QA - 5 : Algebra – 2

Career Launcher

Workshop

Number of Questions : 30

WSP–0005/18

1. Find the range of values of *x* for which the following inequality holds true

$$\frac{4x^2}{(1-\sqrt{2x+1})^2} < 2x+9?$$

(1) $x \in (-45/8, -1/2)$
(2) $x \in (-1/2, 45/8) - \{0\}$
(3) $x \in (-1/2, 45/8)$
(4) $x \in (-45/8, 45/8)$

2. How many natural number triplets (x, y, z) are possible such that when any of these numbers is added to the product of the other two, the result is 2.

(1) None (2) 1
(3) 2 (4) Infinitely many

3. How many integral solutions for the following equation $a^2 + ab + 4a + 2b = 0$ are possible?

(1) None (2) 3
(3) 6 (4) 12

4. Find the value of x such that f(x) will be minimum, where $f(x) = \Sigma|nx - 1|$; for n = 1 to 10.

(1) $x \in (1/5, 1/6)$ (2) $x \in (5, 6)$
(3) $x \in \{1/7\}$ (4) $x \in \{7\}$

5. Find the number of terms in the expansion of $(1 + x^3 + x^5)^{10}$.

(1) 66 (2) 51
(3) 46 (4) 45

6. A quadratic polynomial $f(x) = ax^2 + bx + c$ and $x \neq 0$ satisfies the following conditions:
1. f(– 5) = 0
2. f(14) = f(56)

Find f(0)/(f(10)).

(1) –5/13 (2) 5/13
(3) 15/17 (4) Cannot be determined

7. What will be the coefficient of a^8b^{12} in the expansion of $(a + b)^{21}$?

(1) Cannot be determined
(2) ${}^{21}C_8 \times {}^{13}C_{12}$
(3) (21!)/(8! × 12!)
(4) None of the above

8. $[1^2/100] + [2^2/100] + [3^2/100] + [4^2/100] + \ldots + [k/100] = 45$ where k is a natural number. Find the digital sum of k.
Digital sum is the recursive one digit sum of all digits of a number. For example, digital sum of 235 = 1.
[.] denotes Greatest Integer Function/Floor Function.

(1) 4 (2) 5
(3) 6 (4) 7

9. For how many integral values of x the inequality below holds true?
$\log_{0.3}[\log_x(2x^2 - 15x - 100)] > \log_{0.3}(2)$

(1) 18 (2) 19
(3) 24 (4) Infinite

10. $A = (2 + 1)(2^2 + 1)(2^4 + 1) \ldots (2^{2048} + 1)$. The value of $(A + 1)^{1/2048}$ is
(1) 4 (2) 2048
(3) 2^{4032} (4) 2

11. The 12 numbers $a_1, a_2, \ldots, a_{12}$ are in Arithmetic Progression. The sum of all these numbers is 354.
Let $P = a_2 + a_4 + \ldots + a_{12}$ and $Q = a_1 + a_3 + \ldots + a_{11}$. If the ratio P : Q is 32 : 27, the common ratio of the progression is
(1) 2 (2) 3
(3) 4 (4) 5

12. a and b are positive integers such that $a^2 + 2b = b^2 + 2a + 5$. The value of b is _________.

13. In the sequences 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8, ... the 2016th term is 2^n. Then n = ________.

14. Diophantus's riddle is a poem that encodes a mathematical problem. In verse, it read as follows:

'Here lies Diophantus,' the wonder behold.
Through art algebraic, the stone tells how old:

'God gave him his boyhood one–sixth of his life,
One twelfth more as youth while whiskers grew rife;

And then yet one-seventh ere marriage begun;
In five years there came a bouncing new son.

Alas, the dear child of master and sage,
After attaining half the measure of his father's life chill fate took him.

After consoling his fate by the science of numbers for four years, he ended his life.'

What was sum of the ages Diophantus and his son when the latter died?
(1) 100 years (2) 122 years
(3) 124 years (4) 126 years

15. Mansi counted the total number of vehicles in a traffic jam (consisting of at least one each of motorcycle, 3 wheeler auto rickshaw and car) to be 150 and total number of wheels (not spares) to be 320. If she knew that there were more cars than auto rickshaws, then the number of vehicles having more than 2 wheels can be
(1) 11 (2) 12
(3) 13 (4) All of the above

16. $f(x) = ax^2 + bx + c$, for $a \neq 0$ is a function whose roots do not lie in the interval (–1,1). Which of the following holds true?
(1) $a + c > 0$ (2) $a^2/(b + c) > 1$
(3) $(a + c)^2/b^2 > 1$ (4) $b^2/(a + c) > 1$

17. If all the roots of f(x) are positive integer, and $f(x) = x^{100} - px^{99} + qx^{98} + rx^{97} + \ldots + 1$ then which of the following can be a value of p?
(1) –100 (2) 1
(3) 99 (4) 100

18. What is the minimum value of $a + \dfrac{1}{\{b(a-b)\}}$ where a and b are positive real numbers and a > b.
(1) 2 (2) 3
(3) 4 (4) 5

19. Which of the following statements is true about the following two terms for n > 1?
(a) n!
(b) $[(n + 1)/2]^n$
(1) n! is larger than $[(n + 1)/2]^n$
(2) $[(n + 1)/2]^n$ is larger than n!
(3) Both can be same for some value of n
(4) Nothing can be said about the above two

20. If $a_0 = 1$, $a_1 = 1$ and $a_n = a_{n-1} \times a_{n-2} + 1$ for n > 1 then
(1) a_{999} is odd and a_{1000} is even
(2) a_{999} is odd and a_{1000} is odd
(3) a_{999} is even and a_{1000} is even
(4) a_{999} is even and a_{1000} is odd

21. Fibonacci series is a series where $a_1 = 0$, $a_2 = 1$ and $a_{n+2} = a_{n+1} + a_n$ for $n > 0$. Which of the following is not true about this series?
(1) a_{30} is a multiple of a_2, a_3 and a_5
(2) $a_1 + a_2 + \ldots + a_{28} = a_{30} - 1$
(3) $a_1^2 + a_2^2 + \ldots + a_{29}^2 = a_{29} \times a_{30}$
(4) $a_1 + a_2 + \ldots + a_{30}$ is odd

22. If the sum of n terms of a series is $2n^2 - n$ then find its n^{th} term.
(1) $3n - 4$
(2) $4n - 3$
(3) $2n^2 - 1$
(4) Cannot be uniquely calculated

23. $(6666666666)^2 + (8888888888) =$
(1) $(4444444444)^2$
(2) (44444444444444444444)
(3) (22222222222222222222)
(4) $(8888888888) + (44444)^2$

24. An exam named MEOW (Management Exam for Orphic Women) has 50 questions. For each question correctly answered, a candidate receives + 4 and for every wrong answer, – 2 are awarded. Unattempt questions do not carry any marks. Smarta wants to know the sum of all possible scores that can be scored in the exam which is
(1) 6713
(2) 7361
(3) 7352
(4) 7631

25. Vicky has infinite notes of custom denomination of Rs. 9 and Rs.11. Which is the highest amount he cannot form using notes of these two denominations? Rs. ________
Type 0 if answer is "Cannot be determined" else type in the answer in the blank space provided.

26. f(a, b) = average of a and b; g(a, b) = max of a and b; h(a, b) = min of a and b. Then how many of the following statements is/are true?
(i) f(g(a, b), h(a, b)) > f(a, b)
(ii) g(f(a, b), h(a, b)) < h(a, b)
(iii) f(f(f(a, b), a), b) > h(a, b)
(iv) h(f(a, b),g(a – b, a + b)) < h(a, b)

(1) None (2) One
(3) Two (4) Three

27. Find number of terms in the expansion of $(a + b + c)^4(b + c + d)^3$
(1) 150
(2) 33
(3) 63
(4) None of the above

28. If (a + b – c)/c, (b + c – a)/a and (c + a – b)/b are in AP (a, b, c $\neq$ 0), then which of the following is/are definitely true?
(1) a = (b + c)/2 (2) b = (a + c)/2
(3) c = 2ab/(a + b) (4) a = 2bc/(b + c)

29. f(x) is a polynomial of degree 49 such that f(1) = 4, f(2) =5, f(3) = 6, ... f(49) = 52. Find f(50). (k is positive constant)
(1) 53 + k49! (2) 53!
(3) 54! (4) 49!

30. x, y, z are distinct real numbers, not equals to 0, such that $x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$. The value of $x^2y^2z^2$ is ______.

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Answers and Explanations

1	2	**2**	2	**3**	3	**4**	1	**5**	4	**6**	2	**7**	4	**8**	1	**9**	1	**10**	1
11	4	**12**	–	**13**	–	**14**	4	**15**	4	**16**	3	**17**	4	**18**	2	**19**	2	**20**	2
21	4	**22**	2	**23**	2	**24**	3	**25**	79	**26**	1	**27**	4	**28**	4	**29**	1	**30**	–

1. 2 Let us check its position for x = 0
But x = 0 makes the LHS indeterminate, which discards option (3) & (4).
Now, if we put x = 1, the inequality holds which means it is true for the values also,

So, answer: $-\frac{1}{2} \le x < \frac{45}{8}$, except x = 0.

So, option (1) also not possible.

2. 2 Put natural numbers starting with 1.
We have:
$x + yz = 2$
$y + zx = 2$
$z + xy = 2$
Subtracting first from second, we have,
$(y – x) – z (y – x) = 0$
$(y – x) (1 – z) = 0$
Which gives either y = x and/or z = 1.
Putting z = 1 in third equation, xy = 1 gives x = y = 1.
Hence, one solution is (1,1,1).
Further putting x = y in first/second and third equation,
$x + xz = 2$
and $z + x^2 = 2$
Putting $z = (2 – x^2)$ from second in first equation above, we get
$x + x(2 – x^2) = 2$
$x^3 – 3x + 2 = 0$
$(x – 1)^2(x + 2) = 0$
So, x = 1 or x = – 2.

Which gives the second only possible solution set as (– 2, – 2, – 2) which is not possible.

3. 3 The given equation can be re-arranged to give
$(a + 2)(a + b + 2) = 4$
The number of integral solutions of the above equation in factorized form will be the number of ways in which 4 can be expressed as product of two integers which will be 6. There are three solutions for positive products and another three solutions for negative products.

4. 1 Sum of modulus is least at the middlemost values of zero of modulus when sorted. If there are two middle values, any value from one of them to another will yield minimum.
There are 10 modulus terms in f(x).
Middlemost modulus will be from $|5x – 1|$, $|6x – 1|$

5. 4 All the terms starting from x^0 to x^{50} can be formed except for the following terms x^1, x^2, x^4, x^7, x^{47} and x^{49}. So, out of 51 powers (0 to 50), 6 are missing, hence a total of 45 terms will be formed.

6. 2 $f(x_1) = f(x_2)$ tells us that the heights of symmetric parabola are same at the two points x_1 and x_2. Because of symmetry, these two points x_1 and x_2 will be equidistant from the axis of symmetry given by x = (–b/2a) and so will be the roots. Hence,
– 5 + other root = 14 + 56
∴ Thus other root = 75
Thus, f(x) = a(x + 5)(x – 75). f(0) = a(5)(–75) while f(10) = a(15)(–65).

7. 4 No such term will exist in the expansion. Hence the coefficient will be 0, as $8 + 12 = 20 \neq 21$.

8. 1 Upto 9^2, value of each [.] will be 0. From10^2 to 14^2 each will give 1. 15^2 to 17^2 will give 2 each. 18^2 and 19^2 will give 3 each. 20^2 to 22^2 will give 4 each. 23^2 and 24^2 will give 5 each while 25^2 will give 6 totaling $(9 \times 0 + 5 \times 1 + 3 \times 2 + 2 \times 3 + 3 \times 4 + 2 \times 5 + 1 \times 6) = 45$.
So, $k = 25^2 = 625$ whose digital sum is 4.

9. 1 $\log_{0.3}[\log_x(2x^2 – 15x – 100)] > \log_{0.3}(2)$
$\log_x(2x^2 – 15x – 100) < (2)$
$2x^2 – 15x – 100 < x^2$
$x^2 – 15x – 100 < 0$
$(x + 5)(x – 20) < 0$
$x \in (– 5, 20)$
Only positive integers greater than 1 will satisfy the given log. So, all values from 2 to 19.

10. 1 A = (2 + 1) (2^2 + 1) (2^4 + 1) … (2^{2048} + 1)
(2 – 1) A = (2 – 1) (2 + 1) (2^2 + 1) … (2^{2048} + 1)
A = (2^2 – 1) (2^2 + 1) … (2^{2048} + 1)
⇒ A = (2^{2048} – 1) (2^{2048} + 1).
A = 2^{4096} – 1

$$(A+1)^{\frac{1}{2048}} = (2^{4096}-1+1)^{\frac{1}{2048}} = 2^2 = 4.$$

11. 4 $\frac{P}{Q} = \frac{6a+36d}{6a+30d} = \frac{32}{27}$
2d = 5a.

$$S_{12} = 354 = \frac{12}{2}[a+a+11d]$$

$$354 = 6\left[\frac{2\times 2d}{5}+11d\right]$$

d = 5.

12. $a^2 + 2b = b^2 + 2a + 5$
$a^2 – 2a = b^2 – 2b + 5$
$(a – 1)^2 – (b – 1)^2 = 5$
(a + b – 2) (a – b) = 5
a + b – 2 = 5
a – b = 1 or a + b – 2 = 1 or a – b = 1 not possible as a, b are positive.
2a = 8
a = 4, b = 3.

13. We observed that
term no from 2^1 to 2^2 – 1 = 3 value is 2 = 2^1
term no from 2^2 to 2^3 – 1 = 7 value is 4 = 2^2
term no from 2^3 to 2^4 – 1 = 15 value is 8 = 2^3
⋮
term no from 2^{10} to 2^{11} – 1 = 2047 value is 2^{10} = 1024
so at 2016th term value is = 1024 = 2^n
n = 10

14. 4 If the prose read carefully, the statement "*After attaining half the measure of his father's life chill fate took him*." it can be deduced that sum of their ages will be a multiple of 3. Only option (d) satisfies it.

Diophantus's youth lasted 1/6 of his life. He grew a beard after 1/12 more of his life. After 1/7 more of his life, Diophantus married. Five years later, he had a son. The son lived exactly half as long as his father and Diophantus died just four years after his son's death. All of this totals the years Diophantus lived.

Let *D* be the number of years Diophantus lived, and let S be the number of years his son lived. Then the above word problem gives the two equations

$$D = \left(\frac{1}{6}+\frac{1}{12}+\frac{1}{7}\right)D+5+5+4 \qquad (1)$$

$$S = \frac{1}{2}D. \qquad (2)$$

Solving this simultaneously gives *S* = 42 as the age of the son and *D* = 84 as the age of Diophantus.

15. 4 Give two wheels to each of the 150 vehicles reducing the wheels to 300. Now, 20 wheels are left, if we give one more wheel to each of the 20 vehicles, there will be 20 auto rickshaws. So, there must be less than 20 "more than 2 wheel" vehicles. If we give 2 wheels to 10 more vehicles, those 10 will become cars. So, there must be more than 10 "more than 2 wheel" vehicles. Only (c) satisfies.
m + a + c = 150
2m + 3a + 4c = 320
Subtracting first equation from second, we get
a + 2c = 20
On checking we get for a = 4 and c = 8 or a = 6, c = 7 or a = 2, c = 9, it satisfies the equation.

16. 3 Take an example of f(x) where one of the roots is less than –1 and the other is more than 1 and cross check.
Alternative Method:
If a > 0, f(1)and f(–1) both are negative.
If a < 0, f(1) and f(–1) both are positive.
So, in either case f(1) x f(–1) > 0.
(a + b + c)(a – b + c) > 0
$(a + c)^2 – b^2 > 0$
$(a + c)^2 > b^2$
$(a + c)^2/b^2 > 1$

17. 4 Product of 100 positive integers is 1. Each one being equal to 1 is the only possibility. So, their sum will be 100.

18. 2 We will be trying AM ≥ GM but for that we need the expression containing terms which will cancel each other when taken into product. With a bit of intelligence, if we write it as b + (a – b) + 1/{b(a – b)}, we see that their product will yield 1.
Hence the AM of these three terms will be = product of them.

19. 2 Put random value of n = 2, 3, 4 and $[(n + 1)/2]^n$ will get farther from n!
Using AM ≥ GM
$(1 + 2 + 3 + … + n)/n \ge (1 × 2 × 3 × … × n)^{1/n}$
$[n(n + 1)/2]/n \ge (1 × 2 × 3 × … × n)^{1/n}$
$[(n + 1)/2]^n \ge n!$
Since, n >1, (n + 1)/2 ≠ n, and thus $[(n + 1)/2]^n > n!$

20. 2 Write down first few terms and observe the pattern

- If a_k is even, then a_{k+1} is odd, since it's of the form (even + odd).
- If a_k is odd and a_{k-1} is even, then a_{k+1} is also odd, since it's still of the same form.
- If a_k is odd and a_{k-1} is odd, then a_{k+1} is of the form (odd)(odd) + odd = even, so a_{k+1} is even.

So every third sequence term is even.
So, 998 will be even and hence 999 and 1000 will be odd.

21. 4 Write down the first few terms and check for the options. It is a series where in a group of any 3 consecutive terms, two terms are odd while one term is even. Every other option is a fact about this series.

22. 2 Put n = 1, in the options and sum given in the question and cross check, select and eliminate. Then put n = 2 and get the answer.
$S_n = t_1 + t_2 + \dots + t_n$
$S_{n-1} = t_1 + t_2 + \dots + t_{n-1}$
On subtracting, we get $S_n - S_{n-1} = t_n$,
or $t_n = (2n^2 - n) - [2(n-1)^2 - (n-1)] = 4n - 3$

23. 2 Use last digit check.
$(6666666666)^2 + (8888888888)$
$= [6(9999999999)/9]^2 + 8(9999999999)/9$
$= 4/9[10^{10} - 1]^2 + 8/9[10^{10} - 1]$
$= 4/9[10^{20} - 1]$
$= 4(99999999999999999999)/9$
$= (44444444444444444444)$

24. 3 All the scores possible are even. So, their sum will also be even.
Since the marking scheme is +4 and – 2, all scores from –100, – 98, – 96, …, 194, 196, 200. The score 198 cannot be scored. So, there are a total of 151 even numbers from –100 to 200 whose sum is 151x(–100 + 200)/2. So, sum of all scores is 7550 – 198.

25. 79 Rs. 79 Using two co–prime numbers 'a' and 'b', 'a × b – a – b' is the highest value that cannot be formed using them.

26. 1 Put random values which are equal, of same signs or of opposite signs and check.

27. 4 Any term will be in the form of $ka^xb^yc^zd^w$
Such that $x + y + z + w = 7$ and $w \le 3$ and $y \le 4$
There are total 90 possible solutions of the above equation.

28. 4 Try putting a, b, c from 1, 2 and 3 assuming first or second option is true. Then putting same as 1/1, 1/2 and 1/3 assuming either third or fourth is true.
(a + b – c)/c, (b + c – a)/a and (c + a – b)/b are in AP
(a + b – c)/c + 1, (b + c – a)/a + 1 and (c + a – b)/b + 1 are in AP
(a + b + c)/c, (b + c + a)/a and (c + a + b)/b are in AP
1/c, 1/a and 1/b are in AP
c, a and b are in HP

29. 1 From given data, we can say that
$f(x) = k(x-1)(x-2)(x-3) \dots\dots (x-49) + x + 3$
So, f(50) = k49! + 53.

30. $$x + \frac{1}{y} = y + \frac{1}{z} = z + \frac{1}{x}$$
solving in pair we get
$$x - y = \frac{y - z}{yz} \quad \dots \text{(i)}$$
$$y - z = \frac{z - x}{zx} \quad \dots \text{(ii)}$$
$$z - x = \frac{x - y}{xy} \quad \dots \text{(iii)}$$
(i) × (ii) × (iii)
$$(x-y)(y-z)(z-x) = \frac{(x-y)(y-z)(z-x)}{x^2y^2z^2}$$
$\therefore x^2y^2z^2 = 1.$