Numbers - 6

Career Launcher

QA - 16

CEX-Q-0217/18

Contents

- Base System
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Number of Questions : 30

Base System

1. If $(54)_{10} = (X)_{12}$, what is the value of X?
(1) 46 (2) 44
(3) 51 (4) 38

2. If $(142)_{10} = (X)_{12}$, what is the value of X?
(1) 89 (2) BA
(3) AB (4) None of these

3. If $(12323)_4 = (P)_8$, then find the value of P.
(1) 653 (2) 673
(3) 763 (4) 563

4. If $(ABCD)_{16} = (X)_{10}$, what is the value of X?
(1) 43280 (2) 43521
(3) 43981 (4) None of these

5. If $(1153)_{10} = (X)_{15}$, what is the value of X?
(1) 51D (2) 61E
(3) 51C (4) 61C

6. A number system has 100 as base. How many digits do we need to write 100 in that system?

7. What is the decimal equivalent of $(1234)_6$?
(1) 3100 (2) 3010
(3) 301 (4) 310

8. If $(1555)_{10} = (X)_{16}$, what is the value of X?
(1) 641 (2) 64A
(3) 6A4 (4) 613

9. If $(412)_7 = (X)_8$, what is the value of X?

10. If $(1101.011)_2 = (X)_{10}$, what is the value of X?
(1) 13.357 (2) 12.375
(3) 13.375 (4) 133.75

11. If the number 3402 is converted from base 10 to base x, it becomes 12630. What is the value of x?

12. In decimal system there are 10 digits 0, 1, 2, 3, . . . till 9. Similarly, there is a system which has only 3 digits 0, 1 and 2. Numbers represented in such systems are called base 3 or trinery numbers. $N = 3^{89} + 1$ is a number in the decimal system. M is the trinery equivalent of this number. How many zeros are there in M?
(1) 90 (2) 89
(3) 88 (4) 87

13. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Which of the following may be the value of M?
(1) 31 (2) 63
(3) 75 (4) 91

14. A three digit non-zero number 'abc' is in base 5, when converted to base 7, it becomes 'cba'. Which of the following is necessarily true?
(1) a must be 2 (2) c must be 2
(3) b must be 0 (4) a = 4

15. First 100 natural numbers in the decimal system are converted to base 6 and their product is found. Find the number of consecutive zeros at the end of the product as counted in the base 6 system.
(1) 124 (2) 130
(3) 140 (4) 120

16. If the product of 11 and 44 in decimal system is converted into certain base system it becomes 3413. The number 3111 of this system, when converted to the decimal number system, becomes **[CAT 2001]**
(1) 406 (2) 1086
(3) 213 (4) 691

17. The HCF of $(30)_4$ and $(132)_4$ in base 4 is equal to:
(1) $(8)_4$ (2) $(10)_4$
(3) $(12)_4$ (4) $(15)_4$

18. The value of $(222)_X$ (in base 'X') when converted to base 10 is 'P'. The value of $(222)_Y$ in base 'Y' when converted to base 10 is Q. If $(P - Q)_{10} = 28$, then what is the value of $(Q - X)_{10}$?
(1) 10 (2) 92
(3) 79 (4) Cannot be determined

19. Consider the expression $(xxx)_b = x^3$, where b is the base, and x is any digit of base b. Find the value of 'b'. **[XAT 13]**
(1) 5 (2) 6
(3) 7 (4) None of the above

20. The digits x, y and z of the three-digit natural number xyz satisfy the equation 169x + 13y + z = 786. What is the sum of the digits of the three-digit natural number xyz in base 9?
(1) 15 (2) 20
(3) 21 (4) Data Insufficient

Calendar

21. If it was a Wednesday on 3rd July, 2006, then what day of the week was on 1st July, 2001?
(1) Tuesday (2) Monday
(3) Sunday (4) Friday

22. Which year will have the same calendar as that of 2008?

23. If 25th January 2012 was Friday, then 30th September 2012 was
(1) Monday (2) Tuesday
(3) Sunday (4) Wednesday

24. If the day before yesterday was Monday, what day will it be four days after the day after tomorrow?
(1) Wednesday (2) Monday
(3) Tuesday (4) Sunday

25. The first day of the year 2008 was Tuesday. If Ramesh's birthday was on 21st September, on which day of the week did Ramesh celebrate his birthday in 2008?
(1) Tuesday (2) Monday
(3) Saturday (4) Sunday

26. If 15th June falls 3 days after tomorrow, which is Friday, then what was day on 30th June of the same year?
(1) Monday (2) Tuesday
(3) Wednesday (4) Thursday

27. If 29th February 2004 was a Sunday, which of the following months did not start with a Sunday in that year?
(1) February (2) August
(3) November (4) Both (2) and (3)

28. If 15th September, 1943 was a Wednesday, then what day of the week was on 19th September, 1944?
(1) Tuesday (2) Wednesday
(3) Thursday (4) Friday

29. Mr. and Mrs. Sharma celebrated their wedding anniversary on Wednesday, 25th April 2006. When will they celebrate their next wedding anniversary on the same day?
(1) 25th April 2012 (2) 25th April 2017
(3) 25th April 2013 (4) 25th April 2015

30. Gautam goes to a temple only on the first and the last day of any year. He continues to go to the temple in this fashion till the time he finds out that he has visited the temple at least once on each of the different days of a week. The minimum number of days required to achieve this is

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Answers and Explanations

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1	1	**2**	2	**3**	2	**4**	3	**5**	1	**6**	–	**7**	4	**8**	4	**9**	–	**10**	3
11	–	**12**	3	**13**	4	**14**	3	**15**	4	**16**	1	**17**	3	**18**	3	**19**	4	**20**	2
21	1	**22**	–	**23**	2	**24**	3	**25**	4	**26**	2	**27**	3	**28**	1	**29**	2	**30**	–

1. 1 Consecutively dividing 54 by 12, the remainders are 4 and 6 in the reverse order.
Hence, $(54)_{10} = (46)_{12}$.

2. 2 Consecutively dividing 142 by 12, the remainders are 11(i.e. B) and 10 (i.e. A) in the reverse order.
Hence, $(142)_{10} = (BA)_{12}$.

3. 2 $(12323)_4$

$= \left(\underset{1\ 2\ 3\ 2\ 3}{110111011}\right)_2$

Divide 110111011 into groups of 3 digits from right to left.

$(110\ 111\ 011)_2 = (6\ 7\ 3)_8 = (P)_8$

$\Rightarrow P = 673.$

4. 3 $(ABCD)_{16} = 16^3 \times 10 + 16^2 \times 11 + 16^1 \times 12 + 16^0 \times 13$
$= 40960 + 2816 + 192 + 13 = (43981)_{10}$

5. 1 We divide 1153 by 15 and write the remainders and quotient at appropriate places.
$\therefore$ The number is 51D.

6. $(100)_{10} = (10)_{100}$. Hence, we need two digits.

7. 4 $(1234)_6 = \left(4\times6^0 + 3\times6^1 + 2\times6^2 + 1\times6^3\right)_{10}$

$= (4 + 18 + 72 + 216)_{10} = (310)_{10}$

8. 4 $(1555)_{10} = (?)_{16}$

16	1555	
16	97	$\rightarrow 3$
16	6	$\rightarrow 1$
	0	$\rightarrow 6$

$\therefore (1555)_{10} = (613)_{16}$

9. $(412)_7 = (2 \times 7^0 + 1 \times 7^1 + 4 \times 7^2)_{10} = (205)_{10}$
Now convert it to base 8

8	205	
8	25	5
8	3	1
	0	3

$\therefore (206)_{10} = (315)_8$
Hence, $(412)_7 = (315)_8$

10. 3 Convert the part before the decimal.

$(1101)_2 = 1\times2^0 + 0\times2^1 + 1\times2^2 + 1\times2^3 = (13)_{10}$

For the part after decimal take negative powers of 2

$\therefore\ (0.011)_2 = \left(0\times2^{-1} + 1\times2^{-2} + 1\times2^{-3}\right)_{10} = (0.375)_{10}$

$\therefore$ The answer is 13.375.

11. $(12630)_x = (3402)_{10}$

$\Rightarrow (3402)_{10} = \left(0\times x^0 + 3\times x^1 + 6\times x^2 + 2\times x^3 + 1\times x^4\right)_{10}$

$3402 = 3x + 6x^2 + 2x^3 + x^4$

x = 7 satisfies the above equation.

12. 3 3^{89} in base 3 will be written as 'one followed by 89 zeroes'. When we add one to it, zero in unit's place will become one and we will have 88 zeros.

13. 4 Since the last digit in base 2, 3 and 5 is 1, the number should be such that on dividing by either 2, 3 or 5 we should get a remainder 1. The smallest such number is 31. The next set of numbers are 61, 91.
Among these only 31 and 91 are a part of the answer choices.

Among these, $(31)_{10} = (11111)_2 = (1011)_3 = (111)_5$

Thus, all three forms have leading digit 1.
Hence, the answer is 91.

14. 3 Given that $(abc)_5 = (cba)_7$
or $25a + 5b + c = 49c + 7b + a$
or $24a = 2b + 48c$
or $12a = b + 24c$
As abc is a 3-digit number in base 5, the possible values for a, b and c are 0, 1, 2, 3, 4 only.
The possible solutions of the above equation are:
(a, b, c) = (2, 0, 1) and (4, 0, 2) only
a can take value = 2 or 4 only
b can take value = 0 only
c can take value = 1 or 2 only.

15. 4 The number of zeroes (as counted in the decimal system) is same as the highest power of 6 that divides 100! i.e. 48.
This, when counted in base 6, is 120.

16. 1 The product of 44 and 11 in base 10 is 484.
If base is x, then 3414
$= 3x^3 + 4x^2 + 1x^1 + 4 \times x^0 = 484$
$\Rightarrow 3x^3 + 4x^2 + x = 480$
This equation is satisfied only when x = 5.
So base is 5.
In decimal system, the number 3111 can be written
$3 \times 5^3 + 1 \times 5^2 + 1 \times 5^1 + 1 \times 5^0 = 406$.

17. 3 Converting both the numbers in base 10, we get

18. 3 $P = (222)_X = 2X^2 + 2X + 2$ and
$Q = (222)_Y = 2Y^2 + 2Y + 2$
It is given that $P - Q = 28$.
$\Rightarrow 2X^2 + 2X + 2 - 2Y^2 - 2Y - 2 = 28$

$$\Rightarrow X^2 + X - Y^2 - Y = 14 \Rightarrow \left(X + \frac{1}{2}\right)^2 - \left(Y + \frac{1}{2}\right)^2 = 14$$

$\Rightarrow (X + Y + 1)(X - Y) = 14$
Therefore, the values of X and Y are (7, 6) and (4, 2) in that order. But Y cannot equal to 2 as there exists a number 222 in base Y and digit '2' does not exist in base 2.
$\Rightarrow X = 7, Y = 6$ and $(Q - X) = 79$.

19. 4 Given, $(xxx)_b = x^3$
$\Rightarrow (x \times b^2 + x \times b + x) = x^3$
$\Rightarrow (b^2 + b + 1) = x^2$
$\Rightarrow x \geq b$
But x is a digit in base b, therefore $x < b$
Hence, b cannot assume any value.

20. 2 $169x + 13y + z = 786$
Since x, y and z are single digit natural number, the values of x, y and z must be 4, 8 and 6 respectively.
Hence, the required answer
$= (4 + 8 + 6)_{10} = (18)_{10} = (20)_9$.

21. 1 First we look for the leap years during this period.
$\therefore$ Only 2004 is a leap year and the years 2002, 2003, 2005 and 2006 are the ordinary years.
Number of odd days from 3rd July, 2001 to 3rd July, 2006 = 1 + 1 + 2 + 1 + 1 = 6 odd days.
So, 3rd July, 2001 was 6 days before Wednesday, i.e., on Thursday.
Therefore, 1st July, 2001 was Tuesday.

22. The year 2008 is a leap year and a leap year calendar repeats itself after 28 years. (If there is no non-leap century year in between both the leap years.)
2008 + 28 = 2036
So, 2036 will have the same calendar as that of 2008.

23. 2 Number of odd days for January = 6
Number of odd days for February = 1
Number of odd days for March = 3
Number of odd days for April = 2
Number of odd days for May = 3
Number of odd days for June = 2
Number of odd days for July = 3
Number of odd days for August = 3
Number of odd days for September = 2
Total number of odd days = 6 + 1 + 3 + 2 + 3 + 2 + 3 + 3 + 2 = 25.
$\therefore$ 30th September 2012 will be Tuesday.

24. 3 Day before yesterday was Monday,
$\Rightarrow$ Today is Wednesday.
$\Rightarrow$ Day after tomorrow is Friday.
$\therefore$ Four days after the day after tomorrow will be Tuesday.

25. 4 Number of odd days for January = 2
Number of odd days for February = 1
Number of odd days for March = 3
Number of odd days for April = 2
Number of odd days for May = 3
Number of odd days for June = 2
Number of odd days for July = 3
Number of odd days for August = 3
Total number of odd days = 3 + 1 + 3 + 2 + 3 + 2 + 3 + 3 = 19.
$\therefore$ 21th September will be Sunday.

26. 2 Tomorrow $\Rightarrow$ Friday
Friday + 3 = Monday
Monday = 15th
Other Monday $\Rightarrow$ 22, 29
Therefore, 30th June = Tuesday

27. 3 If February 29th, 2004 was a Sunday, then February 1st was also a Sunday.
1st August is (31 + 30 + 31 + 30 + 31 + 1) 154 days from 29th Feb. 154 is a multiple of 7. So 1st August was again a Sunday.
1st November is (30 + 30 + 31 + 1) = 92 days from 1st August. 92 = 7n + 1.
So, 1st November was a Monday. So, option (3) is the correct choice.

28. 1 It is given that, 15th September, 1943 was Wednesday.
As, 1944 is a leap year.
Therefore, there are two odd days from 15th September, 1943 to 15th September, 1944.
So, 15th September,1944 is two days after Wednesday, i.e., Friday
Hence, 19th September, 1944 is four days after Friday, i.e., Tuesday.

29. 2 25th April 2006 was Wednesday.
For next wedding anniversary to be on Wednesday, number of odd days must be a multiple of 7.
For an ordinary year, there is 1 odd day but for leap year, there are 2 odd days.
Hence, after 11 years, the number of odd days will be 14, which is a multiple of 7.
$\therefore$ On 25th April 2017 there will be Wednesday.

30. If a particular year, which is not a leap year, starts with Monday, it will also end with Monday. For minimum number of days, let's consider a leap year N and assume its first day is Tuesday.
Let Gautam start his visit on the last day of the year (N – 1).

Year	First day	Last Day
N – 1	Monday	**Monday**
N	Tuesday	Wednesday
N + 1	Thursday	Thursday
N + 2	Friday	Friday
N + 3	Saturday	Saturday
N + 4	**Sunday**	Monday

So, on the first day of year (N + 4), Gautam will meet the given condition.
So, the required number of days = 365 × 4 + 3 = 1463.