Numbers - 5

Career Launcher

QA - 15

CEX-Q-0216/18

Contents

- Remainder

Number of Questions : 25

1. What is the remainder when $(37)^{47}$ is divided by 19?
(1) 18 (2) 17
(3) 1 (4) 0

2. The remainder when 25^{1000} is divided by 13 is
(1) 0 (2) 12
(3) 2 (4) 1

3. Find the remainder when 34^{413} is divided by 9.
(1) 6 (2) 8
(3) 4 (4) 1

4. What is the reminder when $(15^{23} + 23^{23})$ is divided by 19?
(1) 4 (2) 15
(3) 0 (4) 18

5. Let N = 1421 × 1423 × 1425. What is the remainder when N is divided by 12? **[CAT 2000]**

6. What is the remainder when 4^{96} is divided by 6? **[2003(R)]**
(1) 0 (2) 2
(3) 3 (4) 4

7. What is the remainder when $723^{243} + 318^{243}$ is divided by 17?
(1) 2 (2) 3
(3) 8 (4) 9

8. P is a prime number greater than 30. When P is divided by 30, the remainder is x. How many different values of x are possible?
(1) 9 (2) 8
(3) 10 (4) 11

9. What is the remainder when $13^{66} - 23$ is divided by 183?
(1) 0 (2) 161
(3) 22 (4) 162

10. What is the remainder when 112123123412345….. up to 36 digits is divided by 36?
(1) 20 (2) 30
(3) 10 (4) None of These

11. A number when divided by 7 leaves a remainder of 5. What will be the remainder when the square of the same number is divided by 7?
(1) 5 (2) 4
(3) 1 (4) 0

12. What is the remainder when n! + (n! + 1) + (n! – 2) + (n! + 3) ..... + (n! – 2006) is divided by 1003 for n = 1003?
(1) 1 (2) 0
(3) 2006 (4) None of these

13. Let $f(y) = y^{y^{y^{y^{y^{y^{\cdots}}}}}}$. What is the remainder when f(2) is divided by 5?
(1) 2 (2) 3
(3) 4 (4) 1

14. V is a positive integer. When V is divided by any of the numbers 2, 11, 13, 71 and 89, the remainder obtained in each case is 1. What is the remainder when V^{256} is divided by 16?

15. When 34369 and 31513 are divided by a certain three digit number, the remainders are equal. Find the remainder.
(1) 79 (2) 97
(3) 87 (4) 78

16. Find the remainder when 5^{4k}, where k is a natural number, is divided by 6.

17. Find the remainder when $(1^3 + 3^3 + 5^3 + ... + 23^3)$ is divided by 4.
(1) 0 (2) 1
(3) 3 (4) 2

18. Find the remainder when 13^{1982} is divided by 100.

19. Find the remainder when 29^{202} is divided by 13.
(1) 5 (2) 3
(3) 12 (4) 0

20. Find the last two digits of 3333^{4442}.
(1) 11 (2) 89
(3) 81 (4) 49

21. Find the remainder when 4^{64} is divided by 21.
(1) 20 (2) 1
(3) 4 (4) 19

22. What is the remainder when 3^{52} is divided by 53?
(1) 1 (2) 2
(3) 52 (4) 38

23. What is the remainder when 4^{67} is divided by 67?
(1) 66 (2) 61
(3) 56 (4) 4

24. What is the remainder when 12! is divided by 13?
(1) 11 (2) 12
(3) 9 (4) 0

25. What is the remainder when 16! is divided by 272?

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QA - 15 : Numbers - 5
Answers and Explanations

CEX-Q-0216/18

1	1	**2**	4	**3**	3	**4**	3	**5**	–	**6**	4	**7**	4	**8**	2	**9**	2	**10**	2
11	2	**12**	2	**13**	4	**14**	–	**15**	2	**16**	–	**17**	1	**18**	–	**19**	2	**20**	2
21	3	**22**	1	**23**	4	**24**	2	**25**	–										

1. 1 We can write $(37)^{47}$ as $(38 – 1)^{47}$ which when divided by 19 leaves a remainder of $(– 1)^{47}$ or 18.

2. 4 $$\text{Rem}\left(\frac{25^{1000}}{13}\right)=\text{Rem}\left(\frac{(26-1)^{1000}}{13}\right)$$

$$=\text{Rem}\left(\frac{(-1)^{1000}}{13}\right)=1.$$

3. 3 $$\text{Rem}\left(\frac{34^{413}}{9}\right)=\text{Rem}\left(\frac{(-2)^{413}}{9}\right)=\text{Rem}\left(\frac{(-1)^{413}\times 2^{413}}{9}\right)$$

$$=\text{Rem}\left(\frac{(-1)^{413}\times(2^3)^{137}\times 2^2}{9}\right)$$

$$=\text{Rem}\left(\frac{(-1)\times(9-1)^{137}\times 2^2}{9}\right)=\text{Rem}\left(\frac{(-1)\times(-1)^{137}\times 2^2}{9}\right)$$

$$=\text{Rem}\left(\frac{(-1)\times(-1)\times 4}{9}\right)=4.$$

4. 3 $15^{23} = (19 – 4)^{23} = 19x + (–4)^{23}$ where x is a natural number.
$23^{23} = (19 + 4)^{23} = 19y + (+4)^{23}$ where y is a natural number.
$15^{23} + 23^{23} = 19 (x + y) + 4^{23} + (–4)^{23} = 19 (x + y)$
The above expression is multiple of 19. Hence, the required remainder is 0.

5. $$\text{Rem}\left(\frac{1421\times 1423\times 1425}{12}\right)$$

$$=\text{Rem}\left(\frac{(118\times 12+5)\times(118\times 12+7)\times(118\times 12+9)}{12}\right)$$

$$=\text{Rem}\left(\frac{5\times 7\times 9}{12}\right)=3.$$

6. 4 $$\text{Rem}\left(\frac{4^1}{6}\right)=4$$

$$\text{Rem}\left(\frac{4^2}{6}\right)=4$$

$$\text{Rem}\left(\frac{4^2}{6}\right)=4$$

$$\text{Rem}\left(\frac{4^4}{6}\right)=4$$

Hence, any power of 4 when divided by 6 leaves a remainder of 4.

7. 4 $\dfrac{(723^{243}+318^{243})}{17}$

$= \dfrac{(731-8)^{243}+(323-5)^{243})}{17}$

As 731 and 323 are divisible by 17, the net remainder is the same as when $(-8)^{243} + (-5)^{243}$ is divided by 17.

$$\text{Rem}\left(\frac{\left\{(-8)^{243}+(-5)^3\times(-5)^{240}\right\}}{17}\right)$$

$$=\text{Rem}\left(\frac{\left\{-2^{729}+(-5)^3\times(625)^{60}\right\}}{17}\right)$$

$$=\text{Rem}\left(\frac{\left\{(-2)\times(2^4)^{182}-(125)\times(612+13)^{60}\right\}}{17}\right)$$

$$=\text{Rem}\left(\frac{\left\{(-2)\times(17-1)^{182}-(125)\times(17\times36k+(170-1)^{30})\right\}}{17}\right)$$

(where k is a natural number)

$$=\text{Rem}\left(\frac{\left\{(-2)\times(17-1)^{182}-(17\times7+6)\times(17\times36k+(17\times10-1)^{30}\right\}}{17}\right)$$

$\therefore$ The required remainder = –2 – 6(0 + 1) = –8 or 9.

8. 2 P = 30 k + x
$\therefore$ P is prime, x cannot be a multiple of 2, 3 or 5. x also has to be less than 30.
Therefore possible values of x are 1, 7, 11, 13, 17, 19, 23 and 29.

9. 2 $13^{66} - 23$ can be written as $(13^3)^{22} - 23$
Remainder when 13^3 divided by 183 is 1.
Remainder when $(13^3)^{22}$ divided by 183 is also 1.
Remainder when $(13^3)^{22} - 23$ divided by 183 is – 22 or 161.

10. 2 The number is a series of 1 natural number, 2 natural numbers, 3 natural numbers and so on. Hence to get 36 digits we need to find n for which 1 + 2 + 3 + … + n $\geq$ 36. For n = 8, the sum is exactly 36 which means the number will have 12345678 at the end.
To divide by 36, we need to divide the number by 4 and 9.
The rule of 4 is to divide the last 2 digits of the number i.e. 78. 78 divided by 4 gives a remainder of 2.
The rule of 9 is to divide the sum of the digits by 9. The sum of digits is 1 × 8 + 2 × 7 + 3 × 6 + 4 × 5 + 5 × 4 + 6 × 3 + 7 × 2 + 8 × 1 = 120 which divided by 9 gives a remainder of 3.
Hence, the remainder by 36 is the smallest number which when divided by 4 and 9 leaves remainders of 2 and 3 respectively i.e. 30. Hence answer 30.

11. 2 Let the number be x = 7p + 5.
$\therefore$ $x^2 = (7p + 5)^2$
$\Rightarrow$ $x^2 = 49p^2 + 70p + 25$
$\Rightarrow$ $x^2 = 7(7p^2 + 10p + 3) + 4$
Hence, the remainder is 4.

12. 2 Given expression is
n! + (n! + 1) + (n! – 2) + (n! + 3) ..... + (n! – 2006)
= 2007 × n! + (1 – 2 + 3 – 4 ..... – 2006)
= 2007 n! + 1003 × (–1)
Now, for n = 1003, the expression 2007 n! – 1003 is clearly divisible by 1003.
Hence, the remainder is zero. Option (2) is the correct choice.

13. 4

= 5 N + 1 (N is a natural number)
When 5N + 1 is divided by 5, the remainder will be1.

14. We have, $V^{256} = (V^{64})^4$
$\therefore$ V is of the form 2K + 1.
$\therefore$ It is odd.
Thus, V^{64} is also odd.
(V^{64}) when divided by 16 will leave a remainder of 1 as fourth power of any odd number divided by 16 leaves a remainder 1.

15. 2 Let V be the number that when divides 34369 and 31513 leaves the same remainder. V must be a factor of 34369 – 31513 = 2856. When 31513 is divided by 2856 it leaves a remainder 97. Hence, any three-digit factor of 2856, when divides 31513 the remainder will be 97.

16. $\text{Rem}\left[\frac{5^{4k}}{6}\right] = \text{Rem}\left[\frac{(-1)^{4k}}{6}\right] = \text{Rem}\left[\frac{1}{6}\right] = 1.$

17. 1 The terms $1^3, 5^3, 9^3, ..., 21^3$ are of the form 4k + 1.
The terms $3^3, 7^3, 11^3, ..., 23^2$ are of the form 4k + 3.

$$= \text{Rem}\left[\frac{6 \times (4k+1) + 6 \times (4k+3)}{4}\right]$$

$$= \text{Rem}\left[\frac{6 \times 1 + 6 \times 3}{4}\right] = 0.$$

18. Since 13 is coprime to 100, it follows a power cyclicity of 20 in its last 2 digits.
So last two digits of 13^{1982} = last two digits of 13^2 = 69.
Hence, the required remainder is 69.

19. 2 $\text{Rem}\left(\frac{29^{202}}{13}\right) = \text{Rem}\left(\frac{3^{202}}{13}\right) = \text{Rem}\left(\frac{(3^3)^{67} \times 3}{13}\right)$

$$= \text{Rem}\left(\frac{(2 \times 13 + 1)^{67} \times 3}{13}\right) = \text{Rem}\left(\frac{1^{67} \times 3}{13}\right) = 3.$$

20. 2 Cyclicity for the last two digits of any number is 20.
Last two digits of 3333^{4442} = Last two digits of 3333^2 = Last two digits of 33^2 = 89.

21. 3 Let x, a and b, where a and b are co-prime to each other. Let r_1 and r_2 be the remainders when x is divided by a and b respectively.
When x is divided by a × b, the remainder obtained will be the smallest numbers which when divided by a and b leaves remainders r_1 and r_2 respectively.
21 = 3 × 7

$$\text{Rem}\frac{4^{64}}{3} = 1$$

$$\text{Rem}\frac{4^{64}}{7} = \text{Rem}\frac{(4^3)^{21} \times 4}{7} = \text{Rem}\frac{(9 \times 7 + 1)^{21} \times 4}{7} = 4$$

The smallest number which when divided by 3 × 7 leaving remainder 1 and 4 respectively is 4.
Hence, the required remainder is 4.

Note: This theorem is known as Chinese Remainder Theorem.

22. 1 This question can easily be solved using Euler's Theorem.
If a and m are co-prime to each other, the Remainder obtained when $a^{\phi(m)}$ where $\phi(m)$ is the number of co-primes to m that are less than m, divided by m is 1.
$\phi(53) = 52$ (As 53 is a prime number)

$$\text{Rem}\left(\frac{3^{52}}{53}\right) = \text{Rem}\left(\frac{3^{\phi(53)}}{53}\right) = 1$$

23. 4 $\text{Rem}\left(\frac{4^{67}}{67}\right) = \text{Rem}\left(\frac{4^{66} \times 4}{67}\right) = \text{Rem}\left(\frac{4^{\phi(67)} \times 4}{67}\right)$

= 1 × 4 = 4.

24. 2 This problem can be easily solved using Wilson's Theorem.
According to this theorem, when (n – 1)! is divided by n, where n is a prime number, the remainder obtained is –1 or n – 1.

$$\text{Rem}\left(\frac{12!}{13}\right) = \text{Rem}\left(\frac{(13-1)!}{13}\right) = -1 \text{ or } 12.$$

25. $\text{Rem}\left(\frac{16!}{272}\right) = \text{Rem}\left(\frac{16!}{16 \times 17}\right)$

$$\text{Rem}\frac{16!}{16} = 0$$

$$\text{Rem}\frac{16!}{17} = \text{Rem}\frac{(17-1)!}{17} = -1 \text{ or } 16$$

Now using Chinese Remainder Theorem the required remainder will be the smallest number that when divided by 16 and 17 gives remainders 0 and 16 respectively. Such smallest number is 16. Hence, the required remainder is 16.