 days e. None of these

24. Two pipes A and B can fill a tank in 4 hr and 5 hr respectively. If they are turned on alternately for 1 hr each, the tank will be filled in (Assume that pipe A is turned on first)

- a. 4 hr 24 min b. 4 hr c. 4.5 hr d. 5 hr e. 4 hr 36 min

Practice Exercise 5 - Level 2

1. If a snail creeps 2 ft 3 inches up a pole in first 12 hr and slips down 1 ft 4 inches in subsequent 12 hr, how many hours will it take to get to the top of the pole, if the height of the pole is 25 ft?

- a. 500 hr
- b. 523.93 hr
- c. 595.2 hr
- d. 611.11 hr
- e. None of these

2. A train travelling at the rate of 18.5 mph started at 7 a.m. on a journey of 148 miles. Another train started from the same station and arrived 15 min after the first one. If the ratio of the speeds of the second train to that of the first is 8 : 5, when did the second train start?

- a. 10 a.m.
- b. 10.15 a.m.
- c. 11 a.m.
- d. 9.30 a.m.
- e. 11.15 a.m.

3. Two places A and B are separated by a distance of 200 m. Ajay and Mallu have to start simultaneously from A, go to B and return to A. In 10 s they meet at a place 10 m from B. If Ajay is faster than Mallu, in how much time, after they start, will Ajay return to A?

- a. 19 s
- b. $\frac{200}{21}$ s
- c. $\frac{400}{21}$ s
- d. 18 s
- e. $\frac{190}{21}$ s

4. An athlete runs from A to B and returns at 10 km/hr, while another athlete simultaneously runs from B to A and goes back at 15 km/hr. If they cross each other for the first time 12 min after they start, they will next cross each other after

- a. 18 min
- b. 30 min
- c. 24 min
- d. 36 min
- e. 42 min

5. At 3 p.m. every day, a man leaves New York for Delhi and at the same time, a man leaves Delhi for New York. Both take the same route. The travel time for each person being 384 hr, how many persons from New York will each man from Delhi meet?

- a. 16
- b. 17
- c. 34
- d. 33
- e. 32

6. The diameter of a roadroller is 42 cm and its length is 100 cm. It takes 400 complete revolutions, moving only once over any stretch, to level the certain stretch of a road. If the cost of levelling is Rs. 10 per square metre, then the total cost of levelling works out to

- a. Rs. 52,80,000
- b. Rs. 6,970
- c. Rs. 5,280
- d. Rs. 4,980
- e. Rs. 8,420

7. A cycled from P to Q at the rate of 10 km/hr and returned at the rate of 9 km/hr. B cycled both ways at 12 km/hr. For the entire journey, A took 10 min longer than B. Thus, the distance PQ is

- a. 7.5 km
- b. 3.5 km
- c. 2.25 km
- d. 1.95 km
- e. 3.75 km

Direction for questions 8 and 9:

Answer the questions based on the following information.

A thief flees from Tihar Jail in a Gypsy towards Jaipur on a stretch of straight road of 300 km long at 60 km/hr. 15 min later a police party (Sigma-Z) leaves Tihar Jail chasing the thief at 65 km/hr.

8. How long, after the chase starts, does it take Sigma-Z to catch the thief?

- a. 2 hr 30 min
- b. 2 hr 45 min
- c. 3 hr
- d. 3 hr 15 min
- e. None of these

9. If another police party (Zigma-S) were to leave Jaipur at 60 km/hr at the same time as Sigma-Z leaves Tihar Jail to catch the thief, then which of the following statement is true?

- a. Sigma-Z takes 37.5 min more than Zigma-S in catching the thief
- b. Zigma-S reaches 37.5 min after Sigma-Z has caught the thief
- c. Sigma-Z and Zigma-S caught the thief together
- d. Sigma-Z was 3.375 km from the thief when the thief was caught
- e. Zigma-S caught the thief after 5 hr of their start.

10. A boat takes 2 hr to go from port A to port B and 3 hr for the return journey. What is the speed of the river? (Assume that speed of the boat in still water and speed of the river remains constant.)

a. 2.5 km/hr b. 1.75 km/hr c. 0.5 km/hr d. 3 km/hr e. Cannot be determined

11. P and Q are two places on the same side of the bank of a uniformly flowing river. A person can row from P to Q in 1 hr and from Q to P in 4 hr. Find the ratio of speeds of the boat in still water to that of the river.

a. 5 : 3 b. 5 : 2 c. 4 : 3 d. 2 : 1 e. 5 : 4

12. A man rows a boat at a speed of 5 km/hr in still water. Find the speed of a river if it takes him 1 hr to row a boat to a place 2.4 km away and return back.

a. 5 km/hr b. 6 km/hr c. 3 km/hr d. 4 km/hr e. 1 km/hr

13. A motor boat sets down a river at the same time as a country raft in the same direction and from the same point. The motor boat travels $\frac{40}{3}$ km and then without stopping it travels $\frac{28}{3}$ km in the reverse direction and meets the raft. What is the ratio of speed of the motor boat in still water to speed of river?

a. 4 b. $\frac{20}{3}$ c. $\frac{17}{3}$ d. $\frac{10}{3}$ e. 3

Direction for questions 14 and 15:

Answer the questions based on the following information.

Two persons, X and Y, start together from the same point and run on a 4 km circular track in a race of 16 km. The ratio of their speeds is 3 : 7.

14. How often do they meet on the track in the race, if they both run in the clockwise direction?

a. Once b. Twice c. Thrice d. Five times e. Cannot be determined

15. How often do they meet in the race if X runs in the clockwise direction and Y in the anticlockwise direction?

a. Twice b. Thrice c. Four times d. Five times e. Cannot be determined

16. At what time between 7 and 8 o'clock are the hour and minute hands of a clock together?

a. $61\frac{1}{11}$ min past 7 b. $43\frac{7}{11}$ min past 7 c. $31\frac{2}{11}$ min past 7

d. $38\frac{2}{11}$ min past 7 e. $38\frac{7}{11}$ min past 7

17. A photocopier copies 1,500 workbooks in 8 hr, while another takes 12 hr to do the same job. How many hours would both photocopiers take working together to complete 1,500 workbooks?

a. 4.8 hr b. 4.4 hr c. 5 hr d. 4.6 hr e. 5.4 hr

Direction for questions 18 and 19:

Answer the questions based on the following information.

Pipe A fills a tank at the rate of $60 \text{ m}^3/\text{hr}$. Pipe B fills the tank in 6 hr. If both the pipes are opened simultaneously, the tank is filled in 4 hr.

18. What is the volume of the tank?

a. 360 m^3 b. 240 m^3 c. 720 m^3 d. 480 m^3 e. Cannot be determined

19. A third pipe C can empty the tank in 8 hr. If all three pipes are opened simultaneously, how long will it take for the tank to be filled?

a. $2\frac{11}{13}$ hr b. $3\frac{5}{11}$ hr c. $6\frac{5}{13}$ hr d. 8 hr e. None of these

20. A and B can do a piece of work in 20 days, B and C in 24 days, and C and A in 15 days. If A works for 4 days, B for 8 days, and C for 8 days, then the fraction of the work completed by them is

- a. $\frac{21}{60}$
- b. $\frac{43}{60}$
- c. $\frac{29}{60}$
- d. $\frac{41}{60}$
- e. $\frac{27}{60}$

Practice Exercise 6 - Level 3

Direction for questions 1 to 4: Answer the questions based on the following information.

Two trains started simultaneously at 9 a.m. from A and B towards B and A. Both of them take 12 hr to reach their respective destinations.

1. At what time will the two trains meet?

- a. 3 p.m.
- b. 3.45 p.m.
- c. 4.20 p.m.
- d. 6 p.m.
- e. 5 p.m.

2. If the first train met with an accident at 1 p.m. and thereafter travels at half of its original speed, when will the two trains meet?

- a. 2.40 p.m.
- b. 3.40 p.m.
- c. 3.20 p.m.
- d. 4.20 p.m.
- e. Cannot be determined

3. The first train met with an accident and travelled at half of its original speed thereafter. It reached the destination 10 hr late. Find the time when the two trains would meet.

- a. 3 p.m.
- b. 2 p.m.
- c. 4.20 p.m.
- d. 4.33 p.m.
- e. None of these

4. The first train met with an accident and travelled at half of its speed thereafter, and the two trains met at 4 p.m. Find the time when the accident occurred.

- a. 3.15 p.m.
- b. 11.00 a.m.
- c. 9.30 a.m.
- d. 12 noon
- e. 12.30 p.m.

5. A and B run a 1,760 m race ending in a dead heat. At first A runs 20% faster than B. B then quickens his pace, and for the remaining distance runs 20% faster than A. When B quickens his pace, A has already run

- a. 800 m
- b. 1,000 m
- c. 790 m
- d. 960 m
- e. 760 m

6. Ram and Shyam are competing with each other in a friendly community competition in a pool of 50 m length and the race is for 1,000 m. Ram crosses 50 m in 2 min and Shyam in 3 min 15 s. Each time they meet/cross each other, they do handshake's. How many such handshake's will happen? (They both start from the same end at the same time.)

- a. 18
- b. 19
- c. 23
- d. 20
- e. 17

Directions for questions 7 to 9:

Answer the questions based on the following information.

The distance between Guntur and Hyderabad is 400 km. Trains P, Q and R travel at 40 km/hr, 60 km/hr and 80 km/hr respectively. P and Q leave from Guntur for Hyderabad and Hyderabad for Guntur respectively at the same time. R leaves from Hyderabad for Guntur 4 hr after Q.

7. When P meets Q, the time is 3 p.m. When did Q start from Hyderabad?

- a. 3 a.m.
- b. 8.20 a.m.
- c. 11 a.m.
- d. 10 a.m.
- e. Data insufficient

8. When will R cross Q on its way to Guntur?

- a. 8 p.m.
- b. 11 p.m.
- c. 3 a.m.
- d. 9 p.m.
- e. Never

9. Q met with an accident x hrs after it had started. It travelled at $\frac{1}{3}$ of its original speed thereafter. If it meets P one hour behind the scheduled meeting time, when did the accident occur?

a. $2\frac{1}{2}$ hr after it starts from Hyderabad b. 2 hr after it starts from Hyderabad

c. 3 hr after it starts from Hyderabad d. 1 hr after it starts from Hyderabad

e. Data insufficient

10. Two trains - a goods train of 490 m long and a passenger train of 210 m long ___ were travelling on parallel tracks towards each other. The driver of the passenger train noticed the driver of the goods train when it was 700 m away; 28 s later the drivers passed each other. Find the respective speeds of each train, if we know that the goods train takes 35 s longer to pass the signal than the passenger train.

a. 36 km/hr and 54 km/hr b. 36 km/hr and 45 km/hr c. 30 km/hr and 50 km/hr

d. 10 km/hr and 15 km/hr e. 36 km/hr and 60 km/hr

11. A train overtakes two persons who are walking in the direction of the train at 2 km/hr and 4 km/hr in 9 s and 10 s respectively. The length of the train and its speed are

a. 50 m and $\frac{55}{9}$ m/s b. 80 m and $\frac{88}{9}$ m/s c. 70 m and $\frac{77}{9}$ m/s

d. 60 m and $\frac{66}{9}$ m/s e. 80 m and $\frac{77}{9}$ m/s

Directions for questions 12 and 13:

Answer the questions based on the following information.

Ram and Suraj start running from point A on a circular track of 1,200 m. Ram runs at a speed of 12 km/hr and Suraj at a speed of 8 km/hr. They start running in opposite directions at the same time. When each of them reaches at point A for the first time, they reverse their directions along the track.

12. In how much time after starting will they meet for the second time?

a. 7.2 min b. 7.8 min c. 10.8 min d. 11.2 min e. 9.4 min

13. If in stead of reversing the directions on reaching A, they reverse the directions when they meet each other, how much time after starting from A, they will meet for the second time?

a. 5.4 min b. 7.2 min c. 10.8 min d. 11.2 min e. 8.2 min

14. Two men, A and B, started a job in which A was thrice as good as B and therefore took 60 days less than B to finish the job. How many days will they take to finish the job, if they start working together?

a. 15 days b. 20 days c. $22\frac{1}{2}$ days

d. 25 days e. 30 days

Direction for questions 14 and 15:

Answer the questions based on the following information.

Bamdev and Manas who are demolition and construction men on a job take 30 days and 10 days respectively to demolish and construct a wall working all alone. Bamdev's son is half as efficient as he is. He is also a demolition man. Bamdev and his son work simultaneously, whereas Manas prefers working alone. They work on alternate days.

15. If Manas starts the work, what fraction of the wall was built by him by the time it was completed?

a. 4 b. $\frac{19}{10}$ c. $\frac{39}{20}$ d. $\frac{37}{20}$ e. $\frac{22}{30}$

16. On the day when Manas started, the job was already 20% complete. After how many days will the job be completed?

- a. 30 days b. 29 days c. 16 days d. 32 days e. 14 days

17. Ganesh prepares 51 glasses of milkshake in 4 min 18 s in his first shift. After taking rest for some time he prepares 73 glasses in 7 min 13 s. After another break, he prepares 112 glasses in 12 min 24 s. The container in which the milkshake is prepared contains a maximum of 9 glasses of the drink at a time. Find his approximate average time of preparation of one container of the drink. (Every time he starts with a fresh container and tries to use completely if possible.)

- a. 48 s b. 50 s c. 40 s d. 51 s e. 49 s

18. Building material was to be delivered from New Delhi railway station to a construction site in 8 hr. The material had to be delivered with 30 three-tonne trucks which were used for 2 hr, and then

9 five-tonne trucks were added 2 hr later to complete the job in time. How many hours will it take for one three-tonne truck alone and one five-tonne truck alone respectively to complete the job? (Assume that it takes very less and equal time for all trucks to make one trip.)

- a. 330 hr and 198 hr b. 198 hr and 330 hr c. 165 hr and 99 hr

- d. Cannot be determined e. None of these

19. If it takes 40 days for 40 cows to graze a park, and 60 days for 30 cows to do the same, how many days will it take 20 cows to graze the park, given that the grass is also growing everyday at the same rate?

- a. 240 days b. 120 days c. 200 days

- d. Cannot be determined e. None of these

20. The time taken by 4 men to complete a job is double the time taken by 5 children to complete the same job. Each man is twice as fast as a woman. How long will 12 men, 10

children and 8 women take to complete a job, given that a child would finish the job in 20 days?

- a. 2 days b. $2\frac{1}{8}$ days c. 4 days d. 1 day e. None of these

Practice Exercise 7 - Level 3

1. A ship is moving at a speed of 30 km/hr. To know the depth of the ocean beneath it, it sends a radiowave which travels at a speed 200 m/s. The ship receives the signal after it has moved 500 m. What is the depth of the ocean?

- a. $\frac{\sqrt{143}}{2}$ km b. 12 km c. 6 km d. 8 km e. None of these

2. Two champion swimmers start a two-length swimming race at the same time, but from opposite ends of the pool. They swim at constant but different speeds. They first pass at a point 18.5 m from the deep end. Having completed one length, each swimmer takes a rest at the edge of the pool for 45 s. After setting off on the return length, the swimmers pass for the second time just 10.5 m from the shallow end. Thus, length of the pool is

- a. 90 m b. 60 m c. 26.5 m d. 40 m e. 45 m

3. The ratio between the speeds of A and B is 2 : 3 and therefore A takes 10 min more than B to reach a destination. If A had travelled at double the speed, he would have covered the distance in

- a. 30 min b. 25 min c. 15 min d. 20 min e. 27 min

4. In a kilometre race, if A gives B a 40 m start, A wins by 19 s. But if A gives B a 30 s start, B wins by 40 m. Find the time taken by B to run 5,000 m?

- a. 150 s b. 450 s c. 750 s d. 825 s e. 650 s

5. Jardin walks 36 km partly at a speed of 4 km/hr and partly at 3 km/hr. If he had walked at a speed of 3 km/hr for the same time required to travel the distance that he had walked at 4 km/hr and walked at 4 km/hr for the same time required to travel the distance that he had walked at 3 km/hr, then he would have walked only 34 km. The time spent by Jardin in walking was

- a. 8 hr b. 12 hr c. 10 hr d. 5 hr e. 9 hr

Direction for questions 6 and 7:

Answer the questions based on the following information.

Sumit left for Kanpur by car. Having travelled 420 km that is 87.5% of the distance, he was stopped due to a traffic jam. The jam was cleared in 9 min. Sumit then increased his speed by 20 km/hr and reached his destination 6 min earlier than he had anticipated.

6. What was the initial speed of the car?

- a. 80 km/hr b. 75 km/hr c. 72 km/hr d. 60 km/hr e. 90 km/hr

7. What is the average speed of the car over the entire journey?

- a. 64 km/hr b. 60 km/hr c. 72 km/hr d. 75 km/hr e. None of these

8. A and B starts together from the same point on a circular track and walk in the same direction till they both again arrive together at the starting point. A completes one complete round in 224 s and B in 364 s. How many times will A have passed B?

- a. 4 b. 5 c. 6 d. 7 e. 8

9. Garfield was a very lazy person. When he went for hunting a job, everyone refused to engage him, except farmer Mehnat Singh. Mehnat Singh engaged Garfield at a wage of Rs. 300 per working day for a month of 28 days. However, he set a condition that Garfield would forfeit Rs. 100 each day if he sits idle. Garfield accepted the job. At the end of the

month, it was found that neither owed the other anything. How many days did Garfield work in that month?

- a. 14 days b. 12 days c. 21 days d. 18 days e. 7 days

10. A can produce one unit in 15 days, while B can do the same in 12 days. After producing one unit, working together, they received Rs. 90, which they distributed amongst themselves in proportion to their efficiency. If they work for 20 days, and sell the produce, then B should receive

- a. Rs. 120 b. Rs. 140 c. Rs. 180 d. Rs. 160 e. Rs. 150

11. Twelve persons go on a picnic. They carry 12 bottles of beer with them, one for each; 2 person mysteriously disappear. So the remaining 10 distributed 2 bottles of beer among themselves equally. After they had consumed 50% of the share, those 2 persons arrive. What fraction of their remaining share would each have to sacrifice in order to give those two their original share?

- a. $\frac{1}{3}$ b. $\frac{1}{4}$ c. $\frac{1}{6}$ d. $\frac{1}{5}$ e. $\frac{1}{2}$

12. Pipes P and Q can fill a tank in 12 min and 16 min respectively. Both are kept open for x min and then Q is closed and P fills the rest of the tank in 5 min. The time x after which Q was closed is

- a. 3 min b. 2 min c. 6 min d. 4 min e. 7 min

13. In 15 days, 9 men and 15 boys can reap a field. In what time could 15 men and 16 boys reap the same field, assuming 3 men can do as much work as 4 boys?

- a. 25 days b. 20 days c. 11.25 days d. 10.5 days e. 12.8 days

14. If 33 untrained labourers can do a work in 15 days of 12 hr each, how many trained labourers can do 50% more work in 11 days of 9 hr each? (It may be assumed that 2

trained labourers do the same amount of work done by 5 untrained labourers in the same time.)

- a. 42 b. 36 c. 90 d. 100 e. 64

15. Pipe A takes 16 min to fill a tank. Pipes B and C cross-sectional circumference is in the ratio 2 : 3. If A has a cross-sectional circumference that is one-third of C, how long will it take for B and C together to fill the same tank? (Assume the rate at which water flows through a unit cross-sectional area is same for all the three pipes.)

- a. $5\frac{1}{13}$ min b. $3\frac{1}{13}$ min c. $1\frac{3}{13}$ min d. $\frac{32}{13}$ min e. $3\frac{3}{13}$ min

16. Ten men, 8 women and 6 boys together can complete a piece of work in 25 days. For how many days will 19 men, 20 women and 30 boys be occupied with three times the work? The work done in a particular time by a man, a woman, and a boy are in the ratio 5 : 3 : 2.

- a. 28 b. 25 c. 30 d. 32 e. 27

Learning Outcomes :

Fill in the blanks using the variables defined in each question and any mathematical operator.

1. If the distance becomes n times (of the usual distance) and the time becomes m times (of the usual time), we can deduce the speed has become _____ times of the usual speed.

2. If one travels at $\frac{x}{y}$ th of regular speed and is t minutes early/late, the usual time taken is _____

3. If ratio of distance covered by A, B and C is a : b : c in time that are in ratio x : y : z respectively, the ratio of speed of A, B and C is _____

4. The time taken by a train of length a and running at speed x to cross another train of length b and running at speed y is _____

5. If S is speed of a stream, B is speed of boat in still water, U is speed of boat moving upstream and D is speed of boat travelling downstream, then

$$U =$$

$$D =$$

$$B = \frac{_}{2} \quad S = \frac{_}{2}$$

6. A train leaves Mumbai for Kolkatta and simultaneously a train leaves Kolkatta for Mumbai. If they meet at Nagpur and distance between Mumbai and Kolkatta is x kms and that between Mumbai and Nagpur is y kms, the ratio of speed of train from Kolkatta and that of train from Mumbai is _____

7. If in t minutes, A runs x rounds of a circular track, B runs y rounds of the same circular track, then they meet for the first time after _____ minutes. (If they are running in the same direction, starting from the same point and at the same time)

8. If A takes a days to build a wall and B takes b days to build a wall, together they would build the wall in _____

9. If A can run around a circular track in a minutes and B can run around the same circular track in b minutes, and they start simultaneously from same starting point, they will meet after _____ for the first time at the starting point.

10. If I travel equal distances at speed of x and y, the average speed for the entire journey is _____

11. If A and B start from opposite ends simultaneously and take t_a and t_b minutes respectively after meeting to reach their respective ends then the ratio of speeds of A and B is _____

4. The time taken by a train of length a and running at speed x to cross another train of length b and running at speed y is _____

5. If S is speed of a stream, B is speed of boat in still water, U is speed of boat moving upstream and D is speed of boat travelling downstream, then

$$U =$$

$$D =$$

$$B = \frac{S + U}{2} \quad S = \frac{D - U}{2}$$

6. A train leaves Mumbai for Kolkatta and simultaneously a train leaves Kolkatta for Mumbai. If they meet at Nagpur and distance between Mumbai and Kolkatta is x kms and that between Mumbai and Nagpur is y kms, the ratio of speed of train from Kolkatta and that of train from Mumbai is _____

7. If in t minutes, A runs x rounds of a circular track, B runs y rounds of the same circular track, then they meet for the first time after _____ minutes. (If they are running in the same direction, starting from the same point and at the same time)

8. If A takes a days to build a wall and B takes b days to build a wall, together they would build the wall in _____

9. If A can run around a circular track in a minutes and B can run around the same circular track in b minutes, and they start simultaneously from same starting point, they will meet after _____ for the first time at the starting point.

10. If I travel equal distances at speed of x and y, the average speed for the entire journey is _____

11. If A and B start from opposite ends simultaneously and take t_a and t_b minutes respectively after meeting to reach their respective ends then the ratio of speeds of A and B is _____

Answer Key

Problems for Practice (Non MCQ)

Level - I

1. 10 s **2.** 22.5 kmph **3.** 15 hr

4. 45 min **5.** 60 kmph **6.** 12 kmph

7. 12 s **8.** 1 : 4 **9.** 20 : 23 : 27; 200 s

10. 125 s **11.** 93.75 s **12.** 9 days

13. $\frac{360}{7}$

Level - II

14. 13.5 min **15.** 15 km **16.** 365 km

17. 4 : 3 **18.** 250 km **19.** 429.2 kms

20. 8.75 hr **21.** 10 kmph **22.** 3 a.m

23. $\frac{28}{3}$, 28, never **24.** $\frac{16}{3}$ min **25.** 72 days

26. $\frac{720}{13}$ min **27.** 18th day **28.** 15th day

29. $25\frac{2}{3}$ days **30.** Cannot complete the job **31.** 25 days

32. $\frac{100}{11}$ min **33.** 25 min **34.** 6.5 hr

Level - III

35. 11 **36.** 10 **37.** 9

38. 5 or 6 **39.** $\frac{1}{2}$ m / s per sec, 67.5m **40.** 30 kmph, 40 kmph

41. $\frac{80}{3}$ kmph, 20 kmph **42.** 10 s, 2 m/s, 25 s **43.** $\frac{9}{14}$ min

44. 40 hr

Practice Exercise 1 - Level 1

1	d	2	a	3	c	4	d	5	d	6	b	7	b	8	d	9	b	10	a
11	d	12	a	13	e	14	b	15	b	16	e	17	c	18	d	19	a	20	b



Practice Exercise 2 - Level 1

1	c	2	d	3	a	4	d	5	c	6	a	7	a	8	c	9	b	10	c
11	a	12	b	13	d	14	b	15	c	16	b	17	b	18	c	19	a		



Practice Exercise 3 - Level 2

1	c	2	e	3	d	4	a	5	d	6	b	7	a	8	e	9	b	10	c
11	d	12	b	13	c	14	d	15	d	16	d	17	a						



Practice Exercise 4 - Level 2

1	d	2	a	3	b	4	a	5	b	6	c	7	e	8	a	9	c	10	a
11	c	12	a	13	c	14	c	15	c	16	b	17	a	18	b	19	c	20	d
21	c	22	a	23	a	24	a												



Practice Exercise 5 - Level 2

1	d	2	b	3	c	4	d	5	d	6	c	7	e	8	c	9	a	10	e
11	a	12	e	13	c	14	b	15	d	16	d	17	a	18	c	19	d	20	c



Practice Exercise 6 - Level 3

1	a	2	b	3	c	4	d	5	d	6	b	7	c	8	e	9	a	10	a
11	a	12	c	13	b	14	c	15	b	16	b	17	d	18	a	19	b	20	d



Practice Exercise 7 - Level 3

1	a	2	e	3	c	4	c	5	c	6	d	7	e	8	b	9	e	10	e
11	a	12	d	13	c	14	b	15	c	16	c								



Explanations: Time, Speed & Distance

Problem for Practice (Non MCQ)

Level - I

1. Distance to be covered = 100 m.

Speed of the person = 36 km/hr = 10 m/s.

$$\text{Therefore, time taken} = \frac{100 \text{ m}}{10 \text{ m/s}} = 10 \text{ s.}$$

2. One revolution means circumference of the wheel = 250 cm = 2.5 m.

In 10 revolutions 25 m is covered by the wheel.

$$\text{Speed of the wheel} = \frac{25}{4} \text{ m/s} = \frac{25}{4} \times \frac{18}{5} \text{ km/hr} = 22.5 \text{ km/hr}$$

3. Method 1:

The man takes 2 hr to travel 14 km, where he stops for 10 min. Therefore, time between the start of journey and start after first stoppage = 2 hr 10 min.

This pattern of his travel repeats. Now, 98 km

= 14 × 7. But when he completes 98 km, the case of his stopping for 10 min does not arise. Therefore, total time taken = 6 (2 hrs 10 min.) + 2 hr = 15 hr

Method 2:

If the man had travelled non-stop he would have covered the distance in 14 hr. But since he stops

6 times on the way he would take an extra $6 \times 10 = 60$ min to cover the same distance. Hence, he would reach the destination after 15 hr.

4. Let the train increase its speed after 't' hours.

Total time taken by the train = 1 hr 15 min = 1.25 hr.

$$\text{Therefore, } 40t + 50(1.25 - t) = 55 \text{ km.}$$

Or $t = 45$ min.

$$5. \text{ Average speed} = \frac{2 \times 75 \times 50}{[75 - 50]} = 60 \text{ km/hr}$$

6. Total relative distance travelled by the car = [40 + 60] m = 100 m.

$$\text{The relative speed} = \frac{100 \text{ m}}{20 \text{ sec}} = 5 \text{ m/s}$$

The relative speed is also equal to $(30 - v)$ km/hr = $(30 - v) \times \left(\frac{5}{18}\right)$ m/s, where 'v' is the speed of the bus.

Therefore, $(30 - v) \times \frac{5}{18} = 5$ or, $v = 12$ km/hr.

7. Speed of first train = $\frac{200}{10}$ m/s = 20 m/s.

Speed of second train = $\frac{200}{15}$ m/s = $\frac{40}{3}$ m/s.

Relative speed of the two trains (when they move in opposite directions) = $20 + \frac{40}{3} = \frac{100}{3}$

Time taken by the two trains to completely pass each other = $\frac{200 + 200}{100/3} = 12$ s

8. Speed downstream = $v + u$ (where, v = Speed of the boat in still water, and u = Speed of the stream).

Speed upstream = $v - u$.

Distance downstream = Distance upstream

$(v + u)45 = (v - u)75$ or, $u : v = 1 : 4$.

9. a. Ratio of distances = Ratio of speeds = 20 : 23 : 27.

b. Let us calculate the time of meeting of A and B for the first time. A will meet B after

$$\frac{200}{23-20} = \frac{200}{3} \text{ s.}$$

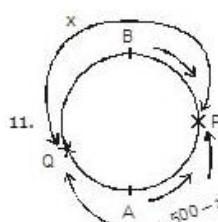
This means that A and B meet every $\frac{200}{3}$ s.

Similarly, B meets C every $\frac{200}{27-23} = \frac{200}{4} = 50$ s.

Therefore, LCM of these time periods will give us the time when all the three meet the first time which is $\text{LCM}\left(\frac{200}{3}, 50\right) = 200$ s or (3 min. 20 s).

10. A's speed is 2 m/s with respect to B. (i.e. relative speed)

So to catch B he has to overcome 250 m because B is 250 m ahead of A. So A will take = $\frac{250}{2} = 125$ s.



At the first time they will meet at point P. After that they meet for the second time at point Q. Up to point P distance travelled by A and B is add up to 250 m. After that when they meet at Q together they have travelled one circle. So total distance travelled by them is 750 m [250 + 500]. So time taken by them to cover this much distance is $(5 + 3)t = 750$

$$t = \frac{750}{6} = 93.75$$

12. They will finish 100% of the work in $\frac{20 \times 30}{20+30} = 12$ days.

Therefore, 75% of the work will be done in 75% of 12 = 9 days.

13. Method 1:

Let a, b and c be the times taken by A, B and C to do the work alone.

$$\text{Also let } x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c}.$$

$$\text{Therefore, } x + y = \frac{1}{20}, y + z = \frac{1}{15}, z + x = \frac{1}{18}.$$

$$\text{From the last two equations, } y - x = \frac{1}{15} - \frac{1}{18} = \frac{1}{90}.$$

$$\text{Therefore, } 2x = \frac{1}{20} - \frac{1}{90} = \frac{7}{180},$$

$$\text{which means } x = \frac{7}{360} \text{ or } a = \frac{360}{7} \text{ days.}$$

Method 2:

Assume that the number of units of work is 180 units.

A and B put in 9 units per day B and C put in 12 units per day; and A and C put in 10 units per day.

So A, B and C working together put in $\frac{(9+10+12)}{2}$ units per day, i.e. 15.5 units per day.

So A alone puts in $15.5 - 12 = \frac{7}{2}$ units per day.

So he takes $180 \times \frac{2}{7} = \frac{360}{7}$ days to complete the job working alone.

Level - II

14. Let the distance travelled by the man be x km.

Therefore, time taken at a speed of 10 km/hr = $\frac{x}{10}$ hr.

Time taken at a speed of 20 km/hr = hr.

$$\text{Hence, } \frac{x}{10} - \frac{x}{20} = \frac{45}{60}$$

Solving, we get $x = 15$ km.

$$\text{Therefore, } \frac{x}{25} - \frac{x}{40} = \frac{15}{25} - \frac{15}{40} = 0.225 \text{ hr} = 13.5 \text{ min}$$

15. Let us say 't' hr is the usual time. When Sanju moves at 10 km/hr, time of travel = $t + \frac{6}{60}$

When he moves at 12 km/hr, time of travel = $t - \frac{9}{60}$

$$\text{Difference in time taken in the two cases} = \left[t + \frac{6}{60} \right] - \left[t - \frac{9}{60} \right] = \frac{1}{10} + \frac{3}{20} = \frac{1}{4} \text{ hr}$$

If x km is the distance between school and house, difference in time taken = $\frac{x}{10} - \frac{x}{12} = \frac{x}{60}$

$$\text{Therefore, } \frac{x}{60} = \frac{1}{4} \text{ hr. or } x = 15 \text{ km.}$$

16. Ratio of distance = Ratio of speed = 32 : 41.

Therefore, distance travelled by both the trains = $32x$ and $41x$.

According to given condition, $41x - 32x = 45$ km

Therefore, $x = 5$. Hence, distance between A and B = $41x + 32x = 365$ km

17. Let the trains meet at some place M between A and B. Time taken from A to M = time taken from B to M.

Therefore, ratio of speed = ratio of distance = 4 : 3.

18. Let x be the distance between A and B.

The train which leaves A travels a total distance of $x + (x - 200) = 2x - 200$.

The train which leaves B travels a total distance of $x + 200$.

The ratio of distances travelled by the trains = Ratio of the speeds.

$$\text{Therefore, } (2x - 200) : (x + 200) :: 40 : 60$$

Or $x = 250$ km.

19. Method 1:

Let the total distance between A and B be x .

Since B had to travel 145 km to reach A, this means they met at a distance of 145 km from A.

Hence, distance between the meeting place and B = $(x - 145)$ km.

Ratio of speed = Ratios of distance

$$25 : 49 :: 145 : (x - 145).$$

Therefore, $x = 429.2$ km.

Method 2:

$$x = \left\{ \frac{(49 + 25)}{25} \right\} \times 145 = 429.2 \text{ km}$$

20. A takes a lead of $(3.5 \text{ km/hr}) (2.5 \text{ hr}) = 8.75 \text{ km}$ over B.

Hence, when B starts he has to travel a relative distance of 8.75 km and at a relative speed of $(4.5 - 3.5) \text{ km/hr} = 1 \text{ km/hr}$, which means B will take 8.75 hr to catch up and then overtake A.

21. Normal day it travels a distance = $25(50 + u)$

On that particular day it completes $\frac{1}{2}$ the journey in same time (12.5 hr).

But for remaining journey, due to engine problem it takes $2\frac{1}{2}$ hr extra.

So it takes 15 hr to complete the remaining half journey.

$$\text{So, } 15(40 + u) = \frac{1}{2} \times 25(50 + u)$$

$$30(40 + u) = 25 \times 50 + 25u$$

$$30u - 25u = 1250 - 1200$$

$$5u = 50; u = 10 \text{ km/hr}$$

22. The thief travels for 2 hr (7.00 p.m. to 9.00 p.m.) and takes a lead of $(4.5 \text{ km/hr})(2\text{hr}) = 9 \text{ km}$.

The policeman is required to cover up a distance of 9 km at a relative speed of

1.5 km/hr. Therefore, the policeman will take 6 hr after 9 p.m. to catch up with the thief, i.e. at 3 a.m.

23. Ratio of speed of Bhim and Arjun = 7 : 4. (Since the ratio of the times = 4 : 7)

a. If the length of circular track = 28m, the speeds of Bhim and Arjun are 7 and 4 m/min.

The time when they are together the first time will be when Bhim (the faster one) has taken one round more than Arjun (the slower one).

Therefore, if time when they meet is 't', then $7t - 4t = 28$, which means $t = \frac{28}{3}$ min.

b. They will meet at the starting place the first time at a time which is the LCM of the times each one of them takes to reach the starting place. Therefore, LCM of 4 and 7 is 28 min.

c. Diametrically opposite point is at a circular distance of 14 m. Bhim reaches this point in $\frac{14}{7} = 2$ min and Arjun reaches this point in

$\frac{14}{4} = 3.5$ min. Bhim reaches this point in the 2nd min, $2 + 4 = 6$ th min, $6 + 4 = 10$ th min ... so on. Arjun reaches after 3.5 min, 10.5 min, 17.5 min. so on.

The time after the start when Bhim reaches the point is a natural number, whereas the time when Arjun reaches this point will always be a non-natural number.

So they will never meet.

Alternatively for (a) and (b):

If the time when they would meet for the first time at the starting point = LCM(4, 7) = 28 min, in this time Bhim does 7 rounds and Arjun completes 4 rounds. So Bhim takes a lead of 3 rounds. Hence, he would take $\left(\frac{28}{3}\right)$ min to take a lead of one round. This is the time when they would meet for the first time.

24. When Bhim gives Arjun a lead of 4 min, in 4 min

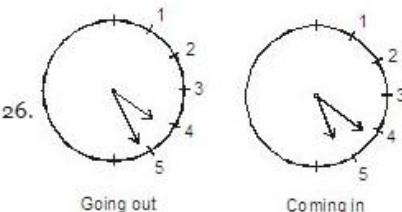
Arjun travels 16 m. Bhim now starts running towards Arjun at a relative speed of 3 m/min. To catch up with a distance of 16 m, Bhim takes $\frac{16}{3}$ min.

25. In one day, the time gap between the two faulty watches is $6 + 4 = 10$ min.

The two watches will show the same time next when the faster watch takes a lead of exactly

12 hr over the slower. This would happen after $\frac{1 \times 12 \times 60}{10} = 72$ days. In 72 days the faster watch would gain $\frac{72 \times 4}{60} = 4.8$ hr = 4 hr and 48 min.

So the time according to the faster watch would be 7:48 p.m. and the slower would be 12 hr behind the faster but would also show the same time.



So the hour hand and minute hand move 360° in this time (together).

As the speed of the hour hand = $\frac{1}{2}^\circ$ per minute and speed of minute hand = 6° per minute.

Let t is the required time in minutes.

$$\text{So } 6t - \frac{1}{2}t = 360^\circ$$

$$\frac{13}{2}t = 360^\circ \quad t = \frac{720}{13} \text{ min.}$$

27. Let us say total work is of 20×20 units.

Each woman does 1 unit per day.

Each man does 2 units per day while each boy does $\frac{1}{2}$ unit per day.

So on first day 20 units are completed.

On second day 21 units, on third day 20.5 units and on fourth day 21.5 units are completed.

Combining two terms, we get $41 + 42 + 43 + \dots + 48 + 49 = 405 > 400$.

So there are 9 such pairs. So on the 18th day work will be complete.

28. Let the total work be of 400 units.

So a woman does 1 unit per day,

while a man does 2 units per day.

So work done on first day = 20 units

and work done on second day = 21 units.

Thus, the pattern goes like $20 + 21 + 22 + \dots + 30 = 275$

$275 + 31 + 32 + 33 = 371$ up to 13th day, 371 units has been completed. On 15th day job will be complete.

29. C can start the work on first day but there is nothing to demolish then. So this day is wasted.

The work effectively starts from second day onwards.

Let the total work be of $\text{LCM}(10, 15, 20) = 60$ units.

Now A can do 6 units in a day.

B can do 4 units in a day.

C can demolish 3 units in a day.

So in 3 days (starting from A) $6 + 4 - 3 = 7$ units will be complete. So in 8×3 days, $8 \times 7 = 56$ units of work will be done by A, B and C. The rest 4 units will be done by A in $\frac{2}{3}$ day. So

total number of days $= 1 + 8 \times 3 + \frac{2}{3} = 25\frac{2}{3}$ days.

30. Let the total job be of $\text{LCM}(10, 15) = 30$ units.

A + B can do 3 units per day.

B + C can do 2 units per day.

A + B + C can do 5 units a day.

A + 2B + C can do 5 units.

Here it can be clearly observed that B's share in the total work is null. Hence, he cannot complete the job by himself.

31. As A alone can do the job in 10 days and A and B together also taking 10 days to complete the job. So $100\% \text{ of } A = 60\% \text{ of } A + B$'s efficiency. So B will take $\frac{10}{0.4} = 25$ days.

32. Let the leakage take 't' minutes to empty the tank completely. When all the three pipes (two inlets and one leakage) are open, then $\frac{1}{10} + \frac{1}{20} - \frac{1}{t} = \frac{1}{25}$
or, $t = 100/11 = 9.09$ minutes (approximately).

33. Method 1:

The outlet pipe will discharge the water in $\frac{2000}{30} = \frac{200}{3}$ minutes alone.

Let the inlet pipe take 't' minutes to fill the tank alone.

Then $\frac{1}{t} - \frac{3}{200} = \frac{1}{40}$. Therefore, $t = 25$ min.

Method 2:

The net inflow is 50 l per minute. Hence, the actual inflow = 80 l per min ($50 + 30$). So the inlet pipe will take $\left(\frac{2000}{80}\right) = 25$ min to fill the tank independently.

34. A → First pipe

B → Second pipe

C → Third pipe

A can fill in t hr.

B can fill in $2t$ hr

C can fill in $\frac{t+2t}{2} = \frac{3}{2}t$ hr.

So,

$$\frac{1}{t} + \frac{1}{2t} - \frac{2}{3t} = \frac{1}{3}$$

$$\frac{1}{t} \times \left(1 + \frac{1}{2} + \frac{2}{3}\right) = \frac{1}{3}$$

$$\frac{1}{t} \times \left(\frac{13}{6}\right) = \frac{1}{3}$$

$$t = 6.5 \text{ hr.}$$

Level - III

35. At 10 a.m. a train from station B is just coming to station A. That train must have started at 5 a.m. from station B. The train that starts at 10 a.m., will reach at 3 p.m. at station B.

So from 5 am to 3 p.m. every train started from station B will cross the train. So total number of train = 11.

36. From A a train is starting at 10 a.m. So from B every train will start at an (integer + $\frac{1}{2}$) hr. interval i.e. 5 : 30, 6 : 30, 7: 30 and so on.

So the 10 a.m train from A will meet first train which is suppose to reach at 10 : 30 starting from B at 5 : 30.

Also, 10 a.m. train from A will reach station B at 3 p.m. So the last train it will cross is the one started at 2 : 30 p.m. from station B. So from 5 : 30 to 2 : 30, we have a total of 10 trains.

37. So at 10 a.m. a train is just reaching station A that started from B at 7 a.m. So, the train that left at 10 a.m. from A will reach at 3 p.m. at B. Thus, from 7 a.m. to 3 p.m. each train will cross the train from B that started at 10 a.m.

So total number of trains = $8 + 1 = 9$ trains.

38. In this case we cannot find the exact number of trains crossing because we do not know the starting time of any train started from station B.

May be possible it will reach station A at 10 a.m. or at 11 a.m. If it reaches at 10 a.m. to station A, then it starts at 5 a.m. and 10 a.m. train will reach at 3 p.m. So total number of trains crossing.

= 5 a.m., 7 a.m., 9 a.m., 11 a.m., 1 p.m., 3 p.m. = 6

Or if train started from B reach A at 11 am should start from B at 6 a.m. So number of crossing trains:

= 6 a.m., 8 a.m., 10 a.m., 12 a.m., 2 a.m. = 5 trains

$39.36 \text{ km/hr} = 10 \text{ m/s}$

$18 \text{ km/hr} = 5 \text{ m/s}$

It is mentioned that the reduction in speed of the faster truck = 1 m/s per second. Note that the brakes are applied successively.

Thus, the faster truck travels 9 m in the 1st second, 8 m in the 2nd second, 7 m in the 3rd second and so on.

Final speed of the faster truck = 0 m/s

= Truck stops at the 10th second.

Since both the trucks stop simultaneously, speed of slower at 10th second = 0 m/s

If the reduction in speed of the slower truck = 'a' m/s per second, $(5 - 10a) = 0$

$= a = \frac{1}{2} \text{ m/s per second.}$

Thus, reduction in the speed of the slower truck = $\frac{1}{2} \text{ m/s per second.}$

Alternative Method:

Ratio of speeds = $18 : 36 = 1 : 2$

Since both the trucks are equally efficient, the reduction in the speed of the slower truck must be $\frac{1}{2}$ m/s per second

Now, distance travelled by the first truck in 10 seconds = $(9 + 8 + 7 + 6 + \dots + 1 + 0) = 45$ m

Distance travelled by the second truck in 10 seconds = $(4.5 + 4 + 3.5 + 3 + \dots + 0.5 + 0) = 22.5$ m

Thus, net separation between the trucks when they notice each other = $45 + 22.5 = 67.5$ m

$$40 \cdot 3t = 6$$

$$\text{Hence, } t = 2$$

Distance travelled by bus is equal to area under the curve, i.e. $\frac{1}{2} \times V_{m_1} \times 2 + V_{m_1} \times 2 + \frac{1}{2} \times V_{m_1} \times 2$

$$= 4V_{m_1} [V_{m_1} = \text{Bus (maximum speed)}]$$

Distance travelled by jeep.

$$\frac{1}{2} V_{m_2} \times 2 + V_{m_2} \times 2 + \frac{1}{2} \times V_{m_2} \times 2 = 4V_{m_2} [V_{m_2} = \text{jeep's maximum speed}]$$

$$\text{So } 4V_{m_1} + 4V_{m_2} = 280$$

$$V_{m_1} + V_{m_2} = 70$$

$$V_{m_1} - V_{m_2} = 10$$

$$V_{m_1} = 40 \text{ km/hr}$$

$$V_{m_2} = 30 \text{ km/hr}$$

$$41. \text{ Average speed by bus} = \frac{\text{Distance travelled by bus}}{\text{Time taken by bus}} = \frac{4V_{m_1}}{6} = \frac{2}{3} \times 40 = \frac{80}{3} \text{ km/hr}$$

$$\text{Average speed by jeep} = \frac{\text{Distance travelled by Jeep}}{\text{Time taken by jeep}} = \frac{4V_{m_2}}{6} = \frac{2}{3} \times 30 = 20 \text{ km/hr}$$

42. As distance covered by faster one is 4 times the distance covered by slower one while their speed ratio is 3 : 2. So faster one is moving downstream and slower one is moving upstream.

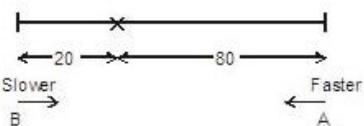
$$\text{So } \left(\frac{6+u}{4-u} = \frac{4}{1} \right)$$

They will meet first time in

$$(6+u)t + (4-u)t = 100;$$

$$\Rightarrow t = 10 \text{ s.}$$

For second time meeting, faster is coming back to its starting point after reaching starting point of slower one.



20 m will cover by A to reach slower's starting point. So he will take $\frac{20}{8} = 2.5$ s to reach B.

In this time slower one has cover $2 \times 2.5 = 5$ m.

Now, A has to cover $(20 + 5 = 25)$ m with respect to B to catch B. Now A is also moving upstream so his speed is now $6 - 2 = 4$ m/s. So relative speed of A with respect to B is $4 - 2 = 2$ m/s.

So A will take $\frac{25}{2} = 12.5$ s.

So in all $= 10 + 2.5 + 12.5 = 25$ s.

They will meet second time after 25 s.

43. Rate of flow for the first pipe $= \lambda(1)^2 = \lambda$.

Rate of flow for the second pipe $= \lambda(2)^2 = 4\lambda$.

Rate of flow for the third pipe $= \lambda(3)^2 = 9\lambda$.

Since rates are inversely proportional to time taken by each to fill the tank, time taken by the 3 pipes are 9 min, $\frac{9}{4}$ min and 1 min respectively. Therefore, when all the three pipes are opened, the fraction of the tank filled in 1 min

$$= \frac{1}{9} + \frac{4}{9} + \frac{1}{1} = \frac{14}{9}$$

The time taken by all the pipes to fill the tank is

$$\frac{9}{14} \text{ min. } (\lambda \text{ is the constant of proportionality})$$

44. Tap 1 can fill the tank in 10 hr.

$$\text{So tap 1 gives } \frac{40}{10} = 4 \text{ lit/hr.}$$

$$\text{Similarly, tap 2 leaks } = \frac{40}{20} = 2 \text{ lit/hr.}$$

$$\text{Evaporation which is similar to a leakage } = \frac{2.5}{100} \times 40 = 1 \text{ lit/hr.}$$

So total leakage is 3 lit/hr.

So added water in an hour is $4 - 3 = 1$

So it will take 40 hr.

Practice Exercise 1 - Level 1

1. d (a) Distance = Speed × Time = $48 \times 10 = 480$ km

(b) To cover the same distance in 8 hr.

$$\text{Speed} = \frac{d}{t} = \frac{480}{8} = 60 \text{ km/hr}$$

∴ Speed must be increased by $60 - 48 = 12$ km/hr

2. a Let his usual time be t hr and his usual speed be s km/hr.

$$\text{Distance, } d = st = \frac{3}{4} s \times (t + 2.5)$$

$$\text{Or, } 4t = 3t + 7.5$$

$$t = 7.5 \text{ hr}$$

3. c By the time the trains crossed each other, one of them had covered 110 km and the other had covered 90 km.

$$\text{Ratio of speeds} = \text{Ratios of the distances covered} = \frac{110}{90} = 11 : 9$$

4. d Since ratio of speeds of A : B = 3 : 4. So, ratio of time taken would be 4 : 3. If A takes 30 min more, then $4x - 3x = 30$ min = $x = 30$ min.

A takes 4x min, or $4 \times 30 = 120$ min = 2 hr.

5. d $A = 2B$, $B = 2C$. C takes 54 min.

So, B would cover the same distance in half of the time = $\frac{54}{2} = 27$ min.

So, time taken by A = $\frac{27}{2}$ min = 13.5 min.

$$6. b \frac{s_A}{s_B} = \frac{3}{4}$$

Speed of A = $3x$ km/min

Speed of B = $4x$ km/min

distance is same.

$$d = s_A \times t_A = s_B \times t_B$$

$$3x \times t_A = 4x \times 36$$

$$t_A = 48 \text{ min}$$

$$7. b \text{ Average speed} = \frac{2xy}{x+y} = \frac{2 \times 24 \times 36}{24+36} = \frac{144}{5} = 28.8 \text{ km/hr}$$

$$8. d \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{2500 + 1200 + 500}{\frac{2500}{500} + \frac{1200}{400} + \frac{500}{250}} = \frac{4200}{10} = 420 \text{ kmph}$$

\therefore in 1 hour A does $\frac{1}{63}$ of work

9. b To cross the platform the train has to cover

$$165 + 110 = 275 \text{ m.}$$

B takes $6 \times 7 = 42 \text{ hr.}$

$$132 \text{ km/hr} = 132 \times \frac{5}{18} = \frac{110}{3} \text{ m/sec}$$

\therefore in 1 hr B does $\frac{1}{42}$ of work

$$t = \frac{275}{\frac{110}{3}} \times 3 = 7.5 \text{ sec}$$

$$A + B \text{ in 1 hr} = \frac{1}{63} + \frac{1}{42}$$

10. a $t = 10 \text{ sec.}$

$$A + B \text{ in } \frac{42}{5} \text{ hr} = \left(\frac{1}{63} + \frac{1}{42} \right) \times \frac{42}{5} = 0.33$$

$$\text{Speed} = 90 \text{ km/hr} = 90 \times \frac{5}{18} = 25 \text{ m/s}$$

\therefore Number of days $= \frac{1}{0.33} = 3$

$$\text{Length} = s \times t = 25 \times 10 = 250 \text{ m}$$

13. e In 1 day, A does $\frac{1}{80}$ part

11. d Total length to be covered = Length of train + Length of platform = $900 + 300 = 1200 \text{ m}$

\therefore 10 days' work $= \frac{10}{80} = \frac{1}{8}$ part

$$\text{Total time} = 60 + 12 = 72 \text{ sec}$$

$$\text{Work to be done by B} = \left(1 - \frac{1}{8} \right) = \frac{7}{8}$$

$$\text{Speed} = \frac{1200}{72} = \frac{50}{3} \text{ m/s} = \frac{50}{3} \times \frac{18}{5} = 60 \text{ km/hr}$$

Let 'B' do 'x' part of the work in 1 day

12. a A takes $7 \times 9 = 63 \text{ hr.}$

$$x \times 42 = \frac{7}{8}, x = \frac{7}{42 \times 8} = \frac{7}{336}$$

$$\text{Number of days required by B} = \frac{336}{7} = 48 \text{ days}$$

$$A + B = \frac{1}{80} + \frac{1}{48} = \frac{128}{80 \times 48}$$

$$\therefore \text{Number of days} = \frac{80 \times 48}{128} = 30 \text{ days}$$

14. b Let us assume that work done by A and B in 1 day are 2 and 1 unit respectively.

Work done in 1 day by $(A + B) = 3$ units

\therefore Work done in 14 days by $(A + B) = 14 \times 3 = 42$

\therefore A finishes the work in $= \frac{42}{2} = 21$ days

15. b Assume total wages $= 21 \times 28$

A's wages for 1 day $= 21 \times 28 / 21 = 28$

B's wages for 1 day $= 28 \times 21 / 28 = 21$

A + B's wages of 1 day $= 49$.

A + B's wages will last for $21 \times 28 / 49 = 12$ days

16. e Assume that units to fill up = LCM of $(20, 15, 12) = 60$.

Units filled up by A, B, C in 1 min $= 3, 4, 5$ respectively.

Time taken by $(A + B + C)$ to fill up $= \frac{60}{12} = 5$ min.

17. c Assume that capacity of the tank = LCM $(10, 12, 20) = 60$ units.

Rate of A, B, C per hr $= \frac{60}{10}, \frac{60}{12}, \left(\frac{-60}{20}\right) = 6, 5$ and (-3) units respectively.

$\therefore A + B = 11$ units, $C = -3$ units

\therefore Per hr intake $= 11 - 3 = 8$ units

Time to fill up $= 60 / 8 = 7.5$ hr.

18. d Capacity in units = LCM $(9, 10) = 90$.

Inflow, when there is no leak $\frac{90}{9} = 10$ units per hr.

Inflow, when there is a leak $= \frac{90}{10} = 9$ units per hr.

\therefore Resultant outflow $= 10 - 9 = 1$ unit per hr.

Total outflow $= 90$ units, Min. $= 90 / 1 = 90$ hr.

19. a Say, capacity of tank = LCM $(12, 15) = 60$ units

1 min total intake in units $= \frac{60}{12} + \frac{60}{15} = 9$

3 min total intake in units $= 9 \times 3 = 27$

Units to be filled $= 60 - 27 = 33$

17. c Assume that capacity of the tank = LCM (10, 12, 20) = 60 units.

Rate of A, B, C per hr = $\frac{60}{10}$, $\frac{60}{12}$, $\left(\frac{-60}{20}\right)$ = 6, 5 and (-3) units respectively.

$\therefore A + B = 11$ units, $C = -3$ units

\therefore Per hr intake = $11 - 3 = 8$ units

Time to fill up = $60/8 = 7.5$ hr.

18. d Capacity in units = LCM (9, 10) = 90.

Inflow, when there is no leak $\frac{90}{9} = 10$ units per hr.

Inflow, when there is a leak $\frac{90}{10} = 9$ units per hr.

\therefore Resultant outflow = $10 - 9 = 1$ unit per hr.

Total outflow = 90 units, Min. = $90/1 = 90$ hr.

19. a Say, capacity of tank = LCM (12, 15) = 60 units

1 min total intake in units = $\frac{60}{12} + \frac{60}{15} = 9$

3 min total intake in units = $9 \times 3 = 27$

Units to be filled = $60 - 27 = 33$

Time taken by B = $33/4 = 8.25$ (8 min 15 sec)

20. b $A + B = 72$ days, $B + C = 120$ days,

$A + C = 90$ days.

Let total units of work = 360 ... (=LCM (72, 120, 90))

$\therefore (A + B)$ do units of work in 1 day = $\frac{360}{72} = 5$.

$(B + C)$ do units of work in 1 day = $\frac{360}{120} = 3$.

$(A + C)$ do units of work in 1 day = $\frac{360}{90} = 4$.

$\therefore 2(A + B + C)$ units in 1 day = $5 + 3 + 4 = 12$.

$A + B + C$ units in 1 day = $\frac{12}{2} = 6$

$\therefore A$ units in 1 day = $6 - 3 = 3$

A will finish 360 units in $\frac{360}{3} = 120$ days.

Practice Exercise 2 - Level 1

1. c Total time = $60 + 24 = 84/60$ hr

At 4 km/hr $t_1 = \frac{2/3d}{4} = \frac{1}{6}d$ hr

At 5 km/hr $t_2 = \frac{1/3d}{5} = \frac{1}{15}d$ hrs

Total time = $\frac{d}{6} + \frac{d}{15} = \frac{84}{60}$ Or, $d = 6$ km

2. d Let normal speed be s km/hr.

Let normal time be t hr.

$$d = st \Rightarrow 40\left(t + \frac{11}{60}\right) = 50\left(t + \frac{5}{60}\right)$$

$$4t + \frac{44}{60} = 5t + \frac{25}{60}$$

$$t = 19/60 \text{ hr} = 19 \text{ min}$$

3. a The time taken to cover 240 km without any stops = 6 hr. Since he stops every 80 km, he would stop twice before reaching the destination.

Hence, total time taken = 6 hr 40 min.

4. d When Sujit runs 100 m, Rishi runs 95 m.

When Rishi runs 100 m, Praveen runs 95 m.

\therefore When Rishi runs 95 m, Praveen runs 90.25 m.

When Sujit runs 100 m, Praveen runs 90.25 m and is beaten by 9.75 m.

Alternative method:

The ratio of speeds of Sujit and Rishi = $100 : 95 = 20 : 19$.

Similarly, the ratio of speeds of Rishi and Parveen = $20 : 19$.

\therefore The ratio of speeds of Sujit and Parveen = $20^2 : 19^2$.

= When Sujit goes 100 m, Parveen goes

$$\frac{361}{400} \times 100 = 90.25 \text{ m.}$$

\therefore The lead that can be given is $100 - 90.25 = 9.75$ m.

5. c Suppose Pallavi takes t seconds to finish the race. It means Anuva and Richa would be taking $(t + 50)$ s and $(t + 90)$ s respectively to finish the race. When Pallavi runs 1,000 m, Richa will run 550 m

\Rightarrow Richa runs 1,000 m in $(t + 90)$ s.

So we have $\frac{t+90}{t} = \frac{1000}{550}$ of $t = 110$ s
 $= 110 + 90 = 200$ s.

6. a Take (+) for the person walking down and (-) for walking up. Therefore, his relative position from the starting point $= +4 - 3 + 6 - 2 - 9 + 2 = -2$.

Hence, the answer is 2 steps above the starting step.

7. a Since they move at the same time on different days, the situation is equivalent to the that of two persons where one moves uphill and the other downhill, both starting at the same time, viz. 7 a.m.

Ratio of speeds of uphill to downhill = Ratio of distances = $10 : 15 = 2 : 3$.

Therefore, $\frac{3}{5}$ of $25 = 15$ km from the top of the hill they will meet or $\frac{2}{5}$ of $25 = 10$ km from the bottom of the hill is the point where they will meet.

8. c $d = s \times t$

$\frac{1}{2}$ journey ($= 40$ km) in $\frac{3}{5}$ of time ($= 6$ hrs)

Total distance = 80 km

Total time = 10 hr

$80 = 40 + S_2 \times 4$, $S_2 = 10$ km/hr

9. b Here the total distance = 180 km.

Total time = $\frac{30}{45} + \frac{60}{90} + \frac{90}{75} = \frac{2}{3} + \frac{2}{3} + \frac{6}{5} = \frac{38}{15}$ hr.

Average speed = $180 - \frac{38}{15} = 71\frac{1}{19}$ km/hr.

10. c Length of train = 110 m,

Speed of train = $58 \times \frac{5}{18}$ m/s = 16.11 m/s

Speed of man = 4 km/hr = $4 \times \frac{5}{18}$ = 1.11 m/s

Time = $\frac{110}{16.11 - 1.11} = \frac{110}{15} = 7.33$ sec

11. a Length of first train = 108 m. Its speed = 50 km/hr

Length of second train = 112 m.

Let its speed = x

time to cross = 6 sec

$$\Rightarrow 6 = \frac{108 + 112}{(50 + x) \times \frac{5}{18}} = \frac{220 \times 18}{5(50 + x)}$$

$$250 + 5x = 660, 5x = 410$$

$$x = 82 \text{ km/hr}$$

12. b Length of the faster train = $(36 + 45) \times \frac{5}{18} \times 8 = 81 \times \frac{5}{18} \times 8 = 180 \text{ m}$

13. d Let x be the rowing speed of the man in still waters

Speed of the river = 2 km/hr = 'y'

Speed upstream = $x - y \text{ km/hr}$

Speed downstream = $x + y \text{ km/hr}$

$$(x - y) 2t = (x + y) t$$

$$2(x - y) = x + y$$

$$x = 6 \text{ km/hr}$$

14. b Let length of train be ℓ m and its speed be s m/s.

It crosses a pole in 15 sec

$$\therefore \ell = 15 \text{ s} \dots (\text{i})$$

It crosses a platform 100-m long in 25 sec

$$\therefore \frac{100 + \ell}{s} = 25 \dots (\text{ii})$$

Solving (i) and (ii)

$$100 + 15s = 25s$$

$$\therefore s = \frac{100}{10} = 10 \text{ m/s}, \therefore \ell = 150 \text{ m.}$$

15. c Let 'l' = Length of train in metres. Then speed of the train = $\frac{l}{15} \text{ m/s} = \frac{l-100}{30} \text{ m/s.}$

$$\therefore l = 100 \text{ m}$$

16. b 10 men in 20 days or 20 women in 15 days.

$\therefore 5$ men in 40 days and 10 women in 30 days.

$$= 5 \text{ men} + 10 \text{ women in } \left(\frac{1}{40} - \frac{1}{30} \right) \text{ days} = \frac{1200}{70} \text{ days} = 17 \frac{1}{7} \text{ days.}$$

17. b Let units of work = 18 units.

In one hour, man completes work = 9 units

$$(\text{=} 6 \text{ units} + 3 \text{ units})$$

\therefore Time taken = 2 hr.

$$18. c \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{15} \dots (\text{i})$$

$$\frac{5}{a} + \frac{25}{b} + \frac{25}{c} = 1$$

$$= \frac{1}{5a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{25} \dots \text{(ii)}$$

$$\text{or } \frac{1}{22} + \frac{1}{33} + \frac{1}{44} = \frac{1}{x}$$

$$\text{or } x = \frac{11 \times 12}{6 - 4 - 3} = \frac{132}{13} = 10 \frac{2}{13} \text{ days}$$

Subtracting (ii) from (i),

$$\frac{4}{5a} = \frac{2}{75} \Rightarrow a = 30 \text{ days}$$

Alternative method:

Let the total work be of 15 units. They complete this work in 15 days. It means one day work is 1 unit.

In 5 days, they will complete 5 units of work.

Remaining 10 unit of work, B and C complete in 20 days. It means one day work of (B + C) = $\frac{1}{2}$ unit

$$\Rightarrow A's \text{ one day work is } 1 - \frac{1}{2} = \frac{1}{2} \text{ unit.}$$

So total days taken by A alone to complete 15 units of work = 30 days.

$$19. \text{ a } \frac{1}{22} + \frac{1}{33} + \frac{1}{44} = \frac{1}{x}$$

when x = Number of days in which they together can finish the work.

Practice Exercise 3 - Level 2

1. c When he travels the entire journey at 5 km/hr, total distance travelled = $35 + 2 = 37$ km.

\therefore Time of travel = $\frac{37}{5} = 7.4$ hr, which is same as the time taken at the original rate.

2. e Let 'w' is the time of walking and 'r' is the time of riding

$$\therefore w + r = 7 \text{ hr.}$$

Riding both ways he gains 2 hr. Therefore, he takes 5 hr to ride both ways or 2.5 hr to ride one way.

$$\therefore r = 2.5 \text{ hr} = w = 4.5 \text{ hr.}$$

Time taken to walk both ways = $2 \times 4.5 \text{ hr} = 9 \text{ hr.}$

$$3. d \text{ Average speed} = \frac{(50)(1) + (48)(2) + (52)(3)}{6} = 50\frac{1}{3} \text{ km/hr.}$$

4. a Let speed of boat be x km/hr.

Let speed of stream be y km/hr.

$$\text{Rowing upstream} = \frac{13}{5} = x - y \dots (\text{i})$$

$$\text{Rowing downstream} = \frac{28}{5} = x + y \dots (\text{ii})$$

$$\text{Solving for } y, y = 1.5 \text{ km/hr}$$

$$5. d \text{ Let speed of boat in still waters be 'x' = 6 km/hr}$$

Let speed of stream be 'y'.

$$x - y = 4.5 \text{ km/hr}$$

$$= y = 1.5 \text{ km/hr}$$

$$\therefore \text{Speed along the stream} = 1.5 + 6 = 7.5 \text{ km/hr}$$

$$6. b \text{ Length of bridge} = 1 \text{ km}$$

$$\text{Length of train} = 0.5 \text{ km}$$

$$\text{Time to clear the bridge} = 2 \text{ min} = \frac{2}{60} \text{ hr}$$

$$\text{Speed} = \frac{1+0.5}{2/60} = \frac{1.5}{2} \times 60 = 45 \text{ kmph}$$

$$7. a \text{ Speed of stream} = 1 \text{ km/hr}$$

$$\text{Let speed of boat in still waters} = x \text{ km/hr}$$

$$\text{Total time} = 12 \text{ hr}$$

$$12 = \frac{35}{x-1} - \frac{35}{x+1}$$

Now put the value of x and check the options, only (a) satisfies.

8. e If the person had not moved towards the source of the gunfire, he would have heard the second shot 12 min after the first shot. Since the person is actually moving towards the source, the shot is now heard after 10 min. It means the sound would have taken 2 min more to reach the initial position of the person; but this very distance was travelled by the train in 10 min. It means it is the speed of the train in 10 min. It means speed of the train is $\frac{1}{5}$ of the speed of the sound, i.e. $\frac{330}{5}$ m/s

$$= 66 \text{ m/s} = \frac{66 \times 3600}{1000} = 237.6 \text{ km/hr}$$

Alternative method:

Solve using the relative velocity concept, V is the speed of train.

$$10(V + 330) = 330 \times 12$$

$$V = \frac{330}{5} = 66 \text{ m/s}$$



9. b From 7 a.m. to 1 p.m., i.e. in 6 hr, the first train travels a distance of $60 \times 6 = 360$ km.

\therefore Distance between the trains at 1 p.m. = $1200 - 360 = 840$ km.

$$\therefore \text{Time taken to cover this relative distance} = \frac{840}{\frac{840}{140}} = 6 \text{ hr, i.e. 7 p.m.}$$

10. c In 6 hr, the second train travels $6 \times 80 = 480$ km.

\therefore They meet at 480 km from Q.

$$11. d \text{ Speed in upstream} = \frac{\frac{3}{4}}{11\frac{1}{4}} \times 60 \text{ km/hr.}$$

$$\text{Speed in downstream} = \frac{\frac{3}{4}}{7\frac{1}{2}} \times 60 \text{ km/hr.}$$

$$\text{Speed of man in still water} = \frac{1}{2} (\text{Speed in downstream} + \text{Speed in upstream})$$

$$= \left(\frac{1}{2}\right) \left(\frac{3}{4} \times 60\right) \left(\frac{1}{11\frac{1}{4}} + \frac{1}{7\frac{1}{2}}\right) = 5 \text{ km/hr.}$$

12. b Let a = Speed in upstream and b = Speed in downstream

$$\frac{40}{a} + \frac{90}{b} = 5 \text{ and } \frac{60}{a} + \frac{60}{b} = 5$$

$$\therefore a = 20 \text{ km/hr and } b = 30 \text{ km/hr.}$$

$$\therefore \text{Speed of the water (current)} = \frac{1}{2}(b - a) = 5 \text{ km/hr}$$

$$\therefore b = V_N + V_E \text{ and } a = V_E - V_N$$

13. c Let the total work be (LCM of 45, 40) = 360 units.

In one day,

$$\text{A does } \frac{360}{45} = 8 \text{ units and}$$

$$\text{B does } \frac{360}{40} = 9 \text{ units}$$

Both A and B together do $8 + 9 = 17$ units of work.

In last 23 days, B does $9 \times 23 = 207$ units of work.

So work done by both A and B together = $360 - 207$ units = 153 units.

$$\text{Number of days} = \frac{153}{17} = 9 \text{ days.}$$

14. d A takes 12 days,

B is 60% more efficient

$$\Rightarrow \text{B will take time} = \frac{12}{1.60} = 7.5 \text{ days}$$

15. d In 10 day's A completes work = $\frac{1}{25} \times 10 = \frac{2}{5}$ of the total work.

Remaining work = $\frac{3}{5}$ of the total work.

$\left(\frac{1}{25} + \frac{1}{20}\right)$ work is done by (A + B) in 1 day

$$= \frac{3}{5} \text{ work is done by them in } \frac{100}{9} \times \frac{3}{5} = 6.66 \text{ days}$$

$$\text{Number of days} = 10 + 6.66 = 16.66$$

16. d A : B = 1 : 2

$$C : (A + B) = 1.5 : 3$$

If C does 1.5 units of work per day,

A does 1 and B does 2.

$$\text{Total units done by C} = 40 \times 1.5 = 60 \text{ units}$$

In a day work done by (A + B + C) = 4.5

$$\Rightarrow \text{Number of days taken by (A + B + C) to complete work} = \frac{60}{4.5} = 13.33$$

17. a Let the job be completed in x days. Then the young man worked for 2 days.

15. d In 10 day's A completes work = $\frac{1}{25} \times 10 = \frac{2}{5}$ of the total work.

Remaining work = $\frac{3}{5}$ of the total work.

$\left(\frac{1}{25} + \frac{1}{20}\right)$ work is done by (A + B) in 1 day

= $\frac{3}{5}$ work is done by them in $= \frac{100}{9} \times \frac{3}{5} = 6.66$ days

Number of days = $10 + 6.66 = 16.66$

16. d A : B = 1 : 2

C : (A + B) = 1.5 : 3

If C does 1.5 units of work per day,

A does 1 and B does 2.

Total units done by C = $40 \times 1.5 = 60$ units

In a day work done by (A + B + C) = 4.5

∴ Number of days taken by (A + B + C) to complete work = $\frac{60}{4.5} = 13.33$

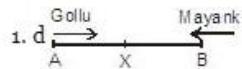
17. a Let the job be completed in x days. Then the young man worked for 2 days.

His father worked for $x - 1$ days and the grandfather worked for x days.

$$\therefore \frac{2}{10} + \frac{x-1}{15} + \frac{x}{20} = 1$$

$$\text{or } \frac{12 + 4x - 4 + 3x}{60} = 1 \text{ or } x = 7\frac{3}{7} \text{ days}$$

Practice Exercise 4 - Level 2



Let X be the point where they meet on the way.

$$\Rightarrow \frac{AX}{XB} = \frac{6}{5} \quad [\text{As their speeds are in this ratio}]$$

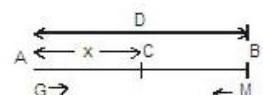
$$\text{Now } \frac{t_m}{t_g} = \frac{d_m/s_m}{d_g/s_g} = \frac{d_m \times s_g}{d_g \times s_m}$$

$$\Rightarrow \text{Since } \frac{d_m}{d_g} = \frac{AX}{XB} = \frac{6}{5} \Rightarrow \frac{t_m}{t_g} = \frac{6}{5} \times \frac{6}{5} = \frac{36}{25}$$

$$t_m = \frac{25 \times 36}{25} = 3.6 \text{ hr} = 3 \text{ hr } 36 \text{ min.}$$

Alternative method 1:

Conventional method of solving



Let Golu and Mayank met at point C which is x kilometres from A, and A and B are D kilometres apart.

$$\text{The ratio of speeds} = \frac{V_G}{V_M} = \frac{6}{5}.$$

They take same time to reach point C.

$$\Rightarrow \frac{x}{6} = \frac{D-x}{5}$$

$$\Rightarrow x = \frac{6}{11}D$$

Now Golu covers $\frac{5}{11}D$ in $\frac{5}{2}$ hr.

$$\text{So } V_G = \frac{5D}{11 \times \frac{5}{2}} = \frac{2D}{11} \text{ km/hr}$$

$$\Rightarrow V_M = \frac{5}{6} \times V_G = \frac{5}{6} \times \frac{2}{11}D = \frac{5D}{33}$$

Mayank covers $5D$ in 33 hr.

$$\text{So he will cover } \frac{6}{11}D \text{ in } \frac{33}{5} \times \frac{6}{11} = 3.6 \text{ hr} = 3 \text{ hr } 36 \text{ min}$$

Alternative method 2:



$$\frac{V_G}{V_M} = \sqrt{\frac{t_m}{t_g}} \Rightarrow \left(\frac{6}{5}\right)^2 = \frac{t_m}{t_g}$$

$$\therefore t_M = \frac{36}{25} \times \frac{5}{2} = 3 \text{ hr } 36 \text{ min}$$

2. a Suppose the winning point is x metres far if the girls finish the race at the same time.
Then Pallavi will have to cover x metres when Richa covers $(x - 350)$ m.

Ratio of their speeds is $20 : 13$.

So we have $\frac{20}{13} = \frac{x}{x-350}$ or $x = 1,000$ m = 1 km.

Alternative method:

Pallavi has to cover 350 m with a relative speed of $7x$.

Required time will be $\frac{350}{7x}$.

In this time, the actual distance covered by Pallavi is $\frac{350}{7x} \times 20x = 1,000$ m, i.e. length of the track.

3. b Let length of the journey = x km and

speed of the train = v km/hr.

Then $\frac{1}{2} + \frac{3}{4} + \frac{x - \frac{v}{2}}{\frac{2v}{3}} - \frac{x}{v} = 1\frac{1}{2}$ [Converting minutes to hours]

$$\text{Also } \frac{\frac{v}{2} + 60}{v} + \frac{3}{4} + \frac{x - \frac{v}{2} - 60}{\frac{2v}{3}} - \frac{x}{v} = 1$$

Solving for x , we get $x = 120$ km and $v = 60$ km/hr.

Alternative method:

Overall, the train was late by 1 hr 30 min.

Out of this for 45 min, the train was stopped. So due to $\frac{2}{3}$ of the speed, the train delayed by 45 min only.

Normal time $\frac{3}{2}t - t = \frac{t}{2} = 45 \text{ min}$ or $t = 90 \text{ min}$.

Had the accident occurred 60 km away, the train would have been delayed by 1 hr only.

Due to $\frac{2}{3}$ of the speed, the delay will be 15 min only (1 hr - 45 min stoppage)

\therefore Normal time to cover the remaining distance = $t = 2 \times 15 = 30 \text{ min}$.

It means the train covers 60 km in 60 min ($90 - 30$ min).

Speed of the train = 60 km/hr.

Total journey = 30 km + 90 km = 120 km.

4. a Vibhor had lost 4 gallons.

Since he travelled at 50 mph for 4 hr = 200 miles, he used 8 gallons ($\frac{200}{25}$) for the journey.

5. b Let's assume the distance to be 24 km. (LCM of 6, 4, 3)

Then time taken to cover that distance will be in the ratio $\frac{24}{6} : \frac{24}{4} : \frac{24}{3} = 4 : 6 : 8 = 2 : 3 : 4$.

Hence, the ratio of their speed will also be 2 : 3 : 4.

Alternative method:

Since the distance travelled by the three is same, the ratio of speeds will be inverse of the ratio of time taken.

Hence, the ratio of speeds = $\frac{1}{6} : \frac{1}{4} : \frac{1}{3} = 4 : 6 : 8 = 2 : 3 : 4$.

6. c Ratio of speeds = 3 : 4.

Distance remaining constant, the ratio of time taken = 4 : 3.

A takes 0.5 hr more than B.

Hence, time taken by A = $4 \times 0.5 = 2$ hr

7. e Let us assume that the distance between the two places be 60 km. This means that to go 120 km at 12 km/hr, she can use $\frac{120}{12} = 10$ hr.

Also, as she goes at 6 km/hr, the time taken for this journey is $\frac{60}{6} = 10$ hr.

Thus, there is no time to come back, which means this situation is not possible.

Alternative method:

If distance from Kashipur to Bareily is D, Nupur takes $\frac{D}{6}$ hr to travel distance D.

Since average speed of journey is 12, total time taken is $\frac{2D}{12} = \frac{D}{6}$.

Hence, there is no time to return, which means this situation is not possible.

8. a Suppose the total time was x hr.

Since distances travelled by both would be same, we have $(8 \times 4) + (x - 4)16 = (13 \times 4) + (x - 4)12$ or $x = 9$

Alternative method:

In 4 hr, lead taken by Shivku is $(13 - 8) \times 4 = 20$ km.

Now speed of Sujeet = 16 km/hr and of Shivku = 12 km/hr.

Sujeet has to cover 20 km with relative speed 4 km/hr in 5 hr. So total time = $4 + 5 = 9$ hr.

9. c Suppose they meet after $(100 + x)$ m. Shivku starts running when Pawan has already run 100 m. Now if Pawan runs x km, Shivku will run $2x$ km.

So we have $100 + x = 2x$ or $x = 100$.

Thus, they will meet at $100 + 100 = 200$ m from the starting point.

If the length of the track is 150 m, they will meet at a distance of 50 m from the starting point.

10. a It happens when both of them meet for first time at the starting point. Ratios of speeds = 4 : 12. Hence, ratios of distances covered = 1 : 3. Hence, when A makes one round, B makes 3 rounds, so the answer is 1.

11. c A can give B a lead of 200 m for every 1,000 m, i.e. every one kilometre. This means that when A covers 1,000 m, B covers 800 m.

Suppose the speed of A is 25 m/s and of B is 20 m/s.

A will meet B for the first time after $\frac{1000}{25 - 20} = 200$ s.

In 200 s, A will have made

$$200 \times \frac{1000}{25} = \frac{200 \times 25}{1000} = 5 \text{ rounds.}$$

Alternative method:

From the given data, in 1,000 m race A can give B a start of 200 m. It means when A has covered 1,000 m, B can cover 800 m. The ratio of their speeds is 5 : 4. From this ratio of speed, it can be easily found that they will meet only at the starting point. So A has covered 5 rounds.

12. a When B covers 800 m A has covered 1,000 m.

So when B has covered 3 rounds = 3,000 m, A will cover 3,750 m.

But to meet at some point, the overall lead should be of one round = 1,000 m.

So there must be an initial lead of 250 m.

13. c We know that in 60 min, the minute hand gains 55 min over the hour hand.

At 3 o' clock, the minute hand and the hour hand are 15 min apart.

To cross the latter, the minute hand must gain 15 min.

Since 55 min is gained by the minute hand in 60 min, 15 min is gained by the minute hand in $\frac{60}{55} \times 15 = 16 \frac{4}{11}$ min.

$$\therefore x = 16 \frac{4}{11} \text{ min}$$

Alternative method:

At 3 o' clock, the angle between the hour hand and the minute hand is 90° . The minute hand will cover the distance with a relative speed of $(6 - 0.5) = 5.5^\circ$ per minute, i.e.
 $\frac{90}{5.5} = 16 \frac{4}{11}$ min.

14. c 3 men = 4 women. 4 women complete the job in 12 days. Hence, 5 women and 3 men, i.e.

9 women can do the job in $\frac{12 \times 4}{9} = 5 \frac{1}{3}$ days.

15. c Earning of 1 girl = Rs. 50.

Earning of 10 girls = Rs. 500.

Earning of 8 boys = Rs. 500.

Earning of 6 boys = Rs. $\frac{6}{8} \times 500 = \text{Rs. } 375$.

Earning of 4 women = Rs. 375.

Earning of 3 men = Rs. 375.

Earning of 1 man = Rs. 125.

Alternative method:

The man earns $50 \times \left[\frac{5}{4} \right] \left[\frac{3}{2} \right] \left[\frac{4}{3} \right] = \text{Rs. } 125$.

16. b Amount of work to be done = 10n,
 where n = Number of workers originally available.

Now $10n = 12(n - 5) = 2n = 60$.

Therefore, n = 30.

17. a $\frac{1}{3}$ of a man = $\frac{1}{2}$ of a woman = 1 child. = 2 men = 3 women = 6 children.

$20m + 30w + 36c = 60c + 60c + 36c = 156c$

If 156 children get Rs. 78, 1 child gets $\frac{78}{156}$.

Now $15m + 21w + 30c = 45c + 42c + 30c = 117c$.

\therefore 117 children should get Rs. $\left(\frac{78}{156} \times 117 \right)$ per day.

\therefore For (18×7) days, they should get = Rs. $(18 \times 7) \left(\frac{78}{156} \times 117 \right) = \text{Rs. } 7371$.

Alternative method:

Let a man, a woman, a child put in 3, 2, 1 units per day.

Then they are paid Rs. 78 for putting in

$(20 \times 3 + 30 \times 2 + 36 \times 1)$ units, i.e. 156 units.

Hence, if $(15 \times 3 + 21 \times 2 + 30 \times 1)$, i.e. 117 units is put in, then the payment = Rs. $\frac{117}{2}$ per day or Rs. 7,371 in 18 weeks.

$$18. b \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{18} \dots (i)$$

$$\frac{1}{a} + \frac{1}{b} = \frac{2}{c} \dots (ii)$$

$$\frac{1}{a} + \frac{1}{c} = \frac{3}{b} \dots (iii)$$

$$(i) \text{ and } (ii) = \frac{1}{18} - \frac{1}{c} = \frac{2}{c} \text{ or } c = 54 \text{ days}$$

$$(i) \text{ and } (iii) = \frac{1}{18} - \frac{1}{b} = \frac{3}{b} \text{ or } b = 72 \text{ days}$$

$$\therefore a = 43.2 \text{ days}$$

Alternative method 1:

Let the number of units of work = 18.

C puts in $\left(\frac{1}{3}\right)$ of that in 18 days = 6 units.

B puts in $\left(\frac{1}{4}\right)$ of that in 18 days = 4.5 units.

Hence, A does 7.5 units in 18 days.

In order to finish it he takes $\left(\frac{18}{7.5}\right) \times 18 = 43.2$ days.

Alternative method 2:

Let the total work be $18 \times 12 = 216$ units.

Ratio of (A and B) : C = 2 : 1.

$\therefore C$ does $\frac{1}{3} \times 12 = 4$ units.

Ratio of (A and C) : B = 3 : 1.

$\therefore D$ does $\frac{1}{4} \times 12 = 3$ units.

$\therefore A$ does $= 12 - (4 + 3) = 5$ units.

Total work will be done in $\frac{216}{5} = 43.2$ days.

19. c $\frac{4}{5}$ of work is done by them in 7 days.

Hence, they take $\frac{7}{\frac{4}{5}} = 8.75$ days to complete the job.

$$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{8.75} \text{ and } \frac{8}{a} + \frac{10}{b} = 1 \Rightarrow \frac{2}{a} = \frac{10}{8.75} - 1 = \frac{1.25}{8.75} = \frac{1}{7}$$

$a = 14$ days

Alternative method 1:

A and B work together for 8 days.

$$W_{A+B} \text{ in 7 days is } \frac{4}{5}W \text{ [W is total work]}$$

$$\Rightarrow W_{A+B} \text{ in 8 days is } \frac{4}{5} \times \frac{8}{7}W = \frac{32}{35}W$$

Now A leaves and B does $\frac{3}{35}W$ in 2 days.

$$\Rightarrow B \text{ will do } W \text{ in } 2 \times \frac{35}{3} = \frac{70}{3} \text{ days.}$$

$$W_A = W_{A+B} - W_B$$

$$= \frac{1}{10} - \frac{3}{70} = \frac{5}{70} = \frac{1}{14}$$

So A will do the work in 14 days.

Alternative method 2:

Let the total work be 35 units (LCM of 7 and 5).

$\frac{4}{6}$ of work = 28 units, is done by A and B in 7 days.

$\therefore 1$ day's work of A + B = 4 units.

B completes $(7 - 4)$ units in 2 days.

(Out of a total of 10 days, $7 + 1 = 8$ days' work = $8 \times 4 = 32$ units)

$\therefore 1$ day's work of B = 1.5 units.

$\therefore A$'s 1 day's work = 2.5 units.

Total time taken by A alone to complete 35 units of work = $\frac{35}{2.5} = 14$ days.

20. d In 30 days, only half the work could be done. Therefore, number of men must be doubled to get the work done in half the time (i.e. half of 30 = 15 days, because only 15 days are remaining).

$$21. c 15m = 24w = 36b$$

x men must be associated.

$$\therefore (x)m + 12w + 6b = \left(\frac{36}{15}x + 18 + 6 \right) \text{ boys.}$$

$$\text{Therefore, } \left(\frac{36x}{15} + 24 \right) \frac{30 \times 6}{2 \frac{1}{4}} = (36) \left(\frac{12 \times 8}{1} \right)$$

$$= x = 8 \text{ men}$$

22. a Suppose the work is of 150 units. So in a day A and B together do 15 units; B and C together do 10 units; and A and C together do 6 units.

We have $A + B = 15$, $B + C = 10$ and $C + A = 6$.

$$\Rightarrow A = 5.5, B = 9.5 \text{ and } C = 0.5$$

In first 4 days, A, B and C together will do

$$4(5.5 + 9.5 + 0.5) = 62 \text{ units.}$$

In next 5 days B and C will do $5(9.5 + 0.5) = 50$ units. So C alone has to do $150 - (62 + 50)$ $= 38$ units, that he will do in $\frac{38}{0.5} = 76$ days.

$$23. a \text{ Work done by A in 2 days} = \frac{2}{6} = \frac{1}{3}.$$

$$\text{Work done by B and C in 2 days} = 2\left(\frac{1}{6} - \frac{1}{10}\right) = 2\left(\frac{5+4}{40}\right) = \frac{9}{20}.$$

$$\text{Remaining work} = 1 - \frac{1}{3} - \frac{9}{20} = \frac{60 - 20 - 27}{60} = \frac{13}{60}.$$

$$\text{Hence, } \frac{13}{60} \text{ part of the job will be done by C in } \frac{\frac{13}{60}}{\frac{1}{10}} = \frac{13}{6} = 2\frac{1}{6} \text{ days}$$

$$\text{Total time taken to complete the job} = 2 + 2 + 2\frac{1}{6} = 6\frac{1}{6} \text{ days.}$$

Alternative method:

Let the total work be LCM (6, 8, 10) = 120 units.

A's 1 day's work = 20 units.

B's 1 day's work = 15 units.

C's 1 day's work = 12 units.

In 2 days A will work = 40 units.

(B + C) will work in 2 days = 54 units.

Work remaining after 4 days = 26 units.

$$\therefore \text{Time taken by C} = \frac{26}{12} = \frac{13}{6}.$$

$$\text{Total time taken} = 4 + \frac{13}{6} \text{ days.}$$

Answer is $6\frac{1}{6}$ days.

$$24. a \text{ Fraction of the tank filled in 2 hr} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}.$$

Total time taken to complete the job = $2 + 2 + 2\frac{1}{6} = 6\frac{1}{6}$ days.

Alternative method:

Let the total work be LCM (6, 8, 10) = 120 units.

A's 1 day's work = 20 units.

B's 1 day's work = 15 units.

C's 1 day's work = 12 units.

In 2 days A will work = 40 units.

(B + C) will work in 2 days = 54 units.

Work remaining after 4 days = 26 units.

$$\therefore \text{Time taken by C} = \frac{26}{12} = \frac{13}{6}.$$

$$\text{Total time taken} = 4 + \frac{13}{6} \text{ days.}$$

Answer is $6\frac{1}{6}$ days.

$$24. \text{ a Fraction of the tank filled in 2 hr} = \frac{1}{4} + \frac{1}{5} = \frac{9}{20}.$$

So in 4 hr, fraction of the tank filled = $\frac{18}{20}$.

The remaining $\frac{2}{20}$ or $\frac{1}{10}$ of the tank will be filled by pipe A.

Pipe A fills $\frac{1}{4}$ of the tank in 1 hr.

So it will fill $\frac{1}{10}$ of the tank in $4 \times \frac{1}{10} = 0.4 = 0.4 \text{ hr} = 24 \text{ min.}$

Hence, the total time taken to fill the tank = 4 hr 24 min.

Practice Exercise 5 - Level 2

1. d In 24 hr, i.e. 1 day, the snail goes up by only 11 inch. height of the pole = $12 \times 25 = 300$ inches.

In 25 days, i.e. 600 hr it will go up (effectively) by $25 \times 11 = 275$ inches.

Now it can creep up 27 inches in 12 hr.

So to climb up 25 inches it will take

$$25 \times \frac{12}{27} = 11.11 \text{ hr.}$$

Hence, the total time = $600 + 11.11 = 611.11$ hr.

2. b The first train takes $\frac{148}{18.5} = 8$ hr, i.e. it reaches 8 hr after 7 a.m., i.e. 3 p.m.

So the second train reaches at 3.15 p.m.

Ratio of speeds of two trains is 8 : 5.

Hence, the ratio of time taken (distance being constant) is 5 : 8.

Therefore, time taken by the second train = $\left(\frac{5}{8}\right) \times 8 = 5$ hr, i.e. it starts 5 hr before 3.15 p.m.

$\Rightarrow 10.15$ a.m.

3. c Since Ajay is faster than Mallu and they start together, to meet at 10 m from B, Ajay would have covered a distance from A to B and would meet Mallu on his way back to A.

Mallu would be on his way from A to B.

So Ajay covers $200 + 10 = 210$ m in 10 s.

Hence, Ajay's speed = 21 m/s.

So he will take $\frac{190}{21}$ s to cover the remaining 190 m.

The time required for Ajay to reach A will be

$$10 + \frac{190}{21} = \frac{400}{21} \text{ s.}$$

4. d Let the total distance covered by them when they first meet is d kilometres.

When they meet for the second time, the total distance covered by them is 3d.

Hence, they will meet after $12 \times 3 = 36$ min.

5. d Duration of the journey = $\frac{384}{24} = 16$ days.

Thus, the person starting from Delhi (at 3 p.m.) is bound to meet the 17 persons already enroute, and 16 more (who will have started after he starts) during his journey.

Therefore, the total number of people he meets = $17 + 16 = 33$.

Alternative method:

It takes $\frac{384}{24} = 16$ days for New York-Delhi travel. Since the speed gets added up, when two people come from opposite sides, anybody going from New York to Delhi will meet two people going from Delhi to New York every $\frac{24}{2} = 12$ hr.

\therefore A person will take $\frac{384}{12} = 32$ intervals to complete his journey.

\Rightarrow He will meet $(32 + 1) = 33$ people during the journey (As he will meet the first person as soon as he leaves New York and last person as soon as he reaches Delhi.)

$$6. \text{c Area per revolution} = (2\pi R) \times L = 1.32 \text{ sq. m}$$

$$\Rightarrow \text{Total area} = 1.32 \times 400 = 528 \text{ sq. m}$$

$$\Rightarrow \text{Total cost} = \text{Rs. } 5280$$

$$7. \text{e Average speed of A} = \frac{2 \times 10 \times 9}{19} = \frac{180}{19} \text{ km/hr}$$

$$\text{Ratio of times of B and A} = \frac{180}{19} : 12 \Rightarrow 15 : 19$$

$$\text{Hence, if A takes 10 min more than Bibek} = \frac{10}{4} \times 19 = \frac{190}{4} \text{ min}$$

$$\text{Hence, total distance covered} = \frac{180}{19} \times \frac{190}{4} \times \frac{1}{60} \Rightarrow 7.5 \text{ km}$$

$$PQ = 3.75 \text{ km}$$

$$8. \text{c In the first 15 min, the thief will cover } \frac{1}{4} \times 60 = 15 \text{ km.}$$

$$\text{Hence, to cover this 15 km, a policeman will take} = \frac{\text{Distance}}{\text{Relative speed}} = \frac{15}{65 - 60} = \frac{15}{5} = 3 \text{ hr.}$$

$$9. \text{a When Zigma-S leaves Jaipur, the thief has covered a distance of 15 km.}$$

$$\text{Distance between them} = 300 - 15 = 285 \text{ km.}$$

$$\text{Relative speed of Zigma-S with respect to the thief} = 60 + 60 = 120 \text{ km.}$$

$$\text{So time taken to catch the thief} = \frac{285}{120} = 2.375 \text{ hr} = 2 \text{ hr } 22.5 \text{ min.}$$

According to question 8, Sigma-Z takes 3 hr to catch the thief. Hence, Sigma-Z takes 37.5 min more than Zigma-S in catching the thief.

10. e Let the speed of the boat in still water be u km/hr and that of the river be v km/hr.

If the distance from A to B is d , then we have

$$\frac{d}{u+v} = 2 \text{ and } \frac{d}{u-v} = 3 \text{ or } \frac{u-v}{u+v} = \frac{2}{3} \text{ or } \frac{u}{v} = \frac{5}{1}.$$

Thus, we see that we can get the ratio of the speeds and not the exact speeds.

11. a Let the speed of the boat in still water is u

Let the speed of the flowing river is v

when boat goes upstream (against the flow of river) speed of the boat = $u - v$

for downstream (with the flow of the river) = $u + v$

$$4 : 1 = (u + v) : (u - v) \Rightarrow u + v = 4u - 4v$$

$$\Rightarrow 3u = 5v$$

$$12. e \text{ Time taken} = \frac{2.4}{5-x} + \frac{2.4}{5+x} = 1 = x = 1 \text{ km/hr}$$

13. c Assume speed of the river and the boat and make an equation by equating their time.

Let M be the speed of the motor boat and R be the speed of the raft/river. Then

$$\frac{40}{3(M+R)} + \frac{28}{3(M-R)} = \frac{4}{R} \Rightarrow \frac{M}{R} = \frac{17}{3}$$

14. b Let the speeds of two persons are $3x$ and $7x$.

V takes $\frac{16}{7x}$ hr to complete the race.

Since they move in the same direction, relative speed = $4x$.

$$\text{Time when they first meet} = \frac{4}{4x} = \frac{1}{x}$$

$$\text{Therefore, number of times they meet in the entire race} = \frac{\frac{16}{7x}}{\frac{1}{x}} = \frac{16}{7} = 2.3.$$

Therefore, they meet twice before the race finishes.

$$15. d \text{ When they move in opposite directions, the time when they meet the first time} = \frac{4}{10x} = \frac{2}{5x}.$$

$$\text{Therefore, they meet } \frac{\frac{16}{7x}}{\frac{2}{5x}} = \frac{80}{14} = 5.7 \text{ times, i.e. 5 times.}$$

16. d The hands of the clock are 35 min ($= 35 \times 6 = 210^\circ$) apart at 7 o' clock.

For the hands to be together, the minute hand has to gain 210° min over the hour hand.

Speed of the minute hand relative to the hours hand = $6^\circ - 1/2^\circ = 5.5^\circ$ per minute.

Hence, the hands will meet after

$$\frac{210}{5.5} = \frac{420}{11} = 38\frac{2}{11} \text{ min}$$

OR

Between x and $(x + 1)$ o' clock, the two hands will be together at $5x \times \left(\frac{12}{11}\right)$ min past x .

i.e. $5 \times 7 \times \left(\frac{12}{11}\right)$ min past 7 = $38\frac{2}{11}$ min past 7.

17. a In 1 hr the first photocopier will complete $\frac{1}{8}$ of the work.

Similarly, the second photocopier will complete $\frac{1}{12}$ of the work in 1 hr.

If both work together, then fraction of the work completed in 1 hr = $\frac{1}{8} + \frac{1}{12} = \frac{3+2}{24} = \frac{5}{24}$.

Hence, both the photocopiers working simultaneously will take $\frac{24}{5} = 4.8$ hr.

18. c In 1 hr, pipe A fills 60 m^3 , and pipe B fills $\frac{1}{6}$ of the tank. Hence, if the capacity of the tank = $V \text{ m}^3$,

$$60 + \frac{V}{6} = \frac{V}{4} \quad (\text{Portion of the tank filled in 1 hr when both A and B are open})$$

$$720 + 2V = 3V \Rightarrow V = 720 \text{ m}^3$$

19. d Pipe A can fill the tank in $\frac{720}{60} = 12$ hr.

Therefore, 1 hr net task of all three pipes, when opened simultaneously, is

$$\frac{1}{12} + \frac{1}{6} - \frac{1}{8} = \frac{2+4-3}{24} = \frac{1}{8}.$$

Hence, the tank will be filled in 8 hr.

20. c Let the job consists of 120 units [i.e. LCM (20, 24, 15)]

$\therefore A + B$ do 6 units per day.

$B + C$ do 5 units per day.

$A + C$ do 8 units per day.

$A + B + C$ do 9.5 units per day.

So $B + C$ in 8 days do 40 units. A in 4 days does 18 units.

Hence, $\frac{58}{120} = \frac{29}{60}$ part of the job is completed.

Practice Exercise 6 - Level 3

1. a Since the two trains have the same speed, they would take same time to cover the same distance (between A and B). Hence, they would meet exactly 6 hr after they start, i.e. at 3 p.m.
2. b If the distance between A and B is assumed to be 12 km, then speed of both the trains = 1 km/hr.

Therefore, by 1 p.m., they would have both covered 4 km each. Distance left between them = 4 km.

Speed of the first train = 0.5 km/hr; speed of the second train = 1 km/hr.

(Relative speed = 1 + 0.5 = 1.5 km/hr)

Hence, time taken to meet, after the accident = $\frac{4}{1.5} = 2\frac{2}{3}$ hr = 2 hr 40 min.

So they meet at 3.40 p.m.

3. c The train is late by 10 hr due to reduction in speed after the accident as the speed is half. So time taken now will be twice that of the original. Hence, the accident must have occurred 10 hr before the actual arrival time at the destination, i.e. at 11 o' clock. Now till 11 o' clock, if we assume the speed of each train to be 1 km/hr and total distance = 12 km,

in 2 hr they together will cover 4 km. The remaining distance of 8 km will be covered with relative speed of 1 km/hr and $\frac{1}{2}$ km/hr.

$$\therefore \text{Time} = \frac{8}{\frac{3}{2}} = \frac{16}{3} \text{ hrs}$$

The two trains would meet after $= \left(\frac{16}{3} + 11\right)$ hr or at 4 : 20 p.m.

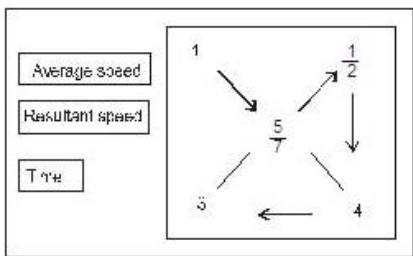
4. d As in question 5,

speeds of the first train before and after the accident would be 1 km/hr and 0.5 km/hr respectively.

Since they meet 7 hr after they leave, the distance covered by the first train till 4 p.m. = 12 - 7 = 5 km.

Average speed of the first train till 4 p.m. = $\frac{5}{7}$ km/hr.

Using alligations,



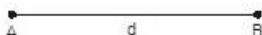
Hence, the time for which the train travels at two speeds is in the ratio $3 : 4$, i.e. after 3 hr of departure, the accident took place.

Hence, the accident took place at 12 noon.

Alternative method:

Assume that the total distance $d = 12$ km.

Speed of the train from A to B and B to A is 1 km/hr.



Now if the accident occurred after t hours from start, the distance covered by the first train in t hours = t km.

And distance covered in $(7 - t)$ hr = $(7 - t) \times \frac{1}{2}$

(Because after the accident the speed is half)

$(9 \rightarrow 4 \rightarrow 7 \text{ hr})$

Distance covered by the second train in 7 hr = 7 km.

To meet at 4 a.m., the total distance = 12 km.

$$\therefore 7 + t + (7 - t) \times \frac{1}{2} = 12$$

$$\therefore t = 3 \text{ hr after start, i.e. at 12 o' clock.}$$

5. $d \xrightarrow{t_1} \xrightarrow{t_2}$

Let t_1 be the time at which B switches the speed and $t_1 + t_2$ be the total time between start and finish.

Let x be the speed of B initially. So A's speed = $1.2x$ and B's final speed = $1.44x$.

Now lag of B in time t_1

$$= (1.2x - x)t_1 = 0.2x \times t_1 \dots (i)$$

Also gain of B in time t_2

$$= (1.44x - 1.2x)t_2 = 0.24x \times t_2 \dots (ii)$$

Since both reach at the same time,

\therefore Lag = lead

$$\Rightarrow \frac{t_1}{t_2} = \frac{0.24}{0.20} = \frac{6}{5}$$

\therefore A covers $\frac{1760 \times t_1}{(t_1 + t_2)}$ of the distance $= \frac{1760 \times 6}{11} = 960$ m.

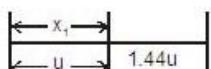
Alternative method 1:

Let the speed of B $\rightarrow u$.

Speed of A $\rightarrow 1.2u$

New speed of

$$B = 20 + 20 + \frac{20 \times 20}{100} = 44\% \text{ more} = 1.44u$$



Since race ended in dry heat, equating time of A and B

$$\frac{x_1}{u} + \frac{1760 - x_1}{1.44u} = \frac{1760}{1.2u} \Rightarrow x_1 = 797.819 \text{ m.}$$

Alternative method 2:

Assume that A runs 20% faster than B for t_1 time and for t_2 time B runs 20% faster than A.

Speed of B initially is V_B for t_1 time.

So $V_A = 1.2 V_B$ for $(t_1 + t_2)$ time

V_B' = 1.44 V_B for t_2 time

$$\therefore 1.2 V_B (t_1 + t_2) = 1760$$

$$V_B + t_1 + 1.44 V_B t_2 = 1760$$

$$\Rightarrow t_1 = \frac{6}{5} t_2$$

Now distance covered by A in t_1 time = $1.2 V_B t_1$

$$1.2 V_B \left(t_1 + \frac{5}{6} t_1 \right) = 1760$$

$$1.2 V_B t_1 = \frac{1760 \times 6}{11} = 960 \text{ m.}$$

6. b When Ram completes second round, they do handshake once. Now for every round which Ram completes, there will be one handshake as the ratio of speed is 13 : 8, Ram and Shyam will meet at the pool end only after

Ram completes 26 rounds. In the 20th round, Ram will finish the race and the total number of handshake's will be = 20 - 1 = 19

7. c The time taken by P and Q to meet after their start is $\frac{400}{40+60} = \frac{400}{100} = 4$ hr

Hence, they must have started at 11 a.m.

Alternative method:

Q covers $\frac{3}{5}$ of the distance, i.e. 240 km.

It takes 4 hr to cover this distance.

8. e R would take $\frac{60 \times 4}{(80-60)} = 12$ hr to meet Q.

By that time, Q would have already reached Guntur.

9. a Normally P and Q meet after 4 hr of journey.

Now they are meeting after 5 hr.

Hence, the distance travelled by

$$Q = 400 - (40 \times 5) = 200 \text{ km}$$

If the accident occurred after x hours, then

$$60(x) + 20(5 - x) = 200 \text{ or } 40x = 100 \text{ or } x = 2\frac{1}{2} \text{ hr}$$

10. a Let the speed of the goods train be X m/s, and that of the passenger train be Y m/s. In 28 s the goods train covered 28X (m), and the passenger train, 28Y (m). Therefore, $28X + 28Y = 700$.

The goods train passes the signal lights in $\frac{490}{X}$ s and the passenger train in $\frac{210}{Y}$ s.

$$\text{Therefore, } \frac{490}{X} - \frac{210}{Y} = 35.$$

Solving the two equations, we get $X = 36$ km/hr and $Y = 54$ km/hr.

11. a x = Length of train, y = Speed of train (m/s)

Relative speed with respect to first one = $y - \frac{5}{9} = \frac{9y-5}{9}$ (As 2 km/hr = $\frac{5}{9}$ m/s)

$$\frac{x}{\frac{9y-5}{9}} = 9 = x = 9y - 5 \dots (\text{i})$$

$$\text{Similarly, for second } \frac{x}{\frac{9y-10}{9}} = 10$$

$$= 90y - 9x = 100 \dots (\text{ii})$$

Solving (i) and (ii),

$$x = 50 \text{ m, } y = \frac{55}{9} \text{ m/s}$$

12. c They reverse directions on reaching A.

Ram with 12 km/hr speed will reach A first and immediately reverse directions.

Suraj will reverse direction only after he reaches A.

Time taken by Ram to complete the circle

$$= 1.2 \text{ km}/12 \text{ km/hr} = 0.1 \text{ hr} = 6 \text{ min}$$

Time taken by Suraj to complete the circle

$$= 1.2 \text{ km}/8 \text{ km/hr} = 0.15 \text{ hr} = 9 \text{ min}$$

In those extra $(9 - 6) = 3$ min, Ram would have already reversed direction and done another half circle in the same direction as Suraj.

$$\text{The effective distance between them} = \frac{1.2}{2} = 0.6 \text{ km}$$

$$\text{Time taken to meet after Suraj reverses direction} = \frac{0.6}{(12+8)} = 0.03 \text{ hr} = 1.8 \text{ min}$$

$$\text{Therefore, time taken to meet for the second time} = 9 + 1.8 \text{ min} = 10.8 \text{ min}$$

$$13. b \text{ Time taken will be double the time taken to meet the first time} = 3.6 \times 2 = 7.2 \text{ min}$$

Since each time the effective distance between them will be 1,200 m.

14. c Let's assume that A takes x days to finish the job.

Then B will take 3x days.

$$3x - x = 60 \Rightarrow x = 30 \text{ days}$$

$$\text{Work done by A in one day} = \frac{1}{30}.$$

$$\text{Work done by B in one day} = \frac{1}{90}.$$

Both combined would do $\left(\frac{1}{30} + \frac{1}{90}\right) = \frac{2}{45}$ of the job in one day. Hence, they together will do the job in $\frac{45}{2}$ days = 22.5 days.

15. b Let the length of the wall be 30 m. Bamdev can demolish 1 unit per day and his son can demolish 0.5 unit per day. Manas can construct 3 units per day. If they work simultaneously with Manas starting they would do effectively 1.5 units per day. So in 36 days, 18×1.5 units is completed, i.e. 27 units is completed. Hence, on the 37th day, Manas would complete the job. Manas would have worked for 19 days and constructed 19×3 m.

$$\text{Hence, the fraction of the job completed} = \frac{57}{30} = \frac{19}{10}.$$

Alternative method:

Manas can construct a wall in 10 days.

\therefore He does $\frac{1}{10}$ work in 1 day.

Also Bamdev and his son can demolish

$$\frac{1}{30} + \frac{1}{60} = \frac{1}{20} \text{ of the wall in next day.}$$

Thus, in 2 days Manas effectively constructs

$$\frac{1}{10} - \frac{1}{20} = \frac{1}{20} \text{ of the wall.}$$

$$\therefore \text{He constructs } \frac{9}{10} \text{ of the wall in } \frac{20 \times 9}{10} \times 2$$

= 36 days and on 37th day, he finishes the construction. Since he has worked for 19 days, he constructed $\frac{19}{10}$ portion of the wall.

16. b If Manas started when there were 24 units to be completed, then working at 1.5 units per 2 days, 21 units would be completed in $\left(\frac{21}{1.5}\right) = 14$ pairs of days or 28 days.

Hence, on 29th day Manas would complete the job.

Alternative method 1:

This is identical to question 7, only that $\frac{2}{10}$ of the wall, i.e. 8 days of work is already done.

\therefore So $37 - 8 = 29$ days are required.

Alternative method 2:

From choices it is evident that there is only one odd choice. We know that the answer has to be odd since on the last day Manas has to complete the job.

17. d In his first shift, he uses the container six times.

(45 glasses in 5 rounds and the remaining 6 glasses in the 6th round)

Similarly, he prepared 73 glasses in 9 rounds, and 112 glasses in 13 rounds.

Hence, the required average

$$\begin{aligned} &= \frac{\text{Total time}}{\text{Total number of rounds}} = \frac{4 \text{ min } 18 \text{ s} + 7 \text{ min } 13 \text{ s} + 12 \text{ min } 24 \text{ s}}{6 + 9 + 13} \\ &= \frac{23 \text{ min } + 55 \text{ s}}{28} \approx 51 \text{ s.} \end{aligned}$$

18. a Let us calculate the tonne-hours done.

$$3 \text{ tonnes} \times 30 \text{ trucks} \times 8 \text{ hr} = 720 \text{ tonne-hours.}$$

$$5 \text{ tonnes} \times 9 \text{ trucks} \times 6 \text{ hr} = 270 \text{ tonne-hours.}$$

$$\therefore \text{Total tonne-hours} = 720 + 270 = 990 \text{ tonne-hours}$$

∴ Hours required for a single three-tonne truck = $\frac{990}{3} = 330$ hr

Hours required for five-tonne truck = $\frac{990}{5} = 198$ hr.

19. b Let the rate of growth of the grass be x straws per day and the initial amount of grass be y straws.

Assuming that one cow grazes on one straw of grass in a day, $(40 \text{ cows}) \times (40 \text{ days}) = 1600 = y + 40x$.

$$(30 \text{ cows}) \times (60 \text{ days}) = 1800 = y + 60x. \Rightarrow y = 1200 \text{ and } x = 10.$$

So for 20 cows,

$$(20 \text{ cows}) \times (n \text{ days}) = y + nx = 1200 + 10n \Rightarrow n = 120 \text{ days.}$$

Alternative method:

Let G be the amount of grass in the field and g be the growth rate of grass per day.

$$\Rightarrow G + 40g = 40 \times 40 \text{ and } G + 60g = 60 \times 30$$

$$\Rightarrow g = 10, G = 1200$$

$$\Rightarrow 1200 + N \times 10 = 20N$$

$$\Rightarrow N = 120 \text{ days}$$

20. d The time taken by 5 children to complete the job = $\frac{20}{5} = 4$ days.

So time taken by 4 men to complete the job = $4 \times 2 = 8$ days.

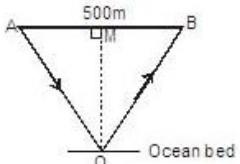
And time taken by 4 women = $8 \times 2 = 16$ days.

Therefore, 12 men + 10 children + 8 women complete = $\frac{3}{8} + \frac{1}{2} + \frac{1}{8} = \frac{(3+4+1)}{8} = \frac{8}{8} = 1$ of the work.

Hence, time taken = 1 day.

Practice Exercise 7 - Level 3

1. a Let ship sends the radiowave from point A and receives it at point B



$$\text{time taken} = \frac{0.5}{30} = \frac{1}{60} = 1 \text{ min}$$

Distance travelled by radiowave in 1 min.

$$= 200 \times 60 = 12000 \text{ m} = 12 \text{ km}$$

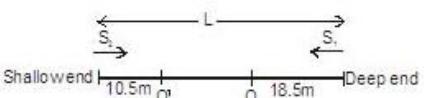
$$\therefore OA = OB = 6 \text{ km}$$

In $\triangle AOM$

$$OM^2 = OA^2 - AM^2 = 6^2 - \left(\frac{1}{4}\right)^2$$

$$= OM \cdot \frac{5\sqrt{23}}{4} \text{ km}$$

2. e



First they meet at point O,

$$\text{for which } \frac{S_2}{S_1} = \frac{L - 18.5}{18.5} \dots (\text{i})$$

as both stay at the ends for equal time, so time taken from the starting up to the second meeting(at O') will be same.

$$\Rightarrow \frac{S_2}{S_1} = \frac{L + (L - 10.5)}{L + 10.5} \dots (\text{ii})$$

$$\text{from (i) and (ii)} \frac{L - 18.5}{18.5} = \frac{2L - 10.5}{L + 10.5}$$

apply componendo & dividendo-

$$\frac{L}{L - 37} = \frac{3L}{L - 21} \Rightarrow L = 45 \text{ m}$$

3. c Ratio of time taken is 3 : 2.

If difference is 1 min, A takes 3 min.

If difference is 10 min, A takes 30 min.

= At double speed A takes 15 min.

4. c If A takes x seconds and B takes y seconds to run 1 km, then

$$x + 19 = \frac{960}{1000}y \text{ and } \frac{960x}{1000} + 30 = y$$

$$\Rightarrow y = 150 \text{ s and } x = 125 \text{ s}$$

$$\Rightarrow \text{answer} = \frac{150}{1000} \times 5000 = 750 \text{ s}$$

5. c Let him walk at 4 km/hr for t hr and at 3 km for h hr

$$\text{Now } 4 \times t + 3 \times h = 36$$

$$\text{Also } 4 \times h + 3 \times t = 34$$

Solving, we get h = 4, t = 6

Therefore, h + t = 6 + 4 = 10 hr

6. d It is given that $\frac{7}{8}$ of the distance = 420 km or total distance = $420 \times \frac{8}{7} = 480$ km.

It is given that he stopped for 9 min and when he increased his speed by 20 km/hr, he reached his destination by 6 min earlier. Hence, there is a difference of $9 + 6 = 15$ min. Now take options into work.

Taking option (d), i.e. 60 km/hr,

$$\text{time taken} = \frac{420}{60} + \frac{60}{60+20} = 7 \text{ hr } 45 \text{ min.}$$

If he travelled the entire 480 km at the rate 60 km/hr, then time taken = 8 hr.

Since there is a difference of 15 min, (d) is the answer.

Alternative method:

Let x be the initial speed. Equating time

$$\frac{420}{x} + \frac{60}{x+20} + \frac{9}{60} = \frac{480}{x} - \frac{6}{60}$$

$$\frac{420}{x} + \frac{60}{x+20} + \frac{1}{4} = \frac{480}{x}$$

$$\Rightarrow \frac{60}{x+20} + \frac{1}{4} = \frac{60}{x}$$

Check from options, x = 60.

$$7. e \text{ Average speed} = \frac{\text{Total distance}}{\text{Total time taken}}$$

$$= \frac{480}{7 \text{ hr } 9 \text{ min } 45 \text{ sec}} = \frac{480}{7 \text{ hr } 54 \text{ min}} = \frac{480}{7.9}$$

$$= 60.75 \text{ km/hr.}$$

$$8. b \text{ LCM of } 224, \text{ i.e. } (2^5 \times 7) \text{ and } 364, \text{ i.e. } (2^2 \times 7 \times 13) = 2^5 \times 7 \times 13$$

$2^5 \times 7$ divides this 13 times.

So A does 13 rounds before meeting again.

$2^2 \times 7 \times 13$ divides this 8 times.

So B does 8 rounds before meeting again.

Thus, the difference in the number of laps is $(13 - 8) = 5$

9. e For every day's work, he can afford to miss 3 days. Hence, to have break even point it has to be 7 days out of 28 days.

10. e In 1 day A and B together produce

$$\frac{1}{15} + \frac{1}{12} = \frac{3}{20} \text{ units}$$

\therefore In 20 days, A and B together will produce $\frac{3}{20} \times 20 = 3$ units, which will fetch them Rs. 270.

$$\text{Now, } \frac{\text{Efficiency of B}}{\text{Efficiency of A}} = \frac{15}{12} = \frac{5}{4}$$

$$= \text{Share of B is } \frac{5}{9} \times 270 = \text{Rs. } 150$$

11. a 12 bottles have to be shared by 10 people, i.e. a man consumes 1.2 bottle and 50% has been consumed

= 6 bottles or (0.6 bottle by each).

2 bottles out of 6 have to be scarified to give them their original share.

$\therefore \frac{2}{6} = \frac{1}{3}$ of the remaining is to be scarified by each.

$$12. d \times \left(\frac{1}{12} + \frac{1}{16} \right) = \frac{28x}{192}$$

$$\text{Left capacity} = 1 - \frac{28x}{192}$$

This is filled by P in 5 min and fills $\frac{1}{12}$ in 1 min

$$= \frac{192 - 28x}{192} = \frac{5}{12} = x = 4 \text{ min}$$

13. c 3 men = 4 boys

Hence, 27 boys can reap a field in 15 days.

So 20 boys + 16 boys = 36 boys will take $\frac{3}{4}$ of the time, i.e. 11.25 days.

= 6 bottles or (0.6 bottle by each).

2 bottles out of 6 have to be scarified to give them their original share.

$\therefore \frac{2}{6} = \frac{1}{3}$ of the remaining is to be scarified by each.

$$12. d \times \left(\frac{1}{12} + \frac{1}{16} \right) = \frac{28x}{192}$$

$$\text{Left capacity} = 1 - \frac{28x}{192}$$

This is filled by P in 5 min and fills $\frac{1}{12}$ in 1 min

$$\Rightarrow \frac{192 - 28x}{192} = \frac{5}{12} \Rightarrow x = 4 \text{ min}$$

13. c 3 men = 4 boys

Hence, 27 boys can reap a field in 15 days.

So 20 boys + 16 boys = 36 boys will take $\frac{3}{4}$ of the time, i.e. 11.25 days.

14. b

Trained labourer	Days	Hours	Work
------------------	------	-------	------

$$\frac{2}{5} \times 33 \quad 15 \quad 12 \quad 1$$

$$y \quad 11 \quad 9 \quad 1.5$$

$$\text{Thus, } y = \frac{2}{5} \times 33 \times \frac{15}{11} \times \frac{12}{9} \times \frac{3}{2} = 36$$

15. c If the cross-sectional circumference of pipe A is 1 unit, that of B would be 2 units and that of C would be 3 units.

So areas would be in the ratio 1 : 4 : 9 or the areas of B and C together is 13 times that of A.

So it takes $\frac{1}{13} \times 16$ min or $1\frac{3}{13}$ min to fill the same time tank.

16. c 1 man = 2.5 boys; 1 woman = 1.5 boys; convert all work in terms of boys.

Thus, group 1 = 43 boys, group 2 = 107.5 boys.

Hence, group 2 would take $25 \times \frac{43}{107.5} \times 3 = 30$ days