Dear Student,

The journey to achieve success has begun. The CL Educate team brings to you an offering,
which incorporates **theme based learning** that revolves around different concepts with **diverse applications**. The outcome is an enriching learning experience.

Our integrated thematic methodology is driven by latest research, undertaken to enhance learning. Numerous practice exercises and tests have been incorporated to reinforce the conviction in one's ability. Our teaching experience coupled with extensive research has lent credence to our conviction that learning is at its best when concept based understanding and applications go hand in hand.

To enhance your learning and assimilation of relevant concepts, our attempt has been to identify the basic concepts (or themes) that are required to solve different questions in MBA entrance examinations. Our class exercises integrate the different types of questions requiring application of these concepts. Each set of concepts along with relevant question types therefore, forms a module. At the end of each module we expect the student to:

1) Clearly understand a concept through its repeated application in different question types.

2) Quickly and effectively apply the relevant concept to different question types in a time-bound examination scenario.

3) Develop long-lasting skills by imbibing each concept that is clearly covered through a module.

Armed with the latest tools for success, along with your diligence and positive attitude, you have begun your march towards success. Have faith in yourself!

The woods are lovely, dark and deep,

But I have promises to keep,

And **miles togo before I sleep**,

And **miles to go before I sleep**

(Robert Frost)

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Number System 1

Learning Objectives

By the end of the chapter, you should be able to

• To understand the classification of numbers.

• Convert recurring numbers to p/q form

• Ascertain if a given number is divisible by any other number

• Explain the meaning and relevance of Factors, Multiples, Factorisation, HCF and LCM and should be able to find the HCF & LCM of given set of numbers

• Application of HCF and LCM.

• Find the number of factors of any given natural number.

• Calculate the remainder when any expression is divided by any natural number.

• Calculate the largest power of a number that divides any given factorial or a number.

• Identify the unit's digit of x^y.

• Understanding the base system.

Did You Know : India's contribution to number system

Not all ancient civilizations based their numbers on a ten base system. In ancient Babylon, a sexagesimal (base 60) system was in use. In India a decimal system was already in place during the Harappan period.

It is generally acknowledged that the concept of zero, crucial to the development of science, is India's contribution to the world, which was given to Europe through the Arabs. The ancient India astronomer Brahmagupta is credited with having put forth the concept of zero for the first time.

Aryabhatta gave the value of p as 3.1416 (circa 500 AD) claiming, for the first time, that it was an approximation.

Number System

What do you mean by the term `number'?

Number is a symbol representing quantity.

In this chapter of Number System, we are going to learn about various types of numbers.

1. **Real Numbers:** Real numbers are those numbers which you can commonly identify and quantify.

For example, -10, -8.33, -1, -0, 1, 2, 5.77 etc.

What is a number line?

Number line is a line on which all the positive and negative numbers can be marked in a sequence. It stretches from negative infinity to positive infinity.

All the numbers which can be represented on the number line are called real numbers.

2. **Imaginary numbers:** Imaginary numbers are those numbers which cannot be represented on the number line. Imaginary numbers are those numbers about which we can just imagine but cannot physically perceive.

For example: $\sqrt{-1}, \sqrt{-2}$, etc. Square roots of all negative numbers are imaginary. $\sqrt{-1}$ is represented by the letter 'i'.

Combination of Real and Imaginary Numbers are called complex numbers.

For example, (2 + 3i) is a complex number.

In this chapter, we will discuss only about the Real Numbers, because imaginary numbers are outside the preview of most management entrance examinations.

Types of Real Numbers:

All the real numbers can be divided into two parts. They are Rational numbers and Irrational numbers.

Rational numbers: All the numbers that can be expressed in the form of $\frac{p}{q}$, where p & q are integers and $q \neq 0$, are called rational numbers.

For example, -1, 2, $-\frac{2}{5}$, 0, 1, 1.7 etc.

We can illustrate the examples mentioned above, in form in the following manner:

$$-1 = \frac{-1}{1} = \frac{p}{q}(q \neq 0)$$

$$\frac{2}{5} = \frac{p}{q}(q \neq 0),\ 0 = \frac{0}{1} = \frac{p}{q}(q \neq 0),\ 1.7 = \frac{17}{10} = \frac{p}{q}(q \neq 0)$$

Irrational Numbers: All the numbers that cannot be expressed in the form of $\frac{p}{q}$ are called irrational numbers.

For example, $\pi, \sqrt{2}, (\sqrt{3}+1)$ etc.

Typically they are having non-terminating and non-recurring decimal part.

Rational numbers can also be classified as integers and fractions.

Integers: All the rational numbers which do not have any decimal or fractional part are called integers.

For example, -3, -2, -1, 0, 1, 2, 3 etc.

Fractions: All the rational numbers which are in the form $\frac{p}{q}$, where $p, q \neq 0$ and p is not a multiple of q are called fractions. The number on the bottom of the fraction is called the denominator and the number on the top of the fraction is called the numerator.

For example, 1.2, $\frac{5}{2}$, $\frac{4}{3}$, 1.7, 0.2 etc.

In the fraction $\frac{5}{2}$, 5 is the numerator and 2 is the denominator.

Fractions are of three types — proper, improper and mixed.

A proper fraction is a fraction whose numerator is smaller than its denominator.

For example, $\frac{2}{5}, \frac{3}{11}$ are proper fractions.

An improper fraction is a fraction whose numerator is greater than its denominator.

For example, $\frac{5}{2}$ and $\frac{8}{3}$ are improper fractions.

A mixed fraction is an integer plus a fraction.

For example, $3\frac{1}{5}$ or $7\frac{1}{3}$. Mixed fraction are sometimes called mixed numbers.

Types of integers

Whole numbers: All the non-negative integers are whole numbers. The set of whole number contains 0, 1, 2, 3, 4 etc.

Natural Numbers: Whole numbers except zero are called natural numbers. Natural numbers start from 1.

Natural Numbers can be classified into different categories based on their property as follows:

a. On the basis of divisor:

Even numbers: All the natural numbers which are multiple of 2 or in other words divisible by 2 are called even numbers such numbers are denoted by 2k (where `k' is a natural number).

For example, 2, 4, 6, 8, ...

Odd Numbers: All the natural numbers which are not a multiple of 2 or in other words which are not divisible by 2 are called odd numbers.

Such numbers are denoted by $2k \pm 1$. (where `k' is a natural number)

Facts about odd and even numbers:

odd $\pm$ odd = even

odd $\pm$ even = odd

even $\pm$ even = even

odd $\times$ odd = odd

odd $\times$ even = even

even $\times$ even = even

b. On the basis of their origin:

Prime numbers: All the natural numbers greater than 1 which are only divisible by 1 and the number itself are called Prime Numbers. In others words we can say that each Prime Number is divisible by only two numbers.

For example, 2, 3, 5, 7, 11, 13, 17, etc.

Prime numbers are randomly distributed in the set of natural numbers. Interestingly all the prime numbers (greater than 3) can be expressed as $6k \pm 1$, the where `k' is a natural number.

Composite Numbers: All the natural numbers greater than 1 which are divisible by at least one more number, other than 1 and the number itself are called composite numbers. In other words the natural
numbers (except 1) which are not prime are composite numbers. Please note that `1' is neither prime nor composite. Can we say that a composite number is always divisible by 3 or more different natural numbers?

How to find whether a number is prime or not?

For small numbers, we could find by checking, if that number is divisible by any other prime number till that number itself.

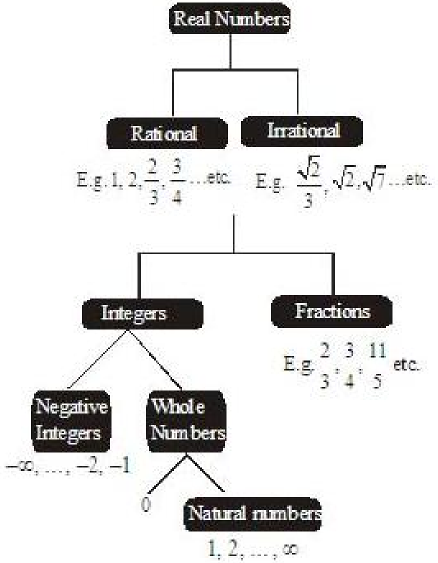
But for the larger numbers like, say 631, there is an alternative method.

Step 1: Find the approximate square root of the given number, i.e. 25.

Step 2: Check if any prime number from 2 to 25 divides 631.

The prime numbers from 2 to 25 are 2, 3, 5, 7, 11, 13, 17, 19 and 23. Since, none of these numbers divides 631 exactly, 631 must be a prime number.

Number Tree



Conversion of recurring decimal into fractions:

What is the $\frac{p}{q}$ form of 0.5555... (also represented as $0.\bar{5}$)?

Let, x = 0.55555... $\Rightarrow 10x = 5.55555\ldots$

$\therefore 9x = 10x - x = (5.5555\ldots) - (0.55555\ldots)$

$\therefore x = \frac{5}{9}$

If x = 0.232323...

$\Rightarrow 100x = 23.232323\ldots$

99x = 100x - x

= (23.232323...) - (0.232323...) = 23

$\therefore x = \frac{23}{99}$

For a purely recurring number (all digits after decimal point recur) we can identify the procedure as:

The $\frac{p}{q}$ form of a purely recurring number

$$= \frac{\text{The recurring part written once}}{\text{As many 9's as the number of digits in the recurring part}}$$

In the number like 0.14333... i.e. $0.14\bar{3}$

Let, x = 0.143333...

$\Rightarrow 100x = 14.3333\ldots$

$\Rightarrow 1000x = 143.3333\ldots$

⇒ 900x = 1000x - 100x = (143.3333...) = 129

$\therefore x = \frac{129}{900}$

Thus, for any recurring number we can identify the procedure as:

The $\frac{p}{q}$ form of any recurring number

$$= \frac{(\text{The non-recurring and recurring part written once}) - (\text{The non-recurring part})}{\text{As many 9's as the number of digits in the recurring part followed by as many 0's as digits in the non-recurring part}}$$

Example 1: Express $0.\overline{643}$ as a fraction.

Solution: Let x = $0.\overline{643}$

$\Rightarrow 1000x = 643.\overline{643}$ or

$\Rightarrow 1000x - x = 643.\overline{643} - 0.\overline{643}$

$\Rightarrow 999x = 643$

$\therefore x = \frac{643}{999}$

Example 2: Arrange the following rational numbers in ascending order: $-\frac{7}{10}, -\frac{5}{8}, -\frac{2}{3}$

Solution:

$-\frac{7}{10} = -0.7, -\frac{5}{8} = -0.625$ and $-\frac{2}{3} = -0.666$

Clearly, - 0.7 < - 0.666 < - 0.625.

So, $-\frac{7}{10} < -\frac{2}{3} < -\frac{5}{8}$

Operations on Numbers

BODMAS — Order of simplification of expression of numbers

B → Bracket

O → Of

D → Division

M → Multiplication

A → Addition

S → Subtraction

In a given expression of numbers, the above order of operations has to be strictly followed.

For example,

$$\frac{3}{5} \text{ of } \frac{5}{9} \div \frac{1}{5} + \left(1 + \frac{1}{3}\right) - \left(\frac{29}{7} - \frac{8}{7}\right) = \frac{3}{9} \div \frac{1}{5} + \frac{4}{3} - \frac{21}{7} = 0$$

Quotient and Remainder:

Suppose, we want to divide 57 by 9 we can do it in the following manner:

$$9\overline{)\begin{array}{c}57\\ \underline{54}\\ 3\end{array}}(6$$

$$\Rightarrow \text{Divisor}\overline{)\text{Dividend}}(\text{Quotient}$$
$$\overline{\text{Remainder}}$$

In the example, given above, 9 is the divisor, 57 is the dividend, 6 is the quotient and 3 is the remainder

Therefore,

Dividend = (Divisor × Quotient) + Remainder.

Note: A number having remainder \`r' when divided by \`d' can be represented as (d × n) + r, where \`n' is a natural number.

Example 3: If dividend is 15968, quotient is 89 and the remainder is 37, then what is the divisor ?

Solution: Divisor $= \left(\dfrac{\text{Dividend} - \text{Remainder}}{\text{Quotient}}\right) = \left(\dfrac{15968 - 37}{89}\right) = 179$

Indices

When a quantity is multiplied by itself a certain number of times, the product thus obtained is called a power of that quantity.

Thus, a^m means `a' is multiplied `m' times consecutively and `m' is called the exponent or the index.

Rules of indices

1. $a^m \times a^n = a^{m+n}$ 2. $\dfrac{a^m}{a^n} = a^{m-n}$ 3. $(a^m)^n = a^{mn}$ 4. $a^{-m} = \dfrac{1}{a^m}$

5. $a^0 = 1$ 6. $(ab)^m = a^m b^m$ 7. $\sqrt[m]{a} = a^{\frac{1}{m}}$ 8. $a^{\frac{p}{q}} = \sqrt[q]{a^p}$

$(x^a)^b \neq x^{a^b}$

$(2^3)^2 = 2^{3\times 2} = 2^6 = 64$

$2^{3^2} = 2^9 = 512$

Which would be greater a^b or b^a, given $b > a$? (a and b are both greater than 3)

Surds

Surds are those numbers which are written as the square root or cube root of the numbers. For
example $\sqrt{2}, \sqrt{3}$. Please note that $\sqrt{4}$ is not a surd.

General form of the surd is $\sqrt[n]{a}$, where a is called radicand and it must be a positive rational
number. n is called the order of the surd and it must be a natural number.

If $\sqrt[n]{a}$ is an integer then it is not a surd.

Properties of Surds:

Addition: $\sqrt{x} + \sqrt{y}$ cannot be written as a single surd.

However, $\sqrt{x} + \sqrt{x} = 2\sqrt{x}$

For example, $\sqrt{5} + 3\sqrt{5} + 6\sqrt{5} = 10\sqrt{5}$

Multiplication: $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$

For example, $\sqrt{3} \times \sqrt{27} = \sqrt{81} = 9$

Sometimes multiplication of surds can give rational numbers $\sqrt{3}$ and $\sqrt{27}$ are surds whereas $\sqrt{81}$ is a rational number.

Conjugate surd: To find conjugate of any surd, for example $\sqrt{2} + \sqrt{5}$, write down the same two terms but replace the sign in between.

For example conjugate surd of $\sqrt{2} + \sqrt{5} = \sqrt{2} - \sqrt{5}$. Conjugate surds are used to rationalise the denominators of surds.

Rationalisation of surds:

Rationalising is a process where the surd is written in a different form. In which denominator finally contains only rational numbers. Here, in this process we multiply the numerator and denominator by the conjugate surd of denominator.

For example:

$$\frac{2+\sqrt{2}}{2-\sqrt{2}} = \frac{2+\sqrt{2}}{2-\sqrt{2}} \times \frac{2+\sqrt{2}}{2+\sqrt{2}} = \frac{4+4\sqrt{2}+2}{2} = \frac{6+4\sqrt{2}}{2} = 3+2\sqrt{2}$$

If one is not comfortable and fluent with indices, it would be fruitful to work on the following examples before proceeding ahead:

1. Find the value of $\frac{1}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{1}{(243)^{-\frac{1}{5}}}$.

2. Simplify: $\left[\frac{a^{-1}b^{2}}{a^{2}b^{-4}}\right]^{7} \div \left[\frac{a^{3}b^{-5}}{a^{-2}b^{3}}\right]^{-5}$.

3. If $a^x = b$, $b^y = c$, $c^z = a$, then find the value of xyz.

4. Which of the following is equal to $3^{-5} + \frac{1}{3^5} + \frac{3^{-4}}{3}$?

a. 3^{-2} b. 3^{2} c. 3^{-4} d. 3^{4} e. 3^{-3}

5. Which of the following is equal to $\sqrt{\sqrt[3]{0.000729}}$?

a. 0.03 b. 0.3 c. 3.0 d. 0.003 e. 0.0003

6. If $a = 2^{44}$ and $b = 2^{22^2}$, then which of the following is true?

a. a > b b. a < b c. a = b

7. If $(6)^{15} \times (10)^5 \times (15)^6 = 2^x \times 3^y \times 5^z$, then find the value of x + y + z.

a. 26 b. 52 c. 42 d. 48 e. 45

8. If $(9)^{3x-5} = (3)^{2x-2}$, then find the value of x.

a. 1 b. 2 c. 3 d. 4 e. 5

9. Ram was supposed to take the cube root of A and then square of that. Instead of that he took the square of A first and then took the cube root. The answer obtained is

a. smaller than expected answer b. larger than expected answer

c. same as expected answer d. depends on value of A

e. None of these

Divisibility Rules

1.Divisibility Rule for 2: If the last digit (units place) of a number is 0, 2, 4, 6, or 8. E.g. 742 is divisible by 2 but 743 is not.

2.Divisibility Rule for 3: If the sum of all the digits is divisible by 3, the number is divisible by 3.

E.g. 1452 (Sum of digits = 12) is divisible by 3 but 763 (Sum of digits = 16) is not divisible by 3. Infact, the remainder when 16 is divided by 3 i.e. 1 will also be the remainder when 763 is divided by 3.

3.Divisibility Rule for 4 : If the last two digits of a number are divisible by 4 or are 00, the number is also divisible by 4.

E.g. 67432 is divisible by 4 (as 32 is divisible by 4) whereas 2146 is not divisible by 4 (as 46 is not divisible by 4). In fact, the remainder when 46 is divided by 4 i.e. 2 will also be the remainder when 2146 is divided by 4. Also note 700 is divisible by 4 as the last two digits are 00.

4.Divisibility Rule for 5: If the last digit (unit's digit) is 0 or 5, the number is divisible by 5.

E.g. 13265 is divisible by 5 whereas 9864 is not.

5.Divisibility Rule for 6: A number is divisible by 6 if the number is divisible by both 2 and
3 simultaneously. E.g. 5424 is divisible both by 2 and by 3 (sum of digits = 15) and thus also divisible by 6. However 3332 is not divisible by 6 as it is not divisible by 3.

6.Divisibility Rule for 8: If the last three digits of a number are divisible by 8 or are 000, the number is also divisible by 8.

E.g. 67432 is divisible by 8 (as 432 is divisible by 8) whereas 2148 is not divisible by 8 (as 148 is not divisible by 8). In fact, the remainder when 148 is divided by 8 i.e. 4 will also be the remainder when 2148 is divided by 8. Also note 13000 is divisible by 8 as the last three digits are 000.

7.Divisibility Rule for 9 : If the sum of all the digits is divisible by 9, the number is divisible by 9.

E.g. 658215 (Sum of digits = 27) is divisible by 9 but 763 (Sum of digits = 16) is not divisible by 9.

In fact, the remainder when 16 is divided by 9 i.e. 7 will also be the remainder when 763 is divided by 9.

8.Divisibility Rule for 11 : A number is divisible by 11, if the difference between the sum of the digits in the even places and the sum of the digits in the odd places is either 0 or is divisible by 11. E.g. 6595149 is divisible by 11 as the difference of 6 + 9 + 1 + 9 = 25 and 5 + 5 + 4 = 14 is 11.

However 27813 is not divisible because sum of digits in the odd places = 13 and the sum of digits in the even places = 8. their difference is neither 0 nor divisible by 11.

If number is divisible by 2 and also by 3, then it is divisible by 2 × 3 = 6. Does this mean that if a number is divisible by 4 and also by 6, then the number is divisible by 4 × 6 = 24? 12, 36 are divisible by 4 and by 6 but not by 24. Why does the rule hold in case of 2 and 3 but not in case of 4 and 6?

If one is not comfortable with divisibility rules, it would be fruitful to work on the following examples before proceeding ahead:

1. If abc4d is divisible by 4, then what is/are the value/s of d?

2. A number 344ab5 is divisible by both 9 and 25. Find the number. [Given (a + b) < 8]

3. A number 1568X35Y is divisible by 88. What are the values of X and Y?

4. If `n' is a positive integer greater than 1, then $n(n^2 - 1)$ is always divisible by

a. 6 b. 12 c. 24 d. 48 e. 96

5. Which of the following numbers is divisible by 99?

a. 32373 b. 37332 c. 32337 d. 23337 e. None of these

6. What is the remainder when 9876532123 is divisible by 9?

a. 1 b. 2 c. 3 d. 4 e. 5

7. The number aaaaaa, where a is a single digit natural number, is divisible by

a. 11 b. 13 c. 143 d. 7 e. All of these

8. How many numbers between 300 and 500 are divisible by both 8 and 5?

a. 3 b. 4 c. 5 d. 6 e. 7

9. What is the remainder when 78X85Y868 is divided by 8?

a. 1 b. 3 c. 4 d. 7 e. 5

10. If the number 786P86Q is divisible by 8 and 9 both, then values of P and Q are

a. 4, 9 b. 6, 4 c. 8, 6 d. 6, 8 e. None of these

Example 1:

What is the least number that must be subtracted from 2000 to get a number which is exactly divisible by 17?

Solution:

On dividing 2000 by 17, we get 11 as remainder.

∴ Required number to be subtracted = 11.

Example 2:

What is the least number that must be added to 3000 to obtain a number exactly divisible by 19?

Solution:

On dividing 3000 by 19, we get 17 as remainder.

∴ Number to be added = (19 - 17) = 2.

Example 3:

Find the number which is nearest to 3105 and exactly divisible by 21.

Solution:

On dividing 3105 by 21, we get 18 as remainder.

∴ Number to be added to 3105 is (21 - 18) = 3.

∴ 3108 is the required number.

Example 4: A number when divided by 342 gives a remainder 47. When the same number is divided by 19, what would be the remainder?

Solution:

On dividing the given number by 342, let k be the quotient and 47 the remainder.

Then, number = 342k + 47

= [(19 × 18k) + (19 × 2 + 9)]

= [19 (18k + 2) + 9]

∴ The given number when divided by 19 gives (18k + 2) as quotient and 9 as remainder.

Alternative method:

342 is a multiple of 19, divide the remainder by the second dividend to get the remainder. 47 when divided by 19 gives 9 as remainder.

Factorial

The continued product of first n natural number is called `n factorial' and is denoted by n! or $\underline{|n}$

n! = 1 × 2 × 3 × ... × (n - 1) × n

E.g. 6! = 1 × 2 × 3 × 4 × 5 × 6 = 720

By definition 0! = 1.

Cyclicity

Unit's place digit of a Number:

The digit at the unit's place of any number is the remainder when the number is divided by 10.

For example, lets consider the number 364. The remainder when 364 is divided by 10 is 4. Hence, `4' is the unit's digit of the number 364.

To find the unit's digit of a number which is the product of two or more numbers, multiply the unit's digit of the numbers and find the unit's digit of the resultant number.

For example, 19×64, the product of the units digit of 19 and 64 is 36 and the unit's digit of 36 is 6, hence the unit's digit of 19×64 is 6.

Unit's digit of higher powers of any Number:

$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$
$2^5 = 32$	$2^6 = 64$	$2^7 = 128$	$2^8 = 256$
$2^9 = 512$	$2^{10} = 1024$	$2^{11} = 2048$	$2^{12} = 4096$

We can see that the unit's digit of 2^1, 2^5, 2^9 is 2, units digit of 2^2, 2^6, 2^{10} is 4, units digit of 2^3, 2^7, 2^{11} is 8 and units digit of 2^4, 2^8, 2^{12} is 6.

Therefore, after every four powers of 2, the units digit of the number starts repeating. Thus we say that cyclicity of unit's digit of higher powers of 2 is 4.

Similarly the digits whose cyclicity is 4 are 2, 3, 7 and 8. The digits whose cyclicity is 2 are 4 and 9.

Any power of numbers whose units digit 1, 5 or 6 always ends in 1, 5 and 6 respectively.

For example,

$11^2 = 121$, $25^2 = 625$ and $16^2 = 256$.

Remainders are cyclic but ideally avoid finding remainder using their cyclic property. It can be very cumbersome.

The cyclicity of the digits are as follows:

Digit Cyclicity

0, 1, 5 and 6 1

2, 3, 7 and 8 4

4 and 9 2

Example 1:

Find the unit's digit of a. 3^{57} and b. 13^{59}.

Solution:

a. The cyclicity of 3 is 4. Hence, $\frac{57}{4}$ gives the remainder 1. So the last digit of 3^{57} is same as the last digit of 3^1, i.e. 3.

b. The number of digits in the base will not make a difference to the last digit. It is the last digit of the base which decides the last digit of the number itself. For 13^{59}, we find $\frac{59}{4}$ which gives a remainder 3. So the last digit of 13^{59} is same as the last digit of 3^3, i.e. 7.

Example 2:

Find the unit's digit of the product $7^{23} \times 8^{13}$.

Solution:

Both 7 and 8 exhibit a cyclicity of 4.

7^{23} ends with the same last digit as 7^3, i.e. 3.

8^{13} ends with the same last digit as 8^1, i.e. 8.

Hence, the product of the two numbers would end with the same last digit as that of 3 × 8, i.e. 4.

The unit's digit of any number N is also the remainder when N is divided by 10.

For example, the unit's digit of the number 1176 is 6. Remainder when 1176 is divided by 10 is also 6.

Ten's place digit of a number:

The tens place digit of a number can be found out by dividing the number by 100.

For example, consider the number 11872. When 11872 is divided by 100, the quotient is 118 and the remainder is 72.

Finding the last two digits of a number

We are going to discuss the way to calculate the last two digits of any power of a natural number. Given below are the cases in which these problems will fall:

I. For a number A^k, where A ends in 0 and k is a natural number

II. For a number A^k, where A ends in 5 and k is a natural number

III. For a number of the form $(2\times m)^{40k+1}$, where m is an odd natural number not ending in 5 and k is a natural number

IV. For all the remaining cases

I. For a number A^k, where A ends in 0 and k is a natural number

When a natural number A ends in 0, A^k will always end in 00 where k is a natural number greater than 1.

II. For a number A^k, where A ends in 5 and k is a natural number

When a natural number A ends in 5, A^k will always end in 25 where k is a natural number greater than 1.

III. For a number of the form $(2\times m)^{40k+1}$, where m is an odd natural number not ending in 5 and k is a natural number

When the condition given above is satisfied, the last two digits of $(2\times m)^{40k+1}$

= the last two digits of (2 × m + 50).

Example 1

Find the last two digits of 26^{81}.

Solution:

26 = 2 × 13, which is of the form 2 × odd number.

Also, the exponent is 40 × 2 + 1, which is of the form 40k + 1. Hence, the last two digits of 26^{81} = the last two digits of (26 + 50) = 76.

Example 2:

Find the last two digits of 94^{561}.

Solution:

$94 = 2 \times 47$, which is of the form $2 \times$ odd number.

Also, the exponent is $40 \times 14 + 1$, which is of the form $40k + 1$. Hence, the last two digits of 94^{561} = the last two digits of $(94 + 50) = 44$.

IV. For all the remaining cases

For all the remaining cases, the last two digits of A^{40k+r} will be equal to the last two digits of A^r.

Example 3:

Find the last two digits of 72^{842}.

Solution:

$72^{842} = 72^{40\times21+2}$. Now we can say that the last two digits of 72^{842} will be equal to the last two digits of 72^2. Hence, the last two digits of 72^{842} are 84.

Example 4:

Find the last two digits of 47^{900}.

Solution:

$47^{900} = 47^{40\times22+20}$. So the last two digits of 47^{900} will be same as the last two digits of 47^{20}.

The table on the right shows the last two digits of a few powers of 47. Using them,

$47^{20} = 47^{16} \times 47^4$

$= 21 \times 81 = 01$.

Hence, the last two digits of 47^{900} are 01.

$47^1 = 47$
$47^2 = 09$
$47^4 = (09)^2 = 81$
$47^8 = (81)^2 = 61$
$47^{16} = (61)^2 = 21$

Example 5:

Find the last two digits of 52^{873}.

Solution:

$52^{873} = 52^{40\times21+33}$. So its last two digits will be same as the last two digits of 52^{33}.

The table on the right shows the last two digits of a few powers of 52.

Using the table,

$52^{33} = 52^{32} \times 52^1 = 96 \times 52 = 92$.

Hence, the last two digits of 52^{873} are 92.

$52^1 = 52$
$52^2 = 04$
$52^4 = (04)^2 = 16$
$52^8 = (16)^2 = 56$
$52^{16} = (56)^2 = 36$
$52^{32} = (36)^2 = 96$

How to make the table of powers of a two-digit natural number

We just need to remember the squares of the first 25 natural numbers to make the table of powers of a number. What we need to look at is the absolute difference of the number from either 50 or 100, whichever is closer.

Study the following examples to understand the process:

Last two digits of 47^2.

The absolute difference of 47 from 50 is 3 and last two digits of 3^2 are 09. So the last two digits of 47^2 are also 09.

Last two digits of 81^2.

The absolute difference of 81 from 100 is 19 and last two digits of 19^2 are 61. So the last two digits of 81^2 are also 61.

Last two digits of 61^2.

The absolute difference of 61 from 50 is 11 and last two digits of 11^2 are 21. So the last two digits of 61^2 are also 21.

Last two digits of 52^2.

The absolute difference of 52 from 50 is 2 and last two digits of 2^2 are 04. So the last two digits of 52^2 are also 04.

Last two digits of 56^2.

The absolute difference of 56 from 50 is 6 and the last two digits of 6^2 are 36. So the last two digits of 56^2 are also 36.

Last two digits of 36^2.

The absolute difference of 36 from 50 is 14 and the last two digits of 14^2 are 96. So the last two digits of 36^2 are also 96.

Last two digits of 88^2.

The absolute difference of 88 from 100 is 12 and the last two digits of 12^2 are 44. So the last two digits of 88^2 are also 44.

Factors, Multiples and Factorisation

For any relation of the type $x \times n = y$ (x, y, n are all natural numbers), x is called a factor of y and y is called a multiple of x.

E.g. Since $4 \times 9 = 36$, 4 is a factor of 36 and 36 is a multiple of 4. But since there does not exist any natural number n such that $4 \times n = 38$, 4 is not a factor of 38 and 38 is not a multiple of 4.

Factors :

a is a factor of b if there exists a relation such that $a \times n = b$, where n is any natural number. Basically it means that a is a factor of b if a can completely divide b.

Thus factors of a number are all those numbers that completely divide the given number.

Needless to say, 1 is a factor of all numbers as $1 \times b = b$. Also it would be obvious that factor of a number cannot be greater than the number (infact the largest factor will be the number itself). Thus the factors of any number will lie between 1 and the number itself (both inclusive) and thus are limited.

If number of factors of a number are limited, is there a way to find the number of factors of a given number ? Yes indeed there is a process and we will see it shortly.

Multiples :

a is a multiple of b if there exists a relation of the type $b \times n = a$. Thus the multiples of 6 are $6 \times 1 = 6$, $6 \times 2 = 12$, $6 \times 3 = 18$, $6 \times 4 = 24$, and so on.

We can easily deduce that the smallestmultiple will be the number itself and the number of multiples would be infinite.

To understand what multiples are, let's just take an example of multiples of 3.

The multiples are 3, 6, 9, 12, ... so on. We find that every successive multiple appears as the third number after the previous.

So if one wishes to find the number of multiples of 6 less than 255, we could arrive at the number through $\frac{255}{6} = 42$ (and the remainder 3).

The remainder is of no consequence to us. So in all there are 42 multiples.

If one wishes to find the multiples of 36, find $\frac{255}{36} = 7$ (and the remainder is 3).

Hence, there are 7 multiples of 36.

Factorisation :

It is the process of splitting any number into the form, where it is expressed only in terms of the most basic prime factors.

For example, $12 = 2^2 \times 3^1$. 12 is expressed in the factorised form in terms of its basic prime factors. This is the factorised form of 12.

If any number x does not have a factor between 2 and $\sqrt{x}$, it is a prime number. Why do we check for factors just till $\sqrt{x}$ and why not beyond that till x?

It is possible to find the number of factors of a composite number without listing all those factors, from its factorised form.

Take 12 for instance, it can be expressed as $12 = 2^2 \times 3^1$.

The factors of 12 are:

$(2^0 \times 3^0), (2^0 \times 3^1), (2^1 \times 3^0), (2^1 \times 3^1), (2^2 \times 3^0), (2^2 \times 3^1)$, .

Here the powers of 2 can be one of (0, 1, 2) and the powers of 3 can be one of (0, 1). So the total possibilities if you take the two as combination is $3 \times 2 = 6$. Each combination of the powers of 2 and 3 gives a distinctly different factor. Hence, since there are 6 different combinations of the powers of 2 and 3, there are 6 distinctly different factors of 12.

In general, for any composite number, C, which can be expressed as $C = a^m \times b^n \times c^p \times ...$, where a, b, c, ... are all prime factors and m, n, p are positive integers, the number of factors is equal to $(m + 1)(n + 1)$
$(p + 1)$ ...

The factors of 12, in increasing order are 1, 2, 3, 4, 6, 12

i.e. a total of 6 factors

We see that,

product of 1^{st} and 6^{th} = 12

product of 2^{nd} and 5^{th} = 12

product of 3^{rd} and 4^{th} = 12.

Same is the case with factors of any number, e.g. factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24

What happens if the number of factors is odd?

Can you extend this logic to determine which numbers will have odd number of factors.

Example 1: Find the total number of factors of 576.

Solution: The factorised form of 576 is $(24 \times 24) = (2^3 \times 3)(2^3 \times 3) = (2^6 \times 3^2)$.

So the total number of factors is $(6+1)(2+1)= 21$.

Example 2: If $N = 12^3 \times 3^4 \times 5^2$ then find the total number of even factors of N.

Solution: The factorised form of N is $(2^2 \times 3^1)^3 \times 3^4 \times 5^2 \Rightarrow 2^6 \times 3^7 \times 5^2$.

Hence, the total number of factors of N is $(6 + 1)(7 + 1)(2 + 1) = 7 \times 8 \times 3 = 168$.

Some of these are odd multiples and some are even. The odd multiples are formed only with the combination of 3s and 5s. So, the total number of odd multiples is $(7 + 1)(2 + 1) = 24$.

Therefore, the number of even multiples = 168 - 24 = 144.

Example 3: A number N when factorised can be written as $N = p_1^4 \times p_2^3 \times p_3^7$. Find the number of perfect squares which are factors of N. (The three prime numbers $p_1, p_2, p_3 > 2$.)

Solution: In order that the perfect square divides N, the powers of p_1 can be 0, 2, or 4, i.e. 3.

Powers of p_2 can be 0, 2, i.e. 2.

Powers of p_3 can be 0, 2, 4 or 6, i.e. 4.

Hence, a combination of these powers gives $3 \times 2 \times 4$, i.e. 24 numbers.

So there are 24 perfect squares that divide N.

Example 4: In how many ways can 36 be written as a product of two natural numbers.

Solution: $36 = 2^2 \times 3^2$ and will have $3 \times 3 = 9$ factors. The factors in increasing order are 1, 2, 3, 4, 6, 9, 12, 18, 36. Product of factors equidistant form the centre will be 36. Thus there are 4 + 1 = 5 ways of writing 36 as a product of 2 natural numbers viz 1×36, 2×18, 3×12, 4×9 and 6×6.

We all know that prime numbers have just two factors. Can you identify which numbers will have exactly 3 factors?

x, a & b are natural numbers. If x is divisible by a & b each then find a condition for which x is definitely divisible by a × b?

Along a long corridor there are 100 doors marked as 1, 2, 3, ... 100. As you know the doors can be in two states - open or close. Initially all doors are closed. Person number 1 changes the state of all doors that are a multiple of 1 i.e. basically all doors. Person number 2 then changes the state of all doors that are a multiple of 2. Person number 3 then changes the state of all doors that are multiple of 3 and so on till the person number 100 changes state of door number 100.

Now how many doors are closed.

***Hint:** Try to find for a particular door, how many persons will change the state?*

How many multiples of x exists from a to b (both inclusive) when

a. neither a nor b is a multiple of x

b. one of a and b is a multiple of x.

c. both a and b are multiples of x.

HCF and LCM

HCF and LCM are one of the basic concepts of mathematics which have variety of applications in our daily life.

Understanding HCF:

Let us take two numbers 15 and 20

Factors of 15 are = 15, 5, 3, 1

Factors of 20 are = 20, 10, 5, 1

To find the HCF, check what is the highest factor common to both the numbers. We can see that it is 5.

Understanding LCM:

Let us take two numbers 15 and 20

Multiples of 15 = 15, 30, 45, 60, 75, 90, 105, 120, 135, etc.

Multiples of 20 = 20, 40, 60, 80, 100, 120, 140, etc.

To find the LCM of these two numbers, check what is the lowest number common to the sets of multiples of both the numbers. We can find that it is 60.

How to find HCF of two numbers?

There are two methods:

a. Division method.

b. Prime factorisation method.

a. Division method:

In this method Divisor becomes dividend and remainder becomes

Divisor and this process continues till one can divide.

The last divisor is your answer.

Now, try to understand the following illustrative examples.

To find to HCF of 15 & 20

```
15) 20(1
   −15
    5) 15(3
      −15
        0
```

So HCF of 15 and 20 is 5.

To find the HCF of 20 & 28.

```
20) 28(1
   −20
    8) 20(2
      −16
        4) 8(2
          −8
           0
```

So the HCF of 20 and 28 is 4.

To find the HCF of 20, 28 & 45

We have seen that HCF of 20 & 28 is 4.

So, we will take HCF of 4 & 45.

```
4) 45 (11
  -44
   1) 4 (4
     -4
      0
```

So, HCF of 20, 28 & 45 is 1.

b. Prime factorization method:

Write the number in terms of prime factors.

$20 = 2^2 \times 5^1 \times 3^0$

$45 = 2^0 \times 3^2 \times 5^1$

For finding out their, HCF, take the lowest power of all prime numbers. The HCF of 20 and 45 is $2^0 \times 3^0 \times 5^1$ i.e. 5.

How to find LCM of two or more numbers?

There are two methods

i. Division method

ii. Prime factorisation method

i. Division method

LCM of 18, 27 and 30.

3	18,	27,	30
3	6,	9,	10
2	2,	3,	10
	1,	3,	5

LCM = $3 \times 3 \times 3 \times 2 \times 5 = 270$

ii. Prime factorisation method

Take two numbers 20 and 45.

Write the numbers in terms of prime factors.

$20 = 2^2 \times 5^1 \times 3^0$

$45 = 2^0 \times 3^2 \times 5^1$

For finding out their LCM, take the highest power of all prime numbers. The LCM of 20 and 45 is $2^2 \times 3^2 \times 5^1$ i.e. 180.

If the LCM of a, b, c is x and that of d, e is y, would the LCM of x, y be the LCM of a, b, c, d, e.

If the HCF of a, b, c is x and that of d, e is y, would the HCF of x, y be the HCF of a, b, c, d, e.

Example 1: Find the HCF of 24 and 72.

Solution: $24 = 2 \times 2 \times 2 \times 3$

$72 = 2 \times 2 \times 2 \times 3 \times 3$

HCF = $2 \times 2 \times 2 \times 3 = 24$

Similarly, you can find the HCF of sets containing more than 2 numbers.

Example 2: Find the largest number that can exactly divide 513, 783 and 1107.

Solution: Required number = HCF of 513, 783 and 1107.

Now, $513 = 3^3 \times 19$, $783 = 3^3 \times 29$, $1107 = 3^3 \times 41$

$\therefore$ HCF $= 3^3 = 27$. Hence, the required number is 27.

Example 3: Find the least number exactly divisible by 12, 15, 20 and 27.

Solution:

Required number = LCM of 12, 15, 20, 27

$\therefore$ LCM $= 3 \times 4 \times 5 \times 9 = 540$

Example 4: Find the least number which when divided by 6, 7, 8, 9 and12 leave the same remainder 1 in each case.

Solution:

Required number = (LCM of 6, 7, 8, 9, 12) + 1

$\therefore$ LCM $= 3 \times 2 \times 2 \times 7 \times 2 \times 3 = 504$

Hence, required number = (504 + 1) = 505

Example 5:

The traffic lights at three different road-crossings, change after every 24 sec, 72 sec and 120 sec respectively. If they all change simultaneously at 10 : 54 : 00 hr, then at what time will they change next simultaneously?

Solution:

Interval of change = LCM of (24, 72, 120) sec = 360 sec.

The lights will change simultaneously after every 360s, i.e., 6 min 00 sec. So, they will change next simultaneously at 11 : 00 : 00 hrs.

Example 6:

How many three-digit numbers are divisible by 6?

Solution:

There are 16 numbers before 100 which are divisible by 6.

There are 166 numbers before 999 which are divisible by 6. Total three-digit numbers divisible by 6 are 166 - 16 =150.

Important results

If two numbers a and b are given, and their LCM and HCF are L and H respectively, then $L \times H = a \times b$.

LCM and HCF of fractions

$$\text{LCM of fractions} = \frac{\text{LCM of numerators}}{\text{HCF of denominators}}$$

$$\text{HCF of fractions} = \frac{\text{HCF of numerators}}{\text{LCM of denominators}}$$

Example:

Find the LCM and HCF of $\frac{25}{12}$ and $\frac{35}{18}$.

$$\text{LCM} = \frac{\text{LCM of 25 and 35}}{\text{HCF of 12 and 18}} = \frac{175}{6}$$

$$\text{HCF} = \frac{\text{HCF of 25 and 35}}{\text{LCM of 12 and 18}} = \frac{5}{36}$$

Note: Do not directly apply the formula if the fractions are not in their simplest form.

Example 6:

How many three-digit numbers are divisible by 6?

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There are 16 numbers before 100 which are divisible by 6.

There are 166 numbers before 999 which are divisible by 6. Total three-digit numbers divisible by 6 are 166 - 16 =150.

Important results

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Note: Do not directly apply the formula if the fractions are not in their simplest form.

Example 7:

The HCF of two numbers is 11 and their LCM is 693. If one of the numbers is 77, then find the other number.

Solution:

The other number = $\frac{11 \times 693}{77} = 99$

Application of HCF and LCM

I. Of the type when would clocks strike together simultaneously

Example 1:

Two cyclists were preparing for the Olympics in the Yamuna Velodrome. The first cyclist takes 10 min to cover one full round, whereas the second takes 9 min (no wonder, India has never won an Olympics medal in cycling!!). Assuming that they have enough stamina to last as long as your answer, find when would they both be together again at the starting block if they both started simultaneously?

Solution:

This might seem to be a problem on time, speed and distance. Yes, but the logic is based on LCM concept. The first cyclist would be at the starting point at every multiple of 10 min. The second would be at the starting point at every multiple of 9 min. If both of them have to be together at the starting block again, then a multiple of 10 min must be equal to a multiple of 9 min. So they would be together at the starting block for the first time after LCM(10, 9) = 90 min.

II. Least number leaving remainder `r' in each case when divided by `x', `y' and `z'.

Example 2:

There is a number greater than 3 which when divided by 4, 5 and 6 always leaves the same remainder 3. Find the following such numbers which satisfy the given condition.

a. Smallest

b. Second smallest

c. Largest number less than 1000

Solution:

a. The smallest number which, when divided by 4, 5 and 6, leaves the remainder 3 in each case is LCM(4, 5, 6) + 3 = 63.

b. All numbers which are of the form {LCM(4, 5, 6)} N + 3 always satisfy the property that when divided by 4, 5 or 6, it leaves a remainder 3. [N is any natural number.]

The numbers that satisfy this property are 63, 123, 183, ..., so on.

Hence, the second smallest number is 123.

c. To solve the problem, we have to find the largest number of the form 60N + 3, that is less than 1000. The largest such number is 963.

Thus least number leaving remainder `r' in each case when divided by `x', `y' and `z'

= (LCM of x, y, z) + r.

The series of such numbers will be (LCM of x, y, z) $\times$ n + r.

III. Least number leaving remainder (x - a), (y - a) and (z - a) when divided by x, y and z respectively.

Find the smallest number which when divided by 5, 6, and 8 leaves remainder 2, 3, and 5?

Here we see that the (divisor - remainder) is same for all divisors, in this case 3. The following number line will make the solution to this problem very transparent.

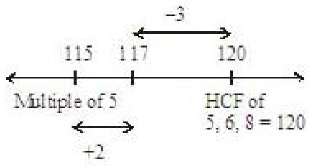
= (LCM of x, y, z) + r.

The series of such numbers will be (LCM of x, y, z) × n + r.

III. Least number leaving remainder (x - a), (y - a) and (z - a) when divided by x, y and z respectively.

Find the smallest number which when divided by 5, 6, and 8 leaves remainder 2, 3, and 5?

Here we see that the (divisor - remainder) is same for all divisors, in this case 3. The following number line will make the solution to this problem very transparent.



Hence, the smallest number is LCM (5, 6, 8) - 3 = 117.

Thus least number leaving remainder (x - a), (y - a) and (z - a) when divided by

x, y, z respectively = (LCM of x, y, z) - a.

The series of such numbers will be (LCM of x, y, z) × n - a.

10 thieves steal x gold coins and escape. While all are asleep, 2 thieves wake up and divide the coins between them and find 1 coin extra. Just then a 3rd thief wakes up. So the 3 thieves divide all the coins among them and find 2 coins extra. Again just then a 4th thief wakes up. Again on dividing the coins among them. 3 coins are extra. This continues till the 10th thief wakes up and on dividing the coins among them 9 coins are extra. What is the least value of x?

In the above problem, if on each division, 1 coin was left, what would have been the least value of x?

If the HCF of A, B and C is x, is the HCF of A - B, B - C and A - C also x? If not always, under what condition will it be equal to x?

IV. Of the type largest measure that measures x, y and z exactly.

Example 3:

A rectangular piece of cloth has dimensions 16 m × 6 m. What is the least number of equal squares that can be cut out of this cloth such that no cloth is wasted?

Solution:

We want to find out the length of the edge of the square piece (largest) to be cut out of this cloth in such a way that no piece of cloth is left over. In other words, we have to find the largest number which completely divides the dimensions 16m and 6m (i.e. the HCF of 16 and 6). This would give the side of the largest squares that satisfy the given conditions.

HCF(16m, 6m) = 2m.

There will be $16 \div 2 = 8$ divisions along the length and $6 \div 2 = 3$ divisions along the breadth. Therefore, total number of square pieces = $8 \times 3 = 24$.

This is the minimum number of squares that can be cut out of this piece of cloth without wasting any part of the cloth.

Example 4:

In a school 437 boys and 342 girls have been divided into classes, so that each class has the same number of students and no class has boys and girls mixed. What is the least number of classes needed?

Solution:

We should have the maximum number of students in a class.

So we have to find HCF (437, 342) = 19.

HCF is also the factor of difference of the numbers.

$\therefore$ Number of classes = $\frac{437}{19} + \frac{342}{19} = 23 + 18 = 41$ classes.

What is the smallest number which when divided by 7, 8, 9 leaves a remainder of 2, 4, 6 respectively?

A set of number with HCF x can be assumed to be $x \times a$, $x \times b$, $x \times c$ ... where a, b and c are co-prime number.

V. Miscellaneous

Example 5:

Manas and his girlfriend met at Nehru Place after a long time. Manas stays at Vivek Vihar and his girlfriend stays in Gurgaon. Both of them commute by bus. They reached the bus stop, and got to know that a bus had left just then for each of their destinations. Neither wanted to leave the other alone at the bus stop. If the frequency of buses to Gurgaon was 7 min and that to Vivek Vihar was 11 min, then

a. how long would they wait at the bus stop?

b. how many buses going to their destinations would each one decide not to board?

Solution:

a. Since both the buses had left for the two destinations just then, their respective buses would be at the bus stop simultaneously after LCM(7, 11) = 77 min.

b. In 77 min, the total number of buses for Gurgaon that would have come to the bus stop is $\frac{77}{7} = 11$. Similarly, the total number of buses to Vivek Vihar that would have arrived at the bus stop is $\frac{77}{11} = 7$.

Hence, Manas and his girlfriend decided not to board 6 and 10 buses respectively.

Example 6:

Mr Tamatar buys some apples at 5 per rupee from one trader, and a similar quantity at 7 per rupee from another trader. He mixes both the varieties, and sells the whole at 6 per rupee. What is the profit or loss percentage that he makes?

Solution:

Assume that Mr Tamatar buys LCM(5, 6, 7) = 210 apples of each variety.

Amount spent on the first variety = $\frac{210}{5}$ = ₹42.

Amount spent on the second variety = $\frac{210}{7}$ = ₹30.

Total amount spent = ₹42 + ₹30 = ₹72.

Now the total (210 + 210) = 420 apples are sold at 6 per rupee.

The total revenue = $\frac{420}{6}$ = ₹70.

Hence, the loss = ₹72 - ₹70 = ₹2.

Loss percentage = $\frac{2}{72} \times 100 = 2\frac{7}{9}\%$.

A difficult one but good for clarity.

How many sets of two numbers will have the LCM as $p_1^a \times p_2^b$ where p_1 and p_2 are prime numbers and a and b are natural numbers.

Highest power dividing a factorial

Example 1:

What is the highest power of 2 that divides 20! completely?

Solution:

$20! = 1 \times 2 \times 3 \times 4 \times \ldots \times 18 \times 19 \times 20 = 1 \times (2^1) \times 3 \times (2^2) \times 5 \times (2^1 \times 3^1) \times 7 \times (2^3) \times \ldots$ so on.

In order to find the highest power of 2 that divides the above product, we need to find the sum of the powers of all 2s in this expansion.

All numbers that are divisible by 2^1 will contribute 1 to the exponent of 2 in the product $\frac{20}{2^1} = 10$.

Hence, 10 numbers contribute 2^1 to the product.

Similarly, all numbers that are divisible by 2^2 will contribute an extra 1 to the exponent of 2 in the product, i.e. $\frac{20}{2^2} = 5$.

Hence, 5 numbers contribute an extra 1 to exponents.

Similarly, there are 2 numbers that are divisible by 2^3 and 1 number that is divisible by 2^4.

Hence, the total 1's contributed to the exponent of 2 in 20! is the sum of (10 + 5 + 2 + 1) = 18.

Hence, group of all 2s in 20! gives $2^{18} \times (N)$, where N is not divisible by 2.

If 20! is divided by 2^x, the maximum value of x = 18.

Example 2:

What is the highest power of 5 that divides 100!?

Solution:

Calculating contributions of the different powers of 5, we have $\frac{100}{5^1} = 20$, $\frac{100}{5^2} = 4$.

Hence, the total contribution to the power of 5 is 24.

Or the number 100! is divisible by 5^{24}.

Thus the approach to find the highest power of x dividing y! is $\left[\frac{y}{x}\right] + \left[\frac{y}{x^2}\right] + \left[\frac{y}{x^3}\right]$...... , where [ ] represents just the integral part of the answer ignoring the fractional part and x is a prime number.

Example 3:

What is the highest power of 6 that divides 9!?

Solution:

If we go by the above process, then we will get the answer as $\frac{9}{6} = 1$ and $\frac{9}{6^2} = 0$.

Thus answer we get is 1 which is wrong. True there is just one multiple of 6 from 1 to 9 but the product 2 × 3 = 6 and also 4 × 9 = 36, can further be divided by 6. Thus, when the divisor is a composite number find the highest power of its prime factors and then proceed.

In this case 9! can be divided by 2^7 and 3^4 and thus by 6^4 (In this case we need not have checked power of 2 as it would definitely be greater then that of 3).

What is the least value of x such that $\frac{60!}{x}$ will be a odd number?

Factorials of 0 to 4 have no trailing zeros (at the end) Factorials of 10 to 14 have 2 trailing zero and 15 to 19 have 3 and so on...

While factorials of 20 to 24 have 4 trailing zeros, those of 25 to 29 have 6 (and not 5) trailing zeros.

Similarly while factorial of 45 to 49 have 10 trailing zeros, factorials of 50 to 54 have 12 trailing zeros.

How many more trailing zeros would 625! have compared to 624!

Remainders

Consider two numbers x and y which when divided by 6 leave remainder 3 and 2 respectively. What will be the remainder when each of the following is divided by 6 :

a. x + y b. x - y c. x × y d. y - x e. y^5

a. x can be written as 6m + 3 and y can be written as 6n + 2.

Thus, x + y = 6(m + n) + 5, i.e. (x + y) is a multiple of 6 plus 5 and hence when divided by 6 will leave remainder of 5 (basically 3 + 2).

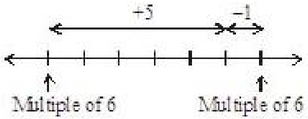
b. Similarly x - y = 6(m - n) + 1, i.e. when (x - y) is divided by 6, the remainder is 1(basically 3 - 2)

c. x × y = (6m + 3)(6n + 2) = 36mn + 12m + 18n + 6 = 6(6mn + 2m + 3n + 1) i.e. it is a multiple of 6 and remainder is 0.

By now it will be clear that whatever is the operation that is performed on the divisors, the same operation has to be performed on the remainders of respective divisors to get the remainder.

d. However when we try to do the same for y -x, the remainder will be 2 - 3 = -1.

What does this mean? Basically it means that the expression y - x will boil down to 6p - 1 = 6(p - 1) + 5 i.e. the remainder is 5. Visually it can be understood from the following number line.



Thus, if remainder, calculated by performing the same operation on respective remainders as that performed on divisors, is negative, all one needs is to reduce the divisor by this negative value to get the effective remainder.

e. In this case the expression is

$y \times y \times y \times y \times y$ and the remainder will be $2 \times 2 \times 2 \times 2 \times 2 = 32$.

But we know the remainder when a number is divided by 6 has to lie between 0 to 5 (both inclusive). This can be understood as the expression would evaluate to $6p + 32$ which is nothing but $6(p + 5) + 2$ i.e. the remainder is 2.

Thus, if one gets a remainder higher than the divisor, just divide this value with the divisor again and find the remainder.

Example 1:

What is the remainder if 7^{25} is divided by 6?

Solution:

$$\frac{7^{25}}{6} = \frac{7 \times 7 \times 7 \ldots 7 (25 \text{ times})}{6}$$

7 divided by 6 leaves remainder 1.

Thus, 7^{25} when divided by 6 will leave remainder

$1 \times 1 \times 1 \ldots (25 \text{ times}) = 1$.

Method 2:Using binomial expansion

$(7)^{25}$ can be written $(6 + 1)^{25}$.

Using binomial theorem, $(a + b)^n$ can be written as $aN + b^n$ i.e. a multiple of a plus b^n.

Thus, $(6 + 1)^{25} = 6N + 1^{25} = 6N + 1$.

Thus, this number when divided by 6 will leave remainder 1.

Using binomial expansion, if we have to find the remainder when y^n is divided by x, express y^n as $(ax + b)^n$ where a is any natural number and b should ideally be -1, 0, 1.

$(ax + b)^n = Nx + b^n$

Thus, if b is -1, 0, 1, b^n can easily be evaluated irrespective of the value of n and the remainder can be found out.

Example 2:

What is the remainder if 7^{25} is divided by 4?

Solution: 7^{25} can be written $(8 - 1)^{25}$.

There are 26 terms in all. All the first 25 terms are divisible by 8, hence also by 4. The last term is $(-1)^{25}$.

Hence, $(8 - 1)^{25}$ can be written as 8X - 1 or 4Y - 1 (where Y = 2X). So 4Y - 1 when divided by 4 leaves the remainder 3.

Example 3:

What is the remainder if 3^{45} is divided by 8?

Solution:

3^{45} can be written as $9^{22} \times 3$. 9 can be written as (8 + 1). Hence, any power of 9 can be written 8N + 1. In other words, any power of 9 is 1 more than a multiple of 8. Hence, $(8N + 1) \times 3$ leaves the remainder 3 when divided by 8.

Example 4:

What is the remainder when $14^{15^{16}}$ is divided by 5?

Solution:

$14^{15^{16}} = (15 - 1)^{odd} = 15n + (-1)^{odd}$, i.e. a (multiple of 5) - 1. Thus when divided by 5 the remainder will be -1, i.e. 4.

144 divided by 60 leaves a reminder of 24. But 144 and 60 have common factor. Lets remove the common factor and see how it affects the remainder. If we divide 144 and 60 by 2, we have 72 divided by 30 and remainder is 12 (original remainder 24 divided by 2)

If we divide 144 and 60 by 3, we have 48 divided by 20 and remainder is 8 (original remainder 24 divides by 3) If we divide 144 and 60 by 4 we have 36 divided by 15 and remainder is 6 (again 24 divided by 4)

Euler's Totient function and the Fermat-Euler theorem

If N is a natural number such that $N = a^p b^q c^r$... where a, b, c etc are prime numbers, then Euler's Totient function is given by $\phi(N) = N\left(1-\frac{1}{a}\right)\left(1-\frac{1}{b}\right)\left(1-\frac{1}{c}\right)$... Here $\phi(N)$ is the number of numbers less than and prime to N.
If P is some other natural number which is prime to N, then the remainder when $P^{\phi(N)}$ is divided by N is 1.

E.g. since $45 = 3^2 \times 5$, $\phi(45) = 45\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = 24$. It means that there are 24 numbers which are less than and prime to 45. As 11 is prime to 45, the remainder when 11^{24} is divided by 45 is 1.

If N is prime in the theorem given above, then $\phi(N) = N\left(1-\frac{1}{N}\right) = N - 1$. So we can say that if N is a prime number and P is some other number which is prime to N, then the remainder when P^{N-1} is divided by N is 1.

Wilson's Theorem

If P is a prime number, then the remainder when (P - 1)! is divided by P is P - 1.

E.g. the remainder when 46! is divided by 47 is 46. Further, since 46! = 45! × 46, the remainder when 45! is divided by 47 should be 1.

Base System

The number system that we work in is called the `decimal system'. This is because there are 10 digits in the system 0-9. There can be alternative systems that can be used for arithmetic operations. Some of the most commonly used systems are: binary, octal and hexadecimal.

These systems find applications in computing.

Binary system has 2 digits: 0, 1.

Octal has 8 digits: 0, 1, 2, ..., 7.

Hexadecimal has 16 digits: 0, 1, 2, ..., 9, A, B, C, D, E, F.

After 9, we use the letters to indicate digits. For instance, A has a value 10, B has a value 11, C has a value 12, ... so on in all base systems.

The counting sequences in each of the systems would be different though they follow the same principle.

For instance, the sequence of the first few numbers on the number line starting with 0 is:

Decimal	Binary	Octal	Hexadecimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
15	1111	17	F
16	10000	20	10
17	10001	21	11
18	10010	22	12

(a) Conversions of numbers from:

(1) base 10 (decimal system) to some other base system,

(2) some other base system to decimal system (base 10).

(b) Arithmetic operations like

(1) addition

(2) subtraction

Base System works like a odometer of your vechicle. (that which record cumulative mileage)

Whenever a wheel starts repeating the cycle, it increases the wheel on the left by 1.

a. Conversion

12 in decimal system coincides with 14 in the octal system. How do we identify this equivalence?

1. Conversion from base 10 to any other base

Example 1:

Convert $(122)_{10}$ to base 8 system.

Solution:

8	122
8	15 - 2
8	1 - 7
	0 - 1

The number in decimal is consecutively divided by the number of the base to which we are converting the decimal number. Then list down all the remainders in the reverse sequence to get the number in that base.

So here $(122)_{10} = (172)_8$.

Example 2:

Convert $(270)_{10}$ to hexadecimal system.

Solution:

16	270
16	16 - (14) E
16	1 - 0
	0 - 1

The solution is not 1014. It is 10E, where E = 14 in the hexadecimal system.

Example 3:

Convert $(1987.725)_{10} \rightarrow (\ )_8$

Solution:

First convert non-decimal part into base 8.

8	1987	
8	248	3
8	31	0
8	3	7
		3

$\therefore (1987)_{10} = (3703)_8$

$(0.725)_{10} \rightarrow (\,)_8$

Multiply

$0.725 \times 8 = 5.8$ 5

$0.8 \times 8 = 6.4$ 6

$0.4 \times 8 = 3.2$ 3

$0.2 \times 8 = 1.6$ 1

$0.6 \times 8 = 4.8$ 4

Keep on accomplishing integral parts after multiplication with decimal part till the decimal part is zero.

$\therefore (.725)_{10} = (.56314\ ...)$

$\therefore (1987.725)_{10} = (3703.56314...)$

2. Conversion from any other base to decimal system

Example 4:

Convert $(231)_8$ into decimal system.

Solution:

In $(231)_8$, the value of the position of each of the numbers (as in decimal system) is:

$1 = 8^0 \times 1$

$3 = 8^1 \times 3$

$2 = 8^2 \times 2$

[This is equivalent to 10^0 (unit's); 10^1 (ten's); 10^2 (hundred's) places in the decimal system.

Hence, $(231)_8 = (8^0 \times 1 + 8^1 \times 3 + 8^2 \times 2)_{10} = (1 + 24 + 128)_{10} = (153)_{10}$

Example 5:

Convert $(761.56)_8 \rightarrow (\,)_{16}$

Solution:

In such conversion which are standard form conversions, it is easier to convert in this manner.

$(761.56)_8 \rightarrow (\,)_2 \rightarrow (\,)_{16}$ Converting every digit in base 8 to base 2, $(111110001.101110)_2 \rightarrow (1F1.B8)_{16}$

Example 6:

Convert $(1AB)_{16}$ into decimal system.

Solution:

$(1AB)_{16} = 16^2(1)+16^1(A)+16^0\,(B)$

$= 16^2(1)+16^1(10)+16^0(11) = 256 + 160 + 11 \Rightarrow 427$

Hence, $(1AB)_{16} = (427)_{10}$

In order to convert from one base x to any other base y (x, y $\neq$ 10), we can convert using the intermediate step of decimal system.

Example 7:

What is the value of the following sum if both the numbers are in octal system?

$$\begin{array}{r} 2\ 4\ 7 \\ +\ 3\ 4\ 5 \\ \hline \\ \hline \end{array}$$

Solution:

There are two methods.

a. Convert both the numbers into decimal numbers. Add the decimal numbers, and convert back this sum into octal system.

b. Direct addition:

Step 1:

7 + 5 = 12 in decimal.

It is equal to 14 in octal.

Hence, keep back 4 and carry over 1 as in decimal addition.

Step 2:

$4 + 4 + 1 = (9)_{10} = (11)_8$

Keep back 1 and carry over 1.

So on final sum = $\begin{array}{r} 2\ 4\ 7 \\ +\ 3\ 4\ 5 \\ \hline (6\ 1\ 4)_8 \\ \hline \end{array}$

Example 8:

Subtract $(247)_8$ from $(345)_8$.

$$\begin{array}{r} 3\ 4\ 5 \\ -\ 2\ 4\ 7 \\ \hline \\ \hline \end{array}$$

Solution:

Step 1: 5 is less than 7. So borrow 1 from the previous digit. Since we are working in octal system, if we borrow 1, then 5 become 5 + 8 = 13. Subtract 7 from 13, i.e. 6.

$$\begin{array}{r} 3\ 4\ 5 \\ -\ 2\ 4\ 7 \\ \hline 6 \\ \hline \end{array}$$

Step 2:

Since we borrow 1, the 4 in the first row has now become 3. Borrow 1 from 3. So it now becomes 3 + 8 = 11.

Subtracting 4 from it, we get 7.

Hence, $\begin{array}{r} 3\ 4\ 5 \\ -\ 2\ 4\ 7 \\ \hline 0\ 7\ 6 \\ \hline \end{array}$

For multiplication and division of numbers in other base systems, convert them into decimal system; and after doing the arithmetic operation, convert the result back into the respective base system.

Perfect square:

A number is said to be a perfect square if and only if the square root of that number is an integer.

Some important facts about perfect squares

(1) The square of an even number is always even.

(2) The square of an odd number is always odd.

(3) Square of an integer cannot end in 2, 3, 7 or 8.

(4) The square of a real number (negative or positive) is always positive.

Some important formulae used in simplification:

(1) $(a + b)^2 = a^2 + b^2 + 2ab$

(2) $(a - b)^2 = a^2 + b^2 - 2ab$

(3) $(a + b)^2 = (a - b)^2 + 4ab$

(4) $a^2 - b^2 = (a - b)(a + b)$

(5) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(6) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

(7) $a^2 + b^2 = \frac{1}{2}[(a + b)^2 + (a - b)^2]$

Example 1: Simplify $\frac{527 \times 527 \times 527 + 183 \times 183 \times 183}{527 \times 527 - 527 \times 183 + 183 \times 183}$.

Solution:

The given expression is equivalent to $\frac{(527)^3 + (183)^3}{(527)^2 - 527 \times 183 + (183)^2}$

We know that, $\frac{a^3 + b^3}{a^2 - ab + b^2} = a + b$

In the above example a = 527 and b = 183

∴ The expression is equal to (527 + 183) = 710

Example 2:

Simplify $\left(\frac{(614 + 168)^2 - (614 - 168)^2}{614 \times 168}\right)$.

Solution:

Expression $\frac{(a+b)^2 - (a-b)^2}{ab} = \frac{4ab}{ab} = 4$

Example 3:

Find the square of (1605).

Solution:

$(1605)^2 = (1600 + 5)^2$

$= (1600)^2 + 2 \times 1600 \times 5 + (5)^2$

$= 2560000 + 16000 + 25 = 2576025$

Example 4:

Find the value of 896 × 896 - 204 × 204.

Solution:

$a^2 - b^2 = (a + b)(a - b)$ (where a = 896 and b = 204)

$= (896 + 204)(896 - 204)$

$= 1100 \times 692 = 761200$

Example 5:

Evaluate $(57)^2 + (43)^2 + 2 \times 57 \times 43$.

Solution:

$(a^2 + b^2 + 2ab) = (a + b)^2 = (57 + 43)^2 = 100^2$

$= 10000$

Example 6:

Simplify $(81)^2 + (68)^2 - 2 \times 81 \times 68$.

Solution:

$(81 - 68)^2 = 13^2 = 169$

Example 7:

Evaluate $(313 \times 313 + 287 \times 287)$.

Solution:

$a^2 + b^2 = \frac{1}{2}[(a+b)^2 + (a-b)^2]$ (where a = 313 and b = 287)

$= \frac{1}{2}[(313+287)^2 + (313-287)^2]$

$= \frac{1}{2}\left[(600)^2 + (26)^2\right] = 180338$

Rules of counting numbers

1. Sum of first n natural numbers = $\frac{n(n+1)}{2}$

2. Sum of first n odd numbers = n^2

3. Sum of first n even numbers = $n(n + 1)$

4. Sum of the squares of first n natural numbers= $\frac{n(n+1)(2n+1)}{6}$

5. Sum of the cubes of first n natural numbers = $\left[\frac{n(n+1)}{2}\right]^2$

Example 8:

If square root of 15 = 3.88, then find the square root of $\frac{5}{3}$.

Solution:

$\sqrt{\frac{5}{3}} = \sqrt{\frac{5 \times 3}{3 \times 3}} = \frac{\sqrt{15}}{3} = \frac{3.88}{3} = 1.29$

Example 9:

A four-digit number divisible by 7 becomes divisible by 3, when 10 is added to it. Find the largest such number.

Solution:

Largest four-digit number is 9999. On dividing 9999 by 7, we get 3 as remainder. Largest four-digit number divisible by 7 is 9996.

Let 9996 - x + 10 be divisible by 3.

By trial and error, we find that x = 7

Required number = (9996 - 7) = 9989

Example 10:

A three-digit number 4a3 is added to another three- digit number 984 to give the four-digit number 13b7 which is divisible by 11. Find the value of (a + b).

Solution:

$$\begin{array}{r} 4\,a\,3 \\ +\,9\,8\,4 \\ \hline 1\,3\,b\,7 \\ \hline \end{array}$$

Here a + 8 = b, if 13b7 is divisible by 11 then (7 + 3) - (b + 1) = 0; b = 9 and a + 8 = b or a = 1.

Hence, a + b = 9 + 1 = 10.

Example 11:

Of the three numbers, the sum of the first two is 45; the sum of the second and the third is 55; and the sum of the third and thrice the first is 90. Find the third number.

Solution:

Let the numbers be x, y and z. Then,

x + y = 45; y + z = 55 and 3x + z = 90.

y = 45 - x and z = 55 - y = 55 - (45 - x) = 10 + x

∴ 3x + 10 + x = 90 or x = 20

y = (45 - 20) = 25 and z = (10 + 20) = 30

∴ Third number = 30

Miscellaneous

Example 1:

There is a two-digit number ab in decimal system. Both a and b are natural numbers. What is the value of ab?

Solution:

From the basic counting rules, we know that a is in ten's place and b is in unit's place. Hence, the value of ab is 10(a) + b.

Example 2:

A two-digit number ab is added to the number formed by reversing the original digits. If their sum is divisible by 11, 9 and 2, find the number of pairs of (a, b).

Solution:

The value of the number = 10a + b.

The number formed by reversing the digits = ba.

Value of this number = 10b + a.

Sum of the two numbers = 11a + 11b = 11(a + b).

Now if the sum is divisible by 11, 9, 2, it means that (a + b) must be divisible by both 9 and 2.

Hence, a + b = 18. So it means a = b = 9.

The original number is 99.

So there is only one pair of (a, b).

Example 3:

In a two-digit prime number, if 18 is added, we get another prime number with reversed digits. How many such numbers are possible?

Solution:

Let a two-digit number be ab.

10a + b + 18 = 10b + a

or -9a + 9b = 18

or b - a = 2

Only two numbers `13' and `79' satisfy the given condition.

Example 4:

If $\begin{array}{r} a\,a \\ +b\,b \\ \hline c\,c\,0 \end{array}$ and a - b = 2, then find a, b.

Solution:

These problems involve basic number theory rules.

a. aa + bb = 11(a + b)

b. aa, bb are two-digit numbers.

Hence, their sum cannot exceed 198.

So c must be 1.

c. Hence, cco = 110.

This implies a + b = 10 or a = 6 and b = 4.

Such problems are part of a category of problems called alphanumerics.

Example 5: If $\begin{array}{rrr} & a & 3 & b \\ (-) & & a & c \\ \hline & a & a & 9 \end{array}$, then find a, b, c if each one of them is distinctly different digit.

Solution:

a. Since the first digit of (a 3 b) is written as it is after subtracting ac from it. It means that there is no carry over from a to 3.

b. There must be a carry over from 3 to b, because if no carry over is there, it means 3 - a = a

$\Rightarrow$ 2a = 3 $\Rightarrow$ a = $\frac{3}{2}$

which is not possible because a is a digit. For a carry over 1, 2 - a = a

$\Rightarrow$ a = 1

c. Now 10 + b - c = 9 $\Rightarrow$ b - c = -1

It means b and c are consecutive digits (2, 3), (3, 4), ..., (8, 9).

Example 6:

$$\begin{array}{rrrr} & 1 & a & 4 \\ & \times & 3 & b \\ \hline & 8 & C & 8 \\ S & 7 & 2 & \\ \hline T & 5 & d & 8 \end{array}$$

Find S, T, b.

Solution: Let us consider 1a4 × 3 = S72.

a × 3 results in a number ending in 6.

As 16 and 26 is ruled out, a is 2.

Thus, S = 3, T = 4.

Now 1 a 4 × b = 8C8; b = 2 or 7

Again 2 is ruled out because in that case, product would be much less than 800.

$\therefore$ b = 7.

Example 7: How many numbers from 100 to 300 are divisible by

a. 5 and 6? b. 5 or 6? c. 5 or 6 or 10?

Solution:

a. All numbers that are divisible by both 5

and 6 are multiples of LCM(5, 6) = 30.

From 0 to 300, there are $\frac{300}{30}$ = 10

numbers that are divisible by 30.

From 0 to 99, there are $\frac{99}{30}$ = 3

numbers that are divisible by 30.

Hence, from 100 to 300, there are (10 - 3) = 7 numbers that are divisible by both 5 and 6.

b. NM (5 or 6) = NM (5)+ NM (6) - NM (5 and 6) [Note: NM is the number of multiples.]

So number of multiples of 5 or 6 from 100 to 300 is

$$\left\{\left[\frac{300}{5}\right]-\left[\frac{99}{5}\right]\right\}+\left\{\left[\frac{300}{6}\right]-\left[\frac{99}{6}\right]\right\}-\left\{\left[\frac{300}{30}\right]-\left[\frac{99}{30}\right]\right\}=41+34-7=68$$

c. The number of multiples of 5 or 6 or 10 would remain the same, since all the multiples of 10 have already been included as multiples of 5.

Example 8: How many times would 3 appear in all the numbers from 255 to 432 (both inclusive)?

Solution:

Let's take from 255 to 454, and then deduct the relevant number of 3s?

Unit's place: Every tenth number will have a 3 appearing once. So from 255 to 454, out of the 200 numbers there would be 20 3s.

Ten's place: Every hundred numbers would have 10 3s in the ten's place. So from 255 to 454, there would be twenty 3's.

Hundred's place: There are hundred numbers in 300s which have a 3 in the hundred's place. So total number of 3s = 20 + 20 + 100 = 140.

From 433 to 454, there are 10 3s.

Hence, there are in all 140 - 10 = 130 3s from 255 to 432.

Example 9: What is the sum of all digits that appear from 1 to 100?

Solution: There are 100 numbers in all. 1 to 9 would appear in the unit's place 10 times. 1 to 9 would appear in the ten's place 10 times. 1 would appear in the hundred's place once.

Hence, the sum of all the digits = (1 + 2 + 3 + ... + 9)(20) + 1 = 45 × 20 + 1 = 901.

Example 10: The value of a number is five times the sum of its digits. Find the number.

Solution:

Solution:

Let's take from 255 to 454, and then deduct the relevant number of 3s?

Unit's place: Every tenth number will have a 3 appearing once. So from 255 to 454, out of the 200 numbers there would be 20 3s.

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Hundred's place: There are hundred numbers in 300s which have a 3 in the hundred's place. So total number of 3s = 20 + 20 + 100 = 140.

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Example 9: What is the sum of all digits that appear from 1 to 100?

Solution: There are 100 numbers in all. 1 to 9 would appear in the unit's place 10 times. 1 to 9 would appear in the ten's place 10 times. 1 would appear in the hundred's place once.

Hence, the sum of all the digits = $(1 + 2 + 3 + ... + 9)(20) + 1 = 45 \times 20 + 1 = 901$.

Example 10: The value of a number is five times the sum of its digits. Find the number.

Solution:

The number can only be a two-digit number.

Let the number be XY.

Then the value of the number = $10X + Y = 5(X + Y)$. So $5X = 4Y$.

Since X and Y are integers, the only possible values are $X = 4$ and $Y = 5$.

[Why cannot it be a three- digit number?]

{Explanation: Sum of three- digits does not exceed 27. So the three-digit number cannot be greater than 135. Hence, the first digit has to be 1. Also five times the sum of the digits cannot exceed 95, which itself is a two- digit number.}

Practice Exercises 2

Introduction

There are 7 practice exercises out of which 2 are of level-1, 3 are of level-2 and 2 are of level 3 apart from Non MCQ Questions to Strengthen you fundamentals. While solving the exercises make sure that each and every concept is understood properly.

Problems for Practice (Non MCQ)

Level 1

1. Convert 4.32323232... into a $\frac{p}{q}$ form.

2. What least number must be added to 8961 to make it exactly divisible by 84?

3. Find the nearest integer to 1834 which is exactly divisible by both 12 and 16.

4. Which is larger, 5^7 or 7^5?

5. If the HCF of 2 numbers is 48, and the HCF of 2 other numbers is 36, what is the HCF of all the four numbers?

6. If there are 8 numbers whose HCF has to be found by the division method, how many steps would be needed in order to find it?

7. What is the smallest number less than 10,000 which when increased by 3 is divisible by 21, 25, 27 and 35?

8. Two athletes were making rounds in a stadium. The first one takes 7 min to complete one full round and the second one takes 4 min to complete one full round. When will they meet for the first time at the start if both of them started simultaneously ?

9. What is the remainder when 732^{732} is divided by 27?

10. A number when divided by 5 and 7 successively, leaves the remainders 2 and 4 respectively. Find the remainder when the same number is divided by 35.

11. What is the last digit of the number $(729)^{59}$?

12. What is the last digit of the number $(123)^7$?

13. How many prime factors are there of the number $(12)^{43} \times (34)^{48} \times (2)^{57}$?

14. A gardner planted saplings in such a way that every row had as many saplings as every column. If in all there were 729 trees, then how many saplings were there in each row?

15. In the previous problem, if he decides to plant one new sapling between every two saplings, how many new saplings would he have to plant?

16. A wealthy businessman gave away $\frac{1}{2}$ of his wealth to the first son; of the remaining to his second son; $\frac{1}{3}$ of the remaining to his third son; and the rest to his youngest son. If the youngest son got ₹6x lakh, where x is an integer, then what is the minimum wealth that the businessman had?

17. The product of 3 positive integers is 36. If one knows the sum of the 3 numbers, but is not able to uniquely identify the numbers, what is the sum of the 3 numbers?

18. Find the largest number that can be formed using four 3's.

19. Convert from decimal into base 12.

a. $(54)_{10}$ b. $(142)_{10}$

20. Convert into decimal from some other base.

a. $(110011)_2$ b. $(ABCD)_{16}$

21. Find the following.

a. $(457)_{12} - (249)_{12}$ b. $(347)_8 - (734)_8$

Level 2

22. The sum of 2 integers X and Y is 19. If 4X + 6Y = 9, then what are the values of X and Y?

23. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place then, find the number.

24. Which is the largest three-digit number which when divided by 6 leaves the remainder 5, and when divided by 5 leaves the remainder 3?

25. What is the largest four-digit number that if added to 2748, makes the sum divisible by 15, 20, 21, 23, 35?

26. Three people A, B and C can finish a job in 6, 7, 8 days respectively. If all the three work on the job simultaneously, how long will it take to complete the job?

27. A number N when divided by 5 leaves the remainder 1, and when divided by 6 leaves the remainder 5. Find the smallest positive N.

28. A number when divided by 145 leaves the remainder 58. What is the remainder, if it is divided by 29?

29. What will be the remainder when $86 \times 293 \times 4919$ is divided by 17?

30. Find the remainder when $7^{13} + 1$ is divided by 6.

31. What is the remainder when 10^{200} is divided by 8?

32. Find the remainder if 30^{40} is divided by 17.

33. On dividing a certain number by 5, 7 and 8 successively, the remainders are 2, 3 and 4 respectively. What would be the remainders if the order of the division is reversed?

34. Find the number of factors of a number

$N = 2^3 \times 3^2 \times 5^3$.

35. How many factors are there of the expression $6^{10} \times 7^{17} \times 11^{27}$? How many of them are prime?

36. The factorised form of a number $N = 2^4 \times 3^2 \times 5^4$. Find the number of unique sets of a and b such that ab = N.

37. If PQRS = (PQR) ×(QS), where P, Q, R, S stand for distinct decimal digits, then find the value of Q.

38. What is the value of $\sqrt{12345678987654321}$?

39. Simplify: $\sqrt{5+\sqrt{5}+\sqrt{3+\sqrt{5}+\sqrt{14+\sqrt{180}}}}$

40. x, y are two different digits. If the sum of the two-digit numbers formed by using both the digits is a perfect square, then find the value of x + y.

Level 3

41. The sum of the digits of a two-digit number is 8. If the digits are reversed, then the number is decreased by 54. Find the number.

42. How many numbers are there between 2000 and 3000, which are divisible by 3 or 4?

43. What is the value of (X + Y) if 789432X64Y is divisible by both 8 as well as 9? X and Y being single digit numbers.

44. The sum of 2 numbers is 144. Their HCF is 24. Find the numbers.

45. What is the remainder in $\frac{2^{643}}{96}$?

46. Prove that $(244)^{1500} - 1$ is divisible by 1001. (1001 can be written in factorised form as $7 \times 11 \times 13$.)

47. Find the last digit of the number $7^{11^{22^{33}}}$.

48. Find the number of zeroes at the end of the product

a. of first 100 multiples of 10.

b. $5 \times 10 \times 15 \times 20 \times 25 \times 30 \times 35 \times 40 \times 45$.

49. Which is the greatest power of

a. 6 that divides the number 34! exactly?

b. 12 that divides the number 34! exactly?

50. What is the total number of positive integer solutions that satisfy the equation $4x + 3y = 120$?

51. A man is climbing a staircase in either steps of 5 or 3. If the flight of the stairs has 100 steps in all, then how many times did he take steps of 5?

52. A boy multiplied 423 by a certain number and obtained 65589 as his answer. If only the 5s are wrong, what is the correct product?

53. The letters in the alphanumatic addition are all different. Find the numbers indicated in codes. All the letters are digits from 0 to 9.

$$\begin{array}{r} \text{F O R T Y} \\ (+)\ \text{T E N} \\ (+)\ \text{T E N} \\ \hline \text{S I X T Y} \\ \hline \end{array}$$

54. You choose any 17 numbers from the set of all successive natural numbers A = {1, 2, 3, ..., 32}. If you find the sum of any pair of numbers from these 17 numbers, there would always be one pair that adds up to P. Find P.

55. Starting with 1, if all positive integers are written one after the another, what is the 10,000th digit that will be written?

56. There are 5 numbers: 20, 22, 23, 25 and 27. How many distinctly different sums would you get taking 2 at a time?

57. Convert:

a. $(13.421)_{10} = (\ )_8$ b. $(13.421)_8 = (\ )_{10}$

c. $(1100011010)_2 = (\ )_8$ d. $(3221301)_4 = (\ )_{16}$

58. Meera offers her prayers to Lord Krishna in a temple everyday. She offers $\frac{1}{4}$ of the number of flowers she has in the first temple, $\frac{1}{4}$ of the remaining in the second temple, $\frac{1}{4}$ of the rest in the third temple and $\frac{1}{2}$ of the rest in the fourth temple. One day, she was left with flowers that were less than 60 and more than 20.

a. What is the minimum number of flowers she carried to the first temple?

b. What is the maximum number of flowers that she carried to the first temple?

Practice Exercise 1 - Level 1

1. The LCM of two numbers is 5200 and their HCF is 40. If one of the numbers is 520, then the other number is

a. 240 b. 560 c. 400 d. 320 e. 520

2. The value of factorial zero (i.e. 0!) is

a. 0 b. $\frac{1}{2}$ c. 8 d. 1 e. Not definied

3. Zero is counted as

a. Whole number b. Positive Integer c. Natural number

d. Both a positive integer and a negative integer e. Both (a) and (d)

4. Which of the following options contains all irrational numbers only?

a. $\sqrt{4}, 8$ b. $\frac{3}{2}, \sqrt{4}, -7, -\frac{9}{8}$ c. $\pi, (\pi-1), \left(3+\sqrt{2}\right), -\sqrt{3}$

d. $\frac{3}{2}, \sqrt{4}, -\frac{7}{\sqrt{2}}, -\frac{9}{8}$ e. All of these

5. Find the digit in the units place in the product 254 × 361 × 159 × 18.

a. 1 b. 6 c. 4 d. 8 e. 2

6. The value of $\sqrt[3]{32} \times \sqrt[3]{250}$ is

a. 50 b. 0.031 c. 20 d. 0.001 e. 2

7. Find the value of 3125 ÷ (25 × 25) - $\sqrt[3]{125}$

a. 5 b. 0 c. 25 d. 100 e. 20

8. Solve: 18.18 ÷ 9 + 2.7 × 3

a. 101.2 b. 27.32 c. 10.12 d. 10.13 e. 101.3

9. Solve: 8127 - 5422 + 1614 - 808

a. 3580 b. 3058 c. 3503 d. 3511 e. 3501

10. The unit's digit of the product (247 × 318 × 577 × 313) is

a. 2 b. 1 c. 4 d. 9 e. 6

11. What least value must be assigned to * so that the number 451*603 becomes exactly divisible by 9?

a. 2 b. 7 c. 8 d. 5 e. 1

12. What least value must be assigned to * so that the number 63576 * 2 is divisible by 8?

a. 1 b. 2 c. 3 d. 4 e. 7

13. Simplify 8756 × 99999

a. 815491244 b. 796491244 c. 875991244 d. 875591244 e. 875951244

14. Evaluate: 1399 × 1399

a. 1687401 b. 1901541 c. 1943211 d. 1957201 e. 1975102

15. Find the value of $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104$.

a. 250001 b. 251001 c. 260101 d. 261001 e. 270101

16. Two rational numbers lying between $\frac{4}{5}$ and $\frac{6}{7}$ are

a. $\frac{65}{84}, \frac{5}{6}$ b. $\frac{29}{35}, \frac{5}{6}$ c. $\frac{29}{35}, \frac{62}{70}$ d. $\frac{28}{34}, \frac{35}{39}$ e. $\frac{31}{35}, \frac{7}{8}$

17. If $x = (6 - \sqrt{35})$, then the reciprocal of x is

a. $\frac{1}{(6+\sqrt{35})}$ b. $(6+\sqrt{35})$ c. $\sqrt{35}$ d. 12 e. $2\sqrt{35}$

18. Which of the following is exactly divisible by 99?

a. 114345 b. 135792 c. 3572404 d. 913464 e. 143098

19. The difference between two numbers is 1365. When larger number is divided by the smaller one, the quotient is 6 and the remainder is 15. The smaller number is

a. 270 b. 360 c. 240 d. 295 e. 380

20. There are four prime numbers written in ascending order of magnitude. The product of first three is 385 and that of last three is 1001. Find the first number.

a. 5 b. 7 c. 11 d. 17 e. 19

21. The number nearest to 99547 which is exactly divisible by 687 is

a. 100166 b. 99615 c. 99579 d. 98928 e. 100302

22. Which is the largest five-digit number that divisible by 99?

a. 99999 b. 99981 c. 99909 d. 99990 e. 99792

23. Which is the smallest six-digit number that is divisible by 111?

a. 111111 b. 110011 c. 100011 d. 100001 e. 100455

24. When n is divided by 4, the remainder is 3. What is the remainder when 2n is divided by 4?

a. 1 b. 6 c. 3 d. 2 e. 0

25. Find the value of $\left(\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}\right)$.

a. $\frac{1}{2}$ b. $\frac{2}{3}$ c. $\frac{1}{2^n}$ d. $\frac{3}{2}$ e. $\frac{1}{2^{n+1}}$

26. Let D be a rational number of the form D = 0. abcd abcd abcd ... , where digits a, b, c and d are integers lying between 0 and 9. At most three of these digits are zero. By what minimum number D be multiplied so that the result is a natural number?

a. 999 b. 9990 c. 9999 d. 49995 e. None of these

27. $N = 144^3 + 22^3 - 166^3$, then N is necessarily divisible by

a. both 11 and 13 b. both 12 and 83 c. both 7 and 19

d. both 13 and 83 e. None of these

28. Dividing by $\frac{3}{8}$ and then multiplying by $\frac{5}{6}$ is equivalent to dividing by what number?

a. $\frac{5}{16}$ b. $\frac{16}{40}$ c. $\frac{9}{20}$ d. $\frac{40}{18}$ e. $\frac{9}{10}$

29. The LCM of two numbers is 72 and their HCF is 12. If one of the numbers is 24, what is the other number?

a. 38 b. 26 c. 36 d. 42 e. 27

Practice Exercise 2 - Level 1

1. P = 441 × 484 × 529 × 576 × 625. The total number of factors of P is

a. 607 b. 5706 c. 1024 d. 6075 e. 2025

2. There are four numbers in a sequence. The average of first three is 6, the average of the last three is 7, and the last number is 3 more than the first. The average of the second and the third numbers is

a. 6 b. 6.5 c. 5.75 d. 7 e. Cannot be determined

3. A number is divided in a way such that the divisor is 12 times the quotient. If the remainder is 48, then find what could be dividend, given that the divisor is 5 times the remainder.

a. 4803 b. 3684 c. 3648 d. 4848 e. 3848

4. 2! + 4! + 6! + 8! + 10! + 100! when divided by 5, would leave remainder

a. 0 b. 1 c. 2 d. 3 e. 4

5. The average of three consecutive prime numbers is $\frac{223}{3}$. What is the difference between the greatest and the smallest number that can be a part of such a set?

a. 8 b. 14 c. 16 d. 10 e. None of these

6. What is the sum of the greatest and the least fraction in the set of following fractions?

$\frac{2}{3}, \frac{13}{21}, \frac{11}{18}, \frac{8}{13}$

a. $1\frac{29}{126}$ b. $1\frac{5}{18}$ c. $1\frac{11}{39}$ d. $1\frac{64}{273}$ e. $1\frac{6}{21}$

7. The sum of the digits of a two-digit number is $\frac{1}{11}$ of the sum of the number and the number obtained by interchanging its digits. What is the difference between the digits of the number?

a. 3 b. 0 c. 1 d. 2 e. Data is insufficient

8. The average of four consecutive even numbers is 27. The largest of these numbers is

a. 24 b. 30 c. 26 d. 28 e. 32

9. Convert 1153 from base 10 to base 15.

a. 51D b. 61E c. 51C d. 61C e. None of these

10. If $x = 17^4$ and $y = 14 \times 16 \times 18 \times 20$, then

a. $x - y > 10000$ b. $y - x > 100$ c. $x - y > 1000$

d. $y - x > 10000$ e. None of these

11. What is the last digit of the number 23457^{194321}?

a. 9 b. 1 c. 3 d. 7 e. 4

12. In how many ways can a number 6084 be written as a product of two different factors?

a. 27 b. 26 c. 13 d. 14 e. None of these

13. A number system has 100 as base. How many digits do we need to write 100 in that system?

a. 1 b. 2 c. 4 d. 100 e. None of these

14. What will be the remainder when $25^{625} + 26$ is divided by 24?

a. 1 b. 2 c. 3 d. 0 e. None of these

15. If $N = 2^3 \times 3^4$, $M = 2^2 \times 3^5$, then find the number of factors of N that are common with the factors of M.

a. 8 b. 15 c. 18 d. 24 e. 20

16. $N = (11111111)^2$. What is the sum of the digits of N?

a. 72 b. 62 c. 64 d. 68 e. None of these

17. What is the value of $\dfrac{\left(1+\frac{1}{3}\right)^4 + \left(1+\frac{1}{3}\right)^{-4} + 1}{\left(1+\frac{1}{3}\right)^2 + \left(1+\frac{1}{3}\right)^{-2} + 1}$?

a. $\frac{193}{144}$ b. $\frac{293}{144}$ c. $\frac{191}{144}$ d. $\frac{291}{144}$ e. None of these

18. Find the value of N, where

$$N = \left(\frac{2\times 8 + 8\times 32 + 18\times 72 \ldots \text{upto } n \text{ terms}}{1+16+81+\ldots \text{up to } n \text{ terms}}\right)^{\frac{1}{4}}$$

a. 2n b. 2^n c. 2 d. 4 e. None of these

19. The average of 2, 7, 6 and x is 5, and average of 18, 1, 6, x and y is 10. Find the value of y.

a. 10 b. 15 c. 20 d. 30 e. 25

20. P + Q + R + S is odd. At least how many of these (is / are) odd?

a. 0 b. 1 c. 2 d. 3 e. Cannot be determined

21. Find the value of $(28+10\sqrt{3})^{\frac{1}{2}}-(7-4\sqrt{3})^{-\frac{1}{2}}$.

a. 3 b. 5 c. 7 d. 14 e. 9

22. What will be the remainder when 13^{36} is divided by 2196?

a. 0 b. 1 c. 12 d. 2195 e. None of these

23. Which of the following statements is not correct about $19^n + 1$?

a. It is never divisible by 18. b. When n is odd it is divisible by 20.

c. When n is even it is not divisible by 20. d. When n is odd it is divisible by 18.

e. None of these

24. How many numbers are there between 200 and 400 which are divisible by 11 and 3 but not by 2?

a. 5 b. 6 c. 4 d. 3 e. None of these

25. Suppose P is a prime number greater than 3, then P can always be written in the form

a. 6k + 1, where k is a natural number

b. 6k - 1, where k is a natural number

c. 13k + 1

d. 13k -1

e. Nothing can be said with certainty about prime numbers

26. A box contains 4 small boxes. Each of the 4 boxes contains 3 smaller boxes, in each of which there are 2 boxes. How many boxes are there altogether?

a. 24 b. 40 c. 41 d. 42 e. 28

27. If m < n, where m and n are real numbers, then which of the following is definitely true?

a. $m^2 < n^2$ b. $m^2 > n^2$ c. $m < n^2$ d. $m^3 < n^3$ e. m = n

28. Ravi and Gopal are playing mathematical puzzles. Ravi asks Gopal, "Which whole numbers, greater than one, can divide all of the nine three-digit numbers 111, 222, 333, 444, 555, 666, 777, 888 and 999?" Gopal immediately gave the desired answer. It was

a. 3, 37 and 119 b. 3, 37 and 111 c. 9, 37 and 111

d. 9, 36 and 1124 e. 3, 9, 37

29. Which of the following statements is false?

a. The product of three consecutive even numbers must be divisible by 48.

b. The numbers $(100)_2$, $(100)_3$, $(100)_4$, $(100)_5$ ... so on, when converted to decimal system are all in an arithmetic progression.

c. The factorial of any natural number greater than 1 cannot be a perfect square.

d. x% of y% of z is same as z% of x% of y.

e. b and c

30. The product of n consecutive positive integers is always divisible by

a. $n^2 - 1$ b. $(n + 1)!$ c. $2n + 1$ d. $n^2 + 1$ e. $n!$

31. When a two-digit number is divided by the sum of its digits, the quotient is 4. If the digits are reversed, the new number is 6 less than twice the original. The number is

a. 12 b. 21 c. 42 d. 24 e. 27

32. Which of the following statements is true?

I. $a^{2n} - b^{2n}$ is divisible by a + b but not by a - b when n is an integer.

II. $a^{n+1} + b^{n+1}$ is always divisible by a + b, n being an integer.

III. 2485035 is the perfect square of an integer.

IV. If A is the AM of 2 positive numbers and M is the GM of the same 2 numbers, then $A \geq M$.

a. I b. II c. III d. IV e. I & II

33. Let `p' be a prime number greater than 3. Find the remainder when $p^2 + 17$ is divided by 12.

a. 6 b. 1 c. 0 d. 8 e. 7

34. How many zeroes will be there at the end of the number N, if N = 100! + 200!?

a. 73 b. 49 c. 20 d. 48 e. 24

35. Between 100 and 200, how many numbers are there in which one digit is the average of the other two?

a. 11 b. 12 c. 10 d. 8 e. 9

Practice Exercise 3 - Level 2

1. The sum of the squares of first ten natural numbers is

a. 281 b. 385 c. 402 d. 502 e. 770

2. The largest number among the following is

a. $(2 + 2 \times 2)^3$ b. $[(2+2)^3]^{1/2}$ c. 2^5 d. $(2 \times 2 - 2)^7$ e. $(2 \times 7 - 2)^2$

3. The smallest number among the following is

a. $(7)^3$ b. $(8.5)^3$ c. $(4)^4$ d. $(6^5)^{3/5}$ e. $(3^{4/5})^5$

4. Find the sum of the first 50 even numbers.

a. 1275 b. 2650 c. 5100 d. 2550 e. 1550

5. Evaluate: $11^2 + 11^4 \div 11^3 - 11 + (0.5)\ 11^2$

a. 302.5 b. 181.5 c. 484.0 d. 121 e. 162.5

6. Which one of the following is incorrect?

a. Square root of 5184 is 72. b. Square root of 15625 is 125. c. Square root of 1444 is 38.

d. Square root of 1296 is 34. e. Square root of 1369 is 37

7. The sum of first 45 natural numbers is

a. 2070 b. 975 c. 1280 d. 1035 e. 1575

8. The greatest fraction among $\frac{2}{5}, \frac{3}{5}, \frac{1}{5}, \frac{7}{15}$ and $\frac{4}{5}$ is

a. $\frac{4}{5}$ b. $\frac{3}{5}$ c. $\frac{2}{5}$ d. $\frac{7}{15}$ e. $\frac{1}{5}$

9. The lowest four-digit number which is exactly divisible by 2, 3, 4, 5, 6 and 7 is

a. 1400 b. 1300 c. 1250 d. 1260 e. 1464

10. The sum of the two numbers is twice their difference. If their product is 27, then the numbers are

a. 5, 15 b. 10, 30 c. 9, 6 d. 9, 3 e. 5, 7

11. The largest fraction among the following is

a. $\frac{17}{21}$ b. $\frac{11}{14}$ c. $\frac{12}{15}$ d. $\frac{5}{6}$ e. $\frac{3}{5}$

12. If the product of three consecutive integers is 720, then their sum is

a. 54 b. 45 c. 18 d. 27 e. 37

13. How many numbers between 200 and 600 are divisible by 4, 5 and 6?

a. 5 b. 6 c. 7 d. 8 e. 4

14. The number $(10^n - 1)$ is divisible by 11 for

a. even values of n b. odd values of n c. all values of n

d. n = multiples of 11 e. No general rule exists

15. Solve: $3+\cfrac{3}{3+\cfrac{1}{3+\cfrac{1}{3}}}$

a. 1 b. 3 c. $\frac{43}{11}$ d. $\frac{63}{19}$ e. $\frac{41}{10}$

16. How many numbers are there between 500 and 600 in which 9 occurs only once?

a. 19 b. 20 c. 21 d. 18 e. 17

17. How many zeros are there at the end of the product $33 \times 175 \times 180 \times 12 \times 44 \times 80 \times 66$?

a. 2 b. 4 c. 5 d. 6 e. 3

18. $N = 56^{56} + 56$. What would be the remainder when N is divided by 57?

a. 0 b. 56 c. 55 d. 1 e. None of these

19. The largest number that always divides the product of 3 consecutive multiples of 2 is

a. 8 b. 16 c. 24 d. 48 e. 36

20. The sum of two natural numbers is 85 and their LCM is 102. Find the numbers.

a. 51 and 34 b. 50 and 35 c. 60 and 25 d. 45 and 40 e. 17 and 68

21. By what smallest number, 21600 must be multiplied or divided in order to make it a perfect square?

a. 6 b. 5 c. 8 d. 10 e. 12

22. If we write down all the natural numbers from 259 to 492 side by side we shall get a very large natural number 259260261262 490491492. How many 8's will be used to write this large natural number?

a. 52 b. 53 c. 32 d. 43 e. None of these

23. $n^3 + 2n$ for any natural number n is always a multiple of

a. 3 b. 4 c. 5 d. 6 e. 8

24. A number when divided by 238 leaves a remainder 79. What will be the remainder when that number is divided by 17?

a. 8 b. 9 c. 10 d. 11 e. 12

25. What is the remainder when 17^{23} is divided by 16?

a. 0 b. 1 c. 2 d. 3 e. 15

26. $9^6 + 1$ when divided by 8, would leave a remainder

a. 0 b. 1 c. 2 d. 3 e. 4

27. $N = 2 \times 4 \times 6 \times 8 \times 10 \times \Lambda\ 100$. How many zeros are there at the end of N?

a. 24 b. 13 c. 12 d. 15 e. None of these

28. It is given that $2^{32} + 1$ is exactly divisible by a certain number. Which one of the following is also divisible by the same number?

a. $2^{96} + 1$ b. $2^{16} - 1$ c. $2^{16} + 1$ d. 7×2^{33} e. $2^{64} + 1$

29. $4^{61} + 4^{62} + 4^{63} + 4^{64} + 4^{65}$ is divisible by

a. 3 b. 5 c. 11 d. 17 e. 13

30. What is the smallest perfect square that is divisible by 8, 9 and 10?

a. 4000 b. 6400 c. 3600 d. 14641 e. 900

Practice Exercise 4 - Level 2

1. What is the least number which must be subtracted from 2024, so that the resultant number when divided by 7, 10 and 15 will leave in each case the same remainder 3?

a. 224 b. 131 c. 24 d. 225 e. 141

2. The product of two numbers is 16200. If their LCM is 216, then find their HCF.

a. 75 b. 70 c. 80 d. 60 e. Data inconsistent

3. 4a56 is a four-digit number divisible by 33. What is the value of a?

a. 3 b. 4 c. 5 d. 6 e. 2

4. A two-digit number is such that cube of its 24th part is the same as the number obtained by interchanging the digit of the number. What is the number?

a. 24 b. 72 c. 48 d. 96 e. None of these

5. A number formed by writing any digit 6 times (say as 444444 or 999999) is always divisible by

a. 1001 b. 7 c. 13 d. 11 e. All of these

6. The positive integer which is nearest to 1000 and divisible by 2, 3, 4, 5 and 6 is

a. 1020 b. 1040 c. 960 d. 1030 e. 1050

7. The difference of the greatest and the least five-digit numbers that can be formed by using the digits 0, 2, 3, 4 and 5 is (Repetition is not allowed)

a. 25694 b. 33975 c. 30870 d. 36246 e. 33946

8. An amount of ₹417 is divided among A, B, C, and D in such a way that A gets ₹13 more than B, B gets ₹9 more than C, and C gets ₹6 more than D. Find A's share.

a. ₹121 b. ₹116 c. ₹120 d. ₹124 e. ₹92

9. Convert 1234 from base 6 to base 10.

a. 3100 b. 3010 c. 301 d. 310 e. None of these

10. N = 148 × 293 × 581 × 874. What will be the remainder when N is divided by 29?

a. 7 b. 6 c. 5 d. 9 e. None of these

11. What is the highest power of 31 in 1000!?

a. 31 b. 32 c. 33 d. 35 e. None of these

12. For any natural number n, $n^4 + n^2 + 1$ is always

a. odd b. even c. odd multiple of 3

d. even multiple of 3 e. Cannot be determined

13. Which among the following is greatest: $\sqrt{5}, \sqrt[3]{11}, \sqrt[6]{123}, \sqrt[4]{1331}$?

a. $\sqrt{5}$ b. $\sqrt[3]{11}$ c. $\sqrt[6]{123}$ d. $\sqrt[4]{36}$ e. $\sqrt[12]{27}$

14. Convert 1556 from base 10 to base 16.

a. 641 b. 64A c. 6A4 d. 614 e. 6B4

15. Convert 413 from base 7 to base 8.

a. 613 b. 362 c. 316 d. 216 e. None of these

16. Which among the following is greatest: $\sqrt{7}+\sqrt{3}, \sqrt{5}+\sqrt{5}, \sqrt{6}+2, \sqrt{2}+\sqrt{8}$?

a. $\sqrt{7}+\sqrt{3}$ b. $\sqrt{5}+\sqrt{5}$ c. $\sqrt{6}+2$ d. $\sqrt{2}+\sqrt{8}$ e. All are equal

17. When 75% of a two-digit number is added to it, the digits of the number are reversed. Find the ratio of the unit's digit to the ten's digit in the original number.

a. 1 : 2 b. 1 : 3 c. 1 : 4 d. 2 : 1 e. 2 : 3

18. If N = $N = \frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+\frac{1}{30}+\cdots+\frac{1}{156}$, then the value of N is

a. $\frac{12}{13}$ b. $\frac{13}{12}$ c. $\frac{1}{13}$ d. $\frac{1}{12}$ e. None of these

19. How many two-digit natural numbers are there so that ten's digit is never less than the unit's digit?

a. 44 b. 55 c. 54 d. 49 e. None of these

20. What is the last digit of the number $3^{5^{7^{9}}}+1$?

a. 1 b. 7 c. 4 d. 3 e. None of these

21. What will be the last digit of $2^{3^{4^5}} - 2^{3^{5^4}}$?

a. 0 b. 2 c. 4 d. 6 e. None of these

22. Simplify: $\frac{1}{\sqrt{1}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{7}} + \ldots$up to 50 teams.

a. $\frac{\sqrt{101}-1}{2}$ b. $\sqrt{109}-\sqrt{99}$ c. $1-\frac{1}{\sqrt{101}}$ d. $\frac{1}{\sqrt{99}}-\frac{1}{\sqrt{101}}$ e. None of these

23. If 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 100, then atleast in how many places you need to change `+' with `×' to make the equality hold good?

a. 2 b. 4 c. 3 d. 1 e. Cannot be determined

24. Find the value of $\sqrt{\frac{(12.12)^2-(8.12)^2}{(0.25)^2+(0.25)(19.99)}} + \frac{\left[(8^{\frac{-3}{4}})^{\frac{3}{2}}\right]^{\frac{8}{15}} \times 16^{\frac{3}{4}}}{\sqrt[3]{\left[\left((128)^{-5}\right)^{\frac{3}{7}}\right]^{\frac{-1}{5}}}}$

(You have to take positive square roots only.)

a. 0 b. 1 c. $\frac{9}{2}$ d. $\frac{3}{2}$ e. None of these

25. For a set of 5 unique integers a, b, c, d, e (in the ascending order), which of the following is always true?

a. $d \times e > c \times b$ b. $\frac{e}{a} > \frac{d}{b}$ c. c > Average of (a, b, c, d, e)

d. $c + d > a + b$ e. None of these

26. Mrs Doubtfire wrote all the numbers from 100 to 200. Then she started counting the number of one's that has been used while writing all these numbers. What is the number that she got?

a. 119 b. 120 c. 121 d. 111 e. None of these

Practice Exercise 5 - Level 2

Direction for questions 1 and 2: Answer the questions based on the following information.

59292564P61Q is divisible by 99, but not by 22. Q is greater than P, then

1. P is

a. 5 b. 2 c. 0 d. 4 e. None of these

2. Q is

a. 5 b. 3 c. 7 d. 9 e. None of these

3. M is the smallest natural number in such a way that when multiplied by 7 it gives a number made up of 6's only. Sum of the digits of M is N. The last digit of N^{36} is

a. 6 b. 7 c. 1 d. 9 e. None of these

4. What is the remainder when 7^{187} is divided by 800?

a. 143 b. 343 c. 243 d. 743 e. 127

5. The number \`a' is exactly divisible by 5. The remainder after the division of the number \`b' by 5 is equal to 1 and the remainder after the division of the number \`c' by 5 is equal to 2. What is the remainder when the number (2a + 3b - 4c) is divided by 5 ?

a. 0 b. 1 c. 2 d. 3 e. 4

6. Let P and Q represent non-zero digits. Let PP represents a two-digit number with identical digits. If Q times the cube of PP is a four-digit number whose ten's digit is 1, then find the numeric value of Q.

a. 8 b. 7 c. 6 d. 10 e. 9

7. When a particular positive number is divided by 5, the remainder is 2. If the same number is divided by 6, the remainder is 1. If the difference between the quotients of division is 3, then find the number.

a. 37 b. 97 c. 67 d. 127 e. 87

8. Seven bells ring at intervals of 2, 3, 4, 6, 8, 9 and 12 min respectively. They started ringing simultaneously at 6 a.m. How many more times would all seven bells ring simultaneously till 5 p.m. on the same day?

a. 8 b. 9 c. 10 d. 12 e. 7

9. How many times does the digit 6 appear when you count from 11 to 100?

a. 9 b. 18 c. 17 d. 20 e. 19

Directions for questions 10 and 11: Answer the questions based on the following information.

The first 25 natural numbers are randomly arranged in the form of a square [(5 × 5) grid].

10. If from each row the smallest number is selected and the smallest of these numbers is called A, then A =

a. 5 b. 1 c. 4 d. 6 e. Cannot be determined

11. If B is the largest of the largest numbers from each column, then B - A = (Use the data from previous question)

a. 0 b. 1 c. 24 d. 2 e. Cannot be determined

12. Which of these is true?

I. $3\sqrt{3}$ is not a rational number.

II. If a is rational and n is an integer greater than 1, then a^n is rational.

III. 7 is not the cube of a rational number.

a. I and II b. I and III c. II and III d. II e. All three

13. A teacher was trying to form groups of students such that every group has equal number of students and that number should be a prime number. She tried for first 5 prime numbers, but on each occasion exactly one student was left behind. How many different possible solutions are there for the total number of students, if the total number of students is in 4 digits.

a. 0 b. 2 c. 3 d. 4 e. 5

14. Juhi and Bhagyashree were playing a simple mathematical game. Juhi wrote a two-digit number and asked Bhagyashree to guess it. Juhi also indicated that the number is exactly thrice the product of its digits. What was the number that Juhi wrote?

a. 36 b. 24 c. 12 d. 48 e. 30

15. $(AA)^2$ = DCBA, where A, B, C and D are distinct digits with B being odd. Find the value of D.

a. 1 b. 6 c. 4 d. 1 or 4 e. 4 or 6

16. $V = 798630 \times 10^{24}$ and $A = 18 \times 10^{24}$. What is the remainder when V is divided by A?

a. 3×10^{24} b. 6×10^{24} c. 12×10^{24} d. 6 e. 8×10^{24}

17. Four bells begin to toll together. After that, they toll at the intervals of 6 s, 7 s, 8 s and 9 s respectively. The maximum number of times they will toll together in any interval of 2 hrs is

a. 14 times b. 15 times c. 13 times d. 11 times e. 9 times

18. If x, y and z are distinct non-zero whole numbers such that $y > x$ and $xy = z$, then which of the following can be true?

a. $y > z$ b. $y = z$ c. $z > x^3$ d. $x > z$ e. $x = z$

19. If $a_1, a_2, a_3, a_4, \ldots, a_n$ are the terms of a series defined by $a_n = a_{n-1} + a_{n-2}$, (here a_n is a positive real number). What can be the value of a_1 if $a_3^2 - a_2^2 = 57$?

a. 19 b. 8 c. 3 d. 0 e. 11

20. Which of the following statements is true?

I. Perfect squares always end in one of {1, 4, 5, 6, 8, 9}.

II. Number of digits in the square of a natural number having n digits is always equal to either 2n or 2n + 1.

III. If a perfect square ends in 9, the second last digit is never even.

a. I b. II c. III d. I and II e. None of these

21. A number `n' is decreased by 4 and the result is multiplied by 4, the operation being repeated four times. The answer after the fourth operation is 4. What is the initial value of `n'?

a. $\frac{341}{64}$ b. 1 c. $\frac{341}{256}$ d. $\frac{89}{64}$ e. $\frac{89}{256}$

22. Find the largest number, smaller than the smallest four-digit number, which when divided by 4, 5, 6 and 7 leaves a remainder 2 in each case.

a. 422 b. 656 c. 12723 d. 748 e. 842

23. The numerator and the denominator of a positive fraction are in the ratio 1 : 5. A new fraction is formed by subtracting 2 from the numerator and adding 5 to the denominator. The difference between the original fraction and the new fraction is $\frac{1}{10}$. What is the numerator of the original fraction?

a. 4 b. 5 c. 25 d. 20 e. 15

24. What is the tens digit of $(51)^{51}$?

a. 0 b. 1 c. 5 d. 4 e. 3

Directions for questions 25 and 26: Answer the questions on the basis of the following information.

25. Among the following find a whole number such that when one of its digit is erased, the resulting number is equal to one-ninth of the original number. The resulting number is also a multiple of 9.

a. 90 b. 83438 c. 10125 d. 70847 e. 62423

26. In the above question, the digit erased must be

a. 9 b. 0 c. 7 d. Any of these e. None of these

27. How many five-digit numbers can be formed using only odd digits such that the number is divisible by 125?

a. 13 b. 100 c. 125 d. 25 e. 50

28. What is the smallest number of ducks that could swim in this formation:

Two ducks in front of a duck, two ducks behind a duck and a duck between two ducks.

a. 4 b. 5 c. 3 d. 6 e. 7

29. A 3 digit number is such that its hundredth digit is equal to the product of the other 2 digits which are prime. Also, the difference between the number & its reverse is 297. Then tens digit of the number is

a. 2 b. 3 c. 7 d. 5 e. 6

30. Ten bags contain 10 coins each. All the coins look alike but the coins in one bag weigh 1 g less than that in other bags. What is the least number of weighings using a spring balance needed to identify the bag containing 9 g coins?

a. 1 b. 2 c. 3 d. 4 e. 5

31. What is the highest power of 5 that divides 90 × 80 × 70 × 60 × 50 × 40 × 30 × 20 × 10?

a. 15 b. 12 c. 9 d. 14 e. 10

32. Shekhar suddenly got angry and started tearing pages from a new copy of his 500-page textbook. On calming down, he realized that the first page he had torn was numbered 123. Further, he had torn successive pages till the page number formed using the same digits as 123. How many pages had Shekhar torn away? (Pages are numbered on both sides)

a. 10 or 199 b. 10 or 190 c. 199 or 190 d. 99 or 10 e. Cannot be determined

33. A certain number when divided by 222 leaves a remainder 35; another number when divided by 407 leaves a remainder 47. What is the remainder when the sum of these 2 numbers is divided by 37?

a. 47 b. 9 c. 17 d. 12 e. 8

34. Between 100 and 300, how many numbers begin or end with 2?

a. 120 b. 20 c. 110 d. 119 e. 88

Practice Exercise 6 - Level 3

1. The least number which on division by 35 leaves the remainder 25 and on division by 45 leaves the remainder 35 and on division by 55 leaves the remainder 45 is

a. 2515 b. 3455 c. 2875 d. 2785 e. 3655

2. A heap of coconuts is divided into groups of 2, 3 and 5, and each time one coconut is left out. The least number of coconuts in the heap is

a. 31 b. 41 c. 51 d. 61 e. 21

3. If $x = 5 - \sqrt{7}$, then find the value of $x + \frac{1}{x}$.

a. $\frac{95+17\sqrt{7}}{18}$ b. $\frac{95-17\sqrt{7}}{18}$ c. $\frac{85-17\sqrt{7}}{18}$ d. $\frac{85+17\sqrt{7}}{18}$ e. $\frac{90+17\sqrt{7}}{18}$

4. 243 has been divided into three parts such that half of the first part, one-third of the second part and one-fourth of the third part are equal. The largest part is

a. 108 b. 86 c. 92 d. 74 e. 96

5. If n is positive integer, then $(3^{4n} - 4^{3n})$ is always divisible by

a. 145 b. 17 c. 112 d. 7 e. 19

6. Six bells commence tolling together and toll at intervals of 3, 6, 9, 12, 15 and 18 s respectively. In 30 min, how many times do they toll together?

a. 4 b. 11 c. 10 d. 15 e. 9

7. When `n' is divided by 5 the remainder is 2. What is the remainder when n^2 is divided by 5?

a. 2 b. 1 c. 3 d. 4 e. 0

8. The expression $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....+\frac{1}{n(n+1)}$ for any natural number n, is

a. always less than 1 b. always greater than 2 c. always equal to 1

d. alway lies between 1 and 2 e. None of these

9. If $y=\sqrt{2}+1$, then value of $y+\frac{1}{y}$ is

a. $\sqrt{\frac{3}{2}}$ b. $\frac{\sqrt{3}}{2}$ c. $\frac{1}{\sqrt{2}}$ d. $2\sqrt{2}$ e. $\sqrt{3}$

10. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit place is 3 more than the digit in the ten's place, what is that number?

a. 25 b. 14 c. 36 d. 69 e. 47

11. If $32^{x-2}=\frac{64}{8^x}$, then find the value of x.

a. -2 b. 3 c. 2 d. -3 e. -4

12. What is the greatest positive power of 5 that divides 30! exactly?

a. 5 b. 6 c. 7 d. 8 e. 9

13. If the sum of two natural numbers is multiplied by each number separately, the products so obtained are 2418 and 3666. What is the difference between the numbers?

a. 16 b. 22 c. 26 d. 35 e. 27

14. How many even numbers from 20 to 2000 are not perfect squares?

a. 1941 b. 1940 c. 970 d. 1171 e. None of these

15. Product of two positive integers is 15210 and their HCF is 39. How many such pairs are possible?

a. 1 b. 2 c. 3 d. 4 e. None of these

16. If the last 2 digits of a four-digit number are interchanged, the new number obtained is greater than the original number by 54. What is the difference between the last two digits of the number?

a. 9 b. 12 c. 6 d. 3 e. Data Inadequate

17. LCM of two numbers x and y is 161. Find the value of (4x - 3y), given that y > x, x >1.

a. - 25 b. - 16 c. - 41 d. - 455 e. Cannot be determined

18. $\begin{array}{r} AA \\ +\underline{BB} \\ CDC \end{array}$ $\quad \begin{array}{l} A > 0 \\ B > 0 \end{array}$

All A, B, C are integers. Find the value of D.

a. 4 b. 3 c. 2 d. 1 e. 5

19. The sum of the digits of a two-digit number is 5. If we put the digits of the number in reverse order, the new number is 41 less than twice the original number. Find 40% of the number.

a. 11.8 b. 12.8 c. 20.8 d. 18.8 e. None of these

20. The HCF and LCM of two numbers are 13 and 455 respectively. If one of the numbers lies between 75 and 125, then that number is

a. 91 b. 78 c. 117 d. 104 e. 65

21. Three consecutive whole numbers are such that the square of the middle number is greater than the product of the other two by 1. Find the middle number.

a. 6 b. 18 c. 12 d. 31 e. All of these

22. $A = 1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots 100^{100}$. How many zeroes will be there at the end of A?

a. 1300 b. 1320 c. 1325 d. 1050 e. None of these

23. If x is a prime such that $(x^2 + 3)$ is also a prime, then x can have

a. two values b. one value c. more than 3 values

d. more than 2 values e. None of these

24. If f(x) = sum of all the digits of x, where x is a natural number, then what is the value of f(101) + f(102) + f(103) + ... + f(200)?

a. 1000 b. 901 c. 999 d. 1001 e. 1111

25. What is the highest power of 54 that divides 31! completely?

a. 2 b. 6 c. 4 d. 5 e. None of these

Practice Exercise 6 - Level 3

1. Convert 1101.011 from base 2 to base 10.

a. 13.357 b. 12.375 c. 13.375 d. 133.75 e. None of these

2. The value of $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\cdots+\frac{17}{5184}+\frac{19}{8100}$ is

a. 0.99 b. 0.98 c. 0.95 d. 0.90 e. None of these

3. If the number 3402 is converted from base 10 to base x, it becomes 12630. What is the value of x?

a. 6 b. 8 c. 9 d. 7 e. None of these

4. In a basket, there are some apples. Sanjay takes half of them but returns one of them. Preeti takes one-third of the remaining and returns two of them. Dharam takes one-fourth of the remaining and returns three of them. No apple is cut. The minimum number of apples left in the basket at the end is

a. 9 b. 28 c. 12 d. 15 e. Cannot be determined

5. abc is a three-digit natural number so that abc = a! + b! + c! . What is the value of $(b + c)^a$?

a. 1296 b. 3125 c. 19683 d. 9 e. None of these

6. abc is a three-digit whole number so that abc = $a^3 + b^3 + c^3$. [$300 < abc < 400$]

What is the value of a + b + c?

a. 10 b. 11 c. 12 d. 9 e. Cannot be determined

7. a and b are two distinct whole numbers less than 10.

N = a × b

N! = a! × b!

How many pairs of a and b are possible?

a. 8 b. 9 c. 10 d. 2 e. None of these

8. Two different numbers when divided by a certain divisor leave remainders 547 and 349 respectively. When the sum of those two numbers is divided by the same divisor, the remainder is 211. Find the divisor.

a. 896 b. 586 c. 685 d. 698 e. Cannot be determined

9. If G(x) = Total number of factors of x, where x is a perfect square, then G(x) is always

a. an even number b. an odd number c. a prime number

d. non-prime number e. Cannot be determined

10. Pawan writes all the numbers from 100 to 999. The number of zeroes that he uses is m, the number of 5's that he uses is n and the number of 8's that he uses is p. What is the value of n + p - m?

a. 280 b. 380 c. 180 d. 80 e. None of these

11. A is the smallest integer which when multiplied with 3 gives a number made of 5's only. Sum of the digits of A is B. Sum of the digits of B is C. What is the value of C^3?

a. 125 b. 64 c. 216 d. 27 e. None of these

12. P is an integer. $P > 883$. If $P - 7$ is a multiple of 11, then the largest number that will always divide $(P + 4)(P + 15)$ is

a. 11 b 121 c. 242 d. 22 e. None of these

13. If p^q is a perfect square as well as a perfect cube, where p and q are natural numbers, then q must be a multiple of

a. 2 b. 3 c. 6 d. 9 e. Cannot be determined

14. What is the highest power of 82 contained in $(83! - 82!)$?

a. 3 b. 2 c. 164 d. 1 e. None of these

15. What is the remainder when $1923^{1924^{1925}}$ is divided by 1924?

a. 1922 b. 1923 c. 1 d. 2 e. None of these

16. If a five-digit natural number is added to a number made by putting the digits of the original number in reverse order, the sum will always be divisible by

a. 11 b. 11111 c. 101 d. 1001 e. None of these

17. If an n-digit natural number is added to a number made by putting the digits of the original number in reverse order, the sum is always divisible by k where n is an even number, then k must be a multiple of

a. 22 b. 111 c. 11 d. 1001 e. None of these

18. ab and cd are two 2-digit natural numbers. $4b + a = 13k_1$ and $5d - c = 17k_2$ where k_1 and k_2 are natural numbers. The largest number that will always divide the product of ab and cd is

a. 13 b. 17 c. 221 d. 663 e. None of these

19. Let $A = \{a_1, a_2, a_3 .... a_n\}$ be a set of n distinct natural numbers and $n \geq 8$. Which of the following will always be a natural number?

a. $a_1 + a_2 + a_3 - a_7$ b. $(a_1 - a_n)^5$ c. $a_n - a_1$ d. $(a_1 - a_n)^7$ e. $(a_{n-1} - a_n)^2$

20. M and N are two distinct natural numbers. HCF and LCM of M and N are K and L respectively. A is also a natural number. Which of the following relations is not possible?

a. $K \times L = A$ b. $K \times A = L$ c. $L \times A = K$ d. $M \times A = N$ e. None of these

Directions for questions 21 and 22: Answer the questions based on the following information.

A defence code is defined by assigning the numbers 1 to 9 to the letters in the grid above such that by adding horizontally, or diagonally the sum of the numbers is the same, i.e.15, and also Y : S is 1 : 4 and S : W is 2 : 1 and W : Q is 2 : 3.

S	Z	Q
X	V	T
W	R	Y

21. Which letter has the highest numerical value?

a. X b. T c. V d. W e. None of these

22. If the code is 1159, then the message will be

a. ZZVR b. RRZV c. XXVT d. a or c e. b or c

Directions for questions 23 and 24: Answer the questions based on the following information.

$V_1, V_2, V_3, V_4, \ldots, V_{999}, V_{1000}$ are all natural numbers and $V_n + V_{n+1} = K$, $1 \leq n \leq 999$. n is also a natural number and K is a constant.

23. If $V_{987} = 987$, what is the value of V_{236}?

a. 236 b. 987 c. K - 987 d. K - 236 e. Cannot be determined

24. If $V_{100} = 100$, what is the value of $V_{10} + V_{11} + V_{12} + V_{13} + V_{14} - V_{15} - V_{16} - V_{17} - V_{18} - V_{19}$?

a. $V_{14} + 100 - K$ b. $V_{14} - 100 + K$ c. 200 - K

d. Both (a) and (c) e. Both (b) and (c)

Direction for questions 25 and 26: Answer the questions based on the following information.

`abcde' is a five-digit number (e is unit's place digit, etc., ...), which has the following characteristics.

I. $a \neq 0$; d is odd

II. $a - b + c = d - e$

III. $d! = a \times b \times c$

25. If $c = a + b$, then c can take values

a. 3 and 6 b. 3 c. 6 and 8 d. 3, 6 and 8 e. 3 and 8

26. Which of the following is/are true?

I. If e = 0, then d has to be equal to 1.

II. If d = 5, then $a + b + c > d + e$

a. I only b. II only c. Both I and II

d. Neither I nor II e. Indeter minate

27. A chain smoker had spent all the money he had. He had no money to buy his cigarettes. Hence, he resorted to join the stubs and to smoke them. He needed 4 stubs to make a single cigarette. If he got a pack of 10 cigarettes as a gift, then how many cigarettes could he smoke in all?

a. 11 b. 12 c. 9 d. 10 e. 13

28. Let X_n denotes the n^{th} element of the sequence {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ...}, where n is a positive integer. How many of the following statements are true?

I. $X_n < n$ for all n if $n > 2$.

II. $(X_n + 1)(X_n) \geq 2n$

III. If $X_{n+1} - X_n = 1$, then n can be written as the sum of first X_n natural numbers.

a. 1 b. 2 c. 3 d. 0 e. Cannot be determined

29. Consider the number n(n + 1)(2n + 1), where n is a positive integer. Then, which of the following is necessarily false?

a. n(n + 1)(2n + 1) is always even

b. n(n + 1)(2n + 1) is always divisible by 3

c. n(n + 1)(2n + 1) is always divisible by the sum of squares of first `n' integers

d. n(n + 1)(2n + 1) is never divisible by 237

e. n(n + 1)(2n + 1) may be a multiple of 5.

30. The number of pairs of positive integers (a, b), where a and b are prime numbers and $a^2 - 2b^2 = 1$ is

a. zero b. one c. two d. eight e. four

31. The number of positive integers not greater than 100, which are not divisible by 2, 3, or 5, is

a. 24 b. 18 c. 31 d. 28 e. 26

32. Rohan sells corn at Nehru Place traffic crossing. The light turns red for 2 minutes after every 3 minutes. Rohan gets the opportunity to sell only when the signal is red. He finds that it takes him 10 s to sell one pack of corn. What is the usual number of packs he sells in one day, considering that he starts at 8 a.m. and goes on till 7 p.m., stopping only for a short lunch from 2 p.m. to 2.30 p.m.?

a. 1,512 b. 1,260 c. 1,080 d. 1,124 e. None of these

Set Theory 1

Introduction

A few disconnected topics make their appearances in the management entrance examinations. Usually, a few questions only are asked from these topics. Among these, set theory is an important topic to study. It is also not very complicated and a lot of day-to-day applications of set theory are there.

The treatment of trigonometry given here is elementary as, usually, problems related to only height and distance are known to be asked in these examinations.

Learning Objectives

At the end of this chapter you would have learnt:

• Basic definitions in set theory

• Venn diagram representation of sets

Miscellaneous Topics

We will cover the following topics in this chapter.

1. Set theory

2. Trigonometry

3. Stocks and shares

Set Theory

Definition: A set is a well-defined collection of objects.

If A is a set and `a' is an element of this set, we say that 'a' belongs to A or $a \in A$. A set `A' which has a finite number of elements is called a finite set. The number of elements in a finite set is denoted by n(A).

The universal set is the set containing all the elements under consideration.

The empty set or null set (ϕ) is the set which has no element.

If a is an element of set A, then we write $a \in A$ (read a belongs to A or a is a member of set A). If a does not belong to A, then we write $a \notin A$. It is assumed that either $a \in A$ or $a \notin A$ and the two possibilities are mutually exclusive.

Some important definitions:

Subset: If every element of A is an element of B, then A is called a subset of B and we write $A \subseteq B$. Every set is a subset of itself and the empty set is a subset of every set. A subset A of set B is called a proper subset of B if $A \neq B$ and we write $A \subset B$. If a set has n elements, then number of its subsets = 2^n.

Superset: If A is a subset of B, then B is known as the superset of A and we write $B \supseteq A$.

Power set: Let A be a set. Then the collection or family of all subsets of A is called the power set of A and is denoted by P(A).

Example: Let A = {1, 2, 3}

Then the subsets of A are ϕ , {1}, {2}, {3}, {1, 2} {1, 3}, {2, 3} and {1, 2, 3}.

Hence P(A) = {ϕ, {1}, {2}, {1, 2} {1, 3}, {2, 3} {1, 2, 3}}

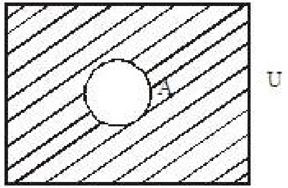
Universal set: A set that contains all the sets in a given context is called the universal set, i.e. It is the super set of all the sets under consideration e.g.

if A = {1, 2, 3} and B = {2, 4, 5, 6}, then a set of all natural numbers (N) can be taken as a universal set.

Introduction to Venn diagrams

The sets can be illustrated by means of Venn diagrams. A universal set U is represented by a rectangle and a subset by a circle within it.

Complement of a set



Let U and A be 2 sets such that $A \subseteq U$, then (U - A) is simply called the complement of A.

It is denoted by $\overline{A}$ or A'.

e.g. U is the set of natural numbers, the complement of odd numbers will be a set of even numbers.

The following letter sets are standard notations:

N: Set of natural numbers

Z: Set of integers

Q: Set of all rational numbers.

R: Set of all real numbers.

C: Set of all complex numbers.

Example 1:

U = {1, 2, 3, 4}, A = {3}. What is the complement of A?

Solution:

A' = {1, 2, 4}

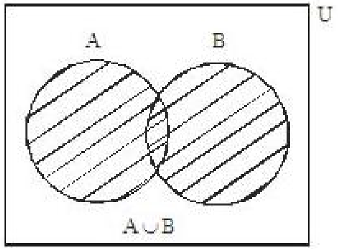
Remember the following

(i) $U' = \phi, \phi' = U$

(ii) $(A')' = A$

Union of Sets

If A and B are 2 sets, then the union of A and B, denoted by $A \cup B$, is the set of all elements which are **either** in A **or** in B or in both A and B.



Union of even and odd non-negative integers is a set of natural numbers.

Example 2:

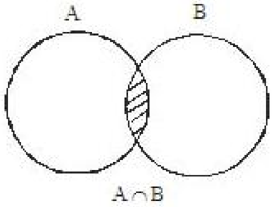
If A = {1, 2, 5, 7, 9} and B = {3, 8, 9, 2, 0}. Find $A \cup B$.

Solution:

$A \cup B$ = {0, 1, 2, 3, 5, 7, 8, 9}

Intersection of Sets

If A and B are sets, then the intersection of A and B, denoted by $A \cap B$, is the set of all elements which belong to **both** A **and** B.



e.g. Intersection of set of prime numbers and set of even numbers is a set having only one element, which is 2, i.e. = {2}

Consider and verify the following identities:

i. $A \cup A = A, \quad A \cap A = A$

ii. $A \cup \phi = A, \quad A \cap \phi = \phi$

iii. $A \cup U = U, \quad A \cap U = A$

iv. $A \cup B = B \cup A, \quad A \cap B = B \cap A$

v. $A \cup A' = U, \quad A \cap A' = \phi$

Example 3:

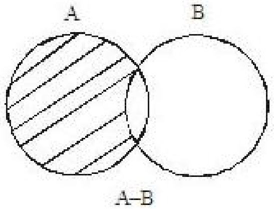
A = {1, 2, 3, 4, 5} and B = {3, 7, 9, 4}. Find $A \cap B$.

Solution:

$A \cap B$ = {3, 4}

If A and B have no elements in common, then they are called **disjoint sets**.

Difference of Sets



If A and B are sets, then the difference of A and B, written as A - B, is the set of all those elements of A which do not belong to B.

Note: $A - B = A - A \cap B = A \cap B'$

Is A - B = B - A?

Find out for the sets A and B given in the example.

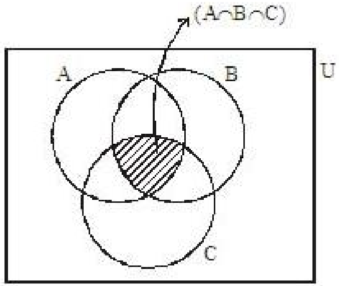
e.g. A = {1, 2, 3, 4, 5} and B = {3, 4, 6, 7}, find A - B.

Solution:

A - B = {1, 2, 5}

Venn Diagrams

For three sets, the following diagram is valid.



$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap C) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$ If $n(A \cup B)$ is not given and $n(A \cap B)$ is to be found, then we get a range of values for both in general. In specific cases, it might be a unique answer.

Example 4:

In a group of 800 persons, 600 can speak English and 400 can speak Telugu. If all the people speak at least one of the two languages, then find

a. how many can speak both the languages?

b. how many can speak exactly one language?

Solution:

a. $n(E \cup T) = n(E) + n(T) - n(E \cap T)$

$800 = 600 + 400 - n(E \cap T) \Rightarrow n(E \cap T) = 200$

b. People speaking exactly one language is equal to $n(E \cup T) - n(E \cap T)$ = 800 - 200 = 600

Example 5:

A survey shows that 63% of the Americans like apples whereas 76% like guns. What percentage of Americans like both apples and guns?

Solution:

The solution for the question cannot be determined. This is because we do not have the information whether all Americans like at least one of these.

(If we assume that 100% Americans like at least one of these)

Then n (A) = 63, n (G) = 76 and $n(A \cup G) = 100$

$\Rightarrow n(A \cap G) = n(A) + n(G) - n(A \cup G)$

$= 63 + 76 - 100 = 39$

Thus, 39% Americans like both guns and apples.

Example 6: In a certain city only 2 newspapers

A and B are published. It is known that 25% of the city population reads A and 20% reads B, while 8% read both A and B. It is also known that 30% of those who read A but not B, look into advertisements and 40% of those who read B but not A, look into advertisements while 50% of those who read both A and B look into advertisements. What percentage of the population look into an advertisement?

Solution: Let A and B denote sets of people who read newspaper A and newspaper B respectively. Then

$n(A) = 25, n(B) = 20, n(A \cap B) = 8;$

$n(A - B) = n(A) - n(A \cap B) = 25 - 8 = 17;$

$n(B - A) = n(B) - n(A \cap B) = 20 - 8 = 12$

Percentage of people reading an advertisement = [(30% of 17) + (40% of 12) + (50% of 8)]% = 13.9%

Concept of maximum or minimum

Type I:

Example 7:

In a school there are 200 students. 100 play cricket, 50 play hockey and 60 play basketball. 30 students play both cricket and hockey, 35 play both hockey and basketball, and 45 play both basketball and cricket.

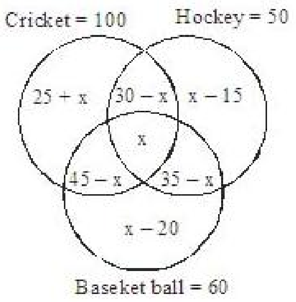
a. What is the maximum possible number of students who play at least one game?

b. What is the maximum possible number of students who play all the 3 games?

c. What is the minimum possible number of students playing at least one game?

d. What is the minimum possible number of students playing all the 3 games?

Solution:



Let x be the number of students playing all the 3 games.

Converting all values in terms of variable x, the number of students cannot be negative in any cell.

$\therefore x - 20 \geq 0$

∴ For minimum possible number of students playing all three games, i.e. x = 20

For maximum possible value of x, again none of the categories should have negative number of students.

$\therefore 30 - x \geq 0$

$x \leq 30$

If x is more than 30, (30 - x) would be negative which is not possible.

$\therefore 20 \leq x \leq 30$

Total number of students playing at least one game.

$= 100 + (x - 15) + (35 - x) + (x - 20) = 100 + x$

$\therefore$ Minimum possible number of students playing at least one game = 100 + 20 = 120

Maximum possible number of students playing at least one game

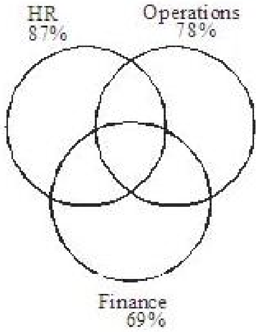
= 100 + 30 = 130

Type II:

Example 8:

In an office, where working in at least one department is mandatory, 78% of the employees are in operations, 69% are in finance and 87% are in HR. What are the maximum and minimum percentages of employees that could have been working in all three departments?

Solution:



Let the total number of employees in the office be 100.

Lets assume that x, y and z number of people are in exactly one, two and three departments of the office respectively.

Therefore, $x + 2y + 3z = 78 + 69 + 87 = 234$ and $x + y + z = 100$

$\Rightarrow (x + 2y + 3z) - (x + y + z) = 134.$

$\Rightarrow y + 2z = 134.$

Maximum possible value of z is $\frac{134}{2} = 67$

Therefore, maximum possible percentage of employees who could be working in all the three departments is 67%.

To minimize the value of z, we need to maximize the value of y, keeping in mind that $x + y + z = 100$.

Maximum possible value of y could be 66 and for this value of y, z = 34 and x = 0.

If we take a value of y greater than 66, lets say 68, then value of z comes out be 33, but here
$x + y + z$ is getting greater than 100, which is not possible.

Therefore, minimum possible percentage of employees who could be working in all the three departments is 34%.

Alternative method:

Minimum: 87% are in HR, it means at least 13% are in operations or finance or operations and finance both. In the same way, 78% are in operations.

So at least 22% are in HR or finance or HR and finance both.

Similarly, 31% are in HR, or operations or HR and operations.

Adding all three, 13% + 22% + 31% = 66%

It means that if there is no intersection among these three sets, 66% would be maximum number of employees in A, B, C alone or $(A \cap B)$, $(B \cap C)$, $(C \cap A)$.

This gives that at least 34% would be in all 3 departments.

The formula is $\left(\overline{A+B+C}\right)$

Alternatively (for Example 8) the minimum value can be found by:

(78 + 69 + 87) - 200 = 34%

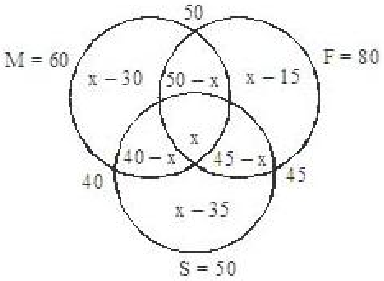
Type III:

Example 9:

There are 3 electives offered to the students in class of 92 (the students have a choice of not choosing any electives).

60 students opted for marketing, 80 for finance and 50 for systems, 40 students opted for both marketing and systems, 50 for both marketing and finance and 45 for both finance and systems. What is the maximum and the minimum possible number of students who opted for all 3 electives?

Solution:



Total number of students = 60 + x - 5 = 55 + x

The minimum possible value of x, so that none of the categories becomes negative = 35

Now applying the same concept, maximum possible value of x = 40. For this value of x, total number of students = 55 + 40 = 95

Which exceeds the total number of students, i.e. 92 by 3, which is not possible.

So to make it equal to 92 the maximum possible value of x = 37

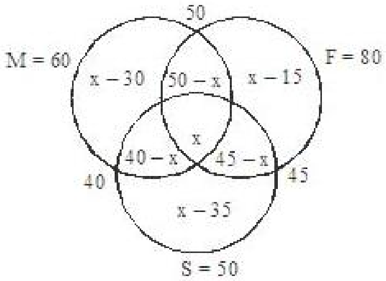
$\therefore 35 \le x \le 37$

Hence minimum possible number of students who opted for all 3 electives is 35. And maximum possible number of students who opted for all 3 electives is 37.

Problems for Practice (Non MCQ)

1. In a group there are 30 guys. 10 of them wear earrings, 15 of them wear ponytails. If the percentage of guys who wear neither is 30%, how many guys wear both earrings and ponytails?

2. Twenty friends, including the host decide to party on a festive Saturday night in a bar. The host knows that 15 of them can have whisky and 10 of them can have rum. He also



Total number of students = 60 + x - 5 = 55 + x

The minimum possible value of x, so that none of the categories becomes negative = 35

Now applying the same concept, maximum possible value of x = 40. For this value of x, total number of students = 55 + 40 = 95

Which exceeds the total number of students, i.e. 92 by 3, which is not possible.

So to make it equal to 92 the maximum possible value of x = 37

$\therefore 35 \leq x \leq 37$

Hence minimum possible number of students who opted for all 3 electives is 35. And maximum possible number of students who opted for all 3 electives is 37.

Problems for Practice (Non MCQ)

1. In a group there are 30 guys. 10 of them wear earrings, 15 of them wear ponytails. If the percentage of guys who wear neither is 30%, how many guys wear both earrings and ponytails?

2. Twenty friends, including the host decide to party on a festive Saturday night in a bar. The host knows that 15 of them can have whisky and 10 of them can have rum. He also knows that each of them have 4 pegs of any drink. None of them has both whisky and rum simultaneously. Every peg of whisky costs ₹60 and every peg of rum costs ₹30. What is the maximum and the minimum budget that the host would have planned for? [Assume that atmost 5 people can go without drink in that party]

3. In a class of 50 students, a test for 2 subjects was conducted. 30 passed in the first subject and 40 passed in the second subject.

(a) What is the maximum number of people who passed in both the subjects?

(b) What is minimum number of people who passed in both the subjects?

4. A survey was conducted on a car brand KHATARA. It was found that 60% of vehicles had a problem with their engines, 60% of vehicles had a problem with the doors, 50% of vehicles had a problem with the tyres. 20% of the vehicles had no problem. What is the minimum percentage of vehicles which had all the 3 problems?

5. In question 4, what is the maximum percentage of vehicles which could have had all the problems?

Practice Exercise

1. In a group of 500 students, selected for admission in a business school, 64% opted for finance and 56% for operations as specialisations. If dual specialisation is allowed, how many have opted for both? Each student opts for at least one of the two specialisations.

a. 200 b. 100 c. 150 d. 125 e. 140

Directions for questions 2 to 4: Read the passage given below and answer the questions.

In a locality, 30% of the residents read *The Times of India* and 75% read *The Hindustan Times*.
3 people read neither of the papers and 6 read both. Only *The Times of India* and *The Hindustan Times* newspapers are available.

2. How many people are there in the locality?

a. 60 b. 120 c. 126 d. 130 e. 90

3. What is the percentage of people who read only *The Times of India*?

a. 15% b. 20% c. 25% d. 30% e. 35%

4. What percentage of residents read only one newspaper?

a. 11% b. 43% c. 85% d. 20% e. 60%

Directions for questions 5 and 6: Answer the questions based on the following information.

In a sports centre 70 students play cricket, 50% play hockey, 25% play both hockey and cricket and 5% play none.

5. How many students are there in the class?

a. 80 b. 85 c. 95 d. 100 e. 110

6. How many students play only one game?

a. 65 b. 70 c. 75 d. 80 e. 90

Directions for questions 7 and 8: Read the following information and answer the questions.

In an examination there are 150 candidates. 40 candidates passed in papers A and B; 40 candidates passed in papers B and C; 30 candidates passed in papers C and A ; and 10 candidates passed in all the 3 papers.

7. How many students passed in paper B only?

a. 40 b. 20 c. 15 d. 25 e. Cannot be determined

8. If no students failed in all the 3 subjects, what is the total number of students who passed in exactly one paper?

a. 25 b. 45 c. 60 d. 50 e. 55

9. In a class of 60 boys, there are 45 boys who play cards and 30 boys play carrom. Find how many boys play both the games. (assuming that every boy plays either cards or carrom or both)

a. 15 b. 17 c. 20 d. 21 e. 16

10. In question number 9, find the number of boys who only play cards.

a. 27 b. 30 c. 32 d. 25 e. 35

11. In question number 9, find the number of boys who only play carrom.

5. How many students are there in the class?

a. 80 b. 85 c. 95 d. 100 e. 110

6. How many students play only one game?

a. 65 b. 70 c. 75 d. 80 e. 90

Directions for questions 7 and 8: Read the following information and answer the questions.

In an examination there are 150 candidates. 40 candidates passed in papers A and B; 40 candidates passed in papers B and C; 30 candidates passed in papers C and A ; and 10 candidates passed in all the 3 papers.

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a. 40 b. 20 c. 15 d. 25 e. Cannot be determined

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a. 25 b. 45 c. 60 d. 50 e. 55

9. In a class of 60 boys, there are 45 boys who play cards and 30 boys play carrom. Find how many boys play both the games. (assuming that every boy plays either cards or carrom or both)

a. 15 b. 17 c. 20 d. 21 e. 16

10. In question number 9, find the number of boys who only play cards.

a. 27 b. 30 c. 32 d. 25 e. 35

11. In question number 9, find the number of boys who only play carrom.

a. 10 b. 12 c. 15 d. 20 e. 14

12. Each student in a class of 40, studies at least one of the subjects namely English, Mathematics and Economics. 16 study English, 22 study Economics and 26 study Mathematics, 5 study English and Economics, 14 study Mathematics and Economics and 2 study English, Economics and Mathematics. Find the number of students who study English and Mathematics.

a. 10 b. 7 c. 17 d. 27 e. None of these

13. In question number 12, find the number of students who study English and Mathematics but not Economics.

a. 8 b. 12 c. 7 d. 5 e. 6

Answers Key Number System

Problems for Practice (Non MCQ)

1. $\frac{428}{99}$ **2.** 27 **3.** 1824

4. 5^7 **5.** 12 **6.** 7

7. 4722 **8.** 28 **9.** 0

10. 22 **11.** 9 **12.** 7

13. 3(viz. 2, 3 and 17) **14.** 27 **15.** 2080

16. 36 lakhs **17.** 13 **18.** $3^{3^{33}}$

19. 46; BA **20.** 51; 43961 **21.** 20A; -365

22. Integer values not possible **23.** 36

24. 983 **25.** 6912 **26.** $\frac{168}{73}$

27. 11 **28.** 0 **29.** 7

30. 2 **31.** 0 **32.** 1

33. 5, 5 and 2 in order **34.** 48 **35.** 60984; 4

36. 38 **37.** Q = 1 **38.** 111111111

39. $1+\sqrt{5}$ **40.** 11 **41.** 71

42. 499 **43.** 2 or 11 **44.** 24, 120

45. 32 **47.** 7 **48.** 124; 7

49. 15; 15 **50.** 9 **51.** 7 different solutions

52. 60489 **53.** $\begin{array}{r} 29786 \\ +\ 850 \\ +\ 850 \\ \hline 31486 \\ \hline \end{array}$ **54.** 33

55. 7 **56.** 8

57. (a) 15.32743... (b) 11.533203 (c) 1432 (d) 3A71 **58.** 128; 256

Practice Exercise 1 - Level 1

1	c	**2**	d	**3**	a	**4**	c	**5**	d	**6**	c	**7**	b	**8**	c	**9**	d	**10**	e
11	c	**12**	c	**13**	d	**14**	d	**15**	b	**16**	b	**17**	b	**18**	a	**19**	a	**20**	a
21	b	**22**	d	**23**	c	**24**	d	**25**	d	**26**	c	**27**	b	**28**	c	**29**	c		

Practice Exercise 2 - Level 1

1	d	**2**	e	**3**	d	**4**	b	**5**	c	**6**	b	**7**	e	**8**	b	**9**	a	**10**	c
11	d	**12**	c	**13**	b	**14**	c	**15**	b	**16**	c	**17**	a	**18**	c	**19**	c	**20**	b
21	a	**22**	b	**23**	d	**24**	d	**25**	e	**26**	c	**27**	d	**28**	b	**29**	b	**30**	e
31	d	**32**	d	**33**	a	**34**	e	**35**	a										

Practice Exercise 3 - Level 2

1	b	**2**	a	**3**	e	**4**	d	**5**	b	**6**	d	**7**	d	**8**	a	**9**	d	**10**	d
11	d	**12**	d	**13**	b	**14**	a	**15**	c	**16**	d	**17**	b	**18**	a	**19**	d	**20**	a
21	a	**22**	b	**23**	a	**24**	d	**25**	b	**26**	c	**27**	c	**28**	a	**29**	c	**30**	c

Practice Exercise 4 - Level 2

42. 499 **43.** 2 or 11 **44.** 24, 120

45. 32 **47.** 7 **48.** 124; 7

49. 15; 15 **50.** 9 **51.** 7 different solutions

52. 60489 **53.**
$$\begin{array}{r} 29786 \\ +\ 850 \\ +\ 850 \\ \hline 31486 \end{array}$$
54. 33

55. 7 **56.** 8

57. (a) 15.32743... (b) 11.533203 (c) 1432 (d) 3A71 **58.** 128; 256

Practice Exercise 1 - Level 1

1	c	**2**	d	**3**	a	**4**	c	**5**	d	**6**	c	**7**	b	**8**	c	**9**	d	**10**	e
11	c	**12**	c	**13**	d	**14**	d	**15**	b	**16**	b	**17**	b	**18**	a	**19**	a	**20**	a
21	b	**22**	d	**23**	c	**24**	d	**25**	d	**26**	c	**27**	b	**28**	c	**29**	c		

Practice Exercise 2 - Level 1

1	d	**2**	e	**3**	d	**4**	b	**5**	c	**6**	b	**7**	e	**8**	b	**9**	a	**10**	c
11	d	**12**	c	**13**	b	**14**	c	**15**	b	**16**	c	**17**	a	**18**	c	**19**	c	**20**	b
21	a	**22**	b	**23**	d	**24**	d	**25**	e	**26**	c	**27**	d	**28**	b	**29**	b	**30**	e
31	d	**32**	d	**33**	a	**34**	e	**35**	a										

Practice Exercise 3 - Level 2

1	b	**2**	a	**3**	e	**4**	d	**5**	b	**6**	d	**7**	d	**8**	a	**9**	d	**10**	d
11	d	**12**	d	**13**	b	**14**	a	**15**	c	**16**	d	**17**	b	**18**	a	**19**	d	**20**	a
21	a	**22**	b	**23**	a	**24**	d	**25**	b	**26**	c	**27**	c	**28**	a	**29**	c	**30**	c

Practice Exercise 4 - Level 2

1	b	**2**	e	**3**	a	**4**	b	**5**	e	**6**	a	**7**	b	**8**	c	**9**	d	**10**	a
11	c	**12**	a	**13**	a	**14**	d	**15**	c	**16**	b	**17**	d	**18**	a	**19**	c	**20**	c
21	c	**22**	a	**23**	c	**24**	c	**25**	d	**26**	b								

Practice Exercise 5 - Level 2

1	c	**2**	a	**3**	c	**4**	b	**5**	a	**6**	b	**7**	b	**8**	b	**9**	e	**10**	b
11	c	**12**	e	**13**	d	**14**	b	**15**	c	**16**	b	**17**	b	**18**	c	**19**	c	**20**	e
21	a	**22**	e	**23**	b	**24**	c	**25**	c	**26**	b	**27**	d	**28**	c	**29**	a	**30**	a
31	e	**32**	b	**33**	e	**34**	c												

Practice Exercise 6 - Level 3

1	b	**2**	a	**3**	b	**4**	a	**5**	b	**6**	b	**7**	d	**8**	a	**9**	d	**10**	c
11	c	**12**	c	**13**	a	**14**	e	**15**	b	**16**	c	**17**	e	**18**	c	**19**	b	**20**	a
21	e	**22**	a	**23**	b	**24**	d	**25**	c										

Practice Exercise 7 - Level 3

1	c	**2**	a	**3**	d	**4**	c	**5**	d	**6**	e	**7**	b	**8**	c	**9**	b	**10**	b
11	a	**12**	c	**13**	e	**14**	a	**15**	c	**16**	e	**17**	c	**18**	c	**19**	e	**20**	c
21	e	**22**	a	**23**	c	**24**	d	**25**	b	**26**	b	**27**	e	**28**	c	**29**	d	**30**	b
31	e	**32**	a																

Answers Key Set Theory

Practice Exercise (Non MCQ)

1. 4

2. ₹4200, ₹2400

3. *(a)* 30 *(b)* 20

4. 10%

5. 45%

Practice Exercise

1	b	**2**	a	**3**	b	**4**	c	**5**	d	**6**	b	**7**	e	**8**	c	**9**	a	**10**	b
11	c	**12**	b	**13**	d														

Explanations: Fundamentals of Numbers

Problems for Practice (Non MCQ)

Level 1

1. Let x = 4.323232..

Then 100 x = 432.323232...

$99x = 428 \Rightarrow x = \frac{428}{99}$

2. The nearest multiple of 84 greater than 8961 is 8988.

Hence, 27 must be added to 8961 to make it divisible by 84.

3. LCM of 12 and 16 = 48

Dividing 1834 by 48, quotient = 38, remainder = 10

Hence, the nearest number is 1834 - 10 = 1824

4. $a^b > b^a$, if a < b and a, b > 3.

5. HCF of 2 numbers is 48 and HCF of the other 2 is 36.

HCF of all 4 is the HCF of 48 and 36 =12.

6. Number of steps would be (n - 1), where n is the number of numbers.

Hence, number of steps = 7.

7. LCM of 21, 25, 27 and 35 = 4725

Therefore, required number = 4725 - 3 = 4722.

8. The first time they would meet at the starting point is LCM (7, 4) = 28 min

9. 732 is nothing but $(27^2 + 3)^{732}$. The last term of the expansion is 3^{732} which again is completely divisible by 27. Thus, the remainder is 0.

10. N = 5 [7x + 4] + 2 = 35x + 22

∴ When divided by 35, remainder = 22.

11. The last digits of

$9^1 = 9$

$9^2 = 1$

$9^3 = 9$

Thus, 9 exhibits a cyclicity of 2.

Therefore, the last digit of $(729)^{59} = 9$.

12. The last digits of

$3^1 = 3$

$3^2 = 9$

$3^3 = 27$

$3^4 = 81$

$3^5 = 243$

Thus, cyclicity of powers of 3 is 4.

Hence, last digit of $123^7 = 7$.

13. Reduce it to factorised form and you will find that there are only three prime bases 2, 3 and 17.

14. Let the number of saplings in each row and column = x.

Then $x^2 = 729$.

Therefore, x = 27.

15. If there were n saplings in each row and column, then the number of new saplings planted in the rows = (n - 1)n

Number of new saplings planted in each column = n(n - 1)

Hence, total number of saplings planted = (n - 1)n + n(n - 1) = 2n(n - 1)

Since n = 27

$\therefore$ Number of saplings along the rows and columns = $2 \times 27 \times 26$

Saplings planted diagonally between any two saplings = $(n - 1)^2$

Hence, total number of saplings will be $2 \times 27 \times 26 + 26^2 = 2080$

16. The youngest son got $6x = \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) \times$ total amount.

Hence, total amount = 36x.

$\because$ x has to be an integer, so minimum wealth the businessman had is 36 lakh.

17. Split the number 36 into 3 factors. Add up all the 3 factors. All the sums are uniquely different except 2 combinations that give 13. They are 6, 6, 1 and 9, 2, 2.

18. The largest number that can be formed is $3^{3^{33}}$

19. a. Consecutively dividing 54 by 12, the remainders are 4 and 6 in the reverse order.

Hence, $(54)_{10} = (46)_{12}$.

b. Consecutively dividing 142 by 12, the remainders are 11(i.e. B) and 10 (i.e. A) in the reverse order.

Hence, $(142)_{10} = (BA)_{12}$.

20. a. $(110011)_2 = 2^5 \times 1 + 2^4 \times 1 + 2^3 \times 0 + 2^2 \times 0 + 2^1 \times 1 + 2^0 \times 1 = 32 + 16 + 2 + 1 = (51)_{10}$

b. $(ABCD)_{16} = 16^3 \times A + 16^2 \times B + 16^1 \times C + 16^0 \times D = 40960 + 2816 + 192 + 13 = (43961)_{10}$

21. a. $(20A)_{12}$

b. $-(365)_8$

Level 2

22. X + Y = 19

4X + 6Y = 9

The two equations do not give an integer as an answer.

23. $\frac{10x + y}{x + y} = 4$ ...(i)

x + 3 = y ...(ii)

Solving the two equations, we get

x = 3 and y = 6

Therefore, the number = 36.

24. If N be the required number, then N = 6x + 5 and N = 5y + 3.

The first such number = 23

The largest number less than 1000 which is a multiple of 30 (LCM of 6 and 5) is 990 and the penultimate number is 960.

Hence, N = 960 + 23 = 983.

25. LCM of (15, 20, 21, 23, 35) = 9660

K × 9660 will be divisible by given numbers

From the given information,

K × 9660 - 2748 = 4 digit number

For this to happen the maximum value K can take is 1.

So the four-digit number = 9660 - 2748 = 6912

26. Let the number of units of work be LCM (6, 7, 8) units, i.e 168 units.

Hence, A does 28 units in 1 day; B does 24 units in 1 day and C does 21 units in 1 day.

Working together they finish 73 units in 1 day.

Hence, time taken = $\frac{168}{73}$ days

27. If N be the required number, then

N = 5x + 1 and N = 6y + 5

When y = 1, we have 11, which satisfies

5x + 1 for x = 2.

∴ N = 11

The next number satisfying this condition would be 11 + LCM(6, 5) = 41.

28. Let the number be x.

Then x ÷ 145 leaves the remainder 58.

∴ x = 145y + 58 = 29 (5y) + 29x 2

Hence, x is totally divisible by 29.

⇒ Remainder = 0

29. 86 × 293 × 4919 = $(17 \times 5 + 1)(17 \times 17 + 4)\ (17^3 + 6)$

When we do the multiplication, all the terms except 1 × 4 × 6 will be divisible by 17.

Hence, the remainder will be 24 - 17 = 7.

30. $7^{13} + 1 = (6 + 1)^{13} + 1$

In the binomial expansion of $(6 + 1)^{13}$, all terms except the last one are multiples of 6.

Last term = 1

Hence, remainder = 1 + 1 = 2.

31. **Method 1:**

10^{200} can be written $(8 + 2)^{200}$. There are in all 201 terms in this expansion. The first 200 terms have a 8 in them. Hence, they can be expressed as a multiple of 8. The last term is 2^{200}, which is $2^3 \times 2^{197}$

This is also a multiple of 8. Hence, 10^{200} is a multiple of 8. Hence, the remainder is 0.

Method 2:

$10^{200} = 2^{200} \times 5^{200} = 2^3 \times 2^{197} \times 5^{200}$. This is obviously divisible by 8. Hence, the remainder will be zero.

32. $(30)^{40}$ can be written $(34 - 4)^{40}$

This in turn can be written $34M + 4^{40} = 34M + 16^{20} = 34M + (17 - 1)^{20} = 34M + 17N + 1^{20}$

(34M is a multiple of 34 and 17N is a multiple of 17)

When this expression is divided by 17, the remainder is 1.

33. N = 5 [7(8x + 4) + 3] + 2 = 35(8x) + 140 + 17 = 280x + 157

When this is divided by 8, quotient = 35x + 19 and remainder = 5.

When (35x + 19) is divided by 7, quotient = 5x + 2 and remainder = 5.

When (5x + 2) is divided by 5, quotient = x and remainder = 2.

34. Let $a = 2^x \times 3^y \times 5^z$

The possible values of x are 0, 1, 2, 3.

The possible values of y are 0, 1, 2.

The possible values of z are 0,1, 2, 3.

Hence, the number of values of a that is possible is 4 × 3 × 4 = 48. Hence 48 factors.

35. Breaking $6^{10} \times 7^{17} \times 11^{27}$ into prime factors $= 2^{10} \times 3^{10} \times 7^{17} \times 11^{27}$

∴ Number of factors = (10 + 1) × (10 + 1) × (17 + 1) × (27 + 1) = 11 × 11 × 18 × 28 = 60984

Number of prime factors = 4 (i.e. 2, 3, 7, 11)

36. Total factors possible are

(4 + 1) (2 + 1) (4 + 1) = 5 × 3 × 5 = 75.

as ab = N

So choosing a out of 75 factors, b will have its value accordingly. But in this (a, b) and (b, a) forms the same set.

So we have to divide it by 2. Out of these 75 sets, one set will be of the form $(\sqrt{N}, \sqrt{N})$ which need not to be divided by 2.

∴ Total sets = $\frac{75-1}{2} + 1 = 37 + 1 = 38$ sets

37. PQRS can be written as PQR × 10 + S which is equal to PQR × QS

PQR × 10 + S = PQR × QS

PQR(QS - 10) = S

PQR is a three-digit number. So even if it is multiplied by a single non-zero digit it will not give S, a single digit number.

So for this to be possible, QS = 10.

Which gives Q = 1

38. This is a property found in integers containing only 1's.

For example, $\sqrt{121} = 11$

$\sqrt{12321} = 111$ and so on

Thus in the given case, there being nine 1's in the number, its square will have the highest number 9, i.e. 12345678987654321

39. $(3+\sqrt{5})^2 = 9+5+2\times3\times\sqrt{5} = 14+6\sqrt{5} = 14+\sqrt{36\times5} = 14+\sqrt{180}$

$\therefore \sqrt{14+\sqrt{180}} = \sqrt{(3+\sqrt{5})^2} = 3+\sqrt{5}$

$3+\sqrt{5}-3+\sqrt{5} = 6+2\sqrt{5} = (1+\sqrt{5})^2 = 6+2\sqrt{5} = (1+\sqrt{5})^2$

Now, the original expression $= \sqrt{5+\sqrt{5}+1+\sqrt{5}} = \sqrt{6+2\sqrt{5}} = \sqrt{(1+\sqrt{5})^2} = 1+\sqrt{5}$

40. The numbers that can be formed are xy and yx.

Hence, $(10x + y) + (10y + x) = 11(x + y)$. If this is a perfect square, then $x + y = 11$.

Level 3

41. $x + y = 8$ ...(i)

$10y + x = 10x + y - 54$

$9y - 9x = -54$

$9x - 9y = 54$ ...(ii)

Solving equation (i) and (ii), we get $x = 7$ and $y = 1$

Therefore, the number = 71

42. Number of multiples of 3 less than 3000 = $\frac{3000}{3} - 1 = 999$ (-1, since 3000 is also a multiple of 3.)

Number of multiples of 3 less than or equal to 2000 = $\frac{2000}{3} \Rightarrow 666.66$

$\Rightarrow$ 666 multiples

Number of multiples of 3 lying between 2000 and 3000 is 999 - 666 = 333 = X ...(i)

Similarly, for number of multiples of 4 less than 3000 = $\frac{3000}{4} - 1 = 750 - 1 = 749$

(-1, since 3000 is also a multiple of 4.)

Number of multiples of 4 less than or equal to 2000 = $\frac{2000}{4} = 500$

Number of multiples of 4 lying between 2000 and 3000 is 749 - 500 = 249 = Y ...(ii)

Similarly, LCM (3, 4), i.e. 12.

Number of multiples of 12 lying between 2000 and 3000 is 249 - 166 = 83 = Z ...(iii)

Hence, number of multiples of 3 or 4 = X + Y - Z = 333 + 249 - 83 = 499.

43. The number is divisible by 8 so last three digits should be divisible by 8.

Thus Y = 0 or 8.

The number is also divisible by 9, sum of the digits must be divisible by 9.

43 + x + y is divisible by 9.

When Y = 0, X = 2

When Y = 8, X = 3

X + Y = 2 or X + Y = 11

44. Let two numbers be x , y

$x + y = 144$

HCF of two numbers is 24

Let $x = 24a$, $y = 24b$

Where a, b are co-prime to each other

$24a + 24b = 144$

$a + b = 6$

Possible values are

$a = 1, b = 5$, or $a = 2, b = 4$ or $a = 3, b = 3$

But a, b are co-prime to each other

So $a = 1, b = 5$

$\therefore x = 24$ and $y = 120$

Alternative method:

Sum = 144 = 24 + 120

Hence, the numbers are 24, 120

Other possibilities are 48 + 96, 72 + 72

But for each of them, HCF is not equal to 24.

45. On simplifying $\frac{2^{643}}{96}$ is reduced to $\frac{2^{643}}{2^5 \times 3} = \frac{2^{638}}{3}$. Thus, the remainder will be the last term of the expression $(3-1)^{638}$ and then multiply it by 32. It is important to multiply it by 32 because the initial expression is $\frac{2^{643}}{96}$ and not $\frac{2^{638}}{3}$.

Hence, final remainder is $1 \times 32 = 32$.

46. $244^{1500} - 1$ is divisible by 7 as the remainder when $\left(\frac{244^{1500}}{7}\right) = 1$ and also $\frac{244^{1500}}{11} = \frac{2^{1500}}{11}$ $= \frac{32^{300}}{11} = \frac{(33-1)^{300}}{11} = +1$ and $\frac{244^{1500}}{13} = \frac{(-3)^{1500}}{13} = \frac{(27)^{500}}{13} = 1$

$\therefore 1 - 1 = 0$ (Net remainder is zero)

So it is divisible by 7, 11, 13 and 1001.

47. End digit of a number will depend upon cyclicity of the number. Cyclicity of 7 is 4.

We will have to determine the remainder when $11^{22^{33}}$ is divided by 4.

The power to 11 is an even number therefore the remainder if $11^{22^{33}}$ is divided by 4 will be 1.

$N = 7^{4N+1}$

Its end digit will be 7.

48. a. $10 \times 20 \times 30 \ldots \times 1000$

$= 10^{100}[1 \times 2 \times 3 \ldots \times 100]$

$= 10^{100}(100!)$

Now $2 \times 5 = 10$

Highest power of 5 in 100!

$= \frac{100}{5} + \frac{20}{5} = 20 + 4 = 24$

$\therefore$ Number of zeros = 100 + 24 = 124

Hint: Need to check for only 5 since it has the smaller power.

b. $(5 \times 10 \times 15 \times 20 \times \ldots \times 45) = 5^9(1 \times 2 \times 3 \times 4 \times \ldots \times 9) = 5^9 \times 9!$

Highest power of 2 in $5^9 \times 9!$, is 7.

$\therefore$ Number of zeros = 7

49. a. $6 = 3 \times 2$

Hence, we have to find the highest power of 3 which divides 34!

Highest power = $\left(\frac{34}{3} + \frac{11}{3} + \frac{3}{3}\right) = 11 + 3 + 1 = 15$

b. $12 = 2^2 \times 3$

We have to check the highest power of 3 and highest power of 2^2.

Highest power of 3 = 11 + 3 + 1 = 15

Highest power of 2 = 17 + 8 + 4 + 2 + 1 = 32

Hence, highest power of 4 = 16

$\therefore 34! = 3^{15} \times 4^{16} \times N$. (N is not divisible by 2 or 3.)

Therefore, highest power of 12 in 34! = 15.

50. Find the positive integer solution satisfying

$4x + 3y = 120$

$3y = 120 - 4x$

$y = 40 - \frac{4}{3}x$ ...(i)

For positive integer value of y, x can take value from 3 to 27, i.e. 9 values

if x = 3, y = 36

if x = 6, y = 32

....

....

if x = 27, y = 4

Hence, total 9 positive integer solutions satisfy the above equation.

51. $5x + 3y = 100$

There are more than one solutions.

All these would be the integer solutions that satisfy the equation.

There are 7 solutions for (x, y): (20, 0), (17, 5),

(14, 10), ..., (2, 30).

52.

```
             4   2   3
          x  a   b   c
          -------------
          4c   2c   3c
      4b  2b   3b   x
  4a  2a  3a   x    x
  ---------------------
  6   --  --   8    9
  ---------------------
  3c = 9; c = 3
  6 + 3b = __8; 3b = __2
```

$\Rightarrow 3b = 12$

b = 4

4a + x = 6, where x maybe the carry-over from 4b +2a

Now, $4 \times 1 + 2 = 6$

$4 \times 2 + 0 = 8$ is not possible. Hence, a = 1

$\therefore 423 \times 143 = 60489$.

53. $$\begin{array}{lr} & \text{FORTY} \\ (+) & \text{TEN} \\ (+) & \text{TEN} \\ \hline & \text{SIXTY} \end{array}$$

(1) Y + N + N ends with Y. Hence, N + N must be either 0 or 10.

(2) T + E + E + (1) or T + E + E ends in T. The first case is not possible since no case of

T + E + E + 1 can end with the same digit as

T because E + E + 1 can neither be 0 nor 10.

(3) So N = 0; E = 5

(4) O must have got a carry-over of 2 and if O = 9, then I = 1. In every other case, I can take only a value
= 0 which already N has taken.

So $$\begin{array}{r} \text{F9RTY} \\ \text{T50} \\ \text{T50} \\ \hline \text{S1XTY} \end{array}$$

From here T must be at least 7 because the second digit of S1xty is 1 and that of F9RTY is 9. We get

$$\begin{array}{lr} & 29786 \\ & 850 \\ (+) & 850 \\ \hline & 31486 \end{array}$$

54. We can write the 32 numbers in the following fashion

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17

The sum of the two numbers in any column is 33. When we have to select 17 numbers, in any selection we will always have a pair of numbers which appear in a single column in the above table.

Thus for any selection their will be a pair of numbers which add up to 33.

55. Single digit numbers are 9.

Two-digit numbers are 90.

Three-digit numbers are 900.

Four-digit numbers are 9000.

If we write one after another, 9 + 180 + 2700 = 2889 digits already written till all three-digit numbers.

Thereafter, 7111 digits are still left and each number after 999 will have 4 digits.

$\frac{7111}{4} = 1777.75$

Thus, we can skip 1777 numbers after 999.

(1777 + 999 = 2776). The third digit of the succeeding number will be the answer.

(The third digit of 2777, i.e. 7.)

56. Five numbers are 20, 22, 23, 25 and 27.

To get distinct sum by taking 2 at a time, take first number and get the sum of taking other numbers.

e.g.

20 + 22 = 42 ...(i)

20 + 23 = 43 ...(ii)

20 + 25 = 45 ...(iii)

20 + 27 = 47 ...(iv)

Here we have 4 distinct sums. Now take the second number, i.e. 22, add with other number and write down only those sums which are not the same as calculated earlier.

i.e. 22 + 27 = 49 ...(v)

concept will remain same for the rest numbers,

23 + 25 = 48 ...(vi)

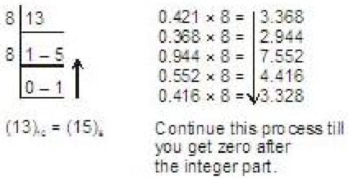
23 + 27 = 50 ...(vii)

25 + 27 = 52 ...(viii)

Hence, 8 distinct sum can be formed.

57.

(a) $(13.421)_{10} = (\ )_8$



$(0.421)_{10} = (0.32743...)_8$

$\therefore (13.421)_{10} = (15.32743...)_8$

(b) $(13.421)_8 = (\ )_{10}$

$(13)_8 = (\ )_{10}$

$1 \times 8^1 + 3 \times 8^0 = 8 + 3 = 11$

$(13)_8 = (11)_{10}$

$(0.421)_8 = (\ )_{10}$

$4 \times 8^{-1} + 2 \times 8^{-2} + 1 \times 8^{-3} = 0.5 + 0.03125 + 0.001953 = 0.533203$

$(0.421)_8 = (0.533203)_{10}$

$(13.421)_8 = (11.533203)_{10}$

(c) $(1100011010)_2 = (\ )_8$

$2^3 = 8$

Make pair of three digits from right hand side of the number and write the decimal equivalent for the pairs.

Insert leading zeroes to make a pair of 3

$$\underbrace{0\ 0\ 1}_{1}\quad \underbrace{1\ 0\ 0}_{4}\quad \underbrace{0\ 1\ 1}_{3}\quad \underbrace{0\ 1\ 0}_{2}$$

$\therefore (1100011010)_2 = (1432)_8$

(d) $(3221301)_4 = (\ )_{16}$

$4^2 = 16$

Make pair of two digits from the right hand side of the number and write the decimal equivalent for the pairs. Insert leading zeroes to make a pair of two

$$\underbrace{0\ 3}_{3}\quad \underbrace{2\ 2}_{A}\quad \underbrace{1\ 3}_{7}\quad \underbrace{0\ 1}_{1}$$

$\therefore (3221301)_4 = (3A71)_{16}$

58. Let Meera have x flowers in the beginning.

Then the number of flowers she had in the end is

$$x\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{2}\right)=\left(\frac{27}{128}\right)x$$

Since this value is between 20 and 60, the value of x must be either 128 or 256.

a. 128

b. 256

Practice Exercise 1 - Level 1

1. c LCM × HCF = Product of Number

$$N_2 = \frac{5200 \times 40}{520} = 400$$

2. d 1

3. a Whole number starts with zero.

4. c According to definition, answer is option (c).

5. d Unit place digit = Unit place digit of $(4 \times 1 \times 9 \times 8)$

Therefore, unit digit of the number = 8.

6. c $\sqrt[3]{32} \times \sqrt[3]{250} = (32)^{1/3} \times (250)^{1/3} = (32 \times 250)^{1/3}$

$= [2^5 \times 2 \times 5^3]^{1/3} = 2^{6\times 1/3} \times 5^{3\times 1/3}$

$= 2^2 \times 5 = 20$

7. b $3125 \div 25$ of $25 - \sqrt[3]{125}$

$= 3125 \div (25 \times 25) - (125)^{1/3}$

$= 3125 \div 625 - 5 = 5 - 5 = 0$

8. c $18.18 \div 9 + 2.7$ of $3 = 18.18 \div 9 + 8.1$

$= 2.02 + 8.1 = 10.12$

9. d Using BODMAS, we get 3511 as answer.

10. e Unit's digit is governed by product of individual unit's digit only. Unit's digit of $(7 \times 8 \times 7 \times 3) = 6$.

11. c $\frac{451*603}{9}$.

$\because$ Sum of all digits must be divisible by 9

$\therefore$ * = 8

12. c Number formed by last 3 digits must be divisible by 8.

$\therefore$ * = 3

13. d $8756 \times 99999 = 8756 \times [100000 - 1]$

$= 875600000 - 8756 = 875591244$

14. d $1399 \times 1399 = (1400 - 1)^2$

$= (1400)^2 + (1)^2 - 2\,(1400)\,(1)$

$= 1960000 + 1 - 2800 = 1957201$

15. b $397 \times 397 + 104 \times 104 + 2 \times 397 \times 104$

$= (397 + 104)^2 = (501)^2$

$= (500 + 1)^2 = (500)^2 + (1)^2 + 2\,(500)\,(1)$

$= 250000 + 1 + 1000 = 251001$

16. b Change the fractions into decimals and then check.

17. b $x = 6 - \sqrt{35}$

$\therefore \frac{1}{x} = \frac{1}{6 - \sqrt{35}} \times \frac{6 + \sqrt{35}}{6 + \sqrt{35}} = \frac{6 + \sqrt{35}}{1} = 6 + \sqrt{35}$

18. a Number should be divisible by 9 and 11 both.

19. a Let the numbers be x and 1365 + x.

Then, $1365 + x = 6x + 15$ or $x = 270$

20. a The best way is to factorize the number, i.e.

$385 = 5 \times 7 \times 11$. Hence, first prime number = 5.

21. b When 99547 is divided by 687, remainder is 619.

$\therefore$ Nearest number = 99547 + 68 = 99615

22. d 99990

23. c 100011

24. d $n = 4 \times Q + 3$ (Q = Quotient)

$2n = 2 \times 4 \times Q + 6$

When 2n is divided by 4, Quotient = 2(Q + 1) and remainder = 2

25. d Given expression $\frac{2^{n-1}\,(2+1)}{2^{n}\,(2-1)} - \frac{3}{2}$

26. c $D = \frac{abcd}{9999}$, $49995 = 9999 \times 5$

Hence, once we multiply D with 9999 we will get a natural number.

27. b $a^3 + b^3 + c^3 = 3abc$ if $a + b + c = 0$

28. c $x \div \frac{3}{8} \times \frac{5}{6} = x \times \frac{8}{3} \times \frac{5}{6} = x \times \frac{40}{18}$

Equivalent to dividing by $\frac{18}{40} = \frac{9}{20}$

29. c $72 \times 12 = 24 \times N = N = 36$

Practice Exercise 2 - Level 1

1. d $P = 2^8 \times 3^4 \times 5^4 \times 7^2 \times 11^2 \times 23^2$

$\therefore$ Total number of factors = $(8 + 1)(4 + 1)(4 + 1)$

$(2 + 1)(2 + 1)(2 + 1) = 6075$

2. e $\frac{(a+b+c)}{3} = 6,\ \frac{(b+c+d)}{3} = 7,\ d - a = 3$

Hence, no unique solution can be obtained as the last equation can be derived from the first two.

3. d Assume the quotient to be x, then divisor = 12x

It is given that $12x = 5 \times 48$, $x = 20$.

Dividend = Divisor × Quotient + Remainder = $(12 \times 20) \times 20 + 48 = 4848$.

4. b All the terms in the series except 2! and 4! are divisible by 5, therefore remainder will be the same as the remainder when (2! + 4!) is divided by 5 i.e. the remainder will be 1.

5. c We can have at least two such sets of prime numbers 71, 73, 79 and 67, 73, 83.

6. b By cross multiplying, we get

$\frac{2}{3} \quad \frac{13}{21} \qquad 42 > 39$

Therefore, $\frac{2}{3} > \frac{13}{21}$

Similarly by doing for the other fractions we get largest and smallest fractions are $\frac{2}{3}$ and $\frac{11}{18}$ respectively. Their sum $= \frac{23}{18} = 1\frac{5}{18}$

7. e $\frac{1}{11}(11x + 11y) = x + y$

So, both sides are equal.

Hence, data given is insufficient.

8. b Since average of 4 even numbers is 27, it shall have 2 numbers on either side.

24, 26, 28 and 30.

You can check this: Average = $\frac{108}{4} = 27$

9. a We divide 1153 by 15 and write the remainders and quotient at appropriate places.

$\therefore$ The number is 51D.

10. c $x = 17^4$

$y = (17 - 3)(17 - 1)(17 + 1)(17 + 3)$

$= (17^2 - 9)(17^2 - 1)$; therefore, $x > y$

$= 17^4 - 10 \times 17^2 + 9$

Therefore, $x\ y > 1000$.

11. d We just need to check the last digit of 7^{194321}

194321 is of the form $4k + 1$, where k is a natural number, so the last digit will be $7^1 = 7$.

12. c $6084 = 2^2 \times 3^2 \times 13^2$

So the total number of factors of 6084 is

$(2 + 1) \times (2 + 1) \times (2 + 1) = 27$

Now these make $\left(\frac{27+1}{2}\right) = 14$ pairs, each giving 6084, as their product.

But the factor $2 \times 3 \times 13 = \sqrt{6084}$ couples with itself hence, we must subtract 1.

Option (c) is correct.

13. b $(100)_{10} = (10)_{100}$. Hence, we need two digits.

14. c When $(24 + 1)^{625}$ is divided by 24 the remainder is 1 When 26 is divided by 24 the remainder is 2.

Hence, the required remainder will be $1 + 2 = 3$.

15. b The factors that are common must also be the factors of the HCF (N, M).

HCF (N, M) $= 2^2 \times 3^4$

Number of factors of $2^2 \times 3^4 = (2 + 1) \times (4 + 1) = 15$

So there are 15 factors that are common to both.

16. c $1^2 = 1$, $11^2 = 121$, $111^2 = 12321$ ...

$11111111^2 = 123456787654321$

Hence, digit sum = 64

17. a The given expression can be written as

$$\frac{\left(\frac{4}{3}\right)^4+\left(\frac{3}{4}\right)^4+1}{\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2+1};\ \frac{\left[\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2+1\right]\left[\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2-1\right]}{\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2+1}=\left(\frac{4}{3}\right)^2+\left(\frac{3}{4}\right)^2-1=\frac{16}{9}+\frac{9}{16}-1=\frac{193}{144}$$

Note: $\frac{a^4+b^4+1}{a^2+b^2+1}=a^2+b^2-1$

18. c $N=\left[\frac{2^4(1^4+2^4+3^4+L\ \text{till } n \text{ terms})}{(1^4+2^4+3^4+L\ \text{till } n \text{ terms})}\right]^{\frac{1}{4}}=(2^4)^{\frac{1}{4}}=2$

19. c $2 + 7 + 6 + x = 20 \Rightarrow 15 + x = 20$; $25 + x + y = 50$ where $15 + x = 20$;

Hence, $10 + y + 20 = 50 \Rightarrow y = 20$.

20. b $\because$ Sum of odd and even is always odd.

$\therefore$ At least one has to be odd and the rest evens to give an odd number.

21. a $(28+10\sqrt{3})^{\frac{1}{2}}-\frac{1}{(7-4\sqrt{3})^{\frac{1}{2}}}$

$$=\left[5^2+(\sqrt{3})^2+2.5.\sqrt{3}\right]^{\frac{1}{2}}-\frac{1}{\left[2^2+(\sqrt{3})^2-2.2.\sqrt{3}\right]^{\frac{1}{2}}}=\left[(5+\sqrt{3})^2\right]^{\frac{1}{2}}-\frac{1}{\left[(2-\sqrt{3})^2\right]^{\frac{1}{2}}}$$

$$=5+\sqrt{3}-\frac{1}{2-\sqrt{3}}=5+\sqrt{3}-(2+\sqrt{3})=3$$

22. b $13^{36}=(13^3)^{12}=(2197)^{12}=(2196+1)^{12}$ So when it is divided by 2196 the remainder will obviously be 1.

23. d $19^n + 1$ can be written in two ways:

(i) $(18 + 1)^n + 1$ (ii) $(20 - 1)^n + 1$

Now check with the options.

24. d We need to find odd multiples of $11 \times 3 = 33$

$\therefore \frac{200}{33} \approx 6.06$ and $\frac{400}{33} \approx 12.1$

$\therefore$ We have 3 such numbers, i.e. $33 \times 7 = 231$

$33 \times 9 = 297$

$33 \times 11 = 363$

25. e Any prime number greater than 3 is always of the form 6k + 1 or 6k - 1.

26. c Number of boxes = 2 × 3 × 4 + (3 × 4) + 4 + 1 = 41

27. d m^3 and n^3 have the same signs as m and n respectively have. So (d) will definitely hold good.

28. b Each of the numbers can be written as a multiple of 111. The factors of 111 are 111, 37, and 3.

29. b Check with the options. Option (b) can be written $2^2, 3^2, 4^2, 5^2$... so on, which is not an arithmetic progression.

30. e The product of any n consecutive numbers would be divisible by the product of the 1st n consecutive natural numbers, i.e. n!

31. d Work with the choices.

$\frac{24}{6} = 4$ and 24 × 2 = 48

Digits when reversed, number is 42.

So the difference is (48 - 42) = 6

32. d I. It is divisible by a - b as well.

II. True if n is even.

III. Not a perfect square. If square must end in 25 at least.

IV. True

33. a Every prime number greater than 3 can be written

6N + 1 or 6N - 1. Hence, $P^2 + 17 = (6N)^2 + 1 \pm 12N + 17$.

So when divided by 12, it must leave a remainder 6.

34. e $\because \left[\frac{100}{5}\right] + \left[\frac{100}{25}\right] = 20 + 4 = 24$

100! has 24 zeroes.

100! + 200! = 100![1 + 101 × 102 × ... × 200]

which will again give 24 zeroes at the end.

35. a The numbers possible are: 102, 111, 120, 123, 132, 135, 147, 153, 159, 174, 195.

Practice Exercise 3 - Level 2

1. b Sum of squares of n natural numbers = $\frac{n(n+1)(2n+1)}{6} = \frac{10 \times 11 \times 21}{6} = 385$

2. a (a) evaluates to 216, the maximum.

3. e $(3^5)^{4/5} = 3^4$ is the smallest.

4. d 2 + 4 + +6 + 8 + ... + 100 (50 even numbers)

50 × (50 + 1) = 2550.

Sum of first n even number is n (n + 1)

5. b Using BODMAS, we get 181.5 as answer.

6. d Square root of 1296 = 36 not 34.

7. d $\because 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$

$\therefore 1 + 2 + 3 + ... + 45 = \frac{45 \times 46}{2} = 1035$

8. a By observing $\frac{4}{5} > \frac{3}{5} > \frac{2}{5} > \frac{1}{5}$

Numerator is decreasing and denominator is same.

Now we have to compare $\frac{4}{5}$ and $\frac{7}{15}$

$\frac{4}{5} = \frac{4\times3}{5\times3} = \frac{12}{15} > \frac{7}{15}$

$\therefore$ $\frac{4}{5}$ is the greatest.

9. d LCM of 2, 3, 4, 5, 6, 7 = 420

Smallest 4 digit-number divisible by LCM = 420 × 3 = 1260.

10. d x + y = 2 (x - y); xy = 27.

Only option (d) gives xy = 27.

11. d The largest fraction is $\frac{5}{6}$.

12. d Numbers are 8, 9, 10

$\therefore$ Sum = 8 + 9 + 10 = 27.

13. b Every such number must be divisible by LCM of 4, 5, 6, i.e., 60.

Such numbers are 240, 300, 360, 420, 480, 540.

There are 6 such numbers.

14. a Put n as 2, 4, 6, etc.

15. c $3 + \frac{3}{3 + \frac{1\times3}{10}} = 3 + \frac{3\times10}{33} = 3 + \frac{10}{11} = \frac{43}{11}$.

16. d Considering the unit's digit only, in numbers like 509, 519, 529 ... 589, 9 occurs 9 times.

Considering the ten's digit, from 590-598, (590, 591, 592 ..... 598) 9 occurs 9 times.

Hence, there are 9 + 9 = 18 numbers between 500 and 600 that contain the digit 9 exactly once.

17. b We know that 2 × 5 = 10 (one zero)

The given product can be written as $33 \times (5^2 \times 7) \times (2^2 \times 5 \times 9) \times (2^2 \times 3) \times (2^2 \times 11) \times (2^4 \times 5) \times (2 \times 11 \times 3)$

The powers of 2 and 5 are $2^{11} \times 5^4$

Only when one 5 is multiplied with one 2 a zero will be produced.

So there are 4 zeros.

18. a $56^{56} + 56$ can be written as $(57 - 1)^{56} + 56$

All the terms except $(-1)^{56}$ of $(57 - 1)^{56}$ will be divisible by 57, so remainder will be 1.

Once 56 is added to $(57 - 1)^{56}$ the sum will be divisible by 57. So remainder will be 0.

19. d Suppose the numbers are 2n, 2n + 2 and 2n + 4, where n is a whole number.

2n(2n + 2) (2n + 4) = 2 × 2 × 2 (n) (n + 1) (n + 2)

Now n(n + 1) (n + 2) is product of three consecutive natural numbers and at least one of them will be divisible by 2 and at least one of them will be divisible by three.

$\therefore$ n(n + 1) (n + 2) is divisible by 6.

$\therefore$ 8(n) (n + 1) (n + 2) will always be divisible by 48.

20. a Since LCM = 102 = $2 \times 3 \times 17$

Therefore, 17 has to be a component of at least one of the numbers. Only choice (a) or (e) will fit. By checking we get the answer as option (a).

Alternative method:

The choices (b), (c) and (d) will have a factor of 5 in the LCM {they have multiples of 5 in numbers} so the only logical choice is (a) or (e).

21. a $21600 = 2^3 \times 3^3 \times 10^2$; we need another pair of 2×3 so that it becomes a perfect square. Hence, it needs to be multiplied by $2 \times 3 = 6$.

22. b From 259 to 458 there are two hundred natural numbers and so there will be $2 \times 20 = 40$ 8's. From 459 to 492 we have 13 more 8's and so answer is $40 + 13 = 53$.

23. a $n^3 + 2n = n(n^2 + 2)$

Any natural number is either of the form 3m or 3m ± 1, where m is a whole number.

If n = 3m, then $n(n^2 + 2)$ is definitely divisible by 3.

If $n = 3m \pm 1$ then,

$n^2 + 2 = (3m \pm 1)^2 + 2 = 9m^2 \pm 6m + 1 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$

$\therefore n(n^2 + 2) = (3m \pm 1)[3(3m^2 \pm 2m + 1)]$, i.e. divisible by 3.

Alternative method:

Try with some natural numbers and get the answer.

24. d $N = 238a + 79 = 17 \times 14a + 17 \times 4 + 11$

Hence, on dividing N by 17, the remainder is 11.

Note: This method would have failed had 17 not been a factor of 238.

25. b All the terms except the last one are multiples of 16 Last term = 1

Hence, remainder when 17^{23} is divided by 16 = 1

26. c $9^6 = (8 + 1)^6$ So remainder is 1.

So in $9^6 + 1$ the remainder is 2.

27. c $N = 2^{50} \times 50!$ The highest power of 5 in 50! is 12, therefore there will be 12 zeroes at the end of N.

28. a If $2^{32} + 1 = a + b$

Since $a^3 + b^3 = (a + b)^3 - 3ab(a + b) = (a + b)[(a + b)^2 - 3ab]$

So $(a^3 + b^3)$ is divisible by same numbers as $(a + b)$.

29. c $4^{61}(1 + 4 + 16 + 64 + 256) = 4^{61}(341)$, 341 is divisible by 11.

30. c The answer should be a multiple of LCM (8, 9,10) = $360 = 2^2 \times 3^2 \times 2 \times 5$

In order to make it a perfect square, we need to multiply (2×5) to it, i.e. $360 \times 2 \times 5 = 3600$

Practice Exercise 4 - Level 2

1. b LCM of 7, 10, 15 = 210

Least multiple of LCM, less than 2024 = 1890 (210×9)

1890 + 3 = 1893 (3 is remainder)

Hence, 2024 - 1893 = 131 is the least number.

2. e The data is inconsistent as the LCM is always a multiple of HCF. Here HCF = 75 and LCM = 216 do not show consistency, i.e. no such numbers are possible.

3. a By divisibility rule of 3, 4 +5 + a + 6 = multiple of 3

⇒ a = 3 or 6 or 9 [since `a' is a single-digit number]

By divisibility rule of 11

(4 + 5) - (6 + a) = multiple of 11 or zero.

⇒ a = 3

∴ Only a = 3, is the right answer.

4. b Best way is to check the options, i.e.

(a) is not possible.

(b) $\frac{1}{24} \times 72 = 3$ cube of 3 = 27 which is the reverse of 72.

(c) $\frac{1}{24} \times 48 = 2$, cube of 2 = 8 which is not a two-digit number.

5. e aaaaaa = aaa × 1000 + aaa = aaa (1000 + 1) = 1001 (aaa) = 7 × 11 × 13 (aaa)

This is obviously divisible by 7, 11 and 13.

6. a LCM of 2, 3, 4, 5 and 6 = 60

On dividing 1000 by 60 remainder is 40

Hence, closest number to 1000 divisible by 2, 3, 4, 5 and 6 = (60 - 40) + 1000 = 1020

Note: (1000 - 40) = 960 is also divisible by 60, but 1020 is closer to 1000.

Alternative method:

Only choice a is divisible by three.

7. b The greatest number is formed by writing the digits in descending order, i.e. 54320

The least number is reverse of this.

(Only the first digit has to be non-zero), i.e. 20345

Now 54320 - 20345 = 33975

8. c Working backwards,

C = D + 6

B = C + 9 = D + 15

A = B + 13 = D + 28

Hence, A + B + C + D = 4D + 49 = 417

D = 92

So, A = 120

9. d To convert from base 6 to base 10, multiply each digit in base 6 number.

$(1234)_6 = (4 \times 6^0 + 3 \times 6^1 + 2 \times 6^2 + 1 \times 6^3)_{10} = (4 + 18 + 72 + 216)_{10} = (310)_{10}$

10. a N=(29× 5+3)× (29×10+3)×(29×20+1) × (29×30+4)

In expansion of this only the last term, i.e.

3 × 3 × 1 × 4 = 36 is not divisible by 29 and will leave a remainder of 7.

11. c The total multiples of 31 in 1000! = $\frac{1000}{31} = 32$

(31, 62, 93, ... 992)

The total number of multiples of $31^2 = 961$

In (1000)! = $\frac{1000}{961} = 1$

∴ Total power of 31 in 1000! = 32 + 1 = 33

12. a If n is even, then even + even + 1 = odd

If n is odd, then odd + odd + 1 = odd

Therefore, $n^4 + n^2 + 1$ is always odd

13. a LCM of powers = 12

The numbers are equal to $(125)^{\frac{1}{6}}, (121)^{\frac{1}{6}}, (123)^{\frac{1}{6}}$

Thus, $\sqrt{5}$ is the greatest amongst these three numbers, comparing $\sqrt{5}$ with other numbers, amongst $(25)^{\frac{1}{4}}$ and $(36)^{\frac{1}{4}}, 36^{\frac{1}{4}}$ is greater.

Finally, comparing $(36)^{\frac{1}{4}}$ and $\sqrt[12]{27} = (3)^{\frac{1}{4}}; \sqrt[4]{36}$ is the greatest of all the numbers.

14. d $(1556)_{10} = (?)_{16}$

$$\begin{array}{r|l}
16 & 1556 \\ \hline
16 & 97 \quad \longrightarrow 4 \\
 & 6 \quad \longrightarrow 1
\end{array}$$

∴ $(1556)_{10} = (614)_{16}$

15. c Follow the instructions given in question 14 and convert base 7 to base 10

i.e. $(413)_7 = (3 \times 7^0 + 1 \times 7^1 + 4 \times 7^2)_{10} = (206)_{10}$

Now convert it to base 8

$$\begin{array}{c|c|c}
8 & 206 & \\ \hline
8 & 25 & 6 \\ \hline
 & 3 & 1 \\ \hline
\end{array}$$

∴ $(206)_{10} = (316)_8$

Hence, $(413)_7 = (316)_8$

16. b Squaring the three expressions given, we get

$10 + 2\sqrt{21}, 10 + 2\sqrt{25}, 10 + 2\sqrt{24}$, $10 + 2 \times 4 = 18$

Therefore, (b) is the greatest.

17. d Suppose the number is ab.

We have $(10a + b) + \frac{75}{100}(10a + b) = 10b + a$

Hence, $\frac{b}{a} = \frac{2}{1}$

18. a $N = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \cdots + \frac{1}{12} - \frac{1}{13} = 1 - \frac{1}{13} = \frac{12}{13}$

19. c If ten's digit is 1, we have two such numbers, viz. 10 and 11.

When ten's digit is 2, we have three such numbers, viz. 20, 21 and 22.

...

...

When ten's digits is 9, we have 10 such numbers, viz. 90, 91...99.

$\therefore$ Total number of such numbers is 2 + 3 + 4 + ... + 10 = 54

20. c Any power of 5 when divided by 4 gives a remainder of 1.

Here the power of 3 is itself a power of 5 and will give a remainder of 1 when divided by 4.

The last digit of the number will be $3^1 = 3$.

And hence, last digit of the given number is 3 + 1 = 4.

21. c All even powers of 3 are of the form 4n + 1

$\therefore$ Last digit of first term is 2 also all odd powers of 3 are of the form 4n + 3

$\therefore$ Last digit of the term is 8 {By cyclicity of last digit}

Thus, the last digit of expression is 2 - 8 = 4

22. a The 50th term = $\frac{1}{\sqrt{99}+\sqrt{101}}$

= The given series

$$= \frac{\sqrt{1}-\sqrt{3}}{(\sqrt{1}-\sqrt{3})\ (\sqrt{1}+\sqrt{3})} + \frac{\sqrt{3}-\sqrt{5}}{(\sqrt{3}-\sqrt{5})\ (\sqrt{3}+\sqrt{5})} + \cdots + \frac{\sqrt{99}-\sqrt{101}}{(\sqrt{99}-\sqrt{101})\ (\sqrt{99}+\sqrt{101})}$$

$$= \frac{\sqrt{1}-\sqrt{3}}{-2} + \frac{\sqrt{3}-\sqrt{5}}{-2} + \frac{\sqrt{5}-\sqrt{7}}{-2} + \cdots + \frac{\sqrt{99}-\sqrt{101}}{-2}$$

$$= \frac{1}{-2}\left[\sqrt{1}-\sqrt{3}+\sqrt{3}-\sqrt{5}+\cdots+\sqrt{99}-\sqrt{101}\right] = \frac{1}{-2}\left[\sqrt{1}-\sqrt{101}\right] = \frac{\sqrt{101}-\sqrt{1}}{2}.$$

23. c 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 100

We know that LHS = 55 thus by changing `+' to `×' we have to increase the LHS by 45 to make it equal to RHS.

If we replace the last `+' sign we increase the sum by 9 × 10 - 10 - 9 = 71 which is too high.

Same way for last but 1 increase is of 55 which is again too high.

Same way for last but 2 increase is of 7 × 8 - 7 - 8 = 41

Thus, by replacing one `+' we cannot get the desired increase.

$\therefore$ We change one more `+'.

We see that by replacing first and third `+' we get the desired increase.

24. c $\sqrt{\frac{(12.12)^2-(8.12)^2}{(0.25)^2+(0.25)(19.99)}} + \frac{\left[\left(8^{-\frac{3}{4}}\right)^{\frac{5}{2}}\right]^{\frac{8}{15}} \times 16^{\frac{3}{4}}}{\sqrt[3]{\left[\left((128)^{-5}\right)^{\frac{3}{7}}\right]^{\frac{-1}{5}}}} = \sqrt{\frac{(12.12+8.12)(12.12-8.12)}{(0.25)(0.25+19.99)}} + \frac{\left[\left((2^3)^{\frac{-3}{4}}\right)^{\frac{5}{2}}\right]^{\frac{8}{15}} \times 16^{\frac{3}{4}}}{\sqrt[3]{\left[\left((2^7)^{-5}\right)^{\frac{3}{7}}\right]^{\frac{-1}{5}}}}$

$$= \sqrt{\frac{(20.24)\ (4)}{\frac{1}{4}\ (20.24)}} + \frac{(2)^{\frac{-45}{8}\times\frac{8}{15}} \times 2^3}{\sqrt[3]{(2^{-5})^{\frac{-3}{5}}}} = 4 + \frac{1}{2} = 4.5 = \frac{9}{2}$$

25. d Option (a) becomes false if all are negative integers.

Option (b) becomes false if e > 0 and rest all are negative.

Option (c) is not necessarily the middle number or need not be the average, e.g. 0, 1, 2, 3, 1000

Option (d) is always true.

26. b From 100 to 200 there are 101 numbers. There are 100 1's in the hundred's place.

10 1's in the ten's place

and 10 1's in the unit's place.

Practice Exercise 5 - Level 2

For questions 1 and 2:

As the number is divisible by 99 it is divisible by 11 also. Now sum of digits at odd and even places is 33 + Q and 16 + P respectively. Thus, 17 + Q - P is either 0 or 11 or 22 ... now Q > P. Thus, Q - P is +ve but < 9 as both are single digit numbers since Q - P > 0 hence 17 + Q - P cannot be 0 or 11 and can only be 22 as Q - P < 9 thus Q - P = 5

Now since it is divisible by 99 it is divisible by 9 too.

⇒ 49 + P + Q is divisible by 9

Thus, P + Q = 5 or 14. But as maximum value of Q is 9 and Q - P is 5.

Thus, maximum value of P + Q = 13 ∴ P + Q = 5

Thus, we get P = 0, Q = 5

1. c

2. a

3. c The smallest number that is made of 6's only and is divisible by 7 is 666666.

$\therefore M = \frac{666666}{7} = 95238$

∴ N = 9 + 5 + 2 + 3 + 8 = 27

Last digit of 27^{36} is 1.

4. b $7^{187} = (7^4)^{46} \times 7^3 = (2401)^{46} \times 343$

Now 2401 = 2400 + 1

So when $(2401)^{46}$ is divided by 800, the remainder must be 1. So the remainder when 7^{187} is divided by 800 is = 1 × 343 = 343

5. a a = 5p; b = 5q + 1; c = 5r + 2 and thus replace in the expression 2a + 3b - 4c to get the answer.

(Hint: Replace only the remainders to get the final remainder or put a = 0, b = 1 and c = 2)

6. b We see that PP must be less than 22, otherwise the cube of PP will have at least 5 digits. Hence, the only possible option for PP = 11 ⇒ PP^3 = 1331

Since Q (PP^3) = a four-digit number with ten's digit as one, hence Q × 3 ends within 1 ⇒ Q has to be 7.

7. b Let quotients obtained when this number is divided by 5 and 6, be x and y respectively.

Number = 5x + 2 = 6y + 1

given that = x - y = 3

5x + 2 = 6y + 1

or (6 -1) x + 2 = 6y + 1

or 6x + (2 - x) = 6y + 1

or 6(x - y) + 1 = x

or 6 × 3 + 1 = x

⇒ x = 19

⇒ Number = 19 × 5 + 2 = 97

Hence (b) is the correct option.

[**Note:** that x will be greater than y as 5 is smaller than 6]

8. b All the bells will be ringing simultaneously after every L min, where L = LCM (2, 3, 4, 6, 8, 9, 12) = 72 min. Then from 6 a.m. till 5 p.m. of the same day, nine times all 7 bells would ring simultaneously. (Except the first one at 6 a.m.)

9. e There are 10 numbers beginning with 6, i.e. 60 to 69. There are 9 numbers ending in 6. So total number of sixes is 19.

Alternative method:

In first 1 - 99 every digit (1 to 9) appears 20 times.

So 6 will not there for one time.

So total number of times when 6 appears = 19

10. b Whatever be the arrangements of numbers, smallest number selected will be 1.

11. c The largest of the largest numbers in each column is 25. Hence, 25 - 1 = 24.

12. e (A) $\sqrt{3}$ is not a rational number, therefore, $3\sqrt{3}$ is also not a rational number.

(B) Rational number multiplied by another rational number gives a rational number.

(C) Let, if possible $7 = x^3$, where x is a rational number. Therefore, $x = \sqrt[3]{7}$. But $\sqrt[3]{7}$ is not a rational number, which is the contradiction.

13. d The number of students will be of the form:

$(2 \times 3 \times 5 \times 7 \times 11)N + 1 = 2310N + 1$, where N is a natural number. For N = 1, 2, 3 and 4, the number of students will be a four-digit number.

14. b The best way to solve this is by reverse substitution. If you look at the answer choices, you will observe that only option (b) satisfies the given condition,

i.e. $24 = 3 \times (2 \times 4)$

15. c Check with digits A = 1, 2, 3, ... 9.

Note that for a perfect square number with ten's place odd, unit's place of the number must be 6.

$(AA)^2 = DCBA$, then $66^2 = 4356$

Then D = 4

16. b Divide 10×10^2, i.e. 1000 by 3×10^2, i.e. 300 remainder is 100, i.e. 1×10^2 and not 1.

so when, 798630×10^{24} is divided by 18×10^{24}, the remainder is 6×10^{24} (and not 6)

17. b LCM of 6, 7, 8 and 9 = 504

$\therefore$ All bells toll after 504 s.

$\therefore$ In 2 hr, number of times they would toll together = $\frac{2 \times 60 \times 60}{504} = 14.28$

$\Rightarrow$ If we start counting just before the moment all of them toll together, we would see them tolling 15 times.

18. c Since all x, y and z are non-zero whole numbers and xy = z, y > x.

So z > x; and $z \geq y$. Since numbers are distinct $y \neq z$.

So $z > x, z > y \Rightarrow z > x^3$ is correct answer.

19. c $a_3^2 - a_2^2 = 57 = 19 \times 3$ or 1×57

Case I: $(a_3 + a_2)(a_3 - a_2) = 19 \times 3$

$\therefore a_3 + a_2 = 19$ and $a_3 - a_2 = 3$

or $a_3 = 11, a_2 = 8$

Hence, $a_1 = 3$

Case II: $(a_3 + a_2)(a_3 - a_2) = 57 \times 1$

$\therefore a_3 + a_2 = 57$ and $a_3 - a_2 = 1$

or $a_3 = 29, a_2 = 28$

Hence, $a_1 = 1$

20. e I. Square never ends in 8.

II. e.g. $10^2 = 100$

III. e.g. 729

Hence, none of these.

21. a Before 4th operation: $\frac{4}{4} + 4 = 5$

Before 3rd operation: $\frac{5}{4} + 4 = \frac{21}{4}$

Before 2nd operation: $\frac{21}{16} + 4 = \frac{85}{16}$

Before 1st operation: $\frac{85}{64} + 4 = \frac{341}{64}$

So n = $\frac{341}{64}$

22. e LCM of 4, 5, 6 and 7 = 420

Smallest four-digit number = 1000

So essentially we need the largest three-digit number, which satisfies 420x + 2 = 842. (For x = 2)

Note:

Solution can also be done worked out the choices.

23. b $\frac{a}{b} = \frac{1}{5}$ ...(i)

As difference between two fraction is $\frac{1}{10}$, second fraction has to be $\frac{1}{10}$

$\left[\because \frac{1}{5} - x = \frac{1}{10}, \quad x = \frac{1}{10}\right]$

$\frac{a-2}{b+5} = \frac{1}{10}$

From (i), b = 5a

$\frac{a-2}{5a+5} = \frac{1}{10}$; $\frac{a-2}{a+1} = \frac{1}{2}$; a = 5

Alternative method:

Use choices to answer the question.

24. c $(51)^{51} = (51)^{50} \times 51 = [(51)^2]^{25} \times 51 = (2601)^{25} \times 51$

Here if we expand 2601 to any power, the last two digits will remain `01' and hence when multiplied by 51, it will give 51 as last two digits of the expression. Hence, ten's digit = 5

25. c Clearly, the original number is a multiple of 9. This eliminates option (b), (d) and (e).

Through elimination process, (c) is the correct answer.

$\frac{10125}{9} = 1125$ (we can get after erasing zero).

1125 = 9 × 1 + 25 (Multiple of 9)

26. b Digit erased is 0.

27. d The number formed by last three digits must be 375. Then for first two places, each place has 5 options.

Thus, 5 × 5 = 25 numbers are possible.

28. c Answer is (c), since there is one duck in-between two ducks both in front and behind, therefore the least number of ducks that could swim is obviously 3.

29. a The 2 digits at unit & tens place can be only 2 or 3.

∴ The hundredth digit must be 6. So the no. can be 623 or 632 on reversing the digits the nos. become 326 or 236. But the difference between 623 & 326 is 297 .

30. a Take 1 coin from bag 1, 2 from bag 2, 3 from bag 3 ... 10 from bag 10. By weighing in a spring balance the deviation from the expected weight will help us identify the defective bag.

e.g. Deviation = 6g, implies it is bag 6.

31. e 90 × 80 × ... × 10 can be expressed as a power of 5. The exponent of 5 would be 5^{10}.

32. b Parity of the first page and last page will not be the same. Since the first page number is 123, i.e. odd, the last page can be 132 or 312, i.e. even.

33. e $N_1 = 222x + 35$, $N_2 = 407y + 47$

$N_1 + N_2 = (37 \times 6 \times x + 35) + (37 \times 11 \times y + 47)$

⇒ Remainder when $N_1 + N_2$ is divided by 37 = $\frac{(35+47)}{37} = 8$

34. c All the numbers from 200 to 299 begin with 2, i.e. 100 numbers and 10 numbers between 100 and 200

(102, 112, ..., 192) end with 2.

So number of such numbers = 100 + 10 = 110.

Practice Exercise 6 - Level 3

1. b Difference between divisor and remainder = 35 - 25 = 45 - 35 = 55 - 45 = 10

LCM of 35, 45, 55 = 5 × 7 × 9 × 11 = 3465

Required Number = 3465 - 10 = 3455

Short cut:

3455 is the only option from which when we subtract 35, we get 3420 which is divisible by 45.

2. a LCM of 2, 3, 5 = 30

∴ number of coconuts = 30 + 1 = 31

3. b $x = 5 - \sqrt{7}$

$\therefore \frac{1}{x} = \frac{1}{5-\sqrt{7}} = \frac{5+\sqrt{7}}{(5-\sqrt{7})(5+\sqrt{7})} = \frac{5+\sqrt{7}}{18}$

$\therefore x + \frac{1}{x} = \frac{95}{18} - \frac{17\sqrt{7}}{18}$

4. a $\frac{A}{2} = \frac{B}{3} = \frac{C}{4} = x \Rightarrow A = 2x,\ B = 3x \text{ and } C = 4x$

$\Rightarrow$ A : B : C = 2 : 3 : 4

Largest part = $\left(243 \times \frac{4}{9}\right) = 108$

5. b Substitute n = 1.

6. b LCM of 3, 6, 9, 12, 15, 18 is 180

So, the bells will toll together after every 180 s, i.e. 3 min.

In 30 min, they will toll together for $\left(\frac{30}{3}\right)+1=11$ times.

7. d n = 5k + 2

$\Rightarrow n^2 = 25k^2 + 10k + 4$

$\frac{n^2}{5} = 5k^2 + 2k + \frac{4}{5}$

Hence the remainder when n^2 is divided by 5 is 4.

8. a Substitute n = 1, 2

9. d y = $\sqrt{2}$ + 1 $\Rightarrow y + \frac{1}{y} = \frac{y^2+1}{y} = \frac{(\sqrt{2}+1)^2+1}{(\sqrt{2}+1)} = \frac{(4+2\sqrt{2})}{\sqrt{2}+1} = \frac{2\sqrt{2}(1+\sqrt{2})}{(\sqrt{2}+1)} = 2\sqrt{2}$

Short cut:

$(\sqrt{2}+1) + \frac{1}{\sqrt{2}+1} = (\sqrt{2}+1) + (\sqrt{2}-1) = 2\sqrt{2}$

10. c Let ten's digit be x. Then, unit's digit = (x + 3)

Sum of the digits = x + (x + 3) = 2x + 3

Number = 10x + (x + 3) = 11x + 3

$\frac{11x+3}{2x+3} = \frac{4}{1} \Leftrightarrow 11x + 3 = 4(2x+3) \Leftrightarrow x = 3$

$\therefore$ Number (11x + 3) = 36

Alternative method:

Check the conditions given in the question in the options.

11. c This question is to be done in two steps:

(a) make the bases equal

(b) equate the powers and solve

By equation, we have

$32^{x-2} = \frac{64}{8^x} \Rightarrow (2^5)^{x-2} = \frac{2^6}{(2^3)^x} \Rightarrow 2^{5x-10} = 2^{6-3x}$

$\therefore$ Equate the powers and solve $\Rightarrow$ 5x - 10 = 6 - 3x

$\Rightarrow$ x = 2

12. c The total number of multiples of 5 in 30! = 6 (5, 10 ... 30)

The total number of multiples of $5^2 = 25$ in 30! = 1 (25 only)

Further powers of 5 cannot be there as $5^3 = 125 > 30$

$\therefore$The greatest power of 5 that divides 30! exactly = 6 + 1 = 7

13. a Suppose numbers are x and y, we have

x(x + y) = 3666 ... (i)

y(x + y) = 2418 ... (ii)

Adding (i) and (ii), we get

$(x + y)^2 = 6084$

$\therefore x + y = 78$ ... (iii)

Subtracting (ii) from (i), we get

$x^2 - y^2 = 1248$

or $(x + y)(x - y) = 1248$

or $78(x - y) = 1248$ from (iii)

$\therefore x - y = 16$

14. e Perfect square just above $20 = 25 = 5^2$

Perfect square just below $2000 = 1936 = 44^2$

Hence, there are $44 - 5 + 1 = 40$ perfect squares

(20 even and 20 odd).

$\Rightarrow$ Out of $2000 - 20 + 1 = 1981$ numbers (990 odd and 991 even), 20 are even perfect squares. The
other 971 even numbers are not perfect squares.

15. b Since HCF is 39

$\therefore$ Let numbers be 39a and 39b

(a and b are coprimes)

We have,

$39a \times 39b = 15210$, or $ab = 10$

There are only two pairs of natural numbers whose product will be 10.

(1, 10) and (2, 5) are the pairs.

16. c We can safely ignore the first 2 digits (as they are lost in subtraction), this gives us {for last 2 digits).

$(10c + d) - (10d + c) = 54$

$\Rightarrow 9(c - d) = 54 \Rightarrow c - d = 6.$

17. e If we factorize the number $161 = 23 \times 7$

Now there are many pairs of x and y.

LCM of which is 161.

(i) 7, 23 (ii) 7,161 (iii) 23,161

Hence, answer is (e).

18. c AA + BB = CDC

As sum of 2 two-digit numbers cannot be greater than 200 so C = 1

Now, taking different combinations of AA and BB, e.g. (44, 77), (88, 33), (99, 22), (66, 55) etc., so that C = 1, we see that in all such cases D = 2.

Hence, the only value of D is 2.

19. b Suppose the number is ab.

We have $a + b = 5$ ... (i) and

$2(10a + b) - (10b + a) = 41$ ... (ii)

Solving (i) and (ii), we get

$a = 3$ and $b = 2$

$\therefore$ 32 is the number and 40% of this is 12.8.

Alternative method:

We can multiply the options by $\frac{100}{40}$ (as it is 40% of the number), and then check for properties.

(a) $11.8 \times \frac{100}{40} = 29.5$ (not a whole number)

(b) $12.8 \times \frac{100}{40} = 32$ (satisfies the given conditions) so the answer is (b).

20. a Let us assume that the numbers are 13x and 13y

$\therefore$ Product of the two numbers = HCF × LCM = 13x × 13y

$13x \times 13y = 13 \times 455 \therefore xy = 35$

The numbers can be

13×1, 13×35 or 13×5, 13×7

Only one number lies within the given limits, $13 \times 7 = 91$. Hence, the answer is (a).

Alternative method:

Only option (a) is a factor of 455.

21. e Let us assume the middle number to be x.

Then the three consecutive numbers are

$(x - 1)$, x and $(x + 1)$

$x^2 = (x - 1)(x + 1) + 1$

$x^2 = x^2 - 1 + 1$ have become equal.

Hence, this condition will hold for every number.

22. a $1^1 \times 2^2 \times 3^3 \times 4^4 \times \ldots 100^{100}$

Zeroes will be formed by multiplication of 2 and 5.

So, we have to calculate the powers of 5 only as 5 is the higher of the two primes.

Now power of $5 = 5^5 \times 10^{10} \times 15^{15} \ldots 25^{25} \ldots 100^{100}$

i.e. $5 + 10 + 15 + 20 + 25 \times 2 + \ldots + 75 \times 2 \ldots + 100 \times 2$

Here, 25×2, 50×2, 75×2 and 100×2 are there because $25 = 5^2$

So total power of 5

$= (5 + 10 + \ldots + 100) + (25 + 50 + 75 + 100) = 1050 + 250 = 1300$

23. b x can have only one value, i.e. 2.

2 is the only even prime number.

The square of an even number is even.

When 3 is added it becomes odd (7 in this case).

For all other prime numbers the square is odd, but on adding 3 to them, the resultant number is a multiple of 2, and hence ceases to be prime.

24. d In unit's place we will have all the numbers from 0 to 9 ten times each. In ten's place again we will have all the numerals from 0 to 9 ten times each.

In hundred's place we will have 99 one's and 1 two's.

$\therefore$ The sum will be $10(0+1+\cdots+9)+10(0+1+\cdots+9)+99\times1+1\times2=1001$

25. c $54 = 2 \times 27$

The highest power of 3 in 31! is 14.

$\therefore$ The highest power of 27 will be 4.

$\Rightarrow$The highest power of 54 will be 4.

Alternative method:

As $54 = 2 \times 27 = 2 \times 3^3$

We count the highest power of 3 (14) in 31! and take one-third of that (14) {only quotient part} to find the highest power of 3^3 in 31!

Practice Exercise 7 - Level 3

1. c Convert the part before the decimal.

$(1101)_2 = 1\times2^0+0\times2^1+1\times2^2+1\times2^3 = (13)_{10}$

For the part after decimal take negative powers of 2

$\therefore\ (0.011)_2 = \left(0\times2^{-1}+1\times2^{-2}+1\times2^{-3}\right)_{10} = (0.375)_{10}$

$\therefore$ The answer is 13.375.

2. a $\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\cdots+\frac{19}{8100}$

$$1-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{3}\right)^2+\left(\frac{1}{3}\right)^2-\left(\frac{1}{4}\right)^2=+\cdots+\left(\frac{1}{9}\right)^2-\left(\frac{1}{10}\right)^2=1-\frac{1}{100}=\frac{99}{100}=0.99.$$

Alternative method:

$$\frac{3}{4}+\frac{5}{36}+\frac{7}{144}+\cdots+\frac{19}{8100}$$

The denominators are of the form

$4 = 2^2\times1^2$, i.e. $n^2\times(n-1)^2$

$36 = 3^2\times2^2$

$144 = 4^2\times3^2$

$\vdots$

$8100 = 10^2\times9^2$

And the numerator is of the form n + (n - 1)

$3 = 2 + 1$

$5 = 3 + 2$

$7 = 4 + 3$

$\vdots$

$19 = 10 + 9$

$$\therefore \frac{n+(n-1)}{n^2\times(n-1)^2}=\frac{1}{(n-1)^2}-\frac{1}{n^2}$$

Thus, the series can be divided as

$$\frac{3}{4}+\frac{5}{36}\cdots\frac{19}{8100}=1-\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2-\left(\frac{1}{3}\right)^2\cdots$$

$$=1-\left(\frac{1}{10}\right)^2=0.99$$

3. d $(12630)_x = (3402)_{10}$

$\Rightarrow (3402)_{10} = \left(0\times x^0+3\times x^1+6\times x^2+2\times x^3+1\times x^4\right)_{10} = \left(3x+6x^2+2x^3+x^4\right)_{10}$

Now we have 3 choices for x. We substitute from the given answer choices and check for last digit. We see that 9 gets eliminated.

If we put $x = 7$, it satisfy Hence correct option will be (d).

4. c Check choices. Working backwards with the smallest choice 9, we get inconsistent data. Working with 12 we get 28 apples in the beginning. Hence, initially there must have been 28 apples. Therefore, taking option (c), we arrive at consistent solution.

5. d $abc = a! + b! + c!$

$= 100a + 10b + c = a! + b! + c!$

Now $6! = 720$ and $7! = 5040$. If 7 is one of digits, then the sum of the factorials become a four-digit number or more. Hence, the numbers 7, 8, 9 can be neglected.

Consider $6! = 720$. But number 7 cannot be there in hundred's place.

Hence, we can neglect 6 also.

Now

$5! = 120$, $4! = 24$, $3! = 6$, $2! = 2$, $1! = 1$ and $0! = 1$. To get a three-digit number, 5 has to be present in the number. But 5 cannot be in hundred's place as then the number becomes greater than 500 which cannot be obtained as the sum of factorials.

Also, maximum possible number is $5! + 4! + 3!$

$= 120 + 24 + 6$. Also, `a' cannot be 0 as it is a three-digit number. Hence, $a = 1$.

Then different possible cases are 154, 153, 152, 125, 135, 145.

From this we find that only 145 satisfies given condition.

$\therefore (b+c)^{a} = (4+5)^{1} = 9$

Alternative method:

$abc = a! + b! + c! = 100a + 10b + c = a! + b! + c!$

Since $6! = 720$ and $7! = 5040$

None of the numbers is equal to or greater than 6.

Also amongst b and c either one has to be 5 as abc is a three-digit number and $5! = 120$ and $4! = 24$

Also $b + c$ can have maximum value of 10 (given that none of them is more than 5) and a can at best be 2 as $a! + 5! + 5! = 240 + a!$ so the only option possible is (d).

6. e Since $300 < abc < 400$

$\Rightarrow 100a + 10b + c = a^3 + b^3 + c^3$

Put $a = 3$ and rearrange, we get

$300 + 10b + c = 27 + b^3 + c^3$

$\Rightarrow (b^3 - 10b) + (c^3 - c) = 273$

Now put $b = 6$ and 7 (as b and c are both less than or equal to 9 and 6^3 and 7^3 are the nearest cubes to 273.) For $b = 6$,

We get $c^3 - c = 117$ and no value of c satisfies this.

For $b = 7$ we get $c^3 - c = 0$ and two values of c, i.e. $c = 0$ and $c = 1$ satisfies this.

Thus, $abc = 370$ or 371

Hence, abc cannot be uniquely determined.

7. b Pairs are (0, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), (7, 1), (8, 1) and (9, 1).

8. c Suppose the numbers are M and N and the divisor

is D.

We have $M = DQ_1 + 547$

$N = DQ_2 + 349$

$M + N = D(Q_1 + Q_2) + 896$

When M + N is divided by D it leaves a remainder of 211, i.e. $D(Q_1 + Q_2) + 896 - 211$.

$= D(Q_1 + Q_2) + 685$ is divisible by D. It means 685 is divisible by D. But we know that D > 547, because divisor is always greater than the remainder. So only possible value of D is 685.

9. b Any perfect square x can always be written as $x = a^m \times b^n \times c^p \times \ldots$ where a, b, c are prime factors and m, n, p are all even. Now total number of factors of x = (m + 1) (n + 1) (p + 1) ...

$= \text{odd} \times \text{odd} \times \text{odd} \times \cdots$ = An odd number.

10. b Concept:

We can see that by symmetry n = p and hence all we need to calculate is n and m.

n = 280 and m = 180

∴ 2n - m = 380

11. a A = 185, B = 14 and C = 5

Hence, $C^3 = 125$

12. c If (P - 7) is a multiple of 11, (P + 4) and (P + 15) must be multiple of 11 as well because P + 4 = (P - 7) + 11 and P + 15 = (P - 7) + 22. Since (P + 4) and (P + 15) are consecutive multiples of 11, so one of them must be an even number. Hence, (P + 4) (P + 15) will always be divisible by 11 × 11 × 2 = 242

13. e Surprised! Many of you must have answered (c) but that will be the case if p is a prime number.

Hence, from given information we cannot say anything about q. It may be the case that p = 64 and q = 1.

14. a 83! - 82! = 83.82! - 82!

= 82! (83 - 1) = 82.82!

Now to check highest power of 82 in 82! we just have to check the highest power of 41 in 82!. So, highest power of 82 in 82! is 2. Hence, highest power of 82 in 82. 82! will be 3.

15. c $(1923)^{1924^{1925}} = (1924 - 1)^{\text{some even power} = x}$

∴ Since all but last term are divisible by 1924.

⇒ Remainder = Last term = ${}^xC_0 \times (1924)^0 \times (-1)^x = 1$

16. e Suppose the number is abcde, when it is added to edcba the sum will be

(10001a + 1010b + 200c + 1010d + 10001e).

Now whether the sum is divisible by 11 or 11111 or 101 completely depends on the values of a, b, c, d and e.

17. c ab + ba = 11(a + b)

abcd + dcba = 1001(n + d) + 110(b + c)

= 11[91 (n + d) + 10(b + c)]

Similarly, we can check for all even digit numbers as in question 37. So k must be multiple of 11.

18. c ab × cd = (10a + b) (10c + d)

$= [40b + 10a - 39b]\,[51d - (50d - 10c)]$

$= (13 \times 10 \times k_1 - 39\,b)\,(51\,d - 17 \times 10 \times k_2)$

$= 13 \times 17\,(10\,k_1 - 3b)\,(3d - 10k_2)$

Hence the largest number will always divide the product of ab and cd is 221.

19. e $A = \{a_1, a_2, a_3, \ldots\, a_n\}$, $n \geq 8$

means there will be atleast 8 natural numbers in the set A. But it doesn't mean that $a_1 < a_2 < a_3 \ldots < a_n$.

i.e. It doesn't give any idea about comparision of those numbers.

$\therefore$ (a), (b), (c), (d) could be negative also.

Thinking about option (d) $(a_{n-1} - a_n)^2$, will be always +ve.

20. c HCF of two distinct natural numbers can never be a multiple of their LCM.

For questions 21 and 22:

As Y : S = 1 : 4

So Y = 1 and S = 4 or Y = 2 and S = 8 but Y = 1 and S = 4 is not possible because in that case V = 10, which is not possible.

8	1	6
7/3	5	3/7
4	9	2

21. e

22. a

For questions 23 and 24:

$V_n + V_{n+1} = K \qquad 1 \leq n \leq 999$

$V_1 + V_2 = K = V_2 + V_3 = V_3 + V_4 = \ldots$

$V_1 + V_2 = V_2 + V_3$ and $V_2 + V_3 = V_3 + V_4$

$\Rightarrow V_1 = V_3$ and $V_2 = V_4$

$\therefore V_1 = V_3 = V_5 = \ldots = V_{999}$ = Odd subscript terms

and $V_2 = V_4 = V_6 = \ldots = V_{1000}$ = Even subscript terms

Now $V_{987} = 987 \Rightarrow V_{odd} = 987$

$\therefore V_{236} = K - V_{237}$, i.e. $K - V_{odd} = K - 987$

Again $V_{100} = 100 \Rightarrow V_{even} = 100$

$V_{odd} = K - 100$

$\therefore V_{10} + V_{11} + V_{12} + V_{13} + V_{14} - V_{15} - V_{16} - V_{17} - V_{18} - V_{19}$

$= [V_{10} + V_{12} + V_{14} - V_{16} - V_{18}] + [V_{11} + V_{13} - V_{15} - V_{17} - V_{19}]$

$(V_{even} - V_{odd})$ [After canceling terms]

$= 100 - (K - 100) = 200 - K$

Also $= V_{14} - (K - 100) = V_{14} + 100 - K$

23. c

24. d

25. b $a = 0$; $d = odd$

d can be equal to 3, 5 only.

$3! = 1 \times 2 \times 3$

$c(3) = a(1) + b(2)$ or $a(2) + b(1)$

$5! = 3 \times 5 \times 8$

$c(8) = a(5) + b(3)$ or $a(3) + b(5)$

But $c = 8$ because for this condition II is violating.

So c cannot take 8.

$5! = 4 \times 5 \times 6$

But in this case $c = a + b$

So $c = 6$

So c can take only 3.

26. b (i) for e = 0, then d = ?

Let d = 1, 1 - 1 + 1 = 1 - 0

a = 1 b = 1 c = 1 d = 1

For d = 5 as $5! = 4 \times 5 \times 6$

4 - 5 + 6 = 5 - 0

So for e = 0, d can take 1 and 5.

So first condition is not true.

(ii) for d = 5 numbers are

a = 3	a = 4	a = 4	a = 5
b = 8	b = 5	b = 6	b = 6
c = 5	c = 6	c = 5	c = 4
e = 5	e = 0	e = 2	e = 2

In all the numbers $a + b + c > d + e$. So second condition is true.

27. e Ten cigarettes give 10 stubs. From 10 stubs 3 more cigarettes can be made (2 stubs would be obtained from 2 cigarattes formed by joining 8 stubs).

28. c I. Except for n = 1 and n = 2 for every other term X_n is less than n.

II. If the nth term is X_n then n has to be less than or equal to the sum of the first X_n natural numbers.

e.g. if $X_n = 5$ $n \leq (1 + 2 + 3 + 4 + 5)$ and also greater than $(1+ 2 + 3 + 4)$.

III. The statement is similar to (II). If $X_{n+1} - X_n = 1$, then X_n must be the last number in a set of X_n, i.e. the total number of terms till the last X_n is $1 + 2 + 3 + ... X_n$.

29. d (a) For any n, the expression is always even.

(b) Sum of the squares of a number of integers = $\frac{n(n+1)(2n+1)}{6}$

So $n(n + 1)(2n +1)$ is divisible by 6.

So it must also be divisible by 3.

(c) The value of quotient is infact equal to 6.

(d) Since $n(n + 1)(2n + 1)$ can be divisible by 237 when either of n or (n + 1) or (2n + 1) is 237 or multiple of 237.

(e) For n = 2, expression is divisible by 5.

Hence, (d) is incorrect.

30. b $a^2 - 2b^2 = 1$

$\Rightarrow a^2 - 1 = 2b^2$

$\Rightarrow (a - 1)(a + 1) = 2b^2$

$\therefore$ a is odd.

Moreover (a + 1)(a - 1) has to be divisible by 8 as product of two consecutive even numbers is divisible by 8.

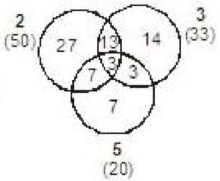
$\Rightarrow 2b^2$ is divisible by 8

$\Rightarrow b^2$ is divisible is 4

$\therefore b = 2$

$\therefore$ (3, 2) is required pair

31. e



The following Venn diagram shows the distribution on numbers between 1 and 100 that are divisible by 2, 3 or 5 or a combination or two or more of them. So we can see that there are 50 numbers that are divisible by 2, 33 numbers by 3 and 20 numbers by 5. There are 3 numbers divisible by 2, 3 and 5, while 7 are divisible by 2 and 5 (only), 13 are divisible with 2 and 3 (only) and 3 that are divisible by 5 and 3 (only). That leaves with 27 of them divisible by 2 only, 14 by 3 only and 7 by 5 only.

So (27 + 13 + 14 + 7 + 3 + 3 + 7) = 74 numbers are divisible by one or more among 2, 3 and 5.

So 26 numbers are not divisible by them.

32. a Total corn sold = $\frac{1}{10} \times$ Time (In seconds)

Time $= 10\frac{1}{2}$ hr $= 630$ min

Productive time = 2 min in every 5 min

Number of 5 min periods $= \frac{630}{5} = 126$

Productive time = 126 × 2 min = 252 min

Corn sold per day = $\frac{252 \times 60}{10} = 1512$

Explanations: Fundamentals of Set Theory

Practice Exercise (Non MCQ)

1. Number of guys wearing at least one of the two = 70% of 30 = 21

Hence, the number of guys wearing both = (10 + 15) - 21 = 4

2. The maximum amount would be when all of them have a drink and there are more number who drink whisky.

Hence, maximum would be when there are 15 who drink whisky and 5 who drink rum in the party.

Hence, total cost = 15 × 4 × 60 + 5 × 4 × 30 = ₹4200

The minimum amount would be when there are only 15 who drink, 10 of whom prefer rum in the party.

Hence, minimum amount = 10 × 4 × 30 + 5 × 4 × 60 = ₹2400

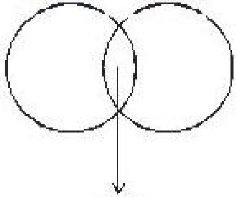
Either way it is a costly party!!

So say no to booze!!

3. (a)The maximum number would happen in the case when all those who passed in the first subject also passed in the second subject. Hence, it would be 30.

Note: In this case, the number of people passing in at least one subject would be minimum, i.e. 40.

(b)The minimum number would happen when the number of people who passed in at least one subject is equal to 50.



Let S_1 and S_2 denote the number of people passing in Subject 1 and Subject 2 respectively.

The union is 50.

Hence, S1 + S2 - Both (S1, S2) = 50

Both $(S1, S2) = 30 + 40 - 50 = 20$

4. Only 80% of the vehicles have the problem.

60% of vehicles had a problem with their engines, it means at least 20% had problem with door or tyre or both.

50% had a problem → With tyre → 30% with door or engine or both

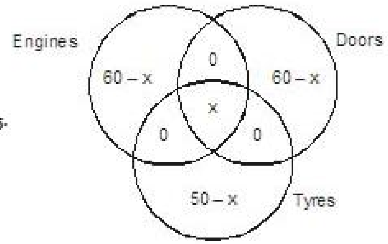
60% had a problem → With doors → 20% with tyre or engine

It means if there is no intersection in these three, then at most (20 + 30 + 20) = 70%

Vehicles will have problem in one or two categories.

Hence, at least 10% vehicles will have all three problems.

5.



Total vehicles = 60 + 110 - 2x = 170 - 2x = 80

$\Rightarrow$ x = 45%

Practice Exercise

1. b For finance = 64% of 500 = 320 = n(A)

For operation = 56% of 500 = 280 = n(B)

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

or $500 = 320 + 280 - n(A \cap B)$

or $n(A \cap B) = 100$

where A is for finance and B is for operation.

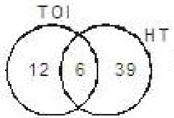
For questions 2 to 4:

Let x = Number of residents

$\therefore\ x - 3 = 0.3x + 0.75x - 6$

x = 60

$\therefore$ Venn diagram will look like.



Reading neither = 3

2. a

3. b

4. c

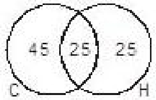
For questions 5 and 6:

Let x = Number of students

x - 0.05x = 70 + 0.5x - 0.25x

$\therefore$ x = 100

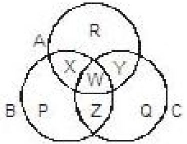
Hence, Venn diagram will look like



5. d

6. b

For questions 7 and 8:

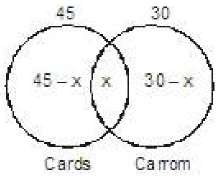


7. e Since we do not know how many students failed in all 3 papers, the answer is indeterminable.

8. c P + Q + R + X + Y + Z + W = 150

or P + Q + R + 110 - 2W = 150 (W = 10)

∴ P + Q + R = 60

9. a 

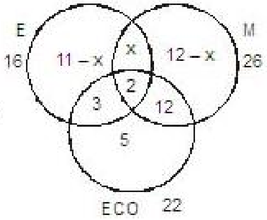
Assume x be the number of boys who play both games.

Then 45 - x + x + 30 - x = 60

⇒ 75 - x = 60 ⇒ x = 15

10. b From solution of 23, only students who play cards is 45 - x, i.e. 45 - 15 = 30

11. c From solution 23, only students who play carrom is 30 - x, i.e. 30 - 15 = 15

12. b 

As given in the question (11 - x) + x + (12 - x) + 22

= 40 and x = 5. Hence number of students studying both English and Maths = x + 2 = 7

13. d From solution 11, those students who study English, Mathematics but not Economics is x, i.e. 5.