Geometry - 2

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Career Launcher

QA - 26

CEX-Q-0227/18

Number of Questions : 25

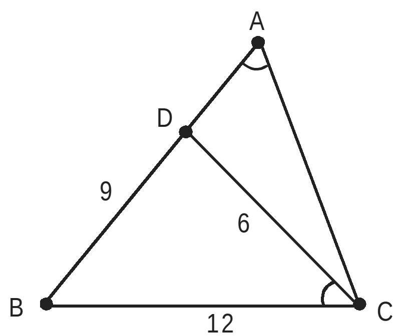
1. Two poles, of height 2 m and 3 m, are 5 m apart. The height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is **[XAT – 2010]**

(1) 1.2 m (2) 1.0 m
(3) 5.0 m (4) 3.0 m
(5) None of these

2. PQRS is a trapezium in which the length of the parallel sides PQ and RS is in the ratio 2 : 3. If the diagonals intersect at O, then find the ratio of area of ΔPOQ to ΔROS.

(1) 9 : 4 (2) 4 : 9
(3) 3 : 2 (4) 2 : 3

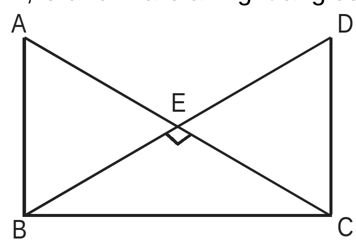
3. Consider the triangle ABC as shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$. What is the ratio of the perimeter of ΔADC to that of the ΔBDC? **[CAT 2005]**



(1) $\frac{7}{9}$ (2) $\frac{8}{9}$

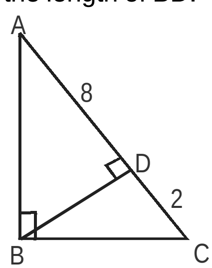
(3) $\frac{6}{9}$ (4) $\frac{5}{9}$

4. AB and BC are sides with integral length. If the length of side AC is 13 cm then find the possible value of the ratio of areas of triangle ABC and triangle BCD. (Given that angles B, C and E are all right angles).

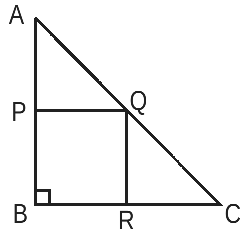


(1) 1 : 1 (2) 25 : 144
(3) 144 : 25 (4) Both (2) & (3)

5. ΔABC is a right-angled triangle, right angled at B. If AD = 8 cm, DC = 2 cm and $BD \perp AC$, then find the length of BD.



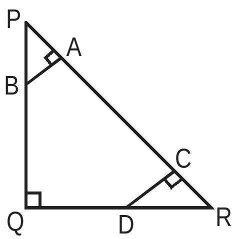
(1) 4 cm (2) 4.5 cm
(3) 5 cm (4) Cannot be determined

6. 

ABC is a right angled triangle with AB = 6 units and BC = 8 units. If PQRB is a square, find its area?

(1) 25 (2) $\frac{625}{36}$

(3) $\frac{576}{49}$ (4) 12.5

7. 

PQR is a right angled triangle with PQ = 15 unit and QR = 20 unit. If PB = 4 unit and DR = 6 unit, find the length of AC?

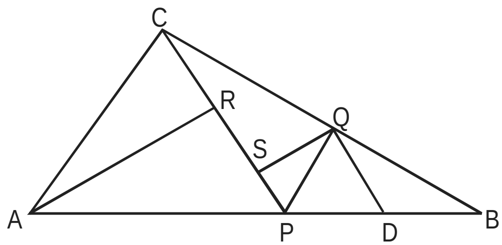
(1) $\sqrt{317}$ (2) 17.8
(3) 20 (4) 19.6

8. PQRS is a square of side 8 units. M, N, O and T are midpoints of sides PQ, QR, RS and SP respectively. Let QT and PO, PO and SN, SN and RM, RM and QT intersect at A, B, C and D respectively. Find the length of AD?

(1) 4 (2) $\frac{8}{\sqrt{5}}$

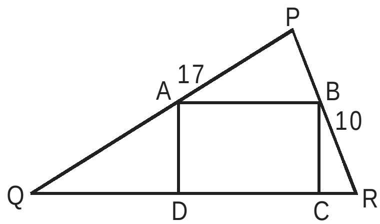
(3) $\sqrt{8}$ (4) None of these

9. In the figure given below, P is a point on AB such that AP: PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, ∠ARC = 90° and in ΔPQS, ∠PSQ = 90°, and length of QS is 6 cms. What is the ratio of AP : PD? **[CAT 2003]**



(1) 10 : 3 (2) 2 : 1
(3) 7: 3 (4) 8 : 3

10. A square ABCD is constructed inside a triangle PQR, having sides 10, 17 and 21 units as shown in the figure. Find the approximate perimeter of the square ABCD.



(1) 28 (2) 23.2
(3) 25.4 (4) 28.8

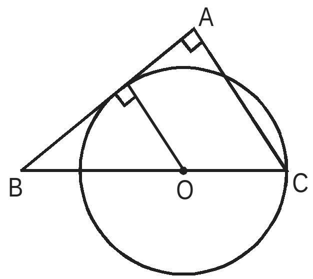
11. In a triangle ABC, the lengths of the sides AB and AC are equal to 17. 5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD = 3 cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC? **[CAT 2008]**
(1) 17.05 (2) 27.85
(3) 22.45 (4) 26.25

12. ABCD is a square of side 4 units. A circle is inscribed in the square. P is the midpoint of AB, and PD intersects the circle at K. Find the length of PK.

(1) $4\sqrt{5}$ (2) $\frac{8}{\sqrt{5}}$

(3) $2\sqrt{5}-1$ (4) $\frac{4}{\sqrt{5}}$

13. If the radius of the given circle with centre O is 4 cm & BC = 9 cm, then find the approximate area of ΔABC



(1) 20 cm (2) 25 cm
(3) 31 cm (4) 30 cm

14. PX and QY are medians of triangle PQR, with lengths 18 and 21 units respectively. If PX and QY intersect at 90 degrees, find the area of triangle PQR?
(1) 126 (2) 252
(3) 256 (4) 128

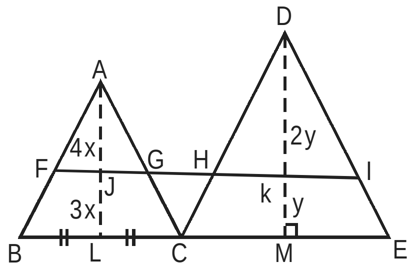
15. XYZ is a triangle with sides XY = 12units, YZ = 10units and XZ = 11units. YP and XQ are angle bisectors intersecting R. Find YR : RP?
(1) 5 : 6 (2) 2 : 1
(3) 4 : 3 (4) 3 : 2

16. In a triangle ABC, let angle C be 90 degrees. a,b and c are sides of triangle opposite to angles A,B and C respectively. If r is in-radius and R is the circumradius of the triangle ABC, the 2(R + r) equals
(1) b + a (2) c + a
(3) a + c (4) a + b + c

17. In ΔPQR, PX and QY are medians which intersect at O and also XZ is parallel to QY such that Z lies on PR. Find the ratio of area of ΔQOX to that of quadrilateral OYZX?
(1) 3 : 4 (2) 4 : 5
(3) 4 : 9 (4) 3 : 8

18. In an equilateral triangle ABC, P, Q and R are mid-points of AB, AC and BC respectively. Join PC and BQ and S is their intersection point. Let AR meet PQ at O. Find the area of triangle ABC if area of triangle POS is 40 square unit.
(1) 480 (2) 960
(3) 240 (4) 720

19.



In the above figure AB || CD, AC || DE and FI || BE. If FI cuts the median AL (at J) and Altitude DM (at K) in the ratio of 4 : 3 and 2 : 1 respectively, what is the ratio of the area of triangle DHI to the area of Quadrilateral BCGF?
(1) 4 : 3 (2) 6 : 5
(3) 8 : 7 (4) 12 : 11

20. A right triangle has a hypotenuse of 10 cm and an altitude to the hypotenuse equal to 6 cm. What is the area of the triangle?
(1) 30 cm^2
(2) 60 cm^2
(3) 25 cm^2
(4) No such triangle exists

21. Unequal side of an isosceles triangle is 2 cm long. The medians drawn to the equal sides are perpendicular to each other. Find the area of the triangle (in cm)^2?

(1) 3 (2) $\sqrt{10}$

(3) $2\sqrt{3}$ (4) $3\sqrt{2}$

22. Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r. If PS and RQ intersect at a point X on the circumference of the circle, then 2 r is always equal to

(1) $\sqrt{(PQ \cdot RS)}$

(2) $\frac{(PQ + RS)}{2}$

(3) $2(PQ \cdot RS)/(PQ + RS)$

(4) $\sqrt{\frac{(PQ^2 + RS^2)}{2}}$

23. In a triangle PQR, PQ = 12 cm and QR = $4\sqrt{3}$ cm. If the measure of $\angle PRQ = 60°$, then what is the ratio of the inradius to the circumradius of the triangle PQR?

(1) $\frac{1}{(\sqrt{3}+2)}$ (2) $\frac{1}{(\sqrt{3}+1)}$

(3) $\frac{2}{(\sqrt{3}+1)}$ (4) $\frac{2}{(\sqrt{3}+2)}$

24. In ΔABC, 'O' is the point of intersection of the altitudes and I is the point of intersection of the angle bisectors. If $\angle BOC = 130°$, then find the measure of $\angle BIC$ (in degrees)?

25. In ΔABC, the internal bisector of $\angle A$ meets BC at D. If AB = 4 cm, AC = 3 cm and $\angle A = 60°$, then the length of AD (in cm) is **[CAT 2002]**

(1) $2\sqrt{3}$ (2) $\frac{12\sqrt{3}}{7}$

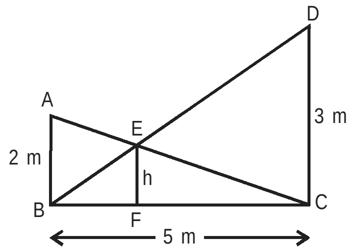
(3) $\frac{15\sqrt{3}}{8}$ (4) $\frac{6\sqrt{3}}{7}$

QA - 26 : Geometry - 2
Answers and Explanations

CEX-Q-0227/18

1	1	**2**	2	**3**	1	**4**	4	**5**	1	**6**	3	**7**	2	**8**	2	**9**	3	**10**	2
11	4	**12**	2	**13**	1	**14**	2	**15**	2	**16**	1	**17**	2	**18**	2	**19**	4	**20**	4
21	1	**22**	1	**23**	2	**24**	115°	**25**	2										

1. 1



In ΔBCD, we have

$$\frac{BF}{BC} = \frac{h}{3} \qquad \text{...(i)}$$

In ΔCAB, we have

$$\frac{CF}{CB} = \frac{h}{2} \qquad \text{...(ii)}$$

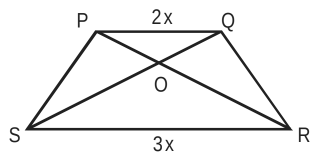
Adding (i) and (ii), we get

$$\frac{BF}{BC} + \frac{CF}{CB} = \frac{h}{3} + \frac{h}{2} \Rightarrow \frac{BF + FC}{BC} = \frac{h}{3} + \frac{h}{2}$$

Now, BF + FC = BC

Hence, $\frac{h}{3} + \frac{h}{2} = 1 \Rightarrow h = 1.2$ meters.

2. 2

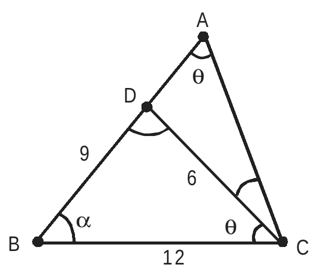


If PQ = 2x, then RS = 3x.

$$\frac{\text{Area of } \Delta POQ}{\text{Area of } \Delta SOR} = \frac{PQ^2}{SR^2} = \left(\frac{2x}{3x}\right)^2 = \frac{4}{9}$$

(Since triangles are similar.)

3. 1



Here $\angle ACB = \theta + [180° - (2\theta + \alpha)] = 180° - (\theta + \alpha)$

So here we can say that triangle BCD and triangle ABC will be similar. $\Delta BCD \sim \Delta BAC$

Hence, from the property of similar triangles

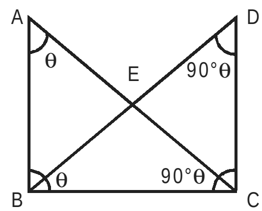
$$\frac{AB}{12} = \frac{12}{9} \Rightarrow AB = 16\ ; \quad \frac{AC}{6} = \frac{12}{9} \Rightarrow AC = 8$$

$\therefore$ AD = 7

$$S_{ADC} = 8 + 7 + 6 = 21\ ;\ S_{BDC} = 27$$

Hence, $r = \frac{21}{27} = \frac{7}{9}$

4. 4



If AC = 13 and AB & BC are integers then their possible values are 5 & 12 or 12 & 5 respectively.
Taking one of the cases

Case I: AB = 5, BC = 12
In ΔABC
if $\angle A = \theta$ then $\angle ACB = 90 - \theta$
In ΔBEC, $\angle EBC = \theta$ ($\because \angle BEC = 90$)
In ΔBCD, $\angle D = 90 - \theta$ ($\because \angle DBC = \theta$)
Therefore, ΔABC & ΔBCD are similar

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BCD} = \left(\frac{5}{12}\right)^2 = \frac{25}{144}$$

Case II:
AB = 12
BC = 5

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BCD} = \left(\frac{12}{5}\right)^2 = \frac{144}{25}.$$

5. 1 $\quad \Delta ADB \sim \Delta BDC$

$$\therefore \frac{AD}{BD} = \frac{BD}{DC}$$

$$\Rightarrow BD^2 = AD \times DC = 8 \times 2 \Rightarrow BD^2 = 16 \Rightarrow BD = 4\text{cm}.$$

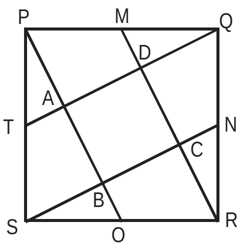
6. 3 Triangle ABC and Triangle QRC are similar. If ABC is a right angle triangle of Pythagoras triplet (3 : 4 : 5) then QRC has be of the same. Let us take QR as 3k so BR will also be 3k as its a square. And RC will be 4k. So BC = 7k. Thus 7k = 8 & k = $\frac{8}{7}$.

So side of square 3k will be 3 × $\frac{8}{7} = \frac{24}{7}$.

So, area = $\left(\frac{24}{7}\right)^2 = \frac{576}{49}$.

7. 2 Triangles PQR, PAB & DCR are similar. Triangle PQR is a Pythagoras triplet of 3 : 4 : 5 and so do the other triangles. So, DR will be 5k = 6 and k = 1.2. So, CR = 4k, CR = 4.8. Similarly, PA = 2.4. Thus, AC = PR – PA – CR = 25 – 4.8 – 2.4 = 17.8.

8. 2



In PQRS, PM = QR and PM || QR,

∴ PMQR is a parallelogram

In ΔSRC

$$\frac{SO}{OR} = \frac{SB}{BC}$$

∴ SB = BC = x (say)

Also AD = DQ = x ⇒ CN = $\frac{x}{2}$

In ΔSRN

$SN^2 = 64 + 16$

$$\left(\frac{5x}{2}\right)^2 = 80 \quad \Rightarrow x = \frac{8}{\sqrt{5}}.$$

9. 3 PQ || AC

$$\therefore \frac{CQ}{QB} = \frac{AP}{PB} = \frac{4}{3}$$

QD || PC

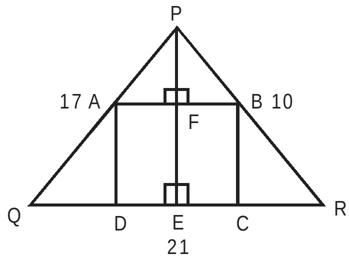
$$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$$

As $\frac{PD}{DB} = \frac{4}{3}$

$$\therefore PD = \frac{4}{7}PB$$

$$\therefore \frac{AP}{PD} = \frac{AP}{\frac{4}{7}PB} = \frac{7}{4} \times \frac{AP}{PB} = \frac{7}{4} \times \frac{4}{3} = 7 : 3.$$

10. 2



PE is an altitude from vertex P to side QR.

Let PE = h, so

$$\frac{1}{2} \times 21 \times h = \sqrt{S(S-a)(S-b)(S-c)}$$

$$= \sqrt{24(24-17)(24-10)(24-21)}$$

$$\Rightarrow \frac{1}{2} \times 21 \times h = 84 \quad \Rightarrow \quad h = 8$$

Let a be side of square ABCD.

ΔPAB ~ ΔPQR

$$\frac{PF}{PE} = \frac{AB}{QR} \Rightarrow \frac{8-a}{8} = \frac{a}{21}$$

$$\Rightarrow 29a = 168 \Rightarrow a = \frac{168}{29}$$

Perimeter of square ABCD = 4a = $\frac{4 \times 168}{29}$

= 23.17 (approx.)

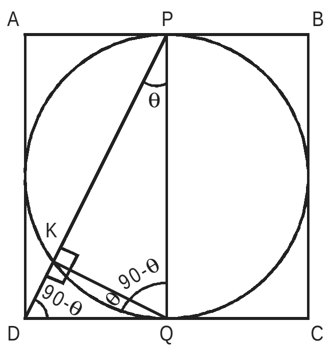
11. 4 We can use the formula for the circum radius of a triangle:

$$R = \frac{a \times b \times c}{4 \times (\text{Area of the triangle})}$$

or $R = \frac{a \times b \times c}{4 \times \left(\frac{1}{2} \times b \times AD\right)} = \frac{a \times c}{2 \times AD}$

$$= \frac{17.5 \times 9}{2 \times 3} = 26.25 \text{ cm.}$$

12. 2



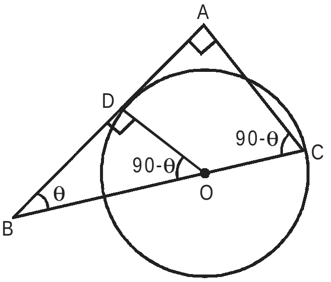
∠PKQ = 90° (Angle in a semi-circle)

ΔPQD ~ ΔPKQ ; $\frac{PQ}{PK} = \frac{QD}{KQ} = \frac{PD}{PQ} \Rightarrow PK = \frac{PQ^2}{PD}$

PQ = 4 units, $PD = \sqrt{PQ^2 + DQ^2} = \sqrt{4^2 + 2^2}$

$$= \sqrt{20} = 2\sqrt{5} \quad \Rightarrow PK = \frac{4 \times 4}{2\sqrt{5}} = \frac{8}{\sqrt{5}}$$

13. 1



$\Delta BDO \sim \Delta BAC$; $\frac{BD}{AB} = \frac{DO}{AC} = \frac{BO}{BC}$

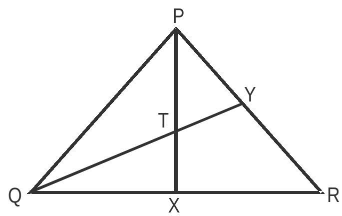
$\Rightarrow$ DO = 4, BC = 9, BO = BC – OC = 9 – 4 = 5

$\Rightarrow$ BD = $\sqrt{BO^2 - DO^2} = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$

$\Rightarrow \frac{3}{AB} = \frac{4}{AC} = \frac{5}{9} \Rightarrow AC = \frac{36}{5}, AB = \frac{27}{5}$

Hence, area of $\Delta ABC = \frac{1}{2} \times AC \times AB = \frac{1}{2} \times \frac{36}{5} \times \frac{27}{5}$ = 19.44.

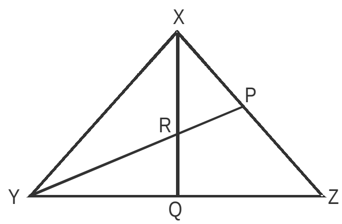
14. 2



Let PX & QY meet at T. Thus T will be centroid and PT:TX = 2 : 1.
So, PT = 12.

Now Area of Triangle PQY = $\frac{1}{2}$ × QY × PT = $\frac{1}{2}$ × 21 × 12 = 126. Area of ΔPQR = 2 × ΔPQY = 252

15. 2



As YP is angle bisector it will divide XZ in ratio of corresponding sides i.e. XY : YZ = 6 : 5.
So, XP = 6 and PZ = 5. As XQ is angle bisector for triangle XYZ, then XR will also be the angle bisector of XYP, so it will divide YP in ratio of corresponding sides i.e. XY : XP = 2 : 1

16. 1 For a right angle triangle in-radius $r = \frac{(a+b-c)}{2}$ and circumradius $R = \frac{c}{2}$.
Using these relation we get 2(R + r) = b + a

17. 2 In ΔPQR, PX divides the triangle into two equal areas.

Area of triangle OQX = $\frac{1}{6}$ (Area of triangle PQR)

Area of triangle QRY = $\frac{1}{2}$ of PQR

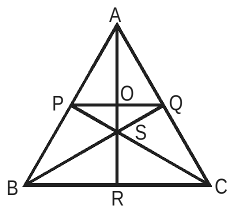
Area of triangle XZR = $\frac{1}{2}$(RQY) = $\frac{1}{8}$ (Area of triangle PQR)

Area of quadrilateral OYZX = $\left(\frac{1}{2} - \frac{1}{8}\right)$of PQR

Area = $\frac{3}{8}$ of PQR

$\frac{\text{Ratio of QOX}}{\text{Ratio of quadrilateral OYZX}}$ = 4 : 5

18. 2



Area of ΔPQS = 80, as AR is the median
Area of ΔPQR = 240 as RO is the median which divides centroid in ratio 2 : 1
Area of ΔABC = 960

19. 4 Since AB||CD, AC||DE and FI || BE the ΔABC ~ ΔAFG ~ ΔDHI ~ ΔDCE.

$\Rightarrow$ Ratio of sides of ΔAFG and ΔABC = $\frac{4}{7}$

$\therefore$ Ratio of area of ΔAFG and ΔABC = $\frac{16}{49}$

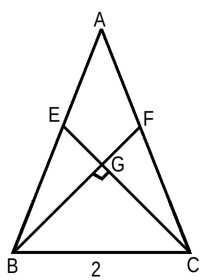
$\Rightarrow$ area of quadrilateral FGCB = 33 square unit
Since JL = KM, so let y = 3 units.

$\Rightarrow$ Ratio of area of ΔDHI and ΔDCE = $\frac{36}{81}$

$\Rightarrow \frac{ar(\Delta DHI)}{ar(\square FGCB)} = \frac{36}{33}$ = 12 : 11

20. 4 No right angled triangle can have an altitude to the hypotenuse more than half the length of the hypotenuse. For example any triangle formed in a semicircle with diameter as the base is always a right triangle. The maximum height it can have is the radius.

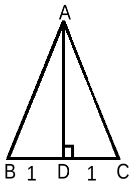
21. 1



As AB = AC, $GB = GC = \sqrt{2}$ cm.

$GE = GF = \frac{1}{\sqrt{2}}$ cm [Since 'G' is the centroid of the Δ ABC and $GE = \frac{1}{2}$ GB] ; $\therefore EB = \sqrt{\frac{5}{2}}$ cm

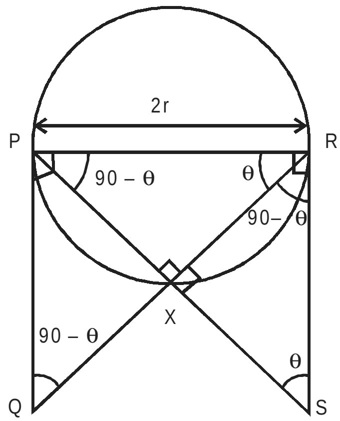
$\Rightarrow AB = AC = \sqrt{10}$ cm



$AD = \text{height} = \sqrt{10-1} = 3$ cm

$\therefore \text{Area} = \frac{1}{2} \times 3 \times 2 = 3\ \text{cm}^2.$

22. 1



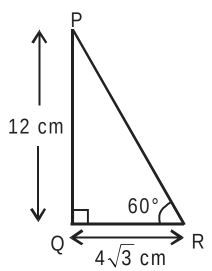
$\angle PXR = 90°$ (Angle in a semi-circle)

$\Delta PQR \sim \Delta RPS$

$\frac{PQ}{PR} = \frac{QR}{PS} = \frac{PR}{RS}$

$\Rightarrow PR^2 = PQ \times RS \Rightarrow PR = \sqrt{PQ \times RS}$

23. 2



Since, the ratio of length of PQ to QR is $\sqrt{3}$ and the measure of angle PRQ is 60 degrees, therefore PQR is a right angled triangle right angled at Q.

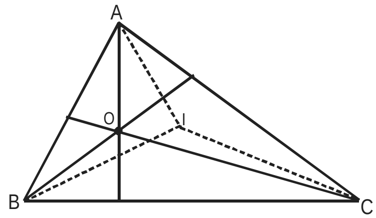
Let the inradius and the circumradius of the triangle be 'r' and 'R' respectively.

$$r = \frac{\Delta}{s} = \frac{\left(\frac{1}{2} \times PQ \times QR\right)}{\frac{1}{2}(PQ + QR + PR)} = \frac{4\sqrt{3}}{\sqrt{3}+1},$$

where 's' is the semi-perimeter of the triangle.

Also, $R = \frac{PR}{2} = 4\sqrt{3} \quad \Rightarrow \frac{r}{R} = \frac{1}{\sqrt{3}+1}$

24. 115°

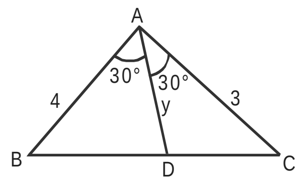


$\angle BOC = 180° - \angle BAC$

$\therefore \angle BAC = 50°$

Hence, $\angle BIC = 90° + \frac{1}{2}\angle BAC = 115°.$

25. 2



Let BC = x and AD = y.

As per Bisector Theorem, $\frac{BD}{DC} = \frac{AB}{AC} = \frac{4}{3}$

Hence, $BD = \frac{4x}{7}$; $DC = \frac{3x}{7}$

In ΔABD, $\cos 30° = \frac{(4)^2 + y^2 - \frac{16x^2}{49}}{2 \times 4 \times y}$

$\Rightarrow 2 \times 4 \times y \times \frac{\sqrt{3}}{2} = 16 + y^2 - \frac{16x^2}{49}$

$\Rightarrow 4\sqrt{3}y = 16 + y^2 - \frac{16x^2}{49}$ ...(i)

Similarly, from ΔADC, $\cos 30° = \frac{9 + y^2 - \frac{9x^2}{49}}{2 \times 3 \times y}$

$\Rightarrow 3\sqrt{3}y = 9 + y^2 - \frac{9x^2}{49}$ ...(ii)

Now (i) × 9 – 16 × (ii), we get

$36\sqrt{3}y - 48\sqrt{3}y = 9y^2 - 16y^2 \quad \Rightarrow y = \frac{12\sqrt{3}}{7}$

Alternative method:

Area of ΔABC = Area of Δ ABD + Area of Δ ADC

$\Rightarrow \frac{1}{2} \times 4 \times 3 \sin 60^o = \frac{1}{2} \times 4 \times y \sin 30^o + \frac{1}{2} \times 3 \times y \times \sin 30^o$

$\Rightarrow 12\sqrt{3} = 4y + 3y \quad \Rightarrow \quad y = \frac{12\sqrt{3}}{7}$.