

Games & Tournaments - 1

LRDI - 10

CEX-D-0282/18

Number of Questions : **25**

Directions for questions 1 to 5: Answer the questions on the basis of the information given below.

Sixteen teams have been invited to participate in the ABC Gold Cup cricket tournament. The tournament is conducted in two stages. In the first stage, the teams are divided into two groups. Each group consists of eight teams, each team playing with every other team in its group exactly once. At the end of the first stage, the top four teams from each group advance to the second stage while the rest are eliminated. The second stage comprises of several rounds. Every round involves exactly one match for every team. The winner of a match in a round advances to the next round, while the loser is eliminated. The team that remains undefeated in the second stage is declared the winner and claims the Gold Cup.

The tournament rules are such that each match results in a winner and a loser with no possibility of a tie. In the first stage, a team earns one point for each win and no points for a loss. At the end of the first stage, teams in each group are ranked on the basis of total points to determine the qualifiers advancing to the next stage. Ties are resolved by a series of complex tie-breaking rules so that exactly four teams from each group advance to the next stage.

1. What is the total number of matches played in the tournament?
 (1) 28 (2) 55
 (3) 63 (4) 56
2. The minimum number of wins needed for a team in the first stage to guarantee its advancement to the next stage is
 (1) 5 (2) 6
 (3) 7 (4) 4
3. What is the highest number of wins for a team in the first stage in spite of which it could be eliminated at the end of first stage?
 (1) 1 (2) 2
 (3) 4 (4) 5
4. What is the number of matches in the second stage of the tournament?
 (1) 1 (2) 2
 (3) 4 (4) 7
5. How many teams got eliminated after winning exactly one match in the second stage of the tournament?
 (1) 4 (2) 5
 (3) 2 (4) 7

Directions for questions 6 to 8: Answer the questions on the basis of the information given below.

Certain number of players participated in a tournament. Each player was categorised as either a 'Pro' or an 'Amateur'. An 'Amateur' would immediately turn 'Pro' once he had played exactly 3 matches with 'Pro' players in the tournament. There were 3 rounds in the tournament. At the end of Round 3 it was found that each player had now turned a 'Pro'. No player left the tournament before the completion of Round 3.

The following table provides the information about the number of matches played in each round and the number of 'Pro' and 'Amateur' players at the beginning of each round.

Round	Number of 'Pro' players	Number of 'Amateur' players	Number of Matches
Round 1	P_1	A_1	14
Round 2	P_2	A_2	N_1
Round 3	12	3	N_2

Here, P_1 , P_2 , A_1 , A_2 , N_1 and N_2 represent missing values in the table.

Further, the following information is given below:

- Each match involved one 'Pro' and one 'Amateur' player.
 - It is not necessary that each player played a match in every Round.
 - At the end of Round 1, the number of 'Amateur' players was half of the number of 'Pro' players.
 - There were exactly 3 more 'Pro' players at the beginning of Round 2 as compared to the beginning of Round 1.
6. How many matches were played in the tournament?
- (1) 30 (2) 21
(3) 24 (4) Cannot be determined

7. What can be the maximum possible difference between any two possible values of N_2 ?
8. Which of the following cannot be TRUE?
- I. $N_1 - N_2 = 4$
II. $N_1 - N_2 = 6$
III. $N_1 - N_2 = 3$

- (1) Only I (2) Only II
(3) Only III (4) Both II and III

Directions for questions 9 to 12: Answer the questions on the basis of the information given below.

In a sports event, six teams (A, B, C, D, E and F) are competing against each other. Matches are scheduled in two stages. Each team plays three matches in stage – I and two matches in Stage – II. No team plays against the same team more than once in the event. No ties are permitted in any of the matches. The observations after the completion of Stage – I and Stage – II are as given below.

Stage-I:

- One team won all the three matches.
- Two teams lost all the matches.
- D lost to A but won against C and F.
- E lost to B but won against C and F.
- B lost at least one match.
- F did not play against the top team of Stage-I.

Stage-II:

- The leader of Stage-I lost the two matches in Stage-II.
 - Of the two teams at the bottom after Stage-I, one team won both matches, while the other lost both matches.
 - One more team lost both matches in Stage-II.
9. The two teams that defeated the leader of Stage-I are:
- (1) F & D (2) E & F
(3) B & D (4) E & D

10. The only team(s) that won both matches in Stage-II is (are):
 (1) B (2) E & F
 (3) A, E & F (4) B, E & F
11. The teams that won exactly two matches in the event are:
 (1) A, D & F (2) D & F
 (3) E & F (4) D, E & F
12. The team(s) with the most wins in the event is (are):
 (1) A (2) A & C
 (3) B & E (4) E

Directions for questions 13 to 16: Answer the questions on the basis of the information given below.

Two players A and B are playing a game of matchsticks. Any player can pick 1, 2 or 3 sticks in his turn. The player who picks the last stick always loses. Assume that both the players play intelligently.

13. If there were 10 matchsticks in all and A starts the game, then how many matchsticks should he pick in his first move in order to always ensure a win?
 (1) 1 (2) 2
 (3) 3 (4) He can never win
14. A starts the game and there are 10 matchsticks. If the option of picking 3 sticks is not exercisable, then how many should he pick in his first move in order to ensure a win?
 (1) 1 (2) 2
 (3) Either (1) or (2) (4) He can never win
15. In question 12, if in his first chance, A picks up 2, then how many would B pick in his first move in order to win?
 (1) 1 (2) 2
 (3) Either 1 or 2 (4) B can never win

16. If there were 16 matchsticks and the rule of the game was that anybody picking the last stick wins then which of the following is always true?
 (1) The person who starts can never win.
 (2) The person who starts will always pick 2 matchsticks.
 (3) The person who plays second can never win.
 (4) The person who plays second always picks 1 matchsticks.
17. Four football teams – Arsenal, Milan, Barcelona and Lyon – played in the Emirates Cup. Each team played all the other teams exactly once during the tournament. A win earns a team 3 points, a draw earns 1 and a loss earns no points. Each team scored 3 goals in the Emirates Cup. Arsenal won the Cup with maximum points followed by Milan and then Barcelona. If no game ended in a draw, then how many goals were scored in the game between Barcelona and Lyon?

Directions for questions 18 to 22: Answer the questions on the basis of the information given below.

The organizers of Indian Wells Tournament decided to seed (rank) all the 128 tennis players as per their ATP rankings. The highest ranked tennis player was seeded 1, the second highest was seeded 2 and so on. The tournament consisted of the conventional 6 knockout (loser of the match gets exited from the tournament) stages – Round I, Round II, Round III, Round IV, Quarter Finals and Semi Finals, before the Finals.

In the first round, Match 1 was between the highest seeded player and the lowest seeded player, Match 2 was between the second highest seeded player and the second lowest seeded player and so on.

In the second round, the winner of Match 1 in the first round plays with the winner of Match 64 of the first round and this match is termed as Match 1 of the Round II.

A similar pattern is followed in the subsequent rounds.

An *upset* is a match where a lower seeded player defeats a higher seeded player.

18. What was the total number of matches played in the tournament?
19. If there were no *upsets* in the tournament, who played against the player seeded 25th in Round III?
(1) 7th (2) 5th
(3) 14th (4) None of these
20. In the third round, match of the player seeded 5th was an *upset*, which of the following cannot be the seed of the player who defeated him?
(1) 37 (2) 69
(3) 92 (4) 101
21. If the player seeded 52nd went on to win the tournament, then what is the minimum number of *upsets* in the tournament?
(1) 2 (2) 4
(3) 6 (4) None of these

22. If the only *upsets* took place in the quarterfinals (for all the matches), what is the seed of the winner of the tournament?
(1) 3 (2) 5
(3) 6 (4) Cannot be determined

Directions for questions 23 to 25: Answer the questions on the basis of the information given below.

In a target shooting competition, a person is allowed to shoot at four targets successively, then followed by the next competitor. When all have finished one such round, the process is repeated. If a target is hit, the shooter is awarded two points. If he misses the target, the others are awarded one point each. The first person who gets 60 points wins the competition. In a contest between A, B and C, the final score card is A = 60, B = 53, C = 43. Out of a total of 78 shots fired, 43 hit the target.

23. Who was the second person to shoot?
(1) A (2) B
(3) C (4) Either A or B
24. How many targets did A hit?
25. How many targets did B miss?

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Answers and Explanations

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1	3	2	2	3	4	4	4	5	3	6	3	7	5	8	4	9	2	10	4
11	2	12	3	13	1	14	4	15	1	16	1	17	5	18	127	19	4	20	2
21	3	22	2	23	1	24	17	25	10										

1. 3 There shall be 8 teams in each group. Each team in a group shall be playing with every other team.

Hence, total number of matches shall be $\frac{(7 \times 8)}{2} = 28$

in one group. Hence, in all there shall be 56 matches. This is for the first stage. Thereafter, there are 8 teams in knockout rounds from which one winner emerges, or 7 losers are identified.

Hence, 7 more matches, i.e. in all 63 matches.

2. 2 If a team, wins 7 matches, it will definitely advance to the next stage. Consider the following case in which exactly five of the eight teams win 5 matches and loose 2. Let the teams be A, B, C, D, E, F, G and H.

Teams	A	B	C	D	E	F	G	H
A	x	W	W	W	W	W	L	L
B	L	x	W	W	W	W	W	L
C	L	L	x	W	W	W	W	W
D	L	L	L	x				
E	L	L	L		x			
F	L	L	L			x		
G	W	L	L	W	W	W	x	W
H	W	W	L	W	W	W	L	x

In the above table, W represents win, L represents loss and X represent no match (as a team cannot play with itself.)

From the table, A, B, C, G and H can win five matches each. As only 4 of the 8 teams can advance to the next stage, we cannot decide which team will qualify for the second round. Therefore, the minimum number of wins that can assure a place in the second stage is 6.

(It is to be noted that total matches will be 28 and hence the case where more than 4 teams win at least

6 matches is not possible. So a team with 6 wins will surely not get eliminated at the end of the first stage)

3. 4 If a team, wins 7 matches, it will definitely advance to the next stage. Consider the following case in which exactly five of the eight teams win 5 matches and loose 2. Let the teams be A, B, C, D, E, F, G and H.

Teams	A	B	C	D	E	F	G	H
A	x	W	W	W	W	W	L	L
B	L	x	W	W	W	W	W	L
C	L	L	x	W	W	W	W	W
D	L	L	L	x				
E	L	L	L		x			
F	L	L	L			x		
G	W	L	L	W	W	W	x	W
H	W	W	L	W	W	W	L	x

In the above table, W represents win, L represents loss and X represent no match (as a team cannot play with itself.)

From the table, A, B, C, G and H can win five matches each. As only 4 of the 8 teams can advance to the next stage, we cannot decide which team will qualify for the second round and hence a team with 5 wins may still get eliminated at the end of the first stage.

The correct answer is 5.

(It is to be noted that total matches will be 28 and hence the case where more than 4 teams win at least 6 matches is not possible. So a team with 6 wins will surely not get eliminated at the end of the first stage)

4. 4 There are 8 teams. Hence, there would be 7 matches in 3 rounds. In the first round 4 teams get eliminated. In the second round 2 teams get eliminated and in the third round, the runner up is eliminated.

5. 3 In the first round of stage two, 4 of the 8 teams win their matches. Out of these 4 teams, exactly two get eliminated in the second round and the other two advance to the next round. Hence correct answer is (3).

For questions 6 to 8:

From the table:

Number of 'Pro' players at the beginning of Round 3 = 12
 Number of 'Amateur' players at the beginning of Round 3 = 3
 Hence, total players participating in the tournament = 12 + 3 = 15.

It is given that each player was categorised as either a 'Pro' or an 'Amateur'. Also, no player left the tournament before the completion of Round 3.

So, total number of players will be the same for each round and can be given by the sum of number of 'Amateur' and 'Pro' players for any round.

$$A_1 + P_1 = A_2 + P_2 = 15$$

From Statement 3: $A_2 = \frac{1}{2}(P_2)$

So, $A_2 = 5$ and $P_2 = 10$

From Statement 4: $P_2 = P_1 + 3$

So, $P_1 = 7$ and $A_1 = 15 - P_1 = 8$

Round 1:

3 of the 6 'Amateur' players in the Round 1, who turned 'Pro' after this round, must have played 3 matches each in this round.

Rest of the 5 matches must have involved three or more of the remaining 5 'Amateur' players.

Round	No of 'Pro' Players	No of 'Amateur' Players	Number of games
Round 1	$P_1(7)$	$A_1(8)$	14
Round 2	$P_2(10)$	$A_2(5)$	N_1
Round 3	12	3	N_2

6. 3 Since, all the players turned 'Pro' in the end, each 'Amateur' player must have played 3 matches each.
 \therefore Total number of matches = $3(A_1) = 24$.

7. 5 Since, total number of matches played = 24
 $\therefore N_1 + N_2 = 10$
 For maximum possible value of N_2 , N_1 must be minimum.

We know that 2 'Amateur' players turned 'Pro' after Round 2.

So, each of these two players must have played at least one match in Round 2.

To minimize N_1 , let two players play 2 matches each in Round 1 and one match each in Round 2. (some third player must have played one match in Round 1 as there were 5 matches left for the rest of the 5 'Amateur' players.)

No other 'Amateur' Player played a match in the Round 2.

So, $N_{2 \max} = 10 - 2 = 8$

Similarly, minimum possible value for N_2 or $N_{2 \min}$ is 3 as 3 'Amateur' players turned "Pro" after Round 3.

(Assuming each of these 3 players required only one match in Round 3 to turn a 'Pro')

So, $N_{2 \max} - N_{2 \min} = 8 - 3 = 5$.

8. 4 $N_1 + N_1 = 10$
 $N_{2 \max} = 8$. So, $N_{1 \min} = 2$
 $N_{2 \min} = 3$. So, $N_{1 \max} = 7$

From I:

$N_1 = 7$ and $N_2 = 3$
 This can be TRUE

From II:

$N_1 = 8$ and $N_2 = 2$
 This cannot be TRUE as $N_{1 \max} = 7$ and $N_{2 \min} = 2$

From III:

$N_1 = 6.5$ and $N_2 = 3.5$
 This cannot be TRUE since N_1 and N_2 must be integers.

For questions 9 to 12:

The given basic information can be collated as below:

- (i) Six teams – A, B, C, D, E, F.
 - (ii) Matches scheduled in two stages – I and II.
 - (iii) No team plays against the same team more than once.
 - (iv) No ties permitted.
- As per the instructions given for Stage – I, we can reach the following conclusions:
- (a) As B lost at least one match, A won all the 3 matches.
 - (b) The two teams who lost all the matches cannot be A (as explained above), B (E lost to B), D (D won against C and F) and B (B won against E and F). Hence, the two teams must be C and F.
 - (c) F did not play against the top team (i.e. A). We get the following table for Stage – I.

(To be read from rows)

	A	B	C	D	E	F
A	X	W	W	W		
B	L	X			W	W
C	L		X	L	L	
D	L		W	X		W
E		L	W		X	W
F		L		L	L	X

As per the instructions given for Stage – II, we can reach the following conclusions.

- (d) A lost both its matches against E and F.
- (e) F won against A, hence is the bottom team (out of C and F) which won both the matches \Rightarrow F won against C as well.

This also means that C lost both its matches against B and F.

- (f) Apart from A and C, one more team lost both the matches in Stage – II.

That team can neither be E (A lost to E), nor B (as C lost to B), nor F (as F won both its matches). Hence, the team must be D.

We get the following table for Stage – II.

(To be read from rows)

	A	B	C	D	E	F
A	X				L	L
B		X	W	W		
C		L	X			L
D		L		X	L	
E	W			W	X	
F	W		W			X

9. 2 E and F defeated A.

10. 4 B, E and F won both the matches in Stage – II.

11. 2 D and F won exactly two matches in the event.

12. 3 B and E have most wins, 4 each.

13. 1 There are 10 matchsticks. The objective of **A** would be to make sure that after he has made his last pick in the game, **B** is left to pick the last remaining stick only. **A** can ensure his win if he leaves **1, 5 or 9** sticks (or if there were more stick available, 13, 17, 21 etc.,...) for B to pick every time A makes his own pick. So, A can ensure his win by picking 1 stick in the start.

14. 4 Here the option of picking 3 sticks is not available and A has to start the game. A can pick either 1 or 2 sticks in the beginning. In either case, A can never win, if B plays intelligently.

A	B	A	B	A	B	A
2	1	2	1	x	3 – x	1

Here value of $x = 1$ or 2

Hence, in this case, A can never win because he picks the last stick.

15. 1 Clear from the above table.

16. 1 In order to pick up the 16th stick, one needs to pick the 12th, 8th and 4th also. The person who starts can only pick upto 3 sticks. So, he will never win.

17. 5 As no game ended in a draw, it can be concluded that the total points earned by the two teams playing a game must be 3. The overall aggregate points of each team must also be a multiple of 3 (or zero). Thus the aggregate points of Arsenal, Milan and Barcelona must be 9, 6 and 3 respectively. Arsenal won all its matches and scored 3 goals in the process. Hence each game must've been won by Arsenal with a score-line of '1-0'. Milan must have won against both Barcelona and Lyon scoring 1 goal against one and 2 against the other (not necessarily in the same order). Barcelona must have defeated Lyon and hence scored more goals than them in that game. Thus it can be concluded that between Lyon and Barcelona, Lyon scored more goals against Milan. Thus the matches 'Milan-Lyon' and 'Milan-Barcelona' ended with score-lines '2-1' and '1-0' respectively. The only game left is Barcelona-Lyon in which a total of 5 goals were scored and it ended with a score-line of '3-2'.

18. 127 The total number of matches played
 $= 64 + 32 + 16 + 8 + 4 + 2 + 1 = 127$.

19. 4 As there were no *upset* in the tournament, the top 32 seeds reached Round III. Hence, the 25th seeded player must have played against the 8th seeded player.

20. 2 In Round I, the 5th seeded player would have defeated the 124th seeded player.
 In Round II, he would have beaten the player seeded 60th or 69th.
 In Round III, he would have played against the player seeded 28th (in case of no upsets), or the player seeded 37th (possible opponent of the 28th seeded player in Round II), or player seeded 101th (opponent of player seeded 28th in Round I) or player seeded 92nd (opponent of player seeded 37th in Round I).
 Hence, any of the players seeded 28th, 37th, 92nd or 101th could have beaten the player seeded 5th in Round III and caused an *upset*.

21. 3 Suppose the *upsets* of the tournament are caused by the winner only. By this we can minimize the number of *upsets* in the tournament. Except for Round I (where he defeats the player seeded 77th), the player ranked 52nd can win the tournament with 6 *upsets*.

22. 2 As there are no *upsets* until the quarterfinals, the top 8 seeds reached the quarter finals.
 So, the players progressing to the semi finals will be seeded 5th, 6th, 7th and 8th. The semifinal lineup will be : 5th seed vs 8th seed and 6th seed vs 7th seed. So, the finalists will be the 5th and 6th seeded players, and the 5th seeded player will be the winner (no *upset* possible).

For questions 23 to 25:

There are three shooters — A, B and C taking part in the competition. Each shooter will aim at 4 targets before the next shooter gets his chance. Thus, one round is completed in every 12 shots. A total of 78 shots are fired out of which 72 shots would complete 6 rounds. This means that the first to shoot will get $(6 \times 4 + 4) = 28$ shots and the next shooter will get $(6 \times 4 + 2) = 26$ shots and the third will get 24 shots.

	Hitting target	Missing target
A	a_1	a_2 (all numbers integers)
B	b_1	b_2
C	c_1	c_2

Of the 78 shots, 43 hit the target and 35 miss the target.

Thus, $a_1 + b_1 + c_1 = 43$... (i)

and $a_2 + b_2 + c_2 = 35$... (ii)

A scored 60 points, $\therefore 2a_1 + b_2 + c_2 = 60$... (iii)

B scored 53 points, $\therefore 2b_1 + a_2 + c_2 = 53$... (iv)

C scored 43 points, $2c_1 + b_2 + a_2 = 43$... (v)

Let us assume that A was the first person to shoot.

Thus, $a_1 + a_2 = 28$

From (ii) and (iii), we have $2a_1 - a_2 = 25$

$3a_1 = 53$, $\therefore a_1 = 17.67$ (Not possible, because a_1 and a_2 must be integers)

Let us assume that B was the first person to shoot.

Thus, $b_1 + b_2 = 28$

From (ii) and (iv), we have

$2b_1 - b_2 = 18$

$3b_1 = 46$

$\therefore b_1 = 15.33$ (Not possible, since b_1 and b_2 must also be integers)

Let us assume that C was the first person to shoot.

Thus, $c_1 + c_2 = 28$

From (ii) and (v), we have

$2c_1 - c_2 = 8$ or $3c_1 = 36$

$c_1 = 12$ and $c_2 = 16$ (Integral values, so possible)

Let us assume that A was the second person to shoot.

Thus, $a_1 + a_2 = 26$

From (ii) and (iii), we have $2a_1 - a_2 = 25$

$3a_1 = 51$

$a_1 = 17$ and $a_2 = 9$

Thus, C was the first person to shoot and A was the second person to shoot.

Also, from the values above

$b_1 = 14$ and $b_2 = 10$.

Alternative method:

Each round = 12 shots

Hence, for 78 shots to be fired,

first shooter fired 28 shots ($4 \times 6 + 4 = 28$).

Second shooter fired 26 shots (ended the competition).

Third shooter fired 24 shots.

As per given conditions, for A to score 60 points, he must fire more than 12 shots at the target (since he can get a maximum of 35 from others' misses and he needs to score balance 25 from hitting the target).

Let A hit 15 targets,

then A's score = $15 \times 2 = 30 + 30$ (from others' misses).

Hence, A misses 5 shots only.

So A's total shots = $15 + 5 = 20$ (not possible).

Let A hit 17 targets, then A's score = $17 \times 2 = 34 + 26$ (from others' misses).

Hence, A misses 9 shots only.

So A's total shots = $17 + 9 = 26$,

i.e. A was the second shooter.

Similarly, for B's point to be 53, he must fire more than 9 shots at target (since he can get a maximum of 35 from others' misses).

Let B hit 12 targets, then B's score = $12 \times 2 = 24 + 29$ (from others' misses).

Hence, B misses 6 shots only

So, B's total shots = $12 + 6 = 18$ (not possible).

Let B hit 14 targets,

then B's score = $14 \times 2 = 28 + 25$ (from others' misses).

Hence, B misses 10 shots only.

So, B's total shots = $14 + 10 = 24$,

i.e. B was the third shooter.

Accordingly, the following table can be compiled:

	First shooter	Second shooter	Third shooter
	C	A	B
Targets Hit	12	17	14
Misses	16	9	10
Total shots	28	26	24
Total points	43	60	53

23. 1

24. 17

25. 10