Algebra - 6

Career Launcher

QA - 22

CEX-Q-0223/18

Contents

- Progressions & Series

Number of questions : 30

1. The series 2, 5, 8, 11,....50 terms is written on a sheet of paper in the ascending order.
   (1) What is the 15^{th} term if the same numbers were written in the descending order?
   (2) How many of the terms are divisible by 15?
   (3) What is the nth term of the series?
   (4) What is the sum of the series?

2. The sum of five consecutive odd numbers a_1 to a_5 is 75. What is the sum of the five consecutive even numbers, whose smallest term is same as $2a_5$.
   (1) 30 (2) 36
   (3) 105 (4) 210

3. What is the 2015th term of a sequence of natural numbers written in the ascending order, that does not have any perfect squares?
   (1) 2058 (2) 2059
   (3) 2060 (4) 2062

4. In an infinite geometric progression, each term is equal to twice the sum of all the terms that follow it. If the sum of first two terms is 12, the sum of the entire progression is
   (1) $\frac{9}{2}$ (2) $\frac{27}{2}$
   (3) $\frac{88}{7}$ (4) 15

5. If a, b, c are in G.P. and $a^x = b^y = c^z$, then:
   (1) $\frac{1}{x}+\frac{1}{z}=\frac{2}{y}$ (2) $\frac{1}{x}+\frac{1}{z}=-\frac{2}{y}$
   (3) $\frac{1}{x}+\frac{1}{y}=\frac{2}{z}$ (4) $\frac{1}{x}+\frac{1}{y}=-\frac{2}{z}$

6. If a, b and c form a G. P. with common ratio r, the sum of the y-coordinates of the points of intersection of the line ax + by + c = 0 and the curve $x + 2y^2 = 0$ is
   (1) $-\frac{r}{4}$ (2) $-\frac{r}{2}$
   (3) $\frac{r}{2}$ (4) $\frac{r}{4}$

7. Compute the infinite sum V, where

$$V=\frac{1}{2}-\frac{2}{4}+\frac{3}{8}-\frac{4}{16}+\frac{5}{32}-\frac{6}{64}+ \ldots+\frac{n}{2^n}(-1)^{n+1}\ldots.$$

   (1) $\frac{2}{3}$ (2) $\frac{2}{9}$
   (3) $\frac{1}{6}$ (4) $\frac{2}{27}$

8. The series of natural numbers is divided into subsets $S_1 = (1)$, $S_2 = (2, 3, 4, 5)$, $S_3 = (6, 7, 8, 9, 10, 11, 12, 13, 14)$, $S_4 = (15, 16, 17, 18, ...30)$ and so on. Find the fifth element starting from the left of the subset S_{37}.
(1) 16207 (2) 16206
(3) 16211 (4) 16210

9. Find the sum of the 37th bracket of the following series.

$$(1)+(7+7^2+7^3)+(7^4+7^5+7^6+7^7+7^8)+(7^9+7^{10}+\cdots+7^{15})\cdots$$

(1) $\frac{7^{37}}{6}(7^{73}-1)$ (2) $\frac{(7^{73}-1)}{6}$

(3) $7^{1260}\left(\frac{7^{71}-1}{6}\right)$ (4) None of these

10. The sum of an infinite geometric series of real numbers is 14, and the sum of the cubes of the terms of this series is 392. Then the first term of the series is
(1) – 14 (2) 10
(3) 7 (4) –5

11. Suppose a; b; c > 0 are in geometric progression and $a^p = b^q = c^r \neq 1$.

Which one of the following is always true?
(1) p; q; r are in geometric progression
(2) p; q; r are in arithmetic progression
(3) p; q; r are in harmonic progression
(4) p = q = r

12. If x > 1, y > 1, z > 1 are in G.P., then

$\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in
(1) A. P. (2) H. P.
(3) G. P. (4) None of the above

13. Let S denotes the infinite sum $2+5x+9x^2+14x^3+20x^4+\ldots$, where |x| < 1 and the coefficient of x^{n-1} is $\frac{1}{2}n(n+3),(n=1, 2, \ldots)$. Then S equals:

(1) $\frac{2-x}{(1-x)^3}$ (2) $\frac{2-x}{(1+x)^3}$

(3) $\frac{2+x}{(1-x)^3}$ (4) $\frac{2+x}{(1+x)^3}$

14. Find the sum of the following series:

$$S=\frac{7}{6}+\frac{13}{12}+\frac{21}{20},\frac{31}{30}+\ldots\ldots\frac{9901}{9900}.$$

(1) $\frac{9749}{100}$ (2) $\frac{9849}{100}$

(3) $\frac{9949}{100}$ (4) $\frac{9649}{100}$

15. Find the value of the following expression:

$$\frac{1}{1\times3\times5}+\frac{1}{3\times5\times7}+\frac{1}{5\times7\times9}+\cdots+\frac{1}{95\times97\times99}$$

(1) $\frac{800}{9603}$ (2) $\frac{700}{9603}$

(3) $\frac{500}{9603}$ (4) $\frac{450}{9603}$

16. For any whole number n, $A_n = (-1)^n \frac{(2n+3)}{(n+1)(n+2)}$. The sum $A_0 + A_1 + A_2 + A_3 + \ldots$, up to infinite terms is equal to

(1) –1 (2) $\frac{1}{2}$

(3) 1 (4) 2

17. If the nth term of a sequence is 3n (n + 2) – 12n, where n = 1, 2, 3 …. Find
A. Smallest term
B. Sum to 10 terms of sequence.

18. Find the sum to 10 terms of the series whose general term is n(n + 1)(n + 2).

19. Find the sum

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots\ldots+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$$

(1) $2008-\frac{1}{2008}$ (2) $2007-\frac{1}{2007}$
(3) $2007-\frac{1}{2008}$ (4) $2008-\frac{1}{2007}$

20. Consider the sequence of numbers a_1, a_2, a_3, ... to infinity where a_1 = 81.33 and a_2 = –19 and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence?
(1) –100.33 (2) –30.00
(3) 62.33 (4) 119.33

21. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals

(CAT 2005)

(1) $3^{99} - 200$ (2) $3^{99} + 200$
(3) $3^{100} - 200$ (4) $3^{100} + 200$

22. Let S be the sum of an arithmetic series. The arithmetic mean of every two consecutive terms and every three consecutive terms of S form the consecutive terms of series S1 and S2 respectively. If the sum of all the terms in series S1 and in series S2 are 1375 and 690 respectively, then find the sum of all the terms in series S.
(1) 1960 (2) 2580
(3) 2060 (4) Cannot be determined

23. Let $\{V_n\}$ be a sequence such that $V_1 = 2$, $V_2 = 1$ and $2V_n - 3V_{n-1} + V_{n-2} = 0$ for n > 2, then find the value of V_9.
(1) $\frac{1}{128}$ (2) $\frac{1}{256}$
(3) $\frac{1}{64}$ (4) $\frac{1}{32}$

24. If $\log 2(5 \times 2^x + 1)$, $\log_4(2^{1-x} + 1)$ and 1 are in A.P., then x equals to
(1) $\log_2 5$ (2) $1 - \log_2 5$
(3) $\log_5 2$ (4) None of the above

25. If the 2nd, 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is
(1) $\frac{4}{3}$ (2) 1
(3) $\frac{7}{4}$ (4) $\frac{8}{5}$

26. In the sequence of five number x, ____, 3,____, 18, each number after the second is obtained by multiplying the two previous numbers. The value of x is
(1) $\frac{2}{3}$ (2) $\frac{3}{2}$
(3) 1 (4) – 9

Challenging

27. The infinite sum

$$1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\cdots \text{ equals}$$

(1) $\frac{27}{14}$ (2) $\frac{21}{13}$
(3) $\frac{49}{27}$ (4) $\frac{256}{147}$

28. Find the sum of the infinite series

$\log_9 3 + \log_{27} 3 - \log_{81} 3 + \log_{243} 3 - \log_{729} 3 + \cdots$

29. If the sum of the first ten terms of the series

$\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \cdots$, is $\frac{16}{5}m$, then m is equal to:

(1) 101 (2) 100
(3) 99 (4) 102

30. A sequence of numbers is written in the following fashion: 1, 7, 1, 1, 7, 7, 1, 1, 1, 7, 7, 7, ... till n terms.

What is the value of $(T_{5040} + T_{5042} + T_{5044} + T_{5046})$?
(T_n is the n^{th} term of the given sequence.)

(1) 4 (2) 28
(3) 10 (4) 22

QA - 22 : Algebra - 6
Answers and Explanations

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1	–	**2**	4	**3**	3	**4**	2	**5**	1	**6**	3	**7**	2	**8**	3	**9**	4	**10**	3
11	3	**12**	2	**13**	1	**14**	2	**15**	1	**16**	3	**17**	–	**18**	–	**19**	1	**20**	3
21	3	**22**	3	**23**	1	**24**	2	**25**	1	**26**	2	**27**	3	**28**	–	**29**	1	**30**	4

1. (1) The given series is in the form of 3n + 2, where n = 0,1,2...,49
15th term from last is same as (50 – 15 + 1) = 36th term from start, which is 3 × 35 + 2 = 107.
(2) For being divisible by 15, a number must be divisible by 3. Since all the numbers of this series are in the form of 3n + 2, which is not divisible by 2. So, no term would be a multiple of 15.
(3) The nth term of the series is (3n + 2)
(4) The sum of the series is

$$\frac{50}{2}\left[2\times2+49\times3\right] = 25 \times 151 = 3775.$$

2. 4 The middle term is the average of all the five numbers

$$= \frac{75}{5} = 15$$

So, a_1, a_2, a_3, a_4, a_5, = 11, 13, 15, 17, 19
Now, smallest term of other series = 2 × 19 = 38
So, the sum is 38 + 40 + 42 + 44 + 46 = 42 × 5 = 210.

3. 3 Since $45^2 = 2025$
So, will have to remove 45 perfect squares of the sequence.
$\Rightarrow x - 45 = 2015$
So, x = 2060.

4. 2 Let a be the first term and r be the common ratio.
Then, a + ar = 12
Also if T_1, T_2 … be the terms of the GP, then
$T_1 = 2(T_2 + T_3 + T_4 + \ldots)$
$\Rightarrow a = 2(ar + ar^2 + \ldots)$

$$\Rightarrow a = \frac{2ar}{1-r} \Rightarrow 3r = 1 \Rightarrow r = \frac{1}{3}$$

$$\therefore a = \frac{12}{1+r} = \frac{12}{1+\frac{1}{3}} = 9$$

$$\text{Hence, } S = \frac{9}{1-\frac{1}{3}} = \frac{27}{2}.$$

5. 1 Given that
$b^2 = ac$ …(1)
$a^x = b^y = c^z$ …(2)
or $a = b^{y/x}$ and $c = b^{y/z}$
Putting there value of a and c in equation (1), we get

$$b^2 = b^{y/x}b^{y/z} = b^{y/x+y/z}$$

Which is possible only when

$$2 = \frac{y}{x}+\frac{y}{z} \Rightarrow \frac{1}{x}+\frac{1}{z}=\frac{2}{y}$$

Hence option (1) is the correct choice.

6. 3 The equation of the given line is
ax + by + c = 0

$$\Rightarrow ax + ary + ar^2 = 0 \Rightarrow x + ry + r^2 = 0 \qquad \ldots \text{(i)}$$

(i) intersects the curve $x + 2y^2 = 0$ at the points whose ordinates are given by
$-2y^2 + ry + r^2 = 0$ or,

$$\Rightarrow 2y^2 - ry - r^2 = 0 \qquad \ldots \text{(ii)}$$

$\therefore$ Required sum of y-coordinates = Sum of the roots of equation (ii) = $\frac{r}{2}$

7. 2

$$V = \frac{1}{2}-\frac{2}{4}+\frac{3}{8}-\frac{4}{16}+\frac{5}{32}-$$

$$\frac{V}{2} = \frac{1}{4}-\frac{2}{8}+\frac{3}{16}-\frac{4}{32}+\ldots.$$

$$V+\frac{V}{2} = \frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}-\ldots..$$

$$\text{or, } \frac{3V}{2} = \frac{\frac{1}{2}}{1-\left(-\frac{1}{2}\right)} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3} \quad \text{or, } V = \frac{1}{3}\times\frac{2}{3} = \frac{2}{9}$$

8. 3 The number of elements in successive subsets are 1, 4, 9, 16 ....(i.e. $1^2, 2^2, 3^2, 4^2, 5^2$, ..)
Sum of squares of first n natural numbers

$$= \frac{n(n+1)(2n+1)}{6}$$

Number of elements in S_1 to S_{36} subsets is

$$\frac{36(37)(73)}{6} = 6 \times 37 \times 73 = 16206$$

So, fifth element of subset S_{37} = 16206 + 5 = 16211.

9. 4 The number of terms in the 1st , 2nd, 3rd and 4th brackets are 1, 3, 5, 7 respectively.
So, the no of terms in the 37th bracket will be
1 + 36 × 2 = 73.
1st term in the 2nd bracket is 7^1, that in the 3rd and 4th are 7^{1+3}, 7^{1+3+5} respectively.
So, the 1st term of the 37th bracket will be
$7^{1+3+5+\cdots+36\text{ terms}} = 7^{(1+35)\times 36} = 7^{1296}$
Hence, the required sum is: $\frac{7^{1296}[7^{73}-1]}{6}$

10. 3 Let, the infinite G. P. be a, ar, ar^2, ...

So, $\frac{a}{1-r} = 14 \Rightarrow \frac{a^3}{(1-r)^3} = 14^3 = 392 \times 7$

and $\frac{a^3}{1-r^3} = 392$

$\Rightarrow \frac{a^3}{(1-r)^3} = \frac{a^3}{(1-r^3)} \times 7 \Rightarrow \frac{1+r+r^2}{(1-r)^2} = 7$

$\Rightarrow r = 2$ or 1/2
Here, $r \neq 2$ (for an infinite G.P.)
So, $r = 1/2 \Rightarrow a = 7$.

11. 3 Let, a = 2, b = 4 and c = 8
For $a^p = b^q = c^r$
$2^p = 4^q = 8^r$
This is possible only if P = 3 q = 3/2 and r = 1
Thus, options (1), (2) and (4) do not satisfy the condition.
So, option (3) is the possible answer.

12. 2 For x, y and z to be in G.P.
$y^2 = xz$
2 log y = log x + log z
i.e. log x. log y and log z are in A.P.
So, (log x + 1), (log y + 1), (log z + 1) are also in A.P.

$\Rightarrow \frac{1}{1+\log x}, \frac{1}{1+\log y}$ and $\frac{1}{1+\log z}$ are in H.P.

13. 1 Coefficient of $x^n = \frac{1}{2}(n+1)(n+4)$

$S = 2 + 5x + 9x^2 + 14x^3 + \ldots.$

$xS = 2x + 5x^2 + \ldots..$

$S(1-x) = 2 + 3x + 4x^2 + 5x^3 + \ldots.$

Let, $S_1 = S(1-x) \Rightarrow S_1 = 2 + 3x + 4x^2 + \ldots$

$xS_1 = 2x + 3x^2 + \ldots$

$S_1(1-x) = 2 + x + x^2 + \ldots.$

$S_1(1-x) = 2 + \frac{x}{1-x}$

$S(1-x^2) = 2 + \frac{x}{1-x}$

$\Rightarrow S = \frac{2-x}{(1-x)^3}$

14. 2 Given $S = \frac{7}{6} + \frac{13}{12} + \frac{21}{20} + \frac{31}{30} + \ldots\ldots\ldots \frac{9901}{9900}$

$= \left(1+\frac{1}{2\times 3}\right) + \left(1+\frac{1}{3\times 4}\right) + \left(1+\frac{1}{4\times 5}\right) + \ldots + \left(1+\frac{1}{99\times 100}\right)$

$= 98 + \left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \ldots.. + \frac{1}{98} - \frac{1}{90} + \frac{1}{99} - \frac{1}{100}\right)$

$= 98 + \frac{1}{2} - \frac{1}{100} = \frac{9849}{100}.$
Hence, (2) is the correct option.

15. 1 $\frac{1}{1\times 3\times 5} + \frac{1}{3\times 5\times 7} + \frac{1}{5\times 7\times 9} + \cdots + \frac{1}{95\times 97\times 99}$

$= \frac{1}{8}\left[\left\{\left(1-\frac{1}{3}\right) - \left(\frac{1}{3} - \frac{1}{5}\right)\right\} + \left\{\left(\frac{1}{3} - \frac{1}{5}\right) - \left(\frac{1}{5} - \frac{1}{7}\right)\right\} + \cdots + \left\{\left(\frac{1}{95} - \frac{1}{97}\right) - \left(\frac{1}{97} - \frac{1}{99}\right)\right\}\right]$

$= \frac{1}{8}\left[\left(1-\frac{1}{3}\right) - \left(\frac{1}{97} - \frac{1}{99}\right)\right] = \frac{1}{8} \times \frac{6400}{9603} = \frac{800}{9603}.$

16. 3 Put the values and note that the sum comes out like
$\frac{3}{2} - \frac{5}{6} + \frac{7}{12} + \cdots$

$= 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \frac{1}{4} - \frac{1}{4} - \frac{1}{5} + \frac{1}{5} + \cdots = 1$

17. A. Since $t_n = 3n(n + 2) - 12n$
and n = 1, 2, 3 …
So, to get the smallest term, we put n = 1.
Thus, $t_1 = 3(1 + 2) - 12 = -3$.
B. $t_n = 3n(n + 2) - 12n = 3n^2 + 6n - 12n$
$t_n = 3n^2 - 6n$
So, their sum upto n terms is given by

$$\sum_{n=1}^{n} t_n = 3\Sigma n^2 - 6\Sigma n$$

$$= 3\left[\frac{n(2n+1)(n+1)}{6}\right] - 6\left[\frac{n(n+1)}{2}\right]$$

$$S_n = \frac{n(n+1)(2n-5)}{2}$$

Now, $S_{10} = \frac{10\times 11\times 15}{2} = 825.$

18. $t_n = n(n+1)(n+2) = n^3 + 3n^2 + 2n$

$$\Rightarrow \Sigma t_n = \Sigma n^3 + 3\Sigma n^2 + 2\Sigma n$$

$$S_n = \left[\frac{n(n+1)}{2}\right]^2 + 3\left[\frac{n(n+1)(2n+1)}{6}\right] + \frac{2n(n+1)}{2}$$

We put n = 10, to get

$S_{10} = (55)^2 + 3(385) + 110 = 3025 + 1155 + 110 = 4290.$

19. 1

$$S = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \ldots + \sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$$

$$T_n = \sqrt{1+\frac{1}{n^2}+\frac{1}{(n+1)^2}} = \sqrt{\frac{n^4+2n^3+3n^2+2n+1}{n^2(n+1)^2}}$$

$$= \frac{n^2+n+1}{n^2+n} = 1 + \frac{1}{n^2+n}$$

$$S = \sum_{n=1}^{2007} T_n = 2007 + \sum_{n=1}^{2007}\left\{\frac{1}{n} - \frac{1}{n+1}\right\} = 2008 - \frac{1}{2008}.$$

Alternative method:

$$S = \sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} + \sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}} + \ldots + \sqrt{1+\frac{1}{(n-1)^2}+\frac{1}{n^2}}$$

we can solve the question by writing the generated form of options

As, option (1) as $n - \frac{1}{n}$

Option (2) is $(n-1) - \frac{1}{(n-1)}$

Option (3) is $(n-1) - \frac{1}{n}$

Option (4) is $n - \frac{1}{(n-1)}$

Now, first term is $\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}} = \frac{3}{2}$

and putting n = 2, only option (1) gives the answer.

20. 3 Given $a_1 = 81.33$ and $a_2 = -19$

Also

$a_i = a_{i-1} - a_{i-2}$, for $j \geq 3$

$\Rightarrow a_3 = a_2 - a_1 = -100.33$

$a_4 = a_3 - a_2 = -81.33$

$a_5 = a_4 - a_3 = 19$

$a_6 = a_5 - a_4 = +100.33$

$a_7 = a_6 - a_5 = +81.33$

$a_8 = a_7 - a_6 = -19$

Clearly onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add $a_1, a_2, a_3 \ldots$ upto a_{6002}, we will eventually be left with $a_1 + a_2$ only i.e. 81.33 – 19 = 62.33.

21. 3 $a_1 = 1$, $a_{n+1} - 3a_n + 2 = 4n$

$a_{n+1} = 3a_n + 4n - 2$

when n = 1 then $a_2 = 3 + 4 - 2 = 5$

when n = 2 then $a_3 = 3 \times 5 + 4 \times 2 - 2 = 21$

from the options, we get an idea that a_n can be expressed in a combination of some power of 3 & some multiple of 100.

(1) $3^{99} - 200$; tells us that a_n could be: $3^{n-1} - 2 \times n$; but it does not fit a_1 or a_2 or a_3.

(2) $3^{99} + 200$; tells us that a_n could be : $3^{n-1} + 2 \times n$; again, not valid for a_1, a_2 etc.

(3) $3^{100} - 200$; tells $3^n - 2n$: valid for all a_1, a_2, a_3.

(4) $3^{100} + 200$; tells $3^n + 2n$: again not valid.

So, (3) is the correct answer.

22. 3 According to the statements given in the question we can write here

$S = a_1 + a_2 + a_3 + \ldots + a_n$ … (i)

Since $a_2, a_3, a_4 \ldots a_{n-1}$ are the arithmetic means of first three consecutive terms, next three consecutive terms(starting with a_2) etc.

$\therefore S_2 = a_2 + a_3 + a_4 + \ldots + a_{n-1}$ … (ii)

Also, $S_1 = \frac{a_1+a_2}{2} + \frac{a_2+a_3}{2} + \ldots \frac{a_{n-1}+a_n}{2}$

$$\Rightarrow S_1 = \frac{1}{2}[a_1 + 2(a_2 + a_3 + \ldots a_{n-1}) + a_n] \quad \ldots \text{(iii)}$$

Note that in equation (i) if a_1 and a_n are excluded, then the rest of the series is nothing but S_2.

Therefore, we can write the series as

$S = a_1 + S_2 + a_n \Rightarrow a_1 + a_n = S - S_2$ … (iv)

$$S_1 = \frac{1}{2}[a_1 + 2S_2 + a_n] \Rightarrow \frac{1}{2}(a_1 + a_n) = S_1 - S_2 \quad \ldots \text{(v)}$$

From (iv) and (v), we have

$S = 2S_1 - S_2 = 2 \times 1375 - 690 = 2060.$

23. 1 $V_1 = 2$, $V_2 = 1$, $V_3 = \frac{3(1)-2}{2} = \frac{1}{2}$

$$\left[\text{as we have } V_n = \frac{3V_{n-1} - V_{n-2}}{2}\right]$$

$$V_4 = \frac{3\left(\frac{1}{2}\right)-1}{2} = \frac{1}{4}, \quad V_5 = \frac{3\left(\frac{1}{4}\right)-\frac{1}{2}}{2} = \frac{1}{8}$$

Thus, we observe that $V_n = \frac{1}{2^{n-2}}$

$$\therefore V_9 = \frac{1}{2^{9-2}} = \frac{1}{2^7} = \frac{1}{128}$$

24. 2 Since the given terms are in A.P.

So, $2\times\log_4(2^{(1-x)}+1)=\log_2(5\times 2^x+1)+1$

$\log_2(2^{(1-x)}+1)=\log_2(5\times 2^x+1)+\log_2 2$

$\Rightarrow 2^{1-x}+1=(5\times 2^x+1)2$

$\Rightarrow \dfrac{2}{2^x}+1=10\times 2^x+2$

Let $2^x = y$

So, $\dfrac{2}{y}+1=10\times y+2$

$2 + y = 10y^2 + 2y$

$\Rightarrow y=\dfrac{2}{5}$ or $\dfrac{-1}{2}$

i.e. $2^x=\dfrac{2}{5}$ $\left(\text{as } 2^x\neq-\dfrac{1}{2}\right)$

$x = 1 - \log_2 5.$

25. 1 $a + d, a + 4d, a + 8d$

$(a + 4d)^2 = (a + d)(a + 8d)$

$a^2 + 16d^2 + 8ad = a^2 + 9ad + 8d^2$

$8d^2 = ad$

$a = 8d, d \neq 0$

$r=\dfrac{a+4d}{a+d}=\dfrac{12d}{9d}=\dfrac{4}{3}$

26. 2 Let the sequence be x, y, 3, z, 18

So, $3z = 18 \Rightarrow z = 6$

and $3y = z \Rightarrow 3y = 6 \Rightarrow y = 2$

and $xy = 3 \Rightarrow x = 3/2$.

27. 3 $S=1+\dfrac{4}{7}+\dfrac{9}{7^2}+\dfrac{16}{7^3}+\dfrac{25}{7^4}+.......$

$\dfrac{S}{7}=\dfrac{1}{7}+\dfrac{4}{7^2}+\dfrac{9}{7^3}+\dfrac{16}{7^4}+\dfrac{25}{7^5}+........$

$\Rightarrow \dfrac{6S}{7}=1+\dfrac{3}{7}+\dfrac{5}{7^2}+\dfrac{7}{7^3}+\dfrac{9}{7^4}+\dfrac{11}{7^5}+..........$

$\Rightarrow \dfrac{6S}{49}=\dfrac{1}{7}+\dfrac{3}{7^2}+\dfrac{5}{7^3}+\dfrac{7}{7^4}+\dfrac{9}{7^5}+\dfrac{11}{7^6}+..........$

$\Rightarrow \dfrac{6S}{7}-\dfrac{6S}{49}=1+\dfrac{2}{7}+\dfrac{2}{7^2}+\dfrac{2}{7^3}+\dfrac{2}{7^4}+..........$

$\Rightarrow \dfrac{6^2}{7^2}\times S=1+\dfrac{2}{7}\left[\dfrac{1}{1-\dfrac{1}{7}}\right]=\dfrac{4}{3}$

$\Rightarrow S=\dfrac{7^2}{6^2}\times\dfrac{4}{3}=\dfrac{49}{27}$

28. $\log_{3^2}3+\log_{3^3}3-\log_{3^4}3+\log_{3^5}3-......$

$=\dfrac{1}{1}-\dfrac{1}{2}+\dfrac{1}{3}-\dfrac{1}{4}+\dfrac{1}{5}-\dfrac{1}{6}+\dfrac{1}{7}-\dfrac{1}{8}+........$

$=\dfrac{1}{1\times 2}+\dfrac{1}{3\times 4}+\dfrac{1}{5\times 6}+\dfrac{1}{7\times 8}+.........$

$\approx 0.5 + 0.083 + 0.03 + 0.017 + 0.01 + ... \approx 0.7.$

29.1 $S_n=\left(\dfrac{8}{5}\right)^2+\left(\dfrac{12}{5}\right)^2+\left(\dfrac{16}{5}\right)^2+\left(\dfrac{20}{5}\right)^2\cdots$

$S_n=\dfrac{4^2}{25}\left[2^2+3^2+4^2+5^2+\cdots 11^2\right]$

$S_n=\dfrac{16}{25}\left[1^2+2^2+3^2+4^2+\cdots 11^2-1\right]$

$=\dfrac{16}{25}\times\left(\dfrac{11\times 12\times 23}{6}-1\right)=\dfrac{16}{25}\times 505=\dfrac{16}{5}\times 101$

$\Rightarrow m = 101.$

30. 4 If we assume (1, 7) as the first block, (1, 1, 7, 7) as the second block and so on till m^th block, then T_{5001} will be in the m^th block.

There are 2 terms in the first block, 4 terms in the second block and so on.

$\Rightarrow 2 + 4 + 6 + 8 + ... + 2m \geq 5001$

$\Rightarrow 2(1 + 2 + ... + m) \geq 5001$

$\Rightarrow \dfrac{2m(m+1)}{2}\geq 5001 \Rightarrow m(m + 1) \geq 5001$

Least possible value of m that satisfy the above is m = 71.

$\Rightarrow T_{5001}$ lies in 71^st block.

Total number of terms in the first 70 blocks = 70 × 71 = 4970 terms.

In the 71^st block, there will be 71 one's and 71 seven's.

Therefore, the sum of first 5001 terms of the series

= (8 + 16 +.... 70 terms) + 1 × 31

= 8(1 + 2 + ... + 70) + 31

$=8\times\dfrac{70\times 71}{2}+31 = 19911$

All the terms from T_{4971} to T_{5041} are '1' and all the terms from T_{5042} to T_{5112} are '7'.

$\Rightarrow T_{5040} + T_{5042} + T_{5044} + T_{5046} = 1 + 7 + 7 + 7 = 22.$