QA - 6 : Geometry

Career Launcher

Workshop

Number of Questions : 25 **WSP–0006/18**

1. A sphere of radius r is cut by a plane at a distance of h from its center, thereby breaking this sphere into two different pieces. The total surface area of these two pieces is 25% more than that of the original sphere. Find h.

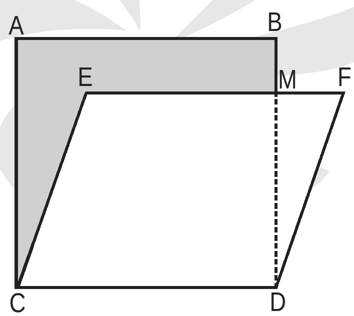
(1) $\frac{r}{\sqrt{2}}$ (2) $\frac{r}{\sqrt{3}}$

(3) $\frac{r}{\sqrt{5}}$ (4) $\frac{r}{\sqrt{6}}$

2. A 150-foot rope is suspended at its two ends from the tops of two 100-foot flagpoles. The lowest point of the rope is 25 feet from the ground. What is the distance between the two flagpoles?

(1) 75 ft (2) 100 ft
(3) 120 ft (4) 87.5 ft
(5) None of these

3. If ABCD is a square with area 625, and CEFD is a rhombus with area 500, then the area of the shaded region is:



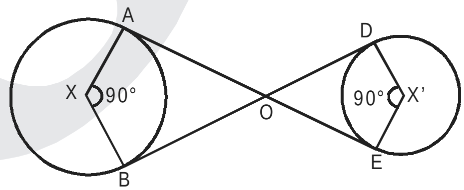
(1) 125 (2) 175
(3) 200 (4) 250
(5) 275

4. Two mutually perpendicular chords AB and CD meet at a point P inside the circle such that AP = 6 cm, PB = 4 cm and DP = 3 cm. What is the area of the circle?

(1) $\frac{125\pi}{4}$ sq cm (2) $\frac{100\pi}{7}$ sq cm

(3) $\frac{125\pi}{8}$ sq cm (4) $\frac{52\pi}{3}$ sq cm

5. A string is wound around two circular disk as shown. If the radius of the two disk are 40 cm and 30 cm respectively. What is the total length of the string?

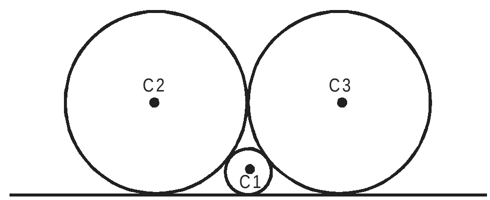


(1) 70 cm (2) $70 + 165\pi$

(3) $70 + 120\pi$ (4) $70 + \frac{165\pi}{2}$

(5) $140 + 105\pi$

6. All three circles are tangent to the same line and to each other. Circles C2 and C3 have equal radii. Find the radius of C2 if the radius of C1 is equal to 10 cm.



(1) 30 cm (2) 40 cm
(3) 45 cm (4) 50 cm

7. Which of the following can be the perimeter of an isosceles triangle with sides 4 cm and 8 cm.
(1) 16 cm
(2) 20 cm
(3) Both (1) and (2)
(4) None of these

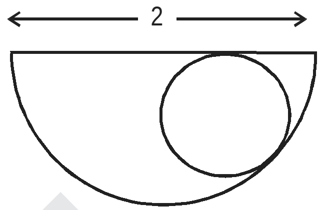
8. Anil grows tomatoes in his backyard which is in the shape of a square. Each tomato takes 1 cm^2 in his backyard. This year, he has been able to grow 131 more tomatoes than last year. The shape of the backyard remained a square. How many tomatoes did Anil produce this year?
(1) 4225 (2) 4096
(3) 4356 (4) Insufficient Data

9. There are 5 concentric circles that are spaced equally from each other by 1.25 cm. The innermost circle has a square of side $\sqrt{(32)}$ cm inscribed in it. If a square needs to be inscribed in the outermost circle, what will be its area?
(1) 324 sq. cm.
(2) $(66 + 40\sqrt{2})$ sq. cm.
(3) 210.125 sq. cm.
(4) 162 sq.cm.

10. a, b and c are three integral sides of an obtuse angled triangle. If ab = 4, find c?
(1) 2
(2) 3
(3) 1
(4) More than 1 value of c can exist

11. What is the distance between the orthocentre and the circumcenter of a triangle whose sides measure 24 cm, 26 cm and 10 cm?
(1) 13 cm (2) 12 cm
(3) 7.5 cm (4) $\sqrt{30}$ cm

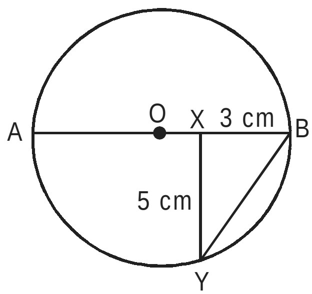
12. A circle is inscribed in the semi-circle as shown. What could be the radius of the circle?



(1) $\sqrt{2} + 12$ (2) $12 - \sqrt{2}$
(3) $\sqrt{2} - 1$ (4) $4 + \sqrt{2}$

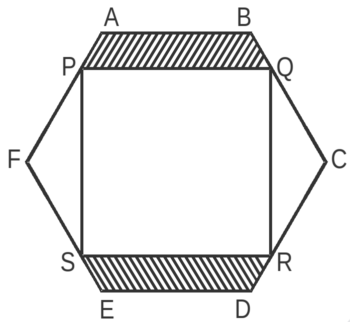
13. There are 2 concentric circles, one big and one small. A square ABCD is inscribed inside the big circle while the same square circumscribes the small circle. The square touches the small circle at points P, Q, R and S. Determine the ratio of circumference of big circle to the polygon PQRS.
(1) $\pi : 2$ (2) $2 : \pi$
(3) $2 : \sqrt{2}$ (4) $\pi : \sqrt{2}$

14. What is the circumference of the below circle given that AB is the diameter and XY is perpendicular to AB?



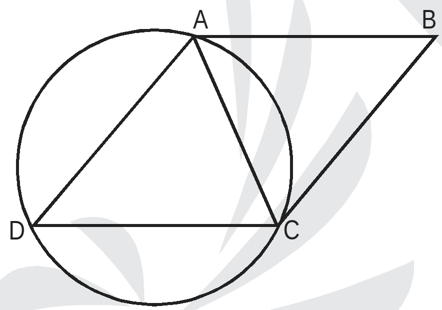
(1) 8π cm (2) $\pi\sqrt{34}$ cm
(3) $\frac{34\pi}{3}$ cm (4) $\frac{\pi\sqrt{31}}{3}$ cm

15. In the following figure, PQRS is the square of side 10 cm and $AP = \frac{1}{3}AF$. Find the ratio of area of shaded region to the whole hexagon. (Given ABCDEF is a Regular Hexagon)



(1) 4 : 9
(2) 7 : 27
(3) 9 : 16
(4) 3 : 13

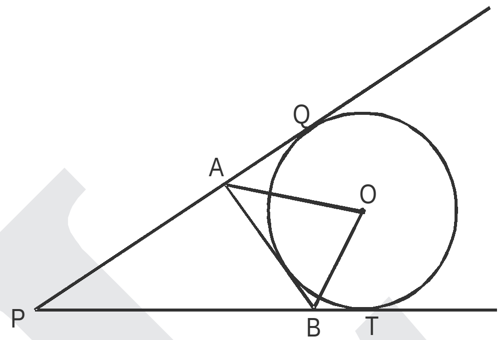
16. In the following figure ABCD is a parallelogram of area 160 cm^2. AD = 10 cm, CD = 20 cm. Find the length of AC?



(1) 10
(2) 15
(3) $\sqrt{250}$
(4) $\sqrt{260}$

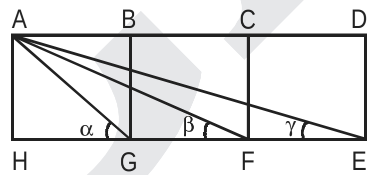
17. The line $y = \frac{2}{\sqrt{3}}x + 4$ is tangent to a circle centre at the origin. Find the radius of the circle.

18. Let PQ and PT be the tangent of the circle whose centre is at O. AB is also a tangent. Find the $\angle AOB$ if $\angle APB = 60°$.



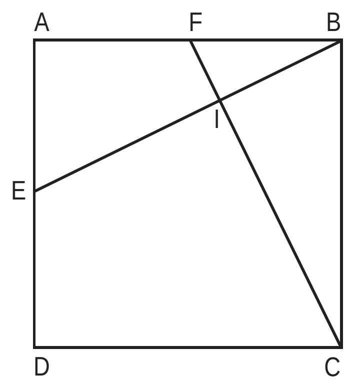
(1) 50°
(2) 60°
(3) 70°
(4) 90°

19. In this figure ABGH, BCFG and CDEF are three identical squares. What is the sum of $\alpha + \beta + \gamma$.



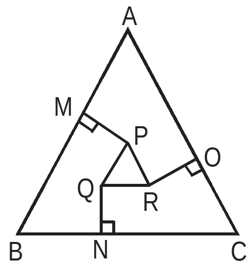
(1) 90°
(2) 120°
(3) 135°
(4) 150°

20. ABCD is a square of length 20 cm. F and E are the mid-points. What is the area of ΔBIF.



(1) 20 cm^2
(2) 25 cm^2
(3) 30 cm^2
(4) 40 cm^2

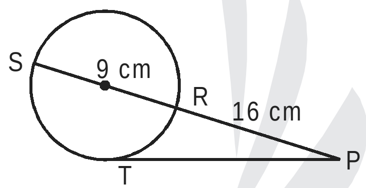
21. In the given figure, ΔABC is an equilateral triangle with sides $\sqrt{3}+1$ cm and ΔPQR is another equilateral triangle with sides 1 cm and is placed anywhere inside the larger triangle such that sides of smaller triangle is parallel to the sides of longer triangle.
Find (PM + QN + RO)



(1) 2 (2) 1

(3) $\frac{3}{2}$ (4) Cannot be determined

22.

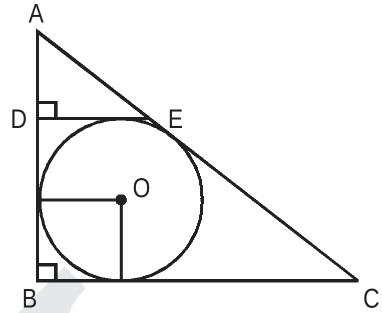


Find the area of ΔRST (approx.), where PT is a tangent and SR is the diameter

(1) 12 cm^2 (2) 16 cm^2

(3) 24 cm^2 (4) 20 cm^2

23.



In the given figure,
DE = 3 cm
r = 2 cm
Area (ΔADE) = ?

(1) 8 cm^2 (2) 6 cm^2

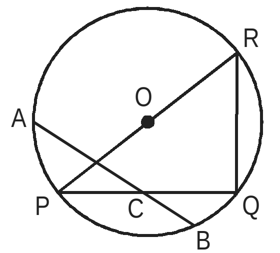
(3) 12 cm^2 (4) 16 cm^2

24. How many right angled triangles having integral sides with base 16 units are possible such that a circle having radius equals to an even integer can be inscribed in it?

(1) 3 (2) 7

(3) 2 (4) Data insufficient

25. In the given figure PR is the diameter of the circle, centered at O, and $\angle PRQ = 30°$ such that area of $\Delta PQR = 50\sqrt{3}\,cm^2$. Chord AB intersect PQ at C such that PC : CQ = 4 : 1. Then find the minimum possible length of chord AB.



(1) 8 cm (2) 12 cm

(3) 16 cm (4) 10 cm

QA - 6 : Geometry
Answers and Explanations

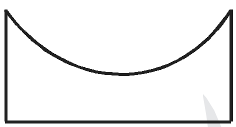
1	1	**2**	5	**3**	5	**4**	1	**5**	5	**6**	2	**7**	2	**8**	3	**9**	4	**10**	2
11	1	**12**	3	**13**	1	**14**	3	**15**	2	**16**	4	**17**	–	**18**	2	**19**	1	**20**	1
21	3	**22**	4	**23**	2	**24**	3	**25**	1										

1. 1 Radius of the sphere is r
From the centre of the sphere, radius of the upper part $= \sqrt{r^2 - h^2}$

Hence, $4\pi r^2 + 2\pi(r^2 - h^2) = \frac{125}{100} 4\pi r^2$

$\Rightarrow 6r^2 - 2h^2 = 5r^2 \Rightarrow h = \frac{r}{\sqrt{2}}$

2. 5



Rope = 150 ft
Half of rope = 75 ft
Pole = 100 ft
Distance above the ground = 25 ft
Here, half of rope = pole – distance above the ground
Nothing left to make the curve

3. 5 Side of square = 25
Area of rhombus = 500

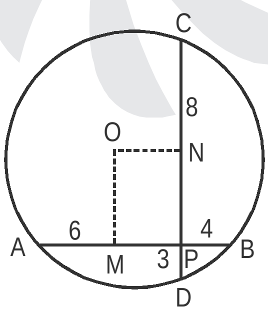
So, altitude, MD $= \frac{500}{25} = 20$

$MF = \sqrt{25^2 - 20^2} = 15$

Area $(\Delta MFD) = \frac{1}{2} \times 15 \times 20 = 150$

$\therefore$ area of shaded region = 625 – (500 – 150) = 275

4. 1



6 × 4 = CP × 3 ⇒ CP = 8
MP = 1
ON = 1
CD = 11
CN = 5.5
$ON^2 + CN^2 = OC^2$ (OC = r)
$\Rightarrow r^2 = 1^2 + (5.5)^2$
$\Rightarrow r^2 = 31.25$

So area $= \pi r^2$

$= \frac{125}{4}\pi$

5. 5 As, tangents from an exterior point make right angle with the radius ⇒ AOBX & ODX'E are squares.
$\therefore$ AO = BO = 40 cm and OD = DE = 30 cm

Length of string wound around the circle $X = \frac{3}{4} \times 80\pi$

$= 60\pi$

Similarly, wound around circle $X' = \frac{3}{4} \times 60\pi$

So, total length $= 40 + 40 + 30 + 30 + 60\pi + 45\pi$

$\Rightarrow 140 + 105\pi.$

6. 2 Let radius of the circles with centres C_1 and C_2 = R and radius of the circle at centre C_1 = r.

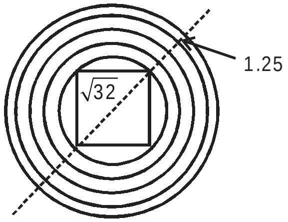
$\because \frac{1}{\sqrt{r}} = \frac{1}{\sqrt{R}} + \frac{1}{\sqrt{R}}$

$\frac{1}{\sqrt{10}} = \frac{2}{\sqrt{R}} \Rightarrow R = 40$ cm

7. 2 The possible triangles are with sides 4, 4, 8 or 8, 8, 4 but triangle one with sides 4, 4, 8 is not possible.
So perimeter is 8 + 8 + 4 = 20

8. 3 Let area of backyard be x^2 this year and y^2 last year.
then $x^2 - y^2 = 131$
$\Rightarrow (x - y)(x + y) = 1 \times 131$
$\Rightarrow x = 66$ and $y = 65$
So, number of tomatoes produced this year is 66^2 = 4356

9. 4



Diagonal $= \sqrt{32} \times \sqrt{2} = 8$

Since, the circles are equally spaced at 1.25 cm, the distance between innermost and outermost circle = 5 cm

$\therefore$ diameter of outermost circle = 5 + 8 + 5 = 18 cm

or $a\sqrt{2} = 18 \Rightarrow a = 9\sqrt{2} \Rightarrow \text{area} = (9\sqrt{2})^2 = 162$

10. 2 ab = 4

$\Rightarrow$ ab = 2 × 2 or 4 × 1

221, 222, 223, 441

these are possible triangles

22x will be right angled if x is 1, 2 or 3

44x is a triangle only if x = 1

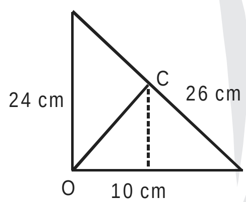
$\Rightarrow$ 221 is acute $1^2 + 2^2$, 222 is equilateral 223 is obtuse

144 is acute.

Only triangle 223 is possible

Hence c = 3

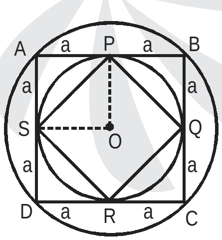
11. 1 Pythagorean triplet 12, 13, 5



$CO = \sqrt{12^2 + 5^2} = 13$

12. 3 Radius of the smaller circle must be less than 1 which is option (3).

13. 1



PQRS would also be a square

radius of big circle = diagonal of the square APOS

$= \sqrt{2}a$

Circumference $= 2\pi\sqrt{2}a$

Now, radius of small circle = side of square APOS = a

Consider $\Delta PSO, PS^2 = OP^2 + OS^2 \Rightarrow PS = \sqrt{2}a$

Circumference of square with side $a\sqrt{2} = 4\sqrt{2}a$

$\Rightarrow \text{Ratio} = 2\pi a\sqrt{2} : 4a\sqrt{2} = \pi : 2$

14. 3 Join AY

L2et AX = a

Diameter = (a + 3)

Since AB is the diameter $\angle AYB = 90°$

$\Rightarrow \angle BXY = 90°$

$\Rightarrow BY = \sqrt{3^2 + 5^2} = \sqrt{34}$

Consider triangles ABY and BXY

$\angle ABY = \angle XBY$ and both triangles are right angled.

$\therefore$ They are similar Δs and sides are proportional

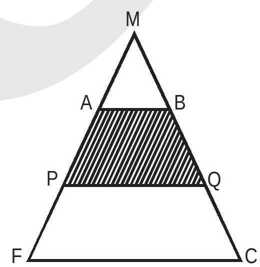
$$\frac{AB}{BY} = \frac{BY}{BX} = \frac{AY}{XY}$$

$$\frac{(a+3)}{\sqrt{34}} = \frac{\sqrt{34}}{3} \Rightarrow a = \frac{25}{3}$$

Diameter $= \frac{25}{3} + 3 = \frac{34}{3}$

$\Rightarrow$ Circumference $= \frac{34\pi}{3}$ cm

15. 2 Extending FA and CB to meet at M as shown below:



AB = 3a (side length of hexagon)

$\Rightarrow$ FC = 6a

from similarity PQ = 4a

and all triangles $\Delta AMB, \Delta PMQ$ and ΔFMC are equilateral and similar to each other.

so, area of trapz. (AFCB) = ar(ΔMFC) – ar(ΔMAB)

$$= \frac{\sqrt{3}}{4}(6a)^2 - \frac{\sqrt{3}}{4}(3a)^2 = \frac{\sqrt{3}}{4} \times 27a^2$$

and shaded area = ar(ΔMPQ) – ar(ΔMAB)

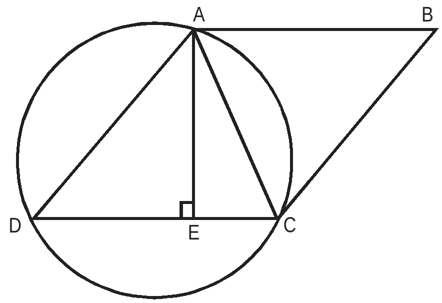
$$= \frac{\sqrt{3}}{4}(4a)^2 - \frac{\sqrt{3}}{4}(3a)^2 = \frac{\sqrt{3}}{4} \times 7a^2$$

So, required ratio $= \frac{(\sqrt{3}/4) \times 7a^2}{(\sqrt{3}/4) \times 27a^2} = \frac{7}{27}$

Alternatively:

Or, for similar Δs, required ratio $= \frac{4^2 - 3^2}{6^2 - 3^2} = \frac{7}{27}$

16. 4



Area = 160 cm^2
DC × AE = 160
AE = 8 cm
and AD = 10 cm
⇒ DE = 6 cm {from Pythagorean triplets}
⇒ CE = 14 cm
Now, $AC^2 = AE^2 + EC^2 = 8^2 + 14^2$

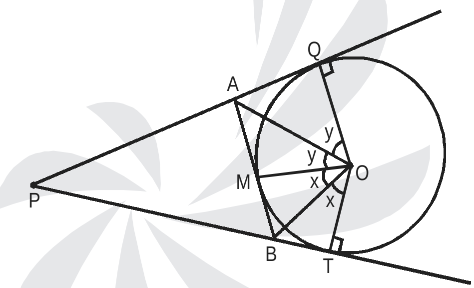
⇒ AC = $\sqrt{260}$

17. Since, we know that here
Radius = R = ⊥r distances from the origin to the tangent
Thus, ⊥r distance from 0(0, 0) to the line

$$2x - \sqrt{3}y + 4\sqrt{3} = 0$$

$$\Rightarrow R = \left|\frac{2(0) - \sqrt{3}(0) + 4\sqrt{3}}{\sqrt{2^2 + (\sqrt{3})^2}}\right| = \frac{4\sqrt{3}}{\sqrt{7}}$$

18. 2



∠QOT = 120°
Let, the tangent AB touches the circle at M
We join MO to get kites OMBT and OMAQ.
Now, we again join BO and AO such that
∠TOB = ∠BOM = x and ∠MOA = ∠AOQ = y
and we know x + x + y + y = 120°

$\Rightarrow x + y = \frac{120°}{2}$

⇒ ∠BOA = 60°

19. 1 $\tan\alpha = 1 \Rightarrow \alpha = 45°$

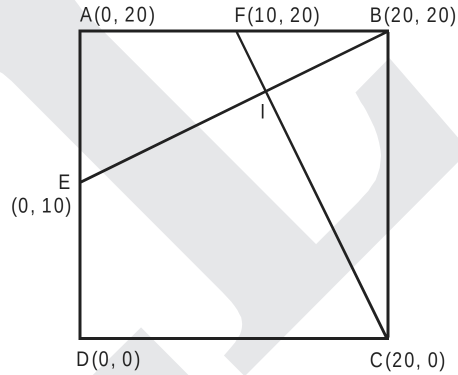
$\tan\beta = \frac{1}{2}$ and $\tan\gamma = \frac{1}{3}$

Now, we know α = 45° and we've to calculate (β + γ)

$$\text{So, } \tan(\beta+\gamma) = \frac{\tan\beta + \tan\gamma}{1 - \tan\beta.\tan\gamma} = \frac{\frac{1}{2}+\frac{1}{3}}{1 - \frac{1}{2}.\frac{1}{3}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$$

⇒ β + γ = 45°
Then α + β + γ = 45° + 45° = 90°.

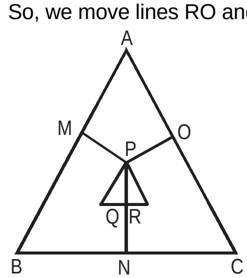
20. 1 On solving by co-ordinate geometry.



We consider the square is placed in a co-ordinate axes
So, equation of CF
2x + y = 40
and equation of BE
x = 2y – 20
On solving both the equations, we get, y-coordinate of point I would be y = 16.
⇒ Height of ΔFBI = 20 – 16 = 4 and Base = 10

So, area $= \frac{1}{2} \times 4 \times 10 = 20$ sq. unit

21. 3 Since, we know that the sum of perpendicular distances from any point inside the triangle to each side of the triangle is equals to the height of the equilateral triangle.
So, we move lines RO and QN to form a new figure as



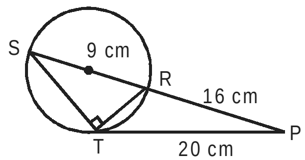
Now $PQ = \frac{\sqrt{3}}{2}$

So, MP + PO + (PQ + QN) $= \frac{\sqrt{3}}{2}(\sqrt{3} + 1)$

$$MP + PO + \frac{\sqrt{3}}{2} + QN = \frac{3}{2} + \frac{\sqrt{3}}{2}$$

$$\Rightarrow PM + QN + RO = \frac{3}{2}$$

22. 4 $PR \times PS = PT^2$
$16 \times 25 = PT^2 \Rightarrow PT = 20$ cm
we join ST and RT
let, $\angle P = \alpha$
and $\angle RTP = \theta$
then $\angle RST = \theta$
$\Rightarrow \Delta PRT \sim \Delta PST$



$$\frac{TR}{ST} = \frac{PR}{TP} \Rightarrow \frac{TR}{ST} = \frac{16}{20} = \frac{4}{5}$$

$\Rightarrow$ 5. TR = 4. ST
Now, from Pythagoras theorem
$ST^2 + TR^2 = 9^2$

$$ST^2 + \left(\frac{4}{5} \times ST\right) = 9^2$$

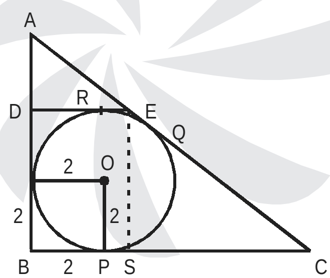
$$\Rightarrow ST = \frac{45}{\sqrt{41}}$$

Now, Area (ΔSTR) = $\frac{1}{2} \times TR \times ST$

$$= \frac{1}{2} \times \frac{4}{5} \times ST^2 = \frac{4}{2 \times 5} \times \frac{45 \times 45}{41}$$

Area (ΔSTR) = 20 cm^2

23. 2 Let the tangents BC and AC touch the circle at P and Q respectively.



So, from tangent property.
DR = 2 $\Rightarrow$ RE = 1 $\Rightarrow$ EQ = 1
let QC = x
$\Rightarrow$ PC = x
Now, we draw a $\perp$r to BC from point E meeting at S
So, EC = (x + 1)
CS = (x – 1) and ES = 4
Now, from Pythagoras theorem $EC^2 = CS^2 + ES^2$
$(x + 1)^2 = (x - 1)^2 + 4^2$
$\Rightarrow$ x = 4cm
So, BC = 6cm
$\because$ $\Delta ADE \sim \Delta ABC$
$\Rightarrow$ AB = 8cm

Thus, Area (ΔADE) = $\frac{1}{2} \times 3 \times 4 = 6$ cm^2.

Alternatively:
Since in radius = 2, which is possible for a right-angled triangle with perimeter 6,8,10
So, (BC = 6 and AB = 8) or (BC = 8 and AB = 6) 2nd pair satisfies the given options.

24. 3 From tangent property for a right-angled triangle, we know

$$r(\text{in-radius}) = \frac{P + B - H}{2} = \frac{P + 16 - H}{2} = \frac{16 - (H - P)}{2}$$

Again, $P^2 + B^2 = H^2$
$\Rightarrow$ $H^2 - P^2 = 16^2$
$\Rightarrow$ (H – P) (H + P) = 16 × 16
So, possible values of (H – P) is {2,4,8}
$\Rightarrow$ possible in-radius would be {7, 6, 4}
Thus, only 2 possible even in-radius are possible

25. 1 We join QO such that ΔPQO becomes an equilateral triangle with area $25\sqrt{3}\,cm^2$

$$\Rightarrow \frac{\sqrt{3}}{4} \times (\text{side})^2 = 25\sqrt{3}\,cm^2$$

side = 10 cm {for ΔOPQ}
$\Rightarrow$ PQ = 10 cm
$\because$ PC : CQ = 4 : 1
$\Rightarrow$ PC = 8 and CQ = 2
Now, AC × BC = PC × CQ
$\Rightarrow$ AC × BC = 16
Now from AM $\geq$ GM

$$\frac{AC + BC}{2} \geq (AC \times BC)^{1/2}$$

$\Rightarrow$ AC + BC $\geq$ 8
Thus, minimum possible length of AB = 8.