Numbers - 3

Career Launcher

QA - 13

CEX-Q-0214/18

Contents

- LCM & HCF
- Factorial

Number of Questions : 25

LCM & HCF

1. Find the least number which when divided by 6, 15 and 17 always leaves a remainder 1, but when divided by 7 leaves no remainder.
(1) 211 (2) 511
(3) 1021 (4) 86

2. The least number which on division by 35 leaves the remainder 25 and on division by 45 leaves the remainder 35 and on division by 55 leaves the remainder 45 is
(1) 2515 (2) 3455
(3) 2875 (4) 2785

3. Find the greatest four-digit number which when divided by 4, 5, 6 and 7 leaves remainders 2, 3, 4 and 5 respectively.

4. A is the set of positive integers such that when divided by 2, 3, 4, 5, 6 leaves the remainders 1, 2, 3,4, 5 respectively. How many integers between 0 and 100 belong to set A? **(CAT 1998)**
(1) 0 (2) 1
(3) 2 (4) None of these

5. At a bookstore, 'MODERN BOOK STORE' is flashed using neon lights. The words are individually flashed at the intervals of $2\frac{1}{2}$s, $4\frac{1}{4}$s and $5\frac{1}{8}$s respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is
(1) 49.5 s (2) 72.5 s
(3) 1744.5 s (4) 855 s

6. The LCM of two numbers is 168 and their HCF is 28. If the sum of the numbers is 140, then the numbers are
(1) 106 and 34 (2) 56 and 28
(3) 112 and 28 (4) 84 and 56

7. The HCF of two natural numbers is 119 and their sum is 833. What are the numbers, if they differ by the minimum possible amount?
(1) 357 and 476 (2) 119 and 714
(3) 238 and 545 (4) 119 and 545

8. The sum of two numbers is 462 and their HCF is 22. What is the maximum possible number of pairs that satisfy these conditions?
(1) 3 (2) 2
(3) 8 (4) 6

9. Let ab and ba be two-digit numbers where a and b are distinct digits from 0 to 9. Difference of ab and ba is HCF of ab and ba. What is the value of a + b?
(1) 6 (2) 7
(3) 8 (4) 9

10. LCM of two numbers is ab^3, where a and b are prime numbers. If one of the numbers is b^3, then which of the following cannot be the other number?

(1) ab^3 (2) ab
(3) ab^2 (4) None of these

11. If (x – m) is the HCF of (x^2 – bx + a) and (x^2 + cx – d), then the value of m is

(1) $\frac{d-a}{c-b}$ (2) $\frac{b+c}{a+d}$

(3) $\frac{b+c}{c-d}$ (4) $\frac{a+d}{b+c}$

12. Three Vice Presidents (VP) regularly visit the plant on different days. Due to labour unrest, VP (HR) regularly visits the plant after a gap of 2 days. VP (Operations) regularly visits the plant after a gap of 3 days. VP (Sales) regularly visits the plant after a gap of 5 days. The VPs do not deviate from their individual schedules. CEO of the company meets the VPs when all the three VPs come to the plant together. CEO is on leave from January 5th to January 28th, 2012. Last time CEO met the VPs on January 3, 2012. When is the next time CEO will meet all the VPs? **(XAT 2012)**

(1) February 6, 2012 (2) February 7, 2012
(3) February 8, 2012 (4) February 9, 2012

13. Sumit committed a mistake in finding the LCM of three distinct positive integers greater than 1 namely A, B and C, and found it to be 840, which is a common multiple of A, B and C all, but is not the lowest. The HCF of A, B and C is 1. Find the maximum possible value of A + B + C.

14. The LCM of three positive integers X, Y and Z is 119^2. Find the total number of ordered triplets (X, Y and Z).

(1) 400 (2) 361
(3) 289 (4) 225

15. For two positive integers a and b define the function h(a,b) as the greatest common factor (G.C.F) of a, b. Let A be a set of n positive integers. G(A), the G.C.F of the elements of set A is computed by repeatedly using the function h. The minimum number of times h is required to be used to compute G is **[CAT 1999]**

(1) 1/2n (2) (n –- 1)
(3) n (4) None of these

Factorial

16. If N = 150!

a. What is the highest power of 2?
b. What is the highest power of 3?
c. What is the highest power of 6?
d. What is the highest power of 48?
e. Find the number of trailing zeros at the end of N.

17. If n is an odd natural number and n! ends with 32 zeros, then how many values of n are possible?

(1) 2 (2) 3
(3) 1 (4) 4

18. The number of trailing zeroes in the number 100! + 10 is

19. The number of positive integers n in the range 2 < n < 20 such that (n – 1)! is not divisible by n is

(1) 7 (2) 8
(3) 9 (4) 10

20. How many real n's are there such that n! is a perfect square?

(1) 1 (2) 2
(3) 3 (4) More than 3

21. What is the highest power of 3 in 87! – 60!?

(1) 28 (2) 42
(3) 14 (4) 27

22. The number of values of 'x' less than 30 for which $50! - x!$ ends in 6 zeroes is:

23. For how many positive integers V less than 15, $V! + (V + 1)! + (V + 2)!$ is an integral multiple of 36?

(1) 12 (2) 11

(3) 9 (4) 10

24. Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \geq 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p + 2$ when divided by $11!$, Leaves a remainder of

(1) 10 (2) 0

(3) 7 (4) 1

25. If $a! + 1 = (b! + 1)!$ and $a = b$, how many solutions are there for (a, b)?

Visit "Test Gym" for taking Topic Tests / Section Tests on a regular basis.

QA - 13 : Numbers - 3
Answers and Explanations

CEX-Q-0214/18

1	2	**2**	2	**3**	–	**4**	2	**5**	2	**6**	4	**7**	1	**8**	4	**9**	4	**10**	4
11	4	**12**	3	**13**	–	**14**	2	**15**	2	**16**	–	**17**	1	**18**	–	**19**	2	**20**	2
21	1	**22**	–	**23**	4	**24**	4	**25**	–										

1. 2 Since dividing the number by 6, 15 and 17, leaves a remainder of 1, the smallest such number is the LCM of 6, 15 and 17 plus 1, viz. 510 + 1 = 511. This number is also divisible by 7 and hence the required answer is 511.

2. 2 Difference between divisor and remainder
= 35 – 25 = 45 – 35 = 55 – 45 = 10
LCM of 35, 45, 55 = 5 × 7 × 9 × 11 = 3465
$\Rightarrow$ Required Number = 3465 – 10 = 3455.

3. The number which when divided by 4, 5, 6 and 7 leaves remainders 2, 3, 4 and 5 respectively will of the form [(LCM of 4, 5, 6, 7) k – (4 – 2)] i.e. 420k – 2, where k is a natural number.
$\therefore$ The required number = 9658.

4. 2 Note that the difference between the divisors and the remainders is constant.
2 – 1 = 3 – 2 = 4 – 3 = 5 – 4 = 6 – 5 = 1
In such a case, the required number will always be [a multiple of LCM of (2, 3, 4, 5, 6) – (The constant difference)].
LCM of (2, 3, 4, 5, 6) = 60
Hence, the required number will be 60n – 1.
Thus, we can see that the smallest such number is (60 × 1) – 1 = 59
The second smallest is (60 × 2) – 1 = 119
So between 1 and 100, there is only one such number, viz. 59.

5. 2 Because each word is lit for a second,

$$\text{LCM}\left(\frac{5}{2}+1, \frac{17}{4}+1, \frac{41}{8}+1\right)-1=\text{LCM}\left(\frac{7}{2}, \frac{21}{4}, \frac{49}{8}\right)-1$$

$$\frac{\text{LCM}(7, 21, 49)}{\text{HCF}(2, 4, 8)}-1=\frac{49\times 3}{2}-1=72.5\,\text{s}$$

6. 4 The product of two numbers is the product of the LCM and HCF of the two numbers. If the numbers are x and y, then x + y = 140, xy = 28 × 168.
Solving for x and y, the numbers are 84 and 56.

Alternative method:
The two numbers are 28X and 28Y, where X and Y are coprimes.
Also 28(X + Y) = 140
So X + Y = 5
Since LCM 28 × 6 = 168, XY = 6
So X = 3, Y = 2

7. 1 Since HCF is 119, the numbers could be taken as 119u and 119v, where u and v are coprimes.
Hence, 119u + 119v = 833, u + v = 7
Since the numbers differ by the minimum possible amounts $\therefore$ (u, v) must be = (3, 4)
$\therefore$ Numbers are 357 and 476

8. 4 Since 22 is the GCF of the two numbers, let the numbers be 22x and 22y respectively such that x and y are relatively prime.
22x + 22y = 462 = 22 × 21
$\Rightarrow$ x + y = 21
We need to find all pairs of numbers such that they are relatively prime and their sum is 21. (1, 20), (2, 19), (4, 17), (5, 16), (8, 13) and (10, 11) are the pairs that satisfy the aforementioned condition.

9. 4 Difference between a two-digit number and the number with its digit reversed is always a multiple of 9.
This difference is equal to the HCF only in case of the number 54 (or 45).
Therefore, the value of a + b = 9.

10. 4 Start checking with option (1). LCM of ab^3 and b^3 will be ab^3. Similarly for option (2) also. LCM of ab and ab^3 is again ab^3. If we examine the options one by one, we can see that all of them individually when paired up with b^3, will give an LCM as ab^3. Therefore the correct choice should be none of these.

11. 4 x – m is the HCF of both expressions, means x – m is the factor for both
$\therefore x - m = 0 \Rightarrow x = m$
Substituting x = m in both expressions
$m^2 - bm + a = m^2 + cm - d$
a + d = (c + b) m

$$\Rightarrow m = \frac{a+d}{b+c}$$

12. 3 Vice Presidents of HR, Operation and Sales visit the plant after a gap of 2 days, 3 days and 5 days respectively. So, VP (HR), VP (Operation) and VP (Sales) visit the plant on every 3rd, 4th and 6th day respectively.
Therefore, CEO meets the VPs on every 12^{th} day (L.C.M. of 3, 4 and 6) i.e. on 3rd, 15th, 27th of January and 8th of February, but CEO is on leave till 28th of January.
Hence, CEO will meet all the three VPs on 8th of February.

13. Largest sub-multiple of 840 is 420.
Now 420 = $2^2 \times 3 \times 5 \times 7$
So those three numbers can be 3, 140 and 420.
So, the maximum possible sum can be 563.
All the other combinations will leas to a lower sum.

14. 2 $119^2 = 17^2 \times 7^2$. Since L.C.M. of (X, Y and Z) is $(17^2 \times 7^2)$, power of $17(17^0, 17^1, 17^2)$ and powers of $7(7^0, 7^1, 7^2)$ have to be distributed among (X, Y, and Z)
Number of ways of distributing powers of 17 among X, Y and Z is as given below:

X	Y	Z
17^0	17^0 or 17^1	17^2
	17^2	17^0 or 17^1 or 17^2
17^1	17^0 or 17^1	17^2
	17^2	17^0 or 17^1 or 17^2
17^2	17^0 or 17^1 or 17^2	17^0 or 17^1 and 17^2

Total number of ways = (2 + 3 + 2 + 3 + 9) = 19.
Similarly, the number of ways of distributing power of 7 among X, Y and Z = 19
$\therefore$ Total number of ordered triplets (X, Y and Z) = 19 × 19 = 361.

Alternative method:
Power of $17(17^0, 17^1, 17^2)$ and powers of $7(7^0, 7^1, 7^2)$ have to be distributed among (X, Y, and Z)
17^2 can be distribeted (X, Y and Z) in 3^3 ways. However we need to subtract the cases who only 17^0 or 17^1 have to be distributed which is 2^3
$\therefore$ 17(0, 1, 2) can be distributed in $(3^3 - 2^3)$ ways.
Similarly for $7^{(0, 1, 2)}$ = 19 × 19
Total number of ordered triplets
= (27 – 8) (27 – 8) = $(3^3 - 2^3) \times (3^3 - 2^3)$.

15. 2 If there are n numbers, the function h has to be performed one time less.

16. a. The highest power of 2 is

$$= \left[\frac{150}{2}\right] + \left[\frac{150}{2^2}\right] + \left[\frac{150}{2^3}\right] + \left[\frac{150}{2^4}\right] + \ \ldots \left[\frac{150}{2^7}\right]$$

= 75 + 37 + 18 + 9 + 4 + 2 + 1 = 146.

b. The highest power of 3 is

$$= \left[\frac{150}{3}\right] + \left[\frac{150}{3^2}\right] + \left[\frac{150}{3^3}\right] + \left[\frac{150}{3^4}\right]$$

= 50 + 16 + 5 + 1 = 72

c. 6 = 2 × 3 and the highest powers of 2 and 3 are 146 and 72 respectively. So the highest power of 6 is 72 as 72 is lower among the two.

d. $48 = 2^4 \times 3$. Highest power of 2^4 is $\frac{146}{4} \approx 25$ which is lower than the power of 3 so the highest power of 48 will be 24.

e. Number of trailing zeroes is equal to the highest power of 10. 10 = 2 × 5. As 5 is a bigger prime than 2, the highest power of 5 will be less than that of 2.

The power of 5 is $\left[\frac{150}{5}\right] + \left[\frac{150}{5^2}\right] + \left[\frac{150}{5^3}\right]$

= 30 + 6 + 1 = 37. So the number of trailing zeroes in 150! will be 37.

17. 1 n can be only 131 and 133. Please check how many times 5, 25 & 125 is there within 131, you will realise that there will be 32 'zeros' in the end of 131!

18. The number of zeroes in the number 100! is 24 since

we have $\left[\frac{100!}{5}\right] + \left[\frac{100!}{5^2}\right] = 24$

But adding 10 leaves only one trailing zeroes in the number 100! + 10.

19. 2 Integers which will not be divisible by n will be all prime numbers 3, 5, 7, 11, 13, 17,19 and the composite number 4.

20. 2 For n = 0 and 1, n! is a perfect square (which is 1). Otherwise n! is never a perfect square.
(Between n and 2n there must be at least 1 prime number)

21. 1 87! – 60!
= 60! [(87 × 86 × ... × 61) – 1]
= 60! (3k – 1)
$\therefore$ Highest power of 3 is highest power of 3 in 60!, i.e.

$$\left[\frac{60}{3}\right] + \left[\frac{20}{3}\right] + \left[\frac{6}{3}\right] = 20 + 6 + 2 = 28$$

(here [x] denotes greatest integer less than or equal to x)

22. For 50! – x! to end in 6 zeroes, x! must also end in 6 zeroes. There are 5 numbers x = 25 to 29 which would all have 6 zeroes in their factorials.
Hence the answer is 5.

23. 4 V! + (V + 1)! + (V + 2)!
= V! + (V + 1) V! + (V + 2)(V + 1) V!
= V!(1 + V + 1 + V^2 + 3V + 2)
= (V^2 + 4V + 4)V! = $(V + 2)^2$V!
Now this will be a multiple of 36 for
V = 4, 6, 7, 8, 9, 10, 11, 12, 13 and 14.
$\therefore$ It is possible for 10 values of V

24. 4 P = 1 + 2.2! + 3.3! + … 10.10!
= (2 – 1)1! + (3 – 1)2! + (4 – 1)3! + … (11 – 1)10!
= 2! – 1! + 3! – 2! + … 11! – 10! = 1 + 11!
Hence the remainder is 1.

25. (0, 0) and (1, 1) satisfy the equation