* **Introduction  
    
  Numbers form an integral part of our lives. In this lesson we will learn about the different types of numbers  
  and the different categories under which they fall. The concepts discussed in this lecture will be your first step  
  towards a general understanding of the mathematics requirements to clear MBA entrance exams. As we  
  proceed with this lecture, you will realise that you have already learnt many of the concepts, included in this  
  lesson, in school. This would further help build confidence in you. Although Number theory is important in the  
  context of all the MBA entrance exams, it gains all the more importance for the students aiming for success in  
  the CAT,  
    
    
  Understanding Numbers  
  A measurement carried out, of any quantity, leads to a meaningful value called the Number. This value may  
  be positive or negative depending on the direction of the measurement and can be represented on the  
  number line.  
    
    
  Natural Numbers (N)  
  The numbers 1, 2, 3, 4, 5…are known as natural numbers. The set of natural numbers is denoted by N.  
  Hence, N = {1, 2, 3, 4…}. The natural numbers are further divided as even, odd, prime etc.  
    
     
    
  Whole Numbers (W)  
  All natural numbers together with ‘0’ are collectively called whole numbers. The set of whole numbers is  
  denoted by W, and W = {0, 1, 2, 3, ……}  
    
    
  Integers (Z)  
  The set including all whole numbers and their negatives is called a set of integers. It is denoted by Z, and Z =  
  {- ∞, … - 3, - 2, - 1, 0, 1, 2, 3, ……. ∞}. They are further classified into Negative integers, Neutral integers  
  and positive integers.  
    
    
  Negative integers (Z-)  
  All integers lesser than Zero are called negative integers.  
  Z − = {- 1, - 2, - 3…- ∞ }  
    
    
  Neutral integers (Z0)  
  Zero is the only integer which is neither negative nor positive and it is called a neutral integer.  
    
    
  Positive integers (Z+)  
  All integers greater than Zero are called positive integers.  
  Z + = {1, 2, 3, …….., ∞ }  
    
    
  Classification of Numbers  
  i. Even Numbers:  
  All numbers divisible by 2 are called even numbers. E.g., 2, 4, 6, 8, 10 …Even numbers can be  
  expressed in the form 2n, where n is an integer. Thus 0, - 2, − 6, etc. are also even numbers.  
    
    
  ii. Odd Numbers:  
  All numbers not divisible by 2 are called odd numbers. e.g. 1, 3, 5, 7,  
  9…Odd numbers can be expressed in the form (2n + 1) where n is any  
  integer. Thus - 1, − 3, − 9 etc. are all odd numbers.  
    
    
  iii. Prime Numbers:  
  A natural number that has no other factors besides itself and unity is a  
  prime number.  
  Examples: 2, 3, 5, 7, 11, 13, 17, 19 …  
    
    
  Important Observation about prime numbers:  
  A prime number greater than 3, when divided by 6 leaves either 1 or 5 as  
  the remainder. Hence, a prime number can be expressed in the form of 6K  
  ± 1. But the converse of this observation is not true, that a number leaving a  
  remainder of 1 or 5 when divided by 6 is not necessarily a prime number. For eg: 25, 35 etc  
    
     
    
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  Ex.1 If a, a + 2, a + 4 are consecutive prime numbers. Then how many solutions ‘a’ can have?  
  (1) one  
    
  (2) two  
    
  (3) three  
    
  (4) more than three  
    
    
  Sol. No even value of ‘a’ satisfies this. So ‘a’ should be odd. But out of three consecutive odd numbers,  
  atleast one number is a multiple of 3.  
  So, only possibility is a = 3 and the numbers are 3, 5, 7. Answer: (1)  
    
    
  iv. Composite Numbers: A composite number has other factors besides itself  
  and unity. e.g. 8, 72, 39, etc. On the basis of this fact that a number with  
  more than two factors is a composite we have only 34 composite from 1 to  
  50 and 40 composite from 51 to 100.  
    
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  v. Perfect Numbers: A number is said to be a perfect number if the sum of ALL its factors excluding itself  
  (but including 1) is equal to the number itself.  
  Or  
  The sum of all the possible factors of the number is equal to twice the number.  
    
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  Example:  
  6 is a perfect number because the factors of 6, i.e., 1, 2 and 3 add up to the number 6 itself.  
  Also Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved = 1 (Unity)  
  Other examples of perfect numbers are 28, 496, 8128, etc. There are 27 perfect numbers discovered  
  so far.  
    
    
  vi. Co-Prime numbers:  
  Two numbers are (prime or composite) said to be co-prime to one another, if  
  they do not have any common factor other than 1. e.g. 35 & 12, since they  
  both don’t have a common factor among them other than 1.  
    
     
    
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  vii. Fractions  
  A fraction denotes part or parts of a unit. Several types are:  
     
    
  1. Common Fraction: Fractions whose denominator is not 10 or a multiple of it. e.g. Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  2. Decimal Fraction: Fractions whose denominator is 10 or a multiple of 10.  
    
  3. Proper Fraction: In this the numerator < denominator e.g. Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved etc. Hence its value < 1.  
    
  4. Improper Fraction: In these the numerator > denominator e.g. Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved etc. Hence its value > 1.  
    
  5. Mixed Fractions: When a improper fraction is written as a whole number and proper fraction it is called  
  mixed fraction. e.g. Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved can be written asBank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
    
    
  Rational Numbers  
  Rational Number is defined as the ratio of two integers i.e. a number that can be represented by a fraction of  
  the form Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved where p and q are integers and q ≠ 0.They also can be defined as the non-terminating recurring  
  decimal numbers. Such as 3.3333…., 16.123123….. are all rational numbers as they can be expressed in the  
  form Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved.  
    
    
  Examples: Finite decimal numbers, whole numbers, integers, fractions i.e.  
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
    
  Irrational Numbers  
  Any number which can not be represented in the form Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved where p and q are integers and q ≠ 0 is an irrational  
  number. On the basis of non-terminating decimals, irrational numbers are non-terminating non recurring  
  decimals. Such as 3.4324546345……. is a non-terminating, non-repeating number.  
    
    
  Examples: π, √5, √7, e  
    
    
  Non-Terminating Decimal Numbers  
     
    
  When we divide any number by other number, either we get a terminating number or a non - terminating  
  number. A non - terminating number on the basis of occurrence of digits after decimal can classified as  
  following.  
    
    
  1. Pure Recurring Decimals:  
     
    
  A decimal in which all the figures after the decimal point repeat, is called a pure recurring decimal.  
     
    
  Examples: 0.6 , 0Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved are examples of pure recurring decimals. (0.6 = 0Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved = 0.666 ……….)  
    
    
  2. Mixed Recurring Decimals: A decimal in which some figures do not repeat and some of them are  
  repeated is called a mixed recurring decimal.  
  Examples: Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  3. Non - Recurring Decimals: A decimal number in which the figure don’t repeat themselves in any  
  pattern are called non-terminating non- recurring decimals and are termed as irrational numbers.  
    
    
  Converting Recurring Decimal as Fraction  
  All recurring decimals can be converted into fractions. Some of the common types can be 0.33….. ,  
  0.1232323…, 5.33…., 14.23636363…. etc.  
    
    
  Pure Recurring to Fractions  
     
    
  FUNDA 1: If a number is of the form of 0.ababab……. then divide the repeating digits with as many 9’s as we  
  have repeated digits.  
    
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  Mixed Recurring to Fractions  
    
  FUNDA 2: If N = 0.abcbcbc…. Then  
    
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
    
  FUNDA 3: If N = a.bcbc…. Then  
  Write N = a + 0.bcbc….  
  Proceed as Funda 1  
  5.3636… = 5 + 0.3636… = Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
    
  Divisibility Test  
    
    
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  FUNDA:  
  How to calculate remainder, when a number is divided by 11, without  
  division?  
    
    
  Step 1: Add all the odd place numbers (O) and even place numbers (E)  
  counted from right to left.  
     
    
  Step 2: If O - E is positive, remainder will be the difference less than 11.  
     
    
  Step 3: If O - E is negative, remainder should be (11 - difference).  
    
    
  Ex.2 What is the remainder when 2354789341 is divided by 11?  
     
    
  Sol. Odd place digit sum (O) = 1 + 3 + 8 + 4 + 3 = 19.  
  Even place digits sum (E) = 4 + 9 + 7 + 5 + 2 = 27.  
  Difference (D) = 19 - 27 = - 8  
  Remainder = 11 - 8 = 3.  
    
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  Ex.3 If 567P55Q is divisible by 88; Find the value of P + Q.  
  (1) 11  
    
  (2) 12  
    
  (3) 5  
    
  (4) 6  
    
  (5) 10  
     
    
  Sol. The number is divisible by 8 means; the number formed by the last 3 digits should be divisible by 8  
  which are 55Q. Only Q = 2 satisfy this. From the divisibility rule of 11, (2 + 5 + 7 + 5) - (5 + P + 6) is  
  divisible by 11. So 8-P is divisible by 11. if P= 8, then only it is possible. So P = 8 and Q = 2.  
  So P + Q = 10. Answer: (5)  
    
    
  Ex.4 If the first 100 natural numbers are written side by side to form a big number and it is divided by  
  8. What will be the remainder?  
  (1) 1  
    
  (2) 2  
    
  (3) 4  
  (4) 7  
    
  (5) cannot be determined  
    
  Sol. The number is 1234…..9899100  
  According to the divisibility rule of 8, we will check only the last 3 digits.  
  If 100 is divided by 8, the remainder is 4. Answer: (3)  
    
    
  Ex.5 What will be the remainder when 4444……..44 times is divided by 7?  
  (1) 1  
    
  (2) 2  
    
  (3) 5  
    
  (4) 6  
    
  (5) 0  
    
  Sol. If 4 is divided by 7, the remainder is 4.  
  If 44 is divided by 7, the remainder is 2.  
  If 444 is divided by 7, the remainder is 3  
  By checking like this, we come to know that 444444 is exactly divisible by 7.  
  So if we take six 4’s, it is exactly divisible by 7. Similarly twelve 4’s is also exactly divisible by 4 and 42  
  4’s will be exactly divisible by 7. So out of 44, the remaining two 4,s will give a remainder of 2.  
  So, answer (2).  
    
    
  Absolute value of a number  
  The modulus of a number is the absolute value of the number or we can say the distance of the number from  
  the origin. The absolute value of a number a is defined as  
  |a| = a, if a ≥ 0  
  = - a, if a ≤ 0  
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  Example: |79| = 79 & | - 45| = - (- 45) = 45  
  Also, | x - 3 | = x - 3, if x ≥ 3  
  = 3 - x, if x < 3.  
    
    
  Always Keep in Mind  
  The number 1 is neither prime nor composite.  
    
  1) 2 is the only even number which is prime.  
    
  2) (xn + yn) is divisible by (x + y), when n is an odd number.  
    
  3) (xn - yn) is divisible by (x + y), when n is an even number.  
  4) (xn - yn) is divisible by (x - y), when n is an odd or an even number.  
    
  5) The difference between 2 numbers (xy) - (yx) will always be divisible by 9.  
    
  6) The square of an odd number when divided by 8 will always give 1 as a remainder.  
    
  7) Every square number is a multiple of 3 or exceeds a multiple of 3 by unity.  
    
  8) Every square number is a multiple of 4 or exceeds a multiple of 4 by unity.  
    
  9) If a square number ends in 9, the preceding digit is even.  
    
  10) If m and n are two integers, then (m + n)! is divisible by m! n!  
    
  11) (a)n / (a + 1) leaves a remainder of Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
  .  
  12) Product of n consecutive numbers is always divisible by n!.  
    
    
  Ex.6 If ‘X’ is an even number; Y is an odd number, then which of the following is even?  
  (1) X2 + Y  
    
  (2) X + Y2  
    
  (3) X2 + Y2  
    
  (4) X2Y2  
    
  (5) None of these  
    
    
  Sol. Since X is even, X2 is even.  
  Y is odd, Y2 is odd  
  So options (1), (2), (3) are even + odd = odd.  
  Option (4) is (even) (odd) = Even. Answer: (4)  
    
    
  Ex.7 What is the difference between 0.343434....…and 0.2343434…… in fraction form?  
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
    
  Sol.  
    
  Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT, Solved  
    
  Ex.8 How many of the following numbers are divisible by at least 3 distinct prime numbers 231, 750,  
  288 and 1372?  
  (1) 0  
    
  (2) 1  
    
  (3) 2  
    
  (4) 3  
    
  (5) 4  
    
    
  Sol. 231 = 3 × 7 × 11 (3 prime factors)  
  750 = 2 × 3 × 53 (3 prime factors)  
  288 = 25 × 32 (only 2 prime factors)  
  1372 = 22 × 73 (only 2 prime factors)  
  So, only 231 & 750 has 3 prime factors. Answer: (3)  
    
    
  Ex.9 n3 + 6n2 + 11n + 6 (where n is a whole no) is always divisible by  
  (1) 4  
    
  (2) 5  
    
  (3) 6  
    
  (4) 8  
    
  (5) 12  
    
    
  Sol. n3 + 6n2 + 11n + 6 = (n + 1) (n + 2) (n + 3).  
  Product of 3 consecutive numbers is always divisible by 3! = 6.  
    
  (or)  
    
  Take n = 0, 1, 2, 3 and check it is always divisible by 6. Answer: (3)  
    
    
  Ex.10 What is the remainder, if 351 × 352 × 353 × - - - - - - - × 356 is divided by 360?  
  (1) 0  
    
  (2) 1  
    
  (3) 2  
    
  (4) 3  
    
  (5) 359  
    
  Sol. Since the given is the product of 6 consecutive numbers, it is always divisible by 6! = 720.  
  ⇒ it is divisible by 360 also. So, the remainder will be 0. Answer: (1)**

**Finding Remainders of a product (derivative of remainder theorem)  
  
  
If ‘a1‘is divided by ‘n’, the remainder is ‘r1’ and if ‘a2’ is divided by n, the remainder is r2. Then if a1+a2 is  
divided by n, the remainder will be r1 + r2  
   
  
If a1 - a2 is divided by n, the remainder will be r1 - r2  
   
  
If a1 × a2 is divided by n, the remainder will be r1 × r2  
  
  
Ex. If 21 is divided by 5, the remainder is 1 and if 12 is divided by 5, the remainder is 2.  
Then if (21 + 12 = 33) is divided by 5, the remainder will be 3 (1 + 2).  
If 9(21 - 12) is divided by 5, the remainder will be 1 - 2 = - 1.  
But if the divisor is 5, - 1 is nothing but 4. 9 = 5 × 1 + 4.  
So, if 9 is divided by 5, the remainder is 4 and 9 can be written as 9 = 5 × 2 - 1.  
So here - 1 is the remainder. So - 1 is equivalent to 4 if the divisor is 5. Similarly - 2 is equivalent to 3.  
If 252(21 × 12) is divisible by 5, the remainder will be (1 × 2 = 2).  
  
  
If two numbers ‘a1’ and ‘a2‘ are exactly divisible by n. Then their sum, difference and product is also exactly  
divisible by n.  
  
i.e., If ‘a1’ and ‘a2’ are divisible by n, then  
a1 + a2 is also divisible by n  
a1 - a2 is also divisible by n  
and If a1 × a2 is also divisible by n.  
  
  
Ex.1 12 is divisible by 3 and 21 is divisible by 3.  
  
Sol. So, 12 + 21 = 33, 12 - 21 = - 9 and 12 × 21 = 252 all are divisible by 3.  
  
   
  
Finding Remainders of powers with the help of Remainder theorem:  
  
  
Ex.2 What is the remainder if 725 is divided by 6?  
   
  
Sol. If 7 is divided by 6, the remainder is 1. So if 725 is divided by 6, the remainder is 125 (because 725 = 7  
× 7 × 7… 25 times. So remainder = 1 × 1 × 1…. 25 times = 125).  
  
  
Ex.3 What is the remainder, if 363 is divided by 14.  
   
  
Sol. If 33 is divided by 14, the remainder is - 1. So 363 can be written as (33)21.  
So the remainder is (- 1)21 = - 1. If the divisor is 14, the remainder - 1 means 13. (14 - 1 = 13)  
By pattern method  
  
  
Ex.4 Find remainder when 433 is divided by 7.  
   
  
Sol. If 41 is divided by 7, the remainder is 4. (41 = 4 = 7 × 0 + 4)  
  
If 42 is divided by 7, the remainder is 2 (42 = 16 = 7 × 2 + 2)  
  
If 43 is divided by 7, the remainder is 1 (43 = 42 × 4, So the  
Remainder = 4× 2 = 8 = 1)  
  
If 44 is divided by 7, the remainder is 4 (44 = 43 × 4, so the  
Remainder = 1× 4 = 4)  
  
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The remainders of the powers of 4 repeats after every 3rd power.  
So, as in the case of finding the last digit, since the remainders are repeating after every 3rd power, the  
remainder of 433 is equal to the remainder of 43 ( since 33 is exact multiple of 3) = 1. (OR)  
If 43 is divided by 7, the remainder is 1. So 433 = (43)11 is divided by 7, the remainder is 111 = 1.  
  
  
Application of Binomial Theorem in Finding Remainders  
  
The binomial expansion of any expression of the form  
  
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There are some fundamental conclusions that are helpful if remembered, i.e.  
  
a. There are (n + 1) terms.  
  
b. The first term of the expansion has only a.  
  
c. The last term of the expansion has only b.  
  
d. All the other (n - 1) terms contain both a and b.  
  
e. If (a + b)n is divided by a then the remainder will be bn such that bn < a.  
  
   
  
Ex.5 What is the remainder if 725 is divided by 6?  
  
  
Sol. (7)25 can be written (6 + 1)25. So, in the binomial expansion, all the first 25 terms will have 6 in it. The  
26th term is (1)25. Hence, the expansion can be written 6x + 1.  
  
6x denotes the sum of all the first 25 terms.  
  
Since each of them is divisible by 6, their sum is also divisible by 6, and therefore, can be written 6x,  
where x is any natural number.  
  
So, 6x + 1 when divided by 6 leaves the remainder 1. (OR)  
  
When 7 divided by 6, the remainder is 1. So when 725 is divided by 6, the remainder will be 125 = 1.  
  
  
Wilson’s Theorem  
  
If n is a prime number, (n - 1)! + 1 is divisible by n.  
  
Let take n = 5  
  
Then (n - 1)! + 1 = 4! + 1 = 24 + 1 = 25 which is divisible by 5.  
  
Similarly if n = 7  
  
(n - 1)! + 1 = 6! + 1 = 720 + 1 = 721 which is divisible by 7.  
  
   
  
Corollary  
If (2p + 1) is a prime number (p!)2 + (- 1)p is divisible by 2p + 1.  
e.g If p = 3, 2p + 1 = 7 is a prime number  
(p!)2 + (- 1)p = (3!)2 + (- 1)3 = 36 - 1 = 35 is divisible by (2p + 1) = 7.  
  
  
Property  
If “a” is natural number and P is prime number then (ap - a) is divisible by P.  
e.g. If 231 is divided by 31 what is the remainder?  
  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT So remainder = 2  
  
  
  
Fermat’s Theorem  
If p is a prime number and N is prime to p, then Np -1 - 1 is a multiple of p.  
  
  
Corollary  
Since p is prime, p - 1 is an even number except when p = 2.  
Therefore ( Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT) = M(p).  
  
  
Hence either Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT -1 is a multiple of p, that is Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT = Kp ± 1, where, K is some positive integer.  
  
  
Base Rule and Conversion  
This system utilizes only two digits namely 0 & 1 i.e. the base of a binary number system is two.  
e.g. 11012 is a binary number, to find the decimal value of the binary number, powers of 2 are used as weights  
in a binary system and is as follows:  
1 × 23 = 8  
1 × 22 = 4  
0 × 21 = 0  
1 × 20 = 1  
Thus, the decimal value of 11012 is 1 × 23 + 1 × 22 + 0 × 21 + 1 × 20 = 13.  
  
  
Conversion from decimal to other bases  
We will study only four types of Base systems,  
1. Binary system (0, 1)  
2. Octal system (0, 1, 2, 3, 4, 5, 6, 7).  
3. Decimal system (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)  
4. Hexa-decimal system (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C D, E, F) where A = 10, B = 11 … F = 15.  
  
  
Let us understand the procedure with the help of an example  
  
  
Ex.6 Convert 35710 to the corresponding binary number.  
  
Sol. To do this conversion, you need to divide repeatedly by 2, keeping track of the remainders as you  
go. Watch below:  
As you can see, after dividing repeatedly by 2, we end up with these remainders:  
  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
These remainders tell us what the binary number is! Read the numbers outside the division block, starting  
from bottom and wrapping your way around the right-hand side and moving upwards. Thus,  
  
  
(357)10 convert to (101100101)2.  
This method of conversion will work for converting to any non-decimal base. Just don't forget to include the  
first digit on the left corner, which is an indicator of the base. You can convert from base-ten (decimal) to any  
other base.  
  
  
Conversion from other bases to Decimal  
We write a number in decimal base as  
345 = 300 + 40 + 5 = 3 × 102 + 4 × 101 + 5 × 100  
Similarly, when a number is converted from any base to the decimal base then we write the number in that  
base in the expanded form and the result is the number in decimal form.  
  
  
Ex.7 Convert (1101)2 to decimal base  
  
  
Sol. (1101)2 = 1 × 23 + 1 x 22 + 0 × 21 + 1 × 20  
= 8 + 4 + 1 = 13  
So (1101)2 = (13)10  
  
  
Ex.8 Convert the octal no 3456 in to decimal number.  
  
Sol. 3456 = 6 + 5 × 8 + 4 × 82 + 3 × 83  
= 6 + 40 + 256 + 1536  
= (1838)10  
  
  
Ex.9 Convert (1838)10 to octal.  
  
Sol.  
  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
= (3456)8  
  
  
Ex.10 What is the product of highest 3 digit number & highest 2 digit number of base 3 system?  
(1) (21000)3  
  
(2) (22200)3  
  
(3) (21222)3  
  
(4) (21201)3  
  
(5) None  
   
  
Sol. The highest 3 digit & 2 digit numbers are 222 & 22  
222 = 2 + 2 × 3 + 2 × 32 = 26  
22 = 2 + 2 × 3 = 8  
∴ Product = 26 × 8 = 208  
Convert back to base  
(21201)3  
  
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Answer: (4)**

**Cyclicity  
  
At times there are questions that require the students to find the units digit in case of the numbers occurring in  
powers. If anyone asks you to find the unit digit of 33, you will easily calculate it also you can calculate for 35  
but if any one ask you the unit digit of 17399, it will be hard to calculate easily.  
  
But it’s very simple if we understand that the units digit of a product  
is determined by whatever is the digit at the units place irrespective  
of the number of digits. E.g. 5 × 5 ends in 5 & 625 × 625 also ends in 5.  
Now let’s examine the pattern that a number generates when it occurs in  
powers of itself.  
See the last digit of different numbers.  
  
   
  
Unit Digit Chart  
  
  
TABLE SHOWING THE UNIT DIGIT OF A NUMBER FOR DIFFERENT EXPONENTS  
  
  
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From the above table we can conclude that the unit digit of a number repeats after  
an interval of 1, 2 or 4. Precisely we can say that the universal cyclicity of all the  
numbers is 4 i.e. after 4 all the numbers start repeating their unit digits.  
  
Therefore, to calculate the unit digit for any exponent of a given number we have  
to follow the following steps  
  
Step 1: Divide the exponent of the given number by 4 and calculate the remainder.  
  
Step 2: The unit digit of the number is same as the unit digit of the number raise to  
the power of calculated remainder.  
  
Step 3: If the remainder is zero, then the unit digit will be same as the unit digit of N4.  
  
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Let us consider an example  
Ex.1 Find the last digit of (173)99.  
  
Sol. We notice that the exponent is 99. On dividing, 99 by 4 we get 24 as the quotient & 3 as the remainder.  
Now these 24 pairs of 4 each do not affect the no. at the units place So, (173)99 ≈ (173)3. Now, the  
number at the units place is 33 = 27.  
  
  
Factors  
  
A factor is a number that divides another number completely. e.g. Factors of 24 are: 1, 2, 3, 4, 6, 8, 12, 24.  
   
  
Number of Factors  
If we have a number, N = pa × qb × rc  
Where p, q, and r are prime numbers and a, b, and c are the no. of times each  
prime number occurs , then the number of factors of n is found by (a + 1) (b +1)(c + 1).  
  
  
Example:  
Find the number of factors of 24 × 32.  
Number of factors = (4 + 1) (2 + 1) = 5(3) = 15  
  
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Number of Ways of Expressing a Given Number as a Product of Two Factors  
When a number is having even number of factors then it can be written as a product of two numbers in  
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But if a number have odd number of factors then it can be written as a product of two different numbers in  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT ways and can be written as a product of two numbers (different or similar) in  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT ways.  
  
  
Examples:  
1. 148 can be expressed as a product of two factors in Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT or 3 ways.  
  
{Because (p + 1) (q + 1) (r + 1) in the case of 148 is equal to 6}.  
  
2. 144 (24.32) can be written as a product of two different numbers in Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT i.e. 7 ways  
  
  
Sum of the factors of a number:  
If a number N is written in the form of N = ap.bq.cr ,where a, b & c are prime numbers and p, q & r are positive  
integers, then the sum of all the factors of the number are given by the formula  
Sum of factors = Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
  
  
Factorial  
Factorial is defined for any positive integer. It is denoted by Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT or !. Thus “Factorial n” is written as n!  
or Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
  
n! is defined as the product of all the integers from 1 to n.  
Thus n! = 1.2.3. …. n. (n! = n(n – 1)!)  
  
  
Finding the Highest power of the number dividing a Factorial  
   
  
Ex.2 Find the largest power of 3 that can divide 95! without leaving any remainder.  
OR  
Find the largest power of 3 contained in 95!.  
  
  
Sol. First look at the detailed explanation and then look at a simpler method for solving the problem.  
  
When we write 95! in its full form, we have 95 × 94 × 93 ….. × 3 × 2 × 1. When we divide 95! by a power  
3, we have these 95 numbers in the numerator. The denominator will have all 3’s. The 95 numbers in  
the numerator have 31 multiples of 3 which are 3, 6, 9….90, 93.Corresponding to each of these  
multiplies we can have a 3 in the denominator which will divide the numerator completely without  
leaving any remainder, i.e. 331 can definitely divide 95!  
  
Further every multiple of 9, i.e. 9, 18, 27, etc. after canceling out a 3 above, will still have one more 3  
left. Hence for every multiple of 9 in the numerator, we have an additional 3 in the denominator. There  
are 10 multiples of 9 in 95 i.e. 9, 18….81, 90. So we can take 10 more 3’s in the denominator.  
  
Similarly, for every multiple of 33 we can take an additional 3 in the denominator.  
Since there are 3 multiples of 27 in 91 (they are 27, 54 and 81), we can have three more 3’s in the  
denominator.  
  
Next, corresponding to every multiple of 34 i.e. 81 we can have one more 3 in the denominator. Since  
there is one multiple of 81 in 95, we can have one additional 3 in the denominator.  
  
Hence the total number of 3’s we can have in the denominator is 31 + 10 + 3 + 1, i.e., 45. So 345 is the  
largest power of 3 that can divide 95! without leaving any remainder.  
  
  
The same can be done in the following manner also.  
Divide 95 by 3 you get a quotient of 31. Divide this 31 by 3 we get a quotient of 10. Divide this 10 by 3  
we get a quotient of 3. Divide this quotient of 3 once again by 3 we get a quotient of 1. Since we cannot  
divide the quotient any more by 3 we stop here. Add all the quotients, i.e. 31 + 10 + 3 + 1 which gives  
45 which is the highest power of 3.  
  
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Add all the quotients 31 + 10 + 3 + 1, which give 45.  
  
{Note that this type of a division where the quotient of one step is taken as the dividend in the  
subsequent step is called “Successive Division”. In general, in successive division, the divisor need  
not be the same (as it is here). Here, the number 95 is being successively divided by 3.  
  
Please note that this method is applicable only if the number whose largest power is to be found out is  
a prime number.  
  
If the number is not a prime number, then we have to write the number as the product of relative  
primes, find the largest power of each of the factors separately first. Then the smallest, among the  
largest powers of all these relative factors of the given number will give the largest power required.  
  
  
Ex.3 Find the largest power of 12 that can divide 200!  
   
  
Sol. Here we cannot apply Successive Division method because 12 is not a prime number. Resolve 12 into  
a set of prime factors. We know that 12 can be written as 3 × 4. So, we will find out the largest power of  
3 that can divide 200! and the largest power of 4 that can divide 200! and take the LOWER of the two  
as the largest power of 12 that can divide 200!.  
  
To find out the highest power of 4, since 4 itself is not a prime number, we cannot directly apply the  
successive division method. We first have to find out the highest power of 2 that can divide 200!. Since  
two 2’s taken together will give us a 4, half the power of 2 will give the highest power of 4 that can  
divide 200!. We find that 197 is the largest power of 2 that can divide 200!. Half this figure-98-will be the  
largest power of 4 that can divide 200!.  
  
  
Since the largest power of 3 and 4 that can divide 200! are 97 an 98 respectively, the smaller of the  
two, i.e., 97 will be the largest power of 12 that can divide 200! without leaving any remainder.  
  
  
Ex.4 What is the last digit of 234 × 334 × 434  
  
  
Sol. Given = (24)34  
Last digit of 4n is 6, if n is even. ⇒ Answer 6  
  
  
Ex.5 What is the right most non zero digit of (270)270  
  
Sol. The required answer is the last digit of 7270.  
Last digit of 7 powers repeat after every 4.  
So, the last digit of 7270 is the last digit of 72 = 9.  
  
  
Ex.6 How many factors do 1296 have?  
  
Sol. 1296 = 4 × 324  
= 4 × 4 × 81  
= 24 × 34  
Number of factors = (4 + 1) (4 + 1) = 25.  
  
  
Ex.7 If x is the sum of all the factors of 3128 and y is the no of factors of x and z is the number of  
ways of writing ‘y’ as a product of two numbers, then z = ?  
  
Sol. 3128 = 4 × 782  
= 4 × 2 × 391  
= 23 × 17 × 23  
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= 15 × (17 + 1) (23 + 1)  
= 3 × 5 × 9 × 2 × 8 × 3  
= 24 × 34 × 5  
∴ y = (4 + 1) (4 + 1) (1 + 1)  
= 2 × 52  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
  
  
Ex.8 How many cofactors are there for 240, which are less than 240?  
  
Sol. 240 = 16 × 15  
= 24 × 3 × 5  
Number of co primes to N, which are less than N  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
if N = ab × bq × - - - - (a, b, - - - - are Prime no.s)  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
  
  
Ex.9 What is the sum of all the co primes to 748? Which are less than N?  
  
  
Sol. 748 = 4 × 187  
= 22 × 11 × 17  
Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
Sum of all the co primes to N. which are less than N is Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT (number of co primes to N, which are less  
than N.  
∴ Sum = Bank PO, Common Admission Test, Mathematics, SSC, Syllabus, GMAT, CAT  
= 119680  
  
  
Ex.10 In how many ways 5544 can be written as a product of 2 co primes?  
  
Sol. If N = ap × bq × - - - -, where a, b, - - - - are prime numbers  
N can be written as a product of two co primes in 2n-1 ways, where n is the number of prime factors to  
N.  
∴ 5544 = 11 × 504  
= 11 × 9 × 56  
= 11 × 9 × 8 × 7  
= 23 × 32 × 7 × 11  
∴ Answer: = 24-1 = 23 = 8. (Because, 2, 3, 7 & 11 are four different prime factors).**