Permutation and Combination - 2

Career Launcher

QA - 31

CEX-Q-0232/18

Contents

- Letters and Geometric Arrangements
- Grouping and Distribution

Number of Questions : 25

Letters and Geometric Arrangements

1. If the letters of the word SWARNALI are arranged in every possible order, for how many times will all the vowels appear together?
(1) 8! – 3! (2) 3 × 6!
(3) 6! (4) 3! × 6!

2. In how many arrangements of the word SLEEPLESS, will no two S appear together?
(1) 2100 (2) $^7C_3$
(3) $\frac{6!}{2!3!}$ (4) $\frac{9!}{2!3!3!}$

3. In how many ways can a maximum of two fruits can be chosen out of 3 apples, 2 oranges, 2 mangoes and 1 peach?
(1) 14 (2) 15
(3) 16 (4) 17

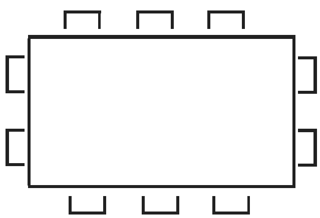
4. In which regular polygon, the number of diagonals is double the number of sides?
(1) Square (2) Pentagon
(3) Octagon (4) Heptagon

5. In how many ways can 4 boys and 4 girls be made to sit around a circular table if no two boys sit adjacent to each other?
(1) 8! (2) 36
(3) 7! (4) 144

6. 15 students are sitting on a circular chair. Every student sings a song with every other student except the adjacent ones. Each song is of two minutes. How long will this chorus be, if there is no any time gap between the songs of two pairs?
(1) 90 (2) 120
(3) 180 (4) 150

7. There are m horizontal parallel lines and n vertical parallel lines in a plane. How many quadrilaterals (rectangles and squares) can be drawn with the help of these lines?
(1) m^2n^2
(2) mn
(3) $\frac{mn(m-1)(n-1)}{4}$
(4) $\frac{mn}{4}$

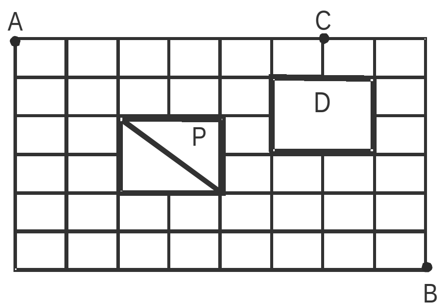
8. In how many ways can 10 persons be seated around a rectangular table with 10 chairs arranged as shown in the figure below?



(1) 5 × 9! (2) 9!
(3) 10! (4) 2 × 9!

9. In how many ways can a cube, placed inside a spherical room, be colored with six distinct colors such that each face of the cube is colored with one of the six paints?
(1) 120 (2) 30
(3) 6! (4) 1

Directions for questions 10 and 11: The figure below shows the plan of a town. The streets are at right angles to each other. A rectangular park (P) is situated inside the town with a diagonal road running through it. There is also a prohibited region (D) in the town.



10. Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is
(1) 60 (2) 75
(3) 45 (4) 90

11. Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path. Then the number of possible shortest paths that she can choose is
(1) 1170 (2) 630
(3) 792 (4) 1200

12. How many different heights can be achieved by arranging ten identical cuboids, having dimension 5cm × 3cm × 2cm, on the top of each other?
(1) 10! × 3 (2) 3^{10}
(3) 31 (4) 30

13. If all the words with 2 distinct vowels and 3 distinct consonants are listed alphabetically, what would be the rank of the word ACDEF?
(1) 4718 (2) 4717
(3) 2892 (4) 2893

14. There are n straight lines in a plane, no two of which are parallel and no three passes through the same point. If the points of intersections of these lines are joined, what would be the maximum number of quadrilaterals that can be formed?
(1) ${}^{28}C_4$ (2) ${}^{56}C_4$
(3) ${}^{24}C_4$ (4) None of these

15. The diagonals of a convex hexagon intersect at n distinct points inside the hexagon. What is the maximum value n can take?
(1) 8 (2) 12
(3) 15 (4) 18

Grouping and Distribution

16. Find the number of ways in which a pack of 52 playing cards can be divided equally among four persons sitting around a circular table.

(1) $\dfrac{52!\times 3!}{4!\times(13!)^4}$ (2) $\dfrac{52!}{4!\times(13!)^4}$

(3) $\dfrac{52!}{(13!)^4}$ (4) $\dfrac{52!\times 3!}{(13!)^4}$

17. In how many ways can a pack of 52 cards be divided into 4 sets, 3 of them having 16 cards each and the fourth one just 4 cards?

(1) $\dfrac{52!}{(16!)^3\,(3!)^2}$

(2) $\dfrac{1}{4}\left[\dfrac{52!}{(16!)^3\,(3!)^2}\right]$

(3) $\left[\dfrac{52!}{(16!)^3\,(4!)^2}\right]\dfrac{1}{4}$

(4) $\dfrac{1}{2}\left[\dfrac{52!}{(16!)^3(3!)^2}\right]$

18. In how many different ways can 3 distinct rings be worn in 5 fingers of a hand?

(1) $3! \times 5!$ (2) 5^3

(3) 3^5 (4) 210

19. What is the total number of ways of selecting 20 balls from an infinite number of Blue, Green and Yellow balls, if each ball of same colour are identical?

(1) 3^{20} (2) 20^3

(3) 231 (4) Cannot be determined

20. Find the total number of terms in the expansion of $(a + b + c + d + e)^{20}$.

(1) ${}^{24}P_4$ (2) 5^{20}

(3) ${}^{24}C_4$ (4) ${}^{25}C_4$

21. In how many ways can 8 identical pens be distributed among Amal, Bimal and Kamal so that Amal gets at least 1 pen, Bimal gets at least 2 pens, and Kamal gets at least 3 pens? **[CAT – 2017]**

22. A train is going from Delhi to Dhanbad, and there are 11 intermediate stations in the route. Find the number of ways in which the train can be made to stop at 4 intermediate stations such that no two stations are consecutive (Consider Delhi and Dhanbad as stations)?

(1) 15 (2) 14

(3) 30 (4) 28

Directions for questions 23 to 25: Anuj appeared for the CAT, a test to get into IIMs, having three papers viz. VARC, LRDI and QA, each of maximum 100 marks. It was found that his scores were integers in each of the three papers, then in how many ways

23. Can he score less than 60 marks?

(1) ${}^{62}C_2$ (2) ${}^{64}C_3$

(3) ${}^{63}C_3$ (4) None of these

24. He can score 240 marks?

25. He can score 140 marks?

(1) ${}^{140}C_2$ (2) ${}^{142}C_2$

(3) 7551 (4) Cannot be determined

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Answers and Explanations

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1	2	**2**	1	**3**	1	**4**	4	**5**	4	**6**	3	**7**	3	**8**	1	**9**	2	**10**	4
11	1	**12**	4	**13**	1	**14**	1	**15**	3	**16**	3	**17**	2	**18**	4	**19**	3	**20**	3
21	6	**22**	1	**23**	3	**24**	1	**25**	3										

1. 2 Since we have to keep the vowels together, we will make all A's and I together in the form of a group.

AAI, W, R, N, L, S

These can be arranged in 6! ways.
Further, AAI can be arranged in 3 ways
So, total number of possible arrangements = 3 × 6!

2. 1 Since we have to keep no two S together, we will first fix other letters as
_L _ E_ E_ P_L_E
There are 7 places in which S can be placed in $^7C_3$ ways. Also, the letters LEEPLE can be arranged in

$\frac{6!}{3!2!}$ ways.

Thus total number of ways = $^7C_3 \times \frac{6!}{3!2!}$

3. 1 There are three cases:
(1) No fruit is selected: Only one way.
(2) One fruit is selected: Four ways.
(3) Two fruits are selected:
(a) Both fruits are identical: Three ways.
(b) Both fruits are distinct: $^4C_2$ = 6 ways.
Hence, required number of ways = 14.

4. 4 Number of diagonals in any polygon = $^nC_2 - n$

$\therefore {}^nC_2 - n = 2n \Rightarrow {}^nC_2 = 3n$

$\Rightarrow \frac{n(n-1)}{2} = 3n$

$\Rightarrow n = 7$
Hence, the polygon is a heptagon.

5. 4 We will first let all the boys occupy 4 seats around the circular table. This can be done in 3! ways.
Now, 4 girls can take the 4 seats between the boys. This can be done in 4! ways.
Thus, total number of ways in which they can be seated = 3! × 4! = 6 × 24 = 144.

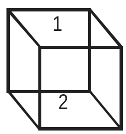
6. 3 Since, we don't have to consider the adjacent boys, each student will sing a song with 12 more student. i.e. we need to find the number of diagonals in a polygon with 15 sides.
So, $^{15}C_2 - 15$ = 90 distinct pairs.
Thus, the chorus will last for 90 × 2 = 180 minutes.

7. 3 To form a quadrilateral, we need to select 2 vertical lines and 2 horizontal lines.
This can be done in $^nC_2 \times {}^mC_2$

$= \frac{mn(m-1)(n-1)}{4}$ ways.

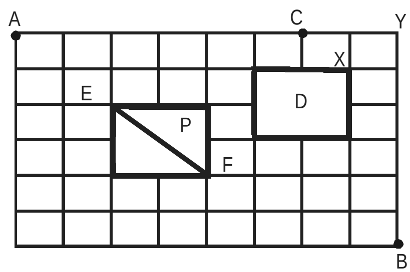
8. 1 There would be 5 different positions for the first person who is going to sit.
Also, the rest 9 people can be seated in 9! ways.
Thus, total number of ways = 5 × 9! ways.

9. 2



We will try to break the symmetry. So, the face 1 can be painted in 1 way, and then the facing face (face 2) can be painted in 5 ways.
Now, we will get a circular arrangement with 4 places, i.e. the remaining faces can be painted in 3! ways.
So, total number of ways = 5 × 3! = 30 ways.

10. 4



For the shortest route, Neelam follows the following path:
$A \rightarrow E \rightarrow F \rightarrow B$

Number of ways to reach from A to E: $\frac{(2+2)!}{2! \times 2!} = 6$

Number of ways to reach from E to F: 1

Number of ways to reach from F to B: $\frac{(4+2)!}{4! \times 2!} = 15$

$\Rightarrow$ Total number of possible shortest paths
= 6 × 1 × 15 = 90

11. 1 Neelam has to reach C via B.
From A to B, the number of paths are 90, as found in the previous question.
From B to C, Neelam follows the route:

Case I: $B \to X \to C$

OR **Case II:** $B \to Y \to C$.

Case I: $B \to X \to C$

Number of ways to reach from B to X: $\frac{(5+1)!}{5! \times 1!} = 6$

Number of ways to reach from X to C : 2
So, total number of paths are 6 × 2 = 12 ways.

Case II: $B \to Y \to C$**:**
There is just one way.
Therefore, from B to C, there are 6 × 2 + 1 = 13 ways
$\therefore$ Total number of ways of reaching from A to C, via B = 90 × 13 = 1170.

12. 4 The minimum height that can be achieved is 20 cm.
The, maximum height that can be achieved is 50 cm.
But, height equals to 49 cm cannot be achieved.
Thus, total 30 distinct heights can be achieved.

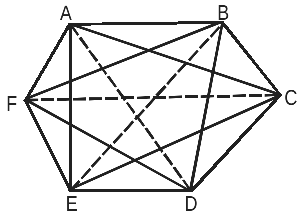
13. 1 The first word with 2 distinct consonants and 3 distinct vowels would be ABCDE.
Now, if we fix AB in the first two positions, there must be 1 vowel and 2 consonants more at the next 3 places, which can be done in $^{20}C_2 \times {^4C_1} \times 3!$ ways.
i.e. the number of words starting with AB = $^{20}C_2 \times {^4C_1} \times 3!$ = 190 × 4 × 6 = 4560.
Next, we move on to words starting with ACB, in the form of ACB_ _ would be $^{19}C_1 \times {^4C_1} \times 2!$ = 152 ways.
Now, the words starting with ACDB, there would be 4 distinct words on this list- ACDBE, ACDBI, ACDBO, ACDBU
Further, the next words would be ACDEB and ACDEF
Thus, rank of ACDEF = 4560 + 152 + 4 + 2 = 4718.

14. 1 Since we have to maximize the number of quadrilaterals, let 1 line is getting intersected by rest 7 lines.
There would be 7 point of intersections on a line.
Thus, total points of intersection on 8 lines = 8 × 7 = 56.
But in this case, all the points are getting counted twice. Thus, total points of intersection = $\frac{56}{2}$ = 28

As we have to form a quadrilateral, we will select 4 points out of 28 points, which can be done in $^{28}C_4$ ways.

15. 3 In any hexagon, there are $n\left(\frac{n-3}{2}\right) = 9$ diagonals.

First let us draw the diagonals and try to visualise this diagram



There are six 'short' diagonals AC, AE, CE, BD, BF, and DF. These intersect with other diagonals at 3 points each.
There are 3 long diagonals – AD, BE and CF. These intersect with other diagonals at 4 points each.
Note that the 'short' diagonals need not be shorter than the 'long' diagonal.
So, the total number of points of intersection should be 6 × 3 + 3 × 4 = 30. But in this case, we would count every point of intersection twice. So, number of points of intersection would be exactly half of this $= \frac{30}{2}$ = 15 points.

16. 3 The number of ways in which a pack of 52 cards can be divided into four persons = $\frac{52!}{(13!)^4}$

4 person sitting around a circular table will have no bearing on the total number of distribution of cards.

17. 2 First we divide 52 cards into 2 groups — 4 cards and 48, i.e. (16 × 3) cards.

This can be done in $\frac{52!}{48!\,4!}$ ways.

Now, a group of 48 cards can be divided into 3 groups of 16 cards each in $\frac{48!}{(16!)^3\,3!}$ ways.

Hence, the required number of ways

$$= \frac{52!}{48!\,4!} \times \frac{48!}{(16!)^3\,3!} = \frac{52!}{(16!)^3\,(4!)(3!)} = \frac{52!}{(16!)^3\,(3!)^2\,4}$$

$$= \frac{1}{4}\left[\frac{52!}{(16!)^3\,(3!)^2}\right]$$

18. 4 Here we have three different cases.

1. All rings are worn in just one finger, this is possible in 5 × 3! = 30 ways.
2. All rings are worn in 2 of the 5 fingers. This is possible in $^5C_2 \times {^2C_1} \times {^3C_2} \times 2 = 120$ ways.
3. All rings are worn in 3 of 5 fingers. This is possible in $^5P_3 = 60$ ways.

∴ Total number of ways = 30 + 120 + 60 = 210.

Alternate method:
If we consider the three rings to be identical and five fingers are F_1, F_2, F_3, F_4 and F_5 then the question gets converted to $F_1 + F_2 + F_3 + F_4 + F_5 = 3$
∴Total number of non-negative solutions = $^7C_3$ = 35
As rings are distinct, required answer = 35 × 3! = 210.
Or,
The first ring can be worn in 5 ways, then there will be 6 more places for next ring. After placing 2nd ring there would be 7 more distinct places for the 3rd ring. Hence, total number of ways = 5 × 6 × 7 = 210

19. 3 Let the number of Blue, Green and Yellow balls be x, y and z respectively.
Then x + y + z = 20, where x, y and z are whole numbers.
The solution of this equation is given by $^{20+3-1}C_2$ = $^{22}C_2$ = 231 ways.

20. 3 Any term of the expansion is given by $a^p b^q c^r d^s e^t$ such that p + q + r + s + t = 20, where p, q, r, s and t are whole numbers.
The solution of this equation is given by $^{20+5-1}C_{5-1}$ = $^{24}C_4$
Thus, there would be $^{24}C_4$ terms.

21. 6 Let Amal, Bimal and Kamal are getting A, B and K number of pens.
So, A + B + K = 8
But $A \geq 1, B \geq 2$ and $k \geq 3$
$\Rightarrow A + B + K = 2$
Where A, B and K are whole numbers.
The number of solution of this equation is $^4C_2$ = 6

22. 1 Delhi $\underbrace{(1)\ (2)\ (3)\ (4) \ldots\ldots\ldots (11)}_{\text{Intermediate Stations}}$ Dhanbad

Firstly, we will select 4 intermediate stations which are not consecutive.
Let us select stations (3), (5), (8) and (10)

D $\underbrace{(1)\quad(2)}_{a}$ (3) $\underbrace{(4)}_{b}$ (5) $\underbrace{(6)\quad(7)}_{c}$ (8) $\underbrace{(9)}_{d}$ (10) $\underbrace{(11)}_{e}$ G

The selection of 4 intermediate station divides the remaining station into five groups a, b, c, d and e such that a + b + c + d + e = 11 – 4 = 7
a + b + c + d + e = 7
Since there must be at least 1 station between any two stoppages,
Number of positive integral solutions of above equation = $^6C_4$ = 15 ways.

23. 3 Let a, b and c represents VARC, LRDI and QA respectively.
Here, a + b + c ≤ 60
where, a, b and c are whole numbers.
We introduce z such that
⇒ a + b + c + z = 60, where z is a whole number.
The number of solution of this is given by $^{63}C_3$.

24. 1 In this case, scoring 240 marks is as good as not scoring 60 marks.
Thus, a + b + c = 60
The number of solution for this equation = $^{62}C_2$.

25. 3 Here, a + b + c = 140 ...(i)
But 0 ≤ a, b, c ≤ 100
So, we will consider all the cases first and then subtract those cases which violates the conditions.
Total number of solutions of equation (i) = $^{142}C_2$
But this also includes those cases in which a, b or c is more than 100.
So, we will assign 101 marks to a such that it could violate the condition.
⇒ 101 + a + b + c = 140
⇒ a + b + c = 39
⇒ $^{41}C_2$ solutions.
To violete the conditions, same can be done with b and c as well.
So, total number of ways in which condition can be violated = 3 × $^{41}C_2$
Thus, the number of ways in which he can score 140 marks = $^{142}C_2$ – 3 × $^{41}C_2$ = 7551 ways.