Career Launcher

Set Theory - 1

LRDI - 08

CEX-D-0280/18

Number of Questions : 22

1. S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}.
   (i) How many subsets of S are there?
   (ii) How many proper subsets of S are there?
   (iii) How many subsets of S contain at least one even number?
   (iv) How many subsets of S are there in which at least one of 2, 3 and 4 is there and at least one of 5, 6 and 7 is not there?
   (v) How many subsets of S are there which contains 2 if 5 is also there.
   (vi) In how many subsets of S are possible such that if 6 is there, then it must be highest?

2. There are certain number of students in a university class. Of all, 25 students play hockey, 12 play cricket, and 18 play football. Of these, 10 students play both hockey and cricket, 9 play both cricket and football, and 14 play both football and hockey. If 8 students play all the three games, how many students are there in the class?
   (1) 28 (2) 30
   (3) 31 (4) Cannot be determined

Directions for questions 3 and 4: Answer the questions on the basis of the information given below.

There are 100 students in a class. If 70 students like Physics and 65 students like Economics.

3. Find the maximum number of students who like both the subjects.
   (1) 40 (2) 65
   (3) 50 (4) 70

4. Find the minimum number of students who like both the subjects.
   (1) 70 (2) 65
   (3) 35 (4) 40

Directions for questions 5 and 6: Answer the questions on the basis of the information given below.

At a certain conference of 100 people, there are 29 Indian women and 23 Indian men. Of these Indian people, 4 are doctors and 24 are either men or doctors. There are no foreign doctors.

5. How many foreigners are attending the conference?
   (1) 51 (2) 55
   (3) 52 (4) 48

6. How many women doctors are attending the conference?

Directions for questions 7 to 10: Answer the questions on the basis of the information given below.

There are 2000 students in a school and they all play atleast 2 of the 5 sports – Cricket, Badminton, Hockey, Table Tennis (TT) and Football. It is also known that no. of students playing in every combination of exactly two sports is 4 times the no. of students playing in every combination of exactly 3 sports. Also, no. of students playing all the 5 sports is $1/4^{th}$ of those playing exactly 4 sports. The number of students playing in every combination of exactly 4 sports is the same.

7. If the number of people playing all the 5 sports is 160, then what is the no. of students playing only cricket & football?
(1) 72 (2) 116
(3) 480 (4) 96

8. If the number of students playing exactly 3 sports is 66.66% more than the number of students playing all 5 sports, then which of the following is true?
I. The number of students playing only Cricket, Hockey and Badminton is 25.
II. The number of students playing all 5 sports is 150.
III. The number of students playing only TT & Hockey is 100.

(1) I & II (2) I & II
(3) II & III (4) I, II & III

9. If the number of students playing cricket, Badminton, Hockey, TT & Football are 1300, 1400, 1500, 1600 & 1700 respectively then find the sum of all the students playing only cricket & TT, playing only Cricket TT & Football and playing only Cricket, TT, Football & Hockey.

10. If in the previous question, number of students playing cricket is not known, then what can be that value?
(1) 1665 (2) 1815
(3) 1792 (4) 1770

Directions for questions 11 to 13: Answer the questions on the basis of the information given below.

There are 50 families in a society. There were three categories of families F, W and FW in the society: category F i.e. families having only a refrigerator, category W i.e. families having only a washing machine and category FW i.e. families having both a refrigerator and washing machine. 25 families were in category F, 10 were in category W and remaining families were in category FW. During Diwali some of the families purchased or sold a refrigerator or a washing machine. Hence, some families changed their category. Further it is known that:

At least 40% of the families having refrigerator changed their category.

At most 50% of the families having washing machine changed their category.

After Diwali too, every family was in one of these three categories only.

11. After Diwali, maximum how many families had a washing machine only?

12. After Diwali, minimum how many families had both of the appliances?
(1) 0 (2) 3
(3) 4 (4) 6

13. What is the maximum difference between the number of families who sold their only appliance i.e. refrigerator to purchase a washing machine and the number of families who sold their only appliance i.e. washing machine to purchase a refrigerator?
(1) 25 (2) 16
(3) 15 (4) 6

Directions for questions 14 and 15: Answer the questions on the basis of the information given below.

There are 170 members in a club. Each one of them belongs to one or more categories out of the four categories viz. Service holder, Professional degree holder, Physically challenged and Married. There are 60 Service holders, 40 Professional degree holders, 50 Physically challenged and 70 Married members.

14. If the number of members who belong to all four categories is maximum possible, then how many members belong to at least three categories?
(1) 18 (2) 17
(3) 16 (4) Either (2) or (3)

15. If there are exactly 10 members who belong to all four categories, then find the maximum possible number of members belonging to only Physically challenged category.
(1) 40 (2) 35
(3) 30 (4) 38

16. A survey was conducted on the eating habits of a group of 1000 people. Results show that 92% of the people surveyed eat south Indian food, 91% eat North Indian food, 82% eat American food, 78% eat Chinese food, 79% eat Italian food and 80% eat Continental food. What must be the minimum number of people who eat all the 6 types of food, if 7 people do not eat any of the six types of food?
(1) 0 (2) 13
(3) 27 (4) 55

Directions for questions 17 to 19: Answer the questions on the basis of the information given below.

In class 5^{th} of DPS there are 100 students. 50 have pens, 45 have pencils, 35 have erasers and 20 have sharpners.

17. Students having pen have no other item with them. All other students have exactly two items. How many students have exactly two items pencil and eraser?

18. If all students have exactly 3 items or no item then how many students are having pencil and sharpner?

19. If maximum number of students are having all 4 items then what is the maximum number of students having exactly two items?
(1) 20 (2) 30
(3) 35 (4) 50

20. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options — air conditioning, radio and power windows were already installed.

Following were the observation of the survey:
I. 15 had air conditioning.
II. 2 had air conditioning and power windows but no radios.
III. 12 had radio.
IV. 6 had air conditioning and radio but no power windows.
V. 11 had power windows.
VI. 4 had radio and power windows.
VII. 3 had all three options.

What is the number of cars that had none of the options?
(1) 4 (2) 3
(3) 1 (4) 2

Directions for questions 21 and 22: Answer the questions on the basis of the information given below.

Eighty-five children went to an amusement park where they could ride on the merry-go round, roller coaster, and Ferris wheel. It was known that 20 of them took all the 3 rides, and 55 of them took at least 2 of the 3 rides. Each ride cost Re 1, and the total receipt of the amusement park was ₹145. [Any children took a ride in any of the rides only once]

21. How many children did not try any of the rides?
(1) 5 (2) 10
(3) 15 (4) 20

22. How many children took exactly one ride?
(1) 5 (2) 10
(3) 15 (4) 20

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LRDI - 08 Answers and Explanations

CEX-D-0280/18

1	–	**2**	4	**3**	2	**4**	3	**5**	4	**6**	1	**7**	4	**8**	4	**9**	293	**10**	4
11	47	**12**	2	**13**	1	**14**	4	**15**	1	**16**	4	**17**	30	**18**	15	**19**	3	**20**	4
21	3	**22**	3																

1. (i) The number of subsets of a set with n elements is 2^n. Here n is 11 so 2^{11} = 2048 subsets are there.
(ii) Proper subsets = total subsets – 1 = 2047.
(iii) Total number of subsets – subsets with only odd numbers = subsets with at least one even number. So, subsets with only odd numbers = 2^6 = 64. Hence subsets with at least one even number = 2048 – 64 = 1984.
(iv) At least one of 2, 3 and 4 can be selected in $2^3 - 1 = 7$ ways. Also if we have to reject at least one of 5, 6 and 7, it can be done in $2^3 - 1 = 7$ ways. For other elements there is no restriction. Hence total number of subsets with above restrictions is 7 x 7 x 2^5 = 1568.
(v) Out of 2 and 5, 4 combinations can be formed i.e. {2}, {5}, {2, 5} and none. Out of these 4 only one combination is not allowed i.e. {5}. Hence total number of subsets is 3 x 2^9 = 1536.
(vi) **Case 1:** 6 is there. Then the other elements must be less than 6. Total number of subsets is 2^5 x 1 = 32.
Case 2: 6 is not there. Total number of subsets is 2^{10} = 1024.
Hence answer 1024 + 32 = 1056.

2. 4 It is not given that all of them play at least one game.

3. 2 Since there are 100 students, to find the maximum number of students who like both the subjects we will assume that all the students who like Economics, also like Physics.
Thus, maximum 65 students like both the subjects.

4. 3 To find the minimum number of students who like both the subjects, we will assume that 70 students like Physics and rest 30 students like Economics. Now, we are still left with 35 students who like Economics, and these 35 students are such that they like both Physics and Economics.

5. 4 Number of foreigners = 100 – Number of Indians = 48

6. 1 $n(M) + n(D) - n(M \cap D) = 24$
[Where, n( ) represents number of elements in a particular set]
$\therefore (M \cap D) = 3$
Hence, there is one woman doctor.

For questions 7 to 10:
II $\rightarrow$ no. of students playing exactly 2 sports.
III $\rightarrow$ no. of students playing exactly 3 sports.
IV $\rightarrow$ no. of students playing exactly 4 sports.
V $\rightarrow$ no. of students playing exactly 5 sports.
II + III + IV + V = 2000 ...(i)
Given, II = 4 × (III) & IV = 4 (V).
Putting those in equation (i), we get
5(III + V) = 2000
or, III + V = 400 ...(ii)
Now, no. of regions of the intersection of the sets,
For exactly 2 sports $\rightarrow$ there will be $5C_2$ regions i.e., 10 regions.
For exactly 3 sports $\rightarrow$ there will be $5C_3$ regions i.e., 10 regions.
For exactly 4 sports $\rightarrow$ there will be $5C_4$ regions i.e., 5 regions.
For exactly 5 sports $\rightarrow$ there will be 1 regions i.e., 1 regions.
It is given that no. of students in every combination of 2 sports (10 regions) is 4 times the each of the region (10 regions; ${}^5C_3$) in exactly 3 sports.
It is only possible when all the regions under II are equal and all the regions under 3 are equal.

7. 4 $\because$ III + V = 400 & V = 160 (given)
$\therefore$ III = 240
and since II = 4 × (III)
= 4 × 240 = 960
As above, students playing only 2 sports will have 10 regions (${}^5C_2$) and all being equal.
$\therefore$ No. of students playing only Cricket & Football
$= \dfrac{960}{10} = 96.$

8. 4 III : V = 5 : 3 & III & V = 400

III = 250

& V = 150

II = 4 × 250 = 1000

IV = 4 × 150 = 600

∴ One region of any 3 sports = $\frac{250}{10} = 25$

& One region of any 2 sports = $\frac{1000}{10} = 100$

Hence, all the statements are correct.

9. 293 I + 2II + 3III + 4IV + 5V = 7500

Now, I = 0 (because all are playing atleast 2 sports)

II = 4(III)

IV = 4(IV)

So, 11(III) + 21(V) = 7500

and III + V = 400

Solving both we get, V = 310

& III = 90

∴ II = 4 × 90 = 360

IV = 4 × 310 = 1240.

Now, One region of any 2 sports = $\frac{360}{10} = 36$

One region of any 3 sports = $\frac{90}{10} = 9$

One region of any 4 sports = $\frac{1240}{5} = 248$

(Since, for 4 sports, $5C_4$ = 5, regions will be there).

∴ Reqd. sum = 36 + 9 + 248 = 293.

10. 4 11(III) + 21(V) = 6200 + x ...(i)

II + V = 400 ...(ii)

Solving, (i) and (ii)

10(V) = 1800 + x

V = 180 + $\frac{x}{10}$

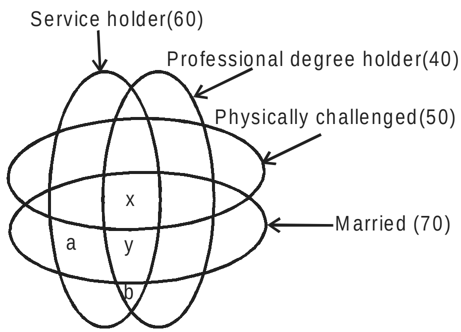
So, it has to be a multiple of '10' among the options, 1770 satisfies.

11. 47 There are already 10 families in category W. All families of category F can shift to category W. Also at most 50% of 25 families i.e. max 12 families having washing machine can change their category. Let all these 12 families are from category FW. So finally category W has 47 families.

12. 2 As not more than 12 families having washing machine can change their category so a maximum of 12 families can shift from FW so minimum 3 families will be left.

13. 1 As minimum 25 families having refrigerators changed their category so let all families from F changed to W. Also, a minimum of 0 families have to shift from W or FW so let no family shifted from W to F. Hence maximum difference is 25 – 0 i.e. 25.

14. 4



Let us define 'x' and 'y' as the number of members belonging to 'all 4' and 'exactly 3, categories respectively. Similarly (a + b) denotes the number of members belonging to 'exactly 2' categories.

We can define a term called 'excess data' as the difference between the sum of number of members belonging to 4 individual categories and the actual number of members in the club.

This 'excess data' will be absorbed with the increasing number of members belonging to exactly 2, 3 or 4 categories.

Case I:

Here excess data is (220 – 170) = 50. On the basis of excess data, we can maximize x. Assume 'x' to be 1, then 3 will be subtracted from the excess data. So the maximum value of x can be 16. Now assume 'y' to be 1, then 2 will be subtracted from the excess data. Therefore excess data will be zero when x = 16 and y = 1.

Hence members belonging to at least three categories = 16 + 1 = 17.

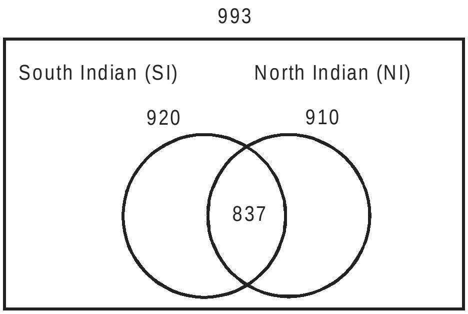
Case II:

Maximum value of x = 16. But if we put y = 0, a = b = 1, even then the excess data gets absorbed.

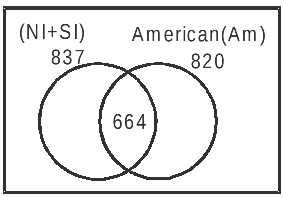
In this case there will be no member belonging to 'exactly 3' categories and 2 members belonging to 'exactly 2' categories. In this case, members belonging to at least 3 categories = 16 + 0 = 16.

15. 1 Here x is 10, then excess data is (50 – 30) = 20. To maximize the number of members belonging to Physically challenged category, 10 members can belong to three categories except Physically challenged.
Therefore excess data will be absorbed when x = 10 and y = 10. Hence maximum number of members belonging to only Physically challenged category = (50 – 10) = 40.

16. 4 There are 7, people who do not eat any of the 6 kinds of foods. Which means there are 993 people who eat at least 1 of the 6 kinds of foods.
Suppose all the people (i.e. all 993) eat either South Indian or the North Indian food.

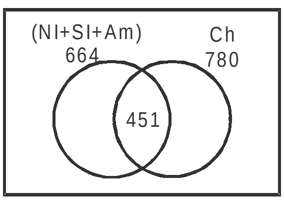


⇒ There are a minimum of 837 people who eat both South Indian & the North Indian food.
Further, 82% people eat American food.

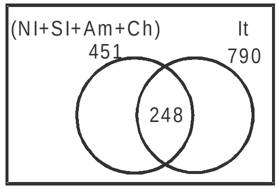


At least 664 people are there who eat (NI + SI + Am) food.

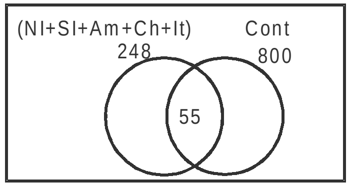
Then, 78% people eat Chinese(Ch) food;



⇒ Atleast 451 people eat (NI +SI + Am +Ch) food
Further, 79% people eat Italian (It) food.



⇒ Minimum 248 people, eat 5 types of foods (NI + SI + Am + Ch + It). And also, 80% people eat Continental food.



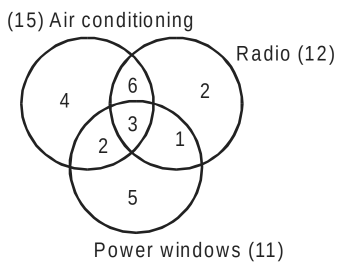
⇒ So, in all, atleast 55 people eat all the 6 types of foods.

17. 30 50 students are having only one item i.e. a pen. Remaining 50 students have a total of 100 items. Let a, b and c be the number of students having exactly two items i.e. pen and pencil, pencil and eraser and pen and eraser respectively.
Hence,
a + b + c = 50, a + b = 35, b + c = 20, a + c = 45.
Solving this we get, a = 30, b = 5, c = 15.

18. 15 If all the students have 3 items, then there must be 50 students having 3 items each. As there are 50 pens, then every student having 3 items must be having a pen. All these students have exactly 2 more items. 35 of them have erasers, which means remaining 15 students have pencil and sharpener.

19. 3 A maximum of 20 students can have all 4 items. The remaining items are 30 pens, 25 pencils, 15 erasers and 0 sharpners. These 70 items are to be distributed among remaining 80 students. A maximum of 35 students can have these 70 items.

20. 4



Total = 4 + 6 + 2 + 2 + 3 + 1 + 5 = 23
∴ Cars having none of the option = 25 – 23 = 2.

For questions 21 and 22:
Let the number of children who took only one ride be ‘x’
Given, $20 \times 3 + 35 \times 2 + x \times 1 = 145$
$\therefore x = 15$
Number of children who did not take any ride
$= 85 - (20 + 35 + 15) = 15$

21. 3

22. 3