Geometry - 4

Career Launcher

QA - 28

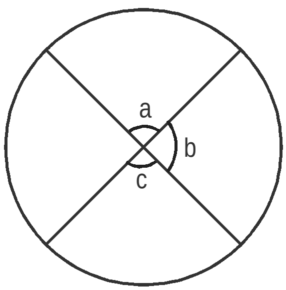
CEX-Q-0229/18

Contents

- Circle

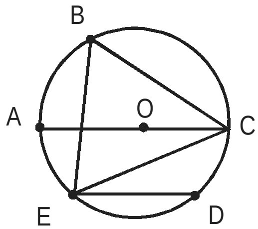
Number of Questions : 25

1. In the figure given below, a, b, and c are angles subtended by the two straight lines that intersect each other at the center of the circle. If b – a = 80°, what portion of the total circumference of the circle is made up by the arc of angle c?



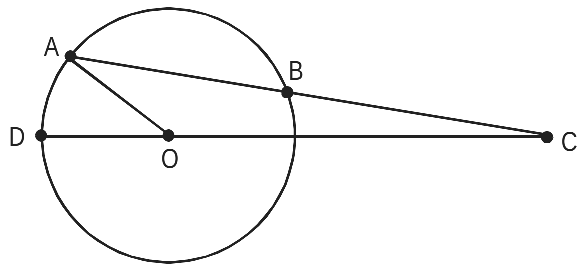
(1) $\frac{5}{36}$ (2) $\frac{1}{6}$ (3) $\frac{5}{18}$ (4) $\frac{1}{3}$

2. In the figure given below, chord ED is parallel to the diameter AC of the circle. If $\angle CBE = 65°$, then what is the value of $\angle DEC$?



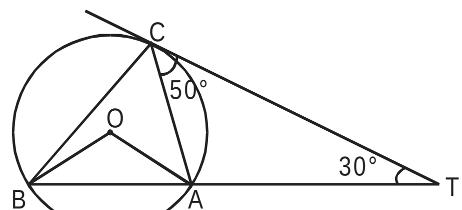
(1) 35° (2) 55°
(3) 45° (4) 25°

3. In the figure given below, AB is a chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If ∠ACD = y° degrees and ∠AOD = x°such that x = ky, then the value of k is



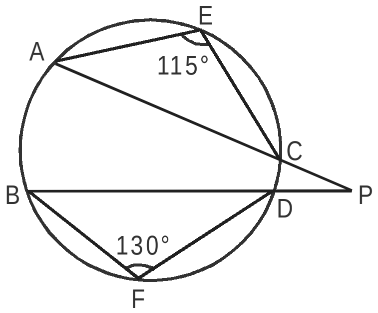
(1) 3 (2) 2
(3) 1 (4) None of these

4. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then measure of ∠BOA is

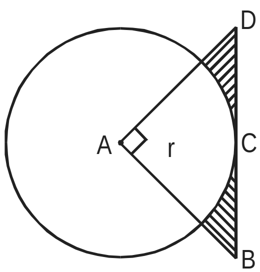


(1) 100° (2) 150° (3) 80°
(4) Not possible to determine

5. In the figure given below, PCA and PDB are two secants. If $\angle AEC = 115°$, $\angle BFD = 130°$ and $\angle P = 30°$, find $m(\text{Arc}(\widehat{AB}))$ in degrees (i.e. angle subtended by arc AB at the center).

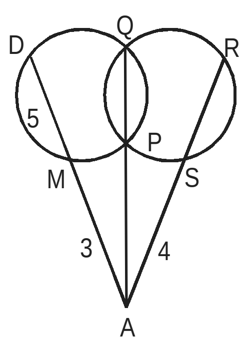


6. A is the centre of the circle with radius 'r' units and BD is the tangent to the circle at point C. If BC = CD, then the area (in sq. units) of the shaded part is



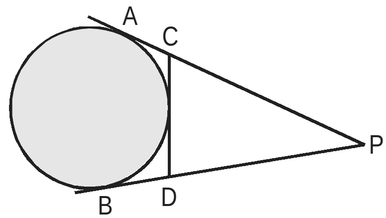
(1) $0.61\ r^2$ (2) $0.214\ r^2$
(3) $0.72\ r^2$ (4) $0.305\ r^2$

7. In the given figure, AMD, APQ and ASR are secants to the given circles. If AM = 3 cm, MD = 5 cm and AS = 4 cm, then find the length of line segment SR.

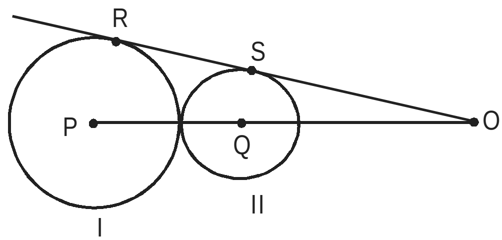


(1) 5 cm (2) 4 cm
(3) 3 cm (4) 2 cm

8. PA and PB are two tangents of a circle with common point P with PA = 11 units. CD is another tangent of circle as shown in the figure. Find the perimeter of triangle PCD.



Directions for questions 9 to 11: Answer the questions on the basis of the information given below. In the adjoining figure I and II, are circles with centres P and Q respectively. The two circles touch each other and have common tangent that touches them at points R and S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4 : 3. It is also known that the length of PO is 28 cm.



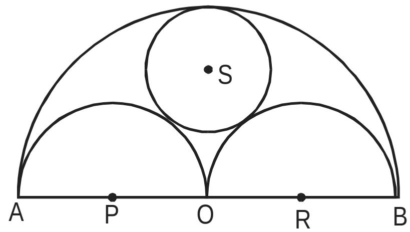
9. What is the ratio of the length of PQ to that of QO?
(1) 1 : 4 (2) 1 : 3
(3) 3 : 8 (4) 3 : 4

10. What is the radius of the circle II?
(1) 2 cm (2) 3 cm
(3) 4 cm (4) 5 cm

11. The length of SO is
(1) $8\sqrt{3}$ cm (2) $10\sqrt{3}$ cm
(3) $12\sqrt{3}$ cm (4) $14\sqrt{3}$ cm

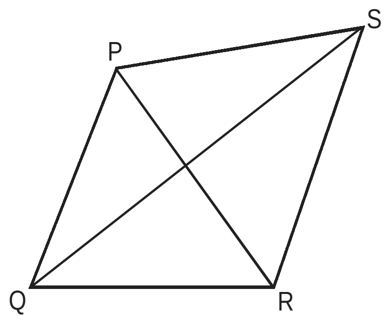
12. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with center at O. Horses are tied up at P, R and S

such that PO and RO are the radii of semi-circles with centers at P and R respectively, and S is the center of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circles and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semi-circle with diameter AB that cannot be grazed by the horses is nearest to

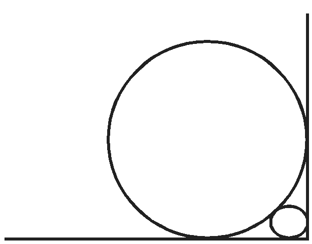


(1) 20 (2) 28
(3) 36 (4) 40

13. In the figure below, ΔPQR is an equilateral triangle, PQRS is a quadrilateral in which PQ = PS. Find angle QSR (in degrees).



14. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



(1) $3-2\sqrt{2}$ (2) $4-2\sqrt{2}$
(3) $7-4\sqrt{2}$ (4) $6-4\sqrt{2}$

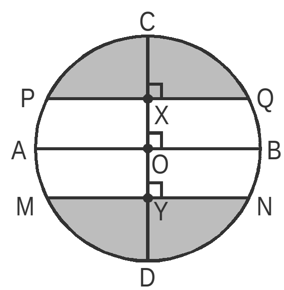
15. Four points A, B, C and D lie on a straight line in the X-Y plane, such that AB = BC = CD, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance (in metres) the ant must traverse to reach the sugar particle is

(1) $3\sqrt{2}$ (2) $1+\pi$
(3) $\frac{4\pi}{3}$ (4) 5

16. Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq. cm.) of the intersecting region (i.e. common to both the circles.?

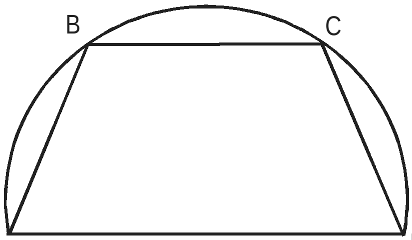
(1) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ (2) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$
(3) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$ (4) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

17. In the figure given below, AB and CD are diameters of a circular sheet with center O and radius 'r' cm. CX = OX = OY = YD. Find the area (in cm^2) of shaded part.



(1) $r^2\left(\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right)$ (2) $r^2\left(\frac{2\pi}{3}-\frac{\sqrt{3}}{2}\right)$
(3) $\frac{r^2}{2}\left(2\pi-3\sqrt{3}\right)$ (4) None of these

18. In a semicircle with diameter AD, chord BC is parallel to diameter AD. If AB = CD = 2 cm, and AD = 8 cm, what is the length of BC?



(1) 7.5 cm (2) 7 cm
(3) 7.75 cm (4) None of these

[CAT 2004]

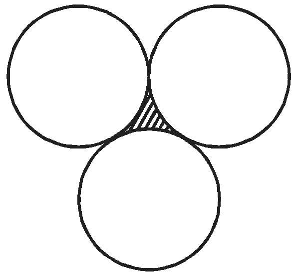
19. A canal is surrounding a circular city of diameter 4 kilometers. Four lampposts are located on the border of the city close to the canal at locations A, B, C and D, such that a direct path from A to C cuts another direct path from B to D perpendicularly at a junction E. The value of $EA^2 + EB^2 + EC^2 + ED^2$ is
(1) 20 sq. km (2) 24 sq. km.
(3) 16 sq. km (4) 32 sq. km

20. A spiral is made up of 13 successive semicircles, with center alternately at A and B, starting with center at A. The radii of semicircles, thus developed, are 0.5 cm, 1.0 cm, 1.5 cm, and 2.0 cm and so on. The total length of the spiral is:
(1) 144 cm (2) 143 cm
(3) 147 cm (4) None of these

21. Two circles with radii 'a' & 'b' respectively touch each other externally. Let 'c' be the radius of a circle that touches these two circles as well as a common tangent to these two circles. Then which of the following is true?
(1) a, b and c are in G.P.
(2) a, b and c are in H.P.
(3) $\frac{1}{c} = \frac{1}{a} + \frac{1}{b}$
(4) $\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$

22. In ΔABC, let AB = 20 cm, AC = 11 cm, BC = 13 cm. The diameter of semicircle inscribed in ABC, (whose diameter lies on AB and side AC & BC are tangents to the semi-circle) is equal to
(1) 13 cm (2) 11 cm
(3) 17 cm (4) 19 cm

23. Three identical circles of area 16π each are touching each other as shown in the figure below. What is the area of the shaded region?



(1) $16\sqrt{3} - 8\pi$ (2) $16\sqrt{3} - 6\pi$
(3) $16\sqrt{3} - 4\pi$ (4) $16\sqrt{3} - \frac{8\pi}{3}$

24. Find the approximate length of the common chord of the circles whose centres are 53 cms apart and radius is 28 cm and 45 cm.
(1) 47.5 cm (2) 50.5 cm
(3) 36.5 cm (4) 30.5 cm

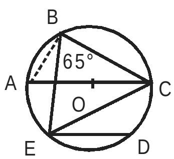
25. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from a square cloth piece of side *a* in the following steps: the original square of side *a* is divided into *n* smaller squares, not necessarily of the same size, then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side *a* and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is
(1) 1 : 1 (2) $\sqrt{2} : 1$
(3) $\frac{n(4-\pi)}{4n-\pi}$ (4) $\frac{4n-\pi}{n(4-\pi)}$

QA - 28 : Geometry - 4
Answers and Explanations

1	1	**2**	4	**3**	1	**4**	1	**5**	95°	**6**	2	**7**	4	**8**	22	**9**	2	**10**	2
11	3	**12**	2	**13**	30°	**14**	4	**15**	2	**16**	4	**17**	2	**18**	2	**19**	3	**20**	2
21	4	**22**	2	**23**	1	**24**	1	**25**	1										

1. 1 $b - a = 80°$ ...(1)
$b + a = 180°$ ... (2) (Sum of angles on a straight line)
Adding (1) & (2), $2b = 260°$
$b = 130°$, $a = 50°$
$c = a = 50°$ (Vertically opposite angles)
Portion of total circumference made up by the arc of angle c = $\frac{50}{360} = \frac{5}{36}$

2. 4



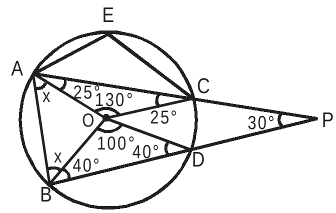
In ΔABC,
$\angle B = 90°$ (Angles in semicircle)
Therefore, $\angle ABE = 90 - 65 = 25°$
Also, $\angle ABE = \angle ACE$ (Angle subtended by same arc AE)
Also, $\angle ACE = \angle CED$ [$AC \parallel ED$]
Therefore, $\angle CED = 25°$

3. 1 If $y = 10°$,
$\angle BOC = 10°$ (opposite equal sides)
$\angle OBA = 20°$ (external angle of ΔBOC)
$\angle OAB = 20$ (opposite equal sides)
$\angle AOD = 30°$ (external angle of ΔAOC)
Thus k = 3

4. 1 $\angle BAC = \angle ACT + \angle ATC = 50° + 30° = 80°$
And $\angle ACT = \angle ABC$ (Angle in alternate segment)
So $\angle ABC = 50°$
$\angle BCA = 180° - (\angle ABC + \angle BAC)$
$= 180° - (50° + 80°) = 50°$
Since $\angle BOA = 2\angle BCA = 2 \times 50° = 100°$

Alternative Method:
Join OC
$\angle OCT = 90°$ (TC is tangent to OC)
$\angle OCA = 90° - 50° = 40°$
$\angle OAC = 40°$ (OA = OC being the radius)
$\angle BAC = 50° + 30° = 80°$
$\angle OAB = 80° - 40° = 40° = \angle OBA$ (OA = OB being the radius)
$\angle BOA = 180° - (\angle OBA + \angle OAB) = 100°$

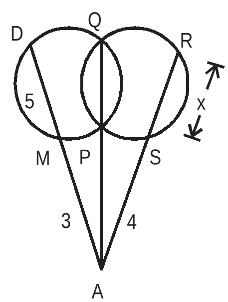
5. 95°



Let the centre of circle be O.
Reflex $\angle AOC = 2 \times 115° = 230°$
$\therefore \angle AOC = 130°$ similarly $\angle BOD = 120°$
$\Rightarrow \angle OAE = \angle OEA = 25°$ (Isosceles triangle)
and $\angle OBD = \angle ODB = 40°$
In ΔAPB
$x + 40° + x + 25° + 30° = 180°$
$\Rightarrow 2x = 85°$
$\therefore \angle AOB = 180° - 85° = 95°$

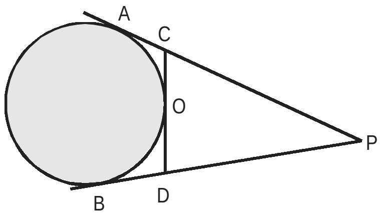
6. 2 Δ ACB is a right triangle. $\angle BAC = \angle CBA = 45°$
$\Rightarrow BC = AC = r$. So that, BD = 2r.
Required area = (Area of triangle) – (Area of sector)
$= \frac{1}{2} r \times 2r - \frac{90}{360} \times \pi r^2 = r^2\left(1 - \frac{\pi}{4}\right) = r^2(0.214)$

7. 4 Let SR = x cm.



AM × AD = AP × AQ = AS × AR
or 3 × 8 = 4 × (4 + x) ⇒ x = 2 cm.

8. 22



Tangent from a common point to a circle are of equal length, so PA = PB

Let O be the point at which tangent CD touches the circle, so AC = OC and OD = BD. Therefore perimeter of triangle PCD is PC + CO + OD + DP = 22 units.

9. 2 $\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

OP = 28

OQ = 21

PQ = OP – OQ = 7

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

Alternative method:

$\frac{OP}{OQ} = \frac{PR}{QS} = \frac{4}{3}$

$\Rightarrow \frac{OP}{OQ} - 1 = \frac{4}{3} - 1 \Rightarrow \frac{OP - OQ}{OQ} = \frac{1}{3} \Rightarrow \frac{PQ}{OQ} = \frac{1}{3}$

10. 2 $PR + QS = PQ = 7$

$= \frac{PR}{QS} = \frac{4}{3} \Rightarrow QS = 3$ cm

11. 3 $SO = \sqrt{OQ^2 - QS^2}$

$= \sqrt{21^2 - 3^2} = \sqrt{24 \times 18} = 12\sqrt{3}$ cm

12. 2 If the radius of the field is r, then the total area of the field = $\frac{\pi r^2}{2}$.

The radius of the semi-circles with centre's P and $R = \frac{r}{2}$.

Hence, their total area = $\frac{\pi r^2}{4}$

Let the radius if the circle with centre S be x.

Thus, OS = (r – x), OR = $\frac{r}{2}$ and RS = $\left(\frac{r}{2} + x\right)$.

Applying Pythagoras Theorem, we get

$(r - x)^2 + \left(\frac{r}{2}\right)^2 = \left(\frac{r}{2} + x\right)^2$

Solving this, we get $x = \frac{r}{3}$.

Thus the area of the circle with centre $S = \frac{\pi r^2}{9}$.

The total area that can be grazed = $\pi r^2\left(\frac{1}{4} + \frac{1}{9}\right)$

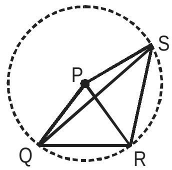
$= \frac{13\pi r^2}{36}$

Thus the area of the field that cannot be

grazed $= \frac{\pi r^2}{2} - \frac{13\pi r^2}{36} = \frac{5\pi r^2}{36}$

The percentage $= \frac{\frac{5}{36}\pi r^2}{\frac{1}{2}\pi r^2} \times 100 \simeq 28.$

13. 30°



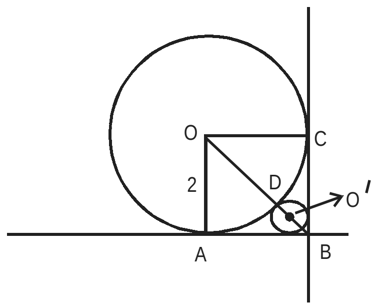
$PQ = PR = PS$

Draw a circle with P as a centre an PQ as radius.

$\angle QSR = \frac{1}{2} \times \angle QPR$ (Angle at center is twice the angle at the circumference)

$= \frac{1}{2} \times 60 = 30°.$

14. 4



Let the radius of smaller circle be r.

$\therefore O'B = r\sqrt{2}$

$\therefore OB = O'B + O'D + OD$

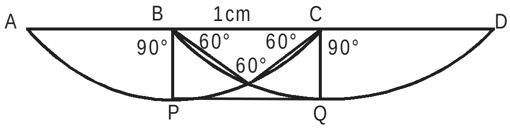
$= r\sqrt{2} + r + 2$

Also OB = $2\sqrt{2}$

$\Rightarrow r\sqrt{2} + r + 2 = 2\sqrt{2}$

$\Rightarrow r = 6 - 4\sqrt{2}$

15. 2



Drawn figure since it have not to be within distance of 1 m so it will go along APQD, which is the path of minimum distance.

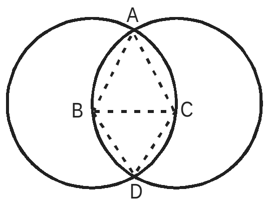
$AP = \frac{90}{360} \times 2\pi \times 1 = \frac{\pi}{2}$

Also, $AP = QD = \frac{\pi}{2}$

So, the minimum distance = AP + PQ + QD

$= \frac{\pi}{2} + 1 + \frac{\pi}{2} = 1 + \pi$

16. 4



It is given that AB = BC = AC = BD = DC = 1 cm.

Therefore, ΔABC is an equilateral triangle.

Hence, $\angle ACB = 60^\circ$

Now area of sector $\overset{\frown}{AB} = \frac{60}{360} \times \pi(1)^2 = \frac{\pi}{6}$

Area of equilateral triangle $\Delta ABC = \frac{\sqrt{3}}{4}(1)^2 = \frac{\sqrt{3}}{4}$

Area of remaining portion in the common region $\overset{\frown}{ABC}$

excluding $\Delta ABC = 2 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

Hence, the total area of the intersecting region

$= 2 \times \frac{\sqrt{3}}{4} + 4 \times \left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ sq. cm.

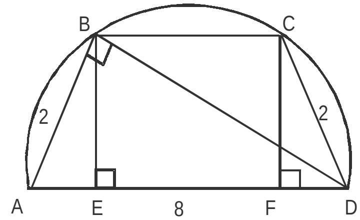
17. 2 Area of shaded portion

= 2 × (Area of sector POQ – area of ΔPOQ)

$= 2\left(\frac{\pi}{3}r^2 - \frac{1}{2} \times r^2 \times r^2 \times \sin 120^\circ\right)$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)r^2$

18. 2



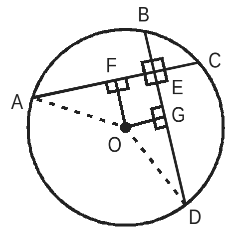
$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$

$\Rightarrow 2\sqrt{8^2 - 2^2} = 8 \times BE \quad \Rightarrow BE = \frac{\sqrt{60}}{4} = \frac{\sqrt{15}}{2}$

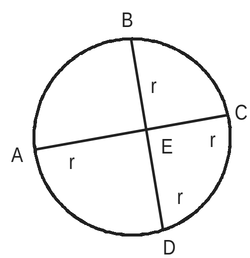
$AE = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$

$BC = EF = 8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$

19. 3

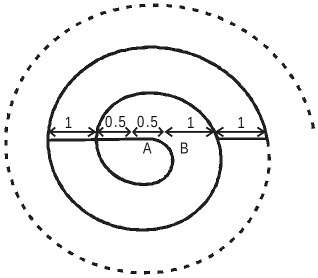


Let AF = x, DG = y and O is the centre of circle.
AF = FC = x, DG = BG = y (perpendicular from centre of a circle to a chord divides it in two equal parts)
FE = OG = x – EC
FO = EG = y – BE
Now, In ΔAFO
$x^2 + (y - BE)^2 = r^2$ .... (1)
In ΔOGD,
$y^2 + (x - EC)^2 = r^2$ .... (2)
Adding (1) & (2),
$2x^2 + 2y^2 + EC^2 + BE^2 - 2xEC - 2yBE = 2r^2$ ......(3)
$EA^2 + EB^2 + EC^2 + ED^2 = (2x - EC)^2 + EB^2 + EC^2 + (2y - BE)^2$
$= 4x^2 + 4y^2 + EC^2 + EB^2 - 4x\,EC - 4yBE$ ....(4)
Using (3) & (4),
$EA^2 + EB^2 + EC^2 + ED^2 = 4r^2 = 4 \times 2^2 = 16$ km
Alternative approach,



Assume AC & BD as diameter of circle, then E is centre of circle
$EA^2 + EB^2 + EC^2 + ED^2 = r^2 + r^2 + r^2 + r^2 = 4r^2 = 4 \times 2^2 = 16$ km.

20. 2

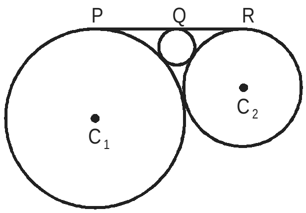


$$\frac{1}{2}\left(2\pi \times \frac{1}{2} + 2\pi \times 1 + 2\pi \times 1.5 + ... + 2\pi \times 6.5\right)$$

$$= \pi\left(\frac{1}{2} + 1 + 1.5 + ... + 6.5\right)$$

$$= \pi \times \frac{7}{2} \times 13 = \frac{22}{7} \times \frac{7}{2} \times 13 = 143 \text{ cm}$$

21. 4



Let C_1, C_2, & C_3 be three centres of three circles of radius a, b & c respectively PQ = $2\sqrt{ac}$ (Length of direct common tangent when two circles touch each other externally)

Similarly,

$$QR = 2\sqrt{bc}, PR = 2\sqrt{ab}$$

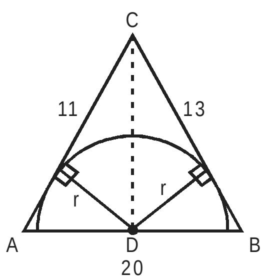
$$PR = QR + PQ$$

$$2\sqrt{ab} = 2\sqrt{bc} + 2\sqrt{ac}$$

Divide by $\sqrt{abc}$ on both sides,

$$\frac{1}{\sqrt{c}} = \frac{1}{\sqrt{a}} + \frac{1}{\sqrt{b}}$$

22. 2



Let D be centre of semi-circle

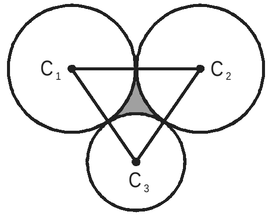
$$\frac{1}{2} \times 11 \times r + \frac{1}{2} \times 13 \times r = \sqrt{S(S-a)(S-b)(S-c)}$$

$$\Rightarrow 12r = \sqrt{22(22-11)(22-13)(22-20)}$$

$$\Rightarrow 12r = 66 \Rightarrow r = 5.5$$

$\therefore$ 2r = 11.

23. 1



$\pi r^2 = 16\pi$

$\therefore$ r = 4

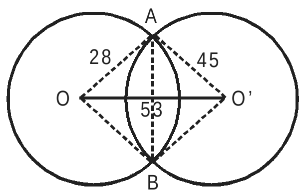
$\Delta C_1C_2C_3$ is an equilateral Δ of side 2r = 8

Area of shaded region = Area of equilateral Δ – 3 ×

Area of a sector = $\frac{\sqrt{3}}{4} \times 8^2 - 3 \times \frac{60}{360} \times \pi \times 4^2$

$= 16\sqrt{3} - 8\pi$

24. 1



OAO'B is a kite and triangle OAO' is a right angle triangle.

Hence, twice the area of triangle OAO' is equal to the area of kite.

$$2\left(\frac{1}{2} \times 28 \times 45\right) = \frac{1}{2} \times 53 \times AB$$

$$AB = \frac{2 \times 28 \times 45}{53} = 47.5 \text{ cm.}$$

25. 1 Consider a square of side x.

Therefore, its area = x^2

Therefore, area of the largest circle which can be cut from square = $\frac{\pi x^2}{4}$.

Therefore, area scrapped $= x^2 - \frac{\pi}{4}x^2 = x^2\left(1 - \frac{\pi}{4}\right)$

$$\therefore \frac{\text{Area scrapped}}{\text{Area of square}} = \frac{x^2\left(1 - \frac{\pi}{4}\right)}{x^2} = 1 - \frac{\pi}{4} = \text{Constant}$$

As this ratio is constant whether we cut a circle from small square or larger square, scrapped area will be a fixed percentage of square. Therefore, in our problem as two squares are of the same size, the ratio will be 1 : 1.