Plane Geometry - Basics & Triangles
Polygons and Quadrilaterals
Circle
Mensuration
Trigonometry
Practice Exercises
Answer Key
Explanations

Plane Geometry - Basics & Triangles

Learning Objectives

At the end of this chapter, you should be comfortable with the following:

• Basics of plane geometry

• Angles

• Triangles

Plane Geometry - Basics

Let us start by revising a few basic geometrical figures that all of us would already be knowing. We would start with plane geometry and towards the end, include 3 dimensional figures. Plane geometry is about figures in a plane and a plane is any 2 dimensional surface such as a table top. Most basic figures that we are acquainted with such as triangle, square, rectangle, circle etc. can be drawn on any plane surface and there is no need for a third dimension i.e. a height or thickness.

Lines

Line is the most elementary figure in geometry, it is a one dimensional figure which only has length. It does not has any height thickness or width. A line consists of infinite number of points.

Types of lines :

Parallel Lines : If the distance between two lines (Like L_3 and L_4) in a plane is constant they are said to be parallel. "Distance" between lines is always equal to perpendicular distance.

Intersecting Lines : Lines which share a common point (line L_1 and L_2 sharing a common point X).

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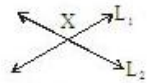
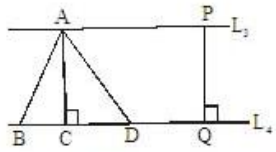


Intersecting lines

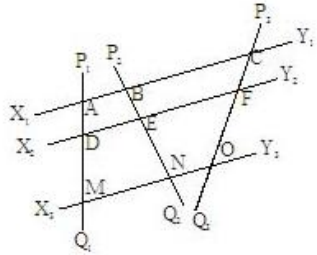
Transversal Lines: A line which intersect two or more parallel lines in distinct points(AD, AC, PQ and AB are the transversals).



Parallel lines

Proportionality Theorem:

This is a very important concept and is used extensively in similarity of triangles. It says, the intercepts made by two (or more) transversals on three (or more) parallel lines are proportional.

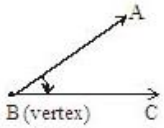


In other words, $\frac{AD}{DM} = \frac{BE}{EN} = \frac{CF}{FO}$

Note : Here X_1Y_1, X_2Y_2 and X_3Y_3 are three parallel lines and P_1Q_1, P_2Q_2 and P_3Q_3 are the transversal lines.

Angles

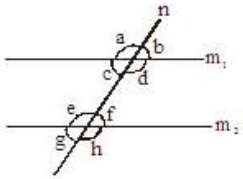
In geometry, angles are of utmost importance . When two rays have the same starting or end-point, they form an angle and the common end point is called VERTEX.



In the figure, rays BA and BC formed the angle ∠ABC.

Relation between the angles

When two parallel lines (m_1, m_2) are cut by another line, n (called TRANSVERSAL), the angles formed, share various relations among them.



I. Corresponding Angles are equal.

a = e, c = g, b = f and d = h

II. Alternate angles are equal.

c = f, d = e

III. Opposite Interior angles are supplementry (i.e. sum of interior angles is 180°).

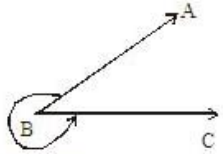
e + c = 180°, d + f = 180°

IV. Vertically Opposite angles are equal.

If a = d, e = h, c = b, g = f

Reflex Angle:

A reflex angle is an angle which is more than 180 degrees in measurement.



Reflex Angle

Reflex angle ∠ABC = 360° - ∠ABC

(∠ABC is less than 180°)

Note : Do not confuse it with obtuse angle, obtuse angle is in **between** 90° and 180°, whereas reflex is **more** than 180°.

Example 1:

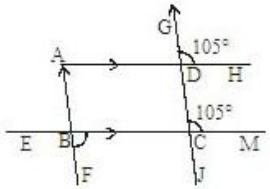
In the given figure, AF is parallel to GJ and AH is parallel to CE. If ∠GDH = 105°, then ∠FBC =?

Solution:

∠GDH = ∠DCM = 105° (corresponding angles)

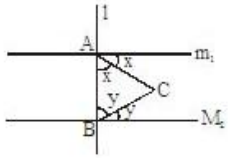
⇒ ∠BCD = 180° - ∠DCM = 180° - 105° = 75°

$\Rightarrow \angle FBC = \angle BCD = 75°$ (Alternate angles)



Example 2:

In the given figure, m_1 is parallel to m_2, AC and BC are angle bisectors. Find the measure of $\angle ACB$.



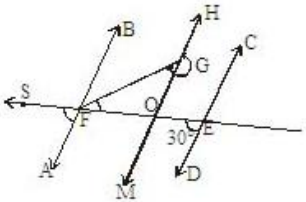
Solution:

$2x = 180° - 2y$ (interior angles) $\Rightarrow 2x + 2y = 180°$ or $x + y = 90°$

In ΔABC $\angle ACB + x + y = 180°$

$\Rightarrow \angle ACB = 180° - (x + y) = 90°$

Example 3:



In the figure above, AB, CD and MH are parallel to each other and FG is the angle bisector of $\angle BFO$. If $\angle FED = 30°$, then reflex $\angle FGH$ =?

Solution:

$\angle FED = \angle BFE$ (Alternate angles)

$\Rightarrow \angle GFE = \frac{1}{2} \angle BFE = 15°$

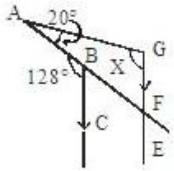
But $\angle BFG = \angle GFO$ (Alternate angles)

$\angle GFE = \angle BFG = \angle FGO = 15°$

$\Rightarrow$ reflex $\angle FGH = 180° + \angle FGO = 195°$

Example 4:

Find the value of X in the figure given below.



Solution:

Assume $\angle AFG = a$, $\angle ABC = \angle BFE = 128°$

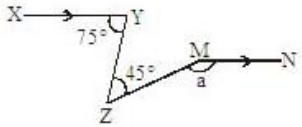
(corresponding angles)

$a = 180° - 128° = 52°$.

In ΔAGF, $a + 20° + X = 180° \Rightarrow X = 108°$

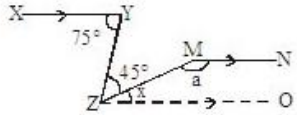
Example 5:

An ant moves in a straight line. After two turns of 75° and 45° as shown in figure, it moves parallel to the initial direction of motion. What is the value of ∠a ?



Solution:

Through Z, construct a line ZO parallel to MN



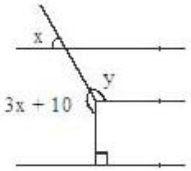
x + 45° = 75° (Alternate angles) ⇒ x = 30°

x + a = 180° (Opposite interior angles)

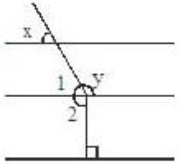
a = 180° - 30° ⇒ a = 150°

Example 6:

Find the values of x, y in the figure given below.



Solution:



∠2 = 90° (Alternate angles),

∠1 = x (Corresponding angles)

∠1 + ∠2 = 3x + 10

x + 90° = 3x + 10

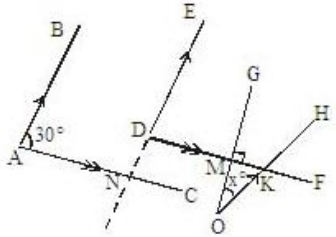
2x = 80° or x = 40°

y = 180° - ∠1 (Adjacent angles)

y = 180° - 40° = 140°

Example 7:

Find the measure of angle x in the figure, where



AB || DE || OH and AC || DF.

Solution:

Let ED be extended and cut AC at N.

∠EDF = ∠DNC = ∠BAC = 30° (corresponding angles)

Again DE || OH.

∴ ∠EDF = ∠DKO = 30° (alternate angles)

Also ∠KMO = 90°

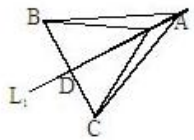
In ΔMKO ∴ x = 180° - 90° - 30° = 60°

Specific Lines and Points in a Triangle:

Perpendicular Bisector of a Line:

Consider the line segment BC. D is the mid-point and L_1 is the perpendicular bisector of BC.

Consider any point A on the perpendicular bisector. The distance of point A from the endpoints of the line BC is AB and AC.



In ΔADB and ΔADC, BD = CD (D is midpoint of BC)

∠ADB = ∠ADC (right angles)

AD = AD (common side)

Thus by S - A - S, the two triangles are

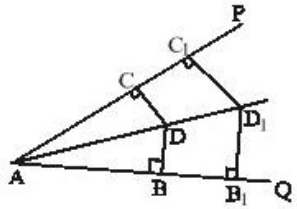
congruent.

Hence AB = AC i.e. the point A is equidistant from the endpoint of BC. This is true for any point A on the perpendicular bisector of the line. Thus,

All points on the perpendicular bisector of a line is equidistant from the endpoints of the line

Angle Bisector:

Consider the angle bisector AD of ∠PAQ as shown in the figure below. Taking any point, say D on the angle bisector. The distance of D from the sides AC and AB are the perpendicular lengths DB and DC.



In ΔADB and Δ ADC,

∠BAD = ∠CAD (angle bisector)

∠ABD = ∠ACD (right angles)

AD = AD (common side)

Thus by A - A - S, the two triangles are congruent.

Thus DB = DC i.e. the point D is equidistant from the sides. This is same for any point on the angle bisector.

Thus,

All points on the angle bisector are equidistant from the sides forming the angle.

Triangles

The next step in the development of our geometric knowledge is to get acquainted with one of the simplest plane figures, with **a triangle**.

Three lines in a plane, of which no two are parallel, either have to intersect at the same point, as shown in figure (i) or would enclose a triangle as shown in figure(ii).

Lines intersecting at the same points are called concurrent lines, [figure (i)].

The closed plane in figure - (ii), formed by the intersection of three lines, no two of which are parallel is a Triangle.

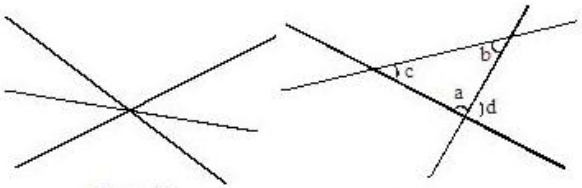


Figure (i) Figure (ii)

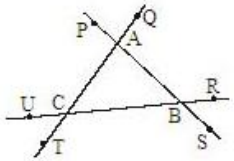
In other words, A **triangle** is a figure that consists of three points that do not lie on the same line and three segments that connect each pair of points. The points are called *vertices* and segments are called *sides* of a triangle.

The following would surely be known by everyone and hence one can go through it rapidly. All new concepts of triangles would be taken later in the chapter.

Classification :

Based on Angles	Based on Sides
Acute angle Triangle: All angles are acute i.e. < 90°	*Equilateral Triangle:* All three sides of the triangle are equal in length.
Right angle triangle: One angle of the triangle is 90°	*Isosceles Triangle:* Two sides of the triangle are equal in length
Obtuse angle triangle: One angle of the triangle is obtuse i.e. > 90°	*Scalene Triangle:* No two sides of the triangle are equal in length

Important Properties/Observation in triangles:



In ΔABC ,

• Sum of three interior angles is 180°

$\angle ABC + \angle ACB + \angle BAC = 180°$

• Exterior angles = Sum of the two interior angles which are not adjacent to it (also called *interior opposite angles*)

$\angle ABR = \angle ACB + \angle BAC$

• If two sides of the triangle are not equal, then the angle opposite to greater side is greater.

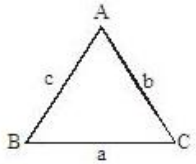
i.e. if in ΔABC, if $AC > AB$,

then $\angle ABC > \angle ACB$

Conversely, if two angles of a triangle are not equal, then side opposite to the greater angle is greater in length.

Sum of sides:

Sum of any two sides is always greater than the third side



Also, if $a + b > c$, then obviously

$a > c - b$ or $c - b < a$.

Thus, 3, 4, 8 would NOT form three sides of a triangle. And nor would 3, 5, 8. The property that difference between two sides has to be less than the third side is just a corollary of this rule. Though this property is known to almost all, its use is not very common. So it would pay to go through the following problems.

Example 8:

What will be the range of perimeter of a triangle with two sides as 4 and 7?

Solution:

For perimeter to be as large as possible, the third side should be as large as possible. But it has to be less than the sum of other two sides. Thus if x is the third side, $x < (4 + 7)$ i.e. $x < 11$. In this case perimeter, $P < 22$

For perimeter to be least, x has to be as small as possible. Can x be 1? No, because in that case $1 + 4$ is not greater than 7. Thus least value of x is such that $(x + 4) > 7$ i.e. $x > 3$. In this case perimeter, $P > (3 + 4 + 7)$

i.e. $P > 14$.

Thus range of perimeter is $14 < P < 22$

Example 9:

How many triangles can be formed such that the sides are natural numbers and the perimeter is equal to 10?

Solution:

Since sum of two sides has to be greater than the third side, the largest side has to be less than 5. Thus the possible triangles can be (4, 2, 4) and (4, 3, 3).

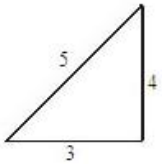
Proportionality of sides and angles

Side opposite to greatest angle will be the greatest side and side opposite to least angle will be the least side.

Even the converse hold i.e. *angle opposite to greatest side will be greatest angle and angle opposite to least side will be least angle.*

But does this mean that the sides are proportion to the angle (measured in degrees)? No not really. A simple example will make this clear.

Consider a triangle with sides 3, 4 and 5.



We all know that this is a right angle triangle with angle opposite to side 5 as 90° (for detail, check Pythagoras triplets at page 108). If angles are proportional to sides, then the angles should be 3x, 4x and 5x.

Since sum of interior angles is 180°, we have 12x = 180° i.e. x = 15 and the angles will be 45°, 60° and 75°. Obviously this is wrong as there is no right angle.

Hence, *sides do show a proportionality with angles opposite to them, but the precise relation is that sides are proportional to the sin of the angle opposite to it.* However, this is never really used and thus students scared of trigonometry can easily ignore it. The mentioned results is as follows:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

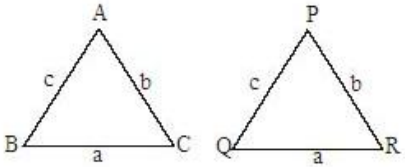
Congruency of Triangles

Two figures are congruent if when placed one over the other, they completely overlap each other i.e. the two figures are identical. They would have the same shape and the same area.

Rules for two triangles to be congruent:

1. S - S - S

If all three sides of one triangle are of the same length as all three sides of the other triangle, then the two triangles are congruent to each other.

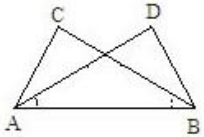


If in two triangles each side of one triangle is equal to a side of the other triangle, then the two triangles are congruent. This rule is S - S - S rule.

2. S - A - S

It states that if an angle between two sides of a triangle is equal to the corresponding angle in the other triangle and the sides containing the angle are equal, then two triangles are congruent to each other by S-A-S rule.

Say, in ΔABC and ΔABD, if BC = AD, then are they congruent?



AB = AB (common side)

∠ABC = ∠ BAD (given)

BC = AD (given)

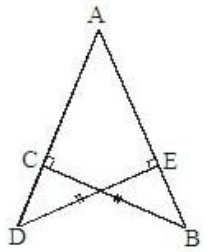
Thus, by rule S-A-S the two triangles are congruent.

Note *that for this rule to hold, the angles that are equal HAS to be the included angle (i.e. the angle should be formed by the two sides that are equal).*

3. A - A - S

It states that if two of the angles and a side of one triangle are equal to the corresponding two angles and side of the other triangle, then two triangles are congruent to each other by A - A - S rule.

Like, In ΔABC and D ADE,



∠ACB = ∠ AED = 90° (given)

∠BAC = ∠DAE (common angle)

BC = DE (given)

Thus by rule A - A - S the two triangles are congruent.

For this rule, the side NEED NOT be the included side.

Similarity

Similarity is a very important topic of geometry and similarity in triangles is extensively tested in CAT and other exams. While similarity as a concept is very simple, problems can be a wee bit difficult if similarity as a concept is ignored because of its simplicity.

Two geometrical figures are similar if they are similar in shape.

In similar figures, it is important to identify corresponding sides. Sides opposite to equal angles are called corresponding sides.

Why is similarity important? What if two figures are similar?

In similar figures, the ratio of corresponding sides are equal

The above should be thoroughly understood as this is what we shall be using after identifying two figures to be similar.

Consider the following shapes:

(a) (i) (ii) (iii)

Which of the figures (i), (ii) and (iii) are similar to figure (a)?

For your convenience the figure (a) is drawn along with each of the figure (i), (ii) and (iii) with dotted lines.

Loosely speaking all the four figures may have the same shape. But then, by using this logic all triangles should be similar because basically all have three vertices and three sides!

In figure (i), the shape has been stretched out horizontally but there is no corresponding change in the vertical dimension. Hence the figure gets distorted in one direction and the shape is not similar to figure (a).

Same is the case in figure (ii) viz. the change in the vertical dimension is disproportionate to the change in horizontal dimension and hence distortion of shape takes place.

In figure (iii), the shape has been enlarged and the enlargement is equal in all possible dimensions. Thus the basic shape remains same and only the size changes. Thus this figure is similar to figure (a).

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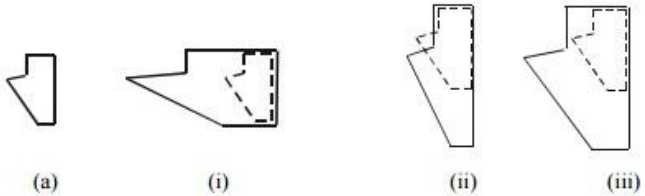

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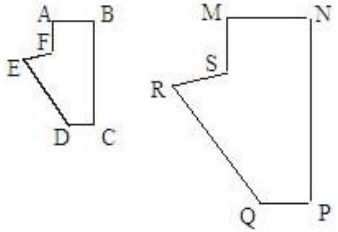
Same is the case in figure (ii) viz. the change in the vertical dimension is disproportionate to the change in horizontal dimension and hence distortion of shape takes place.

In figure (iii), the shape has been enlarged and the enlargement is equal in all possible dimensions. Thus the basic shape remains same and only the size changes. Thus this figure is similar to figure (a).



For no distortion to take place, the change in the dimension along all directions should be the same i.e. the ratio $\frac{MN}{AB}$ should be equal to the ratio $\frac{NP}{BC}$.

In general,

$$\frac{MN}{AB} = \frac{NP}{BC} = \frac{PQ}{CD} = \frac{QR}{DE} = \frac{RS}{EF} = \frac{SM}{FA}$$

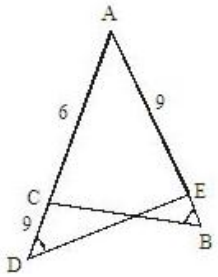
Also important to understand is that ratio of any corresponding linear dimension will be equal to the above ratio i.e. even $\frac{NQ}{DB} = \frac{SP}{FC} = \frac{MN}{AB}$

Rules for two triangle to be similar :

1. A - A rule:

If each of two angles of a triangle are equal to a angle of the other triangle, the third angle has to be equal. Hence all the angles of one figures are equal to the angles in the other figure, the two triangle are similar.

In the figure given below, what is the length of EB?



In ΔABC and ΔADE,

∠ADE = ∠ABC (given)

∠DAE = ∠BAC (common angle)

Hence A - A rule, ΔABC and ΔADE are similar.

Thus we have $\frac{AB}{AD} = \frac{BC}{DE} = \frac{AC}{AE}$

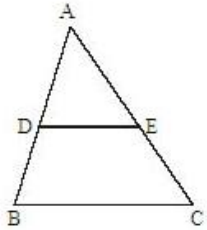
$\frac{AB}{15} = \frac{6}{9} \Rightarrow AB = 10 \ \& \ EB = 1$

(How many of you thought it would be 13.5?)

2.S-S-Srule :

If the ratio of corresponding sides are equal the two figures are similar.

The most common figure of similar triangles will be the adjoining figure.

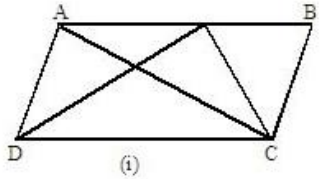


If it is given that $\frac{AB}{BD} = \frac{AC}{CE}$, we can deduce that BC is parallel to DE. Then it is apparent that ΔABC and Δ ADE are similar.

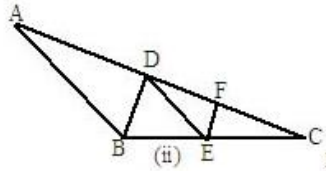
There are a lot of questions based on Similarity of triangles and it would pay off very well for one to get a knack of identifying similar triangles very fast. One simple give-away would be the presence of parallel lines or the presence of a common angle.

In each of the following figures identify pairs of similar triangles: Figure (i) and (ii), each has two pairs of similar triangles and figure (iii) has three pairs of similar triangles.

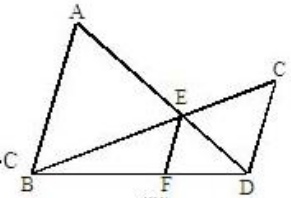
ABCD is a parallelogram.



AB||DE and BD||EF



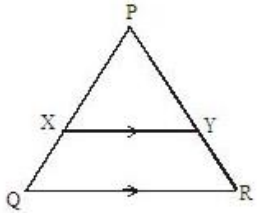
(ii)



(iii)

Basic Proportionality theorem :

It says, *in a triangle, line drawn parallel to any side divides other two sides proportionally.*



In ΔPQR, XY||QR, then according to Basic Proportionality theorem,

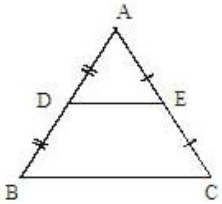
$\frac{PX}{XQ} = \frac{PY}{YR}$ ....(i)

$\frac{PQ}{PX} = \frac{PR}{PY}$ ...(ii)

$\frac{PQ}{XQ} = \frac{PR}{YR}$ ...(iii)

Mid Point Theorem :

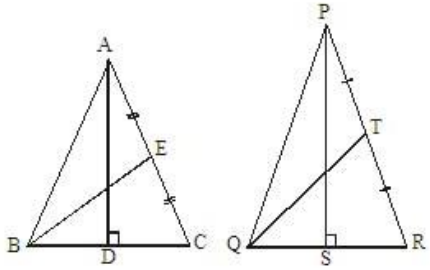
Extension of Basic Proportionality theorem is Mid - Point theorem.



If, D is mid point of AB and E is mid point of AC. Then, DE is parallel to BC and $DE = \frac{1}{2}BC$.

Line joining mid point of any two side is parallel to the third side and equal to half of it.

Area of Similar triangles :



If $\Delta ABC \sim \Delta PQR$, then

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PS} = \frac{BE}{QT} = k$ (say)

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

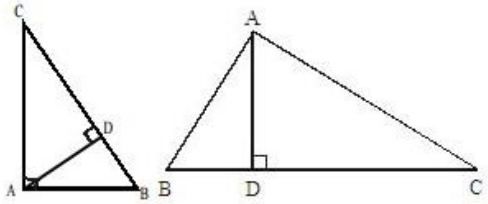
i.e., if $\Delta ABC \sim \Delta PQR$, then

$$\frac{\text{Area }(\Delta ABC)}{\text{Area }(\Delta PQR)} = \frac{\frac{1}{2}\times BC\times AD}{\frac{1}{2}\times QR\times PS} = \left(\frac{BC}{QR}\right)\times\left(\frac{AD}{PS}\right) = k^2$$

$$\frac{\text{Area}(\Delta ABC)}{\text{Area}(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

Another specific case of use of similarity is involving a right-angled triangle as shown in the figure below.

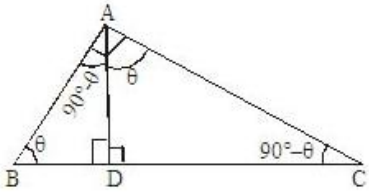
The figure can be drawn in two ways but essentially only the orientation is different and the two figures are the same. Thus, whatever follows hold true for both the figures.



In the figure, all the three triangles are similar as all the three triangles have angles equal to θ, 90°, (90° - θ).

Thus, considering

$\angle ABC = \theta$ and $\angle ACB = (90° - \theta)$,
we get $\angle CAD = \theta$, $\angle BAD = (90° - \theta)$



Now, $\Delta ADB \sim \Delta CDA$ (AA similarity)

Thus $\frac{AD}{DC} = \frac{BD}{AD}$.

So $AD^2 = BD \times DC$

Also $\Delta ABD \sim \Delta CBA$ (AA similarity)

$\frac{BC}{AB} = \frac{AB}{BD}$

$AB^2 = BD \times BC$

Similarly,

$\Delta ADC \approx BAC$

$AC^2 = CD \times BC$

Thus the results

1. $AD^2 = BD \times DC$

2. $AB^2 = BD \times BC$

3. $AC^2 = CD \times BC$

In other words,

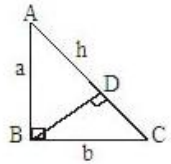
The triangles on each side of the altitude drawn from the vertex of the right angle (B) to the hypotenuse (AC) are similar to the original triangle and to each other. i.e. $\Delta ABC \sim \Delta ADB \sim \Delta BDC$

Also, the altitude from the vertex to the right angle to the hypotenuse is the geometric mean of the segments into which hypotenuse is divided.

Pythagoras Theorem :

In a right angle triangle, the square of the hypotenuse is equal to the sum of the squares of the perpendicular sides.

Pythagorean triplets can be derived if one of the side `n' given is integer. When `n' is odd, then other two sides are $\frac{n^2 \pm 1}{2}$ and when `n' is even, other two sides are $\frac{n^2}{4} \pm 1$.



Three sides which satisfy the relation $a^2 + b^2 = h^2$ are called Pythagorean triplets and can form the sides of a right angle triangle. The most commonly used triplets are (3, 4, 5); (5, 12, 13); and (6, 8, 10). In most of the problems only these triplets are used and one need not do any calculation if one remembers these. Multiples of Pythagorean triplets are also Pythagorean triplets.

E.g. (6, 8, 10); (9, 12, 15); (12, 16, 20) are all multiples of the basic triplet (3, 4, 5). Few other triplets used occasionally are (8, 15, 17);

(7, 24, 25); (9, 40, 41); (11, 60, 61).

Important Observation

If in a triangle,

$c^2 = a^2 + b^2$, *it is a right angled triangle*

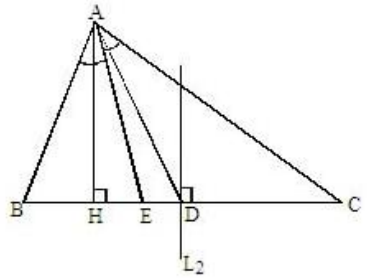
$c^2 < a^2 + b^2$, *it is an acute angled triangle*

$c^2 > a^2 + b^2$, *it is an obtuse angled triangle*

A. Important Lines in a Triangle

Median, Angle Bisector, Perpendicular Bisector of Sides and Altitudes of a Triangle:

With each vertex (or side) as reference, one can draw four specific lines in a triangle viz. median, angle bisector, perpendicular bisector of side and altitude.



Median joins the vertex to midpoint of opposite side. It need not be perpendicular to the side and may also not be the angle bisector. In the figure, AD is the median, where D is the mid-point of BC.

Angle bisector bisects an internal angle of a triangle. It need not pass through the midpoint of opposite side and need not be perpendicular to the opposite side. In the figure AE is the angle bisector.

Perpendicular bisector of side is the perpendicular line passing through the mid-point of a side. It need not pass through the vertex. In the figure, the line L_2 is the perpendicular bisector of the side BC.

Altitude is the line dropped from a vertex and perpendicular to the opposite side. It passes through the vertex but need not pass though the midpoint of the opposite side. In the figure, AH is the altitude from the vertex A.

For that matter the altitude may not even lie within the triangle (in case of an obtuse angle triangle)

In any triangle, the three medians are concurrent. So are the three angle bisectors and also the three perpendicular bisectors. Not surprising that the three altitudes are also concurrent.

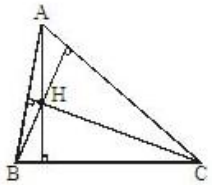
B. Important Points:

In a triangle there are FOUR important points. They are Centroid, Orthocentre, Circumcentre and Incentre.

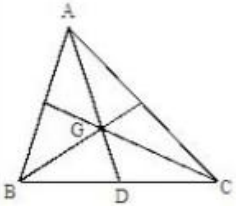
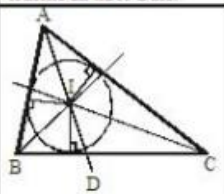
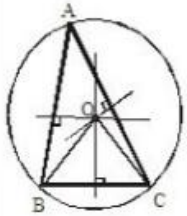
Orthocentre:

The point of concurrency of the altitudes is known as Orthocentre.

There is no `ortho-circle' with center as orthocentre.

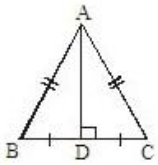


Property: $\angle BHC + \angle A = 180°$

Centroid	In-center	Circum-center
The point of concurrency of medians.	The point of concurrency of the angle bisectors	The point of concurrency of the perpendicular bisector of sides.
	Since point on angle bisector is equidistant from the sides, in-center is equidistant from all three sides. Hence an 'in-circle' can be drawn which touches all three sides.	Since a point on the perpendicular bisector of side is equidistant from the endpoints of the line, circum-center is equidistant from the vertices of the triangle. Thus a 'circum-circle' can be drawn which passes through the vertices
	 Please note the in-radius is the perpendicular distance to the sides and not ID. This is a very common error made.	
Properties: 1. The median divides the triangle in two parts of equal areas (but need not be congruent). Area of ΔADB = Area of ΔADC 2. The centroid divides the median in the ratio 2 : 1 with the larger part towards the vertex. Thus AG : GD is 2 : 1	**Properties:** 1. $\frac{AB}{AC} = \frac{BD}{CD}$ 2. $\angle BIC = 90 + \frac{\angle A}{2}$	**Properties:** 1. $\angle BOC = 2 \times \angle A$ Thus, it can be visualized that for i. **Acute angled triangle →** Circumcentre lies inside the triangle. ii. **Obtuse angled triangle →** Circumcentre lies outside the triangle iii. **Right angled triangle →** Cicrumcentre lies on the mid point of the hypotenuse

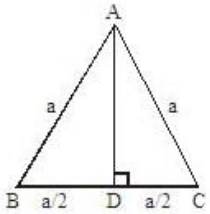
Observations :

In an***isosceles triangle***, ΔABC, where, AB = AC, (see figure)



the angle bisector of the vertex angle $\angle BAC$, AD, is also the perpendicular bisector on the base
BC and the median drawn through the vertex A.

In *an* ***equilateral triangle ΔABC,***



AB = AC = BC = a

Height /altitude = $\frac{\sqrt{3}}{2} \times \text{side} = \frac{\sqrt{3}}{2} a$

Area = $\frac{1}{2} \times \left(\frac{\sqrt{3}}{2} \times \text{side}\right) \times \text{side} = \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} a^2$

In radius = $\frac{1}{3} \times \text{height}(AD) = \frac{a}{2\sqrt{3}}$

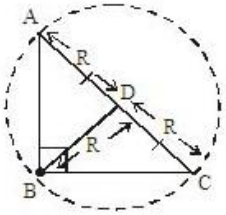
Circumradius = $\frac{2}{3} \times \text{height}(AD) = \frac{a}{\sqrt{3}}$

Perimeter = 3 × side = 3a

• In an equilateral triangle, altitude, median, angle bisector, perpendicular bisector of each base are thesame. Centroid, incentre, orthocentre and circumcentre are also the same.

• In case of triangles, given the perimeter as constant, an equilateral triangle has the maximum area.

In **Right - angled triangle:**



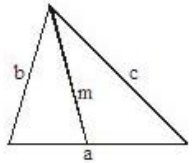
Let D be the mid point of the hypotenuse AC.

Median drawn from the right angular vertex to the hypotenuse in a right angle triangle is equal to the circumradius.

AD = BD = DC = R = Circum radius

Apollonius Theorem:

Let a, b, c be the sides of a triangle and m is the length of the median to the side with length a. Then $b^2 + c^2 = 2\left[m^2 + \left(\frac{a}{2}\right)^2\right]$



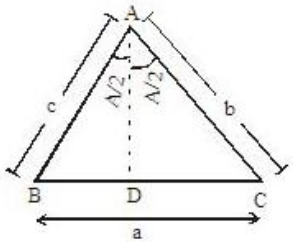
Special case:

If b = c (the triangle is isosceles), then we have

$$2b^2 = 2\left[m^2 + \left(\frac{a}{2}\right)^2\right]$$

$$m^2 = \left(b^2 - \frac{a^2}{4}\right)$$

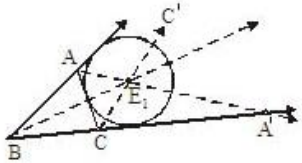
Angle - Bisector Theorem:



Angle bisector divides the opposite side in the ratio of sides containing the angle. So $\frac{BD}{DC} = \frac{AB}{AC}$

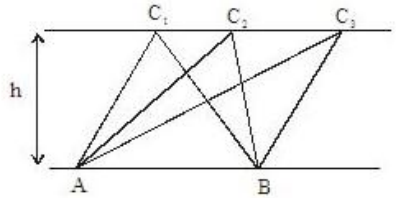
External Angle Bisector:

If we extend two sides of the triangle, we can bisect the exterior angles in the same way. In the figure the exterior angle at A is bisected by the ray AA' and the exterior angle at C is bisected by the ray



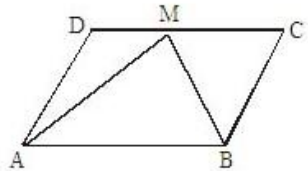
The External bisectors of ∠A and ∠C and the internal bisector of ∠B all intersect in a common point also, and this point is the center of a circle that is tangent to the three sides also. This is called an Excircle, and the center is the excenter E_1. Since there are three sides, this could be done in three ways and we can get three ex-centers. The radius of this circle is also related to the area and perimeter of the triangle.

Triangles on the same base and between the same parallel lines are equal in area



$$\text{Area}(\Delta ABC_1) = \text{Area}(\Delta ABC_2) = \text{Area}(\Delta ABC_3) = \frac{1}{2}(AB) \times h$$

If a parallelogram and a triangle are drawn on the same base and between same parallel lines, area of parallelogram is twice the area of the triangle.



Area of ABCD = 2 × (Area of ΔABM)

Special case:

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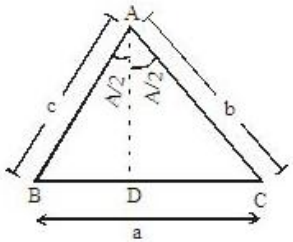
Angle - Bisector Theorem:



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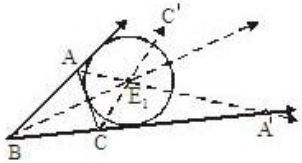

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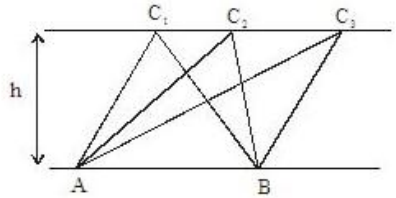

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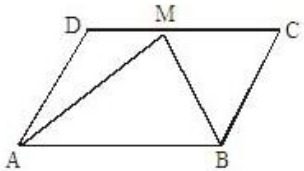


Area of ABCD = 2 × (Area of ΔABM)

Few other formulae related to a triangle:

1. Area of a triangle = $\frac{1}{2}\times b\times h$*, where b is base and h is height*

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$*, where a, b and c are the sides of the triangle and s is the semi-perimeter i.e.* $s=\frac{a+b+c}{2}$.

This formulae of area is known as Hero's formula

3. Area of a triangle = $\frac{1}{2}ab\sin\theta$*, where a and b are the sides of the triangle and q is the included angle i.e. angle between sides of length a and b.*

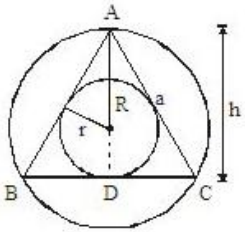
4. Area of a triangle = r × s, where r is the in-radius and s is the semi-perimeter

5. ***Cosine rule:*** *If a, b and c are the three sides of a triangle and if q is the included angle between the sides of length a and b, then*

$\cos\theta=\frac{a^2+b^2-c^2}{2ab}$ *or* $c^2 = a^2 + b^2 - 2ab\cos\theta$

6. $A=\frac{abc}{4R}$ *where R is circum-radius and A is area of the triangle*

Circumference and area of circumscribed and inscribed circles of an equilateral triangle.



If R and r are the radii of the circumscribed and inscribed circles of the triangle, then

(i) $R=\left(\frac{2}{3}\right)h=\frac{a}{\sqrt{3}}$ and $r=\left(\frac{1}{3}\right)h=\frac{a}{2\sqrt{3}}$

(ii) the circumference of the circumscribed circle = $2\pi R=\left(\frac{4}{3}\right)\pi h=\frac{2}{\sqrt{3}}\pi a$

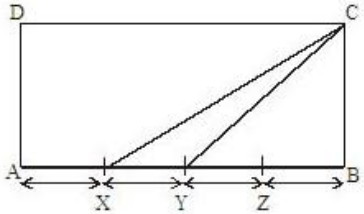
(iii) the area of the circumscribed circle = $\pi R^2=\left(\frac{4}{9}\right)\pi h^2=\frac{\pi a^2}{3}$

(iv) the circumference of the inscribed circle = $2\pi r=\left(\frac{2}{3}\right)\pi h=\frac{\pi a}{\sqrt{3}}$

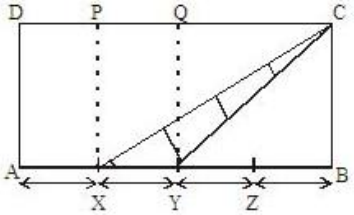
(v) the area of the inscribed circle = $\pi r^2=\left(\frac{1}{9}\right)\pi h^2=\frac{\pi a^2}{12}$

Example 10:

Side AB of rectangle ABCD is divided into four (4) equal parts as shown. Find the ratio $\frac{\text{Area }(\Delta XYC)}{\text{Area (Rectangle ABCD)}}$



Solution:



Let Area of rectangle ABCD = A

Area of rectangle XYQP = A/4

Rectangle XYQP and XYC are on the same base and between the same parallel lines

Thus

$$\text{Area } (\Delta XYC) = \frac{1}{2}\text{Area } (\text{Rectangle } XYQP)$$

$$\text{Area } (\Delta XYC) = \frac{A}{8}$$

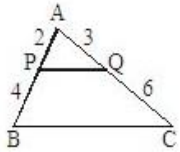
$$\Rightarrow \frac{\text{Area } (\Delta XYC)}{\text{Area } (\text{Rectangle } ABCD)} = \frac{1}{8}$$

Example 11:

P and Q are the points on the sides AB and AC respectively of ΔABC. If AP = 2 cm, PB = 4 cm, AQ = 3 cm, QC = 6 cm, find $\frac{BC}{PQ}$.

Solution:

The two triangles ABC and APQ are similar.

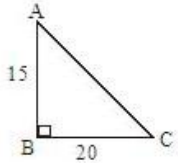


$$\therefore \frac{AB}{AP} = \frac{BC}{PQ} = \frac{AC}{AQ} = \frac{3+6}{3} = 3$$

Example 12:

If the ΔABC is right-angled at B, find its circumradius if the sides AB and BC are 15 cm and 20 cm respectively.

Solution:



ΔABC is a right-angled triangle.

Hence, circumradius $= \frac{1}{2} \times$ (Hypotenuse)

$$AC^2 = AB^2 + BC^2$$

(Using Pythagoras' theorem)

$$\therefore AC = \sqrt{15^2 + 20^2} = 25 \text{ cm}$$

Hence, circumradius = 12.5 cm

Example 13:

Find the ratio of the area of the equilateral triangle drawn with the side of a square as its base and the area of the equilateral triangle described on the diagonal of the square.

Solution:

Let the side of the square be A.

Its diagonal is $A\sqrt{2}$.

Area of the first equilateral triangle = $\frac{\sqrt{3}}{4} \times A^2$

Area of the second equilateral triangle = $\frac{\sqrt{3}}{4} \times (A\sqrt{2})^2$

Required ratio = 1 : 2

Example 14:

The perimeters of two similar triangles are 30 cm and 20 cm respectively. If one side of the first triangle is 12 cm, then find out the proportional side of the other triangle.

Solution:

For similar triangles, ratio of perimeters = Ratio of proportional sides

$\Rightarrow \frac{30}{20} = \frac{12}{X} \Rightarrow X = 8 \text{ cm}$

Example 15:

Two isosceles triangles have equal vertical angles and their areas are in the ratio 9 : 16. Find the ratio of their proportional heights.

Solution:

$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Altitude}$

For similar triangles:

Ratio of areas = Ratio of squares of proportional heights

$\Rightarrow \frac{\text{Height}_1^2}{\text{Height}_2^2} = \frac{9}{16}$

$\Rightarrow \text{Height}_1 : \text{Height}_2 = 3 : 4$

Example 16:

In a ΔABC, $AB^2 + AC^2 = 200 \text{ cm}^2$. Median AD = 10 cm. Find BC.

Solution:

Use the formula for the length of median (Apollonius theorem)

$\Rightarrow 2(AD)^2 + 2(BD)^2 = AB^2 + AC^2$

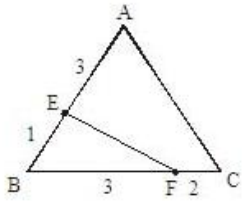
$\Rightarrow 2 \times 100 + 2BD^2 = 200$

$\Rightarrow BD^2 = 0$

$\Rightarrow BD = 0$

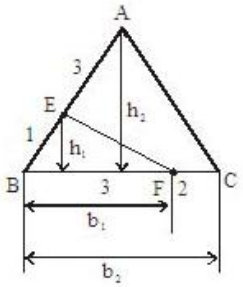
$\Rightarrow$ The triangle does not exist under the given conditions.

Example 17:



In ΔABC, if E divides AB in the ratio 3 : 1 and F divides BC in the ratio 3 : 2, find the ratio of the areas of ΔBEF and ΔABC.

Solution:



In ΔABC, $h_1 : h_2 = 1 : 4$, $b_1 : b_2 = 3 : 5$

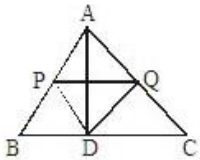
Hence, the ratio of the area of the triangles is

$\Delta EBF : \Delta ABC = b_1 h_1 : b_2 h_2 = 3 : 20$

Example 18:

D is the mid-point of side BC of ΔABC. DP bisects $\angle ADB$ meeting AB at P, and DQ bisects $\angle ADC$ meeting AC at Q. Is PQ || BC?

Solution:



In ΔABC, D is the mid-point of BC, DP is the bisector of $\angle ADB$ and DQ is the bisector of $\angle ADC$. In ΔABD, DP is the angle bisector of $\angle ADB$ (given).

$\therefore \frac{AP}{PB} = \frac{AD}{BD}$

In ΔADC, DQ is the bisector of $\angle ADC$. $\therefore \frac{AQ}{QC} = \frac{AD}{DC}$

But BD = DC (D is the mid-point of BC) $\therefore \frac{AQ}{QC} = \frac{AD}{BD}$

So we get $\frac{AP}{PB} = \frac{AQ}{QC}$

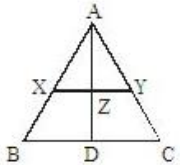
$\therefore$ PQ || BC

Example 19:

In ΔABC, the bisector of $\angle A$ meets BC at D. A straight line parallel to BC meets AB, AC and AD at X, Y and Z respectively. Prove that

(i) AB . CD = AC. BD

(ii) AX . YZ = AY . XZ



Solution:

In ΔABC, AD is the bisector of $\angle BAC$.

XY || BC.

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$,

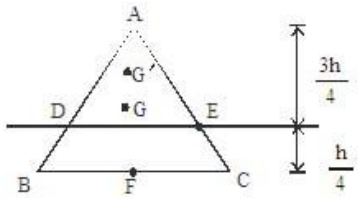
i.e. AC . BD = AB . DC.

In ΔAXY, AZ is the angle bisector of $\angle XAY$.

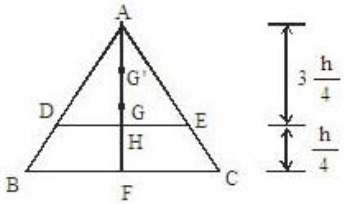
$\therefore \frac{XZ}{ZY} = \frac{AX}{AY}$, i.e. $AY \cdot XZ = AX \cdot ZY$

Example 20:

G is the centroid of ΔABC with height `h' units. If a line parallel to BC, cuts the ΔABC at a height $\frac{h}{4}$ from BC, find the distance GG' in terms of AG if G' is the centroid of ΔADE.



Solution:



AF is the median to BC from A in ΔABC.

$\therefore AG = \frac{2}{3}AF \Rightarrow AF = \frac{3}{2}AG$

AH is the median to DE from A in ΔADE.

$\therefore AG' = \frac{2}{3}AH$

Since DE || BC, $AH = \frac{3}{4}AF$

$\therefore AG' = \frac{2}{3} \times \frac{3}{4}AF = \frac{1}{2}AF = \frac{3}{4}AG$

Hence, $GG' = AG - AG' = AG - \frac{3}{4}AG = \frac{1}{4}AG$

Example 21:

A vertical stick of 12 cm long casts a shadow 8 cm long on the ground. At the same time a tower casts a shadow 40 m long on the ground. Determine the height of the tower.

Solution:

Use similar triangles property:

Height of tower : Height of stick

= Shadow of tower : Shadow of stick

X : 12 cm = 40 m : 8 cm

X = 60 m

Example 22:

The sides of a triangle are in the ratio 5 : 4 : 3. Find the sides of this triangle, given that another similar triangle of sides 30, 24 and 18 cm has an area 9 times the area of the triangle.

Solution:

Use the property of similar triangles. Let the required sides be 5X, 4X, and 3X respectively.

$$\frac{\text{Area of bigger triangle}}{\text{Area of smaller triangle}} = \left[\frac{\text{Side of the bigger triangle}}{\text{Corresponding side of the smaller triangle}}\right]^2$$

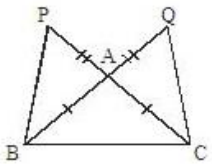
$$\Rightarrow \frac{30^2}{25X^2} = 9 \Rightarrow X = 2$$

Thus, the required lengths of the sides are 10, 8 and 6 cm.

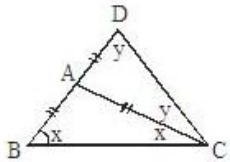
Test Your Understanding

Level - 1

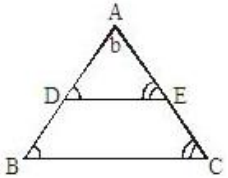
1. In the following figure, equal sides BA and CA of ΔABC are produced to Q and P respectively, so that AP = AQ and AB = AC. Find the ratio PB : QC.



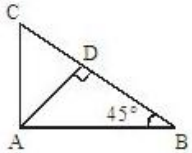
2. ABC is an isosceles triangle such that AB = AC. Side BA is produced to a point D such that AB = AD. Find ∠BCD.



3. In ΔABC, D and E are points on AB and AC such that DE || BC. What is the ratio of the areas of triangle ADE to quadrilateral ΔBCE if DE = $\frac{3}{5}$ BC?



4. In ΔABC, ∠A = 90°, AD ⊥ BC and ∠B = 45°. If AB = x, find AD in terms of x.



5. In an isosceles ΔABC, AD is the median to the unequal side meeting BC at D. DP is the angle-bisector of ∠ADB and PQ is drawn parallel to BC meeting AC at Q. What is the measure of ∠PDQ?

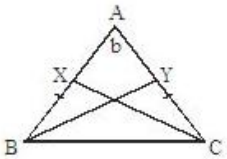
6. ΔABC and ΔDEF are similar. If their areas are 100 cm² and 49 cm² respectively, and AB is 8.2 cm, find DE.

7. ΔABC and ΔPQR are similar. If AB and PQ are 7.5 cm and 3.5 cm respectively, find the ratio of the areas of the two triangles.

8. The medians BE and CF of ΔABC intersect at P. Find the ratio of the area of ΔBPC to that of ΔFPE.

Level - 2

9. If X and Y are two points on equal sides AB and AC of ΔABC, such that AX = AY. Find $\frac{XC}{YB}$.



10. ABC is a triangle with AB = AC. D is the mid-point of BC. What is the ratio of $\angle DAB$ to $\angle DAC$?

11. ABC is an isosceles triangle with $\angle B = \angle C$. The bisectors of $\angle B$ and $\angle C$ meet the opposite sides at P and Q. Is PQ parallel to BC?

12. In ΔABC, AD and BE are perpendiculars from A and B to the sides BC and AC respectively. Show that

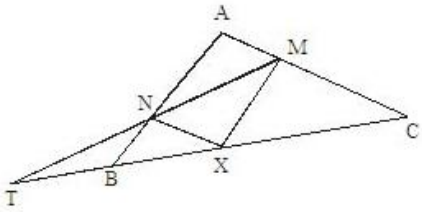
(i) ΔADC is similar to DBEC
(ii) CA × CE = CB × CD

13. In ΔABC, DE || BC. If AD = 5 cm, AE = 6 cm, BC = 12 cm, AB = 15 cm. Find AC and DE.

14. ΔABC and ΔBDC are two right-angled triangles with common hypotenuse BC. The sides AC and BD extended to intersect at P. Then $\frac{AP \times PC}{DP \times PB} = ?$

15. Let X be any point on side BC of a ΔABC. If XM, XN are drawn parallel to BA and CA respectively; MN meets BC produced in T, TX is _____ of TB and TC.

a. AM b. GM c. HM d. None of these



Level -3

16. ABC is an equilateral triangle inscribed in a circle of radius r. What is the area of the largest square that can be inscribed inside it?

17. The bisectors of ∠B and ∠C of ΔABC meet the opposite sides at D and E respectively. BD and CE intersect at O. If ΔABC is isosceles, what kind of triangle is ΔODE?

Polygons and Quadrilaterals

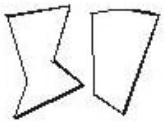
Learning Objectives:

- Basics of Quadrilaterals
- Square, Rectangles, Rhombus, Trapezium
- Pentagon, Hexagon etc.

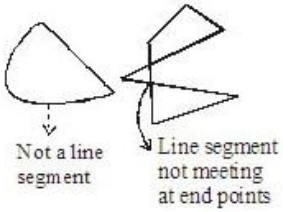
Polygon

It is a simple closed figure in a plane formed by three or more line segments which meet only at their endpoints.

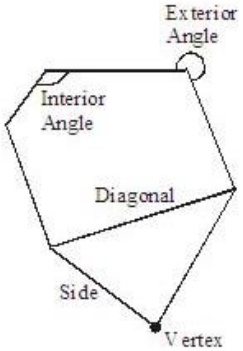
These plane figures are polygons



These plane figures are NOT polygon



Basic terms used in a polygon :



Side - one of the line segments that make up the polygon.

Vertex - point where two sides meet. Two or more of these points are called vertices.

Diagonal - a line connecting two vertices that isn't a side.

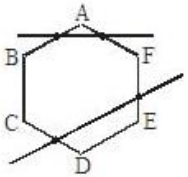
Interior Angle - Angle formed by two adjacent sides inside the polygon.

Exterior Angle - Angle formed by two adjacent sides outside the polygon

Types of Polygon:

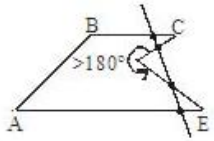
Convex Polygon:

A polygon, in which none of the interior angle is more then 180°, and a straight line drawn through a convex polygon crosses **atmost two sides**.



Concave Polygon:

A polygon, in which *atleast* one angle is more than 180°, and you can draw at least one straight line through a concave polygon that crosses **more than two sides.**

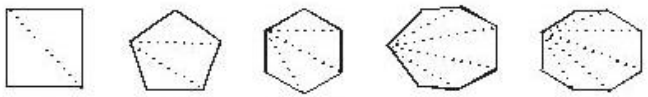


Regular Polygon:

A regular polygon is an n-sided polygon in which the sides are of same length and are symmetrically placed about a common center.

Nomenclature of polygons :

In the polygons shown below, all possible diagonals from **one vertex** are shown. The table shows the number of diagonals drawn, and the number of triangles formed in each polygon.



Square Pentagon Hexagon Heptagon Octagon

	Quadrilateral	Pentagon	Hexagon	Heptagon	Octagon
Sides	4	5	6	7	8
Diagonals	1	2	3	4	5
Triangles	2	3	4	5	6

Important Results :

1. Number of diagonals

In a polygon of `n' sides, (n - 3) diagonals can be drawn from every vertex of the polygon (Why
(n - 3)? Because in the polygon there are n - 1 other vertices, but two of them are adjacent vertices and so don't count towards making diagonals).

$\therefore$ total number of diagonals = (no. of vertices) $\times$ (no. of diagonal from each vertices) = n (n - 3)

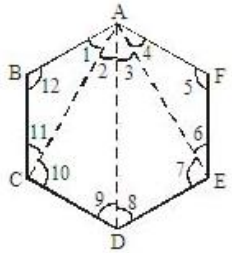
Through this way every diagonal is counted twice, i.e., AC and CA both are the same diagonal

(figure below).

Hence, total number of diagonals in a `n' sided polygon $= \frac{n(n-3)}{2}$

2. Sum of Interior angles

If in a polygon, say hexagon, all possible diagonal from a single vertex are drawn



then sum of interior angles:

$\angle 1 + \angle 2 + \angle 3 + ... + \angle 11 + \angle 12$

or

$$\underbrace{\angle 1+\angle 12+\angle 11}_{\Delta ABC}+\underbrace{\angle 2+\angle 10+\angle 9}_{\Delta ACD}+\underbrace{\angle 3+\angle 8+\angle 7}_{\Delta ADE}+\underbrace{\angle 4+\angle 5+\angle 6}_{\Delta AEF}$$

Four triangles are formed

Sum of interior angles of a triangle = 180°

∴ Sum of interior angles of Hexagon

(containing 4 triangles) = 4 × 180 = 720°

Note: Any `n' sided polygon

→ will have (n - 3) no. of diagonals from a single vertex.

→ And (n - 2) no. of triangles created

by those (n - 3) diagonals drawn from

a particular vertex.

∴ Sum of interior angle of any polygon

of n sides = (n - 2) × 180°

Alternatively :

In general, any polygon of n sides will have n exterior angles. The sum of all exterior angles for any convex polygon is always 360°. Thus for a regular polygon (which has all sides equal and also all angles equal), each exterior angle will be $\frac{360°}{n}$.

Using this, we can find the interior angle of any regular convex polygon as 180° - $\frac{360°}{n}$.

It would be wise to remember the interior (and exterior) angles of the commonly used polygons i.e. pentagon, hexagon and octagon. The interior angles of a regular pentagon, hexagon and octagon are respectively equal to

$$180° - \frac{360°}{5},\ 180° - \frac{360°}{6},\ 180° - \frac{360°}{8}$$

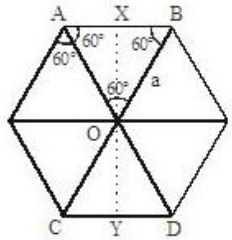
i.e. 180° - 72°, 180° - 60° and 180° - 45°

i.e. 108°, 120° and 135°

***We can also use the formula, sum of* interior angles of a n sided polygon = (2n - 4) × 90°**

Hexagon:

It is one of the mostloved polygon in the context of CAT. A regular hexagon can be considered to be made up of six equilateral triangles as shown in the figure below.



Thus,

The area of a regular hexagon of side a

= 6 × area of equilateral triangle of side a

$$= 6 \times \frac{\sqrt{3}}{4} \times a^2 = \frac{3\sqrt{3}}{2} a^2$$

Further, the distance between two opposite vertices will be equal to 2a. But the distance between two opposite sides (say AB & CD) is not 2a. It would be twice the altitude $(OX + OY = 2OX = XY)$ of the equilateral triangle $(\text{like } \Delta OAB)$ and thus distance between two opposite sides.

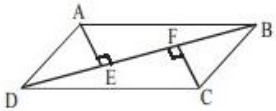
$$= 2 \times \text{height}(OX) = 2 \times \frac{\sqrt{3}}{2} \times a = \sqrt{3}a$$

Therefore, in regular hexagon,distance between two opposite verticesis greater than distance between two opposite sides

Quadrilaterals

Any polygon with n = 4 is a quadrilateral. They are closed plane figures having four sides and four vertices. Quadrilaterals may be concave or convex, but we will deal with convex quadrilaterals only. For a convex quadrilateral, the sum of all interior angles is equal to 360°.

Area of a quadrilateral

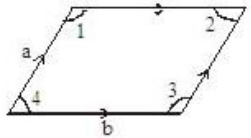


Area = $\frac{1}{2}$ × one of the diagonals × sum of the perpendiculars drawn to that diagonal from the opposite vertices.

$$\text{Area}(\square ABCD) = \frac{1}{2} \times DB \times (AE + FC)$$

The most common quadrilaterals we come across and the basic properties of them are given below:

Parallelogram:



1. The opposite sides are parallel and equal.

2. Opposite angles are equal.

3. The diagonals bisect each other.

[∠1 = ∠3, ∠2 = ∠4]

(The diagonals need not be equal in length and do not necessarily bisect at right angles.)

4. Sum of two adjacent angles = 180°

[∠1 + ∠4 = 180°, ∠1 + ∠2 = 180°,

∠2 + ∠3 = 180° and ∠3 + ∠4 = 180°]

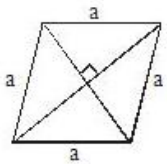
5. Each diagonal of a parallelogram divides it into two congruent triangles.

6. A parallelogram that is inscribed in a circle is a rectangle.

7. A parallelogram that is circumscribed about a circle is a rhombus.

8. Area of a parallelogram = base × height of the parallelogram.

Rhombus



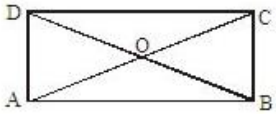
1. If all the sides of a parallelogram are equal then it is called a rhombus.

2. Opposite angles are equal.

3. Diagonals are not necessarily equal but they bisect each other at 90°.

4. Area of a rhombus = $\frac{1}{2}$ × product of the lengths of its diagonals.

Rectangle:



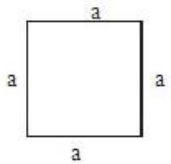
A parallelogram with adjacent sides at right angles. In addition to properties of parallelogram, we have

• All corner angles are right angles.

• Diagonals are equal in length (AC = BD)

Note : Diagonals DO NOT intersect at right angle.

Square



1. It is a rectangle with all sides equal.

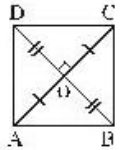
2. Every angle is a right angle.

3. Diagonals are equal and they bisect each other at 90°.

4. The area of a square = (side)^2.

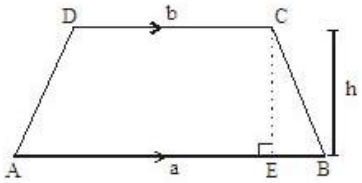
5. The diagonal of a square = $\sqrt{2}$ side.

Thus, square has all the properties of a parallelogram, rectangle and rhombus.



Trapezium:

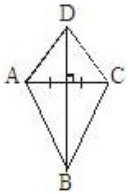
A quadrilateral with one pair of opposite sides parallel to each other but other pair is not so. In the given figure ABCD is a trapezium, where AB is parallel to CD but AD is NOT parallel to BC.



Area = $\frac{1}{2}$ × (sum of parallel sides) × height or $A = \frac{1}{2}(a+b) \times h$

Kite:

A quadrilateral, where diagonals are unequal but intersect at 90°. Moreover exactly one of the diagonals will be bisected but the other diagonal will not.



Particular pairs of adjacent sides are equal. In the diagram, ABCD is a Kite where AB = BC and AD = DC.

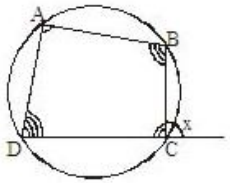
Cyclic quadrilateral

1. The four vertices of the quadrilateral lie on a circle.

2. Opposite angles are supplementary to each other.

∠DAB + ∠BCD = 180°

∠ADB + ∠ABC = 180°



At a glance: (Basics Properties)

Properties	*Parallelogram*	*Rectangle*	*Rhombus*	*Square*
Opposite sides are equal	✓	✓	✓	✓
All sides are equal	×	×	✓	✓
Opposite sides are parallel	✓	✓	✓	✓
Opposite angles are equal	✓	✓	✓	✓
All angles are 90°	×	✓	×	✓
Diagonals are equal	×	✓	×	✓
Diagonals bisect each other	✓	✓	✓	✓
Diagonals bisect each other at 90°	×	×	✓	✓
Diagonals bisect vertex angles	×	×	✓	✓
Diagonals form four triangles of equal area	✓	✓	✓	✓

All (√) indicates this property is necessarily true for the numbered figure:

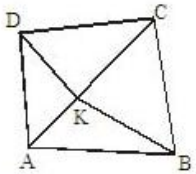
Example 1: If the diagonals of a rhombus are 20 cm and 30 cm respectively, find the side of the rhombus.

Solution: In a rhombus the diagonals bisect at right angles.

Hence, $(\text{Side})^2 = 10^2 + 15^2$.

So side of the rhombus $= \sqrt{325}$ cm $= 5\sqrt{13}$ cm

Example 2:



In a quadrilateral ABCD, the bisectors of ∠ B and ∠D intersect on AC at K.

Show that $\frac{AB}{BC} = \frac{AD}{DC}$.

Solution:

ABCD is a quadrilateral. The angle bisectors of ∠ B and ∠ D intersect on AC at K. In ΔABC, BK is the angle bisector of ∠ABC.

$\therefore \frac{AB}{BC} = \frac{AK}{KC}$.

In Δ ADC, DK is the angle bisector of ∠ ADC. $\therefore \frac{AD}{DC} = \frac{AK}{KC}$.

So, we get $\frac{AB}{BC} = \frac{AD}{DC}$

Test Your Understanding
Level - 1

1. ABCD is a quadrilateral. P, Q, R and S are mid-points of the sides AB, BC, CD and AD respectively. Is PQRS a parallelogram?

2. P and Q are mid-points of sides AB and AC of ΔABC. PQ is produced to R such that PQ = QR. Is PRCB a parallelogram?

3. ABCD is a trapezium with AB || CD. E and F are the mid-points of AD and BC respectively. Is EF || AB?

4. In a trapezium ABCD, AB || DC, AB = 2CD. If the diagonals intersect at O, find $\frac{\text{Area of } \Delta AOB}{\text{Area of } \Delta COD}$.

5. ABCD is a trapezium with AB || CD. If AC and BD intersect at E such that BE : ED = 2 : 3, show that the ratio of areas of ΔAEB and ΔCED is 4 : 9.

6. The diagonal BD of a parallelogram ABCD intersects the segment AE at a point F, where E is any point on BC. Find (DF . EF) : (FB . FA).

Circle

Learning Objectives

• Basic property of circles

• Tangent, Secant etc.

• Cyclic Quadrilateral

Circle

A regular polygon with number of sides `n' tending to infinity becomes a circle. However, please note that a circle is not considered as a polygon. Certain properties of a circle of radius `r' are discussed here.

Chord:

It is a line joining two points (like Q, P) on the circumference.

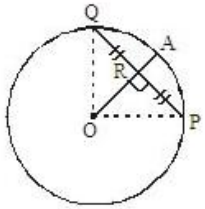


Figure 1.

Properties :

• The longest chord in a circle is the diagonal

• The line joining the center and the mid point of the chord will be perpendicular [in fig(1), OR ⊥ PQ, R being midpoint] to the chord.

Reverse: Perpendicular from the center to the chord *bisects* the chord.

• Also , as the chord moves away from the center, its length decreases.

Result: Chords which are equidistant from the centre are equal in length (AB & CD are equal chords in figure 2, where OM = ON)

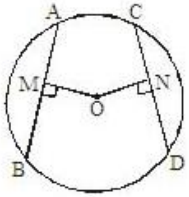


Figure 2.

• Similarly, angle subtended at the centre by equal chords will be equal.

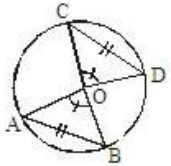


Figure 3.

In the above figure, AB = CD and O is the centre of the circle.

∴ ∠AOB = ∠COD

But why the above mentioned properties hold true?

Following question might help.

In figure 1.

→ Are the triangles ORQ and ORP congruent?

→ If R is given as midpoint of PQ then by which rule are the triangles congruent?

→ In figure 2, if OM = ON is given, can it be proved that AB = CD?

• If two chords, AB and CD intersect inside or outside the circle at a point,

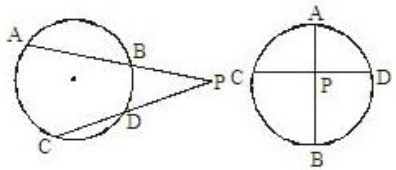


Fig. A Fig. B

Then, $PA \times PB = PC \times PD$

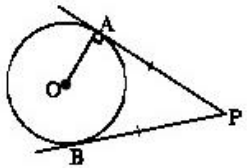
Note: PA and PC are called **SECANT** in figure A.

Tangent :

A tangent is a line which touches the circle at only one point.

• The line joining the center to the point of tangency will be perpendicular to the tangent.

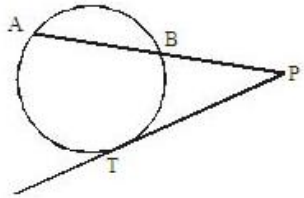
• Two tangents from the same external point will be equal in length.



In the figure drawn above, **PA = PB**

• If AB is any chord of a circle which is extended to P, and PT is a tangent drawn from P on to the circle, then

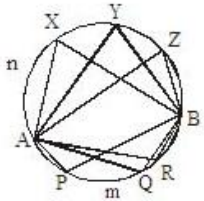
$PA \times PB = PT^2$



Properties related to angle formed at the circumference :

Angles in the same segment of the circle are equal

In the figure below, all angles AXB, AYB, AZB are formed by the segment AmB of the circle. All these angles are equal.

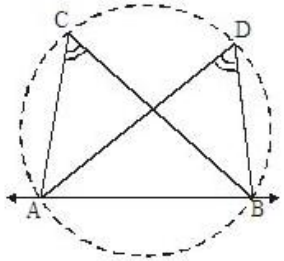


Similarly, the angles APB, AQB, ARB are also formed in the same segment AnB and hence are equal.

In other words,

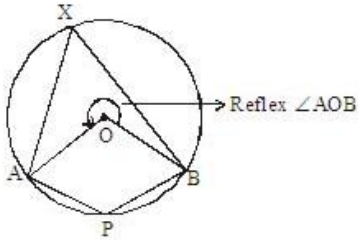
• If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the segment, then the four points lie on the same

circle



Points A, B, C and D lie on the same circle. i.e., they are concyclic.

Angle subtended by the chord at the center of a circle is twice of that subtended at the circumference.

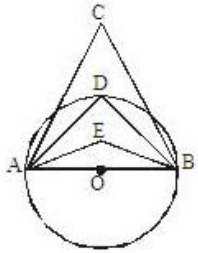


Thus $\angle AOB = 2 \times \angle AXB$

Pay special attention to this property when it is applied for an angle formed in the minor segment. It would be wrong to say that $\angle AOB = 2\times \angle APB$, for the simple reason that $\angle APB$ is an obtuse angle and hence twice its value will be greater than 180°, whereas $\angle AOB$ is less than 180°. When the angle at the circumference is in the minor segment, the angle at the center is the reflex angle formed.

Using the above properties we have two important relations :

1. Angle formed in a semicircle is 90°



• A diameter of a circle, AB subtends an obtuse angle at a point in the interior of the circle. $\angle AEB > 90°$.

• It subtends an acute angle at a point in the exterior of the circle. $\angle ACB < 90°$.

• It subtends a right angle at a point on the circle. $\angle ADB = 90°$.

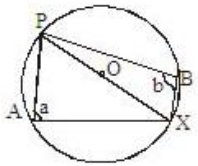
Conversely, if a segment subtends a right angle at a point on the circle, then segment is a diameter of the circle.

If $\angle ADB = 90°$, then AB is a diameter of the circle.

Cyclic Quadrilateral

• A quadrilateral is said to be cyclic if all its vertices lie on a circle. The points lying on a circle are called concyclic.

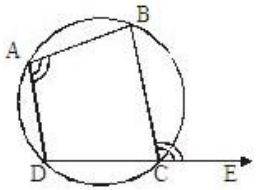
• If the opposite angles of quadrilateral are supplementary, then it is a cyclic quadrilateral.



We have a + b = 180°. Quadrilateral AXBP has all its vertices on a circle. Such a quadrilateral is called a cyclic quadrilateral.

Also carrying the same property a little further, we get a few more results.

An exterior angle of a cyclic quadrilateral is equal to the angle opposite to its adjacent interior angle.

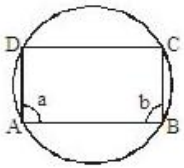


i.e. $\angle BCE = \angle DAB$

Special Case:

What if the cyclic quadrilateral is a parallelogram?

Consider ABCD to be a parallelogram. Thus a + b = 180°



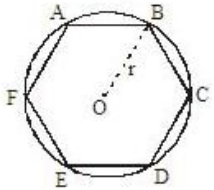
- ***This means that a parallelogram drawn inside a circle is always a rectangle/square.***

- ***Also, when a square or rectangle is inscribed in a circle, the diagonal of the square / rectangle is equal to the diameter of the circle.***

Cyclic Hexagon:

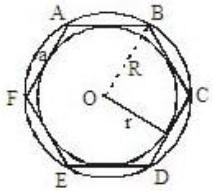
A regular hexagon is also a cyclic hexagon.

The side of the hexagon is equal to the radius of the circle.



Note: Any regular polygon can be inscribed in a circle.

Circumference and area of circumscribed and inscribed circles of a regular hexagon:



Let a be the side of a regular hexagon and R, r be the radii of the circumscribed and inscribed circles respectively of the hexagon, then

(i) $R = a$ and $r = \left(\frac{\sqrt{3}}{2}\right)a$

(ii) the circumference of the circumscribed circle $= 2\pi R = 2\pi a$

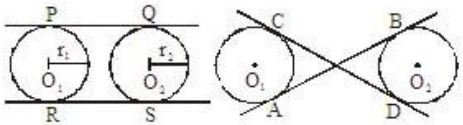
(iii) the area of the circumscribed circle $= \pi R^2 = \pi a^2$

(iv) the circumference of the inscribed circle $= 2\pi r = \sqrt{3}\pi a$

(iv) the area of the inscribed circle $= \pi r^2 = \left(\frac{3}{4}\right)\pi a^2$

Common Tangents for a pair of circles:

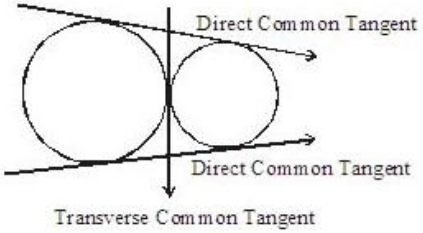
For the two circles with centres O_1 and O_2 and radius r_1 & r_2



PQ, RS are Direct common AB, CD are Transverse common tangents tangents.

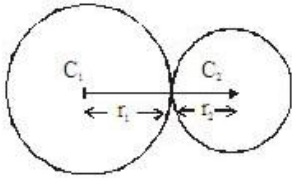
Length of PQ or RS = $\sqrt{(\text{distance between centres})^2 - (r_1 - r_2)^2}$

Length of AB or CD = $\sqrt{(\text{distance between centres})^2 - (r_1 + r_2)^2}$



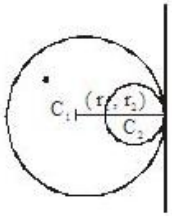
1. When two circles touch externally

Distance between centres $C_1\ C_2 = r_1 + r_2$ and 2 direct common tangents and one transverse common tangents are possible.



2. When two circles touch internally

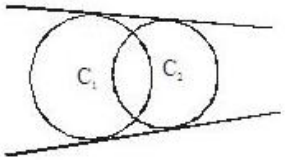
Only one common tangent is possible



$C_1\ C_2 = r_1 - r_2$

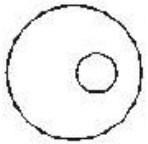
3. **When two circles intersect.**

Two direct common tangents are possible.



4. When one circle is completely inside the other without touching each other.

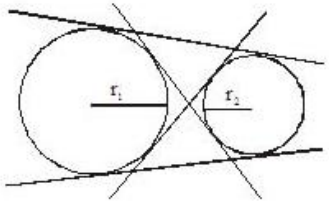
No common tangent is possible



$C_1C_2 < r_1 - r_2$

5. When two circles are apart i.e. not touching each other

Two direct and two transverse tangents are possible.

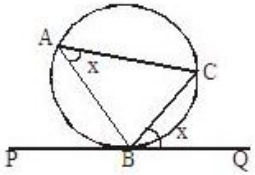


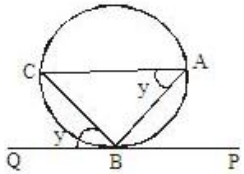
$C_1C_2 > r_1 + r_2$

Important Result :

Alternate segment theorem :

Angle between any chord passing through the tangent point and tangent is equal to the angle subtended by the chord to any point on the other side of circumference (alternate segment)

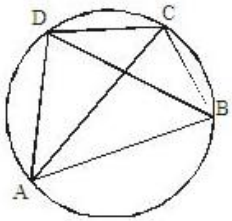




In the above figures, ∠CBQ = ∠CAB.

Ptolmey's theorem:

For a cyclic quadrilateral, the sum of products of two pairs of opposite sides equals the product of the diagonals



AB × CD + BC × DA = AC × BD

Example 1:

The biggest possible circle is inscribed in a rectangle of length 16 cm and breadth 6 cm. Find its area.

Solution:

Diameter of the circle = Breadth of the rectangle

$\Rightarrow$ Required area = $\pi \times \left(\frac{6}{2}\right)^2 = 9\pi \ \text{cm}^2$

Example 2:

With the vertices of the ΔABC as centres, three circles are drawn, each touching the other externally. If the sides of the triangle are 4 cm, 6 cm and 8 cm respectively, find the radii of the circles.

Solution:

Let the radii of the three circles be R_1, R_2 and R_3 respectively.

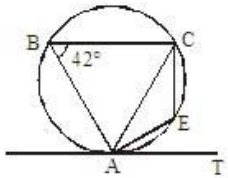
$\Rightarrow R_1 + R_2 = 6, R_2 + R_3 = 8, R_3 + R_1 = 4$

$\Rightarrow R_1 = 1$ cm, $R_2 = 5$ cms, $R_3 = 3$ cm

Example 3:

In the figure, ∠ABC = 42°.

Find ∠ CAT and ∠AEC.



Solution:

AT is a tangent at A and AC is a chord.

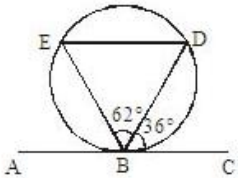
∠ABC is an angle in the alternate segment of $\angle CAT \therefore \angle CAT = 42°$, ∠AEC + ∠ABC = 180°

(Opposite angles of a cyclic quadrilateral)

$\therefore \angle AEC = 180^\circ - \angle ABC = 180^\circ - 42^\circ = 138^\circ$

Example 4:

In the figure, ∠DBE = 62° and ∠DBC = 36°. Find ∠BDE.



Solution:

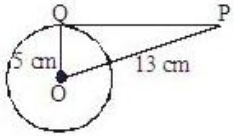
ABC is a tangent at B. BD is a chord through B. Hence, ∠DBC = ∠BED = 36° (Angle between the chord and tangent is equal to the angle in the alternate segment.)

∠DBE + ∠BED + ∠BDE = 180° (Sum of the angles of a triangle)

$\therefore \angle BDE = 180^\circ - \{36^\circ + 62^\circ\} = 180^\circ - 98^\circ = 82^\circ$

Example 5:

Determine the length of the tangent to a circle of radius 5 cm from a point at a distance of 13 cm from the centre of the circle.



Solution:

O is the centre of the circle. P is a point outside the circle such that OP = 13 cm. PQ is a tangent from P to the circle.

Hence, OQ = 5 cm.

Also PQ perpendicular to OQ (radius is perpendicular to the tangent at the point of contact).

Hence, in right-angled Δ PQO, PQ^2

= $OP^2 - OQ^2$ (By Pythagoras' theorem)

= $13^2 - 5^2 = 169 - 25 = 144$, $PQ = \sqrt{144} = 12$ cm.

Example 6:

Two circles of radii 10 cm and 18 cm touch each other internally. Find the distance between their centres. Also, find the distance between their centres if they touch externally.

Solution:

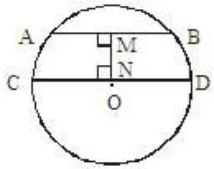
When they touch each other internally, the distance between their centres = 18 - 10 = 8 cm

When they touch each other externally, the distance between their centres = 10 + 18 = 28 cm

Example 7:

AB and CD are parallel chords of a circle with centre O; M and N are the mid- points of these chords. AB = 4 cm, CD = 10 cm and MN = 3 cm. What is the radius of the circle?

Solution:



Let ON = X cm and radius = r cm

AM = MB = $\frac{1}{2} \times 4$ cm = 2 cm

[Perpendicular from centre bisects the chord]

CN = ND = $\frac{1}{2} \times 10$ cm = 5 cm

[Perpendicular from centre bisects chord]

In ΔAMO, $r^2 = 2^2 + (3 + X)^2$ ...(a)

(Pythagoras' theorem)

In ΔCNO,

$r^2 = 5^2 + X^2$ . ..(b)

(Pythagoras' theorem)

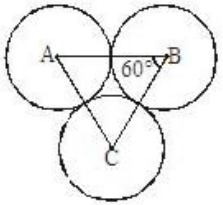
$\Rightarrow 2^2 + (3 + X)^2 = 5^2 + X^2 \Rightarrow X = 2$ cm

Putting X = 2 cm in (b),

$r^2 = 5^2 + 2^2 = 29 \Rightarrow r = \sqrt{29}$ cm

Example 8:

Find the area enclosed when three circles of radius 10 cm each touch each other.



Solution:

Let the centers of three circles be A, B and C.

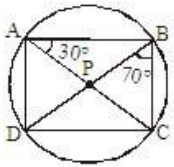
Required area = (Area of the equilateral ΔABC) - (Area of 3 sectors each having central angle as 60°) = $\frac{\sqrt{3}}{4}\times 20^2 - 3\times\left(\frac{60^\circ}{360^\circ}\times\pi\times 10^2\right) = 100\sqrt{3} - 50\pi = 16.12\ \text{cm}^2$

Example 9:

ABCD is a cyclic quadrilateral whose diagonals intersect at P.

If ∠DBC = 70° and ∠BAC = 30°, find ∠BCD.

Solution:



We use the property that the angles subtended by an arc at any point on the circumference are equal.

⇒ Arc DC subtends ∠CBD and ∠CAD at the circumference.

⇒ ∠CBD = ∠CAD = 70°

⇒ ∠DAB = ∠DAC + ∠BAC = 70° + 30° = 100°

Since opposite angles of a cyclic quadrilateral are supplementary, we get ∠DAB + ∠BCD = 180°

⇒ ∠BCD = 80°

Example 10:

The exterior angle of a regular polygon is one-third of its interior angle. How many sides does the polygon have?

Solution:

Each exterior angle of a regular polygon of n sides = $\frac{360^\circ}{n}$

Each interior angle of a regular polygon of n sides = $(2n-4)\times\frac{90^\circ}{n}$

Using the given information, we get

$\frac{360^\circ}{n} = \frac{(2n-4)}{3n}\times 90^\circ$

12 = 2n - 4 ⇒ 2n = 16 ⇒ n = 8

Hence, the given polygon has 8 sides.

Alternative method:

Exterior angle + Interior angle = 180°

⇒ Exterior angle + 3 exterior angles = 180°

$\Rightarrow$ 4 exterior angle = 180°

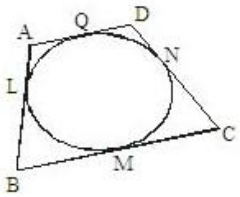
$\Rightarrow$ Each exterior angle = 45°

$\therefore$ Number of sides of the regular polygon = $\frac{360}{45} = 8$

Example 11:

If the sides of a quadrilateral touch a circle, prove that the sum of a pair of opposite sides is equal to the sum of the other pair of sides.

Solution:



ABCD is a quadrilateral touching the circle at L, M, N and Q.

Length of the two tangents from an external point to a given circle are equal in length.

Hence, AL = AQ, BL = BM,

CN = CM, DN = DQ

On adding, we get AL + BL + CN + DN = AQ + BM + CM + DQ

or, AB + CD = AD + BC

Example 12:

In a trapezium the distance between the parallel sides is 10 cm and the length of the line joining the mid-points of the oblique sides is 5 cm. Find the area of the trapezium.

Solution:

Area of a trapezium = $\frac{1}{2}$(Sum of parallel sides) $\times$ (Distance between them)

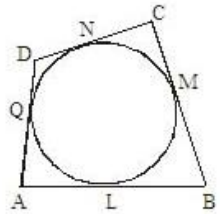
We know that $\frac{\text{Sum of parallel sides}}{2}$ = Length of the segment joining mid-points of the oblique sides

$\Rightarrow$ Required area = $\frac{1}{2}(5 \times 2) \times 10 = 50 \text{ cm}^2$

Example 13:

If all the sides of a parallelogram ABCD touch a circle, what kind of a figure is ABCD?

Solution:



ABCD is a parallelogram.

$\therefore$ AB = CD and AD = BC. But in Example 11, we have already seen that in a similar situation,

AB + CD = AD + BC

$\therefore$ 2AB = 2AD

$\Rightarrow$ AB = AD.

If in a parallelogram adjacent sides are equal, then the parallelogram is a rhombus. Hence, ABCD is a rhombus.

Test Your Understanding

Level - 1

1. Find the length of a chord, which is at a distance of 4 cm from the centre of a circle whose radius is 5 cm.

2. In a circle of radius 5 cm, AB and CD are two parallel chords of length 8 cm and 6 cm respectively. Calculate the distance between the chords, if they are on

(i) the same side of the centre.

(ii) opposite sides of the centre.

3. Two circles touch internally at P and a straight line ABCD meets the outer circle in A and D, and the inner circle in B and C. Is $\angle APB = \angle CPD$?

4. O is the centre of the circle, PA and PB are tangents to the circle from point P. If $\angle PAB = 70^0$, find $\angle PBA$ and $\angle APB$.

5. Three circles with equal radii touch one another externally. Prove that the triangle formed by joining their centres is an equilateral triangle. Find the altitude of the equilateral triangle.

6. AB and CD are two chords intersecting at a point P inside the circle such that AP = CP. Prove that AB = CD.

7. ABCD is a quadrilateral inscribed in a circle and the diagonal BD bisects AC. Prove that AD . AB = CD . CB

8. Two circles having radii r_1 and r_2 respectively such that $r_1 < r_2$, intersects each other at points P and Q. If AB and AC are the tangents to two circles from point A on QP produced then, which of the following is true?

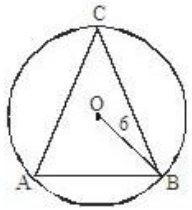
a. AB > AC b. AB < AC c. AB = AC d. Data insufficient

Level - 2

9. O is the centre of a circle of radius 5 cm. P is any point in the circle such that OP = 3 cm.
A is the point travelling along the circumference. X is the distance from A to P. What is the least and greatest values of x in centimetres?

10. Three circles with centres A, B and C touch each other externally at P, Q and R. If AB = 5 cm, BC = 7 cm and CA = 8 cm, find the radius of each circle.

11. An equilateral triangle is inscribed in a circle of radius 6 cm as shown in figure given below. Find its sides.



12. Two circles touch each other externally at P. APC and BPD are drawn through P to meet the two circles in A, B and C, D respectively. Prove that (i) Δ PAB ~ Δ PCD, (ii) AB || CD.

13. CD is a tangent at C to the circumcircle of Δ ABC intersecting AB produced at D. Prove that Δ DBC ~ Δ DCA.

8. Two circles having radii r_1 and r_2 respectively such that $r_1 < r_2$, intersects each other at points P and Q. If AB and AC are the tangents to two circles from point A on QP produced then, which of the following is true?

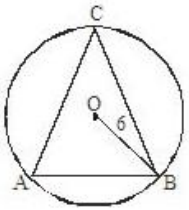
a. AB > AC b. AB < AC c. AB = AC d. Data insufficient

Level - 2

9. O is the centre of a circle of radius 5 cm. P is any point in the circle such that OP = 3 cm.
A is the point travelling along the circumference. X is the distance from A to P. What is the least and greatest values of x in centimetres?

10. Three circles with centres A, B and C touch each other externally at P, Q and R. If AB = 5 cm, BC = 7 cm and CA = 8 cm, find the radius of each circle.

11. An equilateral triangle is inscribed in a circle of radius 6 cm as shown in figure given below. Find its sides.



12. Two circles touch each other externally at P. APC and BPD are drawn through P to meet the two circles in A, B and C, D respectively. Prove that (i) Δ PAB ~ Δ PCD, (ii) AB || CD.

13. CD is a tangent at C to the circumcircle of Δ ABC intersecting AB produced at D. Prove that Δ DBC ~ Δ DCA.

14. ABCD is cyclic quadrilateral. A circle passing through A and B meets AD and BC at E and F respectively. Is EF || DC?

15. A chord of length 10 cm subtends an angle of 120° at the centre of a circle. Calculate maximum distance between the mid-point of such a chord and the mid-point of another chord that subtends 90° at the centre of the circle.

Level - 3

16. Two parallel lines cut two lines AB, AC at points D, B, C and E respectively. Show that the circle drawn through A, B, C and A, D, E touch each other.

17. There are two concentric circles. A particular chord of the larger circle is also tangent to the smaller one. If chord length happens to be 6 cm, find the difference in the radii of two circles if both the radii are integral multiple of a cm.

Mensuration

Learning Objectives

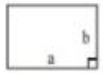
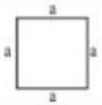
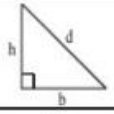
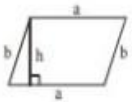
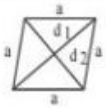
- Right circular cone
- Right circular cylinder
- Spheres
- Prism and Pyramid
- Cubes and Cuboid

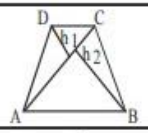
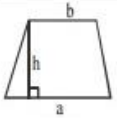
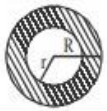
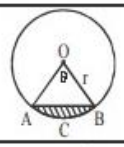
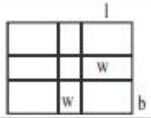
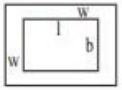
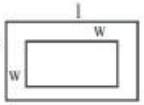
Mensuration

Measurement is fundamental to the science, engineering, construction, other technical fields and to almost all everyday activities. The branch of mathematics that is concerned with measurement of length, surface, and volume is called mensuration.It deals with two and three-dimensional geometrical figures, such as squares, triangles, circles, cubes, spheres, and cylinders.

Following is an exhaustive list of common formulae and relation on which questions are generally asked.

Two dimensional Figures:

S.No.	Name	Figure	Perimeter	Area	Nomenclature
1.	Rectangle		2 (a + b)	ab	a = Length b = Breadth
2.	Square		4a	a^2	a = Side
3.	Triangle		a + b + c = 2s	1. $\frac{1}{2}b \times h$ 2. $\sqrt{s(s-a)(s-b)(s-c)}$	b is the base and h is the altitude. a, b, c are three sides of Δ's is the semiperimeter
4.	Right angled triangle		b + h + d	$\frac{1}{2}bh$	d (hypotenuse) = $\sqrt{b^2+h^2}$
5.	Equilateral triangle		3a	1. $\frac{1}{2}ah$ 2. $\frac{\sqrt{3}}{4}a^2$	a = side h = Altitude = $\frac{\sqrt{3}}{2}a$.
6.	Isosceles right angled triangle		2a + d	$\frac{1}{2}a^2$	d (hypotenuse) = $a\sqrt{2}$, a = Each of equal sides.
7.	Parallelogram		2 (a + b)	ah	a = Side b = Side adjacent to a h = Distance between the parallel sides
8.	Rhombus		4a	$\frac{1}{2}d_1 \times d_2$	a = Side of rhombus, d_1, d_2 are the two diagonals.

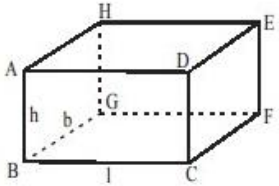
S.No.	Name	Figure	Perimeter	Area	Nomenclature
9.	Quadrilateral		Sum of its four sides	$\frac{1}{2}(AC)(h_1+h_2)$	AC is one of its diagonals and h_1, h_2 are the altitudes on AC from D, B respectively.
10.	Trapezium		Sum of its four sides	$\frac{1}{2}h(a+b)$	a, b are parallel sides and h is the perpendicular distance between parallel sides.
11.	Circle		Circumference $= 2\pi r$	πr^2	r = Radius of the circle $\pi = \frac{22}{7}$ or 3.1416 (approx.)
12.	Semicircle		$\pi r + 2r$	$\frac{1}{2}\pi r^2$	r = Radius of the circle
13.	Ring (shaded region)		$2\pi(R + r)$	$\pi(R^2 - r^2)$	R = Outer radius r = Inner radius
14.	Sector of a circle		$l + 2r$ where $l = \left(\frac{\theta}{360°}\right) \times 2\pi r$	$\left(\frac{\theta}{360°}\right) \times \pi r^2$	θ = Central angle of the sector r = Radius of the sector l = Length of the arc
15.	Segment of a circle		$\left(\frac{\theta}{360°}\right) \times 2\pi r + 2r \sin \theta/2$	Area of segment ACB (Minor segment) $= r^2\left[\frac{\pi\theta}{360°} - \frac{\sin\theta}{2}\right]$	r = Radius θ = Angle subtended at the center by the arc ACB
16.	Pathways running across the middle of a rectangle		- - - -	$A = w(l + b - w)$	l = Length b = Breadth w = Width of the path
17.	Pathways outside		$2[l+b+4w]$	$A = 2w(l + b + 2w)$	
18.	Pathways inside		$2[l+b-4w]$	$A = 2w(l + b - 2w)$	

Solids

Solids are the three dimensional objects, bound by one or more surfaces.

- When plane surfaces bound a solid, they are called **Faces.**
- The lines of intersection of adjacent faces are called its **Edges.**

Cuboid :



It has -

8 *Vertices* : A, B, C, D, E, F, G, H

12 *Edges* : AB, BC, CD, AD ....

6 *Faces* : ABCD, CDEF, ADEH...

4 *Body diagonals* : BE, CH, GD and AF

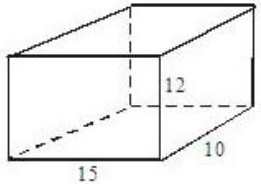
S. No.	Name	Figure	Lateral/curved surface area	Total surface area	Volume	Nomenclature
1.	Cuboid		----	$2(lb+bh+lh)$	lbh	l = Length b = Breadth h = Height
2.	Cube		----	$6a^2$	a^3	a = Edge
3.	Right prism		(Perimeter of base) x Height	2 (Area of base) + Lateral surface area	Area of base x Height	-
4.	Right circular cylinder		$2\pi rh$	$2\pi r(r+h)$	πr^2h	r = Radius of base h = Height of the cylinder
5.	Right pyramid		$\frac{1}{2}$(Perimeter of the base) x (Slant height)	Area of the base + Lateral surface area	$\frac{1}{3}$(Area of the base) x Height	
6.	Right circular cone		πrl	$\pi r(l+r)$	$\frac{1}{3}\pi r^2h$	h = Height r = Radius l = Slant height $=\sqrt{r^2+h^2}$
7.	Sphere		-	$4\pi r^2$	$\left(\frac{4}{3}\right)\pi r^3$	r = Radius
8.	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\left(\frac{2}{3}\right)\pi r^3$	r = Radius
9.	Spherical shell		-	$4\pi(R^2+r^2)$	$\left(\frac{4}{3}\right)\pi(R^3-r^3)$	R = Outer radius r = Inner radius

In many problems, a high level of visualization is required, and generally its always better to visualize the figure, which makes the problem solving much more easier.

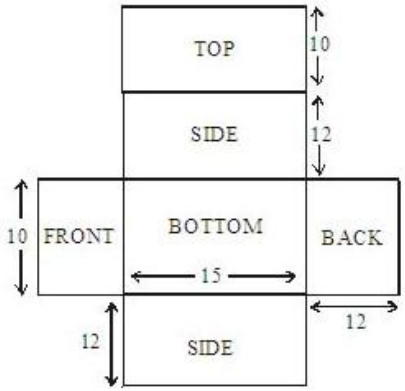
Following is a few opening up or unfolding of 3-dimensional solids in to 2-dimensional plane for better visualization.

Conversion of 3-D figure into 2-D

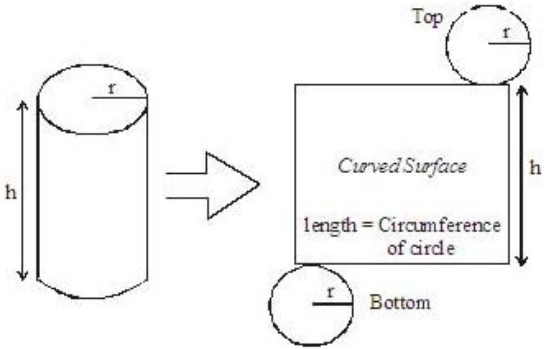
Cuboid : 3-dimensional cuboid box



Unfolding it into 2-dimensional plane.



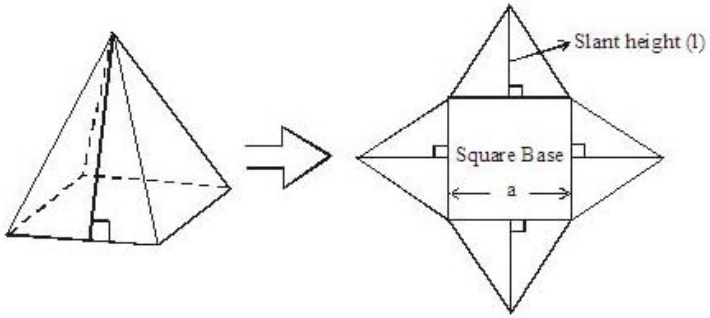
Right Cylinder: If you open the top and bottom of a cylinder and then make a vertical cut in the curved surface. You could lay the cylinder flat.



Square Pyramid:

As done in case of cuboid, upon unfolding square pyramid will look like a star fish, as shown in the figure.

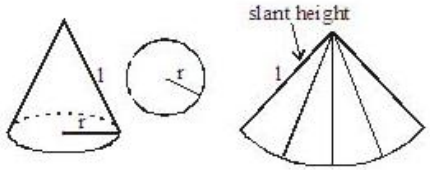
(Square-pyramid is called "square" because of its base, which is of square shape)



Perpendicular height (h) = $\sqrt{l^2 - \left(\frac{a}{2}\right)^2}$

Cone:

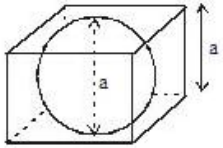
Base of the cone is circle. If its lateral surface (l) is cut vertically and opened, it will be a sector of a circle, with radius = l.



Solids inscribed/Circumscribed in other solids :

1. If largest possible sphere is circumscribed by a cube of edge `a' cm,

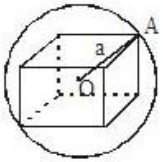
then the radius of the sphere = $\frac{a}{2}$



Remaining empty space in the cube = $a^3 - \frac{\pi a^3}{6}$

2. If the largest possible cube is inscribed in a sphere of radius `a' cm,

then the edge of the cube = $\frac{2a}{\sqrt{3}}$



Let, O → centre of sphere

OA - radius of the sphere

Major diagonal of cube = diameter = 2a

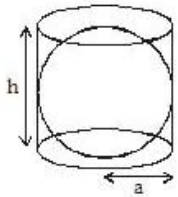
$\therefore$ side = $\frac{2a}{\sqrt{3}}$

Remaining empty space in the sphere = $\frac{4\pi a^3}{3} - \frac{8a^3}{3\sqrt{3}} = \frac{4a^3}{3}\left[\pi - \frac{2\sqrt{3}}{3}\right]$

3. If a largest possible sphere is inscribed in a cylinder of radius `a' cm and height `h' cm, then

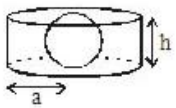
i) For h > a

radius of sphere = a



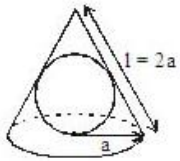
ii) For a > h

radius of the sphere = $\frac{h}{2}$



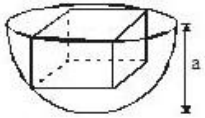
4.If a largest possible sphere is inscribed in a cone of radius `a' cm and slant height equal to diameter of the base.

The radius of the sphere = $\frac{a}{\sqrt{3}}$



5. If a largest possible cube is inscribed in a hemisphere of radius `a' cm,

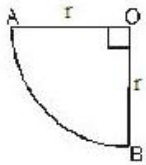
The edge of the cube = $a\sqrt{\frac{2}{3}}$



Example 1:

Find the volume of a right circular cone formed by joining the edges of a sector of a circle of radius r cm where the sector angle is 90°.

Solution:



The circumference of the base of the cone $= 2\pi a$, where `a' is the base radius.

$\therefore 2\pi a = \frac{2\pi r}{4}$ or $a = \frac{r}{4}$ ... (I)

Height of the cone $= \sqrt{r^2 - a^2}$

$h = \sqrt{r^2 - \frac{r^2}{16}} = \frac{\sqrt{15}}{4}\ r$

Volume of the cone $= \frac{1}{3}\pi a^2 h = \frac{1}{3}\pi \frac{r^2}{16} \times \frac{\sqrt{15}}{4} r = \frac{\pi r^3 \sqrt{15}}{192}\ cm^3$

Example 2:

A rectangular sheet of iron foil is 44 cm long and 20 cm wide. A cylinder is made out of it by rolling the foil once along the width. Find the volume of the cylinder.

Solution:

The cylinder obtained from the foil has perimeter 44 cm and height 20 cm

$\Rightarrow 2\pi R = 44 \Rightarrow R = \frac{22}{\pi}\ cm = 7\ cm$

$\Rightarrow$ Volume of the cylinder $= \pi r^2 h = \pi \times 7^2 \times 20 = 3078.8\ cm^3$

Example 3:

A sector of a circle of radius 12 cm has an angle of 120°. If it is made into a cone, find the volume of the cone.

Solution:

Arc of the sector $= \frac{120}{180} \times \pi \times 12\ cm = 8\pi\ cm$

$\Rightarrow$ The circumference of the base of cone $= 8\pi$ cm

Let R cm be radius of the base of the cone

$\Rightarrow 2\pi R = 8\pi \Rightarrow R = 4$ cm

Given L = 12 cm

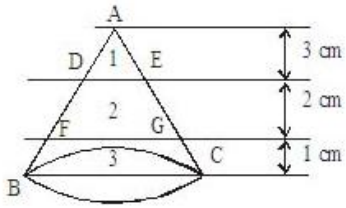
$\Rightarrow$ Height of the cone $= \sqrt{(L^2 - R^2)} = \sqrt{(144 - 16)} = 8\sqrt{2}\ cm$

$\Rightarrow$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 4^2 \times 8\sqrt{2} = 189.5\ cc$

Example 4:

A right circular cone has a height of 6 cm. If two cuts, parallel to the base, are made at height of
1 cm and 3 cm from the base, find the ratio of the volumes of the three parts from the top to the bottom?

Solution:



The front view of the cone looks like the figure.

The volume of the right circular cone with a base radius r and the height h is given by $\frac{1}{3}\pi r^2 h$ or $Kr^2 h$, where K is a constant. Let the volume of the biggest cone be V.

Volume of the cone formed by the combination of (1) and (2) is $\left(\frac{5}{6}\right)^3 (V) = \frac{125}{216} V$.

$$\left(\text{Since the height is } \frac{5}{6} \text{ of the original cone, corresponding radius will be } \frac{5}{6} \text{ of the original radius}\right)$$

Volume of the cone (1) $= \left(\frac{1}{2}\right)^3 V = \frac{V}{8}$.

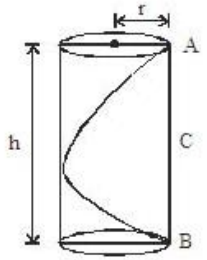
Hence, if V is, say, 216 units.

Then part (1) would have a volume of 27 units; part (2) would have 98 units; and part (3) would have 91 units. Hence, the ratio is 27 : 98 : 91.

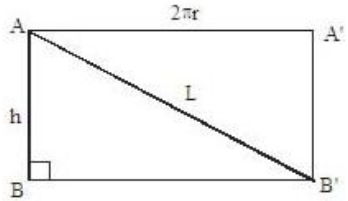
Example 5:

An insect travels from a point A on the rim of a cylindrical glass. It has a constant speed and travels in a spiral on the outer surface of the cylinder. By the time the insect completes one full spiral it reaches the bottom of the cylinder at a point directly below the starting point. Find the total distance travelled by the insect.

Solution:



If we cut the cylinder along ACB and unwrap it, then we get the following figure.



This is a rectangle whose length is $2\pi r$ and breadth is h. The distance travelled by the insect $L = \sqrt{h^2 + (2\pi r)^2}$.

Example 6:

A rectangular plot has an area of 490 m². If the length of the rectangular plot is 24.5 m, then find the perimeter of the rectangular plot.

Solution:

The perimeter of a rectangle = 2(l + b)

Here l = 24.5 and the area is 490 m².

Suppose its breadth is `y' m, then

$l \times y$ = area

$\Rightarrow 24.5 \times y = 490$

$\Rightarrow y = \frac{490}{24.5} = 20$ m

So the perimeter = 2(l + b) = 2(24.5 + 20) = 2 × 44.5 = 89 m.

Example 7:

There is a large room of size 24 ft × 32 ft. The floor of the room is to be paved with square tiles of the same dimension. Find out (i) the minimum number of tiles required and its size and (ii) the maximum number of tiles required.

Solution:

(i) Since the floor of the room is to be paved with minimum number of square tiles, so the size of the square tiles must be maximum. Since the size of the tile is square, so the maximum dimension of each tile is 8 ft × 8 ft (HCF of 24 and 32).

And the number of tiles required = $\frac{\text{Area of the floor}}{\text{Area of 1 tile}} = \frac{24 \times 32}{8 \times 8} = 12$

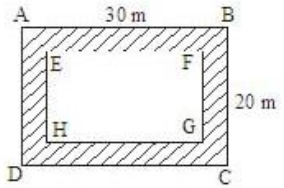
(ii) It's answer is infinity! How? Think over it.

Example 8:

There is a rectangular field having dimension 20 m × 30 m. There is a jogging track inside the field along its perimeter. The width of the track is uniform and it is equal to 2 m. Find the total perimeter of the jogging track and the area of the jogging track.

Solution:

ABCD is the rectangular region field and the shaded in it is the jogging track.



Perimeter of the jogging track = Perimeter of ABCD + Perimeter of EFGH

EF = 30 - (2 × 2) = 26 m because the width of the track is 2 m on either side.

Similarly, FG = 20 - (2 × 2) = 16

Perimeter = 2(30 + 20) + 2(26 + 16) = 100 + 84 = 184 m

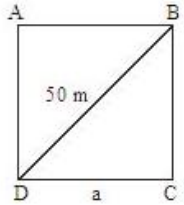
Area of the jogging track = Area of ABCD - Area of EFGH = (30 ×20) - (26 × 16) = 600 - 416 = 184 m^2

Note: Here it is just a chance that the answer of the perimeter and the area of jogging track is coming same.

Example 9:

The diagonal of a square is 50 m. Find the length of its side?

Solution:



We know that diagonal of a square = √2 × side

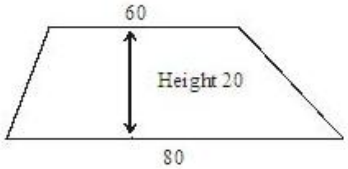
So, BD = √2a = 50

$\Rightarrow a = \frac{BD}{\sqrt{2}} = \frac{50}{\sqrt{2}} = 25\sqrt{2}$ m

Example 10:

There is a flower bed in the shape of trapezium. The length of its parallel sides are 60 m and 80 m. If the distance between them is 20 m, then find the area of the flower bed.

Solution:



Area of a trapezium = $\frac{1}{2}$ (sum of the parallel sides) × height

$= \frac{1}{2}(60 + 80) \times 20 = \frac{1}{2} \times 140 \times 20 = 1400\ m^2$

Example 11:

If the area of square is $3\sqrt{3}$ times the area of an equilateral triangle, then find the ratio of the sides of the square to the side of equilateral triangle.

Solution:

Let the side of the square be `x' m and the side of the equilateral triangle be `y' m, then

$\therefore$ Area of the square x^2 and area of the equilateral triangle $= \frac{\sqrt{3}}{4}y^2$

$\Rightarrow x^2 = 3\sqrt{3}\left(\frac{\sqrt{3}}{4}y^2\right)$

$\Rightarrow \frac{x^2}{y^2} = \frac{3\sqrt{3} \times \sqrt{3}}{4} = \frac{3 \times 3}{4} = \frac{9}{4}$

$\Rightarrow \frac{x}{y} = \sqrt{\frac{9}{4}} = \frac{3}{2}$.

So, ratio of the length of side of a square to the length of side of an equilateral triangle is 3 : 2.

Example 12:

If the length of the rectangle is increased by 100% and the width is increased by 200%, what is the percentage change in the area of the rectangle?

Solution:

Let the length of the rectangle be `x' m and the width of the rectangle be `y' m. Area of the rectangle is xy m^2.

Its new length is 2x and new width is 3y, so its new area is $2x \times 3y = 6xy\ m^2$

So, percentage change in area of the rectangle $= \left(\frac{6xy - xy}{xy}\right) \times 100 = \frac{5xy}{xy} \times 100 = 500\%$

Alternate approach: For successive percentage change we can use the formula

$a + b + \frac{a \times b}{100} = 100 + 200 + \frac{100 \times 200}{100} = 300 + 200 = 500\%$

Example 13:

There is a rope of length 44 m. A circle is made by this rope. Find the diameter of the circle formed.

Solution:

Perimeter of the circle formed = length of the rope 2pr = 44, where `r' is the radius of the circle

$\Rightarrow 2 \times \frac{22}{7} \times r = 44 \Rightarrow r = \frac{44 \times 7}{22 \times 2} = 7$

$\therefore$ Diameter = 2r = 2 × 7 = 14 m

Example 14:

There are 2 spherical balls, each having a radius of 4 m. By melting both balls, a bigger spherical ball is made. Find the radius of the new ball.

Solution: Total volume (of 2 balls) = $2 \times \frac{4}{3} \times \pi (4)^3 = \frac{2}{3} \times \pi (4)^4$

So, $\frac{2}{3} \times \pi (4)^4$ is the volume of the bigger ball.

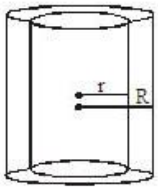
Let the radius of the bigger ball be `r' m.

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{2}{3}\pi (4)^4 \Rightarrow 2r^3 = 4 \times 4 \times 4 \times 4 \Rightarrow r^3 = 2 \times 64$ and r = $\sqrt[3]{128} = 4\sqrt[3]{2}$ m

Example 15:

There is a cylindrical pipe. Its height is 20 m, outer radius is 2 m and inner radius is 1.5 m. Find the volume of the metal used in the pipe.

Solution:



Let the inner radius of the pipe be r and the outer radius of the pipe be R.

So the required volume = Total volume of the cylinder - Volume of hollow cylinder

$= \pi R^2 h - \pi r^2 h = \pi h(R^2 - r^2) = \pi 20\,[(2)^2 - (1.5)^2] = \pi 20(2+1.5)(2-1.5)$

$= \pi 20 \times 3.5 \times 0.5 = 35\pi = 35 \times \frac{22}{7} = 110\ m^3$

Example 16:

A field is in the form of a rectangle having length 20 m and breadth 15 m. There is a square pit having dimension 15 m × 15 m. This pit is to be filled uniformly upto a height of 4 m by the soil taken out by digging the rectangular field. Find out the depth upto which the rectangular field must be dig if the soil is taken out uniformly?

Solution:

To fill the pit, the volume of soil required = 15 × 15 × 4 = 900 m^3

Let `h' be the depth upto which the rectangular field is dug.

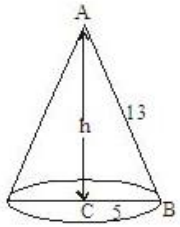
$\therefore$ 20 × 15 × h = 900

$\Rightarrow h = \frac{900}{20 \times 15} = 3$ m

Example 17:

The slant height and diameter of a conical tomb is 13 m and 10 m respectively. Find the cost of constructing tomb at the rate of Rs. 2 per m^3.

Solution:



Height of the cone $= \sqrt{AB^2 - BC^2} = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$ m

Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 5 \times 5 \times 12 = 100\pi m^3$

So, the cost of construction = $100 \times \frac{22}{7} \times 2$ = Rs. 628.57

Example 18:

Find the area of a rectangle whose area is equal to the area of a circle with perimeter equal to 24π.

Solution:

Let the radius of the circle be `r'

$\Rightarrow 2\pi r = 24\pi \Rightarrow r = 12$

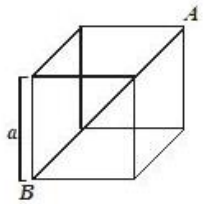
Area of the circle = $\pi(12)^2$ = Area of the rectangle = 144π

Example 19:

The side of a hollow cube is 4 m. What is the length of the largest pole that can be fit into it?

Solution:

The length of the largest pole = largest diagonal of cube, i.e. AB



AB = $\sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2} = a\sqrt{3} = 4\sqrt{3}$ m.

Where a = side of cube.

Example 20:

Three cubes of sides 4 cm, 5 cm and 6 cm are melted together and a bigger cube is made. Find the side of the new cube?

Solution:

Total volume of the cubes $= 4^3 + 5^3 + 6^3 = 64 + 125 + 216 = 405 \text{ cm}^3$

Side of the new cube = $\sqrt[3]{405} = \sqrt[3]{3 \times 3 \times 3 \times 3 \times 5} = 3\sqrt[3]{15}$ cm

Example 21:

A wire bent in the form of a circle of radius 2 cm is cut and bent in the form of a square. Find the ratio of the area of the circle to the area of the square.

Solution:

Area of the circle = $\pi r^2 = \pi(2)^2$

The circle is cut to make a square, so the side of the square `s' = $\frac{2\pi r}{4} = \frac{2 \times \pi \times 2}{4} = \pi$ cm

Area of the square = $\pi \times \pi = \pi^2$ cm^2

Ratio of the area of the circle and square = $\frac{\pi(2)^2}{\pi^2} = \frac{4}{\pi}$

Example 22:

A spherical copper ball, whose diameter is 14 cm, is melted and converted into a wire having diameter equal to 14 cm. Find the length of the wire.

Solution:

Volume of the ball = $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 7 \times 7 \times 7$

Let the length of the wire be `h' m

$\Rightarrow \frac{4}{3}\pi \times 7 \times 7 \times 7 = \pi \times 7 \times 7 \times h\ (\pi r^2 h)$

$\therefore h = \frac{28}{3}$ cm

Example 23:

Two cylindrical jars have their diameter in the ratio 2 : 3 and their heights are in the ratio 3 : 2. Find the ratio of their volumes.

Solution:

Volume of a cylinder = $\pi r^2 h$

So ratio of their volumes = $\frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{\pi(2)^2 \times 3}{\pi(3)^2 \times 2} = \frac{12\pi}{18\pi} = \frac{2}{3}$

Example 24:

The volume of the metal used in a hollow cylindrical pipe is 3300 cm³. If its length is 28 cm and external radius is 21 cm, then find the thickness of the cylindrical pipe.

Solution:

Volume of the metal = $\pi h(R^2 - r^2) = \frac{22}{7} \times 28\{(21)^2 - (r)^2\} = 3300 \Rightarrow 21^2 - r^2 = 37.5$

$\Rightarrow$ r = 20 cm (approx.)

$\therefore$ the thickness of the cylinder = R - r = 21 - 20 = 1 cm (approx.)

Example 25:

There is a room having dimensions 10 m × 15 m × 20 m. Find the cost of painting the four walls at the rate of Rs. 2.5 per m².

Solution:

Total area of the four walls = 2(l + b)h = 2(10 + 15) × 20 = 1000 m²

So, total cost = 1000 × 2.5 = Rs. 2500

Example 26:

A horse is tied with a rope of length 7 m at one corner of a square field having side equal to 10 m. Find the minimum possible area of the square field that is left ungrazed.

Solution:

Total area of the field = 10 × 10 = 100 m²

The area of the field grazed by the horse = $\frac{90°}{360°} \times \pi \times 7 \times 7 = \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2} m^2$

So area of the ungrazed part of the field = Area of the shaded region = $100 - \frac{77}{2} = \frac{123}{2} = 61.5\ m^2$

Example 27:

The length of a rectangular field is
3 times its width. If the perimeter of the field is 24 m, then find the area of the field.

Solution:

Perimeter = $2(l + b) = 24$

$\Rightarrow 2(3b + b) = 24$

$\Rightarrow 8b = 24$

$\therefore b = 3$ and $l = 9$

So area of the field = $l \times b = 3 \times 9 = 27\ m^2$

Example 28:

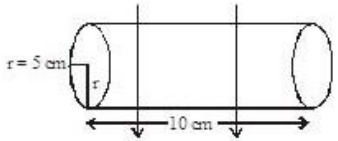
A cube has a side of 4 m. Find its total surface area.

Solution:

Total surface area of a cube = $6 \times side^2 = 6 \times 4 \times 4 = 96\ m^2$

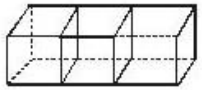
Test Your Understanding
Level - 1

1. Two cuts are made on the right circular cylinder as shown in the figure



What is the percentage increase in the surface area?

2. Six cubes each with 12 cm edge are joined end to end as shown in the figure. Find the surface area of the resulting cuboid.



3. The outer dimensions of a closed box are 42 cm, 30 cm and 20 cm. If the box is made up of wood of thickness 1 cm, determine the volume of wood used.

4. Water flows at the rate of 10 metres per minute from a cylindrical pipe 5 mm in radius. How long would it take to fill a right circular conical vessel whose diameter at the top is 40 cm and depth is 24 cm?

5. A cylinder of minimum height is circumscribed about a hemisphere and a cone is inscribed in the cylinder so as to have its vertex at the centre of the top surface of the cylinder and its base coincides with the base of the cylinder. Find the ratio of the volumes of the cylinder, hemisphere and the cone.

6. A right circular cylinder and a right circular cone have equal bases and equal heights. If their curved surfaces are in the ratio 8 : 5, what is the ratio of the radius of their base to their height?

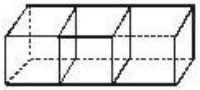
7. A spherical ball of lead has been melted and made into smaller balls of half of the radius of the original one. How many such balls can be made? Find the ratio of the total surface area of a smaller ball to that of the original one.

8. Three spheres of radii 1, 2, 3 cm are melted to form a new sphere. Find the radius of the new sphere

Level - 2

9. Two poles of height `a' and `b' units respectively are `c' units apart on a plane surface. The height `h' of the point of intersection of the lines joining the top of each pole to the

2. Six cubes each with 12 cm edge are joined end to end as shown in the figure. Find the surface area of the resulting cuboid.



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Level - 2

9. Two poles of height `a' and `b' units respectively are `c' units apart on a plane surface. The height `h' of the point of intersection of the lines joining the top of each pole to the foot of opposite pole is _____ of the height of two poles.

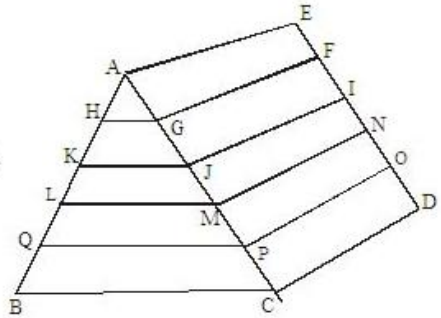
a. AM b. GM c. $\frac{HM}{2}$ d. None of these

10. The height of a right circular cone is 40 cm and the radius of the base is 28 cm. Find the area of cross section of the cone formed by a plane parallel to the base of the cone and at a distance of 10 cm from the vertex of the cone.

11. The radii of the bottom and top ends of a frustum of cone 24 cm high, are 5 cm and 15 cm respectively. Find the surface area.

12. A frustum is that part of a cone that is formed when a cone is cut by a plane parallel to its base and the top portion is removed. If the volume of a frustum is F and volume of the original cone is C and their heights are h_f and h_c, find the ratio of h_f and h_c.

13.



For the figure shown

AG = GJ = JM = MP = PC and AH = HK = KL = LQ = QB

EF = FI = IN = NO = OD

If Area of strip HG JK = 17 sq. units then find the area of strip QP CB

Trigonometry

Learning Objectives

- Basics of Trigonometry
- Height and Distance

Trigonometry

Introduction

Trigonometry is the study of the relationship between the angles and the sides of a triangle.

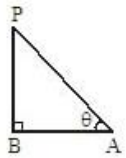
Angle Measures:

Angle are measured in many units viz. degree, minute, seconds, radians. We have

1 degree = 60 minutes, 1 minute = 60 seconds, p radians = 180°

Trigonometrical Ratios:

In a right angled triangle ABP, if θ be the angle between AP and AB we define



1. $\sin\theta = \frac{\text{Height}}{\text{Hypotenuse}} = \frac{PB}{AP}$ 2. $\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AP}$

3. $\tan\theta = \frac{\text{Height}}{\text{Base}} = \frac{PB}{AB}$ 4. $\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Base}}{\text{Height}} = \frac{AB}{PB}$

5. $\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AP}{AB}$ 6. $\text{cosec}\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Height}} = \frac{AP}{PB}$

Important Formulae:

1. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

2. $\sin^2\theta + \cos^2\theta = 1$

3. $1 + \tan^2\theta = \sec^2\theta$

4. $1 + \cot^2\theta = \text{cosec}^2\theta$

Trigonometric measures of certain angles:

Angle	sinθ	cosθ	tanθ
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	∞

Signs of trigonometric ratios

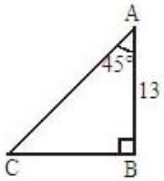
IInd quadrant Here, only sin and cosec are positive.	**Ist quadrant** Here all ratios (sin, cos, tan, sec, cosec, cot) are positive.
IIIrd quadrant Here, only tan and cotare positive.	**IVth quadrant** Here, only cos and secare positive.

You can remember above table as

School	After
To	College

Example 1:

In ΔABC, $\angle A = 45°$, $\angle B = 90°$ and $AB = 13$ cm. Find BC.



Solution:

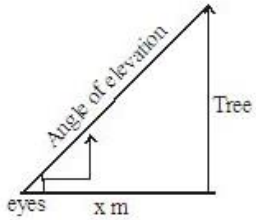
$\tan A = \frac{BC}{AB}$; $\tan 45° = \frac{BC}{13}$

$\Rightarrow BC = 13 \times \tan 45° = 13 \times 1 = 13$

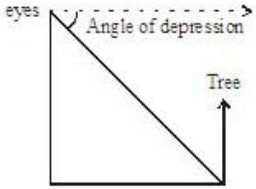
$\therefore BC = 13$ cm

Angle of Elevation and Depression

Angle of elevation: Suppose that there is a big tree and our eyes and the bottom of the tree is in one horizontal. We want to see the top of the tree, so what will happen in that process we will lift up our head to view the top of the tree. In this process our eyes cover certain angle and this angle is called the angle of elevation.

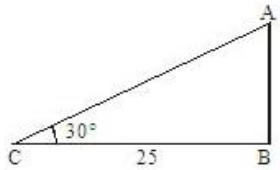


Angle of depression: Suppose we are standing on a tower and there is a tree whose height is less than that of tower. We are facing straight and want to see the bottom of the tree, so what will happen in that process, we will move down our head to view the bottom of the tree. In this process our eyes cover certain angle and this angle is called angle of depression.



Example 2:

A man is standing at a distance of 25 m from the bottom of the tree, and he finds that the angle of elevation of the top of the tree is 30°. Find the height of tree.



Solution:

The man's eyes are at point C.

AB is the height of the tree and BC is given equal to 25 m and ∠ACB = 30°.

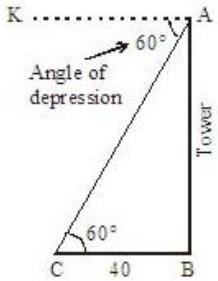
$\tan 30° = \frac{AB}{BC} = \frac{AB}{25}$

So, $\frac{1}{\sqrt{3}} = \frac{AB}{25}$ and $AB = \frac{25}{\sqrt{3}}$

So the height of the tree is $\frac{25}{\sqrt{3}}$ m.

Example 3:

A tower is at a distance of 40 m from an object. From the top of the tower the angle of depression of the object is 60°. Find the height of the tower.



Solution:

AB is a tower and C is the object.

If we draw a line AK parallel to BC, then

∠KAC = ∠ACB (Alternate angles)

$\tan 60° = \frac{AB}{BC} = \frac{AB}{40}$

$\Rightarrow \sqrt{3} = \frac{AB}{40}$

So, AB = $40\sqrt{3}$. So the height of the tower is $40\sqrt{3}$ m.

Cosine Rule:

In the adjoining figure, ABC is a triangle, A, B and C are the vertices of the triangle, side opposite to angle A is denoted by `a', opposite to angle B is denoted by `b' and opposite to C is denoted by `c'. The relationship among their sides and angle is.

$a^2 + b^2 - c^2 = 2ab\cos C$

$b^2 + c^2 - a^2 = 2bc\cos A$

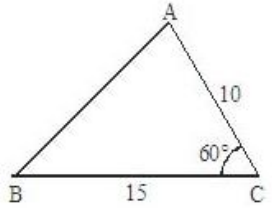
$c^2 + a^2 - b^2 = 2ac\cos B$

Sine Rule:

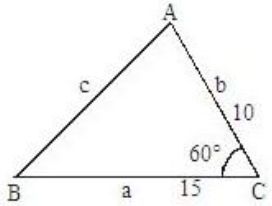
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Example 4:

In the adjacent figure, AC = 10 cm,
BC = 15 cm, and ∠C = 60°, find the side AB.



Solution:



We know that

$a^2 + b^2 - c^2 = 2ab\cos C$

(Cosine rule)

$\Rightarrow (15)^2 + (10)^2 - c^2 = 2 \times 10 \times 15 \cos 60°$

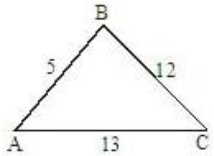
$\Rightarrow 225 + 100 - c^2 = 300 \times \frac{1}{2}$

$\Rightarrow c^2 = 325 - 150 = 175$

$\therefore c = \sqrt{175} = 5\sqrt{7}$ cm

Example 5:

In the figure given below find the measure ∠ABC. where AB = 5 cm, BC = 12 cm and AC = 13 cm.



Solution:

We know that

$a^2 + c^2 - b^2 = 2ac\cos B$

$\Rightarrow (12)^2 + (5)^2 - (13)^2 = 2 \times 12 \times 5 \cos B$

$\Rightarrow 169 - 169 = 120 \cos B$

So, $\cos B = \frac{0}{120} = 0$

Since cosB = 0

So, B = 90° [cos90° = 0]

So, ∠ABC = 90°

Note: Remember that 5, 12, and 13 is a pythagorean triplet.

Hence, $\angle ABC = 90°$

Test Your Understanding
Level - 1

1.(i) The angle of elevation of the top of a lamp post when observed from a point 40 m from the foot of the lamp post is 30°. Find the height of the lamp post.

(ii) The angle of elevation of a tower at a point 60 m from it's base is $\cot^{-1}\left(\frac{3}{5}\right)$. Obtain the height of the tower.

(iii) What is the angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole?

2.(i) From the mast head of a ship 150 m high, the angle of depression of a boat is observed to be 45°. Find its distance from the ship.

(ii) From the top of a light house 60 m high with its base at the sea level, the angle of depression of a boat is 15°. Find the distance of the boat from the foot of the lighthouse.

(iii) From the top of a cliff of 100 m height, a person observes the angle of depression of a boat which is at a distance of $100(2 - \sqrt{3})$ m. Determine the angle of depression of the boat.

[Given that $\tan 15^\circ = (2 - \sqrt{3})$]

3.(i) The angle of elevation of the top of a tower at a point A on the ground is 30°. On walking 20 m towards the tower, the angle of elevation changes to 60°. Find the height of the tower and its distance from A.

(ii) The angle of elevation of the top of an unfinished tower at a point distant 120 m from its base is 45°. By how much height must the tower be raised so that its angle of elevation at the same point may be 60°?

4. (i) Two pillars of equal height stand at a distance of 100 m. At a point on the ground between them, the elevation of their tops are found to be 30° and 60°. Determine the heights of pillars and the position of the point of observation.

(ii) Two towers are standing on a level ground. From a point on the ground midway, between them, the angles of elevation are 60° and 30° respectively. If the height of the first tower is 100 m, find the height of the second tower.

Level - 2

5. Two pillars of equal heights are situated on either side of a roadway which is $200\sqrt{3}$ m wide. At a point on the roadway between the pillars, the elevation of the top of pillars are 45° and 75°. Find their heights and position of the point. [Given that $\tan 75^\circ = (2 + \sqrt{3})$]

6. The angle of elevation of the top of a tower from the bottom of the building is double than that from its top. If the height of the building is 50 m and the height of the tower measured from the level of the bottom of the building is 75 m, find the angle of elevation of the tower from the bottom of the building.

7.(i) The shadow of a tower standing on a level plane is found to be 60 m longer when the sun's altitude is 30° than that when the sun's altitude is 45°. Show that the height of the tower is $30(1 + \sqrt{3})$ m.

(ii) A person standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60°. When he retires 40 m from the bank perpendicular to it, he finds the angle to be 30°. Find the height of the tree and the breadth of the river.

Practice Exercises

Introduction

There are 5 practice exercises out of which 2 are of level-1, 2 are of level 2 and 1 is of level 3 to strengthen you fundamental. While solving the exercises make sure that each and every concept is understood properly.

Practice Exercise 1 - Level 1

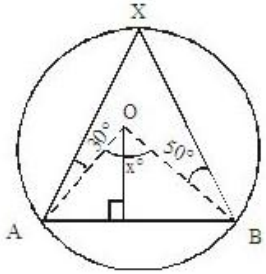
1. From a tower 100 m high, the angle of depression of car is 30°. Find how far is the car is from the tower.

a. $100\sqrt{3}$ m b. 100 m c. $100\sqrt{2}$ m d. $50\sqrt{2}$ m e. $50\sqrt{3}$

2. A tower stands vertically on the ground. At a point on ground 20 m away from the foot of tower, the angle of elevation of the top of tower is 30°. Find the height of tower.

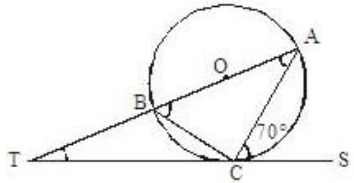
a. $\frac{20}{\sqrt{3}}$ m b. $20\sqrt{3}$ m c. 20 m d. 30 m e. 10 m

3. In the adjoining figure, find the value of x. [`O' is the centre of the circle].



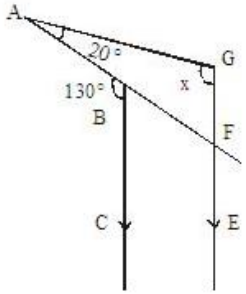
a. 170° b. 180° c. 80° d. 160° e. 90°

4. In the adjoining figure O is the centre of circle. Find the measure of ∠ATS.



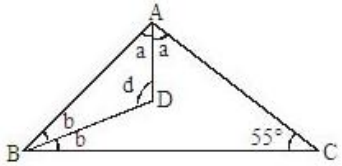
a. 60° b. 80° c. 70° d. 50° e. 20°

5. Find the value of x in the following figure.



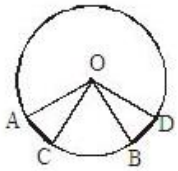
a. 70° b. 80° c. 110° d. 120° e. None of these

6. What is the value of `d' in given figure?



a. 107.5° b. 220° c. 100° d. 117.5° e. 210°

7. In the given figure, chord AB = chord CD, ∠AOB = 60° and ∠BOC = 15° then. Find the measure of ∠BOD.



a. 45° b. 50° c. 40° d. 120° e. 60°

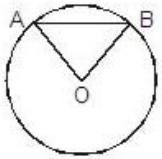
8. The diagonals AC and BD of a parallogram ABCD intersect at O. If ∠DAC = 30°, ∠AOB = 80° then find the measure of ∠DBC.

a. 150° b. 50° c. 120° d. 100° e. 70°

9. Triangle ABC and DEF are similar. If their areas are 64 cm² and 49 cm² and if AB is 7. Find the value of DE. (Assume AB and DE are the corresponding sides in the similar triangle.)

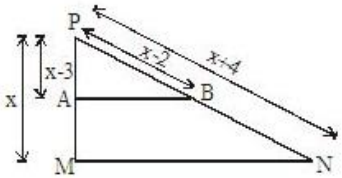
a. 50 cm b. $\frac{25}{3}$ cm c. $\frac{50}{4}$ cm d. $\frac{49}{8}$ cm e. $\frac{40}{7}$ cm

10. In the given circle, O is the centre of circle and ∠AOB = 132°. Find the measure of ∠ABO.



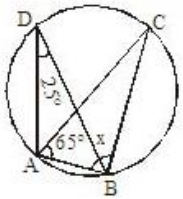
a. 72° b. 24° c. 20° d. 30° e. 25°

11. In the given figure AB | | MN. If PA = x - 3, PM = x, PB = x - 2 and PN = x + 4, then find the value of x.



a. 1 b. 2 c. 3 d. 4 e. 1.5

12. Find the value of ∠ABC in the given figure.



a. 90° b. 85° c. 80° d. 75° e. 70°

13. From the top of 200 m top pole, the angle of depression of two points is 30° and 45°. If one point is directly behind the other, then find the distance between the two points.

a. 150 m b. 100 m c. 142 m d. 146 m e. 146 m

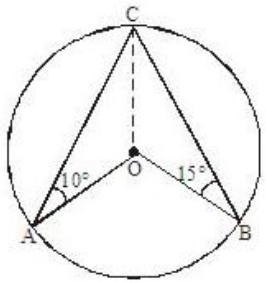
14. The difference between the interior and exterior angles respectively of a regular polygon is 120°. Find the number of sides of polygon.

a. 18 b. 12 c. 8 d. 10 e. 16

15. The angles of the quadrilateral are in ratio of 2: 4: 5: 7. Find the smallest angle.

a. 20° b. 40° c. 10° d. 30° e. 30°

16. Calculate the measure of ∠AOB in the figure given below. [`O' is the center of the circle]



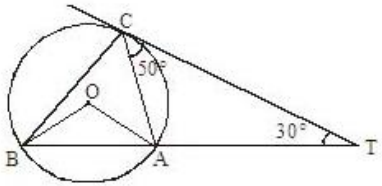
a. 80° b. 70° c. 40° d. 60° e. 50°

17. Two circles having radii 5 cm and 9 cm touch each other externally. Find the distance between their centres.

a. 12 cm b. 16 cm c. 14 cm

d. 18 cm e. cannot be determined

18. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If $\angle ATC = 30°$ and $\angle ACT = 50°$, then the angle $\angle BOA$ is



a. 100° b. 150° c. 80°

d. 60° e. Cannot be determined

19. A kite in the shape of rhombus has one of the diagonal is of length 20 m and one side of the length $5\sqrt{13}$ m. How much will it cost to paint both the faces of the kite at 20 paise/m²?

a. Rs. 150 b. Rs. 240 c. Rs. 60 d. Rs. 120 e. Rs. 180

20. A closed wooden box when measured externally is 9 cm long, 7 cm broad and 6 cm high. If the thickness of the wood is half a centimetre, find the capacity of the box.

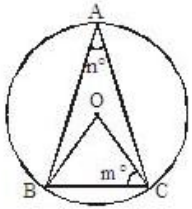
a. 378 cm² b. 315 cm² c. 324 cm² d. 240 cm³ e. 280 cm³

21. In a quadrilateral ABCD, the sides and diagonals are related to each other by

a. AB + BC + CD + DA = AC + BD b. AB + BC + CD + DA < AC + BD

c. AB + BC + CD + DA = AC × BD d. AB + BC + CD + DA > AC + BD e. None of these

22. As shown in the figure, O is the centre of the circle. ∠BCO = m° and ∠BAC = n°. Find out the relationship between m and n.

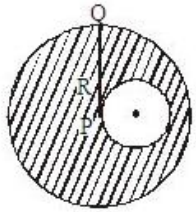


a. m + n = 180° b. 2m + n = 180° c. m + 2n = 180° d. m + n = 90° e. None of these

23. A horizontal pipe for carrying flood water has diameter 1 m. When water in it is 10 cm deep, what is the breadth of the water surface?

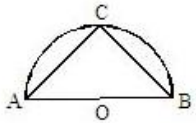
a. 30 cm b. 60 cm c. 50 cm d. 70 cm e. 45 cm

24. In the following figure, if PQ = R (P is the centre of the bigger circle) and the shaded area is equal to A, then
find the radius of the inner circle



a. $(\pi R - A)$ b. $\sqrt{\left(\pi R - \frac{A}{\pi}\right)}$ c. $\sqrt{R - \frac{A}{\pi}}$ d. $\sqrt{R^2 - \frac{A}{\pi}}$ e. None of these

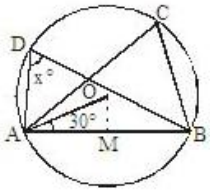
25. AB is a diameter of semicircle ACB, where C is the mid-point of arc ACB. Find the ratio of perimeters of the semicircle and ΔABC.



a. $(\pi+2)(\sqrt{2}-1):2$ b. $(\sqrt{2}+1):(\sqrt{2}-1)$ c. $\sqrt{2}:1$

d. $\pi+2:2\sqrt{2}$ e. $\sqrt{2}+2:1$

26. As shown in the figure, ∠MAO = 30° and the radius of the circle with center O is 4 cm. Find the value of x.



a. 30° b. 45° c. 55° d. 40° e. 60°

Practice Exercise 2 - Level 1

1. XYZ is a given triangle right-angled at the vertex Z. ZM is perpendicular from Z to the side XY. The point M is at distance 3 meters and 4 meters from the sides XZ and YZ respectively. Then the length of XZ is

a. 5.5 m b. 6.6 m c. 5.75 m d. 6.5 m e. 6.25 m

2. One litre of water is poured into two containers, A and B, of equal heights. A has a circular cross section of radius 2 cm and B has a square cross section with side 4 cm. In which container will the water rise to a greater height?

a. A

b. B

c. Equal to both

d. Depends on the rate at which water is poured

e. Cannot be determined

3. In a right-angled triangle, the product of two sides is equal to the half of the square of the third side, i.e. hypotenuse. One of the acute angles may be

a. 60° b. 30° c. 45° d. 15° e. 75°

4. A and B are centres of the two circles whose radii are 5 cm and 2 cm respectively. The direct common tangents to the circles meet AB extended at P. Then P divides AB

a. internally in the ratio 2 : 5

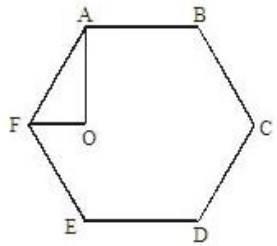
b. internally in the ratio 5 : 2

c. externally in the ratio 5 : 2

d. externally in the ratio 7:2

e. Data Insufficient

5. In the figure below, ABCDEF is a regular hexagon and $\angle AOF = 90^\circ$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?

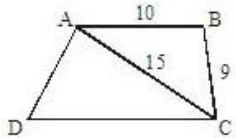


a. 1:12 b. 1:6 c. 1:24 d. 1:18 e. 1:36

6. A sphere has a diameter of $25\sqrt{3}$ mm. What is the surface area of the largest possible cube that would fit in the sphere?

a. 4,200 mm² b. 3,800 mm² c. 3,600 mm² d. 3,750 mm² e. 4,275 mm²

7. As shown in the figure, BA || CD, AB = 10 cm, BC = 9 cm, AC = 15 cm, and ∠CBA = ∠DAC. Find the length of AD.



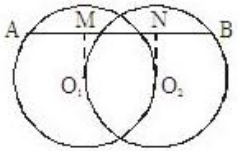
a. 13 cm b. $\frac{27}{2}$ cm c. $16\frac{2}{3}$ cm d. $\frac{25}{2}$ cm e. $\frac{31}{2}$ cm

Directions for question 8: Answer the questions independently of each other.

8. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be

a. 85.5 b. 92.5 c. 90.5 d. 87.5 e. 75

9. In the following figure, two circles of radii 100 ft each intersect as shown. The distance from the centres to the secant AB is 60 ft. Find AB. (O_1 and O_2 are the centres of the circles.)



a. 260 ft b. 180 ft c. 220 ft d. 200 ft e. 240 ft

10. If M is any point in the interior of ΔPQR; MA, MB and MC be the perpendiculars on sides PQ, QR and PR respectively, then

a. $PC^2 + QA^2 + RB^2 = CR^2 + BQ^2 + AP^2$ b. $CR^2 + BQ^2 + PC^2 = AP^2 + RB^2 + QA^2$

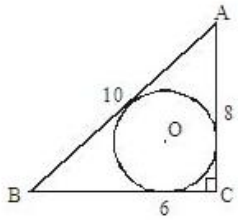
c. $PC^2 + QA^2 = CR^2 + BQ^2 + AP^2$ d. $RC^2 + BQ^2 + PC^2 = AP^2 + RB^2$

e. None of these

11. The co-ordinate of the equilateral triangle's vertices P and Q are (-4, 0) and (4, 0). The co-ordinate of the third vertex could be

a. $(2\sqrt{3}, 2\sqrt{3})$ b. $(4\sqrt{3}, 0)$ c. $(0, 4\sqrt{3})$ d. (0, 0) e. None of these

12. ABC is a right-angled triangle with BC = 6 cm and AC = 8 cm. A circle with centre O is inscribed in ΔACB. The radius of the circle is



a. 1 cm b. 2 cm c. 3 cm d. 4 cm e. 5 cm

13. The circumference of the largest circle that can be drawn on the surface of 2 watermelons is 60 cm and 50 cm respectively. The first is one and half times as costly as the second. Which of the two is more profitable
to buy? (Assume same density watermelons of spherical shape)

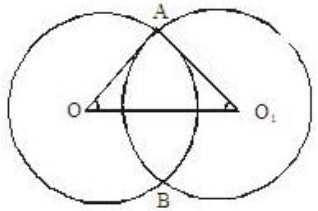
a. First b. Second c. Both are same

d. Depends on the weight e. Data Insufficient

14. An equilateral triangle of side 6 cm is cut into smaller equilateral triangles of side 2 cm. What is the greatest number of the smaller triangles that can be formed?

a. 9 b. 6 c. 12 d. 15 e. 14

15. In the following figure, two equal circles whose centres are O and O_1 intersect at two points A and B where AB = 2 cm and $OO_1 = 2\sqrt{3}$ cm. Find the measure of the angle $\angle OAO_1$.

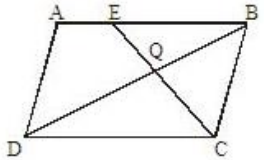


a. 150° b. 90° c. 60° d. 120° e. 75°

16. If the perimeter of a rectangle is p and its diagonal is d, then the difference between the length and width of the rectangle is

a. $\sqrt{\frac{8d^2-p^2}{2}}$ b. $\sqrt{\frac{8d^2+p^2}{2}}$ c. $\sqrt{\frac{6d^2-p^2}{2}}$ d. $\sqrt{\frac{8d^2-p^2}{4}}$ e. $\sqrt{\frac{6d^2+P^2}{2}}$

17. In the given parallelogram ABCD, if 3(BE) = 2(DC) and the area of ΔDQC is 36, then find out the area of ΔBQE.



a. 16 b. 20 c. 24 d. 18 e. 28

18. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

a. 16.665 b. 16.565 c. 15.465 d. 14.365 e. 16.455

19. The sides of a triangle are in the ratio 6 : 8 : 9. Thus,

a. the triangle is acute b. the triangle is right-angled

c. the triangle is obtuse d. the triangle is isosceles e. None of these

20. To make a marriage tent, poles are planted along the perimeter of a square field at a distance of 5 m from each other, and total number of poles used is 20. What is the area of the square field?

a. 500 m² b. 400 m² c. 900 m² d. 625 m² e. 800 m²

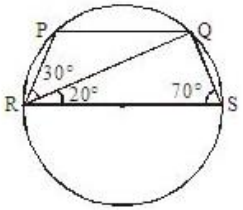
21. In a trapezium, the two parallel sides have lengths a and b (a < b). Each of the two other sides of the trapezium has length c. Then the area of the trapezium is

a. $\frac{1}{2}(a+b)c$ b. $\frac{1}{2}(a+b)\sqrt{c^2-\left(\frac{1}{4}\right)(b-a)^2}$

c. $\frac{1}{2}(a+b)\sqrt{c^2-(b-a)^2}$ d. $\frac{1}{2}(a+b)\left(c+\frac{b-a}{2}\right)$ e. $\frac{1}{2}(a+c)b$

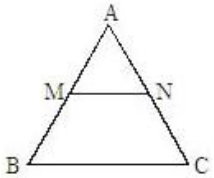
22. In the given figure, find ∠PQR, where RS is diameter and ∠RSQ = 70°,

∠PRQ = 30°.



a. 20° b. 30° c. 50° d. 45° e. 40°

23. In the following figure, if MN || BC, MN divides the triangle into two equal parts, then the value of the ratio of MA and AB will be



a. 500 m^2 b. 400 m^2 c. 900 m^2 d. 625 m^2 e. 800 m^2

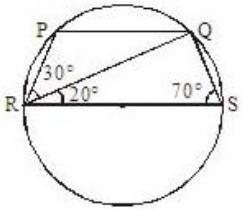
21. In a trapezium, the two parallel sides have lengths a and b (a < b). Each of the two other sides of the trapezium has length c. Then the area of the trapezium is

a. $\frac{1}{2}(a+b)c$ b. $\frac{1}{2}(a+b)\sqrt{c^2-\left(\frac{1}{4}\right)(b-a)^2}$

c. $\frac{1}{2}(a+b)\sqrt{c^2-(b-a)^2}$ d. $\frac{1}{2}(a+b)\left(c+\frac{b-a}{2}\right)$ e. $\frac{1}{2}(a+c)b$

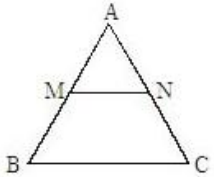
22. In the given figure, find ∠PQR, where RS is diameter and ∠RSQ = 70°,

∠PRQ = 30°.



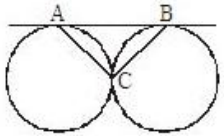
a. 20° b. 30° c. 50° d. 45° e. 40°

23. In the following figure, if MN || BC, MN divides the triangle into two equal parts, then the value of the ratio of MA and AB will be



a. $\sqrt{2}$ b. $\frac{1}{\sqrt{2}}$ c. $\frac{\sqrt{2}+1}{\sqrt{2}}$ d. $\frac{\sqrt{2}-1}{\sqrt{2}}$ e. $\frac{1}{\sqrt{2}+1}$

24. In the following figure, two circles of equal radius touch each other externally at point C, and AB is the common tangent to the circle. Find the value of $\angle CAB$.

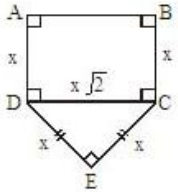


a. 45° b. 30° c. 60° d. 40° e. 50°

25. In the previous question, find the area of ΔABC, if the radius of the circle is 2 cm.

a. 4 cm^2 b. 16 cm^2 c. 8 cm^2 d. 12 cm^2 e. 10 cm^2

26. In the diagram, if the area of the figure is $8 + 16\sqrt{2}$, then find out the value of x.



a. $6-\sqrt{2}$ b. $4+\sqrt{2}$ c. $3\sqrt{2}$ d. 4 e. $2\sqrt{2}$

Practice Exercise 3 - Level 2

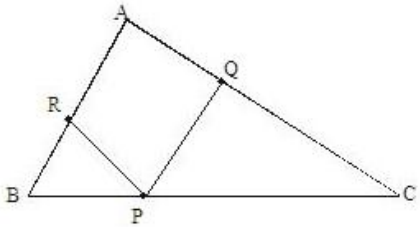
1. The angles of elevation of an aeroplane flying vertically above the ground as observed from the two consecutive stones 2 km apart are 45° and 60°. Find the height of the aeroplane above the ground

a. $\frac{\sqrt{3}+2}{2\sqrt{2}}$ kms b. $\frac{2\sqrt{3}}{\sqrt{3}-1}$ kms c. $\frac{2\sqrt{3}}{\sqrt{3}}$ kms d. $\frac{2\sqrt{3}}{2}$ kms e. $\frac{4\sqrt{3}}{\sqrt{3}-1}$ kms

2. The height of a tower is 200 m. When the angle of elevation of the sun changes from 45° to 60°, the shadow of the tower becomes shorter by x m. Find the value of x.

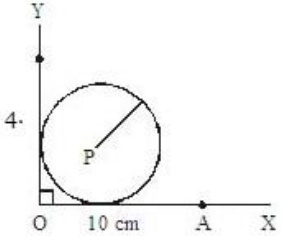
a. $\frac{600-\sqrt{3}}{\sqrt{3}}$ b. $\frac{600+\sqrt{3}}{3}$ c. $\frac{600+\sqrt{3}}{\sqrt{3}}$ d. $\frac{200\left(3-\sqrt{3}\right)}{3}$ e. $\frac{200(3\sqrt{3}-3}{\sqrt{5}}$

3. In the given triangle ABC, points P, Q and R divide the sides BC, CA and AB in the ratios 1 : 2, 3 : 2 and 3 : 2 respectively. Find the ratio of the area of quadrilateral ARPQ to the area of the triangle ABC.



a. 7:15 b. 2:3 c. 2:5 d. 38:75 e. None of these

4.



A circle of radius 3 cm is drawn touching both the axes OX and OY as shown in the figure. Another tangent is drawn to the circle from a point A 10 cm away from O on OX which meets OY at B. Find the length of OB.

a. 10 cm b. $\frac{20}{7}\sqrt{58}$ cm c. $\frac{21}{2}$cm d. 11 cm e. 12 cm

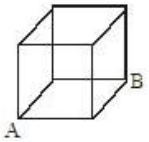
5. The base of a triangle is of length b, and the altitude is of length h. A rectangle of height x is inscribed in the triangle with the base of the rectangle on the base of the triangle. The area of the rectangle is

a. x (b - x) b. $\frac{hx}{b}[b-x]$ c. $\frac{bx}{h}[h-2x]$ d. $\frac{bx}{h}[h-x]$ e. None of these

6. Two straight roads R1 and R2 diverge from a point A at an angle of 120°. Ram starts walking from point A along R1 at a uniform speed of 3 km/hr. Shyam starts walking at the same time from A along R2 at a uniform speed of 2 km/hr. They continue walking for 4 hr along their respective roads and reach points B and C on R1 and R2 respectively. There is a straight line path connecting B and C. Then Ram returns to point A after walking along the line segments BC and CA. Shyam also returns to A after walking along line segments CB and BA. Their speeds remains unchanged. The time interval (in hours) between Ram's and Shyam's return to the point A is

a. $\frac{5\sqrt{19}+26}{3}$ b. $\frac{2\sqrt{19}+10}{3}$ c. $\frac{\sqrt{19}+13}{3}$ d. $\frac{\sqrt{19}+10}{3}$ e. None of these

7. A fly is trapped inside a hollow cube. It moves from A to B along the edges of the cube, taking the shortest possible route. It then comes back to A, again along the edges, taking the longest route (without going over any point more than once). If the total distance travelled is 5,040 m, what is the area of a face of the cube?



a. 99,225 m^2 b. 3,24,000 m^2 c. 3,96,900 m^2 d. 3, 84, 600 m^2 e. 4,24,000 m^2

8. A large solid sphere is melted and moulded to form identical right circular cones with base radius and height same as the radius of the sphere. One of these cones is melted and moulded to form a smaller solid sphere. What is the ratio of the surface area of the smaller sphere to the surface area of the larger sphere?

a. $1:3^{\frac{4}{3}}$ b. $1:2^{\frac{3}{2}}$ c. $1:3^{\frac{3}{2}}$ d. $1:2^{\frac{4}{3}}$ e. None of these

9. The perimeter of a triangle is 8 cm and one of the sides is 3 cm. Area of the triangle is maximum. Then the other two sides are

a. $\left(\frac{5}{2},\frac{5}{2}\right)$ b. (3, 2) c. $\left(\frac{3}{2},\frac{7}{2}\right)$ d. $\left(\frac{3}{2},2\right)$ e. $\left(\frac{5}{2},2\right)$

10. ABC is a triangle . D is a point on AB such that AD : DB = 3:2. E is the midpoint on BC and AE cuts CD at P. Find the ratio of CP : PD.

a. 5:4 b. 2:1 c. 5:3 d. 4:3 e. 5:2

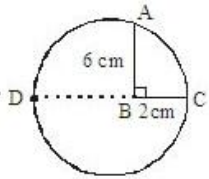
11. The centroid of a triangle is $\left(\frac{3+0-8}{9},\frac{2+0+5}{9}\right)$.

The vertices of the triangle are

a. $\left(\frac{1}{2},\frac{2}{3}\right)$, (o, o), $\left(-\frac{4}{3},\frac{5}{9}\right)$ b. $\left(\frac{3}{2},\frac{2}{3}\right)$, (o,o), $\left(-4,\frac{5}{3}\right)$

c. $\left(1,\frac{2}{3}\right)$, (o, o), $\left(-\frac{8}{3},\frac{5}{3}\right)$ d. Either (a) or (b)

e. Cannot be determined

12. 

In the circle of radius 10 cm; AB = 6 cm, BC = 2 cm and ∠ABC = 90° as shown in the figure. CB is extended to cut the circle at D. Find the length of CD.

a. 12 cm b. 20 cm c. $8\sqrt{10}$ cm d. 16 cm e. $10\sqrt{10}$ cm

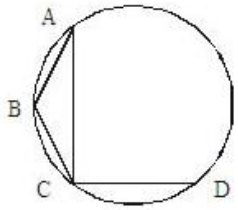
13. The length of the largest possible rod that can be placed in cubical room is $35\sqrt{3}$ m. What is the surface area of the largest possible sphere that fit with in the cubical room?

a. 3,500 m^2 b. 3,850 m^2 c. 2,450 m^2 d. 4, 250 m^2 e. None of these

14. A circle is inscribed in an isosceles triangle whose base is $8\sqrt{3}$ cm and the angle to the base is 30°. The area of the circle measured in cm^2 is

a. 48π b. $48(7-4\sqrt{3})$ c. $48\pi(5-4\sqrt{3})$ d. $48\pi\left(6-2\sqrt{3}\right)$ e. $48\pi\left(9-4\sqrt{3}\right)$

15. In the following figure, AB = BC, AC = CD and ∠ ACD = 90°. If the radius of the circle is "r" units, then find the length of the chord BC.



a. $r\sqrt{\sqrt{2}-1}$ b. $r\sqrt{\sqrt{3}-1}$ c. $r\sqrt{2-\sqrt{2}}$ d. $r\sqrt{2-\sqrt{3}}$ e. None of these

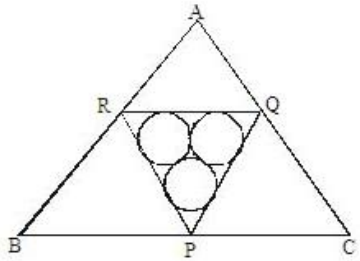
16. Nine dots are placed on a piece of paper. Find the maximum number of right angled triangles which can be formed by joining any three dots.

a. 18 b. 36 c. 40 d. 44 e. 48

17. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then the area (in square centimeters) of the triangle ABC would be

a. $\pi\sqrt{12}$ b. $\frac{9}{\pi}$ c. $\frac{9\sqrt{3}}{\pi}$ d. $\frac{6\sqrt{3}}{\pi}$ e. $\frac{12\sqrt{3}}{\pi}$

18. In the figure given below, radii of the three circles is 1 cm each. P, Q, R are the mid points of the sides BC, CA and AB respectively. Find the area of the triangle ABC.



a. $\left(6+4\sqrt{3}\right)\text{cm}^2$ b. $\left(24+16\sqrt{3}\right)\text{cm}^2$

c. $\left(3+6\sqrt{3}\right)\text{cm}^2$ d. $\left(9+6\sqrt{3}\right)\text{cm}^2$ e. $\left(6+3\sqrt{3}\right)\text{cm}^2$

19. A pyramid with a square base has a height of 12 m. The length of the base of the pyramid is 10 m. If a straight line is drawn from the vertex of the pyramid to any point on the square base, then which of the following cannot be the length of that straight line?

a. 12.5 m b. $\sqrt{194}$ m c. $\sqrt{181}$ m d. 14 m e. None of these

Directions for questions 20 to 22: Answer the questions on the basis of the information given below.

A city has two perfectly circular and concentric ring roads, the outer ring road (OR) being twice as long as the inner ring road (IR). There are also four (straight line) chord roads from E_1, the east end point of OR to N_2, the north end point of IR; from N_1, the north end point of OR to W_2, the west end point of IR; from W_1, the west end point of OR, to S_2, the south end point of IR; and from S_1 the south end point of OR to E_2, the east end point of IR. Traffic moves at a constant speed of 30π km/hr on the OR road, 20π km/hr on the IR road, and $15\sqrt{5}$ km/hr on all the chord roads.

20. The ratio of the sum of the lengths of all chord roads to the length of the outer ring road is

a. $\sqrt{5}:2$ b. $\sqrt{5}:2\pi$ c. $\sqrt{5}:\pi$ d. $\sqrt{5}:1$ e. None of the above

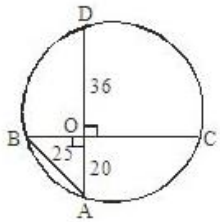
21. Amit wants to reach N_2 from S_1. It would take him 90 minutes if he goes on minor arc S_1 - E_1 on OR, and then on the chord road E_1 - N_2. What is the radius of the outer ring road in kms?

a. 60 b. 40 c. 30 d. 20 e. 50

22. Amit wants to reach E2 from N1 using first the chord N1 - W2 and then the inner ring road. What will be his travel time in minutes on the basis of information given in the above question?

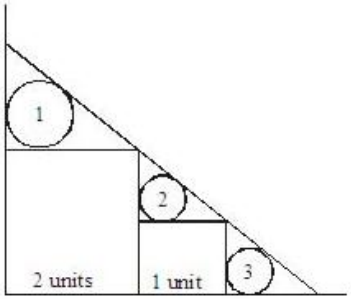
a. 60 b. 45 c. 90 d. 105 e. 75

23. As shown in the figure, OD = 36 cm, OA = 20 cm and AB = 25 cm. Find the length of chord BC.



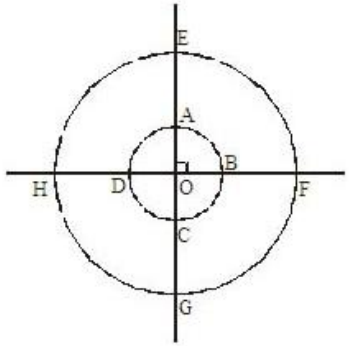
a. 48 cm b. 68 cm c. 56 cm d. 52 cm e. 63 cm

24. Two cubes of edge 2 units and 1 unit are kept side by side as shown in the figure. A wooden plate just covering both the cubes and touching them on the edges is placed such that it rests on the wall as well as floor. Three balls are kept in the free space touching the cube, wall/floor and wooden plate. Find the ratio of radii of balls 1, 2 and 3.



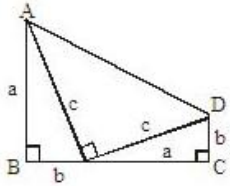
a. 3:2:2 b. 2:1:1 c. 4:3: 3 d. 8:1:1 e. 5:3:3

25. Two concentric circles are drawn with O as the center as shown in the figure given below. The ratio of the area of the annular ring bounded by these two circles and the quadrilateral EBCH is 3 : 2. Find the ratio of the radii of the smaller circle to the larger circle.



a. 1 : 7 b. 2 : 3 c. $\frac{2-3\pi}{2+3\pi}$ d. $\frac{2\pi-3}{2\pi+3}$ e. $\frac{4\pi-3}{4\pi+3}$

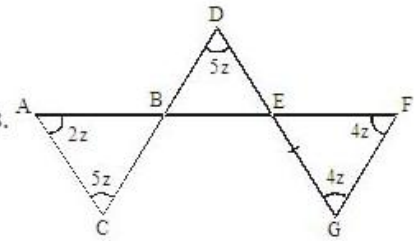
26. As shown in the figure, the area of the trapezium ABCD is 40.50. Find c, where a = 2b.



a. 4.5 b. $3\sqrt{5}$ c. $\sqrt{30}$ d. $2\sqrt{5}$ e. $5\sqrt{5}$

27. In a triangle ABC, AB = 6, BC = 8 and AC = 10. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with center B) is drawn. If the circle cuts AB and BC at P and Q respectively, the AP:QC is equal to

a. 1 : 1 b. 3 : 2 c. 4 : 1 d. 3 : 8 e. 2 : 1

28. 

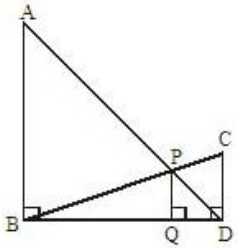
In the given figure, AB : BE : EF = 1 : 2 : 3. What is the measure of z?

a. 10° b. 14° c. 18° d. 20° e. 16°

29. In the diagram given below,

$\angle ABD = \angle CDB = \angle PQD = 90^\circ$.

If AB:CD = 3:1, the ratio of CD: PQ is



a. 1 : 0.69 b. 1 : 0.75 c. 1 : 0.72 d. 1 : 0.65 e. None of the above

30. Two identical squares of side length 11 cm are kept one above the other such that the square above completely overlaps with the square below. Now if the square above is rotated by 45° about its centre, what is the ratio of the area common to both squares to the remaining area?

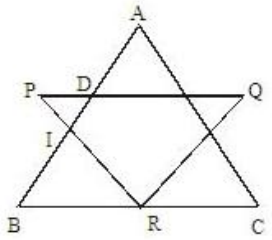
a. $1+\sqrt{2}$ cm² b. $\frac{1}{1+\sqrt{2}}$ cm² c. $\frac{\sqrt{2}-1}{2\sqrt{2}}$ cm² d. $\frac{2\sqrt{2}-1}{2\sqrt{2}}$ cm² e. $\frac{1+\sqrt{2}}{2}$ cm²

Practice Exercise 4 - Level 2

1. A circular ring of radius 3 cm is suspended from a point I by four identical strings tied at equal intervals on its circumference. The ring is in a horizontal plane. The point I is 4 cm vertically above the ring's center. If the angle between any two consecutive strings is θ, then find the value of cos θ.

a. $\frac{9}{25}$ b. $\frac{3}{5}$ c. $\frac{16}{25}$ d. $\frac{4}{5}$ e. $\frac{12}{25}$

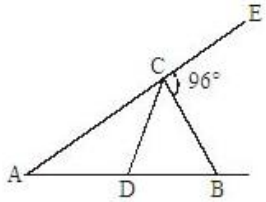
2. ΔABC and ΔPQR are triangles such that PQ || BC, QR || AB and PR || AC as shown in the figure. In ΔPQR, 2PD = DQ and in ΔABC, BI = AD = 2DI. Find the ratio of areas of ΔABC and ΔPQR.



a. $\frac{49}{25}$ b. $\frac{4}{1}$ c. $\frac{25}{9}$

d. $\frac{29}{14}$ e. Cannot be determined

3. In the figure (not drawn to scale) given below, if AD = CD = BC and $\angle BCE = 96^\circ$, how much is the value of $\angle DBC$?



a. 32° b. 64° c. 84°

d. 24° e. Cannot be determined

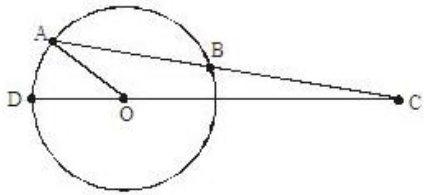
4. A vertical tower OP stands at the center O of a square ABCD. Let h and b denote the height of OP and length AB respectively. Suppose distance from top of the tower to the vertices of the square is equal to the length of the square, then the relationship between h and b can be expressed as

a. $2b^2 = h^2$ b. $2h^2 = b^2$ c. $3b^2 = 2h^2$ d. $3h^2 = 2b^2$ e. $3R^2 = 4b^2$

5. There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is

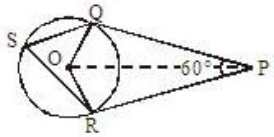
a. 34 b. 38 c. 36 d. 32 e. 40

6. In the figure below, AB is the chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD$ = y degrees and $\angle AOD$ = x degrees such that x = ky, then the value of k is



a. 3 b. 2 c. 1 d. 4 e. None of the above

7. As shown in the figure, PQ and PR are tangents to the circle from an external point P. If ∠QPR = 60°, find out ∠QOR and QSR.

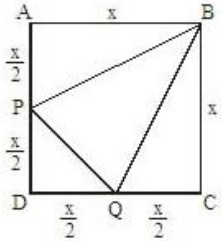


a. 130° and 65° b. 140° and 70° c. 150° and 75° d. 120° and 60° e. None of these

8. The perimeter of an isosceles right-angled triangle is 2p. Find out the area of the same triangle.

a. $(2+\sqrt{2})p^2$ b. $(2-\sqrt{2})p^2$ c. $(3-\sqrt{2})p^2$ d. $(4-2\sqrt{2})p^2$ e. $(3-2\sqrt{2})p^2$

9. ABCD is a square in which P and Q are mid-points of AD and DC respectively. The area of ΔBPQ constitutes what part of the whole area?



a. 50% b. 37.5% c. 66.66% d. 40% e. 45%

10. In the square ABCD, E and F are the two points on its diagonals. If O is the intersection point of the two diagonals and OF = 1, OE = 2, then what is the length of the line segment EF?

I. 3 cm

II. 1 cm

III. $\sqrt{5}$ cm

a. I or II only b. II or III only c. III only d. I and III only e. I, II or III

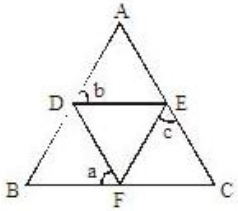
11. Four discs of diameter 10 cm each are cut from the bigger circular disc of radius 20 cm. Find out the percentage of the left-out area with respect to the total area.

a. 75% b. 66.66% c. 33.33% d. 50% e. 80%

12. The interior angles of a convex polygon form an arithmetic progression with a common difference of 5 degrees. If the smallest interior angle is 120°, then the number of sides of the polygon could be

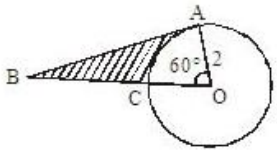
a. 8 b. 9 c. 12 d. 16 e.10

13. It is given that AB and AC are the equal sides of an isosceles ΔABC, in which an equilateral ΔDEF is inscribed. As shown in the figure, ∠BFD = a and ∠ADE = b, and ∠FEC = c. Then



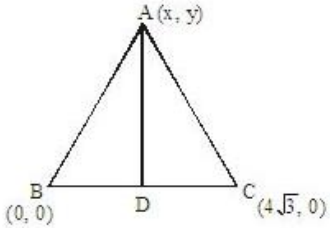
a. $a = \frac{b+c}{2}$ b. $b = \frac{a+c}{2}$ c. $c = 2a + 2b$ d. $a = \frac{b+c}{3}$ e. $b = \frac{a+c}{3}$

14. In the figure, where AB is tangent to the circle with centre O, find the ratio of the area of shaded region to the area of unshaded region of triangle AOB.



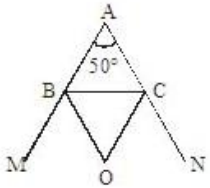
a. $\frac{2\sqrt{3}-2}{\pi}$ b. $\frac{3\sqrt{3}-\sqrt{2}}{\pi}$ c. $\frac{2-\sqrt{3}}{\pi}$ d. $\frac{3\sqrt{3}}{\pi}-1$ e. $\frac{2\sqrt{3}}{\pi}-\frac{1}{2}$

15. The coordinates of an equilateral triangle are as shown in the figure below. What is the length of the median?



a. $4\sqrt{3}$ b. 4.5 c. 6 d. 6.5 e. 5.5

16. In the given figure, BO and CO are the bisectors of ∠MBC and ∠NCB respectively. Find the value of ∠COB.



a. 75° b. 65° c. 90° d. 80° e. 85°

17. A wheel with a rubber tyre has an outside diameter of 25 inches. When the radius has been decreased by a quarter of an inch, the number of revolutions of the wheel in one mile will be

a. increased by about 2% b. increased by about 30%

c. increased by about 20% d. increased by about 0.2% e. None of these

Directions for questions 18 and 19: Answer the questions based on the information given below.

A cube is divided into eight equal small cubes. Each of these small cube is further sub-divided into eight equal smaller cubes.

18. What is the surface area of the smallest cube as a fraction of the surface area of the original cube?

a. 0.625 b. 0.0625 c. 0.0156 d. 0.0039 e. 0.0064

19. If the original cube's sides were painted blue, then what is the probability that exactly two sides of any randomly selected small cube is painted blue?

a. $\frac{3}{8}$ b. $\frac{1}{16}$ c. $\frac{1}{4}$ d. $\frac{3}{4}$ e. $\frac{1}{3}$

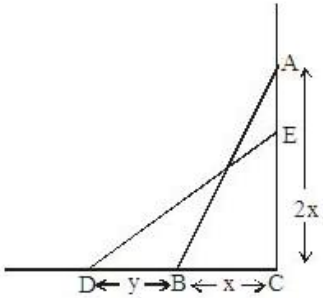
20. Three sides of an isosceles triangle are variably represented as (x + 1), (9 - x) and (5x - 3). How many such triangles are possible?

a. 0 b. 1 c. 2 d. 3 e. More than 3

21. Two vertices of a rectangle lie on the line Y = 2x + λ and coordinate of the rest two vertices opposite to each other are (1, 3) and (5, 1). Find the value of λ .

a. 4 b. 5 c. -4 d. - 3 e. 3

22. A ladder rests against a wall with its lower end at a distance x from the wall and its upper end at a height of 2x above the floor. If the lower end slides through a distance y away from the wall, from its earlier position then by how much distance does the upper end slide?



a. $x\sqrt{5}-\sqrt{5x^2-(x+y)^2}$ b. $x\sqrt{5}-\sqrt{5x^2-(x-y)^2}$

c. $2x-\sqrt{5x^2-(x+y)^2}$ d. $2x-\sqrt{5x^2-(x-y)^2}$

e. $x\sqrt{5}+\sqrt{5x^2-(x-y)^2}$

23. A circle of radius $\frac{R}{2}$ is cut out of another circle of radius R. How much paint is needed to paint this circle (with a hole in it) if the original circle needed 20 L for the painting?

a. 16.66 L b. 18 L c. 12 L d. 15 L e. 20 L

24. The interior angle of the regular polygon exceeds the exterior angle by 132°. The number of sides in the polygon will be

a. 10 b. 16 c. 12 d. 15 e. 14

25. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s, respectively. Then,

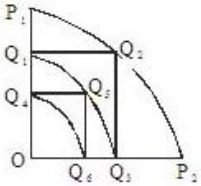
a. s > t > c b. c > t > s c. c > s > t d. s > c > t e. t > s > c

Practice Exercise 5 - Level 3

1. ABC is an equilateral triangle with side $2\sqrt{3}$ cm. A circular arc with radius equal to the inradius of ΔABC is drawn with center at A, which cuts the sides AB & AC at points P & Q respectively. Similarly another circular arc with radius equal to the circumradius of ΔABC is drawn with center at C, which cuts the sides CA & CB at points R & S respectively. Find the area of the triangle PRG, where G is the centroid of the ΔABC.

a. $\frac{(3\sqrt{3}-3)}{2}$ cm^2 b. $\frac{\sqrt{3}(\sqrt{3}-1)}{2}$ cm^2 c. $\frac{\sqrt{3}-1}{2}$ cm^2

d. $\frac{2\sqrt{3}-1}{2}$ cm^2 e. $\frac{(3\sqrt{3}-4)}{2}$ cm^2

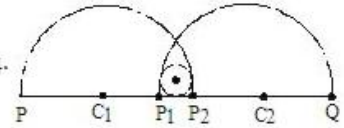
2. 

Arc P_1P_2 is the quadrant of the circle with center O. A square $Q_1Q_2Q_3$ O is drawn and similarly a quadrant Q_1Q_3 with center O is formed. $Q_4Q_5Q_6O$ is another square drawn and this process is continued infinitely. Find the sum of areas of all the squares so formed if the radius of the circle having quadrant P_1P_2O is `r' units.

a. $\sqrt{2}r^2$ sq. units b. $2r^2$ sq. units c. $\frac{r^2}{\sqrt{2}}$ sq. units d. r^2 sq. units e. $2\sqrt{2}r^2$

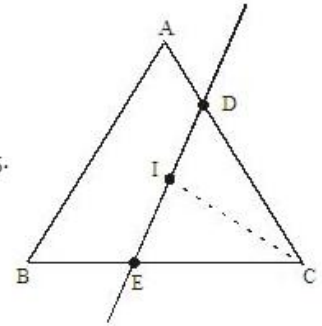
3. 10 parallel lines of length 5 cm each are drawn such that the distance between any two consecutive lines is 1 cm. One line is selected randomly and its ends are marked as A and B. One other line is selected and its midpoint is marked as M. Find the sum of the areas of all the possible triangles ABM.

a. 975 cm² b. 525 cm² c. 675 cm² d. 825 cm² e. 1000 cm²

4. 

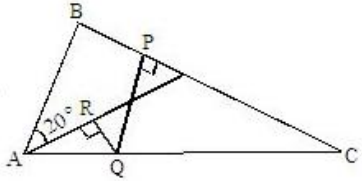
C_1 and C_2 are centres of two identical semicircles with radius `r' units. The ratio of C_1P_1 to PQ is 1 : 7, as shown in the figure. Find the radius of the circle inscribed in the region common to the identical semi-circles, given that the circle touches the two semi-circles and the line segment P_1P_2.

a. $\frac{7r}{32}$ b. $\frac{3r}{16}$ c. $\frac{r}{4}$ d. $\frac{5r}{16}$ e. None of these

5. 

Triangle ABC is an acute angled triangle. A line passing through the incenter of the triangle divides the triangle into two equal areas. If S is the perimeter of ΔABC, then CD + CE =

a. $\frac{S}{2}$ b. $\frac{S}{3}$ c. $\frac{S}{4}$ d. $\frac{2S}{3}$ e. $\frac{3S}{4}$

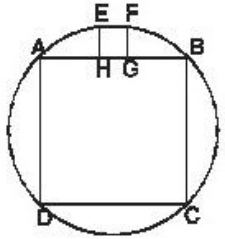
6. 

In the given figure PQ || BA and PQ + QR = AB. What is the measure of ∠BCA?

a. 30° b. 40° c. 45° d. 35° e. 50°

7. ABC is an equilateral triangle of side $\sqrt{3}$ m. P is an external point equidistant from the points A and C, such that the sum of the lengths of the line segments PA, PB and PC is $\sqrt{3}(\sqrt{3}+2)$ m. What is the minimum possible distance one has to cover from P, if one wants to be equidistant from the three vertices of the triangle ABC?

a. $\sqrt{3}$ m. b. $\sqrt{3}(\sqrt{3}-2)$ m c. $\frac{\sqrt{3}}{2}$ m d. 4 m e. 2 m

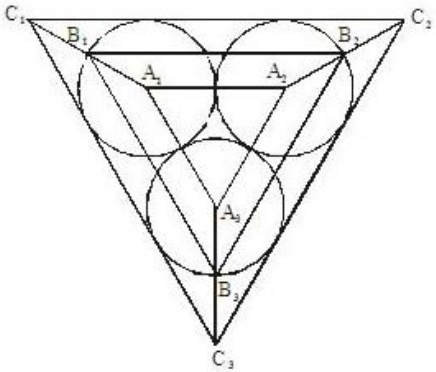
8. 

ABCD is a square and is inscribed in a circle. EFGH is a square such that HG is on AB and the points E & F are on the circumference of the circle. Find the ratio of the area of the bigger square to the area of the smaller square.

a. 4 : 1 b. 25 : 4 c. 25:1 d. 25 : 2 e. None of these

Directions for questions 9 to 11: Answer the questions on the basis of the information given below.

Consider three circular parks of equal size with centres at A_1, A_2, and A_3 respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$,as shown. Three sprinters A, B, and C begin running from points A_1, B_1 and C_1 respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.



9. Let the radius of each circular park be r, and the distances to be traversed by the sprinters A, B and C be a, b and c respectively. Which of the following is true?

a. b - a = c - b = $3\sqrt{3}$ r b. b - a = c - b = $\sqrt{3}$ r

c. b = $\frac{a+c}{2}$ = $2(1+\sqrt{3})$ r d. c = 2b - a = $(2+\sqrt{3})$ r

e. b - a = 2c - b = $\sqrt{3}$ r

10. Sprinter A traverses distances A_1A_2, A_2A_3, and A_3A_1 at an average speeds of 20, 30 and 15 respectively. B traverses her entire path at a uniform speed of $(10\sqrt{3}+20)$. C traverses

distances C_1C_2, C_2C_3 and C_3C_1 at an average speeds of $\frac{40}{3}(\sqrt{3}+1)$, $\frac{40}{3}(\sqrt{3}+1)$ and 120 respectively. All speeds are in the same unit. Where would B and C be respectively when A finishes her sprint?

a. B_1, C_1

b. B_3, C_3

c. B_1, C_3

d. B_1, Somewhere between C_3 and C_1

e. B_3, Somewhere between C_3 and C_1

11. Sprinters A, B and C traverse their respective paths at uniform speeds of u, v and w respectively. It is known that $u^2:v^2:w^2$ is equal to Area A: Area B: Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$ respectively. Where would A and C be when B reaches point B_3?

a. A_2, C_3

b. A_3, C_3

c. A_3, C_2

d. Somewhere between A_2 and A_3, Somewhere between C_3 and C_1

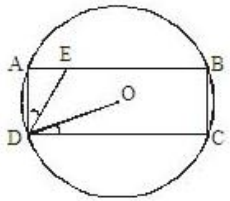
e. A_3, Somewhere between C_3 and C_1

12. Unequal side of an isosceles triangle is 2 cm long. The medians drawn to the equal sides are perpendicular to each other. Find the area of the triangle.

a. $3\ cm^2$ b. $\sqrt{10}\ cm^2$ c. $2\sqrt{3}\ cm^2$ d. $2\sqrt{2}\ cm^2$ e. $3\sqrt{2}\ cm^2$

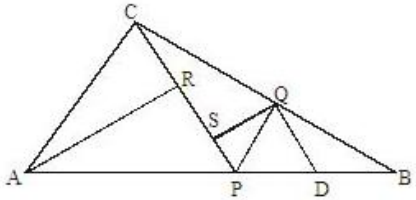
13. In the figure below (not drawn to scale), rectangle ABCD is inscribed in the circle with centre at O. The length of side AB is greater than side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : \sqrt{3}$. The line segment DE intersects AB at E such that $\angle ODC = \angle ADE$

The ratio AE : AD is



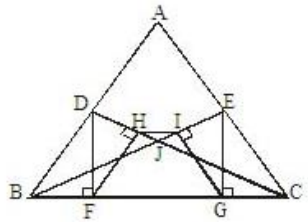
a. $1:\sqrt{3}$ b. $1:\sqrt{2}$ c. $1:2\sqrt{3}$ d. 1 : 2 e. $2:\sqrt{3}$

14. In the figure (not drawn to scale) given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, $\angle ARC = 90°$, and in ΔPQS, $\angle PSQ = 90°$. The length of QS is 6 cm. What is the ratio of AP : PD?



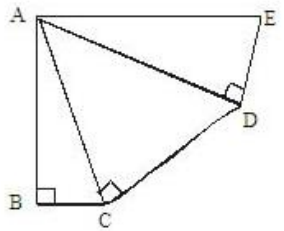
a. 10 : 3 b. 2 : 1 c. 7 : 3 d. 8 : 3 e. None of these

15. In the following figure, ΔABC is an equilateral triangle. Points D and E are the midpoints of sides AB and AC respectively. Find the ratio of area of ΔHIJ to the area of the ΔABC.



a. 1 : 160 b. 1 : 192 c. 1 : 224 d. 1 : 144 e. 1 : 168

16. In the following figure, $\Delta ABC, \Delta ACD \& \Delta ADE$ are right angled triangles. If $\frac{AB}{BC} = \frac{CD}{AC} = \frac{AD}{DE} = k$, then find the value of $\left(\frac{AE}{BC}\right)^2$



a. $\left(1+\frac{1}{k^2}\right)^2$ b. $\frac{(1+k^3)^3}{k^2}$ c. $\frac{(1+k^2)^3}{k^2}$

d. $\frac{k^2+1}{k^2}$ e. Cannot be determined

17. ABCD is a rhombus. Points P, Q and R divides the sides BC, CD and DA of the rhombus in the ratio 1 : 1, 1 : 2 and 1 : 3 respectively. Find the area of the rhombus ABCD, if the area of the triangle ΔPQR is A sq. units.

a. $\frac{34}{7}$ A sq. units b. $\frac{22}{5}$ A sq. units c. $\frac{14}{3}$ A sq. units

d. $\frac{24}{5}$ A sq. units e. $\frac{34}{5}$ A sq. units

Answer key

Chapter 1 - Plane Gemometry - Basics & Triangles

Level - 1

1. 1 : 1 **2.** $\angle BCD = 90^\circ$ **3.** 9 : 16 **4.** $\frac{x}{\sqrt{2}}$

5. $\angle PDQ = 90^\circ$ **6.** 5.74 cm **7.** $\frac{225}{49}$ **8.** 4 : 1

Level - 2

9. 1 **10.** 1 : 1 **11.** Yes **13.** AC = 18 cm, DE = 4 cm

14. 1 **15.** **b** GM

Level - 3

16. $9r^2\left(2-\sqrt{3}\right)^2$ **17.** ΔOED is an isosceles triangle.

Chapter 2 - Polygons and Quadrilaterals

Level - 1

1. Yes **2.** Yes **3.** Yes **4.** 1 : 4 **6.** 1 : 1

Chapter 3 - Circle

Level - 1

1. 6 cm **2.** *(i)* 1 cm, *(ii)* 7 cm **3.** Yes **4.** 70°, 40° **5.** $\sqrt{3}r$ units

8. c AB = AC

Level - 2

9. 2 cm , 8 cm **10.** 3,2,5 cm **11.** $6\sqrt{3}$ cm **14.** Yes **15.** $\frac{5}{3}\left(\sqrt{3}+\sqrt{6}\right)$cm

Level - 3

17. 1 cm

Chapter 4 - Mensuration

Level - 1

1. 66.66% **2.** 3744 cm^2 **3.** 5040 cm^3 **4.** 12.8 min

5. 3 : 2 : 1 **6.** $\frac{3}{4}$ **7.** 8, 1 : 4 **8.** 3.30 cm

Level - 2

9. c $\frac{HM}{2}$ **10.** 154 cm^2 **11.** 545p cm^2 **12.** $1-\sqrt[3]{\frac{C-F}{C}}$

13. 51 sq. units

Chapter 5 - Trigonometry

Level - 1

1. *(i)* $\frac{40}{\sqrt{3}}$m *(ii)* 100 m *(iii)* 30°

2. *(i)* 150 m *(ii)* $\frac{60}{2-\sqrt{3}}$ m or $60\left(2+\sqrt{3}\right)$m *(iii)* 15°

3. *(i)* $10\sqrt{3}$ m, 30 m *(ii)* $120\left(\sqrt{3}-1\right)$ m

4. *(i)* $25\sqrt{3}$ m, 75 m from the tower whose angle of elevation is 30° *(ii)* 33.33 m

Level - 2

5. $100(\sqrt{3}+1)$m , $100(\sqrt{3}-1)$m from the tower whose angle of elevation is 75°

6. 30°

7.(ii)√3 m, 20 m

Chapter 6

Practice Exercise 1 - Level 1

1	a	**2**	a	**3**	d	**4**	d	**5**	c	**6**	d	**7**	a	**8**	b	**9**	d	**10**	b
11	d	**12**	a	**13**	e	**14**	b	**15**	b	**16**	e	**17**	c	**18**	a	**19**	d	**20**	d
21	d	**22**	d	**23**	b	**24**	d	**25**	a	**26**	e								

Practice Exercise 2 - Level 1

1	e	**2**	a	**3**	c	**4**	c	**5**	a	**6**	d	**7**	b	**8**	d	**9**	a	**10**	a
11	c	**12**	b	**13**	a	**14**	a	**15**	d	**16**	d	**17**	a	**18**	d	**19**	a	**20**	d
21	b	**22**	e	**23**	b	**24**	a	**25**	a	**26**	d								

Practice Exercise 3 - Level 2

1	b	**2**	d	**3**	a	**4**	c	**5**	d	**6**	b	**7**	c	**8**	d	**9**	a	**10**	c
11	e	**12**	b	**13**	b	**14**	b	**15**	c	**16**	d	**17**	c	**18**	b	**19**	d	**20**	c
21	c	**22**	d	**23**	e	**24**	b	**25**	e	**26**	b	**27**	d	**28**	c	**29**	b	**30**	a

Practice Exercise 4 - Level 2

1	c	**2**	c	**3**	b	**4**	b	**5**	c	**6**	a	**7**	d	**8**	e	**9**	b	**10**	e
11	a	**12**	b	**13**	a	**14**	d	**15**	c	**16**	b	**17**	a	**18**	b	**19**	a	**20**	b
21	c	**22**	c	**23**	d	**24**	d	**25**	c										

Practice Exercise 5 - Level 3

1	e	**2**	d	**3**	d	**4**	a	**5**	a	**6**	d	**7**	e	**8**	c	**9**	a	**10**	c
11	b	**12**	a	**13**	a	**14**	c	**15**	b	**16**	c	**17**	d						

Explanations: Fundamentals of Geometry

Chapter - 1

Plane Gemometry - Basics & Triangles

Level - 1

1. ΔPAB and ΔQAC are congruent,

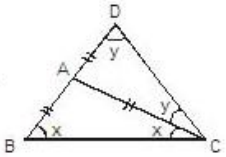
since PA = QA;

BA = CA; and $\angle PAB = \angle QAC$

Hence, PB = QC (Corresponding sides of congruent triangles)

$\therefore$ The required ratio = 1:1.

2.



Method 1:

In ΔABC, AB = AC

So $\angle B = \angle ACB$

In ΔDAC, AD = AC, so $\angle D = \angle DCA$

In ΔBDC, $\angle D + \angle B + \angle C = 180^\circ$

$2x + 2y = 180^\circ$

$x + y = 90^\circ$

$\therefore \angle BCD = 90^\circ$

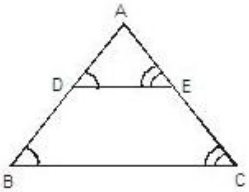
Method 2:

With A as the centre we can draw a circle that passes through the three points B, C, D.

Hence, A is the circum-centre of the ΔABC.

So C must be a right angle.

3.



DE : BC = 3 : 5

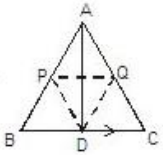
Hence, ratio of proportional sides = 3 : 5

Ratio of area of ΔADE and ΔABC = 9 : 25

Hence, ratio of areas of ΔADE to quadrilateral

ΔECB = 9 : 16

4. If $\angle B = 45^\circ$, then AD = $\frac{x}{\sqrt{2}}$

5. 

AB = AC

AD is the median.

So $\angle ADB = 90^\circ$, $\angle PDA = 45^\circ$ ($\because$ DP is angle bisector)

Now DQ is also angle bisector of $\angle ADC$.

$\therefore \angle PDQ = 90^\circ$

6. Since $\Delta ABC \sim \Delta DEF$

$$\frac{AB^2}{DE^2} = \frac{AC^2}{DF^2} = \frac{BC^2}{EF^2} = \frac{Area(\Delta ABC)}{Area(\Delta DEF)} = \frac{100}{49}$$

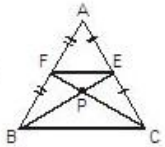
$$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{10}{7}$$

$$\frac{8.2}{DE} = \frac{10}{7}$$

DE = 5.74 cm

7. $\Delta ABC \sim \Delta PQR$

$$\frac{AB^2}{PQ^2} = \frac{Area(\Delta ABC)}{Area(\Delta PQR)} = \left(\frac{7.5}{3.5}\right)^2 = \left(\frac{15}{7}\right)^2 = \frac{225}{49}$$

8. 

Point P divides the median in the ratio of 2 : 1.

So $\frac{CP}{PF} = \frac{BP}{PE} = \frac{2}{1}$

And $\angle FPE = \angle BPC$ (Vertically opposite $\angle S$)

So $\Delta FPE \sim \Delta CPB$.

Hence, $\frac{Area(\Delta FPE)}{Area(\Delta CPB)} = \frac{PF^2}{CP^2} = \frac{1}{4}$

Required ratio = 4 : 1

Level - 2

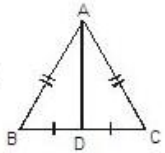
9. Since AB = AC, $\angle B = \angle C$.

In ΔXBC and ΔYBC, BX = CY and $\angle XBC = \angle YCB$.

BC is the common side.

Hence, ΔXBC and $\angle YCB$ are congruent.

XC = YB and $\frac{XC}{YB} = 1$

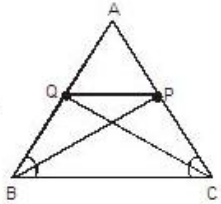
10. 

$\angle ABD = \angle ACD$

$AB = AC$ and $BD = DC$

So ΔABD and ΔADC are congruent.

So $\angle BAD = \angle CAD$ or AD bisects $\angle A$.

11. 

If ABC is an isosceles triangle $AB = AC$

As BP is angle bisector of $\angle B$

P divides AC in the ratio of $\frac{AB}{BC}$

Similarly Q divides AB in the ratio of $\frac{AC}{BC}$

As $AB = AC$, $\frac{AB}{BC} = \frac{AC}{BC}$

As P and Q divides AC and AB in the same ratio respectively $PQ \parallel BC$.

Alternative method:

$\Delta QCB \cong \Delta PBC$ [Similar triangles]

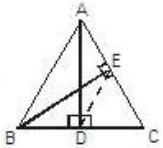
We can see that BC is common

$\angle QCB = \angle PBC$

$\angle QBC = \angle PCB$

$\Delta QCB \approx \Delta PCB \Rightarrow QB = PC$

$\therefore QP \parallel BC$

12. 

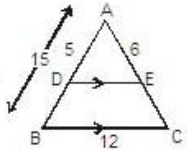
(i) For ΔADC and ΔBEC the angles are all equal.

$\angle E = \angle D = 90^o$; $\angle C$ is common to both.

Hence, $\Delta ADC \sim \Delta BEC$

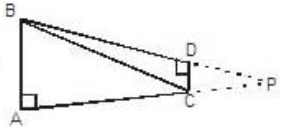
(ii) From (i)

$\frac{CB}{CA} = \frac{CE}{CD} \Rightarrow CB \cdot CD = CA \cdot CE$

13. 

Since $DE \parallel BC$,

$\therefore \frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC} \Rightarrow AC = 6 \times \frac{15}{5} = 18$ cm and $DE = \frac{5}{15} \times 12 = 4$ cm

14. 

In ΔPDC and ΔPBA,

$\angle P$ is common angle.

$\angle BDC = \angle BAC = 90^\circ$

Thus, ΔABP and ΔDCP are similar.

So $\frac{PD}{PC} = \frac{AP}{PB}$ or, $\frac{AP \times PC}{DP \times PB} = 1$

15. b In ΔTXM, XM || BN

$\frac{TB}{TX} = \frac{TN}{TM}$ ... (i)

In ΔTMC XN || CM

$\frac{TX}{TC} = \frac{TN}{TM}$ ... (ii)

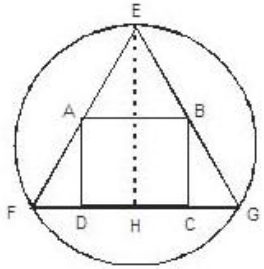
from (i) and (ii)

$TX^2 = TB \times TC$

$\therefore$ TX is GM (Geometric mean) of TB and TC

Level - 3

16.



EFG is an equilateral triangle.

ABCD is a square inscribed in it.

Radius of the outer circle = r.

Hence, side of the equilateral triangle $= r\sqrt{3} = a$

In the figure, let FD = x, $\angle AFD = 60^\circ$

Then side of the square,

$AD = (\tan 60^\circ)x = \sqrt{3}x$

Also $\frac{EH}{FH} = \tan 60^\circ = \sqrt{3}$

$EH = \frac{\sqrt{3}}{2}a;\ FH = x + \frac{\sqrt{3}}{2}x \left[\text{Since } DH = \frac{DC}{2}\right]$

$\Rightarrow FH = x\left(\frac{2+\sqrt{3}}{2}\right)$

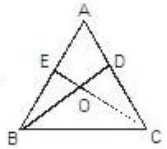
Now $FH = x\left(\frac{2+\sqrt{3}}{2}\right) = \frac{a}{2}$ or $x = \frac{a}{(2+\sqrt{3})} = a(2-\sqrt{3})$

Hence, side of the square $= [(\sqrt{3})(r\sqrt{3})(2-\sqrt{3})] \Rightarrow 3r(2-\sqrt{3})$

Hence, area of the square

$ABCD = 9r^2(2-\sqrt{3})^2$

17.



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Then side of the square,

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Also $\frac{EH}{FH} = \tan 60^\circ = \sqrt{3}$

$EH = \frac{\sqrt{3}}{2}a;\ FH = x + \frac{\sqrt{3}}{2}x\ \left[\text{Since } DH = \frac{DC}{2}\right]$

$\Rightarrow FH = x\left(\frac{2+\sqrt{3}}{2}\right)$

Now $FH = x\left(\frac{2+\sqrt{3}}{2}\right) = \frac{a}{2}$ or $x = \frac{a}{(2+\sqrt{3})} = a(2-\sqrt{3})$

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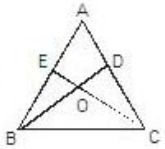
Hence, area of the square

$ABCD = 9r^2(2-\sqrt{3})^2$

17. 

If ABC is an isoceles triangle with AB = AC, then the point D divides AC in the ratio $\frac{AB}{BC}$ and E divides AB in the ratio $\frac{AC}{BC}$.

Since AB = AC, $\frac{AB}{BC} = \frac{AC}{BC}$

D and E divide the sides AC and AB in the same ratio, $\therefore$ ED || BC.

So ΔEDB and ΔCED are congruent.

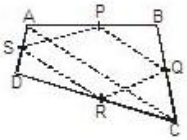
Therefore, $\angle$ BDE = $\angle$ CED

$\therefore$ ΔOED is isosceles triangle.

Chapter - 2

Polygons and Quadrilaterals

Level - 1

1. 

Consider ΔABC and ΔACD.

PQ || AC; RS || AC. Hence, PQ || RS.

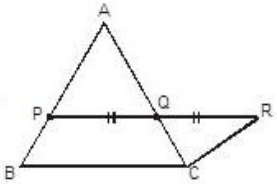
$PQ = \frac{1}{2} AC$

$RS = \frac{1}{2} AC$

∴ PQ = RS

Similarly, from ΔABD and ΔBCD , we get SP || QR and SP = QR

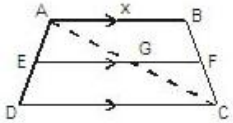
Hence, PQRS is a parallelogram.

2. 

Since P and Q are mid-points of AB and AC respectively.

$PQ = \frac{1}{2} BC \Rightarrow BC = 2PQ = PR$ and PR || BC

So, PRCB is parallelogram.

3. 

Let AC intersects EF at G.

Suppose EF || AB, but AB || CD

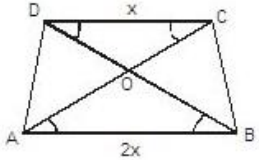
Therefore, EF || CD

Since EG || CD, $\frac{AE}{AD} = \frac{AG}{AC}$

Also GF || AB, $\frac{BF}{CB} = \frac{AG}{CA}$

So $\frac{BF}{CB} = \frac{AE}{AD} = \frac{1}{2}$

So F is the mid-point of BC and E is the mid point of AD. So, our assumption is true, EF || AB.

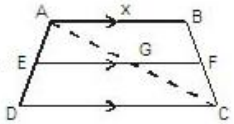
4. 

ΔAOB and ΔCOD are similar.

The ratio of the proportional sides is 2 : 1.

$PQ = \frac{1}{2}BC \Rightarrow BC = 2PQ = PR$ and $PR \parallel BC$

So, PRCB is parallelogram.

3. 

Let AC intersects EF at G.

Suppose $EF \parallel AB$, but $AB \parallel CD$

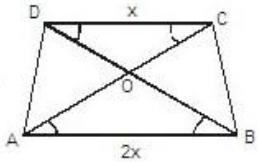
Therefore, $EF \parallel CD$

Since $EG \parallel CD$, $\frac{AE}{AD} = \frac{AG}{AC}$

Also $GF \parallel AB$, $\frac{BF}{CB} = \frac{AG}{CA}$

So $\frac{BF}{CB} = \frac{AE}{AD} = \frac{1}{2}$

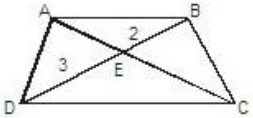
So F is the mid-point of BC and E is the mid point of AD. So, our assumption is true, $EF \parallel AB$.

4. 

ΔAOB and ΔCOD are similar.

The ratio of the proportional sides is 2 : 1.

Hence, the ratio of the areas of ΔDOC and ΔAOB is 1 : 4.

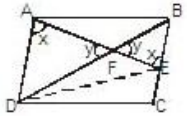
5. 

BE : DE = 2 : 3 and AE : EC = 2 : 3 (By property)

ΔAEB and ΔCED are similar.

The ratio of the proportional sides is 2 : 3.

Therefore, $\frac{Area(\Delta AEB)}{Area(\Delta CED)} = \frac{4}{9}$

6. 

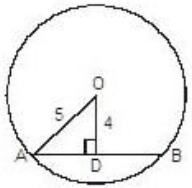
ΔAFD and ΔEFB are similar.

So $\frac{FA}{EF} = \frac{DF}{FB}$ or $FA \cdot FB = DF \cdot EF$

Chapter - 3

Circle

Level - 1

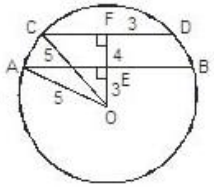
1. 

ΔADO is a right-angled triangle, right-angled at D.

So $AD^2 + OD^2 = OA^2$

AO = 5 cm, OD = 4 cm

Hence, AD = 3 cm . So AB = 6 cm.

2. 

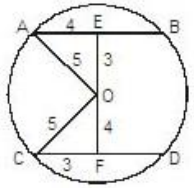
(i) OA = OC = Radius = 5 cm

CF = 3 cm, AE = 4 cm

Hence, OE = 3 cm and OF = 4 cm

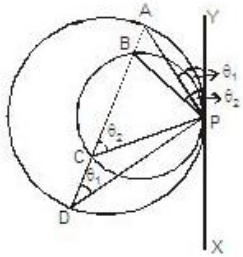
So, the distance between them, EF = 1 cm

(ii)



Similar to the previous problem.

OE = 3 cm, OF = 4 cm. Distance = EF = 7 cm

3. 

Draw a common tangent XY at point P.

Let $\angle YPA = \theta_1$ and $\angle YPB = \theta_2$

Now, $\angle YPA = \angle PDA = \theta_1$

(Alternate segment theorem)

Similarly, $\angle YPB = \angle PCB = \theta_2$

Therefore,

$\angle APB = \angle YPB - \angle YPA = \theta_2 - \theta_1$ ...(i)

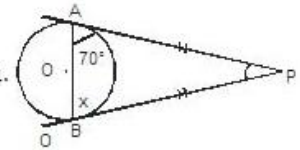
In ΔPCD, $\angle PCB = \angle CPD + \angle CDP$

(Exterior angle theorem)

Therefore, $\angle CPD = \theta_2 - \theta_1$ ...(ii)

From equations (i) and (ii), we get

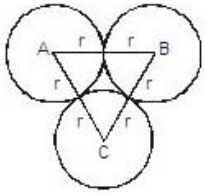
$\angle APB = \angle CPD$. Hence proved.

4. 

PA = PB (Tangents from external point)

So ∠PBA = ∠PAB = 70°

∠APB = 180° - 140° = 40°

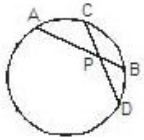
5. 

If A, B and C are the centres of three triangles, each of whose radii is r units, then

AB = AC = BC = 2r units.

Hence, ABC is an equilateral triangle.

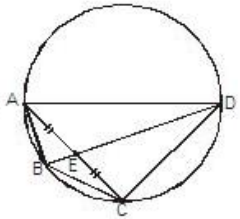
$h = \frac{\sqrt{3}}{2} \times 2r = \sqrt{3}\ r$ units

6. 

In the circle, $AP \cdot PB = DP \cdot PC$ [Chord properties]

Since AP = CP, we have PB = DP

AB = AP + PB = CP + PD = CD

7. 

Let `E' be the point of intersection of AC and BD

$\therefore$ AE = EC (Given)

In ΔAEB and ΔDEC

$\angle BAE = \angle CDE$

(Angles subtended by an arc at the circumference are equal)

Similarly,

$\angle ABE = \angle DCE$

Therefore,

$\Delta AEB \sim \Delta DEC$ (AAA similarity)

$\frac{AE}{DE} = \frac{EB}{EC} = \frac{AB}{DC}$ ...(i)

Similarly, $\Delta AED \sim \Delta BEC$

$\frac{AE}{BE} = \frac{ED}{EC} = \frac{AD}{BC}$ ...(ii)

From equations (i) and (ii), we get

$\frac{AE}{DE} \times \frac{ED}{EC} = \frac{AB}{DC} \times \frac{AD}{BC}$

$\therefore \frac{AB \times AD}{DC \times BC} = 1$ ($\because$ AE = EC)

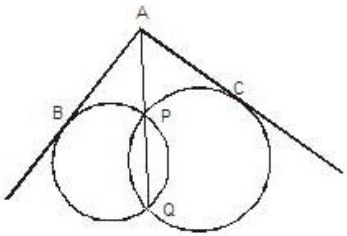
$\Rightarrow$ AB × AD = DC × BC

Hence proved.

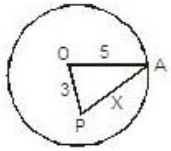
8. c $AB^2 = AP \times AQ$

$AC^2 = AP \times AQ$

Thus AB = AC



Level - 2

9. 

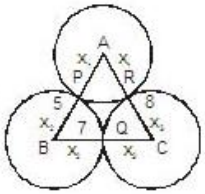
x = 2 is the smallest value of x,

when O, P and A are in a straight line.

{A and P lie on the same side of O.}

x = 8 is the largest value,

when O, P and A lie on a straight line and P and A lie on the opposite sides of O.

10. 

Let x_1, x_2 and x_3 be radii of circles with centres A, B, C respectively.

Then $x_1 + x_2 = 5$

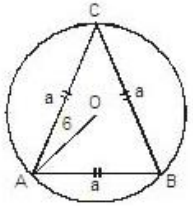
$x_2 + x_3 = 7$

$x_3 + x_1 = 8$

$2(x_1 + x_2 + x_3) = 20$

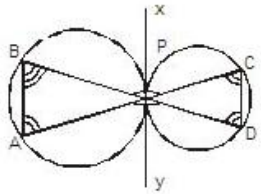
$x_1 + x_2 + x_3 = 10 \Rightarrow x_1 = 3$ cm

$x_2 = 2$ cm; $x_3 = 5$ cm

11. 

$R = \frac{a}{\sqrt{3}} = 6$ cm

$\therefore a = 6\sqrt{3}$ cm

12. 

(i) In ΔPAB and ΔPCD,

∠BPA = ∠CPD (Vertically opposite angles)

∠XPB = ∠A

Similarly, ∠YPD = ∠C

Since ∠XPC = ∠YPA

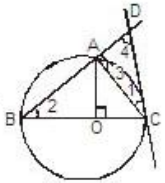
⇒ ∠A = ∠C

∠B = ∠D

Thus, DPABDPCD

(ii) Since ∠D = ∠B

Thus CD || AB (Internal alternate angle test)

13. 

Consider ΔBDC and ΔCDA .

In the circle, DC is a tangent at C.

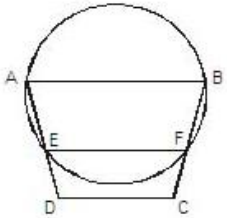
BA is extended and meets the tanget at D.

∠1 = ∠2 (Alternate segment theorem)

∠4 is common to both the triangles.

Since two angles are equal, the third angle must also be equal. By AAA property ΔBDC and ΔCDA are similar.

14. Both ABCD and ABFE are cyclic quadrilaterals.



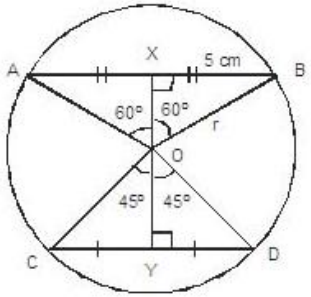
∴ ∠ABF + ∠AEF = 180° (Sum of opposite angles = 180°)

∠ABF + ∠ADC = 180°; ∠AEF - ∠ADC = 0°

$\therefore \angle AEF = \angle ADC$

These are corresponding angles. Since they are equal EF || DC.

15.



AB = 10 cm and subtends an angle 120° at the centre. CD subtends an angle of 90°. If the distance has to be maximum the two chords must be parallel to each other.

$r \sin 60° = 5$ cm.

$\therefore r = \frac{5}{\sin 60°} \Rightarrow \frac{10\sqrt{3}}{3}$ cm

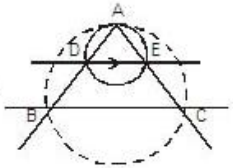
Hence, $OX = r\cos\theta = r\cos 60° = \frac{10\sqrt{3}}{3} \times \frac{1}{2} = \frac{5\sqrt{3}}{3}$ cm

$OY = r\cos 45° = \frac{10\sqrt{3}}{3} \times \frac{1}{\sqrt{2}} \Rightarrow \frac{5\sqrt{6}}{3}$

$\therefore$ The maximum distance between the mid-points of two such chords $= \frac{5}{3}\left(\sqrt{3} - \sqrt{6}\right)$ cm

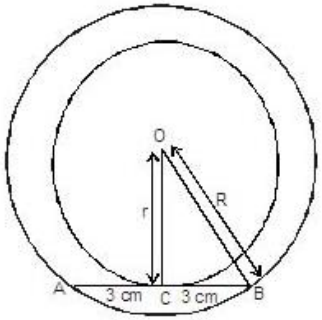
Level - 3

16.



The two circles have a common point A, through which both the circles pass, and point A lies on both the circles passing through A, D, E and A, B, C. Thus, they touch internally at A.

17.



Perpendicular from centre bisects the chord.

(AC = CB = 3 cm)

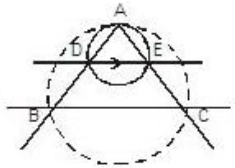
Thus OCB is a right angled triangle.

Where base CB = 3 cm.

Only triplet satisfying it is (3, 4, 5) thus

R = 5, r = 4

So, R - r = 1 cm

16. 

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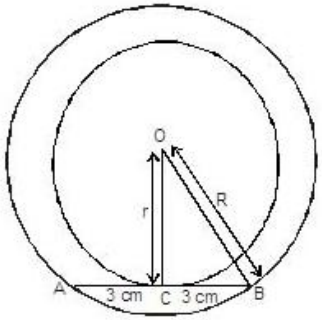
17. 

Perpendicular from centre bisects the chord.

(AC = CB = 3 cm)

Thus OCB is a right angled triangle.

Where base CB = 3 cm.

Only triplet satisfying it is (3, 4, 5) thus

R = 5, r = 4

So, R - r = 1 cm

Chapter - 4

Mensuration

Level -1

1. Two new circles come into picture from every cut.

Thus a total of 4 new circles from two cuts.

Percentage increase in Surface Area = $\frac{\text{Increase in Surface Area}}{\text{Original Surface Area}} \times 100$

$= \frac{4\pi r^2}{2\pi r(r+h)} \times 100 = \frac{2r}{r+h} \times 100 = \frac{2 \times 5}{5+10} \times 100 = \frac{10}{15} \times 100 = \frac{2}{3} \times 100 \approx 66.66\%$

2. If l, b, h are the length, breadth and height of cuboid,

$l = 12 \times 6 = 72$ cm, $b = 12$ cm, $h = 12$ cm

Hence, surface area = 2(lb + bh + lh)

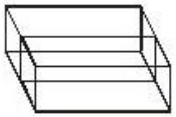
$= 2(864 + 144 + 864) = 2 \times 1872 = 3744$ cm^2

3. Outside volume of the box = $42 \times 30 \times 20$ cm^3

= 25200 cm^3

Inner volume of the box = $40 \times 28 \times 18$ cm^3

= 20160 cm^3



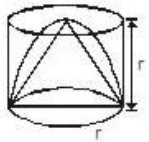
Volume of wood = 25200 - 20160 = 5040 cm^3

4. Volume of water flowing every minute

$= 1000 \text{ cm} \times \pi \times (0.5)^2 \text{ cm}^2 = 250\pi \text{ cm}^3$

Volume of vessel = $\frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \cdot (20)^2 \cdot 24 = 3200\pi$ cm^3

Time taken = $\frac{3200\pi}{250\pi}$ min = 12.8 min

5. 

All have same base radius, i.e. r.

Also height of the cylinder = Height of the cone = Radius of hemisphere = r

$V_{cylinder} : V_{hemisphere} : V_{cone} = \pi r^2 h : \frac{2}{3}\pi r^3 : \frac{1}{3}\pi r^2 h = r^3 : \frac{2}{3} r^3 : \frac{1}{3} r^3 = 3 : 2 : 1$

6. For the cylinder,

let the radius and the height be R and H respectively.

For cone, they are r and h respectively.

Curved surface area for the cylinder : cone = 8 : 5

$\Rightarrow 2\pi RH : pr\sqrt{r^2+h^2} = 8 : 5$

But R = r and H = h

$\Rightarrow \frac{2\ RH}{R\ \sqrt{R^2+H^2}} = \frac{8}{5} \Rightarrow \frac{H^2}{R^2+H^2} = \frac{16}{25} \Rightarrow 3H = 4R$

or $\frac{R}{H} = \frac{3}{4}$

7. Original volume = $\frac{4}{3}\pi r^3$

Volume of new spheres = $\frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi\frac{r^3}{8}$

Hence, 8 such spheres can be made.

$$\frac{\text{TSA big}}{\text{TSA small}} = \frac{4\pi r^2}{4\pi\frac{r^2}{4}} = \frac{4}{1}$$

Where TSA is total surface area.

$\Rightarrow$ Required ratio = 1 : 4

8. The volume so formed after melting will be equal to the combined initial volume of 3 spheres.

Let R be the radii of new sphere.

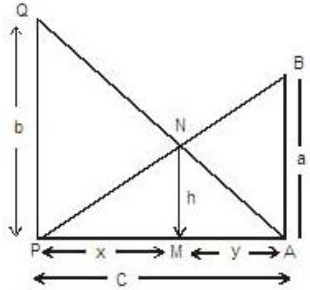
$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3 + \frac{4}{3}\pi r_2^3$$

or directly $R^3 = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$

$R = (36)^{\frac{1}{3}} = 3.30$ cm

Level - 2

9. c



$\Delta PAB \sim \Delta PMN$

$$\frac{AP}{PM} = \frac{AB}{MN}$$

$$\frac{x+y}{x} = \frac{a}{h} \text{ or } \frac{c}{x} = \frac{a}{h}$$

$$x = \frac{ch}{a}$$

Similarly $\Delta APQ \sim \Delta AMN$

$$\Rightarrow \frac{AP}{AM} = \frac{PQ}{MN}$$

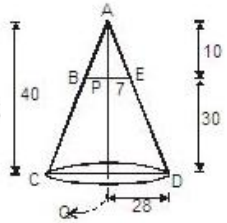
$$\frac{c}{y} = \frac{b}{h} \text{ or } y = \frac{ch}{b}$$

Also x + y = c

$$\frac{ch}{a} + \frac{ch}{b} = c$$

$$h = \frac{ab}{a+b}$$

So, correct answer is (c) $\frac{HM}{2}$

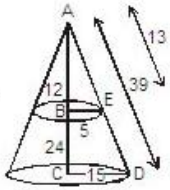
10. 

BE || CD

$\therefore \frac{AB}{AC} = \frac{AE}{AD} = \frac{BE}{CD} = \frac{PE}{QD} = \frac{AP}{AQ}$

So PE = $28 \times \frac{10}{40} = 7$ cm

Area of cross section = $\pi(r)^2 = \frac{22}{7} \times 49 = 154$ cm².

11. 

$\frac{AB}{AC} = \frac{BE}{CD}$

$\frac{AB}{AB+24} = \frac{5}{15} = \frac{1}{3}$

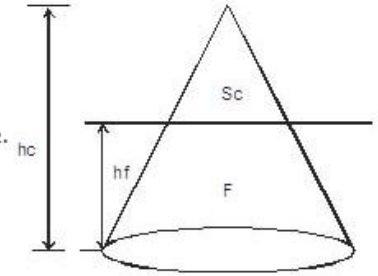
AB = 12

Curved surface area of the cone = $\pi r l = \pi \times 15 \times 39 = 585\pi$

Curved surface area of the cone with radius BE = $\pi \times 5 \times 13 = 65\pi$

$\therefore$ Lateral surface area of bucket = 520π

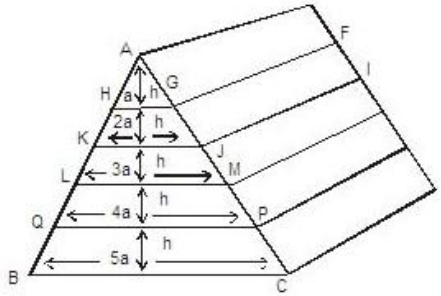
Total surface area = 520π + Base area of the bucket = $520\pi + 25\pi = 545\pi$ cm²

12. 

$\frac{(\text{Vol}) \text{ Smaller cone}}{(\text{Vol}) \text{ Original cone}} = \left(1 - \frac{h_f}{h_c}\right)^3 \therefore \frac{h_f}{h_c} = 1 - \sqrt[3]{\frac{C-F}{C}}$

13. All Δ are similar Δ`s in the cross-section of the figure ΔAGH ~ ΔAKJ ~ ΔALM ~ ΔAQP ≈ ΔABC

so distance between parallel lines is also same. Respective sides will be also proportional.

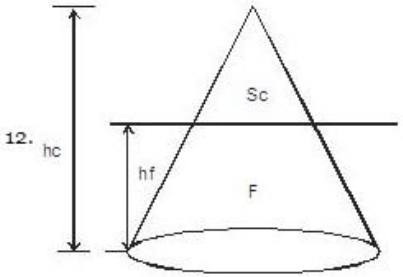


Now Area of strip HGJK = $\frac{1}{2}(a + 2a) \times h = 17$ sq. units

∴ Lateral surface area of bucket = 520π

Total surface area = 520π + Base area of the bucket = $520\pi + 25\pi = 545\pi$ cm^2

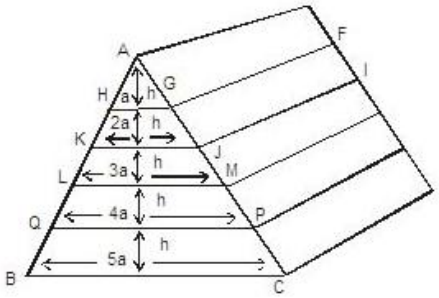
12.



$$\frac{(\text{Vol}) \text{ Smaller cone}}{(\text{Vol}) \text{ Original cone}} = \left(1 - \frac{h_f}{h_c}\right)^3 \therefore \frac{h_f}{h_c} = 1 - \sqrt[3]{\frac{C - F}{C}}$$

13. All Δ are similar Δ`s in the cross-section of the figure ΔAGH ~ ΔAKJ ~ ΔALM ~ ΔAQP ≈ ΔABC

so distance between parallel lines is also same. Respective sides will be also proportional.



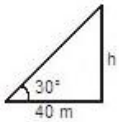
Now Area of strip HGJK = $\frac{1}{2}(a + 2a) \times h = 17$ sq. units

Area of strip QPCB = $\frac{1}{2}(5a + 4a)h = \frac{9a}{2}h = 51$ sq. units .

Chapter - 5

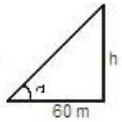
Trigonometry

Level - 1

1. 

(i) $\frac{h}{40} = \tan 30°$

$\Rightarrow h = \frac{40}{\sqrt{3}}$ m

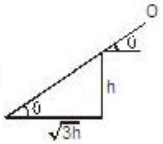
(ii) 

If angle of elevation θ is $\cot^{-1}\left(\frac{3}{5}\right)$, this means

$\frac{60}{h} = \frac{3}{5}$

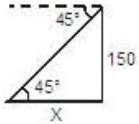
As x = 60 m

$\therefore h = \frac{5 \times 60}{3} = 100$ m

(iii) 

In the figure, $\tan\theta = \frac{h}{\sqrt{3}h} = \frac{1}{\sqrt{3}}$

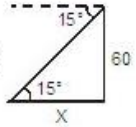
$\therefore \theta = 30°$

2. (i) 

From the above figure,

$\tan 45° = \frac{150}{x} \Rightarrow x = 150$ m

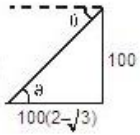
(as $\tan 45° = 1$)

(ii) 

$\tan 15° = \frac{60}{x}$

$(2-\sqrt{3}) = \frac{60}{x}$

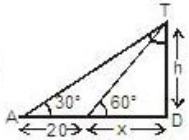
So $x = \frac{60}{2-\sqrt{3}}$ m $= 60(2+\sqrt{3})$ m

(iii) 

$\tan\theta = \frac{100}{100(2-\sqrt{3})}$

$\theta = \tan^{-1}\frac{1}{(2-\sqrt{3})} = 15°$

3. (i)



From the figure, $\tan 60° = \frac{h}{x}$

So $h = \sqrt{3}x$

And in the ΔTAD,

$\frac{h}{x+20} = \tan 30°$

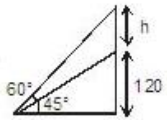
So $h = \frac{(x+20)}{\sqrt{3}} \Rightarrow \sqrt{3}h = (x+20)$

or x = 10 m

Hence, $h = 10\sqrt{3}$ m

Distance AD = 30 m

(ii)



$\frac{120}{x} = \tan 45°$

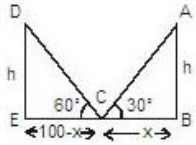
or x = 120 m

Now $\frac{h+120}{x} = \tan 60°$

$h + 120 = 120\sqrt{3}$

$h = 120(\sqrt{3}-1)$ m

4. (i)



In ΔDEC,

$\tan 60° = \frac{h}{(100-x)}$

$h = \sqrt{3}(100-x)$ m

Also, In ΔACB,

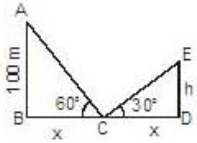
Also $h = x \tan 30° = \frac{x}{\sqrt{3}}$ m

So $\sqrt{3}(100-x) = \frac{x}{\sqrt{3}}$

x = 75 m

Therefore, $h = \frac{75}{\sqrt{3}} = 25\sqrt{3}$ m

(ii)



In ΔABC

$\frac{100}{x} = \tan 60°$

$x = \frac{100}{\sqrt{3}}$ m.

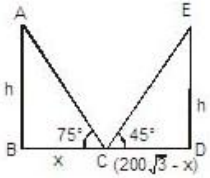
ΔECD

$\frac{h}{x} = \tan 30°$

or $h = \frac{100}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{100}{3} = 33.33$ m

Level - 2

5.



In ΔABC,

$\frac{h}{x} = \tan 75° = 2 + \sqrt{3}$ or $h = x(2 + \sqrt{3})$

In ΔDEC,

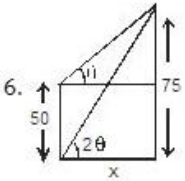
$\theta = 45°$,

$h = (200\sqrt{3} - x)$

So $x = \frac{200}{(1+\sqrt{3})}$ m $= 100\ (\sqrt{3} - 1)$ m

$h = 200\frac{(2+\sqrt{3})}{(1+\sqrt{3})}$ m $= 100\ (\sqrt{3} + 1)$ m

6.



$\frac{75}{x} = \tan 2\theta$ ...(i)

$\frac{25}{x} = \tan\theta$ ...(ii)

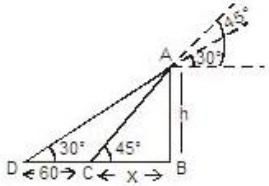
From (i) and (ii),

$\tan 2\theta = 3 \times \frac{25}{x} = 3\tan\theta$

$\therefore \theta = 30°\ \left(\tan 60° = \sqrt{3} = \frac{3}{\sqrt{3}} = 3\tan 30°\right)$

Required angle $= 2\theta = 2 \times 30° = 60°$

7. (i)

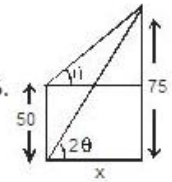


$\frac{h}{x} = \tan 45°$

$\therefore h = x$

Also $\frac{h}{x+60} = \tan 30° \Rightarrow \frac{h}{h+60} = \tan 30°$

$h = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 30(1+\sqrt{3})$ m

6. 

$\frac{75}{x} = \tan 2\theta$ ...(i)

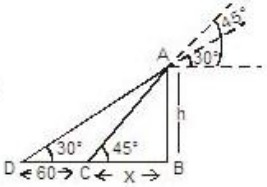
$\frac{25}{x} = \tan\theta$ ...(ii)

From (i) and (ii),

$\tan 2\theta = 3 \times \frac{25}{x} = 3\tan\theta$

$\therefore \theta = 30^\circ \left(\tan 60^\circ = \sqrt{3} = \frac{3}{\sqrt{3}} = 3\tan 30^\circ\right)$

Required angle = $2\theta = 2 \times 30^\circ = 60^\circ$

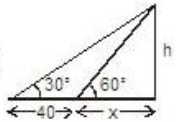
7. (i) 

$\frac{h}{x} = \tan 45^\circ$

$\therefore h = x$

Also $\frac{h}{x+60} = \tan 30^\circ \Rightarrow \frac{h}{h+60} = \tan 30^\circ$

$h = \frac{60}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = 30\left(1+\sqrt{3}\right)$ m

(ii) 

$\frac{h}{x} = \tan 60^\circ$

or $h = \sqrt{3}x$ ... (i)

$\frac{h}{x+40} = \tan 30^\circ$ ... (ii)

From (i) and (ii),

$\frac{\sqrt{3}x}{x+40} = \tan 30^\circ$

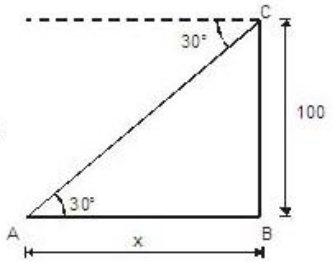
x = 20 m

h = x tan60° = 20 $\sqrt{3}$ m

Chapter - 6

Practice Exercise 1 - Level 1

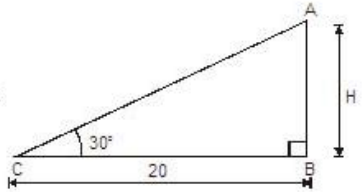
1. a



$\tan 30° = \frac{100}{x}$

$\frac{1}{\sqrt{3}} = \frac{100}{x} \Rightarrow x = 100\sqrt{3}$

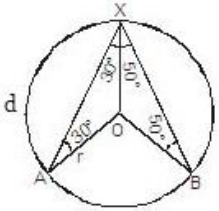
2. a



$\frac{H}{20} = \tan 30°$

$H = \frac{20}{\sqrt{3}}$

3. d



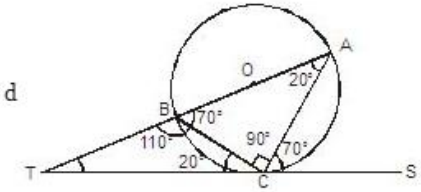
OA = OB = OX (radius of circle)

∴ ΔXOA is isosceles triangle

∴ ∠AXO = ∠OAX = 30° and ∠OBX = ∠OXB = 50°

∴ ∠AXB = 80° and ∠AOB = 2 ∠AXB = 2 × 80° = 160°

4. d



∠ACS = ∠ABC (By alternate segment theorem)

and ∠BCA = 90° (The angle that diameter makes with circumference = 90°)

∠TBC = 180 - ∠CBA = 180 - 70 = 110°

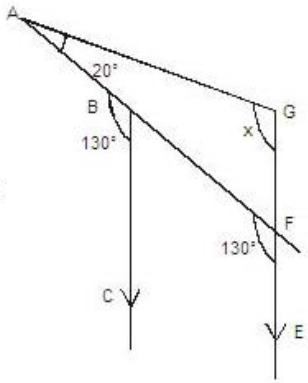
∠BCT = 180 - ∠BCA - ∠ACS

= 180° - 90° - 70° = 20°

∴ ∠BTC = 180 - ∠TBC - ∠BCT

= 180° - 110° - 20° = 50°

5. c



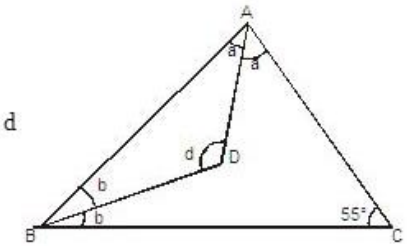
CB || EG

∴ ∠ABC = ∠BFE = 130

∴ ∠BFG = 180 - 130 = 50°

x = 180° - 20° - 50° = 110°

6. d



2a + 2b + 55° = 180°

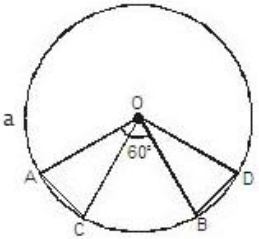
(Sum of all the angles of triangle = 180°)

∴ a + b = 62.5°

∠d = 180 - (a + b) = 180 - 62.5

∠d = 117.5°

7. a



AC = BD

Equal chords makes equal angle with the centre.

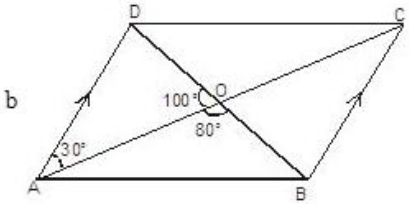
∠AOB = 60

∠BOC = 15

∴ ∠AOC = 45°

∠AOC = ∠BOD = 45°

8. b



∠AOB + ∠DOA = 180

∠DOA = 180 - 80 = 100°

$\angle ADO = 180 - \angle DAO - \angle DOA$

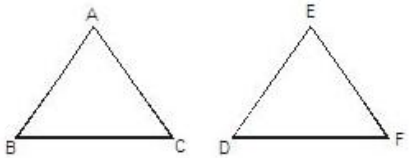
$\angle ADO = 180 - 30 - 100 = 50°$

$\angle ADO = \angle DBC$ (As AD | | BC and DC | | AB)

$\therefore \angle DBC = 50°$

9. d $\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{\text{Square of side}}{\text{Square of side}} = \frac{AB^2}{DE^2}$

(When two triangles ABC and DEF are similar)



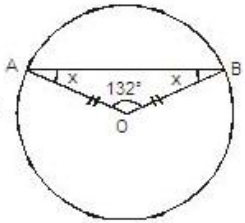
$\frac{AB^2}{DE^2} = \frac{64}{49}$

$\frac{49}{DE^2} = \frac{64}{49}$

$DE = \sqrt{\frac{49 \times 49}{64}}$

$DE = \frac{49}{8}$

10. b



OA = OB = radius = 8

($\therefore$ AOB becomes isosceles triangle)

$\angle OAB + \angle ABO + \angle AOB = 180°$

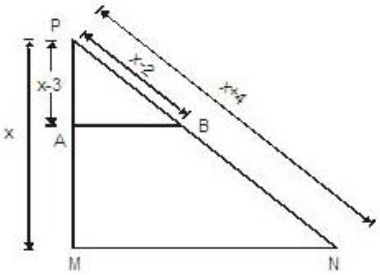
(as $\angle OAB = \angle ABO$)

$x° + x° + 132 = 180$

$2x° = 48°$

$x = 24°$

11. d



As AB | | MN

$\therefore \Delta APB$ and ΔMPN are similar

$\therefore \frac{PM}{PA} = \frac{PN}{PB} \Rightarrow \frac{x}{x-3} = \frac{x+4}{x-2}$

$x(x - 2) = (x - 3)(x + 4)$

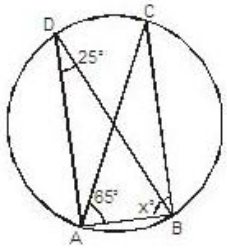
$x^2 - 2x = x^2 + 4x - 3x - 12$

$x = 4$

Alternative method

Since PA = x - 3 > 0 and x > 3 only on option says x > 3, i.e. (d).

12. a



∠ADB = ∠ACB = 25°

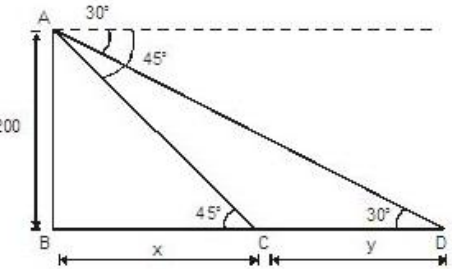
(Angle that chord subtends with circumference are equal)

In ΔABC ∠CAB + ∠x + ∠ACB = 180°

65 + 25 + x° = 180°

x° = 90°

13. e



y = Distance between two points

$\frac{200}{x} = \tan 45° \Rightarrow x = 200$

$\frac{200}{x+y} = \tan 30° \Rightarrow x + y = 200\sqrt{3}$

$y = 200\sqrt{3} - 200 = 200 \times 0.73 = 146$ m

14. b Number of sides of regular polygon $= \frac{360°}{\text{exterior angle}}$

and interior angle + exterior angle = 180° ...(i)

It is given that interior angle - exterior angle

= 120° ...(ii)

So by solving (i) and (ii)

Exterior angle = 30°

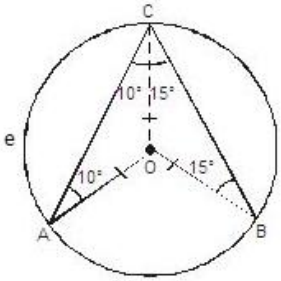
and number of sides $= \frac{360}{30} = 12$

15. b Let's angle of the quadrilateral be 2x, 4x, 5x and 7x. So, sum of angle = 2x + 4x + 5x + 7x = 360°

x = 20°

Hence smallest angle = 2x = 40°.

16. e



AO = OC = OB (radius of same circle)

∴ ΔAOC and ΔBOC are isosceles triangles

In ΔAOC, ∠CAO = ∠OCA = 10°

In ΔCOB, ∠OBC = ∠OCB = 15°

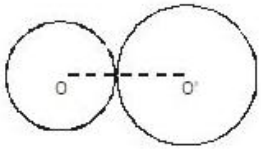
∴ ∠ACB = ∠ACO + ∠OCB = 10° + 15° = 25°

∠AOB = 2 ∠ACB

= 2 × 25 = 50°

(As angle at the center is twice the angles at the circumference)

17. c



If the circle touches each other externally, then distance between their centres is equal to sum of their radius, i.e. 5 + 9 = 14 cm

18. a $\angle BAC = \angle ACT + \angle ATC = 50 + 30 = 80^\circ$

And $\angle ACT = \angle ABC$

(Angle in alternate segment)

So $\angle ABC = 50^\circ$

$\angle BCA = 180 - (\angle ABC + \angle BAC) = 180 - (50 + 80) = 50^\circ$

Since $\angle BOA = 2\angle BCA = 2 \times 50 = 100^\circ$

Alternative Method:

Join OC

$\angle OCT = 90^\circ$ (TC is tangent to OC)

$\angle OCA = 90^\circ - 50^\circ = 40^\circ$

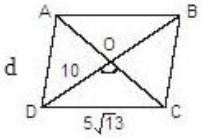
$\angle OAC = 40^\circ$ (OA = OC being the radius)

$\angle BAC = 50^\circ + 30^\circ = 80^\circ$

$\angle OAB = 80^\circ - 40^\circ = 40^\circ = \angle OBA$ (OA = OB being the radius)

$\angle BOA = 180^\circ - (\angle OBA + \angle OAB) = 100^\circ$

19. d



Given BD = 20 or BO = 10 cm = OD

CD = AB = BC = AD = $5\sqrt{13}$

$\angle COD$ is right angle.

Hence,

$OC = \sqrt{(5\sqrt{13})^2 - 10^2} = \sqrt{325 - 100} = \sqrt{225} = 15$

AC = 2(OC) = 2 × 15 = 30 m

Area of the rhombus $= \frac{1}{2} d_1 \times d_2 = \frac{1}{2} 20 \times 30 = 300\ m^2$

Cost of painting on one face will be $\frac{20}{100} \times 300$ = Rs. 60

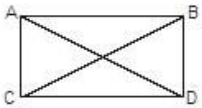
Since both faces are to be painted.

$\therefore$ Rs. 120

20. d Capacity of the box means its volume. So its volume will be V = lbh

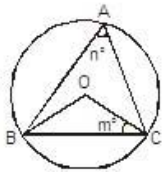
$V = \left(9-\frac{1}{2}\times 2\right)\times\left(7-\frac{1}{2}\times 2\right)\times\left(6-\frac{1}{2}\times 2\right) = (9 - 1)(7- 1)(6 - 1) = 8 \times 6 \times 5 = 240\ cm^3$

21. d There is a rule for quadrilateral that the sum of the sides of any quadrilateral, i.e. the perimeter is greater than the sum of its diagonals.



Hence, AB + BC + CD + DA > AC + BD

22. d



$\because$ OB = OC [Radius of the same circle]

So $\angle OBC = \angle OCB = m^\circ$

$\therefore \angle BOC = 180 - 2m$ ... (i)

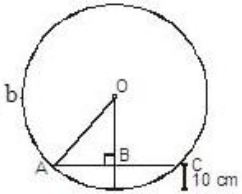
$\because \angle BAC = n^\circ$

$\because \angle BOC = 2\angle BAC = 2n$ ... (ii)

Equalising equation (i) and (ii)

$180 - 2m = 2n$, $2m + 2n = 180^\circ$ or $(m + n) = 90^\circ$

23. b



Since diameter = 1 m or 100 cm,

radius = 50 cm.

$\because$ Water is 10 cm deep, OB = 40 cm

Let us take ΔBAO.

$OA^2 = AB^2 + OB^2$

$50^2 = AB^2 + 40^2$

So AB = 30

$\because$ AB = BC = 30 cm, AC = AB + BC = 60 cm

24. d It is given that PQ = R.

Let us assume r be the radius of the inner circle.

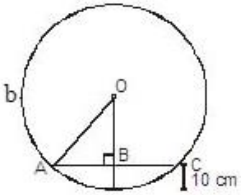
Now $\pi R^2 - \pi r^2 = A$, $\therefore \pi r^2 = \pi R^2 - A$

$r^2 = \frac{\pi R^2 - A}{\pi} = R^2 - \frac{A}{\pi}$ $\therefore r = \sqrt{R^2 - \frac{A}{\pi}}$

25. a Perimeter of the semicircle = r $(\pi + 2)$

Perimeter of $\Delta CAB = 2r + 2\sqrt{2}r = 2r\left(\sqrt{2}+1\right)$

180 - 2m = 2n, 2m + 2n = 180° or (m + n) = 90°

23. b 

Since diameter = 1 m or 100 cm,

radius = 50 cm.

∵ Water is 10 cm deep, OB = 40 cm

Let us take ΔBAO.

$OA^2 = AB^2 + OB^2$

$50^2 = AB^2 + 40^2$

So AB = 30

∵ AB = BC = 30 cm, AC = AB + BC = 60 cm

24. d It is given that PQ = R.

Let us assume r be the radius of the inner circle.

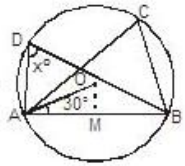
Now $\pi R^2 - \pi r^2 = A$, $\therefore \pi r^2 = \pi R^2 - A$

$r^2 = \frac{\pi R^2 - A}{\pi} = R^2 - \frac{A}{\pi} \therefore r = \sqrt{R^2 - \frac{A}{\pi}}$

25. a Perimeter of the semicircle = r $(\pi + 2)$

Perimeter of ΔCAB = $2r + 2\sqrt{2}r = 2r(\sqrt{2} + 1)$

∴ Required ratio = $\frac{(\pi + 2)r}{2r(\sqrt{2} - 1)} = \frac{(\pi + 2)(\sqrt{2} - 1)}{2}$

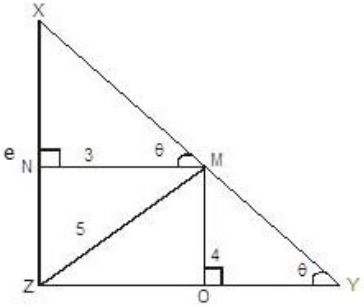
26. e 

AOM = 180 - (90 + 30) = 60°.

So $\angle AOB = 120^\circ$ or $\angle ADB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 120 = 60^\circ$

Practice Exercise 2 - Level 1

1. e



In rt. Δ X NM, XM $= \frac{3}{\cos\phi} = \frac{3\times5}{4} = \frac{15}{4}$

From rt. Δ XMZ, XZ $= \frac{XM}{\sin\phi} = \frac{15}{4}\times\frac{5}{3} = \frac{25}{4} = 6.25$

Hence (e) is correct.

2. a Rise of water in the given two containers depends on the base area of two containers.

Base area of A container $= \pi\,(2)^2 = \frac{88}{7}$

$= 12.57\ cm^2$

Base of B container $= (4)^2 = 16$

Hence, lesser the base area greater the rise of water. Hence, container A.

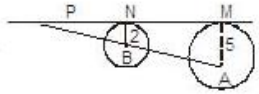
3. c Since we know in right-angled triangle

$c^2 = a^2 + b^2$ and also $c^2 = 2ab$

This can only be possible when a = b.

If a = b, then acute angles are 45° each.

4. c



ΔPAM and ΔPBN are similar.

$\therefore \frac{PA}{PB} = \frac{MA}{NB} = \frac{5}{2}$

So the point P divides AB externally in the ratio 5 : 2.

Even if the circles touch or intersect, the ratio will remain same.

5. a It is very clear, that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle.

Hence the required ratio = 1 : 12

6. d In the largest possible cube, the diameter of the sphere must be equal to $a\sqrt{3}$, where `a' is the side of the cube. So a = 25 mm.

Surface area of the cube $= 6a^2 = 6(25)^2 = 3750\ mm^2$.

7. b Considering ΔABC and ΔCAD

$\angle ABC = \angle DAC$ (given)

$\angle CAB = \angle ACD$ (since BA || CD)

Therefore, $\Delta ABC \sim \Delta CAD$

Now, $\frac{AB}{BC} = \frac{CA}{AD}$

$\frac{10}{9} = \frac{15}{AD} \Rightarrow AD = \frac{15\times9}{10} = \frac{27}{2}$

8. d The surface area of a sphere is proportional to the square of the radius.

Thus, $\frac{S_B}{S_A} = \frac{4}{1}$ (S. A. of B is 300% higher than A)

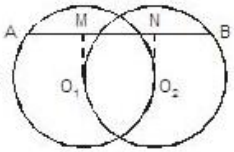
$\therefore \frac{r_B}{r_A} = \frac{2}{1}$

The volume of a sphere is proportional to the cube of the radius.

Thus, $\frac{V_B}{V_A} = \frac{8}{1}$

Or, V_A is $\frac{7}{8}$th less than B i.e. $\left(\frac{7}{8} \times 100\right)$ 87.5%

9. a



It is given that $O_1M = O_2N = 60$ ft and $O_1A = 100$, $AM = \sqrt{100^2 - 60^2} = 80$

So AB = AM + MN + NB = 80 + 100 + 80 or $NB = \sqrt{100^2 - 60^2} = 260$ ft

($\because$ AM = NB)

(Since $MN = O_1O_2 = 100$))

10. a In ΔMQB and ΔMBR, $\angle B$ be right angle.

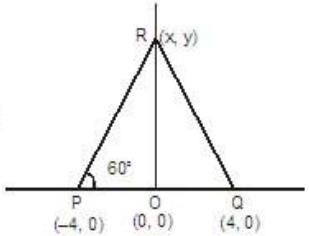
So $MQ^2 = MB^2 + QB^2$ or $MR^2 = MB^2 + BR^2$

So $MQ^2 - MR^2 = QB^2 - BR^2$

Similarly, for the rest of the triangles, make similar relationship, and we can get

$PC^2 + RB^2 + QA^2 = CR^2 + BQ^2 + AP^2$

11. c



To get the coordinate of the point R, it is obvious that `x' coordinate must be `o'.

So y coordinate will be OR = (+4) tan60° = $4\sqrt{3}$

So coordinate of R = (o, $4\sqrt{3}$)

Note: It can be $(0, -4\sqrt{3})$ as well.

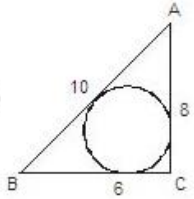
Alternative method:

Alternately, find the slope from the options.

For $(0, 4\sqrt{3})$ and (4, o)

$\tan\theta = \left(\frac{4\sqrt{3} - 0}{0 - 4}\right) = \sqrt{3}$ or $\theta = 60°$

12. b



Radius of the inner circle r = $\frac{A}{S}$

where $s = \frac{a+b+c}{2} = \frac{6+8+10}{2} = 12$

$A = \frac{1}{2} \times 8 \times 6 = 24$

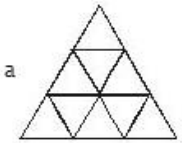
So $r = \frac{24}{12} = 2$ cm

13. a Since ratio of their circumference is directly proportional to their diameter.

So ratio of diameter = 60 : 50 or 6 : 5 and hence ratio of their sizes (volume) $= \left(\frac{6}{5}\right)^3 = \frac{216}{125} \approx 1.73$

So we can conclude the bigger one, If it were priced according to its size (or weight), it should cost 173% as much as the smaller one or 73% more than the small one. Since it is 50% costlier, it is more profitable to buy the bigger melon.

14. a



If you count the number of triangles, there are 9 equilateral triangles of side 2 cm.

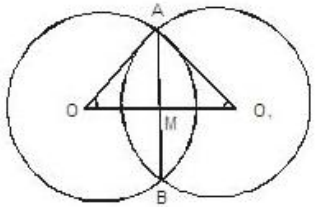
Alternative method:

Area of the big triangle $= \frac{\sqrt{3}}{4} \times (6)^2$

Area of the smaller triangle $= \frac{\sqrt{3}}{4} \times (2)^2$

$\therefore$ Maximum number of small triangles $= \frac{\frac{\sqrt{3}}{4} \times (6)^2}{\frac{\sqrt{3}}{4} \times (2)^2} = 9$

15. d



Length of OM = $\frac{OO_1}{2} = \sqrt{3}$ cm

Length of AM = $\frac{AB}{2} = 1$ cm $\tan(\angle OAM) = \frac{\sqrt{3}}{1}$

$\Rightarrow \tan(\angle OAM) = 60°$

Hence, $\angle OAO_1 = 120°$.

16. d Since perimeter of a rectangle p = 2(l + b) and

diagonal of the rectangle $d = \sqrt{l^2 + b^2}$

$\Rightarrow (l+b) = \frac{p}{2}$ or $(l+b)^2 = \frac{p^2}{4}$

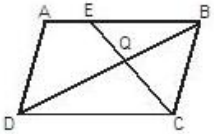
$l^2 + b^2 + 2lb = \frac{p^2}{4}$ or $d^2 + 2lb = \frac{p^2}{4}$

or $2lb = \frac{p^2}{4} - d^2$

$\therefore (l-b)^2 = l^2 + b^2 - 2lb = d^2 - \frac{p^2}{4} + d^2 = \frac{8d^2 - p^2}{4}$

or $(l - b) = \sqrt{\frac{8d^2 - p^2}{4}}$

17. a It is given that 3BE = 2DC or $\frac{BE}{DC} = \frac{2}{3}$



Since ΔDCQ and ΔBQE are similar, $\frac{Ar(\Delta BQE)}{Ar(\Delta DQC)} = \left(\frac{BE}{DC}\right)^2 \therefore \frac{Ar(\Delta BQE)}{36} = \left(\frac{2}{3}\right)^2$

or Ar(ΔBQE) = $36 \times \frac{4}{9} = 16$

18. d The required answer is 34 × 0.65 × 0.65 = 14.365

Because, we get two similar triangles and hence area of both triangles will be proportional to square of its side.

19. a In ΔABC, where 3 sides are a, b and c, and a is the largest side,

if $a^2 < b^2 + c^2$, then the triangle is acute;

if $a^2 = b^2 + c^2$, then it is a right-angled triangle;

if $a^2 > b^2 + c^2$, then the triangle is obtuse.

Here suppose the sides of the triangle are 6k, 8k and 9k.

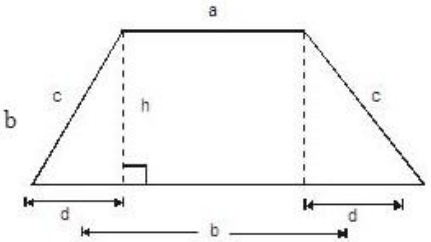
We have $(9k)^2 < (6k)^2 + (8k)^2$

∴ It is an acute triangle.

20. d To calculate the area, first of all calculate the length of the side of the square. Put 4 poles on 4 corners and now you are left with 16 poles.

Hence, 4 poles each between 2 corner poles. Hence, each side of the square contains 6 poles. So distance between 2 extreme poles on one side of the square will be 5 × 5 = 25 m. So area = $(25)^2 = 625\ m^2$

21. b



$d = \frac{b-a}{2} \Rightarrow c^2 = h^2 + \left(\frac{b-a}{2}\right)^2$

$\Rightarrow h = \sqrt{c^2 - \frac{(b-a)^2}{4}}$

⇒ Area of trapezium $= \frac{1}{2}$ (Sum of the parallel sides) × height $= \frac{1}{2}(a+b) \times \sqrt{c - \frac{(b-a)^2}{4}}$

⇒ Option (b) is correct.

22. e ∠RQS = 90°, ∠QRS = 180 - (90 + 70) = 20°,

∠PRS = 50°

∠PSR = 180 - (90 + 50) = 40°

∠PQR = ∠PSR

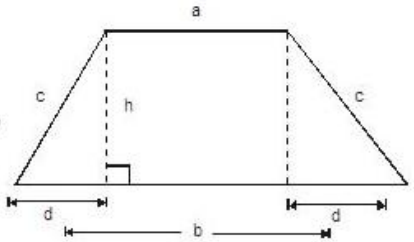
(Angles made by the same segment are equal.)

∠PQR = 40°

23. b It is clear that ΔAMN and ΔABC are similar.

So it's given that area of ΔABC = 2(Area of ΔAMN).

21. b



$d = \frac{b-a}{2} \Rightarrow c^2 = h^2 + \left(\frac{b-a}{2}\right)^2$

$\Rightarrow h = \sqrt{c^2 - \frac{(b-a)^2}{4}}$

$\Rightarrow$ Area of trapezium $= \frac{1}{2}$ (Sum of the parallel sides) $\times$ height $= \frac{1}{2}(a+b) \times \sqrt{c - \frac{(b-a)^2}{4}}$

$\Rightarrow$ Option (b) is correct.

22. e $\angle RQS = 90^\circ$, $\angle QRS = 180 - (90 + 70) = 20^\circ$,

$\angle PRS = 50^\circ$

$\angle PSR = 180 - (90 + 50) = 40^\circ$

$\angle PQR = \angle PSR$

(Angles made by the same segment are equal.)

$\angle PQR = 40^\circ$

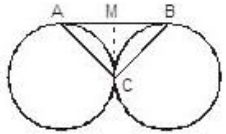
23. b It is clear that ΔAMN and ΔABC are similar.

So it's given that area of $\Delta ABC = 2$(Area of ΔAMN).

So, $\frac{MA^2}{BA^2} = \frac{\text{Area of } \Delta AMN}{\text{Area of } \Delta ABC} = \frac{1}{2}$

So $\frac{MA^2}{BA^2} = \frac{1}{2}$ or $\frac{MA}{BA} = \frac{1}{\sqrt{2}}$

24. a



$\because$ AM = MC (Tangent from the same external points are equal.)

MC = MB

$\angle ACB = 90^\circ$ (Rule)

So $\angle CAM = \angle ACM = 45^\circ$

25. a The area of the triangle $= \frac{1}{2} AB \times MC = \frac{1}{2}(2 + 2) \times 2 = 4\ cm^2$

26. d Area of the figure will be $\frac{x^2}{2} + x \times \sqrt{2}x = 8 + 16\sqrt{2}$

$x^2\left[\frac{1}{2} + \sqrt{2}\right] = 8 + 16\sqrt{2}$ or $x^2\left[2\sqrt{2} + 1\right] = 16\left[2\sqrt{2} + 1\right]$ or $x^2 = 16$

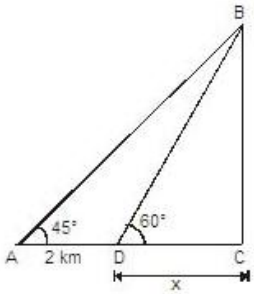
Hence, x = 4 (Since $x \neq -4$)

Practice Exercise 3 - Level 2

1. b Let A and D be the two stones so that AD = 2 km and BC is the height of the aeroplane from the ground.

Let DC = x km and BC = y km

So in ΔABC



$\tan 45° = \frac{BC}{AC}$ . . . (i)

$\Rightarrow 1 = \frac{BC}{2+x}$ and 2 + x = BC

In ΔBCD $\tan 60° = \frac{BC}{CD}$

So $\sqrt{3} = \frac{BC}{x}$ and $BC = \sqrt{3}x$ . . . (ii)

Equating (i) and (ii)

$2 + x = \sqrt{3}x$ and $\sqrt{3}x - x = 2$

So, $x(\sqrt{3}-1) = 2$ and $x = \frac{2}{\sqrt{3}-1}$

and $BC = \sqrt{3}x = \sqrt{3}\left(\frac{2}{\sqrt{3}-1}\right) = \frac{2\sqrt{3}}{\sqrt{3}-1}$

2. d Let BC be the height of the tower and AD = x metres

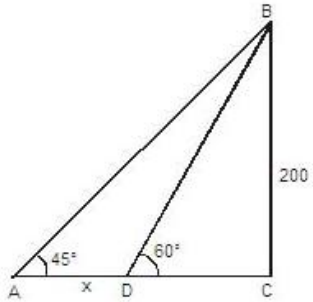
In ΔBAC

$\tan 45° = \frac{BC}{AC}$

$\Rightarrow 1 = \frac{200}{AC}$

⇒ AC = 200

In ΔBDC $\tan 60° = \frac{BC}{DC} = \frac{BC}{AC - AD}$



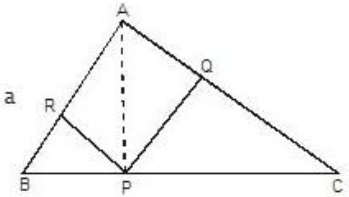
$\Rightarrow \sqrt{3} = \frac{200}{200-x}$

$\Rightarrow 200\sqrt{3} - \sqrt{3}x = 200$

$\Rightarrow \sqrt{3}x = 200(\sqrt{3}-1)$

$\Rightarrow x = \frac{200(\sqrt{3}-1)}{\sqrt{3}} = \frac{200(3-\sqrt{3})}{3}$

3. a



Join A with P. Let the area of the triangle be D.

Now, $\frac{BP}{CP} = \frac{1}{2}, \frac{CQ}{AQ} = \frac{3}{2}$ and $\frac{AR}{BR} = \frac{3}{2}$

area (Δ ABP) = $\left(\frac{1}{1+2}\right)\Delta = \frac{\Delta}{3}$ and

area (Δ ARP) = $\left(\frac{3}{3+2}\right)(\text{area } \Delta \text{ ABP}) = \frac{3}{5} \times \frac{\Delta}{3} = \frac{\Delta}{5}$ ...(1)

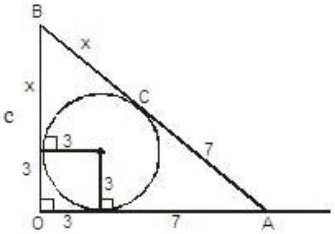
Similarly,

area (Δ APQ) = $\left(\frac{2}{2+3}\right) \times \left(\frac{2}{1+2}\right)\Delta = \frac{4}{15}\Delta$ ... (2)

From (1) and (2)

Area (ARPQ) = (area (Δ ARP) + area (Δ APQ) $= \frac{\Delta}{5} + \frac{4}{15}\Delta = \frac{7}{15}\Delta$.

4. c



AC is the other tangent, which meets OY at C.

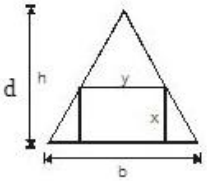
Then the circle becomes the incircle for the right angled ΔOAB.

Then $(x + 7)^2 = 10^2 + (x + 3)^2$

Hence solving we get x = 7.5 cm

So, OB = 3 + x = 10.5 cm.

5. d



Assume that the base of the rectangle be y.

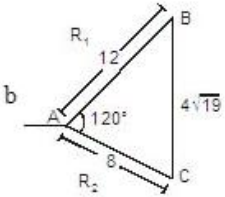
Then because of similar triangle

$\frac{h-x}{y} = \frac{h}{b}$

$y = \frac{b}{h}(h - x)$

Area of the rectangle is xy = $x\frac{b}{h}(h-x)$

6. b



$BC^2 = (12)^2 + (8)^2 - 2 \times 12 \times (8) \times \cos 120°$

$= 144 + 64 - 2 \times 12 \times 8 \times (-\frac{1}{2})$

BC = $4\sqrt{19}$ km

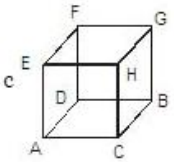
$t_1 = \frac{12+4\sqrt{19}+8}{3}$

$t_2 = \frac{8+4\sqrt{19}+12}{2}$

[where t_1 and t_2 are time taken (in hours) by Ram and Shyam respectively to reach the starting point after moving along the perimeter of the triangle ABC.]

$\therefore$ Required Difference = $20+4\sqrt{19}\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{10+2\sqrt{19}}{3}$ hrs.

7. c



As cube shown in the figure where to reach from A to B by the shortest route will be 2x (if x be the length of the side of the cube), and the longest route from B to A will be 6x. (All of the routes is BGFEHCA)

Hence, 2x + 6x = 5040; x = $\frac{5040}{8} = 630$ m

$\therefore$ Area of a face = $x^2 = (630)^2 = 396900$ m^2

8. d Let the radius of the original sphere = R

and let the number of identical cones = n

So, $\frac{4}{3}\pi R^3 = n\times\left(\frac{\pi}{3}(R)^2\times R\right)$

$\Rightarrow$ n = 4

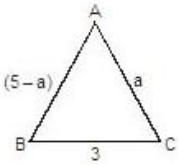
$\Rightarrow$ volume of smaller sphere = $\frac{\pi}{3}R^3$

$\Rightarrow$ radius of smaller sphere = $\left(\frac{R^3}{4}\right)^{\frac{1}{3}} = \frac{R}{(2)^{\frac{2}{3}}}$

$\Rightarrow \frac{\text{surface area of smaller sphere}}{\text{Surface area of larger sphere}} = \frac{R^2}{2^{\frac{4}{3}}}\times\frac{1}{R^2} = 1:2^{\frac{4}{3}}$

Hence, (d) is correct.

9. a



Assume one side is a, then other will be 5 - a,

$s = \frac{a+b+c}{2} = \frac{8}{2} = 4$

Area A = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{4(4-3)(4-a)(4-5+a)}$

A = $\sqrt{4\times1\times(4-a)(a-1)}$

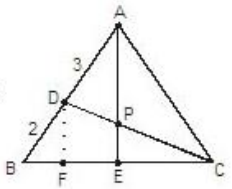
or $A^2 = 4(4 - a)(a - 1) = -4a^2 + 20a - 16$

For maximum area, $\frac{dA}{da} = \frac{-8a+20}{2a} = 0$

$\therefore a = \frac{5}{2}$ also $\frac{d^2A}{da^2} = -8 = (a \text{ is negative})$

Hence, maximum area. Thus, the possible sides are $\frac{5}{2}$ and $\frac{5}{2}$.

10. c



Let BC = 10x . Then BE = 5x and EC = 5x.

Draw a DF parallel to AE such that F is on BC.

In ΔABE, DF is parallel to AE.

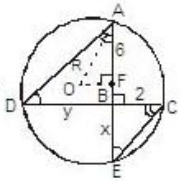
Hence BF : FE = 2:3. (or) BF = 2x.

So FE = 3x And EC = 5x.

Hence CP : PD = CE : EF i.e. 5:3

11. e Since, with the given centroid, many triangles are possible. So, a unique answer cannot be determined.

12. b



In the figure, Δs ABD and CBE are similar.

Hence, $\frac{AB}{BD} = \frac{CB}{BE}$ or $\frac{6}{y} = \frac{2}{x}$ or y = 3x

Also, if F is the mid-point of AE, then

$(AF)^2 + (OF)^2 = R^2$ or $\left[\frac{6+x}{2}\right]^2 + \left[\frac{2+y}{2} - 2\right]^2 = R^2$

Simplifying, we get $(6 + x)^2 + (3x - 2)^2 = 4R^2 = 4 \times 10^2 = 400$

Solving, we get x = 6.

This implies F, B are coincident.

Also y = 18 (i.e. 3x).

Hence CD = 20 cm (which means it is the diameter.)

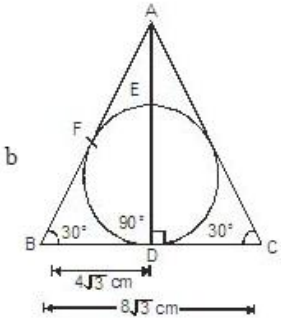
13. b The length of the largest possible rod in the cubical room = $a\sqrt{3}$. So $a\sqrt{3} = 35\sqrt{3}$ or a

= 35, where a = Side of the cube.

So diameter of the sphere that can fit in the cube of side 35 is also 35 m. Radius = $\frac{35}{2}$ m

So surface area = $4\pi r^2 = 4 \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} = 3850\ m^2$

14. b



$\tan 30^\circ = \frac{AD}{BD} = \frac{1}{\sqrt{3}}$

$\therefore AD = 4$ cm

$\sin 30^\circ = \frac{AD}{AB} = \frac{1}{2} = \frac{4}{AB}$

$\therefore AB = AC = 8$ cm.

AB = AC = 8 cm. And AD = 4cm

$(AF)^2 = (AE) \times (AD)$

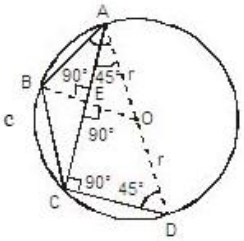
$\Rightarrow (8 - 4\sqrt{3})^2 = (4 - 2r) \times 4$

$\Rightarrow r = 4(2\sqrt{3} - 3)$

$\Rightarrow$ Area $= \pi r^2 = 48\pi(7 - 4\sqrt{3})\,cm^2$.

Hence (b) is correct.

15. c



Let AB = BC = y units and AC = CD = x units,

E, is the midpoint of the chord AC. AD is a diameter of the circle.

From right ΔACD, $x = r\sqrt{2}$ units.

$\Rightarrow AE = \frac{AC}{2} = \frac{r}{\sqrt{2}}$ units.

In the right ΔABE,

$y^2 = BE^2 + AE^2 = (OB - OE)^2 + AE^2 = (r - r\sin 45^\circ)^2 + \left(\frac{r}{\sqrt{2}}\right)^2$

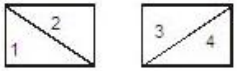
$\Rightarrow y = r\sqrt{2 - \sqrt{2}}$ units.

Hence, (c) is the correct answer.

16. d The way in which we can get maximum right triangles is:

The number of rectangles formed by the horizontal and the vertical lines is ${}^3C_2 \times {}^3C_2 = 9$

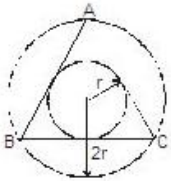
Each rectangle can give 4 right triangles.



Hence number of right triangles from these rectangles = 36.

We also get 4 triangles by joining points like (2, 4, 6); (2, 6, 8); (6, 8, 4) and (2, 4, 8). And 4 more by points (1, 5, 3); (3, 5, 9); (9, 5, 7) and (1, 5, 7). Hence in all 44 right triangles are possible.

17. c



Since the area of the outer circle is 4 times the area of the inner circle, the radius of the outer circle should be 2 times that of the inner circle.

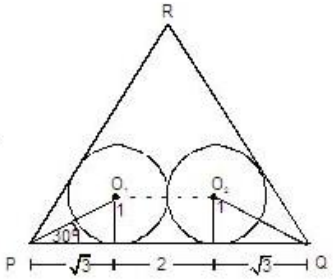
Since AB and AC are the tangents to the inner circle, they should be equal. Also, BC should be a tangent to inner circle. In other words, triangle ABC should be equilateral.

The area of the outer circle is 12. Hence the area of inner circle is 3 or the radius is $\sqrt{\frac{3}{\pi}}$.

The area of equilateral triangle = $3\sqrt{3}\ r^2$, where r is the inradius.

Hence the answer is $\frac{9\sqrt{3}}{\pi}$

18. b



ΔPQR is an equilateral triangle and O_1P and O_2Q are the angle bisectors.

$O_1O_2 = 2$ cm

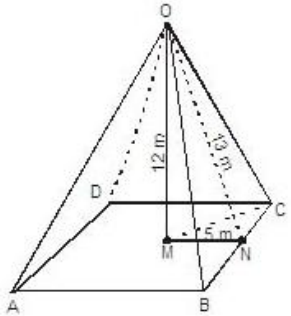
Side of the triangle PQR comes out to be $(2 + 2\sqrt{3})$ cm.

Area of the triangle PQR = $\frac{\sqrt{3}}{4}\left[2+2\sqrt{3}\right]^2$

Area of the triangle ABC = 4 × area of ΔPQR

Hence, the required area = $24 + 16\sqrt{3}$ cm².

19. d



AB = 10 m, MN = $\frac{1}{2}$AB = 5 m

OM is the perpendicular height = 12 m

(shortest distance).

$ON = \sqrt{OM^2 + MN^2} = \sqrt{12^2 + 5^2} = 13\,m$

For maximum length, we have to consider OC.

$OC = \sqrt{ON^2 + NC^2} = \sqrt{169 + 25} = \sqrt{194}\ m < 14\,m.$

Therefore, 12 m < Required Distance < 14 m.

Hence option (d) is the right choice.

20. c Sum of the length of the chord roads = $4\sqrt{5}$ and the length of OR = 4π r.

Thus the required ratio = $\sqrt{5} : \pi$

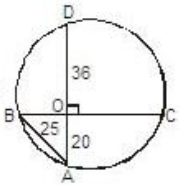
21. c The total time taken by the route given = $\frac{r}{30} + \frac{r}{15} = \frac{3}{2}$ (i.e. 90 min.)

Thus, r = 15 km. The radius of OR = 2r = 30 kms

22. d The total time taken = $\frac{r}{20} + \frac{r}{15} = \frac{7r}{60}$

Since r = 15, total time taken = $\frac{7}{4}$ hr. = 105 min.

23. e



To find the value of chord BC, first of all calculate OC

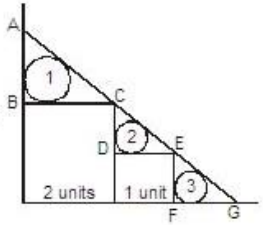
$\because$ OB = $\sqrt{25^2 - 20^2} = \sqrt{225} = 15$

$\because$ OB × OC = OD × OA $\Rightarrow$ 15 × OC = 36 × 20

So $OC = \frac{36 \times 20}{15} = 48$

So BC = 48 + 15 = 63 cm

24. b



Since ΔABC, ΔCDE and ΔEFG are similar

$\therefore \frac{AB}{BC} = \frac{CD}{DE} = \frac{EF}{FG}$

as CD = 1 and DE = 1

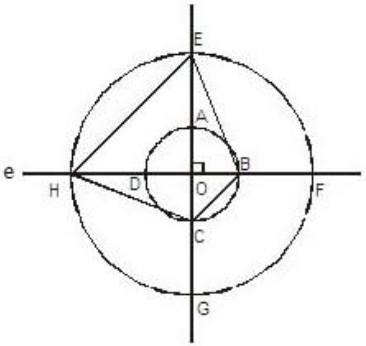
So, all the ratio will be 1 : 1

∴ AB = 2, FG = 1

So the ratio of radii will be 2 : 1 : 1

[The radius of circle 1 is $(2 - \sqrt{2})$ and that of circle 2 and 3 are $\left(1 - \frac{1}{\sqrt{2}}\right)$ each].

25. e



Let us assume the radii of larger and smaller circle be R and r respectively.

Area of annular ring is $\pi(R^2 - r^2)$ and the area of quadilateral HEBC is equal to

$$\text{area}(\square HBEC) = \text{area}(\Delta HOC) + \text{area}(\Delta COB) + \text{area}(\Delta EOB) + \text{area}(AOE)$$

$$= \frac{1}{2}Rr + \frac{1}{2}r^2 + \frac{1}{2}Rr + \frac{1}{2}R^2 = \frac{1}{2}(R + r)^2$$

Now, $\frac{\pi(R^2 - r^2)}{\frac{1}{2}(R + r)^2} = \frac{3}{2}$

or $\frac{2\pi(R - r)}{(R + r)} = \frac{3}{2}$

or $R(4\pi - 3) = r(3 + 4\pi)$

$\frac{r}{R} = \frac{4\pi - 3}{3 + 4\pi}$

26. b Area of the trapezium $= \frac{1}{2}(AB + CD)(BC) = \frac{1}{2}(a + b)(a + b)$

$\therefore 40.5 = \frac{1}{2}(a + b)^2$

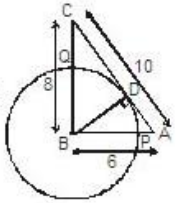
$\therefore a + b = 9$

$b = 3, a = 6$

$c^2 = a^2 + b^2$

$c = \sqrt{45} = 3\sqrt{5}$

27. d



Triangle ABC is a right angled triangle.

Thus, $\frac{1}{2} \times BC \times AB = \frac{1}{2} \times BD \times AC$

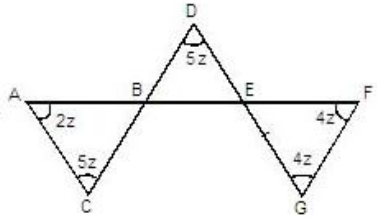
Or, $6 \times 8 = BD \times 10$. Thus $BD = 4.8$.

Therefore, $BP = BQ = 4.8$.

So, $AP = AB - BP = 6 - 4.8 = 1.2$ and $CQ = BC - BQ = 8 - 4.8 = 3.2$.

Thus, $AP : CQ = 1.2 : 3.2 = 3 : 8$.

28. c



$\angle BCA + \angle BAC = \angle BDE + \angle BED$

$\Rightarrow 5z + 2z = 5z + \angle BED$

$\Rightarrow \angle BED = 2z$

Thus, $\angle FEG = 2z$ (Vertically opposite)

In ΔEFG, $10z = 180°$

$\Rightarrow z = 18°$

29. b Using the Basic Proportionality Theorem,

$\frac{AB}{PQ} = \frac{BD}{QD}$ and $\frac{PQ}{CD} = \frac{BQ}{BD}$.

Multiplying the two we get,

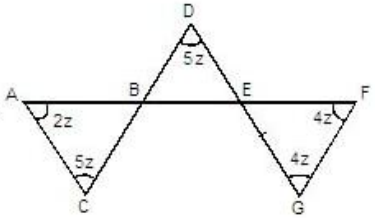
$\frac{AB}{CD} = \frac{BQ}{QD} = 3 : 1$.

Thus $CD : PQ = BD : BQ = 4 : 3 = 1 : 0.75$

Therefore, BP = BQ = 4.8.

So, AP = AB - BP = 6 - 4.8 = 1.2 and CQ = BC - BQ = 8 - 4.8 = 3.2.

Thus, AP : CQ = 1.2 : 3.2 = 3 : 8.

28. c 

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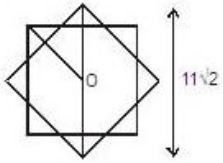
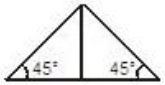
Multiplying the two we get,

$\frac{AB}{CD} = \frac{BQ}{QD} = 3 : 1.$

Thus CD : PQ = BD : BQ = 4 : 3 = 1 : 0.75

30. a 

Taking the section of the square



Area = $\left(\frac{11\sqrt{2}-11}{2}\right)^2$

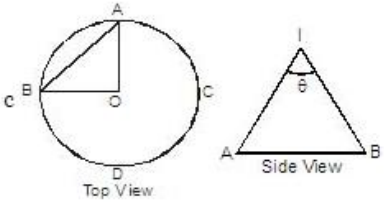
Net area of all uncommon regions = $8\times\left(\frac{11\sqrt{2}-11}{2}\right)^2$

Area of common region = $11^2 - \left(\frac{11\sqrt{2}-11}{2}\right)^2 \times 4$

$\therefore$ Ratio = $\frac{11^2 - 4\times\left(\frac{11\sqrt{2}-11}{2}\right)^2}{8\times\left(\frac{11\sqrt{2}-11}{2}\right)^2} = \frac{11^2}{8\times\left(\frac{11\sqrt{2}-11}{2}\right)^2} - \frac{1}{2} = \frac{1}{2\left(\sqrt{2}-1\right)^2} - \frac{1}{2} = \frac{\left(\sqrt{2}-1\right)^2}{2} - \frac{1}{2} = 1 - \sqrt{2}$ cm²

Practice Exercise 4 - Level 2

1. c



Since the 4 points are equally spaced, the angle between any 2 consecutive radii is $\frac{\pi}{2}$.

Hence OA = 3, OB = 3 and $\angle AOB = \frac{\pi}{2}$.

Hence $AB = \sqrt{3^2 + 3^2} = 3\sqrt{2}$ cm.

Since the vertical distance of the point I is 4 cm, its distance from any of the points on the circle, to which the string is tied, is given by

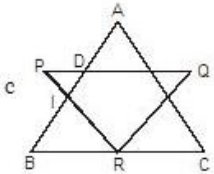
$IA = IB = \sqrt{3^2 + 4^2} = 5$ cm.

Hence in ΔIAB, IA = IB = 5 and AB= $3\sqrt{2}$ cm.

If $\angle AIB = \theta$, applying cosine rule we get

$$\cos\theta = \frac{5^2 + 5^2 - (3\sqrt{2})^2}{2 \times 5 \times 5} = \frac{16}{25}$$

2. c



$\Delta PDI \sim \Delta CBA$ (PD || BC, DI || AB, IP || CA)

and DI : BA = 1:5

$$\Rightarrow \frac{\text{area } \Delta PDI}{\text{area } \Delta CBA} = \frac{1}{25}$$

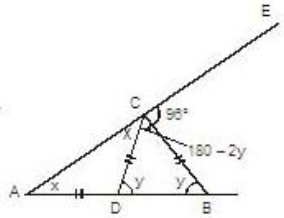
$\Delta PDI \sim \Delta PQR$

and PD : PQ = 1:3

$$\Rightarrow \frac{\text{area } \Delta PDI}{\text{area } \Delta PQR} = \frac{1}{9}$$

Thus, $\frac{\text{area } \Delta ABC}{\text{area } \Delta PQR} = \frac{1/9}{1/25} = \frac{25}{9}$

3. b



Using exterior angle theorem

$\angle A + \angle B = 96$

i.e. x + y = 96 ...(i)

Also x + (180 - 2y) + 96 = 180°

∴ x - 2y + 96 = 0

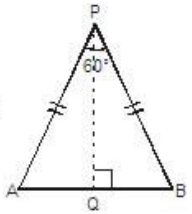
∴ x - 2y = -96 ...(ii)

Solving (i) and (ii),

y = 64° and x = 32°

$\therefore \angle DBC = y = 64^\circ$

4. b



Given $\angle APB = 60^\circ$ and AB = b.

$\therefore PQ = \frac{b}{2} \times \sqrt{3}$

In right angle triangle PQB.

$\frac{b^2}{4} + h^2 = \frac{3b^2}{4} \Rightarrow 2h^2 = b^2$

5. c Assume the number of horizontal layers in the pile be n.

So $\Sigma \frac{n(n+1)}{2} = 8436$

$\Rightarrow \frac{1}{2} [\Sigma n^2 + \Sigma n] = 8436$

$\Rightarrow \frac{n(n+1)(2n+1)}{12} + \frac{n(n+1)}{4} = 8436$

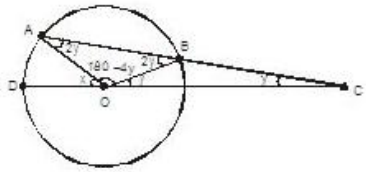
$\Rightarrow n(n+1) \left[\frac{2n+4}{12}\right] = 8436$

$\Rightarrow \frac{n(n+1)(n+2)}{6} = 8436$

$\Rightarrow n(n+1)(n+2) = 36 \times 37 \times 38$

So n = 36

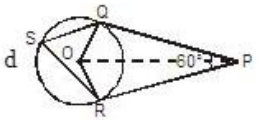
6. a



Therefore, x + 180 - 4y + y = 180

Therefore, x = 3y

7. d



$\because \angle OQP = \angle ORP = 90^\circ$ (Tangent)

$\because \angle QPO = \angle RPO = \frac{60}{2} = 30^\circ$

Hence,

$\angle QOP = \angle ROP = 180 - (90 + 30) = 60^\circ$

So $\angle QOR = 60 \times 2 = 120^\circ$

So $\angle QSR = \frac{1}{2} \angle QOR$ (Angle made by the same chord at the circumference is half of the angle made at the centre by the same chord.)

$= \frac{1}{2} \times 120 = 60^\circ$

Alternative method:

$\angle OQP = \angle ORP = 90^\circ$ (Tangent)

$\angle QOR = 360^\circ - \angle OQP - \angle ORP - \angle QPR$ (Quadrilateral)

$= 360° - 90° - 90° - 60° = 120°$

$\angle QSR = \frac{1}{2}\angle QOR$ (Angle made by the same chord at the circumference is half of the angle made at the centre by the same chord)

$= \frac{1}{2} \times 120° = 60°$

8. e Since in isosceles right-angled triangle, two sides are equal and one angle is 90°, its perimeter will be

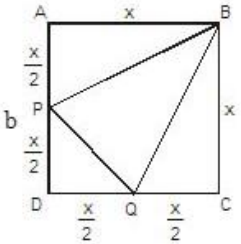
$x + x + \sqrt{2}x$ [If equal side be x] = 2p

$\therefore x(2+\sqrt{2}) = 2p$ or $x = \frac{2p}{2+\sqrt{2}} = \frac{2p}{\sqrt{2}(\sqrt{2}+1)}$

$\therefore x = \frac{\sqrt{2}\ p}{(\sqrt{2}+1)}$

Since area = $\frac{1}{2}x^2 = \frac{1}{2}\frac{2p^2}{(\sqrt{2}+1)^2} = \frac{p^2}{(2+1-2\sqrt{2})} = \frac{p^2}{(3-2\sqrt{2})} = p^2(3-2\sqrt{2})$

9. b



Let us assume the side of the square is x.

So PD = DQ = $\frac{x}{2}$

Area of ΔBPQ = Area of the square - Area of (ΔAPB + ΔPDQ + ΔQBC)

$\because$ Area of ΔAPB = ΔBCQ = $\frac{x^2}{4}$

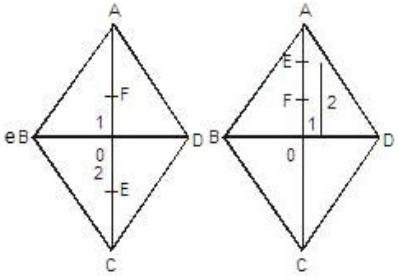
Area of ΔPDQ = $\frac{x^2}{8}$

So area of ΔBPQ =

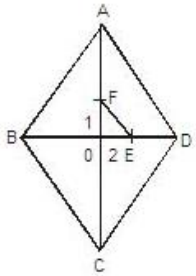
$x^2 - \left(\frac{x^2}{4} + \frac{x^2}{4} + \frac{x^2}{8}\right) = x^2 - \frac{5x^2}{8} = \frac{3x^2}{8}$

So percentage of area of BPQ = 37.5%

10. e



I. EF = 3 cm **II.** EF = 1 cm



III. EF = $\sqrt{5}$ cm

Hence, EF can take any value from I, II or III.

11. a Area of 4 discs = $4\times\frac{\pi}{4}(10)^2 = 100\pi$

Area of the bigger disc = $\pi(20)^2 = 400\pi$

Hence, left-out area = $400\pi - 100\pi = 300\pi$

Hence, percentage of left-out area = $\frac{300\pi}{400\pi}\times 100 = 75\%$

12. b Sum of external angles = 360°

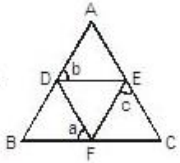
Greatest exterior angle = (180 - 120) = 60°

Common difference = 5°

Hence, 60° + 55° + 50° + ... + (60 - 5(n-1)) = 360°

$\Rightarrow$ n = 9

13. a



Since $b + \angle EDF = a + B$

{Sum of two internal angles of a triangle is equal to third exterior angle.}

So b + 60° = a + B ...(i)

Similarly, $a + \angle DFE = c + C$

So a + 60° = c + C ...(ii)

Subtracting equation (ii) from (i),

(b + 60) - (a + 60) = (a + B) - (c + C)

(b - a) = (a - c) ($\because$ B = C) (Because of isosceles triangle)

$\therefore$ 2a = b + c or $a = \frac{b+c}{2}$

14. d In ΔAOB, AB = 2 tan60° = $2\sqrt{3}$

So area of ΔAOB $= \frac{1}{2}\times 2\times 2\sqrt{3} = 2\sqrt{3}$

Area of sector AOC = $\frac{60}{360}\pi r^2 = \frac{\pi}{6}\times 4 = \frac{2\pi}{3}$

Area of shaded region = $2\sqrt{3} - \frac{2\pi}{3}$

So desired ratio = $\frac{2\sqrt{3} - \frac{2\pi}{3}}{\frac{2\pi}{3}} = \frac{2\sqrt{3}\times 3}{2\pi} - 1 = \frac{3\sqrt{3}}{\pi} - 1$

15. c Since it is given that the triangle is equilateral, length of the sides will remain same,

i.e. AB = BC = CA

$\because$ BC = $\sqrt{(4\sqrt{3}-0)^2 + (0-0)^2} = 4\sqrt{3}$

$\therefore$ AD = h = $\frac{\sqrt{3}}{2}\times BC = \frac{\sqrt{3}}{2}\times 4\sqrt{3} = 6$

16. b Let us assume the value of angle

$\angle ABC = x^\circ$

$\angle ACB = 180 - (50 + x) = 130 - x$

So $\angle MBC = 180 - x$ or $\angle OBC = \frac{180 - x}{2}$

Similarly, $\angle OCB = \frac{180 - (130 - x)}{2} = \frac{50 + x}{2}$

So $\angle BOC = 180 - \left(\frac{180 - x}{2} + \frac{50 + x}{2}\right) = 180 - \left(\frac{230}{2}\right) = 65^\circ$

17. a Since we know the relation between radius and number of revolutions,

$\frac{N_1}{N_2} = \frac{R_2}{R_1}$, i.e. $N_2 = N_1\left(\frac{R_1}{R_2}\right)$

(Where N is number of revolutions and R is radius.)

$\therefore N_2 = N_1\left[\frac{12.50}{12.25}\right]$ i.e. $N_2 = 1.02\,N_1$

(approximately)

Hence, number of revolutions increased by 2% (approximately).

$$\left\{\begin{array}{l} R_1 = \frac{25}{2} = 12.50 \text{ and} \\ R_2 = 12.50 - \frac{1}{4} = 12.25 \end{array}\right\}$$

For questions 18 and 19:

18. b Let side of original cube be 100 units.

Then side of the smallest cube is 25 units.

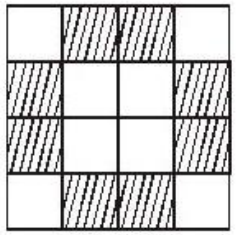
Surface area of the largest cube = $6(100)^2$

Surface area of the smallest cube = $6(25)^2$

∴ Ratio of this surface area is

$\frac{25^2}{100^2} = \frac{1}{16} = 0.0625$.

19. a



If we look at one side of the cube, then 8 blocks have paint on 2 sides. We would find another
8 blocks on the opposite side. From the 4 remaining sides, there are 8 more blocks with paint on 2 sides which are not covered by the face and the back of the cube. Probability of choosing a cube with exactly 2 sides painted

$= \frac{8 + 8 + 8}{\text{Total number of cubes}} = \frac{24}{64} = \frac{3}{8}$

20. b If $(x + 1) = (9 - x) \Rightarrow x = 4$.

Sides of the triangle will be (5, 5, 17), which is not possible as $(a + b > c)$ in any triangle.

If $(x + 1) = (5x - 3) \Rightarrow x = 1$

Sides are (2, 8, 2). Again not possible as in any triangle $(b + c) > a$

If $(9 - x) = (5x - 3) \Rightarrow x = 2$.

In this case (3, 7, 7) are sides of the triangle.

So only one isosceles Δ having sides (3, 7, 7) is possible.

Where, x = 2 is the only possible integral value of x.

21. c Since it is obvious that diagonals of the rectangle bisect each other, the middle point of one diagonal will be the middle point of the other one. That point will be $\left(\frac{5+1}{2}, \frac{3+1}{2}\right) = (3, 2)$

Which lies on the line y = 2x + l also. So substitute the coordinate of this point (3, 2) in

y = 2x + l to get the value of l.

$y = 2x + \lambda$

$2 = 2 \times 3 + \lambda$ or $\lambda = -4$

22. c $AB^2 = x^2 + (2x)^2 = 5x^2$

But AB = DE, therefore, DE = $\sqrt{5}x$

From ΔDCE, $5x^2 = (x + y)^2 + CE^2$

or CE = $\sqrt{5x^2 - (x+y)^2}$

$\therefore$ AE = $2x - \sqrt{5x^2 - (x+y)^2}$

23. d We have to paint $\pi\left[R^2 - \frac{R^2}{4}\right] = \frac{3}{4}\pi R^2$

It is given that the whole disc requires 20 L of paint. The shaded part will require $\frac{\frac{3}{4}\pi R^2 \times 20}{\pi R^2} = 15\,L$ of paint.

24. d Interior angle + Exterior angle = 180°

Let exterior angle be x.

$\therefore x + x + 132 = 180°$

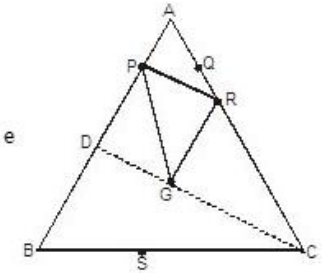
Exterior angle = 24°

Number of sides = $\frac{360°}{\text{Exterior Angle}} = \frac{360}{24} = 15$

25. c It's standard property among circle, square and triangle, for a given parameter, area of circle is the highest and area of the triangle is least whereas area of the square is in-between, i.e. c > s > t.

Practice Exercise 5 - Level 3

1. e



In-radius of the $\Delta ABC = \frac{2\sqrt{3}}{2\sqrt{3}} = 1\,cm$

Circumradius of the $\Delta ABC = \frac{2\sqrt{3}}{\sqrt{3}} = 2\,cm$

Join CG and extend it to intersect the side AB at D.

Area of

$\Delta ADC = \frac{1}{2}\Delta ABC = \frac{1}{2} \times \frac{\sqrt{3}}{4} \times (2\sqrt{3})^2 cm^2 = \frac{3\sqrt{3}}{2} cm^2$

AP = 1 cm $\therefore PD = (\sqrt{3} - 1)\,cm$

CR = 2 cm $\therefore AR = 2(\sqrt{3} - 1)\,cm$

$\therefore$ Area of $\Delta APR = \frac{1}{2} \times AP \times AR \times \sin 60^\circ = \frac{\sqrt{3}(\sqrt{3}-1)}{2} cm^2$

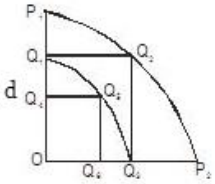
$\therefore$ Area of $\Delta CRG = \frac{1}{2} \times CR \times CG \times \sin 30^\circ = 1\,cm^2$

$\therefore$ Area of $\Delta PDG = \frac{1}{2} \times PD \times DG = \frac{(\sqrt{3}-1)}{2} cm^2$

$\therefore$ Area of ΔPRG = Area of $[\Delta ADC - \Delta APR - \Delta CRG - \Delta PDG]$

$= \frac{3\sqrt{3}}{2} - \left[\frac{\sqrt{3}(\sqrt{3}-1)}{2} + 1 + \frac{(\sqrt{3}-1)}{2}\right] = \left(\frac{3\sqrt{3}-4}{2}\right) cm^2$

2. d



Let the side of the square Q_1Q_2 be `a' units

$\therefore OQ_2 = \sqrt{2}\,a = r \Rightarrow a = \frac{r}{\sqrt{2}}$

Radius of quadrant Q_1Q_3O is a $= \frac{r}{\sqrt{2}}$

Side of square $OQ_4Q_5Q_6 = \frac{a}{\sqrt{2}} = \frac{r}{(\sqrt{2})^2} = \frac{r}{2}$

Required Area

$= \left(\frac{r}{\sqrt{2}}\right)^2 + \left(\frac{r}{2}\right)^2 + \left(\frac{r}{2\sqrt{2}}\right)^2 + \ldots\ldots\infty = \frac{r^2}{2} + \frac{r^2}{4} + \frac{r^2}{8} + \ldots\ldots\infty = \frac{r^2}{2}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\ldots\right] = \frac{r^2}{2} \times \frac{1}{1-\frac{1}{2}} = r^2$

3. d Area = $\frac{1}{2}$ × Base × Height

For all the triangles, Base = 5cm.

Let Δ_h be the sum of areas of all the triangles ABM which have a height of = h cm.

$\Delta_9 = 2 \times \frac{1}{2} \times 5 \times 9$

$\Delta_8 = 4 \times \frac{1}{2} \times 5 \times 8$

$\Delta_7 = 6 \times \frac{1}{2} \times 5 \times 7$

…… ……

$\Delta_3 = 14 \times \frac{1}{2} \times 5 \times 3$

$\Delta_2 = 16 \times \frac{1}{2} \times 5 \times 2$

$\Delta_1 = 18 \times \frac{1}{2} \times 5 \times 1$

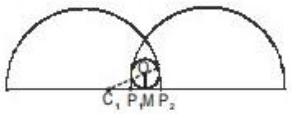
The sum of all the areas

$= \Delta_9 + \Delta_8 + \Delta_7 \ldots\ldots + \Delta_3 + \Delta_2 + \Delta_1 \left(2 \times \frac{1}{2} \times 5\right)[9 + 16 + 21 + 24 + 25 + 24 + 21 + 16 + 9] = 5 \times 165 = 825 \text{ cm}^2$

4. a Let C_1P_1 be `x' units

$\therefore$ $PQ = PC_1 + C_1P_1 + P_1C_2 + C_2Q = r + x + r + r = 3r + x$

$\therefore \frac{x}{3r + x} = \frac{1}{7} \Rightarrow 6x = 3r \Rightarrow x = \frac{r}{2}$



$P_1M = \frac{P_1P_2}{2} = \frac{r}{4}$

Let the radius of the smaller circle be `r_1' units and O be its center.

$\therefore C_1O^2 = r_1^2 + C_1M^2 = r_1^2 + \left(\frac{3r}{4}\right)^2$

$C_1O + r_1 = r \Rightarrow c_1O^2 = (r - r_1)^2$

$\Rightarrow r_1^2 + \frac{9r^2}{16} = r^2 - 2rr_1 + r_1^2 \Rightarrow 2rr_1 = \frac{7r^2}{16} \Rightarrow r_1 = \frac{7r}{32}$

5. a Let AB = c, BC = a, AC = b

Let DC = kb where $0 < k < 1$

EC = ma where $0 < m < 1$

Let h be the perpendicular from incenter to the sides.

So area of $\Delta DIC = \frac{1}{2} \cdot h \cdot kb$

$\Delta EIC = \frac{1}{2} \cdot h \cdot ma$

Also,

$\Delta DCE = \Delta DIC + \Delta EIC = \frac{A}{2} = \frac{1}{2}h[kb + ma]$

Also area of $\Delta ABC = \Delta AIC + \Delta BIC + \Delta CIA$

or $A = \frac{1}{2}bh + \frac{1}{2}ah + \frac{1}{2}ch$

or $A = \frac{1}{2}h(a + b + c) = \frac{Sh}{2}$

or $\frac{A}{2} = \frac{Sh}{4}$

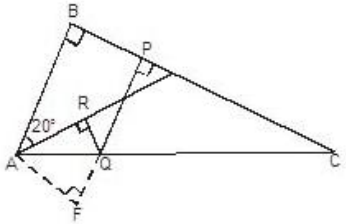
$\therefore \frac{1}{2}h[kb + ma] = \frac{1}{2} \cdot h \cdot \frac{S}{2}$

or kb + ma = CD + EC = semi perimeter = $\frac{S}{2}$.

6. d PQ || BA

$\Rightarrow \angle CBA = \angle CPQ = 90^\circ$

Extend PQ to F such that ABPF becomes a rectangle



PQ + QR = AB ⇒ PQ + QR = PF

⇒ PQ + QR = PQ + QF

⇒ QR = QF

$\therefore \Delta ARQ \cong \Delta AFQ \begin{pmatrix} \angle R = \angle F = 90^\circ \\ QR = QF \\ AQ \text{ common} \end{pmatrix}$

$\Rightarrow \angle FAQ = \angle RAQ = \theta$ (say)

But AF || BC

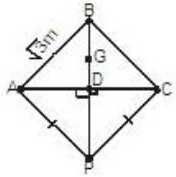
$\Rightarrow \angle FAC = \angle BCA = \theta$

In ΔABC

$90^\circ + 20^\circ + 2\theta = 180^\circ$

$\Rightarrow \theta = 35^\circ = \angle BCA$

7. e



Since ΔABC, is an equilateral triangle

$BD = \frac{\sqrt{3}}{2} \times \sqrt{3} = \frac{3}{2}$m.

Since P is equidistant from points A and C, therefore it will lie on the perpendicular bisector of AC.

Let, PA = PC = x

$\therefore PB = PD + DB = \sqrt{x^2 - DC^2} + DB = \sqrt{x^2 - \frac{3}{4}} + \frac{3}{2}$

$\Rightarrow PA + PB + PC = \sqrt{3}(\sqrt{3} + 2) \Rightarrow \frac{3 + \sqrt{4x^2 - 3}}{2} + 2x = 3 + 2\sqrt{3}$

$\Rightarrow \sqrt{4x^2 - 3} = 3 + 4(\sqrt{3} - x) \Rightarrow 4x^2 - 3 = (3 + 4(\sqrt{3} - x))^2$

$\Rightarrow 3x^2 - x(8\sqrt{3} + 6) + 6\sqrt{3} + 15 = 0$

$\therefore x = \sqrt{3}$ or $2 + \frac{5}{\sqrt{3}}$

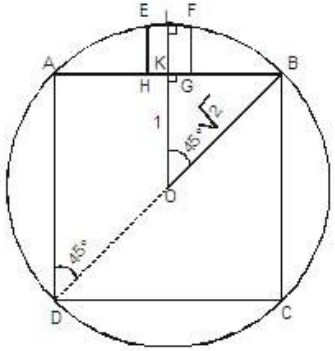
From G (Centroid as well as circumcenter of the ΔABC), the distances from the vertices of the ΔABC will be the same.

$PG = GD + DP = \frac{1}{3} \times \frac{3}{2} + \frac{1}{2}\sqrt{4x^2 - 3} = \frac{\sqrt{4x^2 - 3} + 1}{2}$

Since PG should be minimum possible, x must be equal to $\sqrt{3}$.

$\therefore PG = \frac{\sqrt{4(\sqrt{3})^2 - 3} + 1}{2} = \frac{1+3}{2} = 2m.$

8. c



Let OK = KB = 1 and GB = x

So KG = 1 - x and FG = 2(1 - x)

OL = 3 - 2x

But $OF^2 = OL^2 + LF^2$

$(\sqrt{2})^2 = (3-2x)^2 + (1-x)^2$

$2 = 9 - 12x + 4x^2 + 1 - 2x + x^2$

$2 = 5x^2 - 14x + 10$

So, $5x^2 - 14x + 8 = 0$

$\Rightarrow 5x^2 - 10x - 4x + 8 = 0$

$\Rightarrow 5x(x - 2) - 4(x - 2) = 0$

So, $x = \frac{4}{5}$ or 2

x = 2 is not possible.

$\Rightarrow FG = 2 - 2x = 2 - \frac{8}{5}$

$FG = \frac{2}{5}$

Area of smaller square = $\frac{4}{25}$

Area of bigger square = 4

So, the required ratio = 25 : 1.

For questions 9 to 11:

$A_1A_2 = 2r$, $B_1B_2 = 2r + r\sqrt{3}$, C_1C_2

$= 2r + 2r\sqrt{3}$

Hence, $a = 3 \times 2r$

$b = 3 \times (2r + r\sqrt{3})$

$c = 3 \times (2r + 2r\sqrt{3})$

9. a Difference between (a) and (b) is $3\sqrt{3}r$ and that between (b) and (c) is $3\sqrt{3}r$. Hence, (a) is the correct choice.

10. c Time taken by A = $\frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \left(\frac{2r \times 9}{60}\right) = \frac{3}{10}r$

Therefore, B and C will also travel for time $\frac{3}{10}r$.

Now speed of B = $(10\sqrt{3} + 20)$

Therefore, the distance covered

$= \left(10\sqrt{3}+20\right)\times\frac{3}{10}r = \left(\sqrt{3}+2\right)\times 10\times\frac{3}{10}r = \left(2r+\sqrt{3}r\right)\times 3 = B_1B_2+B_2B_3+B_3B_1$

$\therefore$ B will be at B_1.

Now time taken by for each distance are

$\frac{C_1C_2}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_2C_3}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_3C_1}{120}$ $\frac{3}{40}\times\frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{3}{40}\times\frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{\left(2+2\sqrt{3}\right)r}{120}$

i.e. $\frac{3}{40}\times 2r, \frac{3}{40}\times 2r, \frac{\left(1+\sqrt{3}\right)}{60}r$

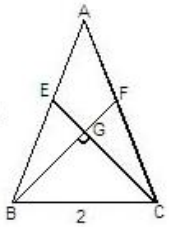
i.e. $\frac{3}{20}r, \frac{3}{20}r, \frac{\left(1+\sqrt{3}\right)}{60}r$

We can observe that time taken for C_1C_2 and C_2C_3 combined is $\frac{3}{20}r+\frac{3}{20}r=\frac{3}{10}r$, which is same as time taken by A. Therefore, C will be at C_3.

11. b In similar triangles, ratio of Area = Ratio of squares of corresponding sides.

Hence, A and C reach A_3 and C_3 respectively.

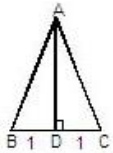
12. a



AB = AC $\Rightarrow GB = GC = \sqrt{2}$ cm

$GE = GF = \frac{1}{\sqrt{2}}$ cm [Since `G' is the centroid of the ΔABC and $GE = \frac{1}{2}GB$] $\Rightarrow EB = \sqrt{\frac{5}{2}}$ cm

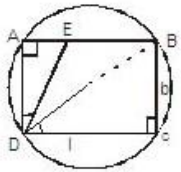
$\Rightarrow AB = AC = \sqrt{10}$ cm



$AD = \text{height} = \sqrt{10-1} = 3$ cm

$\Rightarrow \text{area} = \frac{1}{2}\times 3\times 2 = 3\ cm^2$

13. a



BD = 2r

$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi r^2}{lb} = \frac{\pi}{\sqrt{3}}$

$\frac{r^2}{lb} = \frac{1}{\sqrt{3}}$

$\frac{\frac{d^2}{4}}{lb} = \frac{1}{\sqrt{3}}$

$\therefore \frac{d^2}{4lb} = \frac{1}{\sqrt{3}}$

$\therefore \frac{l^2+b^2}{4lb} = \frac{1}{\sqrt{3}}$

$\therefore \frac{l^2+b^2}{lb} = \frac{4}{\sqrt{3}}$

$\therefore \frac{l}{b} + \frac{b}{l} = \frac{4}{\sqrt{3}}$ ... (i)

Now $\Delta AEB \sim \Delta CBD$

$\therefore \frac{AE}{CB} = \frac{AD}{DC}$

$\therefore \frac{AE}{AD} = \frac{BC}{DC}$

$\therefore \frac{AE}{AD} = \frac{b}{l}$

$\therefore$ We have to find $\frac{AE}{AD}$, i.e. $\frac{b}{l}$.

Let $\frac{b}{l} = x$

Therefore, from (i), we get

$\frac{1}{x} + x = \frac{4}{\sqrt{3}}$

$\frac{1+x^2}{x} = \frac{4}{\sqrt{3}}$

$\sqrt{3} + \sqrt{3}x^2 = 4x$

$\therefore \sqrt{3}x^2 - 4x + \sqrt{3} = 0$

$\therefore x = \frac{-(-4) \pm \sqrt{16 - 4(\sqrt{3})\sqrt{3}}}{2\sqrt{3}}$

$= \frac{4 \pm \sqrt{16-12}}{2\sqrt{3}} = \frac{4 \pm 2}{2\sqrt{3}}$

$= \frac{6}{2\sqrt{3}}$

OR $\frac{2}{2\sqrt{3}}$

$= \frac{\sqrt{3}}{1}$ OR $\frac{1}{\sqrt{3}}$

From options, the answer is $\frac{1}{\sqrt{3}}$,

i.e. $1 : \sqrt{3}$.

14. c PQ || AC

$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$

QD || PC

$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

As $\frac{PD}{DB} = \frac{4}{3}$

$\therefore PD = \frac{4}{7}PB$

$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4} \times \frac{AP}{PB} = \frac{7}{4} \times \frac{4}{3} = 7 : 3$

15. b By the symmetry of the given figure, HI is parallel to BC.

$\angle ECG = 60^\circ = \angle IEG.$

Let the side of the ΔABC be 'a' units.

$$\Rightarrow IE = GE\cos 60^\circ = (EC\sin 60^\circ)\cos 60^\circ = \frac{a}{2}\times\frac{\sqrt{3}}{2}\times\frac{1}{2} = \frac{\sqrt{3}}{8}a$$

$$JE = \frac{1}{3}\times\frac{\sqrt{3}a}{2} = \frac{a}{2\sqrt{3}} \Rightarrow JI = JE - IE = \frac{a}{2\sqrt{3}} - \frac{\sqrt{3}a}{8} = \frac{a}{8\sqrt{3}}$$

$\Delta HIJ \sim \Delta CBJ$

$$\Rightarrow \frac{\Delta HIJ}{\Delta CBJ} = \frac{JI^2}{BJ^2} = \frac{a^2}{\frac{192\times a^2}{3}} = \frac{1}{64}$$

$$\frac{\Delta HIJ}{\Delta ABC} = \frac{1}{3}\left(\frac{\Delta HIJ}{\Delta CBJ}\right) = \frac{1}{192}.$$

16. c In ΔABC,

$AB^2 + BC^2 = AC^2$

or $BC^2\left[1+\left(\frac{AB}{BC}\right)^2\right] = AC^2$

or $AC^2 = (1 + k^2)\, BC^2$

Similarly, in triangle, ΔACD

$AD^2 = AC^2\,(1 + k^2)$

and in triangle; ΔADE;

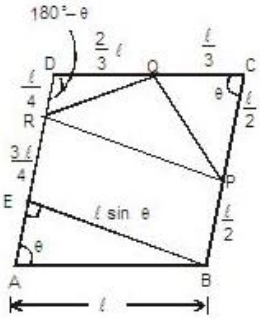
$AE^2 = DE^2\,(1 + k)^2$

$AE^2 = AD^2\left(1+\frac{1}{k^2}\right) = BC^2\,(1 + k^2)^2\left(1+\frac{1}{k^2}\right)$

$$AE^2 = BC^2\frac{(1+k^2)^3}{k^2}$$

$$\therefore \left(\frac{AE}{BC}\right)^2 = \frac{(1+k^2)^3}{k^2}.$$

17. d Let the length of a side of the rhombus be ℓ units and one of the angles be θ.



Area $(\Delta PQR) = A$, (given) ...(i)

Area $(\Delta PCQ) = \frac{1}{2}\times\left(\frac{\ell}{2}\right)\times\left(\frac{\ell}{3}\right)\times\sin\theta$

$= \frac{\ell^2}{12}\sin\theta$ ...(ii)

Area $(\Delta RDQ) = \frac{1}{2}\times\left(\frac{\ell}{4}\right)\times\left(\frac{2}{3}\ell\right)\times\sin(180^\circ - \theta)$

$= \frac{\ell^2}{12}\sin\theta$ ...(iii)

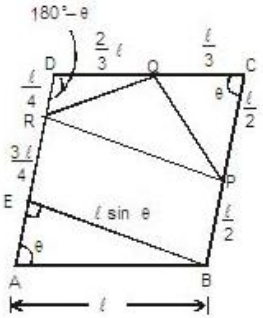
Area (rhombus ABCD)

$= 2\times\left(\frac{1}{2}\times\ell\times\ell\sin\theta\right) = \ell^2\sin\theta$ ...(iv)

$AE^2 = BC^2 \frac{(1+k^2)^3}{k^2}$

$\therefore \left(\frac{AE}{BC}\right)^2 = \frac{(1+k^2)^3}{k^2}$.

17. d Let the length of a side of the rhombus be ℓ units and one of the angles be θ.



Area $(\Delta PQR) = A$ (given) ...(i)

Area $(\Delta PCQ) = \frac{1}{2} \times \left(\frac{\ell}{2}\right) \times \left(\frac{\ell}{3}\right) \times \sin\theta$

$= \frac{\ell^2}{12} \sin\theta$ ...(ii)

Area $(\Delta RDQ) = \frac{1}{2} \times \left(\frac{\ell}{4}\right) \times \left(\frac{2}{3}\ell\right) \times \sin(180° - \theta)$

$= \frac{\ell^2}{12} \sin\theta$ ...(iii)

Area (rhombus ABCD)

$= 2 \times \left(\frac{1}{2} \times \ell \times \ell \sin\theta\right) = \ell^2 \sin\theta$ ...(iv)

Now area of ΔPQR

= area (trapezium RDCP)
− area (ΔPCQ) − area(ΔRDQ)

$A = \frac{1}{2}\left(\frac{\ell}{2} + \frac{\ell}{4}\right) \times \ell \sin\theta - \frac{\ell^2}{12} \times \sin\theta - \frac{\ell^2}{12} \times \sin\theta$

$A = \frac{5}{24} \ell^2 \sin\theta$

= Area of rhombus ABCD, $\ell^2 \sin\theta = \frac{24}{5} A$

Hence (d)