Algebra - 4

Career Launcher

QA - 20

CEX-Q-0221/18

Contents

- Functions
- Greatest Integer Function

Number of questions : 25

Functions

1. Set A = {1, 2, 3, 4, 5}; Set B = { a, b, c, d}
   A. How many elements are there in A × B ?
   B. How many one – one functions can be made from A to B ?
   C. How man onto functions can be made from A to B ?

2. Let X = {a, b, c} and Y = {1, m}. Consider the following four subsets of X × Y.
   F_1 = {(a, 1), (a, m), (b, 1), (c, m)}, F_2 = {(a, 1), (b, 1), (c, 1)}, F_3 = {(a, 1), (b, m), (c, m)} & F_4 = {(a, 1), (b, m)}

   Which one, amongst the choices is a representation of functions from X to Y?
   (1) F_2 and F_3 (2) F_1, F_2 and F_3
   (3) F_2, F_3 and F_4 (4) F_3 and F_4

3. $y = \frac{1}{\sqrt{9 - x^2}}$. Find the range of values of x, if y is real.
   (1) $0 < x < 3$ (2) $x \le -3$ or $x \ge 3$
   (3) $x < -3$ or $x > 3$ (4) None of these

4. $y = \sqrt{15 - x^2 - 2x}$. Find the range of values of x if y is real.
   (1) $x \le -5$ or $x \ge 3$ (2) $-\sqrt{15} \le x \le \sqrt{15}$
   (3) $-5 \le x \le 3$ (4) $-4 \le x \le 4$

5. A function is defined as

$$f(x, y) = \begin{cases} x + y, \text{ if } x + y < 1 \\ 0, \text{ if } x + y = 1 \\ xy, \text{ if } x + y > 1 \end{cases}$$

   where, x and y are real numbers.

   If $f\left(x, \frac{1}{2}\right) = \frac{3}{4}$, then which of the following can be the value of x?

   (1) $\frac{1}{4}$ (2) $\frac{3}{2}$
   (3) $\frac{3}{4}$ (4) Both (1) and (2)

6. Let $f(x) = 2^{10}x + 1$ and $g(x) = 30^{10}x - 1$. If (fog) (x) = x, then x is equal to

   (1) $\frac{3^{10} - 1}{3^{10} - 2^{-10}}$ (2) $\frac{2^{10} - 1}{2^{10} - 3^{-10}}$
   (3) $\frac{1 - 3^{-10}}{2^{10} - 3^{-10}}$ (4) $\frac{1 - 2^{-10}}{3^{10} - 2^{-10}}$

7. Let f(x) be a function satisfying $f(x)f(y) = f(xy)$ for all real x, y. If f(2) = 4, then what is the value of $f\left(\frac{1}{2}\right)$?

(1) 0 (2) $\frac{1}{4}$

(3) $\frac{1}{2}$ (4) 1

8. If $3f(x+2)+4f\left(\frac{1}{x+2}\right)=4x, x \neq -2$, then f(4) =

(1) 7 (2) $\frac{52}{7}$

(3) 8 (4) None of these

9. If $f(x) = \begin{cases} 0, \text{ when } x = 1 \\ -1, \text{ when } x = 2 \\ 1, \text{ when x is an odd prime number} \end{cases}$

and f(xy) = f(x) + f(y), then find the value of f(1995) ?.

(1) 3 (2) 4

(3) 5 (4) None of these

10. Given, g(x) is a function such that g(x + 1) + g(x – 1) = g(x), where x is a positive real number. For what minimum value of p does the relation g(x + p) = –g(x) necessarily hold true?

(1) 2 (2) 3

(3) 5 (4) 6

11. f(x) = max (2x – 6, 4 – 3x).

(1) What is the value of f(x) at x = 10 ?

(2) What is the minimum value of f(x)

(3) What is the maximum value of f(x) if $-5 \leq x \leq 10$

12. If $f(x) = \frac{1}{\sqrt{|x^2 - [x]^2|}}$, where [ ] represents the greatest integer less than or equal to x, then what is the domain of f(x)?

(1) All real numbers

(2) All integers

(3) All rational number

(4) All real numbers except integers

Directions for questions 13 and 14: (CAT 2004)

$f_1(x) = x$, when $0 \leq x \leq 1$

$= 1$, when $x \geq 1$

$= 0$, otherwise

$f_2(x) = f_1(-x)$ for all x

$f_3(x) = -f_2(x)$ for all x

$f_4(x) = f_3(-x)$ for all x

13. How many of the following products are necessarily zero for every x

$f_1(x)f_2(x)$, $f_2(x)f_3(x)$, $f_2(x)f_4(x)$?

(1) 0 (2) 1

(3) 2 (4) 3

14. Which of the following is necessarily true?

(1) $f_4(x) = f_1(x)$ for all x

(2) $f_1(x) = -f_3(-x)$ for all x

(3) $f_2(-x) = f_4(x)$ for all x

(4) $f_1(x) + f_3(x) = 0$ for all x

15. A function F is defined for all the positive integers that satisfy the following condition: F (1) + F (2) + F (3) +…+ F (n) = n^2F (n). If F (1) = 2006, then find the value of F (2005).

(1) $\frac{1}{2005}$ (2) $\frac{2}{2005}$

(3) $\frac{1}{2005!}$ (4) $\frac{2}{2005!}$

16. A 'polynomial f(x) with real coefficients satisfies the functional equation $f(x).f\left(\frac{1}{x}\right) = f(x)+f\left(\frac{1}{x}\right)$. If f(2) = 9, then f(4) is
(1) 82 (2) 17
(3) 65 (4) None of these

Greatest Integer Function

17. If [x] is the greatest integer less than or equal to x then find the value of the following series.
$[\sqrt{1}]+[\sqrt{2}]+[\sqrt{3}]+[\sqrt{4}]+...+[\sqrt{361}]$
(1) 4408 (2) 4839
(3) 3498 (4) 3489

18. If the symbol [x] denotes the largest integer less than or equal to x, then the value of
$\left[\frac{1}{4}\right]+\left[\frac{1}{4}+\frac{1}{50}\right]+\left[\frac{1}{4}+\frac{2}{50}\right]+...\left[\frac{1}{4}+\frac{40}{50}\right]$ is
(1) 40 (2) 28
(3) 3 (4) 0

19. Let {x} and [x] denote the fractional and integral parts respectively of a real number x. If $[x]^2 + 4\{x\} = 2x$, then how many values of x are possible?
(1) 1 (2) 2
(3) 3 (4) 4

20. Find how many positive real values of x satisfy the equation $2[x]^2 = 5x+2$, where [x] denotes greatest integer less than or equal to x.
(1) 0 (2) 1
(3) 2 (4) 3

21. The number of solutions of [x] = x + 1 is
(1) 0 (2) 1
(3) 2 (4) infinite

22. For a real number y, let [y] denote the largest integer less than or equal to y and {y} denotes y – [y]. How many solutions does the equation 11[y] + 23{y} = 250 have?
(1) 0 (2) 1
(3) 2 (4) 3

23. If [.] denotes the greatest integer function, then for how many values of x in the interval [1, 5] will the following equation satisfy?
$x^2 - [x^2] = (x - [x])^2$
(1) None (2) 5
(3) 21 (4) Infinitely many

24. Find the solution set for [x] + [2x] + [3x] + [4x] = 14, where x is a real number and [x] represents the greatest integer less than or equal to x.
(1) $x < \frac{5}{3}$ (2) $\frac{3}{2} \le x < \frac{5}{3}$
(3) $1 \le x < \frac{4}{3}$ (4) None of these

Challenging

25. Let x_n denote the n-th element of the sequence {1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, ⋯}, where n is a positive integer. How many of the following statements are then true?
Statement I: x_n is the largest integer less than $\sqrt{2n+\frac{1}{4}}+\frac{1}{2}$
Statement II: x_n is the largest integer not greater than $\sqrt{2(n-1)+\frac{1}{4}}+\frac{1}{2}$
Statement III: x_n is the smallest integer greater than $\sqrt{2n+\frac{1}{4}}-\frac{1}{2}$
(Consider only the positive values for the square roots in the above statements. For example, $\sqrt{25}$ will given only +5, and not –5)
(1) 3 (2) 2
(3) 1 (4) 0

26. Let $f(x) = \frac{4^x}{4^x + 2}$, then

$f\left(\frac{1}{1997}\right) + f\left(\frac{2}{1997}\right) + \cdots + f\left(\frac{1996}{1997}\right)$ is equal to

(1) 1 (2) 997
(3) 998 (4) 996

Directions for questions 27 and 28: For a real number x, let $f(x) = \begin{cases} \frac{1}{1+x}, \text{ if } x \text{ is non-negative} \\ 1+x, \text{ if } x \text{ is negative} \end{cases}$

and $f^n(x) = f(f^{n-1}(x))$, for n = 2, 3, …

27. What is the value of the product, $f(2)\, f^2(2)\, f^3(2)\, f^4(2)\, f^5(2)$?

(1) $\frac{1}{3}$ (2) 3

(3) $\frac{1}{18}$ (4) None of these

28. r is an integer > 1. Then, what is the value of $f^{r-1}(-r) + f^r(-r) + f^{r+1}(-r)$?

(1) –1 (2) 0
(3) 1 (4) None of these

29. Suppose, a function f is defined over the set of natural numbers as follows: f(1) = 1, f(2) = 1, f(3) = –1, and f(n) = f(n – 1) f(n – 3) for n > 3. Then the value of f(694) + f(695) is

(1) –2 (2) –1
(3) 1 (4) 2

30. A periodic function f satisfies f(x + a) (1 – f(x)) = 1 + f(x) for some constant a. The period of f is

(1) a (2) 2a
(3) 3a (4) 4a

QA - 20 : Algebra - 4
Answers and Explanations

1	–	**2**	1	**3**	4	**4**	3	**5**	4	**6**	4	**7**	2	**8**	4	**9**	2	**10**	2
11	–	**12**	4	**13**	3	**14**	2	**15**	2	**16**	3	**17**	1	**18**	3	**19**	3	**20**	2
21	1	**22**	3	**23**	3	**24**	2	**25**	2	**26**	3	**27**	4	**28**	2	**29**	4	**30**	4

1. A. Number of elements in A × B = 5 × 4 = 20
   B. No one-one function is possible from A to B because number of elements in set A is greater than the number of elements in set B.
   C. Since set A contains 5 elements and set B contains 4 elements, so we will first divide 5 elements of set A into four groups i.e. 10 ways.
   Now, these four groups can be paired with the 4 elements of set B in 4! ways.
   So, number of onto function from A to B = 10 × 4! = 240.

2. 1 In case of F_1, a is paired with 1 and m both, which violates the condition for being a function. So, F_1 is not a function. Whereas in case of F_4, c (an element of x) does not belong to any value of set Y. So, it is not a function. F_2 and F_3 satisfy all the conditions, so option (1) is correct.

3. 4 If y is real, $9 - x^2 > 0 \Rightarrow (3 + x)(3 - x) > 0$

$\Rightarrow -3 < x < 3$

4. 3 If y is real, $15 - x^2 - 2x \ge 0 \Rightarrow x^2 + 2x - 15 \le 0$

$\Rightarrow (x+5)(x-3) \le 0 \Rightarrow -5 \le x \le 3$

5. 4 Substituting x = $\frac{1}{4}$ and $\frac{3}{2}$ from the options, we find that the given condition is satisfied.

$\because f\left(x, \frac{1}{2}\right) = \frac{3}{4}$

So, x + y can be greater than 1 or less than 1 as well. We need to check by options.

6. 4 $f(g(x)) = x \Rightarrow f(3^{10}x - 1) = x$
So, $2^{10}(3^{10}x - 1) + 1 = x$
$\Rightarrow 2^{10} \times 3^{10}x - 2^{10} + 1 = x$

$$\Rightarrow \frac{1-2^{10}}{1-2^{10}3^{10}} = x \quad \Rightarrow x = \frac{1-2^{-10}}{3^{10} - 2^{-10}}.$$

7. 2 f(x).f(y) = f(xy)
Given, f(2) = 4
We can also write,
f(2) = f(2 × 1) = f(2) × f(1)
or f(1) × 4 = 4
$\Rightarrow$ f(1) = 1
Now we can also write,

$$f(1) = f\left(2 \times \frac{1}{2}\right) = f(2) \times f\left(\frac{1}{2}\right)$$

$$\Rightarrow f\left(\frac{1}{2}\right) = \frac{f(1)}{f(2)} = \frac{1}{4}.$$

8. 4 $$3f(x+2) + 4f\left(\frac{1}{x+2}\right) = 4x$$

Putting x = z – 2, we get

$$3f(z) + 4f\left(\frac{1}{z}\right) = 4z - 8 \quad \ldots(i)$$

Now replacing z with $\frac{1}{z}$ in the above equation, we get

$$3f\left(\frac{1}{z}\right) + 4f(z) = \frac{4}{z} - 8 \quad \ldots(ii)$$

From (i) and (ii),

$$f(z) = \frac{1}{7}\left\{\frac{16}{z} - 8 - 12z\right\}$$

$$f(x+2) = \frac{1}{7}\left\{\frac{16}{(x+2)} - 8 - 12(x+2)\right\}$$

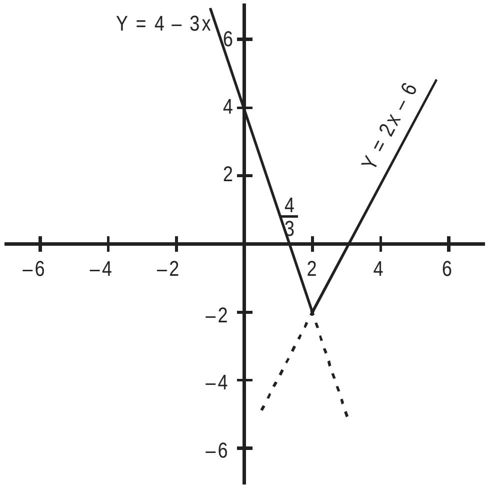
$$f(4) = \frac{1}{7}\left\{\frac{16}{4} - 8 - 12 \times 4\right\} = -\frac{52}{7}.$$

Hence, option (4) is the right choice.

9. 2 f(1995) = f(15 × 133) = f(3) + f(5) + f(7) + f(19)
= 1 + 1 + 1 + 1 = 4.

10. 2 Given that g(x + 1) + g(x – 1) = g(x) ... (i)
So, g(x + 2) + g(x) = g(x + 1) ...(ii)
Adding equations (i) and (ii), we get
g (x + 2) + g (x – 1) = 0
$\Rightarrow$ g(x + 3) + g(x) = 0
$\Rightarrow$ g(x + 3) = –g (x);
So, p = 3.

11. The given function can be plotted as



(1) For x = 10
f(x) = 2x – 6
⇒ f(10) = 2 × 10 – 6 = 14.

(2) The minimum value of f(x) is at x = 2, which is –2.

(3) The maximum value of the given function can either be at x = –5 for f(x) = 4 – 3x or at x = 10 for f(x) = 2x – 6 i.e. we will check both the possibilities.
So, at x = –5
f(–5) = 4 – 3(–5) = 19
and at x = 10
f(10) = 2 × 10 – 6 = 14.
Thus maximum value of f(x) in the given range is 14.

12. 4 If x is an integer, [x] = x.

$\therefore \dfrac{1}{\sqrt{|x^2 - [x]^2|}}$ will not be a real number when x is an integer.

13. 3 $f_1 f_2 = f_1(x) f_1(-x)$

$$f_1(-x) = \begin{cases} -x & 0 \le -x \le 1 \\ 1 & -x \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} -x & -1 \le x \le 0 \\ 1 & x \le -1 \\ 0 & \text{otherwise} \end{cases}$$

$f_1 f_1(-x) = 0$, for all x

Similarly, $f_2 f_3 = -(f_1(-x))^2 \ne 0$ for some x

$f_2 f_4 = f_1(-x).\ f_3(-x)$

$= -f_1(-x)\ f_2(-x)$

$= -f_1(-x)\ f_1(x) = 0$, for all x

14. 2 Checking with options:
Option (2):

$f_3(-x) = -f_2(-x) = -f_1(x) \Rightarrow f_1(x) = -f_3(-x)$, for all x.

15. 2 $F(1) + F(2) + \ldots + F(n) = n^2 . F(n)$
$\Rightarrow F(1) + F(2) + \ldots + F(n-1) = (n^2 - 1) . F(n)$
Also, $F(1) + F(2) + \ldots + F(n-1) = (n-1)^2 . F(n-1)$
Hence we can say $(n-1)^2 F(n-1) = (n^2 - 1) . F(n)$

$$\Rightarrow \frac{F(n)}{F(n-1)} = \frac{(n-1)^2}{n^2-1} = \frac{n-1}{n+1}$$

Now, $\dfrac{F(n)}{F(n-1)} \times \dfrac{F(n-1)}{F(n-2)} \times \ldots \times \dfrac{F(2)}{F(1)}$

$$= \frac{(n-1)}{(n+1)} \times \frac{(n-2)}{n} \times \frac{(n-3)}{n-1} \times \ldots \times \frac{1}{3}$$

$$\Rightarrow \frac{F(n)}{F(1)} = \frac{1 \times 2}{n(n+1)}$$

$$\Rightarrow F(2005) = \frac{1 \times 2}{2005 \times 2006} \times 2006 = \frac{2}{2005}$$

Alternative method:
Putting n = 2, we have F(1) + F(2) = 4 × F(2).

Thus $F(2) = \dfrac{2006}{3}$

Putting n = 3 we have F(1) + F(2) + F(3) = 9 × F(3).

Thus $F(3) = \dfrac{4}{3} \times \dfrac{1}{8} = \dfrac{1}{6}$ of 2006

Putting n = 4 we have F(1) + F(2) + F(3) + F(4)

= 16 × F(4). Thus $F(4) = \dfrac{9}{6} \times \dfrac{1}{15} = \dfrac{1}{10}$ of 2006

Thus in general we can say that

$$F(n) = \frac{1}{(\text{sum of all natural numbers till n})} \times 2006$$

Thus $F(2005) = \dfrac{2}{2005 \times 2006} \times 2006 = \dfrac{2}{2005}$.

16. 3 Assume $f(x) = x^n + 1$

and $f\left(\dfrac{1}{x}\right) = \dfrac{1}{x^n} + 1$

So, $f(x).f\left(\dfrac{1}{x}\right) = \left(x^n + 1\right)\left(\dfrac{1}{x^n} + 1\right)$

$= \left(1 + x^n\right) + \left(\dfrac{1}{x^n} + 1\right)$

$= f(x) + f\left(\dfrac{1}{x}\right)$

Now $f(x) = x^n + 1 = 9$

$\Rightarrow x^n = 8 \Rightarrow n = 3$

$\therefore f(x) = x^3 + 1$

Hence, $f(x) = 4^3 + 1 = 65$.

17. 1 $\left[\sqrt{1}\right]+\left[\sqrt{2}\right]+\left[\sqrt{3}\right]=1\times 3.$

$\left[\sqrt{4}\right]+\left[\sqrt{5}\right]+\left[\sqrt{6}\right]+\left[\sqrt{7}\right]+\left[\sqrt{8}\right]=2\times 5$

$\left[\sqrt{9}\right]+\left[\sqrt{10}\right]+\cdots\left[\sqrt{15}\right]=3\times 7$

n^{th} term is $n \times (2n + 1) = 2n^2 + n$

and $S_n = 2\sum n^2 + \sum n = \frac{n(n+1)(4n+5)}{6}$

Put n = 18

$S_{18} = 4389$

$\left[\sqrt{361}\right]=19$

Total sum = 4389 + 19 = 4408.

18. 3 Only the last 3 terms have numbers greater than or equal to 1 inside the [ ] sign. The last three terms are:

$$\left[\frac{1}{4}+\frac{38}{50}\right]+\left[\frac{1}{4}+\frac{39}{50}\right]+\left[\frac{1}{4}+\frac{40}{50}\right]$$

Each of these terms are equal to 1.

All previous terms have numbers that lie between 0 and 1 and therefore, are equal to zero.

Hence, the sum of the given terms is 3.

19. 3 $[x]^2 + 4\{x\} = 2x$

Let $[x] = I, \{x\} = f$, therefore $x = I + f$

$I^2 + 4f = 2I + 2f \;\Rightarrow 2f = 2I - I^2$

$\therefore 0 \le \frac{2I - I^2}{2} < 1$

Possible values of I = 0, 1 and 2

If I = 0, then f = 0: x = 0

If I = 1, then f = 0.5: x =1.5 and if I = 2, then f = 0: x =2

Therefore, x has three real values.

20. 2 Let x = [x] + {x} = I + f, where [x] = I denotes the integral part of x and {x} = f denotes the fractional part of x.

$2[x]^2 = 5x + 2$

$\Rightarrow 2I^2 = 5I + 5f + 2$

$\Rightarrow f = \frac{2I^2 - 5I - 2}{5}$

$\Rightarrow 0 < \frac{2I^2 - 5I - 2}{5} < 1$

Solving the above inequality the only positive integral value of I that satisfies the equation is I = 3, and the corresponding value of f for this value of I is 0.2.

So, x = 3 + 0.2 = 3.2.

Hence, there is only one integral value of x that satisfies the given equation.

21. 1 Here, if x is an integer, then

[x] = x

So, x = x + 1, which is not possible.

If x is in the form of Integer (I) + Fraction (F)

then [x] = I

So, I = I + F + 1 = (I + 1) + F, which is again not possible.

Hence, no solution.

22. 3 Given that 11[y] + 23{y} = 250 ...(i)

Now 0 < {y} < 1

So, 0 < 23{y} < 23.

Comparing the above with (i) -

227 <11[y]< 250 ...(ii)

As [y] is always an integer the only possible values of [y] in (ii) are 21 and 22. (this is because only multiples of 11 between '227 and 250' are 231 and 242)

when [y] = 21, $\{y\} = \frac{250\text{-}231}{23} = \frac{19}{23}$

Subsequently y = [y] + {y} = 21 + $\frac{19}{23}$ or y = $21\frac{19}{23}$.

Also, when [y] = 22,

$\{y\} = \frac{250 - 242}{23} = \frac{8}{23}$.

Subsequently y = [y] + {y} = 22 + $\frac{8}{23}$ or y = $22\frac{8}{23}$.

So, there are exactly two possible solutions for the equation, y = $21\frac{19}{23}$ and y = $22\frac{8}{23}$.

23. 3 **Case I:**

x is an integer

Then $x^2 = [x^2]$ and x = [x], So x = 1, 2, 3, 4, 5 are five solutions

Case II:

x = 1 + k or 2 + k or 3 + k or 4 + k, where k is a fraction 0 < k < 1

for x = 1 + k

$1^2 + k^2 + 2k - [1 + k^2 + 2k] = k^2$ (or) $2k = [k^2 + 2k]$, k = 0.5

for x = 1.5, this equation is satisfied

for x = 2 + k

$4 + k^2 + 4k - [4 + k^2 + 4k] = k^2$ (or) $4k = [k^2 + 4k]$

k = 0.25, 0.5, 0.75

There are 3 solutions x = 2.25, 2.5, 2.75

Similarly, we get for x = 3 + k (5 solutions)

For x = 4 + k (7 solutions)

In all, 5 + (1 + 3 + 5 + 7) = 21 solutions.

24. 2 Here, x > 1 and x < 2 is obvious.

Now, for $x = \frac{3}{2}$, expression = 1 + 3 + 4 + 6 = 14

$\therefore$ x = 1.5 satisfies for x = 1.5, [3x] = [4.5] = 4

But as soon as $x = \frac{5}{3}$, [3x] = 5 it would not satisfy

So, x should be less than $\frac{5}{3}$.

25. 2 We'll verify the given statements by putting values of n.
So,
Statement I: Put n = 1 So, $x_1 = 1$

So, $\sqrt{2n+\frac{1}{4}}+\frac{1}{2}=\frac{3}{2}+\frac{1}{2}=2$ (true)

Again, n = 3 So, $x_3 = 2$

$\sqrt{2n+\frac{1}{4}}+\frac{1}{2}=\frac{5}{2}+\frac{1}{2}=3$ (true)

It is always true.
Similarly, statement II is also true.
But in case of statement III, if we put n = 1
So, $x_1 = 1$
and x_n is the smallest integer greater than

$\sqrt{2\times1+\frac{1}{4}}-\frac{1}{2}=\frac{3}{2}-\frac{1}{2}=1$, which is not true.

Thus, only two statements are true.

26. 3 Since, f(x) = $\frac{4^x}{4^x+2}$

$\Rightarrow$ f(1 – x) = $\frac{4^{(1-x)}}{4^{(1-x)}+2}=\frac{2}{4^x+2}$

So, f(x) + f(1 – x) = 1

$\Rightarrow f\left(\frac{1}{1997}\right)+f\left(\frac{1996}{1997}\right)=1$

$f\left(\frac{2}{1997}\right)+f\left(\frac{1995}{1997}\right)=1$..... and so on

So, required sum = 998.

27. 4 Since 2 is a non-negative number, so f(x) = $\frac{1}{(1+x)}$

$$f^2(x)=f(f(x))=\frac{1}{1+\frac{1}{(1+x)}}=\frac{(1+x)}{(2+x)}$$

$$f^3(x)=f(f^2(x))=\frac{1}{1+\left(\frac{1+x}{2+x}\right)}=\frac{(2+x)}{(3+2x)}$$

$$f^4(x)=f(f^3(x))=\frac{1}{1+\left(\frac{2+x}{3+2x}\right)}=\frac{(3+2x)}{(5+3x)}$$

$$f^5(x)=f(f^4(x))=\frac{1}{1+\left(\frac{3+x}{5+3x}\right)}=\frac{5+3x}{8+5x}$$

So, f(2) × $f^2(2)$ $f^3(2)$ $f^4(2)$ $f^5(2)$ = $\frac{1+2}{(8+5\times2)}=\frac{1}{6}$

(**Note:** here we don't need to write all the term, as denominator of one term is getting cancelled by the numerator of next term.)

28. 2 Here –r is negative.
So, the given expression becomes
$f^1(-2) + f^2(-2) + f^3(-2)$
= –1 + f(–1) + $f^3(-2)$ = –1 + 0 + f(0)

= –1 + $\frac{1}{1+0}=0$.

Thus option (2) is correct.

29. 4 f(x) = f(x – 1) f(x – 3)
f(4) = –1.1 = –1
f(5) = –1.1 = –1
f(6) = –1.–1 = 1
f(7) = 1.–1 = –1
f(8) = –1.–1 = 1
f(9) = 1.1 = 1
f(10) = 1.–1 = –1
f(11) = –1.1 = –1
f(12) = –1.1 = –1
f(13) = –1.–1 = 1
f(14) = 1.1 = –1
f(15) = –1.–1 = 1
Here, after every multiple of 7, we are getting two one's.
Since 694 = 7k + 1 and 695 = 7k + 2,
So, f(694) + f(695) = 1 + 1 = 2.

30. 4 f(x + a) = $\frac{1+f(x)}{1-f(x)}\Rightarrow f(x)\frac{f(x+a)-1}{f(x+a)+1}$

$x \leftrightarrow (x+a)$

f(x + a) = $\frac{f(x+2a)-1}{f(x+2a)+1}\Rightarrow f(x+2a)=\frac{1+f(x+a)}{1-f(x+a)}$

$\Rightarrow$ f(x + 2a) = $\frac{1+\frac{1+f(x)}{1-f(x)}}{1-\frac{1+f(x)}{1-f(x)}}=\frac{-1}{f(x)}$

Again, $x \leftrightarrow (x+a)$

$\Rightarrow$ f(x + 3a) = $\frac{-1}{f(x+a)}=\frac{f(x)-1}{f(x)+1}$

Again, $x \leftrightarrow (x+a)$

f(x + 4a) = $\frac{f(x+a)-1}{f(x+a)+1}=\frac{\frac{1+f(x)}{1-f(x)}-1}{\frac{1+f(x)}{1-f(x)}+1}$

f(x + 4a) = f(x)
$\Rightarrow$ Period of f(x) is 4a.