Numbers - 2

Career Launcher

QA - 12

CEX-Q-0213/18

Contents

- Factors

Number of Questions : 20

1. Consider $N = 2^5 \times 3^4 \times 5^3$
   a. Find the total number of factors of 'N'.
   b. Find the total number of factors of 'N' which are odd.
   c. Find the total number of factors of 'N' which are even.
   d. Find the total number of factors of 'N' which are composite.
   e. Find the total number of factors of 'N' which are multiple of 5.
   f. Find the total number of factors of 'N' which are multiple of 15.
   g. Find the total number of factors of 'N' which are perfect square.
   h. Find the total number of factors of 'N' which are perfect cube.
   i. Find the total number of factors of 'N' which has unit's digit zero.
   j. Find the total number of factors of 'N' whose last two digits zero.
   k. Find the sum of all factors of 'N'.
   l. Find the product of all factors of 'N'.

2. Consider $N = 3^2 \times 6^2 \times 5^4 = A \times B$
   a. In how many ways 'N' can be written as a product of two numbers?
   b. In how many ways 'N' can be written as a product of two different numbers?
   c. In how many ways 'N' can be written as a product of two numbers which are odd?
   d. In how many ways 'N' can be written as a product of two numbers which are even?
   e. In how many ways 'N' can be written as a product of two numbers such that one of them is even and the other is odd?
   f. In how many ways 'N' can be written as a product of two numbers which are co-prime to each other?

3. Find all the prime factors of 660.
   (1) 2, 3, 5 and 13 (2) 3, 5, 7 and 13
   (3) 2, 3, 5 and 11 (4) 2, 3, 5 and 7

4. How many factors of 350 are even?

5. How many factors of 464 are divisible by 4?

6. How many factors of 1296 have exactly three factors?
   (1) 1 (2) 2
   (3) 3 (4) 4

7. Which of the following numbers has exactly eight factors?
   (1) 999 (2) 1001
   (3) 1010 (4) All of these

8. If n is an integer, for how many values of n will the expression $\frac{16n^2 + 7n + 6}{n}$ be an integer.
   (1) 2 (2) 3
   (3) 4 (4) 5

9. What is the largest possible natural number 'n' for which the following expression would be a natural number?

$$\frac{20n^4 + 15n^3 + 10n^2 + 5n + 10}{5n}$$

10. Find the sum of all the natural numbers from 1 to 100 that are neither a multiple of 2 nor a multiple of 5.

(1) 2000 (2) 2275
(3) 2515 (4) 2845

11. Let N be a natural number such that N = a × b, where a and b are distinct factors of N. How many such sets of (a, b) if N = 24 × 33?

12. How many sets of three distinct factors of the number $N = 2^6 \times 3^4 \times 5^2$ can be made such that the HCF of any two numbers in each sets is 1?

(1) 236 (2) 360
(3) 104 (4) 380

13. A two-digit number has exactly 3 factors, excluding 1 and number itself. What is the sum of all the values of the number?

(1) 97 (2) 96
(3) 359 (4) 1020

14. A "Pure Factor" of a natural number "n" is that factor of n which has only one prime factor. S_n is the sum of all the pure factors of n. What is the value of S_{300}?

(1) 167 (2) 172
(3) 169 (4) 140

15. N_x is a family of natural numbers for different values of x, where x is a natural number. Numbers in the family share a common property of having 24 factors each of the numbers. All the elements of N are arranged in ascending order as follows: $N_1 < N_2 < N_3$ <...and so on. Now, digital sum of a number is defined as the sum of all the digits of number until we get a single digit. What is the digital sum of N_1?

(1) 9 (2) 6
(3) 1 (4) 3

16. Four distinct non-zero single digit numbers a, b, c and d satisfy the following four conditions:

I. The sum of (b, c, d) is a multiple of 9.
II. The sum of (a, c, d) is a multiple of 8.
III. The sum of (a, b, d) is a multiple of 5.
IV. The sum of (a, b, c) is a multiple of 23.

What is the value of a × b?

(1) 24 (2) 35
(3) 40 (4) 48

17. Let S be the set of five-digit numbers formed by digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd position are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?

(1) 228 (2) 216
(3) 294 (4) 192

18. P is the product of first 30 multiples of 30. N is the total number of factors of P. In how many ways N can be written as the product of two natural numbers such that the HCF of these two natural numbers is 19?

19. If $\frac{94}{27} = a + \cfrac{1}{b + \cfrac{1}{c}}$, where a, b and c are positive integers, then find a + b + c.

(1) 18 (2) 20
(3) 12 (4) 15

20. In how many ways 1000 can be written as a sum of two or more consecutive natural numbers?

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QA - 12 : Numbers - 2
Answers and Explanations

CEX-Q-0213/18

1	–	**2**	–	**3**	3	**4**	–	**5**	–	**6**	2	**7**	4	**8**	3	**9**	–	**10**	1
11	–	**12**	1	**13**	2	**14**	2	**15**	1	**16**	4	**17**	2	**18**	–	**19**	1	**20**	–

1. $N = 2^5 \times 3^4 \times 5^3$

The powers of 2, 3 and 5 that are present in the number are (0, 1, 2, 3, 4, 5), (0, 1, 2, 3, 4) and (0, 1, 2, 3).

a. Total number of factors is 6 × 5 × 4 = 120.

b. For odd factors take only 1 power of 2 i.e. 0 power. Hence, required number of factors = 1 × 5 × 4 = 20.

c. Number of even factors = total factors – odd factors = 120 – 20 = 100.

d. N has 3 prime factors i.e. 2, 3, and 5. Also, 1 is neither prime nor composite.
So total number of composite factors = 120 – 4 = 116.

e. A factor will be a multiple of 5 if it contains at least one power of 5. So out of the powers mentioned in N, one power of 5 (i.e. 0 power) is not considered.
So, number of factors 6 × 5 × 3 = 90.

f. $45 = 3^2 \times 5$. To make a number a multiple of 45 we need to take at least 2 powers of 3 and at least 1 power of 5. So **powers** of 2, 3 and 5 now allowed are (0, 1, 2, 3, 4, 5), (2, 3, 4) and (1, 2, 3).
Hence, the number of factors that are divisible by 45 = 6 × 3 × 3 = 54.

g. Perfect squares will have only even powers of the primes. So powers of the primes must be (0, 2, 4), (0, 2, 4) and (0, 2) respectively. So number of factors that are perfect square is 3 × 3 × 2 = 18.

h. Perfect cubes will have only those powers which are divisible by 3. So powers used to make factors as perfect cube are (0, 3), (0, 3) and (0, 3) respectively.
So, the number of factors that are perfect cube = 2 × 2 × 2 = 8.

i. If the unit digit of a number is 0 it must be a multiple of 2 as well as 5. So power 0 will be omitted from powers of 2 and 5.
Hence, number of factors with unit digit 0 = 5 × 5 × 3 = 75.

j. For last 2 digits to be 0 the number must contain at least 2 powers of 2 and 5 both. Hence such factors will be 4 × 5 × 2 = 40.

k. Sum of the factors is $(2^0 + 2^1 + 2^2 + \ldots + 2^5)(3^0 + 3^1 + \ldots + 3^4)(5^0 + 5^1 + 5^2 + 5^3) = 1189188$.

l. Product of the factors = $(N)^{\frac{\text{Number of factors}}{2}} = N^{60}$

2. $N = 3^2 \times 2^2 \times 3^2 \times 5^4 = 2^2 \times 3^4 \times 5^4$

a. Total number of factors of N is (2 + 1) × (4 + 1) × (4 + 1) = 75. If N is written as a product of two numbers, both of them must be factors of N. Here N is a perfect square and thus one of the factors will be of the form $\sqrt{N} \times \sqrt{N}$. For other 74 factors, there will be $\frac{74}{2} = 37$ pairs. So a total of 38 pairs are there.

b. Two different ways will be 37 as $\sqrt{N} \times \sqrt{N}$ will not be allowed now.

c. As N is an even number, it cannot be written as a product of 2 odd numbers.

d. Let N = 2a × 2b. Hence $3^4 \times 5^4 = a \times b$. Again a and b both must be the factors of $3^4 \times 5^4$ and for each value of a there must be a unique value of b. So total of 5 × 5 = 25 ways. Out of these 25 ways, there will be exactly 1 way when both a and b are equal. In other 24 ways, the unordered ways will be $\frac{24}{2} = 12$. Hence total number of ways is 13.

e. As one of them is even so all powers of 2 must be there in that number only. Let $N = (2^2 \times a) \times b$. So our answer would be same as the total number of factors of $3^4 \times 5^4$ i.e. (4 + 1) × (4 + 1) = 25.

f. A number N can be written as a product of 2 co-prime numbers in 2^{P-1} ways where P is the number of prime factors. Hence there are 2^{3-1} = 4 ways.

3. 3 $660 = 2^2 \times 3 \times 5 \times 11$

∴ Prime factors are 2, 3, 5 and 11.

4. Now $350 = 2 \times 7 \times 5^2$

Number of even factors = (Total number of factors) – (Total number of odd factors)

Number of even factors = 12 – 6 = 6.

5. $464 = 2^4 \times 29 = 4(2^2 \times 29)$
So, the number of factors of 464 that are divisible by $4 = 3 \times 2 = 6$.

6. 2 If we factorize 1296 we get $1296 = 2^4 \times 3^4$
Therefore the total number of factors is $(4 + 1)(4 + 1) = 25$
Therefore those numbers that have exactly 3 factors will be in the form of $2^2 \times 3^0$ or $2^0 \times 3^2$
So there are only 2 such numbers 4 and 9.

7. 4 Option (1): $999 = 3^3 \times 37^1$
Number of factors of 999 = $(3 + 1) \times (1 + 1) = 8$
Option (2): $1001 = 7^1 \times 11^1 \times 13^1$
Number of factors of 1001
$= (1 + 1) \times (1 + 1) \times (1 + 1) = 8$
Option (3): $1010 = 2^1 \times 5^1 \times 101^1$
Number of factors of 1010
$= (1 + 1) \times (1 + 1) \times (1 + 1) = 8$
Hence, option (4) is the correct answer.

8. 3 Let $M = \dfrac{16n^2 + 7n + 6}{n} = 16n + 7 + \dfrac{6}{n}$

For $\dfrac{6}{n}$ to be an integer, n has to be a factor of 6 i.e. 1, 2, 3 and 6. Therefore, n can take 4 values.

9. Simplifying the equation, we get
$$\left(4n^3 + 3n^2 + 2n + 1 + \frac{2}{n}\right)$$
To make this a natural number, the maximum value of n is 2.

10. 1 Sum of all numbers from 1 to 100 = 5050
Sum of all multiples of 2 = 2550
Sum of all multiples of 5 = 1050
Sum of all multiples of 10 = 550
Hence, required value = 5050 – 2550 – 1050 + 550 = 2000.

11. The total number of factors = $(3 + 1)(2 + 1)(1 + 1) = 24$.
12 of these factors would give N when multiplied with one of the 12 other factors.

12. 1 $N = 2^6 \times 3^4 \times 5^2$
Case [A]: when none of the elements is 1.
The three factors must be some powers of 2, 3 & 5 respectively.
Total number of such sets = $6 \times 4 \times 2 = 48$
Case [B]: when one of the elements is 1.
The other two factors could be of the form
(2^a), (3^b) – number of sets = $6 \times 4 = 24$
(2^a), (5^b) – number of sets = $6 \times 2 = 12$
(3^a), (5^b) – number of sets = $4 \times 2 = 8$
$(2^a \times 3^b)$, (5^c) – number of sets = $6 \times 4 \times 2 = 48$
$(2^a \times 5^b)$, (3^c) – number of sets = $6 \times 2 \times 4 = 48$
$(3^a \times 5^b)$, (2^c) – number of ses = $4 \times 2 \times 6 = 48$
$\therefore$ Total number of such sets = 188.
Combining both the cases, total sets possible
= 188 + 48 = 236.

13. 2 According to the given condition, the number of factors of the number is 5.
If a number has an odd number of factors, it has to be the square of a natural number. Also 5 can be written as a product of two numbers in only one way i.e. 5×1 and thus the number must be of the form a^4, where a is a prime number. Only two numbers viz. 2^4 and 3^4 satisfy the aforementioned conditions.
Hence, the required sum = 81 + 16 = 97.

14. 2 $n = 3000 = 2^3 \times 3 \times 5^3$
Pure Factors: 2, 2^2, 2^3, 3, 5, 5^2 and 5^3
Hence, the required answer = 2 + 4 + 8 + 3 + 5 + 25 + 125 = 172

15. 1 If N_X is in the form $a^P \times b^Q \times c^R$ ……
Therefore, number of factors will be $(P + 1)(Q + 1)(R + 1)$….
Here $(P + 1)(Q + 1)(R + 1)… = 24$
$360 = 2^3 \times 3^2 \times 5$, is the smallest number which has 24 factors.
Therefore, sum of the digits = 3 + 6 + 0 = 9

16. 4 All of them are distinct non-zero single digits numbers so they can take values 1, 2, 3, 4, 5, 6, 7, 8, 9. The maximum value of the sum of any of any three of the numbers can be 9 + 8 + 7 = 24. Accordingly
b + c + d = 9 or 18 …(1)
a + c + d = 8 or 16 or 24 …(2)
a + b + d = 5 or 10 or 15 or 20 …(3)
a + b + c = 23 …(4)
For their sum to be 23, the numbers, a, b and c must be 9, 8 and 6 only (in any order). Using this (b + c + d) $\neq$ 9, (a + c + d) $\neq$ 8 and (a + b +d) $\neq$ 5 or 10. Now we can write:
b + c + d = 18 …(5)
a + c + d = 16 or 24 …(6)
a + b + d = 15 or 20 …(7)
a + b + c = 23 …(8)
We have deduced (a, b, c) are (6, 8, 9) in any order.
Now (b + c) could be 14 or 15 or 17. Putting these values in equation (5) and (7) we get:
d = 4 or 3 or 1 ...(From equation (5))
d = 1 or 6 ...(From equation (7))
As d has a unique value, it must be consistent with both the above results hence d must be 1. In equation (7), (a + b) is 14 or 15 or 17 and d = 1
$\Rightarrow$ (a + b + d) = 15 only.

$\Rightarrow$ (a + b) = 14

$\Rightarrow$ (a, b) is (6, 8) in any order. ...(9)

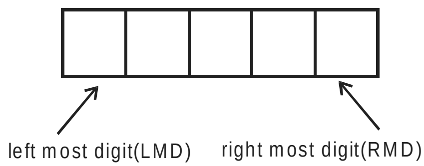
(b + c) = 17 $\Rightarrow$ (b, c) must be (9, 8) ...(10)

From (9) and (10) it can be seen that b must be 8.

$\Rightarrow$ a = 6 and c = 9 ...(11)

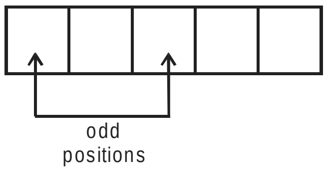
So (a, b, c, d) are (6, 8, 9, 1) in that order.

17. 2

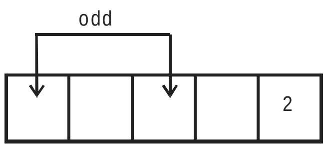


Odd positions can be counted in 2 ways.

(i) Counting from the LMD-end:

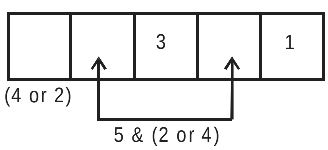


We have 1, 2, 3, 4 & 5 to be filled in these blocks. Odd nos. (1, 3, 5) to be filled in at odd positions. Other places are to be filled by even nos. (2 or 4) Let's count, how many such nos. are there with 2 at the unit's digit.

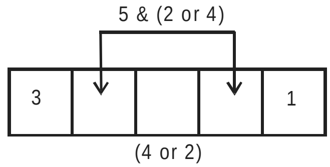


Odd nos. can be filled in $^3P_2$ = 6 ways.
The remaining two places are to be filled by 2 nos. (one odd no. left out of 1, 3, 5 & one even i.e. 4) in = 2 ways.
So, there are 6 × 2 = 12 number with 2 at the rightmost place. Similarly, there are 12 such nos. with 4 at the rightmost digits.
The sum of rightmost digits in all such number = 12(2 + 4) = 72

(ii) Now counting from the RMD-end.
Let's place 1 at the units place and check, how many nos. are possible with (1, 3) at the odd positions:



No. of such cases = 2 × 2 = 4 ways.



Here again no. of ways = 2 × 2 = 4 ways
So, there are 4 + 4 = 8 nos, in which (1, 3) are at odd positions. Similarly there are 8 nos. in which (1, 5) are at odd positions. So, in all there are 16 nos. where 1 is at unit's place. Similarly there are 16 nos. with 3 at unit's place and 16 more with 5 at unit's place.
Summing up all the odd unit's digits = 16(1 + 3 + 5) = 144
From (i) and (ii) we can now, sum up all (even or odd) nos. at units place = 72 + 144 = 216
Hence answer is (2)

18. P = (1 × 30) × (2 × 30) × (3 × 30) × (4 × 30) × … × (29 × 30) × (30 × 30).
P = 30^{30} × 30!

$$P = 2^{56} \times 3^{44} \times 5^{37} \times 7^4 \times 11^2 \times 13^2 \times 17 \times 19 \times 23 \times 29.$$

$$N = 57 \times 45 \times 38 \times 5 \times 3 \times 3 \times 2^4 = 2^5 \times 3^5 \times 5^2 \times 19^2$$

Let the two numbers be '19a' and '19b' respectively such that 'a' and 'b' are relatively prime to each other.

$\Rightarrow$ (19a) × (19b) = N = $2^5 \times 3^5 \times 5^2 \times 19^2$

$\Rightarrow$ ab = $2^5 \times 3^5 \times 5^2$

Possible pairs (a, b) are

$(2^5 \times 3^5 \times 5^2, 1); (2^5, 3^5 \times 5^2); (2^5 \times 3^5, 2^2); (2^5 \times 5^2, 3^5)$.

Therefore, N can be written as the product of two numbers, such that their HCF is 19, in 4 ways.

Short cut method: Here, the answer has to be a power of 2. (Why?)

19. 1 $\dfrac{94}{27} = 3 + \dfrac{13}{27} = 3 + \dfrac{1}{\frac{27}{13}} = 3 + \dfrac{1}{2 + \frac{1}{13}}$

$\therefore$ a = 3, b = 2 and c = 13.
Hence a + b + c = 3 + 2 + 13 = 18.

20. Number of ways in which a number can be written as sum of two or more consecutive natural numbers is one less than total number of odd factors of the number.
Here, 1000 = $2^3 \times 5^3$.
So number of ways = 3 + 1 – 1 = 3.