Set Theory - 2

Career Launcher

LRDI - 09

CEX-D-0281/18

Number of Questions : 25

Directions for questions 1 to 4: Answer the questions on the basis of the information given below.

In class 11th, each of the 72 students study at least one of the three subjects: Math, Physics and Chemistry. In the next year they all promoted to class 12th and some of these students changed their subjects among same three areas. It is further known that:

I. In class 11th the number of students studying only Math, only Physics and only Chemistry were 3 consecutive integers in increasing order but in 12th class the same 3 areas were the same 3 consecutive integers in decreasing order. Exactly the same property with some other 3 consecutive integers was true for three other respective areas i.e. only Physics and Chemistry, only Math and Chemistry and only Math and Physics.

II. In class 11th, 3 students studied all 3 subjects whereas 12 students study Math and Physics.

1. What is the difference between the number of students studying Math in 11th and 12th?
   (1) 0 (2) 2
   (3) 4 (4) 6

2. How many students who were studying only Physics in 11th started studying at least one more subject in 12th?
   (1) 0 (2) 2
   (3) 4 (4) Cannot be determined

3. What is the minimum number of students who gave up or took another subject/s?

4. How many students study Physics and at least one more subject in class 12th?
   (1) 17 (2) 18
   (3) 19 (4) Cannot be determined

Directions for questions 5 to 8: Answer the questions on the basis of the information given below.

Three projects: Lakshay, Moksha and Naksha are handled by a company having 50 employees. Each employee is working with one or more projects. The number of employees working in Lakshay only and Lakshay with exactly one more project is in the ratio 2 : 3. The number of employees working in Naksha only and Moksha only is in the ratio 3 : 4. No employee works in all three projects and exactly 27 employees work in exactly 2 projects. The number of employees working in Naksha and Moksha is thrice the number of employees working in Naksha and Lakshay.

5. How many employees work in Moksha only?

6. How many employees work in Naksha?
   (1) 7 (2) 16
   (3) 41 (4) Cannot be determined

7. How many employees work in Lakshay and at least one more project?
   (1) 5
   (2) 15
   (3) 24
   (4) 40

8. Maximum number of employees work in
   (1) Lakshay
   (2) Moksha
   (3) Naksha
   (4) Lakshay and Moksha both

Directions for questions 9 to 12: Answer the questions on the basis of the information given below.

A and B are two sets (e.g. A = Mothers, B = Women). The elements that could belong to both the sets (e.g. women who are mothers) is given by the set C = A . B. The elements which could belong to either A or B, or both, is indicated by the set D = $A \cup B$. A set that does not contain any elements is known as a null set represented by φ (e.g. if none of the women in the set B is a mother, then C = A .B is a null set, or C = φ).
Let 'V' signify the set of all vertebrates, 'M' the set of all mammals, 'D' dogs, 'F' fish, 'A' alsatian and 'P', a dog named Pluto.

9. Given that X = M .D is such that X = D. Mark all the options which are true?
   (1) All dogs are mammals
   (2) Some dogs are mammals
   (3) X = φ
   (4) All mammals are dogs

10. If Y = F . (D . V) is not a null set, it implies that
   (1) all fish are vertebrates
   (2) all dogs are vertebrates
   (3) some fish are dogs
   (4) None of these

11. If Z = (P . D) $\cup$ M, then
   (1) The elements of Z consist of Pluto, the dog, or any other mammal
   (2) Z implies any dog or mammal
   (3) Z implies Pluto or any dog that is a mammal
   (4) Z is a null set

12. If P . A = φ and P $\cup$ A = D, then which of the following is true?
   (1) Pluto and alsatians are dogs
   (2) Pluto is an alsatian
   (3) Pluto is not an alsatian
   (4) D is a null set

13. There are 240 second year students in a B-School. The Finance area offers 3 electives in the second year. These are Financial Derivatives, Behavioural Finance, and Security Analysis. Four students have taken all the three electives, and 48 students have taken Financial Derivatives. There are twice as many students who study Financial Derivatives and Security Analysis but not Behavioural Finance, as those who study both Financial Derivatives and Behavioural Finance but not Security Analysis, and 4 times as many who study all the three. 124 students study Security Analysis. There are 59 students who could not muster courage to take up any of these subjects. The group of students who study both Financial Derivatives and Security Analysis but not Behavioural Finance, is exactly the same as the group made up of students who study both Behavioural Finance and Security Analysis. How many students study Behavioural Finance only?

Directions for questions 14 to 17: Answer the questions on the basis of the information given below.

In a school, students like four different actors – Aamir Khan, Shahrukh Khan, Salman Khan and Akshay Kumar. 17% of the students in the school don't like any of these four actors. Number of students liking these four actors are

Aamir Khan	385
Shahrukh Khan	335
Salman Khan	335
Akshay Kumar	375

The number of students liking exactly two actors for any combination of two actors is 60. There are 80 students who like all the four actors but there is no such student who likes exactly three out of these four actors.

14. How many students do not like any of these four actors?
(1) 130 (2) 136
(3) 170 (4) 152

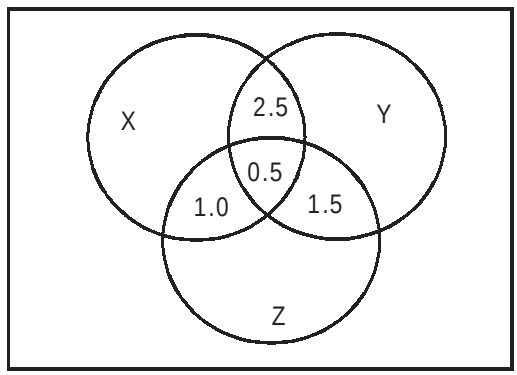
15. Approximately what percentage of the students liking Shahrukh Khan also like atleast one other actor?
(1) 77.6% (2) 77.4%
(3) 77.8% (4) 78%

16. If all the students in the school including these who do not like any actor, start liking atleast one actor, which he/she is not liking at the present, then what could be the least number of students liking all the four actors?
(1) 80 (2) 70
(3) 60 (4) 50

17. What is the number of students liking exactly one actor?
(1) 350 (2) 375
(3) 390 (4) 420

Directions for questions 18 to 21: Answer the questions on the basis of the information given below.

The Venn-diagram given below shows the estimated readership of three daily newspapers (X, Y and Z) in a city.



The total readership and advertising cost for each of these papers is as follows:

Newspaper	Readership (lakhs)	Advertising cost (Rs. Per sq. cm.)
X	8.7	6000
Y	9.1	6500
Z	5.6	5000

The total population of the city is estimated to be 14 million. The common readership (in lakhs) is indicated in the above Venn-diagram.

18. The number of people (in lakhs) who read at least one newspaper is
(1) 4.7 (2) 11.9
(3) 17.4 (4) 23.4

19. The number of people (in lakhs) who read only one newspaper is
(1) 4.7 (2) 11.9
(3) 17.4 (4) 23.4

20. The approximate percentage of population reading at least two newspapers is
(1) 2.9 (2) 3.5
(3) 3.9 (4) None of the above

21. The ratio of readers reading only one newspaper to those reading only two newspapers is
(1) 2.38 : 1 (2) 3.65 : 1
(3) 4.57 : 1 (4) None of the above

Directions for questions 22 to 25: Answer the questions on the basis of the information given below.

Some students appeared for a test having 4 sections: QA, LR, DI and VA. Every student clears the cut-off in one or more of these sections. 43, 42, 36 and 39 students cleared the cut-off of QA, LR, DI and VA respectively. Further it is known that:

I. 11, 12, 13 and 14 students cleared the cut-off in at least 2 more sections apart from DI, LR, VA and QA respectively.

II. The number of students who cleared the cut-off of QA and LR only is equal to those who cleared the cut-off of DI and VA only which is two more than those who cleared the cut-off of QA, LR and DI only.

III. The number of people who cleared the cut-off in QA and exactly one more section is equal to those who cleared cut-off in LR and exactly one more section but is 2 less than those who cleared cut-off in VA and exactly one more section.

IV. 15 students cleared the cut-off of QA only, 6 students cleared the cut-off of VA only, 4 students cleared the cut-off of LR and QA only whereas 6 students cleared the cut-off of LR and DI only.

22. How many students cleared the cut-off in more than 2 sections?

23. How many students cleared the cut-off in QA and LR but not in DI?

24. How many students cleared cut-off in all 4 sections?

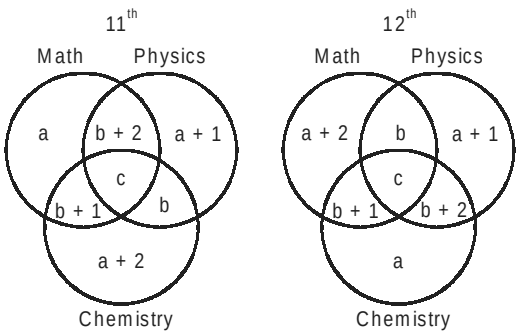
25. What is the difference between the number of students who cleared cut-off in exactly one section and those who cleared cut-off in exactly 3 sections?

LRDI - 09
Answers and Explanations

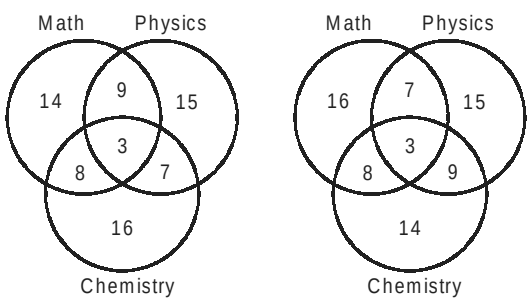
CEX-D-0281/18

1	1	**2**	4	**3**	4	**4**	3	**5**	4	**6**	1	**7**	3	**8**	1	**9**	2, 4	**10**	3
11	1	**12**	3	**13**	29	**14**	3	**15**	1	**16**	1	**17**	3	**18**	3	**19**	2	**20**	3
21	1	**22**	15	**23**	8	**24**	1	**25**	28										

For questions 1 to 4:
Let the number of students studying only Math, only Physics and only Chemistry be a, a + 1 and a + 2 respectively in class 11^{th} and hence a + 2, a + 1 and a in class 12^{th}. Similarly let the number of students studying Physics and Chemistry only, Math and Chemistry only and Math and Physics only be b, b + 1 and b + 2 in class 11^{th} and b + 2, b + 1 and b in class 12^{th}.



Using the statement II and the information above we can solve the data as:
c = 3, b + 2 + c = 12 i.e. b = 7. Also, sum of all numbers is 72.
So, a + a + 1 + a + 2 + b + b + 1 + b + 2 + c = 72. Hence, a = 14.
The final venn diagram looks like:



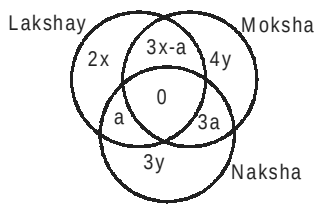
1. 1

2. 4 (As we do not know how many students changed from a subject to other)

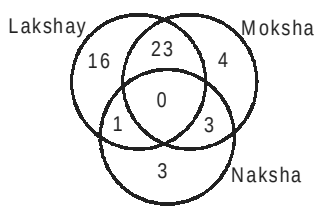
3. 4

4. 3

For questions 5 to 8:
Let 2x number of employees work in Lakshay only. Hence, 3x employees will work in Lakshay and Moksha or Lakshay and Naksha. Also, let a number of employees work in Lakshay and Naksha so, 3x – a work with Lakshay and Moksha. Also, let 4y and 3y number of employees work in Moksha only and Naksha only. The venn diagram looks like:



Now, 2x + 3y + 4y = 23 i.e. 2x + 7y = 23.
Here either x = 1, y = 3 OR x = 8, y = 1.
Also, 3x + 3a = 27 i.e. x + a = 9.
Hence either x = 1, a = 8 OR x = 8, a = 1.
As 3x – a must be positive, x = 8 and a = 1 is the only viable solution. The final venn diagram looks like:



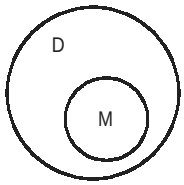
5. 4

6. 1

7. 3

8. 1

9. 2, 4 Here, X = M.D denotes the elements which could belong to either mammals or dogs or both.
Again, X = D denotes the elements which belong to dogs. This can be denoted as



Thus, option (2) and (4) are both correct.

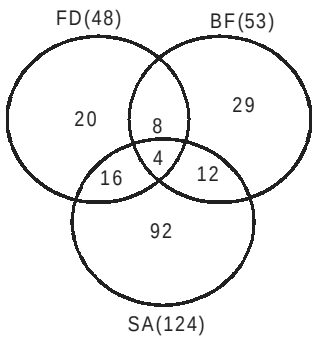
10. 3 $Y = F \cap (D \cap V)$ is not a null set means some F's are D's and sum D's are V's .
This implies some fish are dogs.

11. 1 $Z = (P \cap D) \cup M$
$P \cap D$ means Pluto, the dog.
$(P \cap D) \cup M$ means Pluto, the dog or any other mammal.

12. 3 $P.A = \phi$; $P \cup A = D$

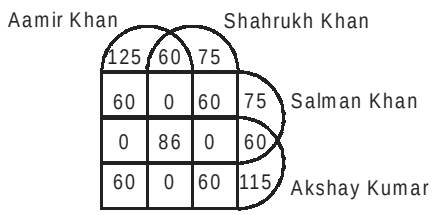
$P \cap A = \phi$ means no alsatian is Pluto or Pluto is not an alsatian where dogs are composed of alsatian or Pluto or both.

13. 29 There are 240 – 59 = 181 students who study at least one of Financial Derivatives (FD), Behavioral Finance (BF) and Security Analysis (SA). The Venn diagram is given below:



So, the answer = 29.

For questions 14 to 17:
Out of 6 actors, liking exactly 2 actors is possible in 6 different combinations i.e. $4C_2 = 6$
They are Aamir and Shahrukh, Aamir and Salman, Aamir and Akshay, Salman and Shahrukh, Salman and Akshay, Shahrukh and Akshay i.e. each actor appears 3 times in these 6 combinations.
Now, number of students liking for each of these pair is 60, so total 60 × 6 = 360 students like exactly 2 actors.
Also, no one likes 3 out of 4 actors and 80 students like all the 4 actors.
Now, to get the number of students liking one particular actor alone, we have to subtract 3 times 60 (as mentioned above that each actor occurs 3 times in total 6 combinations), and also we have to subtract 80, which is the number of students liking all 4 actors.
∴ Number of students liking only Aamir Khan
= 385 – 3 × 60 – 80 = **125**
Number of students liking only Shahrukh Khan
= 335 – 3 × 60 – 80 = **75**
Number of students liking only Salman Khan
= 335 – 3 × 60 – 80 = **75**
Number of students liking only Akshay Kumar
= 375 – 3 × 60 – 80 = **115**
∴ Total number of students liking exactly one actor
= 125 + 75 + 75 + 115 = **390**
Total number of students liking exactly two actors
= 60 × 6 = **360**
Total number of students liking exactly three actors
= 0 (given)
Total number of students liking exactly four actors
= 80
∴ Total number of students liking atleast 1 actor
= 390 + 360 + 80 = **830**
Now, 830 represents 83% of the total number of students.
∴ Total number of students = **1000**
Number of students who do not like any actor
= 1000 – 830 = **170**
So, we can make a Venn-diagram for it.



14. 3 170

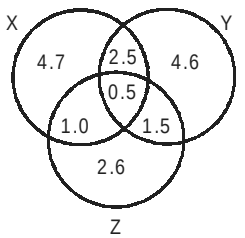
15. 1 Number of students liking Shahrukh Khan and atleast 1 more actor.
= students liking exactly 2 actors (including Shahrukh Khan) + students liking all 4 actors.
= 3 × 60 + 80 = 260

$\therefore$ Required percentage = $\frac{260}{335} \times 100 \approx 77.6\%$

16. 1 Least increase in the number of students who like all four actors will came only if each student likes exactly 1 more actor. But since the number of students who like exactly 3 actors is zero, there will not be any addition to the figure of 80 students who like all 4 actors.
$\therefore$ Hence the answer is 80.

17. 3 As per the above Venn diagram, Number of students liking exactly 1 actor
= 125 + 75 + 75 + 115 = 390

For questions 18 to 21:
As per the direction given, the following Venn diagram can be formed:-



18. 3 Number of people (in lakhs) who read at least one news paper
= 2.5 + 0.5 + 1.5 + 1.0 + 4.7 + 4.6 + 2.6 = 17.4.

19. 2 Number of people (in lakh) who read only one news paper
= 4.7 + 4.6 + 2.6 = 11.9.

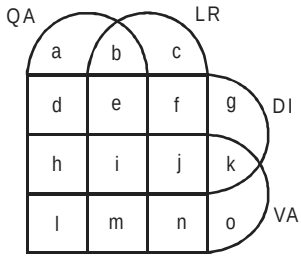
20. 3 Number of people who read at least two news papers
= $10^5 \times (2.5 + 1.5 + 1 + 0.5) = 5.5 \times 10^5$
Total population = 14×10^6.

So, the required percentage $= \frac{55}{14} = 3.9$ (Approx)

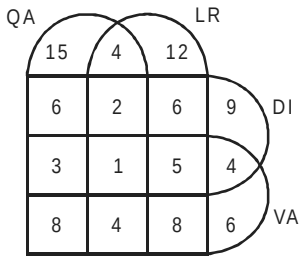
21. 1 The ratio of readers reading only one newspaper to. Those reading only two newspapers

$$= \frac{11.9}{(2.5+1.0+1.5)} = \frac{11.9}{5} = 2.38:1$$

For questions 22 to 25:
Let us make the venn diagram of the four sections as:



Using statement I, e + h + i + j = 11, e + j + i + m = 12, h + i + j + m = 13, e + h + i + m = 14.
Solving this we get, h = e + 1, j = e + 3, m = e + 2, i = 7 – 3e.
Using II, b = k = e + 2.
Using III, b + d + l = b + f + n = k + n + l – 2.
Hence, n = d + 2, f = l -2.
Using IV, a = 15, o = 6, b = e + 2 = 4, f = l-2 = 6.
Solving all, we get the following venn diagram.



22. 15

23. 8

24. 1

25. 28