QA - 4 : Algebra – 1

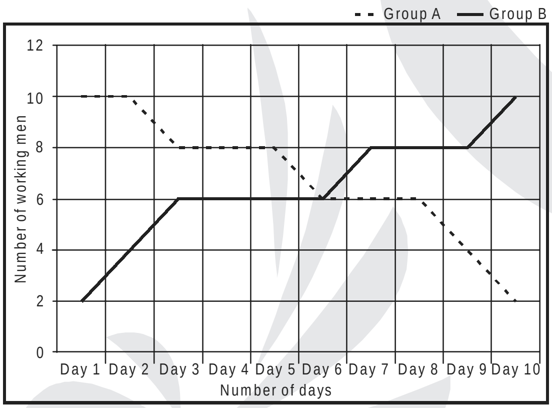
Career Launcher

Workshop

Number of Questions : 30

WSP–0004/18

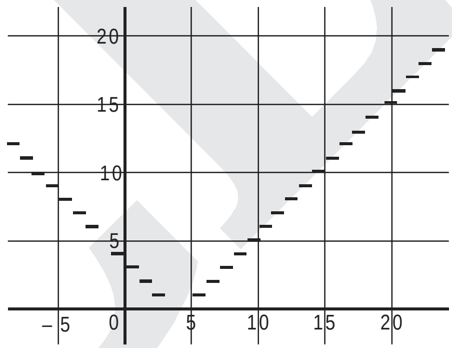
1. Two groups A and B started working on two different projects and finished the projects in a 10 day timeline. The number of men vis-à-vis number of days they worked is shown in the graph below. Calculate the ratio of total amount of work done by the two respective groups using the graph plotted below. Assume that each person in both the groups is equally efficient as any other person in any group.



(1) 12 : 17 (2) 11 : 16
(3) 17 : 16 (4) 31 : 29

2. Solve for x such that $|x + 9| < |2x – 1| + |x + 3|$
(1) $x \in (-\infty, -3.5) \cup (2.5, \infty)$
(2) $x \in (-\infty, -2.5) \cup (3.5, \infty)$
(3) $x \in (-\infty, 3.5)$
(4) $x \in (-\infty, -2.5)$

3. The graph given below is represented by which of the following equations



(1) $f(x) = |[x – 5]|$
(2) $f(x) = |[x – 4]|$
(3) $f(x) = [|x – 5|]$
(4) $f(x) = [|x – 4|]$

Where, [.] denotes the Greatest Integer Function/Floor Function and | . | denotes Modulus Function

4. $f(x) = \Sigma(-i)^{i+1}x^{n-i+1}$ defined for $0 < i < n + 2$, is a polynomial of degree n whose roots are a_i where, $0 < i < n + 1$

Then find $\Sigma\left[\frac{1}{(a_i)}\right]$

(1) –1 (2) 1

(3) $\frac{(n)^{n+1}}{(n+1)^{n+2}}$ (4) $(-n)^{n+1}$

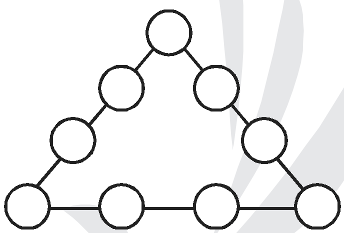
5. If [.] denotes the greatest integer function, then for how many integral values of x in the interval [1, 5] will the following equation satisfy?
$x^2 - [x^2] = (x - [x])^2$
(1) None (2) 5
(3) 21 (4) Infinitely many

6. A rectangular sheet of dimension 2d × d has its four corners cut in the form of squares such that the open box, so formed from the remaining sheet, has the maximum volume. Find the maximum possible value for the volume.

(1) d^3 (2) $\frac{d^3}{2}$

(3) $\frac{d^3}{4}$ (4) $\frac{d^3}{16}$

7. In the given figure, each cell contains distinct digits from 1 to 9 such that the sum of digits along each side is same. Find the sum of the digits at the three vertices.



(1) 5 (2) 6
(3) 7 (4) 8

8. $f(x) = |x - 1| + |x - 2| + ... + |x - 50|$
For how many integral values of x, f(x) has minimum possible value and what is the minimum value of f(x)?

9. Find the area enclosed by graph of
$\frac{|x|}{3} + \frac{|y|}{4} = 1$
(1) 64 (2) 48
(3) 24 (4) 12

10. The minimum value of
$(x + 3y + 5z + 7t) \times \left(\frac{1}{x} + \frac{1}{3y} + \frac{1}{5z} + \frac{1}{7t}\right)$ is
(1) 64 (2) 12
(3) 16 (4) 48

11. The maximum number of negative real roots and positive real roots of $f(x) = x^8 + 3x^5 - 8x^3 + 4x + 1$ are
(1) 3, 2 (2) 3, 4
(3) 4, 2 (4) 2, 1

12. Find the maximum volume of a right circular cylinder, if sum of radius and height is 15?
(1) 500π (2) 250π
(3) 400π (4) 200π

13. $3|K - 1| + K^2 - 7 > 0$
For what values of K will the given inequality be true?
(1) $(-\infty, -5) \cup (2, \infty)$
(2) $(-\infty, -1) \cup (2, \infty)$
(3) $(-\infty, -1) \cup (4, \infty)$
(4) $(-\infty, -5) \cup (4, \infty)$

14. $ax^4 + bx^3 + cx^2 + dx + e$ is divisible by $(x^2 - 3x + 2)$. Find the value of $17a + 9b + 5c + 3d + 2e$?
(1) 117 (2) 38
(3) 0 (4) 10

15. If $-7 < p < 5$ and $-4 < q < 6$, then find maximum possible value of $p^2 - pq + q^2$, where p and q are integers.
(1) 127 (2) 91
(3) 94 (4) 126

16. How many positive integer values are possible for x satisfying $(x - 6)(x - 8)(x - 10)...(x - 82) < 0$
(1) 39 (2) 38
(3) 24 (4) 25

17. How many natural numbers less than 60 can be expressed in the form of 2a + 3b, where a and b are natural numbers?
(1) 59 (2) 54
(3) 39 (4) None of these

18. p, q, r are integers such that $|p| < |q| < |r| < 40$ and p + q + r = 20 then what is the maximum value of pqr?
(1) 3610 (2) 3510
(3) 280 (4) 3500

19. How many real solutions exist for the equation $x^2 - 2^x = 0$
(1) 2 (2) 1
(3) 0 (4) More than 2

20. If $10^K = 12m + 10$, where m and k are integers, what is the value of $4^{(K-1)m}$?
(1) 64 (2) 4
(3) 1 (4) 16

21. If $m^{a \times b} < m^a \times m^b$, a, b are single digit natural numbers and m is a real number. Find the number of possible values of set {a, b}.
(1) 1 (2) 18
(3) 9 (4) 17

22. α and β are roots of the equation $x^2 - px + 12 = 0$. If the difference between the roots is at least 12. What are the possible value for p?
(1) $p > 8\sqrt{3}$
(2) $p < 8\sqrt{3}$
(3) $p \in (-\infty, -8\sqrt{3}] \cup [8\sqrt{3}, \infty)$
(4) $p \in (-\infty, -8\sqrt{3}) \cup (8\sqrt{3}, \infty)$

23. If a, b, c are distinct natural numbers less than 20 what is the maximum possible value of $|a - b| + |b - c| - |c - a|$?
(1) 20 (2) 40
(3) 35 (4) 34

24. In a list of 7 integers, one integer x is unknown. The other six integers are 20, 4, 10, 4, 8 and 4. If the mean, median and mode of these 7 integers are arranged in ascending order, they form an A.P. having all distinct numbers. The sum of all possible values of x is
(1) 40 (2) 34
(3) 38 (4) 32

25. If a > 0, then $2.\log_{a^2} 2 + 3.\log_{a^3} 3 + \ldots + n.\log_{a^n} n = ?$
(1) $\log_a n$ (2) $\log_a \frac{n(n+1)}{2}$
(3) $\frac{n(n+1)(2n+1)}{6}$ (4) $\log_a n!$

26. Sum of n terms of the series:
$$\frac{3}{1^2 . 2^2} + \frac{5}{2^2 . 3^2} + \frac{7}{3^2 . 4^2} + \ldots$$
(1) $\frac{2n+n^2}{(n+1)^2}$ (2) n + 1
(3) $\frac{n^3+2n}{(n^2+1)^2}$ (4) $\frac{n^3+2n^2}{(n^2+1)^2}$

27. Find the solution set for [x] + [2x] + [3x] + [4x] = 14, where x is a real number and [x] represents the greatest integer less than or equal to x.
(1) $x < \frac{5}{3}$ (2) $\frac{3}{2} \le x < \frac{5}{3}$
(3) $1 \le x < \frac{4}{3}$ (4) None of these

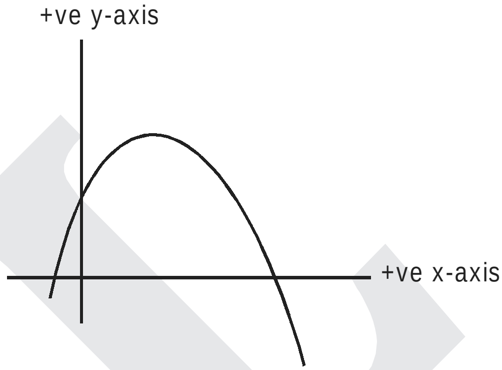
28. An arithmetic series $a_1, a_2, a_3, \ldots, a_{1000}$ exists where no two consecutive terms are same. Find the value of $(a_{999}^2 - a_2^2)/(a_{501}^2 - a_{500}^2)$
(1) 1
(2) Depends on common difference
(3) 499
(4) 997

29. a, b, c are three consecutive integers between –5 and 5. How many different integral values can the equation

$$\frac{a^3+b^3+c^3+3abc}{(a+b+c)}$$ take, if $b \neq 0$?

(1) 0
(2) 2
(3) 1
(4) 3

30. The following curve represents a quadratic function $y = ax^2 + bx + c$. Determine the sign of the coefficient of x^2 and x. Also find the sign of the constrant term.
(Figure drawn on scale)



QA - 4 : Algebra – 1
Answers and Explanations

1	3	**2**	2	**3**	4	**4**	3	**5**	2	**6**	3	**7**	2	**8**	2,625	**9**	3	**10**	3
11	3	**12**	1	**13**	2	**14**	3	**15**	2	**16**	3	**17**	2	**18**	2	**19**	4	**20**	3
21	3	**22**	3	**23**	4	**24**	1	**25**	4	**26**	1	**27**	2	**28**	4	**29**	4	**30**	–

1. 3 If watched closely, the line under Group A is higher by a margin as compared to area under Group B.
Work done is given by (workforce) x (duration) which is the area in this graphical question, but be careful while calculating the area reason being the lines don't start from the base (X axis). If want to calculate the area, use the height from the X axis which will give the area under Group A as 68 sq. units while that under Group B as 64 sq. units.

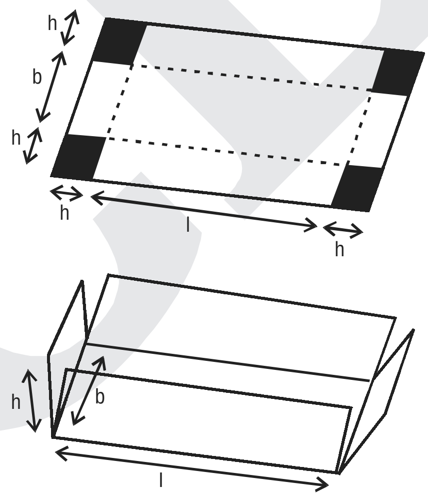
2. 2 Put values from the options.
Rearranging the inequality, we get $|x + 9| - |2x - 1| - |x + 3| < 0$. Putting zeroes of each modulus, we get
for $x = -9$, $y = -25$
for $x = -3$, $y = -1$
for $x = \frac{1}{2}$, $y = 6$
The graph becomes a mountain-like with cape for $x = \frac{1}{2}$, which becomes negative before $x = -2.5$ and after 3.5.

3. 4 Putting the value of x from (3, 5) we can see that only (4) satisfies.

4. 3 $\Sigma\left[\frac{1}{(a_i)}\right]$ will be sum of product of all roots taken except one each time divided by product of all roots. It is given by coefficient of x^1 / constant term.

5. 2 There are 5 integeral values possible.

6. 3 The given rectangular sheet will transform into an open box in the manner given below.



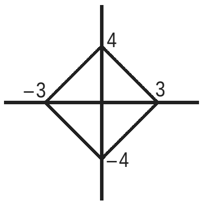
Let h be the side of each square cut from the corners. Then,
$V = x(2d - 2x)(d - x)$
$\Rightarrow 4V = 4x(2d - 2x)(d - x)$
Using AM $\geq$ GM
$\therefore V \leq \frac{d^3}{4}$

7. 2 Let the digits from top vertex in the clockwise direction be represented by a, b, c, d, e, f, g, h and i.
Then, $a + b + c + d = d + e + f + g = g + h + i + a = 17$.
Adding all three equations, we get
$(a + d + g) + (a + b + c + d + e + f + g + h + i) = 51$
$(a + d + g) + 45 = 51$
$(a + d + g) = 6$

8. 2, 625

We consider men standing at each critical point and they are able to move such that the distance covered by all of them in total will represent value of f(x).
So, for minimum value each man has to meet exactly at the middle which is $\frac{1+50}{2} = 25.5.$ But we have to take only integral values this point of minima is at x = 25 or x = 26
Thus only 2 points are possible for minimum value, put x = 25 to get minimum value of f(x)
f(25) = 24 + 23 + 22 + ... + 3 + 2 + 1 + 0 + 1 + 2 + ... + 25 = 25 × 25 = 625

9. 3



$$= \frac{1}{2} \times 6 \times 4 \times 2 = 24$$

10. 3 $AM \geq GM \geq HM$. Here $AM \geq HM$

$$\Rightarrow \frac{(x+3y+5z+7t)}{4} \geq \frac{4}{\left(\frac{1}{x}+\frac{1}{3y}+\frac{1}{5z}+\frac{1}{7t}\right)}$$

11. 3 We will count the number of sign changes

$f(x) = x^8 + 3x^5 - 8x^3 + 4x + 1$

+ + – + +

2 sign changes $\Rightarrow$ 2 +ve real roots

$f(-x) = x^8 - 3x^5 + 8x^3 - 4x + 1$

+ – + – +

4 sign changes $\Rightarrow$ 4 –ve real roots

Alternatively:

whenever imaginary roots exist, they exist in pairs.
Here maximum power of x is 8
$\Rightarrow$ maximum 8 roots will exist
out of which number of imaginary roots will be even i.e. 0 or 2 or 4 or 6 etc
Thus from option, we can say that total real roots (+ve and –ve) must be even which is option (3).

12. 1 r + h = 15

$$v = \pi r^2 h = \pi r^2 (15 - r)$$

$$v = \frac{\pi}{2} \times r \times r \times (30 - 2r)$$

Applying $AM \geq GM$

$$\frac{r + r + (30 - 2r)}{3} \geq \left\{r^2(30 - 2r)\right\}^{1/3}$$

$$\Rightarrow \left\{r^2(30 - 2r)\right\}_{max} = 1000$$

So, $V_{max} = \frac{\pi}{2} \times 1000 = 500\pi$

13. 2 Put the values from the ranges given in the option to get the desired result.
Option (2) satisfies the given condition.

14. 3 As the given function is divisible by $(x^2 - 3x + 2)$, so it must be divisible by its factors $(x^2 - 3x + 2)$ i.e., (x – 1)(x – 2) put x = 1 and x = 2.
So, f(1) = a + b + c+ d + e = 0 and
f(2) = 16a + 8b + 4c + 2d + e = 0
On adding, we get 17a + 9b + 5c + 3d + 2e = 0

15. 2

p p q q

6^2 (-6×5) 5^2

↓ ↓ ↓

$36 - (-30) + (25) = 91$

16. 3 x can take 1, 2, 3, 4, 5 and then 9, 13, 17...81
so total 5 + 19 = 24 values

17. 2 Least such number is 2 + 3 = 5 and 7 onwards every number can be expressed upto 59.
So total 1 + 53 = 54 numbers

18. 2 Let's take p = 39, q = –10, r = –9
So product = 39 × (–10)(–9) = 3510

19. 4 $x^2 - 2^x = 0 \Rightarrow x^2 = 2^x$
Two obvious solution
i.e. x = 2, x = 4
one solution would be for x < 0
$\therefore$ More than 2 solution

20. 3 $10^K = 12m + 10$
$12m = 10^k - 10$
12m could be 90, 990,....
but none of them is a multiple of 12
$\therefore$ only possible value is m = 0

21. 3 ab < a + b, it holds when a = 1 and $1 \leq b \leq 9$
$\therefore$ 9 possible solution as it is given a set and not ordered pair.

22. 3 $\alpha + \beta = p \quad \alpha\beta = 12$

$\therefore (\alpha - \beta)^2 = p^2 - 12 \times 4 = p^2 - 48$

Since $|\alpha - \beta| \geq 12$

$\therefore p^2 - 48 \geq 144 \Rightarrow p^2 \geq 192$

$\therefore p \in (-\infty, -8\sqrt{3}] \cup [8\sqrt{3}, \infty)$

23. 4 To make $|a-b| + |b-c| - |c-a|$ maximum so b should not lie between a and c.
$\therefore$ b should be maximized, so b = 19.
a = 1, c = 2
$\therefore$ Maximum value = 18 + 17 – 1 = 34

24. 1 Mode of these 7 numbers will always be 4
If $x \le 4$ Median is 4 $\Rightarrow$ not possible

$4 < x \le 8$ median x and mean $= \frac{50+x}{7}$

$4, x, \frac{50+x}{7}$ should be in an A.P.

$\therefore \frac{50+x}{7} - x = x - 4$

$\Rightarrow x = 6.$

If x > 8, Median = 8, mean $= \frac{50+x}{7}$

so 4, 8, $\frac{50+x}{7}$ in A.P.

$\Rightarrow 8 - 4 = \frac{50+x}{7} - 8$

$\Rightarrow x = 34$
$\therefore$ Total sum 6 + 34 = 40

25. 4 $\log_a 1 + \frac{2}{2}\log_a 2 + \frac{3}{3}\log_a 3 + \ldots \frac{n}{n}\log_a n$

$= \log_a 1 + \log_a 2 + \ldots + \log_a n$
$= \log_a(1.2.3...n) = \log_a n!$
Alternatively
Put n = 1, 2, 3....

26. 1 put n = 1 $\Rightarrow$ option (2) rejected
put n = 2 $\Rightarrow$ option (3) and (4) rejected

27. 2 x > 1 and x < 2 is obvious

Now for $x = \frac{3}{2}$, expression = 1 + 3 + 4 + 6 = 14

$\therefore$ x = 1.5 satisfies for x = 1.5, [3x] = [4.5] = 4

But as soon as $x = \frac{5}{3}$, [3x] = 5 it would not satisfy

so x should be less than $\frac{5}{3}$.

Hence ans (2)

28. 4 Take the series to be 1, 2, 3,..,1000 and solve.
$a_2 = a + d$
$a_{500} = a + 499d$
$a_{501} = a + 500d$
$a_{999} = a + 998d$
$a^2 - b^2 = (a - b)(a + b)$, so $(a_{999}{}^2 - a_2{}^2)/(a_{501}{}^2 - a_{500}{}^2)$
= [(2a + 999d)(997d)]/[(2a + 999d)(d)] = 997.

29. 4 We will put the integeral values between –5 and 5 in the expression and will get there are three possible values, as $b \ne 0$.

30. Apply the sum of the roots formula to determine sign.