Algebra - 3

Contents

- Binomial
- Inequality

Career Launcher

QA - 19

CEX-Q-0220/18

Number of questions : 25

Binomial

1. How many terms in the expansion of $(a-b)^{30}$ will be negative if a and b are positive real numbers?
(1) 0 (2) 1
(3) 15 (4) 30

2. Which term is independent of x in the expansion of $\left(x^2+\frac{3}{x}\right)^{15}$?
(1) 8th term (2) 9th term
(3) 10th term (4) 11th term

3. The sum of coefficients of the terms which do not contain odd powers of x in the expansion of $(x+y)^{100}+(x-y)^{100}$ is
(1) 2^{100} (2) 2^{99}
(3) $(2^{100}-2)$ (4) $\frac{(2^{100}-1)}{2}$

4. Find the sum of the coefficients of all the powers of x in $\left(4x-\frac{1}{x}\right)^{20}$.
(1) 2^{20} (2) 0
(3) 4^{20} (4) 3^{20}

5. The coefficient of x^7 in the expansion of $(1-x^2+x^3)(1+x)^{10}$ is
(1) 75 (2) 78
(3) 85 (4) None of these

6. In the expansion of $(a+b+c)^{20}$, find
A. Number of terms?
B. Coefficient of $a^{17}b^2c$?
C. Sum of the coefficients

7. In the expansion of $(1+x+x^2)^{20}$, find
A. Number of terms?
B. Coefficient of x^4?

8. Find the coefficient of x^4 in the expansion of $(1-x+x^2)^5$.
(1) 35 (2) 45
(3) 60 (4) 75

Inequality

9. How many of the following statements is/are universally true?
A. If x > a; y > b then x – y > a – b
B. If x > a then $x^2 > a^2$
C. If a < x/c < b then ac < x < bc

(1) 0 (2) 1 (3) 2 (4) 3

10. How many integer values of x satisfy $\frac{x-1}{3}<\frac{5}{7}<\frac{x+4}{5}$?
(1) 0 (2) 1 (3) 2 (4) 4

11. If 20X < Y, 23(X – 1) > Y, and 21X + Y = 500, X and Y are both positive integers, the value of X is
(1) 10 (2) 11 (3) 12 (4) 13

12. If $3 \le p \le 10$ and $12 \le q \le 21$, then the difference between the largest and smallest possible values of $\frac{p}{q}$ is
(1) $\frac{29}{42}$ (2) $\frac{29}{5}$ (3) $\frac{19}{70}$ (4) $\frac{19}{12}$

13. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = \frac{vz}{u}$, then which of the following is necessarily true? **(CAT 2003(L))**
(1) $-0.5 \le w \le 2$ (2) $-4 \le w \le 4$
(3) $-4 \le w \le 2$ (4) $-2 \le w \le -0.5$

14. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys' half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x?
(1) $2 \le x \le 6$ (2) $5 \le x \le 8$
(3) $9 \le x \le 12$ (4) $11 \le x \le 14$

15. Which of the following values of P satisfy the inequality $P(P - 3) < 4P - 12$?
(1) $P > 13, P < 51$ (2) $24 \le P < 71$
(3) $3 < P < 4$ (4) None of these

16. The solution set of $2x^2 - x - 3 < 0$ in the set of all real numbers x such that
(1) $x < 1, x < -\frac{3}{2}$ (2) $\frac{3}{2} > x > 1$
(3) $-1 < x < \frac{3}{2}$ (4) $-1 < x < \frac{5}{2}$

17. Find x, if $x^2 - 5x + 4 \le 0$ and $x \ge 3$.
(1) $x \ge 4$ (2) $1 \le x \le 4$
(3) $3 \le x \le 4$ (4) $x \ge 3$

18. What values of x satisfy $x^{2/3} + x^{1/3} - 2 \le 0$ (x is a real number)?
(1) $-8 \le x \le 1$ (2) $-1 \le x \le 8$
(3) $1 < x < 8$ (4) $1 \le x \le 8$

19. How many integers a are there such that $9x^2 + 3ax + (a + 5) > 0$ for all values of x?
(1) 7 (2) 8 (3) 9 (4) 10

20. The range of values of x which satisfy the inequality $\frac{x^2 - 17x + 72}{2x^2 + x + 18} \ge 0$ is
(1) $x \ge 9$ or $x \le 8$ (2) $8 \le x \le 9$
(3) $x \ge 9$ (4) $x \le 8$

21. What is the solution set for $(x - 1)(x - 2)(x - 3)(x - 4) > 0$?

22. How many integers satisfy the condition $(x + 10)(x + 7)(x + 4)(x - 4)(x - 7) < 0$?

23. If $x < y$, then which of the following is always true?
(1) $x < \frac{(x+y)}{2} < y$ (2) $x < \frac{xy}{2} < y$
(3) $x < y^2 - x^2 < y$ (4) $x < xy < y$

Challenging

24. The value of $\left({}^{21}C_1 - {}^{10}C_1\right) + \left({}^{21}C_2 - {}^{10}C_2\right) + \left({}^{21}C_3 - {}^{10}C_3\right) + \left({}^{21}C_4 - {}^{10}C_4\right) + \ldots + \left({}^{21}C_{10} - {}^{10}C_{10}\right)$ is
(1) $2^{21} - 2^{10}$ (2) $2^{20} - 2^9$
(3) $2^{20} - 2^{10}$ (4) $2^{21} - 2^{11}$

25. $f(x) = (x^2 - 100)(x^2 - 81)(x^2 - 64)\ldots(x^2 - 1) < 0$. How many values of x are possible that satisfy the above inequality, where $x = \frac{p}{q}$, p is any natural number and $q = \pm 2$.
(1) 20 (2) 10
(3) 19 (4) 9

QA - 19 : Algebra - 3

Answers and Explanations

CEX-Q-0220/18

1	3	**2**	4	**3**	1	**4**	4	**5**	2	**6**	–	**7**	–	**8**	2	**9**	1	**10**	4
11	3	**12**	1	**13**	2	**14**	2	**15**	3	**16**	3	**17**	3	**18**	1	**19**	3	**20**	1
21	–	**22**	–	**23**	1	**24**	3	**25**	2										

1. 3 All the terms in which the power of (–b) will be odd, will have a negative sign.
Therefore, the power of (–b) can be 1, 3, 5…29.
Hence, required number of terms is 15.

2. 4 $\left(x^2 + \frac{3}{x}\right)^{15}$

Let the (r + 1)th term be independent of x.

$\therefore T_{r+1} = {}^{15}C_r (x^2)^{15-r} \times \left(\frac{3}{x}\right)^r$

$\Rightarrow 2(15 - r) - r = 0$

$\Rightarrow$ r = 10

$\therefore$ 11th term is independent of x.

3. 1 Expansion of $(x + y)^{100} + (x - y)^{100}$
$= ({}^{100}C_0\, x^{100} y^0 + {}^{100}C_1\, x^{99}y^1 + ... + {}^{100}C_{100}\, x^0y^{100})$
$+ ({}^{100}C_0\, x^{100}(-y)^0 + {}^{100}C_1 x^{99}(-y)^1 + {}^{100}C_1\, x^{98}(-y)^2 + ... + {}^{100}C_{100}x^0(-y)^{100})$
Clearly, all the terms containing (–y), $(-y)^3$ ...$(-y)^{99}$ will get cancelled.
$\Rightarrow (x + y)^{100} + (x - y)^{100} = 2[{}^{100}C_0\, x^{100} + {}^{100}C_2\, x^{98}y^2 ... + {}^{100}C_{100}]$
All terms in the expansion of $(x + y)^{100} + (x - y)^{100}$ contain only non-odd powers of x
$\therefore$ Sum of the coefficients $= (1 + 1)^{100} + (1 - 1)^{100} = 2^{100}$.

4. 4 To find the sum of the coefficients of all the powers of x, we put x = 1 in the given expression i.e. $\left(4x - \frac{1}{x}\right)^{20}$.

Hence, the required sum is 3^{20}.

5. 2 Given expression is $(1 - x^2 + x^3)(1 + x)^{10}$
$= (1 - x^2 + x^3)(1 + {}^{10}C_1 x + {}^{10}C_2\, x^2 + {}^{10}C_3 x^3 + {}^{10}C_4 x^4 + … + {}^{10}C_7 x^7 + … + x^{10})$
$\therefore$ Coefficient of $x^7 = {}^{10}C_7 - {}^{10}C_5 + {}^{10}C_4$

$= \frac{10.9.8}{3.2.1} - \frac{10.9.8.7.6}{5.4.3.2.1} + \frac{10.9.8.7}{4.3.2.1} = 78.$

6. A. Number of terms $= {}^{20+3-1}C_{2-1} = {}^{22}C_2$

$= \frac{22 \times 21}{2} = 231$

B. $(a + b + c)^{20} = \{(a + b) + c\}^{20}$
$= {}^{20}C_0\,(a + b)^{20} \times c^0 + {}^{20}C_1 \times (a + b)^{19} \times c^1 + …$
$= {}^{20}C_0\,(a + b)^{20} \times c^0 + {}^{20}C_1 \times \{{}^{19}C_0 a^{19}b^0 + {}^{19}C_1 a^{18}b^1 + {}^{19}C_2 a^{17}b^2 + … \} \times c + …$
From the above expression the coefficient of $a^{17}b^2c$ is ${}^{20}C_1 \times {}^{19}C_2$ i.e. 3420.

C. To find the sum of the coefficients, we need to substitute a = b = c = 1.
$\therefore$ The required sum $= 3^{20}$.

7. A. All the powers of x starting from 0 to 40 are present in the expansion of $(1 + x + x^2)^{20}$. Hence, the number of terms in the expansion is 41.

B. $\{(1 + x) + x^2\}^2 = {}^{20}C_0(1 + x)^{20} + {}^{20}C_1(1 + x)^{19}x^2 + {}^{20}C_2(1 + x)^{18}x^4 + …$
From the above expression, the coefficient of x^4 is ${}^{20}C_0 \times {}^{20}C_4 + {}^{20}C_1 \times {}^{19}C_2 + {}^{20}C_2 \times {}^{18}C_0$.

8. 2 Here $(1 - x + x^2)^5 = [1 - x(1 - x)]^5$

$= 1 - {}^5C_1 \times 1 \times x(1 - x) + {}^5C_2 \times x^2 (1 - x)^2$

$- {}^5C_3 \times x^3 (1 - x)^3 + {}^5C_4 \times x^4 (1 - x)^4 - ...$

Only 3rd, 4th and 5th terms will give the terms of x^4

In 3rd term, it is ${}^5C_2 \times x^2 \times x^2$

In 4th term, it is $-{}^5C_3 \times x^3 (-3 \times x)$

In 5th term, it is ${}^5C_4 \times x^4 \times 1$

So, the coefficient of x^4 is
10 + (– 10) × (– 3) + 5 i.e. 45

9\. 1 A. Not always true.
Example: If x = 3 & y = 4; a = 0 and b = –10 (false)
But if x = 3 & y= 4; a = -10 and b = 0 (true)
B. Not always true. For example, if x = 1 and a = –10 (false) but if x = 4 and a = 3 (true)
C. True only if c > 0

10\. 4 $\frac{x-1}{3} < \frac{5}{7}$ ⇒ 7x – 7 < 15 ⇒ 7x < 22 ⇒ x < 3.333…

Again, $\frac{5}{7} < \frac{x+4}{5}$ ⇒ 25 < 7x + 28 ⇒ –3 < 7x
⇒ x > –3/7
Hence, x = 0, 1, 2, 3

11\. 3 From the first and the third equations,
20X < 500 – 21X

⇒ 41X < 500 ⇒ X < $12\frac{8}{41}$

From the second and the third equations,
23(X – 1) > 500 – 21X

⇒ 44X > 523 ⇒ X > $11\frac{39}{44}$

Since X is an integer, X = 12

Alternative method:
Use answer choices.
One should pick 2nd option first as it is there in two options. If we try 2nd option first and it gives contradiction then two options gets eliminated.
So, take X = 11 and Y comes out to be 269 which gives contradiction. Now, try the third option which is the answer.

12\. 1 Maximum value of $\frac{p}{q} = \frac{10}{12} = \frac{5}{6}$

Minimum value of $\frac{p}{q} = \frac{3}{21} = \frac{1}{7}$

So, difference = $\frac{5}{6} - \frac{1}{7} = \frac{29}{42}$.

13\. 2 u is always negative. Hence, for us to have a minimum value of $\frac{vz}{u}$, vz should be positive. Also, for the least value, the numerator has to be the maximum positive value and the denominator has to be the smallest negative value. In other words, vz has to be 2 and u has to be –0.5.

Hence, the minimum value of $\frac{vz}{u} = \frac{2}{-0.5}$ = –4.

To get the maximum value, vz has to be the smallest negative value and u has to be the highest negative value. Thus, vz has to be –2 and u has to be –0.5.

Hence, the maximum value of $\frac{vz}{u} = \frac{-2}{-0.5}$ = 4.

14\. 2 Amount of rice bought by the first customer

$= \left(\frac{x}{2} + \frac{1}{2}\right)$kgs

Amount of rice remaining $= x - \left(\frac{x}{2} + \frac{1}{2}\right) = \frac{x-1}{2}$kgs

Amount of rice bought by the second customer

$= \frac{1}{2} \times \left(\frac{x-1}{2}\right) + \frac{1}{2} = \frac{x+1}{4}$ kgs

Amount of rice remaining

$= \left(\frac{x-1}{2}\right) - \left(\frac{x+1}{4}\right) = \frac{x-3}{4}$ kgs

Amount of rice remaining $= \frac{1}{2} \times \left(\frac{x-3}{4}\right) + \frac{1}{2} = \frac{x+1}{8}$ kgs

As per the information given in the question $\frac{x+1}{8} = \frac{x-3}{4}$ because there is no rice left after the third customer has bought the rice.
Therefore, the value of x = 7 kgs.
Hence, option (2) is the correct choice.

Alternative method:
Take the values of x and try. We should take odd values of x to get the integral values.
Take x = 5, which contradicts then take x = 7, which satisfies the condition hence, option (2) is the answer.

15\. 3 P(P – 3) < 4P – 12
P(P – 3) < 4(P – 3)
or P(P – 3) – 4(P – 3) < 0
or (P – 3)(P – 4) < 0
Hence, 3 < P < 4.

Alternative method:
Use answer choices and get the answer.

16\. 3 $2x^2 + 2x - 3x - 3 < 0$

$2x(x+1) - 3(x+1) < 0$

$(x+1)(2x-3) < 0$

$\frac{3}{2} > x > -1$

17. 3 We have

$x^2 - 5x + 4 \le 0$

$x^2 - 4x - x + 4 \le 0$

$(x - 4)(x - 1) \le 0$

$\Rightarrow 1 \le x \le 4$

but, $x \ge 3$

$\Rightarrow$ value of x

$3 \le x \le 4$

Alternative method:

Option (2) cannot be the answer as it is given that $x \ge 3$.

Pick x = 5, which contradicts

So, option (1) and (5) are eliminated

take x = 4, which satisfies hence, option (3) is the answer.

18. 1 $x^{2/3} + x^{1/3} - 2 \le 0 \Rightarrow x^{2/3} + 2x^{1/3} - x^{1/3} - 2 \le 0$

$\Rightarrow \left(x^{1/3} - 1\right)\left(x^{1/3} + 2\right) \le 0 \Rightarrow -2 \le x^{1/3} \le 1$

$\Rightarrow -8 \le x \le 1$

Alternative method:

We can use the options.

take x = –1 which satisfies the equation.

So, option (3) and (4) eliminated.

Now, take x = 8 which contradicts the equation.

So, option (2) got eliminated.

Hence, answer is option (1).

19. 3 If $9x^2 + 3ax + (a + 5) > 0$ for all values of x, then the discriminant of this quadratic expression must be negative.

$\therefore (3a)^2 - 4(9)(a + 5) < 0$

$\Rightarrow a^2 - 4a - 20 < 0$

$\Rightarrow (a - 2)^2 < 24$

$\Rightarrow -2.89 < a < 6.89$

$\Rightarrow a = -2, -1, 0, 1, 2, 3, 4, 5, 6$

Hence, there are 9 such integral values.

20. 1 We have $\dfrac{x^2 - 17x + 72}{2x^2 + x + 18} \ge 0$

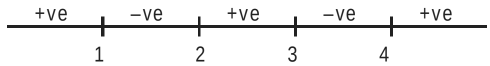
or $x^2 - 17x + 72 \ge 0$ $(\because 2x^2 + x + 18 > 0)$

or $(x - 9)(x - 8) \ge 0$

or $x \ge 9$ or $x \le 8$

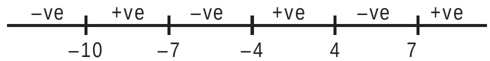
21. Let, $y = (x - 1)(x - 2)(x - 3)(x - 4) > 0$

The points where y = 0 are x = 1, 2, 3, 4



For any value of x in the set $(-\infty, 1) \cup (2, 3) \cup (4, \infty)$, the value of y would be +ve.

22. $(x + 10)(x + 7)(x + 4)(x - 4)(x - 7) < 0$



for –ve value of the given expression, x = 5, 6, – 5, –6 and all the integers less than –10.

So, x can take infinitely many values.

23. 1 It is very apparent that the answer is (a), as $\dfrac{(X + Y)}{2}$ is the average of X and Y and should always lie between X and Y.

Alternative method:

Take any value of X and Y and try with the options.

e.g. try X = 1 and Y = 0

24. 3 Since, ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots + {}^nC_n = 2^n$

and ${}^nC_r = {}^nC_{(n-r)}$

So, ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots$ to middle term $= 2^{(n-1)}$

$\therefore {}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = 2^{20}$

$\Rightarrow {}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10} = (2^{20} - 1)$ $[\because {}^{21}C_0 = 1]$

Again,

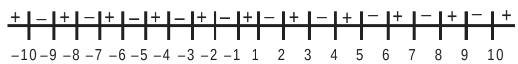
${}^{10}C_0 + {}^{10}C_1 + \ldots + {}^{10}C_{10} = 2^{10}$

$\Rightarrow {}^{10}C_1 + {}^{10}C_2 + \ldots + {}^{10}C_{10} = (2^{10} - 1)$

So, from the given expression in the question

$(2^{20} - 1) - (2^{10} - 1) = (2^{20} - 2^{10})$.

25. 2 $f(x) = (x - 10)(x + 10)(x - 9)(x + 9) \ldots (x - 1)(x + 1) < 0$



$x = \dfrac{p}{q}$.

So, $x = \dfrac{-19}{2}, \dfrac{-15}{2}, \dfrac{-11}{2}, \dfrac{-7}{2}, \dfrac{-3}{2}, \dfrac{3}{2}, \dfrac{7}{2}, \dfrac{11}{2}, \dfrac{15}{2}, \dfrac{19}{2}$.