Geometry - 1

Contents

- Properties of triangle

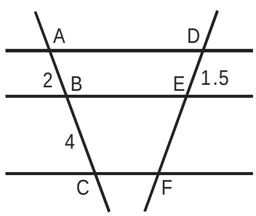
QA - 25

CEX-Q-0226/18

Number of Questions : 25

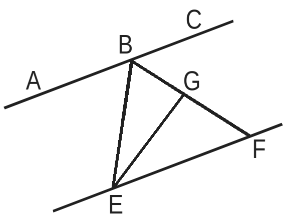
1. Three parallel lines are cut by two transversals as shown in the given figure. If AB = 2 cm, BC = 4 cm and DE = 1.5 cm then the length of EF is:

[SNAP – 2011]



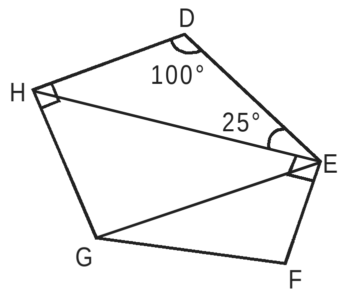
(1) 2 cm (2) 3 cm
(3) 3.5 cm (4) 4 cm

2. Lines AC and EF are parallel. BG and EG are angle bisectors of ∠CBE and ∠FEB respectively. If ∠CBF = 50°, find ∠GEF.



(1) 50° (2) 40°
(3) 30° (4) 20°

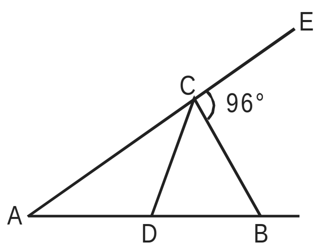
3. Given that HD || GE and GF || HE. Find the measure of the ∠FGE.



(1) 50° (2) 35°
(3) 25° (4) 55°

4. In the figure (not drawn to scale) given below, if AD = CD = BC and $\angle BCE = 96^\circ$, how much is the value of $\angle DBC$?

[CAT 2003 (R)]



(1) 32°
(2) 84°
(3) 64°
(4) Cannot be determined

5. In a triangle ABC, D, E and F are points on AB, BC and CA respectively such that DE = BE and EF = EC. If angle A is 40 degrees then what is angle DEF in degrees?

6. Triangle ABD is right-angled at B. On AD there is a point C for which AC = CD and AB = BC. The magnitude of angle DAB, in degrees, is:

(1) $67\frac{1}{2}$ (2) 60
(3) 45 (4) 30

7. How many differently shaped triangles exist in which no two sides are of the same length, each side is of integral unit length and the perimeter of the triangle is less than 14 units? **[XAT – 2009]**

(1) 3 (2) 4
(3) 5 (4) 6
(5) None of these

8. In a triangle ABC the length of side BC is 295 cm. If the length of side AB is a perfect square, then the length of side AC is a power of 2, and the length of side AC is twice the length of side AB. Determine the perimeter (in cm) of the triangle. **[IIFT – 2010]**

(1) 343 (2) 487
(3) 1063 (4) None of these

9. If the length of the sides of a triangle are in the ratio 3 : 4 : 5, find the ratio of the length of the altitudes to the respective sides.
(1) 3 : 4 : 5
(2) 5 : 4 : 3
(3) 20 : 15 : 12
(4) Cannot be determined

10. Two triangles T_1 and T_2 have three sides of length 10, 10, 12 and 10, 10, 16 respectively. If A_1 and A_2 are the areas of T_1 and T_2 respectively, then the ratio $A_1 : A_2$ is

(1) $\frac{3}{4}$ (2) 1

(3) $\frac{9}{16}$ (4) $\frac{1}{2}$

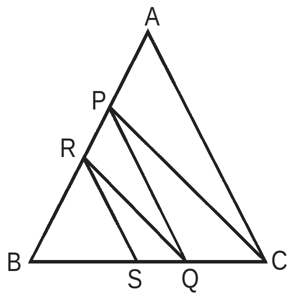
11. The area of a triangle with side length a, b and c ($a \geq b \geq c$) is 1 unit. Then b cannot be less than

(1) $2\sqrt{2}$ (2) 2
(3) $\sqrt{3}$ (4) $\sqrt{2}$

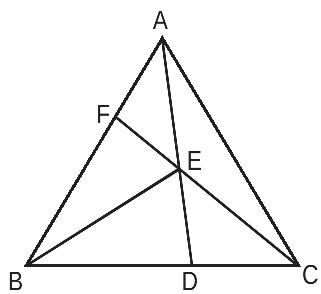
12. In front of a 200 meters long wall, a triangular plot is to be cordoned off using the wall as one of its sides and a fencing of total length 300 meters to form the other two sides. Then the maximum possible area (in square meters) of the plot that can be cordoned off is

(1) 5000 (2) $2500\sqrt{5}$
(3) $5000\sqrt{5}$ (4) None of these

13. What is the minimum possible area (in sq. units) of triangle ABC if the area of triangle BRS and ABC are integers, and AP : PB = CQ : QB = PR : RB = QS : SB = 1 : 2?



14. In the given triangle, BD : DC = 7 : 4, AF : FB = 1 : 2. Find FE : EC.



(1) 1 : 2 (2) 5 : 8
(3) 7 : 12 (4) 6 : 11

15. If p^2, q^2 and r^2 are three sides of a triangle then the triangle with sides p, q and r is necessarily
(1) acute angled
(2) right angled
(3) obtuse angled
(4) Cannot be determined

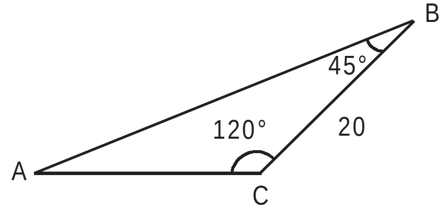
16. Find the area of an equilateral triangle whose height is 12 cm.
[SNAP – 2012]
(1) $24\sqrt{3}\,\text{cm}^2$ (2) $48\,\text{cm}^2$
(3) $48\sqrt{3}\,\text{cm}^2$ (4) $36\sqrt{3}\,\text{cm}^2$

17. An equilateral triangle ABP is formed outside a square ABCD. What is angle APC (in degrees)?

18. One side of an equilateral triangle is 24 cm. The midpoints of its sides are joined to form another triangle whose midpoints are in turn joined to form still another triangle. This process continues indefinitely. Find the sum of the perimeters of all the triangles.
[SNAP – 2011]
(1) 144 cm (2) 72 cm
(3) 536 cm (4) 676 cm

19. The area of a right-angled triangle is 40 sq. cm and its perimeter is 40 cm. The length of its hypotenuse is
(1) 16 cm (2) 17 cm
(3) 18 cm (4) None of these

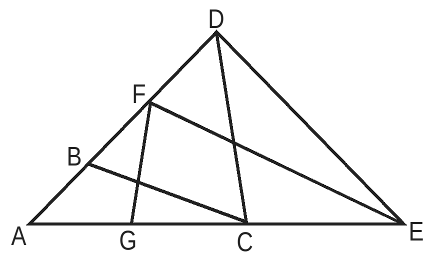
20. In the figure given below, $\angle ABC = 45°$, $\angle ACB = 120°$ and BC = 20 cm. AD is drawn perpendicular on BC such that it meets BC produced at D. Find the length (in cm) of AD.



(1) $10\left(3+\sqrt{3}\right)$ (2) $10\left(3-\sqrt{3}\right)$
(3) $5\left(3+\sqrt{3}\right)$ (4) $5\left(3-\sqrt{3}\right)$

21. A ladder 25 meters long is placed against a wall with its foot 7 meters away from the foot of the wall. How far should the foot be drawn out so that the top of the ladder may come down by half the distance of the total distance by which the foot is drawn out?
[IIFT – 2008]
(1) 6 meters (2) 8 meters
(3) 8.75 meters (4) None of these

22. In the given figure, AB = BC = CD = DE = EF = FG = GA. Find angle A (approximate to nearest integer in degrees).



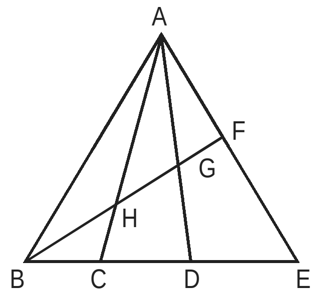
(1) 21 (2) 26
(3) 30 (4) 45

23. Three sides of a triangle are 8, 15 and x where x is an integer. For how many values of x the triangle is obtuse angled?

24. The area of an isosceles triangle is 12 sq. cm. If one of the equal sides is 5 cm long, mark the option which can give the length of the base. **[IIFT – 2006]**

(1) 4 cm (2) 6 cm
(3) 8 cm (4) Both (2) and (3)

25. In the given triangle, AH : HC = 3 : 1, BC : CD : DE = 1 : 2 : 3. Area of triangle BHC is 100 cm^2. Find the ratio of the area of triangle AHG to the area of triangle AGF.



(1) 1 : 1 (2) 5 : 4
(3) 4 : 3 (4) 3 : 2

QA - 25 : Geometry - 1
Answers and Explanations

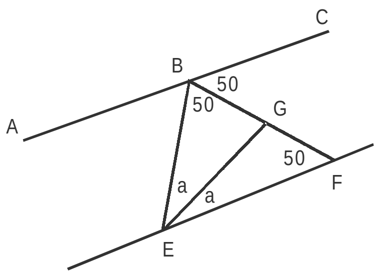
1	2	**2**	2	**3**	4	**4**	3	**5**	100	**6**	2	**7**	3	**8**	3	**9**	3	**10**	2
11	4	**12**	3	**13**	81	**14**	3	**15**	1	**16**	3	**17**	45	**18**	1	**19**	3	**20**	1
21	2	**22**	2	**23**	10	**24**	4	**25**	4										

1. 2 By using basic proportionality theorem,

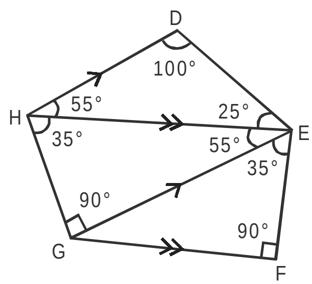
$$\frac{AB}{BC}=\frac{DE}{EF}\Rightarrow\frac{2}{4}=\frac{1.5}{EF}\text{ or EF = 3 cm}$$

2. 2 $\angle CBF = \angle EBF = 50$.
Also, because AC || EF, $\angle CBF = \angle BFE = 50$.
In triangle EBF, $\angle BEF = 180 - 50 - 50 = 80$.

Hence $\angle GEF = \frac{1}{2}$ of $80 = 40°$.



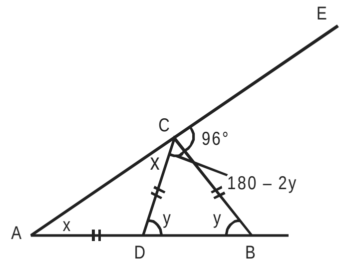
3. 4



Since HD || GE,
$\angle DEG = 80°$
$\therefore \angle HEG = 80° - 25° = 55°$
Also HE || GF,

$\therefore \angle FGE = 55°$.

4. 3

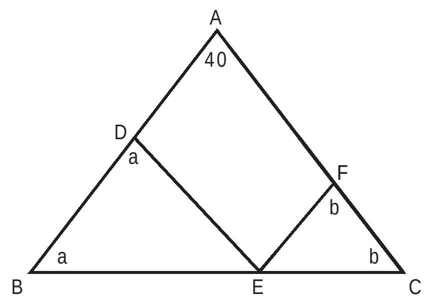


Using exterior angle theorem

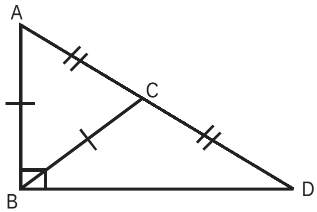
$\angle A + \angle B = 96°$
i.e. $x + y = 96°$ …(i)
Also $x + (180° - 2y) + 96° = 180°$
$\therefore\ x - 2y + 96° = 0$
$\Rightarrow x - 2y = -96°$ …(ii)
Solving (i) and (ii),
$y = 64°$ and $x = 32°$

$\therefore \angle DBC = y = 64°$

5. 100 If DE = BE, then $\angle DBE = \angle BDE = a$ (say)
If EF = EC, then $\angle EFC = \angle ECF = b$ (say)
Hence, $\angle DEB = 180° - 2a$ and $\angle FEC = 180° - 2b$.
Also, $a + b + 40° = 180°$ or $a + b = 140°$.
$\angle DEF = 180° - \angle DEB - \angle FEC$
$= 180° - 180° + 2a - 180° + 2b$
$= 2(a + b) - 180° = 100°$.



6. 2



The above figure represents the case given in the question. If AC = CD and also $\angle ABD = 90°$, we can say that C is the centre of the circle passing through A, B and D with AD as the diameter. So, the radius must be AC, CD or BC. As given that BC = AB, we can conclude that AB = BC = CA.

Hence, ΔABC is an equilateral triangle and $\angle DAB$ must be 60°.

7. 3 Possible sides of different triangles can be as follows: (2, 3, 4), (2, 4, 5), (2, 5, 6), (3, 4, 5) and (3, 4, 6)

8. 3 Let AC = 2x, where x is any natural number.

$\therefore AB = \frac{1}{2}AC = 2^{x-1}$; but AB is a perfect square

$\Rightarrow$ x – 1 is even $\Rightarrow$ x is odd.

Sum of two sides of a Δ is greater than the third side

$\Rightarrow$ AB + AC > BC $\Rightarrow$ 3AB > 295 $\Rightarrow$ AB > 98.33

$\Rightarrow$ 2x – 1 > 98.33 …(i)

Also, AC – AB < BC

$\Rightarrow$ 2x – 2x – 1 < 295

$\Rightarrow$ 2x – 1 < 295 …(ii)

The only satisfying value for equations (i) and (ii) is x = 9

$\therefore$ AB = 2^8 = 256 and AC = 2^9 = 512

$\therefore$ Perimeter = 256 + 512 + 295 = 1063.

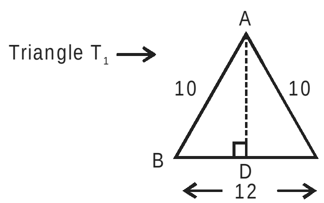
9. 3 Let the three sides are 3a, 4a and 5a.

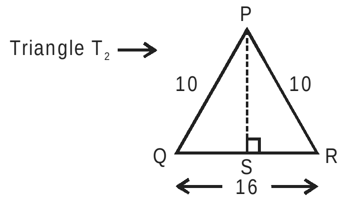
Area of triangle = $\frac{1}{2}$ × side$_1$ × altitude$_1$ = $\frac{1}{2}$ × side$_2$ × altitude$_2$ = $\frac{1}{2}$ × side$_3$ × altitude$_3$

Hence, 3a × altitude$_1$ = 4a × altitude$_2$ = 5a × altitude$_3$.

altitude$_1$: altitude$_2$: altitude$_3$ = $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$ = 20 : 15 : 12.

10. 2





Since both are isosceles triangles, therefore perpendicular dropped from the vertex to the unequal side also bisects the side.

BD = 6 $\Rightarrow$ AD = 8 ($\because \Delta ABD \rightarrow$ right angled triangle)

So, area of $\Delta ABC = \frac{1}{2} \times 12 \times 8$ = 48 sq. unit

Now, PS = 6

$\Rightarrow$ Area of $\Delta PQR = \frac{1}{2} \times 6 \times 16 = 48$ sq. units

$$\frac{\text{Area of } \Delta T_1}{\text{Area of } \Delta T_2} = \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta PQR} = \frac{48}{48} = 1$$

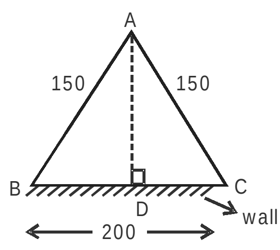
11. 4 Let the angle between sides c and b be α.

So, area of triangle is $\frac{1}{2} \times c \times b \times \sin\alpha = 1$.

Here $b = \frac{2}{c \sin\alpha}$.

To find minimum value of b, c and $\sin\alpha$ must be maximum. $\sin\alpha$ can be maximum 1 and c can maximum be equal to b. So, b^2 = 2 and b = $\sqrt{2}$ is the minimum value of b.

12. 3 For maximum possible area, the figure should be as symmetrical as possible. In this case, it should be an isosceles triangle;



BD = 100 ml ($\because \perp$ from A to BC also bisects BC)

In ΔABD

$AD = \sqrt{150^2 - 100^2} = 50\sqrt{5}$

Area of $\Delta ABC = \frac{1}{2} \times 50\sqrt{5} \times 200 = 5000\sqrt{5}$

13. 81 Let area of ABC is a square units.

The area of BPC will be $\frac{2a}{3}$.

Similarly, area of BPQ will be $\frac{2}{3}$ of $\frac{2a}{3}$ = $\frac{4a}{9}$.

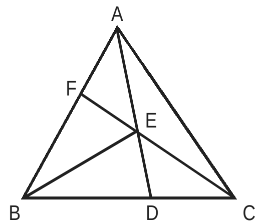
Area of BRQ will be $\frac{2}{3}$ of $\frac{4a}{9} = \frac{8a}{27}$.

Area of BRS will be $\frac{2}{3}$ of $\frac{8a}{27} = \frac{16a}{81}$.

a must be a multiple of 81 for $\frac{16a}{81}$ to be an integer.

So, minimum area of ABC is 81 square units.

14. 3



AF : FB = 1 : 2
BD : DC = 7 : 4
ar(ΔABE) : ar(ΔAEC) = 7 : 4
Let, ar(ΔABE) = 7a
$\Rightarrow$ ar (ΔAEC) = 4a
Again, AF : FB = 1 : 2
So, ar(ΔAFE) : ar(ΔFBE) = 1 : 2

$\Rightarrow$ ar(ΔAFE) = $\frac{7a}{3}$ [$\because$ ar(ΔABE) = 7a]

Thus, ar(ΔAFE) : ar(ΔAEC) = 7a/3 : 4a
$\Rightarrow$ FE : EC = 7 : 12

15. 1 If p^2, q^2 and r^2 are three sides of a triangle then sum of any two sides of the triangle must be greater than third i.e. $p^2 + q^2 > r^2$. This too is a property of acute angles triangle with sides p, q and r. Hence answer (1).

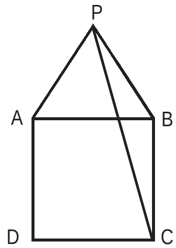
16. 3 The height of an equilateral triangle of side 'a' $= \frac{\sqrt{3}}{2}a$

It is given that, $\frac{\sqrt{3}}{2}a = 12 \Rightarrow a = \frac{24}{\sqrt{3}} = 8\sqrt{3}$ cm

Therefore, area of the equilateral triangle

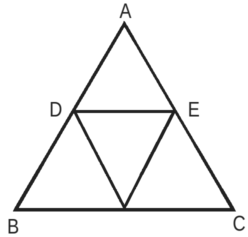
$= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times \left(8\sqrt{3}\right)^2 = 48\sqrt{3}$ cm^2.

17. 45 AP = PB = BC. So, in triangle PBC, $\angle$PBC = 150.
Also, $\angle$BPC = $\angle$PCB = 15.



Hence $\angle$APC = 60 – 15 = 45.

18. 1



Here, DE || BC and DE = $\frac{1}{2}$BC = 12 cm

$\therefore$ Sum of perimeters of triangles so formed

$= 72 + 36 + 18 + \ldots \infty = 72\left\{1 + \frac{1}{2} + \frac{1}{4} + \ldots\infty\right\}$

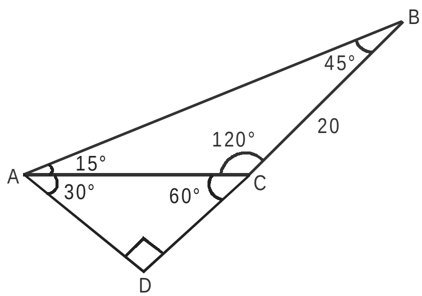
$= 72 \times \frac{1}{1 - \frac{1}{2}} = 72 \times \frac{1}{\frac{1}{2}} = 144$ cm

19. 3 Let P, B and H be the perpendicular, base and hypotenuse of the triangle.

So, $P^2 + B^2 = H^2$. Also $\frac{1}{2}$PB = 40 or PB = 80.

P + B + H = 40 i.e. P + B = 40 - H.
Squaring both sides, we get
$P^2 + B^2 + 2PB = 1600 + H^2 – 80H$.
Putting the value of PB, we get H = 18 cm.

20. 1



In Δ ADC, let AD = x

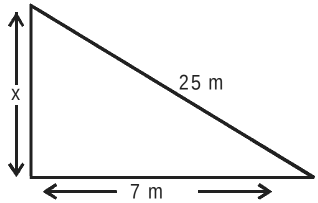
Also tan 60° = $\frac{AD}{CD}$

CD = $\frac{AD}{\sqrt{3}}$ = $\frac{x}{\sqrt{3}}$

Δ ABD is an isosceles right angle triangle, so AD = BD

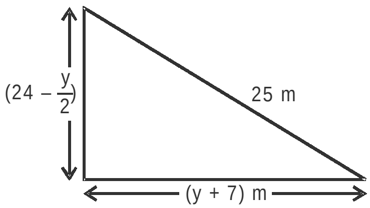
$\Rightarrow x = \frac{x}{\sqrt{3}} + 20,\ x = \frac{20\sqrt{3}}{\sqrt{3}-1} = 10\left(3+\sqrt{3}\right)$

21. 2 **Initial position of the ladder:** If x is the height at which top of the ladder is above the ground.



$\Rightarrow x = \sqrt{25^2 - 7^2} = 24$ meters

Final position of the ladder: Let ladder moves away y meters from the wall.



$(25)^2 = (y+7)^2 + \left(24 - \frac{y}{2}\right)^2$

$\Rightarrow$ y = 8 meters

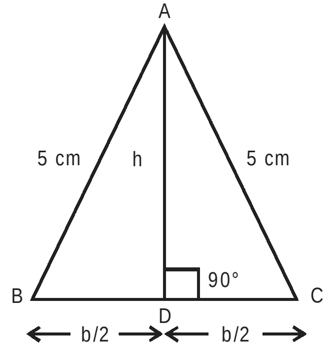
22. 2 Let $\angle$A = a.
In triangle ABC, AB = BC. Hence, $\angle$BCA = a.
$\angle$CBD = $\angle$A + $\angle$BCA = 2a.
(Exterior angle is sum of two remote interior angles).
In triangle BCD, BC = CD.
Hence $\angle$BDC = $\angle$CBD = 2a.
$\angle$DCE = $\angle$A + $\angle$BDC = a + 2a = 3a.
In triangle DCE, $\angle$DEC = $\angle$DCE = 3a.
In triangle AGF, AG = GF. Hence, $\angle$A = $\angle$AFG = a.
$\angle$FGC = $\angle$A + $\angle$AFG = 2a.
In triangle GFE, EF = FG. Hence, $\angle$FGE = $\angle$FEG = 2a.
$\angle$EFD = $\angle$A + $\angle$FEA = a + 2a = 3a.
In triangle FED, $\angle$EFD = $\angle$FDE = 3a.
Now, in triangle ADE,
$\angle$A + $\angle$ADE + $\angle$AED = a + 3a + 3a = 180.

Hence a = $\frac{180}{7}$ = 26 (approx.)

23. 10 **Case I:** Let 15 be the largest side.
15 < 8 + x i.e. x > 7.
For an obtuse angled triangle,
$15^2 > 8^2 + x^2$ i.e. $x^2 < 161$.
Possible values of x are 8, 9, 10, 11 and 12.

Case II: Let x be the largest side.
X < 15 + 8 i.e. x < 23.
For an obtuse angled triangle, $x^2 > 15^2 + 8^2$ i.e. x > 17.
Possible values of x are 18, 19, 20, 21 and 22.
Hence, total possible values of x are 10.

24. 4



$\text{Area}(\Delta ABC) = \frac{1}{2} \times b \times h = 12$ sq.cm

$\Rightarrow$ b × h = 24 …(i)
Also, in right triangle ABD,

$(5)^2 = \left(\frac{b}{2}\right)^2 + h^2$

$\Rightarrow b^2 + 4h^2 = 100$ …(ii)

$\Rightarrow b^2 + \left(\frac{48}{b}\right)^2 = 100$ …(iii)

$\therefore$ b = 6 and 8, are the roots of the above equation.

25. 4 Let area of triangle BHC be a square units. Hence area of BHA will be 3a. Therefore, area of triangle ACD and ADE will be 8a and 12a respectively.
Let area of triangle AHG is x square units. Area of CHD will be 2a and hence area of HDG will be 6a – x.

Now $\frac{BH}{HG} = \frac{\text{Area of ABH}}{\text{Area of AHG}} = \frac{\text{Area of DBH}}{\text{Area of DHG}}$.

Hence, $\frac{3a}{x} = \frac{3a}{6a-x}$. So, x = 3a.

Now, area of triangle DGE will also be 6a.
Now assuming the area of triangle AGF as y, area of GFE will be 6a – y.

Now, $\frac{BG}{GF} = \frac{\text{Area of ABG}}{\text{Area of AGF}} = \frac{\text{Area of BGE}}{\text{Area of EGF}}$.

Hence, $\frac{6a}{y} = \frac{12a}{6a-y}$.

Solving this we get y = 2a. So, required ratio is 3 : 2.