CATapult Courseware

Module 3
Quantitative Ability

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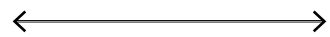
QA-3.1 | TRIANGLES - I

THEORY

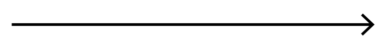
Lines and Planes

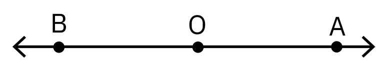
Definition of Terms

Line: A line has length. It has neither width nor thickness. It can be extended indefinitely in both directions.

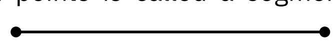


Ray: A line with one end point is called a ray. The end point is called the origin. Two rays, which lie on the same line and have only the origin as a common point are called opposite rays. Rays OA and OB are opposite rays.

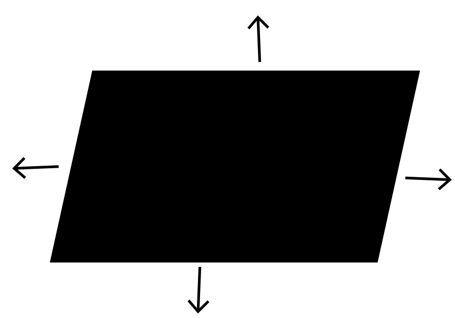




Line Segment: A line with two end points is called a segment.



Plane: A plane is a flat surface. It has length and width but no thickness.



Parallel lines: Two lines, which lie in a plane and do not intersect, are called parallel lines. The distance between two parallel lines is constant. Parallel is indicated by the symbol ||.
Perpendicular lines: Two lines, which lie in a plane and intersect each other at right angles are called perpendicular lines. Perpendicular is denoted by the symbol $\perp$.

Note: The concept of right angles will be covered in Angles later in this chapter.

Properties of Lines and Planes

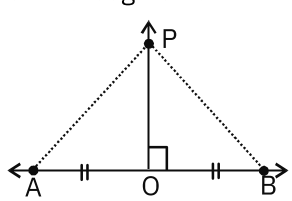
1. There are an infinite number of lines passing through a given point. These lines are called **concurrent lines**.
2. There is one and only one line passing through two distinct points.
3. The intersection of two distinct lines is a **point**.
4. **Three or more points** are said to be **collinear** if they lie on a line, otherwise they are said to be **non-collinear**.

5. There are an infinite number of planes passing through any given line.
6. There is exactly one plane passing through three non-collinear points.
7. A line and a point not on the line lies in one plane.
8. Two distinct intersecting lines lie in a plane.
9. **Two or more lines** are said to be **coplanar** if they lie **in the same plane**, otherwise they are said to be non-coplanar.
10. **Four or more points** are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.
11. If two points of a line lie in a plane, the whole line lies in the plane.
12. A-P-B implies that the points A, P and B are collinear and P lies in between A and B.
13. The intersection of two planes is a line.
14. A line, which intersects two or more given coplanar lines in distinct points, is called a **transversal** of the given lines.

Example

In fig (i) below, line is a transversal of lines a, b, c and line a is a transversal of lines and m

15. A line which is perpendicular to a line segment i.e., intersects at 90° and passes through the midpoint of the segment is called the **perpendicular bisector** of the segment.
16. Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.



Conversely, if any point is equidistant from the two endpoints of the segment, then it must lie on the perpendicular bisector of the segment.

If PO is the perpendicular bisector of segment AB, then, **AP = PB**.

Also, if AP = PB, then P lies on the perpendicular bisector of segment AB.

17. If two lines are perpendicular to the same line, they are parallel to each other.
18. The ratio of intercepts made by three parallel lines on a transversal is equal to the ratio of the corresponding intercepts made on any other transversal by the same parallel lines.

If line a || line b || line c and line ℓ and line m are two transversals, then, $\frac{PR}{RT} = \frac{QS}{SU}$.

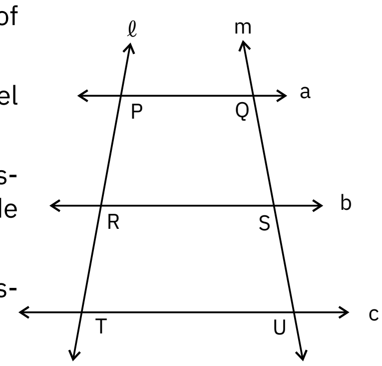
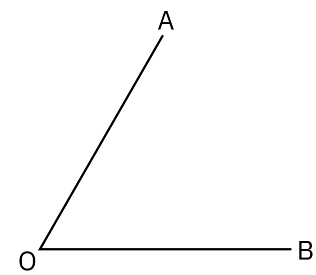


fig (i)

Angles

An **angle** is the union of two non-collinear rays with a common origin. The common origin is called the **vertex** and the two rays are the sides of the angle.

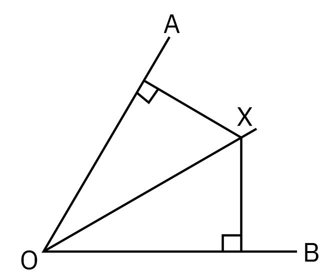


The angle AOB denoted by $\angle AOB$, is formed by rays OA and OB and point O is the common origin. Angles are measured in degrees or radians.

$m\angle AOB$ denotes the measure of $\angle AOB$.

Congruent Angles: Two angles are said to be congruent, (denoted by $\cong$), if their measures are equal.

Bisector of an angle: A ray is said to be the bisector of an angle if it divides the interior of the angle into two angles of equal measure.



Ray OX is the bisector of $m\angle AOB$ if $m\angle AOX = m\angle XOB = \frac{1}{2} m\angle AOB$

Every point on the angle bisector is equidistant from the sides of the angle. Conversely, if any point in the plane of an angle is equidistant from the sides of the angle, then it lies on the angle bisector of the angle.

If OX is the angle bisector of $\angle AOB$ then AX = XB. Also, if AX = XB, then X lies on the angle bisector of $\angle AOB$.

Types of angles

A right angle is an angle of 90° as shown in fig.(i). Ray OB and Ray OA are perpendicular to each other.

An angle less than 90° is called an **acute angle**.

An angle greater than 90° but less than 180° is called an **obtuse angle**.

An angle of 180° is a **straight line**.

An angle greater than 180° but less than 360° is called a **reflex angle**.

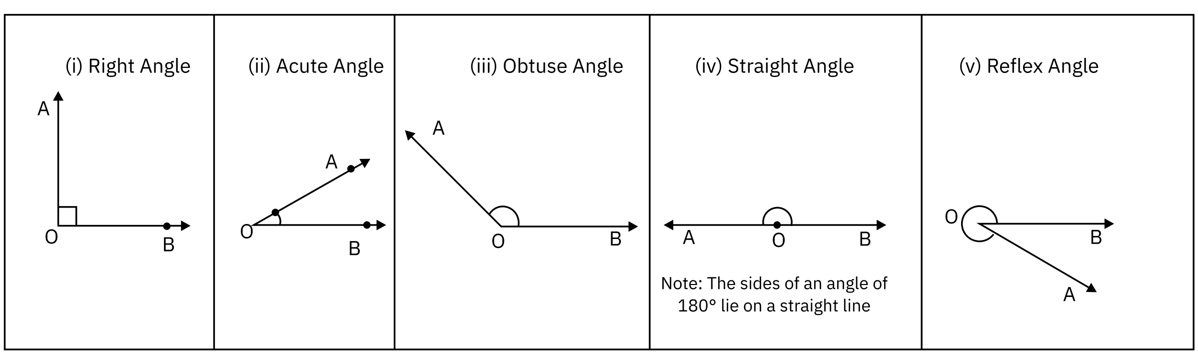
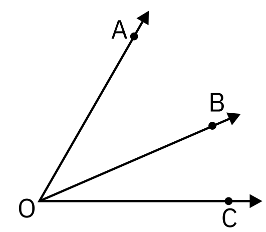


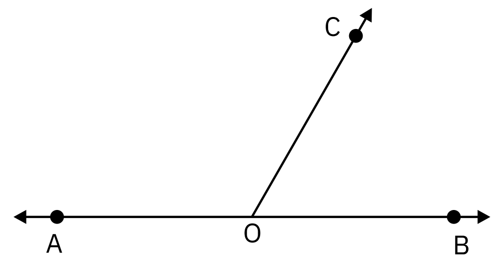
fig (ii)

Pairs of Angles

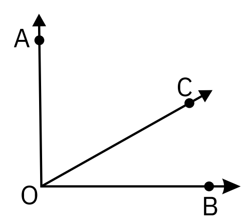
Adjacent Angles: Two angles are called adjacent angles if they have a common side and their interiors are disjoint i.e., separate.
$\angle AOB$ and $\angle BOC$ are adjacent angles.



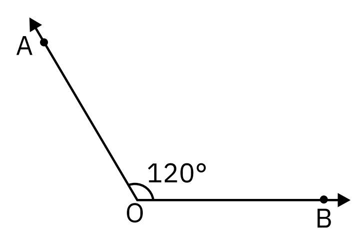
Linear Pair: Two angles are said to form a linear pair if they have a common side and their other two sides are opposite rays. The sum of the measures of the angles is 180°. The angles that form a linear pair are always adjacent.
$\angle AOC$ and $\angle COB$ form a linear pair.
$m\angle AOC + m\angle COB = 180°$

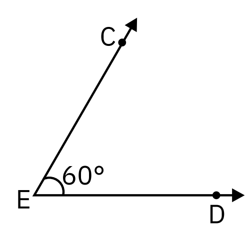


Complementary Angles: Two angles whose sum is 90°, are complementary, each one is the complement of the other. If $m\angle AOC + m\angle COB = 90°$, then $\angle AOC$ and $\angle COB$ are complementary angles.



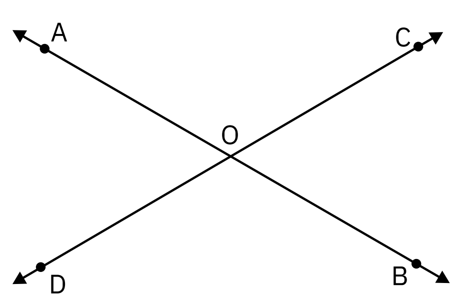
Supplementary Angles: Two angles whose sum is 180°, are supplementary, each one is the supplement of the other. $m\angle AOB + m\angle CED = 120° + 60° = 180°$ therefore these angles are supplementary.





Also, angles of a linear pair are supplementary. (Refer diagram of linear pair as shown above).

Vertically Opposite Angles: Two angles are called vertically opposite angles if their sides form two pairs of opposite rays. In other words, when the two lines intersect, two pairs of vertically opposite angles are formed.



Vertically opposite angles are congruent.
$\angle AOC$ and $\angle DOB$ are vertically opposite.
Also, $\angle AOD$ and $\angle COB$ are vertically opposite.
$\angle AOC \cong \angle DOB$ and $\angle AOD \cong \angle COB$
Also, sum of all angles at a point is 360°. $\Rightarrow m\angle AOC + m\angle DOB + m\angle AOD + m\angle COB = 360°$

Corresponding Angles: When two lines are intersected by a transversal, they form four pairs of corresponding angles.
EF and GH are intersected at points B and C by the transversal AD. The four pairs of corresponding angles are:

(i) ∠ABE, ∠BCG (ii) ∠EBC, ∠GCD

(iii) ∠ABF, ∠BCH (iv) ∠FBC, ∠HCD

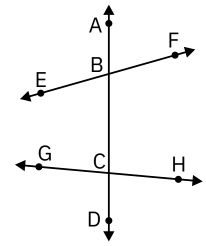


Fig.A

When the two parallel lines are intersected by a transversal, the pairs of corresponding angles so formed are congruent.
If EF is parallel to GH and AD is the transversal then:

(i) ∠ABE ≅ ∠BCG (ii) ∠EBC ≅ ∠GCD

(iii) ∠ABF ≅ ∠BCH (iv) ∠FBC ≅ ∠HCD

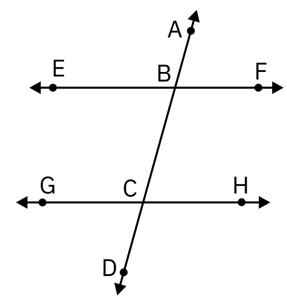


Fig.B

Conversely, if the transversal intersects two lines and if one pair of corresponding angles is congruent then the two lines are parallel. Hence, when one pair of corresponding angles is congruent, then all the pairs of corresponding angles are congruent.

Alternate Angles: When two lines are intersected by a transversal, they form two pairs of alternate angles.
In fig. A, the pairs of alternate angles are:

(i) ∠EBC, ∠BCH (ii) ∠FBC, ∠GCB

When two parallel lines are intersected by a transversal, the pairs of alternate angles so formed are congruent.
In fig. B:

(i) ∠EBC ≅ ∠BCH (ii) ∠FBC ≅ ∠GCB

Conversely, if the transversal intersects two lines and if one pair of alternate angles is congruent, then the two lines are parallel. Hence, when one pair of alternate angles is congruent then the other pair of alternate and all pairs of corresponding angles are congruent.

Interior Angles: When two lines are intersected by a transversal, they form two pairs of interior angles.
In fig.A, the pairs of interior angles are:

(i) ∠EBC, ∠GCB (ii) ∠FBC, ∠BCH

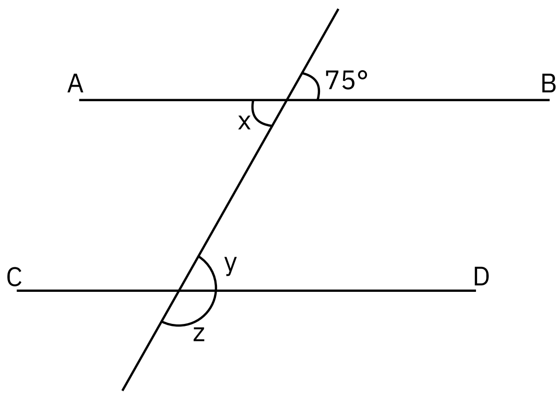
When two parallel lines are intersected by a transversal, the pairs of interior angles so formed are supplementary.
In fig.B:

(i) $m\angle EBC + m\angle GCB = 180°$ (ii) $m\angle FBC + m\angle BCH = 180°$

Conversely, if the transversal intersects two lines and if one pair of interior angles is supplementary then the two lines are parallel. Hence, when one pair of interior angles is supplementary, the other pair is also supplementary and all pairs of alternate and corresponding angles are congruent.

SOLVED EXAMPLES

Q : AB and CD are two parallel lines. Find x, y and z.



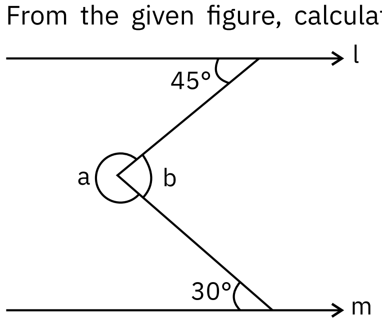
A : $x = 75°$ (vertically opposite angles)

$y = 75°$ (alternate angles)

$y + z = 180°$ (supplementary angles)

$\therefore\ z = 105°$.

Q : From the given figure, calculate a and b. (Given lines l and m are parallel)



A : Draw the construction, as shown alongside.

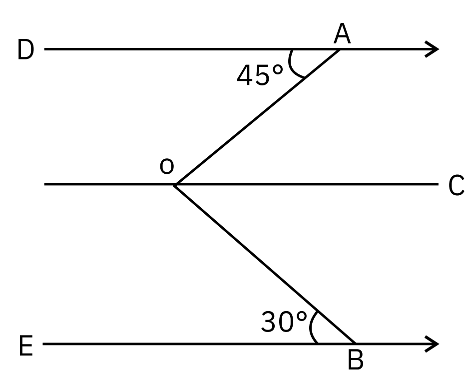
$\angle OAD = \angle AOC$(alternate angle)

Thus, $\angle AOC = 45°$

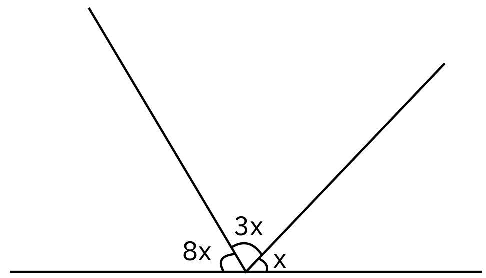
$\angle EBO = \angle BOC = 30°$ (alternate angles)

Thus, $\angle b = 75°$

$\angle a = 360° - 75° = 285°$



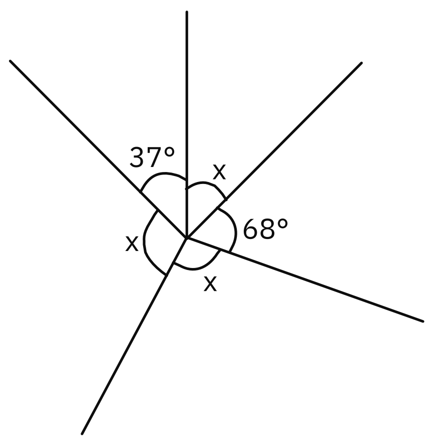
Q : Calculate x.



A : The sum of all angles = 180° ⇒ x + 3x + 8x = 180°

∴ 12x = 180°   x = 15

Q : Find x.

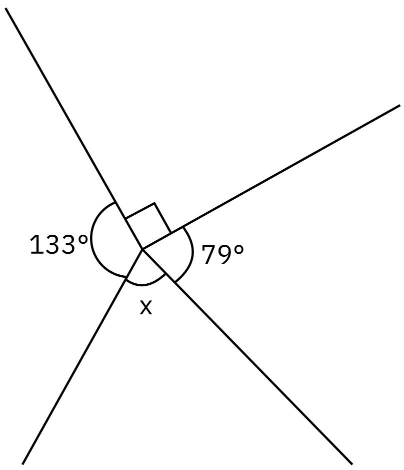


The sum of all the angles is 360°.

⇒ 3x + 37 + 68 = 360°

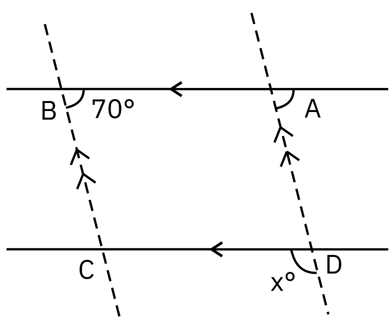
⇒ 3x = 360 − 105 = 255 ⇒ x = 85°

Q : Find x.



A : 133 + 90 + 79 + x = 360°   x = 360 – 302 = 58

Q : In the adjoining figure m∠CBA is 70° determine x°.



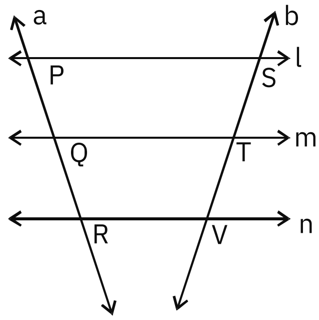
A : AB || CD and BC || AD

m∠CBA + m∠BAD = 180° (interior angles on same side of the transversal) ∴ m∠BAD = 110°

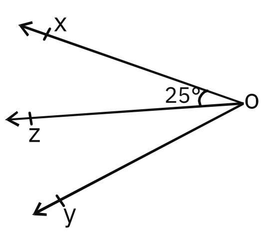
m∠BAD = x° (corresponding angles)   ∴ x° = 110°

Concept Builder 1

1. In the given figure, lines l, m and n are parallel. If PR = 10 and PQ = 4. Find ST : TV.

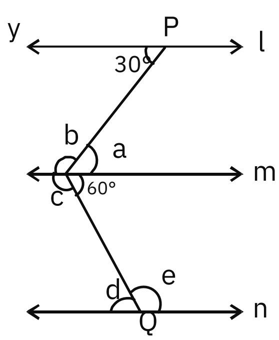


2.



Ray OZ is the bisector of ∠XOY. Find m∠XOY

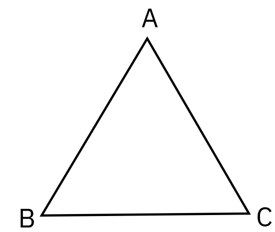
3. Measure of an angle is 55°. Find: a) complement of the angle b) supplement of the angle.
4. Line l || line m || line n. Find angles a, b, c, d, e



Answer key

1. 2 : 3 2. 50°
3. a) 35° b) 125°
4. a = 30°, b = 150°, c = 120°, d = 60°, e = 120°

Triangles

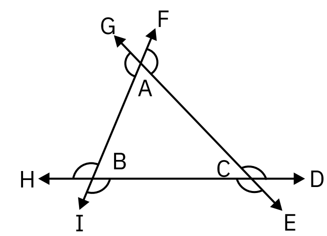


The plane figure bounded by the union of three lines, which join three non-collinear points, is called a **triangle**. A triangle is denoted by the symbol Δ.

The three non-collinear points are called the vertices of the triangle. In ΔABC, A, B and C are the **vertices** of the triangle. Segment AB, segment BC and segment AC are the three **sides** of the triangle.

$\angle$ABC, $\angle$BAC and $\angle$ACB are the three **interior angles** of the triangle. Sides of ΔABC can be denoted by a, b, c; where a = side opposite to vertex A, b = side opposite to vertex B, c = side opposite to vertex C.

The angle formed by extending one side of a triangle with another side is called an **exterior angle** of the triangle. A triangle has six exterior angles.



The exterior angles of ΔABC are $\angle$FAC, $\angle$ACD, $\angle$ECB, $\angle$CBI, $\angle$HBA and $\angle$BAG.
Here, $\angle$HBA = $\angle$CBI, $\angle$BAG = $\angle$FAC and $\angle$ACD = $\angle$ECB ... [Vertically opposite angles]

Properties of Triangles

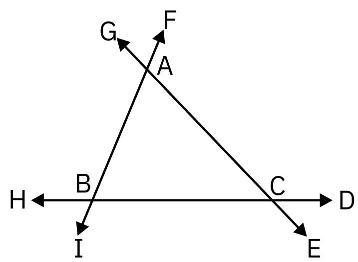


Fig.C

1. The sum of the three interior angles of a triangle is 180°.
2. The sum of an interior angle and the adjacent exterior angle is 180°.

   In fig.C, m$\angle$ABC + m$\angle$ABH = 180° & m$\angle$ABC + m$\angle$CBI = 180°
3. Two exterior angles having the same vertex are congruent.

   In fig.C, $\angle$GAB $\cong$ $\angle$FAC
4. The measure of an exterior angle is equal to the sum of the measures of the two interior angles (called **remote interior angles**) of the triangle, not adjacent to it.

   In fig.C, m$\angle$ABH = m$\angle$BAC + m$\angle$BCA

   Hence, an exterior angle of a triangle is greater than each of the interior angles not adjacent to it.

   e.g., m$\angle$ABH > m$\angle$BAC & m$\angle$ABH > m$\angle$BCA

5. If two sides of a triangle are not congruent, then the angle opposite to the greater side is greater.

   In ΔABC, If $AB > AC$, then $m\angle ACB > m\angle ABC$.

   Conversely, if two angles of a triangle are not congruent, then the sides opposite to the greater angle is greater.

   In ΔABC, if $m\angle ABC > m\angle ACB$ then $AC > AB$ i.e., if the sides are arranged in the ascending order, then the angles opposite the sides will also be in ascending order

6. The sum of lengths of any two sides of a triangle is always greater than the third side. In ΔABC, $\ell(AB) + \ell(BC) > \ell(AC)$, also $\ell(AB) + \ell(AC) > \ell(BC)$ and $\ell(AC) + \ell(BC) > \ell(AB)$.

7. The difference of any two sides is always less than the thirdside. From last result,

   $\ell(AB) + \ell(BC) > \ell(AC)$

   $\therefore\ \ell(AB) > \ell(AC) - \ell(BC)$

   $\therefore\ \ell(AC) - \ell(BC) < \ell(AB)$

   Similarly, $\therefore\ \ell(BC) - \ell(AB) < \ell(AC)$ and $\ell(AC) - \ell(AB) < \ell(BC)$

8. A triangle will have at least two acute angles.

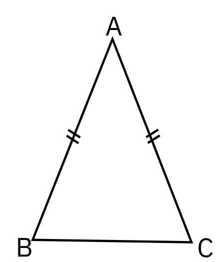
Types of Triangles

1. **With regard to their sides, triangles are of three types:**

   Scalene Triangle: A triangle in which none of the three sides is equal is called a scalene triangle.

   Isosceles Triangle: A triangle in which at least two sides are equal is called an isosceles triangle. In an isosceles triangle, the angles opposite to the congruent sides are congruent. Conversely, if two angles of a triangle are congruent the sides opposite to them are congruent.

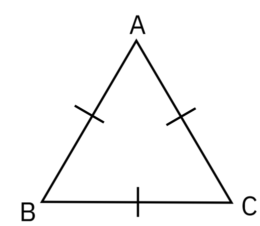
   In ΔABC, $AB = AC$, $m\angle ABC = m\angle ACB$



   Equilateral Triangle: A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

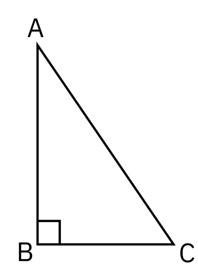
   In ΔABC, $AB = BC = AC$

   $\mathbf{m\angle ABC = m\angle BCA = m\angle CAB = 60°}$



2. **With regard to their angles, triangles are of three types.**

   Right Triangle: If any one angle of a triangle is a right angle, i.e. 90°, then the triangle is a right angled triangle. The other two angles of the right-angled triangle will be acute and complementary. The side opposite to the right angle is called the hypotenuse.



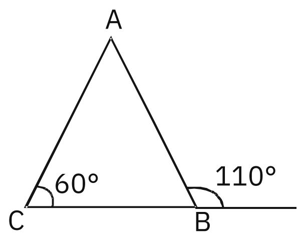
In $\triangle ABC$, AC is the hypotenuse

Acute triangle: If all the three angles of a triangle are acute i.e., less than 90°, then the triangle is an acute-angled triangle.

Obtuse triangle: If any one angle of a triangle is obtuse i.e., greater than 90°,then the triangle is an obtuse- angled triangle. The other two angles of the obtuse triangle will be acute.

SOLVED EXAMPLES

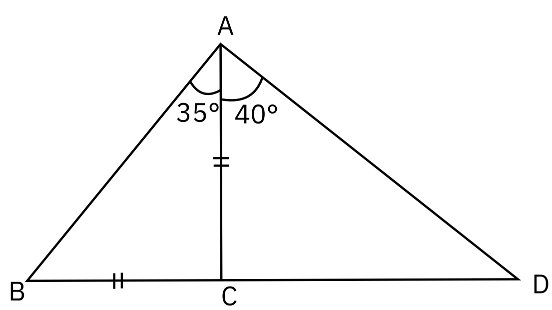
Q : Calculate $\angle ABC$ and $\angle BAC$.



A : $\angle ABC + 110° = 180°$ (straight line) $\therefore \angle ABC = 70°$

$\angle BAC = 180° - (60° + 70°) = 180° - 130° = 50°$ (sum of angles of a trianlge = 180°)

Q : Calculate $\angle ACD$ and $\angle CDA$.

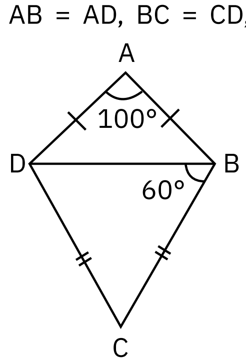


A : ΔABC is isoceles (AC = BC) $\therefore \angle ABC = 35°$

$\angle ACD = \angle ABC + \angle BAC = 70°$ (exterior angle)

$\therefore \angle CDA = 180° - (\angle ACD + \angle CAD) = 180° - (70° + 40°) = 180° - 110° = 70°$

Q : AB = AD, BC = CD, $\angle BAD = 100°$ and $\angle DBC = 60°$. Calculate $\angle ADC$ and $\angle BDC$.



ΔBCD is an isoceles triangle (BC = CD)

$\therefore \angle BDC = 60°$

$\because$ AD = AB $\qquad \therefore \angle ABD = \angle ADB$

In ΔABD, $2\angle ABD + 100° = 180°$

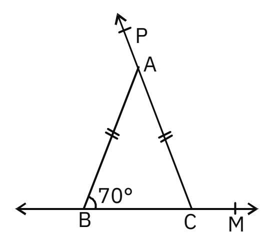
$\therefore 2\angle ABD = 80°$

$\therefore \angle ABD = 40° = \angle ADB$

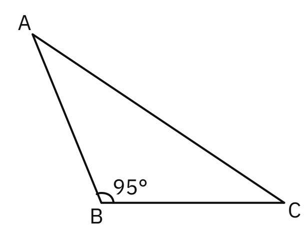
$\therefore \angle ADC = \angle ADB + \angle BDC = 40° + 60° = 100°$

Concept Builder 2

1. In the figure, AB = AC. Find $\angle ACB$, $\angle BAC$, $\angle MCA$ and $\angle PAB$

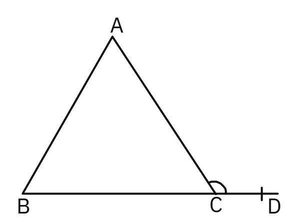


2. In ΔABC, $\angle B = 95°$, $\angle ACB$ can be which of the following?
   a) 90° b) 85°
   c) 75° d) 95°



3. A triangle having sides (5, 5, 10) can be formed (True/False)

4. State whether the following are True or False:
   a. $\angle ACD = \angle ACB + \angle ABC$
   b. $\angle ACD > \angle BAC$
   c. $\angle ACD < \angle CBA$



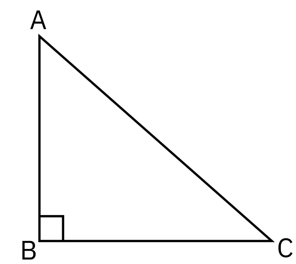
Answer key

1. $\angle ACB = 70°$, $\angle BAC = 40°$, $\angle MCA = 110°$, $\angle PAB = 140°$
2. 75° 3. False
4. a) False b) True c) False

Pythagoras Theorem

Theorem of Pythagoras: In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$(AC)^2 = (AB)^2 + (BC)^2$



Conversely, if the square of one side of a triangle is equal to the sum of the squares of its remaining two sides, then the angle opposite to the first side is a right angle.

The numbers, which satisfy this relation, are called Pythagorean triplets.

Few of the Pythagoras triplets are:

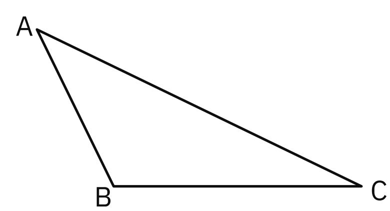
3, 4, 5; 5, 12, 13; 7, 24, 25; 8, 15, 17; 9, 40, 41; 11, 60, 61; 12, 35, 37; 16, 63, 65; 20, 21, 29; 28, 45,53

e.g.,$5^2 = 3^2 + 4^2$; 3, 4 and 5 are called Pythagorean triplets.
e.g., 3 × 0.7, 4 × 0.7, 5 × 0.7 i.e. 2.1, 2.8, 3.5 is a Pythagoras triplet too.

Any multiple of a Pythagoras triplet will also form a Pythagoras triplet.

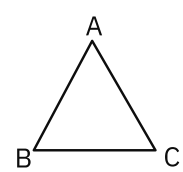
In an **obtuse angled triangle**,

$AB^2 + BC^2 < AC^2$



In an **acute angled triangle**,

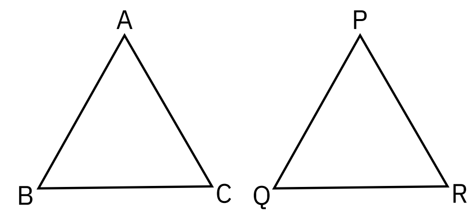
$AB^2 + BC^2 > AC^2$



Congruency of Triangles

One-to-one correspondence between triangles: Pairing of vertices of two triangles is called one-to-one correspondence between vertices, denoted by '≅' :

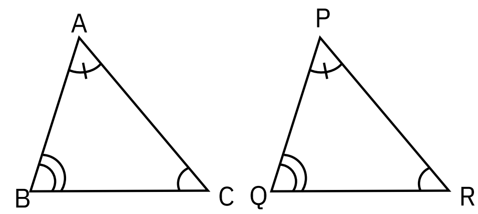
If ΔABC and PQR are any two triangles then there exist six one-to-one correspondences between their vertices.



(i) ABC ↔ PQR (ii) ABC ↔ QPR

(iii) ABC ↔ PRQ (iv) ABC ↔ QRP

(v) ABC ↔ RPQ (vi) ABC ↔ RQP



For a given correspondence ABC ↔ PQR

Side AB ↔ Side PQ ∠ABC ↔ PQR

Side BC ↔ Side QR ∠BAC ↔ QPR

Side AC ↔ Side PR ∠ACB ↔ PRQ

Congruence of triangles

For a given correspondence between two triangles, if the sides and angles of one triangle are congruent to the corresponding sides and angles of the other triangle, then the two triangles are said to be congruent.

If ΔABC ≅ ΔPQR, then

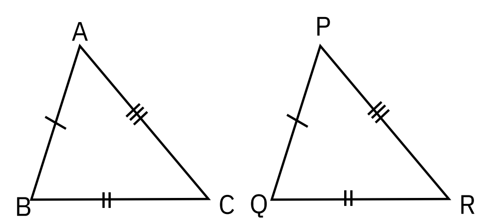
(i) Side AB ≅ Side PQ (iv) ∠ABC ≅ ∠PQR

(ii) Side BC ≅ Side QR (v) ∠BAC ≅ QPR

(iii) Side AC ≅ Side PR (vi) ∠ACB ≅ PRQ

Tests of Congruency

It is not necessary to list all the six conditions i.e., congruence of the three sides and three angles to prove that the two triangles are congruent. If certain selected conditions are satisfied, then the others will necessarily follow. These selected conditions are called the tests of congruence.

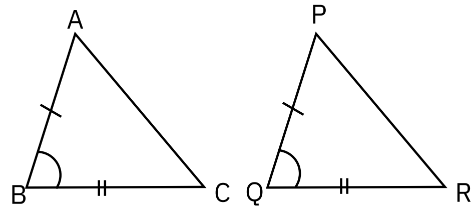


SSS Test: For a given correspondence between two triangles, if the three sides of one triangle are congruent to the corresponding three sides of the other triangle, then the two triangles are congruent.

AB ≅ PQ, AC ≅ PR, BC ≅ QR

∴ ΔABC ≅ ΔPQR by SSS test of congruence.

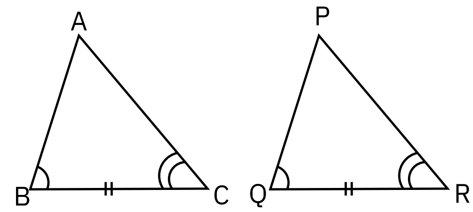
SAS Test: For a given correspondence between two triangles, if two sides and the angle included between them are congruent to the corresponding sides and the included angle of the other triangle, then the two triangles are congruent.



AB ≅ PQ, BC ≅ QR, ∠ABC ≅ ∠PQR

∴ ΔABC ≅ ΔPQR by SAS test of congruence.

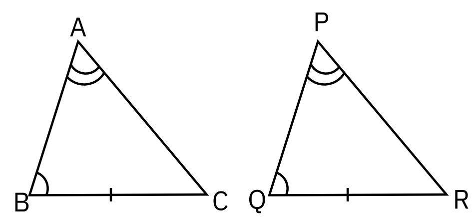
ASA Test: For a given correspondence between two triangles, if two angles and the included side of a triangle are congruent to the corresponding angles and the included side of the other triangle, then the two triangles are congruent.



∠ABC ≅ ∠PQR, BC ≅ QR, ∠ACB ≅ ∠PRQ

∴ ΔABC ≅ ΔPQR by ASA test of congruence.

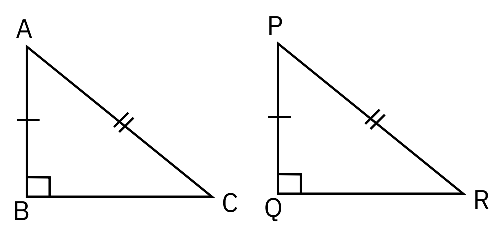
AAS Test: For a given correspondence between two triangles, if two angles and a side other than the included side of a triangle are congruent to the corresponding angles and a corresponding side other than the included side of the other triangle, then the two triangles are congruent.



∠CAB ≅ ∠RPQ, ∠ABC ≅ ∠PQR, BC ≅ QR

∴ ΔABC ≅ ΔPQR by AAS test of congruence.

RHS Test: For a given correspondence between two right-angled triangles, if the hypotenuse and one side of the right triangle are congruent to the hypotenuse and a corresponding side of the other right triangle, then the two right triangles are congruent.



AB ≅ PQ, AC ≅ PR, ∠ABC ≅ ∠PQR

∴ ΔABC ≅ ΔPQR by RHS test of congruence.

Similarity of Triangles

For a given correspondence between two triangles, if the corresponding angles are congruent and their corresponding sides are in proportion, then the two triangles are said to be similar. Similarity is denoted by the symbol '~'.

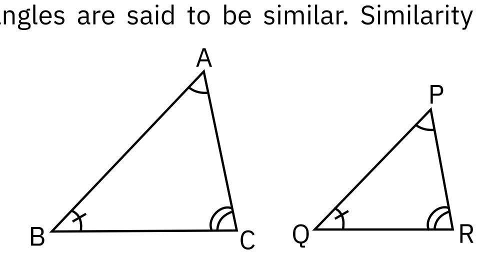
If ΔABC ~ ΔPQR, then,

∠ABC ≅ ∠PQR

∠BAC ≅ ∠QPR

∠BCA ≅ ∠QRP

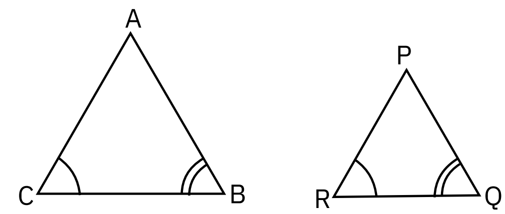
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$



Tests for Similarity

It is not necessary to list all the conditions for similarity i.e., proportionality of sides and congruence of angles to prove that two triangles are similar. If certain selected conditions are satisfied, then the others will necessarily follow. These selected conditions are called the tests for similarity.

AA Test: For a given correspondence between two triangles, if the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangles are similar.

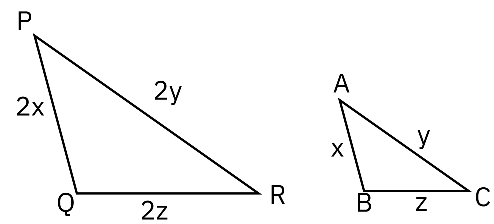


$\angle ABC \cong \angle PQR$

$\angle ACB \cong \angle PRQ$

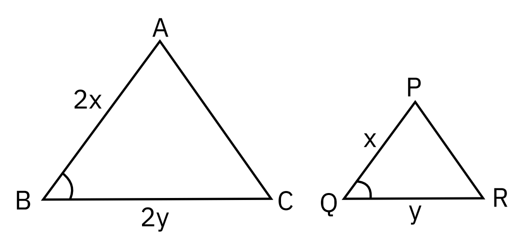
$\therefore$ $\Delta ABC \sim \Delta PQR$ by AA test for similarity.

SSS Test: For a given correspondence between two triangles, if the three sides of one triangle are proportional to the corresponding three sides of the other triangle, then the two triangles are similar.



$\frac{PQ}{AB} = \frac{PR}{AC} = \frac{RQ}{CB}$ $\therefore$ $\Delta PQR \sim \Delta ABC$ by SSS test for similarity.

SAS Test: For a given correspondence between two triangles, if the two sides of one triangle are proportional to the corresponding two sides of the other triangle and the angle included by them are congruent, then the two triangles are similar.



$\frac{AB}{PQ} = \frac{BC}{QR}$

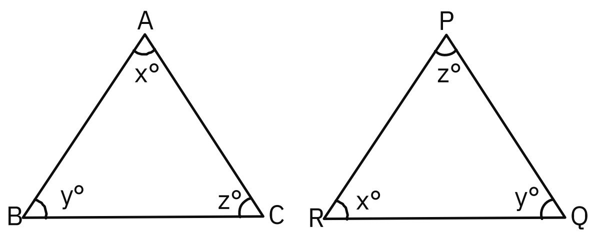
$\angle ABC \cong \angle PQR$

$\therefore$ $\Delta ABC \sim \Delta PQR$ by SAS test for similarity.

Concept Builder 3

1. Two congruent triangles have all corresponding sides and angles equal. (True/False)

2.



In the figure, if $\frac{AB}{PR} = \frac{AC}{PQ} = \frac{RQ}{BC}$ then, $\Delta ABC \sim \Delta PQR$ (True/False)

3. Which of the following are Pythagorean triplets?

a) 24, 10, 26 b) 333, 444, 555 c) .07, 2.4, 2.5

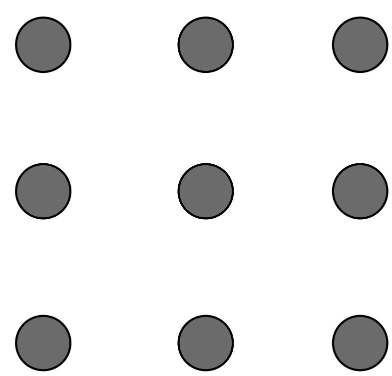
Answer key

1. True 2. False
3. (a) and (b)

CLASS EXERCISE

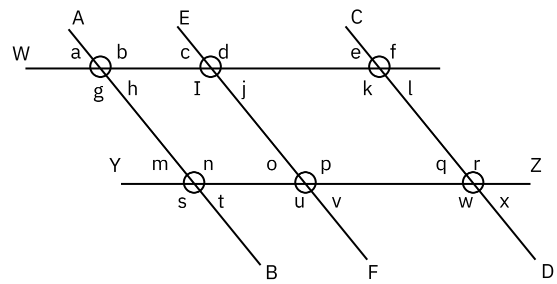
Teaser

4 straight lines (without lifting pen from paper) so that they pass through all the 9 dots below:



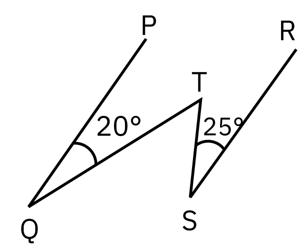
Lines and Angles

1. In the adjacent figure, AB // CD and WX // YZ

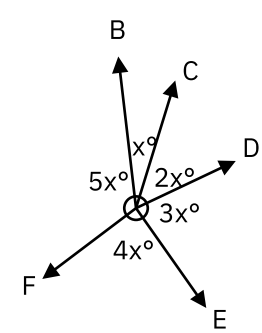


a) If ∠a = 40°, then how much is ∠h?
b) If ∠e = 55°, then how much is ∠b?
c) If ∠f = 145°, then how much is ∠s?
d) If ∠p = 42°, then how much is ∠q?
e) If ∠k = 70°, then how much is ∠d + ∠v?
f) If ∠r = 130°, then how much is ∠m + ∠t?

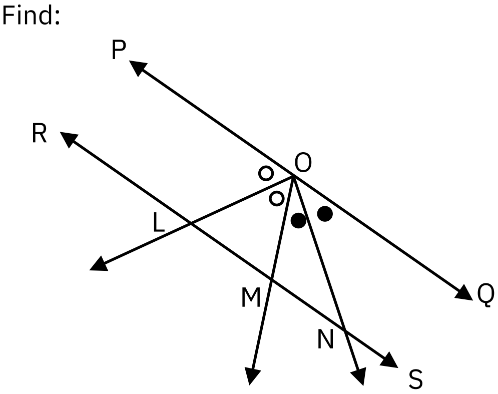
2. In the adjacent figure, PQ // RS, ∠PQT = 20° and ∠TSR = 25°. Find ∠QTS.



3. Find the value of x in the adjoining figure:



4. *In the adjacent figure, PQ // RS and ∠MON = 25°. Find:



a) ∠ LOM

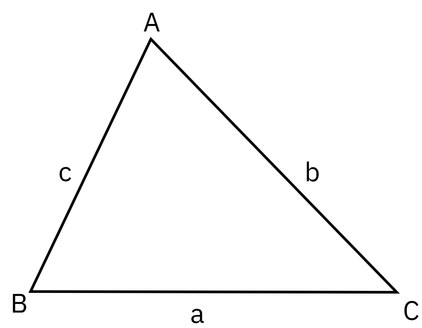
b) ∠ OLN

c) ∠ POM

d) ∠ NOL

Triangles: Properties, Classification, Basic Theorems

5.



Consider a triangle ABC (where a, b and c are the lengths of the sides opposite to $\angle A$, $\angle B$ and $\angle C$ respectively)

a) If $\angle A = 70°$ and $\angle C = 50°$, then find $\angle B$

1) 50°
2) 60°
3) 70°
4) Cannot be determined

b) If $\angle A = 95°$ and $\angle C = 45°$, then find the exterior angle at B

1) 40° 2) 70° 3) 140° 4) Cannot be determined

c) If a = 8 and b = 3.5, how many integer values could c take?

1) 5 2) 7 3) 11 4) Cannot be determined

d) If $\angle A$ is 50° and Δ ABC is isosceles, find the exterior angle at B

1) 80° 2) 100° 3) 115° 4) Cannot be determined

e) If $\angle A$ is 100° and Δ ABC is isosceles, find the exterior angle at C

1) 40° 2) 80° 3) 140° 4) Cannot be determined

f) If a = 8 inches, b = 15 inches and $\angle C = 90°$, then what is the value of c?

1) 13 inches
2) 17 inches
3) 23 inches
4) Cannot be determined

g) If the sides of Δ ABC are 6, 8 and 10, what kind of triangle will Δ ABC be?

1) Acute 2) Right 3) Obtuse 4) Cannot be determined

h) If the sides of Δ ABC are 5, 7 and 9, what kind of triangle will Δ ABC be?

1) Acute 2) Right 3) Obtuse 4) Cannot be determined

i) If $\angle A = 30°$, $\angle B = 80°$, and $\angle C = 70°$, what kind of triangle will Δ ABC be?

1) Equilateral
2) Isosceles
3) Scalene
4) Cannot be determined

6. Find the hypotenuse of a right angled triangle with perpendicular sides as:

a) 6 and 8
b) 5 and 12
c) 10 and 24
d) 14 and 48
e) *39 and 52
f) * 25 and 60

In any triangle ABC (where a, b and c are the lengths of the sides opposite to $\angle A$, $\angle B$ and $\angle C$ respectively) the following properties hold:

- **Sum of Angles:** The sum of all angles of the triangle is 180° i.e. $\angle A + \angle B + \angle C = 180°$
- **Triangle Inequality:** The sum of any two sides exceeds the third ($a + b > c$, $b + c > a$, $c + a > b$)
- Conversely, the difference of any two sides is less than the third ($|a - b| < c$, $|b - c| < a$, $|c - a| < b$)
- **Exterior Angle Theorem:** The exterior angle equals the sum of the remote interior angles
- **Theorem of Pythagoras:** In a right-angled triangle, the square of the hypotenuse equals the sum of squares on the other two sides. Hence if $\angle A = 90°$ then $a^2 = b^2 + c^2$
- Conversely, if $a^2 = b^2 + c^2$ then $\angle A$ is a right angle (Note: if $a^2 > b^2 + c^2$ then $\angle A$ is an obtuse angle while if $a^2 < b^2 + c^2$ then $\angle A$ is an acute angle)

7. *Consider a triangle ABC (where a, b and c are the lengths of the sides opposite to $\angle A$, $\angle B$ and $\angle C$ respectively)

a) If the exterior angles at A and B add up to 240°, find $\angle C$

1) 40° 2) 60°
3) 120° 4] Cannot be determined

b) If the sides of Δ ABC are 6, 8 and 9, what kind of triangle will Δ ABC be?

1) Acute 2) Right
3) Obtuse 4] Cannot be determined

c) If $\angle A = 30°$ and $\angle B = 75°$, what kind of triangle will Δ ABC be?

1) Equilateral 2) Isosceles
3) Scalene 4] Cannot be determined

d) If Δ ABC is right-angled and $a = 12$ and $c = 5$, how much is b?

1) 13 2) 14
3) 15 4] Cannot be determined

e) If $\angle A = 60°$ and $\angle B = \angle C$ what kind of triangle will Δ ABC be?

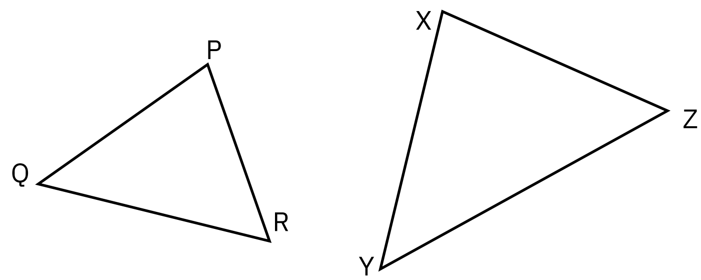
1) Equilateral 2) Isosceles
3) Scalene 4] Cannot be determined

8. * Find the smallest side of a right triangle whose two larger sides are:

1) 40 and 41 2) 50 and 48
3) 36 and 39 4] 45 and 51

Triangles: Similarity and Congruence

9. In each of the following cases, some information is given about the sides and/or angles of two triangles PQR and XYZ. From the given information, identify whether the two triangles PQR and XYZ are congruent, similar, or unrelated.



I	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
	65°	45°		12			65°	45°		12		
Relationship:												
II	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
		32°		7	6			32°		14	12	
Relationship:												
III	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
	81°		55°			12.7		81°	55°		12.7	
Relationship:												
IV	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
	125°	17°						125°	17°			
Relationship:												
V	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
		87°		12		8			87°		12	18
Relationship:												
VI	$\angle P$	$\angle Q$	$\angle R$	l(PQ)	l(PR)	l(QR)	$\angle X$	$\angle Y$	$\angle Z$	l(XY)	l(XZ)	l(YZ)
				15	21	24				21	24	15
Relationship:												

Similarity: Two triangles are said to be similar if they are the same in shape.
Tests for similarity include:

- **SSS:** All three pairs of sides should be in proportion
- **SAS:** Two pairs of sides should be in proportion, and the included angle should be equal
- **AA:** Two pairs of angles should be equal

Congruence: Two triangles are said to be congruent if they are the same in shape as well as size. Tests for congruence include:

- **SSS:** All three pairs of sides should be equal
- **SAS:** Two sides should be equal and the included angle should be equal
- **ASA:** Two angles should be equal and the included side should be equal
- **SAA:** Two angles should be equal and a non-included side should be equal
- Hypotenuse-Side: Hypotenuses and one pair of sides of two right triangles should be equal

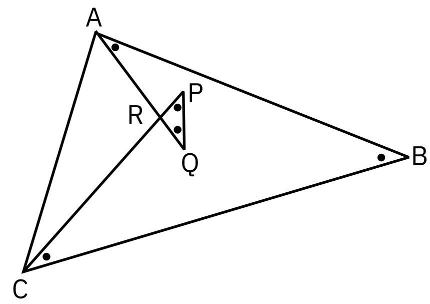
Note that there is no ASS test for congruence

VII*	∠P	∠Q	∠R	l(PQ)	l(PR)	l(QR)	∠X	∠Y	∠Z	l(XY)	l(XZ)	l(YZ)
				11	13	7				39	33	21
Relationship:												
VIII*	∠P	∠Q	∠R	l(PQ)	l(PR)	l(QR)	∠X	∠Y	∠Z	l(XY)	l(XZ)	l(YZ)
			48°		6	8		48°		8		6
Relationship:												
IX*	∠P	∠Q	∠R	l(PQ)	l(PR)	l(QR)	∠X	∠Y	∠Z	l(XY)	l(XZ)	l(YZ)
	33°		44°		6		44°	33°		12		
Relationship:												
X*	∠P	∠Q	∠R	l(PQ)	l(PR)	l(QR)	∠X	∠Y	∠Z	l(XY)	l(XZ)	l(YZ)
	90°			5		13			90°	13		5
Relationship:												

10. In PQR, point S lies on side PQ such that P-S-Q. If m∠SRQ = m∠QPR, l(PQ) = 12 and l(QR) = 8, calculate l(QS).

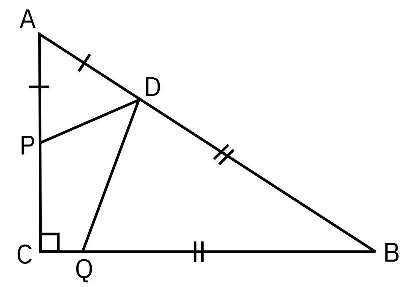
Challengers

1. In the adjacent figure, all the angles marked with a dot are equal. Find the measure of angle ABC.



1) 30° 2) 36°
3) 45° 4) 48°

2. In the adjacent figure, find m PDQ

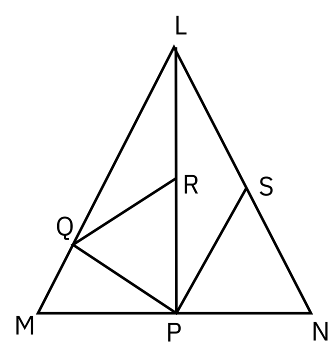


1) 45° 2) 36°
3) 25° 4) 30°

3. All possible obtuse-angled triangles with sides of integer length are constructed, such that two of the sides have length 5 and 12. How many such triangles exist?

1) 5 2) 6 3) 7 4) 8

4. In Δ LMN, l(LM) = l(LN). An altitude LP is drawn from L to MN. PQR is an equilateral triangle such that Q lies on LM and R lies on LP. It is known that S is the midpoint of LN, and that l(QR) = l(RL). Find ∠MPS.



1) 150° 2) 135°
3) 105° 4) 120°

5. How many triangles with integer sides p, q, r are possible such that $p < q < r$ and the perimeter of the triangle is 27?

1) 12 2) 18 3) 15 4) 11

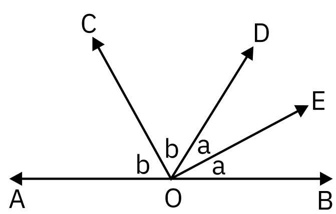
PRACTICE EXERCISE

Directions for 1 to 9: Solve as directed.

1. State the measure of the angle (in terms of x), which is the complement of the angle whose measure is 90° – x.

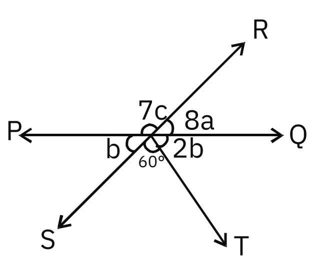
2. An angle is twice its complement. Find the angle.

3.



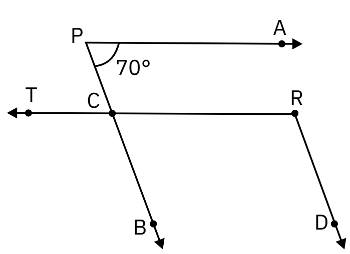
In the figure, OE and OC are the bisectors of ∠BOD and ∠AOD respectively, find m∠EOC.

4.



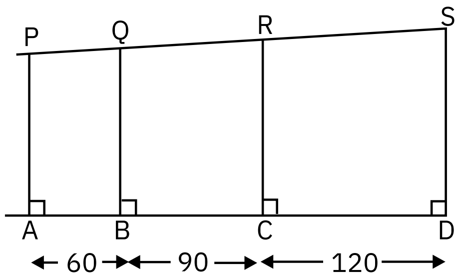
Find a, b and c from the given figure

5.



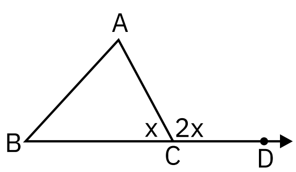
PA || CR and CB || RD. ∠APC = 70°. Find m∠CRD.

6.



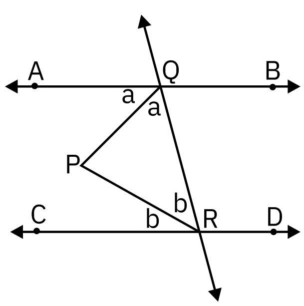
In the figure, if PS = 360, find PQ, QR and RS.

7.



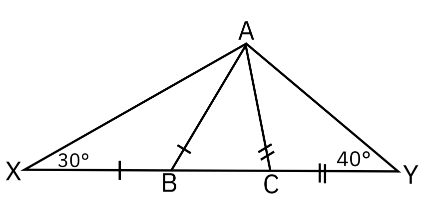
The interior and its adjacent exterior angle of a triangle are in the ratio 1 : 2. What is the sum of the other two angles of the triangle?

8.



In the given figure, find m∠QPR given that AB || CD, PQ and PR are the bisectors of ∠AQR and ∠CRQ respectively.

9.



From the information given in the figure, find ∠BAC and ∠XAY.

Directions for 10 to 20: Choose the correct alternative.

10. One of the exterior angles of an isosceles triangle is 150°. What is the ratio of its unequal interior angles?

1) 1 : 4 2) 5 : 2 3) Either (1) or (2) 4) Can't say

11. In an isosceles triangle, if the vertex angle is increased by 20%, the base angles have to be reduced by 25% each. The vertex angle, in degrees, is:

1) 80° 2) 90° 3) 100° 4) 110°

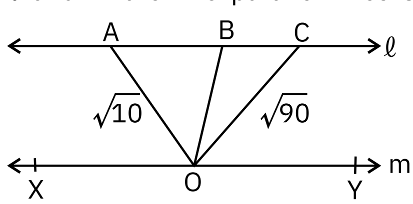
12. The two sides of a ΔABC are 7 and 10. Which of the following cannot be the 3rd side of ΔABC?

1) 3 2) 4 3) 5 4) 6

13. The three sides of a ΔPQR are 10, 12 and 20. Which type of triangle it is?

1) Acute Angled Triangle 2) Right Angled Triangle
3) Obtuse Angled Triangle 4) Cannot be determined

14. ℓ and m are two parallel lines 3 cm apart as shown in the figure.



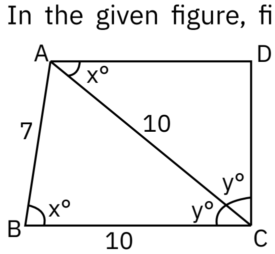
OA is the bisector of $\angle$BOX and OC is the bisector of $\angle$BOY. Find ℓ(BC), if ℓ(AB) = 5 cm.

1) 3 cm 2) 5 cm 3) 4 cm 4) $\sqrt{20}$ cm

15. Perimeter of the right angled triangle is 80. Which of the following can be its sides?

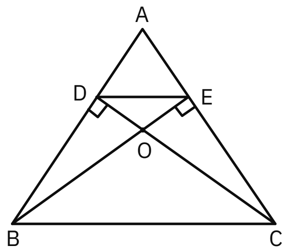
1) 18, 25, 37 2) 15, 31, 34
3) 16, 30, 34 4) 11, 34, 35

16. In the given figure, find the measurement of AD?



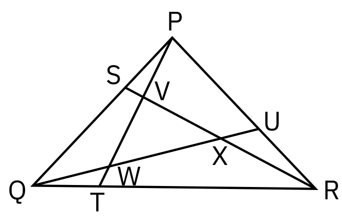
1) 10 2) 8 3) 7 4) 12

17. Which of the following is not necessarily true about this figure?



1) ΔADC and ΔAEB are similar
2) ΔBOD and ΔEOC are similar
3) $AD \times AB = AC \times AE$
4) $BD \times OE = CE \times BO$

18.



In the given figure, ΔPQR is an equilateral triangle.

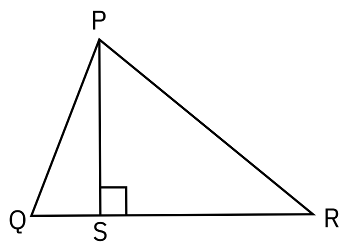
Also $\ell(PS) = \ell(QT) = \ell(RU)$. Then, ΔVWX is:

1) an equilateral triangle
2) an isosceles triangle
3) a scalene triangle
4) Cannot be determined

19. Amit walks 1 km towards North 2 km towards East, 2 km towards North and 1 km again towards East in the given order. Sumit walks double the distance than that initially covered by Amit towards North, half towards East, then half towards North and double towards East following the same order. If the ratio of speeds of Amit and Sumit is 3 : 2, find the approximate ratio of their distances from their individual starting points when Sumit has covered half of the total distance.

1) $\sqrt{13} : \sqrt{3}$
2) $3.2 : \sqrt{5}$
3) 2 : 3
4) Cannot be determined

20. Which of the following is/are true?



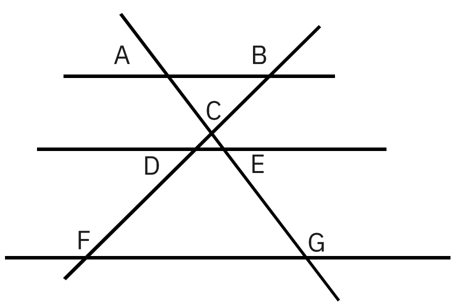
I. If PQ = a, PR = b, QS = c and SR = d, then $(a - b)(a + b) = (c + d)(c - d)$

II. If PQ = $m^2 - n^2$, PR = 2mn and QR = $m^2 + n^2$, then QPR = 90°

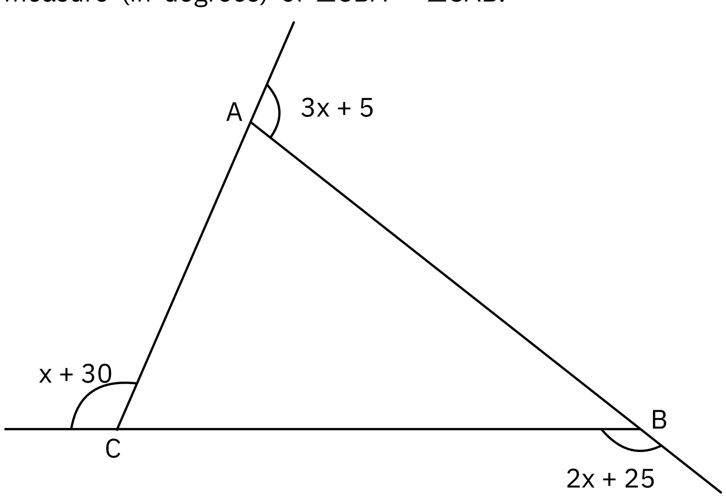
1] Only I 2] Only II 3] Both I and II 4] Either I or II

Directions for 21 and 22: Solve as directed.

21. In the adjoining figure, there are three parallel lines and two transversals. AC = 5, DF = 12, BC = 4. Find EG.



22. The exterior angles of ΔABC are mentioned in the diagram (all angles in degrees). Find the measure (in degrees) of ∠CBA + ∠CAB.

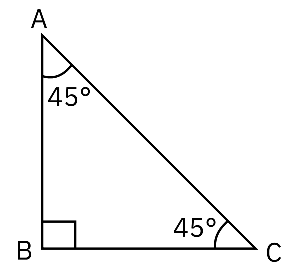


QA-3.2 | TRIANGLES-II

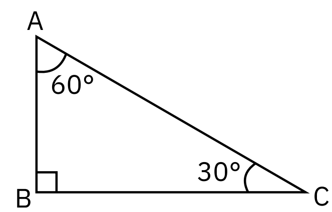
THEORY

Properties and Theorems of Triangles

Theorem of 45°-45°-90° Triangle: If the angles of a triangle are 45°, 45° and 90°, then the perpendicular sides are $\frac{1}{\sqrt{2}}$ times the hypotenuse. In ΔABC, AB = BC = $\frac{1}{\sqrt{2}}$



Theorem of 30°-60°-90° Triangle: If the angles of a triangle are 30°, 60° and 90°, then the sides opposite to 30° is half the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse.

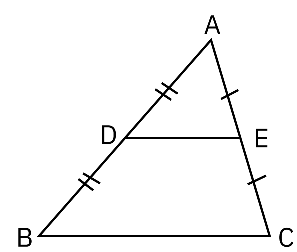


In ΔABC, AB = $\frac{1}{2}$AC and BC = $\frac{\sqrt{3}}{2}$AC.

Ratio of sides: 1 : $\sqrt{3}$: 2

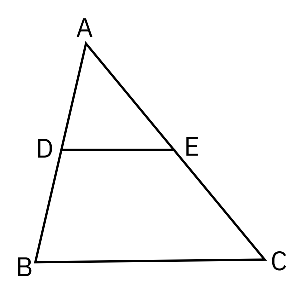
Midpoint Theorem: The segment joining the midpoints of any two sides of a triangle is parallel to the third side and is half of the third side.

If AD = DB, AE = EC, then DE is parallel to BC and DE = $\frac{1}{2}$BC.



Basic Proportionality Theorem: If a line is drawn parallel to one side of a triangle and intersects the other sides in two distinct points, then the other sides are divided in the same ratio by it.

If DE is parallel to BC, then $\frac{AD}{DB}=\frac{AE}{EC}$.



Altitude (height) of a triangle: The perpendicular drawn from the vertex of a triangle to the opposite side (base) is called an altitude of the triangle.

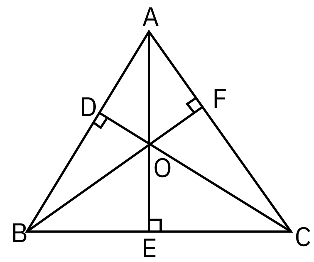


Fig D

A triangle has three altitudes. In ΔABC, the three altitudes are AE, BF and CD.

Orthocentre: The point of intersection of the three altitudes of a triangle is called the orthocentre. The angle made by any side at the orthocentre = 180 − the opposite angle to the side. In fig.D, O is the orthocentre and m∠BOC = 180 − m∠A.

Median of a triangle: The line drawn from a vertex of a triangle to the opposite side such that it bisects the side is called the median of the triangle. A triangle has three medians. In ΔABC, the three medians are AE, BF and DC.

A median divides the triangle into two triangles of equal area.

$A(\Delta ABE) = A(\Delta AEC) = \frac{1}{2} A(\Delta ABC)$

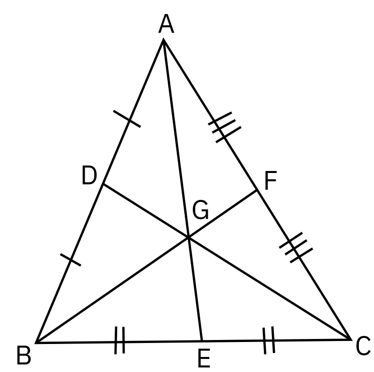


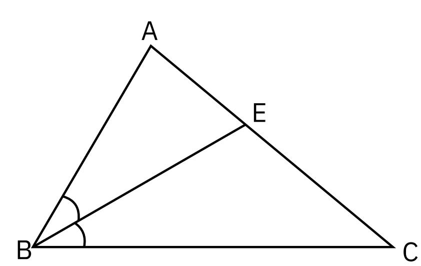
Fig E

Centroid: The point of intersection of the three medians of a triangle is called the centroid. The centroid divides each median in the ratio 2 : 1.

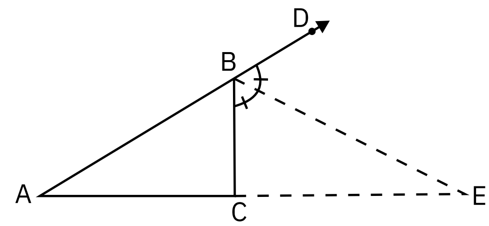
In fig.E, G is the centroid of the triangle and $\frac{AG}{GE} = \frac{BG}{GF} = \frac{CG}{GD} = \frac{2}{1}$

Interior Angle Bisector Theorem: The angle bisector of any angle of a triangle divides the side opposite to the angle in the ratio of the remaining two sides.

In ΔABC, if BE is the angle bisector of ∠ABC, then, $\frac{BA}{BC} = \frac{AE}{CE}$

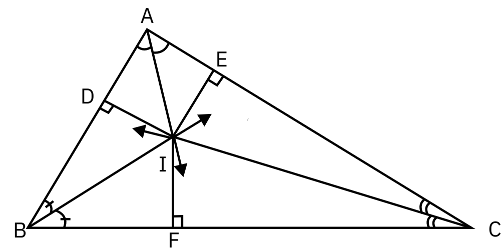


Exterior Angle Bisector Theorem: The angle bisector of any exterior angle of a triangle divides the side opposite to the angle (externally) in the ratio of the remaining two sides.

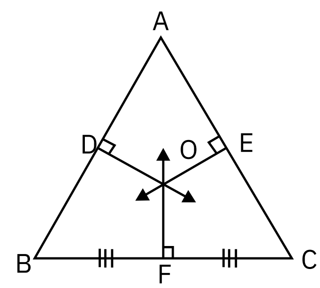


In ΔABC, $\angle DBC$ is an exterior angle and BE is the exterior angle bisector. Here, $\frac{BA}{BC} = \frac{AE}{CE}$

Incentre: The point of intersection of the angle bisectors of a triangle is called the incentre. I is the incentre of ΔABC. The distance of the sides from the incentre is called the **inradius**. IE, ID and IF are the inradii of ΔABC.



Circumcentre: The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre.
The perpendiculat bisector of a side does not necessarily pass through the oppositte vertex in a triangle in general
O is the circumcentre of ΔABC. The distance of the vertex from the circumcentre is called the **circumradius**. OA, OB and OC are the circumradii of ΔABC.



Note: Circumcentre, incentre, circumradii and inradii will be covered later in detail in Chapter of Circles

Some interesting facts about geometric centres

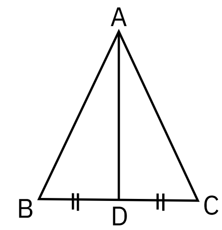
1. In an **acute angled triangle**, the circumcentre and the orthocentre lie within the triangle
2. In a **right angled triangle**, the circumcentre lies on the hypotenuse and is the midpoint of the hypotnuse. Also, the orthocentre is the vertex where the right angle is formed.
3. In an **obtuse angled triangle**, the circumcentre and the orthocentre lie outside the triangle.
4. In a triangle, centroid, orthocentre and circumcentre are collinear. **(Euler's line)**. Centroid divides the line joining orthocentre and circumcentre in ratio 2 : 1.

Perimeter of a Triangle

Perimeter: The perimeter is the sum of all the sides of a triangle.
Semiperimeter: Half of the perimeter is called semiperimeter(s).

If a, b and c are the sides of a triangle, then $s = \frac{a+b+c}{2}$

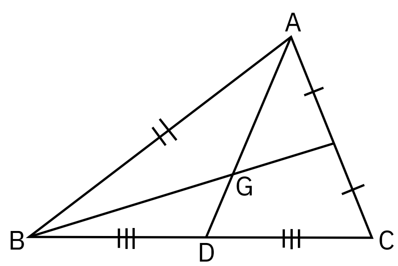
Appollonius Theorem: The sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and the square of half the third side.



AD is the median of ΔABC. $AB^2 + AC^2 = 2(AD^2 + DC^2)$

Solved Examples

Q :



In a triangle ABC, AB = 9, BC = 10, AC = 13. Find the length of median AD. If G is the centroid, find ℓ(GA) and ℓ(GD).

A : By Appollonius theorem,

$AB^2 + AC^2 = 2 \times (AD)^2 + 2 \times (DC)^2$

$\therefore\ 81 + 169 = 2 \times (AD)^2 + 2 \times (5)^2$

$\therefore\ 250 = 2 \times (AD)^2 + 2 \times (5)^2$

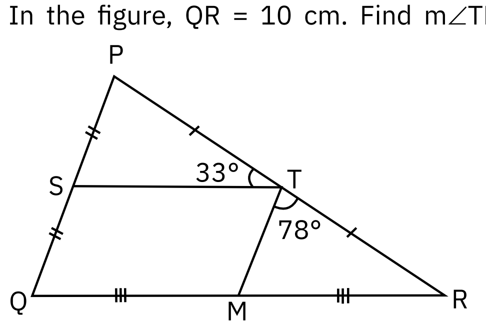
$125 = AD^2 + 25$ ...(Dividing by 2)

$\therefore\ 100 = AD^2 \quad \therefore\ 10 = AD \quad \therefore\ \ell(\text{median}) = 10$

Since G divides AD in the ratio 2 : 1.

$GA = \frac{2}{3} \times AD = \frac{2}{3} \times 10 = \frac{20}{3}; GD = \frac{1}{3} \times 10 = \frac{10}{3}$

Q : In the figure, QR = 10 cm. Find m$\angle$TMR, m$\angle$PST, m$\angle$SQM and ℓ(ST).



A : ST || QR and TM || PQ ... (Midpoint theorem)

m$\angle$PTS = m$\angle$PRQ = 33° ... (Corresponding angles)

In ΔTMR, $\angle$TMR = 180 − (78 + 33) = 69°.

$m\angle SQM = m\angle TMR = 69°$ ...(Corresponding angles)

$m\angle PST = m\angle PQM = 69°$ ...(Corresponding angles)

$ST = \frac{1}{2} \times QR = \frac{1}{2} \times 10 = 5$cm. ...(Midpoint theorem)

Q : In the figure AD is the external bisector of $\angle EAC$ intersects BC produced in D. If AB = 8, AC = 6, BC = 3, find CD.

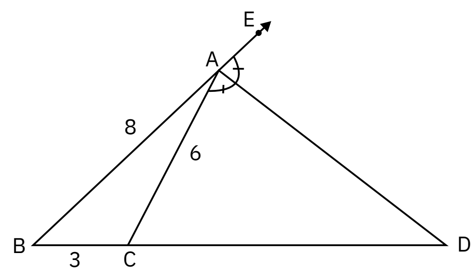
A : $\frac{AB}{AC} = \frac{BD}{DC}$

Let CD = x

$\frac{8}{6} = \frac{3+x}{x}$

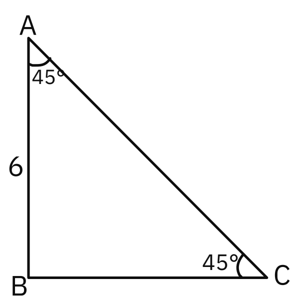
8x = 18 + 6x; 2x = 18

$\therefore$ x = 9 $\quad \therefore$ CD = 9



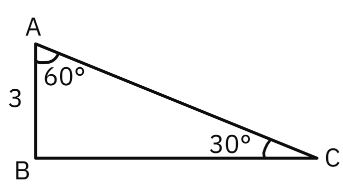
Concept Builder 1

1.



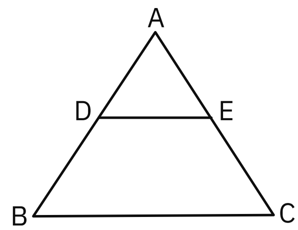
In the figure AB = 6, Find the BC, AC

2.



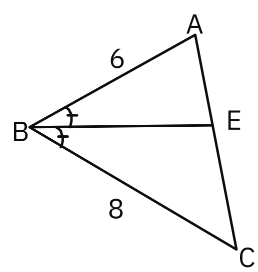
In the figure, AB = 3. Find BC, AC

3.



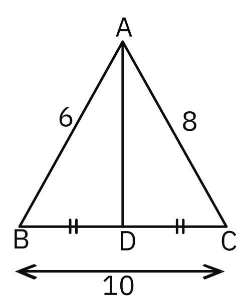
If DE || BC and D is the mid point of AB, where AB = 10, AE = 6, BC = 8. Find AD, AC, DE

4.



BE is the angle bisector of $\angle ABC$. Find $\frac{EC}{EA}$

5.



AD is the median of side BC. Find the length of AD.

Answer key

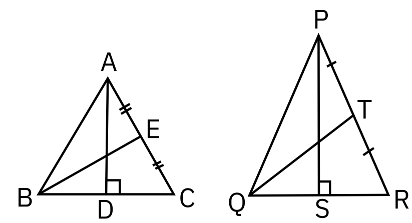
1. BC = 6, AC = $6\sqrt{2}$
2. BC = $3\sqrt{3}$, AC = 6
3. AD = 5, AC = 12, DE = 4
4. 4 : 3
5. AD = 5

Properties of Similar Triangles

1. If two triangles are similar, Ratio of sides = Ratio of heights = Ratio of Medians = Ratio of angle bisectors = Ratio of inradii = Ratio of circumradii = Ratio of perimeters

   If $\Delta ABC \sim \Delta PQR$

   $\frac{AB}{PQ} = \frac{AD}{PS} = \frac{BE}{QT}$



2. The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

   If $\Delta ABC \sim \Delta PQR$, then

   $$\frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

   also, $\frac{A(\triangle ABC)}{A(\triangle DQA)} = \frac{AD^2}{PS^2} \left[\because \frac{AB}{PQ} = \frac{AD}{PS}\right]$

3. The triangles on each side of the altitude drawn from the vertex of the right angle to the hypotenuse are similar to the original triangle and to each other. $\Delta ABC \sim \Delta ADB \sim \Delta BDC$

   The altitude from the vertex of the right angle to the hypotenuse is the geometric mean of the segments into which the hypotenuse is divided.

   In fig.G, $(DB)^2 = AD \times DC$

   Also, $(CB)^2 = CA \times CD$

   $(AB)^2 = AD \times AC$

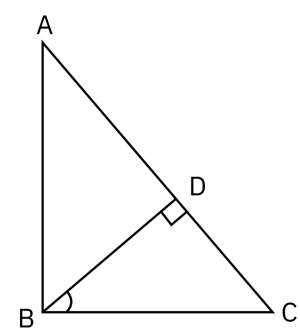
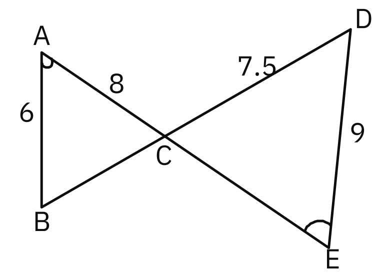


Fig G

SOLVED EXAMPLES

Q : In the figure $\angle BAC = \angle DEC$

find BC, CE, $\frac{A(\Delta ABC)}{A(\Delta CDE)}$



A : $\Delta ACB \sim \Delta ECD$

$\because$ $\angle ACB = \angle ECD$ [Vertically opposite angles]

$\angle BAC = \angle CED$ [Given]

$\therefore$ $\frac{AC}{EC} = \frac{CB}{CD} = \frac{AB}{ED}$

$\therefore$ $\frac{8}{EC} = \frac{CB}{7.5} = \frac{6}{9}$

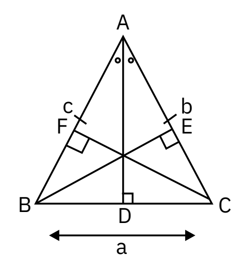
$\Rightarrow$ EC = 12 and CB = 5

Now, $\frac{A(\Delta ABC)}{A(\Delta CDE)} = \frac{AB^2}{ED^2} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$

Area of a Triangle

The region enclosed within a triangle is called the **area of the triangle**, denoted by the symbol A(Δ).

Some of the general formulae used in calculating area of a triangle are as below.



1. Area of a triangle = $\frac{1}{2}$ × base × height. Area is measured in square units.
2. Heron's formula: When three sides a, b, c are given,

   Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where semiperimeter, s = $\frac{a+b+c}{2}$
3. Area = r × s, where r = inradius

   s = semiperimeter
4. Area = $\frac{abc}{4R}$, where R = circumradius

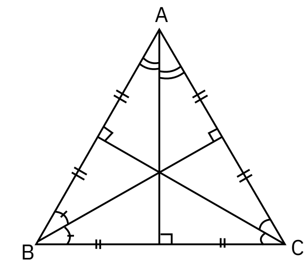
Area of Isosceles Triangle

If ΔABC, is an **isosceles triangle** with AB ≅ AC, then the angle bisector of BAC is the perpendicular bisector of the base BC and is also the median to the base.

Area of an isosceles triangle = $\frac{a}{4}\sqrt{4c^2 - a^2}$, Where c is the measure of equal sides and a is the third unequal side.

The altitudes on the congruent sides are equal i.e., BE = CF.

Area of Equilateral Triangle



For an equilateral triangle,

height = $\frac{\sqrt{3}}{2}$ × side; area = $\frac{\sqrt{3}}{4}$ × (side)^2

inradius = $\frac{1}{3}$ × height; circumradius = $\frac{2}{3}$ × height

perimeter = 3 × side

Also, the altitude, median, angle bisector, perpendicular bisector of each base are the same and the orthocentre, centroid, incentre and circumcentre are the same.

In case of triangles, given the perimeter, an equilateral triangle has maximum area.

Area of Right-angled Triangle

For a **right-angled triangle**, the median to the hypotenuse = $\frac{1}{2}$ × hypotenuse.

The median to the hypotenuse is also the circumradius of the triangle.

Area = $\frac{1}{2}$ × Product of perpendicular sides.

Relation between 2 triangles on the basis of height and base

1. The ratio of the areas of two triangles is equal to the ratio of the products of base and its corresponding height.

   $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AD \times BC}{PS \times QR}$

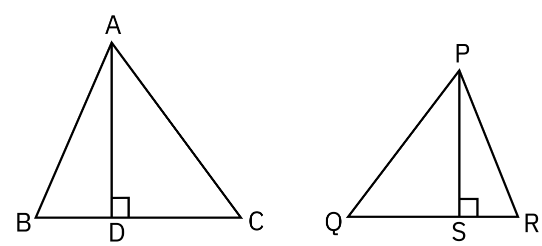


Fig F

2. Triangles of equal heights have areas proportional to their corresponding bases.

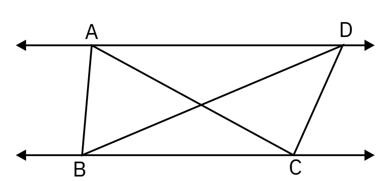
   In fig.F, if AD = PS, then $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{BC}{QR}$

3. Triangles of equal bases have areas proportional to their corresponding heights.

   In fig.F if BC = QR, then $\frac{A(\Delta ABC)}{A(\Delta PQR)} = \frac{AD}{PS}$

4. Areas of two triangles having the same base and lying between the same parallel lines will be equal.

   A(△ABC) = A(△BCD)



SOLVED EXAMPLES

Q : The sides of a triangle are 6 cm., 8 cm. and 10 cm. Find the area, inradius and circumradius of the triangle.

A : s = $\frac{6+8+10}{2}$ = 12

Area = $\sqrt{s(s-a)(s-b)(s-c)} = \sqrt{12\times6\times4\times2}$ = 24 sq.cm.

Let π, R be inradius & circumradius of the triangle.

A = r × s, ∴ 24 = r × s; 24 = r × 12; r = 2

A = $\frac{abc}{4R}$, 24 = $\frac{6\times8\times10}{4R}$; 4R × 24 = 6 × 8 × 10

∴ R = $\frac{6\times8\times10}{4\times24}$ = 5 cm.

Q : ΔABC is right angled at A and AD is the altitude to BC. If ℓ(AB) = 8 cm and ℓ(AC) = 15 cm, find ℓ(BC) and altitude AD. If M is the midpoint of BC, find ℓ(AM).

A : By the theorem of Pythagoras

ℓ((BC)^2) = 8^2 + 15^2 = 64 + 225 = 289 cm ∴ ℓ(BC) = $\sqrt{289}$ = 17 cm

Area of the triangle = $\frac{1}{2}$ × Product of perpendicular sides

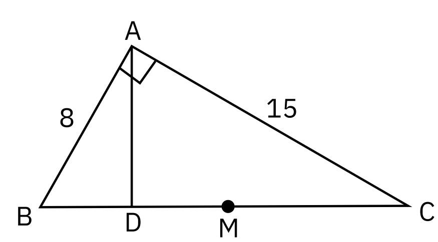
= $\frac{1}{2}$ × 8 × 15 = 60 cm^2

Also area = $\frac{1}{2}$ × BC × AD = 60

= $\frac{1}{2}$ × 17 × (AD) = 60 ∴ ℓ(AD) = $\frac{120}{17}$ cm

Again, AM is the median to the hypotenuse.

∴ ℓ(AM) = $\frac{1}{2}$ × hypotenuse = $\frac{1}{2}$ × 17 = 8.5 cm



Q : Each side of an equilateral triangle is $2\sqrt{3}$ cm. Find its height, area, inradius and circumradius.

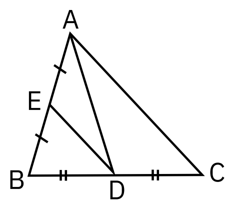
A : Height = $\frac{\sqrt{3}}{2}\times\text{side} = \frac{\sqrt{3}}{2}\times2\sqrt{3}$ = 3cm.

Area = $\frac{\sqrt{3}}{4}\times(\text{side})^2 = \frac{\sqrt{3}}{4}\times4\times3 = 3\sqrt{3}$ cm^2.

Inradius = $\frac{1}{3}$ × height = 1 cm; Circumradius = $\frac{2}{3}$ × height = 2 cm.

Q : Area of ΔABC = 18 sq. cm. D is the midpoint of BC and E is the midpoint of AB. Find A(ΔBDE).

A :



AD bisects the area of ΔABC.

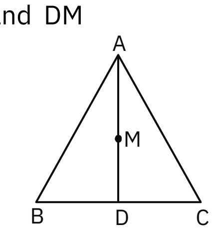
$$A(\Delta ADB) = \frac{1}{2} \times 18 = 9 \text{ sq.cm.}$$

DE bisects the area of ΔBDA

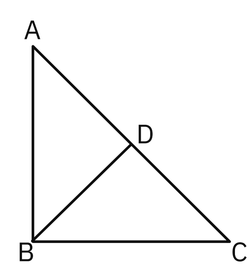
$$A(\Delta BDE) = \frac{1}{2} \times 9 = 4.5 \text{ sq.cm.}$$

Concept Builder 2

1. In ΔABC, AD is the median and M is the centroid and AD = 15. Find AM and DM



2. In ΔABC, $\angle B = 90°$, AC = 10. BD is the median to AC. Find AD, DC and BD.



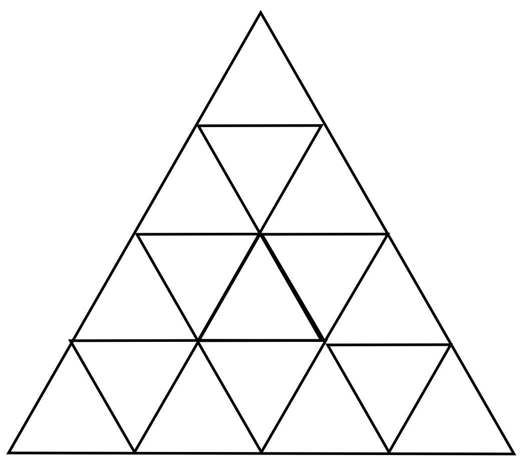
Answer key

1. 10 and 5
2. AD = DC = BD = 5cm

CLASS EXERCISE

Teaser

How many triangles are there in the figure below?



- **Basic Proportionality Theorem:** A line parallel to one side of a triangle divides the other two sides in the same ratio.
- **Midpoint Theorem:** A segment joining the midpoints of two sides of a triangle is parallel to the third side and half its length.

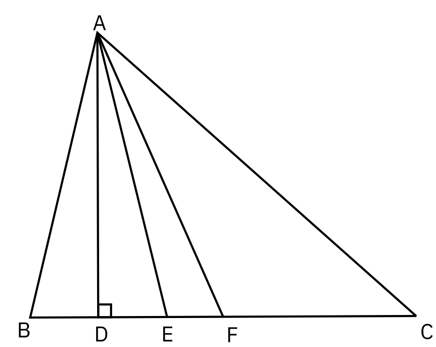
1. In a triangle ABC, AB = 12, AC = 9 and BC = 13. D and E are points on AB and AC respectively such that DE is parallel to BC. If l(AE) = 6, find l(BD)

Triangles : Elements

- **Median** : It is a line segment joining the vertex of a triangle with the midpoint of the side opposite to it. Three medians of a triangle are concurrent and their point of intersection is called 'centroid'.
- **Altitude** : It is a perpendicular drawn from a vertex of a triangle to the side opposite to it. Three altitudes of a triangle are concurrent and their point of interesction is called 'orthocentre'.
- **Angle bisector** : It is a ray originating at a vertex of a triangle passing through the interior of the triangle and dividing the angle of a triangle into two angles having equal measure. Three angle bisectors of a triangle are concurrent and their point of intersection is called 'incentre'.
- **Perpendicular bisector** : It is a line perpendicular to the side of a triangle that divides the side of the triangle into two equal halves. Three perpendicular bisectors of a triangle are concurrent and their point of interesction is called 'circumcentre'.

Triangles: Other useful theorems

2. In the triangle shown in the figure, l(AB) = 6, l(BC) = 7 and l(AC) = 8. Also AD is a perpendicular to BC, AE is an angle bisector of $\angle$A, and AF is a median of Δ ABC.



a) Find l(BD)

1) 1 2) 1.5
3) 2 4) 2.5

b) Find l(AD)

1) $\sqrt{25}$ 2) $\sqrt{31.25}$
3) $\sqrt{37.75}$ 4) $\sqrt{33.75}$

c) Find l(AF)

1) $\sqrt{41}$ 2) $\sqrt{31.25}$
3) $\sqrt{37.75}$ 4) $\sqrt{33.75}$

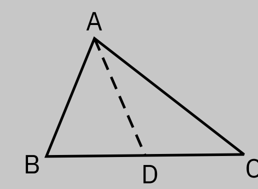
d) Find l(AE)

1) $\sqrt{36}$ 2) $\sqrt{37.25}$ 3) $\sqrt{39.75}$ 4) $\sqrt{33}$

In any triangle ABC

- **Apollonius' Theorem:** If AD is the median from A to BC, then

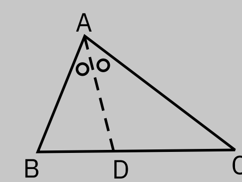
  $AB^2 + AC^2 = 2(AD^2 + BD^2)$



- **Internal Angle Bisector Theorem:** If AD is the internal angle

  bisector of $\angle$A, then

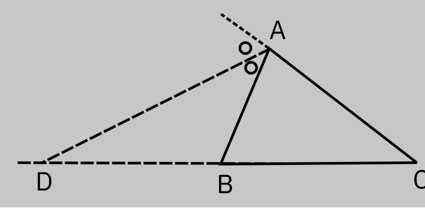
  $$\frac{\ell(AB)}{\ell(AC)} = \frac{\ell(DB)}{\ell(DC)}$$



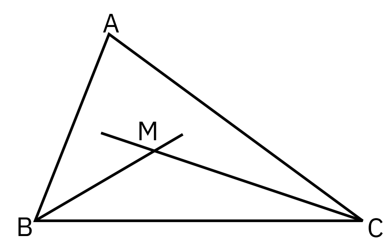
- **External Angle Bisector Theorem:** if AD is the external angle

  bisector of $\angle$A, then

  $$\frac{\ell(AB)}{\ell(AC)} = \frac{\ell(DB)}{\ell(DC)}$$



3. * In the adjoining figure, BM and CM are angle bisectors and m BAC = 70°. Find the measure of BMC.



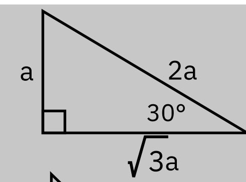
Special Triangles

4. A triangle VWX has $\angle V = 30°$ and $\angle W = 60°$. If l(VW) = 10, find l(VX) and l(WX)

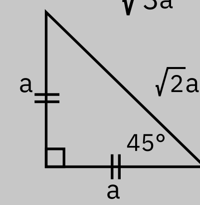
5. In the right-angled triangle LMN, l(LM) = l(MN)= 6 cm. Find l(LN) and the area of Δ LMN.

6. An equilateral triangle has perimeter 12 cm. Find its altitude and area.

The 30° - 60° - 90° Triangle: If the angles of a triangle in order are 30°, 60° and 90°, then the sides opposite to them will be in the ratio $1 : \sqrt{3} : 2$



The 45° - 45° - 90° Triangle: If the angles of a triangle in order are 45°, 45° and 90°, then the sides opposite to them will be in the ratio $1 : 1 : \sqrt{2}$. This is also called an isosceles right-angled triangle.



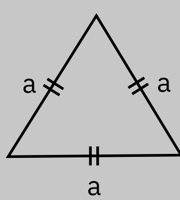
The Equilateral Triangle: If the side of an equilateral triangle is of length 'a' then:

The altitude of the triangle is $\frac{\sqrt{3}}{2}a$

The area of the triangle is $\frac{\sqrt{3}}{4}a^2$

The circumradius of the triangle is $\frac{1}{\sqrt{3}}a$

The inradius of the triangle is $\frac{1}{2\sqrt{3}}a$



7. * In Δ FGH, ∠F = 30°, ∠G = 90° and l(FG) = 5.19. What is the area of the triangle?

8. * In Δ PQR, ∠P : ∠Q : ∠R = 3 : 2 : 1. If l(PQ) = 5, what is l(QR)?

9. * Find the ratio of circumradius and inradius of an equilateral triangle of side m.

10. * In a triangle HIJ, ∠H = 30°. If l(HI) = l(IJ) = 12, find the length of HJ.

Triangles: Area

11. Find the area of triangle ABC if
    a) AB = 7, AC = 25 and $\angle B = 90°$
    b) AB = 8, AC = 12 and altitude BD = 5
    c) AB = 5, BC = 12, AC = 13
    d) AB = 13, BC = 14, AC = 15
    e) AB = 13, AC = 13 and median AD = 12
    f) AB + AC + BC = 24 and inradius r = 2
    g) AB = AC = 10, BC = 12 and circumradius R = 6.25

Area of a triangle:

Standard Formula: $A = \frac{1}{2} \times \text{base} \times \text{height}$ (from the opposite vertex)

Heron's Formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$ where s = semi-perimeter = $\frac{a+b+c}{2}$

For a Right-Angled Triangle: $A = \frac{1}{2} \times$ product of perpendicular sides

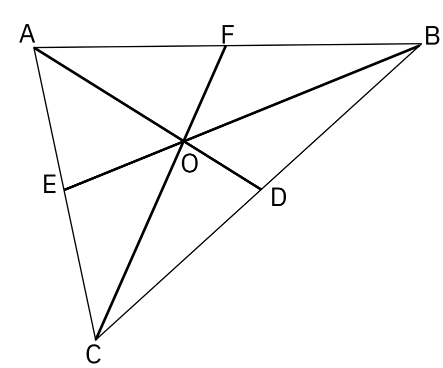
In terms of Inradius: $A = r \times S$ where r is the inradius and S the semi-perimeter = $\frac{a+b+c}{2}$

In terms of Circumradius: $A = \frac{abc}{4R}$ where R is the circumradius

Note that two triangles with the same base will have areas proportional to their heights, while two triangles with the same height will have areas proportional to their bases.

In terms of sine of angle: $A = \frac{1}{2}$ ab.sinθ, where θ is the angle between two sides having lengths a and b.

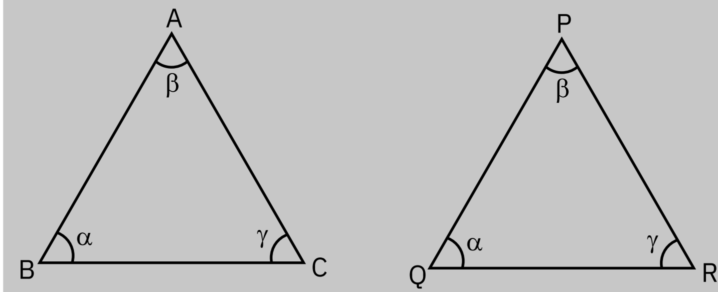
12. ABC is a triangle and AD, BE and CF are the medians from the vertices A, B and C respectively. If the area of ΔABC is 24, then compute the area of:



a) Δ ABD	b) Δ AOB
c) Δ AOF	d) Δ AEF
e) Δ DEF	f) Δ AED
g) Δ EOF	h) $\diamond$ AEOF

Areas of similar triangles

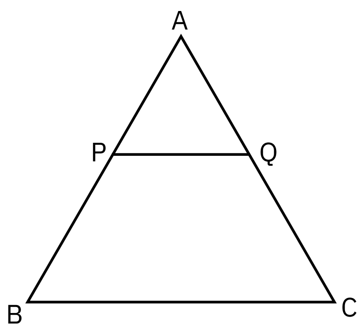
Areas of similar triangles are proportional to the square of the ratio of their corresponding sides.



$\Delta ABC \sim \Delta PQR$

$$\therefore \frac{A(\triangle ABC)}{A(\triangle PQR)} = \frac{\ell(AB)^2}{\ell(PQ)^2} = \frac{\ell(AC)^2}{\ell(PR)^2} = \frac{\ell(BC)^2}{\ell(QR)^2}$$

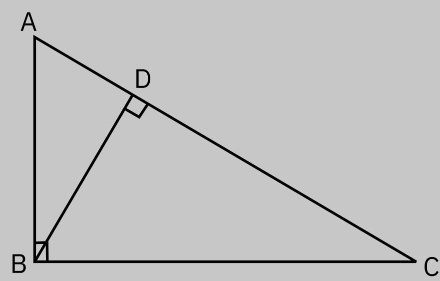
13. In the following figure, PQ || BC. Also, $2\ell(AB) = 3\ell(AP)$.



If $A(\Delta APQ) = 40$ sq.units, calculate $A(\Delta ABC)$ and $A(\square\ PQCB)$

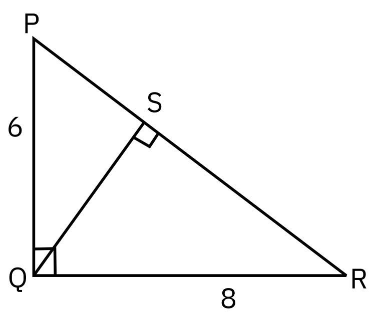
14. Suppose ΔABC and ΔPQR are equilateral triangles such that $A(\Delta ABC) : A(\Delta PQR) = 1 : 2$. If $\ell(AB) = 10$, calculate $\ell(PQ)$.

Advanced properties of a right angled triangle

- Orthocentre of a right angled riangle is at a vertex where two perpendicular sides of the triangle meet.
- Circumradius of a right angled triangle is half the hypotenuse. Circumcentre of a right angled triangle is the midpoint of its hypotenuse.
- 

  When perpendicular BD is drawn on hypotenuse AC of right angled triangle ABC,
  1) ABD ~ ACB ~ BCD
  2) $BD^2 = AD \times DC$; $AB^2 = AD \times AC$; $BC^2 = AC \times DC$

15. In right angled ΔPQR, QS is a perpendicular dropped on hypotenuse PR from vertex Q, as shown. If l(PQ) = 6 and l(QR) = 8



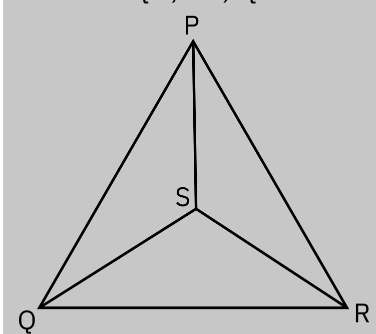
1) Calculate l(PS), L(SR) and l(QS)
2) Calculate circumradii of ΔPQR, ΔPQS and ΔQSR

16. If three sides of a triangle are 8, 15 and 17 cm, calculate the distance of the midpoint of the hypotenuse from the vertex opposite the hypotenuse.

17. What is the ratio of the circumradius to inradius of an isosceles right angled triangle?

Other advanced properties of triangles

- In ΔPQR, PS, QS and RS are angle bisectors, then:



1) $m\angle QSR = 90 + \frac{m\angle QPR}{2}$

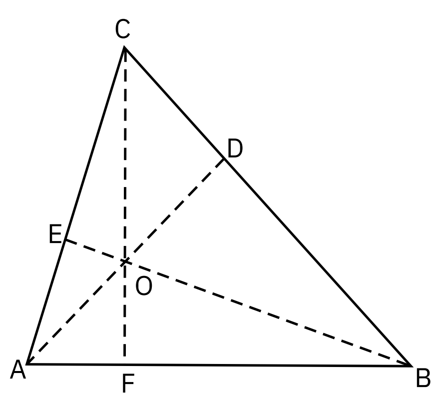
2) $m\angle PSQ = 90 + \frac{m\angle PRQ}{2}$

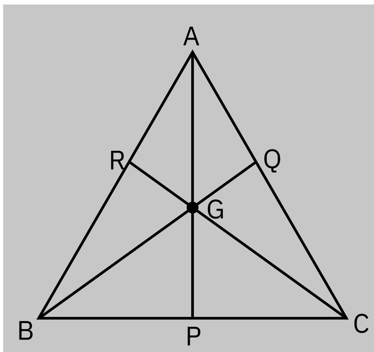
3) $m\angle PSR = 90 + \frac{m\angle PQR}{2}$

18. I is the incentre of a scalene triangle ΔPQR. $m\angle PQR = 50°$, $m\angle QRP = 60°$ and $m\angle RPQ = 70°$. Calculate

    1) $m\angle QIR$ 2) $m\angle PIR$ 3) $m\angle PIQ$

19. In the figure, AD, BE and CF are altitudes meeting at point O. If $m\angle ACB = 70°$ find $m\angle AOB$





- Centroid divides the medians of triangles in the ratio 1:2

If three medians AP, BQ and CR intersect in point G (which is centroid of ABC),

then $\frac{\ell(PG)}{\ell(GA)} = \frac{\ell(QG)}{\ell(GB)} = \frac{\ell(RG)}{\ell(GC)} = \frac{1}{2}$

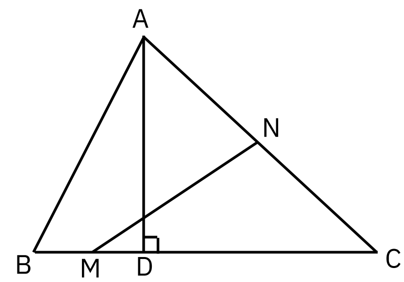
- Three medians divided a triangle into six small triangles having equal area i.e. A(ΔARG) = A(ΔBRG) = A(ΔBGP) = A(ΔPGC) = A(ΔCGQ) = A(ΔAGQ) = A(ΔABC)

20. ΔABC is a right angled triangle such that l(AB) = 6, l(BC) = 8 and l(AC) = 10. AD, BE and CF are the medians of the triangle that intersect at point G. Calculate –

    1) l(GD) 2) A(ΔAGC)

Challengers

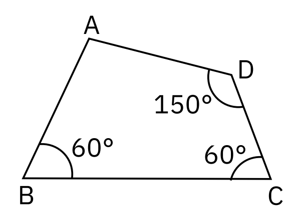
1. In the adjacent figure, N and M are midpoints of AC and BD respectively. BC = 8 units and AD = 6 units. Find length (MN)



1) 3
2) 3.5
3) 5
4) 5.25

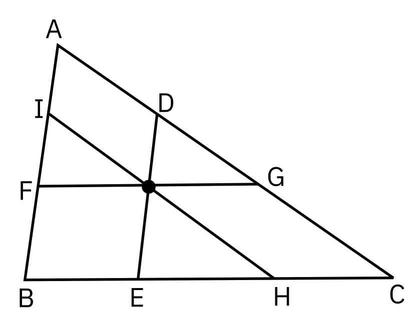
2. In triangle ABC, the medians BP and CQ are perpendicular to each other and intersect at R. If BP = 8 cm and CQ = 12 cm, find the area of triangle ABC.

3. In the given figure ABCD is a quadrilateral with BC = 4 cm and AD = 2 cm. What is the length of AB (in cm)?



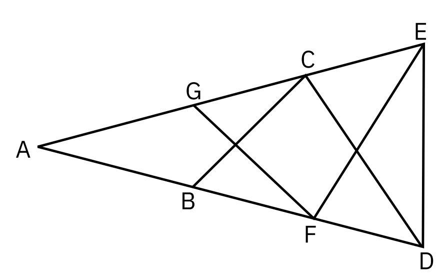
1) $4 - \frac{1}{\sqrt{3}}$
2) $\frac{4\sqrt{3} - 2}{\sqrt{3}}$
3) $\frac{2\sqrt{3} + 1}{2}$
4) $\frac{2\sqrt{3} - 1}{2}$

4. In the figure given below, P is a point inside the triangle ABC. Line segments DE, FG and HI are drawn through P, parallel to the sides AB, BC and CA respectively. The areas of the three triangles DPG, FPI and EPH are 9, 16, and 25 respectively. What is the area of the triangle ABC? (All the areas are in sq cm).



1) 100
2) 144
3) 150
4) 196

5. In the figure,l(AB) = l(BC) = l(CD) = l(DE) = lEF) = l(FG) = l(GA). If m(∠DAE) is x°, find the approximate value of x:



1) 20
2) 25
3) 30
4) 35

PRACTICE EXERCISE

For question 1, use the following information:

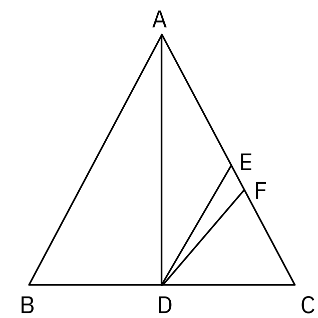
Consider the triangle ABC shown in the figure.

AB = AC = 17 and ℓ(BC) = 16

AD is an altitude drawn from A to BC.

E is the midpoint of AC

F is a point on AC such that ℓ(DF) = 8.



a) Find ℓ(DE) and ℓ(EF)?

b) What will be the perpendicular distance from E to AD and from E to BC?

c) By how much does the altitude AD exceed the altitude of an equilateral triangle with base BC ?

d) What will be the ratio of areas of AED and DEC?

Directions for Questions for 2 to 19: Choose the correct alternative.

2. In ΔPQS, the bisector of $\angle$QPS meets QS in R. If PQ = 12, PS = 16 and QS = 21, find QR and SR.

   1) 9, 12 2) 8, 22 3) 10, 11 4) 7, 14

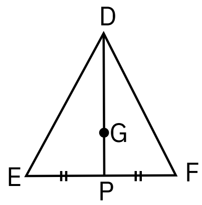
3. An altitude of a triangle of area 25 sq. cm. is equal to an altitude of a triangle of area 30 sq. cm. Find the ratio of their corresponding bases.

   1) 1 : 2 2) 3 : 2 3) 2 : 3 4) 5 : 6

4. ΔABC ~ ΔPQR and $\frac{AB}{PQ} = \frac{2}{5}$ = . If A(ΔPQR) = 175 sq. cm., find A(ΔABC).

   1) 70 sq. cm. 2) 437.5 sq. cm.
   3) 28 sq. cm. 4) 1093.75sq. cm.

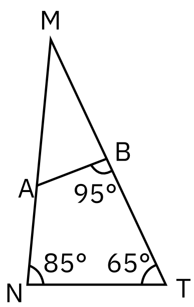
5.



In ΔDEF, G is the centroid, DE = 7, EF = 8 and DF = 9. Find GP.

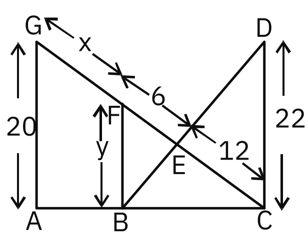
1) 7　　2) $\frac{7}{3}$　　3) $\frac{7}{2}$　　4) $\frac{14}{3}$

6. In the figure, if $\frac{NT}{AB} = \frac{9}{5}$ and if MB = 10, find MN.



1) 5.5　　2) 4.5　　3) 28　　4) 18

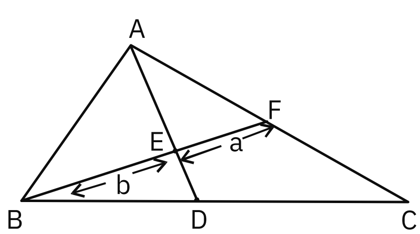
7.



In the given figure AG, BF and CD are parallel. Then xy = ?

1) 162　　2) 360　　3) 198　　4) $\frac{283}{11}$

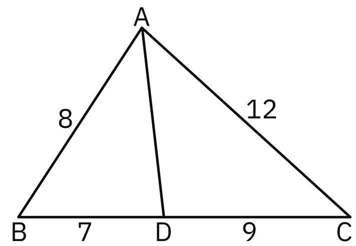
8.



If D and E are mid points of BC and AD in the figure then $\frac{a}{b}$ = ?

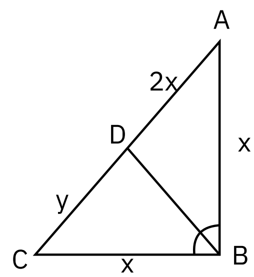
1) $\frac{1}{3}$　　2) $\frac{1}{2}$　　3) $\frac{2}{3}$　　4) $\frac{3}{5}$

9. Find the ratio of perimeters of ΔABD and ΔADC.



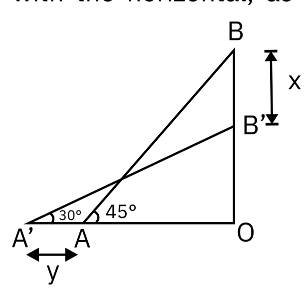
1) 7 : 9
2) 5 : 7
3) 2 : 3
4) Cannot be determined

10. ΔABC is as shown below. BD is the angle bisector of $\angle$ABC. Which of the following can be concluded?



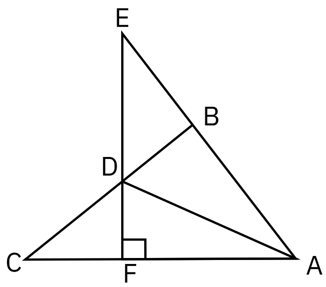
1) y = 2x
2) y < 2x
3) y = 0
4) None of these

11. A ladder resting along the wall at an angle of 45° slides such that it makes an angle of 30° with the horizontal, as shown. Find $\frac{x}{y}$.



1) $\frac{\sqrt{2}-1}{\sqrt{3}-\sqrt{2}}$
2) $\frac{1-\sqrt{2}}{\sqrt{6}-\sqrt{3}}$
3) $\frac{\sqrt{2}+1}{\sqrt{3}+\sqrt{2}}$
4) $\frac{1+\sqrt{2}}{\sqrt{6}+\sqrt{3}}$

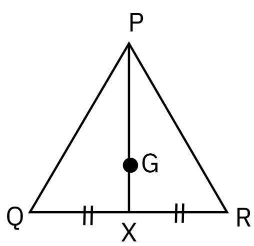
12.



ΔABC is right angled triangle at B. EF $\perp$ CA. Find m$\angle$AEC, if m$\angle$ADB = 55°.

1) 110° 2) 55° 3) 125° 4) 35°

13.



In ΔPQR, G is the centroid,

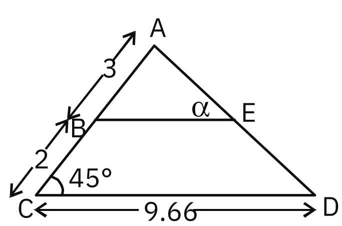
PQ = 7.5, QR = 8.0 and PR = 6.5. Find GX ?

1) 1.4 2) 2.5 3) 1.9 4) 1.2

14. Find the maximum area that can be enclosed in a triangle of perimeter 12 cm.

1) $4\sqrt{3}$ cm^2 2) $\frac{\sqrt{3}}{4}$ cm^2 3) $\sqrt{3}$ cm^2 4) $2\sqrt{3}$ cm^2

15.



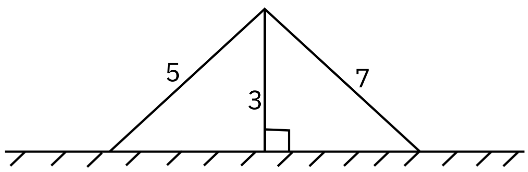
In the given figure BE is parallel to CD. Find area (in sq. units) of the ΔABE approximately.

1) 12.3 2) 41 3) 6.15 4) 82

16. In ΔABC, length of side AB is 9 units, that of side BC is 6 units and that of side AC is 5 units. BA is extended to D such that ℓ(AB) = ℓ(AD) and BC is extended to E such that ℓ(BC) = ℓ(CE). What is the area of ΔDCE?

1) $20\sqrt{2}$ sq. units 2) $15\sqrt{3}$ sq. units 3) $40\sqrt{3}$ sq. units 4) $18\sqrt{2}$ sq. units

17. A flat piece of cardboard is placed on a mirror as shown below:



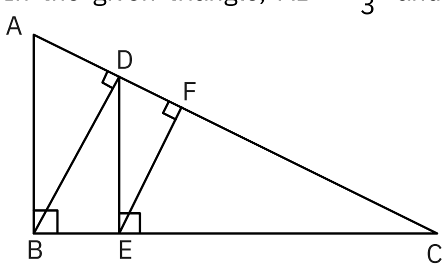
What is the area of the figure formed by combining the above triangle and its image?

1) $3(2 + \sqrt{10})$ sq. units 2) $6(2 + \sqrt{10})$ sq. units
3) $4(2 + \sqrt{10})$ sq. units 4) None of these

18. Consider an isosceles triangle with integer sides and perimeter 20. Which of the following cannot be the length of a side of such a triangle?

1) 4 2) 5 3) 6 4) 7

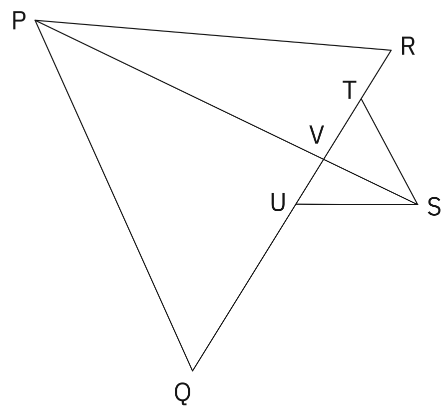
19. In the given triangle, AB = $\frac{8}{3}$ and AC = $\frac{16}{3}$. Find EF.



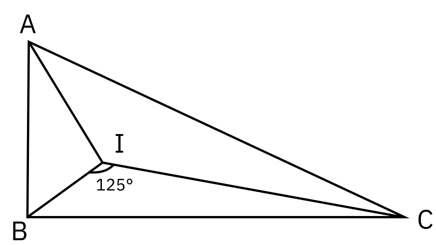
1) $\frac{\sqrt{3}}{2}$ 2) 1 3) $\sqrt{3}$ 4) 2

Directions for Questions for 20 and 21: Solve as directed.

20. ΔPQR is an equilateral triangle. T and U are points on QR such that TU = $\frac{1}{3}$ QR. Another equilateral triangle STU is drawn with TU as base. If PS intersects QR at V and SV = 3, find PS.



21. In the given triangle, right-angled at B, the angle bisectors of all the angles intersect at I. Find m $\angle$BAI.



Directions for Questions for 22 to 24: Choose the correct alternative.

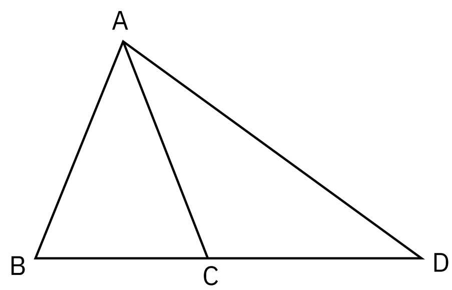
22. In ΔABC, which is right-angled at B, BD is drawn perpendicular to AC. BC = a, BD = b and the area of ΔABC = R. Which of the following represents the ratio of the areas of ΔADB and ΔCDB?

1) $\frac{R^2}{a^4}$ 2) $\frac{4R^2}{a^4}$ 3) $\frac{4a^2}{R^4}$ 4) $\frac{a^2}{R^4}$

23. ΔABC is right-angled at B. D is the midpoint of BC and E is the midpoint of AB. If CE = 15 and AC = 16, find AD.

1) 10 2) $\sqrt{95}$ 3) $\sqrt{85}$ 4) 9

24 . In the given figure, $\angle ABC = \angle ACB = \angle DAB$. If AB = 8 and BC = 4, find CD.



1) 8 2) 10 3) 12 4) 16

QA-3.3 | QUADRILATERALS & POLYGONS THEORY

Polygons

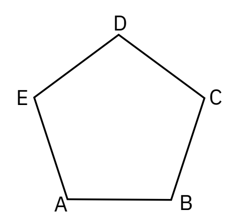
Definition of terms

A plane figure formed by three or more non-collinear points joined by line segments is called a polygon.

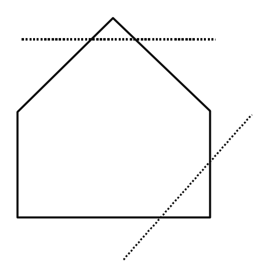
A polygon with: 3 sides is called a triangle.
4 sides is called a quadrilateral.
5 sides is called a pentagon.
6 sides is called a hexagon.
7 sides is called a heptagon.
8 sides is called an octagon.
9 sides is called a nonagon.
10 sides is called a decagon.

Convex polygon

A polygon, in which none of the interior angles is more than 180° is called a convex polygon.

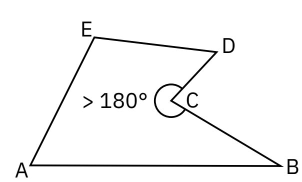


Any straight line drawn cutting a convex polygon passes through only two sides of the polygon.

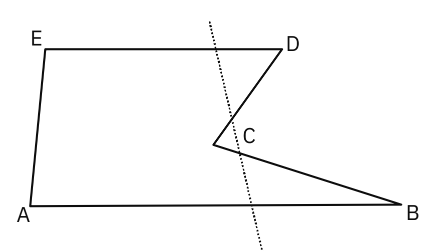


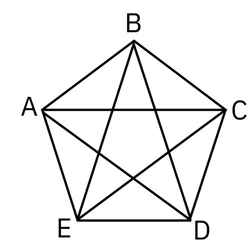
Concave polygon

A polygon, in which at least one angle is more than 180°, (i.e., a reflex angle) is called a concave polygon.



In a concave polygon, it is possible to draw lines passing through more than two sides.





Regular polygon: A convex polygon which has all **its sides and angles equal** is called a regular polygon.
Perimeter: The perimeter of a figure is the sum of the lengths of all its sides.
Area: The region enclosed within a figure is called its area.

Diagonal

The segment joining any two non-consecutive vertices is called a diagonal.
In the pentagon ABCDE, AC, AD, BE, BD and CE are the diagonals.

Number of diagonals of a polygon with n-sides = $\frac{n(n-3)}{2}$

In the given figure, number of diagonals in ABCDE = $\frac{n(n-3)}{2}$ = $\frac{5 \times 2}{2}$ = 5

Properties of a Polygon

1. Sum of all interior angles of a n-gon (polygon of side n) is given by (2n – 4)90° i.e.,(n – 2)180°.

   Hence, each interior angle of a regular n-gon = $\frac{(n-2)180°}{n}$

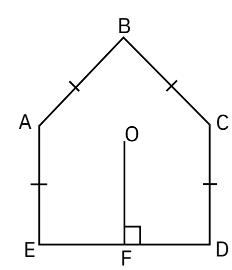
   Example

   Equilateral triangle is a regular polygon, with 3 sides where each angle = $\frac{3-2}{3}$ × 180 = 60°
2. Sum of an interior angle and its adjacent exterior angle is 180°.
3. Sum of all exterior angles of a polygon is 360°.
4. For a regular polygon, each exterior angle = $\frac{360°}{n}$
5. Area of a regular polygon

   = $\frac{1}{2}$ × perimeter × perpendicular from centre to any side.

   Area of pentagon ABCDE

   = $\frac{1}{2}$ × (AB + BC + CD + DE + EA) × OF

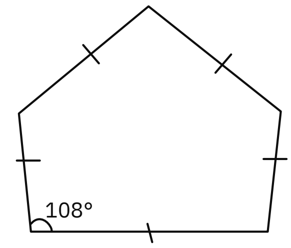


Types of Polygons

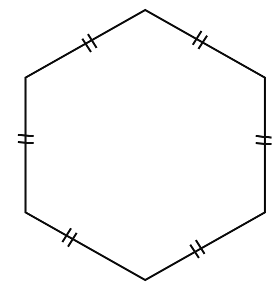
Quadrilateral

As defined earlier, a convex polygon of four sides is called a quadrilateral. It is denoted by the symbol '◻'. The sum of the measures of all angles of a quadrilateral is 360°. Quadrilaterals will be discussed in detail in the next section of this chapter.

Pentagon: A convex polygon of five sides is called pentagon. In a regular pentagon each interior angle is 108° and each exterior angle is 72°.



Hexagon: A convex polygon of six sides is called a hexagon. In a regular hexagon, each interior angle is 120° and each exterior angle is 60°.

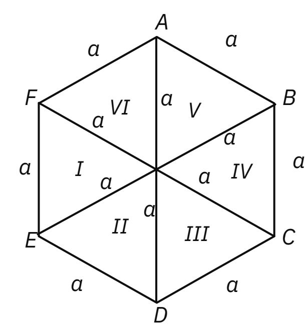


A regular hexagon is a combination of six equal equilateral triangles

$\therefore$ Area of regular hexagon

$= 6 \times$ Area of each triangle $= 6 \times \frac{\sqrt{3}}{4} \times a^2$

$= \frac{\sqrt{3}}{2} \times a^2 = \frac{3\sqrt{3}}{2} \times (\text{side})^2$



SOLVED EXAMPLES

Q : The sum of the measures of the angles of regular polygon is 2160°. How many sides does it have?

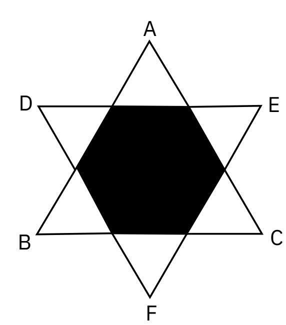
A : Sum of all angles = 90(2n − 4)

2160 = 90(2n − 4)

2n = 24 + 4

$\therefore$ n = 14

The polygon has 14 sides.



Concept Builder 2

1. Find the number of diagonals of

   a) Nonagon b) Decagon

2. Find the interior angle of a

   a) Regular Nonagon b) Regular Decagon

3. Sum of all the exterior angles of hexagon is 720° (True/False)

4. Side of a hexagon is 6 units. Find area of the hexagon and length of the diagonal

Answer key

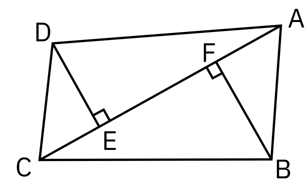
1. a) 27 b) 35
2. a) 140 b) 144
3. False
4. Area = $54\sqrt{3}$ sq. units

   Diagonal = 12 units

Quadrilaterals

Area of a quadrilateral = $\frac{1}{2}$ × one of the diagonals × sum of the perpendicular drawn to that diagonal from the opposite vertices.

A(□ABCD) = $\frac{1}{2}$ × AC × (BF + DE)



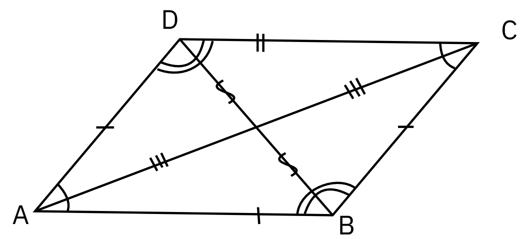
Types of Quadrilaterals

The different kinds of quadrilaterals are Square, Rectangle, Rhombus, Parallelogram, Kite and Trapezium.

Parallelogram

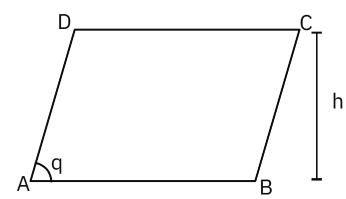
A quadrilateral is called a parallelogram if its opposite sides are parallel.

Properties of a parallelogram



1. Opposite sides are parallel and congruent.
2. Opposite angles are congruent.
3. Diagonals bisect each other.
4. Sum of any two adjacent angles is 180°.
5. Each diagonal divides the parallelogram into two triangles of equal area.
6. Straight lines joining the midpoints of the adjacent sides of any quadrilateral form a parallelogram.
7. **Area of parallelogram** = base × height.

   AB × h



8. Sum of the squares of the diagonals is equal to the sum of the squares of the 4 sides of a parallelogram.

   $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + AD^2$

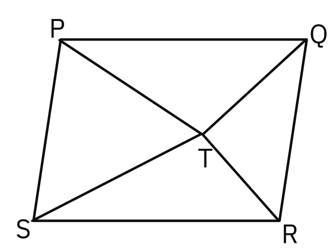
   $= 2(AB^2 + BC^2)$

9. Parallelograms that lie on the same base and between the same parallel lines are equal in area.
10. If a triangle and parallelogram lie on the same base and between the same parallel lines,

    Area(triangle) = $\frac{1}{2}$ Area(parallelogram)
11. Diagonals of a parallelogram need not be the angle-bisectors

    If any point inside a parallogram is taken and is joined to the 4 vertices the 4 resulting triangles will be such that the sum of the areas of the opposite triangles is equal.

    i.e., Area of ΔSTR + Area of ΔTPQ = Area of ΔTSP + Area of ΔRTQ = Half the area of the parallelogram



Example

□ABCD is a parallelogram. m∠BAD = 30°. BC = 5 cm and DC = 10 cm. Find the area of the parallelogram.

Let DP be perpendicular to AB.

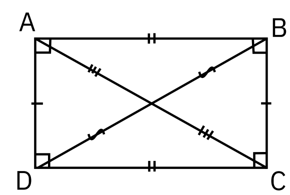
△ADP is a 30° - 60° - 90° triangle.

DP = $\frac{1}{2}$ × AD ... (side opposite to 30° is $\frac{1}{2}$ of hypotenuse)

= $\frac{1}{2}$ × 5 = 2.5. Area of □ABCD = base × height = 2.5 × 10 = 25 sq. cm.

Rectangle

A parallelogram, in which each angle is a right angle is called a rectangle. Obviously, every rectangle is a parallelogram.



Properties of Rectangles

1. Opposite sides are parallel and congruent.
2. Each angle is equal to 90°.
3. Diagonals are congruent and bisect each other.
4. **Perimeter** = $2(\ell + b)$, where ℓ is the length and b is the breadth.
5. **Area** = $\ell \times b$
6. **Diagonal** = $\sqrt{\ell^2 + b^2}$

Note:

- A parallelogram is a rectangle if its diagonals are congruent.
- The quadrilateral formed by the points of intersection of the angle bisectors of a parallelogram is a rectangle.

Example

The consecutive angles of a parallelogram are (2x + 30)° and (x + 60)°. What type of quadrilateral is the parallelogram?

2x + 30 + x + 60 = 180

3x + 90 = 180

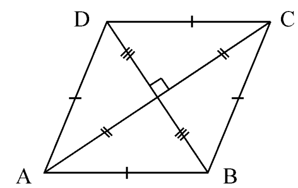
∴ x = $\frac{90}{3}$ = 30°

∴ (2x + 30)° = 90° and (x + 60)° = 90°

∴ The parallelogram is a rectangle.

Rhombus

A parallelogram in which all sides are congruent is called a rhombus.



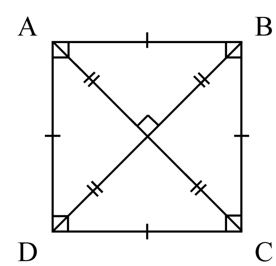
Properties of Rhombus

1. Opposite sides are parallel.
2. All sides are congruent.
3. Opposite angles are congruent.
4. Diagonals bisect each other at right angles (but they are not congruent).
5. Area = $\frac{1}{2}$ × Product of the diagonals.
6. Side^2 = $\left(\frac{1}{2}\text{one diagonal}\right)^2 + \left(\frac{1}{2}\text{other diagonal}\right)^2$

Note: A parallelogram is a rhombus if its diagonals are perpendicular to each other.
Rhombus is not a regular polygon as its angles are not equal.

SQUARE

A rectangle in which all sides are congruent is called a square. Obviously, every square is a rhombus, rectangle and parallelogram.



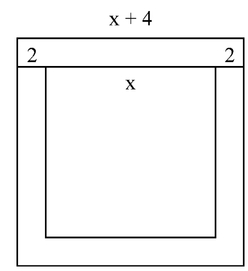
Properties of Square

1. All sides are congruent and opposite sides are parallel.
2. All angles are 90°.
3. The diagonals are congruent and bisect each other at right angles.
4. **Perimeter** = 4 × side.
5. **Area** = (side)^2 = $\frac{1}{2}$ × (diagonal)^2
6. **Diagonal** = $\sqrt{2}$ × side

Note: A parallelogram is a square if its diagonals are congruent and bisect each other at right angles.

Example

A square swimming pool is surrounded by a footpath 2 m wide. The area of the footpath is $\frac{5}{4}$ times that of the swimming pool. Find the area of the swimming pool.



Let the side of the swimming pool be x m.

Area of the swimming pool = x^2

Area of footpath = $(x + 4)^2 - x^2$

$\frac{5}{4}x^2 = (x + 4)^2 - x^2$

$5x^2 = 4(x + 4)^2 - 4x^2$

$9x^2 = 4(x + 4)^2$

$\therefore$ $3x = 2(x + 4)$ ...(taking square roots)

$3x - 2x = 8$ $\therefore$ $x = 8$

$\therefore$ Area of swimming pool = 8^2 = 64 sq. m.

TRAPEZIUM

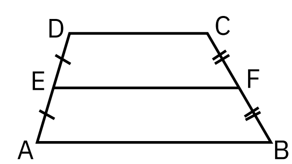
A quadrilateral is called a trapezium if two of the opposite sides are parallel but the other two sides are not parallel.

Properties of Trapezium

1. The segment joining the midpoints of the oblique (non-parallel) sides is called the median of the trapezium.

   Median = $\frac{1}{2}$ × sum of the parallel sides

   □ABCD is a trapezium, $\ell(EF) = \frac{1}{2} \times [\ell(DC) + \ell(AB)]$



2. **Area of a trapezium** = $\frac{1}{2}$ × sum of the parallel sides × height

Note: The trapezium is said to be an isosceles trapezium if the two non-parallel sides are congruent.

Example

The angles of a □ABCD are in the ratio 2 : 3 : 6 : 7. What type of a quadrilateral is it?

$2x + 3x + 6x + 7x = 360^\circ$

$18x = 360^\circ$

$\therefore$ $x = 20^\circ$

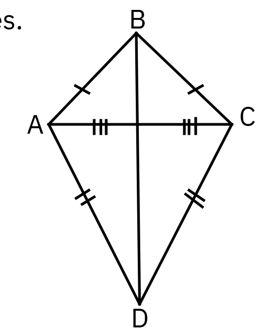
$\therefore$ Measures of angles are 40°, 60°, 120° and 140°.

$\therefore$ AB || DC and AD is not parallel to BC

$\therefore$ □ABCD is a trapezium.

KITE

A quadrilateral is called a kite, if it has two pairs of equal and adjacent sides.

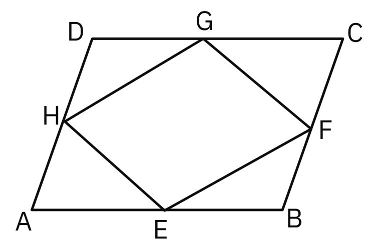


Properties of a Kite

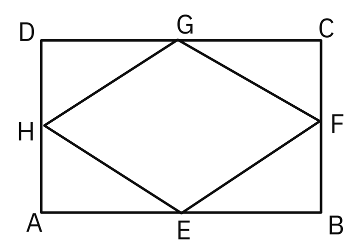
1. Two pairs of adjacent sides are congruent.
2. The diagonals intersect at right angles.
3. The longer diagonal bisects the shorter diagonal.
4. Area = $\frac{1}{2}$ × product of diagonals

Some interesting results

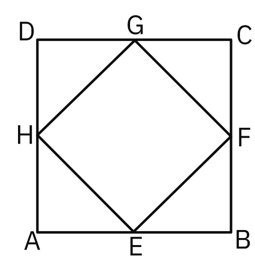
1. If the midpoints of the adjacent sides of a **parallelogram** are joined, **a parallelogram** is formed.



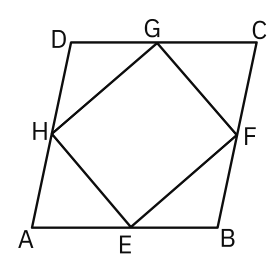
2. If the midpoints of the adjacent sides of a **rectangle** are joined, it forms a **rhombus**



3. If the midpoints of the adjacent sides of a **square** joined, it forms a **square**.



4. If the midpoints of the adjacent sides of a **rhombus** are joined, it forms a **rectangle**



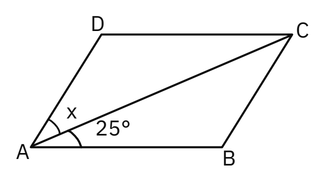
Summarizing what we have learnt so far:

Properties	Parallelogram	Rectangle	Rhombus	Square
Opposite sides are congruent	✓	✓	✓	✓
All sides are congruent	–	–	✓	✓
Opposite sides are parallel	✓	✓	✓	✓
Opposite angles are congruent	✓	✓	✓	✓
All angles are 90°	–	✓	–	✓
Diagonals are congruent	–	✓	–	✓
Diagonals bisect each other	✓	✓	✓	✓
Diagonals bisect each other at right angles	–	–	✓	✓
Diagonals bisect vertex angles	–	–	✓	✓
Diagonals form four triangles of equal area	✓	✓	✓	✓
Diagoanls form four congruent triangles	–	–	✓	✓

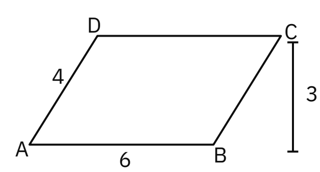
Concept Builder 2

1. Rectangle is a regular polygon (True/False)

2. In parallelogram ABCD, $\angle x = 25°$ (True/False)

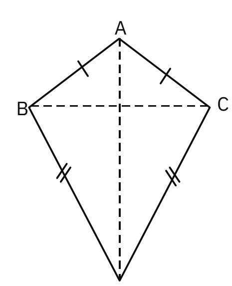


3. Find a) Perimeter of parallelogram ABCD
   b) Area of parallelogram ABCD

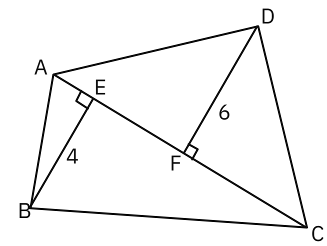


4. State true or false:
   a) Every square is a rhombus
   b) Every kite is a parallelogram
   c) Every rhombus is a parallelogram

5. In kite ABCD,
   BC = 4 units, AD = 8 units
   Find area of the kite.



6. In the figure DF = 6, BE = 4 and the area of quadrilateral ABCD = 40 sq.units. Find the length of the diagonal AC.



Answer key

1) False 2) False

3) a) Perimeter = 20 units

   b) Area = 18 sq. units

4) a) True b) False c) True

5) Area = 16 sq. units 6) 8 units

SOLVED EXAMPLES

Q : If P and Q are the midpoints of sides AB and BC of square ABCD of area 4 cm^2. Find the area of ΔDPQ.

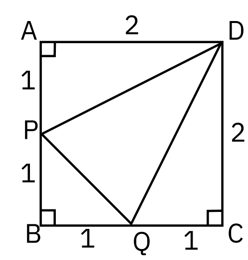
A : Area of square ABCD = 4 cm^2

$\therefore$ Side of square = 2 cm.

(ΔDPQ) = A($\square$ABCD) − A(ΔAPD + ΔPBQ + ΔDQC)

$= 4 - \left(\frac{1}{2}\times 1\times 2+\frac{1}{2}\times 1\times 1+\frac{1}{2}\times 1\times 2\right)$

$= 4 - \left(1+\frac{1}{2}+1\right)$ = 1.5 sq. cm.



Q : $\square$ABCD is a parallelogram. E is the midpoint of the diagonal DB. DQ = 10 cm, DB = 16 cm. Find PQ.

A : $\angle$EDQ $\cong$ $\angle$EBP ... (alternate angles)

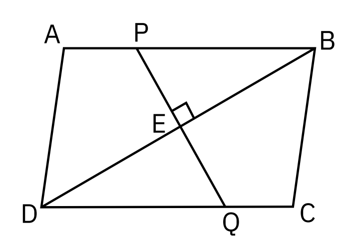
ΔDEQ $\cong$ ΔBEP ... (ASA test of congruence)

$\therefore$ ℓ(PE) = ℓ(EQ)

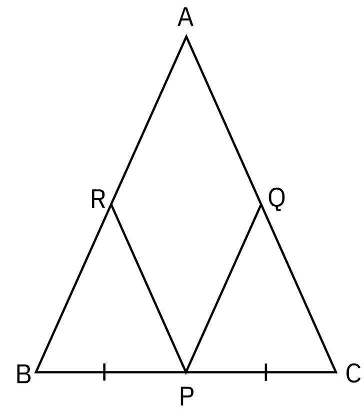
$[\ell(EQ)]^2 = [\ell(DQ)]^2 - [\ell(DE)]^2$

$= 10^2 - 8^2 = 100 - 64 = 36$

$\therefore$ ℓ(EQ) = 6 $\therefore$ ℓ(PQ) = 12 cm.



Q : In ΔABC, $AB = AC = 8$, PR and PQ are parallel to lines AC and AB respectively. P is the midpoint of BC. Find the perimeter of $\square$PRAQ.



A : R is the midpoint of side AB and Q is the midpoint of side AC...(midpoint theorem)

$\therefore\ RP = \frac{1}{2}AC = \frac{1}{2}AB$

$PQ = \frac{1}{2}AB$

Perimeter of $\square$PRAQ $= 2(RP + PQ) = 2\left(\frac{1}{2}AB + \frac{1}{2}AB\right) = 2AB = 2 \times 8 = 16$

Q : In parallelogram PQRS, the angle bisectors of $\angle P$ and $\angle S$ intersect at T. Find $\angle PTS$?

A : In parallelogram PQRS,

let $\angle P = \angle R = 2x$

$\angle S = \angle Q = 2y$

now, $\angle TPQ = \angle TPS = x$

$\angle TSR = \angle TSP = y$

In ΔTPS,

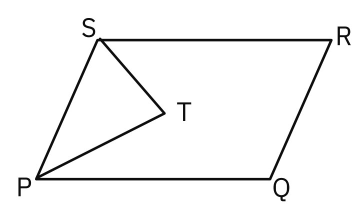
$x + y + \angle STP = 180°$ [angles of the triangle] ....(1)

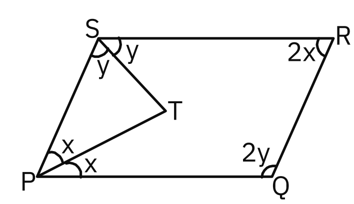
But $2x + 2y = 180°$ [adjacent angles of parallelogram]

$x + y = 90°$

Now (1) $\Rightarrow 90 + \angle STP = 180°$

$\Rightarrow \angle STP = 90°$

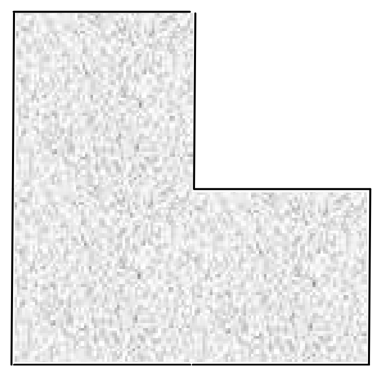




CLASS EXERCISE

Teaser

A farmer has a large L-shaped field which is formed of three square plots as shown below. He wishes to divide it into four parts such that all four parts are congruent (i.e. of the same shape and size).



Is it possible to divide the field as required?

Quadrilaterals and polygons

1. State whether the following statements are True or False:

a) The diagonals of a parallelogram bisect each other

b) A quadrilateral whose sides are equal must be a square

c) A quadrilateral whose angles are equal must be a square

d) The diagonals of an isosceles trapezium must be congruent

e) The diagonals of a rectangle bisect the angles at the vertices

f) A quadrilateral whose diagonals intersect at right angles must be a rhombus

g) A quadrilateral whose opposite angles are supplementary must be a rectangle

h) The longer diagonal of a kite is the perpendicular bisector of the shorter diagonal

i) A quadrilateral whose adjacent angles are supplementary must be a parallelogram

j) On joining the midpoints of the sides of any random quadrilateral in order, we get a parallelogram

k) The area of a parallelogram with diagonals 6 units and 8 units is 24 sq. units.

l) * The diagonals of a kite bisect the angles at the vertices

m) * A quadrilateral whose diagonals are equal must be a rectangle

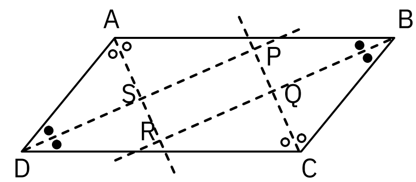
n) * If the diagonals of a parallelogram intersect at right angles, it must be a rhombus

o) * Of all rectangles with a given perimeter, the square has the largest area

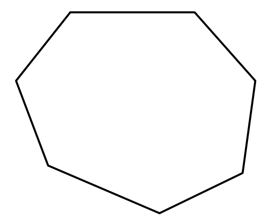
p) * The area of a kite with diagonals 10 cm and 14 cm is 70 sq cm

q) * The area of a parallelogram, having one side of length 7 inches, and a perpendicular distance of 6 inches between that side and its opposite side, is 42 square inches

2. In a parallelogram ABCD (with adjacent sides not equal), the angle bisectors are drawn. They intersect in points P, Q, R and S as shown. What kind of quadrilateral will PQRS be?



3. In the given septagon



a) Find the sum of the interior angles

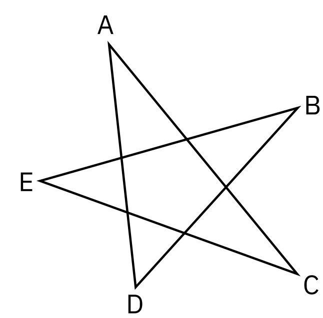
b) Find the sum of the exterior angles

4. If each interior angle of a regular polygon with n sides measures 144°, find n

5. If three angles of a pentagon are right angles, and the other two measure z° each, find the value of z.

6. Find the area of a regular hexagon of side 6 cm

7. Find the sum of measures of all the angles A, B, C, D, E at the vertices of the figure shown alongside.



Sum of all interior angles of a polygon with n sides = 180 (n − 2)°

Measure of each interior angle of a regular polygon with n sides = $\frac{180(n-2)°}{n}$

Sum of all exterior angles of a polygon with n sides = 360° (independent of n!)

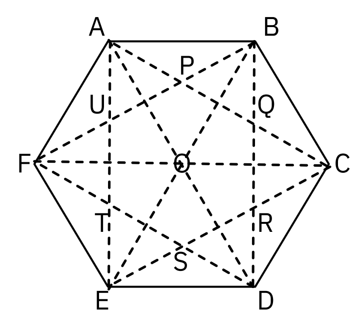
8. * A regular hexagon PQRSTU has a side of 4 cm. Find

a) The perimeter of ⬡PQRSTU b) The longest diagonal of ⬡PQRSTU

c) The shortest diagonal of ⬡PQRSTU d) The area of ⬡PQRSTU

9. * If each exterior angle of a regular polygon with n sides measures 20°, find n

10. A regular hexagon ABCDEF has its centre at O. All the non-adjacent vertices are joined to form the diagonals as shown in the adjacent figure. Identify the shapes formed by joining the following sets of points:



a) ACE b) ACDF

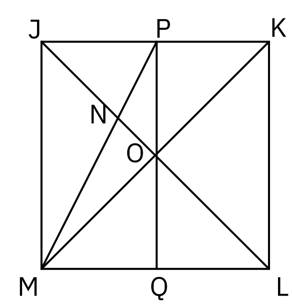
c) AQDT d) BOF

e) ABEF f) ACF

g) PQRSTU h) APDS

i) AUP j) ABDF

11. In a square JKLM, P and Q are the midpoints of JK and LM as shown in the adjoining figure. O is the point of intersection of the diagonals JL and KM, while N is the point of intersection of JO and PM.



If the area of ΔPON = 1, find

a) Area of ΔMON

b) Area of ΔPOJ

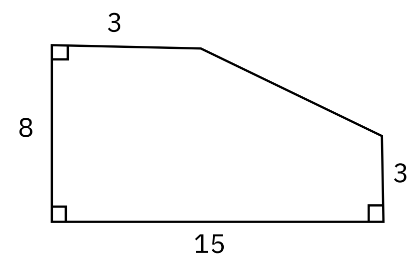
c) Area of ΔKOL

d) Area of ☐JKLM

e) Area of ΔJNM

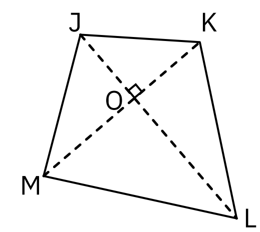
f) Area of ☐JOQM

12. * Find the area and perimeter of the adjoining figure:



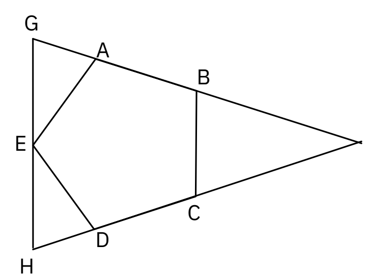
Challengers

1. In the quadrilateral JKLM shown alongside, JL ⊥ KM. If the area of the triangular regions JOM, JOK and KOL are 12, 9 and 15 respectively, find the area of ΔMOL.



1) 18 2) 20 3) 25 4) 21

Data for questions 2 and 3: A regular pentagon ABCDE has sides of length 1 cm. Sides AB and DC are extended to meet in a point F as shown in the figure. HG is drawn parallel to BC such that it meets the sides BA and CD (extended) in G and H respectively.



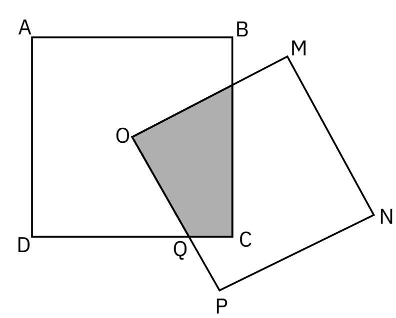
2. Which of the following is true?

1) m(∠AEB) > m(∠GEA) 2) m(∠DEH) > m(∠ECD)
3) m(∠BFC) > m(∠BEC) 4) None of the above

3. If l (AG) = x, find l (BF)

1) 3x 2) x^2 3) $\frac{1}{x}$ 4) x − 1

4. ABCD and OMNP are squares of sides 8cm and 7cm respectively. O is the centre of square ABCD. PQ = 2cm. Find the area of the shaded region.

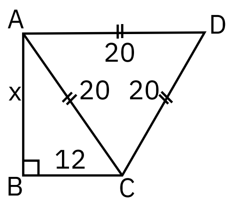


1) 16 cm^2 2) 14 cm^2
3) 18 cm^2 4) 12 cm^2

PRACTICE EXERCISE-1

Directions for Questions for 1 to 13: Choose the correct alternative.

1.



Find the area of the □ABCD.

1) 256 sq. units 2) 352 sq. units 3) 269 sq. units 4) 376 sq. units

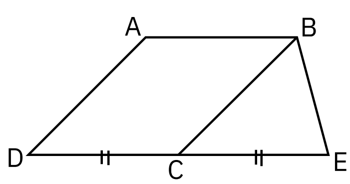
2. One diagonal of a rhombus is 24 cm and its side is 13 cm. Find the area of the rhombus.

1) 25 sq. cm. 2) 312 sq. cm. 3) 125 sq. cm. 4) 120 sq. cm.

3. Diagonals of a parallelogram are 6 cm and 8 cm respectively. If one side is 5 cm, find its area.

1) 48 sq. cm. 2) 30 sq. cm. 3) 24 sq. cm. 4) 40 sq. cm.

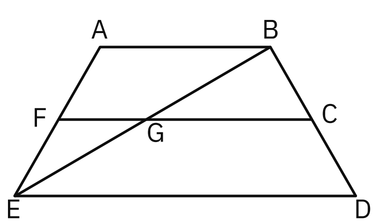
4.



C is the midpoint of DE. Area of parallelogram ABCD = 16 sq. cm. Find the area of ΔBCE.

1) 8 sq. cm. 2) 16 sq. cm. 3) 32 sq. cm. 4) 24 sq. cm.

5. In the adjoining figure: If AB, FC and ED are parallel, BC : BD = 2 : 5 and ED = 2AB then FC : AB = ?

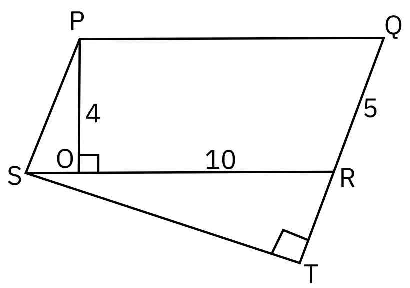


1) 5 : 7 2) 7 : 5 3) 3 : 2 4) 2 : 3

6. A square and a rhombus have the same base and the rhombus is inclined at 45°. What is the ratio of area of the square to that of the area of the rhombus?

1) 1 : 1 2) 1 : 2 3) $\sqrt{2}$: 1 4) 1 : $\sqrt{2}$

7.



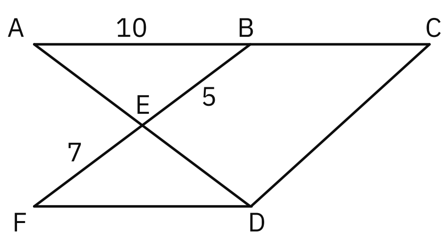
In the above figure PQRS, is a parallelogram with PQ = 10, QR = 5 and PO = 4. Find ST and area of PQTS.

1) 8, 12 2) 8, 64 3) 6, 48 4) 6, 24

8. The sum of the measures of the angles of regular polygon is 2340°. How many sides does it have?

1) 13 2) 14 3) 15 4) 16

9.



In the given figure, AC is parallel to FD and FB is parallel to DC. Area of $\square$BCDE is 238 sq. units. ℓ(AB) = 10 units, ℓ(BE) = 5 units and ℓ(EF) = 7 units. Find the area of ΔABE.

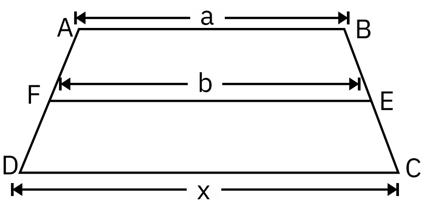
1) 40 sq. units 2) 25 sq. units 3) 50 sq. units 4) 35 sq. units

10. Two squares, with side lengths A and B, where A > B are placed together such that the right side of the square with side A touches the left side of the square with side B and their bases are collinear. A line is drawn from the bottom left corner of square A to the top right corner of squareB. What is the area below the line in the square with side A?

1) $\frac{1}{2}(A + B)B$ sq. units 2) $\frac{A^2B}{2(A+B)}$ sq. units

3) $\frac{1}{2}(A + B)A$ sq. units 4) $\frac{B^2A}{2(A+B)}$ sq. units

11.



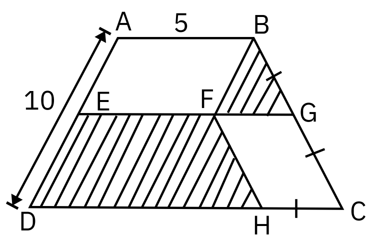
AB || FE || DC. Find the value of x if A(□ABEF) = A(□ECDF).

1) $\sqrt{2b^2 - a^2}$ 2) $\frac{2b - a}{2}$ 3) $\sqrt{\frac{4a^2 - b^2}{a^2}}$ 4) $\frac{a + b}{2\sqrt{ab}}$

12. Alternate vertices of a regular octagon are joined to form a quadrilateral. Find the ratio of the area of the quadrilateral formed to that of the octagon.

1) $1 : \sqrt{2}$ 2) $1 : 2\sqrt{2}$ 3) $1 : 4\sqrt{2}$ 4) $1 : 2$

13. □ABCD is an isosceles trapezium. Given that EG || DC, AE || BF, FH || GC and m∠AEG = 60°, find the area of the shaded region.

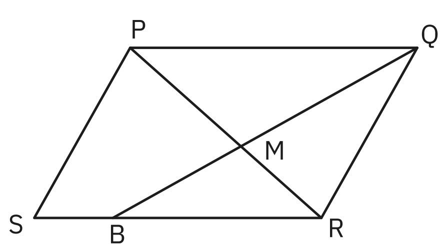


1) $25\sqrt{3}$ sq. units 2) $\frac{25}{4}\sqrt{3}$ sq. units 3) $75\sqrt{3}$ sq. units 4) $75\frac{\sqrt{3}}{4}$ sq. units

Directions for Questions for 14 and 15: Solve as directed.

14. The midpoints of all the sides of a regular hexagon are joined to form another smaller hexagon. What percentage of the area of the outer larger hexagon is the area of the inner smaller hexagon?

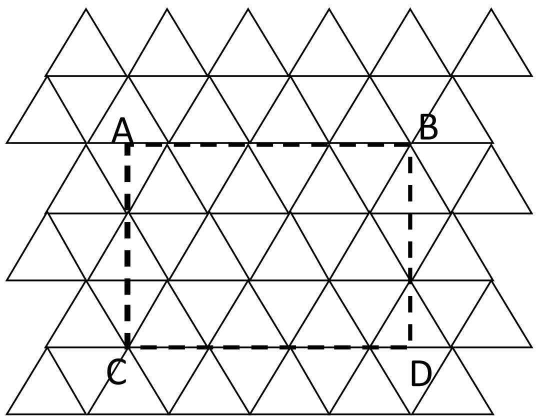
15.



Consider the following figure. PQRS is a parallelogram. Point B is on side RS. Diagonal PR meets segment QB at M. Ratio RB:BS is 4:1. Calculate the area of triangle RMQ (in square units) if the area of PQRS is 126 sq. units.

Directions for Questions for 16 to 19: Choose the correct alternative.

16. Consider the triangular grid shown below where each side of each of the small triangles is of one unit length.



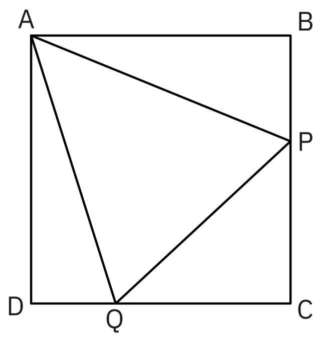
What is the area of rectangle ABCD?

1) $\frac{21\sqrt{3}}{4}$ sq. units
2) 12 sq. units
3) $\frac{15\sqrt{3}}{4}$ sq. units
4) 10.5 sq. units

17. ABCDEF is a regular hexagon. Point P is a point on side AB such that $\frac{\ell(AP)}{\ell(PB)} = \frac{1}{3}$. What is the ratio of the area of the hexagon to the area of triangle PED?

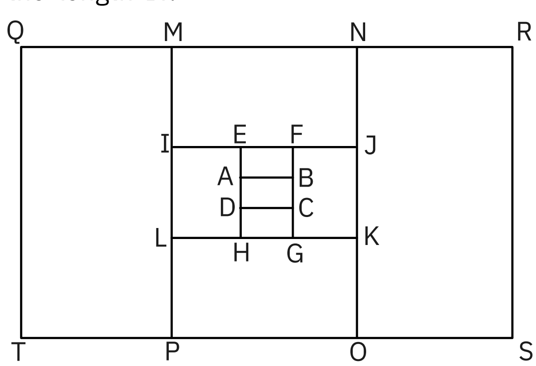
1) $3\sqrt{3} : 2$
2) $3 : 1$
3) $3 : 2\sqrt{2}$
4) Cannot be determined

18. In the following diagram, ABCD is a square while points P and Q lie on sides BC and CD respectively such that APQ is an equilateral triangle. What is the ratio of the area of square ABCD to that of equilateral triangle APQ?



1) $\frac{1+\sqrt{3}}{\sqrt{3}}$ 2) $\frac{4+2\sqrt{3}}{\sqrt{3}}$ 3) $\frac{4+\sqrt{3}}{2\sqrt{3}}$ 4) $\frac{2+\sqrt{3}}{\sqrt{3}}$

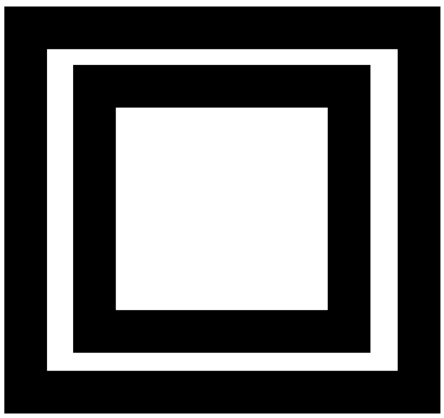
19. In the given figure containing all rectangles, AB = 2 cm and BC = 1 cm. Also, the ratio of the length and breadth of rectangles ABCD, EFGH, IJKL, MNOP and QRST is the same. Find the length IT.



1) $\sqrt{65}$ 2) $2\sqrt{61}$ 3) $2\sqrt{63}$ 4) $2\sqrt{65}$

Directions for Question for 20: Solve as directed.

20. The given figure consists of 4 concentric squares whose sides differ by 2 units. The area (in square units) of the shaded region is 64 square units. What is the difference in the areas (in square units) of the largest and smallest squares?

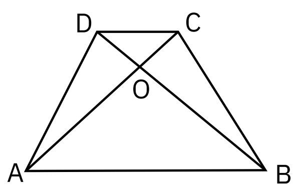


PRACTICE EXERCISE-2

Directions for questions 1 to 16 : Choose the correct alternative.

1. ☐ABCD is a parallelogram, BD = 26, DC = 17, DA = 11. Find AC.

   1) 12 2) 6 3) 7 4) 10.95

2. 

   ☐ABCD is a trapezium, AB = 3DC. If A(ΔOCD) = 6 sq. cm, find A(ΔOAB).

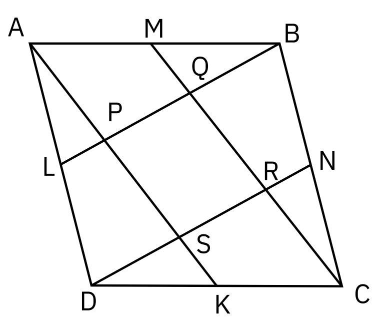
   1) 54 sq. cm. 2) 27 sq. cm. 3) 18 sq. cm. 4) 108 sq. cm.

3. The rectangular courtyard ABCD of side 50 ft and 42 ft encloses a lawn EFGH surrounded by a 6ft wide gravel path. Find the cost of spreading gravel along the path if gravelling cost is Rs.10 per sq. ft.

   1) Rs.96 2) Rs.480 3) Rs.51.60 4) Rs.9600

4. The perimeter of a regular hexagon is 36 cm. Find its area.

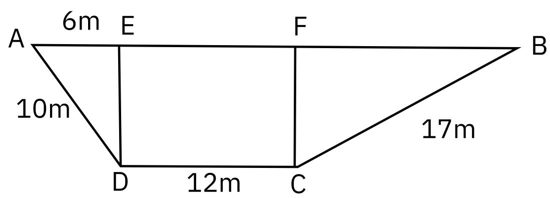
   1) $18\sqrt{3}$ sq .cm. 2) $36\sqrt{3}$ sq. cm. 3) $27\sqrt{3}$ sq. cm. 4) $54\sqrt{3}$ sq. cm.

5. 

   ABCD is a parallelogram. M, N, K and L are the midpoints of the sides respectively. PQRS is the quadrilateral formed by the intersections of AK, BL, CM and DN. Determine the area of ☐PQRS if the area of quadrilateral ABCD is 3000 square units, and the areas of ☐AMQP and ☐CKSR are 513 and 388 sq. units, respectively.

   1) 599 sq. units 2) 799 sq. units
   3) 2099 sq. units 4) 2567 sq. units

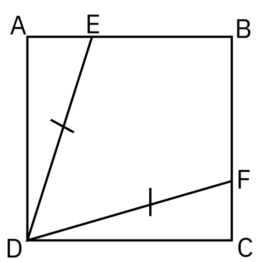
6.



AB and DC are two opposite sides of a road parallel to each other. To cross the road there are zebra crossings at DE and CF. Somu who is standing at point A is unable to cross the road to reach point D. So, he crosses the road from E. E is 6 m from A. If he crosses the road from A, he covers 10 m to reach D. Kanu who is standing at point B wants to meet Somu, so she walks 17 m, to reach point C. Now, if Somu and Kanu are 12 m apart, what is the distance AB and what is the area between the two zebra crossings?

1) 33 m, 96 m^2
2) 15 m, 48 m^2
3) 27 m, 54 m^2
4) None of these

7.



In the square ABCD, E and F are two points on sides AB and BC respectively such that DE = DF, then which of the following is false?

1) EB = BF
2) AE = CF
3) Area of □DEBF = Area of □ABCD − 2 Area of ΔADE.
4) Area of ΔADE = $\frac{1}{8}$ Area of □ABCD.

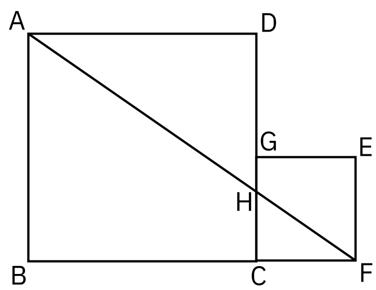
8. P, Q, R, S are mid-points of sides AB, BC, CD, AD respectively. PK, QL, RM and SN are bi-sectors of ∠P, ∠Q, ∠R and ∠S of □PQRS thus formed by joining the mid-points in cyclic order. K and N are points on the side RQ and L and M are the points on the side SP. If (RM) = ℓ(SR) = 12 units, find m∠SRM.

1) 60°
2) 120°
3) 90°
4) 30°

9. Two regular polygons have their number of sides in ratio 5:3. The difference between their angles is 8°. How many sides do the polygons have?

1) 25 and 15
2) 30 and 18
3) 35 and 21
4) 20 and 12

10.

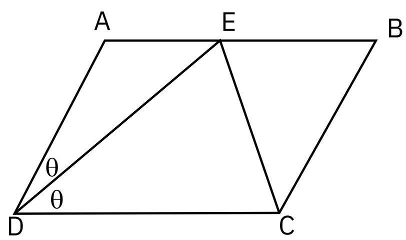


□ABCD and □CFEG are squares. ℓ(HF) = 5 cm, ℓ(AH) : ℓ(HF) = 3 : 1

Which of the following could be the value of A(□ABCD)? (All sides are integers)

1) 324 cm^2 2) 81 cm^2 3) 144 cm^2 4) 256 cm^2

11.



□ABCD is a parallelogram, ℓ(AB) = 8 cm, ℓ(AD) = 4 cm and ℓ(EC) = 4 cm. If DE is an bisector of ∠ADC then find ℓ(ED).

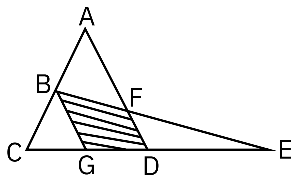
1) $(2 + 2\sqrt{3})$cm 2) $4\sqrt{3}$ cm 3) $6\sqrt{3}$ cm 4) $(4 + \sqrt{3})$cm

12. A regular polygon of 12 sides is formed by cutting off each corner of a hexagon with side 10. Find the perimeter of the 12 sided polygon.

1) $\frac{120}{2+\sqrt{3}}$ 2) $\frac{120\sqrt{3}}{2+\sqrt{3}}$ 3) $\frac{60\sqrt{3}}{2+\sqrt{3}}$ 4) None of these

13. Two congruent rhombi intersect each other such that exactly one pair of parallel sides of one rhombus is parallel to that of the other. One of their intersection points is the mid-point of the intersecting sides of both rhombi. If the rhombus has its side equal to 10 cm and one of its angles is equal to 60°, then find the common area of the quadrangle formed due to the intersection.

1) $\frac{25}{4}\sqrt{15}$ cm^2 2) $\frac{25}{4}\sqrt{3}$ cm^2 3) $\frac{75}{4}\sqrt{3}$ cm^2 4) None of these

14. 

In the figure, B and D are midpoints of side AC and side CE respectively and FD || BG. Find the area of the shaded region if A(ΔACD) = 40 sq. units

1) 30 sq. units
2) $\frac{50}{3}$ sq. units
3) 35 sq. units
4) Cannot be determined

15. What is the approximate area of the largest regular hexagon that can be inscribed in an equilateral triangle of each side 9 cm?

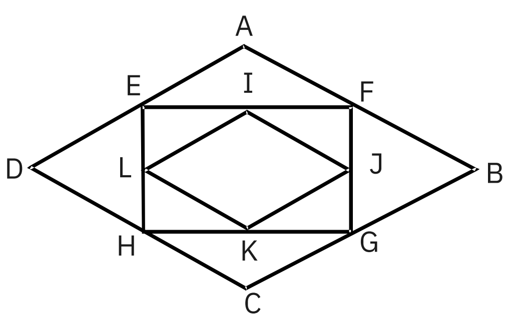
1) $12\sqrt{3}$ cm^2
2) $18\sqrt{3}$ cm^2
3) $\frac{27}{2}\sqrt{3}$ cm^2
4) $\frac{39}{2}\sqrt{3}$ cm^2

16. ABCD and PQRS are two rhombuses. The lengths of their smaller diagonals are in the ratio 3:5. The lengths of the diagonals of ABCD are in the ratio 4:7 and the lengths of the diagonals of PQRS are in the ratio 4:5. What is the ratio of the area of ABCD and the area of PQRS?

1) 13:17
2) 39:47
3) 63:125
4) None of these

Directions for question 17: Solve as directed.

17. ABCD is a rhombus. E, F, G and H are the midpoints of DA, AB, BC and CD respectively. I, J, K and L are the midpoints of EF, FG, GH and HE respectively. The area of the ELI is 2.25 cm^2. What is the area (in cm^2) of rhombus ABCD?

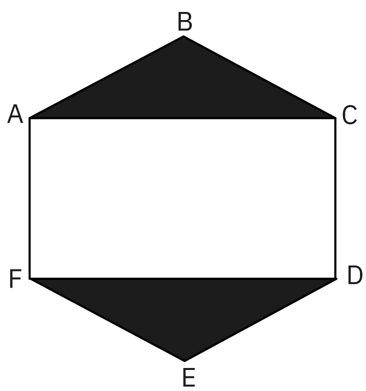


Directions for questions 18 to 21 : Choose the correct alternative.

18 . ABCD is a rhombus with $\angle DAB = 120°$ and side ‘x’. If a rectangle PQRS is drawn such that the diagonal of the rectangle is equal to the longer diagonal of the rhombus, and if the sides of the rectangle are in the ratio 3 : 1, what will be the smaller side of the rectangle in terms of x?

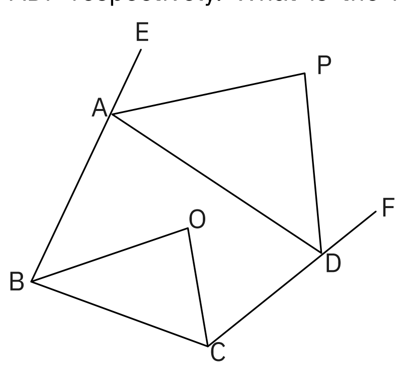
1) 2x 2) $\sqrt{\frac{3}{10}}\,x$ 3) $x\sqrt{5}$ 4) $\frac{x}{\sqrt{3}}$

19. In the following diagram, ABCDEF is a regular hexagon. What is the ratio of the area of the shaded region to the area of the hexagon?



1) $1 : 2\sqrt{2}$ 2) 1 : 3 3) $1 : 3\sqrt{2}$ 4) $1 : 2\sqrt{3}$

20. In quadrilateral ABCD, BO, CO, AP and DP are the angle bisectors of ABC, BCD, EAD and ADF respectively. What is the relation between BOC and APD?



1) They are equal.
2) They are complementary.
3) They are supplementary.
4) Nothing conclusive can be said

21. Each of the points P, Q, R and S divides a different side of a rectangle ABCD in the ratio 1 : 2 in such a way that PR and QS intersect at a point inside ABCD. Which of the following cannot be the ratio of the areas of quadrilateral PQRS and rectangle ABCD?

1) 1 : 2 2) 5 : 9 3) 4 : 9 4) 2 : 3

QA-3.4 | CIRCLES - I

THEORY

Circle

The set of points in a given plane, which lie at a fixed distance from a fixed point, forms a circle. The fixed point is called the **centre** of the circle and the fixed distance is called the **radius (r)** of the circle. In the figure, O is the centre of the circle and OR is the radius.

A circle is uniquely determined by three non-collinear points i.e. only one circle passes through three non-collinear points.

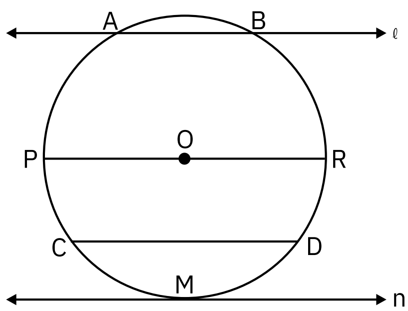


Fig. A

Chord

A chord is a segment whose endpoints lie on the circle. In fig.A, CD is a chord of the circle.

Diameter (d)

The diameter is the chord passing through the centre of the circle. The length of the diameter of a circle is twice the radius of the circle. In fig.A, PR is a diameter of the circle. The diameter is the largest chord of a circle.

Circumference (c)

The circumference of a circle is the distance around the circle. It contains 360° at the circle. The value of the circumference is equal to $2\pi r$.

$c = 2\pi r$, where $\pi = \frac{22}{7}$ or 3.14

Semicircle

Half of a circle cut off by a diameter is called the semicircle. In fig.A, PAR is a semicircle. The measure of a semicircle is 180°.

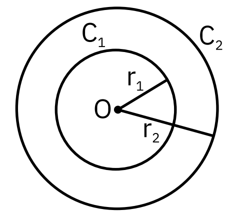
Arc

An arc is a part of a circle. A minor arc is an arc less than the semicircle and a major arc is an arc greater than a semicircle. In fig.A, arc CMD is a minor arc and arc CAD is a major arc formed by the chord CD.

Congruent Circles

Circles with equal radii are called congruent circles.

Concentric Circles



Circles lying in the same plane with a common centre are called concentric circles.
C_1 and C_2 are concentric circles with the same centre O and radius r_1 and r_2, respectively.

Tangent

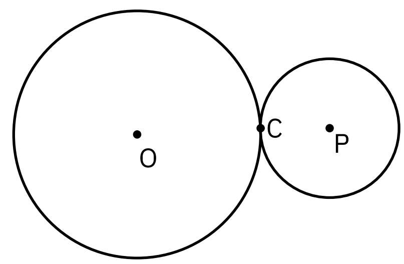
Tangent is a line in the plane of a circle and having one and only one point common with the circle. The common point is called the point of contact. In fig.A, line n is a tangent to the circle and M is the point of contact.

Secant

A secant is a line, which intersects the circle in two distinct points. In fig.A, line ℓ is a secant, which intersects the circle in points A and B.

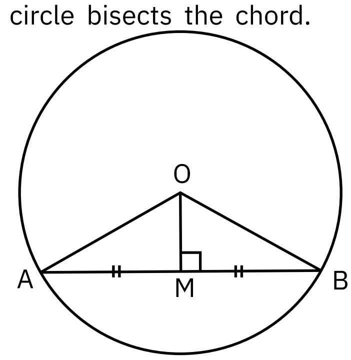
Tangent Circles

Circles lying in the same plane and having one and only one point in common are called tangent circles.
In the given figure, C is the common point of the circles with centres O and P.

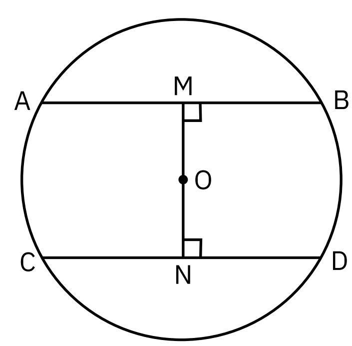


Properties of Chords

1. The perpendicular from the centre of a circle to a chord of the circle bisects the chord.
   If OM ⊥ AB, then AM = MB.
   Conversely, the line joining the centre of the circle and the midpoint of a chord is perpendicular to the chord.
   If AM = MB, then OM ⊥ AB.



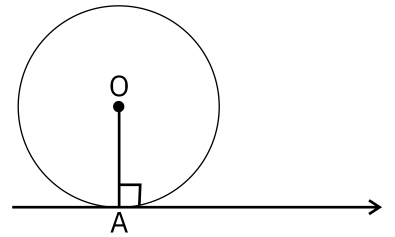
2. Equal chords of a circle or congruent circles are equidistant from the centre.
   If AB = CD, then OM = ON.
   Conversely, two chords of a circle or congruent circles that are equidistant from the centre are equal.
   If OM = ON, then AB = CD.



Properties of Tangents

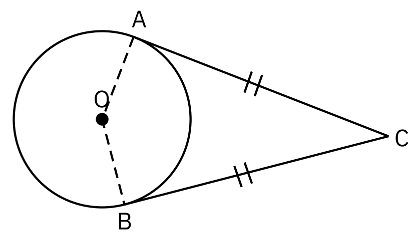
1. The tangent at any point of a circle and the radius through that point are perpendicular to each other. (**Tangent Perpendicularity Theorem**)

   If O is the centre of the circle, A is the point of contact of the tangent ℓ, then $OA \perp \ell$.



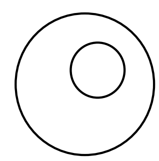
2. Given a point on the circle, there is one and only one tangent to the circle passing through that point.
3. From an exterior point of the circle, two tangents can be drawn onto the circle.
4. The lengths of two tangent segments, from the exterior point to the circle, are equal. AC = BC.

   Now, $OA \perp AC$, $OB \perp BC$. So here we get a **kite OACB**

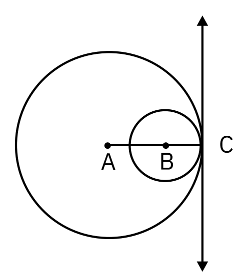


Common Tangents

1. If two circles are such that one lies completely inside the other without touching each other then there will be no tangent common to these circles.



2. If the two circles touch internally, then they have only one common tangent. Distance between their centres = difference of the radii.
   Distance between centres = AC – BC.



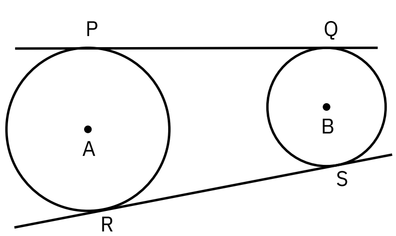
3. For the two circles with centres A and B, PQ and RS are the direct common tangents, and CD and EF are the transverse common tangents. (Only two of both transverse common tangents and direct common tangents are possible.) Where r_1 and r_2 are the radii of the two circles.

   Length of direct common tangent

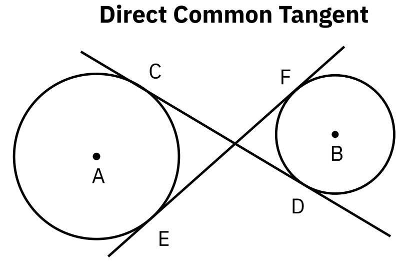
   $$= \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$$

   Length of transverse common tangent

   $$= \sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$$

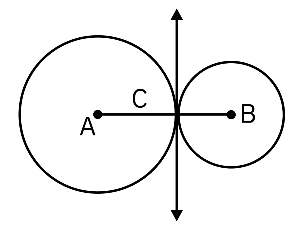


Direct Common Tangent



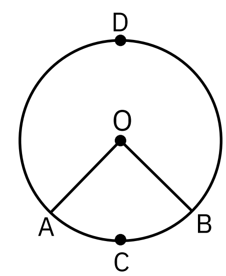
Transverse Common Tangent

4. If the two circles touch externally, distance between their centres = sum of their radii. **Distance between centres = AC + CB.**



Central Angle

An angle in the plane of the circle with its vertex at the centre is called a central angle. In figure shown above, ∠AOB is a central angle subtended by arc ACB at the centre.



Measure of an arc

(i) The measure of a semicircle is 180°.

(ii) The measure of a minor arc is equal to the measure of its central angle.
m(arc ADB) = m∠AOB

(iii) The measure of a major arc = 360° – (measure of corresponding minor arc)
m(arc ACB) = 360° – m(arc ADB)

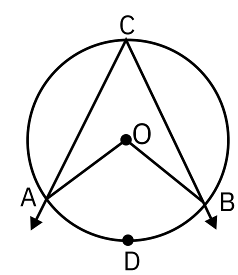


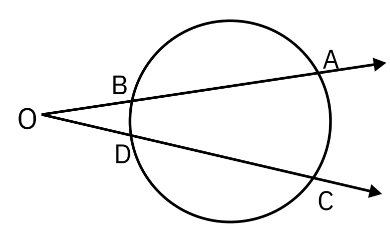
Fig. B

Inscribed Angle

An angle with vertex (C) as a point of the arc other than its endpoints (A and B) and each side of the angle containing one endpoint (A and B) of the arc is called the angle inscribed in the arc.
∠ACB is inscribed in the arc ACB.

Intercepted Arc

An arc is said to be intercepted by an angle if each side of the angle contains an endpoint of the arc, and the arc but for its endpoints, lies in the interior of the angle.
Arc BD and arc AC are intercepted by ∠AOC.

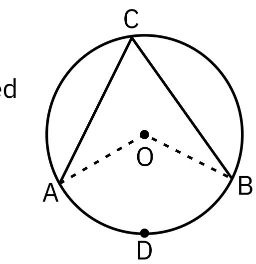


Inscribed Angle theorem and its corollaries

1. **Inscribed Angle theorem**

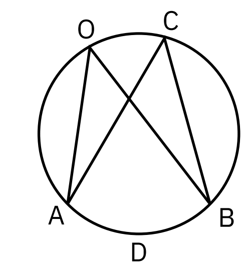
   The measure of an inscribed angle is half the measure of its intercepted arc. (Inscribed angle theorem).

   m∠ACB = $\frac{1}{2}$ m(arc ADB) = $\frac{1}{2}$ m∠AOB

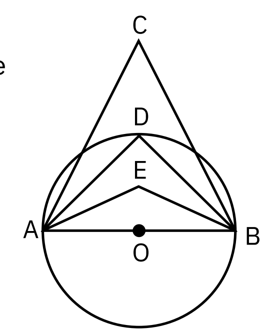


2. Angles inscribed in the same arc are equal.

   m∠AOB = m∠ACB as they are inscribed in the same arc ADB.



3. The angle subtended by a diameter.
   a) A diameter of a circle subtends **an obtuse angle** at a point in the interior of the circle. **m∠AEB > 90°.**
   b) A diameter of a circle subtends **an acute angle** at a point in the exterior of the circle. **m∠ACB < 90°**.
   c) A diameter of a circle subtends **a right angle** at a point on the circle. **m∠ADB = 90°.**

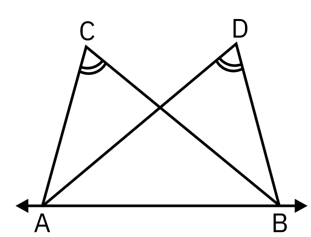


Conversely, if a chord subtends a right angle at a point on the circle, then the chord is a diameter of the circle.

If m∠ADB = 90°, then AB is a diameter of the circle.

4. If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle.

   Points A, B, C and D lie on the same circle. i.e., they are concyclic.

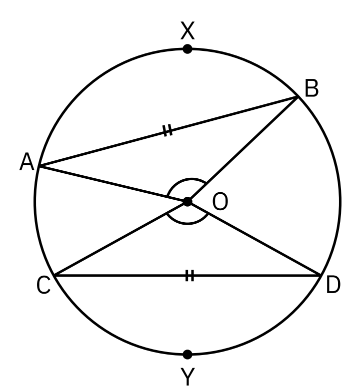


5. Equal arcs of a circle or congruent circles have equal chords.

   If m(arc AXB) = m(arc CYD), then AB = CD.

   Conversely, equal chords of a circle or congruent circles have their corresponding arcs equal.

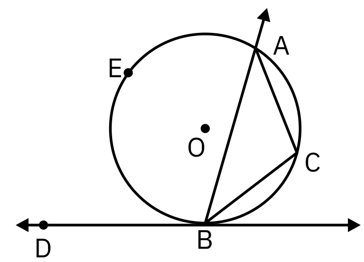
   If AB = CD, then m(arc AXB) = m(arc CYD)



Alternate-Segment theorem

The angle formed by a tangent and secant is half the intercepted arc.

m∠DBA = $\frac{1}{2}$ m(arc BEA) = m∠ACB



Important formulae

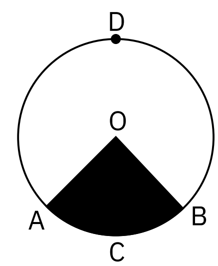
Area of a circle: The area of a circle = πr^2

Length of an arc: The length of the arc is given by the formula:

$\ell = \frac{\theta}{360} \times 2\pi r$

where, θ is the angle subtended at the centre by the arc in degrees. Also, $\ell = r\theta$, where θ is in radians.

Sector of a circle: An angle subtended at the centre of the circle divides the circular region into two parts called the sectors of the circle. The sector bounded by a minor arc is called a minor sector and the sector bounded by a major arc is called a major sector.
Sector (O-ACB) is a minor sector and sector (O-ADB) is a major sector.

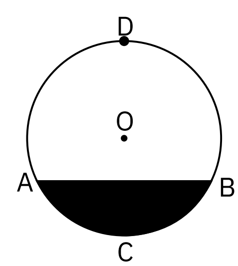


Area of the sector = $\frac{\theta}{360} \times 2\pi r^2$, where θ is the angle subtended at the centre, in degrees.

Area of the sector = $\frac{1}{2} \times r^2\theta$, where θ is in radians.

Area of the sector = Length of arc × $\frac{\text{Radius}}{2}$ = $\ell \times \frac{r}{2}$

Segment of a circle: A chord of the circle divides the circular region into two parts called the segments. The part, which contains the centre of the circle, is called the major segment and the other part is called minor segment.
ACB is a minor segment and ADB is a major segment.



Area of segment = $r^2\left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2}\right]$**, where θ is the measure of** the corresponding arc in degrees.

Note: $\sin\theta$ will be discussed in the chapter on Trigonometry of this module.

Also, Area of minor segment = Area of corresponding sector – Area of triangle formed

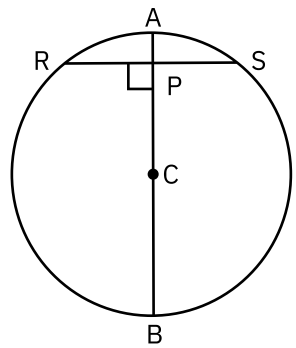
∴ **Area of segment ACB = A(O-ACB) – A(△AOB)**

Area of major segment = Area of corresponding sector + Area of triangle formed

∴ **Area of segment ADB = A(O-ADB) + A(△AOB)**

SOLVED EXAMPLES

Q : In the figure given alongisde, RS = 12 and radius of the circle is 10. If C is the centre of the circle, the find PB.



A : RP = PS = 6

$(CS)^2 = (PC)^2 + (PS)^2$

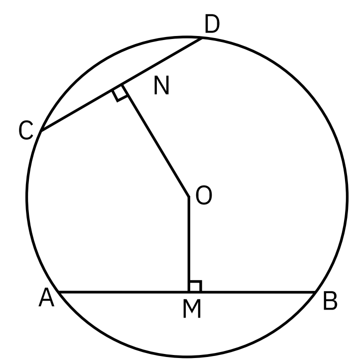
$10^2 = PC^2 + 6^2$

$(PC)^2 = 100 - 36 = 64$

PC = 8

$\therefore$ PB = 8 + 10 = 18

Q : In the figure, AB = 16, CD = 12 and OM = 6. Find ON.



A : MB = $\frac{1}{2}$ × AB = 8 ... (Perpendicular from the centre of the circle bisects the chord)

$\ell(OB)^2 = \ell(OM)^2 + \ell(MB)^2$

$\ell(OB)^2 = 6^2 + 8^2 = 36 + 64 = 100 \therefore \ell(OB) = 10$

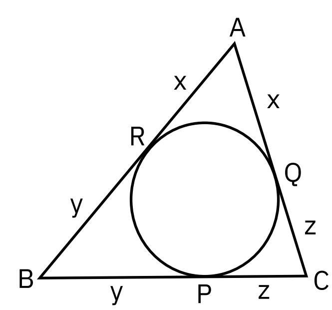
$\ell(OB) = \ell(OD) = 10$ ... (Radii of the same circle)

$\ell(OD)^2 = \ell(ON)^2 + \ell(ND)^2$

$10^2 = (ON)^2 + 6^2$

$\therefore \ell(ON)^2 = 100 - 36 = 64 \qquad \therefore \ell(ON) = 8$

Q : In the figure, ℓ(AB) = 16 cm, ℓ(BC) = 11 cm and ℓ(CA) = 19 cm. Find ℓ(BP), ℓ(CQ) and ℓ(AR).



A : Tangents from external points are equal.

Let, ℓ(AR) = ℓ(AQ) = x; ℓ(BR) = ℓ(BP) = y and ℓ(CP) = ℓ(CQ) = z

x + y = 16 ... (i)

y + z = 11 ... (ii)

z + x = 19 ... (iii)

Adding (i), (ii) and (iii), 2x + 2y + 2z = 46

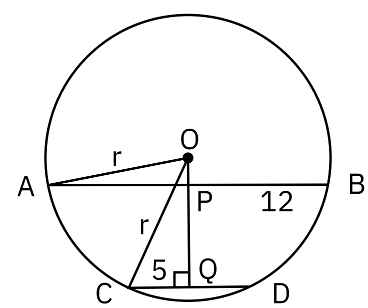
$\therefore$ x + y + z = 23 ...(iv)

Subtracting (i) from (iv), z = 7

Similarly, x = 12 and y = 4.

ℓ(BP) = 4 cm; ℓ(CQ) = 7 cm; ℓ(AR) = 12 cm

Q : Two parallel chords of a circle are 10 cm and 24 cm in length and they are on the same side of the centre. If the distance between them is 7 cm, find the radius of the circle.



A : ℓ(AP) = 12, ℓ(CQ) = 5 (perpendicular from the centre bisects the chord)

$\ell(OQ)^2 = r^2 - 25$ ... (i)

$\ell(OP)^2 = r^2 - 144$ ... (ii)

$\ell(OQ)^2 - \ell(OP)^2 = 119$ ... [subtracting (ii) from (i)]

$\ell(OQ + OP)(OQ - OP) = 119$

$\ell(OQ + OP) \times 7 = 119$

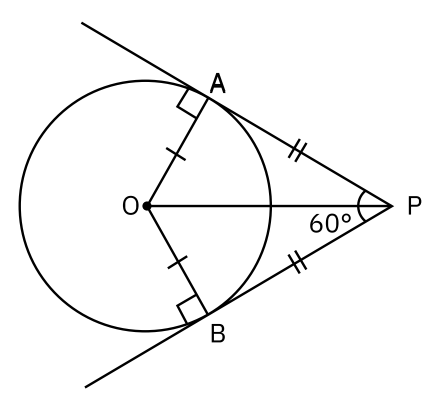
$\therefore$ $\ell(OQ) + \ell(OP) = 17$ and $\ell(OQ) - \ell(OP) = 7$

$\therefore$ $\ell(2(OQ)) = 24$ $\therefore$ $\ell(OQ) = 12$

$\therefore$ $144 = r^2 - 25$ $\therefore$ $169 = r^2$

$\therefore$ r = 13 cm.

Q : Two tangents of length 21 inches from a point P to the circle with centre O are inclined at an angle of 60°. Find the circumference of the circle. $\left(\pi = \frac{22}{7}\right)$



A : $\Delta AOP \cong \Delta BOP$ ... (SSS test of congruence)

$\therefore$ m$\angle$APO = m$\angle$OPB = 30° $\therefore$ ΔOAP is a 30°-60°-90° triangle.

Side opposite to 60° = $\frac{\sqrt{3}}{2}$ of hypotenuse

$21 = \frac{\sqrt{3}}{2} \times OP \therefore OP = \frac{42}{\sqrt{3}}$

$OA = \frac{1}{2} \times OP = \frac{1}{2} \times \frac{42}{\sqrt{3}} = \frac{21}{\sqrt{3}}$

$\therefore$ Circumference $= 2 \times \frac{22}{7} \times \frac{21}{\sqrt{3}} = 44\sqrt{3}$ inches.

Q : In a circle of radius 7 cm a rectangle is inscribed such that its area is maximum. Find the area of the rectangle.

A : The area will be maximum when the rectangle is a square.

The diagonal of the square = Diameter of the circle = 14 cm.

$\sqrt{2}\ \times$ side = 14

$\therefore$ side = $\frac{14}{\sqrt{2}}$ $\quad\therefore$ Area = $\left(\frac{14}{\sqrt{2}}\right)^2$ = 98 sq. cm.

Alternatively

Area of square = $\frac{1}{2} \times (\text{diagonal})^2 = \frac{1}{2} \times 14 \times 14 = 98$ sq. cm.

Q : The circumference of the front wheel of a wagon is 2π ft and that of the back wheel is 3π ft. Find the distance travelled when the front wheel has made 10 more revolutions than the back wheel.

A : Distance travelled in one revolution = Circumference of the wheel

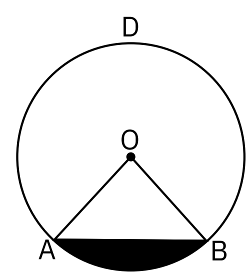
If the back wheel makes n revolutions

Distance travelled = $2\pi \times (n + 10) = n \times 3\pi$

$\therefore\ 2(n + 10) = 3n \qquad \therefore\ n = 20$

$\therefore$ Distance travelled = $20 \times 3\pi = 60\pi$ ft.

Q : Find the radius of the circle which has an arc making an angle of 90° at the centre of the circle and the area of the segment corresponding to this arc is 28.5 sq cm. (sin90° = 1)



A : Area of segment

$= r^2\left[\frac{\pi \times 90}{360} - \frac{1}{2}\right]$

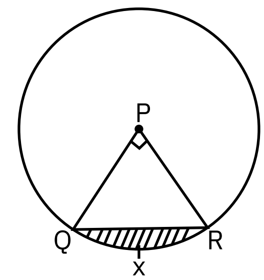
$28.5 = r^2\left[\frac{\pi}{4} - \frac{1}{2}\right] = r^2\left[\frac{\pi - 2}{4}\right] = r^2\,\frac{1.14}{4}$

$\therefore\ r^2 = \frac{4 \times 285 \times 100}{114 \times 10}$

$= \frac{2 \times 285 \times 10}{57} = 2 \times 5 \times 10 = 100$

$\therefore\ r = 10$ cm.

Q : In the figure, P is the centre of a circle of radius 40 cm. If m(arc QXR) = 90°, find the area of the shaded region.



A : m(arc QXR) = 90° $\therefore$ m$\angle$QPR = 90°

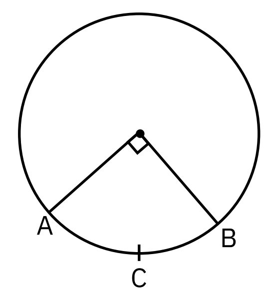
$\therefore$ A(ΔPQR) = $\frac{1}{2}$ × 40 × 40 = 800 sq. cm.

A(P-QXR) = $\frac{\theta}{360} \times \pi r^2 = \frac{90}{360} \times \frac{22}{7} \times 40 \times 40 \approx 1257$ sq.cm.

Area of shaded region = A(P-QXR) − A(ΔPQR)

= 1257 − 800 = 457 sq. cm.

Q : Find the length of the arc ACB if radius of the circle is 7 cm.



A : ℓ(arc ACB) = $\frac{\theta}{360} \times 2\pi r$

= $\frac{90}{360} \times 2 \times \frac{22}{7} \times 7 = 11$ cm.

Concept Builder

1. The circumference of a circle is 16π. Find the radius of the circle.

2. In the figure OM = ON = 3 units
If CD = 8 units
Find:
a) ND b) AB
c) AM d) Radius of the circle

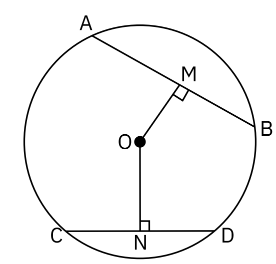
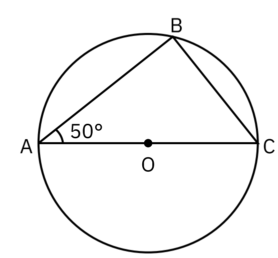


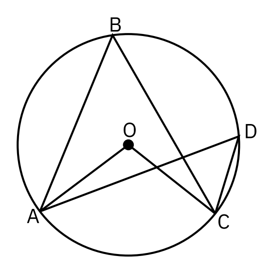
Fig (i)

3. In the figure (i).
If$\angle AOB = 120°$, Find $\angle COD$, $\angle MOB$

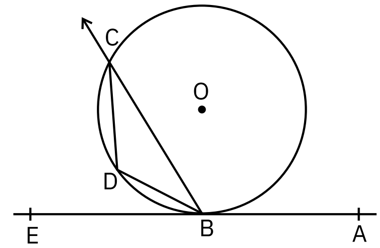
4. In the figure O is the centre of the circle.
If $\angle BAC = 50°$, Find $\angle BCA$



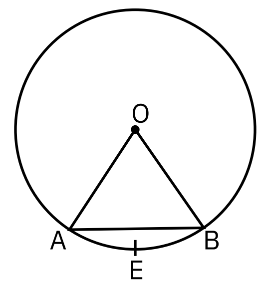
5. In the figure O is the centre of the circle
If $\angle ABC = 25°$. Find $\angle ADC$ and $\angle AOC$



6. If $\angle CBA = 110°$ and $\angle DCB = 30°$
   Find $\angle CDB$, $\angle CBD$, $\angle DBE$



7. AB is a chord of a circle of a radius 14 units. The central angle of the arc is 90°.
   Find:
   a) length of the arc AEB
   b) Area of the sector OAEB
   c) the area of minor segment AEB



Answer Key

1. Radius = 8 units
2. a) ND = 4
   b) AB = 8
   c) AM = 4
   d) Radius = 5
3. $\angle COD = 120°$, $\angle MOB = 60°$
4. $\angle BCA = 40°$
5. $\angle ADC = 25°$, $\angle AOC = 50°$
6. $\angle CDB = 110°$, $\angle CBD = 40°$, $\angle DBE = 30°$
7. a) 22 units
   b) 154 sq. units
   c) 56 sq. units

CLASS EXERCISE

Teaser

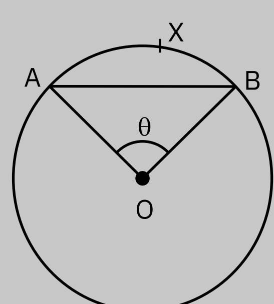
Preeti has a circular birthday cake. While cutting it, she decides to make only 3 straight cuts. What is the maximum number of pieces she can cut the cake into?

Circles

Circles: Basic elements

- Circle is a bounded geometric figure lying in one plane such that the distance of each point on the figure from a fixed point called 'centre' is equal.
- The boundary of the circle is called 'Circumference' (denoted by 'C')
- Line segment joining any point on the circle with the centre of the circle is called 'radius' of the circle (denoted by 'r')
- Line segment joining any two points on the circle is called 'chord' of a circle.
- A chord of the circle passing through the centre of the circle is called 'diameter' of a circle (denoted by 'd').
- Diameter is the longest chord of a circle.
- Diameter of a circle is two times the radius of the circle.
- Each circle has infinite radii and diameters.
- The ratio of the length of the circumference of a circle to its diameter is a constant, denoted by 'π'. Approximately $\pi \approx \frac{22}{7}$ or 3.14.

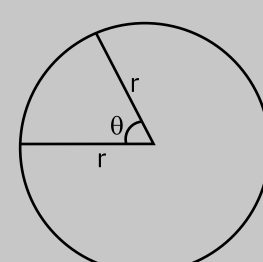
  $C = \pi d = 2\pi r$
- Area of a circle = πr^2
- Any two points on the circumference of a circle divide the circle into two arcs. The longer arc is called 'major arc' and the shorter arc is called 'minor arc'.
- The angle formed at the centre of the circle by joining the end points of an arc is called 'central angle'.



Central angle of minor arc 'AXB' is θ.

Length of an arc with central angle $\theta = \frac{\theta}{360} \times 2\pi r$

Area of a sector with central angle $\theta = \frac{\theta}{360} \times \pi r^2$

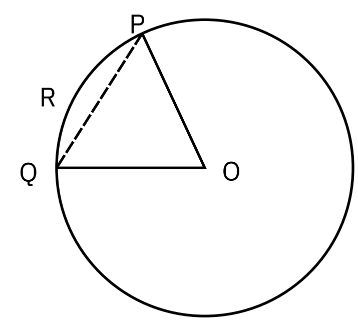


- Area of segment AXB = Area of sector OAXB – Area of ΔOAB

1. Diameter of each wheel of a horse-cart is 1m. If the wheel completes 200 revolutions, what is the distance travelled by the horse-cart?

2. A chariot has two front wheels and two rear wheels. The radius of each rear wheel is 25% more than that of the front wheel. If the front wheels makes 100 revolutions to travel certain distance, how many revolutions will the rear wheels make to cover the same distance?

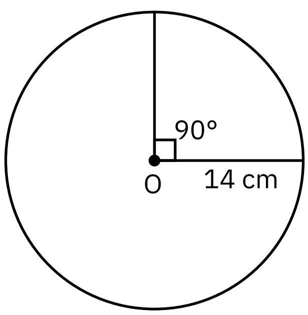
3. The circle shown in the figure has a radius of 3.5 cm. O is the centre of the circle and PQ is a chord such that $\angle POQ = 60°$. Find



   a) The circumference of the circle
   b) The area of the circle
   c) The length of the minor arc PRQ
   d) The area of the sector O-PRQ
   e) * The area of the segment PRQ

   (take $\pi = \frac{22}{7}$ and $\sqrt{3}$ = 1.73 where necessary)

4. Find the area and perimeter of the sector of the circle shown below:



Theorems on chords:

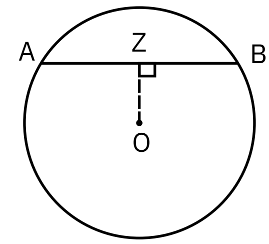
Given a circle with centre O and chords PQ and RS as shown:

- If OA $\perp$ PQ, then l (PA) = l (QA)
  The perpendicular from the centre to a chord bisects the chord
- If l (PQ) = l (RS), then l (OA) = l (OB) (where OA and OB are perpendiculars)
  Equal chords of a circle are equidistant from the centre
- If l (PQ) = l (RS), then $\angle POQ = \angle ROS$
  Equal chords of a circle subtend equal angles at the centre.

For each of the above theorems, the converse is also true. Hence:

- A line joining the midpoint of the chord to the centre is perpendicular to the chord
- Chords of a circle which are equidistant from the centre are equal in length
- Chords which subtend the same angle at the centre are equal in length

5. A circle has radius 10 and centre O. A chord AB is drawn as shown. A perpendicular is dropped from O to AB, meeting AB at point Z.



a) If l(OZ) = 6, find l(AB)

b) If l(AB) = 12, find l(OZ)

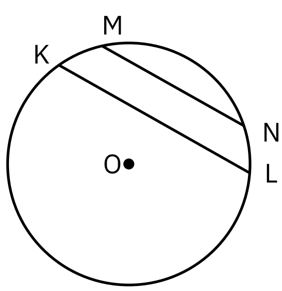
c) If l(OZ) = 5, find ∠AOB

d) If AO ⊥ OB, find l(AB)

6. * In a circle of radius 65, find the distance from the centre of a chord of length:

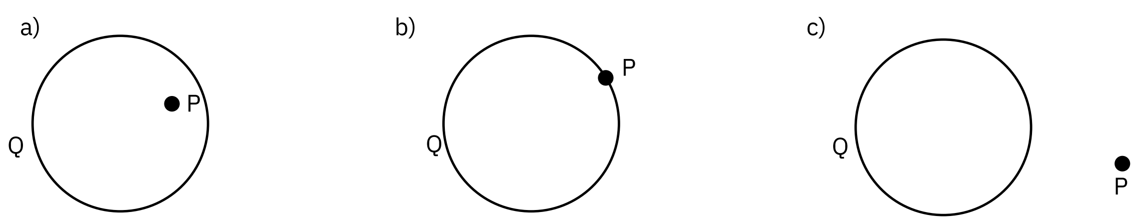
1) 32 2) 50 3) 66 4) 78 5) 126

7. In the adjacent figure, two parallel chords KL and MN are drawn in a circle with centre O and radius 25. If l(KL) = 48 and l(MN) = 40, find the distance between the two chords.

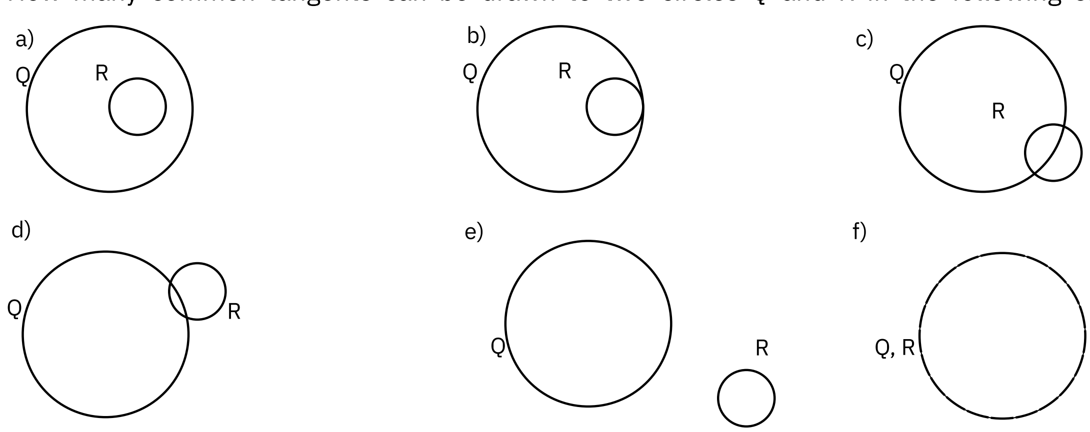


Tangent : If a line touches a circle in one point, that line is said to be a 'tangent' to the circle.
Secant : If a line interescts a circle in two points, that line is said to be a 'secant' to the circle.

8. How many tangents can be drawn from point P to circle Q in the following cases?



9. How many common tangents can be drawn to two circles Q and R in the following cases?

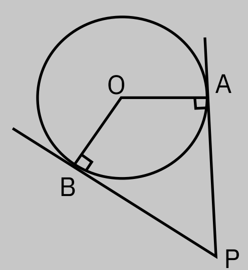


Theorems on tangents:

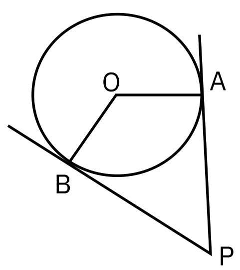
From an external point, two tangent segments can be drawn

Given a circle with centre O and tangents PA and PB:

- OA $\perp$ AP and OB $\perp$ BP i.e. the tangent to a circle is perpendicular to the radius at the point of contact
- Δ OAP $\cong$ ΔOBP. As a result, m$\angle$OAP = m$\angle$OBP and l(PA) = l(PB) i.e. the two tangent segments drawn to a circle from an external point are equal in length.



10. The circle in the adjacent figure has a radius of 5 and is centred at O. From an external point P, tangents PA and PB are drawn (A and B are points on the circle)



a) If l(AP) = 7, how much is l(PB)?

b) If l(AP) = 12, how much is l(OP)?

c) If l(AP) = 5, how much is $\angle$APB?

d) If l(AP) = $5\sqrt{3}$, how much is $\angle$APB?

e) If l(AP) = $\frac{5}{\sqrt{3}}$, how much is $\angle$APB?

11. Two tangents XY and XZ are drawn from a point X to a circle with centre W. If the length of XZ is 156 and the length YZ is 120, find the radius of the circle.

1) 65 2) 75 3) 60 4) 72

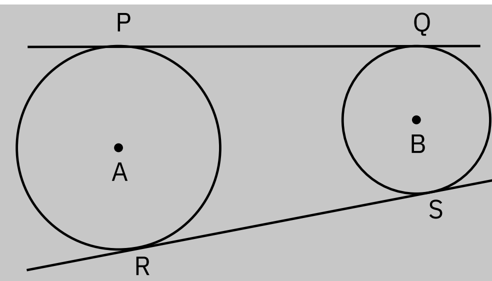
For the two circles with centres A and B, PQ and RS are the direct common tangents, and CD and EF are the transverse common tangents. (Only two of both transverse common tangents and direct common tangents are possible.) Where r_1 and r_2 are the radii of the two circles.

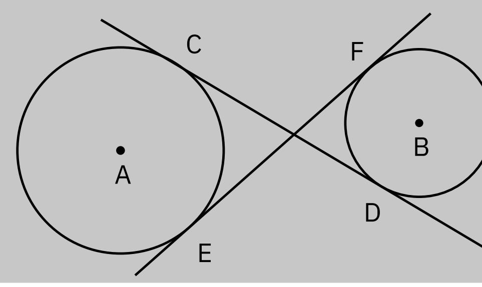
Length of direct common tangent

$= \sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$

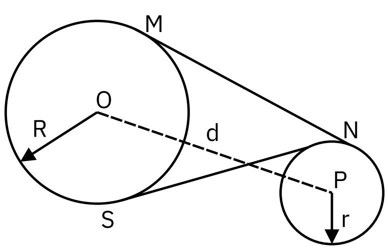
Length of transverse common tangent

$= \sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$

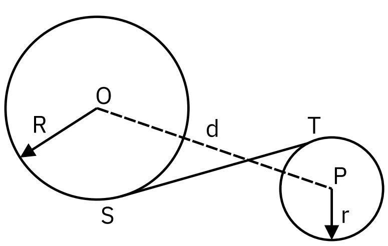




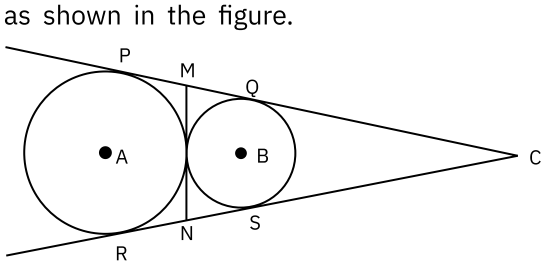
12. In the adjacent figure, find the length of the direct common tangent MN to the two circles with centres O and P and radii 'R' and 'r', given that the distance between their centres is 'd'.



13. * In the adjacent figure, find the length of the transverse common tangent ST to the two circles with centres O and P and radii 'R' and 'r', given that the distance between their centres is 'd'.



14. Two circles with radii 9 and 6 and centres at A and B respectively touch each other externally as shown in the figure.



Two direct common tangents PQ and RS are drawn and they meet at C.

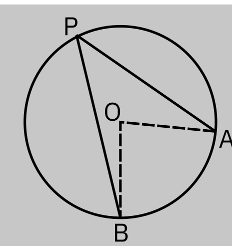
The transverse common tangent MN is also drawn (where M and N are points on PQ and RS respectively)

a) Find l(AC)
b) Find l(PQ)
c) Find the ratio l(MN) : l(PQ)
d) In the region enclosed by the circle centred at B and the two tangents, another circle of the largest possible size is drawn. Find its radius.

15. An ant is trying to reach a drop of honey 4 m away. At 1 and 3 m on the straight line joining the two, there are drops of insect repellant. The and cannot come within 1 m of these. What is the minimum distance it will have to travel to get the honey?

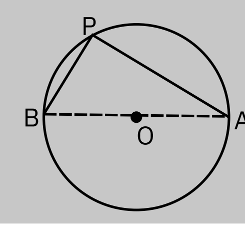
Inscribed Angle Theorem:

- The measure of an inscribed angle is half that of its intercepted arc
  m($\angle$APB) = $\frac{1}{2}$ m(arc AB) = $\frac{1}{2}$ m($\angle$AOB)

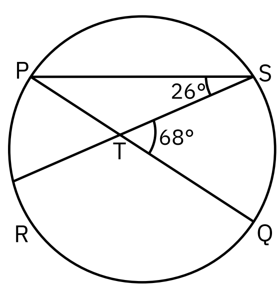


Corollaries of Inscribed Angle Theorem:

- Angles inscribed in the same arc are congruent.
- Angle inscribed in a semi-circle is always a right angle.

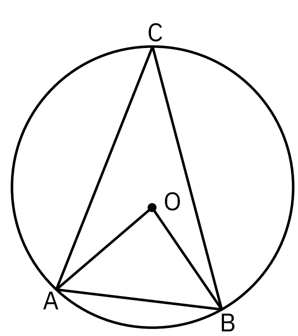


16.



Calculate m(arc SQ)

17.

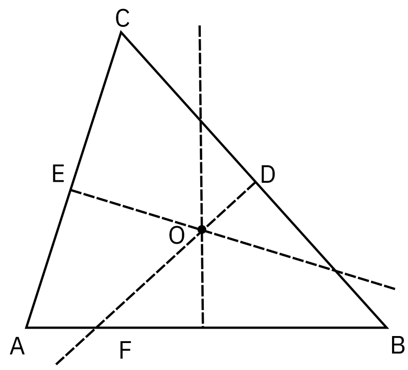


If length of chord AB is equal to the radius of the circle, calculate m$\angle$ACB.

18. Arc ADC is a semicircle and DB ⊥ AC. If AB = 9 and BC = 4, find DB.

19. Find the length of the common chord of the two circles of radii 6 cm and 8 cm with their centres 10 cm apart.

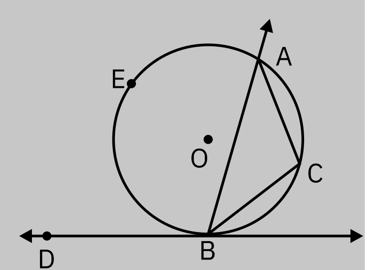
20. In the figure, the perpendicular bisectors of AB, BC and AC meet at point O. If m∠ACB = 70° find m∠AOB



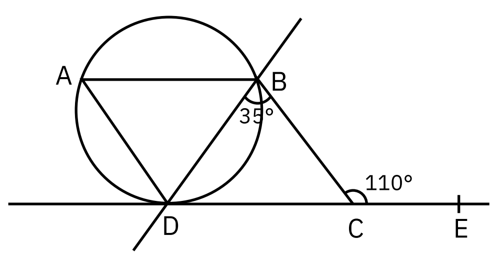
Alternate-Segment theorem

The angle formed by a tangent and secant is half the intercepted arc.

m∠DBA = $\frac{1}{2}$ m(arc BEA) = m∠ACB



21. Line DC is a tangent to a circle touching the circle at D, while DB is a secant, intersecting the circle at points D and B, as shown. Point E is on line DC extended such that D-C-E, as shown. If m∠BCE = 110° and m∠DBC = 35°, calculate m∠DAB, if point A lies on the circumference of the circle.

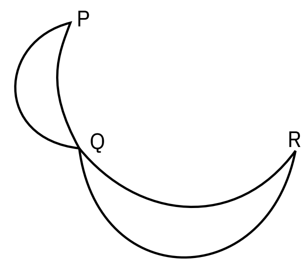


Challengers

1. A circular table is pushed into the corner of a rectangular room in such a way that it just touches two adjacent walls. An ant crawling on the table rim (on the minor arc between the two points of contact) observes that it is 8 inches from one wall and 1 inches from the other. Find the diameter of the table.

   1) 13 inches 2) 5 inches 3) 26 inches 4) either (1) or (2)

2. Three points P, Q and R are taken such that the lengths PQ, QR and PR are 5, 12 and 13 cm respectively. Semicircular arcs are constructed using the segments PQ, QR and PR as diameters (as shown in the figure). Find the area enclosed by the 3 arcs.



   1) 30π cm^2 2) 36 cm^2
   3) 30 cm^2 4) $12\pi - 6\sqrt{3}$ cm^2

3. Two circles of radii 13 and 15 centered at O and P have a common chord MN of length 24. Find the distance between O and P.

   1) 14 2) 13 3) 15 4) Cannot be determined

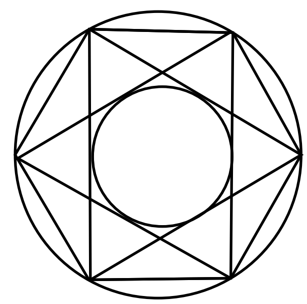
4. A regular pentagon ABCDE is inscribed in a circle. Point Z lies somewhere on the minor arc AB. Find m$\angle$AZB

   1) 120° 2) 126° 3) 144° 4) 150°

5. The two adjacent sides of a quadrilateral are 6 cm and 8 cm long. What is the maximum possible area of the quadrilateral (in sq cm) if it is inscribed in a circle of radius 5 cm?

   1) 45 2) 48 3) 49 4) 50

6. In the figure, a regular hexagon is inscribed in a circle and a smaller circle is inscribed in the hexagon formed by joining its diagonals. If x is the ratio of the area of the bigger circle to that of the smaller circle, find x.



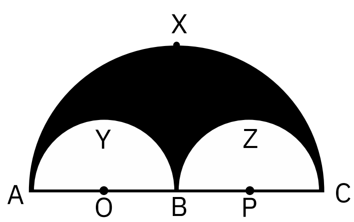
PRACTICE EXERCISE

DIRECTIONS for questions 1 to 19: Choose the correct alternative.

1. The diameter of the rear wheel of a cart is 1.5 m. In travelling a certain distance the rear wheel makes 80 revolutions while the front wheel makes 240 revolutions. Find the diameter of the front wheel.

1) 0.9 m 2) 0.75m 3) 0.5 m 4) 0.66 m

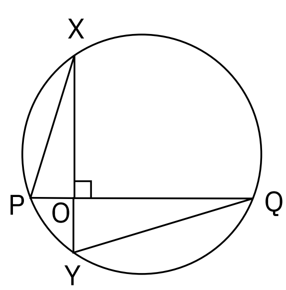
2.



B, O and P are centres of semicircles AXC, AYB and BZC respectively. AC = 14 cm. Find area of shaded region.

1) 77sq.cm 2) 154 sq.cm 3) 57.35sq.cm 4) 38.5 sq.cm

3.



Chords PQ and XY intersect at right angles.

Find m∠QPX + m∠PQY.

1) 60° 2) 90°
3) 120° 4) Cannot be determined

4. If length of chord AB is $\sqrt{3}$ times the length of radius of the circle then what will be the angle inscribed in the arc AB?

1) 60° 2) 120° 3) 60° or 120° 4) None of these

5. A wire of length ℓ cm can form a circle of area 154 cm^2 when joined end to end. If a certain end portion of the wire is cut, it forms a circle of area 38.5 cm^2. Find the length of the portion of the wire which was cut.

1) 11 cm 2) 20 cm 3) 22 cm 4) 44 cm

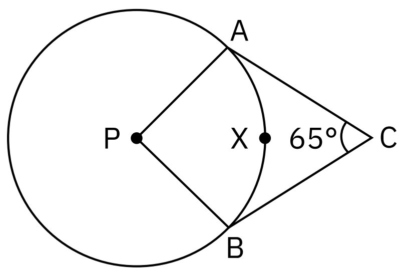
6. In a circle with radius 7 units and centre O, the length of a minor arc is 14 units. The area of the (minor) sector constituted by this arc, is same as the area of sector (O – ACB) of a circle with centre O of radius 14. Find the length of the arc ACB.

1) 7 units 2) 14 units 3) 18 units 4) None of these

7. Two circles with diameters 16 cm and 12 cm touch each other internally. Find the distance between their centres.

1) 10 cm 2) 2 cm 3) 14 cm 4) 3 cm

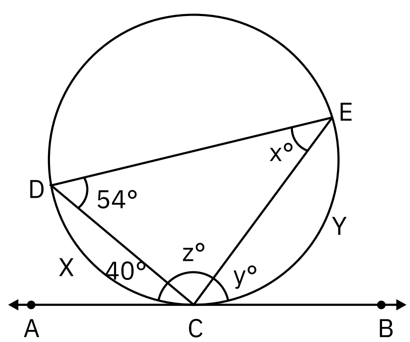
8.



P is the centre of the circle m∠ACB = 65°. Find m(arc AXB).

1) 105° 2) 115° 3) 65° 4) 245°

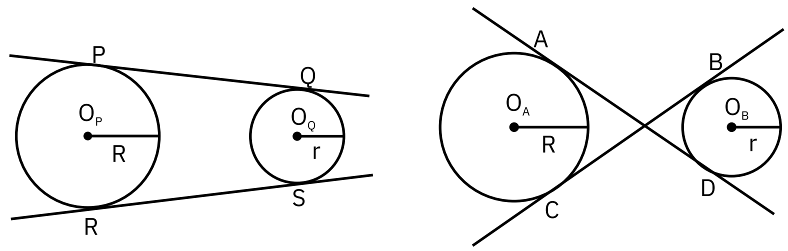
9.



In the given figure, m∠EDC = 54°, m∠DCA = 40°. Find x, y and z.

1) 20°, 27°, 86° 2) 40°, 54°, 86° 3) 20°, 27°, 43° 4) 40°, 54°, 43°

10. For the figures shown below:

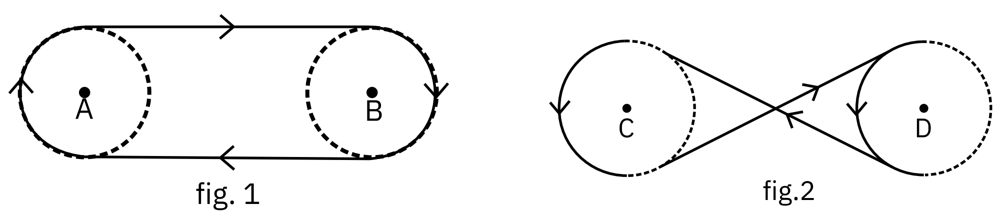


O_PO_Q = O_AO_B = 15

If R = 9 and r = 2, which one of the following relations is true?

1) PQ = 13.3 2) AD = 10.2 3) PQ > AD 4) All of these

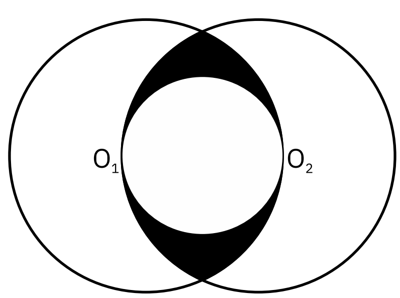
11.



A, B, C and D are wheels of equal diameter. A and B are tied with a rope as in figure 1. C and D are also tied as in figure 2. AB = CD = 2 × diameter of wheels. Two ants are moving on the marked path as in figure 1 and 2. Find the ratio of the distance covered by the ants in one round (i.e. ants travelling paths in figure 1 and 2 respectively). (take π = 3.14)

1) Less than 0.5
2) Between 0.5 and 1
3) Between 1 & 1.5
4) Greater than 1.5

12. Find the approximate area of the shaded region if the two circles are identical and $\ell(O_1O_2)$ = 3 cm. O_1 and O_2 are the centres of the circles (O_1O_2 is the diameter of the circle in the common area of intersection of circles having centres O_1 and O_2) .

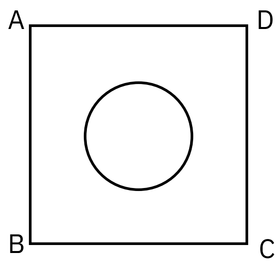


1) 4.5 cm^2 2) 6 cm^2 3) 4.0 cm^2 4) 3.5 cm^2

13. Four 50 paise coins are arranged in a way such that each coin touches two others and the centres of the coins when joined, form a square. What is the area of the square so formed that is not covered by the coins? (The maximum distance between any two centres is $\sqrt{392}$ cm.)

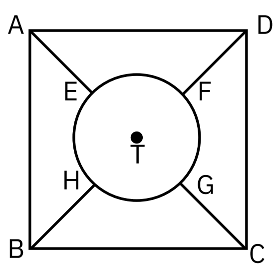
1) 84 cm^2 2) 42 cm^2 3) 14π cm^2 4) 28 cm^2

14. A square gymnasium has a circular swimming pool of area 154 sq. units, at it's centre. A, B, C and D (the corners) are four entrances to the gym. The shortest distance from any of the four entrances to the swimming pool is $10\sqrt{2} - 7$ units. Find the area of the gymnasium, not covered by the swimming pool.



1) 400 sq. units 2) 154 sq. units 3) 246 sq. units 4) None of these

15.



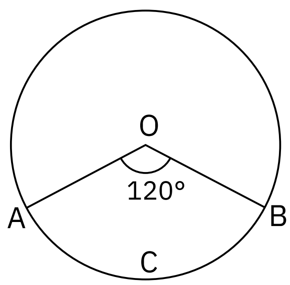
In the figure, square ABCD is divided into 5 equal parts all having same area. 'T' is central part is circular and lines AE, BF, CG, DH lie along the diagonals AC and BD of the square. If ℓ(AB) = 11 cm. Find radius of the circle of central part.

1) $\frac{11}{\sqrt{5\pi}}$ cm 2) $\frac{21}{\sqrt{5\pi}}$ cm 3) $\frac{\sqrt{5\pi}}{11}$ cm 4) $\frac{\sqrt{5\pi}}{21}$ cm

16. If a circle is inscribed in a □ABCD, then which of the following is true?
1) ℓ(AD) + ℓ(BC) = ℓ(AB) + ℓ(CD) 2) ℓ(AD) + ℓ(AB) = ℓ(CD) + ℓ(CB)
3) ℓ(AC) = ℓ(BD) 4) $m\angle D + m\angle B = 180°$

17. Four dogs are tied with chains of equal length such that the points where they are tied, when joined, form a square whose side is twice the length of a chain. If the length of a chain is ℓ units then what is the area enclosed within the square which cannot be accessed by any dog?
1) $(4 - \pi)\ell^2$ sq. units 2) $(2\pi - 4)\ell^2$ sq. units
3) $\left(\frac{2\pi}{3} - 1\right)\ell^2$ sq. units 4) None of these

18. An ant is standing at the centre O of the circle shown here. It needs to reach point C which is exactly midway between A and B. The ant can move only along the circumference of the circle and radii OA and OB. What is the ratio of the shortest distance to the longest distance that the ant can travel in order to reach point C from point O if it is known that it doesn't travel along any stretch twice?



1) 1 : 5 2) 3 : 5 3) 21 : 110 4) 43 : 131

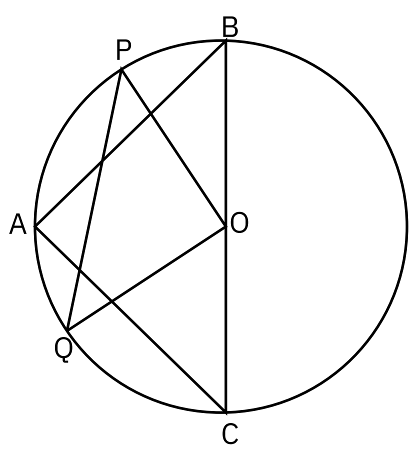
19. A circle has a chord of length 20 cm that subtends an angle of 60° at the centre of the circle. Another circle is drawn with the chord as diameter. Find the approximate area (in cm^2) of the region common to both the circles.

Use $\pi = \frac{22}{7}$ and $\sqrt{3} = 1.73$.

1) 366.67 2) 293.67 3) 193.67 4) 155.67

DIRECTIONS for question 20: Solve as directed.

20. In the given figure, BC is a diameter of the circle with centre O. AB and AC are two equal chords. Also, $\angle POQ = 90°$. If the area of ΔABC = 50 square units, find the area of ΔPOQ (in square units).

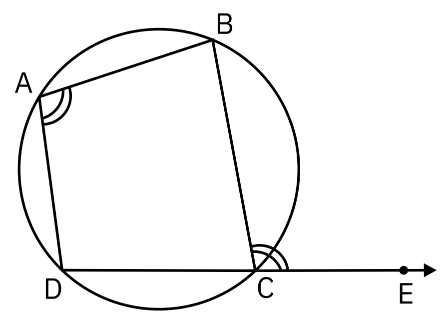


QA-3.5 | CIRCLES - II

THEORY

Cyclic Quadrilateral

A quadrilateral is said to be cyclic if all its vertices lie on a circle. The points lying on a circle are called concyclic.



1. **The opposite angles of a cyclic quadrilateral are supplementary.**

   Conversely, if the opposite angles of a quadrilateral are supplementary then it is a cyclic quadrilateral.

   $m\angle \mathbf{A} + m\angle \mathbf{C} = \mathbf{180°}$

   $m\angle \mathbf{B} + m\angle \mathbf{D} = \mathbf{180°}$

2. An exterior angle of a cyclic quadrilateral is equal to the angle opposite to its adjacent interior angle. **m∠BCE = m∠DAB.**

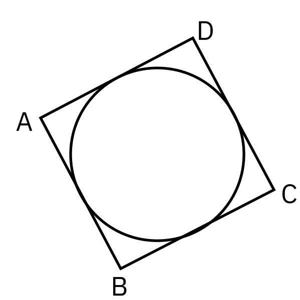
3. **Ptolemy's theorem:**

   In a cyclic quadrilateral the product of diagonals is equal to sum of products its opposite sides

   In the above figure, **AC × BD = AB × CD + AD × BC**

   The converse is also true

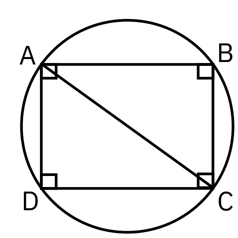
4. If a circle touches all the four sides of a quadrilateral the sum of the two opposite sides is equal to sum of the other two AD + BC = AB + CD



5. **Area of a cyclic quadrilateral** =

   $\sqrt{(s-a)(s-b)(s-c)(s-d)}$; where s is the semiperimeter

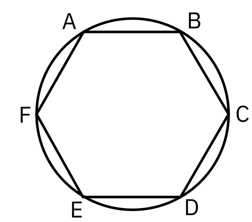
   $s = \dfrac{a+b+c+d}{2}$ where a, b, c and d are the sides of the quadrilateral.



6. Area of a cyclic quadrilateral in which a circle can be inscribed
$= \sqrt{a \times b \times c \times d}$, where a, b, c and d are the sides of the quadrilateral.

7. When a square or rectangle is inscribed in a circle, the diagonal of the square/rectangle is equal to the diameter of the circle.

8. **Cyclic Hexagon:** A regular hexagon is also a cyclic hexagon. The side of the regular hexagon is equal to the radius of the circle.



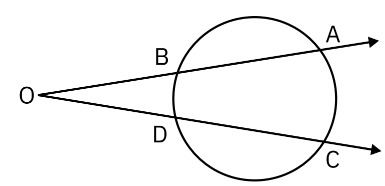
Note: A regular polygon can be inscribed in a circle.

Properties of Secants

A secant is a line, which intersects the circle in two distinct points.

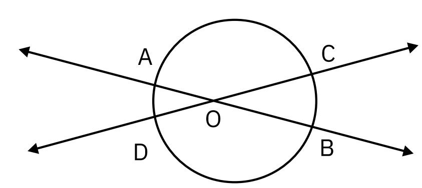
1. If two secants intersect in the exterior of a circle, the angle so formed is equal to half the difference of the measures of the arcs intercepted by them.

m∠AOC = $\frac{1}{2}$[m(arc AC) − m(arc BD)]



2. If two secants intersect in the interior of the circle, the angle so formed is equal to half the sum of the measures of the arcs intercepted by them.

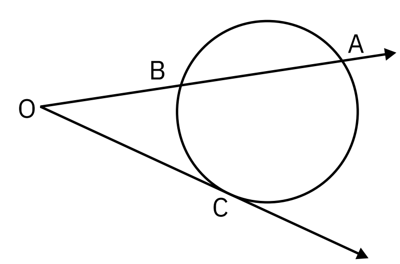
m∠AOC = $\frac{1}{2}$[m(arc AC) + m(arc BD)]



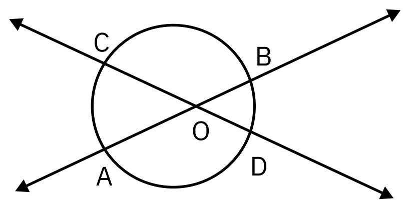
Note: O need not be the origin.

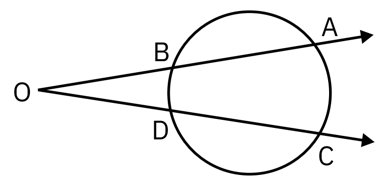
3. If a tangent and a secant intersect in the exterior of a circle, the angle so formed is equal to half the difference of the measure of the arcs intercepted by them.

m∠AOC = $\frac{1}{2}$[m(arc AC) − m(arc BC)]



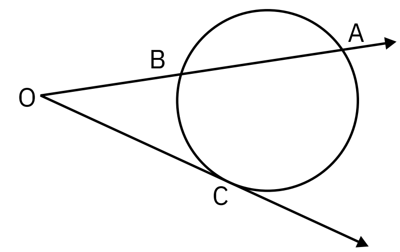
4. If AB and CD are two secants intersecting at point O then, **AO × BO = CO × DO.**





5. If OC is a tangent and OA is a secant intersecting the circle at B, then **OC^2 = OA × OB.**

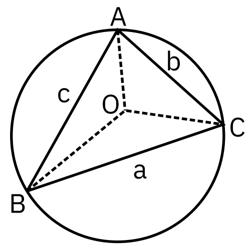
   This is called **Tangent Secant theorem**



Circles and Triangles

Circumcircle: A circle passing through the three vertices of a triangle is called the circumcircle of the triangle. The centre of the circumcircle is called its circumcentre and the radius is called the circumradius.

The circumcentre is also the point of intersection of the perpendicular bisectors of the sides of the triangle.



In the figure, O is the circumcentre and OA, OB and OC are the circumradii of ΔABC

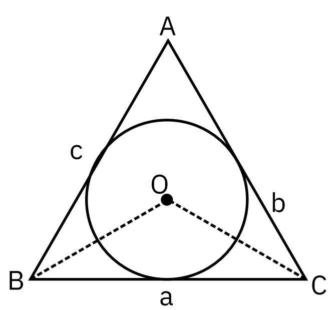
By definition area of triangle:

$A = \frac{abc}{4R}$, where $R \rightarrow$ circumradius and a, b and c are lengths of sides of a triangle.

$\therefore$ circumradius $(R) = \frac{abc}{4A}$

Incircle: A circle touching three sides of a triangle internally is called an incircle of the triangle. The centre of the incircle is called incentre and the radius is called inradius. The incentre is also the point of intersection of the angle bisectors of the triangle.

The three sides of the triangle are tangents to the circle. The angle subtended at the centre of the circle by any side of a triangle is equal to the sum of 90^o and half the measure of the angle opposite to that side.



$m\angle BOC = \frac{1}{2} m\angle A + 90°$

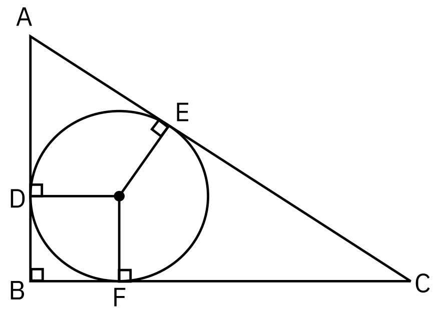
The incentre, O divides the bisector of $\angle A$ in the ratio (b + c) : a where a, b and c are the length of BC, AC and AB.

By definition, area of triangle:

$A = r \times s$, where r = inradius and s = semiperimeter

$\therefore$ Inradius $(r) = \frac{A}{s}$

Incentre of a right angled triangle:



In right angled triangle ABC, the incircle is touching the triangle at D, E, F on AB, AC, BC respectively.
Now, let AD = x, BD = y and CF = z
then AE = x, BF = y, CE = z
Now, inradius, OD = OF = y

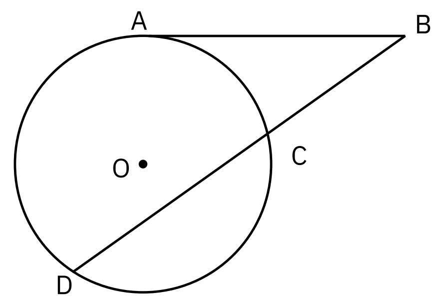
Now AB + BC + AC = 2x + 2y + 2z

$\Rightarrow \dfrac{AB + BC + AC}{2} = x + y + z$

Now, $\dfrac{AB + BC + AC}{2} - AC = (x + y + z) - (x + z) = y =$ inradius

$\therefore$ If the measure of the sides of a **right angled triangle** are given
Inradius = Semiperimeter – hypotenuse

SOLVED EXAMPLES



Q : In a circle with centre 'O' shown alongside, OB = 15 and the radius of the circle is 9.

Find BC × BD.

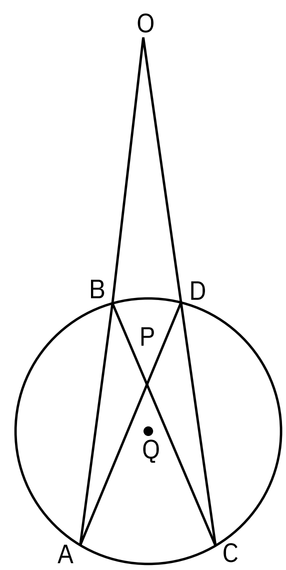
A : $(OB)^2 = (OA)^2 + (AB)^2$

$15^2 = 9^2 + (AB)^2$

$(AB)^2 = 225 - 81 = 144$

$(AB)^2 = BC \times BD$

$\therefore$ BC × BD = 144



Q : In the figure, m∠APC = 45°, m∠AOC = 15°. Find m∠AQC.

A : m∠APC = $\frac{1}{2}$[m(arc AC) + m(arc BD)]

2 × 45° = m(arc AC) + m(arc BD)

m(arc AC) + m(arc BD) = 90° ... (i)

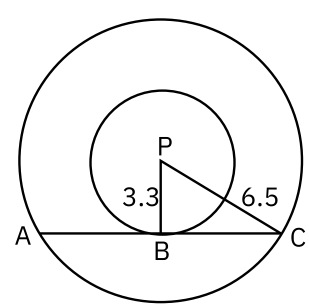
m∠AOC = $\frac{1}{2}$[m(arc AC) − m(arc BD)]

2 × 15° = m(arc AC) − m(arc BD)

m(arc AC) − m(arc BD) = 30° ... (ii)

Adding (i) and (ii),

2m(arc AC) = 120° $\therefore$ m(arc AC) 60° $\therefore$ m∠AQC = 60°



Q : Two concentric circles with centre P have radii 6.5 cm and 3.3 cm. Through a point A of the larger circle, a tangent is drawn to the smaller circle touching it at B. Find ℓ(AC).

A : m∠PBC = 90°

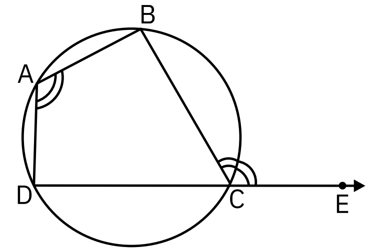
(A tangent is perpendicular to the radius at the point of contact)

$\therefore (6.5)^2 = (3.3)^2 + (BC)^2$

$(BC)^2 = (6.5)^2 - (3.3)^2 = (6.5 + 3.3)(6.5 - 3.3) = 9.8 \times 3.2$

$\therefore$ BC = 5.6 $\therefore$ AC = 2 × 5.6 = 11.2 cm.

Q : Find the exterior angle at point C i.e., $\angle BCE$, of a cyclic quadrilateral ABCD inscribed in a circle such that the m(arc BCD) is 240°.



A : m(arc BCD) = 240°

$\therefore$ $m\angle BAD = \frac{1}{2}$ m(arc BCD) = 120°

$\therefore$ $m\angle BCE = 120°$ ... (exterior angle of a cyclic quadrilateral is equal to the angle opposite to its adjacent interior angle).

Q : In a ΔABC, the sum of $\ell(AB)$ and $\ell(AC)$ is 8 cm. The radius of the incircle drawn in the ΔABC, with centre O, is 2 cm and $\ell(AO)$ is 4 cm. Find the length of the side BC.

A : Let the incircle touch the sides of the triangle AB, AC & BC in points P, Q & R respectively.

Since $OP \perp AB - AP^2 + OP^2 = AO^2$ or $AP^2 + (2)^2 = (4)^2$ or $AP^2 = 12$ or $AP = 2\sqrt{3}$

$\therefore$ $AQ = 2\sqrt{3}$.

Also BP = BR and CQ = CR.

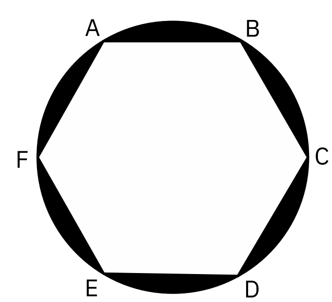
BC = BR + CR = BP + CQ

AB + AC = 8 or AP + PB + AQ + QC = 8

$\therefore$ $BP + CQ = 8 - (AP + AQ) = 8 - 4\sqrt{3}$

$\therefore$ $BC = 8 - 4\sqrt{3}$

Q : A regular hexagon is inscribed in a circle of radius 10 cm. Find the area of the shaded portion. (π = 3.14)



A : Area of shaded region

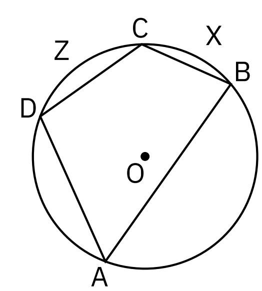
= Area of circle − Area of hexagon

$= \pi r^2 - \frac{3\sqrt{3}}{2} \times (\text{side})^2$

$= 3.14 \times 10 \times 10 - \frac{3\sqrt{3}}{2} \times 10 \times 10$

$= 314 - \frac{3 \times 1.73 \times 100}{2} = 314 - 259.5 = 54.5$ sq. cm.

Q : In the cyclic quadrilateral ABCD, $m\angle BCD = 120°$, m(arc DZC) = 70°, find $\angle DAB$ and m(arc CXB).



A : $m\angle DAB = 180° - 120° = 60°$

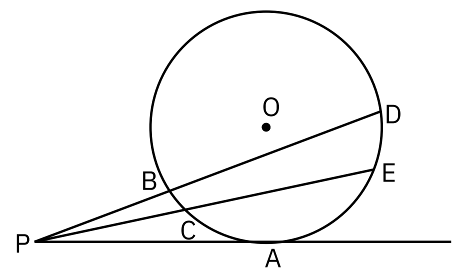
... (opposite angles of a cyclic quadrilateral)

m(arc BCD) = 2 $m\angle DAB$ = 120°

$\therefore$ m(arc CXB) = m(arc BCD) − m(arc DZC) = 120° − 70° = 50°

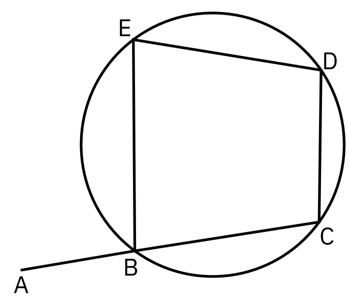
Concept Builder

1. If PA = 6, PB = 3, CE = 5
   Find a) BD b) PE



2. A triangle has sides 24, 10, 26. Find the inradius

3. In the fig ∠D = 95°. Find a) ∠ABE b) ∠EBC



4. Find the product of diagonals of the above cyclic quadrilateral, if EB = 8, BC = 10, CD = 6, ED = 7

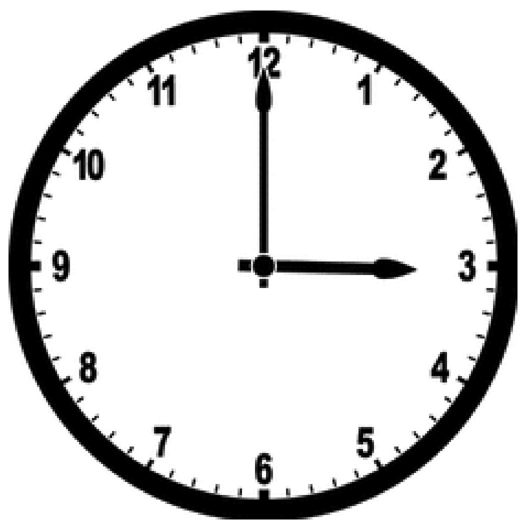
Answer Key

1. BD = 9
   PE = 9
2. 4 units
3. ∠ABE = 95°, ∠EBC = 85°
4. 118

CLASS EXERCISE

Teaser

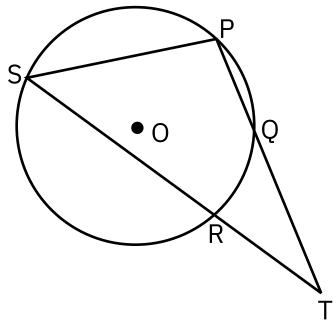
In a clock, a mouse standing at the 12 o'clock position runs in a straight line across the face of the clock to the 7 o'clock position. Then it runs in a straight line from there to the 3 o'clock position. What angle must it have turned through?



Cyclic Quadrilateral

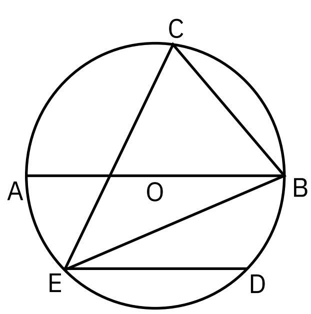
- A quadrilateral is said to be cyclic if all its vertices lie on a circle
- Opposite angles of a cyclic quadrilateral are supplementary
- Exterior angle of a cyclic quadrilateral is equal to the remote interior angle
- Area of a cyclic quadrilateral with sides a, b, c and d = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$ where s is the semi-perimeter = $\frac{a+b+c+d}{2}$

1. PQRS is a cyclic quadrilateral in a circle with centre O. PQ and SR, when extended, meet at T. If $\angle$POR = 100° and $\angle$PTR = 30°, find $\angle$QRS



2. PQRS is a cyclic quadrilateral. Point T lies between points P and S on the circle such that m$\angle$STQ = 50°. Calculate m$\angle$QRS.

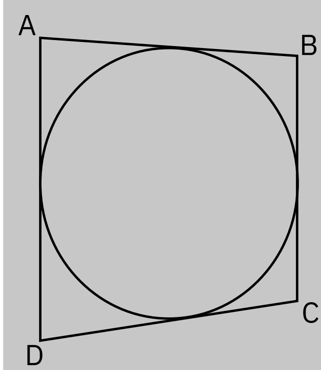
3. In the adjoining figure, chord ED is parallel to the diameter AB of the circle. If $\angle$BCE = 75°, then what is the value of $\angle$DEB?



4. In a circle of radius 28 cm, the largest possible hexagon and the largest possible triangle are inscribed. What is the ratio of their areas?

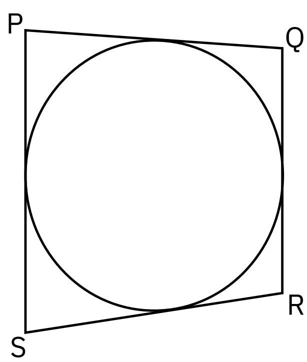
Tangential quadrilateral

If all the four sides of a quadrilateral are tangent to a circle, that quadrilateral is called 'tangential quadrilateral'.



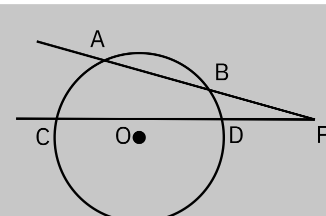
The sum of the lengths of opposite sides of a tangential quadrilateral is constant.

5. In the following quadrilateral, l(PQ) = 8, l(QR) = 10, l(RS) = 12. Calculate l(PS).



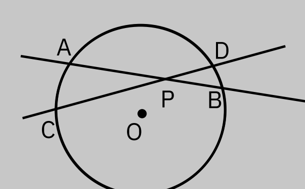
Theorems on Secants:

If two secants AB and CD intersect each other at a point P outside the circle as shown, then



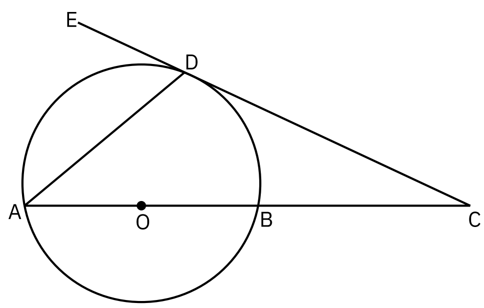
- l(PA) × l(PB) = l(PC) × l(PD)
- $\angle APC = \frac{1}{2}$ m(arc AC) – $\frac{1}{2}$ m(arc BD)

If two secants AB and CD intersect each other at a point P inside the circle as shown, then



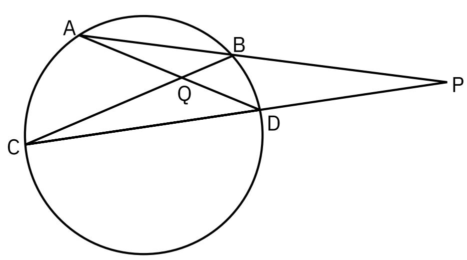
- l(PA) × l(PB) = l(PC) × l(PD)
- $\angle APC = \frac{1}{2}$ m(arc AC) + $\frac{1}{2}$ m(arc BD)

6. In the adjoining figure, AB, a diameter of the given circle, is extended to a point C. A tangent CE is drawn, touching the circle at D. The measure of ∠BAD is 36°. Find the following angles:



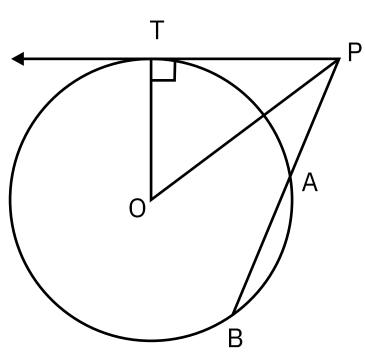
a) ∠ADO	b) ∠DOA	c) ∠DOB	d) ∠DBA	e) ∠ADB
f) ∠ODC	g) ∠BDC	h) ∠DBC	i) ∠BCD	j) ∠ADE

7. In the adjoining figure, secants AB and CD meet at P, a point outside the circle. AD and BC intersect inside the circle, in point Q.



a) If ∠QCD = 15° and ∠QDC = 25°, find ∠BPD
b) If ∠QCD = 15° and ∠QDC = 25°, find ∠AQC
c) If ∠BQD = 60° and ∠BPD = 12°, find ∠ADC
d) If l(AB) = 5, l(BP) = 4 and l(PD) = 3, find l(DC)
e) If l(AB) = 5 and l(BP) = 4, find the length of a tangent drawn from P

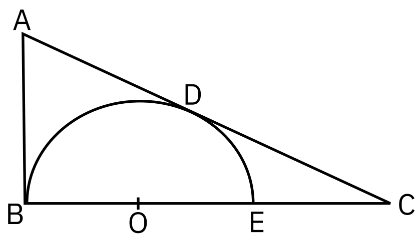
8.



O is the centre of the circle. OP = 7.5 cm, AB = 5 cm and radius OT = 4.5 cm. Find PB.

1) 14 cm. 2) 9 cm. 3) 7 cm. 4) 8 cm.

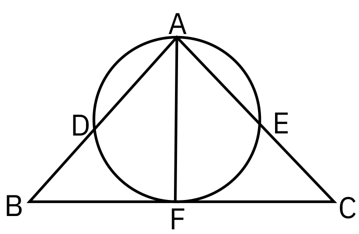
9.



ΔABC is a right angled triangle. A semicircle with centre O is inscribed inside the triangle as shown in the figure. Find A($\square$ABOD) if ℓ(AB) = 30 units and m$\angle$C = 30°.

1) 300 sq. units
2) $\frac{300}{\sqrt{3}}$ sq. units
3) $150\sqrt{3}$ sq. units
4) $300\sqrt{3}$ sq. units

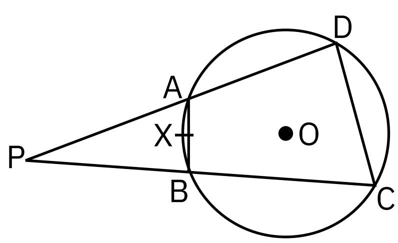
10.



In ΔABC, ℓ(AB) = ℓ(AC) = 16 cm. Points D, E and F are the midpoints of sides AB, AC and BC respectively. Find the ratio of area of $\square$ADFE to the area of ΔABC.

1) 1 : 1
2) 1 : 3
3) 1 : 2
4) 2 : 3

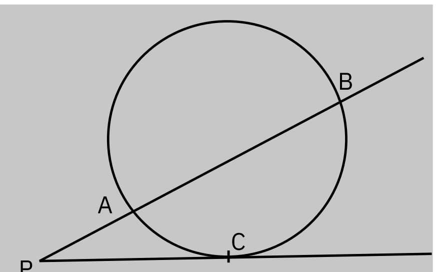
11.



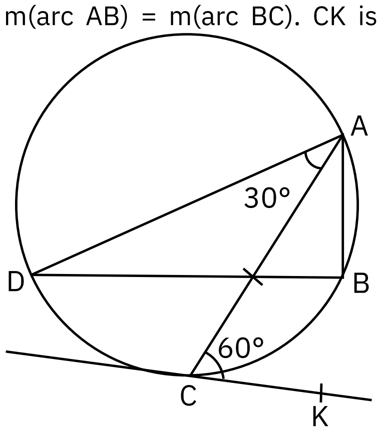
Point P is outside the circle of diameter 14 cm as shown in the figure. m(arc AXB) = 60°, ℓ(DC) = 10 cm, ℓ(PC) = 24 cm, ℓ(PA) = 12 cm and ℓ(PB) = 15 cm. Find the approximate value of A($\square$ABCD).

1) 120 cm^2
2) 131 cm^2
3) 97 cm^2
4) Data insufficient

Tangent-secant theorem: If line PC is a tangent and line PB is a secant to a circle such that P-A-B, then $PC^2 = PA \times PB$

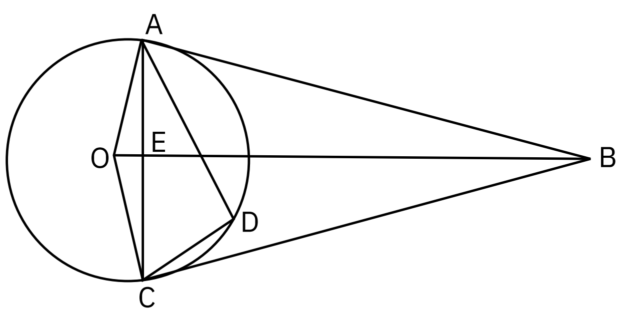


12. In the figure, the diameter of the circle is 8 units.
m(arc AB) = m(arc BC). CK is tangent to the circle at C. What is the length of DB?



1) $2\sqrt{3}$ units 2) $4\sqrt{2}$ units 3) $4\sqrt{3}$ units 4) $6\sqrt{2}$ units

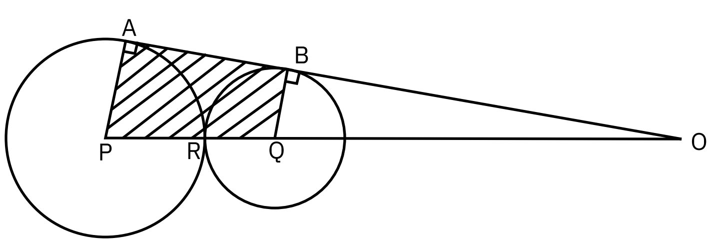
13.



If $\angle ADC$ is 120°, O is the centre of the circle and OE = $\frac{\sqrt{3}}{4}$ and AB and BC are tangents to the circle, what is the length of OB?

1) $\sqrt{3}$ 2) $\frac{\sqrt{3}}{2}$ 3) 1 4) 2

14.

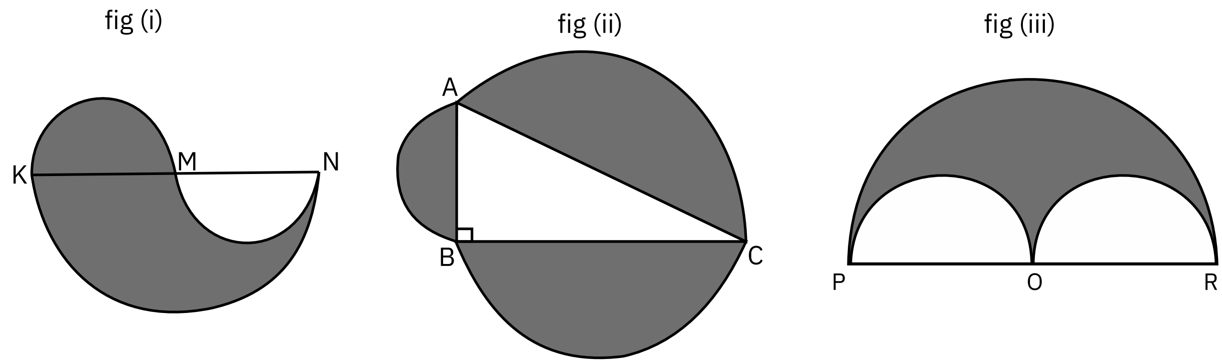


In the figure, two circles with centres P and Q touch each other and the common tangent from O meets the circles at points A and B. The diameters of the circles are in the ratio 5 : 2 and ℓ(OP) = 35 cm. Find the area of the shaded region.

1) $63\sqrt{10}$ cm^2 2) $53\sqrt{10}$ cm^2 3) 90 cm^2 4) $40\sqrt{10}$ cm^2

Circles: Miscellaneous

For questions 15 to 17, refer to the figures below:

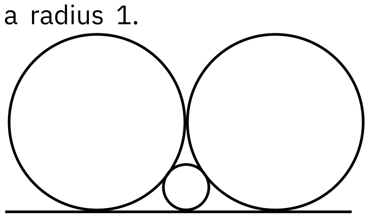


15. Find the area of the shaded region in figure (i). Note that semicircles are drawn with KM, MN and KN as diameters and KM = MN = 4 cm.

16. Find the area of the shaded region in figure (ii). Note that semicircles are drawn with AB, BC and AC as diameters and ABC is a right angled triangle with AB = 3 cm, BC = 4 cm.

17. Find the area of the shaded region in figure (iii). Note that semicircles are drawn with OP, OR and PR as diameters and OP = OR = 4 cm.

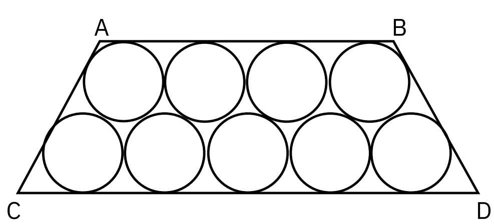
18. In the given figure, find the radius of the smallest circle if the two larger circles each have a radius 1.



19. * A square of side 1 is taken. It is circumscribed by a circle. The circle in turn is circumscribed by another square. This process is repeated till there are 5 squares. What is the side of the largest square?

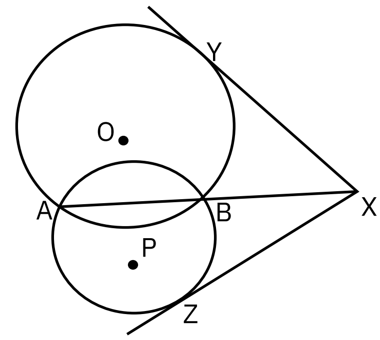
20. * A hexagon of side 1 is taken. It is circumscribed by a circle. The circle in turn is circumscribed by another hexagon. This process is repeated till there are 5 hexagons. What is the side of the largest hexagon?

21. In the adjacent figure, ABCD is an isosceles trapezium. All the circles are of radius = 1 cm and touch the sides of the trapezium. Find the height of the trapezium.



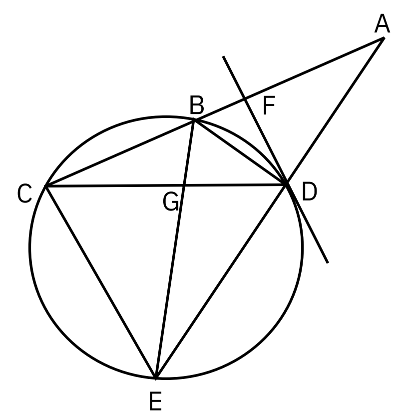
Challengers

1. Two circles with centres O and P and radii R and r respectively (where R > r) intersect in points A and B as shown. AB is extended to a point X. Tangents XY and XZ are drawn to the two circles. Which of the tangents is greater in length?



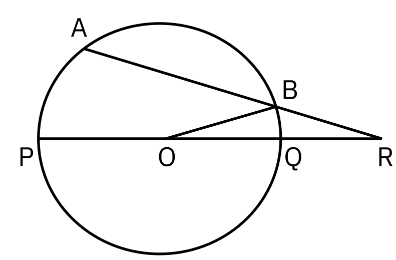
1) XY 2) XZ
3) XY = XZ 4) Can't say

2. Two secants CB and ED of a circle meet in point A outside the circle. DF is a tangent to the circle at D. $\angle BDF = \angle FAD = 30°$. Which of the following statements is necessarily true?



1] G is the centre of the circle
2] BE is perpendicular to CD
3] $\angle CEB$ measures 45°
4] l(AB) + l(BE) = l(AD) + l(DC)

3. In the figure, AB is a chord of the circle with centre O. It is extended to meet the diameter PQ at R, such that l(OB) = l(BR). If $\angle ARP = x°$, and $\angle AOP = kx°$, then what is the value of k?

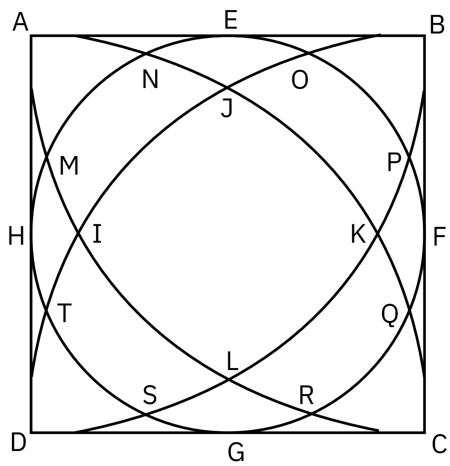


1] 3 2] 4
3] 1 4] 2

4. ABCD is a square of side 4. A semicircle is drawn inside it with diameter CD. A tangent AE is drawn from A to the semicircle touching it at E, where E is a point inside the square. AE is extended to F, where F is a point on side BC. If l(AF) = y, what can be said about y?

1] $3.75 < y \le 4.25$ 2] $4.25 < y \le 4.75$
3] $4.75 < y \le 5.25$ 4] $5.25 < y \le 5.75$

5. * In the adjacent figure, ABCD is a square of side 1. A circle is drawn such that it just touches all the 4 sides of the square (at points E, F, G and H). Also, four circular arcs are drawn with centres A, B, C and D, and radius 1, as shown in the figure.



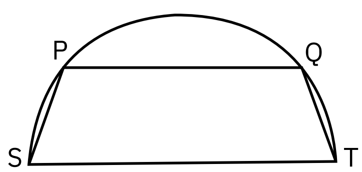
a) Find $\angle CJD$ b) Find $\angle JBK$

c) Find $\angle JDK$ d) Find $\angle IJK$

e) Find $\angle JIK$ f) Find $\angle AJB$

g) Find l(EJ) h) Find l(IK)

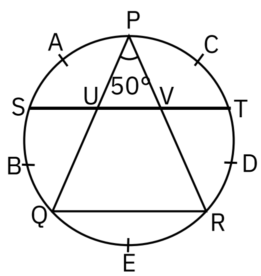
6. In the adjacent figure, PQ is a chord of a semi-circle with diameter ST.
If l(ST) = 9 and l(PS) = l(QT) = 3, find l(PQ).



PRACTICE EXERCISE-1

DIRECTIONS for questions 1 to 13: Choose the correct alternative.

1.

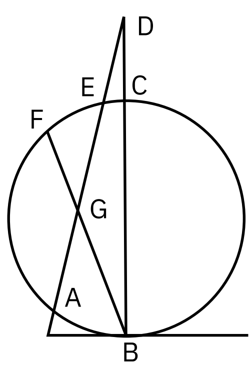


In the given figure, m(arc PAS) = m(arc SBQ), m(arc PCT) = m(arc TDR) and m∠QPR = 50°. Find m∠UVR.

1) 100° 2) 85° 3) 115° 4) Data insufficient

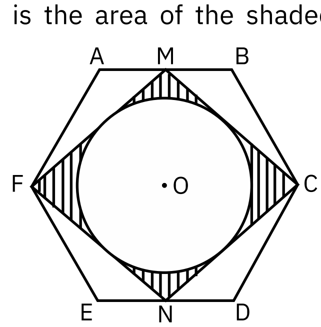
2. A, B, C, E and F are points on the circle as given in the figure.

ℓ(FG) = 5 cm, ℓ(AG) = 8 cm, ℓ(BG) = 24 cm, ℓ(ED) = 7 cm and ℓ(CD) = 5 cm. What is length of BC?



1) 12 cm 2) 24 cm 3) 35 cm 4) 37 cm

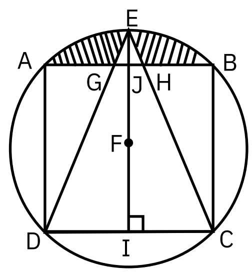
3. ABCDEF is a regular hexagon with sides ‘a’ units. M and N are midpoints of AB and ED. What is the area of the shaded region?



1) $\frac{3}{2}a^2\left(1-\frac{\pi}{6}\right)$ sq. units 2) $\frac{2}{3}a^2\left(1-\frac{\pi}{6}\right)$ sq. units

3) $\left(\sqrt{3}-\frac{7}{16}\pi\right)a^2$ sq. units 4) $3a^2\left(1-\frac{\pi}{4}\right)$ sq. units

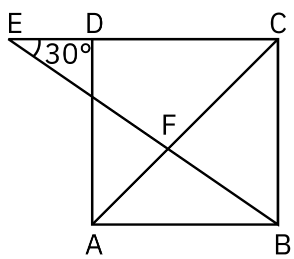
4.



In the figure, □ABCD is a square of side 'a'. What will be the area of shaded region? ℓ(ED) = ℓ(EC) and F is the center of the circle. (Take $\sqrt{2}$ = 1.5)

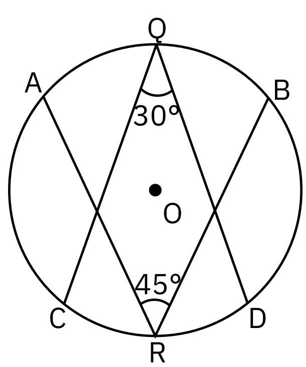
1) $\frac{a^2}{4}(3\pi - 2)$ 2) $\frac{a^2}{8}(\pi - 2)$ 3) $\frac{a^2}{40}(5\pi - 11)$ 4) $\frac{a^2}{3}(2\pi - 1)$

5. In the figure not drawn to scale, □ABCD is a cyclic quadrilateral while ΔABF and ΔCEF are similar triangles. Which of the following, cannot be the value of m∠BAC, if m∠BEC = 30°?



1) 20° 2) 40° 3) 50° 4) 60°

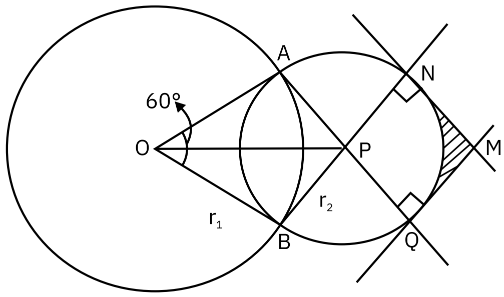
6.



AB and CD are two arcs of the circle with centre O. AC is joined and extended to meet the line, which is an extension of BD at P. Find m∠CPD.

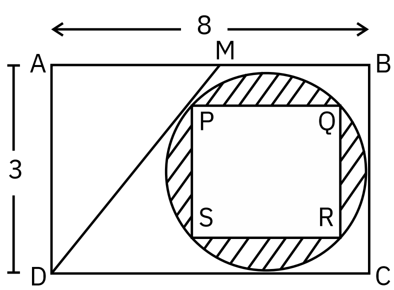
1) 5° 2) 10° 3) 15° 4) 30°

7. Two circles with centres at O and P intersect each other at points A and B such that m∠AOB = 60°, r_1 = 21 units and r_2 = $7\sqrt{3}$ units. NM and QM are tangents to the smaller circle. What is the area of the shaded region?



1) $(126\sqrt{3} - 56\pi)$ sq. units
2) $(147\sqrt{3} - 49\pi)$ sq. units
3) $(126\sqrt{3} - 49\pi)$ sq. units
4) $(147\sqrt{3} - 56\pi)$ sq. units

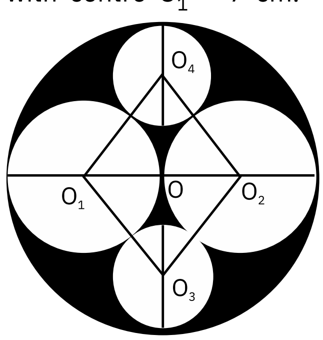
8.



In rectangle ABCD, M is the midpoint of AB. A square PQRS is inscribed in a circle such that it touches MD, BC and DC. What is the area of the shaded region?

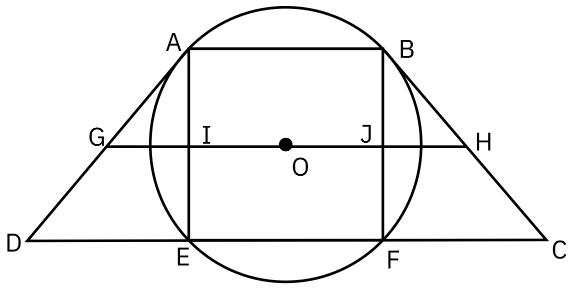
1) $(4\pi - 8)$ sq. units
2) $(4\pi - 6)$ sq. units
3) $(8\pi - 4)$ sq. units
4) $(8\pi - 6)$ sq. units

9. If the areas of circles with centres O_1 and O_2 are equal and that of circles with centres O_3 and O_4 are equal, then calculate the area of the shaded region if the radius of the circle with centre O_1 = 7 cm.



1) 54.4π cm^2
2) 32.5π cm^2
3) 58.9π cm^2
4) None of these

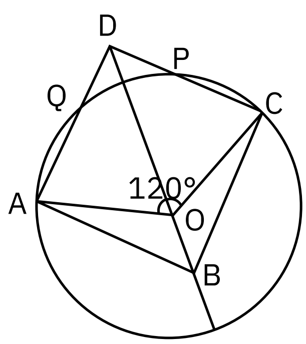
10.



□ABCD is an isosceles trapezium with AB || CD. A circle with centre 'O' lying at the centre of the median of the trapezium as shown in the figure. AE and BF are perpendiculars to DC. If AB = 8 cm, DC = 24 cm and AI = 4 cm. Find the approximate area of the circle enclosed within the trapezium.

1) $16(\pi - 2)$ cm^2
2) $16(\pi + 2)$ cm^2
3) 16π cm^2
4) Data insufficient

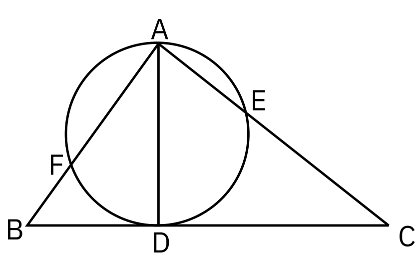
11.



In the given figure, not drawn to scale, m∠AOC = 120°. O is the centre of the circle and □ ABCD is a square. Find ℓ(DP) : ℓ(PC).

1) $\sqrt{2} : (1+\sqrt{3})$
2) $(\sqrt{3}+1) : 3$
3) $1 : (\sqrt{3}-1)$
4) $\sqrt{3} : (1+\sqrt{2})$

12.



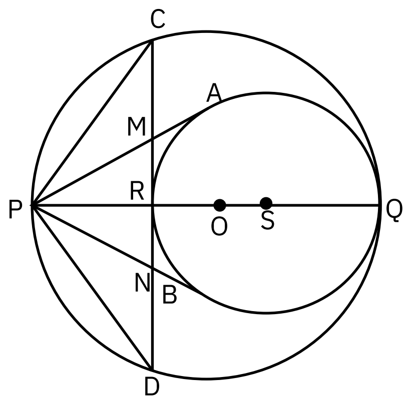
In the given figure, not drawn to scale, BC is tangent to the circle at D and AD passes through the centre of the circle such that ℓ(AD) : ℓ(BD) : ℓ(DC) = $\sqrt{3}$: 1 : 3. Find the ratio of area of △AEF to that of △ABC.

1) 3 : 16
2) 1 : 4
3) 3 : 8
4) 2 : 5

13. Let AB be a diameter of a circle. Point C divides diameter AB in the ratio 1 : 2. Let DE be another diameter of the circle such that DE is perpendicular to AB. What is the ratio of the area of ΔDCE to area of ΔABD?

1) 1 : 2 2) 1 : 4 3) 1 : 6 4) 1 : 3

14. In the given figure, the two circles touch each other internally such that the diameter of the inner circle is two-thirds the diameter of the outer circle. CD, PA and PB are tangents to the inner circle. What is the ratio of the areas of ΔPMN and the smaller circle?



1) $\sqrt{3}\,\pi$ 2) $\frac{\pi}{\sqrt{3}}$ 3) $\frac{\sqrt{3}}{\pi}$ 4) $\frac{1}{\sqrt{3}\,\pi}$

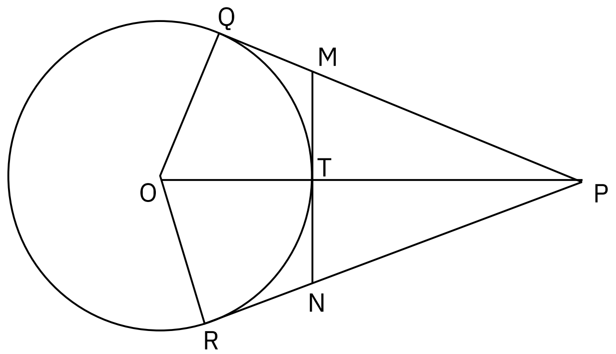
15. AB = 12 cm is a diameter of a circle with centre O. Another circle touches the given circle internally at B and passes through O. If C is a point on the larger circle such that AC is a tangent to the smaller circle at D, then find AC (in cm).

1) $6\sqrt{2}$ 2) 9 3) $8\sqrt{2}$ 4) $3\sqrt{15}$

16. The largest possible square is drawn inside a circle of radius $\sqrt{6}$ units. An equilateral triangle is drawn inside the square with one of the sides of the square as its base. The largest possible circle is drawn inside this equilateral triangle. What is the distance between the centres of the two circles?

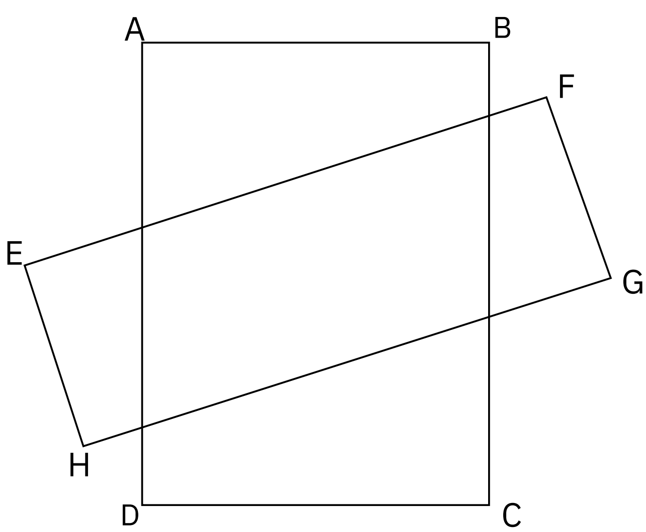
1) $\sqrt{2}+1$ 2) $\sqrt{3}$ 3) $\sqrt{3}-1$ 4) $\sqrt{2}$

17. In the following diagram, two tangent segments PQ and PR of length 12 cm each are drawn from an external point P to a circle of radius 9 cm. O is the center of the circle. Segment MN, which is perpendicular to OP, touches the circle at T, as shown. What is ℓ(MT)?



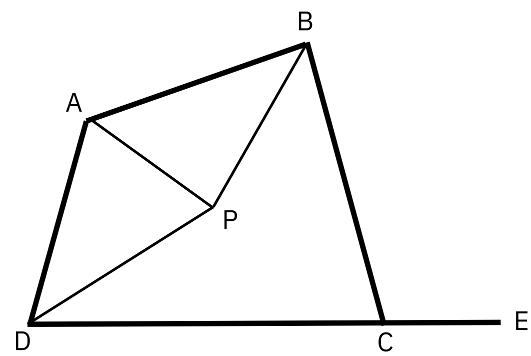
1) 7.5 cm
2) 4.5 cm
3) 6 cm
4) Cannot be determined

18. ABCD and EFGH are two rectangles such that the points A, B, C, D, E, F, G and H all lie on a circle. The ratio of the areas of rectangles ABCD and EFGH is 75 : 42. If AB = 15 and EH = 7, find the area (in square units) of the circle that passes through A, B, C, D, E, F, G and H.



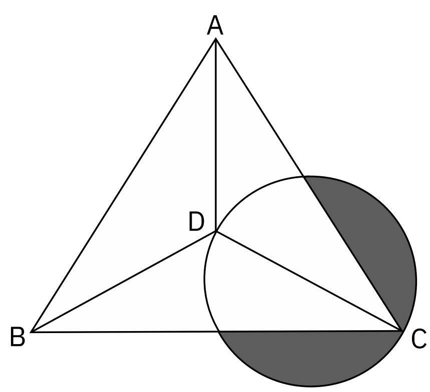
1) $\frac{625}{4}\pi$
2) 100π
3) 144π
4) 625π

19. In the following diagram, point E is on the extended side DC of the quadrilateral such that m∠DAB = m∠BCE = 76°. Point P lies in the interior of the quadrilateral such that ℓ(PA) = ℓ(PB) = ℓ(PD) = 4 cm. What is ℓ(PC)?



1) $4\sqrt{3}$ cm
2) 2 cm
3) 4 cm
4) More information is needed to answer this question

20. In the given figure, the angle bisectors of all the three angles of the equilateral triangle ABC meet at D. A circle is drawn with CD as diameter. Find the area (in square units) of the shaded region if the length of each side of ΔABC = $14\sqrt{3}$ units.

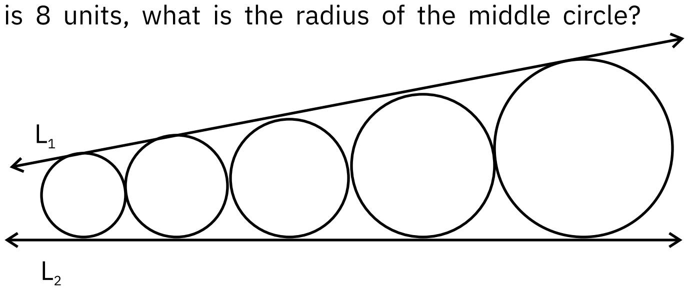


1) $\frac{49}{2}\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$ 2) $49\left(\frac{2\pi}{3}-\sqrt{3}\right)$ 3) $49\left(\frac{2\pi}{3}-\frac{1}{2}\right)$ 4) $49\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$

PRACTICE EXERCISE-2

DIRECTIONS for questions 1 to 3: Choose the correct alternative.

1. In the following figure, five circles are adjacent to one another and have lines L_1 and L_2 as common tangents. If the radius of the largest circle is 18 units and that of the smallest circle is 8 units, what is the radius of the middle circle?

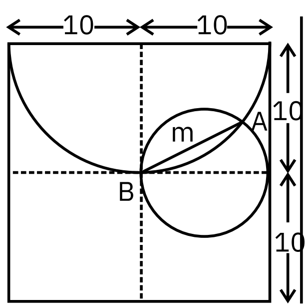


1) 11 units 2) 11.5 units 3) 12 units 4) 12.5 units

2. Find the ratio of the area of the largest triangle that can be inscribed in a circle of radius 7 cm to the area of the largest triangle that can be inscribed in a semicircle of radius 7 cm.

1] 3 : 4 2] $\sqrt{3}$: 4 3] $3\sqrt{3}$: 2 4] None of these

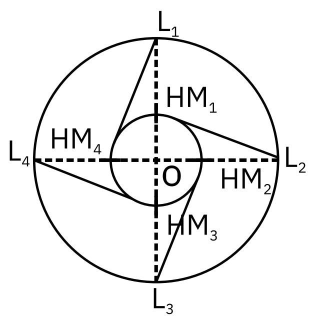
3. AB is the line joining the points of intersection of a semicircle and a circle as shown in the figure. ℓ(AB) = m. Find the area of a circle with radius AB.



1) 40π sq. units 2) 80π sq. units 3) 120π sq. units 4) 160π sq. units

DIRECTIONS for questions 4 and 5: Refer to the data below and answer the questions that follow.

In a village, there are two circular and concentric ring roads as shown. The inner ring road has a circumference equal to half that of the outer ring road. The village has four headmen (HMs) and four chiefs (Ls). The positions of their houses are marked on the figure.
A vehicle can move at the rate of 40π kmph on the inner ring road, 60π kmph on the outer ring road and $30\sqrt{5}$ kmph on the chord roads.



4. I am in L_3's house and want to go to HM_1's house. If I take the route from L_3 to L_2 along the outer ring road and then take the chord road from L_2 to HM_1, I will take a total of 3 hours to reach my destination. What is the radius of the outer ring road in km?

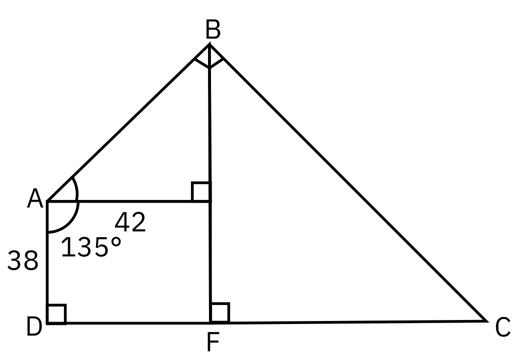
   1) 30 km 2) 40 km 3) 60 km 4) 80 km 5) 120 km

5. A car wants to reach HM_2 from L_1 using first, the chord L_1HM_4 and then, the inner ring road. How many minutes will it take? (Use the data from the previous question)

   1) 120 min 2) 90 min 3) 180 min
   4) 210 min 5) None of these

DIRECTIONS for questions 6 to 20: Choose the correct alternative.

6. Mohanlal has a field having dimensions as given in the figure. He has four horses and four ropes of lengths 12, 16, 24, 28 metres. What can be the minimum area which will remain ungrazed, if they are tied to the corners A, B, C and D using the four ropes such that no two horses ever come in contact?



   1) $(5678 - 520\pi)$ m^2 2) $(5678 - 480\pi)$ m^2
   3) $(5678 - 314\pi)$ m^2 4) $(5678 - 494\pi)$ m^2

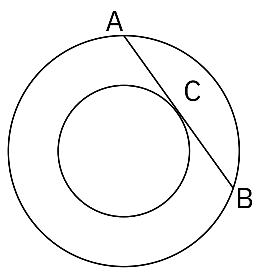
7. A circle is inscribed in a given square and another circle is circumscribed about the square. What is the ratio of the area of the inscribed circle to that of the circumscribed circle?

1) 2 : 3 2) 3 : 4 3) 1 : 4 4) 1 : 2

8. A one-rupee coin is placed on a table. The maximum number of similar one-rupee coins which can be placed on the table, around it, with each one of them touching it and only two others is _____.

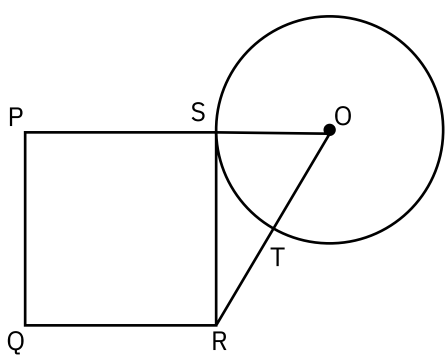
1) 8 2) 6 3) 10 4) 4

9. The line AB is 6 m in length and is tangent to the inner one of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer is:



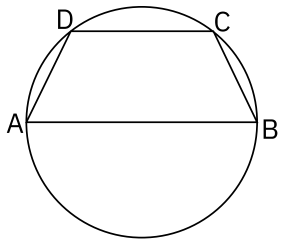
1) 5 m 2) 4 m 3) 6 m 4) 3 m

10. PQRS is a square. SR is a tangent at point S to the circle with centre O and TR = OS. Then, the ratio of area of the circle to the area of the square is _____.



1) $\frac{\pi}{3}$ 2) $\frac{11}{7}$ 3) $\frac{3}{\pi}$ 4) $\frac{7}{11}$

11. In the given figure, AB is diameter of the circle and the points C and D are on the circumference such that $\angle CAD = 30°$ and $\angle CBA = 70°$.



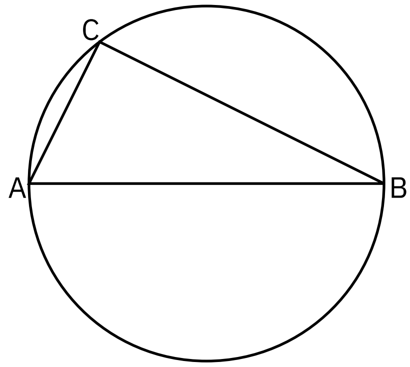
What is the measure of $\angle ACD$ in degrees?

1) 40 2) 50 3) 30 4) 90

12. From a circular sheet of paper of radius 20 cm, four circles, each of radius 5 cm are cut out What is the ratio of the areas of the uncut to the cut portion of the sheet?

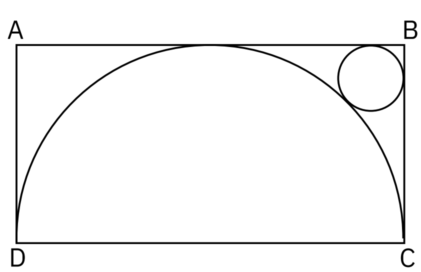
1) 1 : 3 2) 4 : 1 3) 3 : 1 4) 4 : 3

13. The figure shows a circle of diameter AB and radius 6.5 cm. If chord CA is 5 cm long, find the area of the triangle ABC.



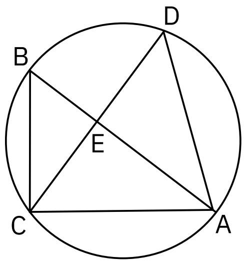
1) 60 sq. cm 2) 30 sq. cm 3) 40 sq. cm 4) 52 sq. cm

14. The figure shows the rectangle ABCD with a semi-circle and a circle inscribed inside it. What is the ratio of the area of the circle to that of the semi-circle?



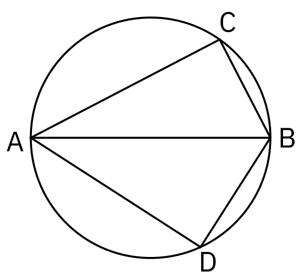
1) $(\sqrt{2} - 1)^2$ 2) $2(\sqrt{2} - 1)^2$ 3) $\frac{(\sqrt{2} - 1)^2}{2}$ 4) None of these

15. In the adjoining figure, points A, B, C and D lie on the circle. AD = 24 and BC = 12. What is the ratio of the area of the triangle CBE to that of the triangle ADE.



1) 1 : 4 2) 1 : 2 3) 1 : 3 4) Insufficient data

16. AB is the diameter of the given circle, while points C and D lie on the circumference as shown. If AB is 15 cm, AC is 12 cm and BD is 9 cm, find the area of the quadrilateral ACBD.

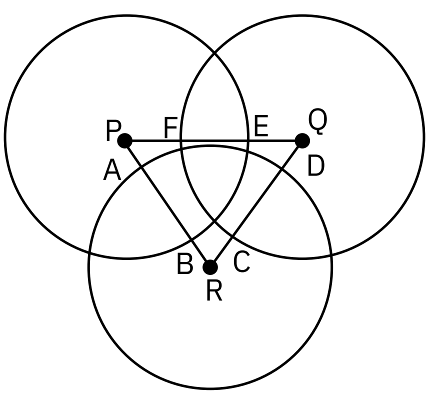


1) 54 sq. cm 2) 216 sq. cm 3) 162 sq. cm 4) None of these

17. The sum of the areas of two circles which touch each other externally is 153π. If the sum of their radii is 15, find the ratio of the larger to the smaller radius.

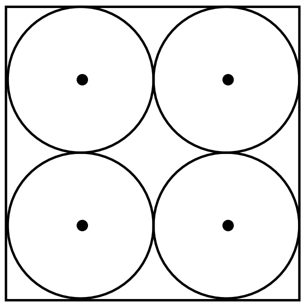
1) 4 : 1 2) 2 : 1 3) 3 : 1 4) None of these

18. Three circles, each of radius 20 cm and have their centres at P, Q and R. Further, AB = 5 cm, CD = 10 cm and EF = 12 cm. What is the perimeter of the triangle PQR?



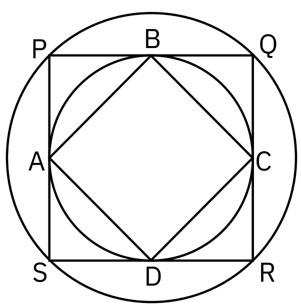
1) 120 cm 2) 66 cm 3) 93 cm 4) 87 cm

19. Four identical coins are placed in a square. For each coin the ratio of numerical value of area (in sq. units) to numerical value of circumference (in units) is the same as the ratio of numerical value of circumference (in units) to numerical value of area (in sq. units). Find the area of the square that is not covered by the coins.



1) $16(\pi - 2)$ 2) $16(\pi - 2)$ 3) $16(4 - \pi)$ 4) $16\left(4 - \frac{\pi}{2}\right)$

20. The figure below shows two concentric circles with centre O. PQRS is a square, inscribed in the outer circle. It also circumscribes the inner circle, touching it at points B, C, D and A. What is the ratio of the perimeter of the outer circle to that of polygon □ABCD?



1) $\frac{\pi}{4}$ 2) $\frac{3\pi}{2}$ 3) $\frac{\pi}{2}$ 4) π

QA-3.6 | 3-DIMENSIONAL FIGURES AND MENSURATION

THEORY

Definition of Terms

Mensuration deals with the measurement of areas and volumes of plane and solid figures.

The areas of plane figures have been covered in previous chapters.

Solids: Solids are three-dimensional objects, bound by one or more surfaces. When plane surfaces bound a solid, they are called its **faces**. The lines of intersection of adjacent faces are called its **edges**. The points of intersection of the edges are called **vertices**.

Euler's Formula

For any regular solid,

Number of faces + Number of vertices = Number of edges + 2

This formula is called **Euler's formula**.

However this does not hold for a solid with curved surface.

Different solids and their areas and volumes

For solids, two different types of areas are defined:

1. Lateral Surface Area **(L.S.A)** or Curved Surface Area **(C.S.A)** and
2. Total Surface Area **(T.S.A)**.

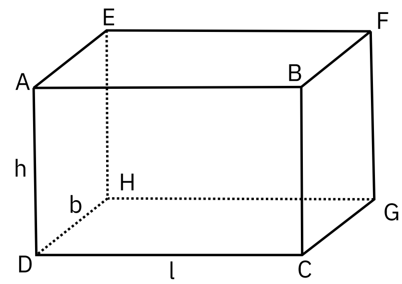
Lateral surface area is the area of the vertical faces of the solid. In solids, where the lateral surface is curved, the lateral surface is referred to as the Curved Surface Area.
Total surface Area = Lateral/Curved S.A + Area of top face + Area of Bottom face

Volume of a solid figure is the amount of space enclosed by its bounding surfaces. Volume is measured in cubic units.

Note:
- Weight = Volume × Density.
- 1cu. m = 100 × 100 × 100 = 10^6 cu. cm.
- If a solid is melted into another solid, volume remains the same.

Rectangular Parallelopiped or CUBOID: A rectangular parallelopiped is bound by 6 rectangular faces. The opposite faces of a rectangular solid are equal rectangles lying in parallel planes.

A cuboid has,
8 vertices i.e., A, B, C, D, E, F, G and H.
12 edges i.e., AE, AB, AD, BF, BC, FE, FG, EH, DH, DC, CG and GH.
2 horizontal faces; ☐AEFB, ☐DHGC.
4 vertical faces; ☐AEHD, ☐ABCD, ☐BFGC and ☐EFGH.
4 body diagonals; DF, EC, AG and HB.
DC is the length of the cuboid (ℓ).
DH is the breadth of the cuboid (b).
DA is the height of the cuboid (h).

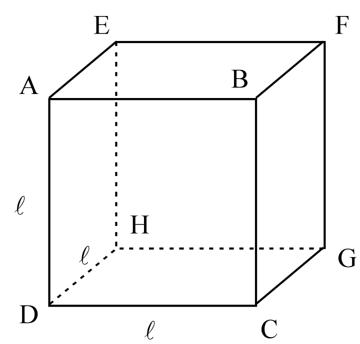


Surface area of vertical faces of cuboid = perimeter of base × height = $2(b + \ell)\ h$
T.S.A of cuboid = L.S..A + 2 × Base Area = $2(b + \ell)h + 2 \times \ell \times b = 2[\ell \times b + b \times h + h \times \ell]$
Volume of cuboid = Area of base × height = $\ell \times b \times h = \ell bh$
Length of base diagonal, AF = $\sqrt{\ell^2 + b^2}$
Length of body/ longest diagonal, AG = $\sqrt{\ell^2 + b^2 + h^2}$

CUBE: A cube is a special case of a parallelopiped in which the length, breadth and height are equal i.e., it is bound by six square faces.

Surface area of vertical faces of a cube = $4\ell^2$
Total surface area of a cube = $6\ell^2$

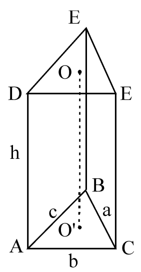
Volume of a cube = Area of base × height
= $\ell^2 \times \ell = \ell^3$
Body diagonal = $\sqrt{3}\ \ell$



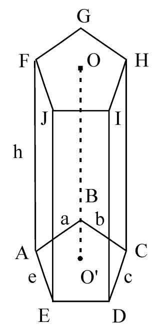
Right PRISM: A prism is a solid, whose top and bottom faces are identical polygons and parallel. Their vertical faces are rectangular.

A prism is said to be triangular prism, quadrilateral prism pentagonal prism, hexagonal prism, octagonal prism according to the number of sides of the polygon that form the base.
In a prism with a base of n sides, **Number of vertices = 2n**, **Number of faces = n + 2.**

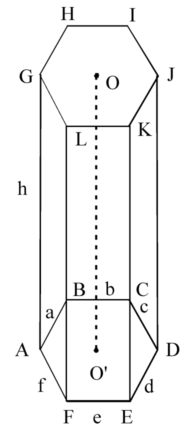
Cube and Cuboids are prisms with a square and rectangle as a base respectively



Triangular Prism



Pentagonal Prism



Hexagonal Prism

In the figure above, OO' is the perpendiclar height (h) of the prism

Surface area of vertical faces of a prism = perimeter of base × height (h)
Total surface area of a prism = perimeter of base × height + 2 × area of base
Volume of a prism = area of base × height.

Example

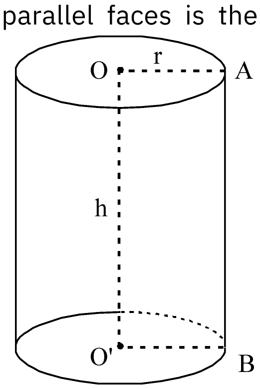
In a equilateral triangular prism, the side of base is equal to 6 units. The height of the prism is 8 units. Find the volume, L.S.A. and T.S.A. of the prism.

Lateral Surface Area = Base perimeter × height
= (3 × 6) × 8
= 144 sq. units
Total Surface Area = L.S.A + 2 × Area of base
= $144 + 2 \times \frac{\sqrt{3}}{4} \times 6^2$
= $144 + 18\sqrt{3}$ sq.units
Volume = Area of base × height
= $18\sqrt{3} \times 8$
= $144\sqrt{3}$ cubic. units

Right Circular CYLINDER: The base and upper face of a right circular cylinder are equal circular regions lying in parallel planes. The perpendicular distance between these parallel faces is the height of the cylinder.

If r is the radius of the base and h is the height of the cylinder

The vertical surface of the cylinder is a curved surface.



C.S.A of a cylinder = perimeter of base × height = $2\pi rh$
T.S.A of a cylinder = C.S.A + 2 × B.ase Area = $2\pi rh + 2 \times \pi r^2 = 2\pi r(r + h)$
Volume of a cylinder = Area of base × height = $\pi r^2 h$
Volume of material of a hollow cylinder = $\pi(R^2 - r^2)h$
where, R is the outer radius and r is the inner radius of the cylinder.

Example

The radius of a right circular cylinder is increased by 50%. Find the percentage increase in volume.

Let the original radius be r and height be h.

$\therefore$ New radius = 1.5r

Percentage Increase in volume = $\frac{\text{Inscrease in volume}}{\text{Original volume}} \times 100$

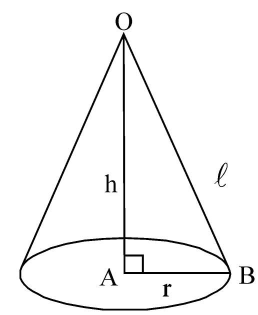
$= \frac{\pi(1.5r)^2h - \pi r^2 h}{\pi r^2 h} \times 100 = \frac{2.25\pi r^2 h - \pi r^2 h}{\pi r^2 h} \times 100$

$= \frac{\pi r^2 h(2.25 - 1)}{\pi r^2 h} \times 100 = 1.25 \times 100 = 125.$

Right Circular CONE: The base of a right circular cone is a circular region.

A is the centre of the circle and OA $\perp$ AB where O is the vertex of the cone and AB is the radius (r).

OA is the perpendicular height (h) of the cone (i.e., the segment joining the vertex of the cone and the centre of the base). OB is the slant height (ℓ) of the cone (i.e., a segment joining the vertex of the cone and any point on the circumference of the base).



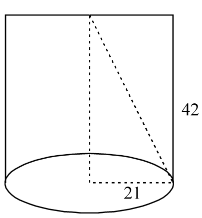
Slant height, $\ell = \sqrt{r^2 + h^2}$

Curved surface area = $\pi r \ell$

Total surface area = $\pi r (r + \ell)$

Volume of cone = $\frac{1}{3}$ × Volume of cylinder = $\frac{1}{3} \times \pi r^2 h$.

Example



Find the volume of the largest right circular cone that can be cut out of a cube of edge 42 cm.

The base of the cone will be a circle inscribed in a face of the cube and its height will be equal to the edge of the cube.

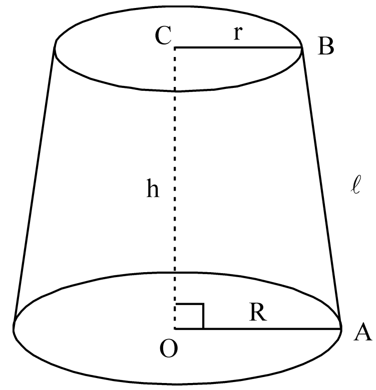
Radius of cone = 21 cm.

Height = 42 cm.

Volume of cone = $\frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 42 = 19404$ cu. cm.

FRUSTUM OF A CONE: A frustum is the lower part of a cone, containing the base, when it is cut by a plane parallel to the base of the cone.



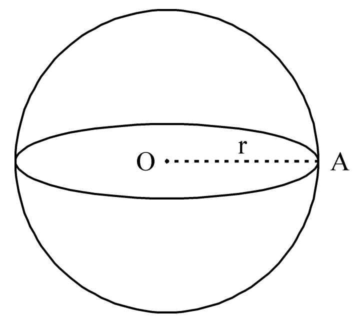
Slant height, $\ell = \sqrt{h^2 + (R - r)^2}$

Curved Surface area of frustum = $\pi(R + r)\ \ell$

Total surface area of frustum = Base area + Area of upper circle + Area of lateral surface = $\pi(R^2 + r^2 + R\ell + r\ell)$

Volume of frustum = $\dfrac{\pi h}{3}[R^2 + r^2 + Rr]$

SPHERE: The set of all points in space, which are at a fixed distance from a fixed point, is called a sphere.



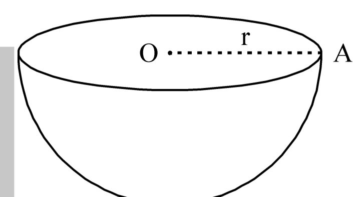
The fixed point is the centre of the sphere and the fixed distance is the radius of the sphere.

The entire surface of a sphere is curved.

Surface area of sphere = $4\pi r^2$

Volume of a sphere = $\dfrac{4}{3}\pi r^3$

HEMISPHERE: A sphere cut by a plane passing through its centre forms two hemispheres. The upper surface of a hemisphere is a circular region.

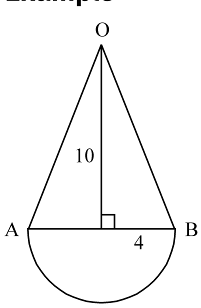


Curved surface area of a hemisphere = $2\pi r^2$

Total surface area of a hemisphere = C.S.A + Area of circular base = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Volume of a hemisphere = $\dfrac{1}{2}$ volume of sphere = $\dfrac{2}{3}\pi r^3$

Example



A toy has a hemispherical base and a conical top as shown in the figure. The perpendicular height of the cone is 10 cm and radius of the hemisphere is 4 cm. Find the volume of the toy.

Volume of toy = Volume of cone + Volume of hemisphere

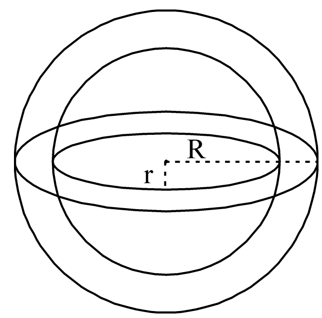
$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2(h + 2r)$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4(10 + 2 \times 4)$

$= \frac{1}{3} \times \frac{22}{7} \times 16 \times 18 \approx 302$ cu. cm.

SPHERICAL SHELL: If R and r are the outer and inner radii respectively of a hollow sphere, then

Volume of material in a Spherical Shell = $\frac{4}{3}\pi(R^3 - r^3)$.

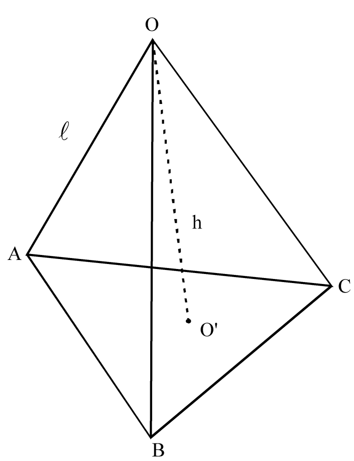


SOLID RING: If R and r are the outer and inner radii of a solid ring (can be considered as a cylindrical rod joined end to end), then

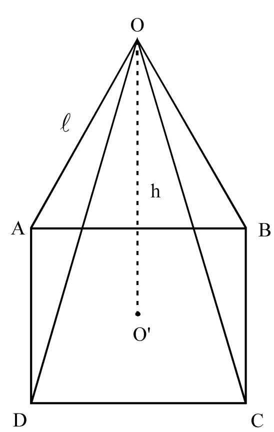
Volume = $\frac{\pi^2}{4}(R - r)^2(R + r)$.
Curved Surface area = $\pi^2(R^2 - r^2)$.

PYRAMID: A pyramid is a solid, whose base is a polygon and lateral faces are triangular with a **common vertex.**

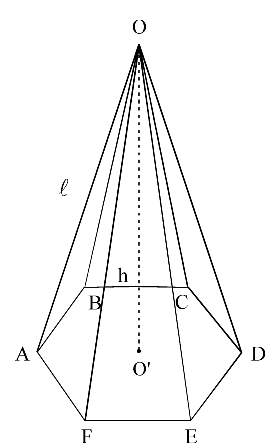
A pyramid is said to be tetrahedron (triangular base), square pyramid, hexagonal pyramid etc., according to the number of sides of the polygon that form the base.



Tetrahedron



Square Pyramid



Regular Hexagonal Pyramid

In the above figures, O is the common vertex and OO' is the perpendicular height (h) of the pyramid. The distance measured along the lateral face from the base to the apex or common vertex (O) along the center of the face is called the **slant height of the pyramid (ℓ)**. In other words, it is the altitude of the triangle comprising the lateral face.

In a pyramid with a base of n sides,

Number of vertices = n + 1; Number of faces including the base = n + 1.

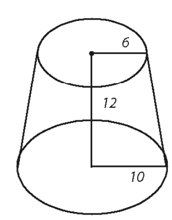
Surface area of lateral faces = $\frac{1}{2}$ × perimeter of base × slant height

Total surface area of pyramid = Base area + $\frac{1}{2}$ × perimeter of base × slant height

Volume of pyramid = $\frac{1}{3}$ × Base area × height.

Concept Builder

1. A solid with 8 faces has 16 edges. What are the number of vertices of the solid
2. The height of a right prism is 8 units.
   a) Find the L.S.A if perimeter of base is 10 units
   b) Area of base, if volume = 320 cubic units.
3. A cuboid of dimensions 2 unit × 4 unit × 8 units is melted in to a cube. Find
   a) Volume of cuboid
   b) Surface area of cuboid
   c) Length of the diagonal of cuboid
   d) Volume of cuboid = Volume of the new cube (T/F)
   e) Side of the new cube
   f) Surface area of the cube
   g) Length of diagonal of the cube
4. The radius and height of a cylinder are 7cm and 10cm respectively. Find
   a) C.S.A b) T.S.A c) Volume
5. A tetrahedron (with equilateral triangular base) with side 6 cm has slant height of 7cm. Find a) L.S.A b) T.S.A
6. Find the volume, C.S.A, T.S.A of a right circular cone of height and radius 4 and 3 respectively.
7. The radius of sphere is 9 and that of a hemisphere is 6 units. Find the ratio of their volumes and ratio of their T.S.A's
8. Find the Slant height, volume, curved S.A., T.S.A of the given frustum



Answer Key

1. 10
2. a) 80 b) 40
3. a) 64 b) 112 c) $2\sqrt{21}$ d) T e) 4 f) 96 g) $4\sqrt{3}$
4. a) 140π b) 238π c) 490π
5. a) 63 b) $63 + 9\sqrt{3}$
6. CSA = 15π, TSA = 24π, Volume = 12π
7. Volume = $\frac{27}{4}$
   TSA = 3 : 1
8. Slant height = $4\sqrt{10}$, V = 784π, C.S.A = $64\sqrt{10}\,\pi$
   T.S.A = $(64\sqrt{10} + 136)\pi$

SOLVED EXAMPLES

Q : A road roller of diameter 1.75 m and length 1 m has to press a ground of area 1100 sq. meter. How many revolutions does it make?

A : Area covered in one revolution = Curved surface area

$\therefore$ Number of revolutions made $= \dfrac{\text{Total area to be pressed}}{\text{Curved surface area}}$

$= \dfrac{1100}{2\pi rh}$

$= \dfrac{1100}{2 \times \dfrac{22}{7} \times \dfrac{1.75}{2} \times 1}$

$= 200.$

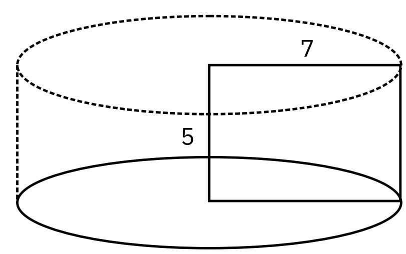
Q : A rectangle 7 cm × 5 cm is rotated about its smaller edge as axis. Find the curved surface area and volume of solid generated.

A : Curved Surface area $= 2\pi rh = 2 \times \dfrac{22}{7} \times 7 \times 5$

$= 220$ sq. cm.

Volume of solid $= \pi r^2 h$

$= \dfrac{22}{7} \times 7 \times 7 \times 5 = 770$ cu. cm.



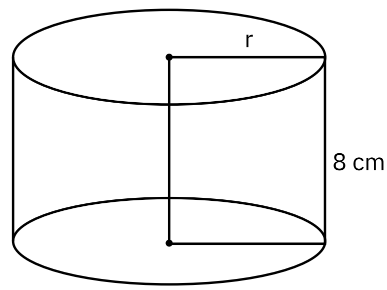
Q : The perpendicular sides of the base of a right triangular metallic pyramid are 6 cm and 8 cm. It weighs 810 g. Find its height, if density of the metal is 13.5 g/cc.

A : Volume $= \dfrac{\text{Weight}}{\text{Density}} = \dfrac{810\text{g}}{13.5\text{g/cc}} = 60$ cc

Volume $= \dfrac{1}{3}$ Base area × height

$60 = \dfrac{1}{3} \times \dfrac{1}{2} \times 6 \times 8 \times \text{height}$ $\therefore$ Height = 7.5 cm.

Q : A rectangular sheet of paper of length 8 cm and breadth 22 cm is rolled end to end to form a right circular cylinder of height 8 cm. Find the volume of the cylinder.



A : Circumference = 22 cm i.e., $2\pi r = 22$ cm.

$\therefore\ r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}$ cm

$\therefore$ Volume $= \pi r^2 h = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 8 = 308$ cu. cm.

Q : A regular hexagonal prism has perimeter of its base as 600 cm and height equal to 200 cm. How many litres of petrol can it hold? Find the weight of petrol if density is 0.8gm/cc.

A : Side of hexagon $= \frac{\text{Perimeter}}{\text{Number of sides}} = \frac{600}{6} = 100$ cm.

Area of regular hexagon $= \frac{3\sqrt{3}}{2} \times 100 \times 100 = 25950$ sq. cm.

Volume = Base Area × height = 25950 × 200 = 5190000 cu. cm = 5.19 cu.m.

Weight of petrol = Volume × Density = 5190000 cc × 0.8 gm/cc = 4152000 gm = 4152 kg.

Q : A solid metallic right circular cone of height 45 cm and radius 15 cm is melted and two solid cylinders of height 15 cm are prepared. If the volume of one is 8 times that of the other, find the radius of smaller one.

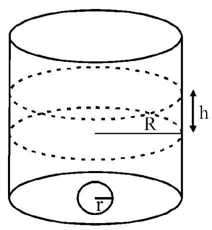
A : Volume of cone = Volume of smaller cylinder + 8 times of volume of the smaller cylinder = 9 times volume of smaller cylinder.

$\frac{1}{3}\pi \times 45 \times 15 \times 15 = 9 \times \pi \times r^2 \times 15$

$\therefore\ r^2 = \frac{45 \times 15 \times 15}{9 \times 3 \times 15}$

$\therefore\ r = 5$ cm.

Q :



An iron ball of diameter 6 inches is dropped into a cylindrical vessel of diameter 1 ft filled with water. Find the rise in water level. (Assume density of water and material used in sphere is the same)

A : Radius of vessel = $\frac{1 \text{ feet}}{2} = \frac{12 \text{ inches}}{2}$ = 6 inches.

Volume of water that has risen = Volume of sphere

$\Rightarrow \pi R^2 h = \frac{4}{3}\pi r^2$

$\Rightarrow (6)^2 \times h = \frac{4}{3} \times 3^3$

$\Rightarrow 36 \times h = \frac{4}{3} \times 3^3$

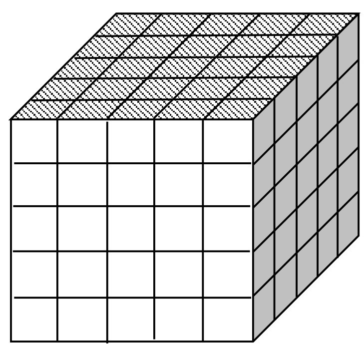
$\therefore$ h = 1 inch.

CLASS EXERCISE

Teaser

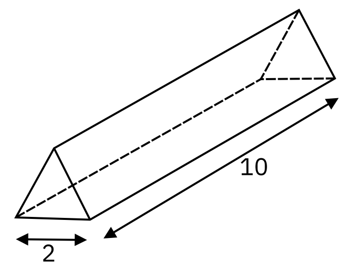
A cube is painted with six different colours on its six faces (red, yellow, blue, green, black and purple). Then it is cut into 125 identical smaller cubes. How many of these cubes will have:

a) No face painted?
b) One face blue and exactly one other face painted?
c) One face purple and at least one other face painted?
d) One face red and one face yellow?

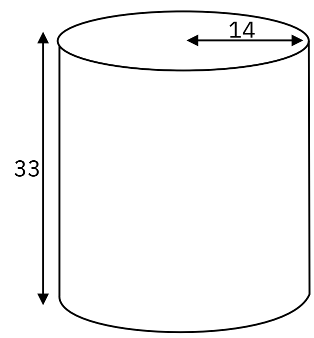


Prisms

1. A physics laboratory has a triangular prism made of glass. The base is an equilateral triangle of side 2 cm, while the distance between the two triangular faces is 10 cm.
   a) What is the volume of glass used in making the prism?
   b) If the entire surface of the prism is to be coated with black paint, what area needs to be painted?



2. An open (hollow) cylindrical barrel has a base radius of 14 cm and a height of 33 cm
   a) Find the volume and the surface area of the barrel.
   b) If a fly is at a point on the bottom inner rim of the barrel, how much distance (minimum)
      will it have to travel to reach the farthest point on the barrel?
   c) If an ant is at a point on the bottom inner rim of the barrel, how much distance (minimum) will it have to travel to reach the farthest point on the barrel?



A **prism** is a polyhedron formed by connecting the corresponding vertices of two congruent polygonal figures (the bases) in parallel planes. (A cylinder also can be considered prismatic).

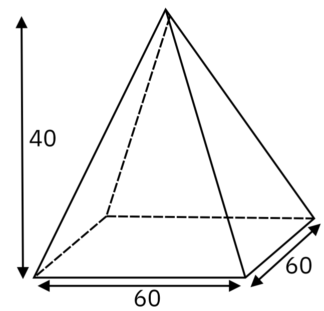
The following table gives the general formulae for a right prism and some examples of prisms

	Volume	**Surface Area**
General Right Prism	Area(base) x Height	2 × Area(base) + Perimeter(base) × Height
Cuboid (length l, breadth b, height h)	(l x b) x h	2lb + 2(l +b) h
Regular Hexagonal Prism (side s, height h)	$\frac{3\sqrt{3}\,s^2}{2} \times h$	$3\sqrt{3}\,s^2 + 6sh$
Equilateral Triangular Prism (side s, height h)	$\frac{\sqrt{3}\,s^2}{4} \times h$	$\frac{\sqrt{3}\,s^2}{2} + 3sh$
Cylinder (radius r, height h)	$\pi r^2 \times h$	$2\pi r^2 + 2\pi rh = 2\pi r(r + h)$

3. * An open jewelry box in the shape of a square-based prism has a base side of length 5 inches and a height of 4 inches. The walls are of negligible thickness.
   a) What is the capacity of the box in cubic inches?
   b) What is the cost of painting the surface of the box at a cost of 20 paise per square inch?

Pyramids

4. A square pyramid is discovered in Egypt with a base of 60 m and a height of 40 m.
   a) If the pyramid is solid stone, what is the volume of stone used in the construction?
   b) If on further examination, it is found that the interior of the pyramid contains a hollow chamber of similar shape with a square base of 45 m and a height of 30 m, then how much stone is used in the construction?



5. A conical vessel (with its vertex at the bottom) has a height of 24 units and a volume of 216 litres. What will be the volume of water in it if:
   a) water is filled to a height of 8 units?
   b) water is filled to a height of 12 units?
   c) it is turned upside down and water is filled to a height of 8 units?
   d) it is turned upside down and water is filled to a height of 12 units?

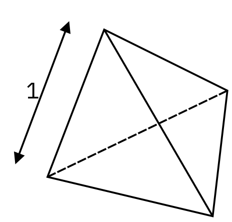
A **pyramid** is a polyhedron formed by connecting the vertices of a polygonal figure (the base) with a point (the vertex) outside the plane of the base. (A cone also can be considered pyramidal in nature).

The following table gives the general formulae for a right pyramid (with a regular polygonal base), and some examples of pyramids

	Volume	Surface Area
General Right Pyramid	$\frac{1}{3} \times$ Area(base) $\times$ Height	Area(base) + $\frac{1}{2} \times$ Perimeter(base) $\times$ Slant Height
Regular Hexagonal Pyramid (side s, height h, slant height l)	$\frac{\sqrt{3}\,s^2}{2} \times h$	$\frac{3\sqrt{3}\,s^2}{2} + 3sl$
Equilateral Triangular Prism (side s, height h, slant height l)	$\frac{1}{3} \times \frac{\sqrt{3}\,s^2}{4} \times h$	$\frac{\sqrt{3}\,s^2}{2} + 3sh$
Cone (radius r, height h, slant height l)	$\frac{1}{3} \times \pi r^2 \times h$	$\pi r^2 + \pi rl = \pi r(r + l)$

6. * An ice-cream cone has a radius of 42 mm and a height of 100 mm. The seller claims that the cone contains 200 ml of ice-cream. Is he telling the truth? (Note: 1 ml = 1 cubic cm)

7. *A tetrahedron has all faces as equilateral triangles of side 1, as shown. Find its volume.



Spheres and Frustums

8. A hemisphere of radius 14 is melted down and formed into 4 smaller identical spheres.
   a) What is the radius of each smaller sphere?
   b) What is the ratio of the new total volume to the original total volume?
   c) What is the ratio of the new total surface area to the original surface area?

9. A cone of base radius 7 cm and height 24 cm is cut parallel to the base to form a frustum or truncated cone. If the cut is made:
   a) at a distance of 6 cm from the vertex, find the volume of the frustum formed.
   b) at a distance of 4 cm from the base, find the ratio of the volumes of the two parts formed.
   c) at a distance of 12 cm from the vertex, find the upper radius of the frustum formed.
   d) at a distance of 4.8 cm from the base, find the slant height of the frustum so formed

Other standard figures which are neither pyramidal nor prismatic include the **sphere** (and hemisphere), and the **frustum** or truncated pyramid.

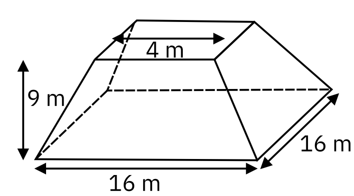
The following table gives general formulae for spheres, hemispheres and conical frustums:

	Volume	**Surface Area**
Sphere (radius r)	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Hemsphere (radius r)	$\frac{2}{3}\pi r^3$	$3\pi r^2$
Frustum of cone (radii R and r, height h, slant height l)	$\frac{1}{3} \times \pi h(R^2 + Rr + r^2)$	$\pi(R^2 + r^2 + Rl + rl)$

Note that in many cases it is far easier to use the basic principles of similarity of figures to solve problems involving frustums, rather than applying a cumbersome formula.

10. * An Aztec temple made of stone is shown in the figure.

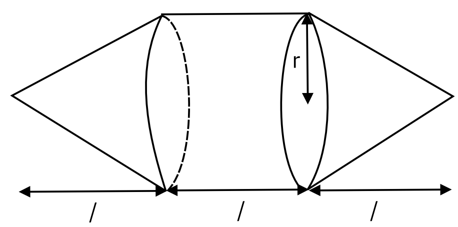
    It is in the shape of the frustum of a square pyramid. The bottom surface is a square of side 16 m, while the top surface is a square of side 4 m. The height of the pyramid is 9 m. Find the volume of the pyramid.



Compound Figures

11. A cube of side 3 inches is taken and a hole of circular cross-section, with diameter 1 inch, is drilled right through from the centre of one face to the centre of the opposite face. Find the volume and surface area of the resultant block.

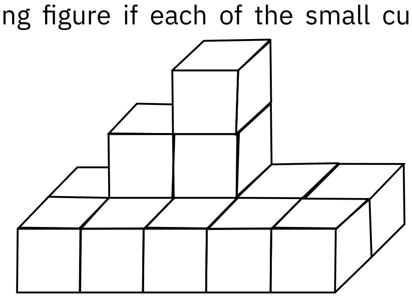
12. A child's spinning top is made in the shape of a cylinder of radius 'r' and length 'l', topped at each end by a cone of base radius 'r' and height 'l' as shown in the figure. Find:
 a) The volume of the top
 b) The surface area of the top



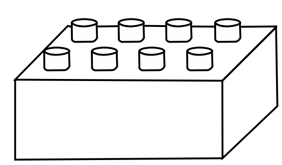
13. What is the maximum possible volume of a sphere that can fit inside:
 a) A cube of side 6 cm
 b) A prism of height 8 cm whose base is an equilateral triangle of side 6 cm

14. * What is the maximum possible volume of a cube that can fit inside:
 a) A sphere of radius 6 cm
 b) A cylinder of radius 6 cm and height 8 cm

15. * Find the volume and surface area of the adjoining figure if each of the small cubes has a side of length 1 cm, and each cube in an upper layer has a cube below it.



16) * A molded plastic toy is made as shown in the adjacent figure. The base is a cuboid of dimensions 16 mm x 8 mm x 4 mm, while the projections are cylinders of diameter 2 mm and height $\frac{5}{\pi}$ mm. Find the volume of the block. If the cost of painting such blocks is 2 paise per sq cm, find the cost of painting a box of 100 such blocks.



Challengers

1) A metal cylinder of radius 12 cm and height 16 cm is melted down and formed into identical smaller spheres of radius 3 cm.

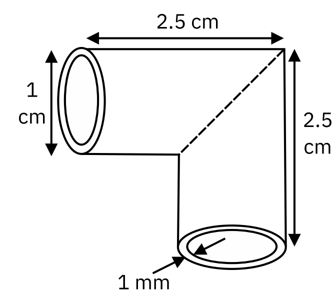
a) How many smaller spheres are formed?

1) 48 2) 54 3) 72 4) 64

b) What is the percentage change in the total surface area?

1) 143% increase 2) 243% increase
3) 343% increase 4) No change

2) A right-angled connector made of rubber can be used to connect two hose-pipes. If the connector has an outer diameter of 1 cm, a length of 2.5 cm and a thickness of 1 mm, find the volume of rubber used in its construction.



1) 1.44 π cc 2) 18 π cc
3) $\frac{(5+2\sqrt{3})\pi}{16}$ cc 4) 0.36 π cc

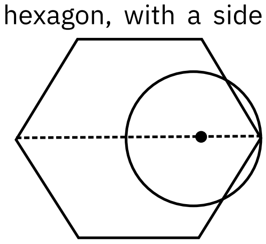
3. There is a cube with sides of length 5 units. An ant on one vertex wants to reach the farthest (opposite) vertex, traversing the minimum distance in the process. How far will it have to crawl?

1) 15 2) $5(\sqrt{2}+1)$ 3) $5\sqrt{5}$ 4) $5\sqrt{3}$

4. A cone and a sphere have equal heights and equal volumes. Find the ratio of their radii.

1) 1 : 1 2) $\sqrt{2}$: 1 3) $\sqrt{3}$: 1 4) 2 : 1

5. * The ticket for the Chess World Cup final is in the shape of a regular hexagon, with a side of 6 inches. A circular stamp, of radius 2 inches, is meant to be used to stamp the ticket in the centre. By mistake, the stamp get misaligned and stamps the ticket in such a way that the edge of the stamp exactly touches a vertex of the hexagon, while the centre of the stamp lies on the line joining that vertex to the opposite vertex (as shown in the figure). Find the portion of the stamp, in square inches, which falls outside the ticket.



1) $\frac{4}{3}\pi - 2\sqrt{3}$ 2) $4\pi - \frac{\sqrt{3}}{2}$ 3) $\frac{4}{3}\pi - \sqrt{3}$ 4) $2\pi - \frac{\sqrt{3}}{2}$

PRACTICE EXERCISE-1

DIRECTIONS for questions 1 to 14: Choose the correct alternative.

1. Find the length of the longest rod that can be fit into a box of dimensions 6 cm × 3 cm × 2 cm.

   1) 45 cm   2) 7.8 cm   3) 7 cm   4) 8 cm

2. The lower part of a tent is a circular cylinder and its upper part is a right cone. The diameter of the base is 70 m. The total height is 15 m and the height of the cylindrical part is 3 m. Find the cost of canvas required at Rs.10 per sq. m.

   1) Rs.47300   2) Rs.162800
   3) Rs.16280   4) Rs.44000

3. A thermal chimney of circular cross section has outer and inner radii 3 m and 2 m respectively. Find the cost of cement finishing for the the inner and outer surface at Rs.20 per sq. meter, if the height of the chimney is 56 m.

   1) Rs.80000   2) Rs.45760   3) Rs.35200   4) Rs.7040

4. Water flows out at the rate of 10 m/minute from a cylindrical pipe of diameter 5 mm. Find the time taken to fill a conical tank whose diameter at the base is 40 cm and depth is 24 cm.

   1) 153.6 minutes   2) 51.2 minutes   3) 12.8 minutes   4) 128 minutes

5. The sides of a cube were uniformly increased to increase the total surface area by 44%. What was the percentage increase in the volume of the cube?

   1) 33.1%   2) 57.6%   3) 66.3%   4) 72.8%

6. A spherical iron ball of radius 56 m is to be painted. However, it is realised that the requirement is for balls of radii 14 m. So, the bigger ball is melted and made into smaller balls of radii 14 m each. The rate per sq. meter for painting the smaller balls is reduced to $\frac{3}{8}$th of the original rate. What is the impact on the original painting cost?

   1) decreases by 50%   2) increases by 50%
   3) decreases by 37.5%   4) doubles

7. A bowl in the shape of a hemisphere and of inner radius $\sqrt[3]{33}$ inch is filled with soup of volume 18π inch^3. At the most how many Manchurian balls, of 1 inch radius, can be added to the soup so that it does not spill over?

   1) 1   2) 2   3) 3   4) 4

8. 1 m^3 of metal is used to make four wires of equal length whose cross-sectional area is 250 cm^2 each. Find the length of the wires made.

   1) 0.05 m   2) 0.5 m   3) 0.01 m   4) 10 m

9. A right circular cone, a right circular cylinder and a hemisphere, all have the same radius, and the heights of the cone and the cylinder equal their diameters. Then their volumes are proportional, respectively to _____.

1) 1 : 3 : 1 2) 2 : 1 : 3 3) 3 : 2 : 1 4) 1 : 2 : 3

(Past CAT question)

10. If a right circular cone of height h is cut by a plane parallel to the base and at a distance $\frac{h}{3}$ from the base, then the volumes of the resulting cone and the frustum are in the ratio _____.

1) 1 : 3 2) 8 : 19 3) 1 : 4 4) 1 : 7

(Past CAT question)

11. A wooden box (open at the top) of thickness 0.5 cm and length 21 cm, width 11 cm, and height 6 cm, is painted on the inside. The expenses of painting are Rs. 70. What is the approximate rate of painting in rupees per sq cm?

1) 0.72 2) 0.51 3) 0.13 4) 0.26

(Past CAT question)

12. A cube is painted red on all sides. It is then cut into 8 identical smaller cubes. All unpainted areas are painted blue. Then each cube is further cut into 8 identical smaller cubes. All unpainted areas are painted green now. What is the ratio of the total areas painted red, blue and green?

1) 1 : 2 : 1 2) 1 : 2 : 2 3) 2 : 1 : 1 4) 1 : 1 : 2

13. A right angled triangle ABC of perpendicular sides AB and BC is rotated about AB as the axis. If AB = 12 and BC = 5. Find the total surface area of solid generated.

1) 90π 2) 45π 3) 30π 4) 85π

14. A right circular cone is immersed in clear water to $\frac{1}{4}$th of its height, such that its tip is in water and the circular base is at the top. Find the volume of the portion of the cone above water if the full height of the cone is 12 cm and the radius of the base of the cone is 7 cm.

1) $\frac{4851}{8}$ cm^3 2) $\frac{77}{4}$ cm^3 3) 616 cm^3 4) $\frac{4851}{4}$ cm^3

DIRECTIONS for question 15: Solve as directed.

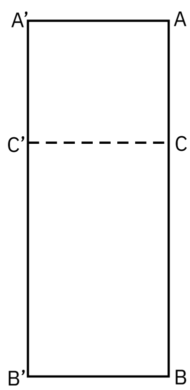
15. 2112 m^3 of mud is dug out from the middle of a circular playground to build an 8 metre-deep rectangular swimming pool. The remaining 3586 m^2 area of the playground is covered in grass. What is the radius (in metres) of the playground?

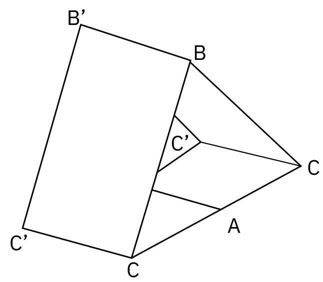
DIRECTIONS for questions 16 to 21: Choose the correct alternative.

16. A hollow hemispherical bowl of outer radius 6 cm and uniform thickness 2 cm is remolded to form a hollow spherical ball of same outer radius. By what percentage will the thickness reduce?

1) Between 35% and 40%
2) Between 45% and 50%
3) Between 55% and 60%
4) Between 65% and 70%

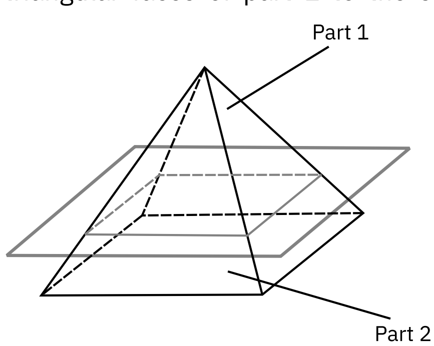
17. Two identical rectangular sheets of the form A'ABB' are folded along C'C to form a triangular prism whose bases are equilateral triangles. It is known that AC = AA'. If the volume of the prism is $27\sqrt{3}$ cubic units, find the area of the rectangle A'ABB' (in square units).





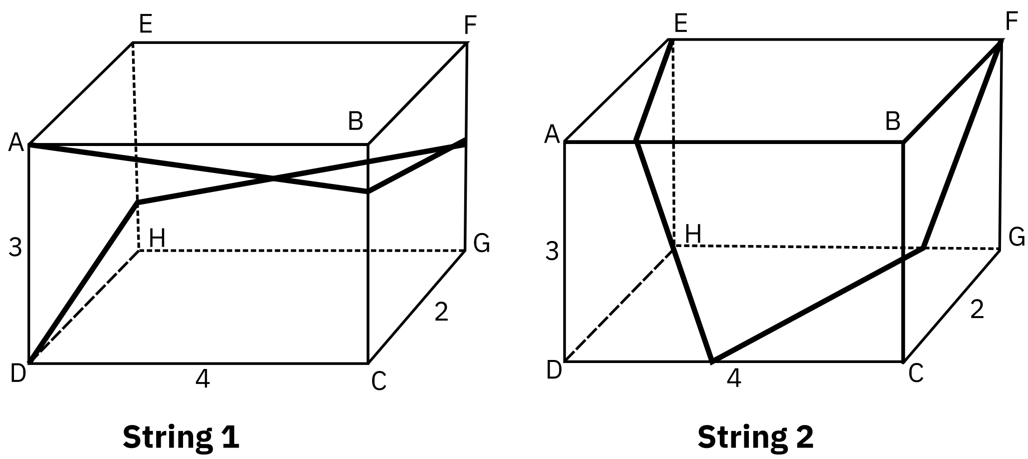
1) 18
2) 24
3) 27
4) 36

18. A square based pyramid is cut horizontally into 2 parts as shown in the diagram. The ratio of height of the original pyramid to that of the new pyramid is the same as the ratio of side of square of original pyramid to that of new pyramid, and the volume of new pyramid is $\frac{1}{8}$th of the volume of original pyramid. What is the ratio of the sum of surface areas of the triangular faces of part 1 to the sum of the area of the inclined faces of part 2?



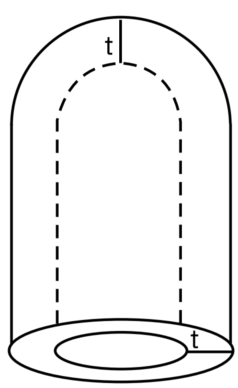
1) 2:1
2) 3:1
3) 1:3
4) 2:3

19. ABCDEFGH is a cuboid. String 1 is wound around four faces of the cuboid starting from point A and ending at point D, as shown in the figure. String 2 is wound around four faces of the cuboid starting from point E and ending at point F, as shown in the figure. What is the ratio of the smallest possible lengths of string 1 and string 2?



1) $\sqrt{\frac{145}{104}}$ 2) $\sqrt{\frac{148}{109}}$ 3) $\sqrt{\frac{153}{116}}$ 4) $\frac{153}{116}$

20. A toy is made up of a hollow cylindrical base and a hollow hemispherical top as shown in the figure. The toy is open only on one end and has uniform thickness "t" throughout. The outer curved surface area of the toy is 2013 cm^2 and the inner curved surface area of the toy is 1188 cm^2. The total height of the toy is 30.5 cm. Find the thickness of the toy. (Take $\pi = \frac{22}{7}$)



1) 2.8 cm   2) 3.15 cm   3) 3.5 cm   4) 4.2 cm

21. Three cones of base radii 1 cm, 2 cm and 3 cm and same height 5 cm are placed next to each other such that the base of each cone touches the bases of the other two cones. What is the circumference of the circle that passes through the tips of all the three cones?

1) 10π 2) 7.5π 3) 5π 4) 3π

PRACTICE EXERCISE-2

DIRECTIONS for question 1: Solve as directed.

1. If the surface area of a blue-coloured sphere is 800% more than that of a red-coloured sphere, by what percent is the volume of the blue-coloured sphere more than that of the red-coloured sphere?

DIRECTIONS for questions 2 to 20: Choose the correct alternative.

2. A slab of ice 8 inches in length, 11 inches in breadth, and 2 inches thick was melted and re-solidified in the form of a rod of 8 inches diameter. The length of such a rod, in inches is nearest to:

   1) 3 2) 3.5 3) 4 4) 4.5

   (Past CAT question)

3. The diameter of a hollow cone is equal to the diameter of a spherical ball. If the ball is placed at the base of the cone, what portion of the ball will be outside the cone?

   1) 50% 2) Less than 50%
   3) More than 50% 4) Cannot be determined

   (Past CAT question)

4. A solid square pyramid having base area of 49 cm^2 is cut by a plane parallel to the base. The upper part of the pyramid is melted to form a sphere. If the diameter of the sphere is equal to the side of base of melted part of the pyramid then, what was the height of the solid square pyramid?

   1) 7 cm 2) 22 cm 3) 11 cm 4) 15 cm

5. A room with dimensions 24 m, 20 m and 14 m is ventilated by a rectangular opening with area $\frac{4}{3}$ m × $\frac{5}{4}$ m. At what rate should air pass through the opening per unit area, so that $\frac{1}{5}$th of the air in the room would change every hour?

   1) 14.5 m/min 2) 15 m^3/min 3) 13.4 m^3/min 4) 13.44 m/min

6. A metal ball, 6 cm in diameter, is dropped into a cylindrical vessel of diameter 12 cm and height 20 cm. The water level, inside the cylinder, is 19.5 cm from the bottom face of the cylinder. Find the volume of the water that spills out.

   1) 18π cm^3 2) 36π cm^3 3) 100π cm^3 4) No water spills out.

7. A regular tetrahedron of length 12 cm is accommodated in a sphere such that all its vertices touch the sphere. Find the radius of the sphere.

   1) 7.2 cm 2) 7 cm 3) 7.5 cm 4) 6.5 cm

8. An ice-cream, in the shape of right circular cone is cut (parallel to its base) into 5 slices, all of equal heights. What is the ratio of the volume of middle slice to that of the volume of the biggest slice?

   1) 4 : 25 2) 41 : 93 3) 19 : 61 4) 27 : 125

9. A hollow cone with base radius 9 cm and height 12 cm is filled with water up to a level of 8 cm height when kept on closed circular base. What height does the water come upto, if this cone is kept in an inverted position such that the apex of the cone will be facing down?

   1) $3\sqrt[3]{26}$ cm 2) 8 cm 3) $7\sqrt[3]{13}$ cm 4) $4\sqrt[3]{26}$ cm

10. An iron pillar has some part in the form of a right circular cylinder and the remaining part in the form of a right circular cone. The radius of the base of the cone as well as the cylinder is 8 cm. The cylindrical part is 240 cm high and the conical part is 36 cm high. Find the weight of the pillar, if 1 cm^3 of iron weighs 7.8 gm.

    1) 3.6 kg 2) 163.7 kg 3) 253 kg 4) 395 kg 5) 300 kg

11. A glass showpiece in the shape of a sphere of radius 5 cm is packed into a cardboard cubic box whose sides are of length are 12 cm. To prevent the showpiece from rolling around, 8 identical rubber spheres are placed in the corners of the box. What is the radius of the rubber spheres?

    1) $\frac{6\sqrt{3}-5}{1+\sqrt{3}}$ cm 2) $\frac{12\sqrt{3}-10}{2+\sqrt{3}}$ cm3) $\frac{6\sqrt{3}-5}{1+\sqrt{2}}$ cm 4) $\frac{12\sqrt{3}-10}{2+\sqrt{2}}$ cm

12. The ratio of the volume of a hexagonal pyramid, P, to that of a hexagonal prism, Q, is 4 : 25. If the heights of P and Q are interchanged, then the ratio of their respective volumes becomes 9 : 100. What is the ratio of their respective volumes, if the sides of P and Q are interchanged?

    1) 100 : 9 2) 100 : 81 3) 100 : 27 4) 23 : 36

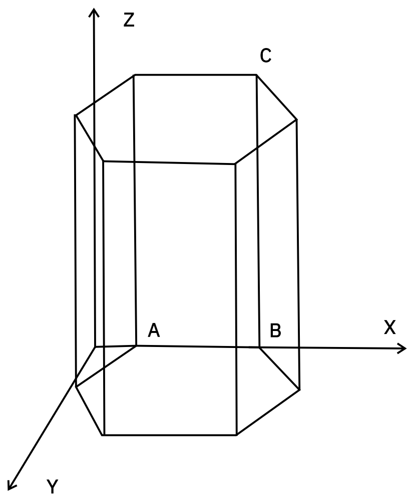
13. What is the length of side of a cube, if the area of the largest equilateral triangle that can perfectly fit inside the cube is $50\sqrt{3}$ cm^2?

    1) 5 cm 2) $5\sqrt{3}$ cm 3) $10\sqrt{3}$ cm 4) 10 cm

14. One of the base edges of a metallic plate, which is in the form of a right triangle is soldered to a wire. The wire is rotated around its own axis. What is the maximum volume that can be generated by the metallic plate given that the smallest side of the triangular plate is 8 cm and the largest side is 2 cm greater than the second largest side?

    1) 600π sq.cm 2) 320π sq.cm 3) 60π sq.cm 4) Data insufficient

15. The given three-dimensional figure has bases which are regular hexagons. One of the bases is placed on the x-y plane. The coordinates of the points A, B and C are (1, 0, 0), (3, 0, 0) and (3, 0, 4). What will be the coordinates of the point on the 3-D figure that is farthest from A ?



1) $(3, 2\sqrt{3}, 4)$ 2) $(3, 2\sqrt{3}, 3)$ 3) $(4, 2\sqrt{3}, 3)$ 4) $(3, \sqrt{3}, 4)$

16. A fish tank in the shape of a cuboid has length, breadth and height of 60 cm, 40 cm and 80 cm respectively and it is filled with water. The base of the tank is tilted by 45° along the breadth and water coming out from the tank is collected in an empty vessel. The volume of water collected in the empty vessel is 32 litres. What was the height of water that was initially there in the cuboid tank? [Assume thickness of the water tank to be negligible].

1) 80 cm 2) 60 cm 3) 63.33 cm 4) 76.66 cm

17. A solid sphere is cut along three planes perpendicular to each other to form 8 identical parts. The curved surface of each part is painted red and the flat surfaces are painted blue. Find the ratio of the area of the red surface to the area of the blue surface in each part.

1) 2 : 3 2) 3 : 8 3) 1 : 2 4) 1 : 3

18. A right circular cylinder of radius 10 cm and height 40 cm is placed on the ground. A sphere of largest possible size is enclosed in the cylinder and is on the base of the cylinder. A second sphere having half the radius of the first sphere is stuck to the first sphere. Continuing this, a number of spheres are stuck to one another such that each sphere touches the two adjoining spheres and each sphere has radius equal to half the radius of the earlier sphere. If the line joining the centres of all the spheres is parallel to the axis of the cylinder and the total height of the "tower of spheres" is equal to the height of the cylinder, what is the sum of the surface areas of all the spheres?

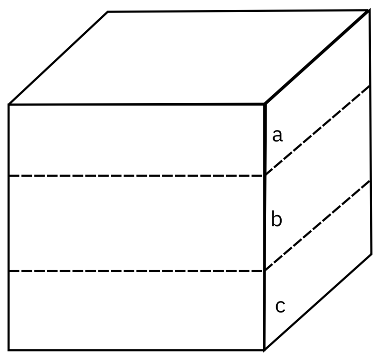
1) $\frac{1600}{3}\pi$

2) $\frac{800}{3}\pi$

3) $\frac{3200}{3}\pi$

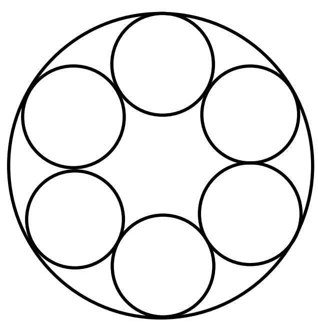
4) More information is needed to answer the question

19. A cube is cut into three parts as shown below, such that the ratio of the total surface areas of the resulting cuboids is 13 : 17 : 15 (a, b and c are integers). Which of the following can be the volume of the original cube, given that the length of the side of the cube is an integer value?



1) 343 cu. units 2) 1000cu. units 3) 512 cu. units 4) 729 cu. units

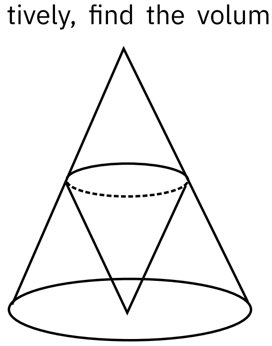
20. Seven identical solid spheres are fit inside a larger hollow sphere such that they touch each other and the outermost spheres touch the larger sphere. The centres of all the spheres lie on the same plane. Find the ratio of the volume inside the larger sphere that is occupied by the smaller spheres to the volume of air inside the larger sphere.



1) 7 : 27 2) 1 : 3 3) 7 : 20 4) 7 : 10

DIRECTIONS for question 21: Solve as directed.

21. A cone of radius 21 cm is inserted inside a larger cone (as shown in the figure) such that when it can't go further in, the vertex of the smaller cone is on the same plane as the base of the larger cone. If the radius and height of the larger cone are 35 cm and 45 cm respectively, find the volume of the larger cone (in cm^3) that is not occupied by the smaller cone.



QA-3.7 | TIME, SPEED AND DISTANCE — THEORY

Time, Speed & Distance

Definition of Terms

The speed of a body is defined as the distance covered by it in unit time.

Speed = $\frac{\text{Distance}}{\text{Time}}$;

Time = $\frac{\text{Distance}}{\text{Speed}}$;

Distance = Time × Speed

Units of measurement

- Time is measured in seconds (s), minutes (min) or hours (hr).
- Distance is usually measured in metres (m), kilometres (km), miles, yards or feet.
- Speed is usually measured in metres/sec. (m/s), kilometres/hour (km/hr) or miles/hr.

Conversion of units

1. 1 hour = 60 minutes = 60 × 60 seconds.
2. 1 kilometer = 1000 metres
3. 1 kilometre = 0.6214 mile
   1 mile = 1.609 kilometre
   i.e., 8 kilometres 5 miles
4. 1 yard = 3 feet
5. $\frac{\text{km}}{\text{hr}} = \frac{5\text{m}}{18\text{s}}$
6. $\frac{\text{m}}{\text{s}} = \frac{18\text{km}}{5\text{hr}}$ =
7. $\frac{\text{km}}{\text{hr}} \approx \frac{5\,\text{miles}}{8\text{hr}}$
8. $\frac{\text{miles}}{\text{hr}} = \frac{22}{15} = \frac{\text{ft}}{\text{sec}}$

Relation between Time, Speed & Distance

1. If distance is constant, Speed α $\frac{1}{\text{Time}}$

Example

Walking at $\frac{3}{4}$th of his usual speed, a man is $1\frac{1}{2}$ hrs late. Find his usual travel time.
If the usual time is t hrs, and usual speed is x kmph, then,

Distance travelled = xt = $\frac{3}{4} \times x\left(t+\frac{3}{2}\right) \Rightarrow t = \frac{3}{4}\left(t+\frac{3}{2}\right)$

$\therefore$ $4t = 3t + \frac{9}{2} \Rightarrow t = 4\frac{1}{2}$ hrs

$\therefore$ Original time taken for journey = $4\frac{1}{2}$ hrs and new time taken for journey = 6 hrs
With distance being the same, decrease in speed leads to increase in time i.e., with distance being constant, speed is inversely proportional to time.

2. If time is constant, Distance α Speed

Example

A car travels at 30 kmph for the first 2 hours of a journey and then travels at 40 kmph for the next 2 hours of the journey. Find the ratio of the distances travelled at the two speeds.
Since Time is constant in both the intervals, ratio

$$\Rightarrow \frac{\text{Distance Travelled in the } 1^{st} \text{ interval}}{\text{Speed during the } 1^{st} \text{ interval}} = \frac{\text{Distance Travelled in the } 2^{nd} \text{ interval}}{\text{Speed during the } 2^{nd} \text{ interval}}$$

$$\Rightarrow \frac{\text{Distance Travelled in the } 1^{st} \text{ interval}}{\text{Distance Travelled in the } 2^{nd} \text{ interval}} = \frac{\text{Speed during the } 1^{st} \text{ interval}}{\text{Speed during the } 2^{nd} \text{ interval}}$$

$$= \frac{30}{40} = \frac{3}{4}$$

i.e., we can say that ratio of the distance travelled is equal to the ratio of the speed of the vehicle in two equal time intervals or that distance is directly proportional to speed if time is constant.

3. If speed is constant, Distance α Time

Example

A car moves for 3 hours at the speed of 20 kmph and a truck moves for 4 hours at the same speed. Find the ratio of the distances covered by the car and the truck.
Since Speed is constant for both the vehicles

$$\Rightarrow \frac{\text{Distance Travelled by the car}}{\text{Time taken by the car}} = \frac{\text{Distance Travelled by the truck}}{\text{Time taken by the truck}}$$

$$\Rightarrow \frac{\text{Distance Travelled by the car}}{\text{Distance Travelled by the truck}} = \frac{\text{Time taken by the car}}{\text{Time taken by the truck}}$$

$$= \frac{3 \times 20}{4 \times 20} = \frac{3}{4}$$

i.e., we can say that ratio of the distance travelled is equal to the ratio of the time taken by the two vehicles or that distance is directly proportional to time if speed is constant.

Average speed

If a body travels d_1, d_2, d_3... d_n distances, with speeds s_1, s_2, s_3... s_n in time t_1, t_2, t_3 ... t_n respectively then the average speed of the body through the total distance is given by:

$$\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

$$= \frac{d_1 + d_2 + d_3 + \ldots d_n}{t_1 + t_2 + t_3 + \ldots t_n} = \frac{s_1 t_1 + s_2 t_2 + s_3 t_3 + \ldots + s_n t_n}{t_1 + t_2 + t_3 + \ldots + t_n} = \frac{d_1 + d_2 + d_3 + \ldots + d_n}{\frac{d_1}{s_1} + \frac{d_2}{s_2} + \frac{d_3}{s_3} + \ldots + \frac{d_n}{s_n}}$$

Cases of an object travelling with different speeds during different time intervals

1. If a body covers the distance d_1 and d_2 at a speed of s_1 and s_2 km/hr, respectively, in time t_1 and t_2 then the total time taken T is given by:

   $T = t_1 + t_2 = \frac{d_1}{s_1} + \frac{d_2}{s_2}$.

   The total distance covered is given by: $D = d_1 + d_2 = s_1 t_1 + s_2 t_2$.
2. While travelling a certain distance d, if a man changes his speed in the ratio m : n, then the ratio of time taken becomes n : m.
3. If a certain distance (d), say from A to B, is covered at 'a' km/hr and the same distance is covered again say from B to A in 'b' km/hr, then the average speed during the whole journey is given by:

   Average speed = $\left(\frac{2ab}{a+b}\right)\frac{km}{hr}$ ... (which is the harmonic mean of a and b)

   Also, if t_1 and t_2 is time taken to travel from A to B and B to A, respectively, the distance 'd' from A to B is given by:

   $d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$

   $d = (t_1 - t_2)\left(\frac{ab}{a-b}\right)$

   $d = (a - b)\left(\frac{t_1 t_2}{t_1 - t_2}\right)$

4. If a body travels a distance 'd' from A to B with speed 'a' in time t_1 and travels back from B to A i.e., the same distance with $\frac{m}{n}$ of the usual speed 'a', then the change in time taken to cover the same distance is given by:

   Change in time $= \left(\frac{n}{m} - 1\right) \times t_1$; for $n > m$

   $= \left(1 - \frac{n}{m}\right) \times t_1$; for $m > n$

SOLVED EXAMPLES

Q : If a boy goes to school at 6 km/hr and returns home at 4 km/hr, find his average speed.

A : Average speed = $\frac{2\times6\times4}{6+4}=\frac{48}{10}$ = 4.8 km/hr

Q : A man starts from B to K, another from K to B at the same time. After passing each other they complete their journeys in $3\frac{1}{3}$ and $4\frac{4}{5}$ hours, respectively. Find the speed of the second man if the speed of the first is 12 km/hr.

A : $\frac{1^{st}\text{man's speed}}{2^{nd}\text{man's speed}} = \frac{\sqrt{b}}{\sqrt{a}}=\sqrt{\frac{b}{a}}=\sqrt{\frac{4\frac{4}{5}}{3\frac{1}{3}}}=\sqrt{\frac{24}{5}\times\frac{3}{10}}=\sqrt{\frac{36}{25}}=\frac{6}{5}$

$\therefore\ \frac{12}{2^{nd}\text{ man's speed}}=\frac{6}{5}$ $\therefore$ 2^{nd} man's speed = $\frac{60}{6}$ = 10 km/hr.

Q : I shall be 40 min late to reach my office if I walk from my house at 3 km/hr. I shall be 30 min early if I walk at 4 km/hr. Find the distance between my house and the office.

A : Let the usual time taken be 't' hours and speed be x km/hr.

Distance = xt = $3\left(t+\frac{40}{60}\right)=4\left(t-\frac{30}{60}\right)$

$\therefore$ 3t + 2 = 4t − 2

$\therefore$ 4 = t

$\therefore$ Distance = $3\left(4+\frac{2}{3}\right)$ = 14 km.

Q : A man travels 120 km by ship, 450 km by rail and 60 km by horse taking altogether 13 hrs 30 min. The speed of the train is 3 times that of the horse and 1½ times that of the ship. Find the speed of the train.

A : If the speed of the horse is x km/hr; that of the train is 3x and that of the ship is $\frac{3x}{1\frac{1}{2}}$ = 2x km/hr

$\therefore\ \frac{120}{2x}+\frac{450}{3x}+\frac{60}{x}=\frac{27}{2}$ $\therefore\ \frac{60}{x}+\frac{150}{x}+\frac{60}{x}=\frac{27}{2}$

$\therefore\ \frac{270}{x}=\frac{27}{2}$ $\therefore$ x = 20 $\therefore$ Speed of the train = 60 km/hr

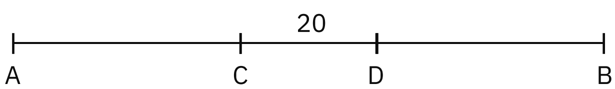
Q : A and B walk from P to Q, a distance of 21 km at 3 and 4 km/hr. B reaches Q, and immediately returns and meets A at R. Find the distance from P to R.

A : When they meet, both together have walked 2 × 21 = 42 km Since their speeds are as 3 : 4, distances travelled are also as 3 : 4

$\therefore$ Distance travelled by A = PR = $\frac{3}{7}$ × 42 = 18 km.

Q : A train after travelling 50 km from A meets with an accident and proceeds at $\frac{4}{5}$th of the former speed and reaches B, 45 min late. Had the accident happened 20 km further on, it would have arrived 12 min sooner(than if the accident occured at C). Find the original speed and the distance.

A : Let the speed be x km/hr.



When the speed becomes $\frac{4}{5}$th of the usual, time taken would become $\frac{5}{4}$th the usual, i.e., $\frac{1}{4}$th more of the usual time.

So, $\frac{1}{4}$th of the usual time taken to travel CB = 45 min.

$\therefore$ $\frac{1}{4}$th of usual time taken to travel CD (i.e. 20 km) = 12 min

$\therefore$ Usual time to travel 20 km = 48 min.

$\therefore$ Usual speed = $20 \times \frac{60}{48}$ = 25 km/hr.

Usual time taken to travel CB = 45 × 4 = 3 hrs.

$\therefore$ Distance CB = 25 × 3 = 75 km.

$\therefore$ Total distance = 50 + 75 = 125 km.

Q : A man travelling from A to B at 3 mph, takes half an hour rest at B, and returns to A at 5 mph. Total time taken is 3 hrs 26 min. Find the distance from A to B. (m ≡ miles)

A : Total time taken for travelling = 3 hrs 26 min. – 30 min. = 2 hrs. 56 min.

$= 2\frac{56}{60} = \frac{176}{60}$ hrs.

$\therefore$ Distance from A to B

$= \frac{176}{60}\left(\frac{3 \times 5}{3+5}\right) = \frac{176}{60} \times \frac{15}{8} = 5.5$ miles

Q : Walking $\frac{8}{7}$th of his original rate, a man reaches his office 3 minutes early. Find the usual time he takes to reach office.

A : $\left(1 - \frac{7}{8}\right)$ × Usual time = Change in time

∴ Usual time = 3 × 8 = 24 minutes.

Q : The ratio between the speed of Meena and Teena is 2 : 3. Meena takes 20 minutes more than Teena to walk from A to B. If Meena had walked at double her speed, find the time she would take to walk from A to B.

A : Ratio of speed of Meena and Teena is 2 : 3.

∴ Ratio of time taken = 3 : 2

If Teena takes x minutes to walk from A to B, then Meena takes x + 20 minutes.

∴ $\frac{x+20}{x} = \frac{3}{2}$ ∴ 2x + 40 = 3x ∴ x = 40 minutes.

∴ Meena takes 60 minutes walking at her usual speed.

∴ At double the speed, she would take 30 minutes.

Concept Builder 1

1. A distance is covered by man in 2 hrs and 45 minutes at 4 kmph. How much time will be taken to cover same distance at 16.5 kmph?
2. The ratio of speeds of A and B is 4 : 5. If the time taken by B to cover a certain distance is 40 minutes, then the time taken by A to cover the same distance is:
3. A man performs $\frac{4}{7}$ of the total journey by train, $\frac{5}{21}$ by car and the remaining 8km on foot. His total journey is:
4. A person goes from X to Y at 20 kmph and comes back to X at 30 kmph. Find his average speed?
5. The ratio between the speeds of two trains is 4 : 5. If the first train runs 200 km in 2 hours, then the speed of the second train is:
6. A person starts at 6 km/hr from a particular city Q towards city R. After travelling 4 hours he realises that he will not reach in an estimated time and for this he increases his speed by 2 km/hr and travels for the next 4 hours and reaches city R in the estimated time. If next day he wants to travel from city Q to city R in the estimated time then at what speed should he travel?
7. Travelling at a speed of 4 km/hr from his home Rakesh reaches his office 45 minutes late. Next day, Rakesh travels at a speed of 5 km/hr from his home and reaches his office on time. What is the distance between his home and office?

Answer key

1. 40 mins.
2. 50 minutes.
3. 42 km.
4. 24 kmph.
5. 125km/hr.
6. 7 km/hr.
7. 15 km.

Relative speed

The word 'relative' means one with respect to another. Relative Speed means the speed of an object A with respect to another object B, which may be stationary, moving in the same direction as A or in the opposite direction as A.

Cases related to Relative Speed

Case 1

When one object is stationary and the other is moving.

Consider a boy standing on a platform and a train passing by. Here, the boy is stationary, while the train is moving. The relative speed of the train and the boy will be the speed of the train.

Relative speed of a stationary object and a moving object = Speed of the moving object.

Case 2

When the two objects are moving in the opposite direction.

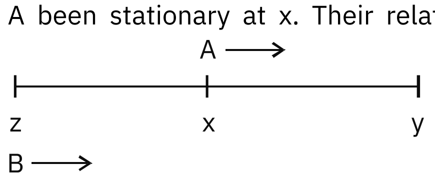
Consider two boys, A and B, standing at two opposite ends of a ground. Now, if they start walking towards each other in a straight line, they would meet sooner than had one of them been stationary and their relative speed will be the sum of their speeds.

Relative speed of two objects moving in opposite direction = Sum of their speeds.

Case 3

When the two objects are moving in the same direction.

Consider a boy 'A' walking from x to y. Now, if another boy 'B' walks from a point z which is behind x, in the same direction as A at a speed greater than A's, they would meet later than had A been stationary at x. Their relative speed is the difference of their speeds.



Relative speed of two objects moving in the same direction = Difference of their speeds.

Rules and Formulae for Relative Speed

1. Time taken by a moving object 'x' metres long in passing a stationary object of negligible length from the time they meet is same as the time taken by the moving object to cover 'x' meters with its own speed.

2. Time taken by a moving object 'x' metres long in passing a stationary object 'y' metres long from the time they meet, is same as the time taken by the moving object to cover 'x + y' metres with its own speed.

3. If two objects of length 'x' and 'y' metres move in the same direction at 'a' and 'b' m/s, then the time taken to cross each other from the time they meet

   $= \dfrac{\text{Sum of their length}}{\text{Relative speed}}$ i.e., $\dfrac{x+y}{a-b}$ if a > b or else, $\dfrac{x+y}{b-a}$.

4. If two objects of length 'x' and 'y meters, move in the opposite direction at 'a' and 'b' m/s, then the time taken to cross each other from the time they meet

   $= \dfrac{\text{Sum of their length}}{\text{Relative speed}} = \dfrac{x+y}{a+b}$

5. If the speed of a boat in still water is x km/hr. and the speed of the stream is y km/hr Speed while travelling with the stream i.e., speed downstream = (x + y) km/hr.

   Speed while travelling against the stream i.e., speed upstream = (x – y) km/hr.

6. Also, speed of the boat in still water = $\frac{1}{2}$(Speed with stream + Speed against stream)

   Speed of the river = $\frac{1}{2}$(Speed with stream – Speed against stream)

SOLVED EXAMPLES

Q : I row from A to B against the current in 8 hrs. and from B to A in 2 hrs. If the speed of the river is 9 m/sec., what is the speed of the boat in still water?

A : Let the speed of the boat in still water be x

$$\frac{\text{Time taken for up - journey}}{\text{Time taken for down - journey}} = \frac{8}{2} = \frac{4}{1}$$

$\therefore \frac{\text{Speed for up - journey}}{\text{Speed for down - journey}} = \frac{1}{4}$ (inverse)

$\therefore \frac{x - 9}{x + 9} = \frac{1}{4}$ (x is the speed of the boat)

$\therefore$ 3x = 45 $\therefore$ x = 15 m/sec.

Q : A train travelling at 25 km/hr, leaves Delhi at 9 a.m. and another leaves Delhi at 35 km/hr. at 2 p.m. in the same direction. How many kms from Delhi do they meet?

A : The first train has a start of 5 × 25 = 125 km. Relative speed = 35 – 25 = 10 km/hr.

$\therefore$ Time taken to meet = $\frac{\text{Distance}}{\text{Relative speed}} = \frac{125}{10}$ = 12.5 hrs. from 2 p.m.

$\therefore$ Distance from Delhi = $12\frac{1}{2} \times 35 = 437\frac{1}{2}$ kms.

Q : A man rows 27 km. with the stream and 15 km. against the stream taking 4 hrs each time. Find his speed in km/hr in still water and the speed in km/hr at which the stream flows.

A : Speed with the stream = $\frac{27}{4} = 6\frac{3}{4}$ km/hr.

$\therefore$ Speed against the stream = $\frac{15}{4} = 3\frac{3}{4}$ km/hr.

$\therefore$ Speed of the man in still water = $\frac{1}{2}\left(6\frac{3}{4} + 3\frac{3}{4}\right) = 5\frac{1}{4}$ = km/hr.

$\therefore$ Speed of the stream = $\frac{1}{2}\left(6\frac{3}{4} - 3\frac{3}{4}\right)$ = 1.5 km/hr.

Q : Two trains 121 metres long and 99 metres long are running in opposite directions, the first at 40 km/hr. and the second at 32 km/hr. In what time will they completely clear each other from the moment they meet?

A : Total distance to be travelled = 121 + 99 = 220 metres.

Relative speed = Sum of speeds = 72 km/hr. = $72 \times \frac{5}{18}$ = 20 m/s.

$\therefore$ Time required = $\frac{220}{20}$ = 11 seconds.

Q : How long does a train 110 metres long running at 36 km/hr. take to cross a bridge 132 metres in length?

A : Distance to be covered = 110 + 132 = 242 metres

Speed = $36 \times \frac{5}{18}$ = 10 m/sec. $\therefore$ Time taken = $\frac{242}{10}$ = 24.2 seconds.

Q : A car which was driven in fog passed a man walking at 3 km/hr. in the same direction. He could see the car for 4 minutes and upto a distance of 100 m. What was the speed of the car?

A : Distance travelled by the man in 4 mins. = $\frac{4}{60} \times 3000 = \frac{12000}{60}$ = 200 m.

Distance travelled by the car in 4 mins. = 200 + 100 = 300 m.

$\therefore$ Speed of the car = $\frac{300}{4}$ m/minute = $\frac{300}{4 \times 1000} \times 60 = 4\frac{1}{2}$ km/hr.

Q : A person can row $7\frac{1}{2}$ km/hr. in still water. It takes him twice as long to row up a distance as to row down the same distance. Find the speed of the stream.

A : Speed up-stream + Speed down-stream = $2 \times 7\frac{1}{2}$ = 15 km/hr.

Since the times taken are in the ratio 2 : 1, the speeds will be in the ratio 1 : 2.

$\therefore$ Speed up-stream = $\frac{1}{3} \times 15$ = 5 km/hr.

Speed down-stream = $\frac{2}{3} \times 15$ = 10 km/hr.

Speed of stream = $\frac{1}{2}(10 - 5)$ = 2.5 km/hr.

Alternatively,

If the speed of the stream is x km/hr., $= \frac{7\frac{1}{2} + x}{2} = \frac{7\frac{1}{2} - x}{1}$

$\therefore$ x = 2.5 km/hr.

Q : A hare sees a dog 100 metres away from her, and scuds off in the opposite direction at a speed of 12 km/hr. A minute later, the dog sees the hare and chases the hare at a speed of 16 km/ hr. After how much time does the dog catch up with the hare?

A : 12 km/hr. = $12 \times \frac{1000}{60}$ = 200 metres/min.

Distance of the hare from the dog when the dog sees the hare

= (100 + 1 × 200) = 300 metres

Since both are running in the same direction,

Relative speed (16 – 12) = 4 km/hr. = $4 \times \frac{1000}{60}$ m/min. = $\frac{200}{3}$ m/min.

∴ Time required to overtake = $\frac{300}{\frac{200}{3}} = \frac{300 \times 3}{200}$ = = $4\frac{1}{2}$ mins.

Q : A train leaving L at 3:10 p.m. reaches W at 5:00 p.m. One leaving W at 3:30 p.m. arrives in L at 5:50 p.m. At what time do they pass each other?

A : Let the distance be d and let them meet t mins after 3:10 p.m. or (t – 20) mins. after 3:30p.m.

3:10 p.m. ⟶ d km. 5:00 p.m.

L t W

5:50 p.m. ⟵ 3:30 p.m.

Their speeds are $\frac{d}{110}$ km/mins. and $\frac{d}{140}$ km/mins.

∴ Distance = $\frac{d}{110} \times t + \frac{d}{140}(t - 20) = d$ ∴ $\frac{t}{110} + \frac{t-20}{140} = 1$

Solving t = 70.4 mins.

∴ They meet at 3:10 + 70.4 mins. = 4 hrs 20.4 mins.

Q : A train moving at uniform speed takes 20 secs. to pass a cyclist riding in the same direction at 11 km/hr but only 9 secs. to pass a post. Find the length of the train.

A : If the length of the train is ℓ km. and its speed x km/hr., then,

$\frac{\ell}{x-11} = \frac{20}{3600}$ ... (i)

and

$\frac{\ell}{x} = \frac{9}{3600}$ ... (ii)

Dividing (i) by (ii), $\frac{\ell}{x-11} \times \frac{x}{\ell} = \frac{20}{9}$ ∴ $\frac{x}{x-11} = \frac{20}{9}$

∴ x = 20 km/hr.

∴ Length = 0.05 km = 50 metres.

Concept Builder 2

1. Two trains which are running in oppositite direction meet each other after 2 mins, total distance travel by both trains is 1.8 km. Find average of speed of both trains (in m/s).
2. A goods train runs at the speed of 54 km/hr and crosses a 110 metres long platform in 20 seconds. What is the length of the goods train?
3. Two trains running at the rate of 30 kmph and 36 kmph respectively in the same direction. The length of second train is 130 metres and the time taken by them to cross each other is 150 seconds (from the time they meet). Then the length of first train is:
4. Two trains of 400 m and 475 m in length runs at the speed of 45 km/hr and ‘X’ km/hr respectively in opposite directions on parallel tracks. The time taken by them to cross each other (from the time they meet) is 42 seconds, then find ‘X’.
5. A man can row at a speed of 8 km/hr in still water to a certain upstream point and back to the starting point in a river which flows at 4 km/hr. Find his average speed for the total journey.
6. A man can row upstream at 7 km/hr and downstream at 15 km/hr. Then the speed of the stream is:
7. A boat takes 7 hours for travelling downstream from point ‘A’ to point ‘B’ and coming back to point ‘A’ travelling upstream. If the speed of the stream is 3 kmph and the speed of the boat in still water is 7 kmph, what is the distance between A and B?
8. The speed of a river is 3 kmph. If a man takes twice as long to row up as to row down the river, the rate of man in still water is:

Answer Key

1. 7.5 m/s.
2. 190 metres.
3. 120 meters.
4. 30 m/s.
5. 6 km/hr.
6. 4 kmph
7. 20 km
8. 9 kmph

CLASS EXERCISE

Teaser

"That clock is going to strike noon"
Old Grandpa said to little Pete
"The two hands will be meeting soon
How often, each day, will they meet?"

"Twelve times, of course!" said Pete at once
Grandpa replied "No - try once more?"
"Oh dear" cried Pete "I'm such a dunce
The answer should be twenty-four!"

Is Pete correct?

Time, Speed and Distance: Basics

1. Raghu walks at 4 kmph and reaches his school in 1hr 45 min. How far is his school?
2. Ramu strolls at 4 kmph to reach his school, 1.4 km away. How long will he take?
3. Raju takes 4 sec to cover certain distance at 72 kmph. What is the total distance covered by him?
4. A man goes for a jog at 6 a.m. He jogs for 15 minutes and covers 2.8 km. He then walks backs to his starting point at a speed 25 % lower than his jogging speed. At what time will he reach back?
5. Akbar, Babar and Chandragupta set out for a certain place. Akbar reached in 20 minutes at 45 kmph. If Babar and Chandragupta reached 5 and 10 minutes later than Akbar respectively, find their speeds.
6. "The Great Indian Endurance Rally" is a new TV show which requires participants to run, cycle and swim. Jignesh swims for 3 hours covering 4 km. He then cycles for an hour at 12 kmph and runs 20 km in 8 hours. What is his average speed over the entire event?
7. At 7:42 a.m., Lucky gets into a bus to go to school. The bus travels at 42 kmph, and drops Lucky a short way away from school at 7:51 a.m. He then walks the remaining distance at 6 kmph, thereby reaching school exactly at 8 a.m. What is his average speed for the journey?
8. In a 2-man relay race, the first man covers his 400 m stretch at 18 kmph while the second covers his 400 m stretch at 22 kmph. What is their average speed for the entire race?

Time, Speed and Distance are related by the formula **Speed =** $\frac{\textbf{Distance}}{\textbf{Time}}$

Average speed can be found, in general, by the formula **Average Speed =** $\frac{\textbf{Total Distance}}{\textbf{Total Time}}$

Specifically, **if the time is the same** in two legs of a journey, the average speed is the **Arithmetic Mean** while **if the distance is the same** in two legs of a journey, the average speed is the **Harmonic Mean**

Some useful conversions: 1 kmph = $\frac{5}{18}$ m/s, 1 m/s = $\frac{18}{5}$ kmph, 1 mile = 1.6 km, 1 km = 1000 m

9. * A boy reaches his school, 1.4 km away, in 1hr 40 min. At what speed does he travel?
10. * A boy leaves for school at 10:40 a.m. He spends 5 hrs in school and comes back at 5:20 pm on the same day. He travels at a constant speed of 1.4 km/hr. How far away is his school?
11. * A boy travels at 4 kmph to reach school. How many metres does he travel per second?
12. * A boy travels to school at 4 kmph and returns back at the speed of 6 kmph. What is his average speed?
13. Ravi usually takes 1.4 hrs to reach school. If he walks at 75% of his usual speed, how much time will he take?
14. If Raj walks at 5/4th of his usual speed, he reaches 10 min early. How long does he usually take?

15. Akbar and Birbal start towards each other from their homes, 15 km apart, at 10 a.m. Akbar walks at a constant speed of 4 kmph, while Birbal walks at a constant speed of 6 kmph. When will they meet?
16. In the previous question, after Akbar and Birbal meet, they chat for half an hour and then leave for their respective homes. At what time will they reach home?
17. Daisy Duck and Donald Duck start moving towards each other from their homes, 24 km apart, at noon. Daisy walks at a constant speed of 5 kmph, while Donald walks at a constant speed of 7 kmph. How far away from each other will they be 10 minutes before they meet?
18. Ricky snatches Vicky's wallet and starts running away at a speed of 6 m/s. After 5 seconds, Vicky starts chasing Ricky at 8 m/s. After how much more time will Ricky be caught?
19. Mike is showing off his new car. He drives along the road at a speed of 20 m/s, overtaking a truck 12 m long which is moving in the same direction with a speed of 15 m/s. If he takes 3 seconds to overtake the truck, what is the length of his car?

Relative Speed:
The relative speed of two objects is the speed with which one object moves with respect to the other.
When two objects are moving in **opposite** directions, their relative speed is the **sum** of their speeds
When two objects are moving in the **same** direction, their relative speed is the **difference** of their speeds
However, the **lengths** of the objects involved always **add**!

20. * One fine day Anil started late by half an hour for work. By what % should he increase his speed to reach office in time if he usually takes 2 hrs to reach office?
21. * The ratio of time taken by P and Q is 7: 9 to cover the same distance. What is the ratio of their speeds?
22. * A bus 6 m in length takes two seconds to pass a truck, 8 m long, going in the opposite direction. If the bus driver is driving at 4m/s, at what speed is the truck moving?
23. * A deer sees a tiger 100m away and starts running away from it at the speed of 15 mps. The tiger sees the deer 5 seconds later and pursues it at the speed of 40 mps. How much later will the tiger catch the deer?

Directions for questions 24 – 26: Each of the following sets of questions is based on an independent scenario. Information given in a question is valid for all later questions in the set.

24. A 180 m long train is travelling at 36 kmph:
    a) How long will it take to pass an electric pole 6 m tall?
    b) If it is initially 100 m away from a signal, at what time will it have completely passed the signal?
    c) How long will it take to completely pass a 300 m long platform?
    d) A man is standing in the middle of the above platform. How long will the train take to cross him?

25. The Howrah Mail is a 180 m long train that travels at 54 kmph and the Coromandel Express is a 200 m long train that travels at 72 kmph. These trains run on parallel tracks.

a) How long will the Howrah Mail take to cross the Coromandel Express if they are running towards each other?

b) How long will the Howrah Mail take to cross the Coromandel Express if they are running in the same direction?

c) How long will the Coromandel Express take to cross the Howrah Mail if they are running in the same direction?

d) At 10 a.m., the Coromandel Express is 100 m behind the Howrah Mail and running in the same direction. At what time will it have completely crossed the latter?

26. A boat travels at the speed of 18 kmph in still water. It travels between points A and B, 80 km apart.

a) If the boat goes from A to B, downstream in 4 hours, how long will it take to return?

b) What is the normal speed of the current in the river?

c) If the speed of the river triples one day due to a rainstorm, how long will the round trip from A to B and back take on that day?

d) If the boat develops engine trouble and can consequently go at only one-third of its normal rate in still water, how long will the round trip take on a normal day?

e) At what speed must the current flow so that the round trip would take 22.5 hours with the boat in normal condition?

27) * If a train 200 m long crosses a bridge 350 m long in 11 seconds, what is the speed of the train?

28) * Find the time taken by a 240-foot roller-coaster moving at 56 ft/sec to pass through a 40-foot tunnel.

Challengers

1. Wilson starts for office at the same time every day. If he walks at 5 kmph he is 2 min late. If he walks at 10 kmph he is 4 min early. Find his correct speed to reach on time, and the distance to his office.

   1) 6 kmph, 1 km 2) 6.33 kmph, 0.8 km
   3) 6 kmph, 0.8 km 4) 7.5 kmph, 1 km

2. Prashant can row $7\frac{1}{2}$ kmph in still water. It takes him twice as long to row upstream as to row downstream. Find the speed of the stream.

   1) 2.5 kmph 2) 5 kmph 3) 3 kmph 4) 1.5 kmph

3. A train had engine trouble and hence its speed got reduced to 2/3rd of normal. It reached the destination at 3:12 p.m. Had the engine trouble occurred 48 km further on, the train would have reached at exactly 3 p.m. What is the normal speed of the train?

   1) 60 kmph 2) 80 kmph 3) 100 kmph 4) 120 kmph

4. Two trains A and B, whose speeds are in the ratio 4 : 3, cross each other in 18 seconds when they travel in opposite directions. Train A crosses a pole in 24 seconds. What is the time taken (in seconds) by train B to cross a pole?

PRACTICE EXERCISE-1

Directions for questions 1 to 3: Solve as directed.

1. Rohan travels at 40 kmph for 2 hrs, 60 kmph for the next 2 hrs and 80 kmph for the last 2 hrs. Find the average speed for the entire trip?
2. P covers a certain distance in 4.8 hrs. Q covers twice the distance in 8 hours. How much time will P take to cover the distance that Q covers in 4.5 hours?
3. Raman covers 3/7th of the distance by train, 5/14th of the distance by car and the remaining 9 km on foot. What is the total distance that he covered?

Directions for questions 4 to 17: Choose the correct alternative.

4. A train travelling at 42 km/hr. passes a cyclist going in the same direction in 9 secs.; if the cyclist had been going in the opposite direction, the train would have passed him in 5 secs. Find the length of the train.

   1) 75 metres 2) 60 metres 3) 90 metres 4) 80 metres

5. A train overtakes 2 persons walking at 2 km/hr and 4 km/hr respectively in the same direction and completely passes them in 9 seconds and 10 seconds. Find the length of the train and its speed in km/hr.

   1) 75 m, 18 km/hr 2) 80 m, 21 km/hr
   3) 60 m, 20 km/hr 4) 50 m, 22 km/hr

6. A policeman sees a thief 300m away and starts chasing him at a speed of 16 kmph. The thief is running at a speed of 12 kmph but after 4 minutes, the thief gets exhausted and falls down. What is the distance between the two now?

   1) 10 m 2) 25 m 3) 33.33 m 4) 66.67 m

7. A man rows upstream 13 km. and downstream 28 km. taking 5 hrs. each time. Find the speed of the current.

   1) 2 km/hr. 2) 1.5 km/hr. 3) 3 km/hr. 4) 3.5 km/hr.

8. A place B lies exactly midway on the path from A to C such that all the places are connected by roadway, waterway and railway. The average speeds by road, water and rail are 10 kmph, 5 kmph and 15 kmph respectively. Which of the following routes will take the maximum time to reach C from A?

   1) A to B by road and B to C by road
   2) A to B by water and B to C by rail
   3) A to B by rail and B to C by road
   4) A to B by road and B to C by water

9. Two trains, both 120 metres in length, move in the same direction. The faster train completely overtakes the slower one in 15 seconds. If the slower train were to move at half its speed, the overtaking would take 10 seconds. At what speeds are the two trains moving (in mps)?

1) 48 and 16 2) 32 and 16 3) 48 and 32
4) 64 and 48 5] 64 and 32

10. Two trains A and B start from two points P_1 and P_2 respectively at the same time and travel towards one another. The difference between their speeds is 10 kmph and train A takes one hour more to cover the distance between P_1 and P_2 as compared to train B. Also by the time they meet, train B has covered $\frac{200}{9}$ km more as compared to train A. What is the distance between P_1 and P_2?

1) 150 km 2) 200 km 3) 250 km 4) Data insufficient

11. A train travelling at 10 m/sec from A to B at 7 a.m. meets a train leaving B at 7:20 a.m. and coming to A at a speed $\frac{1}{3}$ times faster than the first train. If the distance from A to B is 68 km. then, at what distance from A will the two trains meet?

1) 72 km 2) 36 km 3) 60 km 4) 50 km

12. A car after travelling 18 km from a point A developed some problem in the engine and its speed became 4/5 of its original speed. As a result, the car reached point B 45 minutes late. If the engine had developed the same problem after travelling 30 km from A, then it would have reached B only 36 minutes late. The original speed of the car (in km per hour) and the distance between the points A and B (in km) is _____.

1) 25, 130 2) 30, 150 3) 20, 90 4) None of these

(Past CAT question)

13. Two trains are travelling in opposite direction at uniform speed 60 and 50 km/hr respectively. They take 5 seconds to cross each other. If the two trains had travelled in the same direction, then a passenger sitting in the faster moving train would have overtaken the other train in 18 seconds. What are the lengths of trains (in metres)?

1) 112.78, 45 2) 97.78, 55 3) 102.78, 50 4) 102.78, 55

(Past CAT question)

14. A man travels from A to B at a speed of x km/hr. He then rests at B for x hours. He then travels from B to C at a speed of 2x km/hr and rests for 2x hours. He moves further to D at a speed equal to twice of that taken to travel between B and C. He thus reaches D in 16 hours. If the distances A-B, B-C, C-D are all equal to 12 km, the time for which he rested at B could be _____.

1) 3 hours 2) 6 hours 3) 2 hours 4) 4 hours

(Past CAT question)

15. The distance between A and B is 72 km. Two men started walking from A and B at the same time towards each other. The person who started from A travelled uniformly with an average speed of 4 km/hr. The other man travelled with varying speeds as follows: In the first hour his speed was 2 km/hr, in the second hour it was 2.5 km/hr, in the third hour it was 3 km/hr, and so on. When will they meet each other?

    1) 7 hours   2) 10 hours
    3) 35 km from A   4) Midway between A and B

(Past CAT question)

16. A train travelled a certain distance at a uniform speed. Had the speed been 8 kmph more, the journey would have taken 3 hours less and had the speed been 10 kmph less, the journey would have taken 6 hours more. Find the distance travelled by the train.

    1) 480 km   2) 720 km   3) 800 km   4) 640 km

17. A boat sails 24 km downstream of a river stretch in 3 hours. How long will it take to cover the same distance upstream, if the speed of the current is one-third the speed of the boat in still water?

    1) 4 hours   2) 6 hours   3) 8 hours   4) 12 hours

Directions for questions 18 to 20: Solve as directed.

18. A motorboat travels 23 kmph in still waters. If it goes downstream from X to Y, a distance of 120 km, in four hours, how long will it take to return?

19. The life guards at Aksa beach spotted an unconscious man flowing downstream along the water towards the beach at 1m/s. They immediately set sail on their power boat upstream at a speed of 25m/s. As soon as they reached the man, they stopped the boat. It took them 14 seconds to bring him to consciousness inside their boat while the engine of the boat stopped and the boat itself was flowing downstream towards the shore at the speed of the water current. Immediately after the man regained his conciousness, they sailed at their original speed downstream and were able to return to the shore in 41 seconds. At what distance had the lifeguards initially spotted the man?

20. At 7:00 AM, two brothers Ajay and Vijay started from their home to their school to appear for an examination. Ajay walked at the speed of 6 km/hr while Vijay rode a bicycle at the speed of 18 km/hr. On reaching school, Vijay realized that he forgot his hall-ticket. So he immediately headed back home riding at the same speed. On his way back, at 7:15 AM, he met Ajay, who was on his way to school. What is the distance between the home and the school (in metres)?

PRACTICE EXERCISE-2

Directions for questions 1 and 2: Solve as directed.

1. A boy travels for 40 minutes at the speed of 4 km per hour and the next 40 minutes at the speed of 6 kmph. What is his average speed?

2. The ratio of speeds of two brothers Jenish and Jinesh is 3: 2. Jinesh takes 20 min more than Jenish to walk from home to school. If Jinesh walks at twice his usual speed, find the time he will take to walk from home to school.

Directions for questions 3 and 4: Refer to the data below and answer the questions that follow.

It takes Kim 4 hours to drive from her house to her office at her normal speed. If she reduces her speed by 3kmph, she takes 40 minutes extra.

3. Find the distance between Kim's house and her office.

   1) 24 km 2) 84 km 3) 93 km 4) None of these

4. If Kim's car breaks down after 3 hours, while driving at her normal speed, and then she decides to walk to the office, what should be her walking speed so that she is just 2 hours late to the office than the anticipated time?

   1) 3.5 kmph 2) 5 kmph 3) 3 kmph 4) 7 kmph

Directions for questions 5 to 17: Choose the correct alternative.

5. The speeds, of a sports car in four different practice runs on the same track, are 100, 120, 80 and 120 miles/hour. Find its average speed (approximately) in miles/hour.

   1) 100 2) 102 3) 96 4) 110

6. Two trains A and B of length 88 yds. and 132 yds. respectively, are approaching each other from opposite directions at 60 and 45 mph, respectively. How long do they take to pass each other?

   $\left(1 \text{ yard} = 3 \text{ feet}, \frac{\text{miles}}{\text{hr}} = \frac{22 \text{ feet}}{15 \text{ sec.}}\right)$

   1) 5 secs. 2) $3\frac{1}{4}$ sec. 3) $4\frac{2}{7}$ secs. 4) $6\frac{3}{8}$ secs.

7. A train 150m long travels at a speed of 100 kmph. A man standing on top of the train runs from one end of the train to the other at a speed of 20 kmph in the same direction as the train. How many seconds will he take to reach the other end?

1) 27 2) 22.5 3) 2.25 4) None of these

8. A policeman, who is 80m away from a thief, starts chasing him. The thief is running at a speed of 10m/s. After exactly 8 second, the distance between them is 40m. Find the policeman's speed.

1) 10.5 m/s. 2) 16.67 m/s. 3) 15 m/s. 4) None of these

9. If the current flows at 2 mph and it takes me 3 hrs. to row 9 miles upstream, how long will it take to return?

1) $1\frac{2}{7}$ hrs. 2) $2\frac{5}{7}$ hrs. 3) $2\frac{2}{3}$ hrs. 4) $1\frac{5}{7}$ hrs.

10. Train A started from Mumbai to Calcutta. At the same time, train B started from Calcutta to Mumbai. The two trains are sent for a testing wherein they travelled at constant speed, reached the destination and returned to the source without halting. The trains meet each other for the first time in 18 hours after starting. The two trains reached their respective sources i.e., A to Mumbai and B to Calcutta in 17 hours 4 minutes and 19 hours 16 minutes respectively after they cross each other for the second time. After how many hours from the starting point, did they cross each other second time?

1) 36 hours 2) 45 hours 3) 54 hours 4) 72 hours

11. A hunter sees a deer 200m away and immediately shoots an arrow which can travel at a speed of 30 m/sec for 300 metres after which it drops down (immediately). At what speed should the deer run so as to miss the arrow?

1) more than 10 m/sec 2) more than 30 m/sec
3) more than 5 m/sec 4) None of these

12. I take 4 hours less to row down a 12 mile stream than I take to row upstream. For this 24 mile roundtrip, if I double my rowing speed, I would take half an hour less to row downstream than to row upstream. Find the speed of the stream in miles/hr.

1) 6 2) 8 3) 0 4) 10 5) None of these

13. Two trains A and B start from stations X and Y towards each other. B leaves station Y half an hour after train A leaves station X. Two hours after train A has started, the distance between trains A and B is $\frac{19}{30}$th of the distance between stations X and Y. How much time would it take each train (A and B) to cover the distance X to Y, if train A reaches half an hour later to its destination as compared to B?

1) 8 hrs, 7 hrs 2) 5 hrs, 4 hrs 3) 10 hrs, 9 hrs 4) 9 hrs, 8 hrs

14. A man starts cycling at 12.00 a.m. at a speed of 5 kmph. A train, A, starts running on a track with a speed of 100 kmph at 5.00 a.m. Another train, B, starts running on the adjacent parallel track with a speed of 120 kmph at 7.00 am from the same station and in the same direction. Find the distance travelled by the man when the two trains meet.

1) 68 km 2) 85 km 3) 90 km 4) 92 km

15. Every day Neera's husband meets her at the city railway station at 6.00 p.m. and drives her to their residence. One day she left early from the office and reached the railway station at 5.00 p.m. She started walking towards her home, met her husband coming from their residence on the way and they reached home 10 minutes earlier than the usual time. For how long did she walk?

    1) 1 hour 2) 50 minutes 3) $\frac{1}{2}$ an hour 4) 55 minutes

    (Past CAT question)

16. I started climbing up the hill at 6 a.m. and reached the temple at the top at 6 p.m. The next day I started coming down at 6 a.m. and reached the foothill at 6 p.m. I walked on the same road. The road is so short that only one person can walk on it. Although I varied my pace along the way, I never stopped on my way. On the basis of this, which of the following must be true?

    1) My average speed downhill was greater than that uphill.
    2) At noon, I was at the same spot on both the days.
    3) There must be a point which I reached at the same time on both the days.
    4) There cannot be a spot which I reached at the same time on both the days.

    (Past CAT question)

17. A bus started from the bus stand at 8.00 a.m. After staying for 30 minutes at its destination, it returned back to the bus stand. The destination is 27 miles from the bus stand while the speed of the bus was 18 mph. During the return journey, the bus traveled 50% faster than the original journey. At what time did it return to the bus stand?

    1) 11:30 a.m. 2) 11:00 a.m. 3) 12:30 p.m. 4) 12:00 noon

Directions for question 18: Solve as directed.

18. If the speed of a car is increased by 20 km/h, it reaches its destination 2 hours earlier. If the speed is further increased by 20 km/h, the car reaches 3 hours earlier than its usual time. What will be the speed (in km/h) of the car if it reaches 2 hours late?

Directions for questions 19 and 20: Choose the correct alternative.

19. A hare and a tortoise decided to race each other. They began running at the same time. After the hare gained a lead of 35 m, he stopped to rest while waiting for the tortoise to at least reach the point where he had reached. But he lost track of time and when he woke up after $4\frac{2}{3}$ minutes, he saw that the tortoise had gained a lead of 35 m. So he immediately started running and lost the race by 7 m. What is the ratio of the time that the hare ran before going to sleep and the time he ran after waking from sleep?

    1) 4 : 5 2) 5 : 4 3) 1 : 1 4) Data insufficient

20. A cruise carries passengers from the port to a certain point upstream and brings them back to the port. It travels upstream for 1.5 hours and downstream for half an hour. However, on one occasion, after travelling 1.5 hours upstream, the ship developed a technical error, because of which its speed reduced. At the same time, the speed of the stream suddenly increased due to a thunderstorm. The ship managed to return to the port in time. What was the ratio of the % increase in the speed of the stream to the % decrease in the speed of the ship?

1) 1 : 2 2) 2 : 1 3) 3 : 2 4) 3 : 4

QA-3.8 | WORK, PIPES AND CISTERN THEORY

Work

When we say, A has worked for an hour, we actually refer to the amount of job that is done in one hour. The same job can probably be done by B in less time, if the speed of doing the job is more than that of A. Therefore, we can say B is more efficient than A.
This chapter introduces you to the different concepts in Work and problems based on the application of the concepts.

Definition

Work is defined as the amount of job assigned or the amount of job actually done.
Work is always considered as a whole or 1. There exists an analogy between the time-speed-distance problems and work.
Work ≡ 1 ≡ Distance.
Rate at which the work is done = Speed
Number of days required to do the work = Time

Rules and Formulae for Work related problems

1. If A can do a piece of work in 'a' number of days, then in one day $\frac{1}{a}$th of the work is done. Conversely, if a man does $\frac{1}{a}$th of a work in 1 day, then he can complete the work in $1 \div \frac{1}{a} = a$ days.

2. If A is 'x' times as good a workman as B, then he will take $\frac{1}{x}$th of the time taken by B to do the same work.

3. If A and B can do a piece of work in 'x' and 'y' days respectively, then working together, they will take $\frac{xy}{x+y}$ days to finish the work and in one day, they will finish $\frac{x+y}{xy}$th part of the work.

4. To compare the work done by different people, first find the amount of work each can do in the same time.

5. If the number of men to do a job is changed in the ratio a : b, then the time required to do the work will be in the ratio b : a, assuming the amount of work done by each of them in the given time is the same, or they are identical.

6. If two men A and B together can finish a job in 'x' days and if A working alone takes 'a' days more than A and B working together and B working alone takes 'b' days more than A and B working together then $x = \sqrt{ab}$.

7. To do a piece of work, the number of men employed and the number of days required to do the work are in inverse proportion, also, the number of men employed and the hours worked per day are in inverse proportion.

SOLVED EXAMPLES

Q : A group of labourers do a piece of work in 10 days, but five of them are absent and so the rest do the work in 12 days. Find the original number of labourers.

A : More men, less days

Men	Days
x	10
x – 5	12

$\frac{x}{x-5} = \frac{12}{10}$ $\quad\therefore\ 12x - 60 = 10x \quad \therefore\ x = 30$ men

Q : If 3 men or 5 women take 26 days to do a work, how long will 7 men and 10 women take?

A : 10 women are equivalent to 6 men.

More men, fewer days

Men	Days
3	26
(7 + 6)	x

$x = 26 \times \frac{3}{13} = 6$ days

Q : If 8 men can reap 80 hectares in 24 days, how many hectares can 36 men reap in 30 days?

A : More men, more hectares; more days, more hectares.

Men	Days	Hectares
8	24	80
36	30	x

$\frac{x}{80} = \frac{36}{8} \times \frac{30}{24}$

$\therefore$ Number of hectares $= 80 \times \frac{36}{8} \times \frac{30}{24} = 450$

Q : If 30 men working 7 hours per day can do a work in 18 days in how many days will 21 men working 8 hours a day do the same work?

A : Fewer men, more days; more hrs., fewer days.

Men	Hrs./day	Days
30	7	18
21	8	x

$\frac{x}{18} = \frac{30}{21} \times \frac{7}{8}$

Number of days $= 18 \times \frac{30}{21} \times \frac{7}{8} = 22.5$

Q : Two men and 7 boys can do a piece of work in 14 days. 3 men and 8 boys can do it in 11 days. In how many days can 8 men and 6 boys do a work 3 times as big as the first?

A : 2 men + 7 boys in 14 days $\Rightarrow$ 28 men + 98 boys in 1 day

3 men + 8 boys in 11 days $\Rightarrow$ 33 men + 88 boys in 1 day

$\therefore$ 28 men + 98 boys = 33 men + 88 boys $\quad\therefore$ 2 boys $\equiv$ 1 man

Now, 2 men + 7 boys = 11 boys; 8 men + 6 boys = 22 boys

More boys, fewer days; more work, more days

Boys	Days	Work
11	14	1
22	x	3

$\therefore \dfrac{x}{14} = \dfrac{11}{22} \times \dfrac{3}{1}$ $\quad\therefore$ Number of days = 21 days.

Q : A, B and C can do a work in 6, 8 and 12 days respectively. B and C work together for 2 days, then A takes C's place. How long will it take to finish the work?

A : Work done by B and C in 2 days = $2\left(\dfrac{8+12}{96}\right) - 2 \times \dfrac{20}{96} = \dfrac{5}{12}$ th part.

Remaining work = $\dfrac{7}{12}$ th part.

Work done by A and B in 1 day = $\dfrac{6+8}{48} = \dfrac{14}{48} = \dfrac{7}{24}$

$\therefore$ Number of days required = $\dfrac{7}{12} \div \dfrac{7}{24}$ = 2 days.

Q : 2 women, A and B can mow a field in 8 and 12 hours respectively. They work for an hour alternately, A beginning at 9 a.m. When will the work be completed?

A : In the first 2 hours, A and B will mow $\left(\dfrac{1}{8} + \dfrac{1}{12}\right) = \dfrac{5}{24}$ th of the field.

In 4 hrs, $\dfrac{10}{24}$ th, in 6 hrs $\dfrac{15}{24}$ th, in 8 hrs $\dfrac{20}{24}$ th and now the work left is $\dfrac{4}{24}$ th.

Now, A works for 1 hour and does $\dfrac{1}{8}$ th work; Work left = $\dfrac{1}{24}$ th which B can do in $\dfrac{1}{2}$ hr.

$\therefore$ Total time taken = $8 + 1 + \dfrac{1}{2} = 9\dfrac{1}{2}$ hrs.

So the work will be over at 6:30 p.m.

Q : To do a piece of work, B takes 3 times as long as A and C together, and C twice as long as A and B together. If the three together can complete the work in 10 days, how long would each take by himself?

A : 3 times B's daily work = (A + C)'s daily work.

$\therefore$ 4 times B's daily work = (A + B + C)'s daily work = $\dfrac{1}{10}$ (adding B's daily work to both sides)

$\therefore$ B's daily work = $\frac{1}{40}$th $\quad\quad \therefore$ B takes 40 days.

Similarly, 2 times C's daily work = (A + B)'s daily work

$\therefore$ 3 times C's daily work = (A + B + C)'s daily work = $\frac{1}{10}$

$\therefore$ C's daily work = $\frac{1}{30}$th work. $\quad \therefore$ C takes 30 days.

$\therefore$ A's daily work = $\frac{1}{10} - \left(\frac{1}{40} + \frac{1}{30}\right) = \frac{1}{10} - \frac{7}{120} = \frac{5}{120}$

$\therefore$ A takes $\frac{120}{5}$ = 24 days.

Concept Builder 1

1. P does the job in 11 days while Q takes 12 days and R takes 22 days for the same job. P and Q started working together. But, after two days they left and R had to complete the job. Find the number of days required to complete the job?
2. A and B can do a piece of work in 10 days, B and C in 12 days, and C and A in 15 days. In how many days will they finish it together?
3. A can do a piece of work in 10 days. If B is 25% more efficient than A, then find the number of days required by B to do the same piece of work.
4. A and B can do a piece of work in 12 days. B and C can do it in 15 days; A and C can do it 20 days. Who among these will take the least time if put to do it alone?
5. A can do a piece of work in 7 days working 9 hours each and B can do it in 6 days working 7 hours each. How long will they take to do the work together, working $8\frac{2}{5}$ hours a day?
6. A can do a piece of work in 15 days which B can do in 18 days. In how many days will they finish the work, both working together?
7. A and B together complete a piece of work in 40 days while B alone can complete the same work in 60 days. A alone will be able to complete the same work in:
8. X is half as good a workman as Y and together they finish a piece of work in 24 days. The number of days taken by X alone to finish the work is:
9. P can do a piece of work in 10 days, which Q can finish in 15 days. If they work at it on alternate days with P begining in how many days the work will be finished.

Answer Key

1. $16\frac{1}{3}$ days. 2. 8 days. 3. 8 days. 4. B
5. 3 days. 6. $8\frac{2}{11}$ days. 7. 120 days. 8. 72 days.
9. 12 days.

Pipes & Cisterns

In the previous section, you have learnt about the concept of work. Pipes and Cisterns is a special application of the same. Filling or emptying a cistern can be considered as work done.
This section introduces you to problems based on work with relation to pipes and cisterns.
A pipe connected with a cistern is called an inlet, if it fills the cistern.
A pipe connected with a cistern is called an outlet, if it empties the cistern.

Important formulae for pipe & cisterns related problems

1. If an inlet pipe fills a cistern in 'a' hours, then $\frac{1}{a}$th part is filled in 1 hour.

   The **concept of negative work** implies work that is destructive in nature. For example, if an outlet pipe empties a cistern in 'a' hours, then $\frac{1}{a}$th part is emptied in 1 hour. In this case, the outlet pipe is doing negative work.

2. If pipe A is 'x' times bigger than pipe B, then pipe A will take $\frac{1}{x}$th of the time taken by pipe B to fill the cistern.

3. If A and B fill a cistern in 'm' and 'n' hours, respectively then together they will take $\frac{mn}{m+n}$ hours to fill the cistern and in one hour $\frac{m+n}{mn}$th part of the cistern will be filled.

   Similarly, 'A' and 'B' empty a cistern in 'm' and 'n' hours, respectively, then, together they will take $\frac{m+n}{mn}$ hours to empty the cistern and in one hour $\frac{m+n}{mn}$th part of the cistern will be empty.

4. If an outlet pipe empties the cistern in 'n' hours and an inlet pipe fills a cistern in 'm' hours then the net part filled in 1 hour when both the pipes are opened is $\left(\frac{1}{m}-\frac{1}{n}\right)$ i.e., $\frac{n-m}{mn}$ and the cistern will get filled in $\left(\frac{mn}{n-m}\right)$ hours.
   For the cistern to get filled, it is necessary that $m < n$. If $m > n$, the cistern will never get filled.
   In general, Net part filled of a cistern = (Sum of work done by inlets) – (Sum of work done by outlets)

5. If an inlet pipe fills a cistern in 'a' minutes, takes 'x' minutes longer to fill the cistern due to a leak in the cistern, then the time in which the leak will empty the cistern is given by a $\times \left(1+\frac{a}{x}\right)$.

6. If two pipes A and B can fill a cistern in 'x' minutes and if A alone can fill it in 'a' minutes more than 'x' minutes and B alone can fill it in 'b' minutes more than 'x' minutes then $x = \sqrt{ab}$.

SOLVED EXAMPLES

Q : A tank 9 ft. by 5 ft. by 2 ft. has a supply pipe pouring in 576 in^3 of water in a minute and an exhaust pipe emptying it in 3 hours. If the tank is full, and both pipes are open, how many hours will it take to empty it?

A : Volume of the tank = $9 \times 5 \times 2 \times 12 \times 12 \times 12$ in^3

Volume of water exhausted in 1 min = $\frac{90 \times 12 \times 12 \times 12}{3 \times 60}$ = 864 in^3

$\therefore$ The combined effect of the two pipes in 1 min is (864 – 576)

i.e. 288 in^3 of water is removed in 1 min.

$\therefore$ Time required to empty the tank = $\frac{90 \times 12 \times 12 \times 12}{288 \times 60}$ = 9 hrs.

Q : Pipes A and B can fill a cistern in 20 and 30 minutes and C can empty it in 15 minutes. If the three are opened and closed one after the other successively for 1 min each in that order, how soon will the cistern be filled?

A : Part filled in 3 min. = $\frac{1}{20} + \frac{1}{30} - \frac{1}{15} = \frac{1}{60}$th

Part filled in 55 × 3 min. = $\frac{55}{60}$th

Part filled in 165 + 1 min. = $\frac{55}{60} + \frac{1}{20} = \frac{58}{60}$th

Part filled in 166 + 1 min. = $\frac{58}{60} + \frac{1}{30} = \frac{58+2}{60} = \frac{60}{60}$ = full

It takes 167 minutes to fill the cistern.

Q : A bath can be filled by the cold water pipe in 10 minutes and by the hot water pipe in 15 minutes. A person leaves the bathroom after turning on both. He returns just when the bath should have been full. Finding however, the waste pipe has been open, he closes it. In 4 mins. more, the bath is full. In what time will the waste water pipe empty it?

A : Time taken by the two pipes to fill it = $\frac{ab}{a+b} = \frac{150}{25}$ = 6 minutes.

$\therefore$ For six minutes, the three taps worked, and for the last 4 minutes, the first two taps worked and the bath was full. If x is the time taken to empty the bath, then,

$\frac{6}{10} + \frac{6}{15} - \frac{6}{x} + \frac{4}{10} + \frac{4}{15} = 1$ $\qquad \therefore \frac{10}{15} = \frac{6}{x}$ $\qquad \therefore x = 9$ mins.

Q : 4 pipes can fill a reservoir in 15, 20, 30 and 60 hrs. respectively. The first was opened at 6 a.m., second at 7 a.m., third at 8 a.m. and fourth at 9 a.m. When will the reservoir be full?

A : Let the time be t hours after 6 a.m.

$$\therefore \frac{t}{15} + \frac{(t-1)}{20} + \frac{(t-2)}{30} + \frac{(t-3)}{60} = 1$$

$$\therefore 4t + 3(t - 1) + 2(t - 2) + (t - 3) = 60$$

$$\therefore t = 7 \text{ hrs.}$$

$\therefore$ It is filled at 1 p.m.

Q : A barrel contains 36 gallons of beer at 12 noon. One tap draws a pint in every 4 minutes and another draws a quart every 6 minutes. How much beer will be left at 12 minutes past 8 p.m.? (4 quarts = 8 pints = 1 gallon)

A : Capacity of the barrel = 288 pints.

Beer removed by the first tap = $\frac{8 \times 60 + 12}{4}$ = 123 pints

Beer removed by the second tap in 492 minutes = $\frac{2}{6}$ × 492 = 164 pints

Total beer removed = 287 pints

$\therefore$ Quantity of beer remaining = 1 pint.

Concept Builder 2

1. A cistern has a leak which would empty it in 10 hours. A tap is turned on, which admits 4 litres a minute into the cistern, and it is now emptied in 12 hours. How many litres does the cistern hold?
2. A barrel contains 56 litres of kerosene. It has two taps. One tap draws 500 ml in every 6 minutes. After first 5 litres are drawn from barrel, the second tap starts. It draws 1 litre in every 5 minutes. How many hours will be taken by both taps to empty the tank?
3. A man fills up a vessel with a pipe in 10 minutes. Incidentally, there is a leak in the vessel which would empty the vessel in 20 minutes. After how many minutes can the vessel be filled up, if the pipe and the leak function simultaneously?
4. To fill a cistern, pipes A, B and C takes 2 hrs, 3 hrs and 6 hrs respectively. The time in hours that the three pipes together will take to fill the cistern is:
5. Two pipes A and B can fill a tank in 6 hrs and 9 hrs respectively. While a third pipe ‘C’ empties the full tank in 3 hours. If all the three pipes operate simultaneously, in how much time will the tank be emptied?
6. Two pipes A and B can fill a tank in 3 hours and 6 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?
7. A tap can fill a tank in 45 minutes and anothers tap can empty it in 3 hours. If both taps are opened at 11.30 am. Then the tank will be filled by:
8. Two pipes A and B can fill a cistern 15 min and 20 min. respectively. Both pipes are opened the cistern will be filled in just 12 min, if the pipe B is turned off after:
9. A water tank is three-fifth full. Pipe A can fill a tank in 8 minutes and pipe B can empty it in 5 minutes. If both the pipes are open, how long will it take to empty/fill the tank completely?

Answer Key

1. 14400 litres. 2. 4 hours 3. 20 minutes. 4. 1 hr
5. 18 hrs. 6. 2 hours. 7. 12.30 p.m 8. 4 min.
9. 8 mins to empty.

CLASS EXERCISE

Teaser

A sage stays atop a mountain in the Himalayas. Every morning he starts at 5:00 and reaches the foothills at 9:00. He travels non-uniformly, walking leisurely at times and at times breaking into a trot. He also takes rest at some points, but never digresses from his path. In the evening he starts at 5:00 to climb up in the same manner on the exact same path and reaches the top exactly at 9:00.

On any given day, will the sage be at any point on his route at the same time in the morning and evening?

1] Certainly 2] Certainly Not 3] Cannot Say

A piece of work

1. 5 men complete a piece of work in 25 days. In how much time will 15 men complete the work?

2. If a man takes 15 days to complete a piece of work, what part of the work will 2 men complete in 5 days?

3. Some workers complete a piece of work in 60 days. If 8 more workers are employed, they take 10 days less. What is the original number of workers?

4. Twelve men complete a piece of work in 8 days. Three days after they start, another 3 men join them. How many days later will they complete the work?

5. Three men can complete a piece of work in 10, 12 and 15 days respectively. They worked on it together and received a total amount of Rs.4500 for the job. How should they distribute the amount amongst themselves?

More work

6. Calvin's father asks Calvin to clear the snow from the sidewalk. Calvin says, "Dad it'll take me 8 hours to complete the work! Why don't you ask Hobbes instead? He is twice as fast; he'll finish it in just 4 hours." "I have a better idea" says Calvin's father, "Both of you work at it together."

   a) How long will it take Calvin and Hobbes to complete the work together?

   b) Calvin and Hobbes start on the job together at 9 am but after 2 hours Calvin gets bored and leaves. At what time will Hobbes complete the work?

7. Miss Wormwood asks Calvin to complete an 8-page exercise set in Math. Susie blurts out, "M'am, shall I do that too? I can complete it in just 2 hours!" Miss Wormwood replies, "Well, if you help him out, the two of you will finish it in just 1hr 40 min." Calvin thinks to himself, "If I take Hobbes' help, I will finish the task in 4 hours".

   a) If the three of them work at it together, how long will it take them to finish the task?

   b) Calvin glances at the exercise and realises he has no clue how to solve it. So he dumps the task completely on Hobbes. How long will Hobbes take to finish it alone?

8. Calvin and Hobbes together take 20 minutes to build a snowman. Hobbes and Susie together take 30 minutes for the same while Calvin and Susie together take 40 minutes.

   a) How long will Susie take to build a snowman alone?

   b) If Calvin and Hobbes together make 24 snowmen, how many snowmen will Susie alone make in the same time?

Work on pipes

For questions 9 and 10: Refer to the following data and answer the questions. Do not carry additional data provided in a question to the next questions.

A 500-litre water storage tank is situated at the top of a building at a height of 300 feet. The tank, which is completely full, has two outlet pipes and one inlet pipe. Veeru climbs up to the tank and opens an outlet which can empty a full tank in 10 minutes. At the same time Jay opens an inlet pipe which can fill an empty tank in 12 minutes.

9. In how much time will the tank be empty?

10. When the tank becomes half empty, Veeru opens the other outlet pipe which can empty a full tank in 15 minutes. In how much time (from the start) will the tank be empty?

11. There are 3 pipes P, Q and R, which could fill a certain container in 38, 44 and 28 minutes respectively. The container is initially empty. Pipe P is opened at 9:41 a.m. 8 minutes later, pipe Q is opened. How many minutes after this should pipe R be opened so that the container will get filled at exactly 10 a.m.?

12. A cistern has 3 pipes P, Q and R. P and Q can fill it in 6 and 5 hours respectively and R can empty it in 2 hours. If these pipes are opened in order (i.e. P, Q and R) at 6 a.m., 7 a.m. and 8 a.m. respectively, then when will the cistern be empty?

Still more work

13. 32 workers working 10 hours a day can complete one-third of the work in 8 days. After the 1st day, 32 additional workers were employed and all of them (i.e. the original set of workers plus the additional workers employeed) worked 9 hours per day. In how many days will the remaining work be completed?

14. * 20 workers were employed to do a task in 40 days. But some of the workers had to leave after 16 days. The remaining workers completed the work in 46 days from the day the work was started initially. How many workers left after 16 days?

15. A man, a woman and a boy can complete a piece of work in 20, 30 and 60 days respectively. How many boys should join 2 men and 8 women to complete the work in 2 days?

16. 8 women or 6 boys do the work of 4 men. 15 boys and 5 men are employed together to complete a job in 12 days. How long will 3 men, 4 women and 6 boys take to complete the job together?

17. * In 10 days, 6 horses and 2 mules complete a task which takes 4 horses and 4 mules 12 days to complete. What is the ratio of rate of work of a horse to that of a mule?

Challengers

1. Ten men started on a job. After some days four men left. So, instead of 40 days the job took 50 days in all. After how many days did the men leave?

2. Two taps fill a tank in 6 hours and 8 hours respectively. If they are opened turn by turn, each for an hour starting with the 1st tap, then after how many hours will the tank be full?

3. A king is planning to build a tomb that would take ten masons ten days. Unfortunately, on the first day only 1 mason is available. On the second day two more join. This continues, with 2 more masons joining in the work every day till the work is complete. How many days will it take to build the tomb?

4. There is an empty tank to be filled with water using the three pipes ‘x’, ‘y’ & ‘z’. Pipe ‘x’ & ‘y’ alone can fill the tank completely in 30 hours and 45 hours respectively. If the pipes ‘x’ & ‘y’ are opened in alternate hours along with ‘z’ (i.e. in the first hour pipes ‘x’ & ‘z’ fill the tank, in the second hour pipes ‘y’ & ‘z’ fill the tank & so on), then the tank is 70% filled in 15 hours. If the supply of through pipe ‘z’ is 75% of the original supply when it is opened along with pipe ‘x’ or pipe ‘y’, then what is the time taken by pipe ‘z’ alone to fill the tank completely?

   1) 45 hours 2) 40.5 hours 3) 56.25 hours 4) 50 hours

5. 11 men and 8 women can complete a piece of work in 5 days, whereas 11 boys and 3 women can complete the same work in 6 days. In how many days will 1 man, working along-with 1 woman and 1 boy complete the work?

   1) 11 days 2) 30 days 3) 26 days 4) 21 days

PRACTICE EXERCISE-1

DIRECTIONS for questions 1 to 8: Choose the correct alternative.

1. A is thrice as good a workman as B and so takes 60 days less than B for doing a job. Find the time in which they can do it together.

   1) 20 days 2) $21\frac{1}{2}$ days 3) $22\frac{1}{2}$ days 4) $23\frac{3}{4}$ days

2. Two pipes can fill a cistern in 10 and 12 hours respectively. A third pipe can empty the cistern in 15 hours. If all of the three pipes are opened simultaneously how much time will they take to completely fill the cistern?

   1) $8\frac{4}{7}$ hours 2) $4\frac{3}{7}$ hours 3) $7\frac{4}{8}$ hours 4) $4\frac{7}{8}$ hours

3. 3 pipes can fill a reservoir in 10, 15 and 20 hours respectively. If the three taps are opened one after another in the given order, with a certain fixed time gap between them, the reservoir fills in 5 hours. Find the time gap.

   1) $\frac{1}{2}$ hr 2) $\frac{1}{4}$ hr 3) $\frac{3}{4}$ hr 4) 1 hr

4. A alone completes some work in 18 days and B alone can do it in 16 days. They work together for 5 days, then B quits and rejoins after 3 days, when A quits the work completely. Find the number of days required to finish the job.

   1) 9 days 2) 10 days 3) 13 days 4) None of these

5. A can paint a wall in 11 days. How long would it take (approximately) for B and C to paint the same wall together if it is known that to paint a smaller wall, individually, A took 7 days, B took 10 days and C took 12 days?

   1) 9 days 2) 15 days
   3) 60 days 4) Cannot be determined

6. 8 men and 3 women finished a piece of work in 8 days. 2 men and 12 women can also finish the work in 8 days. How many days will 6 men and 6 women take to finish the same work?

   1) 10 days 2) 8 days 3) 6 days 4) None of these

7. There are 16 workers in a coal mine factory. All of them start working for a project. After working for $\frac{1}{3}$rd of the estimated time, they realise that only $\frac{1}{4}$th of the work is done. If they finish the entire work in $\frac{2}{3}$rd of the original time, how many extra workers they have employed?

   1) 40 workers 2) 20 workers 3) 32 workers 4) 24 workers

8. An amount of Rs.148.39 is to be distributed as wages amongst a man, a woman and a boy. The man worked for 26 days, the woman for 22 days and the boy for 21 days. If in a fixed time 5 times the work done by the boy is equivalent to 3 times the work done by the woman and 7 times the work done by the woman is equivalent to 5 times the work done by the man, how much would the man earn?

   1) Rs.76.076 2) Rs.81.33 3) Rs.73.99 4) Rs.57.42

DIRECTIONS for questions 9 and 10: Refer to the data below and answer the questions that follow.

Sita can complete a project in 4 days and if the same project is done by Gita she needs 5 days. Their sister Rita can do a project in 4 days while for the same project Gita requires 3 days. All of them working together can finish a particular project together in 25 days.

9. How many days are required by Rita to complete the project alone?

   1) 200 days 2) 145 days 3) 150 days 4) 100 days

10. How many days are required by Gita if she works alone to complete the project?

    1) 175 days 2) 75 days 3) 100 days 4) 150 days

DIRECTIONS for questions 11 to 18: Choose the correct alternative.

11. Three pipes can fill a pond in 4, 6 and 8 hours respectively. In addition to these three pipes, a fourth pipe was used and all these 4 pipes used together filled the pond in 2 hours. It is also known that the first two pipes were opened one hour prior to opening the other two pipes. Find the number of hours required for the fourth pipe to fill the pond individually.

    1) 24 2) 8 3) 4 4) None of these

12. Two pipes A and B can separately fill a cistern in 15 and 20 mins respectively and the waste pipe C can carry off 10 litres per minute. If all the pipes are opened when the cistern is full, it is emptied in 2 hours. How many litres does the cistern hold?

    1) 80 litres 2) 90 litres 3) 56 litres 4) 40 litres

13. Mr.X has to build a wall 1000 metres long in 50 days. He employs 56 men but at the end of 27 days finds that only 448 metres are built. How many more men must be employed so that the work may be finished in time?

1) 81 2) 25 3) 38 4) 58

14. A supply of water lasts for 150 days, if 7.5 gallons leak out every day, but only for 100 days if 15 gallons leak out daily. What is the total quantity of water in the supply?

1) 2250 gallons 2) 1125 gallons 3) 3350 gallons 4) 1250 gallons

(Past CAT question)

15. If 200 soldiers eat 10 tonnes of food in 200 days, how much will 20 soldiers eat in 20 days?

1) 1 ton 2) 10 kg 3) 100 kg 4) 50 kg

(Past CAT question)

16. A group of workers was put on a job. From the second day onwards, one worker was withdrawn each day. The job was finished when the last worker was withdrawn. Had no worker been withdrawn at any stage, the group would have finished the job in two thirds the time. How many workers were there in the group?

1) 2 2) 3 3) 5 4) 10

(Past CAT question)

17. P, Q and R are the three workers assigned to a particular work. They can finish the work individually in 8, 12 and 16 days respectively. On any given day, all the three do not work together. Also no worker can work on more than two consecutive days. What is the minimum number of days in which the work will be completed?

1) $4\frac{4}{7}$ 2) $5\frac{3}{5}$ 3) $5\frac{5}{9}$ 4) $5\frac{3}{7}$

18. Three groups—A, B and C—are required to complete the same job individually. Group A consists of 1 man and 3 women, group B consists of 2 men and 3 women and group C consists of both men and women. The number of days required by groups A, B and C to individually complete the job are in the ratio 6 : 4 : 3. If, instead of three groups, all the men work together and all the women work together to do the same job, which of the following cannot be the ratio of the number of days taken by the group of men and women?

1) 1 : 2 2) 4 : 5 3) 2 : 1 4) 5 : 4

DIRECTIONS for questions 19 to 21: Solve as directed.

19. Hitler and Mussolini are working separately on two assignments that are identical in all respects. Hitler takes 8 days to complete the assignment while Mussolini takes 12 days to complete the assignment while working alone. If the both of them start working on their assignments at the same time, after how many days will the remaining work to be done by Mussolini be exactly double the remaining work to be done by Hitler?

20. Person A was given a certain job to be completed in 20 days. But after working for 5 days, he was asked to finish the job in total 15 days. For this, he included person B to join him from the 6th day onwards such that they would be able to finish the job exactly at the end of 15 days. But after the 10^{th} day, they were asked to finish the job in total 13 days. So, A included another person C who would join them from the 11^{th} day onwards such that the work would be completed exactly at the end of 13 days. Had all three been working together from the beginning, how many days would they have taken to finish the job?

21. Three friends - A, B and C - were assigned to do a certain job together. A alone can do it in 15 days, B alone can do it 10 days and C alone can do it in 12 days. Initially, all three started working together on the job. After some time, C left the job and A and B worked together to finish the job. They received a total of Rs.12000 for finishing the job and A got Rs. 1300 more than C. How much did B earn (in Rs.)?

PRACTICE EXERCISE-2

DIRECTIONS for questions 1 to 12: Choose the correct alternative.

1. 3 women and 4 boys together can finish a piece of work in $2\frac{1}{2}$ days. 4 men and 2 boys together can finish the same work in 2 days. Also, 2 men and 3 women together can finish it in $2\frac{1}{2}$ days. How long would it take 1 man, 1 woman and 1 boy working together at double their efficiencies to complete the work?

   1) $1\frac{4}{13}$ days 2) $2\frac{4}{13}$ days 3) $3\frac{8}{13}$ days 4) $4\frac{8}{13}$ days

2. An amount of Rs.121.55 is to be distributed as wages amongst a man, a woman and a boy. If the man worked for 16 days, the woman for 14 days and the boy for 10 days with the ratio of the work done by them per day being $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$, how much would the man earn?

   1) Rs.59.74 2) Rs.48.82 3) Rs.59.84 4) Rs.59.27

3. A, B and C can finish a project in 9, 12 and 15 days respectively. They decide to take turns and complete the work with A alone working on Monday, B alone on Tuesday, followed by C alone on Wednesday and so on. What fraction of the work is done by C?

   1) $\frac{2}{9}$ 2) $\frac{4}{15}$ 3) $\frac{1}{3}$ 4) $\frac{16}{45}$ 5) $\frac{4}{45}$

4. In a shoe manufacturing factory, 18 men manufacture 20 shoes in 10 days working for 7.5 hours a day. How long will it take for 36 men to manufacture 40 shoes working 6 hours a day? It is also known that 4 men in the latter case do as much work as 6 men in the former.

   1) $\frac{20}{3}$ days 2) $\frac{25}{3}$ days 3) $\frac{30}{3}$ days
   4) $\frac{35}{3}$ days 5) Cannot be determined

5. Taps A and C can fill a tank in 5 and 7 hrs respectively. Taps B and D can drain a full tank in 6 and 8 hrs. respectively. Taps A and B are opened at 6 a.m. and 6.30 a.m. respectively, till 65% of the tank is filled. Then, C and D are also opened. At what time will the tank get filled?

   1) $\frac{1025}{43}$ hrs 2) $\frac{1134}{43}$ hrs 3) $\frac{1249}{43}$ hrs 4) $\frac{1312}{43}$ hrs

6. A tank has four taps A, B, C, D. A and C are inlets and can fill the tank in 4 and 6 hours respectively. B and D are outlets and can drain the completely filled tank in 5 and 7 hours respectively. A is opened at 12:00 noon and B, C, D are opened after a gap of one hour each, in that order. In how many hours, will the tank be full?

1) $\frac{269}{31}$ 2) $\frac{296}{31}$ 3) $\frac{267}{31}$ 4) $\frac{324}{31}$

7. There are ten water tanks T_1, T_2,..., T_{10} and another tank U each of capacity 6000 litres. A common inlet pipe giving 100 litres/min fills the tanks according to the following scheme. While T_1 gets filled, half the inlet water goes into it, half of the remaining inlet water goes into T_2, half of the remaining goes into T_3 and so on till T_{10}. Whatever is left out of the inlet water goes into U. After T_1 gets filled, no more water goes into it and half the inlet water goes into T_2, half of the remaining goes into T_3, and so on till T_{10} and then all the remaining inlet water goes into U. This procedure goes on as each tank gets filled. How long does it take to fill U?

1) 5 hours 2) 10 hours 3) 11 hours 4) None of these

8. There are three pipes in a tank which can work as both inlet and outlet pipes. Their rate of filling or emptying a tank are in the ratio 2 : 3 : 6. The largest pipe can fill the tank alone in 3 hrs. Initially the tank is empty. The first pipe is used as an outlet and second and third pipe are used as inlets for the first hour. In the second hour, the second pipe works as an outlet pipe and the first and the third are used as inlet pipes. In the third hour the third pipe is used as an outlet and the first two as inlet and so on. The process is continued till the tank is full. The total time required to fill the tank is:

1) 5 hrs 2) 4 hrs 3) 3.5 hrs 4) None of these

9. The work done by a man, a woman and a child is in the ratio of 3 : 2 : 1. There are 20 men, 30 women and 36 children in a factory. Their weekly wages amount to Rs.780, which is divided in the ratio of work done by the men, women and children. What will be the wages of 15 men, 21 women and 30 children for 2 weeks?

1) Rs.585 2) Rs.292.5 3) Rs.1170 4) Rs.900

10. A can do a piece of work in 8 days which B and C can do in 12 and 16 days respectively. If they work on alternate days, then which of the following is true?

I. If A started the work on the first day followed by B and C, then the work will be finished on the 10th day.

II. If B started the work on the first day followed by C and A, then the work will be finished on the 11th day.

III. If C started the work on the first day followed by A and B, then the work will be finished on the 11th day.

1) I 2) I & II 3) II 4) III

11. Pipes P, Q and R are attached to a tank and each can act as either an inlet or outlet pipe. Pipes P, Q and R respectively take 8, 10 and 12 hours to fill the empty tank or empty the full tank. In the first hour, pipes P and R work as inlet and Q work as outlet. In the second hour, pipes P and Q work as inlet and pipe R as outlet. In the third hour pipes Q and R work as inlet and pipe P as outlet and the process goes on like this. When will the cistern be filled?

    1) In the 8th hour. 2) In the 9th hour.
    3) In the 10th hour. 4) In the 11th hour.

DIRECTIONS for questions 12 and 13: Refer to the data below and answer the questions that follow.

The boiler tank in a chemical factory holds 105 litres. 5 tanks working together each having one-fifth the capacity of the boiler tank can fill in ‘hard water’ at same rates in the boiler tank in 2 hours. Let us suppose the outlet of the two of the small tanks work as inlet pipes, two other small tanks work as outlet pipes and the fifth small tank fills in the main ‘boiler’ at half its efficiency.

12. What is the number of hours taken to fill in the main boiler when all the pipes are opened at once?

    1) 1 hour 2) 5 hours 3) 20 hours 4) 25 hours

13. Only three pipes are working, two at their full efficiency and the third one at half its efficiency and all three are acting as inlet pipes. If all the three pipes are opened alternately (with only one pipe operating at a time) to fill the boiler such that each small tank fills ‘hard water’ in the main boiler for equal time, then the number of hours taken to fill the boiler to half its capacity is:

    1) 2 hours 2) 4 hours 3) 6 hours 4) 8 hours

DIRECTIONS for questions 14 to 19: Choose the correct alternative.

14. A water tank has three taps A, B and C. A fills 4 buckets in 24 minutes, B fills 8 buckets in 1 hour and C fills 2 buckets in 20 minutes. If all the taps are opened together, a full tank is emptied in 2 hours. If a bucket can hold 5 litres of water, what is the capacity of the tank?

    1) 120 litres 2) 240 litres 3) 180 litres 4) 60 litres

    (Past CAT question)

15. There is a leak in the bottom of the tank. This leak can empty a full tank in 8 hours. When the tank is full, a tap is opened into the tank which admits 6 litres per hour and the tank is now emptied in 12 hours. What is the capacity of the tank?

    1) 28.8 litres 2) 36 litres 3) 144 litres 4) Indeterminate

    (Past CAT question)

16. One man can do as much work in one day as a woman can do in 2 days. A child does one-third the work done by a woman in a day. If an estate-owner hires 39 pairs of hands - men, women and children in the ratio 6 : 5 : 2 and pays them a total of Rs. 1,113 at the end of the day's work. What must the daily wages of a child be, if the wages are proportional to the amount of work done?

    1) Rs. 14 2) Rs. 5 3) Rs. 20 4) Rs. 7

    (Past CAT question)

17. 3 men and 4 women together finish a certain job in 11 days. However, all of them did not work for all the 11 days. The ratio of the number of days that the 3 men and 4 women did not work is same as the ratio of the efficiency of 1 man and 1 woman. What is the ratio of the number of days that the 3 men and 4 women worked?

    Note: Either all three men worked together or didn't work together. Similarly, either all four women worked together or didn't work together. It is also known that 5 men working together can complete the entire job in 3 days and 2 women working together can complete the entire job in 12.5 days.

    1) 5 : 3 2) 1 : 5 3) 3 : 5 4) 5 : 1

18. B is 20% less efficient than C and A is 20% more efficient than C. The number of days taken by A and B together to complete a particular work is one less than the number of days taken by B and C together to complete the same work. If C works alone for as many days as the number of days required for A and B to complete the work together, what percentage of work will C complete in those many days?

    1) 40% 2) 50% 3) 60% 4) 75%

DIRECTIONS for questions 19 and 20: Solve as directed.

19. 5 men can mow 500 m^2 of a lawn in 10 days whereas 15 women can mow 1500 m^2 of the lawn in 20 days. In how many days can 6 men and 2 women mow 2100 m^2 of the lawn?

20. Jericho started working at a certain rate and finished half of the work in 16 days. Then he increased his efficiency by 100% each day as compared to the previous day. On which day will he complete the work, counting from the day he began?

QA-3.9 | TSD APPLICATIONS - I THEORY

Races

Basic terminology in races

Races is an application of Time, Speed and Distance.
A contest of speed between participants is called a race.
The point from where a race begins is called the starting point and the point where the race finishes is called the winning post or finishing point or a goal.
If all the persons contesting a race reach the winning post at exactly the same time, then the race is said to be a dead-heat race.

Important Concepts and Formulae related to Races

1. **Start distance:** 'A gives B a start of x metres', implies that, if the distance between the starting point and finishing point is L metres, A covers L metres while B covers L – x metres. e.g., In a 100 metre race, A gives B a start of 10 metres means, while A runs 100 metres, B runs 100 – 10 = 90 metres.

2. **Beat distance:** 'A beats B by x metres', implies that, if the distance between the starting point and finishing point is L metres, A wins the race by covering L metres, while B covers L – x metres only.

3. **Start time:** 'A gives B a start of t seconds', implies that, A starts the race t seconds after B starts from the starting point.

4. **Beat time:** 'A beats B by t seconds', implies that, A and B start together from the starting point, but A reaches the finishing point t seconds before B finishes.

   Note: (3) & (4) both imply that B takes t seconds more than A to finish the distance.

5 A beats B by 'x' metres or 't' seconds means, B runs 'x' metres in 't' seconds.

6. Winner's distance = Length of the race.

7. Distance covered by loser = Winner's distance – (Beat distance + Start distance)

8. Time taken by winner = Time taken by loser – (Beat time + Start time)

9. $\dfrac{\text{Winner's Time}}{\text{Loser's Distance}} = \dfrac{\text{Loser's Time}}{\text{Winner's Distance}} = \dfrac{\text{Beat Time + Start time}}{\text{Beat Distance + Start distance}}$

10. If a race ends in a dead heat, beat time = 0 and beat distance = 0.

11. Two persons starting at the same time and from the same point along a circular path will be together again for the first time, when the faster gains one complete round over the other.
    Time taken by faster person to complete one round over the other = $\dfrac{\text{Length of race course}}{\text{Relative Speed}}$

12. Two persons, starting at the same time from the same point along a circular path, will be together again for the first time at the same starting point, at a time which is the LCM of the time taken by each to complete a round.

13. Three persons, starting at the same time and from the same point along a circular path, will be together for the first time after the start at a time which is equal to the LCM of the time taken by the fastest to gain a complete round over each of the other two.

14. A overtakes B $\frac{1}{n}$th of x^{th} round means, when A has completed $\left(x - \frac{1}{n}\right)$ rounds, B has completed $\left[(x-1) - \frac{1}{n}\right]$ round.

Also, $\frac{\text{A's Speed}}{\text{B's Speed}} = \frac{\text{Rounds completed by A in a given time}}{\text{Rounds completed by B in the same time}}$

Example

A overtakes B in the middle of the 4^{th} round implies, when A has completed 3½ rounds, B has completed 2½ rounds. $\frac{\text{A's Speed}}{\text{B's Speed}} = \frac{3\frac{1}{2}}{2\frac{1}{2}}$

SOLVED EXAMPLES

Q : A can give B a 40 metres start and C a 70 metres start, in a one km race. How many metres start can B give C in a 1 km race?

A : A runs 1000 m, while B runs (1000 – 40) or 960 m.

A runs 1000 m, while C runs (1000 – 70) or 930 m.

$\therefore$ B runs 960 m, while C runs 930 m.

$\therefore$ B runs 1000 m, while C runs $\frac{1000 \times 930}{960} = 968\frac{3}{4}$ m.

$\therefore$ B can give C $\left(1000 - 968\frac{3}{4}\right)$ or $31\frac{1}{4}$ m. start.

Q : In a km race, A beats B by 40 metres or 7 seconds. Find A’s time over the course.

A : Here B runs 40 metres in 7 seconds.

$\therefore$ B runs 1000 m in $\frac{1000 \times 7}{40}$ = 175 seconds.

Hence, A’s time over the course = (175 – 7) = 168 seconds.

Alternatively,

By formula, $\frac{\text{Winner's Time}}{\text{Loser's Distance}} = \frac{\text{Beat Time + Start time}}{\text{Beat Distance + Start distance}}$

$\frac{\text{A's time}}{1000 - 40} = \frac{7+0}{40+0}$ $\therefore$ A’s Time = $\frac{7}{40} \times 960$ = 168 seconds.

Q : In a km race, if A gives B a 40 m start, A wins by 19 seconds, but if A gives B 30 seconds start, B wins by 40 m. Find the time that each takes to run a km?

A : Suppose A takes x seconds and B takes y seconds to run 1000 m.

Then $x + 19 = \frac{960}{1000}y$ and $\frac{960x}{1000} + 30 = y$

Solving for x and y: x = 125 seconds and y = 150 seconds.

A takes 125 seconds and B takes 150 seconds.

Q : Two men, A and B, walk around a circle 1200 m in circumference. A walks at the rate of 150 m/min. and B at the rate of 80 m/min. If they both start at the same time from the same point, and walk in the same direction,

(a) When will they first be together again at the starting point?

(b) When will they be together again?

A : (a) A makes one complete round of the circle in $\frac{1200}{150}$ = 8 minutes,

B in $\frac{1200}{80}$ = 15 minutes

That is, after every 8 minutes, A is at the starting point and after every 15 minutes B is at the starting point.

Now, A and B will be together again at the starting point at the end of the time during which each can make an exact number of rounds. Hence, the required time is the LCM of 8 and 15 minutes, i.e., 120 minutes or 2 hours.

(b) A and B will be together again for the first time when A has gained one complete round over B.

Now, A gains (150 – 80) or 70 metres on B in 1 minute.

$\therefore$ A will gain 1200 metres in $\frac{1200}{70}$ or $17\frac{1}{7}$ minutes

$\therefore$ A and B will be together in $17\frac{1}{7}$ minutes.

Q : Three men A, B and C walk around a circle, 1760 metres in circumference, at the rate of 160, 120 and 105 m/minute, respectively. If they all start together and walk in the same direction, when will they first be together again?

A : A, the quickest man gains one complete round on C, the slowest man, in $\left(\frac{1760}{160-105}\right)$ or 32 min. A gains one complete round on B, the next slowest man, in $\left(\frac{1760}{160-120}\right)$ or 44 mins.

Thus, A and C are together after every 32 minutes and A and B are together after every 44 minutes. Hence, A, B and C will be together in the time which is the LCM of 32 and 44, i.e., 352 minutes or 5 hours 52 minutes.

Concept Builder 1

1. In a 100 m race, A can give B a start of 10 m and C 19 m. In the same race, B can give C a start of:
2. A and B start from the same point to run in opposite directions round a circular path 550 yards in length, A giving B a start of 100 yards. They pass each other when A has run 250 yards. Who will come first to the starting point and at what distance will they be apart?
3. A runs $1\frac{3}{8}$ times faster than B. A gives B a start of 120 metres. How far must the winning post be so that it may be a dead heat?
4. How much time does a racer X take running on a circular track of 440 metres in anticlock-wise direction to meet racer Y who runs at double the speed of X in the anticlockwise direction. They start running in the same race from the diametrically opposite ends given that the speed of X is 19.8 kmph?
5. Red and black ants are running on a rectangular frame of length 7 cm and breadth 3 cm. Red ant is running with a speed of 60 cm/min and black ant is running with a speed of 40 cm/min. How much time (in seconds) will it take for both of them to meet at starting point if both of them start running from the same point?
6. Two boys running in opposite directions meet each other after 10 minutes on travelling a distance of 2.4 km. Their speeds are in the ratio 3 : 5. Find the time required for both of them to meet each other on a circular track of 2.4 km, for the first time if both of them start at the same time from the same point and move in the same direction.
7. The speeds of A and B are 100 m/s and 75 m/s, respectively. If A and B runs a race of 500 metres, then A beats B by what distance?

Answer Key

1. 10m
2. A, 10 yards
3. 440 metres
4. 40 sec.
5. 60 sec.
6. 40 minutes.
7. 125 metres

Clocks

This section introduces you to the application of the concepts of Time, Speed, Distance & races to the problems on clocks.

Basic Terminolgy

The face or dial of a clock is divided along the circumference into 60 equal spaces called minute spaces. The minute hand moves around the whole circumference of the clock once in one hour. The hour hand moves around the whole circumference of the clock once in 12 hours. Thus, the minute hand is twelve times faster than the hour hand.

The minute hand passes over the 60 minute spaces, while the hour hand goes over the 5 minute spaces.

That is, in 60 minutes, the minute hand gains 55 minutes over the hour hand or $\frac{55}{60}$ minute spaces in 1 minute.

Important formulae for clock related problems

1. 1 minute space = $\frac{360°}{60}$. (As 360° of the circle is divided into 60 minutes)

2. In one minute, the minute hand moves 6°.

3. In one minute, the hour hand moves $\frac{360}{12 \times 60} = \frac{360}{720} = \frac{1°}{2}$

   (As there are 12 hours of 60 minutes each) Thus, in one minute the minute hand gains $5\frac{1°}{2}$ over the hour hand.

4. In every hour,
   (a) the hands coincide once (0° apart).
   (b) the hands are twice at right angles (90° apart) and in these positions the hands are 15 minute spaces apart.
   (c) the hands point in opposite directions (180° apart) once and in this position, the hands are 30 minute spaces apart.

5. The hands are in the same straight line when they are coincident or opposite to each other.

6. The hands coincide 11 times in every 12 hours (between 11 and 1 o'clock there is a common position at 12 o'clock). Hence, the hands coincide 22 times in a day.

7. If both the hands start moving together from the same position, both the hands will coincide after $\frac{360 \times 2}{11} = 65\frac{5}{11}$ minutes.

8. The hands of a clock are at right angles twice in every hour, but in 12 hours they are at right angles 22 times since there are two common positions in every 12 hours.

9. Interchangeable positions of minute hand and hour hand occur when the original interval between the two hands is $\frac{60}{13}$ minute spaces or a multiple of this.

Incorrect clocks

A clock which gains or loses time is called an incorrect clock. In incorrect clocks both hands coincide at an interval which is **not** equal to $65\frac{5}{11}$ minutes.

In a slow clock, i.e., a clock that loses time:

Total time lost in T hours = $(T \times 60)\left(\frac{x - 65\frac{5}{11}}{x}\right)$ minutes,

where x is the time in which the hands of the incorrect clock coincide.

Also, for a fast clock, i.e., a clock that gains time:

Total time gained in T hours = $(T \times 60)\left(\frac{65\frac{5}{11} - x}{x}\right)$ minutes,

where x is the time in which the hands of the incorrect clock coincide.

SOLVED EXAMPLES

Q : At what time between 4 and 5 will the hands of a watch coincide?

A : At 4 o'clock, the hands are 20 minutes apart. Clearly, the minute hand must gain 20 minutes before the hands can be coincident. But the minute hand gains 55 minutes in 60 minutes. Hence, it will gain 20 minutes in:

$\frac{20 \times 60}{55} = \frac{240}{11} = 21\frac{9}{11}$ minutes.

$\therefore$ The hands will be coincident at $21\frac{9}{11}$ minutes past 4.

Q : At what time between 4 and 5 will the hands of a watch be at right angles?

A : At 4 o'clock the hands are 20 minutes apart. They will be at right angles when there is a space of 15 minutes between them. This will happen twice (i) when the minute hand has gained (20 – 15) or 5 minutes; (ii) when the minute hand has gained (20 + 15) or 35 minutes.

The minute hand gains 5 minutes in $\frac{5 \times 60}{55} = 5\frac{5}{11}$ minutes.

i.e., $5\frac{5}{11}$ minutes past 4.

and

The minute hand gains 35 minutes in $\frac{35 \times 60}{55} = 38\frac{2}{11}$ minutes.

i.e., $38\frac{2}{11}$ minutes past 4.

Q : At what time between 4 and 5 will the hands of a watch point in opposite directions?

A : They will be opposite to each other when there is a space of 30 minutes between them.

This will happen when the minute hand gains (30 + 20) minutes = 50 minutes.

The minute hand gains 50 minutes in $\frac{50 \times 60}{55} = 54\frac{6}{11}$ minutes,

i.e., they will point in opposite directions at $54\frac{6}{11}$ minutes past 4.

Q : My watch, which gains uniformly, is 2 minutes slow at noon on Sunday, and is 4 minutes 48 seconds fast at 2 p.m. on the following Sunday. When was it correct?

A : From Sunday noon to the following Sunday 2 p.m. = 7 days 2 hours = 170 hours.

The watch gains $2 + 4 + \frac{48}{60} = 6 + \frac{48}{60}$ minutes or $6\frac{4}{5}$ minutes in 170 hours.

The watch will show the correct time when it has gained 2 minutes.

The watch gains 2 minutes in $\frac{2}{6\frac{4}{5}} \times 170$ or 50 hours.

Now, 50 hours = 2 days and 2 hours. The watch will show the correct time after 2 days and 2 hours from Sunday noon, i.e., at 2 p.m. on Tuesday.

Concept Builder 2

1. From noon, by how many degrees has the minute hand moved to 2 : 40 p.m.?
2. In 36 hours, how many times will the hands of the clock coincide?
3. How many minutes does a watch gain per day, if its hands coincide after every 60 minutes?
4. How many degrees does an hour hand move in 20 minutes?
5. What is the angle between the minute hand and the hour hand of a clock when the time is 7:30?

Answer Key

1. 960°
2. 33 times
3. $130\frac{10}{11}$ mins
4. 10°
5. 45°

Calendar

The Solar Year consists of 365 days, 5 hours, 48 minutes. In the calendar known as Julian Calendar, arranged in 47 BC by Julius Caesar, the year was taken as being of $365\frac{1}{4}$ days and in order to get rid of the odd quarter of a day, an extra or intercalary day was added once in every fourth year and this was called Bissextile or Leap Year. But as the Solar Year is 11 minutes 12 seconds less than a quarter of a day, it followed in a course of years that the Julian Calendar became inaccurate by several days and in 1582 AD, this difference amounted to 10 days. Pope Gregory XIII determined to rectify this, and devised the calendar now known as the Gregorian Calendar. He dropped or cancelled these 10 days – October 5th being called October 15th and made centurial years leap years only once in 4 centuries – so that whilst 1700, 1800 and 1900 were to be ordinary years, 2000 was a leap year. This modification brought the Gregorian System into such close exactitude with the Solar Year that there is only a difference of 26 seconds which amounts to a day in 3323 years. This is the new style. It was ordered by an Act of Parliament to be adopted in England in 1752, 170 years after its formation and is now used throughout the civilized world.

The following simple facts should be remembered:

1. An ordinary year contains 365 days, i.e. 52 weeks and 1 odd day.

2. A leap year contains 366 days, i.e. 52 weeks and 2 odd days.

3. 100 years (a century) contain 76 ordinary years and 24 leap years
   $= (76 \times 52)$ weeks + 76 odd days + (24×52) weeks + 48 odd days
   $= [(76 \times 52) + (24 \times 52)]$ weeks + 124 odd days
   $= [(76 \times 52) + (24 \times 52) + 17]$ weeks + 5 odd days,
   i.e. 100 years contain 5 odd days.
   200 years contain 10 and therefore 3 odd days. Similarly, 300 years contain 1 odd day, 400 years will have (20 + 1) odd days i.e. 0 odd days. Similarly, the years 800, 1200, 1600, 2000 each contain no odd days.

4. First January 1 AD was Monday. Therefore, we must count days from Sunday, i.e. Sunday for 0 odd days, Monday for 1 odd day, Tuesday for 2 odd days and so on.

5. February in an ordinary year gives no odd day, but in a leap year gives one odd day.

THEORY
CLASS EXERCISE
PRACTICE EXERCISE

SOLVED EXAMPLES

Q : What day of the week was 20^{th} June 1837 ?

A : 20^{th} June 1837 means 1836 complete years + first 5 months of the year 1837 + 20 days of June.

1600 years give no odd days.

200 years give 3 odd days.

36 years give (36 + 9) or 3 odd days.

1836 years give 6 odd days.

From 1^{st} January to 20^{th} June there are 3 odd days.

Odd days :

January	:	3
February	:	0
March	:	3
April	:	2
May	:	3
June	:	<u>6</u>
		17

Therefore, the total number of odd days = (6 + 3) or 2 odd days.

This means that the 20^{th} of June fell on the 2^{nd} day commencing from Monday. Therefore, the required day was Tuesday.

Q : How many times does the 29^{th} day of the month occur in 400 consecutive years?

A : In 400 consecutive years there are 97 leap years. Hence, in 400 consecutive years February has the 29^{th} day 97 times and the remaining eleven months have the 29^{th} day 400 × 11 or 4400 times.

Therefore, the 29^{th} day of the month occurs (4400 + 97) or 4497 times.

Q : Today is 3^{rd} November. The day of the week is Monday. This is a leap year. What will be the day of the week on this date after 3 years?

A : This is a leap year. So, none of the next 3 years will be leap years. Each year will give one odd day so the day of the week will be 3 odd days beyond Monday i.e. it will be Thursday.

Q : What will be the day of the week on 19^{th} April, 2020?

1] Sunday 2] Saturday 3] Friday 4] Monday

A : 2000 years has 0 odd day.

19 years = (4 leap years + 15 ordinary years) = [(4 × 2) + (15 × 1)] odd days = 23 odd days ⇒ 2 odd days

Days of months (Jan, Feb, Mar, Apr) 31 + 29 + 31 + 19 = 110 days = 15 weeks + 5 days = 5 odd days

Total number of odd days = (0 + 2 + 5) odd days = 7 odd days ⇒ 0 odd days

∴ 19^{th} April 2020 is Sunday. Hence, [1].

CLASS EXERCISE

Teaser

You have two wooden sticks each of which burns completely in an hour. You have a match box with many matches to light the sticks. The sticks are of uneven density so they don't burn evenly (for instance, in half an hour not necessarily half a stick is burnt).

How can you use these sticks to measure exactly 45 minutes?

Races

Data for questions 1 – 4:

In a 100 m race, Harry the hare beats Terry the tortoise by 20 m.

1. When Harry runs half a kilometre, how far will Terry have run, if both start together?

2. If both start together, how much distance will Harry have to run to gain a lead of 150 m?

3. If Terry starts 250 metres ahead, how much further will he reach before Harry catches him?

4. If Harry beats Terry by 10 seconds in a 200 m race, find their speeds.

5. A gives B a start of 100 m in a 600 m race. If the ratio of their speed is 5:4 who will win and by how much?

6. A beats B by 20 m in a 100 m race. B beats C by 50 m in a 200 m race. C beats D by 150 m in a 500m race. By how much meters A beats D in a km race?

7. * In a race, if Shahid gives Saif a start of 100 metres, Shahid wins by 30 sec. If he gives Saif a start of 150 metres, he still wins but by 15 sec. Find Saif's speed (in m/min).

8. * In a 100m race, if Raj gives Rohit a start of 10 seconds, Raj wins by 5m. If he gives Rohit a start of 10m, he wins by 5 seconds. What is Rohit's speed?

Data for questions 9 – 15:

Every morning A, B and C go for a jog on a circular racing track 300 m long.

9. If A runs clockwise at 60 m/min and B runs clockwise at 100 m/min, starting from the same point, when and where will they meet for the first time?

10. If A runs clockwise at 60 m/min and B runs clockwise at 100 m/min, starting from the same point, when will they meet at the starting point for the first time?

11. If A runs clockwise at 60 m/min and C runs anti-clockwise at 40 m/min, starting from the same point, when and where will they meet for the first time?

12. If A runs clockwise at 60 m/min and C runs anti-clockwise at 40 m/min, starting from the same point, where will A be when they meet for the first time?

13. If A runs clockwise at 60 m/min and C runs anticlockwise at 40 m/min, starting from diametrically opposite points, when and where will they meet for the second time?

14. If A runs clockwise at 60 m/min and B runs clockwise at 100 m/min, starting from the same point and if they keep running infinitely, at how many points will they keep meeting?

15. If A runs clockwise at 60 m/min and C runs anti-clockwise at 40 m/min, starting from the same point and if they keep running infinitely, at how many points will they keep meeting?

16. A, B and C start from the same point at the same time in the same direction and move along a circular track of length 120 m at speeds of 5 m/s, 3 m/s and 2 m/s respectively.
    a. After how much time will they meet for the first time?
    b. After how much time will they meet for the first time at the starting point?
    c. If they keep running infinitely, at how many points will they keep meeting.

17. A, B and C start from the same point at the same time and move along a circular track of length 112 m at speeds of 5 m/s, 3 m/s and 2 m/s respectively. B and C move in the same direction and A moves in the opposite direction.
    a. After how much time will they meet for the first time?
    b. After how much time will they meet for the first time at the starting point?
    c. If they keep running infinitely, at how many points will they keep meeting.

18. * A, B and C start driving at speeds of 15, 24 and 42 kmph, from the same point and in the same direction, around a circular track of length 360 km. When and where will all three meet again?

Clocks

Answer questions 19 – 27 assuming a standard clock with 12 hours marked on its face:

19. What are the speeds (in degrees/minute) of the hour and the minute hand of a clock?

20. When will the hour and the minute hand of a clock first be together after 4 o'clock?

21. When will the hour and the minute hand of a clock first be at right angles after 8 o'clock?

22. When will the hour and the minute hand of a clock first point in opposite directions after noon?

23. When will the hour and the minute hand of a clock first be together after 11 o'clock?

24. What is the smaller angle between the hour and the minute hand at 9.20?

25. *What is the smaller angle between the hour and the minute hand at 3.40?

26. *A watch was running 7 minutes behind time on Monday at noon. It was running 10 minutes ahead of time next Monday at 2.00 pm. If it showed the correct time exactly once during the week, on which day and at what time did that happen?

27. * When will the hour and minute hands of a clock be at right angles for the second time after 2 pm?

Calendar

28. In 2010, Valentine's Day was a Sunday.

    a) In which two years after 2010 will Valentine's Day (14th Feb) again be a Sunday?

    b) Independence Day (15th Aug) would have fallen on which day in the same year (2010)?

    c) What was the day on Bal Din (14th Nov) in the year 2000?

29. * In a certain year, January had exactly 4 Mondays and 4 Fridays. What was the day on Gandhi Jayanti (2nd Oct) the previous year?

Challengers

1. In a 2 km cross-country race, if Rachit gives Suraj a start of 120 m, Rachit wins by 70 seconds. But if Rachit gives Suraj a start of 120 seconds in the same race, Suraj wins by 100 m. What is the time taken by Rachit to complete the race?

   1) 4 min 10 s　　2) 5 min　　3) 6 min 40 s　　4) 7 min 30 s

2. Hritik, Abhishek and John run a race starting from the same point. They run at the speeds of 200, 300 and 400 m/min respectively. Abhishek being faster than Hritik, starts 10 minutes after Hritik. John being the fastest starts even later. Abhishek and John overtake Hritik at the same time, how many minutes after Abhishek does John start?

3. The rim of a clock is divided into 60 equal divisions. At a certain point in time, the second hand is exactly 1 division ahead of the minute hand. After how long will they next meet?

   1) 58 sec　　2) 59 sec　　3) 60 sec　　4) 61 sec

4. A and B start jogging at the same moment, from the same point (but in opposite directions) along a circular track of length 500 metres. Initially, A jogs at 90 m/min while B jogs at 135 m/min. Every time they meet each other, they exchange speeds and reverse directions. When they meet at the starting point for the first time, how much distance will A have jogged in total?

   1) 1200 m　　2) 1500 m　　3) 1 km　　4) Cannot be determined

PRACTICE EXERCISE

DIRECTIONS for questions 1 to 14: Choose the correct alternative.

1. Ram and Shyam can run at the speeds of 45 m/min and 54 m/min respectively. They start from a point on a circular track of circumference 1980 m in the opposite direction. On meeting, they turn around and run in opposite directions. Which of the following is a possible distance on the track between their two meeting points?

   1) 1080 m 2) 980 m 3) 1100 m 4) 1030 m

2. A and B run a km race and A wins by 60 seconds. A and C run a km race and A wins by 375 metres. B and C run a km race and B wins by 30 seconds. Find the time each takes to run a km.

   1) $2\frac{1}{2}$ min., $3\frac{1}{2}$ min., 4 min. 2) 3 min., 4 min., $4\frac{1}{2}$ min.
   3) $3\frac{1}{2}$ min., $4\frac{1}{2}$ min., 5 min. 4) $4\frac{1}{2}$ min., $5\frac{1}{2}$ min., 6 min.

3. Two men, A and B, run a 4 km race on a circular course of $\frac{1}{4}$ km. If their speeds are as 5:4, how often does the winner pass the other?

   1) Once 2) Twice 3) Thrice 4) Four times

4. Two cyclists start a race at 3 p.m. around a circular track. The first goes round once in 3 minutes 12 seconds and the second in 3 minutes 30 seconds. Find at what time they will again meet at the starting point.

   1) 4:52 p.m. 2) 3:06 p.m. 3) 9:12 p.m. 4) 3:50 p.m.

5. I want my watch which gains 1 minute every 15 hours to show the correct time after exactly 24 hours. How many minutes slow should I set my watch at the beginning of the 24 hours?

   1) 1.6 2) 2 3) 1.06 4) 1.55

6. A man, who went out between five and six o'clock and returned between six and seven o'clock, found that the hands of the watch had exchanged places. When did he go out?

   1) $32\frac{4}{13}$ minutes past 5 o'clock. 2) $27\frac{2}{13}$ minutes past 5 o'clock.
   3) $37\frac{3}{13}$ minutes past 5 o'clock. 4) $31\frac{5}{13}$ minutes past 5 o'clock.

7. Determine the time between 7 a.m. and 8 a.m. when the hands of a clock will be in the same straight line but not coincide.

   1) 7 hours $5\frac{5}{11}$ min 2) 7 hours $3\frac{5}{11}$ min
   3) 7 hours $7\frac{3}{11}$ min 4) 7 hours $7\frac{7}{11}$ min

8. Find the time between 5 and 6 o'clock when the hands of a clock are 3 minutes apart.

1) 24 minutes past 5
2) $30\frac{6}{11}$ minutes past 5
3) Both (1) and (2)
4) $2\frac{6}{11}$ minutes past 5

9. A clock loses 5 seconds an hour and is set right on Sunday at noon. What time will it indicate on the following Monday at noon?

1) 11.56 a.m.
2) 11.58 a.m.
3) 12.02 p.m.
4) 12.04 p.m.

10. Ram was born on the first Monday of March 1952. Then on which date was he born?

1) 1^{st}
2) 3^{rd}
3) 5^{th}
4) 2^{nd}

11. Which of the following is true?

1) 21^{st} February 2001 is a Saturday.
2) 27^{th} February 1999 is a Saturday.
3) 23^{rd} February 2002 is a Sunday.
4) 22^{nd} February 1998 is a Saturday.

12. A race is designed such that the person who completes twenty rounds of the circular field track of 50 meter radius first wins the race. Irrespective of the time taken everybody has completed the race. A, B and C were among the participants. A has overtaken B (for the first time) after A completed $\frac{4}{5}$th of his 10th round and B has overtaken C (for the first time) in exactly the middle of B's 12th round. If C has taken 30 minutes to complete the race then what was the speed of A? It is assumed that the running speed of participants remains constant throughout the race. [m/m = meter/minute]

1) 236 m/m
2) 246 m/m
3) 256 m/m
4) 266 m/m

13. If the hands of the clock coincide after every $66\frac{9}{11}$ minutes, then how many minutes do the clock gain or lose in an entire day?

1) 30 minutes, gains
2) 30 minutes, loses
3) $32\frac{8}{11}$ minutes, gains
4) $32\frac{8}{11}$ minutes, loses

14. In a 2 km race on a circular course of $\frac{1}{4}$ of a km, A overtakes B in the middle of his 6^{th} round. By what distance will A win at the same rate of running?

1) $\frac{2}{9}$ km
2) $\frac{18}{11}$ km
3) $\frac{4}{11}$ km
4) $\frac{11}{9}$ km

Directions for questions 15 to 18: Answer the following questions based on the information given.

A and B are running along a circular course of radius 7 km in opposite directions such that when they meet, they reverse their directions and when they meet, A will run at the speed of B and vice versa. Initially, the speed of A is thrice the speed of B. Assume that they start from M_0 and they first meet at M_1, then at M_2, next at M_3, and finally at M_4.

15. What is the point that coincides with M_0 along the course?

    1) M_1 2) M_2 3) M_3 4) M_4

(Past CAT question)

16. What is the shortest distance between M_1 and M_2 along the course?

    1) 11 km 2) $7\sqrt{2}$ 3) 7 km 4) 14 km

(Past CAT question)

17. What is the shortest distance between M_1 and M_3 along the course?

    1) 22 km 2) $14\sqrt{2}$ 3) $22\sqrt{2}$ 4) 14 km

(Past CAT question)

18. What is the total distance travelled by A when they meet at M_3?

    1) 77 km 2) 66 km 3) 99 km 4) 88 km

(Past CAT question)

DIRECTIONS for questions 19 to 23: Choose the correct alternative.

19. In a watch, the minute hand crosses the hour hand for the third time exactly after every 3 hours, 18 minutes, 15 seconds of watch time. What is the time gained or lost by this watch in one day?

    1) 14 min 10 sec, lost 2) 13 min 50 sec, lost
    3) 13 min 20 sec, gained 4) 14 min 40 sec, gained

(Past CAT question)

20. In a mile race, Akshay can be given a start of 128 metres by Bhairav. If Bhairav can give Chinmay a start of 4 metres in a 100-metre dash, then who out of Akshay and Chinmay will win a race of one and half miles, and what will the final lead given by the winner to the loser be (in terms of miles)? (One mile is 1600 metres).

    1) Akshay, $\frac{1}{12}$ 2) Chinmay, $\frac{1}{32}$ 3) Akshay, $\frac{1}{24}$ 4) Chinmay, $\frac{1}{16}$

(Past CAT question)

21. If Raghu and Ram are running along a circular path after starting from the same point and running in the same direction with speeds 9 m/s and 11 m/s respectively, how many times will Ram overtake Raghu before they meet at the starting point for the first time?
    1) 1
    2) 2
    3) 9
    4) Depends on the circumference of the circular track

22. In a 100 metres walking race, Abbas gives Mustaan a headstart of 4 metres and beats him by 19 seconds. If Abbas gives Mustaan a headstart of 30 seconds, by how much will the winner win over the loser, if it is known that the ratio of Abbas' speed to that of Mustaan is 6 : 5?
    1) Abbas will win by 5 seconds.
    2) Mustaan will win by 5 seconds.
    3) Abbas will win by 10 seconds.
    4) Mustaan will win by 10 seconds.

23. Three participants—A, B and C—ran a race of total length 192 m. A won the race ahead of B by 48 m and ahead of C by 72 m. By what distance (in metres) did B win the race ahead of C?
    1) 30
    2) 32
    3) 36
    4) 40

DIRECTIONS for question 24: Solve as directed.

24. Karan and Arjun are running a race between points A and B. If both start from point A, Karan beats Arjun by 20 m. If Karan starts from 10 m behind A and Arjun starts from point A, Karan beats Arjun by 12 m. What is the distance between points A and B (in metres)?

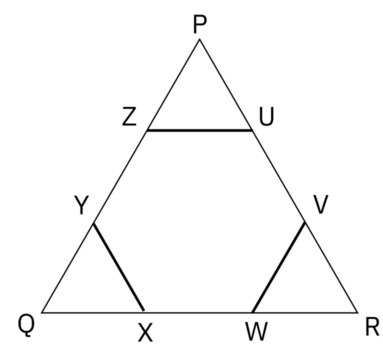
QA-3.10 | TSD APPLICATIONS - II CLASS EXERCISE

Teaser

Tony plans to go on a tour of South India in his car. All the 4 tyres in his car have been recently replaced and in addition he has a new spare tyre. From experience he knows that a tyre wears out after 2500 km of travel. However, the route he plans to take is 3000 km long. Will he be able to manage the tour without needing to buy fresh tyres?

Answer questions 1 to 3 based on the following information:

A jogging track in a park is in the shape of a regular hexagon inscribed in an equilateral triangle as shown in the adjoining figure. Two people M and N start jogging along this track along different paths. The speeds of M and N are in the ratio 1 : 2 and M can complete the distance PR in 9 minutes.



1. If both of them start jogging from Y and M follows the clockwise path round the hexagon while N follows the clockwise path around the equilateral triangle, where will they meet for the first time? After how long?

2. If after the meeting in the previous question, M reverses his direction of jogging, where and when will they meet for the second time?

3. If both of them start together at Y as in the first question, by what percent will M have to change his speed so that both of them complete one round and returns to Y at the same time?

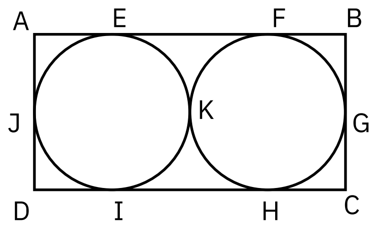
4. Passenger trains ply between Kochi and Kollam throughout the day at 30 minutes interval. The first train of the day starts at 7 AM while the last train starts at 11.30 PM from both Kochi and Kollam. The distance between the two cities can be covered in four hours. Muralidharan boarded a train at Kochi at 9 AM. How many trains travelling in the opposite direction will he see (including the trains at either station) till he reaches Kollam?

5. Two trains A and B start simultaneously at 10 a.m from cities P and Q towards each other at speeds of 60 and 80 kmph respectively. They meet at point R at 10 pm. After how long will they reach their respective destinations Q and P?

6. Two people W and X start from two points Y and Z walking towards each other's starting points. They meet along the way and then W takes 18 minutes more while X takes 50 minutes more to reach their destinations. How much time after starting did they meet? What is the ratio of their speeds?

7. * At 9 a.m., A starts driving from city P to city Q at a constant speed. At 10 a.m., B starts driving from city Q to city P, at a different constant speed. The two of them meet along the way at 10:48 a.m., and eventually reach their destinations simultaneously. At what time did they reach?

8.



* A robotics design competition requires remote-controlled cars to move along the paths in the adjacent figure (where ABCD is a rectangle circumscribing two circles touching externally). One car moves along the circular path JEKHGFKIJ while a second moves along the path JAEFBGCHIDJ. If both start from J at the same time, and also finish simultaneously, what is the ratio of their speeds?

9. At 9:00 AM, Ajay and Vijay started walking together at the same speed for Udaipur railway station in order to reach just in time to catch a train there at 11:00 AM. When they were exactly midway, Vijay realized that he forgot his ticket. Therefore he immediately rushed back home in a horse-cart, while Ajay continued walking towards the railway station. Vijay collected his ticket from home and took the same horse-cart to the railway station and reached the railway station just in time to catch the train. What is the ratio of the speeds of the horse cart to the walking speed of Ajay?

10. Donald was supposed to reach Washington railway station in a train from New York at 10 AM and his brother, Mike was suppose to leave home at 8 AM in a car to reach Washington railway station exactly at 10 AM to receive Donald. However, Donald got ticket on an earlier train and reached Washington at 7 AM. On reaching Washington, Donald immediately called Mike and Mike left home right away. In the meantime, Donald took a horse-cart and started travelling towards home. Mike met Donald and both Mike and Donald drove back home. If they reached home at 10.30 AM, what is the ratio of the distance travelled by Donald in a horse cart to the distance travelled by him in a car?

11. Mr. Slow drives at a speed of 25 kmph while Mr. Fast drives at a speed of 50 kmph. Both drivers drove from point P to point Q and then immediately return to point P. Mr. Slow started at 8 AM while Mr. Fast started at 10 AM. Mr. Fast reached point Q and on his way back to point P, he met Mr. Slow, who was still on his way to point Q at 12:40 PM. Calculate the distance between points P and Q.

12. Two friends named Ajay and Vijay simultaneously start swimming from points A and B respectively towards points B and A respectively with constant speed. On reaching points B and A respectively, they immediately turn back and run towards their starting points with same speed. They continue swimming this way. On their way, they meet each other at a point 500 m away from A during their first meeting and at a point 300 m from B in their second meeting.

Calculate:
1. The distance of their third meeting point from point A.
2. The distance of their third meeting point from point B.

13. Two cats named Amy and Pamy simultaneously start running in opposite directions from vertex A of a running track ABC in the shape of an equilateral triangle of side 500 m. They keep running indefinitely. If their first meeting point is along side BC at a distance 200 m from vertex B, what would be the location of their 20th meeting?

    1] Between point A and point B
    2] Point B
    3] Between point B and point C
    4] Between point C and point A

14. Mr. One and Mr. Two simultaneously started driving towards each other at the speed of 25 kmph from two points separated by 1000 km. At the same time, Mr. Three, who was exactly at the midpoint of the segment joining Mr. One and Mr. Two, started driving towards Mr. One at 100 kmph. On meeting Mr. One, Mr. Three immediately reversed his direction and drove towards Mr. Two at the same speed of 100 kmph. On meeting Mr. Two, Mr. Three immediately reversed his direction and drove towards Mr. One at the same speed. Mr. Three continued driving this way at the same speed till Mr. One and Mr. Two met. Calculate:

    1. Total distance driven by Mr. Three
    2. Distance driven by Mr. Three in the direction towards Mr. One
    3. Distance driven by Mr. Three in the direction towards Mr. Two

15. Everyday I go for a morning walk to a hill at a distance of 3 km from my home. Today, my dog, Rocky also accompanied me. He left home with me and ran to the hill at a speed three times that of my walking speed. On reaching the hill, he immediately reversed his direction and ran towards me at the same speed. On meeting me, he again ran towards the hill at the same speed. This process continued till I reached the hill.

    Calculate:

    1. The total distance run by Rocky
    2. The distance run by Rocky towards the hill
    3. The distance run by Rocky towards me

16. Two trains, named A and B are at a distance of 200 km and they simultaneously start towards each other at equal speed of 100 km. A bird, who is perched on the engine of train A starts flying towards train B at a speed 200 kmph. The bird keeps flying between the two trains at the same speed without stopping till two trains meet.

    Calculate:

    1. Total distance travelled by the bird
    2. The distance travelled by the bird in the direction from train A to train B
    3. Total distance travelled by the bird in the direction from train B to train A

17. Two local trains simultaneously start from Bandra and Andheri towards each other with speeds 30 km/hr and 42 km/hr respectively. A crow is perched on the engine of the train starting from Bandra while a parrot is perched on the engine of the train starting from Andheri. Both crow and the parrot start flying towards each other with speeds 48 km/hr and 60 km/hr respectively from the moment the two trains start. The crow and the parrot meet each other and then immediately return to the trains that they started from. They again fly towards each other and on meeting, immediately fly back towards the train they started from. This process continues till the two trains meet. If the distance between Bandra and Andheri is 24 km, what is the total distance flown by the crow?

    1] 20 km
    2] 16 km
    3] 32 km
    4] 40 km

18. Rajesh started walking towards his home from a point 6 km away. At the same instant, his dog started running from home towards him at twice the speed of Rajesh. The dog met Rajesh, then immediately reversed his direction and ran towards the home at the same speed. The dog continued running between home and Rajesh till the time Rajesh reached home.

    Calculate:
    1. Total distance run by the dog
    2. The distance run by the dog towards the home
    3. The distance run by the dog towards Rajesh

19. Tushar takes 80 seconds to climb down an escalator that is moving downwards but he takes 100 seconds to climb down an escalator that is moving upwards. If the escalator is stationary, how many seconds would he take to climb down the escalator?

20. Tushar is climbing down an escalator that is moving downwards from the first floor to the ground floor. He takes 100 seconds to reach the ground floor and traverses 60 steps if he decides to walk. On the other hand, he takes 60 seconds and traverses 90 steps if he decides to run. How many steps are there on the escalator?

21. Tushar and Vishal are climbing on an escalator that is moving up. Tushar takes 25 seconds while Vishal takes 20 seconds to reach the top because Vishal is faster than Tushar. Vishal takes 2 steps per second, while Tushar takes 1 step per second. What is the total number of steps on the escalator?

22. Tushar is climbing up an escalator that is going up, while Vishal is climbing down the same escalator. For every one step Tushar takes, Vishal takes two steps. Both start together and reach their destinations together. If Tushar walks 48 steps up the escalator, what is the total number of steps of the escalator?

23. Raghav is walking at a constant speed beside a busy highway that connects Ahmedabad and Vadodara. Buses ply between the two cities at equal intervals in both directions. He encounters a bus going from Ahmedabad to Vadodara after every 12 minutes and a bus going from Vadodara to Ahmedabad after every 10 minutes. What is the time interval between the two consecutive buses going from Ahmedabad to Vadodara if it is known that the buses in both directions run at the same speed?

Challengers

For questions 1 to 3 refer to the data below:

There is a railway line in East-West direction. Three railway stations—Rampur, Laxmanpur and Sitapur are on the railway line, with Rampur being towards West and Sitapur being towards East and Laxmanpur between Rampur and Sitapur. Ajay is standing exactly to the south of Laxmanpur railway station at a distance of 2.4 km. He is 3 km away from Rampur station and 4 km away from Sitapur station. A train travelling eastward is approaching Rampur station. The speed of train and Ajay are such that if Ajay had walked at his normal speed, he would have just caught the train either at Rampur or Sitapur. However Ajay decides to walk up north towards Laxmanpur station and board the train there. On reaching Laxmanpur station, he realizes that the train does not halt there. Therefore he decides to walk towards Sitapur station. As Ajay is on his way to Sitapur, he receives a call from his brother saying that the train has just left Rampur and he should better run lest he would miss the train.
(Note: Halting time of the train at Rampur, time spent by Ajay at Laxmanpur and time spent for the phone call may be considered negligible)

1. How much faster should Ajay run as a percentage of his normal walking speed in order to just catch train at Sitapur?
   1) He should run 200% faster as compared to his normal walking speed
   2) He should run 260% faster as compared to his normal walking speed
   3) He should run 160% faster as compared to his normal walking speed
   4) Even if he walks at his normal walking speed, he would be catch the train

2. If Ajay continues to walk from Laxmanpur to Sitapur at his normal walking speed, where would he be when the train reaches Sitapur?
   1) 1.2 km short of Sitapur
   2) 1.6 km short of Sitapur
   3) 1.8 km short of Sitapur
   4) At Sitapur station

3. How much faster does the train run as compared to Ajay's normal walking speed?
   1) 500% faster
   2) 250% faster
   3) 300% faster
   4) 400% faster

4. An escalator in a mall has 30 steps. Mithun steps on it and climbs along with it and climbs 10 steps by the time he reaches the top. Rajni climbs at twice the speed Mithun does. How many steps will he need to reach the top?
   1) 12 2) 15 3) 18 4) 20

5. Shanti's school normally finishes at 4 pm. Her mom drives from home to pick her up, reaching the school exactly at 4 pm. One day, a half-holiday is announced and the school finishes for the day at 1 pm. Rather than sitting and waiting, Shanti decides to start walking towards home. Her mother meets her along the way and as a result they reach home an hour earlier than normal. What is the ratio of Shanti's walking speed to her mother's driving speed?
   1) 1 : 4 2) 1 : 2 3) 1 : 5 4) 1 : 8

PRACTICE EXERCISE

DIRECTIONS for questions 1 and 2: Answer the questions on the basis of the information given below.

Cities A and B are in different time zones. A is located 3000 km east of B. The table below describes the schedule of an airline operating non-stop flights between A and B. All the times indicated are local and on the same day.

Departure		Arrival	
City	Time	City	Time
B	8:00 AM	A	3:00 PM
A	4:00 PM	B	8:00 PM

Assume that planes cruise at the same speed in both directions. However, the effective speed is influenced by a steady wind blowing from east to west at 50 km per hour.

1. What is the time difference between A and B?

   1) 2 hours and 30 minutes 2) 1 hour 3) 1 hour and 30 minutes
   4) 2 hours 5) Cannot be determined

   (Past CAT question)

2. What is the plane's cruising speed in km per hour?

   1) 600 2) 500 3) 700
   4) 550 5) Cannot be determined.

   (Past CAT question)

DIRECTIONS for questions 3 and 4: Choose the correct alternative.

3. A jogging park has two identical circular tracks touching each other and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, **A** and **B**, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. **A** jogs along the rectangular track, while **B** jogs along the two circular tracks in a figure of eight. Approximately, how much faster than **A** does **B** have to run, so that they take the same time to return to their starting point?

   1) 3.88% 2) 4.22% 3) 4.44% 4) 4.72%

   (Past CAT question)

4. On a rectangular track, points A and B are diagonally opposite. If X starts running from point A towards point B along rectangular track, he takes 12 minutes more than he would have taken had he run along the straight path. This difference is 4 minutes for Y. Z travels half the distance with X's speed and remaining half with Y's speed taking 10 minutes to travel 9 km. Find the speed of Y.

   1) 10 m/s 2) 15 m/s 3) 12 m/s
   4) 9 m/s 5) None of these

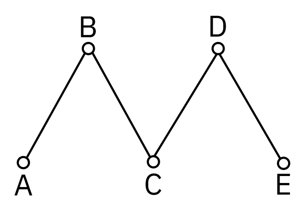
DIRECTIONS for questions 5 and 6: Refer to the data below and answer the questions that follow.

Kedar and Vishnu are moving along a rectangular track ABCD. Kedar starts cycling at 6 kmph from point A at 8:00 am on the route A-D-C-B-A. Vishnu starts walking at 8:50 am from point C at the speed of 1 kmph and takes the route C-B-A-D-C.
AB = 6 km and BC = 4 km.

5. At what time does Kedar overtake Vishnu?
   1) 10:15 am 2) 9:00 am 3) 9:50 am
   4) 10:00 am 5) None of these

6. How far away from each other are the two at 10:50 am?
   1) 5 km 2) 4 km 3) 10 km 4) 17 km 5) 6 km

DIRECTIONS for questions 7 and 8: Refer to the data below and answer the questions that follow.



K is travelling from A to E through four different roads of equal length i.e., AB, BC, CD and DE. He covers this distance in 10 minutes at a speed of 1.2 kmph.

7. Find the length AD, if AC = BD = CD.
   1) 50 m 2) $50\sqrt{3}$ m 3) 100 m
   4) $100\sqrt{3}$ m 5) $25\sqrt{6}$ m

8. If there is a direct road joining A and D, find the approximate time taken by K to reach E, if he travels directly via D. (use data from the previous question)
   1) 5 minutes 2) 7 minutes 3) 8 minutes
   4) 9 minutes 5) Cannot be determined

DIRECTIONS for questions 9 to 12: Choose the correct alternative.

9. A wolf spots a rabbit 100 m away and starts chasing it. The rabbit realizes this after two minutes and starts running away from the wolf towards a burrow which is 40 m from where the rabbit is right now. The rabbit covers 1 m in 3 leaps whereas the wolf covers 3 m in 4 leaps. At what speed should the rabbit run so that it just manages to reach the burrow if the wolf is running at 20 m per minute? (The wolf, rabbit and burrow are in a straight line.)
   1) 24 leaps/min 2) 18 leaps/min 3) 16 leaps/min 4) $\frac{180}{11}$ leaps/min

10. Two cats named Amy and Pamy are standing at points A and B respectively. They simultaneously start running towards each other and meet after 't' seconds. After meeting each other, Amy takes 6.4 seconds to reach point B whereas Pamy takes 10 seconds to reach point A. What is the value of 't' (in seconds)?
    1) 8
    2) 6
    3) 7.2
    4) More information is needed to answer the question

11. A and B are standing at the opposite ends of a 200-metre long stretch of road. They start driving towards each other. The one who reaches the half way first wins the race, however they continue driving towards the opposite point even after the race is over. It is known the A won 1.33 seconds before meeting B along the way and B takes 6.66 seconds more to travel the entire distance than A. What is the speed of A?
    1) 10 m/s 2) 12 m/s 3) 15 m/s 4) 18 m/s

12. Basanti is walking beside a railway track between Pune and Baramati at a constant speed towards Baramati. Local trains ply between the two cities at equal intervals in both directions. She encounters a train going from Pune to Baramati after every 8 minutes and a train going from Baramati to Pune after every 6 minutes. What is the time interval between the two consecutive trains going from Pune to Baramati?
    1) 7 minutes
    2) $6\frac{6}{7}$ minutes
    3) $6\frac{1}{2}$ minutes
    4) More information is needed to answer this question

DIRECTIONS for questions 13 and 14: Solve as directed.

13. Karan and Arjun are standing at points A and B respectively, separated by 500 meters. Karan starts running towards Arjun while Arjun starts running away from Karan, both along the same line. Since Karan runs faster than Arjun, Karan catches up with Arjun at a point beyond B. On meeting Arjun, Karan immediately reverses his direction and starts running towards point B and then continues towards point A. If Arjun has run 1200 meters by the time Karan reaches point A, how much distance has Karan run (in meters)?

14. Gaurav starts running along a straight line at uniform speed from point A to point B, which is located at a distance of 3000 m from point A. At the same instant, his pet dog starts running from point B to point A at a speed twice that of Gaurav. After the dog meets Gaurav at a point between A and B, the dog reverses his direction and starts running towards point B. On reaching point B, the dog again reverses his direction and meets Gaurav. On meeting Gaurav, the dog again reverses his direction and starts running towards point B. This process continues till Gaurav reaches point B. What is the distance run by the dog in the direction opposite to Gaurav (in meters)?

DIRECTIONS for questions 15 to 18: Choose the correct alternative.

15. Every morning, Rajesh goes for morning walk from his house to a temple located at a distance of 6 km from his house at a uniform speed of 6 km/hr. Today his pet dog, who was exactly midway on the way to temple started running towards the temple the same instant when Rajesh started from his house. The dog reached the temple, immediately turned back and started running towards Rajesh (who was on his way to temple). On meeting Rajesh, his dog again reversed his direction and started running towards the temple. This process continued till Rajesh reached the temple. What was the distance run by the dog against the direction of Rajesh (i.e. in the direction from the temple to the house) if the speed of the dog was 11 km/hr?

    1) 7 km 2) 4 km 3) 11 km 4) 8 km

16. There is an escalator connecting the foot of a hill and the hilltop that goes up from the foot to the top. While going up, Sachin took the escalator and reached the top in 20 seconds after taking 120 steps. While returning, he took the same escalator but walked down on the escalator that is moving up. He reached the foot of the hill in 40 seconds after having taken 240 steps. If the escalator is stationary, how many steps would he have to take to climb from the foot to the top?

    1) 200 2) 180 3) 160 4) 144

17. Rahim plans to drive from city A to station C, at the speed of 70 km per hour, to catch a train arriving there from B. He must reach C at least 15 minutes before the arrival of the train. The train leaves B, located 500 km south of A, at 8:00 am and travels at a speed of 50 km per hour. It is known that C is located between west and northwest of B, with BC at 60° to AB. Also, C is located between south and southwest of A with AC at 30° to AB. The latest time by which Rahim must leave A and still catch the train is closest to

    1) 6:15 am 2) 6:30 am 3) 6:45 am 4) 7:00 am 5) 7:15 am

    (Past CAT question)

18. There is a race track ABCDEFGH in the shape of a regular octagon. Two runners start running from point A along the race track in opposite directions at speeds which are in the ratio 9 : 7. While running along the track, they meet each other multiple times. At which of the following meetings will the two be the farthest from point A?

    1) 12^{th} 2) 15^{th} 3) 20^{th} 4) 24^{th}

DIRECTIONS for questions 19 and 20: Solve as directed.

19. There is a ground ABC in the shape of an equilateral triangle with A, B and C as its vertices. Amy, a cat, starts running from vertex A in the direction towards C along side AC. At the same instant, Pamy, another cat, starts running from vertex C in the direction towards A along side CA. The two cats meet for the first time after 12 seconds. After meeting, they continue running in their respective directions along the sides of the triangle. After how many seconds will they meet for the third time?

20. Rohit climbs up an escalator which is going up. If he climbs 35 steps, he will reach the top in 28 seconds. If he climbs 44 steps, he will be able to reach the top in only 16 seconds. How many steps are visible on a stationary escalator?

QA-3.11 | TRIGONOMETRY

THEORY

Definition of Trigonometrical ratios

Trigonometry is the study of the relationship between the sides and angles of a right-angled triangle.

Angles are measured in degrees or radians.

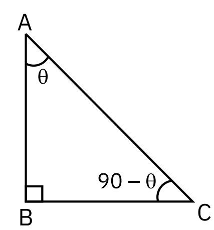
1 degree = 1° = 60 minutes

One radian = $1^c = \left(\frac{180}{\pi}\right)^\circ$ and $1^\circ = \left(\frac{\pi}{180}\right)^c$

$\therefore$ Radian Measure = $\frac{\pi}{180^\circ}$ × Degree Measure

Degree Measure = $\frac{180^\circ}{\pi}$ × Radian Measure

Degree	30°	45°	60°	90°	180°	270°	360°
Radian	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π



In a right angled triangle ABC, if m$\angle$ABC = 90°, then AC is the hypotenuse. **With respect to $\angle$ACB,** AB is the opposite side and BC is the adjacent side. **With respect to $\angle$BAC**, BC is the opposite side and AB is the adjacent side.

In a right angled triangle for a given acute angle, $\angle$BAC there are six possible ratios of sides. The values of all these ratios are constant. These ratios are called trigonometric ratios.

Sine θ i.e., $\sin\theta = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \frac{BC}{AC}$

Cosine θ i.e., $\cos\theta = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \frac{AB}{AC}$

Tangent θ i.e. $\tan\theta = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{BC}{AB}$

Cosecant θ i.e., $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \frac{AC}{BC}$

Secant θ i.e., $\sec\theta = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \frac{AC}{AB}$

Cotangent θ i.e., $\cot\theta = \frac{\text{Adjacent side}}{\text{Opposite side}} = \frac{AB}{BC}$

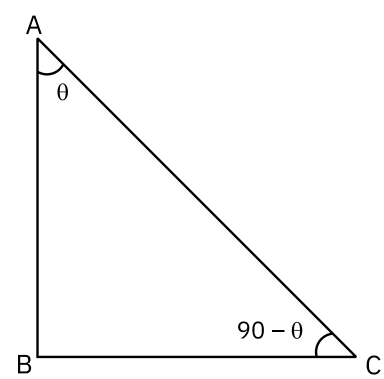
Relation between the trigonometrical ratios of complementary and supplementary angles

1. $\sin\theta = \dfrac{\text{Opposite side}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

   $\cos(90-\theta) = \dfrac{\text{Adjacent side}}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

   $\therefore \sin\theta = \cos(90-\theta)$ for $0^o \le \theta \le 90^o$
2. $\cos\theta = \sin(90-\theta)$
3. $\tan\theta = \cot(90-\theta)$
4. $\cot\theta = \tan(90-\theta)$
5. $\text{cosec}\,\theta = \sec(90-\theta)$
6. $\sec\theta = \text{cosec}(90-\theta)$
7. $\tan\theta \times \tan(90-\theta) = 1$
8. $\cot\theta \times \cot(90-\theta) = 1$
9. $\cos(90+\theta) = -\sin\theta$
10. $\sin(90+\theta) = \cos\theta$
11. $\sin(180-\theta) = \sin\theta$

12 $\cos(180-\theta) = -\cos\theta$



Example

(i) Find the value of $\dfrac{\sin 25}{\cos 65}$.

$\sin 25 = \cos(90-25) = \cos 65$; $\therefore \dfrac{\sin 25}{\cos 65} = \dfrac{\cos 65}{\cos 65} = 1$

(ii) If $\cos 50 = 0.6428$. Find $\sin 40$.

$\sin 40 = \cos(90-40) = \cos 50 = 0.6428$

Trigonometrical Identities

1. $\sin\theta = \dfrac{BC}{AC}$, $\text{cosec}\,\theta = \dfrac{AC}{BC}$

   $\therefore \sin\theta \times \text{cosec}\,\theta = 1$
2. $\cos\theta \times \sec\theta = 1$
3. $\tan\theta \times \cot\theta = 1$

Note:
cosecant, secant and cotangent ratios are the reciprocal of sine, cosine and tangent ratios respectively.

4. $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

5. $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

6. $\sin^2\theta + \cos^2\theta = 1$

7. $\sec^2\theta = 1 + \tan^2\theta$

8. $\text{cosec}^2\theta = 1 + \cot^2\theta$

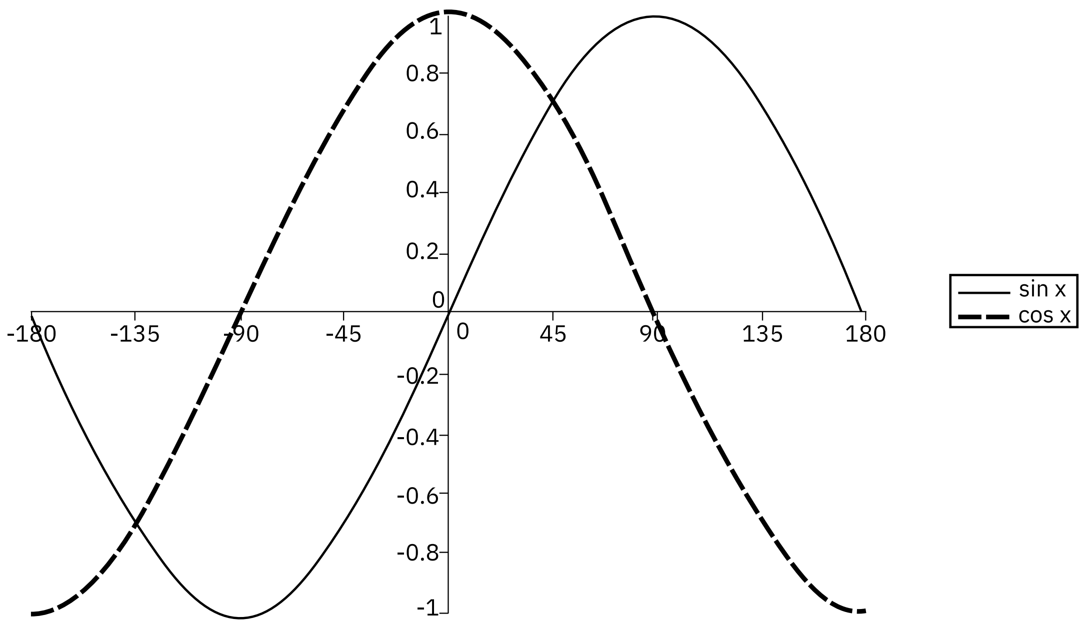
9. $\sin(-\theta) = -\sin\theta$

10. $\cos(-\theta) = \cos\theta$

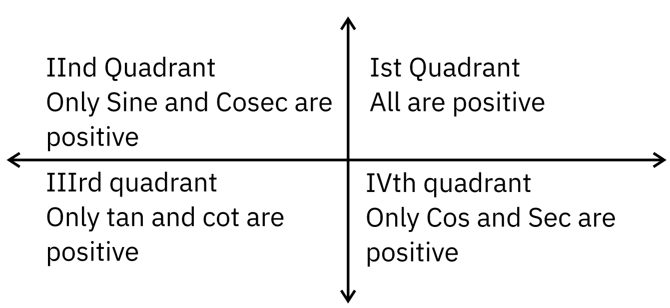
Trigonometrical ratios of some angles

Measure of θ		sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
θ^o	θ^c						
0^o	0	0	1	0	undefined	1	undefined
30^o	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
45^o	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
60^o	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$
90^o	$\frac{\pi}{2}$	1	0	undefined	0	undefined	1
180^o	π	0	-1	0	undefined	-1	undefined
270^o	$\frac{3\pi}{2}$	-1	0	undefined	0	undefined	-1
360^o	2π	0	1	0	undefined	1	undefined

Graphs of trigonometric functions (sinusoidal curves)



Signs of Trigonometrical ratios in the Cartesian Plane



This can be easily remembered as:

Students	All
Take	Coffee

Note:

The sine, cosine and tangent ratios of 0°, 30°, 45°, 60° and 90° are most important and need to be learned by heart.

The values of sine ratios are the square root of the fraction $\frac{0}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$ and $\frac{4}{4}$.

The values of cosine ratio are written in the reverse order of sine ratios and the values of tangent ratios are obtained by dividing the respective sine ratio by the cosine ratio.

Example

Find the value of cos 30 . cos 60 + sin 30 . sin 60

cos 30 . cos 60 + sin 30 . sin 60

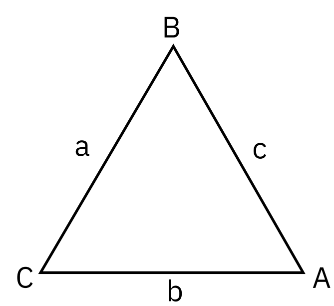
$= \frac{\sqrt{3}}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$

Appplications of Trigonometry in Geometry

1. Area of a triangle = $\frac{1}{2}$ × Product of 2 sides × sine of included angle.
2. Area of a parallelogram = Product of 2 sides × sine of included angle.
3. Area of a quadrilateral = $\frac{1}{2}$ × product of diagonals × sine of angle between them.
4. In a ΔABC, where a, b and c are the lengths of the opposite sides of $\angle$BAC, $\angle$ABC and $\angle$ACB.

   $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

   This is called the **sine rule**.



5. In ΔABC,

   $a^2 = b^2 + c^2 - 2bc \cos A$

   $b^2 = c^2 + a^2 - 2ca \cos B$

   $c^2 = a^2 + b^2 - 2ab \cos C.$

   This is called **cosine rule**.

   In case of a right angled triangle, if C = 90°

   $c^2 = a^2 + b^2 - 2ab \cos 90°$

   $\Rightarrow c^2 = a^2 + b^2$ [Pythagoras theorem]

Example

(i) In ΔABC, BC = 13 cm, AC = 8 cm and AB = 7 cm. Find $\angle$BAC.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 7^2 - 13^2}{2 \times 8 \times 7} = \frac{64 + 49 - 169}{2 \times 8 \times 7} = -\frac{1}{2};$

$\therefore \angle BAC = 120°$

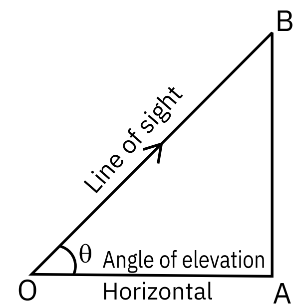
(ii) Find the area of ΔABC, if AB = 10 cm, BC = 8 cm and $\angle$ABC = 30°.

A(ΔABC) = $\frac{1}{2}$ × product of two sides × sine of included angle

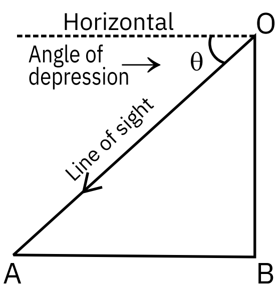
$= \frac{1}{2} \times 10 \times 8 \times \sin 30 = \frac{1}{2} \times 10 \times 8 \times \frac{1}{2} = 20$ sq. cm.

Angles of Elevation & Depression

If a person on a lower level looks up at an object at a higher level, the line of sight makes an angle with the horizontal called the **angle of elevation**.



If a person standing at a higher level observes an object at a lower level, the line of sight makes an angle with the horizontal called the **angle of depression**.



Note:
Numerically angle of elevation is equal to the angle of depression.
The angle of elevation and angle of depression are measured with the horizontal.

Example

A boy is standing on the road at a distance of 50m from the bottom of a tower, and his angle of elevation of the tower is 30°. Find the height of the tower.

Let AB be the height of the tower and let the boy be standing at position C.

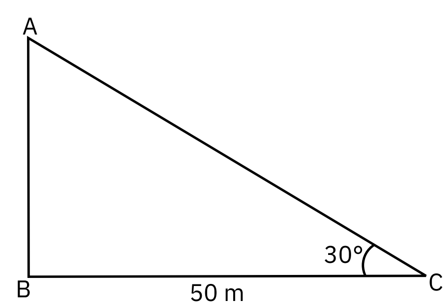
Now, AB = height of the tower and BC = 50m

In ΔABC, $\angle C = 30°$

$\therefore \tan 30° = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{50}$

$AB = \frac{50}{\sqrt{3}}$ m



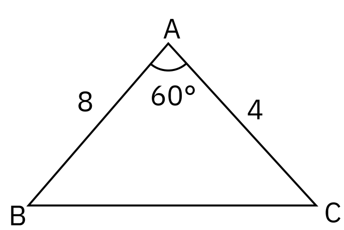
Advanced Trigonometrical Formulae

Some of these formulae are used in management entrance exams like XAT, IIFT etc.

1. $\cos(A + B) = \cos A \cos B - \sin A \sin B$
2. $\cos(A - B) = \cos A \cos B + \sin A \sin B$
3. $\sin(A + B) = \sin A \cos B + \cos A \sin B$
4. $\sin(A - B) = \sin A \cos B - \cos A \sin B$
5. $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$
6. $\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$
7. $\sin 2x = 2\sin x.\cos x$
8. $\cos 2x = \cos^2 x - \sin^2 x$

Concept Builder 1

1. Convert $\left(\dfrac{\pi}{4}\right)^c$ into degrees
2. Convert 60° into radians
3. If $\sin\theta = \dfrac{1}{\sqrt{3}}$. find a) $\cos\theta$ b) $\tan\theta$
4. The bottom of a tree is at a horizontal distance of 60m from an object lying on the ground. From the top of the tree, the angle of depression with respect to the object is 60°. Find
   a) The angle of elevation of the object with respect to the top of the tree
   b) The height of the tree
5. In the fig. Find BC.



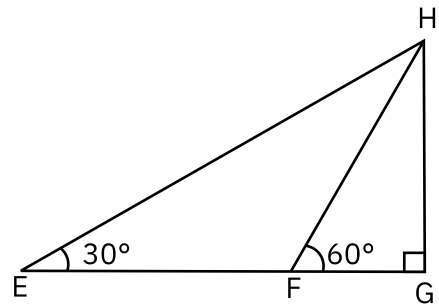
6. Find Sin 15° [Use Sin (A − B) formula]

Answer Key

1. 45°
2. $\dfrac{\pi}{3}$
3. a) $\cos\theta = \sqrt{\dfrac{2}{3}}$, b) $\tan\theta = \dfrac{1}{\sqrt{2}}$
4. a) 60° b) $60\sqrt{3}$
5. $4\sqrt{3}$
6. $\dfrac{\sqrt{3} - 1}{2\sqrt{2}}$

SOLVED EXAMPLES

Q : In the figure, EF = 100 cm, m∠HEF = 30° and m∠HFG = 60°. Find HG.



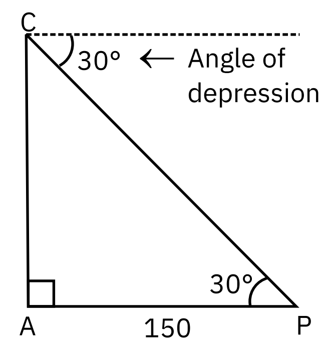
A : m∠HFE = 180 – 60 = 120°.

∴ m∠FHE = 30° ∴ FH = EF = 100 cm ...[Isosceles triangle]

ΔHFG is a 30° – 60° – 90° triangle

HG = side opp. 60° = $\frac{\sqrt{3}}{2}$ × FH = $\frac{\sqrt{3}}{2}$ × 100 = $50\sqrt{3}$ cm.

Q : From the top of a cliff the angle of depression of a point on the ground 150 feet away from the bottom of the cliff is 30°. Find the height of the cliff.

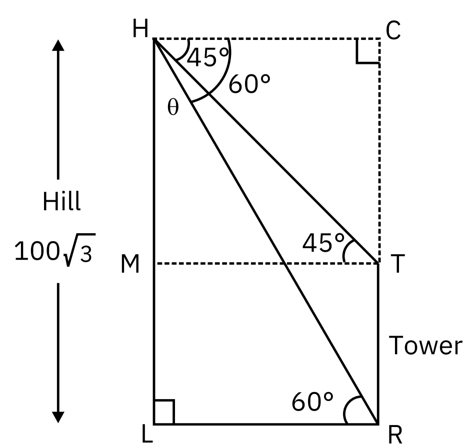


A : Let CA be the cliff.

tan 30 = $\frac{CA}{AP}$

$\frac{1}{\sqrt{3}} = \frac{CA}{150}$; ∴ CA = $\frac{150}{\sqrt{3}} = 50\sqrt{3}$ feet

Q : From the top of a hill the angle of depression of the top and the bottom of a tower are observed to be 45° and 60° respectively. If the height of the hill is $100\sqrt{3}$ m, find the height of the tower.



A : tan 60 = $\frac{HL}{LR}$; $\sqrt{3} = \frac{100\sqrt{3}}{LR}$

∴ LR = 100

LR = HC = 100 m

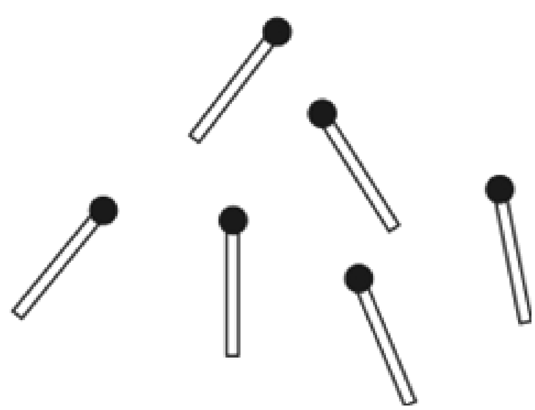
ΔHCT is a 45° - 45° - 90° triangle. ∴ HC = CT = 100 m

∴ TR = $100\sqrt{3}$ – 100 = 100(1.732 – 1) = 73.2 m

CLASS EXERCISE

Teaser

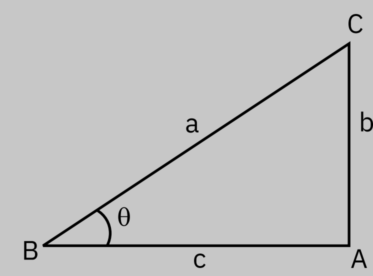
How many equilateral triangles having length of side equal to one unit can be formed by using 6 matchsticks of length one unit each?



Trigonometry

Consider the right-angled triangle shown in the adjoining figure.

Trigonometric ratios for the angle θ may be defined as follows:



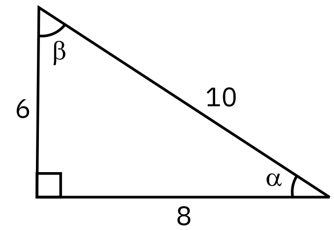
$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{b}{a}$, $\text{cosec}\ \theta = \dfrac{\text{Hypotenuse}}{\text{Opposite}} = \dfrac{a}{b}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{c}{a}$, $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Adjacent}} = \dfrac{a}{c}$

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{b}{c}$, $\cot\theta = \dfrac{\text{Adjacent}}{\text{Opposite}} = \dfrac{c}{b}$

Some useful rules: • $\sin^2\theta + \cos^2\theta = 1$ • $\sin\theta = \cos(90 - \theta)$ • $\sin(2\theta) = 2\sin\theta\cos\theta$

1. In the adjoining triangle, calculate the following ratios



sin α =
cos α =

tan α =

sin β =
cos β =
tan β =

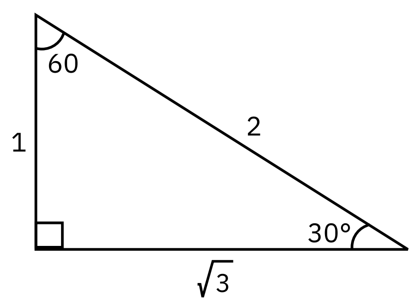
cosec α =
sec α =
cot α =

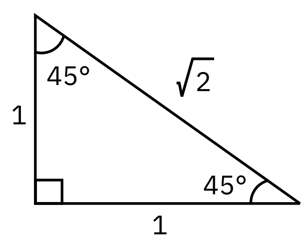
cosec β =
sec β =
cot β =

2. *Consider a right angled triangle with sides 9 cm, 40 cm and 41 cm. Calculate—

a) sin of the angle opposite to side of length 9 cm
b) cos of the angle opposite to side of length 40 cm
c) tan of the angle opposite to side of length 9 cm.

3. Calculate sin, cos and tan of 30, 45 and 60

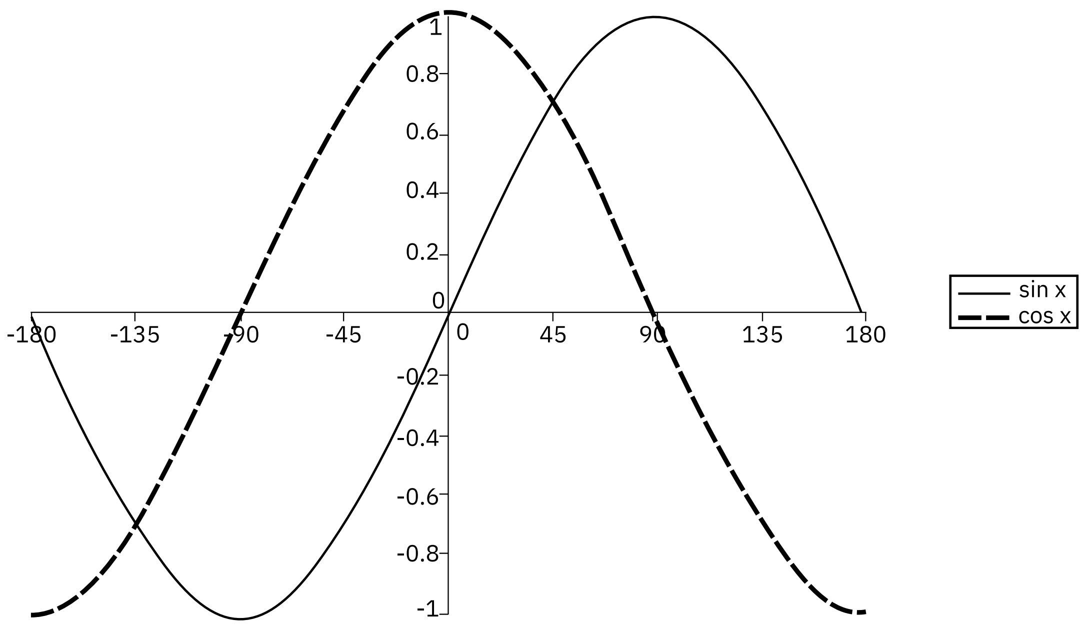




Now fill in the following table:

	30	45	60
Sin			
Cos			
Tan			

Graphs of trigonometric functions (sinusoidal curves)



4. In triangle BAC, BD is an altitude on side AC, which meets side AC in D such that A-D-C. Suppose side AB = 6, side BC = 10 and angle C = 30 degrees. Calculate lengths of AD, DC and BD.

5. Calculate the value of $\frac{\text{sin1.sin2.sin3............sin89}}{\text{cos1.cos2.cos3..........cos89}}$

Advanced trigonometric formulae

1. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$
2. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$
3. $\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$
4. $\sin 2x = 2 \sin x \cos x$
5. $\cos 2x = \cos^2 x - \sin^2 x$

6. If each of a, b and c is a positive acute angle such that $\sin(a + b - c) = \frac{1}{\sqrt{2}}$, $\operatorname{cosec}(b + c - a) = \frac{2}{\sqrt{3}}$ and $\tan(c + a - b) = \frac{1}{\sqrt{3}}$, what are the values of a, b and c?
1) 37.5, 52.5, 45 2) 37,53,4 3) 45,37.5,52.5 4) 34.5,55.5,45

7. If $\cot^2\theta - (1 + \sqrt{3})\cot\theta + \sqrt{3} = 0$, then what is the value of θ ?
1) 90, 60 2) 45, 60 3) 45, 30 4) 90, 180

8. Given that 2 sin x cos x = sin 2x and 1 – 2 $\sin^2 x$ = cos 2x, which for the following is always true about f(x) = sin2x tan x – cos2x cotx, for 45 < x < 90 ?
   1) f(x) is always positive
   2) f(x) is always negative
   3) f(x) intersects the X-axis at only one point
   4) None of the above

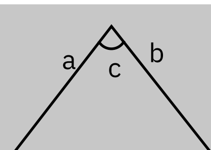
9. If x = $\sin^2 15° + \sin^2 30° + \sin^2 45° + \sin^2 60° + \sin^2 75° + \sin^2 90°$, what can be said about x?
   1) 1 < x < 2   2) 2 < x < 3   3) 3 < x < 4   4) 4 < x < 5

10. Find the approximate value of (sin 34° + cos 56°)(sin 56° – cos 34°)

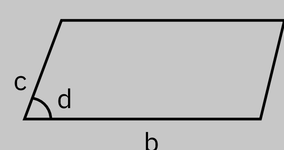
11. *If sin (3x) = cos (7x), where x is an angle between 0° and 90°, what will be the value of tan (5x)?

Applications of trigonometric ratios

1. Area of a triangle: $\frac{1}{2}$ab sinC



2. Area of a parallelogram: bc sin d



3. Suppose a regular polygon of n sides is divided into n congruent triangles by joining the vertices of the polygon with the center of the polygon, area of the polygon: $\frac{nr^2}{2}$ sinθ, where r is radius of the circumcircle and $\theta = \frac{360}{n}$

12. Suppose in triangle ABC, side AB = 20 cm and side BC = 10 cm. If angle B = 30°, what is area of triangle ABC?

13. Calculate area of parallelogram PQRS if SR = 20 cm, PS = 12 cm and angle Q = 45.

14. Calculate area of a regular octagon inscribed in a circle of radius 10 cm.

15. *Calculate area of a regular polygon with 20 sides inscribed in a circle of radius 10 cm., given that sin18 = 0.3090

16. *Calculate value of $(\sin 10 + \cos 10)^2$ (1 – sin 20) – $\cos^2 20$

Angle of elevation and depression

17. Sagar is standing in front of a building at a distance of 400 feet. The angle of elevation for the top of the building is 30°. There is one electric pole between Sagar and the building. Sagar can see the top of the building just beyond the top of the pole. If Sagar is standing at a distance of 20 feet from the pole, what is the height of the pole?

18. There are two buildings, one on each bank of a river, opposite each other. From the top of one building, which is 60 m high, the angles of depression of the top and the foot of the other buildings are 30 and 60 respectively. What is the height of the other building?

    1) 30 m   2) 18 m   3) 40 m   4) 20 m

19. A car is being driven, in a straight line and at a uniform speed towards the base of a vertical tower. The top of the tower is observed from the car and in the process, it takes 10 minutes for the angle of elevation to change from 45 to 60 degrees. After how much more time(in minutes) will this car reach the base of the tower?

    1) $5\left(\sqrt{3}+1\right)$ 2) $6\left(\sqrt{3}+\sqrt{2}\right)$ 3) $7\left(\sqrt{3}-1\right)$ 4) $8\left(\sqrt{3}-2\right)$

20. Raju is a naughty boy. He climbs a tree of height 50 m and reaches the top. There is a pillar in front of the tree. The angle of depression for the top of the pillar is 30 degrees. Raju can also see that just beyond the top of the pillar, a dog is sleeping on the ground at a distance of $10\sqrt{3}$ m from the bottom of the pillar and $50\sqrt{3}$ m from the bottom of the tree. An eagle is flying exactly above the pillar such that Raju's angle of elevation for the eagle from the top of the tree is 60 degrees. What is the distance between the eagle and the top of the pillar?

    1) 120 meters   2) $120\sqrt{3}$ meters   3) 160 meters   4) 130 meters

21. The angles of depression from a 150m tower to the top and bottom of a chimney are 30° and 60° respectively. Find the height of the chimney.

    1) $50\sqrt{3}$ m   2) 75 m   3) $75\sqrt{3}$ m   4) 100 m

22. *A soldier standing at the top of a tower of height 10 sees three enemies approaching along a straight road. The angles of depression made by the three are 35°, 45° and 55°, while their distances from the foot of the tower are a, b and c respectively. What is the value of a × b × c?

Challengers:

1. For all real numbers x, except x = 0 and x = 1, function F is defined by $F\left(\frac{x}{x-1}\right) = \frac{1}{x}$. If $0 < \alpha < 90$, what is the value of $F(\text{cosec}^2\alpha)$?

 1) $\sin^2\alpha$ 2) $\cos^2\alpha$ 3) $\tan^2\alpha$ 4) $\cot^2\alpha$ 5] $\sec^2\alpha$

2. In triangle ABC, angle A = 60 degrees. AD is an angle bisector of angle A. Side AB = 8 and side AC = 10 cm. Calculate length of AD.

 1) $4\sqrt{3}$ 2) $\frac{40\sqrt{3}}{9}$ 3) $5\sqrt{3}$ 4) $\frac{50\sqrt{3}}{9}$

3. A tower is at corner B of a rectangular field ABCD. From corner D, the top E of the tower is at an angle of elevation of 30°, while from the corner C, it is at an angle of elevation of 45°.

 Find the ratio of the lengths of the sides of the field.

 1) $\sqrt{3} : 1$ 2) 2 : 1 3) $\sqrt{2} : 1$ 4) 1 : 1

4. A warship and a submarine (completely submerged in water) are moving horizontally in a straight line. The captain of the warship observes that the submarine makes an angle of depression of 30 degrees and the distance between them from the point of observation is 50 km. After 30 minutes, the angle of depression becomes 60 degrees. Find the distance(in kms) between them after 30 minutes from the initial point of reference.

 1) $\frac{50}{\sqrt{3}}$ 2) 25 3) $\frac{25}{\sqrt{3}}$ 4) $25\sqrt{3}$

5. If $\sin\alpha + \sin\beta = a$, $\cos\alpha + \cos\beta = b$, $\tan\left(\frac{\alpha}{2}\right) * \tan\left(\frac{\beta}{2}\right) = c$ and $a \neq b \neq c \neq 0$, $c \neq 1$, $\frac{1-c}{1+c}$ is equal to

 1) $\frac{b}{a^2+b^2}$ 2) $\frac{2a}{a^2+b^2}$ 3) $\frac{2b}{a^2+b^2}$ 4) $\frac{a}{a^2+b^2}$

PRACTICE EXERCISE

DIRECTIONS for questions 1 to 6: Choose the correct alternative.

1. If $\cot\theta + \tan\theta = p$ then which of the following is true?

   1) $p\sqrt{\sin^2\theta - \sin^4\theta} = 1$
   2) $1 = p\tan\theta\cos^2\theta$
   3) Both of these
   4) None of these

2. If θ & ϕ are supplementary angles then $\tan^2\theta - \sec^2\phi$ will be equal to:

   1) $\sin^2\theta + \cos^2\phi$
   2) $\sin^2\theta - \cos^2\phi$
   3) $\cos^2\theta + \sin^2\phi$
   4) $-\sin^2\theta - \cos^2\phi$

3. Find the simplified form of:

   $[\sin\theta\cos\theta + \sin(90 - \theta)\cos(\theta + 90)]^2 - [\sin\theta\cos\theta - \sin(90 + \theta)\cos(\theta - 90)]$.

   1) $2\sin\theta\cos\theta$
   2) $(\sin\theta + \cos\theta)^2 - 3\sin\theta\cos\theta$
   3) 1
   4) 0

4. If $\operatorname{cosec}\theta - \sin\theta = a$, $\sec\theta - \cos\theta = b$ & $\tan\theta + \cot\theta = c$ then which of the following is not true?

   1) $abc = 1$
   2) $(a^2b) + (ab^2) = 1$
   3) $ab = \sin\theta\cos\theta$
   4) $\dfrac{1 + \sin\theta + \cos\theta}{ab} - (\sin\theta + \cos\theta) = a + b + c$

5. If $\operatorname{cosec}\theta - \sin\theta = p$ and $\sec\theta - \cos\theta = q$ then which of the following is true?

   1) $p^2q^2 + pq^2 = 1$
   2) $pq = 1$
   3) $(p^2q)^{2/3} + (pq^2)^{2/3} = 1$
   4) $p/q = 1$

6. Which of the following cannot be the value of $\operatorname{cosec}^2x + \sec^2x$?

   1) $2\sqrt{3}$ 2) $3\sqrt{3}$ 3) $3\sqrt{2}$ 4) $2\sqrt{5}$

DIRECTIONS for question 7: Solve as directed.

7. What is the maximum value that 4sinA + 3cos A can take?

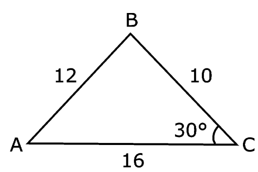
DIRECTIONS for questions 8 to 20: Choose the correct alternative.

8. If $5^0 \le x^0 \le 15^0$, then the value of sin 30^0 + cos x^0 – sin x^0 will be:

1) Between -1 and - 0.5 inclusive
2) Between - 0.5 and 0 inclusive
3) between 0 and 0.5 exclusive
4) between 0.5 and 1 inclusive
5) None of the above

(Past XAT question)

9. In the given figure, what is the value of sin B – sin A?



1) 0.5
2) 1
3) 0.33
4) 0.25

10. A dog moving towards a wall, observes a lizard moving up a wall. At the particular instant the angle of elevation is 30°. When the lizard reaches the top of the wall, the dog is 15 metres away from the wall.

What is the distance travelled by the dog in this period if the lizard has travelled $5\sqrt{3}$ m (during the same period) and the angle of elevation of the dog's eye with the lizard becomes 60^o?

1) 16.5 m
2) 15 m
3) $16.5\sqrt{3}$ m
4) $15\sqrt{3}$ m

11. A crow, sitting on the ground, sees the top of a building at an angle of elevation of 30° and starts flying towards it along a straight line. At the same time, a pigeon, who is sitting at the bottom of the building, flies straight up towards the top of the building. They both fly at the same speed. The pigeon reaches 20 seconds earlier than the crow. If they were moving along the ground from their initial positions towards each other at the same speed, after how much time (in seconds) will they meet?

1) 5
2) $5\sqrt{3}$
3) 10
4) $10\sqrt{3}$

12. A tower of height 100 m is installed at a point in an open ground. A cat named Amy is standing at a point from where the angle of elevation of the top of the tower is 30°. Another cat named Pamy is standing at a point on the line joining Amy's position and the foot of the tower (on the same side as Amy) such that the angle of elevation of the top of the tower from Pamy's position is 60°. If Amy and Pamy simultaneously start running towards each other, they meet at a point from where the angle of elevation of the top of the tower is 45°. If Amy and Pamy both simultaneously start running towards the tower from their original positions, how far from the foot of the tower would Amy be when Pamy reaches the foot of the tower (in m)?

1) $100(\sqrt{2} - 1)$
2) $100(\sqrt{3} + 1)$
3) 100
4) $100(\sqrt{3} - 1)$

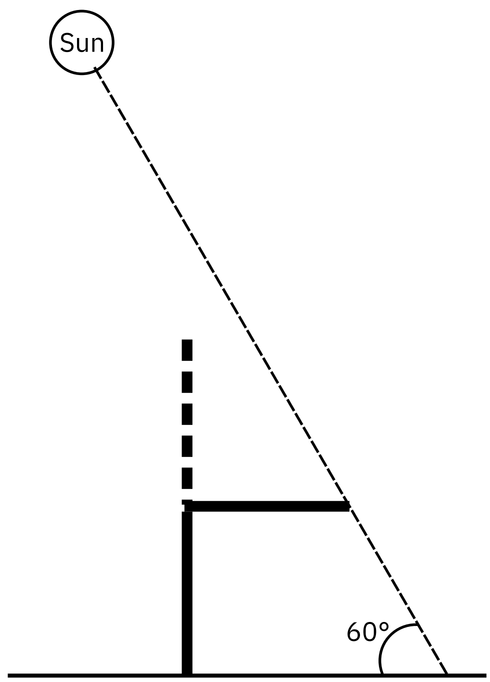
13. There are two towers AB and CD (A and C being their tops and B and D being their feet). The angle of elevation of C from B is 30° and the angle of elevation of A from D is 60°. An eagle and a kite start flying at the same moment from the tops of towers AB and CD respectively towards the foot of the other tower along a straight line and at uniform speeds. If they meet at one point along their flight, find the ratio of their speeds.

1) $\sqrt{3}$: 1   2) $3\sqrt{3}$: 2   3) 3 : 1   4) $3\sqrt{3}$: 1

14. On a regular octagon ABCDEFGH, two towers of heights h_1 and h_2 are set on the points D and F respectively. The angle of elevation of the top of the tower at F from A is 45° while the angle of depression of the point A from the top of the tower at D is 30°. Find the ratio $h_1 : h_2$.

1) $\sqrt{3}$ 2) $\frac{2}{\sqrt{3}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $\frac{\sqrt{3}}{2}$

15.



The upper half of a 20 metre high pole got bent away from the sun by 90° at its midpoint such that the upper half and lower half of the pole are now at right angles to each other, as shown in the figure. The sunrays fall at an angle of 60° with respect to the horizontal. Find the distance between the shadows cast by the topmost point of the pole before and after it got bent.

1) $10\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right)$m   2) $\frac{10}{\sqrt{3}}$m   3) $10\sqrt{3}$ m   4) $10\left(\sqrt{3}-1\right)$m

16. The angle of elevation of the top of a building from point A on the ground is 30°. The angle of elevation of the top of a tower from the top of the building is 60°. Point A and the tower are on either side of the building. If the angle of elevation of the top of the tower from point A is 45°, what is the ratio of the distances of the base of the building from point A and the base of the tower respectively?

1) $\sqrt{3} : 1$ 2) $1 : \sqrt{3}$ 3) 1 : 2 4) 2 : 1

17. The angle of elevation of the top of a tower from point P on the ground is β. The angle of elevation of a certain point on the tower above the ground from point P is α. If the distance of the point P from the foot of the tower is 15, find the height of the tower. Given: $\tan \alpha = \frac{1}{3}$; $\beta = 2\alpha$.

1) 11.25 2) 13.75 3) 15 4) 16.25

18. A lizard is lying stationary at point A on the floor. It observes an insect on a vertical wall at point B which is at an angle of elevation of 30° from A. As soon as the lizard starts moving along the floor towards the vertical wall in the direction of the insect, the insect starts moving upwards along the vertical wall. Once the lizard reaches the edge of the wall, it starts moving upward along the vertical wall towards the insect. The lizard is able to catch the insect when it reaches point C on the vertical wall, which is at an angle of elevation of 60° with respect to A. What is the ratio of the speeds of the insect and the lizard?

1) $\frac{1}{\sqrt{3}}$ 2) $\frac{2}{1+\sqrt{3}}$ 3) $\frac{1}{3+\sqrt{3}}$ 4) $\frac{2}{3+\sqrt{3}}$

19. Buildings A and B are parallel to each other. The height of B is less than that of A. I am standing in such a way that building B is between building A and me. I am standing at a distance of $200\sqrt{3}$ from building B and can see only 20 floors of building A. After travelling a distance of $\frac{400}{3} \times \sqrt{3}$ m towards the buildings, I am able to see only the top of building A. Building B has 20 floors.

(Height of floors in buildings A and B are the same. Also assume my height to be of negligible value).

What is the distance between the buildings A and B?

1) $100\sqrt{3}$ m 2) $200\sqrt{3}$ m 3) 100 m 4) 200 m

20. A person is standing at a distance of 1800 meters facing a giant clock at the top of a tower. At 5.00 p.m., he can see the tip of the minute hand of the clock at 30 degree elevation from his eye-level. Immediately, the person starts walking towards the tower. At 5.10 pm., the person noticed that the tip of the minute hand made an angle of 60 degrees with respect to his eye-level. Using three dimensional vision, find the speed at which the person is walking. The length of the minute hand is $200\sqrt{3}$ meters ($\sqrt{3}$ = 1.732).

1) 7.2 km/hour
2) 7.5 km/hour
3) 7.8 km/hour
4) 8.4 km/hour
5) None of the above

(Past XAT question)