QA - 3 : Numbers

Career Launcher

Workshop

Number of Questions : 25 **WSP–0003/18**

1. M is the product of ten consecutive two-digit natural numbers. Y is the highest power of 2 in M. How many different values of M are possible such that the value of Y is maximum given?
(1) 1 (2) 2 (3) 3 (4) 4

2. N bells toll at different intervals which are multiples of two minutes. If all the bells toll together for the first time at 11:00 a.m. and 9th time at 3:00 a.m. on the next day, what is the maximum possible value of N?
(1) 60 (2) 24 (3) 16 (4) 12

3. A student wrote all the natural numbers from 2 to 10000 on a blackboard. Another student came and erased all the perfect squares. A second student came and erased all the perfect cubes. If students come this way and erase all the higher powers, find the number of students who erased at least one number?
(1) 6 (2) 7
(3) 12 (4) None of these

4. What is the unit digit of the following product: $(8-5)(8^2-5^2)(8^3-5^3)\ldots\ldots\ (8^{2005}-5^{2005})$
(1) 1 (2) 3
(3) 7 (4) 9

5. How many integers between 1 and 500 are divisible by exactly two of the three numbers 3, 5 and 7?
(1) 62 (2) 66
(3) 70 (4) 58

6. How many multiples of 8 from 10^6 to 10^{10} (both inclusive) are perfect squares?
(1) 24751 (2) 24750
(3) 12376 (4) 12375

7. For how many ordered pairs (x, y), where x and y are non-negative integers, is the equation $\sqrt{x}+\sqrt{y}=\sqrt{650}$ satisfied?
(1) 0 (2) 2 (3) 6 (4) 7

8. $2^{28}-1$ is exactly divisible by two numbers between 120 and 130. The sum of these two numbers is:
(1) 255 (2) 256 (3) 253 (4) 257

9. You are selecting 5 numbers randomly out of the first 150 odd numbers. Sum of these 5 odd numbers is A. How many different values of A are possible?
(1) ${}^{150}C_5$ (2) 150
(3) 725 (4) 726

10. A box contains 100 tickets, numbered from1 to 100. A person picks out three tickets from the box, such that the product of the numbers on two of the tickets yields the number on the third ticket. How many tickets can never be picked up?
(1) 10 (2) 11 (3) 25 (4) 26

11. Find the last two digits of 26^{1979} ?

12. Find the remainder when 198^{79} is divided by 27?

13. If $x^{2a+1} + y^{2b}$ always ends in 0 and a, b are positive integers, then the values of x, y respectively are
(1) 21, 27 (2) 27, 29
(3) 21, 29 (4) 29, 21

14. Jaswant Singh had N number of chocolates with him. He started to distribute the chocolates among his kids according to ascending order of their age. First he gives x number of chocolates to the youngest kid, then x+ 5 chocolates to second youngest, x + 10 chocolates to third youngest and so on. After distributing among certain number of kids in the above manner he left with 7 chocolates and he realized that had he started with 15 more chocolates, he could have distributed chocolates to one more child in the same manner. Which of the following can be the value of N and x respectively?
(1) 34, 4 (2) 30, 9
(3) 26, 7 (4) All of the above

15. If the “n” th day of August is the same day of week as the “2n” th day of October, then how many values of n are possible?
(1) 4 (2) 5 (3) 2 (4) 0

16. M is a natural number ($3 < M < 500$). How many values of M are possible such that HCF of M and 1250 is 1?
(1) 200 (2) 198
(3) 150 (4) 197

17. How many numbers below 100 are co-prime to 100 but not co-prime to 1500?
(1) 11 (2) 14 (3) 17 (4) 26

18. The micromanometer in a certain factory can measure the pressure inside the gas chamber from 1 unit to 999999 units. The problem with the instrument is that it always skips the digit 5 and hence moves directly from 4 to 6. What is the actual pressure inside the gas chamber if the micromanometer displays 003016?
(1) 2201 (2) 2202
(3) 2600 (4) 2960

19. The digits x, y and z of a three-digit natural number xyz satisfy the equation $121x + 11y + z = 567$. What is the sum of the digits of the three-digit natural number xyz?
(1) 17 (2) 18
(3) 21 (4) Data Insufficient

20. In how many zeroes does $(10!)^{100!} \times (100!)^{10!}$ end in?
(1) 96 (2) $2^{24} + 24^{2}$
(3) $2^{100!} \times 24^{10!}$ (4) $2 \times (100!) + 24 \times (10!)$

21. There is an integer consisting of 2000 digits beginning with 3 as the leftmost digit. This integer is such that any two consecutive integers taken together is divisible either by 17 or 23. The last digit can be 'a' or 'b'. What is the value of 'a + b'?
(1) 4 (2) 5
(3) 7 (4) Cannot be determined

22. Take any 2-digit number and multiply the digits together. If this process is continued, all 2-digit numbers will become a single digit number. How many two digits will finish with zero?
(1) 13 (2) 20
(3) 24 (4) 19

23. Which factorial needs to be removed to make it a perfect square ?
$1! \times 2! \times 3! \times \ldots\ldots\ldots\ldots\ldots\ldots \times 119! \times 120!$
(1) 1! (2) 59!
(3) 60! (4) 120!

24. What is the sum of all positive integers less than 10^4 which are relatively prime to 10^4?
(1) 1.25×10^6 (2) 1.25×10^7
(3) 2×10^7 (4) 2×10^6

25. XX + YY + ZZ = XYZ where each of X, Y and Z represent distinct non zero digits, Z can be?
(1) 6 (2) 7
(3) 8 (4) 9

QA - 3 : Numbers
Answers and Explanations

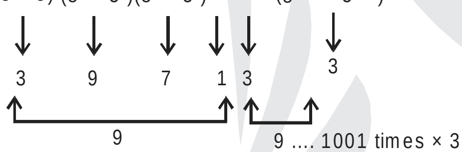
1	4	**2**	4	**3**	1	**4**	3	**5**	4	**6**	1	**7**	3	**8**	2	**9**	3	**10**	2
11	76	**12**	0	**13**	4	**14**	4	**15**	3	**16**	2	**17**	2	**18**	1	**19**	3	**20**	4
21	3	**22**	3	**23**	3	**24**	3	**25**	3										

1. 4 The number of 2s will be maximum if 64 (i.e. 2^6) is present and a multiple of 8 (i.e 2^3) is also present in the range of there ten consecutive natural numbers.
so, 56 64 72
Thus, only 4 possible ranges – 55 to 64, 56 to 65, 63 to 72 and 64 to 73.

2. 4 This is same as LCM of N numbers which are multiples of 2 is 16 × 60 (because, from 11 a.m. to 3 a.m. next day is 16 hours)
So, 16 × 60 = 960 minutes.
Since, it bell 9th time, i.e. there are eight intervals and each interval of $\frac{960}{8} = 120$ minutes.
So, $120 = 2^3 \times 3 \times 5$
Thus, ultimately we've to find the number of even factors of 120.
Which is 3 × 2 × 2 = 12.

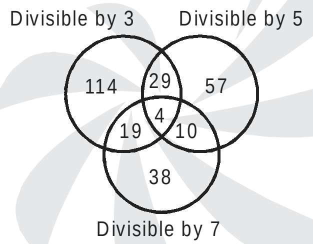
3. 1 Apply hit and trial method by assuming numbers.

4. 3 $(8-5)\ (8^2-5^2)(8^3-5^3) \cdots\cdots\cdots (8^{2005}-5^{2005})$



Last digit $(9^{1001} \times 3) = 3^{2003} = 7$

5. 4



Hence, from the above venn diagram we can say that 29 + 19 + 10 = 58 numbers will satisfy the given condition.

6. 1 $\sqrt{10^6} = 10^3$ and $\sqrt{10^{10}} = 10^5$ and we have to make all the perfect squares, a multiple of 8.
Ultimately, we've to find all the numbers from 1000 to 100000 which are multiple of 4 (because a number of 4k type, will become $16k^2$ on squaring and will also become a multiple of 8). So, from 1000 to 100000, there are $\frac{100000-1000}{4}$ = 24750 perfect square which are multiple of 8 and including 10^6, there are a total of 24751 such numbers.

7. 3 $\sqrt{x} + \sqrt{y} = \sqrt{650} = \sqrt{25 \times 26} = 5\sqrt{26}$ and also, x. y $\in$ whole number
So, there would be 6 possible solutions.

8. 2 $(2^{28} - 1)$ is an odd number.
So, $(2^{28} - 1)$ = odd × odd × odd × .... and so on.
Thus, sum of any two factors must be even which is option (2)

9. 3 Since, we've selecting 5 numbers randomly.
So, minimum possible sum of these five numbers = first 5 odd numbers = 5^2 = 25
and maximum possible sum = 291 + 293 + 295 + 297 + 299 = 295 × 5 = 1475
Thus, different possible sums $= \frac{1475-25}{2} = 725$
(Dividing by 2 because sum will increase in the steps of 2, as only odd numbers between 25 and 1475 will be the answer)

10. 2 All the prime numbers greater than 50 i.e. 10 numbers and the number 1. Thus total 11 numbers cannot be picked up.

11. Let us try to find the pattern.
$(26)^2 = 676 \rightarrow$ ends in 76
$(26)^3 = 17576 \rightarrow$ ends in 76
$(26)^4 = \quad \rightarrow$ ends in 76
So, $(26)^{1979}$ will also end in 76.

12. $\frac{(198)^{79}}{27} \equiv \frac{9^{79}}{27} \equiv 0$

13. 4 $x^{(2a+1)} + y^{2b}$; Here power of x, i.e. (2a + 1) is always odd while power of y is always even.
Now from option (4) we can say that $x^{(2a+1)}$ will always end in 9 and y^{2b} will always end in 1. Thus the entire sum will end with 0. So, (4) is the correct answer.

14. 4 Among the given options, (1), (2) and (3) satisfy the given condition.

15. 3 For 'nth' day of August to be the same week-day as '2nth' day of October, the total number of days between these two dates has to be a multiple of 7.
i.e. $\underbrace{31-x}_{\text{remaining days of August}} + \underbrace{30}_{\text{Days in September}} + \underbrace{2n}_{\text{Required No. of days in October}} = 7k$
$\Rightarrow 61 + n = 7k$
Also, $1 \le n \le 15$ as 2n has to be less than 31.
$\therefore$ n = 7 k – 61
$\Rightarrow$ n = 2 and 9 only
$\therefore$ Only two values are possible for n.

16. 2 Here we've to calculate all the number which are in the range 3 < M < 500 and not a multiple of 2 or 5, as 1250 = $5^4 \times 2$.

Thus, $500\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=200$, such number in the rang M < 500.
So, to satisfy the given range we'll remove two more numbers i.e. 1 and 3.
Thus answer is 200 – 2 = 198 which is option (2)

17. 2 To make number co-prime with 100, there should not be any. multiple of 2 or 5, for this $100\left(\frac{1}{2}\right)\left(\frac{4}{5}\right)=40$ number are possible. Now to make not co-prime with 1500 there should be a multiple of 3 and every 3 number out of 40 numbers would be a multiple of 3 thus $\frac{40}{3}$ = 13 or 14 numbers are possible option (2) is correct.

18. 1 Skipping a digit means the machine is working in base 9 system. So, we'll simply convert the given number from base 9 to base 10. i.e. $(003016)_9 = (2202)_{10}$
But minimum pressure is 1 unit inside the chamber. (Not zero). Thus, actual pressure would be 2202 – 1 = 2201 unit.

19. 3 121x + 11y + z = 567 thus is same as $(xyz)_{11} = 567$ convert it to base 10 and then sum up the digits of the number thus obtained i.e 6 + 7 + 8 = 21.

20. 4 As we've to count the number of zeroes, so we'll count the highest power of 5 present in $(10!)^{100!}$ and $(100!)^{10!}$ which is 2 × (100!) and 24 × (10!) respectively. Thus total number of zero would be 2 × (100!) + 24 × (10!) which is option (4)

21. 3 Only possibilities for any two consecutive digits in the number having 2000 digits which are multiple of either 17 or 23 can be,

$\underbrace{17, 34, 51, 68, 85}_{\text{2-digit multiples of 17}},\ \underbrace{23, 46, 69, 92}_{\text{2-digit multiples of 23}}$

Now, as it is given that the leftmost digit is 3, we can generate the number in the following manner:

3 4 6 ⟨ 8 5 1 7 – × not possible
3 4 6 ⟨ 9 2 3 4 6 92 ....

Now, we can have only two possible number,
I. 3469234692 .... 3469234692.
II. 3469234692 .... 3469234685.
So, a = 2, b = 5
∴ Required value of a + b = 2 + 5 = 7.

22. 3 To get final as zero, second last number must be multiple of 10 i.e., (10, 20, 30, …, 90)

Now, taking the second last number as
(i) 10 can be obtained by multiplying the digits of 25 or 52, further 25 can be obtained by 55.
(52 and 55 cannot be obtained by multiplying two single digit number)
Similarly, for numbers 20, 30, …, 90 are explained below.

(ii) 20 → 45 → 95, 59
20 → 54 → 96, 69

So, there are 7 numbers 20, 45, 54, 59, 95, 96, 69.

(iii) 30 → 65 — ×
30 → 56 → 78 — ×, 87 — ×

So, there are 5 numbers 30, 56, 65, 87, 78.

(iv) 40 → 85 — ×
40 → 58 — ×

So, there are 3 numbers 40, 58, 85
(v) The numbers 50, 60, 70, 80, 90 = 5 numbers
∴ Total = 4 + 7 + 5 + 3 + 5 = 24

23. 3 $1! \times 2! \times 3! \times 4! \times 5! \times ...... \times 119! \times 120!$
$(1!)^2 \times 2 \times (3!)^2 \times 4 \times (5!)^2 \times 6 \times ...... \times (119!)^2 \times 120$
$(1!)^2 \times (3!)^2 \times (5!)^2 \times ...... \times (119!)^2 \times 2^{60} \times (60!)$ thus option (3) is correct

24. 3 Prime factors of 10^4 are 2 and 5.
The required sum can be obtained by
Sum of all numbers till 10^4 – sum of multiple 2 or 5 till 10^4.

$$\frac{10000\times10001}{2}-\frac{2(1+5000)\times5000}{2}-\frac{5(1+2000)\times2000}{2}+\frac{10\times(1+1000)\times1000}{2} = 2\times10^7.$$

Alternative method:
The sum of multiple of 2 or 5 can be obtained by Average × forms of number
Similarly the sum of number co-prime to 10^4 can be obtained by the same way.
To get the number of forms, we'll use the totient function

i.e. $10000\times\left(1-\frac{1}{2}\right)\left(1-\frac{1}{5}\right)=4\times10^3$

∴ The required sum $=\frac{(1+9999)}{2}\times4\times10^3 = 2\times10^7$.

25. 3 XX + YY + ZZ = XYZ
Here, LHS is a multiple of 11. So, RHS must be a multiple of 11. Thus, (X + Z – Y) = 11K and another condition is X cannot be greater than 2 (because, the maximum value of XYZ would be 264)
So, X = 1, Z = 8 and Y = 9 satisfy the given condition.
Thus, Z = 8 which is option (3)