Algebra - 7

Career Launcher

QA - 23

CEX-Q-0224/18

Contents

- Maxima and Minima
- Miscellaneous

Number of questions : 30

Maxima and Minima

1. $x^2 + ax + 1 = 0$ has both roots real and negative. The minimum possible value of a is
(1) 0 (2) 1
(3) 2 (4) 3

2. The mean of 15 terms of a series consisting of positive integers is 13. What is the maximum possible value of the largest of the numbers?
(1) 180 (2) 90
(3) 100 (4) 80

3. The sum of 10 distinct natural numbers is S. M is the maximum of those numbers, and is an integer. What would be the minimum value of S/M ?
(1) 4 (2) 3
(3) 2 (4) 1

4. If x, y and z are real numbers such that x + y + z = 5, and xy + yz + xz = 3. What is the largest value which x can have?
(CAT 2002)
(1) $3\sqrt{13}$ (2) $\sqrt{19}$
(3) $\frac{13}{3}$ (4) None of these

5. The number 15 is to be divided into two parts such that the square of one multiplied with the cube of the other is maximum. The parts are
(1) (10, 5) (2) (9, 6)
(3) (8, 7) (4) None of these

6. If a + b + c = 10, where a, b, and c are non-negative real numbers. Find the maximum value of abc.

7. If a + b + c = 10, find the maximum value of (a – 1) (b – 2)(c – 5).Given that $a \geq 1$, $b \geq 2$ and $c \geq 5$.

8. If a + b + c = 6, find the maximum value of a^3b^2c, where all a, b and c are positive real numbers.

9. If x + y + z + w = 29, where x, y, z and w are real numbers greater than 2, then find the maximum possible value of (x – 1) (y + 3) (z – 1) (w – 2).
(1) 625 (2) 256
(3) 1296 (4) 2401

10. If x, y and z are distinct positive integers, and x + y + z = 11, then the maximum value of (xyz + xy + yz + zx) is
(1) 84 (2) 78
(3) 72 (4) 58

11. If $a^2bc^3d^2 = 256$ and the value of $2a + b + 3c + 2d$ is minimum possible, where a, b, c and d are positive real numbers, then find the value of $(a + 2b + 3c + 4d)$.

(1) 16 (2) 18
(3) 24 (4) 20

12. p, q, r, s are any four positive real numbers, the minimum possible value of

$\frac{p}{q}+\frac{q}{r}+\frac{r}{s}+\frac{s}{p}$ is

(1) 1 (2) 2
(3) $2\sqrt{2}$ (4) 4

13. If a, b, c and d are four positive real numbers such that abcd = 1, what is the minimum value of; $(1 + a)(1 + b)(1 + c)(1 + d)$?

(1) 4 (2) 1
(3) 16 (4) 18

(CAT 2001)

14. Let x, y be two positive numbers such that x + y = 1. Then, the minimum value of

$\left(x+\frac{1}{x}\right)^2+\left(y+\frac{1}{y}\right)^2$ is

(1) 12 (2) 20
(3) 12.5 (4) 13.3

(CAT 2001)

15. Find the minimum value of the following function
$f(x) = (x - 5)^2 + (x - 7)^2 - (x - 4)^2 - (x - 8)^2 + (x - 3)^2 + (x - 9)^2$

(1) 11 (2) 10
(3) 13 (4) 12

16. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is **(CAT 2006)**

(1) $\frac{1}{3}$ (2) $\frac{1}{2}$
(3) $\frac{2}{3}$ (4) $\frac{4}{3}$
(5) $\frac{5}{3}$

17. Find the maximum possible value of the function $f(x) = \min(x^2 - 4, 4 - x^2)$, where $-3 \le x \le 3$.

(1) 4 (2) – 4
(3) 0 (4) 2

18. If the positive real numbers a, b and c are in Arithmetic Progression, such that $abc = 4$, then minimum possible value of b is

(1) $2^{\frac{3}{2}}$ (2) $2^{\frac{2}{3}}$
(3) $2^{\frac{1}{3}}$ (4) None of these

Miscellaneous

19. Let T be the set of integers {3, 11, 19, 27, ... 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is

(1) 32 (2) 28
(3) 29 (4) 30

(CAT 2003(L))

20. If pqr = 1, the value of the expression

$\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$ is equal to

(1) $p + q + r$ (2) $\frac{1}{p+q+r}$
(3) 1 (4) $p^{-1}+q^{-1}+r^{-1}$

21. If $\sqrt{x} - \sqrt{1-x} = \frac{1}{5}$ (x is a real number), then find the value of $\sqrt{x} + \sqrt{1-x}$.

(1) $\frac{2}{5}$ (2) $\frac{3}{5}$

(3) $\frac{6}{5}$ (4) $\frac{7}{5}$

22. How many pairs of positive integers m and n satisfy $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}$, where n is an odd integer less than 60? **(CAT 2007)**

(1) 6 (2) 4

(3) 7 (4) 5

(5) 3

23. The total number of integer pairs (x, y) satisfying the equation x + y = xy is **(CAT 2004)**

(1) 0 (2) 1

(3) 2 (4) None of these

24. Let f(x) = $\max\left(\left|\frac{3}{2}x - 3\right|, \left|\frac{3}{5}x + 3\right|\right)$, where x is any real number and max (A, B) is equal to larger number between A & B. Find the value of f(–7) × f(7).

(1) $\frac{486}{5}$ (2) $\frac{216}{25}$

(3) $\frac{-108}{2}$ (4) $\frac{405}{4}$

Challenging

25. The minimum value of V for which $Vy^2 - 6Vy + 5V + 1 > 0$ for all real y is

(1) $\frac{1}{4}$ (2) 0

(3) –4 (4) $-\frac{1}{4}$

26. From four positive real numbers a, b, c and d, 4 combinations of three numbers are chosen such that their sums are S_1, S_2, S_3 and S_4. If a × b × c × d = 5, then find the minimum value of the product of $S_1 \times S_2 \times S_3 \times S_4$.

(1) 40 (2) 135

(3) 405 (4) 1080

27. In the equation $x^2 + y^2 - 14x - 6y - 7 = 0$, x and y are integers. What is the largest possible value of 3x + 4y?

(1) 61 (2) 68

(3) 70 (4) 73

28. Davji Shop sells samosas in boxes of different sizes. The samosas are priced at Rs. 2 per samosa up to 200 samosas. For every additional 20 samosas, the price of the whole lot goes down by 10 paise per samosa. What should be the maximum size of the box that would maximise the revenue?

(1) 240 (2) 300

(3) 400 (4) 250

Directions for questions 29 and 30: Answer the questions on the basis of the information given below.

(CAT 2007)

Shabnam is considering three alternatives to invest her surplus cash for a week. She wishes to guarantee maximum returns on her investment. She has three options, each of which can be utilized fully or partially in conjunction with others.

Option A : Invest in a public sector bank. It promises a return of +0.10%.

Option B : Invest in mutual funds of ABC Ltd. A rise in the stock market will result in a return of + 5% while a fall will entail a return of –3%.

Option C: Invest in mutual funds of CBA Ltd. A rise in the stock market will result in a return of –2.5%, while a fall will entail a return of +2%.

29. The maximum guaranteed return to Shabnam is

(1) 0.25% (2) 0.10%
(3) 0.20% (4) 0.15%
(5) 0.30%

30. What strategy will maximize the guaranteed return to Shabnam?

(1) 100% in option A
(2) 36% in option B and 64% in option C
(3) 64% in option B and 36% in option C
(4) 1/3 in each of the three options
(5) 30% in option A, 32% in option B and 38% in option C

QA - 23 : Algebra - 7
Answers and Explanations

CEX-Q-0224/18

1	3	**2**	2	**3**	4	**4**	3	**5**	2	**6**	–	**7**	–	**8**	–	**9**	4	**10**	2
11	4	**12**	4	**13**	3	**14**	3	**15**	4	**16**	5	**17**	3	**18**	2	**19**	4	**20**	3
21	4	**22**	5	**23**	3	**24**	4	**25**	2	**26**	3	**27**	4	**28**	2	**29**	3	**30**	2

1. 3 Here, from options.

if a = 0

$\Rightarrow \quad x^2 + 1 = 0$ (not possible)

if a = 1 $\Rightarrow x^2 + x + 1 = 0$

$\Rightarrow \quad x = \dfrac{-1 \pm \sqrt{3}\,i}{2}$, which is imaginary (not possible)

if a = 2

$\Rightarrow \quad x^2 + 2x + 1 = 0 \Rightarrow (x + 1)^2 = 0$

So, roots are – 1 and – 1.

This is a possible minimum value.

Alternative method:

For real roots,

$a^2 - 4 \ge 0$

$\Rightarrow \quad (a - 2)(a + 2) \ge 0$

$a \in (-\infty, -2] \cup [2, \infty)$

Since, $x^2 + ax + 1 = x^2 - (\alpha + \beta)x + \alpha\beta = 0$

and α and β are negative

So, a must be positive $\Rightarrow a_{min.} = 2$.

2. 2 As we have to maximise one integer.

So, we will assume the numbers are 1, 2, 3, 4....,14, a (where a is maximum)

$\Rightarrow \quad 1 + 2 + 3 + ... + 14 + a = 15 \times 13$

$\Rightarrow \quad 7 \times 15 + a = 15 \times 13$

$\Rightarrow \quad a = 15(13 - 7) = 15 \times 6 = 90.$

3. 4 To get the minimum value, we will take first 9 natural numbers and 10th number (M) will be maximum such that $\dfrac{S}{M}$ could be maximized.

S = 1 + 2 + 3 + ... + 9 + M

S = 49 + M

Here, if we take M = 100

$\Rightarrow \quad \dfrac{S}{M} = \dfrac{149}{100} = 1.49$

if we further increase the value of M,

$\dfrac{S}{M} \to 1$

So, minimum value of $\dfrac{S}{M} \approx 1$.

4. 3 xy + yz + zx = 3

$\Rightarrow xy + (y + x)z = 3$

$\Rightarrow xy + (y + x)(5 - x - y) = 3$

$\Rightarrow x^2 + y^2 + xy - 5x - 5y + 3 = 0$

$\Rightarrow y^2 + (x - 5)\,y + x^2 - 5x + 3 = 0$

As it is given that y is a real number, the discriminant for above equation must be greater than or equal to zero.

Hence, $(x - 5)^2 - 4(x^2 - 5x + 3) \ge 0$

$\Rightarrow 3x^2 - 10x - 13 \le 0 \Rightarrow 3x^2 - 13x + 3x - 13 \le 0$

$\Rightarrow x \in \left[-1, \dfrac{13}{3}\right]$

Largest value that x can have is $\dfrac{13}{3}$.

Alternative method:

x + y + z = 5

On squaring both sides, we get

$x = \sqrt{19 - (y^2 + z^2)}$

Here, we have to maximise x and $y \ne z \ne 0$ because xy + yz + xz = 3.

So, x must be less than $\sqrt{19}$, only option (3) represents a number less than $\sqrt{19}$.

5. 2 Let, it is divided into two parts x and y.

So, x + y = 15

$$\Rightarrow \quad \frac{\frac{x}{2} + \frac{x}{2} + \frac{y}{3} + \frac{y}{3} + \frac{y}{3}}{5} \ge \left(\frac{x^2 y^3}{2^2 \times 3^3}\right)^{\frac{1}{5}}$$

to maximise R. H. S.

$\dfrac{x}{2} = \dfrac{y}{3}$

$\Rightarrow \quad x : y = 2 : 3$

Hence, x = 6 and y = 9.

6. a + b + c = 10

We know that

AM(a, b, c) $\ge$ GM(a, b, c)

$\Rightarrow \dfrac{a + b + c}{3} \ge (abc)^{1/3}$

$\Rightarrow \dfrac{10}{3} \ge (abc)^{1/3} \Rightarrow \dfrac{1000}{27} \ge abc$

Hence, the maximum possible value of abc is $\dfrac{1000}{27}$.

7. $a + b + c = 10$
$(a - 1) + (b - 2) + (c - 5) = 2$
We know that
$AM(a - 1, b - 2, c - 5) \geq GM(a - 1, b - 2, c - 5)$

$$\Rightarrow \frac{(a-1)+(b-2)+(c-5)}{3} \geq \{(a-1)\times(b-2)\times(c-5)\}^{1/3}$$

$$\Rightarrow \frac{2}{3} \geq \{(a-1)\times(b-2)\times(c-5)\}^{1/3}$$

$$\Rightarrow \frac{8}{27} \geq \{(a-1)\times(b-2)\times(c-5)\}$$

Hence, the maximum possible value of $(a - 1)(b - 2)(c - 5)$ is $\frac{8}{27}$.

8. Since, $a + b + c = 6$
and $AM \geq GM$

$$\text{So, } \frac{\frac{a}{3}+\frac{a}{3}+\frac{a}{3}+\frac{b}{2}+\frac{b}{2}+c}{6} \geq \left(\frac{a^3b^2c}{3^32^2}\right)^{\frac{1}{5}}$$

$$\Rightarrow \frac{6}{6} \geq \left(\frac{a^3b^2c}{3^32^2}\right)^{\frac{1}{5}}$$

$$\Rightarrow 3^32^2 \geq a^3b^2c$$

So, maximum value of given expression is 108.

9. 4 $x + y + z + w = 29$
$(x - 1) + (y + 3) + (z - 1) + (w - 2) = 28$
Applying A.M. $\geq$ G.M.,

$$\frac{(x-1)+(y+3)+(z-1)+(w-2)}{4} \geq ((x-1)(y+3)(z-1)(w-2))^{1/4}$$

Maximum value of $(x - 1)(y + 3)(z - 1)(w - 2)$

$$= \left(\frac{28}{4}\right)^4 = 2401.$$

10. 2 The possible sets for distinct integers (x, y, z) are (1, 2, 8); (1, 3, 7); (1, 4, 6); (2, 3, 6); (2, 4, 5).
So, the maximum value of $(xyz + xy + yz + zx)$ occurs for $x = 2$, $y = 4$ and $z = 5$.
Hence, the maximum value of given expression $= 2 \times 4 \times 5 + 2 \times 4 + 4 \times 5 + 5 \times 2 = 78$.

11. 4 $2a + b + 3c + 2d = a + a + b + c + c + c + d + d$
Applying A.M. $\geq$ G. M.,

$$\frac{a+a+b+c+c+c+d+d}{8} \geq \left(a^2bc^3d^2\right)^{\frac{1}{8}}$$

$$2a+b+3c+2d \geq 16$$

For $2a+b+3c+2d$ to be minimum possible
$a = b = c = d = 2$
$\Rightarrow a + 2b + 3c + 4d = 20$.

12. 4 $\frac{p}{q}, \frac{q}{r}, \frac{r}{s}$ and $\frac{s}{p}$ are all positive numbers and for positive numbers.
A.M. $\geq$ G.M.

$$\text{So, } \frac{\frac{p}{q}+\frac{q}{r}+\frac{r}{s}+\frac{s}{p}}{4} \geq \left(\frac{p}{q}\cdot\frac{q}{r}\cdot\frac{r}{s}\cdot\frac{s}{p}\right)^{\frac{1}{4}}$$

$$\text{or, } \frac{p}{q}+\frac{q}{r}+\frac{r}{s}+\frac{s}{p} \geq 4$$

13. 3 Since a, b, c and d are positive real numbers.

$$\text{So, } \frac{a+b+c+d}{4} \geq (abcd)^{\frac{1}{4}}$$

$\Rightarrow a + b + c + d \geq 4$
Again,

$$\frac{(a+1)+(b+1)+(c+1)+(d+1)}{4} \geq [(a+1)(b+1)(c+1)(d+1)]^{1/4}$$

$$\Rightarrow \frac{a+b+c+d}{4}+1 \geq [(a+1)(b+1)(c+1)(d+1)]^{1/4}$$

Since, we have to minimise RHS, so LHS must be minimum for minimum value of LHS
$a + b + c + d = 4$

$$\Rightarrow \frac{4}{4}+1 = [(a+1)(b+1)(c+1)(d+1)]^{1/4}$$

$$16 = [(a+1)(b+1)(c+1)(d+1)]_{min.}$$

Alternative method:
For minimum value $a = b = c = d = 1$
So, $(1 + a)(1 + b)(1 + c)(1 + d) = 2 \times 2 \times 2 \times 2 = 16$.

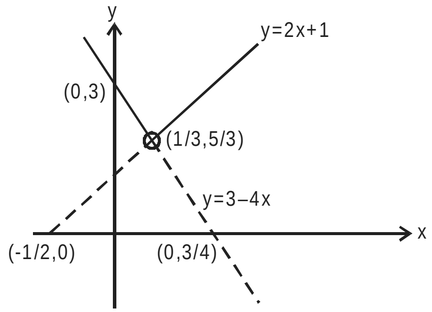
14. 3 Since $x + y = 1$
So, for the minimum value of given expression $x = y = 1/2$
We put this value on the given expression.

$$\text{So, } \left(\frac{1}{2}+2\right)^2 + \left(\frac{1}{2}+2\right)^2 = (2.5)^2 + (2.5)^2 = 12.5$$

15. 4 f(x) is a quadratic function and the coefficient of x^2 is $1 + 1 - 1 - 1 + 1 + 1 = 2 > 0$.
Therefore, the graph of f(x) is an upward-pointing parabola, and the minimum value of the function is attained at its vertex. The given function is also symmetric about $x = 6$.
So, the vertex must be at $x = 6$.
Therefore,
$f(6) = 1 + 1 - 4 - 4 + 9 + 9 = 2(1 + 9 - 4) = 12$
Therefore the minimum value of the function is 12.

16. 5 $f(x) = \max(2x + 1, 3 - 4x)$
So, the two equations are $y = 2x + 1$ and $y = 3 - 4x$
Their point of intersection would be
$2x + 1 = 3 - 4x$

$\Rightarrow 6x = 2 \Rightarrow x = \frac{1}{3}$



So when $x \le \frac{1}{3}$, then $f(x)_{max} = 3 - 4x$

and when $x \ge \frac{1}{3}$, then $f(x)_{max} = 2x + 1$

Hence, the minimum of this will be at $x = \frac{1}{3}$

i.e. $y = \frac{5}{3}$.

17. 3 $f(x) = \min(x^2 - 4, 4 - x^2)$

Case 1: $x^2 - 4 > 4 - x^2 \Rightarrow 2x^2 > 8$
$x < -2$ or $x > 2$

Case 2: $x^2 - 4 < 4 - x^2$
$-2 < x < 2$

Therefore,

$$f(x) = \begin{Bmatrix} 4 - x^2 & -3 \le x \le -2 \text{ or } 2 \le x \le 3 \\ x^2 - 4 & -2 \le x \le 2 \end{Bmatrix}$$

$\Rightarrow$ maximum value of the function is 0.

18. 2 It is given that abc = 4.
We know, (a + b + c) will be minimum when a, b, c are constants and a = b = c.
As a, b, c are in arithmetic progression,
$\Rightarrow a + b + c = 3b$
This will be minimum when a = b = c, $b^3 = 4$.
$\Rightarrow b = 4^{1/3} = 2^{2/3}$

19. 4 T = {3, 11, 19, 27, ..., 451, 459, 467}
Here, we will be getting 470 if we add first term and last term, or second term and second last term and so on.

Now, number of elements in the set $= \frac{(467 - 3)}{8} + 1 = 59$.

So, $\frac{59 + 1}{2} = 30$ elements are such that they will not give their sum 470.

20. 3 Given pqr = 1

$\Rightarrow pq = \frac{1}{r}$ and $\frac{1}{p} = qr$

$$\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$$

$$= \frac{q}{1+q+pq} + \frac{r}{1+qr+r} + \frac{1}{1+r+qr}$$

$$= \frac{qr}{1+qr+r} + \frac{r}{1+qr+r} + \frac{1}{1+r+qr} = \frac{1+r+qr}{1+r+qr} = 1.$$

Alternative method:
Putting x = y = z = 1, we get

$$\frac{1}{1+p+q^{-1}} + \frac{1}{1+q+r^{-1}} + \frac{1}{1+r+p^{-1}}$$

$$= \frac{1}{1+1+1} + \frac{1}{1+1+1} + \frac{1}{1+1+1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1.$$

21. 4 $\sqrt{x} - \sqrt{1-x} = \frac{1}{5}$

$\sqrt{x} + \sqrt{1-x} = a$, say

On squaring and adding, we get

$2(x + 1 - x) = \frac{1}{25} + a^2$

$\Rightarrow a^2 = 2 - \frac{1}{25} = \frac{49}{25}$

$\Rightarrow a = \frac{7}{5}$ (as $a > 0$)

22. 5 $\frac{1}{m} + \frac{4}{n} = \frac{1}{12}, n < 60$

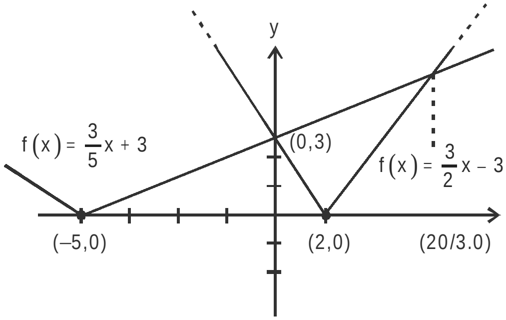
$\Rightarrow \frac{1}{m} = \frac{1}{12} - \frac{4}{n} = \frac{n - 48}{12n}$

$\Rightarrow m = \frac{12n}{n - 48}$

Positive integral values of m for odd integral values of n are for n = 49, 51 and 57.
Therefore, there are 3 integral pairs of values of m and n that satisfy the given equation.

23. 3 Given equation is x + y = xy
$\Rightarrow xy - x - y + 1 = 1$
$\Rightarrow (x - 1)(y - 1) = 1$
$\therefore x - 1 = 1$ and $y - 1 = 1$ or $x - 1 = -1$ & $y - 1 = -1$
Clearly, (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

24. 4



The graph of f(x) is shown in bold in the above figure. We can break-up the definition of f(x) in different regions of the x-axis.

$$f(x)=\begin{cases} -\frac{3}{2}x+3 & x \le 0 \\ \frac{3}{5}x+3 & 0 < x \le \frac{20}{3} \\ \frac{3}{2}x-3 & x \ge \frac{20}{3} \end{cases}$$

Now; f(7) = $\frac{3}{2}(7) - 3 = \frac{15}{2}$ and f(–7) = $\frac{27}{2}$

$$\Rightarrow f(7)\times f(-7)=\frac{405}{4}.$$

Alternative method:

$$f(-7)=\max\left[\left|\frac{-21}{2}-3\right|,\left|\frac{-25}{5}+3\right|\right]=\max\left(\frac{27}{2},2\right)=\frac{27}{2}.$$

$$f(7)=\max\left[\left|\frac{21}{2}-3\right|,\left|\frac{21}{5}+3\right|\right]=\max\left(\frac{15}{2},\frac{36}{5}\right)=\frac{15}{2}.$$

$$\therefore f(-7)\times f(7)=\frac{27}{2}\times\frac{15}{2}=\frac{405}{4}.$$

25. 2 $Vy^2 - 6Vy + 5V + 1 > 0$

$\Rightarrow V(y^2 - 6y + 5) + 1 > 0$

$\Rightarrow V(y^2 - 6y + 9 - 4) + 1 > 0$

$\Rightarrow V(y - 3)^2 - 4V > (-1)$

$\Rightarrow V \times Y > (-1)$, where $Y = (y - 3)^2 - 4$

It should be noted that

$-4 \le Y < \infty$ for any real value of y.

Objective is to find the minimum value of V. If we take any negative value of V and multiply it with a very high positive value of Y, it would be lesser than –1(means even more negative). Thus, V must be non-negative. for V = 0, V × Y = 0 for any Y.

Thus, V = 0 is the minimum value.

26. 3 S_1, S_2, S_3 and S_4 are (a + b + c), (b + c + d), (a + d + c) and (a + b + d). As all the sums are positive, using the AM ≥ GM property, we get,

$$\frac{a+b+c}{3} \ge (abc)^{1/3} \quad \ldots \text{(i)}$$

$$\frac{b+c+d}{3} \ge (bcd)^{1/3} \quad \ldots \text{(ii)}$$

$$\frac{c+d+a}{3} \ge (cda)^{1/3} \quad \ldots \text{(iii)}$$

$$\frac{a+d+b}{3} \ge (adb)^{1/3} \quad \ldots \text{(iv)}$$

Multiplying (i), (ii), (iii) and (iv), we get

$(a + b + c)(b + c + d)(c + d + a)(a + d + b) \ge 3^4 (abcd)$

$S_1 \times S_2 \times S_3 \times S_4 \ge 405$.

27. 4 Simplifying the expression, we get

$(x - 7)^2 + (y - 3)^2 = 8^2 + 1^2$ or $7^2 + 4^2$

To get the maximum value of 3x + 4y we find that x will be equal to

x	y	3x + 4y
15	4	61
14	7	70
8	11	68
11	10	**73**

We are neglecting negative solutions since we are looking for largest value. Hence, the maximum value is 73.

28. 2 Number of samosas = 200 + 20n,

n is a natural number.

Price per samosa = Rs. (2 – 0.1n)

Revenue = $(200 + 20n)(2 - 0.1n) = 400 + 20n - 2n^2$

For maxima, n = 5

⇒ Maximum revenue will be at (200 + 20 × 5)

= 300 samosas

For questions 29 and 30:

To maximise Shabnam's return we need to evaluate all the given options in the question number 7. Assume Shabnam had one rupee to invest. Let the return be denoted by 'r'.

Consider the option (30% in option A, 32% in option B and 38% in option C): If the stock market rises, then

r = 0.1 × 0.3 + 5 × 0.32 – 2.5 × 0.38 = 0.653

If the stock market falls, then

r = 0.1 × 0.3 – 3 × 0.32 + 2 × 0.38 = – 0.197

Consider option (100% in option A): This will give a return of 0.1%.

Consider option (36% in option B and 64% in option C): If the stock market rises, then

r = 5 × 0.36 – 2.5 × 0.64 = 0.2

If the stock market falls, then

r = – 3 × 0.36 + 2 × 0.64 = 0.2

Consider option (64% in option B and 36% in option C): If the stock market rises, then

r = 5 × 0.64 – 2.5 × 0.36 = 2.1

If the stock market falls, then

r = – 3 × 0.64 + 2 × 0.36 = –1.2

Consider option (1/3 in each of the 3 options): If the stock market rises, then

r = 0.1 × 0.33 + 5 × 0.33 – 2.5 × 0.33 = 0.858

If the stock market falls, then

r = 0.1 × 0.33 – 3 × 0.33 + 2 × 0.33 = –0.297

We can see that only in option **(36% in option B and 64% in option C)**, Shabnam gets an assured return of 0.2% irrespective of the behaviour of the stock market. So right option for questions number 13 is **(0.20%)** and question number 14 is **(36% in option B and 64% in option C)**.

29. 3

30. 2