Trigonometry

Career Launcher

QA - 33

CEX-Q-0234/18

Contents

- Trigonometric Identities
- Height and Distance

Number of questions : 30

1. Find the degree measure corresponding to the $\left(\frac{\pi}{9}\right)^c$.

   (1) 24° (2) 20°
   (3) 22°30' (4) 18°

2. Find the length of an arc of a circle of radius 6 cm subtending a central angle measuring 18°.

   (1) $\frac{5\pi}{12}$ cms (2) $\frac{20\pi}{3}$ cms
   (3) $\frac{3\pi}{5}$ cm (4) $\frac{6\pi}{7}$ cm

3. The value of the expression $\tan 1^\circ \tan 2^\circ \tan 3^\circ ... \tan 89^\circ$ is ____.

4. If $\sin(\theta + 36^\circ) = \cos\theta$, where $\theta + 36^\circ$ is an acute angle, then θ is

5. Evaluate: $\frac{3\sin 62^\circ}{\cos 28^\circ} - \frac{\sec 42^\circ}{\operatorname{cosec} 48^\circ}$

6. The value of the expression $\tan 5^\circ \tan 25^\circ \tan 30^\circ \tan 45^\circ \tan 65^\circ \tan 85^\circ$ is:

   (1) $\sqrt{3}$ (2) $\frac{1}{\sqrt{3}}$
   (3) 1 (4) $\frac{1}{2}$

7. The value of the expression $2\left(\cos^4 60^\circ + \sin^4 30^\circ\right) - \left(\tan^2 60^\circ + \cot^2 45^\circ\right) + 3\sec^2 30^\circ$ is :

   (1) $\frac{4}{3}$ (2) $\frac{1}{3}$
   (3) $\frac{1}{4}$ (4) $\frac{1}{2\sqrt{3}}$

8. In a right angled ΔABC, right angled at B. If $\angle ACB = \theta$, AB = 1cm, BC = 2 cm, then the value of $\cos^2\theta + \cot^2\theta$ is _____ cm.

   (1) 10/23 (2) 5/24
   (3) 24/5 (4) 23/10

9. If $\cot B = \frac{12}{5}$, then $\tan^2 B - \sin^2 B$ = ?

   (1) $\cot^2 B - \operatorname{cosec}^2 B$ (2) $\sin^2 B \tan^2 B$
   (3) $\sin^2 B \cot^2 B$ (4) None of these

10. If $\sin\theta = \frac{m^2 - n^2}{m^2 + n^2}$, determine the value of $\tan\theta$.

    (1) $\frac{m^2 - n^2}{\sqrt{2}\sqrt{m^4 - n^4}}$ (2) $\frac{m^2 - n^2}{2mn}$
    (3) $\frac{m^2 + n^2}{2mn}$ (4) $\frac{m^2 + n^2}{\sqrt{2}\sqrt{m^4 - n^4}}$

11. Find the value of cos240° cos120° – sin240° sin120°.
(1) 0 (2) –1
(3) 4 (4) 1

12. The value of $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is:
(1) $\sec\theta - \cot\theta$ (2) $\cot\theta - \sec\theta$
(3) $\sec\theta + \tan\theta$ (4) $\sin\theta + \cos\theta$

13. Find the value of $\cot^4 A - 1$.
(1) $\text{cosec}^4 A - 2\text{cosec}^2 A$
(2) sec A + tan A
(3) $2\text{cosec}^2 A - 4\text{cosec}^4 A$
(4) None of these

14. What is the value of $\sin(\theta + 45°)$?
(1) $\frac{3}{\sqrt{2}}(\sin\theta + \cos\theta)$ (2) $\sqrt{2}(\sin\theta + \cos\theta)$
(3) $\frac{\sqrt{3}}{2}(\sin\theta - \cos\theta)$ (4) None of these

15. $\frac{\cot^2\theta - 1}{\cot^2\theta + 1} = ?$
(1) $-\cos 2\theta$ (2) $\tan 2\theta$
(3) $\cos 2\theta$ (4) $\sin\theta$

16. Find the value of $\sin(60° + \theta) - \sin(60° - \theta)$.
(1) $\sqrt{2}\cos\theta$ (2) $\sqrt{3}\sin\theta$
(3) $\sin\theta$ (4) $\cos\theta$

17. What is the value of 2sin75°.cos15°?
(1) $\frac{2+\sqrt{3}}{2}$ (2) $\frac{1+\sqrt{3}}{2}$
(3) $\frac{2+\sqrt{2}}{\sqrt{3}}$ (4) None of these

18. If $\tan A = \frac{7}{8}$ and $\tan B = \frac{1}{15}$, then the value of A + B =
(1) $\pi/3$ (2) $\pi/4$
(3) $\pi/2$ (4) π

19. Find the value of $\frac{1+\cos 2A + \sin 2A}{1-\cos 2A + \sin 2A}$.
(1) cot A (2) –cot A
(3) cot 2A (4) tan A

20. A circus artist is climbing from the ground along a rope, stretched from the top of vertical pole and tied at the ground. The height of the pole is 8 m and the angle made by the rope with ground level is 30°. The distance covered by the artist in climbing to the top of the pole is equal to ____.
(1) $\frac{24\sqrt{3}}{3}$ m (2) $\frac{16\sqrt{3}}{3}$ m
(3) 24 m (4) 16 m

21. Find the angle of elevation of the sun when the length of the shadow of a vertical pole is equal to its height.
(1) 30° (2) 60°
(3) 40° (4) 45°

22. A ladder leaning against a wall makes an angle of 60° with the ground. The distance between the foot of the ladder and wall is 36 m. Find the height of the wall.
(1) 66. 45 mts. (2) 92. 75 mts
(3) 78. 85 mts (4) None of these

23. A building 56 m high, has the angle of depression 30° and 60°, for the top and bottom of a vertical pole, respectively. Find the horizontal distance between the pole and the building.
(1) $\frac{56}{\left(\sqrt{3}+1\right)}$ mts (2) $\frac{56\sqrt{3}}{3}$ mts
(3) $\frac{56}{\left(\sqrt{3}-1\right)}$ mts (4) None of these

24. From the top of a building 60 m high, the angles of depression of the top and the bottom of a tower are observed to be 30° and 60° respectively. The height of the tower is ____.
(1) 50 m (2) 40 m
(3) 30 m (4) 20 m

25. Rohit was moving towards the hill to have a sight scene from its top. At a certain point in his way he found that the angle of depression to top of the hill was 45°. After travelling 30 m towards the hill he found that the angle of depression to the top of the hill how become 60°. Find the height of the hill.
(1) 69.80 mts (2) 70.98 mts
(3) 68.47 mts (4) None of these

26. The shadow of a flag staff is three times as long as the shadow of the flag staff when the sun rays meet the ground at an angle of 60°. The angle between the sun rays and the ground at the time of longer shadow is ____.
(1) 45° (2) 75°
(3) 15° (4) 30°

27. A beach rescue helicopter at an altitude of 250 m from the surface of the sea finds two persons sinking in the sea. If the angle of depression for the persons in opposite directions are 60° and 30°, find the distance between the two persons.
(1) $\frac{1000}{\left(\sqrt{3}-1\right)}$ mts (2) $\frac{1000}{\left(\sqrt{3}+1\right)}$ mts
(3) $\frac{1000}{3}$ mts (4) $\frac{1000}{\sqrt{3}}$ mts

28. A boy is flying a kite whose string is 125 m long and it makes an angle of 30° with the horizontal. Find the height of the kite above the ground.
(1) 67.5 m (2) 88. 5 m
(3) 69.5 m (4) None of these

29. A ladder 40 m long rests against a wall. If the feet of the ladder is 20 m from the wall, then the angle of elevation is ____.
(1) 30° (2) 45°
(3) 60° (4) 90°

30. The angle of elevation of the top of a tower from a point on the ground is 30°. After walking 200 m towards the tower, the angle of elevation becomes 60°. The height of the tower is ____.
(1) $100\sqrt{3}$ m (2) $200\sqrt{3}$ m
(3) 100 m (4) 200 m

Solutions
QA - 33 : Trigonometry

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1	2	**2**	3	**3**	1	**4**	27°	**5**	2	**6**	2	**7**	3	**8**	3	**9**	2	**10**	2
11	4	**12**	3	**13**	1	**14**	4	**15**	3	**16**	3	**17**	1	**18**	2	**19**	1	**20**	4
21	4	**22**	4	**23**	2	**24**	2	**25**	2	**26**	4	**27**	4	**28**	4	**29**	3	**30**	1

1. 2 $\left(\frac{\pi}{9}\right)^c = \left(\frac{\pi}{9}\times\frac{180}{\pi}\right)^0 = 20°$

2. 3 Let l be the length of the arc subtending an angle θ radian at the centre of a circle of radius r. Then $\theta = \frac{l}{r}$. We have, r = 6 cm and $\theta = 18° = \left[18\times\frac{\pi}{180}\right]^c$

$\theta = \left(\frac{\pi}{10}\right)^c$

$\therefore \theta = \frac{l}{r} \Rightarrow \frac{\pi}{10} = \frac{l}{6} \Rightarrow l = \frac{6\pi}{10} = \frac{3\pi}{5}$

3. $\tan1°\tan2°\tan3°...\tan89°$

$= \tan(90°-89°)\tan(90°-88°)\tan(90°-87°)$
$...\tan87°\tan88°\tan89°$

$= \cot89°\cot88°\cot87°...\tan87°\tan88°\tan89°$

$= (\cot89°\tan89°)(\cot88°\tan88°)(\cot87°\tan87°)$
$...(\cot44°\tan44°)\tan45°$

$= 1 \times 1 \times 1 ... \times 1 \times 1 = 1$

4. $\sin(\theta+36°) = \cos\theta$

or $\cos[90°-(\theta+36°)] = \cos\theta$

$\therefore 90°-(\theta+36°) = \theta \Rightarrow 2\theta = 54° \Rightarrow \theta = 27°$

5. $\frac{3\sin62°}{\cos28°} - \frac{\sec42°}{\text{cosec}48°}$

$= \frac{3\cos(90°-62°)}{\cos28°} - \frac{\text{cosec}(90°-42°)}{\text{cosec}48°}$

$= \frac{3\cos28°}{\cos28°} - \frac{\text{cosec}\,48°}{\text{cosec}\,48°} = 3(1) - 1 = 2$

[$\because$ sin (90° – θ) = cos θ, cosec (90° – θ) = sec θ]

6. 2 $\tan5°\tan25°\tan30°\tan45°\tan65°\tan85°$

$= (\tan5°\tan85°)(\tan25°\tan65°)\tan30°$

$= [\tan5°\tan(90°-5°)][\tan25°\tan(90°-25°)]\tan30°$

$= (\tan5°\cot5°)(\tan25°\cot25°)\tan30°$

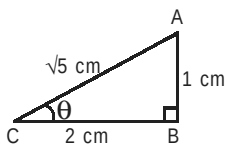
$= 1\times1\times\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}}$

7. 3 $2(\cos^4 60° + \sin^4 30°) - (\tan^2 60° + \cot^2 45°) + 3\sec^2 30°$

$= 2\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \left[\left(\sqrt{3}\right)^2 + (1)^2\right] + 3\left(\frac{2}{\sqrt{3}}\right)^2$

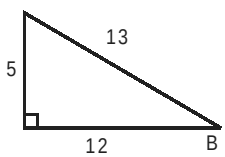
$= 2\left(\frac{1}{16} + \frac{1}{16}\right) - (3+1) + 3\times\frac{4}{3} = 2\times\frac{1}{8} - 4 + 4 = \frac{1}{4}$

8. 3



$\cos^2\theta + \cot^2\theta = \left(\frac{2}{\sqrt{5}}\right)^2 + \left(\frac{2}{1}\right)^2 = \frac{4}{5} + \frac{4}{1} = \frac{24}{5}$

9. 2



$\tan^2 B - \sin^2 B = \left(\frac{5}{12}\right)^2 - \left(\frac{5}{13}\right)^2$

$= \frac{25\times169 - 25\times144}{144\times169} = \frac{25\times25}{144\times169}$

$\sin^2 B\tan^2 B = \left(\frac{5}{13}\right)^2\left(\frac{5}{12}\right)^2 = \frac{25\times25}{169\times144}$

10. 2 $\sin\theta = \frac{m^2-n^2}{m^2+n^2} \Rightarrow \text{cosec}\,\theta = \frac{m^2+n^2}{m^2-n^2}$

Squaring both side

$$\text{cosec}^2\theta = \frac{\left(m^2+n^2\right)^2}{\left(m^2-n^2\right)^2}$$

$$\cot^2\theta = \text{cosec}^2\theta - 1 = \frac{m^4+n^4+2m^2n^2}{m^4+n^4-2m^2n^2} - 1$$

$$\cot^2\theta = \frac{4m^2n^2}{(m^2-n^2)^2}$$

$$\tan\theta = \frac{m^2-n^2}{2mn}$$

11. 4 $\cos240° \cos120° - \sin240° \sin120°$

we can also write this as

$\Rightarrow \cos(270° - 30°).\cos(90° + 30°)$

$- \sin(270° - 30°) \sin(90° + 30°)$

$\Rightarrow (-\sin30°)(-\sin30°) - [-\cos30°.\cos30°]$

$$\Rightarrow \frac{1}{2}.\frac{1}{2} + \frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$$

12. 3 $$\frac{\sqrt{1+\sin\theta}}{\sqrt{1-\sin\theta}} \times \frac{\sqrt{1+\sin\theta}}{1+\sin\theta} = \frac{1+\sin\theta}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{1+\sin\theta}{\sqrt{\cos^2\theta}} = \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

13. 1 $\cot^4 A - 1 = (\cot^2 A)^2 - 1 = (\cot^2 A - 1)(\cot^2 A + 1)$

$= (\cot^2 A - 1).\text{cosec}^2 A \quad [\because \text{cosec}^2\theta = 1 + \cot^2\theta]$

$= (\text{cosec}^2 A - 1 - 1)\,\text{cosec}^2 A = \text{cosec}^4 A - 2\,\text{cosec}^2 A$

14. 4 $\sin(\theta + 45°) = \sin\theta.\cos45° + \cos\theta.\sin45°$

[using sin(A + B) = sinA cosB + cosA sinB]

$$= \sin\theta.\frac{1}{\sqrt{2}} + \cos\theta.\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}[\sin\theta + \cos\theta]$$

15. 3 We have $$\frac{\cot^2\theta - 1}{\cot^2\theta + 1} = \frac{\frac{\cos^2\theta}{\sin^2\theta} - 1}{\text{cosec}^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} \times \frac{\sin^2\theta}{1} = \cos^2\theta - \sin^2\theta = \cos2\theta$$

$(\because \cos2A = \cos^2 A - \sin^2 A)$

16. 3 We have, $\sin(60° + \theta) - \sin(60° - \theta) = 2\cos60°.\sin\theta$

$[\because 2\cos A.\sin B = \sin(A + B) - \sin(A - B)]$

$$= 2.\frac{1}{2}.\sin\theta = \sin\theta$$

17. 1 $2\sin75°.\cos15° = \sin(75° + 15°) + \sin(75° - 15°)$

By using [2sinA.cosB = sin(A + B) + sin(A – B)]

$$= \sin90° + \sin60° = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$$

18. 2 $$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A + \tan B}$$

We have $\tan A = \frac{7}{8}$ and $\tan B = \frac{1}{15}$

$$\tan(A + B) = \frac{\frac{7}{8} + \frac{1}{15}}{1 - \frac{7}{8}.\frac{1}{15}} = \frac{105 + 8}{120 - 7}$$

$$\tan(A + B) = \frac{113}{113} = 1 = \tan\frac{\pi}{4} \qquad \left[\because \tan\frac{\pi}{4} = 1\right]$$

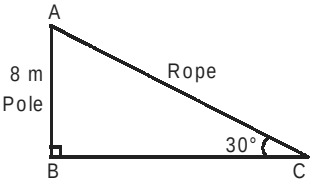
$$\therefore A + B = \frac{\pi}{4}$$

19. 1 $$\frac{(1+\cos2A) + \sin2A}{(1-\cos2A) + \sin2A} = \frac{2\cos^2 A + 2\sin A\cos A}{2\sin^2 A + 2\sin A\cos A}$$

$\left[\because (1 + \cos2\theta = 2\cos^2\theta) \text{ and } (1 - \cos2\theta = 2\sin^2\theta)\right]$

$$= \frac{2\cos A[\cos A + \sin A]}{2\sin A[\sin A + \cos A]} = \frac{\cos A}{\sin A} = \cot A$$

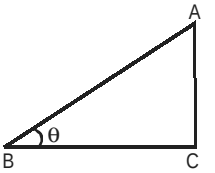
20. 4



In right-angled triangle ABC,

$$\sin30° = \frac{AB}{AC} \Rightarrow \frac{1}{2} = \frac{8}{AC} \Rightarrow AC = 16 \text{ m}$$

21. 4



AC = height and BC = length of the shadow

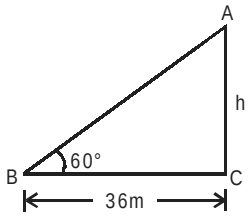
Let θ be the angle of elevation

$\therefore \tan\theta = \frac{AC}{BC}$ But AC = BC

$\therefore$ $\tan\theta = 1 \Rightarrow \tan\theta = \tan 45°$

$\therefore$ $\boxed{\theta = 45°}$

22. 4



Let the height of the wall (AC) = h mts.

$\because \tan 60° = \frac{h}{36}$

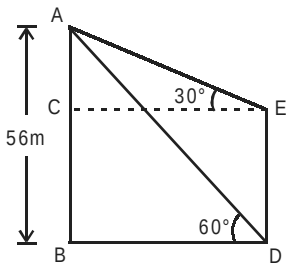
$\therefore$ h = 36 × tan 60°

$h = 36\sqrt{3}$

h = 36 × 1.732

h = 62. 353 $\simeq$ 62.35 mts.

23. 2

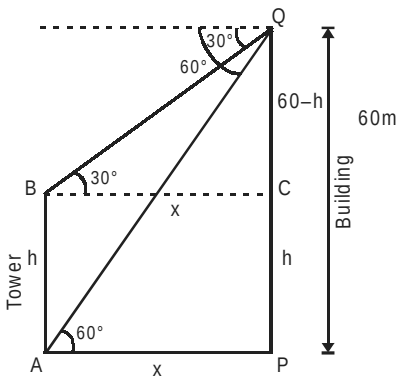


In ΔABD

$\tan 60° = \frac{AB}{BD} \Rightarrow BD = \frac{AB}{\tan 60°} = \frac{56}{\sqrt{3}}$

$= \frac{56}{\sqrt{3}}$ mts $= \frac{56}{3}\sqrt{3}$ mtr.

24. 2



In ΔBCQ,

$\tan 30° = \frac{60-h}{x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{60-h}{x}$

$\Rightarrow x = (60-h)\sqrt{3}$ ... (i)

In ΔAPQ,

$\tan 60° = \frac{60}{x}$

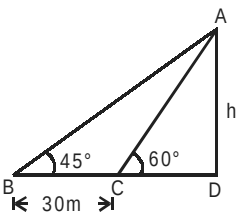
$\Rightarrow \sqrt{3}x = 60 \Rightarrow (60-h)\sqrt{3} \times \sqrt{3} = 60$

$\Rightarrow 60 - h = 20 \Rightarrow h = 40$ m

25. 2 Let height of the hill be AD mts.

In Δ ABD

$\tan 45° = \frac{AD}{30 + CD}$



$\Rightarrow$ AD = 30 + CD

In Δ ACD

$\tan 60° = \frac{AD}{CD} \Rightarrow CD = \frac{AD}{\tan 60°} = \frac{AD}{\sqrt{3}}$

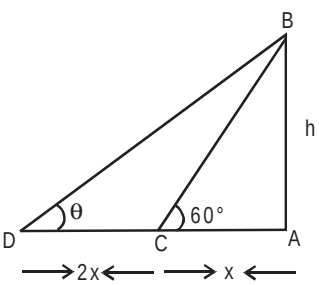
Combining the two equations we have

$AD = CD + 30 \Rightarrow AD = \frac{AD}{\sqrt{3}} + 30$

$AD\left(\sqrt{3}-1\right) = 30\sqrt{3} \Rightarrow AD = \frac{30\sqrt{3}}{\sqrt{3}-1}$

$\Rightarrow AD = 15\,(3 + \sqrt{3}) = 15 \times 4.732 = 70.980 = 70.98$ mts

26. 4



Let AB be the flag staff and let θ be the angle

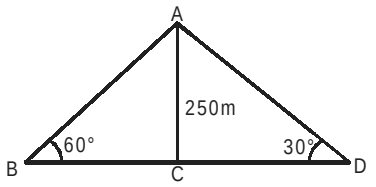
In ΔABC,

$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow h = \sqrt{3}x$

In ΔABD,

$\tan\theta = \frac{h}{3x} \Rightarrow \tan\theta = \frac{\sqrt{3}x}{3x} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = 30°$

27. 4



Let A be the point where rescue helicopter was and AC be the height of helicopter from surface.

In ΔABC

$$\tan 60° = \frac{AC}{BC} \Rightarrow AC = \tan 60° \times BC = \sqrt{3} \times BC$$

$$BC = \frac{AC}{\sqrt{3}} = \frac{250}{\sqrt{3}} \text{ m.}$$

In ΔACD

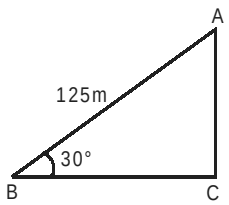
$$\tan 30° = \frac{AC}{CD} \Rightarrow CD = \frac{AC}{\tan 30°} = \frac{250}{\frac{1}{\sqrt{3}}} = 250\sqrt{3} \text{ m.}$$

$\because$ Distance between the two person = BC + CD

$$= \frac{250}{\sqrt{3}} + 250\sqrt{3} = 250\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right)$$

$$= 250\left(\frac{1+3}{\sqrt{3}}\right) = \frac{1000}{\sqrt{3}} \text{ mts.}$$

28. 4

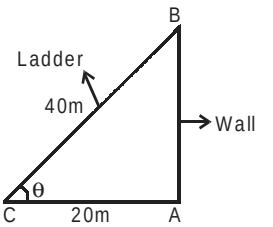


Let AC be the height of the kite above the ground.

$$\because \sin 30° = \frac{AC}{AB}$$

$$\Rightarrow AC = AB \times \text{Sin } 30° = 125 \times \frac{1}{2} = 62.5 \text{ m}$$

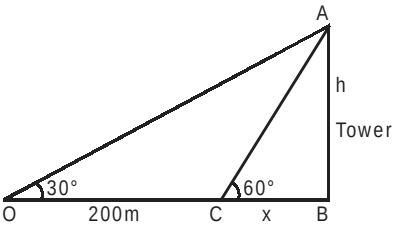
29. 3



Let θ be the angle of elevation. Then

$$\cos\theta = \frac{AC}{CB} \Rightarrow \cos\theta = \frac{20}{40} = \frac{1}{2} \Rightarrow \theta = 60°$$

30. 1



In ΔABC,

$$\tan 60° = \frac{h}{x} \Rightarrow \sqrt{3} = \frac{h}{x} \quad \text{... (i)}$$

In ΔABO,

$$\tan 30° = \frac{h}{200 + x} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{200 + x} \quad \text{... (ii)}$$

From (i) and (ii),

$$h = 100\sqrt{3} \text{ m}$$