Numbers - 1

Career Launcher

QA - 11

CEX-Q-0212/18

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Number of Questions : 30

Classification of Numbers

1. If n^3 is odd for $n > 1$, then which of the following is true?
   I. n^4 is even.
   II. n^2 is even.
   III. $(n – 1)(n + 1)$ is even.
   IV. n is odd.

   (1) Only I (2) Only IV
   (3) III and IV (4) Only III

2. If $n = 1 + x$, where x is the product of four consecutive positive integers, then which of the following is/are true? **[CAT 1999]**
   A. n is odd
   B. n is prime
   C. n is a perfect square

   (1) A and C only (2) A and B only
   (3) A only (4) None of these

3. Let x, y and z be distinct integers, that are odd and positive. Which one of the following statements cannot be true? **[CAT 2000]**
   (1) xyz^2 is odd
   (2) $(x – y)^2 – z$ is odd
   (3) $(x + y – z)^2 (x + y)$ is even
   (4) $(x – y)(y + z)(x + y – z)$ is odd

4. If x and y are integers such that $x > y > 0$, how many integers are there between x and y, excluding x and y?
   (1) $x – y$ (2) $x – y – 1$
   (3) $x + y$ (4) $x + y – 1$

5. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?

6. Consider four-digit numbers for which the first two digits are equal and the last two digits are also equal. How many such numbers are perfect squares?
   (1) 3 (2) 2
   (3) 0 (4) 1

7. Prashant typed a 4-digit number on his calculator. He multiplied it by 12. He added 39 to it. He subtracted 36 dozens from that number and he multiplied the resultant number with 24. The number that he got was 25x2536. He cannot see digit x because the calculator was malfunctioning. What could be digit x from amongst the choices given?
   (1) 2 (2) 4
   (3) 6 (4) 5

8. Let x, y and z be distinct positive integers satisfying x < y < z and x + y + z = k. What is the smallest value of k that does not determine x, y and z uniquely?
(1) 9 (2) 7
(3) 10 (4) 8

9. If cb × c × 3 = ccc, then what is the value of b × c?

10. Two prime numbers P_1, P_2 ($P_1 < P_2$) are called twin primes if they differ by 2. (e.g. 11, 13 or 41, 43 etc.) If P_1 and P_2 are twin primes with $P_2 > 23$ then which of the following numbers would always divide $P_1 + P_2$?
(1) 12 (2) 8
(3) 24 (4) 16

11. The product of two positive real numbers is 1. Their sum is
(1) ≥ 1 (2) ≥ 2
(3) ≤ –1 (4) ≤ –2

12. All the two-digit natural numbers that have their unit's digit greater than their ten's digit are selected. If all these numbers are written one after the other in a series, how many digits are there in the resulting number?

13. The sequence 2, 3, 5, 6, 7, 10, … consists of all natural numbers that are neither perfect squares nor perfect cubes. Find the 76th term of this sequence.
(1) 89 (2) 87
(3) 86 (4) 88

14. P denotes the sum of two three-digit numbers such that P is 606. How many of the 10 digits from 0 to 9 cannot appear as the last digit of the product of these two three-digit numbers?
(1) 4 (2) 3
(3) 5 (4) 2

15. When a natural number "p" is multiplied by 4, it gives a perfect cube and when it is multiplied by 9, it gives a perfect square. Find the minimum possible value of the expression $p^2 - 16p + 1$.
(1) 8 (2) 12
(3) 16 (4) None of these.

16. A three digit number is such that its hundredth digit is equal to the product of the other 2 digits which are prime. Also, the absolute difference between the number and the number formed by its digits in reverse order is 297. Then tens digit of the number is
(1) 2 (2) 3
(3) 7 (4) 4

17. When a certain two-digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two-digit numbers are there?
(1) 4 (2) 6
(3) 8 (4) 10

18. A and B play a game of 'Dingo' in which each player in his turn has to choose a positive integer that is less than the previous number but at least half the previous number. The player who chooses 1, loses and the game ends there. A starts by choosing 2021 and after that both the players (A & B) continue to play with the best strategy. Which integer should B choose immediately after A has chosen 2021, to ensure his own victory?
(1) 1025 (2) 011
(3) 2049 (4) 1535

19. Each of the 20 persons seating in a row is assigned a number from 1 to 20 corresponding to his seating position counting from left to right. Every time, a count is made on the seats, every person seating on a seat corresponding to a prime number is removed and the seat numbers are rearranged beginning with one. This procedure is repeated until only 3 persons are left. What is the original seat number of the person seating on the seat number 3?
(1) 8 (2) 14
(3) 16 (4) 18

20. When the digits of a three-digit number N are written in the reverse order, we get a three-digit number M. Which of the following statements is/are correct about the number S, which results from the addition of numbers N and M?

A. At most, only two of the three digits of number S can be even.
B. Number S cannot be a prime number.
C. Each of the three digits in number S, is greater than 2.

(1) A & B (2) B & C
(3) A & C (4) None of these

21. If u, v, w and m are natural numbers such that $u^m + v^m = w^m$, then which one of the following is true? **[CAT-2002]**

(1) $m \geq \min(u, v, w)$
(2) $m \geq \max(u, v, w)$
(3) $m < \min(u, v, w)$
(4) None of these

22. The product of three positive integers is 6 times their sum. One of these integers is the sum of the other two integers. If the product of these three numbers is denoted by P, then find the sum of all distinct possible values of P.

23. Let D be a recurring decimal of the form D = 0. a1 a2 a3 a1 a2 a3 a1 a2 a3 …, where digits a1 and a2 lie between 0 and 9. Further, at most one of them is zero. Which of the following numbers necessarily produces an integer, when multiplied by D?

(1) 1800 (2) 1080
(3) 1998 (4) 2898

Surds & Indices

24. Which among $2^{1/2}$, $3^{1/3}$, $4^{1/4}$, $6^{1/6}$ and $12^{1/12}$ is the largest?

(1) $2^{1/2}$ (2) $3^{1/3}$
(3) $4^{1/4}$ (4) $6^{1/6}$

25. Find the value of $\sqrt{8+2\sqrt{7}}$.

26. $\dfrac{3^{x^2} \times 3^{-xy}}{3^{-y^3}} = 81$ and $\dfrac{2^{x^3}}{2^{-y^3}} = 256$. Find the value of (xy)?

(1) 0 (2) – 2
(3) 4 (4) None of these

27. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?

(1) $1 \leq m \leq 3$ (2) $4 \leq m \leq 6$
(3) $7 \leq m \leq 9$ (4) $10 \leq m \leq 12$

28. How many integral values of 'x' are possible for which ($2^{70} + 2^{1039} + 2^x$) equals the square of a whole number?

(1) 0 (2) 1
(3) 2 (4) 3

29. Find the value of $\dfrac{3^{n+2} + 3^{n-1}}{3^{n+5} + 3^{n+2}}$.

(1) $\dfrac{1}{3}$ (2) $\dfrac{1}{27}$

(3) $\dfrac{(n-1)}{(n+1)}$ (4) $\dfrac{1}{9}$

30. Suppose n is an integer such that the sum of digits on n is 2, and $10^{10} < n < 10^{11}$. The number of different values of n is

Space for Rough Work

QA - 11 : Numbers - 1
Answers and Explanations

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1	3	**2**	1	**3**	4	**4**	2	**5**	–	**6**	4	**7**	2	**8**	4	**9**	–	**10**	1
11	2	**12**	–	**13**	2	**14**	1	**15**	4	**16**	1	**17**	3	**18**	4	**19**	3	**20**	4
21	4	**22**	–	**23**	3	**24**	2	**25**	–	**26**	1	**27**	1	**28**	2	**29**	2	**30**	–

1. 3 Since $n^3 = n \times n \times n$ is odd, it implies that n is odd. This is because any number when multiplied by an even number gives even number while product of two odd numbers is odd. By the same logic, both n^2 and n^4 are odd. Now $(n – 1)(n + 1) = n^2 – 1$ is even since the difference of two odd numbers is even. Thus, both III and IV are true. Hence, (3) is the most appropriate answer.

2. 1 Let the four consecutive positive integers be x – 1, x, x + 1 and x + 2.
$x(x + 1)(x – 1)(x + 2) + 1$ (odd)
$= (x^3 – x)(x + 2) + 1$
$= x^4 + 2x^3 – x^2 – 2x + 1$
$= (x^2 + x – 1)^2$.
Hence, statements A and C are true.

3. 4 Take any three odd positive numbers and veryfy the options.

4. 2 The number of integers between any two integers is always one less than the obsolute difference between the integers.
E.g. 3 > 1 > 0. Where 3 = x, 1 = y.
There is one number between 3 and 1 (excluding 3 and 1)
i.e. 3 – 1 – 1 = 1
i.e. x – y – 1.

5. If XY is the number, the value of the number is 10X + Y. The value of the number formed by reversing the digit is 10Y + X.
The difference in the value is 9 (Y – X) which is equal to 18.
So Y – X = 2. There are 7 possible pairs of (X, Y) … (3, 1) to (9, 7). So, apart from 13, there are 6 possibilities.

6. 4 Let the four-digit number be denoted by aabb
= 11 × (100a + b)
Now since aabb is a perfect square 100a + b should be a multiple of 11.
The only pair of values of a and b that satisfy the above mentioned condition is a = 7 and b = 4. Now 7744 is a perfect square.

7. 2 Let the number be N, then we get
25x2536 = [N × (3×4) + 3×13 – 3×4(12×3)] × (3×8)
= 72[4N –131].
So, the number has to be multiple of 9. Among the choices only (2) is satisfies the condition.

8. 4 Since x < y < z, then x + y + z = k.
For values of k = 6 or k = 7, there are unique values of (x, y, z)which are (1, 2, 3) and (1, 2, 4).
For k = 8, (x, y, z) can be either (1, 2, 5) or (1, 3, 4).

9. $cb \times 3 = \frac{ccc}{c} = 111 = 37 \times 3$
Hence cb = 37
$\therefore$ c × b = 21

10. 1 If P_1 and P_2 are twin-primes then P_1 will be of the form 6K – 1, where K is a natural number.
$P_1 = 6K - 1$ and $P_2 = 6K + 1$.
$P_1 + P_2 = 6K - 1 + 6K + 1 = 12K$
$\therefore$ 12 would always divide $P_1 + P_2$.

11. 2 Let the two numbers be x and y.
It is given that x × y = 1.
We know that AM ≥ GM
$$\Rightarrow \frac{x+y}{2} \geq \sqrt{xy} \Rightarrow x + y \geq 2 \times \sqrt{1} \Rightarrow x + y \geq 2.$$

12. In such numbers we cannot have 0 or 1 at unit’s place.
When we have 2 in unit's place, we have one such number i.e., 12.
When we have 3 in unit’s place, we have two such number 13 and 23.
Similarly we can say that When we have 9 in unit’s place, we have eight such numbers. So number of such numbers is
(1 + 2 + ... + 8) = 36
Hence the resulting number has 72 digits

13. 2 76th natural number = 76
We have 7 perfect squares and 3 perfect cubes from 2 to 75 in which 64 occur twice (because of being both a perfect square and a perfect cube)
Hence, 9 numbers must have been removed.
The number 76, if we start a series of natural numbers from 2, will be the 75th number.
If we do not include the above 9 numbers in this series then 76 becomes the 75 – 9 = 66th number.
Subsequently 80 will be the 70th number.
But 81 being a perfect square cannot be included.
Hence, 82 will be the 71st number and subsequently the 76th number will be 87.

14. 1 The last digits of the 2 three digit numbers can be (0, 6), (1, 5), (2, 4), (3, 3), (8, 8) an (7, 9)
$\therefore$ The last digit of product of these numbers can be 0, 5, 8, 9, 4 and 3.
$\therefore$ Four digits i.e. 7, 6, 2 and 1 cannot be the last digit of the product of these numbers.

15. 4 $9p = 3^2p$ is a perfect square.
As 3 is a prime number p must also be a perfect square. Let $p = k^2$
Now, 4p is a perfect cube and $4p = (2k)^2$
Minimum possible value of $p = 4(2^2) = 16$
The expression $p^2 - 16p + 1$ will have minimum possible value at p = 16 only as any other value of p is much greater than 16 and correspondingly the expression will carry a higher value.
$p^2 - 16p + 1$ for p = 16 is equal to $(16)^2 - 16(16) + 1 = 1$.

16. 1 The 2 digits at unit's & tens place can be only 2 or 3.
$\therefore$ The hundredth digit must be 6. So the number can be 623 or 632. On reversing the digits, the numbers become 326 or 236. But the difference between 623 & 326 is 297. Therefore, the tens digit of the number will be 2.

17. 3 Let the two digit number be ab.
Value of ab = 10a + b
We have,
10a + b + 10b + a = 11(a + b) is a perfect square.
$\because$ 11(a + b) is a perfect square.
$\therefore$ (a + b) should also be a multiple of 11 but a + b can not be equal to 22 or more as a and b cannot be more than 9.
$\therefore$ a + b = 11
Now we have these 8 solutions.
(2, 9), (3, 8), … …, (9, 2)

18. 4 It is obvious that one who chooses the integer '2' wins.
So winning sequence is
2, (2 × 2 + 1), 2(2 × 2 + 1)+1….
or, 2, 5, 11, 23, 47, 95, 191, 383, 767, 1535
$\therefore$ B should choose 1535 to ensure his victory.

19. 3

Initial count	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Original seat No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

After count 1	1	2	3	4	5	6	7	8	9	10	11	12
Original seat No.	1	4	6	8	9	10	12	14	15	16	18	20

After count 2	1	2	3	4	5	6	7
Original seat No.	1	8	10	14	15	16	20

After count 3	1	2	3
Original seat No.	1	14	16

Original seat no of the person sitting in the 3rd seat at the end is 16.

20. 4 Let N = abc & M = cba.

$$\begin{array}{ccc|c|c} N & = & a & b & c \\ M & = & c & b & a \\ \hline S & = & a+c & 2b & a+c \end{array}$$

For S to be a 3-digit number, a + c must be less than or equal to 9. As N and M, both are 3-digit numbers, neither a nor c can be zero.
So, the minimum value of (a + c) is 2. The ten's digit of the number S is 2 × b. So, it can be 0, 2, 4, 6 or 8. The prime number 383 qualifies to be S hence, S can be prime also.
Option (4) as correct.

21. 4 $u^m + v^m = w^m$
$u^2 + v^2 = w^2$
Taking Pythagorean triplet 3, 4 and 5, we see
m < min(u, v, w)
Also 1' + 2' = 3' and hence m $\leq$ min(u, v, w)

22. Let the numbers be K, L, K + L,
$\Rightarrow$ KL(K + L) = 12(K + L)
$\Rightarrow$ KL = 12.
$\Rightarrow$ {K, L} = {1, 12}, {2, 6} or {3, 4}
Hence, product P = 156, 96 or 84
$\therefore$ Required sum = 156 + 96 + 84 = 336.

23. 3 $999 \times D = a1a2a3.$

$$\Rightarrow D = \frac{a1a2a3}{999}$$

So D must be multiplied by 1998 as 1998 is a multiple of 999.

24. 2 LCM of 2, 3, 4, 6, 12 = 12
$\sqrt[12]{2^6}$ $\sqrt[12]{3^4}$ $\sqrt[12]{4^3}$ $\sqrt[12]{6^2}$ $\sqrt[12]{12^1}$
$\therefore 3^4$ is greatest.

25. Let $8 + 2\sqrt{7} = \left(a + \sqrt{b}\right)^2$ or $a^2 + b + 2a\sqrt{b} = 8 + 2\sqrt{7}$

$a^2 + b = 8$ and $a\sqrt{b} = \sqrt{7}$. a = 1 and b = 7 satisfies it.

Hence, answer is $1 + \sqrt{7}$.

26. 1 $\dfrac{2^{x^3}}{2^{-y^3}} = 256$ or $2^{x^3+y^3} = 256$. Hence $x^3 + y^3 = 8$.

$\dfrac{a^{x^2} \times a^{-xy}}{a^{-y^2}} = 81$ or $3^{x^2+y^2+xy} = 81$ or $x^2 + y^2 + xy = 4$.

x = 2 and y = 0 satisfies this. Hence xy = 0

27. 1 Let the three consecutive positive integers be equal to 'n – 1', 'n' and 'n + 1'.

$\Rightarrow n - 1 + n^2 + (n+1)^3 = (3n)^2$

$\Rightarrow n^3 + 4n^2 + 4n = 9n^2$

$\Rightarrow n^2 - 5n + 4 = 0$

$\therefore n = 1$ or $n = 4$

Since the three integers are positive, the value of 'n' cannot be equal to 1, therefore the value of 'n' = 4 or m = n – 1 = 3.

Hence, the three consecutive positive integers are 3, 4 and 5.

Hence, option (1) is the correct choice.

28. 2 Here $2^{70} + 2^{1039} + 2^{x}$ or $2^{70}(1 + 2^{969} + 2^{x-70})$

But 2^{70} is the square of a number

Then $1 + 2^{969} + 2^{x-70}$ has to be the square of an odd number.

So $1 + 2^{969} + 2^{x-70} = (2n + 1)^2$

$\Rightarrow 2^{969} + 2^{x-70} = 4n^2 + 4n$

or $2^{967} + 2^{x-72} = n(n + 1)$

So it is the product of two consecutive numbers.

Case I: $2^{967}(1 + 2^{x-1039}) = n(n + 1)$

Now $2^{967} = 2^{x-1039}$

$\Rightarrow x = 2006$

Case II: $2^{x-72}(1 + 2^{1039-x}) = n(n + 1)$

Now $2^{x-72} = 2^{1039-x}$

$\Rightarrow 2x = 1039 + 72$

$\Rightarrow x \neq$ integral value.

Hence, x has only one value.

29. 2 $\dfrac{3^{n+2} + 3^{n-1}}{3^{n+5} + 3^{n+2}}$

$= \dfrac{3^{n-1}\left[3^3 + 1\right]}{3^{n+2}\left[3^3 + 1\right]}$

$= \dfrac{\frac{3^n}{3}}{3^n.3^2} = \dfrac{1}{3.3^2} = \dfrac{1}{27}$

30. We have

(1) $10^{10} < n < 10^{11}$

(2) Sum of the digits for 'n' = 2

Clearly-

(n)min = 10000000001 (1 followed by 9 zeros and finally 1)

Obviously, we can form 10 such numbers by shifting '1' by one place from right to left again and again.

Again, there is another possibility for 'n'

n = 20000000000

So finally : No. of different values for n = 10 + 1 = 11 ans.